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Selecting fast-moving consumer goods product lines

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"He says you eat greens like the goat and pork like the pigs. Your wives are shameless and show their faces - he has seen them. He says you never pray. He says, what good are your airplanes and wireless... if you do not possess the Truth?"

Kemal's translation of Mouyan's thoughts,
Wind, Sand and Stars, Cap. VII,
Antoine de Saint-Exupéry

Acknowledgments

This five-year research project was eventful and life-changing. It was marked by evictions, broken relationships, and several deaths. On the other end of the scale, I became engaged and moved in together with my wife, attended a dance party between two three-story yachts on a paradisaical island, and I am going to be a father. Also, my family's cat is doing well, with no teeth and twenty-one years of age. Therefore, I shall begin by acknowledging the cat for her commendable confidence and composure. Only a couple of months ago, she was calmly striding through a rowdy dogfight which included a labrador retriever and a dalmatian. A raise of her paw managed to intimidate the labrador twenty times her weight.

On a more serious note, I acknowledge my supervisors' role in making this project happen. Luís' conjoint marketing-operation planning vision was this dissertation's starting point. His scientific guidance and optimization expertise were fundamental throughout this project's development. Gonçalo's thoroughness and attention to detail are also reflected throughout this document, in each chapter's quality. I believe his perfectionism is a fundamental trait of every prime researcher. Together, they form a demanding team of uncompromising negotiators with strong technical and analytical minds. Regarding my supervision, I am also thankful to José Fernando and Maria Antónia, whose advice should never be overlooked.

Moreover, I am grateful for the conditions provided to me at my research institution. It is hard to overstate the importance of being surrounded by hard-working colleagues in achieving the mindset and skills to earn a doctoral degree. Such an environment leads by example, driving each person involved to dedicate themselves to honing its research. I am especially thankful to Eduardo for his suggestions involving this dissertation's fourth chapter and funk carioca. Along with my colleagues, I must recognize that my research builds on top of an immense body of research. Mainly, I am indebted to the lot-sizing research community, being the one I worked the closest with.

Most importantly, I thank family for their guidance and support. Their presence was vital for this dissertation's completion. My parents' industriousness, unwavering through adversity, is inspiring. After the first couple of years, their advice turned away from me completing the doctoral program. Nevertheless, they supported my decision to continue pursuing this degree. The stability they have provided me makes them this dissertation's backbone. I am particularly thankful for my sister's drawing tips, which made composing my elevator pitches enjoyable. Also, my wife pulled me out clean from the dirty place I fell into throughout this project's roughest periods. I am truly grateful for that.

My paternal grandparents passed away within these five years. They were exemplary to me and everybody around them. My grandfather was a paragon of intelligence and potency. By twenty, he had already helped assemble a manufacturing plant that he was in charge of running. At that age, I was still taking production management classes to learn what a production line was. My grandmother's pureness of heart was admirable. When she was sixteen, she stood against her family to marry my grandfather. From the day she left her parents' house until the day of her demise she led him through the narrow path. This lesson in spirit they have imprinted on me is my most valuable blessing.

Abstract

Products with slight variations flood supermarket shelves, with introductions being overused for acquiring market share. Variety harms producers by increasing operational complexity and forecasting difficulty (also note that competitor introductions make market share benefits temporary). Additional product variants induce changeovers, and businesses require more inventory to safeguard against forecasting errors. Production-demand mismatches force unplanned production runs, further increasing setup numbers. Moreover, excessive variety can negatively affect consumers by difficulting their product searches and comparisons. Motivated by a real case, this research studies fast-moving consumer goods manufacturers' selection of their product lines.

Fast-moving consumer goods businesses (e.g., bottled beverages, packaged cheese) work with large volumes in a highly competitive sector. Therefore, inventory costs represent a significant margin chunk. Capacity extensions are expensive, with the additional setups resulting from unplanned production runs having dire consequences on delivery. Recent trends (e.g., direct-to-consumer channels, manufacturer-retailer cooperation) promote manufacturer access to point-of-sales data, providing them consumer choice information (e.g., sales and substitution effects). Despite this recent information availability, manufacturers lack the quantitative tools to manage their product lines. Consequently, our research aims to endow fast-moving consumer goods producers with the mathematical models needed to manage their product lines.

Product line selection literature disregards fast-moving consumer goods specificities. Filling this gap drives this dissertation's research. Essentially, we employ mixed-integer programming creatively to explore using a tactical production planning model to estimate the strategic product line decision's performance metrics. Particularly, we study how using capacitated lot-sizing models to compute complexity costs improves product line decision making. We also define the conditions potentiating our approach's benefits and propose a safety stock model extension. Furthermore, we introduce a stochastic programming approach incorporating consumer behavior prediction errors. Considering uncertainty increases the reliability of mathematical optimization solutions, thus fomenting practitioner adoption. We propose combining Benders decomposition with a scenario reduction algorithm to handle the stochastic programming model's complexity. We conclude by analyzing the approach's value under different consumer choice models and showing the benefits of capacity surpluses in handling stochasticity.

Resumo

Durante as últimas décadas, os produtores de bens de consumo introduziram produtos com o intuito de ganhar quota de mercado. Consequentemente, produtos similares, apenas com pequenas alterações dos seus atributos, populam as prateleiras dos supermercados. A variedade excessiva dificulta o processo de comparação e escolha pelos compradores. No entanto, vendo aumentadas a sua complexiade operacional e incerteza na previsão da procura, são os produtores que sofrem as piores consequências (notando que as introduções dos competidores anulam os aumentos de quota de mercado). Variantes adicionais implicam preparações das linhas de produção e inventário de segurança acrescidos. Ademais, corridas de produção extra são necessárias quando o inventário acabado não corresponde à procura, implicando mais tempo gasto na preparação de máquinas. Motivada por um caso real, esta dissertação investiga a selecção de linhas de bens de consumo.

Os produtores de bens de consumo (p.ex., bebidas engarrafadas, queijo embalado) operam em volumes elevados num setor extremamente competitivo. Consequentemente, os custos de inventário representam uma porção considerável da margem. Aumentar a capacidade é incomportavelmente dispendioso, pelo que corridas não planeadas de produção afetam gravemente a entrega das encomendas. Tendências recentes (p.ex., canais diretos ao consumidor, cooperação com retalhistas) promovem o acesso dos produtores a dados de vendas. Tendo acesso à informaçõe necessária, faltam aos produtores as ferramentas quantitativas para gerir as suas linhas de produtos. Desta forma, esta investigação pretende munir os produtores de bens de consumo com os modelos matemáticos necessários para o fazerem.

Para isto, direcionamos os nossos eforços para colmatar as lacunas na literatura sobre seleção de linhas de produtos relativas às necessidades de modelação específicas dos produtores deste sector. Essencialmente, usamos programação mista-inteira criativamente para explorar como um modelo de planeamento de produção tático pode ser usado para avaliar o desempenho da decisão da escolha da linha. Particularmente, usamos um modelo de dimensionamento de lotes com capacidade limitada para contabilizar os custos da complexidade de uma seleção. Adicionalmente, determinamos as condições que mais beneficiam o nosso modelo, em detrimento dos presentes na literatura, e propomos uma extensão para considerar os custos com o inventário de segurança. Também introduzimos um modelo de programação estocástica que incorpora erros na previsão do comportamento do consumidor. Desta forma, pretendemos melhorar a fiabiliade das soluções propostas pelos nossos modelos matemáticos, facilitando a sua adoção em casos reais. Para conseguirmos resolver o modelo estocástico, acoplamos uma decomposição de Benders com um algorítmo de redução de cenários. Concluímos com uma análise do valor da nossa abordagem sob vários modelos de comportamento do consumidor, mostrando também os benefícios que sobredimensionar a capacidade na luta contra a incerteza.

Résumé

Les producteurs de biens de consommation emballés ont souvent introduit des produits afin de gagner part de marché, ce qui a amené des produits similaires, avec seulement de mineurs changements dans leurs attributs, à remplir les rayons des supermarchés. Une variété excessive rend la comparaison et le choix difficile pour les acheteurs. Cependant, en voyant augmenter leur complexité opérationnelle et leur incertitude dans la prévision de la demande, ce sont les producteurs qui en subissent les pires conséquences (notant que les introductions de sus concurrents annulent les accroissements de la part de marché). Des variantes adicionelles impliquent la préparation des lignes de production et l'inventaire de sécurité. De plus, des cycles de production supplémentaires sont nécessaires lorsque les stocks finis ne correspondent pas à la demande, ce qui implique plus de temps passé à préparer les machines. Motivés par un cas réel, notre étude examine la sélection la sélection de lignes de biens de consommation.

Les producteurs de biens de consommation (p.ex., les boissons en bouteille, le fromage emballé) opèrent en gros volumes et le secteur est extrêmement compétitif. Par conséquent, les coûts d'inventaire représentent une part considérable de la marge. L'accroissement de la capacité est coûteux et inabordable, de sorte que les cycles de production non planifiés affectent gravement la livraison des produits. Les tendances récentes (p.ex., les canaux de vente directe aux consommateurs, la coopération avec les détaillants) favorisent l'accès des producteurs aux données sur les ventes. Bien qu'ils aient accès aux informations nécessaires, les producteurs manquent d'outils quantitatifs pour gérer leurs lignes de produits. Ainsi, cette recherche vise à fournir aux producteurs de biens de consommation les modèles mathématiques nécessaires pour gérer leurs gammes de produits.

Pour cela, nous orientons nos efforts dans le sens de combler les lacunes de la littérature en ce qui concerne la sélection des lignes de produits par rapport aux besoins de modélisations spécifiques des producteurs de ce secteur. Essentiellement, nous utilisons l'optimisation mixte de manière créative pour explorer comment un modèle de planification tactique peut être utilisé pour évaluer des performances de la décision stratégique de choix de ligne. En particulier, nous utilisons un modèle capacité de dimensionnement des lots pour tenir compte des coûts de la complexité d'une sélection. De plus, nous déterminons les conditions qui profitent le plus à notre modèle, au détriment de celles de la littérature, et nous proposons une extension pour considérer les coûts avec l'inventaire de sécurité. Nous introduisons également un modèle de programmation stochastique qui intègre des erreurs dans la prédiction du comportement des consommateurs. De cette façon, nous entendons améliorer la fiabilité des solutions proposées par nos modèles mathématiques, facilitant leur adoption dans des cas réels. Afin de résoudre le modèle stochastique, nous avons couplé une décomposition de Benders à un algorithme de réduction de scénarios. Nous concluons par une analyse de la valeur de notre approche sous différents modèles de comportement des consommateurs, montrant également les avantages que le surdimensionnement de la capacité apporte pour faire face à l'incertitude.

Acronyms

ATO	Assembly-to-order
CLSP	Capacitated lot-sizing problem
CV	Coefficient of variation
FMCG	Fast-moving consumer good
MCI	Mutiplicative competitive interaction
MNL	Multinomial logit
MOI	Marketing-operations interface
MP	Multi-period
MTO	Make-to-order
MTS	Make-to-stock
PLS	Product line selection
PVM	Product variety management
SKU	Stock-keeping unit
SMED	Single-minute exchange of die
SP	Single-period
SS	Safety stock

Contents

CHAPTER 1

Introduction

Fast-moving consumer goods (FMCG) producers lack the quantitative tools to manage their product lines. The sector is strenuously competitive, with low margins requiring players to operate efficiently regarding both cost and delivery (i.e., service level). Considerable research has been devoted to optimizing production (Jans and Degraeve, 2008; Glock et al., 2014; Copil et al., 2017) with several papers applying the resulting methodologies to real cases (Almada-Lobo et al., 2015). Nevertheless, these approaches optimize the production plan assuming the product line is unchangeable. We extend current research by questioning such assumption.

This dissertation explores optimizing the set of products a manufacturer offers – product line selection. Finding the profit-maximizing selection requires weighting products' sales net contributions against the line's complexity costs. Previous product line selection research contains models capable of reproducing consumers' response to assortment changes (Schön, 2010). Still, no approach fully considers the impact of those changes on manufacturing. For FMCG especially, cost estimates must be accurate since margins are thin. Therefore, we determine complexity costs using production planning models.

Most FMCG manufacturers have overextended their product lines since both internal and external stimuli incentivize introductions. Internally, marketing department performance is measured by sales numbers which temporarily increase after launching new products. Externally, firms must match competitor introductions to recover their market share. Repeated introductions increase complexity costs significantly, but not category market size, resembling a prisoner's dilemma. In this chapter's Appendix, we show that even without complexity costs, competing introductions lead to mutually disadvantageous equilibria.

Restructuring the product line becomes notably difficult. Production quickly notices the complexity: variety-induced congestion and increasing changeovers and safety stock (Quelch and Kenny, 1994; Guimarães, 2013). However, marketing and production are different functional silos which, regarding variety, have conflicting objectives (Shapiro, 1977; Crittenden et al., 1993). Cost-reduction incentives make production propose delistings which marketing opposes to avoid short-term sales reductions. This dissertation proposes mathematical optimization models for product line selection. Our models consider the overall profit-maximizing goal to support decision making, effectively interfacing the conflicting departments.

This thesis's developments gain particular importance with current direct-to-consumer channel trends among FMCG manufacturers and retailers-manufacturer cooperation initiatives (Li and Zhang, 2015; Li and Chen, 2018). Besides retailer orders, manufacturers have increasing access to consumer-purchase data. This research introduces mixed-integer and stochastic models capable of using such data to optimize manufacturers' product lines. Therefore, this exposition's timing is opportune since manufacturers have the need and means to implement our suggested approaches. Besides its direct scientific and practical value, handling the overextended product lines reduces the cognitive overload grocery shoppers feel and guarantees they find their favorite products (Chernev et al., 2012).

Initially, we drew our motivation from a beverages producer suffering from stock-keeping unit proliferation and the general supermarket shelf overcrowding that shoppers face. Chapter 2 explores this hypothesis and shows that FMCG producers lack the quantitative tools to select their products. We survey such tools' requirements and highlight the resulting research opportunities. Chapter 3 introduces a product line selection model using a tactical production planning problem to account for complexity costs. Besides tackling the most critical opportunity, we include a sensitivity analysis evaluating the approach's benefits over current research and practice. Chapter 4 studies the advantages of incorporating demand uncertainty into our model. Figure 1 overviews and relates this dissertation's core chapters.

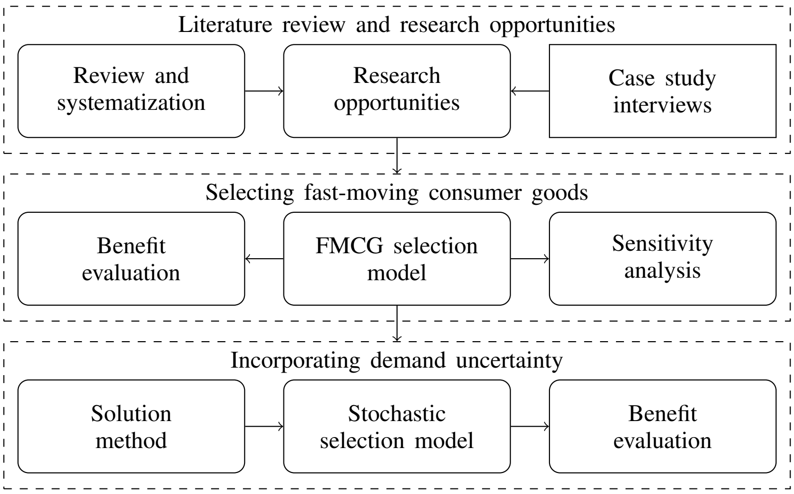

Figure 1 – Dissertation overview. Each of the dissertation body's chapters are enclosed in dashed lines. Square corners refer to tasks, rounded corners refer to contributions.

This introduction's remainder surveys each chapter. We describe the chapter's contents and contribution and present the chapter's methodology. We also review how it answers this project's research questions, discussing its purpose within this dissertation. Chapters prepare the reader for its contents with an introduction and literature review. They are research articles and can be read standalone. Finally, Chapter 5 finishes this dissertation by summarizing its contribution and positing future research directions.

1.1. Literature review and research opportunities

Chapter 2 comprehensively reviews product line selection research. We provide short surveys of marketing-operations interface research and product variety management, serving as the review's background. We introduce a product variety decisions diagram to clarify product line selection's scope. Additionally, we notice the problem lacks a unified definition and address this by developing a generic formulation. These contents prepare the reader to interpret product line selection models. The review follows, where we organize work by models and solution methods. Our synthesis makes literature gaps evident.

Besides introducing the reader to product line selection, the chapter aspires to direct researcher effort to the most rewarding opportunities. Therefore, we probed FMCG manufacturers' needs and developed a framework relating their decisions and trade-offs. It enables us to find potential decisions to consider simultaneously. We present cases guaranteeing our research direction's practical relevance. Finally, we combine research gaps and directions (obtained from manufacturer needs) to select pertinent research opportunities. We find incorporating uncertainty, improving solution methods, and considering variety-induced congestion to be particularly relevant.

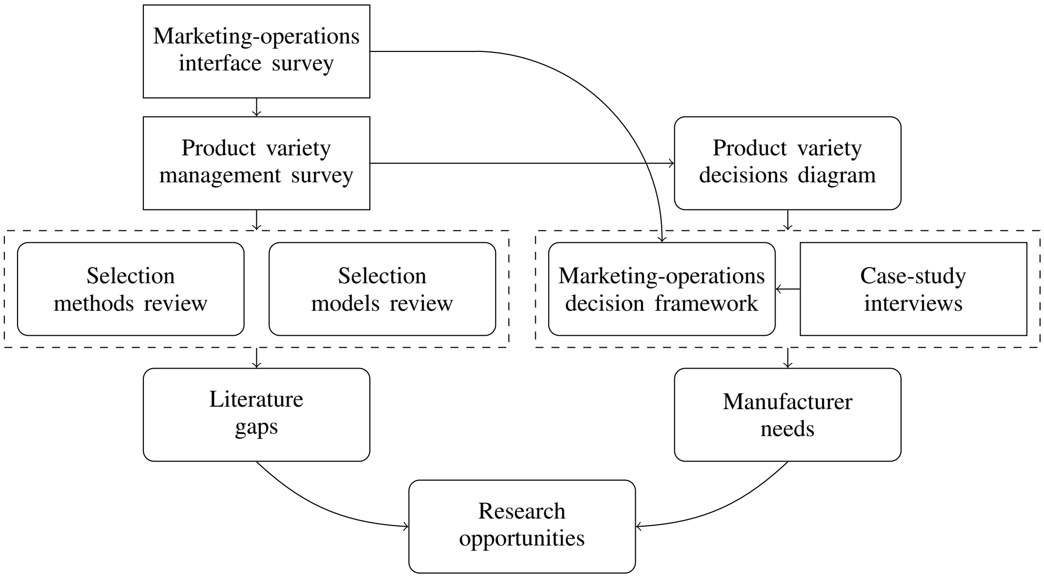

Figure 1.1 – First chapter's methodology summary. Square corners refer to tasks, rounded corners refer to contributions.

Figure 1.1 schematizes the chapter's methodology. We contribute to product line selection research by reviewing and systematizing the literature, culminating in research gaps. Additionally, we present two FMCG cases and propose a decision framework leading to research directions aligned with the sector's needs. Both produce opportunities with proved practical relevance. Further contribution arises from introducing the problem didactically and providing its general definition, increasing the topic's accessibility to managers and new researchers. The chapter answers the following research questions:

1. *How are product lines currently selected?*

   Our experience shows that most practitioners rely on myopic heuristics. Nevertheless, researchers have invested considerable attention into product line selection optimization models. We answer this question by categorizing current approaches by their models' decisions and features and by solution procedures used.

2. *Are current practices adequate to select FMCGs?*

   FMCG businesses operate with low margins and high volumes. Moreover, FMCG categories frequently have seasonal demand patterns. Only Andrade et al. (2020) (this dissertation's third chapter) present a multi-period approach capable of estimating inventory and setup costs satisfactorily. Moreover, industry-specific adjustments must precede implementation.

Overall, the chapter makes the need to research product line selection models for FMCG evident. It enlightens readers on this dissertation's motivation and scientific relevance. Since we wrote Chapter 3 before Chapter 2, the latter considers the former's advances.

1.2. Selecting fast-moving consumer goods

Chapter 3 introduces a mixed-integer optimization model for selecting FMCGs. The model incorporates attraction consumer response, fitting the sector's low-involvement purchases, with capacitated lot sizing, to estimate inventory and setup costs under dynamic demand conditions. Additionally, the model accommodates product categories and families. We postulate no substitution between different categories' products (e.g., between beer and alcohol-free beer). Also, manufacturing products within a family requires a single-family setup (e.g., among same-sized bottles). We further extend the formulation to regard safety stock.

The previous chapter's review shows that FMCG producers require precise cost accounting (i.e., correctly quantifying setups and inventory) to select products correctly. This chapter quantifies the benefits of using capacitated multi-period production planning to estimate complexity costs. We show 9.4% profit improvements relative to state-of-the-art production modeling and conclude that myopic marketing-driven approaches should be avoided. Furthermore, we present a sensitivity analysis showing the conditions that favor our approach.

Figure 1.2 shows how this chapter's research was carried out. Our formulation integrates the attraction consumer response and production planning models (extended to consider safety stock). We compare it with state-of-the-art production modeling in product line selection research and two greedy heuristics practitioners use. The comparison quantifies our model's value using two real cases. Plus, using generated instances, we perform an extensive sensitivity analysis to find the conditions favoring our model. That is, we answer the following research question:

3. *What are the benefits of including FMCGs' specificities?*

   With the real-case comparison, we manage profit increases of 9.4% relative to state-of-the-art approaches. The generated-instance sensitivity analysis determines that our approach's benefits increase with capacity utilization, consumer price-insensitivity, demand seasonality, competitive position strength, and required service level.

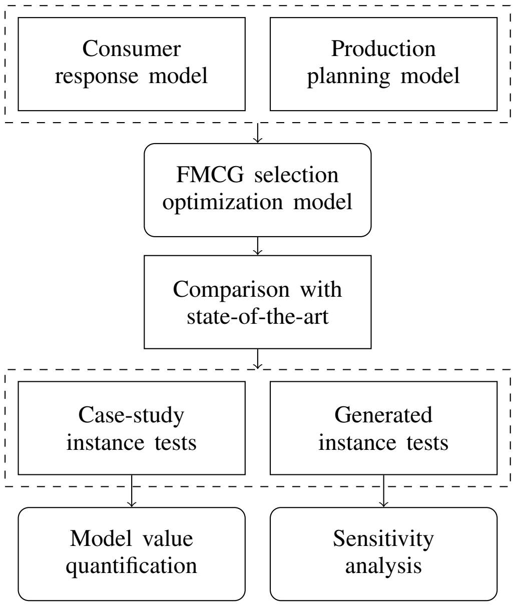

Figure 1.2 – Second chapter's methodology summary. Square corners refer to tasks, rounded corners refer to contributions.

This chapter proves that proper inventory and setup cost accounting is essential to select FMCG product lines. Moreover, researchers can build upon the mathematical formulation presented, adapting them to their particular contexts. Chapter 4 exemplifies this by extending the model with market size and consumer response stochasticity.

1.3. Incorporating demand uncertainty

Chapter 4 proposes a stochastic formulation for selecting fast-moving consumer goods, positing both market sizes and product utilities as uncertain. Product line selection is a high-impact decision, crucial for business success. Therefore, incorporating uncertainty can avoid exposing decision-makers to suggestions resulting from erroneous predictions and flawed parameterization. In turn, managers are more receptive to implementing proposed solutions, that being this research's ultimate purpose.

Hence, we introduce the stochastic problem and its deterministic equivalent formulation. We manage to reduce the number of scenarios needed for solution stability and decompose the problem, further decreasing its computational load. We assess its performance considering two widespread random-utility models (i.e., multiplicative competitive interactions and multinomial logit) for consumer choice, leading to different market-share error distributions. We find that accounting for a multitude of demand scenarios leads to considerable profit increases and value-at-risk reductions concerning both random-utility models. With a sensitivity analysis, we also demonstrate the importance of capacity surpluses in handling uncertainty.

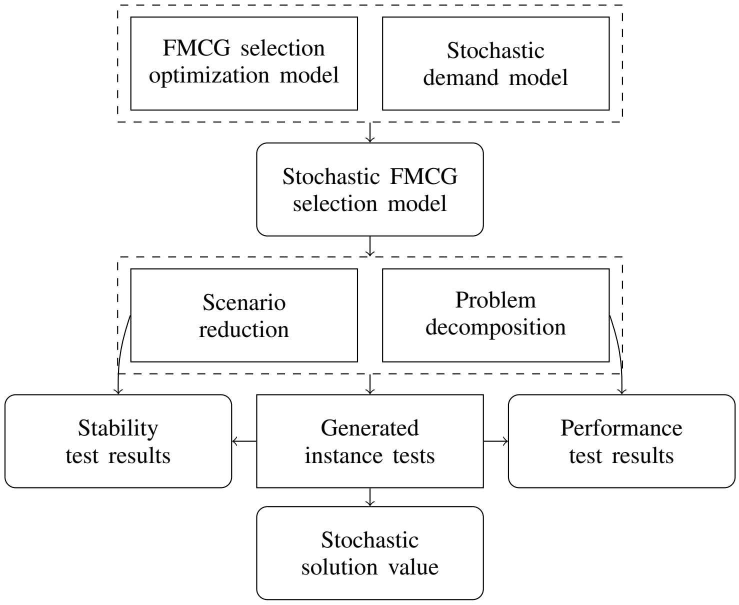

Figure 1.3 – Third chapter's methodology summary. Square corners refer to tasks, rounded corners refer to contribution.

Figure 1.3 presents the chapter's methodology. Chapter 3 provides the optimization model extended with stochastic market sizes and consumer response. Its resulting complexity motivates us to apply a scenario reduction procedure and decompose the resulting problem. We test the scenario reduction procedure for objective value stability and compare decomposition's solution times with the deterministic equivalent formulation. Lastly, we assess the stochastic approach's value for various dispersion levels and random utility model combinations under multiple capacity conditions. We contribute to answering the following research questions:

4. *Can the problem be solved efficiently?*

   The formulation's complexity forced us to develop a solution strategy. We derive valid cuts for making a Benders decomposition (Benders, 1962) of the problem into selection and production planning. Also, we implement a scenario reduction procedure, particular to stochastic programming. This strategy is valid for the expected-value problem. Nonetheless, its efficiency is limited by the production planning model's complexity.

5. *How can manufacturer adoption be stimulated?*

   We found that disregarding the uncertainty this decision involves is a critical adoption drawback. Therefore, this chapter introduces stochastic demand to product line selection. Given the stakes involved, considering a multitude of scenarios ensures managers trust the approach. Moreover, our model fits a large portion of real cases since it accommodates multiple independent categories and product families. These pertinent features address manufacturer reluctance in implementing selections resulting from optimization solutions.

Chapter 4 explores the most prominent research opportunity the literature review identifies to extend the preceding chapter's model. It safeguards manufacturers from demand uncertainty by proposing an approach considering multiple scenarios. Its value-at-risk reductions contribute to manufacturers accepting optimization solutions.

Bibliography

B. Almada-Lobo, A. Clark, L. Guimarães, G. Figueira, and P. Amorim. Industrial insights into lot sizing and scheduling modeling. *Pesquisa Operacional*, 35(3):439–464, 2015.

X. Andrade, L. Guimarães, and G. Figueira. Product line selection of fast-moving consumer goods. *Omega*, page 102389, 2020. ISSN 0305-0483. doi: https://doi.org/10.1016/j.omega.2020.102389. URL `https://www.sciencedirect.com/science/article/pii/S030504832030743X`.

J. F. Benders. Partitioning procedures for solving mixed-variables programming problems. *Numerische mathematik*, 4(1):238–252, 1962.

A. Chernev et al. Product assortment and consumer choice: An interdisciplinary review. *Foundations and Trends in Marketing*, 6(1):1–61, 2012.

K. Copil, M. Wörbelauer, H. Meyr, and H. Tempelmeier. Simultaneous lotsizing and scheduling problems: a classification and review of models. *OR spectrum*, 39(1):1–64, 2017.

V. L. Crittenden, L. R. Gardiner, and A. Stam. Reducing conflict between marketing and manufacturing. *Industrial Marketing Management*, 22(4):299–309, 1993.

C. H. Glock, E. H. Grosse, and J. M. Ries. The lot sizing problem: A tertiary study. *International Journal of Production Economics*, 155:39–51, 2014.

L. Guimarães. Advanced production planning optimization in the beverage industry. 2013.

R. Jans and Z. Degraeve. Modeling industrial lot sizing problems: a review. *International Journal of Production Research*, 46(6):1619–1643, 2008.

T. Li and H. Zhang. Information sharing in a supply chain with a make-to-stock manufacturer. *Omega*, 50:115–125, 2015. doi: 10.1016/j.omega.2014.08.001.

W. Li and J. Chen. Backward integration strategy in a retailer stackelberg supply chain. *Omega*, 75:118–130, 2018. doi: 10.1016/j.omega.2017.03.002.

J. A. Quelch and D. Kenny. Extend profits, not product lines. *Make sure all your products are profitable*, 14, 1994.

C. Schön. On the optimal product line selection problem with price discrimination. *Management Science*, 56(5):896–902, 2010.

B. Shapiro. Can marketing and manufacturing coexist? *Harvard Business Review*, 55: 104–114, 1977.

Appendix

This appendix demonstrates how successive introductions to gain and recoup market share can lead to mutually disadvantageous equilibria if the new products do not significantly increase market size. It sustains some affirmations the introduction presents, but is not essential to understand this dissertation.

Consider two competing firms attempting to maximize profits, each offering a single product to a given market category. Assume that category's demand d does not increase with the number of products offered. Rationally, both firms offer maximum-margin products since those maximize profit, unknowing its product share. For any firm, a product introduction increases its market share and possibly its profit. Still, it reduces the overall welfare since it sells for a smaller margin, hence reducing the market's average margin. To recoup its profit, the other firm also introduces a less profitable product, further reducing welfare.

Let p_j be the margin of product j, a_j its attractiveness, and A the attractiveness sum of all firms' current products. Assume an attraction model for consumer response, implying share $S_j = a_j/A$, and negligible complexity costs. One of the firms ponders the introduction of a new product k besides j. We know that $p_j a_j/A < p_k$ for the introduction to be temporarily advantageous. Also $p_k \leq p_j$ since j has the highest margin. Market share would increase to $(a_j + a_k)/(A + a_k)$ resulting in a $p_k a_k/(A + a_k) - p_j a_j a_k/(A^2 + A a_k)$ profit change. It can occur that $p_k = p_j$ since k may fit some consumers better, and the firm's margin would remain unchanged.

Say the competing firm responds by introducing a product with considerable attractiveness a_0. The first firm's share advantage would be mitigated. It can be easily observed that, if $a_0 > a_k$, if $a_0 = a_k$ and $p_k < p_j$, and if $a_0 > 0$ and $p_k = p_j a_j/A$, the first firm is worst off. Also, if $p_j a_j/A \leq p_k \leq p_j a_j/(A + a_0)$ the firm introduces k and removes it after the competitor responds, ending up with less profit. The disadvantageous equilibrium occurs when $p_j a_j/(A + a_0) < p_k < p_j$. Notice that, for the second firm to keep the introduction, these conditions must also apply to its products.

CHAPTER 2

Product line selection: opportunities in the fast-moving consumer goods sector

Product line selection: opportunities in the fast-moving consumer goods sector

Xavier Andrade[*] · **Luis Guimarães**[*] · **Gonçalo Figueira**[*]

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Abstract Product line selection – weighing sales and complexity costs to determine the profit-maximizing assortment from a set of products – is a crucial problem for manufacturing firms. The recent adoption of direct-to-consumer distribution channels allows these firms easy access to point-of-sale data. This data allows manufacturers to estimate sales response to assortment changes and, by determining how the change influences production costs, manage their product lines. Despite its current relevance, practitioners seem unaware of this line of research. We tackle this issue on three fronts: (1) we provide the first literature review dedicated to this problem, (2) we make this review didactic, to ease the entry of new researchers into this topic, and (3) we derive our research directions from fast-moving consumer goods manufacturer needs. This paper defines the product line selection problem within its research landscape by discriminating it from other variety management decisions and proposing a generic formulation. Furthermore, it comprehensively reviews and systematizes work on optimization models for this problem, delineating this topic's research frontier. Considering two manufacturer cases, we identify research directions relevant to the fast-moving consumer goods sector. The frontier and directions enable us to guide the efforts of researchers to the most prominent opportunities. We find incorporating uncertainty, developing solution methods, and researching production modeling strategies to consider variety-induced congestion to be particularly relevant.

Keywords product line selection; fast-moving consumer goods; marketing-operations interface; product variety management; literature review

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2.1. Introduction

Selecting the right product line is a determining factor of firm sustainability (Zhu et al., 2018). Still, to satisfy consumers and retailers, manufacturers have stretched their product lines past the point where variety sales compensate for operational complexity (McKinsey & Company, 2016). Consultants recommend these firms separately optimize their assortment for revenue and simplify operations by eliminating unnecessary sourcing and product variety (McKinsey & Company, 2020). On the other hand, manufacturers are getting closer to consumers (McKinsey & Company, 2017), with some brands opting for exclusive online or brick-and-mortar stores (e.g., Soylent and Nespresso, respectively). Direct-to-consumer (D2C) channel adoption make integrated approaches ever-so pertinent and beneficial (Andrade et al., 2020).

