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EFECTS OF NEGATIVE INTEREST RATES IN FIRMS NET WORTH DISTRIBUTION: AN AGENT-BASED MODEL APPROACH

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Abstract: This study intends to investigate, mainly, if unconventional monetary policies had an unequal effect on income distribution across households. In order to do so we will use an Agent-Based Model to simulate real data in order to understand the influence of the monetary policy in inequality. The key advantage of an Agent-Based Model is that it permits to have heterogeneous agents and it does not require rationality to describe the monetary system. Agent-based models are one of the most well-known methods used in Computational Economics. Initially, in its first applications, agent-based models were mainly concerned with Microeconomics but now they are being applied more and more to Macroeconomics, e.g. the study of monetary policy. These agent-based models can create artificial societies and economies of interacting heterogeneous agents and are normally associated with the study of complexity and complex systems, with applications ranging from Physics, Neurosciences and Biology to Economics or Political Science. We implement an Agent-Based Model, that simulates economies where two industrial sectors, one of capital-goods and the other of final consumption goods, where firms are heterogenous and interact between themselves and with a bank to produce goods, invest and satisfy the demand created by workers. In this context, we observe how this model can reproduce stylized facts and the impact of negative interest rates on firms net worth.

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Contents

List of Figures

List of Tables

1. Introduction

This study intends to investigate, mainly, if unconventional monetary policies have an unequal effect on income distribution. In order to do so we will use an Agent-Based Model (ABM from now on) to simulate and compare with real data in order to understand the influence of the adopted unconventional monetary policy on income distribution. The key advantage of an ABM is that it permits to have heterogeneous agents and it does not require rationality to describe the monetary system. Agent-based models are one of the most well-known methods used in Computational Economics. Initially, in its first applications, agent-based models were mainly concerned with Microeconomics but now they are being applied, more and more, to Macroeconomics, e.g. the study of monetary policy. These agent-based models can create artificial societies and economies of interacting heterogeneous agents and are normally associated with the study of complexity and complex systems, with applications ranging from Physics, Neurosciences and Biology to Economics or Political Science. We implement an Agent-Based Model, that simulates economies where two industrial sectors, one of capital-goods and the other of final consumption goods, where firms are heterogenous and interact between themselves and with a bank to produce goods, invest and satisfy the demand created by workers. In this context, we observe how this model can reproduce stylized facts and the impact of negative interest rates on firms net worth.

1.1. What's the deal with inequality?

One could indeed ask 'Why inequality is such a problem?'. If the answer is one such 'the poorest don't have enough to satisfy their basic needs', than the real problem is an absolute one, poverty. But if we added, 'while the top richest persons in the world spend money on things they don't even know they need or care about' or simply 'while some individuals accumulated more wealth than some countries with millions of inhabitants are capable to produce in one year', then, is a relative problem, that is, a problem when compared to the richest. It would be a question of justice if existed fraud/crime in the obtainment of that wealth. Not a matter of justice, in the sense of giving to a person what is theirs (which is different than giving what one has a right to) as it seems to be matter a of broader consent that it would be "good" that some part of higher incomes and earnings should be redistributed to the lowest percentiles of income or wealth by the state in the form of transfers and public services that would even provide some people with conditions to ascend in the social ladder and create wealth and jobs, that would again benefit the poorest.

However, admitting a scenario characterized by high economic inequality but where the poorest percentiles have more than their necessities guarantied, it would still be a problem? If yes, then why? It seems that the reasons, for which inequality is a problem, are more profound than what we normally hear. Perhaps because of social and political instability (Alesina, & Perotti, 1996). If the public, think "the game is unfair", if there is a notion that corruption is the rule or simply because psychologically people feel envy and want to enjoy a standard of living more similar of the upper deciles, the public will demand political change and if political change does not bring about it, perhaps the probability of social tensions increases. That is why, social justice it's almost a synonym for fighting inequality, whatever kind of inequality we are dealing with. It's always useful to remember why some phenomena is relevant and trace back to its theoretical roots and see the big picture. But this, it's up to other scientists to investigate. So it becomes self-evident that if public independent authorities as Central Banks are, conducts policies that increases inequality, it would be relevant to know.

Inequality is a well and long theme discussed in academia, we could even say, one of the most discussed in economics and in general in the social sciences. The literature associates inequality with a series of problems, such as being harmful to economic growth (Persson & Tabellini, 1994) or social problems (Alesina, & Perotti, 1996), for example, increases in criminal reports such homicide rates in a most extreme situation like Brazil (Menezes et al., 2013) and even political implications where indicators of inequality are high. In terms of wealth, which we can look at, as stock of income or others earnings from the past periods, inequality has consistently rising up (Piketty, 2015) to a point where the richest decile of adults is estimated to own 70.7% of global wealth while the poorest half of the adult population owns 3.7% of global wealth (Davies et al., 2011). And of course, the theme brings attention to Europe as well in a time of expected economic crisis.

1.2. About Monetary Policy

The economic motivation for studying this theme has many faces. Perhaps the most unsuspicious one is the relevance and the contemporaneity of unconventional monetary policy. Not neglecting the short history of the European Central Bank (ECB), relatively to other central banks, the truth is that the first time ECB was forced to intervene in a financial crisis adopting unconventional measures, celebrated a decade since it begun.

The ECB debuted what is now called "non-standard" or "unconventional" measures, already in the beginning of the financial crisis, for example, with longer-term refinancing operations (LTRO's) that had maturities bigger than the usual limit of 3 months, operations that eventually extended up to 36 months and main refinancing operations (MRO's) with full allotment (Cour-Thimann & Winkler, 2013). In June 11th of 2014, ECB annunciated for the first time a negative key interest rate, the "offered remuneration" of its deposit facility was lowered -0.1%. In the same year, started the Asset Purchase Program (APP), which is being associated with ECB's quantitative easing program, was suspended in the end of 2018, merely maintaining the cumulative net purchases and already restarted again in November 2019.

Therefore, the first long term economic and social consequences are now beginning to take form with data from the last years, that permit social scientists to evaluate the main and consistent effects of UMP in the long run. And, as we can see this can be corroborated by the year of publication of some of the literature and the presence of working papers that are cited in this document are immediate to this theme are recent articles specially with respect to Euro Area (EA), United Kingdom (UK) and United States of America (USA).

Because of covid-19 pandemic, in March 2020, ECB decided to amplify APP and together launched the Pandemic Emergence Purchase Program (PEPP) assuring proper monetary expansion to relieve the negative pressures of low economic activity resulted from lockdowns. Since BCE can only utilize UMP to react against the consequences of the pandemic, is very convenient what are effects that can arise from using QE or the prolonged maintenance of interest rates close to zero especially in inequality, a theme that recently has regained attention from part of economists and other social scientists and relevance from part of the public.

There are many articles studying the impact of monetary policy in inequality but few regarding the specificities of unconventional measures of monetary policy. The contribution of this work to the literature is to focus only in the effects that unconventional monetary policies have on inequality. To see if quantitative easing and negative interest rates have harmful distributional effects in income distribution across the Eurozone.

This article is divided in four sections: this very same section 1., a general introduction of inequality, monetary policy and of the tools we could have used to construct an Agent-Based Model; followed by section 2, which presents the historical context and economic framework in which the research question is inserted; section 3, is where the methodology is exposed, the model description, the differences and difficulties in the implementation of the model; in the section 4, we show that our implementation even with differences preserves most of stylized facts and the answers to our questions; in section 5, we do a brief conclusion about the strategy of using Agent-Based Models and about our main findings; in section 6, we describe the model following the latest update of the ODD protocol.

2. Literature review

Traditionally, the goals of ECB with respect to monetary policy are: to maintain price stability with an inflation rate close but below 2% over the medium term and help governments to keep low levels of unemployment. But with the advent of the European sovereign debt crisis caused by the 2007 financial crash and the consequential entering of the majority of advanced economies in the Great Recession, monetary policy was used also to assure financial stability of a banking system that was near collapsing in 2012.

In recent statements Christine Lagarde, the current President of the European Central Bank, intends to take in account new issues such as "inequality and climate change" (Lagarde, 2020). Hence, it will be very relevant, even for future determinations of monetary policy and in a environment of zero lower bound (ZLB) restrictions, what are the consequences of UMP in the distribution of income.

2.1. Monetary policy

UMP will be used to refer to all or a group of the following policies: asset purchase programs QE, negative, close to zero, interest rates and forward guidance. Mainly the first two. Despite the fact, that the subject of this article is assess the impact of non-standard monetary policy in inequality of income in the EA, we found that it can be very helpful, to somewhat contrast the consequences in income inequality with wealth inequality and in EA with other countries, the impacts of UMP with conventional monetary policy to provide a greater definition of each one.

It is inevitable to talk about UMP without referring Japan's case, since it was the first country to implement a QE program, in 2001. In a first attempt to access the effects of UMP, Saiki & Frost (2014) found out that Bank of Japan's (BoJ) quantitative easing program widened income inequality via portfolio channel since are those households with higher income and savings invested in financial markets which tend to be most benefited by QE via the positive effect on asset prices. The authors also suggest that in the other advanced economies such as the Eurozone, these effects might be even bigger due to holdings of higher risk assets. In Saiki & Frost (2020), the article confirms the last conclusion and adds that UMP benefits may be constrained to the older and richer parts of the population, letting working-age groups with harmful consequences of monetary policy. Still, in the case of Japan, the results of Israel and Latsos (2020) contradict in some way those of Saiki and Frost (2014, 2020) and found evidence of the impact of UMP in income inequality between high-skilled and low-skilled workers but no significant impact in income inequality between age groups. In addition, the results show that expansionary monetary policy widens education pay gap and insignificant role in diminishing the gender pay gap, which, is mostly explained by improved education and more hours worked by women. Inui, Sudo, and Yamada (2017) find in contrast with both, evidences for increasing inequality before 2000 and after no statistical significance and reveal that job market flexibility is crucial for monetary policy do not have perverse effects in earnings inequality.

In Hafemann, Rudel, & Schmidt (2018), when dealing with the impact expansionary monetary shocks in the Gini coefficients of gross income, all countries had an increase in its measure of inequality. However, when dealing with net income, countries with lower redistribution mechanisms have notable responses in Gini coefficient, that is to say, that increase income dispersion, while in countries with higher redistribution policies does not verify significant changes in inequality of net income. This led the authors to conclude that the effects of monetary policy on inequality should be addressed through redistributive policies.

The a Heterogeneous Agent New Keynesian (HANK) model of Kaplan, Moll, and Violante (2018) addresses precisely that problem because the indirect effects of monetary policy, through the intertemporal substitution channel outweighs the direct effects. The inverse would occur in Representative Agent New Keynesian (RANK) model. The main motivation in utilizing a ABM was the possibility of incorporating heterogeneous and irrational agents and incomplete markets to satisfy two of limitations of DSGE (Dynamic Stochastic General Equilibrium) models (Dawid & Delli Gatti, 2018).

For the UK, Pugh, Bunn, and Yeates (2018) considering the period 2008-2014, state that overall, non-standard monetary policy had very small effects in income and wealth inequality. If for one side, younger households gained with lower interest rates and older households because they are net savers, lose in savings redistribution, for other side older households have seen asset prices increase in relation to 2007, not including housing prices while younger households had decreased real incomes during recession. One important conclusion they argue is that it is important to truly, evaluate the effects of monetary policy, have also, a holistic approach to transmission mechanism instead of only consider one separately. Evgenidis and Fasianos (2020) aimed with recent data, oppose the previous conclusions regarding wealth inequality and find that UMP cause more aggressive consequences on wealth distribution via the portfolio composition channel and confirm the conclusions of Domanski and Zabai (2016).

Analyzing the portfolio composition channel, Albert, Gómez-Fernández, and Ochando (2019) in the USA and EA, the authors ascertain that UMP enhanced income and wealth inequality in the USA but not in the EA, considering only this channel. But that can be explained by, in the EA can be smaller portfolio diversification and participation in financial markets by households. Perhaps, if the analysis contain more distributional channels results would be different for EA.