Most managers are unaware of product line selection (PLS) research despite its current relevance since it is disorganized and multidisciplinary, hence difficult to delve into. Papers are spread between the marketing and management journals, and the problem lacks a unified definition and terminology. Recent work introducing PLS formulations is categorized as product line design (Bertsimas and Mišić, 2016, 2019), and vice-versa (Fruchter et al., 2006; Kumar and Chatterjee, 2013), emphasizing the necessity of such unification. Moreover, the survey of profit-optimizing models for product line selection and design by Yano and Dobson (1998) is the latest review paper covering PLS. It excludes more recent advances such as the introduction of attraction models to PLS and their linear reformulation (Chen and Hausman, 2000; Schön, 2010a).

This paper introduces, reviews, and positions research on the PLS problem whose objective is to find the profit-maximizing product subset among a range of options. Selecting a product line involves maximizing variety sales, accounting for cannibalization, while minimizing complexity costs – working in frequently opposing directions. Since marketing and operations performance measures are sales and cost-based, respectively, the departments conflict in respect to variety management decisions. To contextualize PLS, we first overview marketing-operations interface (MOI) research. This top-down view of related literature enables positioning PLS within product variety management (PVM), a pressing MOI issue. Figure 2.1 presents our review's approach.

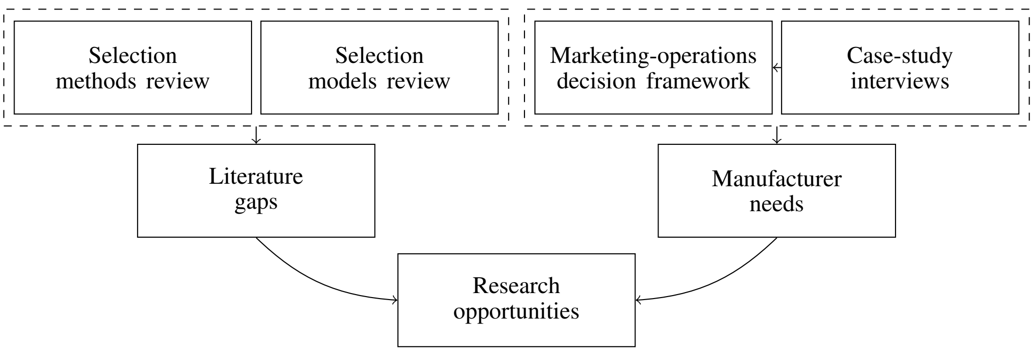

Figure 2.1 – Paper's methodology and contribution summary.

Considering that classifying PLS formulations has been a source of confusion, we introduce a general mathematical problem definition and enumerate the distinct modeling features previous work presents. Then, we comprehensively review and categorize PLS optimization models and solution methods, allowing us to identify research gaps. We expose beverages and dairy industry cases from our consulting experience. They make fast-moving consumer goods (FMCG) manufacturers' PLS modeling needs tangible, resulting in research directions with confirmed practical relevance. Along with our overview of marketing-operations conflict sources, the cases enable us to construct a framework for manufacturers' MOI decisions. The framework assists in finding decisions that should be taken together, and the impact of changing an involved variable. Lastly, we consider both literature gaps and manufacturer needs to determine the most promising research opportunities.

Our contribution is threefold and targets three audiences. First, we provide a structured review of PLS optimization models and methods, delineating the research frontier and enabling us to posit research opportunities. Hence, we ensure researchers direct their efforts at pertinent opportunities. Second, we present a didactic top-down PLS introduction and propose a general problem definition. We expect to expand the PLS research community by making the topic more orderly and accessible. Third, our cases make the motivation to research PLS tangible and FMCG manufacturers' variety management needs known, ensuring valuable research opportunities. We hope our problem contextualization and real-case analyses bring this body of literature closer to FMCG managers.

In the paper's remainder, Section 2.2 positions the product line selection problem in marketing and operations planning and overviews research intersecting the two subjects. We introduce PVM within the MOI topic and establish the relationship between these parent topics and PLS. In Section 2.3 we provide a general PLS problem definition. Section 2.4 presents an exhaustive review of PLS models and solution methods culminating into research gaps. Section 2.5 presents two discussions with FMCG business managers, among with the framework relating the MOI decisions of high-volume low-margin make-to-stock producers. Its contents enable us to find research directions of practical relevance. Finally, Section 2.6 makes some concluding remarks and enumerates the principal research opportunities.

2.2. Positioning

PLS is a cross-functional problem. Currently, its positioning is unclear, as the respective literature spreads throughout marketing and management journals. The scope of the problem also lacks a unified definition, as its terminology has been used ambiguously with product line design. PLS was initially a marketing problem – choosing the assortment that maximized profit or utility based on consumer preferences. Then, researchers introduced fix costs and resource sharing to mathematical models. Additional computational power and optimization techniques enabled models to base the strategic product program decisions on its production planning effects (Morgan et al., 2001a). Breaking the vertical decision hierarchy, featuring products' effects on complexity costs facilitates better product line decisions and reinforces selections' validity.

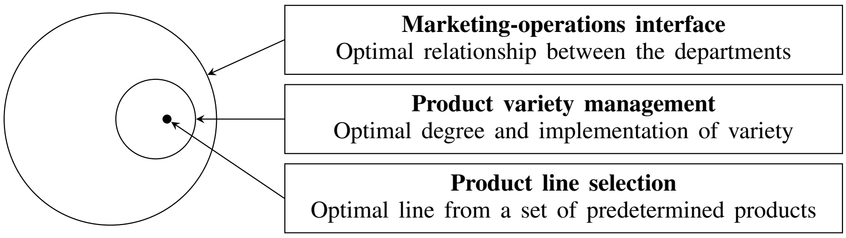

Figure 2.2 – Venn diagram positioning the PLS literature in PVM and the MOI, with the main objective of each stream.

Figure 2.2 shows PLS as a critical PVM decision. For long, marketing-operations conflict arises from variety decisions: marketing seeks market share through variety purchases and differentiation, and production seeks cost reductions through homogeneous manufacturing and predictable plans. Finding the most favorable compromise between both objectives is a determinant of firm success. MOI research studies the coordination of these departments to achieve the compromise. We start this section by familiarizing the reader with the interdepartmental conflict sources and the MOI. Then, we organize variety management research, providing a clear picture of PLS's scope. We use this picture to shed light on the problem's current pertinence.

2.2.1 Marketing-operations interface

Marketing conceives, promotes, prices, and distributes goods and services. Operations manages resources to produce the goods and services marketing distributes. The first seeks to maximize revenue, net of advertising and sales costs. The second, to minimize production costs. These complementary functions have adversarial incentives. Several studies classify potential conflict arising from decisions involving both functions. Shapiro (1977) observes that trade-offs involving lead time, inventory, product variety and introductions, price, and post-sale service potentiate conflict. Crittenden et al. (1993) classify these trade-offs as diversity (dealing with product variety), conformity (dealing with the fit between orders and production capabilities), or dependability related (dealing with service level and quality). Piercy (2009, 2007) includes facility location and job design.

Table 2.1 summarizes the classifications these authors make by hierarchical decision order (e.g., product variety decisions depend on the facility planning). The first column respects this paper's terminology, which aims to align conflicts with interdepartmental trade-offs. We overlook cost control from Shapiro (1977), and facility location and job design from Piercy (2007) as nor marketing nor operations are responsible for these decisions. While cost control is under the financial department, facility location, and job design are decisions taken before operations are running, hence before operations being managed. Each term's description follows.

Facility planning Consists of determining resource needs to satiate long-range demand estimates and selecting the production system. Overshooting would increase both cap-

Table 2.1 – Sources of conflict in the MOI (below depends on above).

This research	**Piercy (2007)**	**Crittenden et al. (1993)**	**Shapiro (1977)**
Facility planning	Capacity and demand level	Capacity/facility planning	Capacity and long--range forecasting
Product variety management	Variety New product introduction Standardisation	Product line length/breadth Product line changes Product customization	Breadth of product line New product introduction
Service	Tertiary and support costs		Adjunct services (installing, repairs)
Quality and throughput	Managing expectations	Quality control	Quality assurance
Inventory and service level	Lead-time and inventory	Delivery	Delivery and distribution
Demand and resource management	Effective production Capacity and demand changes	Product scheduling	Scheduling and sales forecasting

ital and operating expenditures, and undershooting can limit the plant's sales potential. Although marketing and operations conflict they work as consultants for the surrogate decision-maker, usually the financial department. All other MOI decisions are dependent on facility planning.

Product variety management Being the following subsection's topic, it encompasses design, selection, timing, and customization decisions. Marketing wants distinct designs, large assortments, frequent introductions, and customizable products. Production seeks to reduce entropy in manufacturing facilities: simple designs, standardized products, and predictable sales numbers.

Service Refers to post-sale support and maintenance decisions. It depends on product variety decisions, particularly product design. Divergences result from marketing wanting close customer support, which is expensive, and from manufacturing expending resources supporting clients on problems marketing could not predict that would arise from product usage.

Quality and throughput Amounts to quality's trade-off with production costs. It depends on the production line and influences inventory levels and resource management. Lower quality, in the sense of conformity with specifications, usually enables additional throughput and implies lower utility, which reduces demand for a given price. In the long term, quality impacts sales by influencing how consumers value the brand.

Inventory and service level Includes dimensioning the safety stock to handle short-term forecast uncertainty, which is highly dependent on the production lead time – WIP divided by throughput (Little, 1961). Throughput depends on the desired quality. Batch sizes can be manipulated to manage lead time. Hence, this category's controllable decisions are service level and lot size (when applicable), with lead time and inventory depending on them.

Demand and resource management Defines the production and promotion plan under the established capacity, quality and desired service level. Here, marketing can coordinate pricing and advertising with production scheduling to appropriately utilize manufacturing resources and take advantage of consumer willingness to pay (Khouja and Robbins, 2003).

The MOI was the subject of several reviews. Parente (1998) and Sombultawee and Boon-itt (2018) organize the research into the strategic tactical and operational levels. Tang (2010) extensively reviews and introduces a classification framework for quantitative MOI models. The topic has also been the focus of three special issues (Malhotra and Sharma, 2002; Ho and Tang, 2004, 2009). Ultimately, marketing-operations research aspires to improve firm performance (Shapiro, 1977; Crittenden et al., 1993). Empirical evidence of marketing-operations integration's benefits arose only decades after its importance was identified (Weir et al., 2000) and, since then, has been repeatedly confirmed (Hausman et al., 2002; Sawhney and Piper, 2002; O'Leary-Kelly and Flores, 2002; Paiva, 2010). Product development was a particularly prominent interface topic (Balakrishnan et al., 1997; Kong et al., 2015; Feng et al., 2018). Its research shows that uncertainty increases the advantages of interfunctional coordination (Tatikonda and Montoya-Weiss, 2001; Calantone et al., 2002). Additionally, there are several relevant case study articles. Spencer and Cox III (1994) find that price strategies require more coordination than product strategies and Marques et al. (2014) analyze marketing strategy's influence on service level. Finally, Piercy (2009, 2010) establishes guidelines for favorable marketing-operations relationships.

Several attempts were made at interfacing the departments. Karmarkar (1996) enumerates product mix, competitive positioning and simultaneous pricing, inventory, and production as key joint-decision areas. Some analytical studies use transfer price to coordinate production with pricing (Eliashberg and Steinberg, 1987), and advertising (Erickson, 2012). Erickson (2011) adds backlogs, modeling the interface as a differential game. Other authors derive and compare centralized with decentralized models for economic order quantity, pricing (Eliashberg and Steinberg, 1993) and marketing expenditure (Lee and Kim, 1993) to assess the integration's benefits. By developing a framework mapping marketing-manufacturing interactions, Chakravarty and Ghose (1992) propose an iterative information pull-push strategy for coordinating the functions. Ghose and Mukhopadhyay (1993) introduce a conceptual model positing the quality decision as the departments' interface. Motivated by practical cases, Karmarkar and Lele (2005) cover analytic models for inventory deployment and location in manufacturing, and price-promotion policy, forecasting, and market intelligence in marketing.

Since beverages and dairy producer cases motivate this research, we pay special attention to FMCGs, a low-margin high-volume sector generally with capital-intensive make-to-stock production. We consider facility planning as too expensive to change. Service is irrelevant since FMCGs are consumables. Demand and resource management has already been extensively studied (Kim and Lee, 1998; Karimi et al., 2003). Likewise, inventory and service level theory is at a developed stage (Chopra et al., 2004; Kulp et al., 2004; Silver et al., 1998). Therefore, the most promising research topics for interfacing marketing and operations are quality and throughput, and PVM. We study the latter since it was particularly relevant to our case studies and, having a higher hierarchical position should lead to greater benefits. The following section position PLS by categorizing PVM research.

2.2.2 Product variety management

Variety is crudely defined as the number of offered SKUs. Still, consumers' variety perception is primarily dependent on favorite product availability and category shelf space (Broniarczyk et al., 1998; Sloot et al., 2006). When searching for specific products, they report more variety if the shelf is organized. The opposite occurs while browsing (Hoch et al., 1999). Overall, they prefer shelves displaying no duplicates and varying few attributes between neighboring products. Having asymmetrical but organized shelves increases variety purchases (Kahn and Wansink, 2004). Broader assortments make brands less vulnerable to new competitors, hedge against consumer preference uncertainty (Lancaster, 1998), and reduce satiation (Kahn, 1998). They match consumer tastes and capture impulsive purchases better. On the other hand, brands lose their focus and products their strategic value, which can harm consumer loyalty (Berger et al., 2007). For an integrative review of the effects of variety on consumer choice, the reader is referred to Chernev et al. (2012).

Needless variety can hamper customer experience and raise reservation prices by increasing product search costs (Kuksov and Villas-Boas, 2010). Most readers should be able to remind themselves of the mental exhaustion felt when purchasing groceries. Overchoice (Scheibehenne et al., 2010; Chernev et al., 2015) is its cause, with product evaluations involving multiple attributes being especially tiresome. It is a form of cognitive overload occurring when an excessive number of comparisons is made, and its magnitude is moderated by purchase intent (to buy or to browse) and preference uncertainty. Tversky and Shafir (1992) discovered overchoice by noticing assortment improvements were sometimes followed by choice deferral. Gourville and Soman (2005) further study its causes. Retailers can use the effect to erode customer self-control (Vohs et al., 2008), seasoning consumers for irrational purchases (Simonson, 1989).

Variety generally leads to shorter production runs, increased uncertainty from complexity, and less effective research and development (Quelch and Kenny, 1994). The magnitude of these effects depends on the plant's production system (MacDuffie et al., 1996; Kekre and Srinivasan, 1990). Predominantly, PVM studies the trade-off between operational complexity and sales. As an example, De Groote (1994) analyzes the trade-off between further matching consumer preferences and incurring more setups, concluding that modeling the marketing and manufacturing functions separately seldom yields optimal solutions. Other general PVM studies show how variety can lead to demand mismatch costs (Wan and Sanders, 2017; Moreno and Terwiesch, 2016), and the detrimental effects of overlooking

Table 2.2 – Decisions within product variety management research.

Decision	Definition	Keywords	Example
Design	What features compose a product?	Share-of-choice; Platform	Simpson et al. (2001)
Selection	Which products to produce?	Assortment; Production	Dobson and Kalish (1993)
Timing	When to introduce a product?	Rollover; Difusion	Moorthy and Png (1992)
Customization	What should be customizable?	Variegation; Decoupling	Lee and Tang (1997)

setups when computing capacity utilization (Menezes et al., 2021). Table 2.2 displays the four main variety management decisions. It is an adaptation of the classification of variety creation decisions made by Ramdas (2003) materializing variety and product architecture decisions into selection and design.

Design Products should share productive process stages and please consumers (Chhajed and Raman, 1992). With this objective, design literature uses mostly two types of models: analytic target cascading (Michalek et al., 2005; Li et al., 2008; Michalek et al., 2011) and mixed-integer programming (Raman and Chhajed, 1995; Dobson and Yano, 1995; Kohli and Sukumar, 1990). Share of choice is a design problem consisting of finding product attribute levels that maximize the number of consumers that find the product useful (Shocker and Srinivasan, 1974; Alexouda, 2004). A design can satisfy the requirements of less demanding consumers (one-way substitution) and can provide different utilities for different segments (two-way substitution; Rutenberg, 1971). The study of commonalities, modularity, and product platforms has also received considerable attention (Fisher et al., 1999; Simpson et al., 2001; Wong et al., 2021), especially regarding product platforms for different quality levels (Desai et al., 2001; Kim and Chhajed, 2000; Heese and Swaminathan, 2006). These technologies allow for delayed product differentiation, taking advantage of economies of scope (Lee and Tang, 1997).

Selection Choosing the optimal set of products to manufacture, out of a predefined assortment of possible offers, is this review's focus. As in the profit-maximizing problem covered by Dobson and Kalish (1993), there is a trade-off between product profitability and complexity costs since both respond to assortment changes. Although costs are sometimes simplified as fix, they are composed of the indirect costs required to offer the product. Making profitable selections requires knowing how product inclusions reverberate on the whole line's production costs and gross profit. Detailed production modeling results in costing accuracy (Morgan et al., 2001a). Likewise, consumer response models must estimate how demand shifts with product line changes, which the attraction model does efficiently (Chen and Hausman, 2000).

Timing A designed and selected product should be introduced when revenues can be kept high and cannibalization low. As a general timing rule, premium products should be introduced before their low-end counterparts (e.g., hard-cover and soft-cover books), as the latter cannibalizes the high-margin sales of the former (Moorthy and Png, 1992). Specifically, timing covers product rollovers and information diffusion (Billington et al., 1983; Wilson and Norton, 1989; Krider and Weinberg, 1998). Rollover frequency (Souza et al., 2004; Klastorin and Tsai, 2004) and marketing-manufacturing integration's impact on development time (Morgan et al., 2001b; Swink and Song, 2007) have also been a subject of study, as firms compete for introduction times. Albeit unethical, time-based competition can be mitigated by sharing monopoly time within each period (e.g., Apple released the iPhone XR in September 2018 and Samsung released the Galaxy S10 in March 2019, iPhone 11 and Galaxy S20 were released in September 2019 and March 2020).

Customization Empowering consumers with product design participation ensures variety is aligned with demand. It protects the manufacturer from preference uncertainty and buyer remorse returns. Overall, high degrees of customization raise production costs make purchases difficult (Huffman and Kahn, 1998) but attach consumers to their creation (Franke et al., 2010). Customization requires assembly-to-order (ATO) production (i.e., MTS up to a decoupling point and make-to-order onward). As a time-postponement strategy, ATO synergizes with standardisation and modularization (Lee and Billington, 1994; Lee and Tang, 1997), deviating firms from customization and massification extremes (Lampel and Mintzberg, 1996). Storage efficiency (e.g., sodas are stored as a concentrate and diluted when served) and perishability (e.g., frozen peeled-and-cut vegetables significantly reduce lead time without spoiling) also motivate ATO production systems. Where customization is concerned, decoupling points are usually points of variegation, where product designs diverge. The earlier they occur in the productive process, the higher the degree of customization, complexity of the sourcing, and costs.

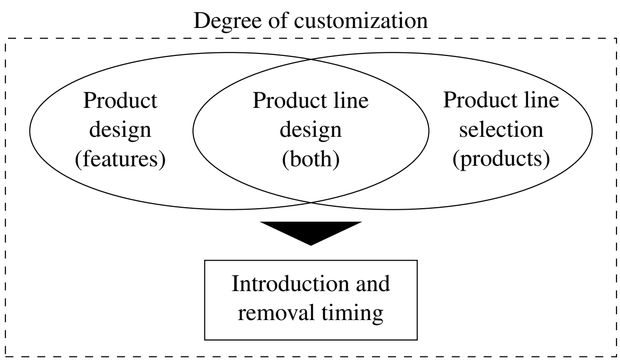

Figure 2.3 – Conceptual framework relating the decisions in PVM. The timing decision is preceeded by a product or product line design or selection. Every other decision depends on the degree of customization that the firm offers for a given product family.

PVM literature does not clearly distinguish between design and selection. How these decisions relate to the introduction timing and the degree of customization is also uncertain. Figure 2.3 describes the dependencies between PVM decisions. The degree of customization is directly associated with the production system. Therefore, it influences every other decision. Design and selection decisions follow. Product line design refers to defining the features composing the set of products that a firm offers (i.e., simultaneous product design and product line selection). Given the decision's complexity, its applicability is usually limited to greenfield situations and complete product line overhauls. Raman and Chhajed (1995) and Steiner and Hruschka (2002) exemplify product line design formulations considering fixed costs associated with each product's attribute levels with first-choice and attraction models, respectively. Product design concerns a single product for a potential introduction. Selection aims to delist some of the current products or relist familiar products (i.e., similar or previously sold). It revises a season's products and should be done frequently. By this definition, product design and selection problems are special cases of product line design. Determining introduction and removal timings occurs after design and selection decisions. Nevertheless, design and timing can be integrated (Lacourbe, 2012). Since FMCG are not customizable, and players in the sector seldom need complete line overhauls, it is unnecessary to delve into these decisions further. We intend to deal with overextended FMCG lines. Hence, the remainder of the paper focuses on PLS exclusively.

2.3. Generic model

A manufacturer wants to select its product line for the season (i.e., a planning horizon of three months to a year) – the profit-maximizing subset $W \in \mathscr{J}$ from a set of $\mathscr{J} = \{1, ..., J\}$ options. He has either offered these options in previous seasons or extensively studied them. Therefore, he knows their market and impact on operations. The selection generates $\mathrm{R}(W)$ gross profit, depending on how consumers respond to the assortment, and induces $\mathrm{C}(W)$ complexity costs, determined by the assortment's needs for development and publicity and the required production changes.

$$\max_{W} \quad \mathrm{R}(W) - \mathrm{C}(W) \tag{2.1}$$

Expression (2.1) formulates the general PLS problem. Gross profit, $\mathrm{R}(W)$ in the expression, depends on the margin p_j of each product j, and on that product's sales $D_j(W)$, meaning that $\mathrm{R}(W) = \sum_j p_j D_j(W)$. Besides product demand, which is a function of the offered selection, capacity limitations can constrain sales. Subsection 2.3.1 covers the consumer response models used in PLS, used to encode product demand's dependency on the selection. Moreover, a line's complexity costs are contingent on manufacturing facilities variety perception. They include specific product development, advertising, and the administrative costs each product may require (e.g., acquiring contracts with suppliers and buyers). Moreover, there are indirect costs of producing the demanded amounts (i.e., holding and setup costs). We enumerate the production modeling features used to account for these plant limitations and costs in Subsection 2.3.2.

2.3.1 Consumer response modelling

Determining product demand requires knowing the way purchases distribute themselves throughout a selection W. Consumer choice and market share models, from marketing science, study this aspect (Cooper, 1993; Huang et al., 2013). Consumer choice literature seeks to determine an option's utility for a consumer segment and predict the probability of a sale using that measure. Market share models focus on working out the portion of sales each product manages. PLS research uses market share models to determine how demand for existing products shifts when some are unavailable (i.e., consumer response). We categorize the different kinds of consumer response models as parametric (or exogenous), ranking (with the notable case of locational choice models), and attraction (or probabilistic), decreasing in generality. Let k and j be two products (we consider k for removal), and d the size of the market segment. Figure 2.4 describes consumer response models and their behaviour.

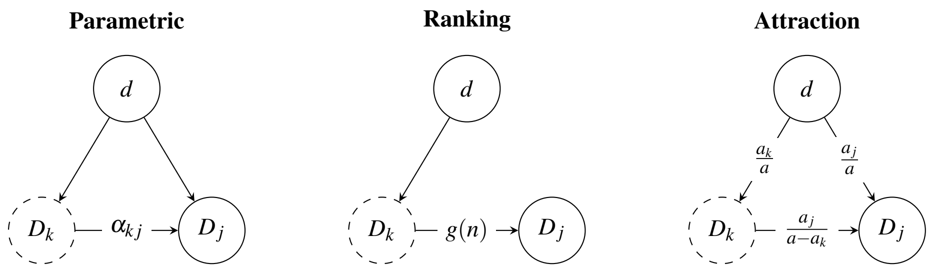

Figure 2.4 – Categories for marketing share models. Demand redistribution, when a product is removed (dashed circle), is represented by the bottom arrow.

Parametric These models are the general case of the ranking and attraction models and use previously determined measures to directly model demand, disregarding any underlying pattern in consumer behavior. Also known as exogenous demand models, they populate assortment and production planning models, including product substitution and complementarity relationships. A notable example (as shown in the figure) is to calculate $D_j(W) \leq d_j + \sum_{k \notin W} \alpha_{kj} d_k$ for all $j \in W$, where α_{kj} is the fraction of the demand for k that is recaptured by option j (Smith and Agrawal, 2000). There is recursion potential in the recapture of an unavailable option's demand by another unavailable option. For the sake of simplicity, this effect is usually curtailed at the first substitute.

Ranking In n^{th}-choice models, a consumer segment will choose the offer that generates him the most utility, provided it exceeds the product's price. If that offer is unavailable, consumers may opt for the next best alternative. The figure represents a case where option k becomes unavailable and the segment shifts to j. Let $g(\cdot)$ be the consumer retention function, decreasing throughout the utility-ordered preference rankings. With n_j being the rank of option j, $D_j(W) \leq dg(n_j)$ if $n_j < n_k$ for all $k \in W$, and $D_j(W) = 0$ otherwise. The single-choice model, popular in PLS, belongs to this category and corresponds to the particular case where $g(n_j) = 1$ for every j, implying perfect consumer retention. Locational choice models are based Hotelling's (1990) example where consumers go, *ceteris paribus*, to the closest competing seller. These models are a particular case of first-choice where the utility of product j for consumer segment i is the difference between how closely product attributes z_j match consumer preferences z_i and its price, or $u_{ij} = g(z_i, z_j) - p_j$ with $g(\cdot)$ being a proximity function. The model provides an intuitive quantification of the substitution, guaranteeing similar segments go for the same alternative, and penalization, occurring when the closest product is not worth its price.

Attraction Attraction models are based on Luce's choice axiom (Luce, 1959), stating that j's purchase probability is computed as $a_j / \sum_k a_k = a_j / a$, with a_j being the weight (or attractiveness) of j, and k going through all available options. Congruently, the ratios between the available options' shares are maintained (IIA). Early market share models define a_j as the marketing effort directed to a product or brand. While the complexity of research on quantifying marketing effort increased (e.g., requiring each parcel of the marketing mix to be accounted for and multiplied by its effectiveness), Bell et al. (1975) posit redefining this measure as offer attractiveness. With attraction models, $D_j(W) \leq da_j/a$ for all $j \in W$, as exemplified in the figure. If k is removed, then $a := a - a_k$ and $D_k(W) \leq 0$. Let β_b be the appreciation consumers have for attribute b and $z_{j_b} = 1$ if attribute b is present in product j. Random utility models such as the multinomial logit and the multiplicative competitive interactions model are particular instances of the attraction model where $a_j = \exp \sum_b \beta_b z_{j_b}$ and $a_j = \prod_b (z_{j_b})^{\beta_b}$ respectively (Lau et al., 1997).

2.3.2 Production planning

Production modeling in PLS accounts for the selection's complexity costs $\mathrm{C}(W)$ and limits amounts sold $\mathrm{D}_j(W)$ to the manufacturing plant's capacity. There are three main ways of simulating the aforementioned limitations. One way is to impose an upper limit K on the cardinality of W, or $|W| \leq K$. In these cases, the line will select K products unless cannibalization by products with lower margins makes it more profitable to offer less. Another way is to exclusively limit the size of a line by the losses resulting from cannibalization (e.g., Kraus and Yano, 2003), without considering any limitation the manufacturing process may impose on the selection. The third way is to compute the indirect costs associated with introducing each product and let the line's economic viability be the limiting factor. Additionally, constraints making the capacity of each resource explicit can accompany all three ways of limiting line size. The production modelling features used in PLS research follow.

Fix costs Fixed costs are the simplest way to account for negative introduction effects. A developed product needs to be sourced (e.g., a new beverage could need a new bottle and label, and would benefit from timely advertising) and requires additional changeovers in the production plan. These costs can be considered by adding $\sum_j f_j W_j$ to $\mathrm{C}(W)$, with $W_j = 1$ if $j \in W$ and f_j being the fixed cost of selecting product j. For single-period analyses, disregarding production frequency, costs s_j relative to changeovers Y_j serve the same purpose as fix costs. Modeling complexity costs this way naturally limits line size to what is economically viable.

Family setups Setups shared across products model economies of scope, occurring when multiple products from the same family are selected. Sometimes products share some part of the changeover process (e.g., it is quicker to switch between two beverages with the same bottle size than if sizes were different). Producing such products does not require further family changeover costs. Let $\mathcal{N}_m$ be the set of products that share the family setup m, s^m family m's changeover costs, and $Y^m = 1$ if any $j \in W \cap \mathcal{N}_m$ is produced. Adding $\sum_m s^m Y^m$ to $\mathrm{C}(W)$ implies that, if at least one of family m's products are selected, the manufacturer incurs the family setup costs.

Capacity Productive processes have bottlenecks with c available time. Producing a unit of j takes r_j bottleneck time. With X_j being the production quantity for product j ($X_j > 0$ only if $Y_j = 1$), necessarily $\sum_j r_j X_j \leq c$. If the impact of changeovers cannot be neglected, the number of production cycles T in a planning horizon is directly proportional to the number of changeovers (Morgan et al., 2001a). Assuming additive setup times, with product changeovers taking τ_j time and family changeovers taking τ^m time, $\sum_m \tau^m T Y^m + \sum_j \tau_j T Y_j + \sum_j r_j X_j \leq c$.

Resource sharing Capacity distributes itself across multiple resources, which products share (e.g., beers fermented in different places are bottled in the same production line). For these cases, the operation's bottleneck (i.e., the resource whose available time is exhausted) can change with the selection. Let r_{jk} represent the time it takes for resource k to produce a unit of product j and c_k is that resource's available time, resource sharing implies $\sum_j r_{jk} X_j \leq c_k$ for all k. Likewise, if changeovers must be considered, $\sum_m \tau_k^m T Y^m + \sum_j \tau_{jk} T Y_j + \sum_j r_{jk} X_j \leq c_k$.

Setup and inventory costs Setup and holding costs incurred throughout a planning horizon can be estimated using the number of production cycles. Discriminating these costs provides a representation more accurate than considering a fix complexity cost per product. It approximates the true complexity costs by estimating inventory and setup numbers. With h_j being the costs of carrying a unit of j through a given time period, the indirect production costs can be computed as $\sum_j s_j T Y_j + \sum_j h_j X_j / (2T)$ (linear inventory consumption). Alternatively, setups and inventory can be estimated by determining a feasible production plan.

2.4. Literature review

This section provides an exhaustive review of PLS optimization models and methods. We start by reviewing problem modelling advances, analysing in detail some key models, and systematizing research by how it considers consumer response and production costs. Then we summarize solution approaches. While the topic started mainly as a marketing issue, we observe that, in time, research separates into models with attraction consumer response and single choice models including additional production features. We also review innovative approaches considering uncertainty and consumer irrationality (compromise effects). Delving into PLS solution methods, we categorize advances in both heuristics and exact methods. The use of genetic algorithms for PLS has also received considerable attention. Recently, researchers provided efficient reformulations for multiple variants of both single-choice and attraction PLS.

2.4.1 Optimization models

Primordial PLS research intended to incorporate the cost and demand interdependencies between products to assess the total influence of product introductions and withdrawals. Through the parametrized pairwise interactions between products, Monroe et al. (1974) manage to capture the effect that a product being present has in the demand for, and production cost of, others. Shugan et al. (1977) deliver the first PLS formulation considering a single-choice process (i.e., consumers purchase the highest-utility available product), where products can be offered at different price levels. To the best of our knowledge, this work initiated the PLS research stream. Green and Krieger (1985) formalize PLS into two problems with single-choice consumer behavior: considering the buyer's perspective and seller's. While buyers want a certain number of products, maximizing their overall utility, sellers seek the profit-maximizing set of products. The seller's problem can be constrained by the line's size or organically, including fixed costs and reservation prices. McBride and Zufryden (1988) extend the seller's problem with the suggested fixed costs and solve it as a combinatorial problem. Pricing is tackled continuously by Dobson and Kalish (1988). Later, Dobson and Kalish (1993) conclude that fixed costs make the problem NP-hard, by analogy with the uncapacitated plant location problem. Formulation (2.2) describes the PLS problem as tackled in the work of Dobson and Kalish (1993).

$$\max_{X,W} \quad \sum_{i,j} p_j X_{ij} - \sum_j f_j W_j \tag{2.2a}$$

$$\text{s.t.} \quad (u_{ij} - p_j) W_j \leq \sum_b (u_{ib} - p_b) X_{ib} \quad \forall i, j \tag{2.2b}$$

$$\sum_j X_{ij} = 1 \quad \forall i \tag{2.2c}$$

$$X_{ij} \leq W_j \quad \forall i, j \tag{2.2d}$$

$$p_0 = s_0 = u_{i0} = 0 \quad \forall i \tag{2.2e}$$

$$X_{ij}, W_j \in \{0, 1\} \quad \forall i, j \tag{2.2f}$$

Let $W_j = 1$ if product j is offered and $X_{ij} = 1$ when consumer segment i purchases it, granting p_j margin. Also, consider $j = 0$ the no-purchase option. Expression (2.2a) defines the objective as maximizing the difference between the revenue and fixed costs for the selected products. Constraints (2.2b) and (2.2c) capture the single-choice consumer behaviour. The first constraint set ensures consumers purchase the product providing them the most after-price utility u_{ij}, and the second constraint set defines that each consumer segment chooses one product. Single-choice models match consumer segments and products. Constraints (2.2d) and (2.2e) ensures purchases must involve selected products and are useful to the segment's consumers. Finally, Constraints (2.2f) define the binary variables. Onwards, research on the topic diverged into two streams: one focuses on manufacturing and incorporates it in the model in a more detailed manner, the other focuses on consumer choice and uses attraction models for market share instead of ranking-based ones.

Research on PLS was first dominated by marketing and, with time, crept into the management science literature. After this, it eventually caught the eye of production management researchers and practitioners. It is common practice in capital-intensive industries for products to share part of the production procedure. Instead of product changeovers, Dobson and Yano (1995) propose process changeovers which require setting up a group of resources. Yano and Dobson's (1998) review covers PLS and product line design optimization models up to this point. Morgan et al. (2001a) overlook pricing but add considerable detail to production modeling. Besides including family setups, their work manages to capture inventory costs by considering the average production frequency. Nichols et al. (2005) study solution procedures for Dobson and Kalish's (1993) problems with an objective weighting profit and welfare. Fruchter et al. (2006) address the profit-maximizing problem using a genetic algorithm with customized operators and an optimization step for pricing. Day and Venkataramanan (2006) tackle PLS with pricing and resource sharing. Moon et al. (2017) introduce a formulation with discrete pricing, inventory capacity, and lost sales. Recently, Bertsimas and Mišić (2019) present a new formulation for single-choice PLS providing stronger linear relaxations. Formulation (2.3) recreates Morgan et al.'s 2001a model.

$$\max_{T,X,Y} \quad \sum_{i,j} p_j d_i X_{ij} - \frac{1}{2T}\sum_{i,j} d_i h_j X_{ij} - T\sum_m s^m Y^m - T\sum_j s_j W_j \tag{2.3a}$$

$$\text{s.t.} \quad u_{ij} W_j \leq \sum_{b\in W} u_{ib} X_{ib} \quad \forall i, j \tag{2.3b}$$

$$\sum_{j\in W} X_{ij} = 1 \quad \forall i \tag{2.3c}$$

$$\sum_i X_{ij} \leq M W_j \quad \forall j \tag{2.3d}$$

$$\frac{1}{T}\sum_j q_j^m W_j \leq Y^m \quad \forall m \tag{2.3e}$$

$$T > 0; \quad W_j, Y^m, X_{ij} \in \{0,1\} \ \forall i, j, m \tag{2.3f}$$

Expression (2.3a) defines the objective function. Here, W_j work both as selection and product-setup variables. With d_i being the size of segment i, the first sum refers to the selection's gross profit. Let T be the production frequency and h_j the unitary holding cost for product j. The second sum computes the mean inventory costs. Noting that s^m refers to the setup costs of family m and $Y^m = 1$ if any product in that family is produced, the third and fourth sums account for setup costs. Constraints (2.3b) and (2.3c) define single choice. With W being the set of selected products, the constraints ensure each segment chooses the utility-maximizing product. Hence, $j \in W \Rightarrow W_j = 1$. Constraints (2.3d) define setups for selected products. Constraints (2.3e) set the category setup variables, with $q_j^m = 1$ if product J belongs to family m. Finally, Constraints (2.3f) define the domain of the variables. The paper includes extensions for multiple planning horizons and capacity constraints. The authors suggest optimizing the product line over different values for T.

Modeling consumer behavior as a single choice can be limiting. One can bypass this by modeling the market with a large number of tiny homogeneous consumer segments. Still, this can vastly increase the model's 0-1 decision number, making the problem intractable. Single-choice is adequate for sporadic high-involvement purchases, such as cars or desktop computers. For recurrent low-involvement purchases, some authors opt for probabilistic choice (Baier and Gaul, 1998), namely attraction models, as they predict market share more accurately and intuitively. In a mathematical note, Chen and Hausman (2000) show that, without fixed costs, solving the linear relaxation of PLS with attraction consumer response and cardinality constraints leads to the integer optimum. Kraus and Yano (2003) compute attraction as the surplus between utility and price. Schön (2010a) provides the first PLS formulation where linear constraints define the attraction model, which we analyze below. In later work, Schön (2010b) proposes a model with continuous pricing and resource sharing, taking advantage of the problem structure to devise an efficient solution method. Formulation (2.4) respects Schön's 2010a model where, for simplicity, we consider a single market segment.

$$\max_{Y,\hat{W}} \quad \sum_{j,s\in\mathscr{P}_j} p_{js} a_{js} \hat{W}_{js} - \sum_m s^m Y^m \tag{2.4a}$$

$$\text{s.t.} \quad \sum_{s\in\mathscr{P}_j} \hat{W}_{js} \leq vY^m \quad \forall m, j \in \mathscr{N}^m \tag{2.4b}$$

$$\sum_{j,s\in\mathscr{P}_j} a_{js}\hat{W}_{js} + a_0 V = 1 \tag{2.4c}$$

$$\hat{W}_{js} \leq V \quad \forall j, s \in \mathscr{P}_j \tag{2.4d}$$

$$V, \hat{W}_{js} \geq 0 \; \forall j, s \in \mathscr{P}_j; \quad Y^m \in \{0,1\} \; \forall m \tag{2.4e}$$

Products have different price levels and fixed costs are process-based. Let a_0 be the no-purchase attractiveness, $V = 1/\left(\sum_j a_j W_{js} + a_0\right)$ with the $W_{js} = 1$ if j is selected at price level s, $\hat{W}_{js} = VW_{js}$, and $v \geq V$. Expression (2.4a) denotes the objective. The first term computes gross profit, with $\mathscr{P}_j$ containing j's potential price. The second term refers to resource setups. Constraints (2.4b) control the value of the setup binaries and ensure products are not offered at two different prices. Constraints (2.4c) and (2.4d) define the attraction model. The first set of constraints ensures the selection's market share plus that of the no-purchase option equals one. The second set limits the ratios between product and competition market shares to their attractiveness ratios (more clearly seen if each term is multiplied by $a_0 a_{js}$, also present in Gallego et al. (2014)). Let S_{js} be the market share of product j at price level s and S_0 that of the no-purchase option. Constraints (2.4d) imply that $S_{js}/a_{js} \leq S_0/a_0$ for all j and s. Constraints (2.4c) would be stated as $\sum_{js} S_{js} + S_0 = 1$, $\forall j, s \in \mathscr{P}_j$ in this notation. The constraint sets cooperate to emulate attraction consumer response.

Some authors present extraordinarily innovative PLS models and applications. Yücel et al. (2009) consider quality costs and supplier selection in a model with parametric consumer response accommodating both substitution costs and demand loss. Tang and Yin (2010) analyze how costs, capacity, and competition influence selection, prices, and quantities of two substitutable products, considering random reservation prices. Bertsimas and Mišić (2016) tackle the mismatch between choice modeling and consumer with robust optimization. They protect retailer profits from both imprecise parameter estimates and model selection. Qi et al. (2020) derive closed-form solutions for PLS with fix costs under marginal moment consumer choice model, originally used for data uncertainty (Bertsimas et al., 2006). Bechler et al. (2021) present an optimization model with attraction consumer response considering the compromise effect. To the best of our knowledge, two works apply PLS optimization models to real scenarios. Bernales et al. (2017) apply a model with a fixed production quantity, holding costs, and capacity, considering parametric one-step substitution to a technological-products distributor case. Shunko et al. (2018) develop an approach for a construction-equipment manufacturer. The authors consider single-choice consumer response with pricing and model complexity costs through the differential impact an inclusion has in each department.

Table 2.3 synthesizes mathematical formulations for PLS based on the decisions and features that approaches consider. For each paper, we classify the pricing decision as discrete (D) or continuous (C) if price is an index or a variable in the model. We denote fixed (F) production when there is a production frequency decision. We categorize demand as parametric (P), single-choice (S), and resulting from an attraction model (A). We further categorize approaches by the features each model captures (Y, meaning yes). We consider fixed costs, family setups, inventory costs, capacity, and resource sharing. Lastly, recent research created the urge to discriminate approaches that account for uncertainty by proposing robust (R) solutions. Tang and Yin's (2010) model only serves the purpose of their analysis: derive the conditions under which the two products are offered. Bechler et al.'s (2021) approach considers design decisions. Therefore, they were excluded from the summary.

2.4.2 Solution methods

With a clear picture of PLS models, we seek the most efficient procedures and their current limitations. The buyer problem from Green and Krieger (1985) is not adopted by subsequent research and is neglected. The authors obtain their best results for the seller's problem using a greedy insertion heuristic without any performance guarantee. The formulations studied by Green and Krieger (1985) and McBride and Zufryden (1988) constrain the number of products explicitly. In the most recent paper, the authors apply mathematical programming methods, guaranteeing optimal integer solutions. Shugan et al. (1977) use fixed costs to limit selection size. The author solves linear relaxations with up to nine

Table 2.3 – Literature summary and classification on product line selection.

Paper	Ancillary decisions			Modeled features					
	Pricing	Production	Demand	Fixed costs	Family setups	Inventory costs	Capacity	Shared resources	Uncertainty
Monroe et al. (1974)	-	-	P	-	-	-	-	-	-
Shugan et al. (1977)	D	-	S	Y	-	-	-	-	-
Green and Krieger (1985)	-	-	S	-	-	-	-	-	-
McBride and Zufryden (1988)	-	-	S	Y	-	-	-	-	-
Dobson and Kalish (1988)	-	-	S	Y	-	-	-	-	-
Dobson and Kalish (1993)	C	-	S	Y	-	-	-	-	-
Dobson and Yano (1995)	-	-	S	Y	Y	-	-	-	-
Morgan et al. (2001a)	-	F	S	Y	Y	Y	Y	-	-
Nichols et al. (2005)	C	-	S	Y	-	-	-	-	-
Fruchter et al. (2006)	C	-	S	Y	-	-	-	-	-
Yücel et al. (2009)	-	-	P	Y	Y	Y	Y	-	-
Day and Venkataramanan (2006)	C	-	S	Y	Y	-	-	-	-
Moon et al. (2017)	D	F	S	Y	-	Y	Y	-	-
Shunko et al. (2018)	C	-	S	Y	-	Y	Y	-	-
Bertsimas and Mišić (2019)	-	-	S	-	-	-	-	-	-
Aydin and Ryan (2000)	C	-	A	-	-	-	-	-	-
Chen and Hausman (2000)	D	-	A	-	-	-	-	-	-
Kraus and Yano (2003)	C	-	A	-	-	-	-	-	-
Schön (2010a)	D	-	A	Y	Y	-	-	-	-
Schön (2010b)	C	-	A	Y	Y	-	Y	Y	-
Bertsimas and Mišić (2016)	-	-	P	-	-	-	-	-	Y
Bernales et al. (2017)	-	F	P	Y	-	Y	Y	-	-
Qi et al. (2020)	C	-	P	Y	-	-	-	-	-
Andrade et al. (2020)	-	V	A	Y	Y	Y	Y	-	-

Decision or feature is included (Y); continuous (C) or discrete (D) pricing; production is fixed (F) or variable (V); consumer response is modelled parametrically (P), as a single choice (S), or by an attraction model (A).

products and consumer segments, using the technology available at the time. Dobson and Kalish (1988) further consider product pricing. Their strategy is to start by selecting the utility-maximizing line and make the profit-maximizing product removal at each iteration (a backward procedure). Products can be priced efficiently by decomposing the pricing subproblem into a network of shortest path problems. In Dobson and Kalish (1993), the authors suggest that introducing the welfare-maximizing product and optimizing profit at each iteration (a forward procedure) can lead to better results. The model Morgan et al. (2001a) propose does not cover pricing. Besides modeling resource sharing, they decide on production cycle duration and use an implicit enumeration algorithm to solve problems with up to 20 products and 40 consumer segments. Recently, Bertsimas and Mišić (2019) present a stronger single-choice formulation and a Benders decomposition approach.

Several papers consider genetic algorithms. Nichols et al. (2005) address PLS weighting both profit and consumer welfare objectives using three solution procedures. In the first, a genetic algorithm seeks both the selection and product prices. The second genetic algorithm guesses prices, while branch and bound determines product-segment correspondences. The third procedure uses the genetic algorithm to select products and their respective segments. Then, it solves pricing using the shortest path algorithm. The authors find that the second and third procedures scale better with problem size. Day and Venkataramanan (2006) tackle the profit-maximizing problem with resource sharing. The authors compare the performance of using mathematical optimization on the discrete-pricing problem with the two former genetic algorithms from Nichols et al. (2005). They find that the discrete-pricing reduction performs best for small instances and the first genetic algorithm for larger ones. Fruchter et al. (2006) propose three deterministic operators. The first lowers prices attempting to increase profit by reducing cannibalization. The second checks for profitable fits between unselected products and non-purchasing segments. The third raises prices without breaking any previous product-segment association. Moon et al. (2017) include dynamic mutation probabilities and a catastrophe operator removing low-quality solutions from the pool.

Kraus and Yano (2003) devise a heuristic for PLS with attraction consumer response. The authors use simulated annealing to find the best hypercube (selection) and steepest ascent for pricing within the hypercube. Chen and Hausman (2000) find that integrality can be relaxed in cardinality-constrained attraction PLS since the objective is strictly quasi-concave. Schön (2010a) formulates attraction PLS as a sum of linear ratios. This technology leap enables formulating attraction models using linear constraints. The author addresses the problem with fix costs, resource sharing, and discrete pricing. Schön (2010b) also tackles the problem with continuous pricing. By making market share the decision variable (which depends on attractiveness, which is contingent on price), the author finds the problem having concave objective functions for the major attraction consumer choice models. Park and Mo (2019) apply optimal-price closed-form expressions (Li and Huh, 2011) to reduce PLS with pricing under MNL consumer response (a mixed-integer fractional model) to an integer program with a concave objective.

Table 2.4 overviews the advances in PLS solution methods. Works are organized by consumer response model as they define the problem structure and the solution procedure. The inclusion of the pricing decision is also crucial in determining the adequate solution

Table 2.4 – Work containing advances in solution methods for PLS problems.

Paper	Price	Constraint	Size	Method
Single-choice consumer response				
Green and Krieger (1985)	-	S	5/32, 117	H
McBride and Zufryden (1988)	-	S	10/16, 100	E
Shugan et al. (1977)	D	F	8, 9	H
Dobson and Kalish (1988)	C	F	4, 5	H
Dobson and Kalish (1993)	C	F	80, 800	H
Morgan et al. (2001a)	-	F	20, 40	E
Nichols et al. (2005)	C	F	100, 1000	H
Fruchter et al. (2006)	C	F	144, 61	H
Day and Venkataramanan (2006)	C	F	40, 120	H
Moon et al. (2017)	D	F	100, 1000	H
Bertsimas and Mišić (2019)	-	-	10/500, 1000	E
Attraction consumer response				
Kraus and Yano (2003)	C	M	5, 10	H
Chen and Hausman (2000)	D	S	-	E
Schön (2010a)	D	F	5000, 4	E
Schön (2010b)	C	F	10000, 10	E
Park and Mo (2019)	C	F	300, 1	E

Continuous (C) or discrete (D) price variable; line size is limited by size (S), by fix cost (F), or by margin (M); size of the largest instance is presented as $\{size/products, segments\}$ for products with a line size constraint and as $\{products, segments\}$ otherwise; the employed solution method can be exact (E) or heuristic (H).

method. Continuous pricing (C) can make PLS non-linear, and the inclusion of discrete pricing (D) multiplies the number of selection binaries by the number of price levels. The third column specifies what limits product line size. There can be explicit constraints on the number of products (S), fixed costs (F), or cannibalization from products with thinner margins (M), making extensions detrimental. The table shows that there were recent advances in exact solution methods for both single-choice and attraction PLS. With attraction consumer response and fixed costs, the problem can be solved optimally for realistically sized instances.

2.4.3 Research gaps

Organizing the literature sheds light on several research gaps regarding production planning, uncertainty modeling, solution methods, pricing, and shared resources. We detail these below, ordered by practical relevance.

Uncertainty modeling Considering that PLS is a high-stakes decision, knowing that uncertainty is accounted for in the proposed solution makes it more enticing for practitioner adoption. Nevertheless, research including it is scarce and only regards the consumer response aspect of PLS. There is a gap for robust approaches that incorporate production modeling features. Also, stochastic programming and complexity cost uncertainty remain unexplored.

Production planning Complexity costs are oversimplified in PLS. Models seldom account for more than fixed product and family costs. Notably, only Andrade et al. (2020) consider multiperiod production planning, which correctly estimates inventory and setups under dynamic demand. This same level of detail is absent from single-choice models. Moreover, variety's impact on plant congestion has not been addressed.

Solution methods There were recent advances in exact approaches for PLS with both single-choice and attraction consumer response. Nevertheless, approaches managing optimal solutions for real-sized instances (Bertsimas and Mišić, 2019; Schön, 2010a; Bernales et al., 2017; Shunko et al., 2018) could benefit from additional detail. Incorporating uncertainty and manufacturing constraints into them requires further computational effort. The added complexity motivates solution method development.

Pricing From approaches considering production decisions, only Moon et al. (2017) consider discrete pricing. Synergies can arise from using pricing to handle demand variations. Joint pricing and uncertainty modeling are still unexplored. Furthermore, multiperiod pricing and production decisions (i.e., with variable production) would be advantageous in capacity-constrained scenarios with dynamic demand.

Shared resources Resource sharing is usually associated with design decisions. Nevertheless, controlling non-bottleneck resource utilization can enhance a selection's ability to capitalize on economies of scope. It also protects the approach against the appearance of new bottlenecks. The issue is underexplored in PLS since only Schön (2010b) addresses it.

2.5. Fast-moving consumer goods

This section describes two cases regarding manufacturers of beverages and cheeses. The beverages producer needs to trim its overextended product lines. The cheese producer wants to find the right extensions to utilize his capacity excess. Both cases require PLS models for decision support. Our case descriptions result from two-hour semi-structured interviews with high-ranking officers, having a deep knowledge of and clout over product line decisions. We ensured each talk's subject was the firm's product offer. We also introduce a framework relating FMCG manufacturers' MOI decisions. The framework and firm manager interviews enable us to identify research directions with proved practical relevance.

2.5.1 Beverages industry

The Chief Operations Officer of a beverage firm confronted us, stating that they had no established procedures for planning their product mix, a yearly task. Like others in the FMCG sector, the firm had been adding different products to satisfy the whims of its clients, primarily retailers. By adding different beverage flavors or packing and palletizing the products differently, complexity piled up, and production became congested. Moreover, the increased number of products made forecasting less accurate, and the firm, more often than before, found itself holding the wrong finished goods. This demand mismatch meant additional production runs would be needed to restitute the stocks of the goods the predictions undershot. The larger the number of these products, the larger the number of setups that would ensue, and the more capacity became constrained. The complexity would then move downstream to the logistics function, which would exceed material storage and movement capacities frequently.

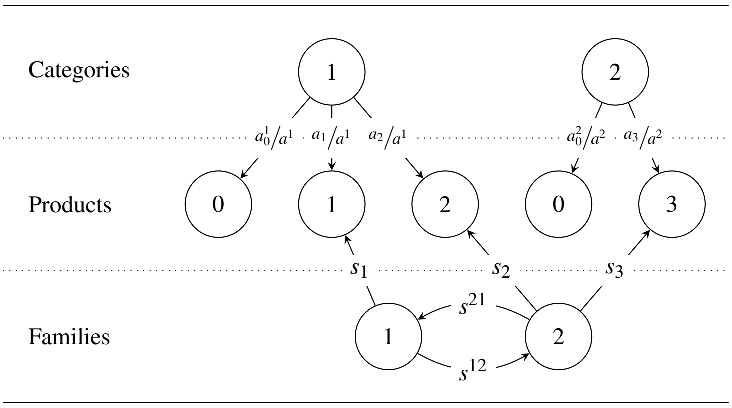

Figure 2.5 – Illustrative market and setup structure for a PLS problem with attraction consumer response, non-interactive demand categories, and resource sharing. If $s^{21} \neq s^{12}$ then there are sequence-dependent setups between the families.

The beverages industry, among others in the FMCG sector, has several peculiarities. For starters, it relies on large sales volumes to break even, with consumption being highly variable and seasonal (Guimarães et al., 2012). Single-choice models the low-involvement purchases of beverages inappropriately, as consumers often vary their choices. Variety is perceived differently by production lines and consumers since the latter care more about different flavors, and the plant is more affected by different sizes as exemplified in Figure 2.5 (i.e.,categories are flavors and families are bottle sizes). Capacity is a hard constraint, as extensions are expensive and not short-term viable. Additionally, beverages can be manufactured (bottled, as it is the bottleneck of the operation) interchangeably between the various production lines, with different throughput rates. Products are grouped in families, and changeovers (e.g., machine adjustments, cleaning) are sequence-dependent. Both production and changeovers consume the limited time available at the respective production lines.

At the time, we were collaborating with the firm to improve its tactical distribution planning and production scheduling. We were optimizing operations, considering that sales and product choices were beyond reach. While it resulted in significant operational cost reductions (Guimarães et al., 2014a), we did not deal with the root causes of all the clutter. With over 390 SKUs, there was an evident product portfolio fragmentation that the company had no quantitative support for tackling. While, in some businesses (i.e., where production systems are not product-focused), capacity extensions are cheap, the capital-intensive nature of beverage manufacturing does not allow it. Therefore, managers must shift demand to core products by removing those adding too much complexity for their strategic relevance. While this makes sense from an operational standpoint, marketing and salespeople are prone to oppose removing any SKU to avoid client dissatisfaction.

2.5.2 Dairy industry

After several automation projects at its main plant, the Marketing and Sales Manager of a dairy products firm found himself with a generous amount of unused capacity. To increase its performance, the firm had two options: to explore additional automation, further reducing labor costs and leaving the demand side of the equation untouched, and to expand its consumer base by introducing additional products and covering different channels. Low-hanging fruit had already been picked for the first option. Therefore, we discussed the introduction of non-innovative products (with similar options being offered by competitors) to gather consumers from the most profitable categories. The firm's main production line outputs three-pound balls and ten-pound bars of Edam and other curated cheeses. We examined the introduction of grated and sliced cheeses, organic cheese trends, and the requirements to offer low-fat cheese.

The first option regards a competitive category and requires purchasing additional machinery to process and package the goods. The slicing and dicing do not interfere with the cheese production itself and could reutilize deformed products. Organic cheeses require organic suppliers and additional cleaning cycles after producing the regular product. Still, given the high quality of the regular cheese, few competitions dare to market an organic product. Lastly, producing low-fat cheese simply requires minor package design changes. Competition is milder than in the sliced and grated cheese category. Potentially, it could require researching recipes since full-fat cheese is considerably better tasting.

We analyzed the advantages of each option qualitatively, but quantitative support was required to make a decision. We need to compute the required cheese output (before slicing and dicing) and account for deformities for the first option. Additional cheese would be required to serve sliced and grated cheese consumers, but quality would not be an issue. For the second option, the contributions from demanded amounts must surpass the additional setup costs. The third option seems safest, but there can be damaging long-term impacts of offering less flavorful cheese. Our manager craves the quantitative tools to answer these questions.

2.5.3 Discussion

The beverages manufacturer is facing a twofold variety management problem. First, the product mix's extent overburdens finding a favorable removal. Second, operations recognize the problem, but marketing is skeptical about the benefits of delisting products. Moreover, greedy heuristics can lead to crassly suboptimal selections (Andrade et al., 2020). PLS optimization approaches make impartial decisions. They handle large SKU numbers, maximizing the common profit objective hence quantify the delistings' benefits. These approaches also evaluate gains resulting from introducing known products, weighting our dairy manufacturer's options. Below, we introduce a framework relating MOI decisions to identify the modeling features required by our manufacturers to adopt PLS optimization approaches.

Figure 2.6 exhaustively enumerates MOI trade offs. The framework is general to MTS producers with unchanging facilities offering products without post-sale service. Our man-

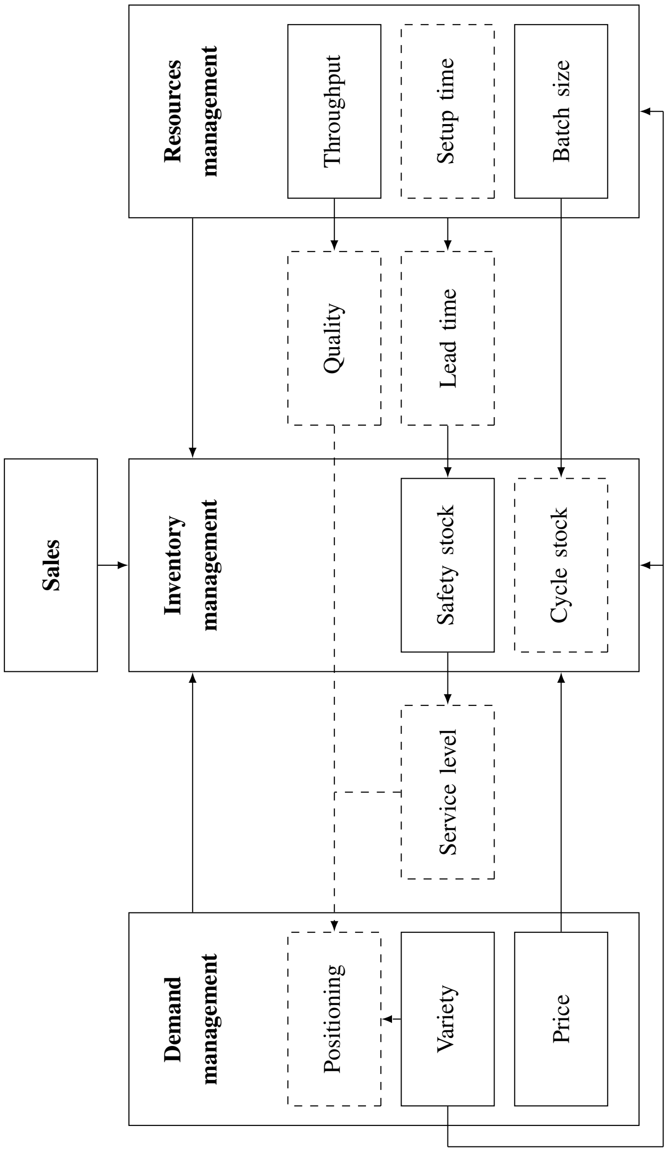

Figure 2.6 – Constructivist framework of the MOI in a capital-intensive MTS producer. Controllable variables are presented in full squares, uncontrollable in dashed squares. Direct influence is represented by arrows with full lines, indirect by arrows with dashed lines.

ufacturers fit in such category. Arrows point from controllable to dependent variables. MTS systems require sales to come from existing inventory. Demanded amounts depend on brand positioning, assortment, and pricing (hence we consider inventory dependent on sales).

Brand positioning determines how consumers value product unknowns (e.g., a consumer not knowing the exact taste and texture of a natural yogurt before eating it, relies on brand image when purchasing), and is built on quality and consistency. Even though quality is a dependent variable, the manufacturer controls throughput, and consequently, inspections. Similarly, the safety-stock policy influences service level. Furthermore, and regardless of the forecasting technology, demand uncertainty directly depends on the production lead time. Lead time can be regulated through batch size and throughput, affecting WIP inventory and the speed that it is processed. Production lead time can account for setup times, but these cannot be changed significantly (considering SMED has already been implemented).

A congruent selection can strengthen brand image. It also affects resources management as a broader product line will, as a rule, require more changeovers. Furthermore, the complementarity and substitution among selected products influence sales and, consequently, inventory management. Increasing prices decrease sales for elastic consumable products. Hence, price can manipulate sales numbers at the operational level, even though FMCG firms have limited power over price (Quante et al., 2009). Lastly, cycle stock depends on batch size directly since it computes as the production and depletion rates difference integrated throughout the planning period.

2.5.4 Research directions

Through our case studies and the trade-offs that the framework enumerates, we establish research directions. We find that incorporating consumer unpredictability is valued by both manufacturers. Interesting questions arise from trade-offs involving quality and throughput. Product line change's brand impact can be a promising research topic. Safety stock's dependency on the selection is particularly significant for FMCG producers. Lastly, we identify low-hanging fruit in production modeling.

Uncertainty modelling Practitioners must trust optimization solutions to adopt them. Since PLS is a high-stakes decision, managers must accept model parameterization to embrace its decisions. Accounting for consumer response prediction errors is particularly valued. Work addressing this direction propose robust selections (Bertsimas and Mišić, 2016) and use stochastic programming models to consider a range of possibilities. Considering uncertainty is essential for both manufacturers.

Quality We consider throughput affects product conformity and, in time, how consumers view a brand. By raising the throughput, a manufacturer can benefit from additional capacity without considerable investment. The associated quality sacrifices may reduce consumer willingness to pay. Our dairy manufacturer can include sliced and grated cheeses, which reuse deformed cheese. Considering this trade-off, and products mitigating defect impact on consumer brand perception, are novel PLS issues.

Positioning The dairy manufacturer worries that offering low-fat cheese can damage the brand. Quantifying product line change's impact on brand positioning is particularly important for managers assuring the business's long-term sustainability. Investigating the issue and incorporating it into PLS models is innovative and pertinent. Such research should quantitatively show how incongruent product lines weaken brand image. The beverage manufacturer's marketing department would benefit from its conclusions.

Safety stock Holding costs become significant in high-volume low-margin settings. Both beverages and cheeses are seasonal products with considerable demand variability. Making large batches, anticipating demand peaks increases lead time and, consequently, demand uncertainty. Chong et al. (2005) and Andrade et al. (2020) consider market share's impact on demand predictability. Further incorporating risk pooling and demand complementarity effects is needed to wholly consider a product line change's impacts on safety stock costs.

Production modeling Our beverage producer's production planning problem can be modeled as a capacitated lot-sizing problem with product families and sequence-dependent setups (Almada-Lobo et al., 2015; Guimarães et al., 2014b). Considering shared resources is required if bottling stops being the bottleneck. Andrade et al. (2020) study integrating consumer response with lot sizing, handling demand variations with limited capacity, which is key for process industries (Clark et al., 2011). Nevertheless, extending this integration with industrial constraints (Glock et al., 2014) is low-hanging fruit for researchers. Furthermore, new models should incorporate the plant congestion variety creates.

2.6. Conclusion and research opportunities

This research's ultimate purpose is to endow managers with PLS optimization tools. Towards this purpose, we potentiate the research community by clearly defining the problem's scope, background, state-of-the-art, and research gaps. Further, we increase this line of research's visibility to managers by making a didactic problem introduction, suggesting a marketing-operations decision framework (assuming make-to-stock production and no capital investments), and exposing FMCG producers' needs respecting PLS optimization. For researchers, these needs become opportunities. Therefore, considering research gaps and producer needs enables us to propose practically and scientifically relevant opportunities for future research.

We hope our introduction is didactic enough to bring more researchers into the topic and that its contents are sufficient to ready a researcher familiar with optimization to tackle this problem. Our formulation is rigorous enough to serve as a unified definition of the problem currently. Further developments in PLS research can steer the problem away from its present form, outdating the definition. Likewise, we present the first exhaustive review focusing on PLS exclusively, which, in time, will need updating. With the beverages and dairy cases, we employ our practical experience to identify the modeling requirements of FMCG manufacturers inductively. We intend that the cases we present bring the body of literature closer to FMCG practitioners. Below, we conclude this paper by presenting the six future research opportunities we found most pertinent.

Modeling uncertainty

Modeling uncertainty is both a prominent research gap and manufacturer modeling requirement. Practitioners seldom implement solutions being skeptical of input parameter exactitude. Proposing robust selections, as studied by Bertsimas and Mišić (2016), is a viable alternative to impeccable parameter estimation. This option is still unexplored when models consider production decisions. Additionally, PLS research neglects any source of uncertainty besides consumer behavior. One possibility is the development of solutions robust to changes in capacity and setup times. Also, no stochastic programming PLS approach exists. The selection's impact on demand variability is a pertinent question, as Chong et al. (2005) study for trimming a single product. For capital-intensive industries, dealing with demand uncertainty may require large investments in manufacturing flexibility or significant safety stock costs. A selection otherwise thought profitable is potentially made unviable by the magnitude of such costs.

Including industrial decisions and constraints

Existing models neglect important production modeling features, limiting their practical use. Figure 2.6 shows us potential model extensions relevant for FMCG producers. Throughput relates negatively with quality, being a means to extend capacity and reduce lead times. Moreover, pricing can be a tool for short-term demand management, even with FMCG producers having limited pricing power (Quante et al., 2009). The inclusion of pricing is overlooked when production decisions can vary throughout the planning horizon, as required for FMCGs. Additionally, this research opportunity suggests the inclusion of industry-specific constraints into current PLS models. Real application examples of PLS optimization are welcome. The application of production modeling strategies capable of considering variety-induced congestion is also decisive.

Developing solution methods

Advances in first-choice optimization (Bertsimas and Mišić, 2019) change the trade-off between attraction model efficiency and ranking model detail (ranking models can emulate attraction consumer response with the minimum common multiple between attractiveness values as the number of segments). More importantly, the modeling features practitioners require for PLS optimization adoption imply computational complexity increases. Complexity motivates researching heuristics and decomposition strategies with efficient methods for solving problem partitions. For instance, we can decompose Andrade et al.'s (2020) problem into selection and production planning. Still, the subproblems become attraction PLS with fix costs and the capacitated lot-sizing problem. Both problems are difficult but were extensively studied. We suggest using the best-performing methods from each problem's research.