Casiraghi, Gaiotti, Rodano, and Secchi, (2018) use three main distributional channels: earnings composition, savings remuneration and asset price channels to measure UMP. Their assessment, which confirms the hypothesis of Bernanke (2015), is that expansionary monetary policy, in this case unconventional, do not have "reverse Robin Hood effect", since households at the beginning of income distribution benefit more from the economic boost, job creation and wage increments than high-income households benefit from booming financial asset prices. Hence, because the response is U-shaped, middle-income households are leaved unchanged, as the overall impact in is negligible. Yet, an advertence must be given, they analyze only short-term effects, once the period they cover is 2011-2013. Also, they argue that savers are not hurt by the decrease in interest rates because they are compensated by the hike in financial asset prices, but this can misleading as there are much more savers (middle-income) than those who invest in financial assets (high-income), which are a minority.

Coibion, Gorodnichenko, Kueng, and Silvia (2017) using three different indicators of economic inequality (Gini coefficient, standard deviation, and the difference between the 90^{th} and 10^{th} percentiles of log levels), find that contractionary (when it's not referred will be considered as conventional) monetary policy shocks in the USA, result in more inequality across households in income, labor earnings, consumption and total expenditures. However, these monetary shocks do not significantly explain the increase in income inequality verified in the period 1980-2008, despite being significant for other inequalities. They identify two main distributional channels of monetary policy that impact economic inequality:

composition of household incomes and unexpected inflation and interest rate changes. Areosa and Areosa (2016) share the consequences of contractionary monetary policy in inequality while Davtyan (2016) found that contractionary monetary policy actually decreases income inequality in the USA.

In Dosi et al. (2013), the authors consider the markup over costs an indicator of functional income inequality in the sense that separates income of firms between profits and wages and that empirically are a proxy for income inequality for households.

2.2. Distributional channels

Colciago, Samarina, and de Haan (2019), after an extensive analysis, understands that effects of both conventional monetary policy and UMP on inequality are probably mixed. But there are some conclusions that seem to be more or less consensual such as, that, when considering conventional monetary policy, higher inflation tends to increase inequality and that UMP, in one hand, can generate economic activity reducing income inequality but by other hand, that effect can be eliminated by growth of financial asset prices and hence, increasing income inequality. Moreover, they assemble some of the relevant distributional channels, which is very useful our research. In case of monetary easing:

- Savings Redistribution Channel: money becomes cheaper for borrowers while savers (or lenders), have a decrease in the return of their liquid assets and vice versa in case of monetary tightening. In other words, there is a transmission of expected wealth, from savers to borrowers;

- Unexpected Inflation Channel: (or as Auclert (2019) puts it, the Fisher channel) all kind of debts that aren't indexed to inflation such as nominal ones causes a depreciation of debt in real terms. Creditors involuntarily (it is believed) transfer wealth to debtors;

- Interest Rate Exposure Channel. Normally, a fall in real interest rates increases financial asset prices. But, Auclert (2019) uses another measure to assess the real impact in households of real interest rates. The difference between all maturing assets and liabilities or unhedged interest rates exposure (URE), is perhaps a better way of measuring the exposure to interest rates. The heterogeneity happens between households and not by each type of relation with financial type. This matters because one household can detain more than one type of financial assets, or being creditor and borrower at the same time, and the overall impact in those households could not be measured if we are measuring only one type of asset or liability separately. A drop in the real interest rate benefits households with positive URE's at the expense of households with negative exposure;

- Portfolio Composition Channel. The portfolio composition of households can origin transfers of wealth from different asset holders to others, due to interest rate changes (Coibion et al., 2017; Inui et al., 2017). In case of decrease in interest rates, equity prices who would be relevant to high-income households, tend to increase generating capital gains but in the other hand, real estate prices, who would be more relevant to households with house ownership, would also increase tend to increase and this would possibly cancel the first effect but only if house ownership is generalized, once high income households can own houses, possibly more houses than middle or low income households. The overall difference will have to consider the weight of each asset in the portfolio.

- Earnings Heterogeneity Channel. (Heathcote et al., 2010)(Amaral, 2017). Expansionary monetary policy can have effects that increase inequality in earnings lowering the share of income respected to labor for low-skilled workers and shock increases earnings inequality by lowering the labor share of income for low-skilled workers and raising it for high-skilled workers (Dolado et al., 2021).

- Income Composition Channel. Households can receive their incomes from different sources, may them be from labor, from transfers, from their businesses or from capital gains and each of which may respond differently to changes in monetary policy. Typically, low-income households tend to depend more on transfers, while middle-income households mainly rely on labor income and those at the end of the income distribution rely relatively more on business and capital income (Amaral, 2017). If a fall in interest rates stimulates economic activity, expansionary monetary policy may result in higher wages and lower unemployment, thereby increasing inequality at the lower end of the distribution. On the other hand, lower interest rates decrease interest income, which may reduce inequality at the top of the distribution.

In Ampudia et al. (2018) those distributional channels through which monetary policy affects are divided according to nature of the effects they produce, direct and indirect effects. Savings redistribution channel, unexpected inflation channel, interest rate exposure channel and portfolio composition channel are considered direct effects because they are immediate to the monetary policy change and have no intermediate effect in other variables to affect households. Indirect effects are those who affect first other variables like wages and princes as the earnings heterogeneity channel and income composition channel. In addition to this, their results show, they argue, that both conventional and unconventional monetary policy have equalizing effects both on income and wealth distribution. An important conclusion they make is that indirect effects are most impactful than direct effects, because indirect effects the economy through more interactions and possibly during much more time and that seems more realistic although more difficult to prove. We think that this idea could question the hypothesis that the effects of monetary policy are temporary.

2.3. Agent-Based Models

The reason for which we choose to do an Agent-Based Model has to do with the possibility of doing something that normally we can not do in social sciences, that is experimental studies. The kind of experimental economics we are able to do depends almost solely on our capacity to know the attributes of the different agents, of establishing the due relations between them and organizing them. As "creators" of artificial economies we get what we put.

The economic motivation of studying inequality fits one the advantages of dealing with Agent-Based Models, that is the possibility do it repeatedly, in different conditions with the hope of tracing back to its origins and causes.

Since the first time economists seriously became to look at economies as Complex Adaptive Systems (CAS) in the 90's, there have been a series of developments in different streams of ABM's applied to macroeconomics with microeconomic foundations (Dawid & Delli Gatti, 2018). This feature became relevant since the burst of the Great Depression, as DSGE models were accused of being incapable of predicting such crisis (Colciago et al., 2019). One of those streams is the K+S model, which already was the laboratory for the study of different kinds of policies, although normally related to the labor market (G. Dosi et al., 2017, 2018, 2019, 2020; Giovanni Dosi et al., 2017; Giovanni Dosi & Roventini, 2019). The K+S refers to Keynes and Schumpeter respectively and the first of the kind was Dosi, Fagiolo and Roventini (2010). In the years before the last, there were already attempts of those authors (Giovanni Dosi et al., 2006, 2008) to the level of tuning of the 2010's model.

The baseline model we used, in its essence is the object of 15 years of an exhaustive analysis, covering many themes and giving rise to one stream of Agent-Based Models inside computational economics.

2.3.1. Tools for Agent-Based Models

The Python ecosystem for working with ABM's is based on the usual scientific Python toolbox: Numerical Python, Pandas, Matplotlib and other associated packages. There are already several packages/frameworks that allow us to deal with ABM's, tools with a purpose similar to NetLogo[1]. Two of the most promising are:

1) AgentPy[2]

2) Mesa[3]

In particular, Mesa has several packages that build on mesa with applications to Economics: "Economics Agent Resource toolkit"[4] and "Agent Based Computational Economy Platform"[5]. These packages are built on top of the usual suspects from the Scientific Python tools. That means that they use those packages and through the use of appropriated interfaces allow a seamless integration. In this work we have not used these frameworks, that are constantly improving, and instead we have used the bare essentials.

The option was to use the usual scientific Python packages since the available data types are quite rich and allow us to express agents and their relations using Python's object-oriented features. The use of Numpy and Pandas as the data members make the expression of the relations intuitive. We made also use of list comprehensions to transverse the different agents in the simulations. The analysis of the simulations, together with the development of the code and their results was done using Python's data analysis tools. As we have seen above, that is in the same spirit as other more complete frameworks.

Our choice also resulted from the model formulation. If, for example, the model implied some kind of spatial implementation of the capital good firms then the added complexity of

[1] https://ccl.northwestern.edu/netlogo/
[2] https://agentpy.readthedocs.io/en/latest/index.html
[3] https://github.com/projectmesa/mesa
[4] https://github.com/econ-ark
[5] https://github.com/DavoudTaghawiNejad/abce

using one of the frameworks described above would have paid off. The alternative would be to reinvent the wheel integrating NetworkX[6] with our proto framework.

[6] https://networkx.github.io/

3. Methodology

To implement and run the model we used a code interpreter program called JUPYTER that runs on the high complexity code language, PYTHON.

As in all K+S models, we are dealing with a model whose main driver is technological change to create endogenous growth and business cycles together with the Keynesian engine that fuels growth through consumption of workers and investment and in the relations between the macroeconomic aggregate variables. This one in particular, has a simple credit system from which we can derive some monetary policy implications related to interest rates.

3.1. Description of the baseline model

In this work, we adopt the model of Dosi, Fagiolo, Napoletano and Roventini (2013) for being microfundational model which already had many extensions and applications in other articles as said above. The baseline model that we study is itself an extension of the original version of Dosi, Fagiolo and Roventini (2008) and models endogenous growth and business cycles. This in particular, is one, which combines the Keynesian accounting for macroeconomics variables founded on microeconomic precepts with a 'Schumpeterian' way for expulsion of weak companies from the market and the possibility of entry of new ones and even, a 'Mishkian' credit system. It is a relatively complex and complete model in terms of semblance to reality. The model has three main types of agents: capital-good firms (F1), consumption-good firms (F2), a bank and a supply of labor (Ls) that represents both consumers and workers even if they do not make any actual decisions. Implicitly also has a state who can set a unemployment benefit and collect taxes and a central bank who sets the policy rate and the minimum of rate of reserve required for banks to hold.

Capital-good firms invest in R&D with funded by previous year sales and produce heterogeneous machines. The productivity of capital-good as well as the productivity of the machine to be produced will have on consumption-good firms depends on the success of imitation and/or innovation processes. Consumption-good firms receive the price and productivity of machine tools and place their orders according to their capital needs. If the firms liquidity stock (NW*j*) is not sufficient to meet production and investment costs, the bank can provide credit to firms. Credit is allotted to firms on a pecking-order basis according to their liquidity-to-sales ratio. Then, combine machine tools bought to capital-good firms and labor in order to produce a final product for consumers. Firms deposit in the bank their profits. The Central Bank affects the supply of loans and the actual dynamics of debt of the firms in the economy fixing the interest rate and the credit multiplier. Finally, the public sector levies taxes on firms' profits and pays unemployment benefits.

In what follows we will describe how the microeconomic decisions take place in Dosi et al. (2013), that will be our baseline model. In each period, *t*, the following sequential order:

1. Policy variables are determined (interest rates; tax rate; unemployment subsidy);
2. Capital-good firms perform R&D to innovate or to imitate other firms, to get the best pair of technologies A_i^τ, B_i^τ that minimizes equation 3.10 and advertise their machines to consumption-good firms;
3. Consumption-good firms compute their desired quantity to produce and the machines necessary to produce that quantity. In addition, firms verify the obsolescence of their machines and count the machines that need replacement. The model distinguishes both types of investment, the substitution investment (RId) and the expansion investment desired (EId). As external funds are expensive and conditioned, firms will spend first their funds and if necessary will borrow from banks. Firms that need credit, calculate their credit demand (CD) summing their investment and production necessities since both labor and the machine-tools (or machines) are payed in advance.
4. The total amount of disposable credit in the economy (equation 3.29) is distributed by the bank to each firm, satisfying firstly the firms with highest liquidity-to-sales ratio which means that if the sum of the all credit demands exceeds the maximum credit available, firms can be rationed;
5. In both sectors firms hire workers according to their plans or the funds that they have specially in what respects to consumption-good firms;
6. The market for consumption goods "opens". The demand for each firm is determined by the market share (*f*) of the firm which in turn depends on price competitiveness;
7. Firms in both sectors compute their profits and net liquid assets after paying back their due loans to the bank to the extent that they have liquidity to do that and deposit their savings, if any;
8. In both sectors, firms with near-zero market shares and/or negative net liquid assets are expelled (Schumpeterian part) from their industry and replaced by new and identical firms (number of firms is maintained fixed);

9. Machines that were ordered at the beginning of the period are delivered in the end of the same period but only become part of the capital stock in *t*+1.