Improving consumer behavior modeling

Product utility (and attractiveness) can account for brand strength. Still, it disregards the effect of assortment congruity. These effects can shed light on viable product line strategies and ensure the strategic purpose of each product. Furthermore, current consumer response models emphasize product substitution but not complementarity and disregard demand elasticity. PLS research should incorporate these effects. Assortment planning (Kök et al., 2015) studies consumer response in the retail context. It proposes more flexible solutions such as nested attraction models with a general number of levels (Li et al., 2015), sequential multinomial choice (Flores et al., 2019), exponomial choice (Alptekinoğlu and Semple, 2016), and the general attraction model (Gallego et al., 2014) which has both the attraction model and independent demand model as special cases. PLS should draw from these examples to overcome current limitations.

Integrating introduction and removal timing

Product introduction and removal timing operationalizes the product program PLS defines. It bridges knowing the profit-maximizing product line with the ability to market it. Overwhelming consumers with introductions and removals can damage the brand. Other stakeholders (i.e., suppliers and retailers) may find dealing with drastic product line changes difficult. Moreover, the optimal product line may differ throughout the year. Seasonal demand profiles may induce the manufacturer to alternate product lines between peaks and troughs. Therefore, timing would be the most pertinent PVM decision to integrate with PLS. It potentiates the demand management aspect of selecting specific products (i.e., sell fewer products with higher margins when capacity is insufficient to handle a demand peak).

Assessing the long term implications

Optimizing a selection can shift demand to products with higher margins and achieve cost reductions, leading to significant gains in profit. Still, these advantages may be temporary, depending on brand impact and other firms' responses. An exciting line of research would be to determine selections that preemptively defend from competitor reactions. Accounting a selection's impact on supplier negotiating power (and retailer, if that is the case) is another interesting research question. Finally, considering the market equilibrium into product line decisions as a function of the selection and its pricing would heighten PVM and PLS research's societal impact. As Lancaster (1998) posits, sellers having incomplete information about consumer preferences results in above-optimal degrees of variety. Hence, an optimal degree of variety would reduce overchoice among consumers and grant them a share from the improved margins.

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Bibliography

G. Alexouda. An evolutionary algorithm approach to the share of choices problem in the product line design. *Computers & Operations Research*, 31(13):2215–2229, 2004.

B. Almada-Lobo, A. Clark, L. Guimarães, G. Figueira, and P. Amorim. Industrial insights into lot sizing and scheduling modeling. *Pesquisa Operacional*, 35(3):439–464, 2015.

A. Alptekinoğlu and J. H. Semple. The exponomial choice model: A new alternative for assortment and price optimization. *Operations Research*, 64(1):79–93, 2016.

X. Andrade, L. Guimarães, and G. Figueira. Product line selection of fast-moving consumer goods. *Omega*, page 102389, 2020. ISSN 0305-0483. doi: https://doi.org/10.1016/j.omega.2020.102389. URL https://www.sciencedirect.com/science/article/pii/S030504832030743X.

G. Aydin and J. K. Ryan. Product line selection and pricing under the multinomial logit choice model. Working paper. Stanford University, 2000.

D. Baier and W. Gaul. Optimal product positioning based on paired comparison data. *Journal of Econometrics*, 89(1-2):365–392, 1998.

N. Balakrishnan, A. K. Chakravarty, and S. Ghose. Role of design-philosophies in interfacing manufacturing with marketing. *European Journal of Operational Research*, 103 (3):453–469, 1997.

G. Bechler, C. Steinhardt, J. Mackert, and R. Klein. Product line optimization in the presence of preferences for compromise alternatives. *European Journal of Operational Research*, 288(3):902–917, 2021.

D. E. Bell, R. L. Keeney, and J. D. Little. A market share theorem. *Journal of Marketing Research*, 12(2):136–141, 1975.

J. Berger, M. Draganska, and I. Simonson. The influence of product variety on brand perception and choice. *Marketing Science*, 26(4):460–472, 2007.

P. J. Bernales, Y. Guan, H. P. Natarajan, P. S. Gimenez, and M. X. A. Tajes. Less is more: Harnessing product substitution information to rationalize skus at intcomex. *Interfaces*, 47(3):230–243, 2017.

D. Bertsimas and V. V. Mišić. Robust product line design. *Operations Research*, 65(1): 19–37, 2016.

D. Bertsimas and V. V. Mišić. Exact first-choice product line optimization. *Operations Research*, 67(3):651–670, 2019.

D. Bertsimas, K. Natarajan, and C.-P. Teo. Persistence in discrete optimization under data uncertainty. *Mathematical programming*, 108(2):251–274, 2006.

P. J. Billington, J. O. McClain, and L. J. Thomas. Mathematical programming approaches to capacity-constrained mrp systems: Review, formulation and problem reduction. *Management Science*, 29(10):1126–1141, 1983.

S. M. Broniarczyk, W. D. Hoyer, and L. McAlister. Consumers' perceptions of the assortment offered in a grocery category: The impact of item reduction. *Journal of Marketing Research*, 35(2):166–176, 1998.

R. Calantone, C. Dröge, and S. Vickery. Investigating the manufacturing–marketing interface in new product development: does context affect the strength of relationships? *Journal of Operations Management*, 20(3):273–287, 2002.

A. K. Chakravarty and S. Ghose. Supporting manufacturing-marketing interface management with a partially automated knowledge base: a precursor to computer-integrated business enterprise. *Journal of Intelligent Manufacturing*, 3(6):347–362, 1992.

K. D. Chen and W. H. Hausman. Mathematical properties of the optimal product line selection problem using choice-based conjoint analysis. *Management Science*, 46(2): 327–332, 2000.

A. Chernev, U. Böckenholt, and J. Goodman. Choice overload: A conceptual review and meta-analysis. *Journal of Consumer Psychology*, 25(2):333–358, 2015.

A. Chernev et al. Product assortment and consumer choice: An interdisciplinary review. *Foundations and Trends in Marketing*, 6(1):1–61, 2012.

D. Chhajed and N. Raman. An integrated approach to product design and process selection. *BEBR faculty Working Paper; no. 92-0178*, 1992.

J.-K. Chong, T.-H. Ho, and C. S. Tang. Demand modeling in product line trimming: Substitutability and variability. In *Managing Business Interfaces*, pages 39–62. Springer, 2005.

S. Chopra, G. Reinhardt, and M. Dada. The effect of lead time uncertainty on safety stocks. *Decision Sciences*, 35(1):1–24, 2004.

A. Clark, B. Almada-Lobo, and C. Almeder. Lot sizing and scheduling: Industrial extensions and research opportunities. *International Journal of Production Research*, 49(9): 2457–2461, 2011.

L. G. Cooper. Market-share models. *Handbooks in operations research and management science*, 5:259–314, 1993.

V. L. Crittenden, L. R. Gardiner, and A. Stam. Reducing conflict between marketing and manufacturing. *Industrial Marketing Management*, 22(4):299–309, 1993.

J. M. Day and M. Venkataramanan. Profitability in product line pricing and composition with manufacturing commonalities. *European Journal of Operational Research*, 175(3): 1782–1797, 2006.

X. De Groote. Flexibility and marketing/manufacturing coordination. *International Journal of Production Economics*, 36(2):153–167, 1994.

P. Desai, S. Kekre, S. Radhakrishnan, and K. Srinivasan. Product differentiation and commonality in design: Balancing revenue and cost drivers. *Management Science*, 47(1): 37–51, 2001.

G. Dobson and S. Kalish. Positioning and pricing a product line. *Marketing Science*, 7(2): 107–125, 1988.

G. Dobson and S. Kalish. Heuristics for pricing and positioning a product-line using conjoint and cost data. *Management Science*, 39(2):160–175, 1993.

G. Dobson and C. Yano. Product line and technology selection with shared manufacturing and engineering design resources. *William School of Business Administration Working Paper*, 1995.

J. Eliashberg and R. Steinberg. Marketing-production decisions in an industrial channel of distribution. *Management Science*, 33(8):981–1000, 1987.

J. Eliashberg and R. Steinberg. Marketing-production joint decision-making. *Handbooks in Operations Research and Management Science*, 5:827–880, 1993.

G. M. Erickson. A differential game model of the marketing-operations interface. *European Journal of Operational Research*, 211(2):394–402, 2011.

G. M. Erickson. Transfer pricing in a dynamic marketing-operations interface. *European Journal of Operational Research*, 216(2):326–333, 2012.

T. Feng, Y. Huang, and E. Avgerinos. When marketing and manufacturing departments integrate: The influences of market newness and competitive intensity. *Industrial Marketing Management*, 75:218–231, 2018.

M. Fisher, K. Ramdas, and K. Ulrich. Component sharing in the management of product variety: A study of automotive braking systems. *Management Science*, 45(3):297–315, 1999.

A. Flores, G. Berbeglia, and P. Van Hentenryck. Assortment optimization under the sequential multinomial logit model. *European Journal of Operational Research*, 273(3): 1052–1064, 2019.

N. Franke, M. Schreier, and U. Kaiser. The “i designed it myself” effect in mass customization. *Management Science*, 56(1):125–140, 2010.

G. Fruchter, A. Fligler, and R. Winer. Optimal product line design: Genetic algorithm approach to mitigate cannibalization. *Journal of optimization theory and applications*, 131(2):227–244, 2006.

G. Gallego, R. Ratliff, and S. Shebalov. A general attraction model and sales-based linear program for network revenue management under customer choice. *Operations Research*, 63(1):212–232, 2014.

S. Ghose and S. K. Mukhopadhyay. Quality as the interface between manufacturing and marketing: a conceptual model and an empirical study. *MIR: Management International Review*, pages 39–52, 1993.

C. H. Glock, E. H. Grosse, and J. M. Ries. The lot sizing problem: A tertiary study. *International Journal of Production Economics*, 155:39–51, 2014.

J. T. Gourville and D. Soman. Overchoice and assortment type: When and why variety backfires. *Marketing Science*, 24(3):382–395, 2005.

P. E. Green and A. M. Krieger. Models and heuristics for product line selection. *Marketing Science*, 4(1):1–19, 1985.

L. Guimarães, D. Klabjan, and B. Almada-Lobo. Annual production budget in the beverage industry. *Engineering Applications of Artificial Intelligence*, 25(2):229–241, 2012.

L. Guimarães, P. Amorim, F. Sperandio, F. Moreira, and B. Almada-Lobo. Annual distribution budget in the beverage industry: a case study. *Interfaces*, 44(6):605–626, 2014a.

L. Guimarães, D. Klabjan, and B. Almada-Lobo. Modeling lotsizing and scheduling problems with sequence dependent setups. *European Journal of Operational Research*, 239 (3):644–662, 2014b.

W. H. Hausman, D. B. Montgomery, and A. V. Roth. Why should marketing and manufacturing work together?: Some exploratory empirical results. *Journal of Operations Management*, 20(3):241–257, 2002.

H. S. Heese and J. M. Swaminathan. Product line design with component commonality and cost-reduction effort. *Manufacturing & Service Operations Management*, 8(2):206–219, 2006.

T. H. Ho and C. S. Tang. Introduction to the special issue on marketing and operations management interfaces and coordination. *Management Science*, 50(4):429–430, 2004.

T. H. Ho and C. S. Tang. Introduction to the special issue on marketing and operations management interfaces and coordination. *Production and Operations Management*, 18 (4):363–364, 2009.

S. J. Hoch, E. T. Bradlow, and B. Wansink. The variety of an assortment. *Marketing Science*, 18(4):527–546, 1999.

H. Hotelling. Stability in competition. In *The Collected Economics Articles of Harold Hotelling*, pages 50–63. Springer, 1990.

J. Huang, M. Leng, and M. Parlar. Demand functions in decision modeling: A comprehensive survey and research directions. *Decision Sciences*, 44(3):557–609, 2013.

C. Huffman and B. E. Kahn. Variety for sale: mass customization or mass confusion? *Journal of Retailing*, 74(4):491–513, 1998.

B. Kahn. Variety: From the consumer's perspective. In *Product Variety Management*, pages 19–37. Springer, 1998.

B. E. Kahn and B. Wansink. The influence of assortment structure on perceived variety and consumption quantities. *Journal of Consumer Research*, 30(4):519–533, 2004.

B. Karimi, S. F. Ghomi, and J. Wilson. The capacitated lot sizing problem: a review of models and algorithms. *Omega*, 31(5):365–378, 2003.

U. S. Karmarkar. Integrative research in marketing and operations management. *Journal of marketing Research*, 33(2):125–133, 1996.

U. S. Karmarkar and M. M. Lele. The marketing/manufacturing interface: strategic issues. In *Managing Business Interfaces*, pages 311–328. Springer, 2005.

S. Kekre and K. Srinivasan. Broader product line: a necessity to achieve success? *Management Science*, 36(10):1216–1232, 1990.

M. Khouja and S. S. Robbins. Linking advertising and quantity decisions in the single-period inventory model. *International Journal of Production Economics*, 86(2):93–105, 2003.

D. Kim and W. J. Lee. Optimal joint pricing and lot sizing with fixed and variable capacity. *European Journal of Operational Research*, 109(1):212–227, 1998.

K. Kim and D. Chhajed. Commonality in product design: Cost saving, valuation change and cannibalization. *European Journal of Operational Research*, 125(3):602–621, 2000.

T. Klastorin and W. Tsai. New product introduction: Timing, design, and pricing. *Manufacturing & Service Operations Management*, 6(4):302–320, 2004.

R. Kohli and R. Sukumar. Heuristics for product-line design using conjoint analysis. *Management Science*, 36(12):1464–1478, 1990.

A. G. Kök, M. L. Fisher, and R. Vaidyanathan. Assortment planning: Review of literature and industry practice. In *Retail Supply Chain Management*, pages 175–236. Springer, 2015.

T. Kong, G. Li, T. Feng, and L. Sun. Effects of marketing–manufacturing integration across stages of new product development on performance. *International Journal of Production Research*, 53(8):2269–2284, 2015.

U. G. Kraus and C. A. Yano. Product line selection and pricing under a share-of-surplus choice model. *European Journal of Operational Research*, 150(3):653–671, 2003.

R. E. Krider and C. B. Weinberg. Competitive dynamics and the introduction of new products: The motion picture timing game. *Journal of Marketing Research*, 35(1):1–15, 1998.

D. Kuksov and J. M. Villas-Boas. When more alternatives lead to less choice. *Marketing Science*, 29(3):507–524, 2010.

S. C. Kulp, H. L. Lee, and E. Ofek. Manufacturer benefits from information integration with retail customers. *Management science*, 50(4):431–444, 2004.

S. Kumar and A. K. Chatterjee. A profit maximizing mathematical model for pricing and selecting optimal product line. *Computers & Industrial Engineering*, 64(2):545–551, 2013.

P. Lacourbe. A model of product line design and introduction sequence with reservation utility. *European Journal of Operational Research*, 220(2):338–348, 2012.

J. Lampel and H. Mintzberg. Customizing customization. *Sloan Management Review*, 38 (1):21–30, 1996.

K. Lancaster. Markets and product variety management. In *Product Variety Management*, pages 1–18. Springer, 1998.

K. Lau, A. Kagan, and G. Post. Market share modeling within a switching regression framework. *Omega*, 25(3):345–353, 1997.

H. L. Lee and C. Billington. Designing products and processes for postponement. In *Management of Design*, pages 105–122. Springer, 1994.

H. L. Lee and C. S. Tang. Modelling the costs and benefits of delayed product differentiation. *Management science*, 43(1):40–53, 1997.

W. J. Lee and D. Kim. Optimal and heuristic decision strategies for integrated production and marketing planning. *Decision Sciences*, 24(6):1203–1214, 1993.

G. Li, P. Rusmevichientong, and H. Topaloglu. The d-level nested logit model: Assortment and price optimization problems. *Operations Research*, 63(2):325–342, 2015.

H. Li and W. T. Huh. Pricing multiple products with the multinomial logit and nested logit models: Concavity and implications. *Manufacturing & Service Operations Management*, 13(4):549–563, 2011.

Y. Li, Z. Lu, and J. J. Michalek. Diagonal quadratic approximation for parallelization of analytical target cascading. *Journal of Mechanical Design*, 130(5), 2008.

J. D. Little. A proof for the queuing formula: L= λ w. *Operations research*, 9(3):383–387, 1961.

R. D. Luce. *Individual choice behavior.* John Wiley, 1959.

J. P. MacDuffie, K. Sethuraman, and M. L. Fisher. Product variety and manufacturing performance: evidence from the international automotive assembly plant study. *Management Science*, 42(3):350–369, 1996.

M. K. Malhotra and S. Sharma. Spanning the continuum between marketing and operations. *Journal of Operations Management*, 20(3):209–219, 2002.

A. Marques, D. P. Lacerda, L. F. R. Camargo, and R. Teixeira. Exploring the relationship between marketing and operations: Neural network analysis of marketing decision impacts on delivery performance. *International Journal of Production Economics*, 153: 178–190, 2014.

R. D. McBride and F. S. Zufryden. An integer programming approach to the optimal product line selection problem. *Marketing Science*, 7(2):126–140, 1988.

McKinsey & Company. Simpler is (sometimes) better: Managing complexity in consumer goods, 2016.
`https://www.mckinsey.com/industries/consumer-packaged-goods`.

McKinsey & Company. Should CPG manufacturers go direct to consumer – and, if so, how?, 2017.
`https://www.mckinsey.com/industries/consumer-packaged-goods`.

McKinsey & Company. Harnessing the power of simplicity in a complex consumer-product environment, 2020.
`https://www.mckinsey.com/business-functions/marketing-and-sales/our-insights`.

M. B. Menezes, H. Jalali, and A. Lamas. One too many: product proliferation and the financial performance in manufacturing. *International Journal of Production Economics*, page 108285, 2021.

J. J. Michalek, F. M. Feinberg, and P. Y. Papalambros. Linking marketing and engineering product design decisions via analytical target cascading. *Journal of product innovation management*, 22(1):42–62, 2005.

J. J. Michalek, P. Ebbes, F. Adigüzel, F. M. Feinberg, and P. Y. Papalambros. Enhancing marketing with engineering: Optimal product line design for heterogeneous markets. *International Journal of Research in Marketing*, 28(1):1–12, 2011.

K. B. Monroe, S. Sunder, W. A. Wells, and A. Zoltners. *A Multi-period Integer Programming Approach to the Product Mix Problems*. Columbia University, Graduate School of Business, 1974.

I. Moon, K. S. Park, J. Hao, and D. Kim. Joint decisions on product line selection, purchasing, and pricing. *European Journal of Operational Research*, 262(1):207–216, 2017.

K. S. Moorthy and I. P. Png. Market segmentation, cannibalization, and the timing of product introductions. *Management Science*, 38(3):345–359, 1992.

A. Moreno and C. Terwiesch. The effects of product line breadth: Evidence from the automotive industry. *Marketing Science*, 36(2):254–271, 2016.

L. O. Morgan, R. L. Daniels, and P. Kouvelis. Marketing/manufacturing trade-offs in product line management. *Iie Transactions*, 33(11):949–962, 2001a.

L. O. Morgan, R. M. Morgan, and W. L. Moore. Quality and time-to-market trade-offs when there are multiple product generations. *Manufacturing & Service Operations Management*, 3(2):89–104, 2001b.

K. Nichols, M. Venkataramanan, and K. W. Ernstberger. Product line selection and pricing analysis: Impact of genetic relaxations. *Mathematical and computer modelling*, 42(13): 1397–1410, 2005.

S. W. O'Leary-Kelly and B. E. Flores. The integration of manufacturing and marketing/sales decisions: impact on organizational performance. *Journal of Operations Management*, 20(3):221–240, 2002.

E. L. Paiva. Manufacturing and marketing integration from a cumulative capabilities perspective. *International Journal of Production Economics*, 126(2):379–386, 2010.

D. H. Parente. Across the manufacturing-marketing interface classification of significant research. *International Journal of Operations & Production Management*, 1998.

J. Park and J. Mo. An efficient method for joint product line selection and pricing with fixed costs. *Optimization Letters*, 13(2):367–378, 2019.

N. Piercy. Framing the problematic relationship between the marketing and operations functions. *Journal of Strategic Marketing*, 15(2-3):185–207, 2007.

N. Piercy. Positive management of marketing-operations relationships: the case of an internet retail sme. *Journal of Marketing Management*, 25(5-6):551–570, 2009.

N. Piercy. Improving marketing–operations cross-functional relationships. *Journal of Strategic Marketing*, 18(4):337–356, 2010.

W. Qi, X. Luo, Y. Yu, and J. Tang. Product line optimisation based on semiparametric choice model. *International Journal of Production Research*, 58(18):5676–5692, 2020.

R. Quante, H. Meyr, and M. Fleischmann. Revenue management and demand fulfillment: matching applications, models and software. In *Supply Chain Planning*, pages 1–32. Springer, 2009.

J. A. Quelch and D. Kenny. Extend profits, not product lines. *Make sure all your products are profitable*, 14, 1994.

N. Raman and D. Chhajed. Simultaneous determination of product attributes and prices, and production processes in product-line design. *Journal of Operations Management*, 12(3-4):187–204, 1995.

K. Ramdas. Managing product variety: An integrative review and research directions. *Production and Operations Management*, 12(1):79–101, 2003.

D. P. Rutenberg. Design commonality to reduce multi-item inventory: Optimal depth of a product line. *Operations Research*, 19(2):491–509, 1971.

R. Sawhney and C. Piper. Value creation through enriched marketing–operations interfaces: an empirical study in the printed circuit board industry. *Journal of Operations Management*, 20(3):259–272, 2002.

B. Scheibehenne, R. Greifeneder, and P. M. Todd. Can there ever be too many options? a meta-analytic review of choice overload. *Journal of Consumer Research*, 37(3):409–425, 2010.

C. Schön. On the optimal product line selection problem with price discrimination. *Management Science*, 56(5):896–902, 2010a.

C. Schön. On the product line selection problem under attraction choice models of consumer behavior. *European Journal of Operational Research*, 206(1):260–264, 2010b.

B. Shapiro. Can marketing and manufacturing coexist? *Harvard Business Review*, 55: 104–114, 1977.

A. D. Shocker and V. Srinivasan. A consumer-based methodology for the identification of new product ideas. *Management Science*, 20(6):921–937, 1974.

S. M. Shugan, V. Balachandran, et al. A mathematical programming model for optimal product line structuring. Discussion paper No. 265, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1977.

M. Shunko, T. Yunes, G. Fenu, A. Scheller-Wolf, V. Tardif, and S. Tayur. Product portfolio restructuring: methodology and application at caterpillar. *Production and Operations Management*, 27(1):100–120, 2018.

E. A. Silver, D. F. Pyke, R. Peterson, et al. *Inventory management and production planning and scheduling*, volume 3. Wiley New York, 1998.

I. Simonson. Choice based on reasons: The case of attraction and compromise effects. *Journal of Consumer Research*, 16(2):158–174, 1989.

T. W. Simpson, J. R. Maier, and F. Mistree. Product platform design: method and application. *Research in Engineering Design*, 13(1):2–22, 2001.

L. M. Sloot, D. Fok, and P. C. Verhoef. The short-and long-term impact of an assortment reduction on category sales. *Journal of Marketing Research*, 43(4):536–548, 2006.

S. A. Smith and N. Agrawal. Management of multi-item retail inventory systems with demand substitution. *Operations Research*, 48(1):50–64, 2000.

K. Sombultawee and S. Boon-itt. Marketing-operations alignment: A review of the literature and theoretical background. *Operations Research Perspectives*, 5:1–12, 2018.

G. C. Souza, B. L. Bayus, and H. M. Wagner. New-product strategy and industry clockspeed. *Management Science*, 50(4):537–549, 2004.

M. Spencer and J. Cox III. Sales and manufacturing coordination in repetitive manufacturing: characteristics and problems. *International Journal of Production Economics*, 37 (1):73–81, 1994.

W. J. Steiner and H. Hruschka. A probabilistic one-step approach to the optimal product line design problem using conjoint and cost data. *Review of Marketing Science Working Paper*, 441, 2002.

M. Swink and M. Song. Effects of marketing-manufacturing integration on new product development time and competitive advantage. *Journal of Operations Management*, 25 (1):203–217, 2007.

C. S. Tang. A review of marketing–operations interface models: From co-existence to coordination and collaboration. *International Journal of Production Economics*, 125(1): 22–40, 2010.

C. S. Tang and R. Yin. The implications of costs, capacity, and competition on product line selection. *European Journal of Operational Research*, 200(2):439–450, 2010.

M. V. Tatikonda and M. M. Montoya-Weiss. Integrating operations and marketing perspectives of product innovation: The influence of organizational process factors and capabilities on development performance. *Management science*, 47(1):151–172, 2001.

A. Tversky and E. Shafir. Choice under conflict: The dynamics of deferred decision. *Psychological Science*, 3(6):358–361, 1992.

K. Vohs, R. Baumeister, B. Schmeichel, J. Twenge, N. Nelson, and D. Tice. Making choices impairs subsequent self-control: a limited-resource account of decision making, self-regulation, and active initiative. *Journal of Personality and Social Psychology*, 94 (5):883–898, 2008.

X. Wan and N. R. Sanders. The negative impact of product variety: Forecast bias, inventory levels, and the role of vertical integration. *International Journal of Production Economics*, 186:123–131, 2017.

K. Weir, A. Kochhar, S. LeBeau, and D. Edgeley. An empirical study of the alignment between manufacturing and marketing strategies. *Long Range Planning*, 33(6):831–848, 2000.

L. O. Wilson and J. A. Norton. Optimal entry timing for a product line extension. *Marketing Science*, 8(1):1–17, 1989.

H. Wong, K. Kim, and D. Chhajed. Reducing channel inefficiency in product line design. *International Journal of Production Economics*, 232:107964, 2021.

C. Yano and G. Dobson. Profit-optimizing product line design, selection and pricing with manufacturing cost consideration. In *Product Variety Management*, pages 145–175. Springer, 1998.

E. Yücel, F. Karaesmen, F. S. Salman, and M. Türkay. Optimizing product assortment under customer-driven demand substitution. *European Journal of Operational Research*, 199(3):759–768, 2009.

Q. Zhu, P. Shah, and J. Sarkis. Addition by subtraction: Integrating product deletion with lean and sustainable supply chain management. *International Journal of Production Economics*, 205:201–214, 2018.

CHAPTER 3

Product line selection of fast-moving consumer goods

Product line selection of fast-moving consumer goods

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Abstract The fast-moving consumer goods sector relies on economies of scale. However, its assortments have been overextended as a means of market share appropriation and top-line growth. This paper studies the selection of the optimal set of products for fast-moving consumer goods producers to offer, as there is no previous model for product line selection that satisfies the requirements of the sector. Our mixed-integer programming model combines a multi-category attraction model with a capacitated lot-sizing problem, shared setups and safety stock. The multi-category attraction model predicts how the demand for each product responds to changes within the assortment. The capacitated lot-sizing problem allows us to account for the indirect production costs associated with different assortments. As seasonality is prevalent in consumer goods sales, the production plan optimally weights the trade-off between stocking finished goods from a long run with performing shorter runs with additional setups. Finally, the safety stock extension addresses the effect of the demand uncertainty associated with each assortment. With the computational experiments, we assess the value of our approach using data based on a real case. Our findings suggest that the benefits of a tailored approach are at their highest in scenarios typical fast-moving consumer goods industry: when capacity is tight, demand exhibits seasonal patterns and high service levels are required. This also occurs when the firm has a strong competitive position and consumer price-sensitivity is low. By testing the approach in two real-world instances, we show that this decision should not be made based on the current myopic industry practices. Lastly, our approach obtains profits of up to 9.4% higher than the current state-of-the-art models for product line selection.

Keywords product line selection; lot sizing; fast-moving consumer goods; attraction model; mixed-integer linear programming

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3.1. Introduction

Fast-moving consumer goods (FMCG) producers are offering suboptimal assortments. The sector is composed of capital-intensive plants that manufacture low-margin high-volume products, such as beverages and toiletries. Investment sizes associated with these kinds of plants make capacity a hard-constraint. Additionally, the demand for goods such as beverages exhibits a clear seasonal pattern, further complicating capacity allocation. Nevertheless, product introductions have been widely used for market share appropriation and entry deterrence (Quelch and Kenny, 1994), and broad product lines cut economies of scale short, nibbling at the already thin margins practised within the sector. It is also common for manufacturers to find themselves producing multiple categories (e.g. beer, water and juices), which further complicates operations.

Firms matching the introductions of their rivals are led into a competitive trap where operational costs are high, the size of the market does not catch up with the size of the line, and delisting the wrong products potentially results in ruin. Consultants currently recommend such firms to focus on their core product portfolio while taking advantage of high-margin consumer trends, such as organic packaged goods (McKinsey & Company, 2016). The competitive trap creates an opening for a manufacturer to become price and quality leader by selecting the optimal product line and concentrating its efforts on the products that the consumers really seek. Practitioners are aware of this fact and a recent trend for stock-keeping-unit (SKU) rationalization. Still, in the absence of models to assist their product line decisions, some FMCG firms myopically make product removals only taking share or margin into account, and not its impact on operations and the sales of other products. With the increasing cooperation between retailers and manufacturers (Li and Zhang, 2015; Li and Chen, 2018) along with several FMCG brands opting for direct-to-consumer (D2C) channels, such as brand-owned online stores (McKinsey & Company, 2017) or even brick-and-mortar stores exclusively for their products, the urgency for these models increases.

Optimal lot sizing and scheduling can only do so much, and product line selection (PLS) can only do so much without it. There is ample research on planning the production of FMCG (Almada-Lobo et al., 2015). However, with the dissemination of these approaches, higher-level decisions need to be optimized for substantial competitive advantages over other firms to arise. Still, current product line selection models disregard some subtleties required to assess the performance of a FMCG line accurately. Even the approaches with the most sophisticated production planning models (Morgan et al., 2001; Dobson and Yano, 2002) are single-period. They consider a fixed production frequency, neglecting how inventory levels can be adjusted to compensate for variations in demand (i.e., dealing with seasonality) which is critical for FMCG (Quante et al., 2009). Furthermore, these approaches tend to oversimplify consumer response to a single-choice process. Although it fits the high-involvement purchases of hard goods (e.g., cars, computers), it is unreasonable to assume that a consumer segment will wholly converge to a single FMCG option. Attraction models define the share of a product as its attractiveness divided by that of the whole assortment (including competition), which corresponds better to consumer behaviour on such low-involvement purchases. Consumer choice was first tackled this way in PLS by Schön (2010a), with no work contemplating multiple product categories. Still, the gap for PLS models with attraction consumer response and multi-period production planning remains unfilled.

In this paper, we propose a mixed-integer programming (MIP) model for the PLS of FMCG and quantify its benefits when compared with the state-of-the-art models and heuristics that practitioners use. For consumer response, we develop a linear formulation for the multi-category attraction model, which can simulate low-involvement purchases even if the manufacturer produces different categories within a plant. While the demanded amounts depend on the selection and are defined by the multi-category attraction model, production costs and quantities are estimated based on the well-known capacitated lot-sizing problem. As required for high-margin low-volume products, the model accounts for the right number of setups and the amounts of stock that are carried over, thus resulting in a more accurate cost estimation than previous PLS models. Moreover, the model also takes into account the costs of an inventory build-up when anticipating an increase in demand, and the potential benefits of carrying inventory instead of having to set up for a product when demand is at a trough. While it is not our objective to determine the production plan, it serves to assess the indirect costs associated with a product line and hence, obtain a feasible and cost-efficient product line.

The MIP model is the central contribution of this work. We complement it by proposing a safety stock modelling approach, addressing the demand uncertainty associated with a product line and its impact on holding costs. By using our model to perform computational experiments, we assess its value and derive managerial insights. Using generated instances, we show the conditions under which an integrated and multi-period model outperforms the state-of-the-art approach and the typical heuristics used by practitioners. The results make evident that our approach has the most advantage when capacity is tight, consumers seek quality, service level requirements are high, and the firm is a market leader seasonal categories. With real instances, we assess the magnitude of the benefits that our approach can bring to a real production line, relative to previous methods. They suggest that the decision should not be left to myopic sales-driven heuristics, as the profit resulting from an optimal selection surpasses that of the heuristics by 125.3% in a real case. They further show that our multi-period approach, which accounts for the setup-inventory trade-offs, beats the state of the art by up to 9.4%.

The remainder of this paper is structured as follows. Section 3.2 provides a focused review of the PLS literature, emphasizing the different models and reflecting the gap for our work. In Section 3.3, we present the model for the PLS of FMCG, an extension to include the costs of safety stock, and an assessment of the computational performance of the model. Section 3.4 describes and exposes the results of the computational experiment designed to evaluate each contribution. Section 3.5 exhibits the results of the application of the model to real-world instances. We consolidate the insights derived from the results of the application of the model to both test and real-world instances in Section 3.6. Finally, Section 3.7 summarizes the conclusions and contributions of this research and makes future issues evident to explore.

3.2. Literature Review

Interest in PLS arose from marketing research, with the realization that there are complex reactions from the market to the offered assortment. The literature on the problem traces back to Monroe et al. (1974), who introduce a model for the addition and removal of products over a planning horizon, of which the resulting revenue depends on the pairwise interactions between the on-shelf products. Locander et al. (1976) introduce a model to rank products for elimination using the covariances between the returns of each product for accounting synergies. A mathematical model for product selection, assuming a single-choice process and considering setup costs, is provided by Shugan et al. (1977). Green and Krieger (1985) consolidate the problem by presenting models for the selection of a product line of a specified size, in the perspectives of both buyer and seller. With the buyer as the decision-maker, the objective is to maximize the utility of the assortment, given a single-choice model. In the seller's problem, the value of each segment is weighted in the decision, and a product is sold only provided its utility for the consumer exceeds that of the *status quo* (having a reservation value). McBride and Zufryden (1988) formally apply mixed-integer programming to solve the seller's problem and extend it by incorporating fixed costs. Likewise, Dobson and Kalish (1988, 1993) consider fixed costs associated with each product and add the pricing decision to the perspective of the seller. Additionally, the authors compare heuristics for the buyer's problem and propose decomposing the seller's problem into selection and pricing for tractability. From this point on, research on this topic either attempts to incorporate detailed manufacturing implications into models with single-choice or aims to model consumer behaviour more efficiently and meaningfully with attraction consumer response.

The importance of detailed manufacturing modelling derives from the weight of the underlying upstream costs associated with the variety in the assortment. Using single-choice consumer response, some authors developed approaches that make more sensible selections by increasing the detail in which operational costs are estimated. Dobson and Yano (1995) substitute the simple fixed and variable costs in their version of the seller's model for ones associated with resource and process choice. Raman and Chhajed (1995) propose a model integrating the design decision (choosing the features and attribute levels that compose a product) and price, and the assignment of production processes to attribute levels, given associated fixed and variable costs. Without pricing and product design, Morgan et al. (2001) take on resource sharing by including family setups and decide on the production frequency, which enables a rough estimate of the trade-off between setup and production inventory costs. As extensions to their model, the authors provide capacity constraints and a framework to include multiple planning horizons with no inventory carry-over. The PLS problem with pricing and the make-to-order or make-to-stock decision is introduced by Dobson and Yano (2002). In this formulation capacity is explicitly modelled, with demand being parametric and affected by delivery time at a linear rate. However, the demanded amounts do not depend on the offered assortment. Day and Venkataramanan (2006) extend the profit-maximizing problem from Dobson and Kalish (1993) with family setups, segment sizes and a reservation price for consumers to make a purchase. The authors also compare the performance of a discrete-pricing reduction approach with genetic algorithms to solve this problem.

Single-choice can be limiting in both tractability and modelling power. As it states that a homogeneous segment will holistically go for the same alternative, it requires a large number of segments to represent a market realistically, especially if purchases require little engagement. Some PLS research tries to mitigate these implications with the use of attraction models. The multinomial logit (MNL) can be seen as a particular case of attraction models. It was introduced to PLS by Aydin and Ryan (2000), who decide on pricing and divide the purchase process into arrival and choice. Chen and Hausman (2000), on a technical note, undertake the product line selection problem with attraction consumer response, when the number of products to choose from is bounded. The authors show that, without fix costs, the optimal line is the solution to the linear relaxation of the problem. Steiner and Hruschka (2002) present a nonlinear program for product and product line design and selection, using MNL choice probability. They consider fixed and variable costs, dependent on the attribute levels of each product, and solve the problem using a genetic algorithm. Kraus and Yano (2003) present a formulation with continuous pricing that computes attractiveness as the utility-price surplus.

More recently, some authors have made developments in manufacturing modelling with attraction consumer response. Hopp and Xu (2005) study a product line design, selection and pricing problem without economies of scale. The authors study the influence of reducing product introduction costs through modularity in the overall assortment strategy of a company. Schön (2010a) extends the PLS and pricing problem with attraction consumer response is extended by with resource sharing and fixed costs associated with resources. The model supports multiple segments, allowing price discrimination, and is the first work in PLS using linear constraints to implement the attraction model. In a technical note, Schön (2010b) extends her work by considering price as a bounded continuous variable and associating fix costs with capacitated attribute levels. In both works, the author takes advantage of the problem structure to make the solution tractable.

In a different research stream, Bertsimas and Mišić (2016) tackle the uncertainty caused by the mismatch between choice modelling and consumer response by developing a robust approach for PLS. The authors consider that both distinct parameter estimations and structurally distinct models can constitute the uncertainty set. Hence, the approach can protect retailers from losses due to both incorrectly modelling and parameterizing consumer behaviour. In Bertsimas and Mišić (2019), the authors present a new formulation for the PLS problem with single-choice and a decomposition approach for solving it. The authors prove that the formulation provides a stronger linear relaxation than previous ones, and provide computational experiments to demonstrate the magnitude of the benefits of their approach. By not covering manufacturing issues, this research moves away from the needs of the FMCG sector. Therefore, two papers emerge as the state-of-the-art PLS research. Morgan et al. (2001), with single-choice consumer response, propose the most thorough model to account for manufacturing costs. With attraction consumer response, Schön (2010b) has the most advanced production modelling.

Despite these advances, there are still gaps in both marketing and manufacturing fronts holding back the use of the aforementioned research for decision making, notably, by FMCG producers. Given the extent of the lines, one FMCG plant can manufacture products that do not fall under the same market category, and therefore do not compete with each other. These categories have negligible demand interactions between them, so basic attraction models, as Schön (2010b) uses, do not describe well such a line. This work extends the marketing front by introducing a linear formulation for a multi-category attraction model for consumer response. The model makes it possible to represent the behaviour of several independent categories sharing the capacity of a single plant.

Moreover, the current manufacturing cost estimates are simplistic for capital-intensive low-margin industry cases, where capacity cannot be relaxed, and inventory management is critical. Morgan et al. (2001) use a production frequency variable to assess the significance of the setup and inventory costs on the selection of the optimal product line. Nevertheless, extending a single-period model with production frequency is not enough to account for demand variations in a capacity-constrained scenario. Ghoniem and Maddah (2015) introduce a (single-choice) multi-period assortment problem with order and inventory costs. Our model extends the manufacturing modelling front of PLS literature by considering this multi-period setting with capacity constraints, enabling it to correctly account for setup and holding costs under demand variations. This is crucial, as the sales of many FMCG categories frequently exhibit seasonal patterns.

Lastly, and as explored by Chong et al. (2005) for delisting a single product, demand predictability depends on the products that are selected. Therefore, we introduce a safety stock modelling approach that can be used to estimate the inventory needed to handle the demand uncertainty associated with a product line. In the assortment literature, Bernales et al. (2017) apply a single-period assortment planning model with safety stock to rationalize the stock-keeping-units of an information technology retailer. However, their approach to safety stock considers a fixed coefficient of variation, which does not depend on the share that the product gets.

3.3. Mathematical model

With the objective of maximizing profit, a fast-moving consumer goods manufacturer has to choose the subset $W \subseteq \{1, \ldots, J\}$ of products to make and sell. In a given planning horizon (i.e., throughout a year), a selection generates $\mathrm{R}(W)$ gross profit, determined by the choices consumers make. On the other hand, the manufacturer needs to incur $\mathrm{C}(W)$ indirect costs to produce and sell the selected products, which depend on the production decisions made for each planning period (i.e., monthly). Expression (4.1) is the general objective function of this problem.

$$\max_{W} \quad \mathrm{R}(W) - \mathrm{C}(W) \tag{3.1}$$

Each product j has a contribution margin p_j and, during period t, $D_{jt}(W)$ units are sold, leading to $\mathrm{R}(W) = \sum_t \sum_j p_j D_{jt}(W)$. The demand for a product depends on W and, while it may not be profitable or even feasible for a manufacturer to satisfy the demand fully, the amounts sold $D_{jt}(W)$ will never exceed it. At each period t, the manufacturer will carry inventory I_{jt} with a unitary holding cost h_j. Additionally, a setup for j, denoted by binary variable Y_{jt} and resulting in a cost θ_j, is incurred to allow a quantity X_{jt} to be produced (with p_j including the variable production cost). The manufacturing plant has a capacity c_t available at each period t that is spent either producing or on changeovers. Setting up for a product j uses up τ_j available time, and each unit of the product takes r_j time to be produced. By computing the amounts to produce and stock that enable $D_{jt}(W)$ to be made available for sale, the manufacturer estimates the costs associated with selection $\mathrm{C}(W) = \sum_t \sum_j h_j I_{jt} + \sum_t \sum_j \theta_j Y_{jt}$.

Throughout the following subsections, we introduce the model for the PLS of FMCG. In Subsection 3.3.1 we provide a consumer response model able to handle both the substitution within the product categories that a manufacturer produces and the independence between them. The integration of the demand management model with the constraints in Subsection 3.3.2, adds the multi-period production planning needed to account for the costs of hedging demand variations with inventory, which is crucial in such capital-intensive businesses. We also provide extensions for our formulation to include fixed administrative costs and shared setup. Subsection 3.3.3 provides an approach to safety stock modelling, and to include the respective costs in the selection problem. Then, in Subsection 3.3.4, we display the bounds of each parameter for the generation of the virtual instances and make a brief assessment of the performance of the formulation. We have centralized the notation used throughout this section in Appendix B.

3.3.1 Consumer response

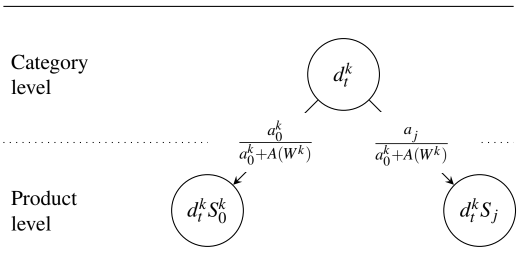

Figure 3.1 – Demand structure for this problem. The category demand is determined exogenously. Inside subcategories, product share results from an attraction model.

Among all J products that the manufacturer can offer, some do not compete with each other for the same consumers. Let $\mathcal{N}^k$ be the subset of products that belong to an independent category k. A product $j \in \mathcal{N}^k$ can be only be substituted by others in $\mathcal{N}^k$ and options offered by competitors (denoted by $j = 0$). Considering d_t^k as the demand for category k in period t, and $S_j(W^k)$ the probability of a consumer selecting product j as function of the selection within its category $W^k = W \cap \mathcal{N}^k$, the demanded amount for the product will be computed as $d_t^k S_j(W^k)$. We assume that, within each category, consumer response follows an attraction model, which is a generalization of the MNL and the multiplicative competitive interaction model (MCI) (Lau et al., 1997). As Figure 3.1 represents, the share of a product is computed as its attractiveness a_j divided by that of the assortment $A(W^k) = \sum_j a_j$, with $j \in W^k$, and the competition a_0^k in that subcategory. Hence, $S_j(W^k) = a_j/(a_0^k + A(W^k))$ for all k and $j \in \mathcal{N}^k$. Also, consumers cannot purchase products that are not selected, implying that $S_j = 0$ for any $j \notin W$. The following constraints capture these relations, with $D_{jt}(W)$ is noted as D_{jt} since the formulation makes the dependence on W explicit.

$$D_{jt} \leq d_t^k S_j \quad \forall t, k, j \in \mathcal{N}^k \tag{3.2}$$

$$S_j \leq W_j \quad \forall j \tag{3.3}$$

$$a_0^k S_j = a_j S_0^k - F_j \quad \forall k, j \in \mathcal{N}^k \tag{3.4}$$

$$F_j \leq a_j S_0^k \quad \forall k, j \in \mathcal{N}^k \tag{3.5}$$

$$S_0^k + \sum_{j \in \mathcal{N}^k} S_j = 1 \quad \forall k \tag{3.6}$$

$$W_j \in \{1,0\} \quad \forall j \tag{3.7}$$

$$D_{jt}, S_j, S_0^k, F_j \in \mathbb{R}_0^+ \quad \forall t, k, j \in \mathcal{N}^k \tag{3.8}$$

Constraints (3.2)-(3.8) model consumer response. Constraints (3.2) set the demanded amounts as an upper limit on sales. Let $W_j = 1$ if $j \in W$ and $W_j = 0$ otherwise, constraints (3.3) only allow selected products to be demanded. Constraints (3.4) define that the ratios between the shares behave according to an attraction model, within each category. Similarly to the scale constraints in the sales-based linear program of Gallego et al. (2014), they ensure that the ratio between the share of a product and the share S_0^k of the competing options in its category is, at most, their attractiveness ratio a_j/a_0^k (i.e., $S_j/S_0^k \leq a_j/a_0^k$, where the equality is obtained by adding the slack variables F_j). Constraints (3.5) and (3.6) establish that, if the manufacturer does not offer any products in a subcategory, the competition will absorb all of the market share. In this situation, F_j in constraints (3.4) relax the relationship between the attractiveness and share ratios. Given that our objective encourages us to maximize the shares of selected products, and that the sum of all shares in a subcategory is limited by constraints (3.6), variables F_j will be null otherwise. Finally, constraints (3.7) and (3.8) set the domain of the variables. Let U_j denote the utility of product j, with its observable V_j and unobservable ε_j components. For the MCI consumer behavior $a_j = V_j$, since $U_j = V_j \varepsilon_j$ and $\ln \varepsilon_j$ follows the Type I extreme value distribution. For the MNL $a_j = \exp V_j$, since $U_j = V_j + \varepsilon_j$ and ε_j follows the same Type I extreme value distribution. The problem of maximizing $\mathrm{R}(W)$ subject to constraints (3.3)-(3.8) is a simple one. The optimal selection can be determined by starting with an empty set, and adding the product with the highest margin to the choice set until no increase in revenue is possible. This was first shown by Talluri and Van Ryzin (2004).

3.3.2 Production planning

The following constraints capture the multi-period production planning, which greatly increases the complexity of the model.

$$X_{jt} \leq Y_{jt} K_{jt} \quad \forall t, j \tag{3.9}$$

$$\sum_{j} (\tau_j Y_{jt} + r_j X_{jt}) \leq c_t \quad \forall t \tag{3.10}$$

$$D_{jt} = I_{jt-1} + X_{jt} - I_{jt} \quad \forall t, j \tag{3.11}$$

$$Y_{jt} \in \{1, 0\} \quad \forall j, t \tag{3.12}$$

$$X_{jt}, I_{jt} \in \mathbb{R}_0^+ \quad \forall j, t \tag{3.13}$$

Constraints (3.9) - (3.13) model manufacturing as a capacitated lot sizing problem (Karimi et al., 2003). They impact the objective function both directly, with inventory and setup costs, and indirectly, by constraining the amounts that can be produced and, consequently, sold. Constraints (3.9) express that the plant must be setup for a product if any given quantity is to be manufactured. The upper bound for variables X_{jt} is the minimum between the maximum possible demand for the product j in remainder of the horizon, and the maximum possible output of j in period t. This implies that, in order for the formulation to be valid, constants K_{jt} need to satisfy $K_{jt} \geq \min\left\{ \sum_{t_2} d_{t_2}^k a_j / (a_j + a_0^k);\ (c_t - \tau_j)/r_j \right\}$ $\forall t, j, t_2 \in \{t, ..., T\}$ and $k : j \in \mathcal{N}^k$. Constraints (3.10) model the plant's capacity, which is shared among all products and split between production and setups. Constraints (3.11) ensure inventory is balanced between periods by requiring sold products to be either produced in that period or stocked from a previous one. Together with the capacity limitations, these constraints impose a limit on the sales the manufacturer can make, so $D_{jt} = \min\{d_t^k S_j, I_{jt-1} + X_{jt}\}$, given that the objective will try to maximize D_{jt}. Lastly, constraints (3.12) and (3.13) define the domain of the variables.

With objective (3.1), subject to constraints (3.2)-(3.13), we introduce the mixed-integer linear formulation for the selection of fast-moving consumer goods. The formulation may be extended with implementation-based considerations. For cases where product development and administrative costs are not negligible, a fixed cost per product throughout the planning horizon should be considered. Let f_j be the fixed development and introduction cost associated with product j, the term $\sum_j f_j W_j$ should be added to $\mathrm{C}(W)$. It is also common for FMCG to share part of the changeover across a set of products. Similarly to the categories defined for the market structure, manufacturing can have families that share a portion of the setup procedure.

$$X_{jt} \leq Y_t^m K_{jt} \quad \forall m, t, j \in \mathcal{N}_m \tag{3.14}$$

$$\sum_m \left[\tau^m Y_t^m + \sum_{j \in \mathcal{N}_m} (\tau_j Y_{jt} + r_j X_{jt}) \right] \leq c_t \quad \forall t \tag{3.15}$$

With $\mathcal{N}_m$ being the set of products that belong to family m and $Y_t^m \in \{1,0\}\ \forall j,t$, constraints (3.14) ensure $Y_t^m = 1$ whenever a product of that family is produced in that period. Constraints (3.15) modify constraints (3.10) to include the setup times τ^m for the families. The constants in constraints (3.9) are also used in constraints (3.14) and the conditions for the formulation to be valid become $K_{jt} \geq \min\left\{\sum_{t_2} d_{t_2}^k a_j/(a_j + a_0^k);\ (c_t - \tau_j - \tau^m)/r_j\right\}\ \forall t, j$, with $t_2 \in \{t, ..., T\}$ and $(k,m) : j \in \mathcal{N}^k \cap \mathcal{N}_m$. To finally incorporate the effect of resource sharing, category setup costs θ^m are accounted for by adding the term $\sum_m \theta^m Y_t^m$ to $\mathrm{C}(W)$.

3.3.3 Safety stock

Since our model is deterministic, it disregards the inventory costs that are incurred to deal with demand unpredictability and how the selection affects them. To counteract this, we simulate the most widespread safety stock policies by requiring inventory levels to be a number l of standard deviations above the average demand for a product (α service level).

We assume that the number of consumers N_t^k that purchase a unit of category k in time period t is a normally distributed random variable with average d_t^k and standard deviation σ_t^k. We also consider that the selection of a given product $j \in \mathcal{N}^k$ by a consumer is the result of a binomial trial with $S_j(W^k)$ probability of success, as proposed by Chong et al. (2005). For a given selection within category k, W^k, the variance of the Bernoulli variable B_j that denotes that a consumer selects product $j \in \mathcal{N}^k$ is computed as $S_j(1-S_j)$, with its expected value being S_j. Then, the amount of product $j \in \mathcal{N}^k$ sold in time period t is the product of the two random variables $N_t^k B_j$, with $\mathrm{E}[N_t^k B_j] = d_t^k S_j$ and $\mathrm{Var}(N_t^k B_j) = {\sigma_t^k}^2 S_j(1-S_j) + {\sigma_t^k}^2 {S_j}^2 + S_j(1-S_j){d_t^k}^2$. With the plant having a production lead time of Λ periods, non-linear constraints (3.16) enforce the safety stock policy.

$$I_{jt} \geq l\sqrt{\left({\sigma_t^k}^2 S_j(1-S_j) + {\sigma_t^k}^2 {S_j}^2 + S_j(1-S_j){d_t^k}^2\right)\Lambda} \quad \forall t,k,j \in \mathcal{N}^k \tag{3.16}$$

In order to keep the advantages of a linear formulation, we break each constraint into N linear segments, using $N+1$ borders. Implementing this approximation requires the addition of constraints (3.18)-(3.21) to the original model.

$$I_{jt} \geq y_{0t}^{kn} + \delta y_t^{kn} S_j - M_t^{kn}\left(1 - Z_j^n\right) \quad \forall t,n,k,j \in \mathcal{N}^k \tag{3.17}$$

$$S_j \leq 1 + \left(x_0^{n+1} - 1\right) Z_j^n \quad \forall j,n \tag{3.18}$$

$$S_j \geq x_0^n Z_j^n \quad \forall j,n \tag{3.19}$$

$$\sum_n Z_j^n = 1 \quad \forall j \tag{3.20}$$

$$Z_j^n \in \{1,0\} \quad \forall j,n \tag{3.21}$$

Let y_{0t}^{kn} denote the intersection of the n^{th} linear segment with the vertical axis and δy_t^{kn} its slope for subcategory k and period t. Constraints (3.17) impose the linear approximation of the safety stock policy. Working with constants M_t^{kn}, binary variables Z_j^n ensure that, for product j, the only potentially active constraint is the one relative to the segment where S_j is placed. The formulation requires that $M_t^{kn} \geq y_{0t}^{kn} + \delta y_t^{kn}$ for segments where the slope is positive and $M_t^{kn} \geq y_{0j}^{n}$ otherwise. Constraints (3.18) and (3.19) set $Z_j^n = 1$ if $S_j \in \left[x_0^n, x_0^{n+1}\right]$ and $Z_j^n = 0$ otherwise. Constraints (3.20) ensure that S_j is placed at one segment. Finally, constraints (3.21) define Z_j^n as binary variables.

3.3.4 Computational performance

To assess the tractability and scalability of the approach, we have generated problem instances with different numbers of products and planning periods. We compare the quality of the solutions reached by the MIP model with those reached by its linear relaxation, along with the time elapsed to obtain each solution. We evaluate the impact of the number of products by using five sets of ten instances with the number of periods $T = 12$ being constant across the sets and the number of products $J \in \{8, 10, 12, 14, 16\}$. Likewise, to measure impact of the number of periods we use sets of instances with $J = 12$ and $T \in \{4, 6, 8, 10, 12\}$. In this experiment, capacity is constrained to the maximum time that it can take to produce the whole line without setup times, or $\max_t\{\sum_k d_t^k \sum_{j\in\mathcal{N}^k} r_j a_j/(a_0^k + A(W^k))\}$ with $W = \{1, ..., J\}$.

Table 3.1 – Parameters for instance generation, and bounds for the uniform distribution.

Parameter	Description	Lower bound	Upper bound
d_t	Market size	400.0	500.0
σ_t^k	Standard deviation	$\sqrt{s_t^k} \times 60.0$	$\sqrt{s_t^k} \times 150.0$
p_j	Margin	6.0	12.0
h_j	Holding cost	1.6	3.6
f_j	Fix cost	22.5	110
θ_j	Setup cost	6.0	32.4
θ^m	Family setup cost	12.0	72.0
τ_j	Setup time	1.0	1.0
τ^m	Family setup time	3.0	3.0
a_j	Attraction	12.0	48.0
r_j	Resource utilization	0.4	1.0
a_0^k	Competition	$\lvert\mathcal{N}^k\rvert \times 24.0$	$\lvert\mathcal{N}^k\rvert \times 96.0$

For all these instances, we sample the values of the model parameters using uniform distributions. The bounds of the distributions are based on real-world data and are displayed in Table 3.1. Product families (with respect to setups) correspond to the demand categories, thus sets $\mathcal{N}^k = \mathcal{N}_m$ for every $k = m$. To generate these sets, we assign to each product j a value of a random variable uniformly distributed between zero and one, $\mathrm{U}(0,1)$. Then, we iterate through the set of products and make partitions when the value is less than ⅓. The resulting partitions correspond to the subcategories. This process generates a non-deterministic number of categories while averaging three products per subcategory. Additionally, we generate category demand $d_t^k = d_t s_t^k$ where, for each period, s_t^k is obtained by dividing K random $\mathrm{U}(0,1)$ variables by their sum.

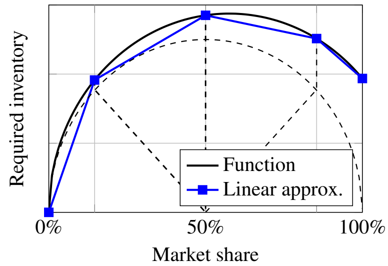

Figure 3.2 – Illustrative example of piecewise linearization. An inner linear approximation is made for the required inventory, as a function of the share of a product within its subcategory.

Regarding safety stock, we place borders at $(1-\cos n\pi/N)/2$, with $n \in \{1,\ldots,N\}$. Since the standard deviation of a Bernoulli variable, as a function of the probability of a successful trial, has the shape of a semicircle, this division would cut it into equally sized slices. The linear constraints are defined by the connections between adjacent intersections of the function with a border, resulting in an inner approximation. This is shown by the dashed lines in Figure 3.2, where we illustrate this procedure. Throughout this work production lead time $\Lambda = {}^1\!/\!_8$ and the number of segments in the piece-wise linearization $N = 4$, which guarantees an approximation (i.e., area below the non-linear function covered by the linear one) of at least 90.6% for the instance generation parameters. The resulting test instances are available online (Andrade, 2019).

All experiments were conducted on four cores of an Intel Xeon E5-2450 processor and 16 GB of memory allocated from a computer cluster running Version 7 of the Scientific Linux distribution. The optimization problems were solved using IBM ILOG CPLEX 12.8, and the experiments were implemented in the C++ programming language using the Concert Technology library as an interface with the optimizer.

Table 3.2 – Results for the tractability tests without safety stock.

T	J	Fractional solution time (s)	Integer solution time (s)	Integrality gap
4	12	0.00	168.53	54.0%
6	12	0.01	197.76	43.4%
8	12	0.01	194.05	41.3%
10	12	0.01	229.47	38.2%
12	8	0.00	81.67	28.9%
12	10	0.01	155.74	29.8%
12	12	0.01	295.70	38.0%
12	14	0.01	913.73	42.5%
12	16	0.01	4664.29	53.2%

Table 3.2 exhibits the results of the experiment for the fully extended model without safety stock. The first and second columns regard the number of products and periods.

The continuous and integer solution times, in seconds, follow. In the last column, we display the integrality gap as the ratio between the difference in objective values of the fractional and the integer solutions, and the value obtained by the integer solution ($\pi_f/\pi_i - 1$, with π_f and π_i being the profit values for the fractional and integer solutions). As it stands, the model contains $2J + 3TJ + K$ positive continuous variables and $2J + TJ + M$ binaries. Their behaviour is delimited by $3J + 3TJ + T + K$ constraints, besides those that define the domain of the variables. The results show that integer solution times scale exponentially with both the number of products and periods, with fractional solution times being negligible. The integrality gap increases with the number of products and decreases with the number of periods. In the linear relaxation of the problem, fractional product selections and setups reduce the fixed costs associated with the inclusion of each product and the setup costs needed to produce them. Moreover, a part of the capacity that, in integer solutions, would be used for setups becomes free to be used for production. As the fixed costs are one-shot and setups are usually concentrated in early periods, both these effects get diluted over longer planning horizons.

Table 3.3 – Results for the tractability tests with safety stock.

T	**J**	**Fractional solution time (s)**	**Integer solution time (s)**	**Integrality gap**
4	12	0.01	352.58	133.1%
6	12	0.02	1098.75	107.8%
8	12	0.02	1187.78	98.3%
10	12	0.02	1573.56	94.8%
12	8	0.01	94.98	79.3%
12	10	0.02	249.95	81.3%
12	12	0.02	2994.95	97.6%
12	14	0.05	3110.34	97.7%
12	16	0.06	19087.91	105.4%

The previous model needs to be supplemented with JN binary variables and $J + 2JN + TJN$ constraints (excluding those defining the domain of the new variables) to include safety stock. Table 3.3 displays the computational performance results for the fully extended model with safety stock. Overall, the inclusion of safety stock makes the problem harder to solve with the table showing higher integer solution times. The fractional values of the segment selection variables for the linearization lower the value of the right-hand side of constraints (3.17), potentially deactivating them. This leads to integrality gaps higher across the board than when safety stock is not accounted for.

3.4. Model assessment

In this section, we expose and analyse the results of the experiment aimed to measure the performance of our approach. We test the value of the integration of the decisions, of considering multiple production periods, and of safety-stock modelling by comparing our results with those of approaches that disregard these features. The comparison enables us to study the impact of production and demand conditions on the benefits of each model or solution method. To estimate the value of our contribution, and gauge the influence of the external parameters on our conclusions, we conduct computational experiments on 480 instances resulting from 20 templates generated using the bounds from Table 3.1. For all templates, $J = 12$ and $T = 6$, which correspond to a relatively small line and planning horizon when compared to a real-world case. The templates can be found online (Andrade, 2019). Throughout the tests, we change how limiting the plant's capacity is, how intense the competition is, the demand pattern that categories exhibit, and the α service level that the company is required to guarantee. Furthermore, customers can exhibit different behavioural patterns according to their priorities. For luxury items, a higher price tag can translate into more demand for a product, as consumers are sensitive to quality. The opposite is expected for common consumption goods. Varying the distribution of the attractiveness throughout the product line can simulate this effect. With this analysis, we extend our conclusions, as well as test how robust each method is to deviations against the predicted behaviour.

Table 3.4 – Computational experiment design including the number of product selection decisions.

Test	**Condition**	**Number of levels**	**Number of instances**	**Number of decisions**
Integration	Capacity	6	120	1440
	Market priority	3	60	720
	Dynamicity	3	60	720
Multi-period	Capacity	6	120	1440
	Seasonality	3	60	720
Safety-stock	Competitiveness	6	120	1440
	Service level	3	60	720

Table 3.4 outlines the experiment. The first column regards the contribution for which we are testing. For each test, we compare the resulting profit and product selections made by each method. The second column displays the external and internal parameters for which sensitivity analysis was conducted, with the considered number of levels exhibited in the third column. The fourth column is the number of instances, computed as the number of templates multiplied by the number of levels and the last column, relative to the number of decisions, allows the comparison of the selections of each approach to be clearer in the following subsections. In the table, we denote period-to-period variations in demand as dynamicity.

3.4.1 The value of the integrated approach

Historically, sales managers dictate which products a company sells. To assess the value of an integrated decision, we test the performance of two heuristics used in practice against that of the integrated optimisation model. In these product line strategies, marketing dictates the products to sell and the target market shares. Production seeks to cost-efficiently (optimally) comply with marketing's manufacturing needs, given the sales target for each product. We consider marketing only has access to administrative and development cost data. To make the performance of the lines comparable, we inject the selection made by the heuristics into the integrated model, so that it computes the profit that the line generates.

Algorithm 1 Pseudo-code for marketing-driven optimization.

```
procedure MARKETINGOPTIMIZATION()
    W ← marketingOptimization();
    profit ← integratedOptimization(W);
```

In the marketing-driven optimization heuristic, marketing determines the product line that maximizes gross profit after fixed costs by solving the problem with the objective $\sum_t \sum_j p_j D_{jt} - \sum_j f_j W_j$ subject to constraints (3.2)-(3.8). The solution is then injected into the integrated approach to evaluate profit. Algorithm 1 (Marketing, in Figure 3.3), describes this procedure where marketing commands production. Trimming the lame-duck (Chong et al., 2005) is a heuristic that, starting from a full line, removes the worst performing product according to a myopic criterion (e.g., margin, share or revenue). It is a common product removal strategy that firms opt for when attempting to deal with cannibalisation and manufacturing issues. This strategy is conservative, as it can choose a product line containing all of the manufacturer's products if none are worth to remove. In this implementation, we consider the choice criterion to be the gross profit of a product j after its fixed administrative and development costs, $\sum_t p_j D_{jt} - f_j$.

Algorithm 2 Pseudo-code for iteratively removing the worst performers.

```
procedure REMOVEWORSTPERFORMERS(criterion)
    W ← 1⃗;
    currentProfit ← integratedOptimization(W);
    repeat
        incumbentProfit = currentProfit;
        W ← removeWorst(W, criterion);
        currentProfit ← integratedOptimization(W);
    until currentProfit < incumbentProfit;
    profit ← incumbentProfit;
```

After choosing which product to trim out, marketing consults production to understand the operational impact of the removal. If an increase in profit is expected, the removal is accepted, and marketing will search for another product to remove. Otherwise, the removal is cancelled, and the local optimum is reached. The pseudo-code in Algorithm 2 (Lame-duck, in Figure 3.3) shows the procedure of trimming the lame duck. By fixing product selection W to a vector of ones, the algorithm starts by evaluating the performance of the full line. Then, we remove the product with the worst performance from the line and recalculate profit using the integrated model. If profit increases, this procedure is repeated. If not, the line is reverted to that of the previous iteration, and the procedure ends. The heuristic depicts some degree of cooperation between the departments as, although marketing still commands, at each step of the PLS process, the feedback from production has the final say on the decision.

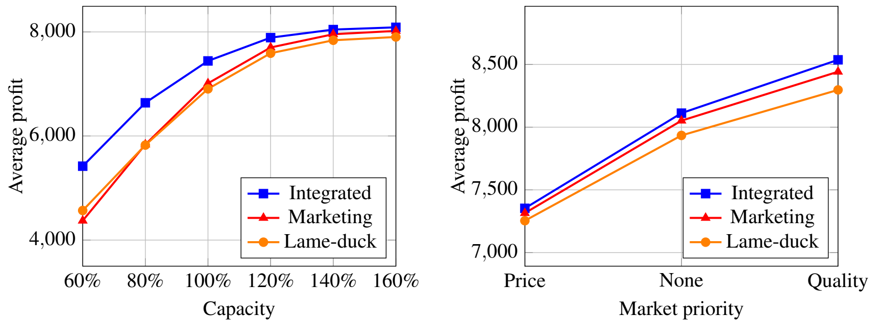

Figure 3.3 – Results for the integrated approach versus gross-profit optimisation and iterative removal (trimming the lame duck). On the left, the average profit is presented as a function of capacity. On the right, as a function of the priority of the market.

Figure 3.3 exposes the results of each approach as a function of capacity and the evaluation that consumers make on price and quality. On the left, the performance of each solution method as a function of capacity is plotted. The integrated approach corresponds to the model exposed in Section 3.3 with development and administrative costs and resource sharing. Our base unit for capacity (100%) is computed as described in Subsection 3.3.4. To isolate this effect, a random attractiveness-price distribution was considered. Similarly, to isolate the effects of the priorities of the market, as represented on the right side of the figure, production is not constrained by capacity. We simulate quality-prioritising markets by ensuring that a product with a higher margin has a greater attractiveness value, and reverse this association to mimic price-sensitive markets. In the case with no market priority, the association between attractiveness and price is random.