10. At the end of each time step, aggregate variables are computed, summing over the corresponding microeconomic quantities.

3.1.1. Capital-good firms

In the capital-good sector there are F1 firms denoted by the subscript *i*. The technology of capital-good firms is defined by the vector (A_i^τ, B_i^τ), where the former coefficient is the productivity of the manufactured machine tool in the consumption-good industry, the latter coefficient is the efficiency of the production technique employed by the firm, and the positive integer τ denotes the current technology vintage. Given the monetary nominal wage $w(t)$, the unit cost of production of capital-good firms is:

Equation 3.1 $$c_i(t) = \frac{w(t)}{B_i^\tau}$$

With a fixed mark-up $(\mu_1 > 0)$ pricing rule, prices $(p_i(t))$ are defined as $p_i(t) = (1 + \mu_1)c_i(t)$

The unit labor cost of production entailed by a machine in the consumption-good sector is

Equation 3.2 $$c(A_i^\tau, t) = \frac{w(t)}{A_i^\tau}$$

Capital-good firms invest in R&D a fraction of their past sales (S_i)

Equation 3.3 $$RD_i(t) = vS_i(t-1)$$

with $0 < v < 1$. R&D expenditures are used to hire researchers paying the market wage $w(t)$. Firms split their R&D efforts between innovation (IN) and imitation (IM) according to the parameter $\xi \in [0,1]$

Equation 3.4 $$IN_i(t) = \xi RD_i(t)$$

Equation 3.5 $$IM_i(t) = (1-\xi)RD_i(t)$$

They model innovation as a two-step process. The first step determines whether a firm successfully innovates or not through a draw from a Bernoulli distribution, whose parameter $\theta_i^{\text{in}}(t)$ is given by:

Equation 3.6 $\theta_i^{\mathrm{in}}(t) = 1 - e^{-\zeta_1 IN_i(t)}$

with $0 < \zeta_1 \leq 1$. If a firm innovates, it gets the new technology $\left(A_i^{\mathrm{in}}, B_i^{\mathrm{in}}\right)$ according to:

Equation 3.7 $A_i^{in}(t) = A_i(t)\left(1 + x_i^A(t)\right)$

Equation 3.8 $B_i^{in}(t) = B_i(t)(1 + x_i^B(t))$

where x_i^A and x_i^B are two independent draws from a Beta (α_1, β_1) over the support $\left[\underline{x}_1, \bar{x}_1\right]$ with $\underline{x}_1$ belonging to the interval [-1,0] and $\bar{x}_1$ to $[0,1]$ Imitation follows a two steps procedure as well. The set of successfully imitating firms is formed sampling from a Bernoulli $\left(\theta_i^{im}(t)\right)$:

Equation 3.9 $\theta_i^{\mathrm{im}}(t) = 1 - e^{-\zeta_2 IM_{\mathrm{i}}(t)}$

With $0 < \zeta_2 \leq 1$. Firms accessing the second stage are able to copy the technology of one competitor $\left(A_i^{im}, B_i^{im}\right)$.

Knowing that consumption-good firms invest following a payback period routine, capital-good firms select the machine to produce according to the rule:

Equation 3.10 $\min\left[p_i^h(t) + bc^h\left(A_i^h, t\right)\right],\ h = \tau, in, im$

where b is a positive payback period parameter. Capital-good firms send then a "brochure" with the price and the productivity of their machines to both their historical ($client\_hist_i$) and a random sample of potential new customers.

Capital-good firms' profit is defined below:

Equation 3.11 $\Pi_i(t) = S_i(t) - c_i(t)Q_i(t) - RD_i(t)$

$S_i(t)$ being sales, $c_i(t)Q_i(t)$ total labor costs and $RD_i(t)$ R&D expenditures for each firm *i*.

The stock of liquid assets is defined as:

Equation 3.12 $NW_i(t) = NW_i(t-1) + \Pi_i(t)$

3.1.2. Consumption-good firms

The consumption-good sector is composed of F_2 firms labelled by the subscript j.

Consumption-good firms plan their production (Q_j) following the simplest adaptive demand expectations (D_j^e) :

Equation 3.13 $D_j^e(\mathrm{t}) = D_j(\mathrm{t}-1)$

where $D_{\{}(t-1)$ is the past demand actually faced by firm j. The desired level of production (Q_j^4) is computed adding the desired inventories (N_j^d) and the actual stock of inventories (N_j) to the expected demand:

Equation 3.14 $Q_j^d(t) = D_j^e(t) + N_j^{\mathrm{d}}(t) - N_j(t-1)$

with $N_j^d(t) = lD_j^e(t), l \in [0,1]$.

To simplification purposes, we have added a new variable that represents maximum capacity installed ($instcap_j$) and is simply the productivity of each type of machine times number of machines with the correspondent technology. Given that, the firms only receive their ordered machines in the next period, firms in our interpretation compare the quantity they desire to produce with their installed capacity and compute the required machines to fill the gap (K_j^d):

Equation 3.15 $K_j^d(t) = \frac{Q_j^d(t) - instcap_j(t)}{best\_buy(A_i^\tau)}$

If the desired additional capital is bigger than 0, firms invest (I_j^d) in order to increase their capital stock (K_j)

Equation 3.16 $I_j^d(t) = best\_buy_j K_j^d(t)$

We define as $machines_j(t)$ as the set of all machine tools belonging to firm j at time t. Moreover, if we measure machines in terms of their production capacity (normalized to one), the capital stock is defined as:

Equation 3.17 $K_j(t) = \sum machines_j(t)$

where $g_j(A_i^\tau, t)$ is the absolute frequency of machine A_i^t Fims scrap machines according to a payback period routine. More specifically, firm j replaces machine with $A_i^\tau \in$ $\mathrm{machines}_j(t)$ according to its technology obsolescence as well as to the price of new machines:

Equation 3.18 $RS_j(t) = \left\{A_i^t \in machines_j(t) \colon \frac{p^*(t)}{c(A_{i,\tau},t) - c^*(t)} \leq b\right\}$

where p^* and c^* are the price and unit cost of production of new machines. Firms compute their replacement investment summing up the machine-tools. Moreover, they also scrap the machines older than η periods (positive integer). Summing up expansion and replacement investment one gets the gross investment of each firm (I_j).

Given the "brochures" received by (a subset of) capital-good firms, consumption-good firms send their investment order to the supplier with the lowest price and unit cost of production.

Consumption-good firms compute average productivity (π_j) and unit cost of production (c_j):

Equation 3.19 $\pi_j(t) = \frac{instcap_j(t)}{K_j(t)}$

Equation 3.20 $c_j(t) = \frac{w(t)}{\pi_j(t)}$

Prices are set applying a variable markup, μ_j, on unit costs of production:

Equation 3.21 $p_j(t) = \big(1 + \mu_j(t)\big)c_j(t)$

The competitiveness (E_j) of consumption-good firms depends both on the average price and on the past level of unfilled demand (l_j):

Equation 3.22 $E_j(t) = -\omega_1 avgP_j(t) - \omega_2 l_j(t)$

where $\omega_{1,2}$ are positive parameters. Weighting the competitiveness of each consumption-good firms by its past market share $(f_j(t-1))$, one can compute the average competitiveness of the consumption-good sector:

Equation 3.23 $\bar{E}(t) = \sum_{j=1}^{F_2} E_j(t) f_j(t-1)$

Firms' market shares evolve according to a replicator dynamics:

Equation 3.24 $f_j(t) = f_j(t-1)\left(1 + \chi \frac{E_j(t-1) - \bar{E}(t-1)}{\bar{E}(t-1)}\right)$

with $\chi > 0$.

Aggregate consumption C is allocated to consumption-good firms proportionally to their market shares:

Equation 3.25 $$D_j(t) = f_j(t)C(t-1)$$

The mark-up applied over the unit cost of production is:

Equation 3.26 $$\mu_j(t) = \mu_j(t-1)\left(1 + v\frac{f_j(t-1) - f_j(t-2)}{f_j(t-2)}\right)$$

The profits (Π_j) of each consumption-good firm read as:

Equation 3.27 $$\Pi_j(t) = S_j(t) - PC_j(t) - rDeb_j(t)$$

With sales, $S_j(t) = p_j(t)D_j(t)$, total production costs $PC_j(t) = c_j(t)Q_j(t)$ and Deb_j denotes the stock of debt.

The stock of liquid assets is defined as:

Equation 3.28 $$NW_j(t) = NW_j(t-1) + \Pi_j(t) - cI_j(t)$$

Where $cI_j(t)$ are the internal funds used to finance production and investment, (with $cI_j(t) \leq NW_j(t-1)$).

3.1.3. Banking sector and closing of the model

Total production and investment expenditures ($EI_j^d(t)$ is expansion investment, $RI_j^d(t)$ is replacement investment) must not exceed the amount of liquid assets plus the credit demanded by the firm:

Equation 3.29 $$c_j(t)Q_j(t) + EI_j^d(t) + RI_j^d(t) \leq NW_j(t-1) + CD_j(t)$$

The credit demand of each firm is limited by sales of period $t-1$ according to a loan-to-value ratio:

Equation 3.30 $$CD_j(t) \leq \lambda S_j(t-1)$$

The maximum credit available in the economy at time *t*, MTC(*t*) is:

Equation 3.31 $$MTC(t) = k\left(\sum_{i=1}^{F1} NW_i(t-1) + \sum_{j=1}^{F2} NW_j(t-1)\right), k > 0$$

Equation 3.32 $$\sum_{j=1}^{F2} Deb_j(t) = \mathrm{Loan}(t) \leq MTC(t)$$

The Central Bank fixes *r* and the commercial bank puts a mark down ψ_D in interests paid on deposits r_D and a mark up ψ_L for the interest rate charged on loans r_L.

Equation 3.33 $$r_D = (1-\psi_D)r, \quad 0 \leq \psi_D \leq 1$$

Equation 3.34 $$r_L = (1+\psi_L)r, \quad 0 \leq \psi_L \leq 1$$

The wage-setting function in the labor market is:

Equation 3.35 $$w(t) = w(t-1)\left(1 + \psi_1 \frac{\Delta\overline{AB}(t)}{\overline{AB}(t-1)} + \psi_2 \frac{\Delta cpi(t)}{cpi(t-1)} + \psi_3 \frac{\Delta U(t)}{U(t-1)}\right)$$

where $\overline{AB}$ is the average labor productivity, *cpi* is the consumer price index, and U is the unemployment rate.

The public sector levies taxes on firm profits and worker wages (or on profits only) and pays to unemployed workers a subsidy, that is a fraction (φ) of the current market wage.

Since we presuppose active population have hand-to-mouth behavior, aggregate consumption is computed by the wages employed population earns and if not in a work-or die scenario, the unemployment benefits:

Equation 3.36 $$C(t) = w(t)Ld(t) + \varphi w(t)(Ls - Ld(t))$$

The total output function, where ΔN is represents the total variation in inventories is:

Equation 3.37 $$\sum_{i=1}^{F_1} Q_i(t) + \sum_{j=1}^{F_2} Q_j(t) = Y(t) \equiv C(t) + I(t) + \Delta N(t)$$

3.2. Difficulties in interpreting and implementing the model

We successfully managed to replicate the model of Dosi et al. (2013) but not without a series of problems and hundreds of little nuances that is impossible describe here. Implementing the model was a seriously difficult and time-consuming task. We consider two, to be the main challenges: first, understanding, the concepts of the model (i.g. A_i^τ, B_i^τ), to look at the model, as much as possible, through the eyes of the authors; second, the sequential ordering of tasks, in other words, understand what comes first, what are the foundational variables upon which the model is founded. And understanding the precedence order in an Agent-Based Model is crucial.

There were also implementing differences that we had to overcome. The first was already in the beginning, we needed a wage in period *t=0*, to obtain the unit cost of production in the capital-good sector, when the same wage was dependent on the relative variation of average productivity in the same period. The wage was also needed before the computation of total costs because capital-good firms have to offer their price to consumption industry. For this to be resolved, we lagged average productivity one period, as well as *cpi* and *U* that were variables only known in the end of the period.