The plot on the left shows that an integrated approach is most beneficial when capacity is the most constrained. We note that when capacity is not constraining, and the market does not have a specific priority, keeping the full line results in 31.5% less profit than using the product lines recommended by our model. For this set of instances, when capacity is at 160% of the production time of the full line, profits obtained from the optimisation for gross profit are, on average, 0.9% lower than those obtained by the integrated approach. For the iterative removal of the worst products, this difference is of 2.3%. These values rise to 19.3% and 15.7% when capacity is at 60% of the full line, noting that the iterative worst removal outperforms gross-profit optimisation when capacity is below 80%. The difference between both non-integrated heuristics diminishes with how constraining capacity is. The iterative removal manages to beat gross-profit optimisation in fifteen instances and tie in one, for the scenario where capacity is most constrained. When capacity is most loose, the iterative removal heuristic is worse in eight cases and ties in nine.

On the right, we observe that the profit difference between the approaches increases with how much quality is prioritised over price. In price-prioritising markets, while product lines resulting from gross-profit optimisation manage to profit 0.5% below the optimal profit value, lines chosen by the iterative marketing-led removal result in a 1.3% difference. In quality-prioritising markets, the same metrics present the values of 1.1% and 2.8%. The marketing-driven heuristic is not dominant across all instances, however the number of instances in which it outperforms the iterative removal increases with quality prioritisation.

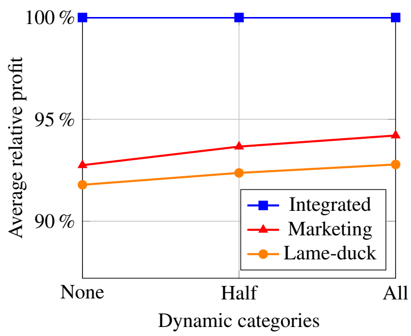

Figure 3.4 – Results for the integrated approach versus gross-profit optimization and iterative removal (trimming the lame duck). Average relative profit presented as a function of the number of dynamic categories, with capacity at 100%.

Figure 3.4 shows the evolution of the benefits of the integrated approach with the number of categories for which demand varies throughout the planning horizon, with capacity being constrained to the 100% level. The relative profit of the approaches is displayed for static demand (none of the categories is dynamic), for uniform demand (all categories are dynamic, which corresponds to the unaltered templates), and for the half level (where demand in half of the categories, rounded up, is static). The results show that, unintuitively, the heuristics perform relatively better if demand is dynamic. In this case, trimming the lame-duck leads to profits of 1.0 *p.p.* higher than if demand were static. This difference, when optimising for gross profit is of 1.5 *p.p.*. To justify these results, we recall that, to assess the performance of each heuristic, we inject its product selection into the integrated approach with the profit it generates being calculated assuming an optimal production plan. With dynamic demand, the approach manages to make better trade-offs between piling up inventory and making changeovers, as opportunities will arise to setup at a demand peak, produce, and hold additional product to cover a demand trough without additional setups. Moreover, gross profit slightly increases with demand variations, since these trade-offs open up some capacity that otherwise would be lost setting up, with this effect being constant throughout the approaches. Therefore, the benefits of the integrated approach decrease with how dynamic demand is because, by finding better fits in the production plan, indirect production costs become a smaller slice of gross profit.

Both heuristics increasingly overestimate product line breadth with capacity, with this effect being more prominent in the solutions provided by the marketing-driven heuristic. This is expected as the heuristic does not account for capacity (does not affect the solution), and the iterative removal only accounts for it indirectly. Moreover, for all cases, the optimal product line was contained in the solution provided by gross-profit optimisation. Concerning both heuristics, the more quality is prioritised by the consumers, the bigger the divergence from the optimal product line. For the lame-duck, while 15.8% of the 720 product choices differ when the market prioritises price, 24.6% of the choices differ when the market seeks quality. Although with lesser differences from the optimal line, this effect still manifests when optimising for gross profit, with the respective values being of 5.0% and 10.8%. The differences between the product lines proposed by the heuristics are most significant when the market has no specific priority, and demand is static. While the size of the lines selected by the gross-profit optimisation heuristic does not vary with the number of dynamic categories, the lines selected by the lame-duck heuristic and the integrated approach become slightly larger. This is expected since setups can be better utilised in the dynamic scenario, and the two approaches have some insight into operations. At 42.1% of different selections, the lines of lame-duck and the integrated approach for the half level are the ones that differ the most. When demand is static, 41.3% of the choices the lame-duck heuristic makes are suboptimal. When demand is dynamic, the proportion lowers to 39.6%. Regarding the marketing-driven heuristic, the percentages become 32.1% and 29.6% for the static and dynamic scenarios, respectively.

3.4.2 The value of multi-period planning

The value of a multi-period approach is connected to the trade-off between setups and inventory, given non-stationary demand. If capacity is considered, additional profit can be generated from more detailed production planning by fitting the right products into the line. In this subsection, we measure the impact that dividing the horizon of the production plan into multiple periods can have on the selection of the product line. We compare the performance of and choices made by a single-period approach to those of the multi-period approach corresponding to the model exposed in Section 3.3 with development and administrative costs, and resource sharing.

For the single-period approach, one period encompasses the whole planning horizon and might include multiple production cycles. We evaluate the optimal solution W for each possible production frequency by inputting it into the multi-period model, with the maximum number of cycles being the initial number of periods. Finally, we pick out the best and worst-performing production frequencies ω to contend with the selection from the multi-period approach. To convert the multi-period instance into single-period, demand and capacity are reevaluated as $d = \sum_t d_t$ and $c = \sum_t c_t$. Furthermore, $s^k = \sum_t s_t^k / T$. In each production on cycle, the necessary setups for each product in W must be executed, and the average inventory is half of the cycle's production (linear depletion rate). We implement this by considering setup costs and times as $\omega T \theta_j$ and $\omega T \tau_j$ for each product and $\omega T \theta^m$ and $\omega T \tau^m$ for each manufacturing category. Additionally, holding costs are reformulated as $2h_j T/\omega$. After the conversion, the model exposed in Section 3.3 can solve the single-period instance. Nevertheless, the single-period model, where production planning is similar to that of Morgan et al. (2001) and our multi-category attraction model estimates demand response, is provided in Appendix A.

Capacity and the demand pattern are the most relevant factors when considering the

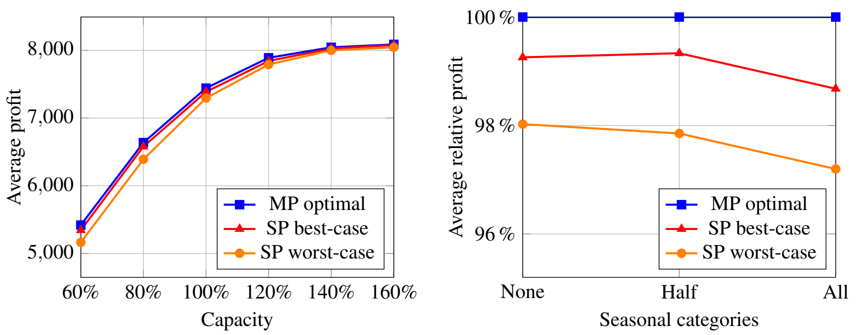

Figure 3.5 – Results for the multi-period approach versus the single period approaches with the best and worst production frequencies. On the left, average profit is presented as a function of capacity. On the right, average relative profit is shown as a function of the number of seasonal categories, with capacity at 100%.

trade-off between stocking and setting up in a situation where demand is not stationary. Figure 3.5 shows the results of this analysis as a function of these factors. We exhibit the profit resulting from the multi-period optimal product line, against that of the single-period approach. We include profit for the best and worst production frequencies.

The left side of the figure deals shows profit as a function of capacity. As in the previous subsection, the base capacity level is the maximum time that the full line could take to produce, without setups. Having a time-between-runs $1/\omega = 1$ consistently results in the worst profits. As expected, for cases where capacity is tightly constrained, higher times between runs lead to better results. When the capacity is at 60% and 80% of its baseline, a $1/\omega = 5$ and $1/\omega = 3$ are the best performers, whereas $1/\omega = 2$ leads to the best profits in less restricted cases. Comparing the best and worst cases for production frequency, the multi-period model leads to improvements in profit inferior to 0.5% in the situation with the loosest capacity. In a constrained scenario, we estimate the multi-period approach to lead to product lines that are 1.4% to 4.7% more profitable. This denotes the increasing value of multi-period PLS with a decreasing capacity, which stems from the augmented importance of managing the setup-inventory trade-off. This value is strictly increasing for a given time-between-runs.

The right side of the figure exhibits profit for three proportions of seasonal categories, with a capacity level of 100%. When none of the categories is seasonal, instances correspond to the unaltered templates. In the half level, half of the categories (rounded up) have a seasonal pattern that is implemented by moving half of the demand from the first two periods into the fourth and fifth ones. In the last level, all of the categories exhibit this seasonal pattern. For all demand patterns, $1/\omega = 1$ is to the worst single-period production frequency and $1/\omega = 2$ the best. For all approaches, holding costs increase steeply with seasonality. The results show that the benefits of a multi-period approach increase with the number of seasonal categories. For the case where every category is seasonal, the profit advantage of the model is of 1.3% for the best frequency and of 2.8% for the worst. The advantage becomes 0.7% and 2.0% for the respective best and worst production frequencies.

When half of the categories exhibit the seasonal pattern, the best production frequency obtains similar results to when demand is uniform, and the worst frequency leads to results 0.2 *p.p.* lower.

In the worst-case, 11.8% of the choices the single-period method makes differ from the optimal multi-period approach across the tests involving capacity. This amounts to 134 product exclusions and 34 product inclusions that are suboptimal. By considering setting up every period, this scenario is undershooting the number of products that the line withstands. The opposite is done by the best-case single-period approach, which slightly overshoots the number of products, in all scenarios but the most constrained. This translates into 74 inclusions and 54 exclusions that differ from the multi-period approach, totaling of 5.1% of different choices. The best-case includes all products worst-case approach does for the two situations where capacity is most lax. More so, it only does not include 30 products that the worst-case approach does the total of 1440 product choices. Inversely, it includes 152 products that the worst-case single-period method does not. While the single-period approaches maintain the size of the line with an increase in the number of seasonal categories, the multi-period approach selects smaller lines. With 15.0% of the choices being made differently, the worst single-period and the multi-period approaches lead to the most distinct solutions. Then, with 13.8% of different choices, stand the single-period approaches. With 9.9% of the choices made differently, the multi-period and the best single period approaches select the most identical lines. The half level increases the differences between the lines chosen by the worst single-period and the multi-period by 1.5 *p.p.*. It also decreases the differences between single-period approaches by 1.3 *p.p.* and by 1.6 *p.p.* between the best single-period and the multi-period approaches. Again, the single-period with the worst frequency undershoots the line and the one with the best production frequency slightly overshoots it.

3.4.3 The value of safety stock modelling

Safety stock must be taken into consideration as service levels must be guaranteed. In this subsection, we compare how considering a safety-stock policy influences product selection. Concerning the piece-wise linearization used to model safety stock, we established $N = 4$, trading the added complexity of the binary variables over the fit of the linearised function, and define the lead time parameter $\Lambda = {}^{1}/_{8}$. To appraise the value of including safety stock in the model, we compare the profits obtained from the product lines chosen when the model regards and does not regard safety stock, in a situation where it must be assured. Here, advantages arise from taking the variability of the sales and the costs associated with the additional inventory into account.

The most influential parameters to consider in this assessment are service level and market competitiveness. While the service level directly influences the amount of inventory needed to fulfil the policy's requirements, market competitiveness influences the share of the company, which can affect the variability of the demand for its products. On the left of Figure 3.6, the results of both approaches are compared as a function of competitiveness, considering a base service level of 95%. In the right side of the figure, profit is plotted as a function of the α service level requirements, with competitiveness at its base level (corresponding to a market share of 33% while offering the full product line and meaning that $\sum_k a_0^k = 2\sum_j a_j$).

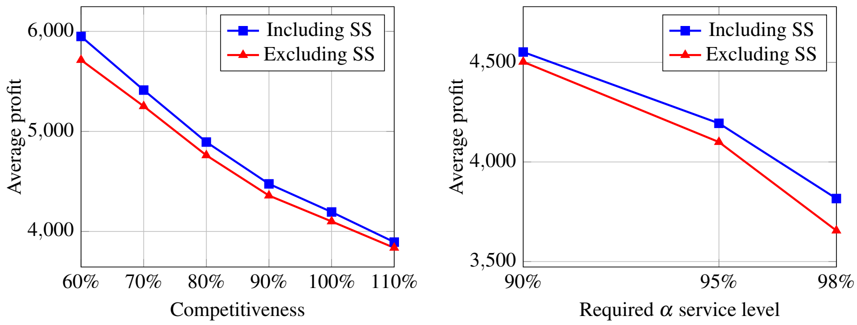

Figure 3.6 – Results for the integrated approach with and without accounting for safety-stock (SS). On the left, the average profit is presented as a function of market competitiveness. On the right, as a function of the required α service level.

The importance of accounting for safety stock in the decision decreases with the strength of the competition and the consequent reduction in the market share of the product line. Revenue is asymptotically decreasing with the strength of the competition, and this is reflected in profit. Regarding the base case for both factors, excluding safety-stock leads to product lines 2.2% less profitable. Furthermore, the expected profit difference between both approaches for the situation when competitiveness is at 60% is 4.0%. This value falls to 1.5% when the competition is 110% of the basis. Contrarily, the importance of including safety-stock policies in the model increases with the service level requirements. When a 90% α service level is required, including safety-stock constraints in the model leads to 1.1% better results. This value rises to 4.2% with a 98% service level requirement.

Expectedly, the number of products included in the line is overestimated when not considering the safety-stock policy. In a total of 1920 decisions, 165 inclusions and 7 exclusions separated the selection made by each approach. Congruently with the variation in profit, the percentage of different product choices decreases with competitiveness, from 15.0% at 60% of the base level to 6.3% at 110%. More so, this value increases with the service level requirements from 7.5% at 90% of the base level to 11.7% at 98%.

3.5. Application to real-world instances

Our computational tests made evident the conditions in which our proposed model provides its utmost utility. Still, the results from its application to instances generated by uniform distributions with parameters based on a real case do not accurately quantify the benefits that would transfer a real production line. For this reason, we conduct the tests on two real production lines from the beverages industry. In both cases, the planning horizon comprises 12 periods, and the manufacturing categories differ from the consumer perceived ones. We estimate attractiveness values using point-of-sale data. Setup costs are null, and fixed costs were neglected as they were not made available by the firm. Nevertheless, the firm provided setup and production times, which we account for. In this section, we refer to the real production line with 21 potential products as instance A, and to the one with 13 as instance B. Again, we start by assessing the value of an integrated optimization model by comparing its solutions with those heuristics that the industry frequently uses.

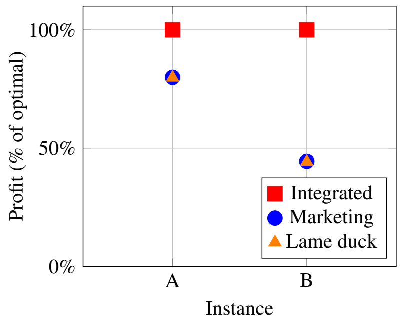

Figure 3.7 – Results for the integrated approach versus gross-profit optimization and iterative removal (trimming the lame duck) for the real production lines.

Figure 3.7 shows the profit of the heuristics as a percentage of the profit of the integrated optimal, for the real instances. The absence of fixed costs makes it easy for both heuristics to converge to the same solution, that being the product line that generates the most revenue. Notwithstanding, the solutions could diverge if several high-margin low-attractiveness products options, which generate little revenue, were available. The lame-duck heuristic could remove these products, and it would stray from the parsimonious approach exposed in Subsection 3.3.1. The benefits of the integrated approach are a 25.3% increase in profit for instance A and a 125.3% increase for instance B. This demonstrates how removal strategies that are used in practice can be sub-optimal. Furthermore, the disparity between the benefits achieved in both cases sheds light on the risks of using myopic heuristics with no performance guarantee. We have shown the potential increases in profit relative to the practices of the industry. We now follow up with a comparison of our approach with the current state of the art in production modelling for PLS, resulting in an assessment of the value of considering multiple periods.

Figure 3.8 presents the profit of the single-period solutions compared to those obtained by our multi-period formulation. The figure contrasts with the results obtained for the virtual instances, where the benefits of the multi-period model seem slim. This is a consequence of market size, in the templates, being generated according to a uniform distribution, thus exhibiting no intentional seasonality and of, in this case, holding costs representing a larger portion of revenue. Nevertheless, the results for the real cases reflect the stark seasonality that the market for beverages presents. For instance A, the increase in profit resulting from the use of the multi-period ranges from 4.9% to 6.4%. For instance B, the increase is situated in the 7.9% to 9.4% range. Lastly, after estimating the benefits of our model relative to the state of the art in PLS, we evaluate the potential of accounting for the costs of safety stock when making the selection.

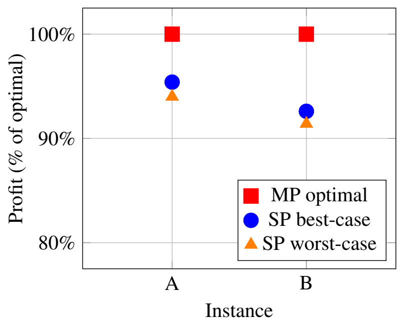

Figure 3.8 – Results for the multi-period approach versus the single period approaches with the best and worst production frequencies for the real production lines.

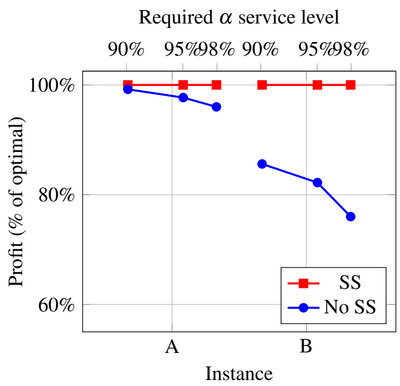

Figure 3.9 – Results for the integrated approach with and without accounting for safety-stock (SS) for the real production lines.

Figure 3.9 shows the difference in the performance of the choices made by the integrated model with and without the safety stock extension, in a scenario where, by policy, safety stock is required. While, for instance A, the benefits increase from 0.9% to 4.1% higher profits with the increase in the required service level, for instance B, the benefits range from a 16.8% to a 31.6% profit increase. Although the product lines that both solution methods chose will differ depending on the market share of each product and the inherent variability of that category, the results emphasize the desirability of accounting for safety stock in the decision, especially when high service levels are required.

3.6. Managerial insights

The insights derived from the computational experiment are summarized in Table 3.5. The first column presents the coupling of approaches in each comparison. The second and third columns show if the first approach leads to higher profits and larger lines than the second, respectively (plus sign for higher and larger, minus sign for lower and smaller). As the approaches in each couple are ordered by the profit they generate, the second column is populated with plus signs. The fourth column states the independent variable of the sensitivity analysis, the fifth and sixth display its results. While the second column states if the profit of the first approach is higher or lower than the one of the second, the fifth column shows an arrow pointing up if this difference increases with an increase in the parameter. The sixth column is analogous to the fifth but respecting size. In these last two columns, a flat line represents that there is no clear trend. The table shows that the value of our approach is highest when capacity is tight, consumers prioritize quality, the company has weaker competition and high service level requirements. As an example of interpretation, in the comparison between marketing-driven optimization and the iterative lame-duck removal heuristics, marketing-driven optimization is generally the best performer (plus sign) with its profit advantage increasing with capacity (arrow pointing up). Furthermore, the lines that this approach chooses are usually longer than the ones chosen by its counterpart (plus sign), with this effect diminishing as capacity increases (arrow pointing down). On another note, although no clear trend was found for the difference in line size throughout the different capacity levels when comparing the single-period approaches, the lines that each selects become more similar as capacity increases.

Table 3.5 – Summary results of the computational experiments.

Comparison	Profit	Size	Parameter	Profit (trend)	Size (trend)
Marketing Lame-duck	+	+	Capacity	↑	↓
	+	+	Priority	↑	↓
	+	+	Dynamicity	↑	↓
Integrated Marketing	+	-	Capacity	↓	↓
	+	-	Priority	↑	↑
	+	-	Dynamicity	↓	↓
Integrated Lame-duck	+	-	Capacity	↓	↓
	+	-	Priority	↑	↑
	+	-	Dynamicity	↓	-
SP best-case	+	+	Capacity	↓	-
SP worst-case	+	+	Seasonality	↑	-
MP optimal	+	-	Capacity	↓	↓
SP best-case	+	-	Seasonality	↑	↑
MP optimal	+	+	Capacity	↓	↓
SP worst-case	+	+	Seasonality	↑	↓
Including SS	+	-	Competitiveness	↓	↓
Excluding SS	+	-	Service level	↑	↑

An FMCG producer can benefit from an integrated multi-period approach for PLS. In the virtual instances, the integrated approach leads to profits up to 19.3% higher than those of the current industry practices. The improvements increase with capacity tightness and with the quality sensitiveness of the consumers. We have seen that dynamic categories can allow for capacity to be better utilized, even for sub-optimal selections. Additionally, we exhibited that optimizing for gross profit leads to higher profits than iteratively removing the worst performing product unless the plant is under very tight capacity constrictions. Lastly, the results from the real cases show that PLS should not be left to myopic heuristics with no performance guarantee, as the performance of their solutions obtained can be very disparate from the optimal.

Furthermore, we demonstrate the potential of a multi-period approach. The advantages over the single-period approach, which materialize in a difference of up to 4.7% in average profit, as well as the variation in performance between the best and worst production frequencies, increase with capacity tightness. If a planner manages to foretell the best production frequency, the difference between the single-period approach the multi-period optimal was only of up to 1.4% throughout the tests. We also show that the benefits of considering multiple periods increase with the number of seasonal product categories. The real cases demonstrate the full potential of modelling production as a capacitated lot-sizing problem with shared setups, validating the need for this contribution in cases with predictable demand variations (e.g., seasonality). Especially in FMCG, the inventories needed to handle seasonality and holiday peaks can influence the optimal selection.

When safety stock is a requirement, we showed its inclusion into the decision can lead to significant increases in the profitability of the resulting product lines. These increases become more prominent in markets where competition is weaker, with differences of up to 4.0% in average profit, and in cases with more demanding service levels, these differences can reach 4.2%. The extension leads to different decisions as, it accounts for demand variability and the additional inventory needed to fulfil the policy. This additional component in inventory costs is influenced by the competitive position of the products of the firm and the variability of the category and is particularly relevant when high service levels are required.

The inclusion of the results for the real production lines confirms the existence of the untapped benefits from an approach to PLS that accounts for the specificities of FMCG. In sum, tight capacity, low price sensitivity, and pronounced seasonality in demand maximize the need for our integrated multi-period approach. Moreover, the safety-stock extension is most required when the firm has products that serve around 50% of the market for a whole category with a lot of variability and high demanded service levels. Optimizing PLS can effectively lead to additional profits relative to those of optimizing production planning. Nevertheless, the high stakes involved in the decision make the approach vulnerable to the risks of improper parameterization.

3.7. Conclusions

In this paper, we deliver a mixed-integer programming model for PLS to be used by FMCG producers. Adding to the literature, the share of each product is obtained from a multi-category attraction model. Capacity is explicitly modelled, and the approach is multi-period, which allows variations in demand to be managed by the trade-off between stocking and setting up. Furthermore, we incorporate safety stock into the mixed-integer programming model in a new fashion. Employing a set of instances generated with real-world parameters and two real cases, we assessed the value of our research by making three performance comparisons. We contrast (1) the heuristics used in practice with the integrated approach, (2) single-period planning with the integrated approach, and (3) the integrated model with the integrated model considering safety stock. To strengthen our conclusions and derive managerial insights, we vary the conditions under which each comparison is made. The results of our computational experiments in virtual instances identify the conditions in which the integrated multi-period model is most beneficial. Furthermore, the results obtained from applying the approach to the real production lines made the potential profit increases appreciable and provided some key insights. They make evident that the decision should not be left to heuristics, and that a multi-period approach is needed when the demand has predictable variations (e.g., seasonality).

Three main streams of future research follow from this work. Although we showed the potential benefits of the approach using a real-world inspired instance set and validated the existence of these benefits with real cases, we are still unable to solve larger real-world instances. With safety stock, our approach took two days to reach the optimum of the larger instance. The magnitude of this solution time is impractical for a decision-maker to iterate over different parameter settings. Firstly, solution methods for this problem need to be further researched. Specifically, this problem integrates the attraction revenue management with fixed costs and capacitated lot-sizing problems which have been extensively studied. Therefore, looking into heuristics combining the solution methods for these problems is a promising direction. Safety stock was modelled as a linear approximation, avoiding the computational stress of uncertainty modelling. Secondly, with better methods, this approximation should be possible to overcome. Furthermore, providing robust product line selections can hedge against poor parameterization, which is much needed for such a high stakes decision. Lastly, demand interactions between the products are limited, and brand loyalty is still not accounted for. For a more realistic consumer choice model, fully incorporating the effects of substitutability and complementarity is vital.

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Bibliography

B. Almada-Lobo, A. Clark, L. Guimarães, G. Figueira, and P. Amorim. Industrial insights into lot sizing and scheduling modeling. *Pesquisa Operacional*, 35(3):439–464, 2015. doi: 10.1590/0101-7438.2015.035.03.0439.

X. A. Andrade. Product line selection of fast-moving consumer goods (instances), 2019. URL `10.6084/m9.figshare.7874639.v1`.

G. Aydin and J. K. Ryan. Product line selection and pricing under the multinomial logit choice model. Working paper. Stanford University, 2000.

P. J. Bernales, Y. Guan, H. P. Natarajan, P. S. Gimenez, and M. X. A. Tajes. Less is more: Harnessing product substitution information to rationalize skus at intcomex. *Interfaces*, 47(3):230–243, 2017. doi: https://doi.org/10.1287/inte.2017.0889.

D. Bertsimas and V. V. Mišić. Robust product line design. *Operations Research*, 65(1): 19–37, 2016. doi: 10.1287/opre.2016.1546.

D. Bertsimas and V. V. Mišić. Exact first-choice product line optimization. *Operations Research*, 67(3):651–670, 2019. doi: 10.1287/opre.2018.1825.

K. D. Chen and W. H. Hausman. Mathematical properties of the optimal product line selection problem using choice-based conjoint analysis. *Management Science*, 46(2): 327–332, 2000. doi: 10.1287/mnsc.46.2.327.11931.

J.-K. Chong, T.-H. Ho, and C. S. Tang. Demand modeling in product line trimming: Substitutability and variability. In *Managing Business Interfaces*, pages 39–62. Springer, 2005. doi: 10.1007/0-387-25002-6_2.

J. M. Day and M. Venkataramanan. Profitability in product line pricing and composition with manufacturing commonalities. *European journal of operational research*, 175(3): 1782–1797, 2006. doi: 10.1016/j.ejor.2005.05.013.

G. Dobson and S. Kalish. Positioning and pricing a product line. *Marketing Science*, 7(2): 107–125, 1988. doi: 10.1287/mksc.7.2.107.

G. Dobson and S. Kalish. Heuristics for pricing and positioning a product-line using conjoint and cost data. *Management Science*, 39(2):160–175, 1993. doi: 10.1287/mnsc.39.2.160.

G. Dobson and C. Yano. Product line and technology selection with shared manufacturing and engineering design resources. Working paper., Simon School of Business, 1995.

G. Dobson and C. A. Yano. Product offering, pricing, and make-to-stock make-to-order decisions with shared capacity. *Production and Operations Management*, 11(3):293–312, 2002. doi: 10.1111/j.1937-5956.2002.tb00188.x.

G. Gallego, R. Ratliff, and S. Shebalov. A general attraction model and sales-based linear program for network revenue management under customer choice. *Operations Research*, 63(1):212–232, 2014. doi: 10.1287/opre.2014.1328.

A. Ghoniem and B. Maddah. Integrated retail decisions with multiple selling periods and customer segments: optimization and insights. *Omega*, 55:38–52, 2015. doi: 10.1016/j.omega.2015.02.002.

P. E. Green and A. M. Krieger. Models and heuristics for product line selection. *Marketing Science*, 4(1):1–19, 1985. doi: 10.1287/mksc.4.1.1.

W. J. Hopp and X. Xu. Product line selection and pricing with modularity in design. *Manufacturing & Service Operations Management*, 7(3):172–187, 2005. doi: 10.1287/msom.1050.0077.

B. Karimi, S. F. Ghomi, and J. Wilson. The capacitated lot sizing problem: a review of models and algorithms. *Omega*, 31(5):365–378, 2003. doi: 10.1016/S0305-0483(03)00059-8.

U. G. Kraus and C. A. Yano. Product line selection and pricing under a share-of-surplus choice model. *European Journal of Operational Research*, 150(3):653–671, 2003. doi: 10.1016/S0377-2217(02)00522-2.

K. Lau, A. Kagan, and G. Post. Market share modeling within a switching regression framework. *Omega*, 25(3):345–353, 1997. doi: 10.1016/S0305-0483(96)00054-0.

T. Li and H. Zhang. Information sharing in a supply chain with a make-to-stock manufacturer. *Omega*, 50:115–125, 2015. doi: 10.1016/j.omega.2014.08.001.

W. Li and J. Chen. Backward integration strategy in a retailer stackelberg supply chain. *Omega*, 75:118–130, 2018. doi: 10.1016/j.omega.2017.03.002.

W. B. Locander, R. W. Scannell, et al. Modeling the product elimination decision. *Omega*, 4(6):741–742, 1976. doi: 10.1016/0305-0483(76)90103-1.

R. D. McBride and F. S. Zufryden. An integer programming approach to the optimal product line selection problem. *Marketing Science*, 7(2):126–140, 1988. doi: 10.1287/mksc.7.2.126.

McKinsey & Company. Western Europe's consumer-goods industry in 2030, 2016. https://www.mckinsey.com/industries/consumer-packaged-goods.

McKinsey & Company. Should CPG manufacturers go direct to consumer – and, if so, how?, 2017. https://www.mckinsey.com/industries/consumer-packaged-goods.

K. B. Monroe, S. Sunder, W. A. Wells, and A. Zoltners. *A Multi-period Integer Programming Approach to the Product Mix Problems*. Columbia University, Graduate School of Business, 1974.

L. O. Morgan, R. L. Daniels, and P. Kouvelis. Marketing/manufacturing trade-offs in product line management. *IIE Transactions*, 33(11):949–962, 2001. doi: 10.1080/07408170108936886.

R. Quante, H. Meyr, and M. Fleischmann. Revenue management and demand fulfillment: matching applications, models, and software. *OR Spectrum*, 31(1), 2009.

J. A. Quelch and D. Kenny. Extend profits, not product lines. *Make sure all your products are profitable*, 14, 1994.

N. Raman and D. Chhajed. Simultaneous determination of product attributes and prices, and production processes in product-line design. *Journal of Operations Management*, 12(3-4):187–204, 1995. doi: 10.1016/0272-6963(95)00013-I.

C. Schön. On the optimal product line selection problem with price discrimination. *Management Science*, 56(5):896–902, 2010a. doi: 10.1287/mnsc.1100.1160.

C. Schön. On the product line selection problem under attraction choice models of consumer behavior. *European Journal of Operational Research*, 206(1):260–264, 2010b. doi: 10.1016/j.ejor.2010.01.012.

S. M. Shugan, V. Balachandran, et al. A mathematical programming model for optimal product line structuring. Discussion paper No. 265, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1977.

W. Steiner and H. Hruschka. A probabilistic one-step approach to the optimal product line design problem using conjoint and cost data. *SSRN Electronic Journal*, 1, 08 2002. doi: 10.2139/ssrn.319463.

K. Talluri and G. Van Ryzin. Revenue management under a general discrete choice model of consumer behavior. *Management Science*, 50(1):15–33, 2004. doi: 10.1287/mnsc.1030.0147.

Appendix A

We hereby present the single-period model that can be used to reproduce the experiment in Subsection 3.4.2 with changes in the objective function and capacity constraints as postulated.

$$\max_{S,I,Y,X} \quad \sum_{m} \sum_{j \in \mathcal{N}^m} p_j D_j - 2h_j D_j T/\omega - (f_j + \omega T \theta_j) W_j - \omega T \theta^m Y^m \tag{3.22}$$

$$\text{s.t.} \quad D_j \leq d^k S_j \quad \forall k, j \in \mathcal{N}^k \tag{3.23}$$

$$a_0^k S_j = a_j S_0^k - F_j \quad \forall k, j \in \mathcal{N}^k \tag{3.24}$$

$$F_j \leq a_j S_0^k \quad \forall k, j \in \mathcal{N}^k \tag{3.25}$$

$$S_0^k + \sum_{j \in \mathcal{N}^k} S_j = 1 \quad \forall k \tag{3.26}$$

$$S_j \leq W_j \quad \forall j \tag{3.27}$$

$$S_j \leq Y^m \quad \forall m, j \in \mathcal{N}^m \tag{3.28}$$

$$\sum_m \omega \tau^m T Y^m + \sum_{j \in \mathcal{N}^m} \omega \tau_j T W_j + r_j D_j \leq c \tag{3.29}$$

$$W_j, Y^m \in \{1, 0\} \quad \forall j, m \tag{3.30}$$

$$D_j, S_j, S_0^k \in \mathbb{R}_0^+ \quad \forall j, k \tag{3.31}$$

Hence, expression 3.22 and the capacity constraints (3.29) accommodate the assumption that every production cycle setups are incurred. Again, constraints (3.23)-(3.26) ensure the market behaves as desired. Constraints (3.27) and (3.28) activate the binary variables relative to product and manufacturing category setups. We note that, in this single-period model, W_j is used both for product setups and the presence of the product in the assortment. Constraints (3.24) limit the production to the demand, noting that here we use D_j both as production quantity and sales. Lastly the domain of the variables is defined in expressions (3.30) and (3.31).

Appendix B

Table 3.6 – Table of notation.

Indexes and sets	
$j \in \{1,..,J\}$	Product index.
$k \in \{1,..,K\}$	Category index.
$m \in \{1,..,M\}$	Family index.
$t \in \{1,..,T\}$	Planning period index.
$n \in \{1,..,N\}$	Piece index.
$\mathcal{N}^k$	Set of products belonging to category k.
$\mathcal{N}_m$	Set of products belonging to family m.
Parameters	
d_t^k	Demand for category k in period t.
σ_t^k	Standard deviation of category k in period t.
p_j	Margin of product j
h_j	Cost of holding a unit of j.
f_j	Fixed administrative cost for producing j.
θ_j	Setup cost for product j.
θ^m	Setup cost for family m.
τ_j	Setup time for product j.
τ^m	Setup time for family m.
a_j	Attractiveness of product j.
r_j	Time that a unit of j takes to produce.
c_t	Capacity in period t.
a_0^k	Attractiveness of the outside competition in category k.
K_{jt}	Maximum production of j in period t.
x_0^n	Beginning of piece n.
y_{0t}^{kn}	Intersection for the straight line of piece n.
δy_{0t}^{kn}	Slope for the straight line of piece n.
M_t^{kn}	Maximum value that linear piece n can take.
Variables	
W	Set of the selected products.
W_j	1 if product j is selected, 0 otherwise.
D_{jt}	Sales of product j in period t.
X_{jt}	Production of product j in period t.
I_{jt}	Inventory of product j in period t to be carried over.
Y_t	1 if machines are setup in period t, 0 otherwise.
S_j	Share of product j.
S_0^k	Share of the outside competition in category k.
F_j	Slack for the ratios between attraction and shares for j.
Z_j^n	1 if share of j is on piece n, 0 otherwise.
Functions	
$\mathrm{R}(W)$	Gross profit generated by product line W.
$\mathrm{C}(W)$	Indirect costs of supplying product line W.

CHAPTER 4

Product line selection of fast-moving consumer goods under stochastic demand

Product line selection of fast-moving consumer goods under stochastic demand

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Abstract Selecting fast-moving consumer goods product lines requires deep insight into projected production costs and demand volumes, given the sector's thin margins. Demand volumes are difficult to estimate, because both the total market size and each individual product utility are subject to considerable uncertainty. This paper tackles product line selection for fast-moving consumer goods producers, incorporating market sizes and product utility uncertainty. We propose a stochastic formulation to determine profit-optimizing product lines. Sequentially, our approach makes a selection and, when demand reveals itself, computes complexity costs by solving a capacitated lot-sizing problem. We address this model's complexity on two fronts: using Bender's decomposition and reducing the number of scenarios to consider. The decomposition method drastically cuts solution times, particularly for instances with small subproblems. The scenario reduction procedure achieves equivalent stability to random draws using almost one-third of the scenarios. Lastly, we analyze the value of the stochastic solutions under different random utility models, attractiveness coefficients of variation, and capacity conditions. The stochastic approach leads to clear benefits in expected profit, managing up to 5.1% mean profit increases and 39.4% expected shortfall reductions compared to deterministic solutions.

Keywords product line selection; stochastic programming; multinomial logit; multiplicative competitive interaction; lot sizing

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4.1. Introduction

Product line selection (PLS) – weighing sales and complexity costs to choose the profit-maximizing assortment from a set of options – is a critical strategic problem for manufacturers. It is non-trivial since beyond the sales and costs of a single product, interactions, such as customer preferences, sales cannibalization, and shared changeovers, account for a significant gross margin chunk. Throughout almost five decades, researchers have sought to optimize product lines. Monroe et al. (1974) model the effects that products have on others' demand and production costs with explicit pairwise interactions. Several approaches consider first-choice consumer response – that a consumer segment will wholly purchase the most attractive product (Green and Krieger, 1985).

Marketing executives usually decide on which products compose the line. Nevertheless, they must consider introductions' impact on operational complexity and costs. Congruently, later work makes single-choice PLS models more realistic with the introduction of fixed costs (McBride and Zufryden, 1988; Dobson and Kalish, 1988) and the choice between different prices (Shugan et al., 1977). Dobson and Kalish (1993) present heuristics capable of handling PLS with fixed costs and continuous pricing. Further research adds lot-sizing considerations (i.e., setup and inventory costs) (Morgan et al., 2001), and experiments applying genetic algorithms to the increasingly difficult problems (Day and Venkataramanan, 2006). Recently, Bertsimas and Mišić (2019) propose a stronger formulation for the first-choice rule.

Concomitantly, Chen and Hausman's (2000) linearization of the attraction model – where product share is in proportion with its attractiveness – enabled random utility models to penetrate the PLS research stream. These models provide more conservative estimates of segment-leading products' shares than first choice models which overestimate them. Kraus and Yano (2003) propose a formulation with attraction consumer response and continuous pricing. Schön (2010a) introduces fixed costs shared across production resources, and later (Schön, 2010b) adds continuous pricing and capacity constraints. Finally, Bechler et al. (2021) includes compromise effects in attraction consumer response. For a comprehensive review and introduction to PLS, see Chapter 2.

Product introductions considerably impact the manufacturing plan in low-margin high-volume businesses with high changeover costs, such as fast-moving consumer goods (FMCG). Andrade et al. (2020a) determine product line profitability by integrating multiple independent attraction models (Bradley and Terry, 1952; Luce, 1959) with a capacitated lot-sizing problem (CLSP) (Karimi et al., 2003). Attraction models determine how consumers respond to product line changes (assortment-based substitution), and lot sizing estimates the impact of a selection on manufacturing costs. They demonstrate the value of marketing-operations integration in PLS models. Moreover, recent e-commerce and direct-to-consumer trends allow manufacturers to easily access point-of-sale data (McKinsey & Company, 2017), enabling them to estimate how consumers respond to assortment changes. Notwithstanding, a critical issue hinders the adoption of mathematical programming approaches for product line decisions: a failing assortment can damage short-term profits and even the brand.

Managers can be skeptical about deterministic approaches given the high stakes involved in PLS and the firms' almost irreversible commitment to selected products. Indeed, considering a single demand estimate is insufficient, as a wrong forecast can make an unprofitable line seem profitable (and vice-versa). Consumer behavior is also prone to mismodeling. Deterministic approaches exclusively consider the expected-value scenario. Although optimal for that expected situation, they provide solutions that are not the optimal strategy to tackle the array of possibilities arising from demand uncertainty. Therefore, since demand parameters are not known precisely, the uncertainty needs to be incorporated into the model. Bertsimas and Mišić (2016) propose a robust optimization approach for PLS. The approach intends to protect retailers' product line selection decisions from erring in estimating parameters and choosing the wrong consumer response model. Its objective is to mitigate worst-case losses.

Alternatively, stochastic optimization finds the best decisions to make under uncertain parameters, given that their distribution is known. Applications of this technique to assortment planning, which aims to determine the profit-maximizing retail category, are emerging (Goyal et al., 2016; Alqahtani and Shaikh, 2019). Assortment planning is the retail counterpart of the PLS problem, hence neglects manufacturing implications. Our work uses a CLSP with shared setups to model manufacturing costs. The lot-sizing literature (Karimi et al., 2003) has extensively studied the problem under uncertainty (Aloulou et al., 2014). Lot-sizing research considers uncertainty both in demand (Brandimarte, 2006; Tempelmeier and Hilger, 2015; Guan and Liu, 2010) and production (Taş et al., 2019). For our purposes, uncertainty in production would overcomplicate the problem, so we focus on demand uncertainty. We propose a stochastic programming PLS approach since it protects firms from uncertainty without robust optimization's conservatism. By handling a stochastic version of a problem that encompasses both assortment and lot-sizing decisions, our work is relevant to both lines of research.

Building on Andrade et al.'s (2020a) work, this paper introduces a two-stage stochastic formulation for selecting FMCG product lines. Uncertainty is present in both market size predictions and consumer response. The product line is selected in the first stage, and when demand values are revealed, its real costs are computed as a function of the manufacturing plan. Our work should therefore contribute to manufacturer adoption of mathematical programming approaches for PLS. Complexity cost estimates are made by solving a tactical planning problem. This implies that our solution provides a concrete production plan besides the optimal selection, emphasizing its viability. Also, by considering various demand scenarios, our approach reassures manufacturers that the solution resulting from the mathematical program is reliable.

This research has three outstanding contributions to PLS. Firstly, it proposes the first stochastic formulation for PLS, which considers both uncertainty and complexity costs. The formulation is compatible with the most frequent random-utility models (Lau et al., 1997) and satisfies FMCG manufacturer needs for detailed cost accounting. Secondly, we propose a solution approach to address the problem's computational load. The deterministic version of the problem is already NP-hard since it contains the CLSP (Florian and Klein, 1971). Therefore, we illustrate Benders decomposition (Benders, 1962) and derive its respective valid cuts. The decomposition is also effective in solving variations of this problem (particularly regarding production planning modeling), given that substitution is assortment-based. We further apply the forward scenario reduction procedure from Heitsch and Römisch (2003) to enhance the approach's efficiency. Thirdly, our analysis quantitatively shows the benefits of using stochastic formulations over deterministic ones under various consumer response and capacity conditions.

The remainder of this paper is structured as follows. Section 4.2 introduces the two-stage stochastic PLS of FMCG problem. Section 4.3 presents the mixed-integer programming formulation of the deterministic equivalent problem. Our solution strategy consists of decomposing the problem and summarizing the scenario space. Section 4.4 contains the valid cuts and computational performance test results for the decomposition approach. Section 4.5 summarizes the scenario reduction procedure and contrasts it with sample average approximation. Section 4.6 assesses the benefits of the stochastic approach versus the deterministic one under different consumer choice and capacity conditions. Finally, Section 4.7 summarizes this work's the conclusions and managerial insights, and proposes directions for future research.

4.2. Stochastic selection of fast-moving consumer goods

A manufacturer wants to select the profit-maximizing product line $W \subseteq \{1,...,J\}$ for a given planning horizon (e.g., throughout the next year). The product line generates $\mathrm{R}(W,\xi)$ gross profit, and its production requires $\mathrm{C}(W,\xi)$ indirect costs to be incurred for the horizon. Even though the future demand $\xi \in \Xi$ is not known precisely, the performance of the selection depends on it. A selection requires that an amount $\mathrm{F}(W) = \sum_j f_j W_j$ is spent regarding the development and marketing of each included product, with $W_j = 1$ if product $j \in W$ and f_j being its fixed cost. Expression (4.1) presents this problem's objective, subject to the random vector ξ.

$$\max_{W} \quad \mathbb{E}_{\xi}\left[\mathrm{R}(W,\xi) - \mathrm{C}(W,\xi)\right] - \mathrm{F}(W) \tag{4.1}$$

For each product j, gross profit is computed by multiplying its contribution margin p_j by its sales $D_{jt}(W,\xi)$ throughout periods $t \in \{1,...,T\}$. The dependence of sales on the selection is threefold. No sales of j occur if $W_j = 0$. Because of assortment-based substitution, demand for j is a function of W. Lastly, since the production line has limited capacity, the manufacturer may not satisfy demand totally. Substitution and capacity sharing make j's sales dependent on ξ as a whole instead of its demand realization exclusively. Equation (4.2) expresses these relations.

$$\mathrm{R}(W,\xi) = \sum_{t,j} p_j D_{jt}(W,\xi) \tag{4.2}$$

Before detailing capacity limitations, we delve into product demand's contingencies on uncertainty and selection. The plant produces product categories targeting distinct uses. With $\mathscr{N}^k$ being the subset of products up for consideration in category k, a product $j \in \mathscr{N}^k$ only influences demand from others in that independent category. Random variable N_t^k is the demand for products within category k for period t.

The probability that j is purchased, among those within $\mathcal{N}^k$, follows a random utility model (Baltas and Doyle, 2001). It depends on the observable utility V_j that a product provides its consumers and on its unobservable component ε_j. Given some distributional assumptions about ε_j (detailed in Section 4.5), market share follows an attraction model (Cooper, 1993). We also consider V_j as a random variable since its estimates are not trivial. Therefore, $\xi = (N, V)$. Constraints (4.3) state that sales $D_{jt}(W, \xi)$ are, at most, the demand for the product, with $W^k = W \cap \mathcal{N}^k$, with product line attractiveness $A(W^k, V^k) = \sum_{j \in W^k} A_j(V_j)$, and $A_0^k(V_0^k)$ denoting the attractiveness of the competition. The fraction defines share as the ratio between the product and available option attractiveness within a category. Every term in the fraction is contingent on its utility realization.

$$D_{jt}(W, \xi) \leq N_t^k \frac{A_j(V_j)}{A_0^k(V_0^k) + A(W^k, V^k)} \tag{4.3}$$

In sum, product-wise demand is dependent on market size and attraction realizations, with the latter depending on the real observable utility. Production times and costs are deterministic and modeled as a CLSP with shared setups (Karimi et al., 2003). Nevertheless, demand realization ξ affects sales amounts, altering the optimal production plan and, consequently, complexity costs $\mathrm{C}(W, \xi)$.

A concise production plan consists of quantities of each product to produce $X_{jt}(W, \xi)$ and to keep $I_{jt}(W, \xi)$, at each period. To balance inventory, sold and held amounts must equal the production from that period and amounts held from previous ones. Equations (4.4) express this condition.

$$I_{jt-1}(W, \xi) + X_{jt}(W, \xi) = D_{jt}(W, \xi) + I_{jt}(W, \xi) \tag{4.4}$$

Amounts sold are limited both by demand and available capacity. To produce any product, the manufacturer needs to set the production line up for it $Y_{jt}(W, \xi)$ using τ_j of the available time. If the product belongs to family m, that is $j \in \mathcal{N}_m$, family setup $Y_t^m(W, \xi)$ must occur at least once in that period, using τ^m. Producing a unit of j takes r_j time. Equations (4.5) show that capacity at each period c_t is shared between production and setups.

$$\sum_m \left\{ \tau^m Y_t^m(W, \xi) + \sum_{j \in \mathcal{N}_m} [\tau_j Y_{jt}(W, \xi) + r_j X_{jt}(W, \xi)] \right\} \leq c_t \tag{4.5}$$

Since p_j is the contribution margin of j, it already accounts for the direct unitary production costs. The indirect costs, which scale with complexity, consist of inventory and setup costs. Holding a product j unit for one period costs h_j. Moreover, setting up machines for product j induces setup cost s_j, and the respective family changeover cost s^m. Equation (4.6) computes complexity costs of a production plan throughout its horizon as a function of the selection and demand realization.

$$\mathrm{C}(W, \xi) = \sum_{t,m} \left\{ s^m Y_t^m(W, \xi) + \sum_{j \in \mathcal{N}_m} [h_j I_{jt}(W, \xi) + s_j Y_{jt}(W, \xi)] \right\} \tag{4.6}$$

Selecting a product line W is the here-and-now decision of this problem. Fixed costs are directly affected by the selection and are independent of demand realizations – these are the costs of keeping each product in the line. After making a selection, uncertainty is revealed, and the production plan, composed of the D, I, X, and Y recourse decisions, is determined. Since the manufacturer can adjust the production plan to the demand realization, it is crucial to select a product line allowing for viable production plans throughout Ξ.

4.3. Deterministic equivalent

Solving the problem numerically requires a finite number C of scenarios. Each scenario $\xi_c = (n_c, v_c)$, where n_c and v_c are the T by K and J-sized observation matrix and vector of random variables N_t^k and V_j. Additionally, P_c is the probability of ξ_c occurring. If scenario space Ξ is not discrete, or the possible outcomes of a discrete scenario space are not totally considered, this approach becomes an approximation.

$$\mathbb{E}_\xi \left[\mathrm{R}(W,\xi) - \mathrm{C}(W,\xi)\right] = \sum_c \mathrm{P}_c \left[\mathrm{R}(W,\xi_c) - \mathrm{C}(W,\xi_c)\right] \tag{4.7}$$

Equation (4.7) computes the expected value of the difference between gross profit and indirect costs. By substituting its right-hand side, along with equations (4.2) and (4.6), into expression (4.1), we construct the objective function for the deterministic equivalent of our stochastic problem.

$$\min_{WDIXY} \quad \sum_j f_j W_j + \sum_c \mathrm{P}_c \sum_{t,m} \left\{ s^m Y_t^{mc} + \sum_{j \in \mathcal{N}_m} \left[s_j Y_{jt}^c + h_j I_{jt}^c - p_j D_{jt}^c \right] \right\} \tag{4.8}$$

Expression (4.8) represents the profit-maximizing (or loss-minimizing) objective of our problem. The first term accounts for fixed costs, with $W_j = 1$ if $j \in W$. The remainder of the expression regards recourse decisions. We consider family and product setups, inventory costs, and the gross profit sales generate by order of appearance. Recourse problem terms are conditional on their respective scenario c and weigh P_c on the objective.

$$D_{jt}^c \leq d_t^{kc} S_j^c \quad \forall c, t, k, j \in \mathcal{N}^k \tag{4.9}$$

$$S_j^c \leq W_j \quad \forall c, j \tag{4.10}$$

Constraints (4.9) set product demand as an upper limit for sales, with d_t^{kc} being the market size for category k at period t. Note that each d_t^{kc} is an element of n_c. Market shares S_j^c are the fraction of demand each product j captures given demand realization c. Constraints (4.10) state $S_j^c = 0$ for every scenario c if the product is not offered. Together, the constraint sets ensure only selected products are sold.

$$S_0^{kc} + \sum_{j \in \mathcal{N}^k} S_j^c = 1 \quad \forall c, k \tag{4.11}$$

$$a_0^{kc} S_j^c = a_j^c S_0^{kc} - F_j^c \quad \forall c, k, j \in \mathcal{N}^k \tag{4.12}$$

$$F_j^c \leq a_j^c S_0^{kc} \quad \forall c,k,j \in \mathcal{N}^k \tag{4.13}$$

Condition (4.3) requires share to behave like an attraction model. Equations (4.11) certify that, within a category, shares sum to one. The attraction model is defined by constraint sets (4.12) and (4.13). The first set of constraints ensures that the maximum ratio between our products' shares S_J^c and competition's share S_0^{kc} is a_j^c/a_0^{kc}. Variables F_j relax the equality when j is excluded. Since the program benefits from maximizing D_{jt}^c, which $d_t^{kc} S_j^c$ bounds, $F_j^c = 0$ if $j \in W$.

$$I_{jt-1}^c + X_{jt}^c = D_{jt}^c + I_{jt}^c \quad \forall c,t,j \tag{4.14}$$

$$X_{jt}^c \leq Y_{jt}^c K_{jt}^c \quad \forall c,t,j \tag{4.15}$$

$$X_{jt}^c \leq Y_t^{mc} K_{jt}^c \quad \forall c,m,t,j \in \mathcal{N}_m \tag{4.16}$$

$$\sum_m \left\{ \tau^m Y_t^{mc} + \sum_{j \in \mathcal{N}_m} \left[\tau_j Y_{jt}^c + r_j X_{jt}^c \right] \right\} \leq c_t \quad \forall c,t \tag{4.17}$$

Equations (4.14) state, D_{jt}^c amounts must be manufactured in period t or earlier, expressing condition (4.4). Constraints (4.15) and (4.16) enforce the respective product and family setups in productive periods. Constants $K_{jt} \geq \min\left\{ \sum_{t_2} d_{t_2}^{kc} a_j^c/(a_j^c + a_0^{kc});\ (c_t - \tau_j - \tau^m)/r_j \right\}$ $\forall t,j$, with $t_2 \in \{t,...,T\}$ and $(k,m) : j \in \mathcal{N}^k \cap \mathcal{N}_m$ to guarantee no viable solution is excluded. Constraints (4.17) handle limitations on capacity which, as condition (4.5) states, must be shared between production an setups. Finally, constraints (4.18) and (4.19) define the domain of the variables.

$$W_j, Y_{jt}^c, Y_t^{mc} \in \{1,0\} \quad \forall c,m,t,j \in \mathcal{N}_m \tag{4.18}$$

$$D_{jt}^c, S_j^c, S_0^{kc}, F_j^c, X_{jt}^c, I_{jt}^c \in \mathbb{R}_0^+ \quad \forall c,t,k,j \in \mathcal{N}^k \tag{4.19}$$

We remark that constraints (4.9) separate sales from shares, implicating that insufficient capacity to satisfy a product's demand totally does not influence the share of consumers seeking it. Nonetheless, unsatisfied demand is lost. Therefore, assortment-based substitution is defined by constraints (4.11)-(4.13), and no stockout-based substitution takes place. Additionally, lost sales imply that every selection has feasible production and sales plans.

4.4. Decomposition strategy

Our stochastic program's computational difficulty is remarkable. Each additional $\xi_c \in \Xi$ increases the number of binary variables by $T(J+M)$ and the number of constraints by $K+T+J(3+4T)$. This solution space explosion makes the problem intractable even for small instances (e.g., selecting twelve products over six periods, with ten demand scenarios).

Bender's decomposition (Benders, 1962) is a recurring strategy when one attempts to solve large-scale two-stage stochastic programs. The algorithm exploits the problem structure by dividing it into master and subproblems. The master problem usually optimizes first-stage variables, given an approximation of the second-stage objective values. The subproblem assumes the master problem's solution and attempts to optimize the remaining variables. If the resulting problem is infeasible, a feasibility cut (corresponding dual's unbounded ray) is added. If an optimal solution is found, we constrain the objective value approximation with an optimality cut (corresponding to dual's optimal extreme point). Iteratively, the procedure approximates the second-stage objective value polytope as a function of first-stage decisions.

$$\max_{\eta W}\{\eta - \mathrm{F}(W)\colon\, \eta \leq \mathbb{E}_{\xi}\left[\mathrm{R}(W,\xi) - \mathrm{C}(W,\xi)\right]\} \tag{4.20}$$

Expression (4.20) restates our stochastic PLS problem with variable η accommodating the second-stage objective value. In essence, optimality cuts iteratively define the domain of η (i.e., the right-hand side of the constraint, in the expression), and feasibility cuts define the domain of W. Since our model considers lost sales, every binary first-stage solution is second-stage feasible. Therefore, only optimality cuts must be regarded. Also, the decomposition approach must be adapted to the mixed-integer second stage. Conventionally, this is done using Gomori cuts or branch-and-bound (Gade et al., 2014). Since the second-stage problems in our formulation are usually of manageable size, we use branch-and-bound. De-

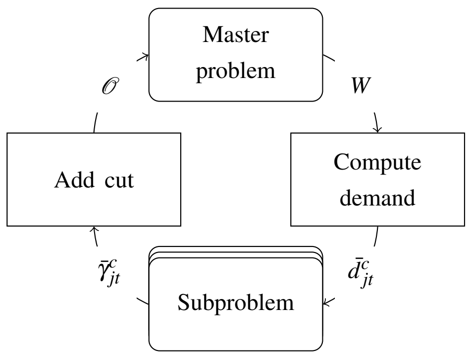

Figure 4.1 – Bender's feedback loop. Rectangles with rounded corners denote optimization steps. The overlapping rectangles represents that each scenario's subproblem is solved standalone.

termining the W which maximizes $\eta - \mathrm{F}(W)$ subject to optimality-cut set $\mathscr{O}$ is the master problem for our formulation. Constraints (4.10)-(4.13) are included to define variables S^c_j and their dependency on W (i.e., the attraction model). Let $\bar{S}^c_j$ denote the parameter corresponding to S^c_j for an iteration's W. Maximizing the objective in Equation (4.7), given product demand values $\bar{d}^c_{jt} = d^{kc}_t \bar{S}^c_j$, is the second stage problem. Constraints (4.14)-(4.17) apply, plus the lost sales condition $D^c_{jt} \leq \bar{d}^c_{jt}$ for all c,t and j. Constraints (4.9), linking the first and second-stage problems, reproduce the condition for a given W. Hence, fixing W makes the second-stage problem partitionable by scenario, easing its computational burden. At each iteration, from the second-stage objective value z^* and lost-sales constraints' dual-variable values $\bar{\gamma}^c_{jt}$ we derive the valid cut:

$$\eta \leq z^* - \sum_{c,t,k} d_t^{kc} \sum_{j \in \mathcal{N}^k} \bar{\gamma}^c_{jt} \left(\bar{S}^c_j - S^c_j \right) \tag{4.21}$$

Figure 4.1 describes the algorithm's feedback loop. The loop initiates with any feasible W. We compute $\bar{d}^c_{jt}$ for that iteration's selection, and solve the respective C subproblems. We determine $z^* = \sum_c \mathrm{P}_c z^*_c$, where z^*_c is the optimal objective value for scenario c's subproblem, and extract the values of dual variables $\bar{\gamma}^c_{jt}$. Cut (4.21), with that iteration's $\bar{\gamma}^c_{jt}$ and $\bar{S}^c_j$, is added to $\mathscr{O}$, further approximating the η's domain. Finally, we resolve the master problem, obtaining a new W and computing its respective $\bar{d}^c_{jt}$. The procedure repeats until $\eta - \mathrm{F}(W)$ and $z^* - \mathrm{F}(W)$ (i.e., the upper and lower bounds) meet. Constraints (4.9) having slack implies the respective $\bar{\gamma}^c_{jt} = 0$. Cut (4.21) then can be loose. Under these conditions, we apply the cardinality-based cut from Laporte and Louveaux (1993).

Table 4.1 – Computational performance test results.

T	J	C	Det. equivalent Gap	Det. equivalent Time (s)	Decomposition Gap	Decomposition Time (s)
8	12	4	0.1%	1792.7	0.0%	8.1
8	12	8	0.3%	1804.9	0.0%	15.6
8	12	12	0.7%	1804.1	0.0%	17.6
8	12	16	2.2%	1804.2	0.0%	24.0
8	12	20	3.4%	1804.0	0.0%	29.8
8	8	12	0.1%	1806.1	0.0%	5.4
8	10	12	0.3%	1629.3	0.0%	9.4
8	14	12	4.3%	1805.3	0.0%	22.8
8	16	12	5.4%	1803.5	0.0%	45.0
4	12	12	0.1%	1810.6	0.0%	13.2
6	12	12	0.5%	1804.2	0.0%	6.9
10	12	12	2.1%	1804.3	0.0%	47.0
12	12	12	4.1%	1810.9	0.0%	310.7

Table 4.1 demonstrates our decomposition's computational advantage over the deterministic equivalent formulation for different problem sizes. We conducted the experiment on four cores of an Intel Xeon E5-2450 processor with 16 GB of memory allocated from a computer cluster running Version 7 of the Scientific Linux distribution. Algorithms are implemented in the C++ programming language using the Concert Technology library to interface with the IBM ILOG CPLEX 12.8 optimizer. We report the ten-instance average gaps and elapsed times for each method and T, J, C combination. Optimization time was limited to thirty minutes. When the optimum is not proved, gaps report the distance to the optimal solution, computed as $\pi^*/\pi - 1$ with π being the best integer profit and π^* the optimal (determined by the decompostition). We have used the tractability test instances from Andrade et al. (2020a) and generated the scenarios in R 4.0.3 (R Core Team, 2020). Both are available online (Andrade, 2021), in this section's folder.

The deterministic equivalent formulation manages low gap values for the smaller problems, with average times approximating the limit. Under these conditions, it frequently reached unproved optimality. Decomposition algorithm times scale linearly with C and exponentially with J and T. Each subproblem is a profit-maximizing CLSP with lost sales, hence NP-hard. Large J and T values make the algorithm unviable by producing intractable subproblems. Nevertheless, production planning does not require large T values since it is merely a costing tool for PLS. Our decomposition dramatically outperforms the commercial solver for the tested instance sizes and can solve twelve-product six-period instances with up to five hundred scenarios in under two hours.

4.5. Scenario space

This section defines the scenario space. Seeing it is high-dimensional we discretize it using Monte Carlo sampling. To reach solution stability with a parsimonious scenario amount we employ a scenario reduction algorithm.

Frequently, FMCG manufacturers find multiplying market size and share predictions to result in better product-wise demand estimates than direct forecasting, since data is usually noisy from promotions and cannibalization. From point-of-sale data, obtainable through its direct-to-consumer channels or Nielsen retail-scanner data, the manufacturers can estimate market sizes N_t^k – normally distributed around $\mathbb{E}\left[N_t^k\right] = d_t^k$ with Var $\left[N_t^k\right]^{1/2} = \sigma_t^k$. Random utility models make bottom-up market share predictions (Baltas and Doyle, 2001). Observable utility V_j is also assumed to be normally distributed around the estimate $\mathbb{E}\left[V_j\right] = v_j$ with Var $\left[V_j\right]^{1/2} = \sigma_j^v$. In practice, these distributions correspond to model adjustment errors. Alternative, the empiric error distributions could be considered. We analyze two specifications widely used throughout marketing science: multinomial logit (MNL) and multiplicative competitive interactions (MCI) (Lau et al., 1997).

Observable utility is contingent on the importance β_a consumers put on attribute a and the level Z_{ja} in which a product j includes that attribute. Imagine j maximizes observable utility V_j within a consumer segment. In random utility models, only part of consumers will purchase j, depending on the distribution of its unobservable component ε_j. For the MNL model, utility $U_j = V_j + \varepsilon_j$, with $V_j = \sum_a \beta_a Z_{ja}$. For the MCI model, $U_j = V_j \varepsilon_j$, with $V_j = \prod_a (Z_{ja})^{\beta_a}$. Assuming ε_j are independent and identically distributed Gumbel random variables, MNL choice probability is an attraction model, with $A_j = \exp V_j$. Similarly, the MCI model choice probability is an attraction model when $\ln \varepsilon_j$ are Gumbel distributed and $A_j = \max\left\{V_j, 0\right\}$ (consumers purchase useful products). Since V_j's residuals are normally distributed, so are $\ln A_j$ for the MNL and A_j for the MCI.

Consequently, Ξ is a multivariate normal space with $J + KT$ dimensions, continuous and with infinite support. Discretization grids compute finite-discrete distribution estimates. Nevertheless, the procedure scales exponentially with $J + KT$ and, unless for exceptionally small problems sizes, is intractable. Since Ξ is high-dimensional, we use Monte-Carlo sampling (Shapiro and Philpott, 2007). However, solving problems with large numbers of scenarios (i.e., well over fifty) is time-consuming and impractical. Therefore, to fully capitalize on the employed scenarios, we apply the fast-forward scenario selection algorithm presented by Heitsch and Römisch (2003). Using a large scenario sample from Ξ and a distance measure, the algorithm selects the most representative scenario from the sample. The process repeats for the unselected scenarios until C are selected. Although optimal choices at each iteration seldom result in an optimal scenario set, the algorithm is computationally convenient.

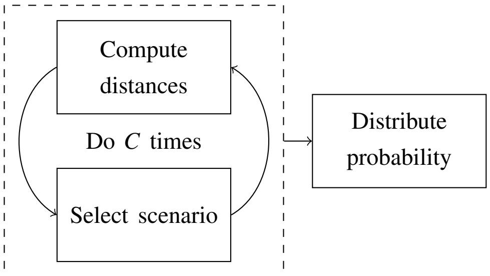

Figure 4.2 – Fast-forward selection algorithm outline.

Figure 4.2 outlines the scenario reduction procedure. We store selected scenarios in $\mathscr{C}$. At initialization, its complement $\bar{\mathscr{C}}$ contains the full sample. We start by calculating the euclidean distances between scenarios.

$$\delta(b,a) = \sqrt{\sum_{j,t} \left(\bar{d}^{a}_{jt} - \bar{d}^{b}_{jt} \right)^2} \tag{4.22}$$

Let $\bar{d}^{c}_{jt}$ be product demands assuming the full line is selected. Equation (4.22) exemplifies computing the distance $\delta(\cdot)$ between scenarios a and b. Then, we select the scenario $s \in \bar{\mathscr{C}}$ that minimizes the weighted distance to the closest between the other scenarios $b \in \bar{\mathscr{C}}$ and the most similar $c \in \mathscr{C}$.

$$s := \arg\min_{a \in \bar{\mathscr{C}}} \left\{ \sum_{b \in \bar{\mathscr{C}}} \mathrm{P}_b \min_{c \in \mathscr{C}} \{\delta(b,a), \delta(b,c)\} \right\} \tag{4.23}$$

Expression (4.23) conveys this condition. Since we randomly draw from Ξ, probabilities P_b are equal for every scenario. After, we update $\mathscr{C} := \mathscr{C} \cup \{s\}$ and $\bar{\mathscr{C}} := \bar{\mathscr{C}} \setminus \{s\}$. Instead of comprehensively computing the distances at each iteration (other than initialization), updating them with the newly added scenario s saves considerable effort. That is, $\delta(b,a) := \min\{\delta(b,a), \delta(b,s)\}$ for each $a, b \in \bar{\mathscr{C}}$ with s being previous iteration's selected scenario. We repeat this process until $|\mathscr{C}| = C$, the desired number of scenarios. Finally, the probability of each scenario in $\bar{\mathscr{C}}$ is absorbed by the closest $c \in \mathscr{C}$.

To evaluate the algorithm's effectiveness, we compare it with the standard sample average approximation. That is, we draw a number C of scenarios, between ten and fifty, random-sampling from Ξ and solve the problem using the previous section's method. This process repeats over thirty random-sampling observations for each Andrade et al.'s (2019) twenty instances, totaling six hundred observations. The results enable us to evaluate the stability of the approximation. A similar process is done concerning scenario reduction.

The difference is that we draw ten thousand scenarios from Ξ and extract C from them using the reduction algorithm. The samples used for the sample average approximation, the scenario reduction results, and the twenty instances are available online (Andrade, 2021), in this section's folder. They consider $A_j = \max\{V_j, 0\}$ (i.e., MCI utility) with coefficient of variation CV$= 20\%$.