Another one, had to do with machines. In the first period, *0*, we admit that consumption firms have only one type of machine. We also admit that the desired stock of capital, *Kd*, is in fact the number of new desired machines and made a distinction with a variable, *core_machines*, that are those will not be scrapped until the new machines be incorporated in period *t+1*. Because we thought it would not be relevant to the questions we were going to study and because of how we defined *Kd,* we do not separate desired expansion investment, *EId*, from desired replacement investment, *RId*, like the authors did and created a new variable, *Ij,* where both types of investment are reflected.

The way equation 3.22 is formulated, relating competitiveness, *Ej,* unfilled demand is a relative value while price, *P*, is an absolute number, so that *Ej,* is dominated by price and unfilled demand actually did not have any influence. Therefore, we created a new variable, *avgP,* which respects to the average price of the previous period to divide *P* and make the equation a operation of relative values.

Relatively to demand, *D,* in equation 3.25, we had to divide the product of the market and aggregate consumption by the price since demand is a quantity and aggregate consumption, *C,* is a value in monetary units and more importantly, we lagged *C* one period, since the aggregate consumption of the period because of precedent variables, can only be known at the end of the period. Also, it would not make sense to determine it *ex ante*.

One of the things that were not referred or less evident in Dosi et al. (2013) was the way firms in the consumption-good market are expelled and substituted by new ones and if capital-good firms also should get expelled. Our assumption to the last was that no, capital-good firms are not expected to be expelled from the market since the equations seem to refer only to consumption firms and the description of capital firms does not prescribe it. The interpretation that, we had was that firms can be expelled of the economy when their market share is below a certain parameter, when their stock of liquid assets is below zero or, and here is one of the innovations, when the firm doesn't have machines. Important to notice, that this happens because of the demand expected since many of the times they do have financial capability to invest. When we substitute the considered bankrupt firms by existing firms, these new ones steal some variables of these. Debt and inventories return to zero while the stock of liquid assets, the market share, machines, competitiveness, expected demand and unfilled demand are mimicked from another firm for the model to function.

In respect to the banking sector, the borrowing of credit was the only aspect of the model, we think that we were not completely able to recreate. But with almost certainty we could say that the problem is not in the banking sector itself, but in consumption sector since the investment desired by the firms is really low and rare.

One of the other issues, was calculating the Gini coefficient when some firms, have in average, negative profits. And with negative values the Gini coefficient can be bigger that one (Chen et al., 1982) and with absurd variations. Nevertheless, the general impression of the inequality in the distribution of profits.

Certainly of the biggest limitations of model and one of reasons capital and consumption firms is, at least in what one can withdraw from Dosi et al. (2010, 2013), that capital firms do not invest in machines (only R&D) and more importantly, they cannot go to bankrupt and ask for credit. For future work, this should be completed because we it would give the model more accuracy as well as an opportunity to flatten the distribution of profits and sales and cure its concentration.

The model has limitations, that we'll not try to hide. As listed above, certainly, this replication is not completely loyal to the original model. There are many ways, in which the model could evolve but most of limitations are inherent to the model. In this context, where we only have one bank, there is no central bank as an agent, no implication of government pursuing high deficit budgets, no financial assets, no heterogeneous workers, without openness to an international market, etc…, the degree of likeness to reality is good.

Finally, the model we choose doesn't seem to be the most adequate to study the impact of monetary policy in inequality. Perhaps one more adequate and that is an extension of this is that of Dosi et al. (2020) where even workers are heterogeneous and different wages but also lacks other mechanisms for example for quantitative easing purposes, lacks bonds. But that would permit us to study inequality at the level of households. So, we had to make choices. And in this case were only able to study inequality in firms' distribution of profits and net worth.

4. Results

4.1. Model properties

Microeconomic level	Macroeconomic level
Skewed firm size distribution	Endogenous self-sustained growth with persistent fluctuations
Fat-tailed firm growth rate distribution	Fat-tailed GDP growth rate distribution
Productivity heterogeneity across firms	Relative volatility of GDP, consumption and investment
Persistent productivity differentials among firms	Cross-correlation of macro variables
Lumpy investment rates at firm-level	Pro-cyclical aggregate R&D investment
	Persistent unemployment

Table 1 - Stylized Facts adapted from Dosi et al. (2017)

The replicated model matched many of the stylized facts of empirically corroborated by real data in opposition to a simulated one produced by model. It means that the model we implemented, in this version can reproduce at least these facts and be a proxy for the economic reality.

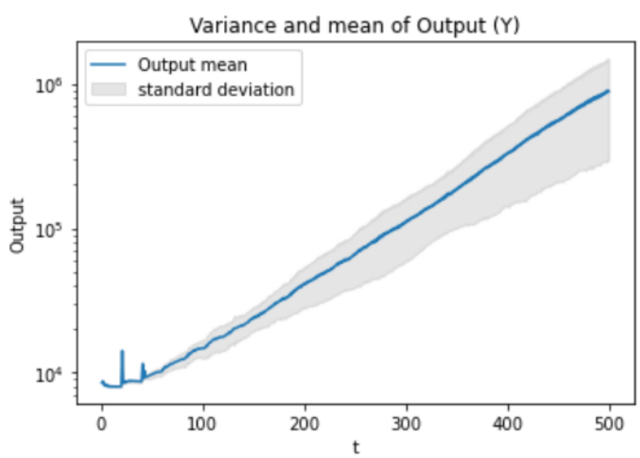


Figure 1 – Evolution of the mean and variance of the Output

First in section, are being shown results from the closest interpretation of Dosi et al. (2013) we have made. They were achieved by making a simulation of 50 economies with 500 periods. The baseline conditions for which we plot the graphics are the presence of fiscal and monetary policy: baseline interest rate of 2.5%, tax over firms' profits of 10% and unemployment subsidy of 40% of the prevailing nominal in period t. The last two function effectively as automatic stabilizers.

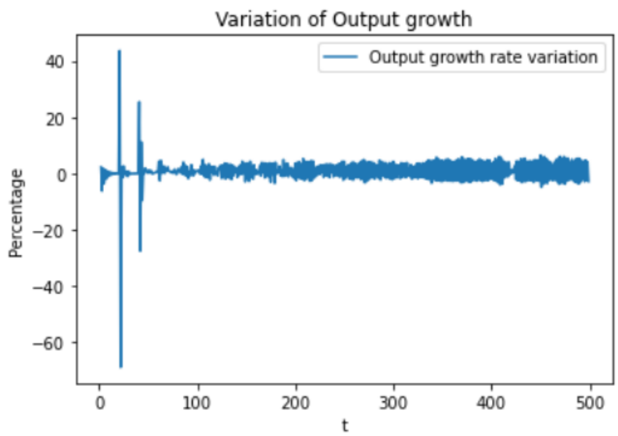


Figure 2 - Variation of Output growth rates

In Figure 1, we can observe the Output mean with consistent exponential growth of 1.02% in average and relative volatility (Figure 2) ranging from 40% to -60% in the beginning but in majority of periods from 6% to -4%. Notice the perturbations of Y in the first periods

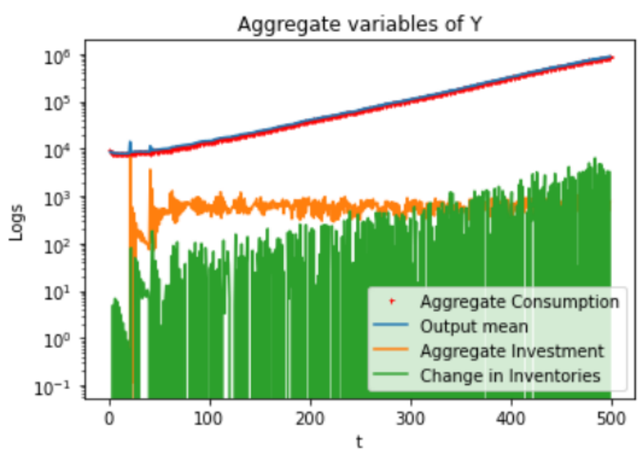


Figure 3 - Aggregate variables of Output

and the variations in the end of the x-axis are increasingly volatile as it gets closer to t=500 in both graphics.

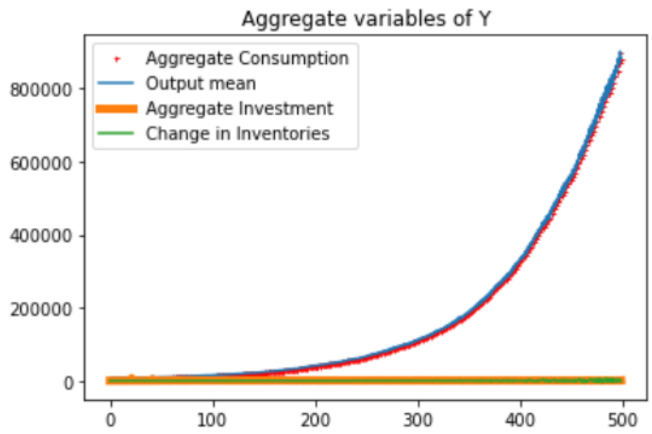


Figure 4 - Aggregate variables of Output

Almost all of the output is due to aggregate consumption. Aggregate investment and the variation of inventories are virtually null, relative to output, though are very volatile. We can see that investment is more volatile than consumption and output and confirms that stylized fact.

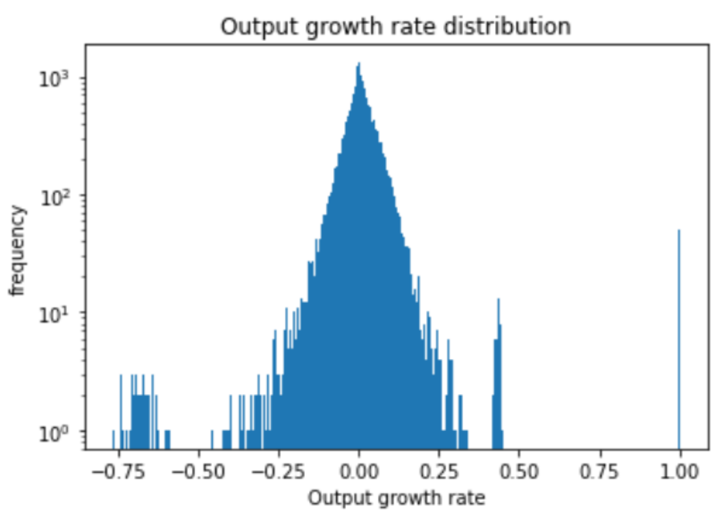


Figure 5 - Histogram of Output growth rates

Another stylized fact that we can confirm is that the output growth rate distribuition have a fat-tailed distribuition. In the histogram above, the most extreme observations should not occur as frequently as if the output growth rate distribution were normal. But to be sure of that we've made an Kolmogorov-Smirnov test, where the null hypothesis is if the distribution is normal and it gave a p-value of aproximately 0.001. So we reject the hypotysis that Output growth rate variations follow a normal distribution for α=5% and 1% at least.

In what respect wages, its main driving force is productivity. It increases exponentially, similarly, to output and consumption. It is assumed that the work force is homogeneous, so the salary is equal to each worker in both sectors.

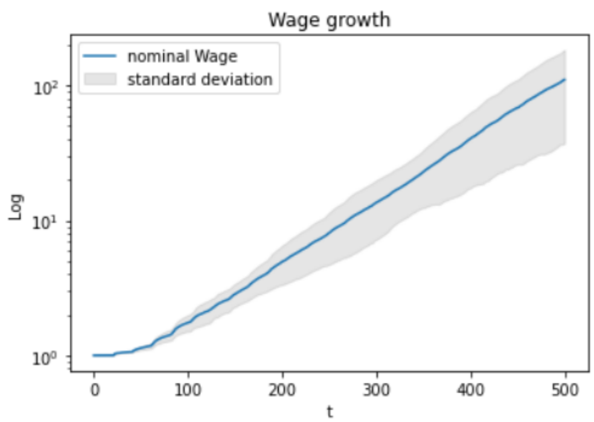


Figure 7 - Wage growth in log scale

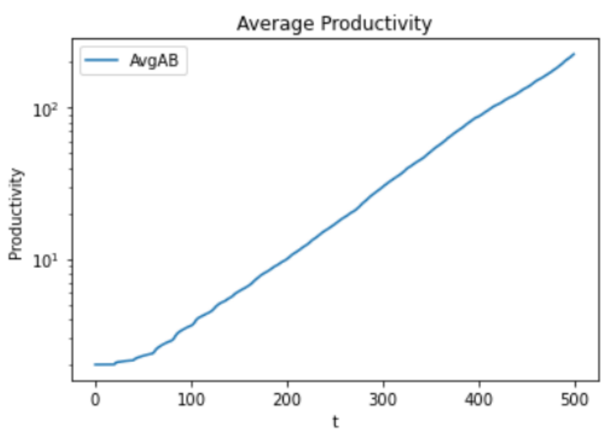


Figure 6 - Average productivity in logarithmic scale

Prices of final consumer goods seem to be very dominated by periods where the obsolescence of machines is evaluated and decide if they need investment in new

Figure 8 - Consumer Price Index variation

machines because the spikes in the consumer price index have a gap of 20 periods that corresponds to parameter η. In average, consumer prices have virtually zero changes from period to period.

Unemployment stabilizes fluctuates around 30%, between 25% and 35% and in average 31,13%. This also confirms the persistence of unemployment. One of the

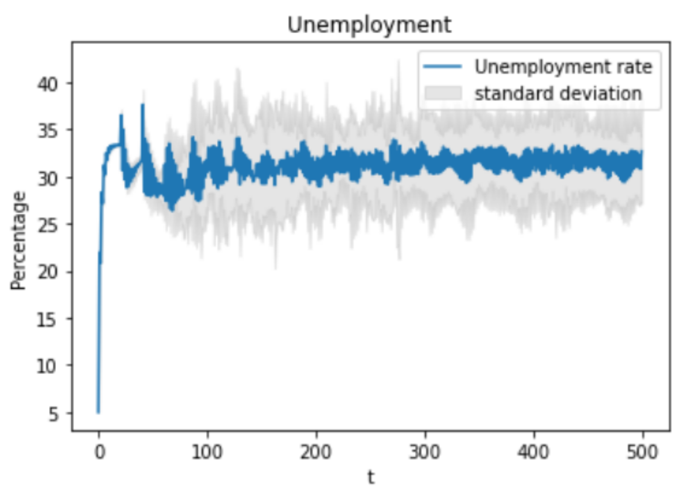


Figure 9 - Unemployment evolution

presuppositions of this model is that labor supply is exogenous, inelastic and constant and that corresponds also to the only consumers.

The bankruptcy rate is relatively low, in average 1.67% of consumption good firms go to bankrupt in each period.

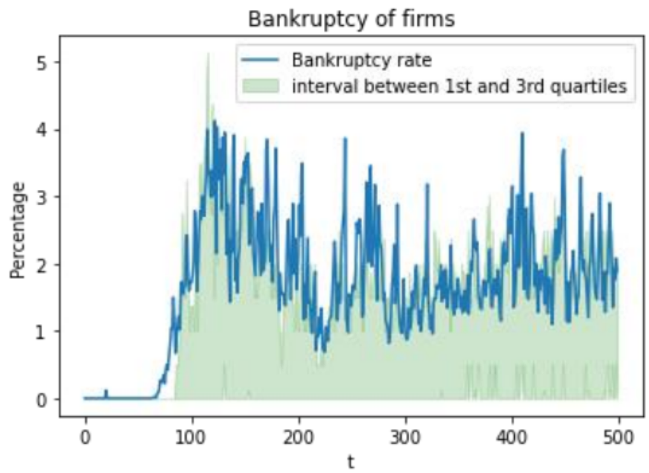
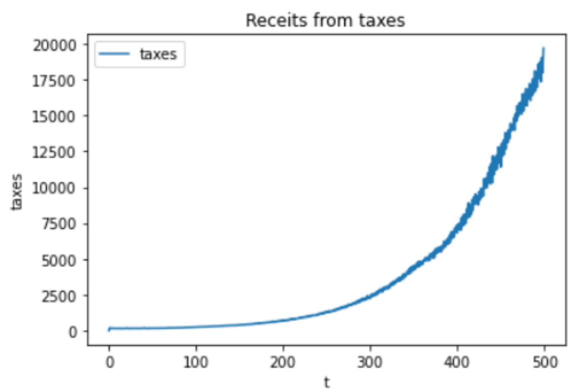


Figure 10 - Bankruptcy of firms rate (mean)

In other words the market has a tendency to concentrate profits and market shares



in a few companies in consumption sector and monopolize the capital sector. In the last, one firm manages to monopolize the sector very early in the timeline until the end. For one side,

Figure 11 - evolution of tax collects

the inability of capital firms to bankrupt can be one the reasons for monopolization, for other the distribution of information about prices and productivities is imperfect. In the consumption sector, few firms alternately switch places in the oligopoly phenomenon.

Taxes behave expectedly since the tax rate is fixed and the output grows exponentially. We assume that the state has infinite capacity of borrowing credit since it is an unmodeled state. Nevertheless, is good to see the evolution of the balance between taxes and subsidies where the tax rate over firms' profits is 10% and unemployment subsidy of 40% of the prevailing wage in period t. A very simple and limited welfare state although also consistently with deficit. It's no surprise, that the behavior of accumulated debt be the one

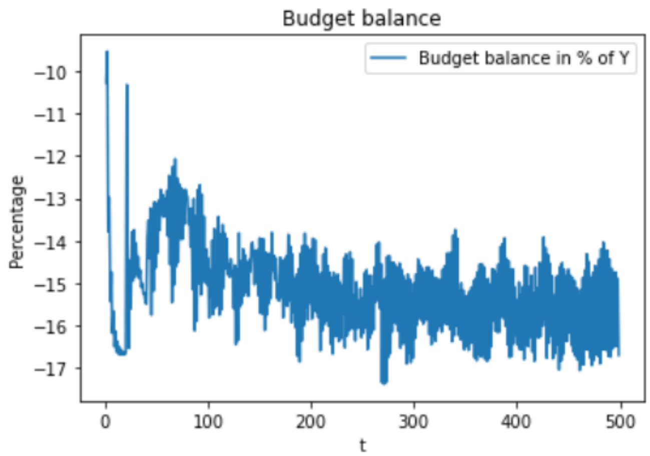


Figure 122 - Budget balance mean in relative to Output

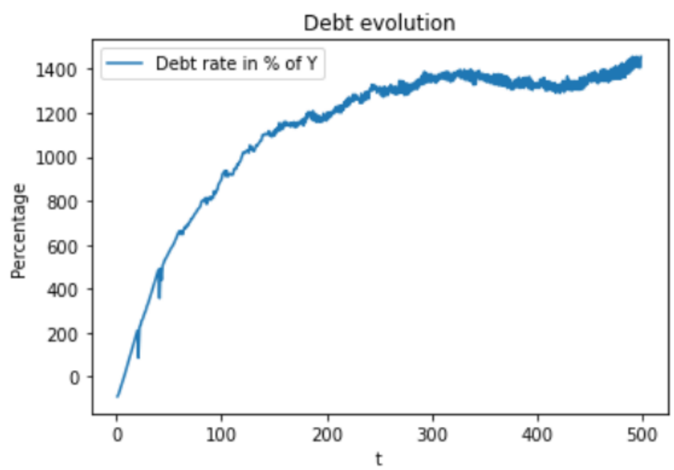


Figure 133 - Evolution of debt stock relative to Output

in the figure below. Those parameter values were chosen to be in conformity with the baseline model.

Relatively to the banking sector, the results are somewhat disappointing because the loans are incredibly rare events. There is only credits in the last quartile of loans distribution, which signifies that in more than 75% of the periods firms do not borrow from banks.

Additionally, the borrowings are relatively low, even more, in the end, when prices of machines are much higher. The reason to this can only be, not in the banking sector itself which is relatively simple, but in the consumption sector where firms do not even ask for credit and when they borrow is probably because their machines become obsolete. More profoundly the reason in the mechanism of evolution consumption firms follow where

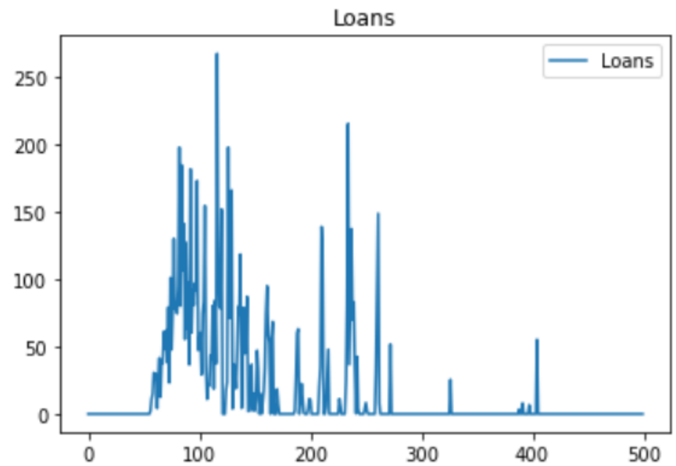


Figure 14 - Mean of loans evolution

competitiveness brings down the market shares and if market share declines than their demand will be lower and hence the decreased necessity of investment, which we saw above is very low, and naturally credit.

One thing we can be sure, it's not because the lack of credit capacity of the bank, because the maximum credit available in the economy is always more than enough, which is a reflex of total deposits of firms.

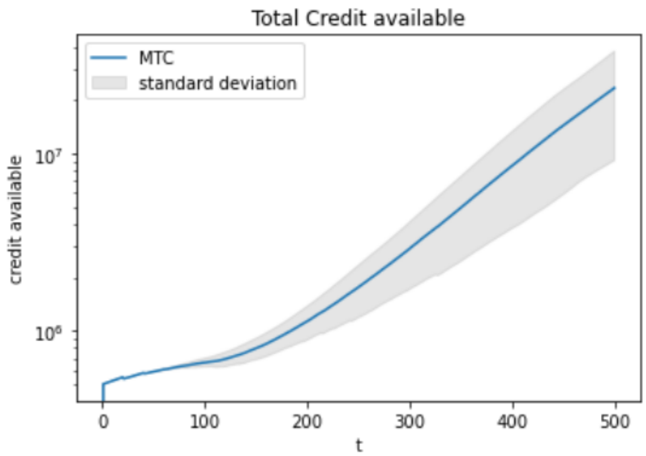


Figure 16 - Total credit available in logarithmic scale

The authors referred the existence of two transmission channels of monetary policy instead of the distributional channels, in this model: the balance sheet channel and a bank lending channel. However, in the model we've had recreated, the two are not strong. Because even if interest rates (if positive) make borrowing costlier, reduce the level of net worth, via profits, and hence the possibilities of production and investment if firms do not borrow, even more because don't need to, means that possibly monetary policy is not going to be effective. What restrains consumption firms and make them go to bankruptcy are those have to do with demand and competitiveness we discuss above.

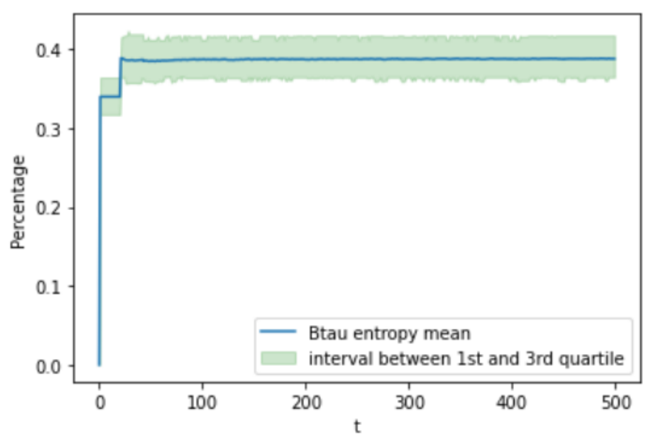


Figure 15 – entropy of productivity in the capital sector

To see how much heterogeneity there is in each sector in terms of productivity, we measure the Shannon's entropy, which is a measure of heterogeneity from Information

Theory. This concept comes from the corresponding Statistical Physics measure with the same name. Shannon's entropy measures the level of information inherent in the variable's outcome. From its definition the entropy is always either positive or zero.

In the case of a finite set of states the entropy is minimal (zero) when all the states are equal and has the maximum when each state has a different value (the surprise is also maximum). The maximum value for a set of N states is log(N). In this work in order to simplify the analysis of the results we will divide the entropy by the logarithm of the number of possible states and thus this measure varies between 0 (all the states are equal) and 1 (all the states are different).