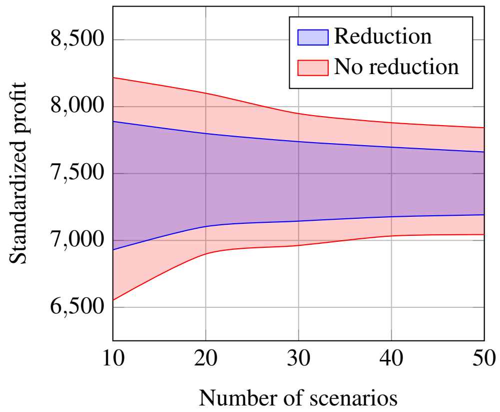

Figure 4.3 – Stability of scenario generation with (inner) and without (outer) scenario reduction. Shaded areas represent 95% credible intervals.

Figure 4.3 displays the 95% credible intervals (i.e., the interval containing 95% of observations), enabling us to visualize algorithm effectiveness from solution stability. Narrower intervals mean stabler – and therefore closer to full-space stochastic solution – solutions. We standardize profit values by instance to make results comparable. For ten scenarios, the reduction algorithm narrows the 95% credible interval from 22.4% of the mean to 12.9%. For fifty scenarios, interval size reduces from 10.8% to 6.3% of the mean profit. For every instance, both approaches reach the same modal selection. Using the law of large numbers, we estimate that an interval size of 5% requires using 231 random-sampling scenarios. Fast-forward selection reduces this number to eighty. Throughout the remainder of the paper, we approximate the occurrence space using fifty scenarios.

4.6. Stochastic solution value

This section evaluates the stochastic approach's benefits. We compare solution performance from the deterministic expected-value model and the stochastic programming model using fifty scenarios (post reduction procedure from ten thousand). With this aim, we generate one thousand demand realizations and solve the respective second-stage problem, assuming each model's selections. That is, we fix selection variables and compute the final objective value. We make this comparison under different random utility models and uncertainty magnitudes (i.e., attractiveness CVs). Scenario sets, scenario reduction results, and solutions are available online (Andrade, 2021). Expected-value cases correspond to Andrade et al.'s instances (2019), generated considering a beverages producer's parameter ranges. We make the generation procedure explicit in the data description file, within the repository. Further, we test how capacity extensions increase the performance delta. For each instance, $\max_t\{\sum_k d_t^k \sum_{j\in\mathcal{N}^k} r_j a_j/(a_0^k + A(W^k))\}$ with $W = \{1,\ldots,J\}$ (i.e., full line's longest production period without setups) computes base-case capacity. Again, we present comparable profit values by standardizing our results by instance.

4.6.1 MCI consumer choice

Our first tests occur under MCI consumer choice, hence $A_j = \max\{V_j, 0\}$. We use two scenario sets, with A_j's CVs of 20% and 40%, being congruent with product market share variability in beverages retail categories. We compare the stochastic approach's solutions to the expected-value selection. On average, the stochastic approach selects lines one to two products longer, depending on capacity. Below, we analyze base-capacity benefits. Those of increasing throughput follow.

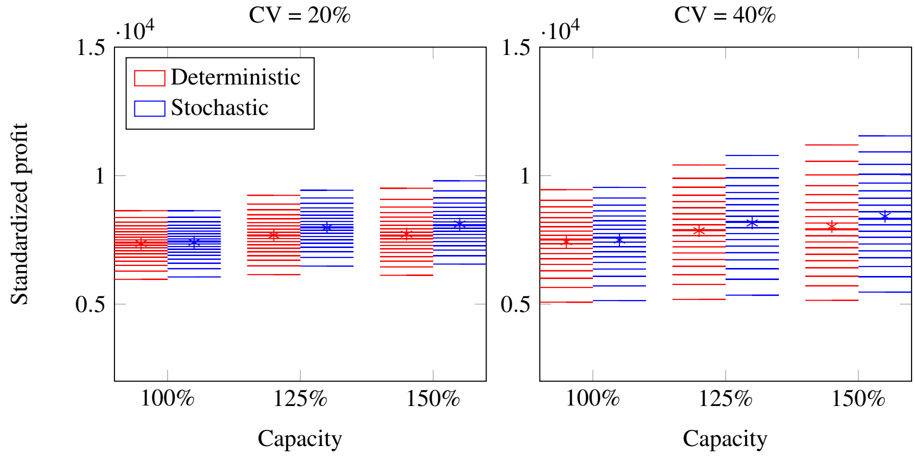

Figure 4.4 – Standardized profit demi-deciles for different capacity levels (maximum and minimum excluded). Thick lines signal medians and asterisks represent means.

Figure 4.4 presents the standardized profit demi-deciles for the deterministic and stochastic approaches, under MCI consumer choice with a CV of 20% (left) and 40% (right). In the first case, mean profit differs by 0.6% for the base capacity. The stochastic selection outperforms the expected-value one for 68.3% of the test set. When the CV is 40% there is a 0.9% profit difference, with stochastic selection being better for 70.0% of realizations. The Z-test scores for the expected value difference (paired samples) between the profit resulting from both approaches are well over thirty for both realization sets, implying p-values approximate zero, thus rejecting the null hypothesis of equal expected values. Therefore, under MCI consumer response, there is strong statistical evidence of the stochastic approach's superiority. Results show the stochastic approach reduces 5% value at risk in 1.5% and 1.2% for scenario sets with CVs of 20% and 40%. The reductions in 5% expected shortfall are of 1.8% and 0.5%, respectively.

Furthermore, the figure shows capacity's influence on standardized profit. A 25% extension increases the mean profit difference to 3.8% for both CV levels. The 5% value at risk reductions (represented by the respective plot's bottom line), versus the expected-value solution, become 5.4% and 3.1% for CVs of 20% and 40%. Likewise, reduction in 5% expected shortfall increase to 5.6% and 2.1%, respectively. Further extending capacity to 150% increases mean profit benefits to 5.1% and 4.9%, respectively. Using the stochastic approach results in 7.1% and 7.8% reductions in 5% value at risk and expected shortfall for an A_j's CV of 20%. With a CV of 40% the reductions become 6.2% and 6.2%. With MCI consumer choice, results are negatively skewed for the base-capacity case, and, with a 50% extension, the skew becomes evidently positive (the asterisk is above the thick line in the plot). In practice, capacity allows the manufacturer to capitalize on favorable realizations more effectively (especially when the market is larger and competition is less attractive than expected).

4.6.2 MNL consumer choice

Our second test concerns MNL consumer choice, with $\ln A_j = V_j$. Therefore, V_j were drawn from a log-normal distribution with variance parameters guaranteeing 20% and 40% CVs and that the median remains unchanged. The stochastic approach selects one product longer lines, on average.

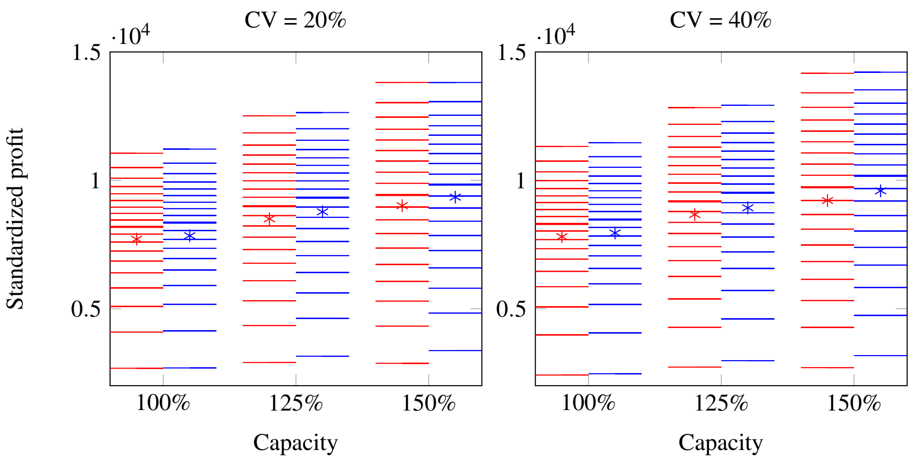

Figure 4.5 – Standardized profit demi-deciles for different capacity levels (maximum and minimum excluded). Thick lines signal medians and asterisks represent means.

Figure 4.5 shows standardized profit demi-decile plots for MNL consumer choice and both CVs. Again, the stochastic approach outperforms the expected-value approach for both coefficient of variance values. For base-case capacity and an attractiveness CV of 20%, mean profit differs by 1.7%, and the stochastic selection is better for 63.0% of the test set. When the CV is 40%, the difference increases to 1.8%, and the stochastic selection is the best performer for 61.9% of the instances. Expected profit difference's Z-test scores are over twenty for both realization sets, implying p-values approximate zero, thus pointing to a clear benefit of the stochastic model. The stochastic solutions manage to reduce 5% value at risk by 0.5% and 2.2% under the MNL model for attractiveness CVs of 20% and

40%. Adversely, the stochastic solutions increase expected shortfall by 1.3% and 0.2%, respectively. Under MNL consumer choice, there is a wider spread of profit and pairwise profit differences. The shape of the log-normal distribution (one-tailed) leads to more considerable disparities in attractiveness ratios and, consequently, market share for the same CV.

The stochastic approach outperforms the expected-value approach for all capacity levels. With 125% of base capacity profit differences increase to 3.2% and 3.0% for 20% and 40% CVs. The reductions in 5% value at risk become 8.3% and 9.3%, respectively. The stochastic solutions manage 14.4% and 15.5% reductions in 5% expected shortfall, instead of the slight increases obtained under base-case capacity. Extending capacity to 150% augments profit benefits to 3.7% and 4.1% for CVs of 20% and 40%. The 5% value at risk reduction improves to 17.4% for both CV levels. Moreover, 5% the stochastic solutions manage 31.0% and 39.4% reductions in expected shortfall, respectively. For all presented capacity levels, the standardized profit distribution is negatively skewed. Even under unlimited capacity, the outcome distribution's skew is negative. Figures 4.6 and 4.7, in the Appendix, show outcome distributions' histograms for base-case capacity.

4.7. Discussion and conclusions

Product line selection is typically a controversial decision for companies. Marketing seeks to increase sales through introductions. On the other hand, operations seek cost reductions through delistings. The problem requires a holistic and integrated decision. The conflicting incentives of these functional silos emphasize the need for mathematical support for the product line decision-making, notably since its repercussions spread throughout the whole value chain. This paper introduces a methodology assisting managers in determining profit-optimizing lines impartially. Modeling demand uncertainty takes some clout from error-prone forecasting and consumer surveys. It increases expected product line profitability and reduces expected shortfall. Hence, the stochastic approach reassures managers of the selection's quality. As a mild limitation, it requires demand and consumer response estimation error distributions to be known or assumed.

Our work proposes the first stochastic programming model for product line selection. Our model considers manufacturing costs with a reasonable level of detail, being of utmost importance to FMCG producers. Also, it accounts for uncertainty in market size and individual product share predictions. Given the decision's complexity, the paper presents a twofold solution approach: decomposing the problem and selecting the right scenarios. The decomposition approach consists of solving the first-stage and second-stage problems separately, for which we derive the valid cuts using a Benders decomposition framework. On the other front, we pick the most representative scenario from a large set of realizations for the desired number of times, and distribute probabilities according to scenario proximity. With computational tests, we demonstrate the effectiveness of both fronts. Then, we show the value of the stochastic solutions under different random utility models and CVs. Table 4.2 summarizes our test results.

The table highlights several relations between consumer response, capacity, and the

Table 4.2 – Results summary. Values indicate average relative differences between stochastic and expected value solution across all instances and observations. Wins indicate the percentage of scenarios where the stochastic outperforms the expected-value solution.

Consumer response		Capacity	Profit	Wins	Value at risk	Expected shortfall	Line size
MCI	CV=20%	100%	0.6%	68.3%	-1.5%	-1.8%	10.1%
		125%	3.8%	93.6%	-5.4%	-5.6%	15.0%
		150%	5.1%	95.8%	-7.1%	-7.8%	27.2%
MCI	CV=40%	100%	0.9%	70.0%	-1.2%	-1.5%	7.4%
		125%	3.8%	86.1%	-3.1%	-2.1%	13.9%
		150%	4.9%	87.1%	-6.2%	-6.4%	23.7%
MNL	CV=20%	100%	1.7%	63.0%	-0.5%	1.3%	5.4%
		125%	3.2%	72.0%	-8.3%	-14.4%	5.2%
		150%	3.7%	64.7%	-17.4%	-31.0%	15.6%
MNL	CV=40%	100%	1.8%	61.9%	-2.2%	0.2%	5.4%
		125%	3.0%	70.3%	-9.3%	-15.5%	10.4%
		150%	4.1%	67.2%	-17.4%	-39.4%	15.0%

stochastic approach's relative performance. Stochastic solutions beat expected-value ones most reliably under the scenario set with the least variable outcome (MCI consumer response with CV= 20%) and loose capacity conditions. Capacity surpluses tend to favor the stochastic approach, and outcome variability seems to attenuate this effect. Our results suggest that variability increases profit differences between stochastic and expected value solutions. On the other hand, it may reduce the number of observations won. Further, MNL consumer response results suggest a non-injective relationship between capacity and wins. Value at risk and expected shortfall reductions are particularly sizeable when capacity is loose under MNL consumer response. The stochastic solution's risk reduction benefits present an increase with capacity, which variability accentuates. Generally, optimal product lines become longer with capacity and shorter with variability. This is congruent with the practices of offering broader lines to hedge against uncertainty in consumer tastes, and leaving a capacity surplus for capturing excess demand.

Under all test conditions, the stochastic approach increases expected profits significantly and outperforms the expected-value approach for 65.8% of all base-case capacity realizations. Besides the profit benefits, its imperious to consider the risk reductions achieved by the stochastic approach. It manages reductions in 5% value at risk of up to 17.4% and up to 39.4% in 5% expected shortfall (MNL demand, CV of 40%, capacity at 150% of the full-line production time for the longest period). Our results show that chance favors the prepared. The stochastic approach beats the expected-value solutions under all tested conditions. Nevertheless, the most sizeable benefits arise from having enough throughput to capitalize on fortuitous realizations fully. We observe this in the negative profit distribution sks under all random utility models and CV combinations for base-case capacity. Increasing throughput under MCI demand leads to a positive profit distribution skew. Additionally, extending capacity under MNL demand reduces the negative profit skew but does not eliminate it (i.e., the outcome distribution for the uncapacitated problem is negatively skewed).

Therefore, the first research direction we propose is to further analyze the interaction between capacity surpluses and uncertainty. Research should price additional throughput, considering that inventory costs and setup times can influence capacity use. Hence, the analysis should isolate the effects of these parameters. The other research direction we propose regards our solution method. The decomposition approach for our formulation can be improved by applying specific CLSP methods to solve the subproblems. More importantly, our solution method is general to PLS problems with production planning subproblems, deterministic and stochastic. Therefore, it paves the way for subproblems to take form from production planning modeling techniques other than lot sizing. Cost estimates that are easier to compute should be explored (e.g., linear), as do more holistic production planning models capable of handling the complexity-caused congestion.

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Bibliography

M. A. Aloulou, A. Dolgui, and M. Y. Kovalyov. A bibliography of non-deterministic lot-sizing models. *International Journal of Production Research*, 52(8):2293–2310, 2014.

K. N. Alqahtani and N. I. Shaikh. A two-stage stochastic programming model for assortment optimisation. *International Journal of Applied Management Science*, 11(3): 185–209, 2019.

X. Andrade. Product line selection in the FMCG industry (instances), 2019. URL 10.6084/m9.figshare.7874639.v1.

X. Andrade. Product line selection of fast-moving consumer goods under stochastic demand: instances, 2021. URL 10.17632/844ktyf9pd.2.

X. Andrade, L. Guimarães, and G. Figueira. Product line selection of fast-moving consumer goods. *Omega*, page 102389, 2020a.

X. Andrade, L. Guimarães, and G. Figueira. Product line selection of fast-moving consumer goods. *Omega*, page 102389, 2020b. ISSN 0305-0483. doi: https://doi.org/10.1016/j.omega.2020.102389. URL https://www.sciencedirect.com/science/article/pii/S030504832030743X.

G. Baltas and P. Doyle. Random utility models in marketing research: a survey. *Journal of Business Research*, 51(2):115–125, 2001.

G. Bechler, C. Steinhardt, J. Mackert, and R. Klein. Product line optimization in the presence of preferences for compromise alternatives. *European Journal of Operational Research*, 288(3):902–917, 2021.

J. F. Benders. Partitioning procedures for solving mixed-variables programming problems. *Numerische mathematik*, 4(1):238–252, 1962.

D. Bertsimas and V. V. Mišić. Robust product line design. *Operations Research*, 65(1):19–37, 2016.

D. Bertsimas and V. V. Mišić. Exact first-choice product line optimization. *Operations Research*, 67(3):651–670, 2019.

R. A. Bradley and M. E. Terry. Rank analysis of incomplete block designs: I. the method of paired comparisons. *Biometrika*, 39(3/4):324–345, 1952.

P. Brandimarte. Multi-item capacitated lot-sizing with demand uncertainty. *International Journal of Production Research*, 44(15):2997–3022, 2006.

K. D. Chen and W. H. Hausman. Mathematical properties of the optimal product line selection problem using choice-based conjoint analysis. *Management Science*, 46(2):327–332, 2000.

L. G. Cooper. Market-share models. *Handbooks in operations research and management science*, 5:259–314, 1993.

J. M. Day and M. Venkataramanan. Profitability in product line pricing and composition with manufacturing commonalities. *European Journal of Operational Research*, 175(3):1782–1797, 2006.

G. Dobson and S. Kalish. Positioning and pricing a product line. *Marketing Science*, 7(2):107–125, 1988.

G. Dobson and S. Kalish. Heuristics for pricing and positioning a product-line using conjoint and cost data. *Management Science*, 39(2):160–175, 1993.

M. Florian and M. Klein. Deterministic production planning with concave costs and capacity constraints. *Management Science*, 18(1):12–20, 1971.

D. Gade, S. Küçükyavuz, and S. Sen. Decomposition algorithms with parametric gomory cuts for two-stage stochastic integer programs. *Mathematical Programming*, 144(1):39–64, 2014.

V. Goyal, R. Levi, and D. Segev. Near-optimal algorithms for the assortment planning problem under dynamic substitution and stochastic demand. *Operations Research*, 64(1):219–235, 2016.

P. E. Green and A. M. Krieger. Models and heuristics for product line selection. *Marketing Science*, 4(1):1–19, 1985.

Y. Guan and T. Liu. Stochastic lot-sizing problem with inventory-bounds and constant order-capacities. *European Journal of Operational Research*, 207(3):1398–1409, 2010.

H. Heitsch and W. Römisch. Scenario reduction algorithms in stochastic programming. *Computational optimization and applications*, 24(2):187–206, 2003.

B. Karimi, S. F. Ghomi, and J. Wilson. The capacitated lot sizing problem: a review of models and algorithms. *Omega*, 31(5):365–378, 2003.

U. G. Kraus and C. A. Yano. Product line selection and pricing under a share-of-surplus choice model. *European Journal of Operational Research*, 150(3):653–671, 2003.

G. Laporte and F. V. Louveaux. The integer l-shaped method for stochastic integer programs with complete recourse. *Operations research letters*, 13(3):133–142, 1993.

K. Lau, A. Kagan, and G. Post. Market share modeling within a switching regression framework. *Omega*, 25(3):345–353, 1997.

R. D. Luce. Individual choice behavior. 1959.

R. D. McBride and F. S. Zufryden. An integer programming approach to the optimal product line selection problem. *Marketing Science*, 7(2):126–140, 1988.

McKinsey & Company. Should CPG manufacturers go direct to consumer – and, if so, how?, 2017.
https://www.mckinsey.com/industries/consumer-packaged-goods.

K. B. Monroe, S. Sunder, W. A. Wells, and A. Zoltners. *A Multi-period Integer Programming Approach to the Product Mix Problems*. Columbia University, Graduate School of Business, 1974.

L. O. Morgan, R. L. Daniels, and P. Kouvelis. Marketing/manufacturing trade-offs in product line management. *Iie Transactions*, 33(11):949–962, 2001.

R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2020. URL https://www.R-project.org/.

C. Schön. On the optimal product line selection problem with price discrimination. *Management Science*, 56(5):896–902, 2010a.

C. Schön. On the product line selection problem under attraction choice models of consumer behavior. *European Journal of Operational Research*, 206(1):260–264, 2010b.

A. Shapiro and A. Philpott. A tutorial on stochastic programming. *Manuscript. Available at www2.isye.gatech.edu/ashapiro/publications.html*, 17, 2007.

S. M. Shugan, V. Balachandran, et al. A mathematical programming model for optimal product line structuring. Discussion paper No. 265, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1977.

D. Taş, M. Gendreau, O. Jabali, and R. Jans. A capacitated lot sizing problem with stochastic setup times and overtime. *European Journal of Operational Research*, 273(1):146–159, 2019.

H. Tempelmeier and T. Hilger. Linear programming models for a stochastic dynamic capacitated lot sizing problem. *Computers & Operations Research*, 59:119–125, 2015.

Appendix

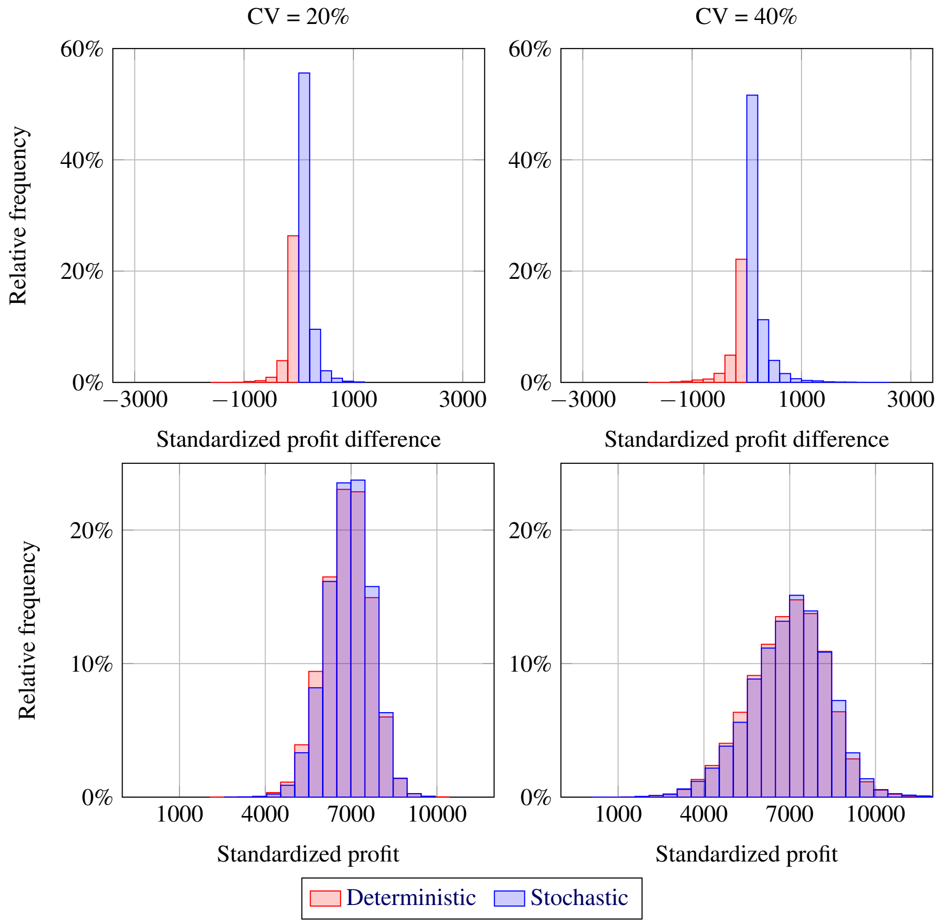

Figure 4.6 – Pairwise standardized profit difference (top) and standardized profit (bottom) histograms for MCI consumer response under base-case capacity.

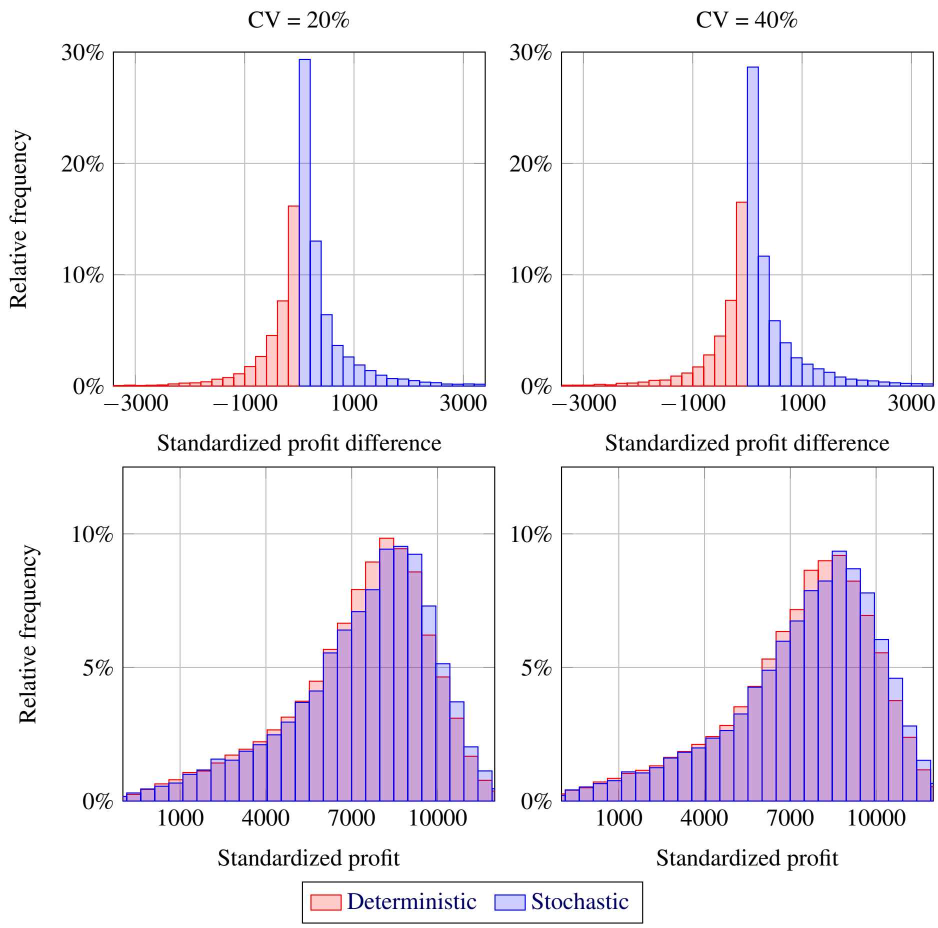

Figure 4.7 – Pairwise standardized profit difference (top) and standardized profit (bottom) histograms for MNL consumer response under base-case capacity.

CHAPTER 5
Conclusions and future work

This dissertation investigates product line selection theory and practice. Realizing the selection process can result in marketing-operations conflict, we suggest aligning the departments with mathematical optimization. We improve current models with better complexity cost estimates by integrating production planning decisions. To hedge against forecasting and parameterization errors, we introduce a stochastic approach for selecting FMCG product lines. Our cumulative work convincingly answers this dissertation's fundamental question: *How should FMCG manufacturers manage their product lines?* Below, we summarize each chapter's contributions.

In Chapter 2 we motivate computerized decision support to interface marketing and operations since their objectives regarding selection decisions conflict. Our literature review identifies the modeling requirements needed to handle the sector's specificities. Particularly, we assert that approaches must consider setup and inventory costs since these are significant for low-margin high-volume production. Additionally, we acknowledge that uncertainty modeling is valued by manufacturers and lacking in product line selection research. Our research resulted in the following research paper:

> X. Andrade, L. Guimarães, and G. Figueira. Product line selection: opportunities in the fast-moving consumer goods sector. *Submmited to International Journal of Production Economics*, 2021.

Besides, our paper makes a didactic problem introduction and contextualization. It provides the first comprehensive review dedicated to product line selection, providing a unified problem definition and organizing the research field. This dissertation's fourth chapter tackles the gap for uncertainty modeling. Hopefully, our review bolsters interest in product line selection research, and, in the years to come, researchers explore the opportunities we have not addressed. This chapter's last paragraphs reexamine these opportunities.

In Chapter 3 we introduce a product line selection formulation considering the sector's modeling requirements. Our multi-period approach outperforms the state-of-the-art by 9.4% efficacy in a real case. Our results show that myopic heuristics, such as removing the worst performer, lead to suboptimal selections, having no performance guarantee. We introduce a safety stock modeling extension, incorporating the selection's effect on demand uncertainty and, with sensitivity tests, identify the conditions that favor our formulation. The following research paper describes our work:

> X. Andrade, L. Guimarães, and G. Figueira. Product line selection of fast-moving consumer goods. *Omega, Volume 102, Article 102389*, 2021.

Specifically, we integrate the attraction consumer response model, fitting FMCGs' low-involvement purchases with capacitated lot-sizing, to estimate inventory levels and changeovers.

The model accommodates multiple independent categories and family setups. We find that using multi-period capacitated production planning to determine complexity costs leads to better FMCG selections than the previous single-period approaches. Overall, integrating consumer response with production planning models opens up new possibilities for operations researchers.

Chapter 4 proposes a stochastic product line selection approach, exploring the most prominent research direction our review identifies. We consider stochastic market sizes and product utilities. Also, we consider that attractiveness can be computed assuming either a multiplicative competitive interactions model or a multinomial logit define consumer choice. Our approach managed mean profit improvements of up to 5.1% versus the expected-value model and expected shortfall reductions of up to 39.4%. The research culminated in the following research paper:

> X. Andrade, L. Guimarães, and G. Figueira. Product line selection of fast-moving consumer goods under stochastic demand. *Submmited to European Journal of Operational Research*, 2021.

By recommending solutions considering a multitude of scenarios, the formulation addresses practitioner skepticism about computerized decision support. The complexity of such formulation required us to develop a solution approach consisting of decomposing the problem and applying a scenario reduction procedure. The decomposition method drastically improves the computational times for problems with relatively simple production planning. Still, the size of the production planning problem limits model performance. This limitation should be addressed by inquiring into alternative production planning modeling strategies.

This dissertation lays the foundation for selecting FMCG product lines through mathematical optimization approaches. Its societal welfare consists of matching the manufacturer's offer with consumer preferences. We frequently observed that FMCG product lines are overextended. Cutting their size can extend profits, reduce production time wasted on changeovers, and improve forecasting. Producers will share these benefits with consumers either voluntarily or through competitive pressures. Furthermore, it reduces the cognitive overload consumers suffer when choosing their purchases. Having lean product lines reduces production-demand mismatch, among other wastes, ultimately raising the system's welfare. The proposed models are tailored to capital-intensive low-margin manufacturing and require product attraction to be known. Therefore, they must be preferably applied within such scope.

Our review chapter shows interesting and pertinent research questions regarding this topic, suggesting much research is still needed. We emphasize modeling variety-induced plant congestion, studying selections' long-term implications, and integrating timing decisions as particularly interesting. We consider operations through a particular capacitated lot-sizing problem – a leap in complexity cost accounting accuracy for product line selection. Nevertheless, the problem overlooks the relationship between batch sizes, work-in-process inventory and lead time, among others depending on the production system. Consequently, our approach neglects congestion effects and plant responsiveness when finding a product line candidate. Queue theory can address these shortcomings. Likewise, our stochastic formulation's study brings up a compelling question: *How much capacity surplus is needed to capitalize on luck?* We found capacity limitations to curtail the benefits from favorable demand realizations. Future research should weigh capacity costs against the benefits of preparedness.

The short-term market share reduction (and occasionally profit increase) that follows delistings is a key reason for the overextension of the current general product line. In fact, it may lead to negotiating power losses, freeing up supplier capacity and shelf space to competitors. Additionally, competitor responses can nullify the advantages our delistings earned. From these issues, three lines of research become evident. The first regards ways of forearming selections against competitor response. It should also determine introduction and removal timings that mitigate cannibalization and vulnerability to competitive action. The second line respects maintaining supplier (and retailer) proximity. Market share reductions imply power losses. Still, that must be weighed against other benefits such as purchase order predictability (potentially narrower raw material needs and improved forecasting of reuced line products). The third line of research should attempt to maximize a market's societal benefit through product line selection. We have shown how product introductions can result in a prisoner's dilemma. Given firms' natural inertia to leave their local optima, increasing the overall system's welfare is particularly interesting for cartels and regulators. Therefore, such research must cover policies and incentives for assortment complexity reduction.

We finish this dissertation highlighting the importance of transferring knowledge to practitioners. We hope our case-study producers find it helpful in solving their product line selection problems. We also provide our insight on the future of operations research practice. Throughout this thesis, we reiterate market share appropriation as the primary product line breadth motivator. However, myopic utilization-based production management also incites proliferation. It incentivizes introducing products until there is no capacity surplus. With this in mind, the attentive reader should notice the fractal nature of the cooperation benefits resulting from our dissertation's line of thinking. Our research shows how marketing and operations must cooperate to avoid locally optimum selections. It shows that production must be managed for throughput, instead of resource utilization. Finally, as future research, we propose establishing policies for brand coordination to mend the overall variety problems markets face. Hence, our perspective is that operations research will turn to farsighted optimization: coordinating the addends of the various stakeholders a decision involves and becoming an effective tool for supporting high-level decisions.