What we can say about productivity in the capital-good, is that firms start from a point where the productivity of all firms in sector is the same and that makes all the sense because we initialize firms with the same technology, hence with the same productivity. Then in the first period, firms begun to innovate and that is the why of that big jump in entropy where 35% of firms have different technologies. For the rest of the timeline it stabilizes because there is another force that wrestles with innovation, that is imitation and because firms who do not sell, stagnate in the terms of R&D which leads to bigger and bigger concentration of sells. Except when consumption-good firms have the first scrapping of machines, where the big demand for machines causes sales and consequently an increase in R&D in the next period.

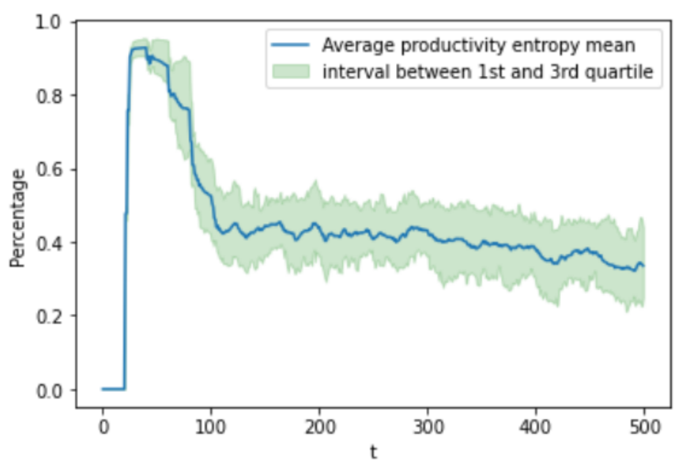


Figure 17 – entropy of productivity in the consumption sector

In terms of the consumption good sector, productivity variations are more dynamic. In the first periods is the same because capital-good firms only had one technology for the first machines sold. There are at least two factors that explain that variability: final goods firms can multiple types of technologies after the beginning and they go bankrupt and are substituted. However, average productivity entropy in the sector when stabilized is comparable to what happens in the capital-good sector, but more unstable.

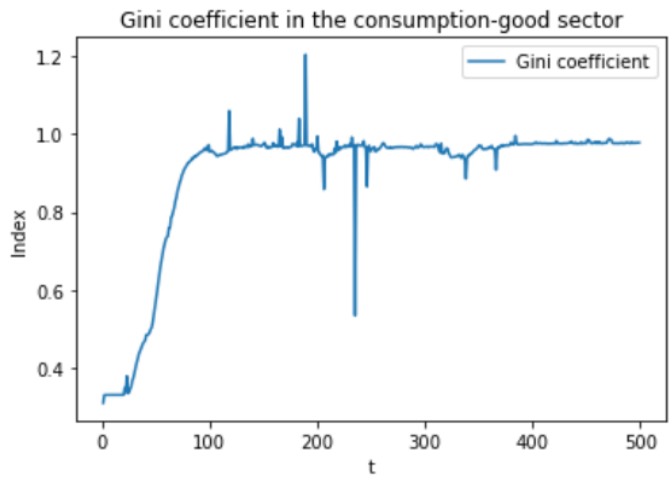


Figure 19 - Gini coefficient mean of profits in consumption-goods sector

The concern in this work is to study the impact of monetary policy in inequality. In particular how unequal profit distribution of firms is. Because some firms profits, in average, are negative, the Gini

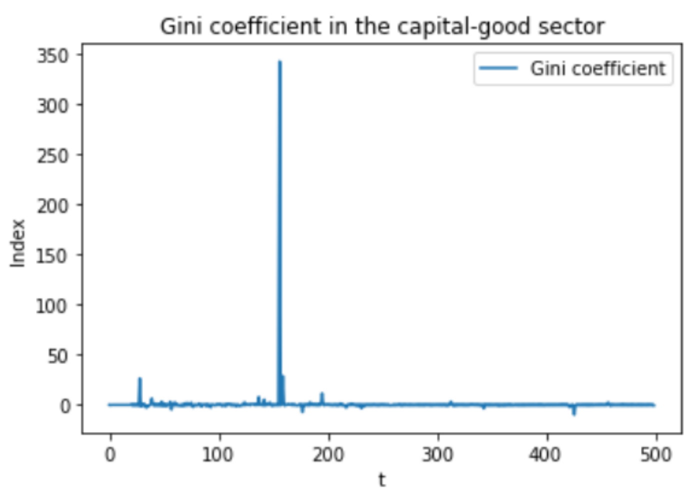


Figure 18 - Gini coefficient mean of profits in capital-goods sector

coefficient can be bigger than one resulting in those peculiar forms in the graphics. Capital firms have in average have a Gini coefficient of 0.8 while consumption-good firms have 0.9.

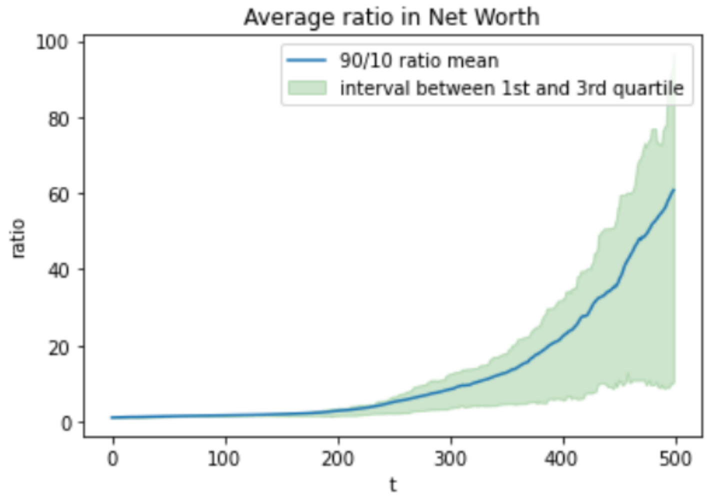


Figure 20 – Distribution of 90/10 ratio in Consumption firms

Another way of measuring inequality can be 90/10 ratio that is an indicator that tells how much profits, net worth, sales the 10% most well performed firms (in this case) of an distribution in relation to the 10% firms with worst performance.

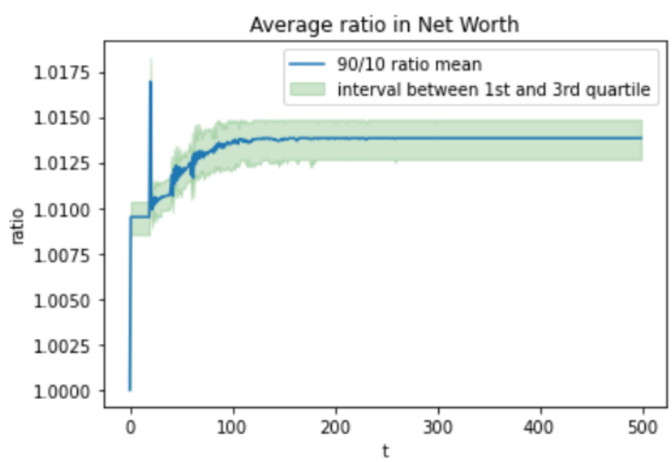


Figure 21 - Distribution of 90/10 ratio in Capital firms

The inequality in consumption-good sector is much larger and much sooner than in the capital sector. The 10% wealthiest firms have more liquid assets than the 10% poorest while in the consumption sector (Figure 20 it's not in percentage scale) the 10% richest have can have more than 80 times the poorest 10%

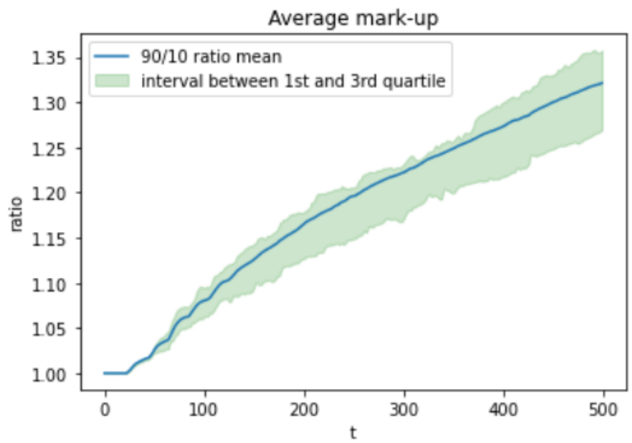


Figure 22 – average mark-up in the Consumption sector

Mark-ups increase throughout the time, always bigger in last decile than in first, except in the beginning where is the same because of initialization.

4.2. Change in initial values

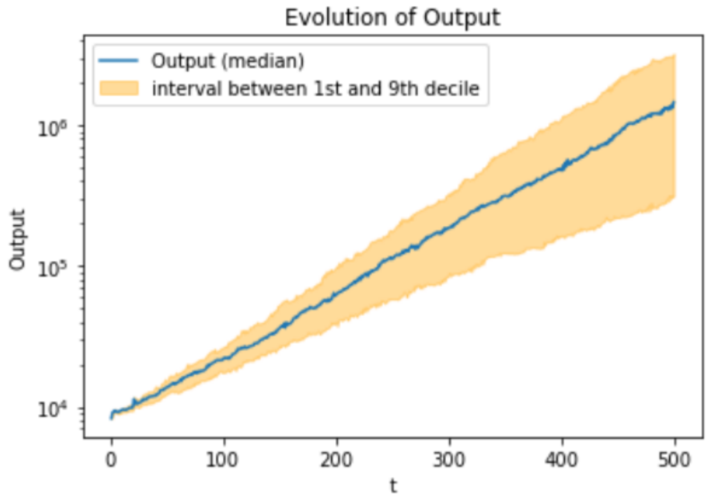


Figure 23 - Evolution of Output median

Next, we run the same model, with the difference that the initial values for the productivities in the capital sector are not the same, and consequently, neither in the consumption sector because the productivity of the machines is determined by capital firms and neither the values of the net worth (*NW*) of the two types of companies. Both productivities and net worth, variate inside an interval, following an uniform distribution. We introduce this to better describe the heterogeneity between firms although arguably there could be other ways to induce it. Also, we'll be able to see if inequality of the initial conditions of the firm has an impact in its evolution although we're mainly interest in the effects in the inequality, not the other way around, the effects of inequality on other variables. The firms substituted in consumption sector begin with *NWj* respecting the same mechanism.

Instead of the mean and standard deviation, above is represented the output median and its variation between the first and the nineth deciles because standard deviation values were going below 0 which means that some output values were high enough to distort standard deviation. Which is the same reason, we present the bankruptcy rate variance also with quartiles instead of standard deviation, which is a better indicator when distributions approximate to normal ones.

Note the difference of Figure 16 and Figure 23. In the happens the inverse of what happens in the first. Capital-good firms begin all with same productivity, 100% of the firms have different technologies. It's the imitation processes that make firms have a higher degree of homogeneous technologies between them. In the consumption-good sector (Figure 24) maintains entropy maintains it self more or less with a little tendency to decrease in the of the timeline.

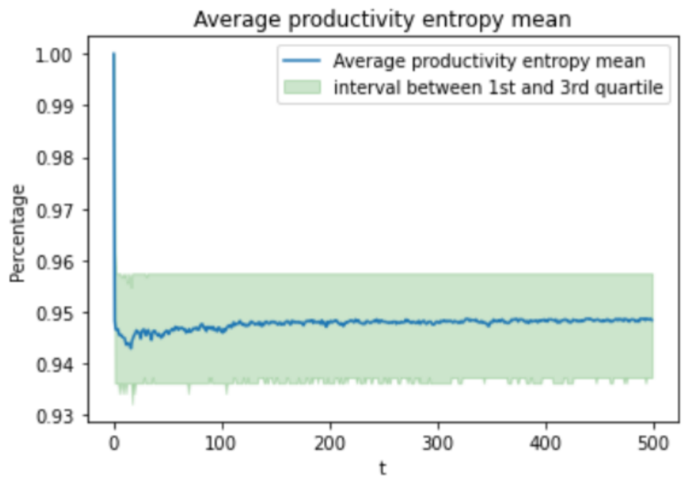


Figure 25 – entropy in Capital firms

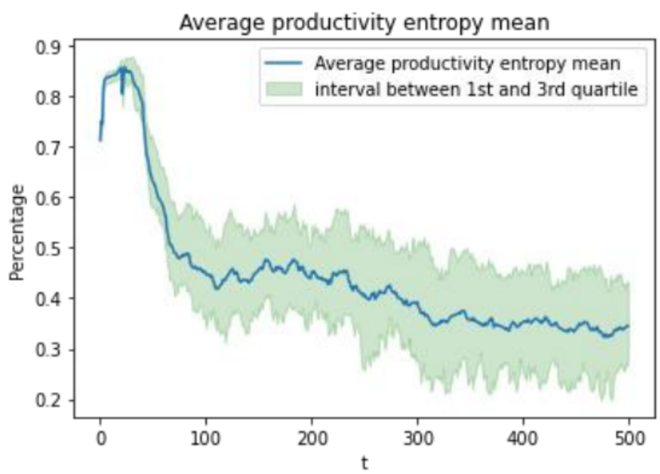


Figure 24 – entropy in Consumption firms

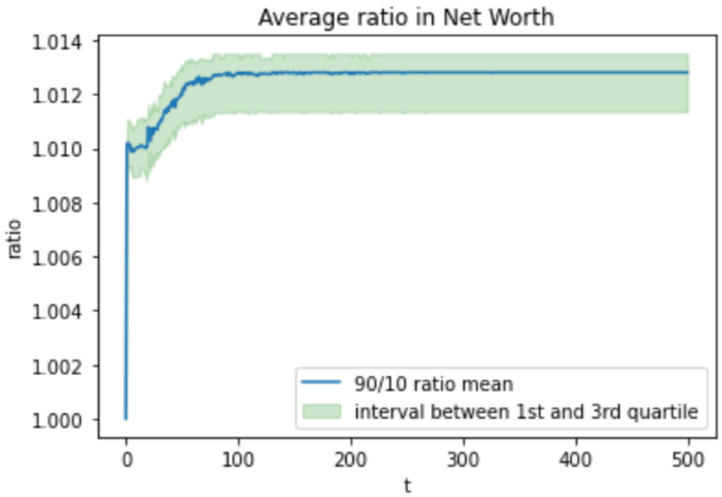


Figure 26 - Distribution of 90/10 ratio in Capital firms

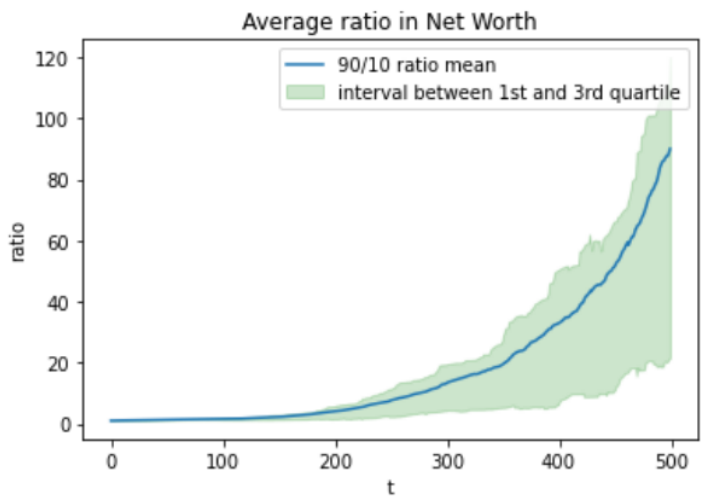


Figure 276 - Distribution of 90/10 ratio in Consumption firms

We can see that changing the initial values provoked a higher dispersion of net worths, (we can call it net wealth also) in consumption firms inequality. The last decile reaches as high as 100 times richer than the first decile.

1.1. Negative Interest rates

To be able to study the effects of negative interest rates more easily and to be possible that the policy interest rate be 0, we changed the formulas of interest rates on loans and deposits to simply: $r_D = r - \psi_D$ and $r_L = r + \psi_L$. We set r in 0, interest rate on deposit in -2.5% and interest rate on loans of 1%.

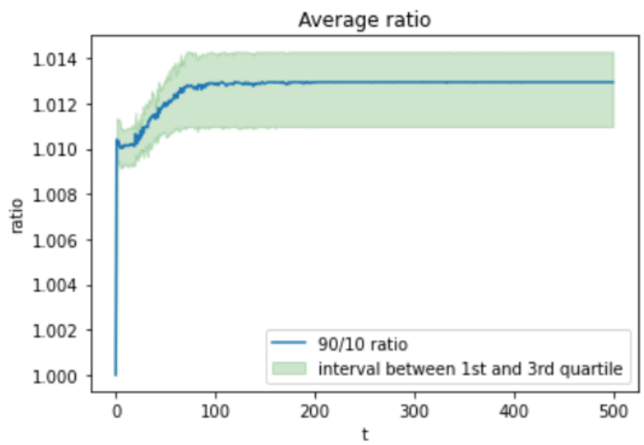


Figure 28 - Distribution of 90/10 ratio in Capital firms

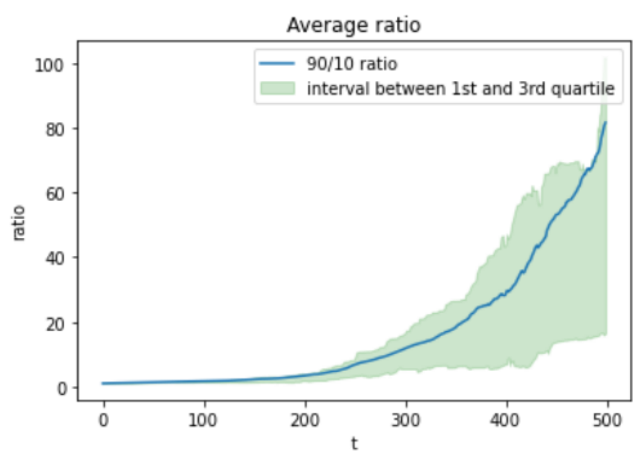


Figure 29 - Distribution of 90/10 ratio in Consumption firms

The decrease in interest rates under the zero lower bond provoked some redistributive effects. In Figure 27 we observe that interval between quartiles is wider. It surely has to do with the payment of interest rates for the deposits held at the bank while in the case of consumption firms, negative interest rates have the a balance sheet effect. Not only paying for deposits but decreasing liquid assets which restrains them from liquidity. Because, in terms of loans, they continue low and are, not benefiting from negative interest rates on credit borrowing. Also firms they lose some liquidity because of the interest rates that left receiving.

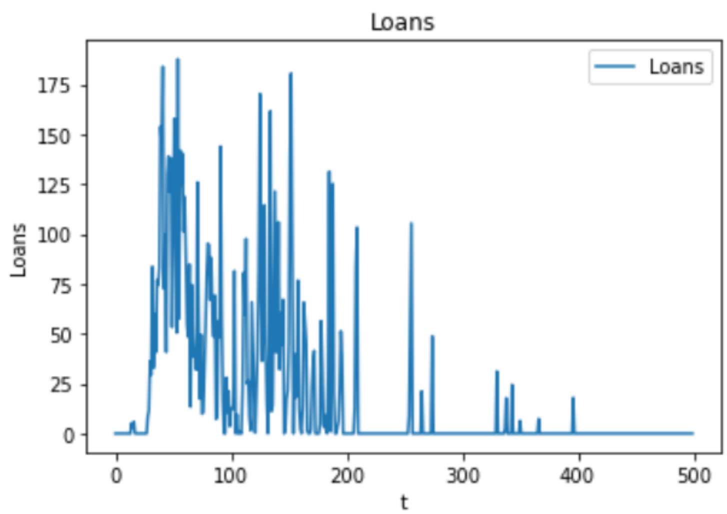


Figure 30 - Loans in interest rate scenario

5. Conclusion

We managed to conclude that negative interest rates do really have redistributive effect on firms net worth through the balance sheet channel, but not a positive one. In the real economy, firms that really pay interests on deposits but they do not have access to credit, they end up worse off because there is no canceling effect.

The model was successfully replicated and proved robust in reproducing most of the stylized facts empirically verified, for a model with these characteristics.

The model has limitations, that we'll not try to hide. As listed above, certainly, this replication is not completely loyal to the original model. There are many ways, in which the model could evolve but most of limitations are inherent to the model. In this context, where we only have one bank, there is no central bank as an active agent, no implication of government pursuing high deficit budgets, no financial assets, no heterogeneous workers, without openness to an international market, etc…, the degree of likeness to reality is good. But, we must admit, this is not the most adequate model to study inequality in either way.

For example, we could have done the same studies of inequality we did with net worth distributions, with profits but there were no reasonable instruments that we could use to measure (and it is important the we measure inequality makes sense and that we can withdraw conclusions from it) in this model with tools we know for studying any kind of inequality distribution.

The initial purpose for this work was to focus on the effects of unconventional monetary policy in income inequality of households. The strategy was to find a model which were complete enough to do it. And we this one. We relied on the hypothesis of extending it, to incorporate heterogeneous workers like in Dosi et al. (2020) but even so we have to had another extension with a more sophisticated financial market that would permit to the Central to make asset purchases with impact in real economy.

It was too of a larger task for us considering there is university consortiums of investigators working on this kind of models, with different streams and multiple ways of implementation. However, economic Agent-Based Models is a methodological tool that is growing fast in complexity as in possibilities, inside the ever-growing computational science.

Appendices

Baseline model code used:

```
class Economy:
  def __init__(self):
    self.W        =  np.zeros(T)            # wage setting function
    self.Y        =  np.zeros(T)            # agregate product
    self.C        =  np.zeros(T)            # agregate consumption
    self.I        =  np.zeros(T)            # agregate investment
    self.deltaN   =  np.zeros(T)            # variation in inventories
    self.avgAB    =  np.zeros(T)            # average productivity
    self.CPI      =  np.zeros(T)            # change in consumer price index
    self.U        =  np.zeros(T)            # change in unemployment rate
    self.avgE     =  np.zeros(T)            # average competitiveness
    self.Ls       =  Ls0                    # labor supply
    self.Ld       =  np.zeros(T)            # labor demand
    self.sub      =  np.zeros(T)            # unemployment subsidy
    self.avgP     =  np.zeros(T)            # average price for consumption firms
    self.broked_firms=  [[] for t in range(T)]
    self.taxes    =  np.zeros(T)
    self.bruptcyr =  np.zeros(T)
    self.balance  =  np.zeros(T)

    self.bank     =  [Bank() for i in range(N_banks)]
    self.cap      =  [Capital_good_firms(i) for i in range (N_capital_firms)]
    self.con      =  [Consumption_good_firms(j) for j in range(N_consumption_firms)]

    self.gini_con =  np.zeros(T)
    self.gini_cap =  np.zeros(T)
    self.W[0]     =  w0
    self.CPI[0]   =  CPI0
    #self.Ls[0]   =  Ls0
    self.U[0]     =  U0
    self.Ld[0]    =  (1 - self.U[0]) * self.Ls

    self.C[0]     =  self.W[0] * self.Ld[0] + self.U[0] * self.Ls * self.sub[0]
```

```
self.deltaN[0]  =   sum(self.con[j].D[0] - self.con[j].Q[0] for j in range (N_consumption_firms))
f0              =   np.random.random_sample(N_consumption_firms)
f0             /=   sum(f0)
for j,firm in enumerate (self.con):
    firm.f[0]   =   f0[j]


consumption_machines = np.random.randint(N_capital_firms, size = N_consumption_firms)
client_list = [[] for i in range(N_capital_firms)]

for j,i in enumerate(consumption_machines):
    client_list[i].append(j)
    self.con[j].pi[0]= self.cap[i].Atau[0]            # in the intial moment we only have one type of
machines
    self.con[j].c[0] = self.W[0] / self.con[j].pi[0]
    self.con[j].P[0] = self.con[j].c[0] * (1 + mu_0)
    self.con[j].D[0] = self.con[j].f[0] * self.C[0] / self.con[j].P[0]

    self.con[j].K[0] = int(np.ceil( self.con[j].D[0] / self.cap[i].Atau[0] ))
    self.con[j].machines[0].append((self.cap[i].P[0],   self.cap[i].cAtau[0],   self.cap[i].Atau[0],   0,
self.con[j].K[0]))
    self.con[j].Q[0] = self.con[j].K[0] * self.con[j].pi[0]
    self.con[j].N[0] = (1 + iota) * self.con[j].Q[0] - self.con[j].D[0]
    self.con[j].S[0] = self.con[j].D[0] * self.con[j].P[0]

    self.cap[i].Q[0]+= self.con[j].K[0]
    self.cap[i].client_hist.append(j)

for i in range (N_capital_firms):
    self.cap[i].S[0] = self.cap[i].Q[0] *  self.cap[i].P[0]
    self.cap[i].client_list[0] = client_list[i]
self.avgP[0]    =   np.mean([firm.P[0] for firm in self.con])
for firm in self.con:
    firm.E[0]   =   -omega1 * firm.P[0] / self.avgP[0]
self.avgE[0]    =   sum(self.con[j].E[0] * self.con[j].f[0] for j in range (N_consumption_firms))
```

```
        self.avgAB[0]    =    np.mean([firm.Btau[0] for firm in self.cap]) + np.mean([m[2] for firm in self.con for m in firm.machines[0]])
        self.credit_list=                  sorted([(self.con[j].S[0]    /    self.con[j].NW[0],j)    for    j    in range(N_consumption_firms)], reverse=True)



    def update(self,t):
        if self.broked_firms[t-1]:
            for j in self.broked_firms[t-1]:
                while True:
                    emulate_firm=self.con[np.random.randint(N_consumption_firms)]
                    if not emulate_firm.broked or len(self.broked_firms[t-1]) == N_consumption_firms:
                        break
                if DEBUG:
                    print("firm {} replaced by firm {}".format(j, emulate_firm.brand))
                    with   pd.option_context('display.max_rows',   None,   'display.max_columns',   None, 'display.width', 1000):
                        print(econ.con[j].table()[t-4:t])
                self.con[j].reset(emulate_firm, t-1)
            Sf=sum(firm.f[t-1] for firm in self.con)
            for firm in self.con:
                firm.f[t-1] /= Sf

        if t<2:
            self.W[t]    =   w0
        else:
            self.W[t]    =   self.W[t-1] * (1 + psi1 * ((self.avgAB[t-1] - self.avgAB[t-2]) / self.avgAB[t-2]) +
                        psi2 * ((self.CPI[t-1] - self.CPI[t-2]) / self.CPI[t-2]) +
                        psi3 * ((self.U[t-1] - self.U[t-2]) / self.U[t-2]))
        self.avgAB[t]    =    np.mean([firm.Btau[t-1] for firm in self.cap]) + np.mean([m[2] for firm in self.con for m in firm.machines[t-1]])          ####
        self.avgP[t]    =   np.mean([firm.P[t-1] for firm in self.con])
        self.sub[t]     =   self.W[t] * phi
```

```
    def update2(self,t):
        self.Ld[t]      =   sum(firm.Ld_i[t] for firm in self.cap) + sum(self.con[j].Ld_j[t] for j in range
(N_consumption_firms))
        self.U[t]       =   (self.Ls - self.Ld[t]) / self.Ls
        self.avgE[t]    =   sum(self.con[j].E[t] * self.con[j].f[t-1] for j in range (N_consumption_firms))
        self.C[t]       =   self.W[t] * self.Ld[t] + self.U[t] * self.Ls * self.sub[t]
        self.CPI[t]     =   np.mean([self.con[j].P[t] / self.con[j].P[t-1] for j in range
(N_consumption_firms)]) ### necessário atualizar
        self.credit_list=   sorted([[firm.ltsr[t], firm.brand] for firm in self.con], reverse = True)

        self.deltaN[t]  =   sum((self.con[j].N[t] - self.con[j].N[t-1]) * self.con[j].P[t] for j in range
(N_consumption_firms))
        self.I[t]       =   sum(self.con[j].Idj[t-1] for j in range (N_consumption_firms))
        self.Y[t]       =   self.C[t] + self.I[t] + self.deltaN[t]



    def update3(self,t):
        self.taxes[t]   =   sum(firm.taxi[t] for firm in self.cap) + sum(firm.taxj[t] for firm in self.con)
        self.bruptcyr[t]=   len(self.broked_firms[t]) / N_consumption_firms
        self.balance[t] =   self.taxes[t]-self.U[t] * self.Ls * self.sub[t]
        self.gini_con[t]=   gini_coefficient(np.array([firm.Pi[t] for firm in self.con]))
        self.gini_cap[t]=   gini_coefficient(np.array([firm.Pi[t] for firm in self.cap]))

    def update_step(self,t):
        self.update(t)
        self.bank[0].update(t,self)
        for firm in self.cap:
            firm.update(t,self.cap,self.con)
        for firm in self.con:
            firm.update(t,self)

        self.update2(t)
        self.bank[0].update2(t,self)
        for firm in self.con:
            firm.update2(t,self)
```

```
        for firm in self.cap:
            firm.update2(t)

        self.update3(t)
                #
class Bank:
    def __init__(self):
        self.MTC      =  np.zeros(T)            # maximum total credit available
        self.rL       =  (1 + (rhoL)) * r        # interest rate on loans
        self.rD       =  (1 - (rhoD)) * r        # interest rate on deposits
        self.Loans    =  np.zeros(T)            # total loans
        self.Deps     =  np.zeros(T)            # aggregated deposits
        self.Res      =  np.zeros(T)            # reserves
        self.Pi       =  np.zeros(T)            # profit of the bank
        self.NW       =  np.zeros(T)            # net worth of the bank
                #
    def update(self,t,econ):
        self.MTC[t]    =  k * (sum(firm.NW[t-1] for firm in econ.cap) + sum(firm.NW[t-1] for firm in
econ.con))
        self.available_credit =  self.MTC[t]
        self.Deps[t]    =   sum(firm.NW[t-1] for firm in econ.cap) + sum(firm.NW[t-1] for firm in
econ.con)
        self.Res[t]    =  self.Deps[t] * Mr

    def update2(self,t,econ):
        if DEBUG:
            print("credit decisions: credit available {}".format(self.MTC[t]))
        cfirms = 0
        for _,j in econ.credit_list:
            if DEBUG:
                if econ.con[j].CD[t] > 0:
                    print("consumption firm {} is requesting {}".format(j,econ.con[j].CD[t]))
            if econ.con[j].CD[t] > 0 and self.available_credit > 0 and econ.con[j].NW[t-1]:
                if self.available_credit > econ.con[j].CD[t]:
                    if DEBUG:
                        print("Loan {}  to consumption firm {}".format(econ.con[j].CD[t],j))
```

```
                print()
            econ.con[j].CO[t] = econ.con[j].CD[t]
            self.available_credit -= econ.con[j].CD[t]
            cfirms += 1
        else:
            break
    if DEBUG:
        print("bank gave credit to {}/{}, available credit =
{}".format(cfirms,len(econ.con),self.available_credit))
    self.Loans[t]  =  self.MTC[t] - self.available_credit
    self.Pi[t]     =  self.Loans[t] * self.rL + self.Res[t] * rr - self.Deps[t] * self.rD   # profit of the
bank
    self.NW[t]     =  self.NW[t-1] + self.Pi[t]

class Capital_good_firms :
  def __init__(self,i):
    self.S       =  np.zeros(T)            # sales
    self.Q       =  np.zeros(T)            # quantity produced
    self.RD      =  np.zeros(T)            # amount spent on research and development
    self.c       =  np.zeros(T)            # unit cost of production
    self.cAtau   =  np.zeros(T)            # labor unit cost in consumption good industry
    self.IN      =  np.zeros(T)            # funds for innovation
    self.IM      =  np.zeros(T)            # funds for imitating other firms' technology
    self.xA      =  np.zeros(T)            # independent draw of a beta distribution for
productivity A
    self.xB      =  np.zeros(T)            # independent draw of a beta distribution for
productivity B
    self.P       =  np.zeros(T)            # price
    self.Atau    =  np.zeros(T)            # productivity of the machine currently produced
by a F1 in F2
    self.Btau    =  np.zeros(T)            # current productivity in a capital-good firm
    self.Ain     =  np.zeros(T)            # productivity of a new innovative machine
    self.Bin     =  np.zeros(T)            # productivity of the new technology in producing
machines
    self.Aim     =  np.zeros(T)            # productivity of a imitated machine
```

```
        self.Bim        =    np.zeros(T)                    # productivity of technology imitated frpm other
firm
        self.THETAin    =    np.zeros(T)                        # draw from a Bernoulli distribution in the
innovation process
        self.THETAim    =    np.zeros(T)                        # draw from a Bernoulli distribution in the
imitation process
        self.brand      =    i                          # each firm
        self.NW         =    np.zeros(T)                    # liquid assets in the period t
        self.Pi         =    np.zeros(T)                    # total profit of period t
        self.client_list=    [[]for i in range (T)]         # list of clients
        self.client_hist=    []                             # list of the historical clients
        self.PC         =    np.zeros(T)                    # production costs
        self.Ld_i       =    np.zeros(T)                    # labor demand
        self.taxi       =    np.zeros(T)                    # firm's tax

        self.xA         =    scpy.beta.rvs( alpha, beta, loc = lowx, scale = 2*upx, size = T)
        self.xB         =    scpy.beta.rvs( alpha, beta, loc = lowx, scale = 2*upx, size = T)
        self.Atau[0]    =    At0min
        self.Btau[0]    =    Bt0min
        self.NW[0]      =    NW0mini
        self.P[0]       =    (1 + mu_fix) * w0 / self.Btau[0]
        self.cAtau[0]   =    w0 / self.Atau[0]

    def update(self,t,cap,con):
        self.RD[t]      =    nu * self.S[t-1]
        self.IN[t]      =    xi * self.RD[t]
        self.IM[t]      =    ( 1 - xi ) * self.RD[t]
        tech_candidate  =    [(self.Atau[t-1], self.Btau[t-1])]
        self.THETAin[t] =    - np.expm1( max( - zeta * self.IN[t], -100 ))          ### the same as 1 -
np.exp(...)
        self.THETAim[t] =    - np.expm1( max( - zeta * self.IM[t], -100 ))          ### the same as 1 -
np.exp(...)
        ins             =    np.random.binomial( 1, self.THETAin[t] )
        ims             =    np.random.binomial( 1, self.THETAim[t] )
                #
        if ins:
```

```
            self.Ain[t] =   self.Atau[t-1] * ( 1 + self.xA[t] )
            self.Bin[t] =   self.Btau[t-1] * ( 1 + self.xB[t] )
            tech_candidate.append(( self.Ain[t], self.Bin[t] ))
        else:
            self.Ain[t] =   0
            self.Bin[t] =   0
        if ims:
            self.Aim[t],self.Bim[t]   =   closest_tech(cap,t-1, self.brand)
            tech_candidate.append((self.Aim[t], self.Bim[t]))
        else:
            self.Aim[t] =   0
            self.Bim[t] =   0
                    #
        self.Atau[t],self.Btau[t] = min(tech_candidate, key=lambda t:  (1 + mu_fix ) / t[1] + b / t[0])
        self.c[t]       =   econ.W[t] / self.Btau[t]
        self.P[t]       =   ( 1 + mu_fix ) * self.c[t] #como alteramos W já não é preciso w[t-1]
        self.cAtau[t]   =   econ.W[t] / self.Atau[t]

        self.client_list[t]= self.client_hist + list(np.random.randint(N_consumption_firms, size= 20))
        for cgf in self.client_list[t]:
            con[cgf].catalog[t].append((self.P[t], self.cAtau[t], self.Atau[t], self.brand))

    def update2(self,t):
        self.Ld_i[t]    =   self.Q[t] / self.Btau[t]
        self.S[t]       =   self.Q[t] * self.P[t]
        self.PC[t]      =   self.c[t] * self.Q[t]
        if self.S[t] - self.PC[t] - self.RD[t] > 0:
            self.Pi[t]      =   (1-tr)*(self.S[t] - self.PC[t] - self.RD[t])
            self.taxi[t]    =   tr*(self.S[t] - self.PC[t] - self.RD[t])
        else:
            self.Pi[t]      =   self.S[t] - self.PC[t] - self.RD[t]        # Capital-good firms' profit
            self.taxi[t]    =   0
        self.NW[t]      =   self.NW[t-1] + self.Pi[t]                             # Capital-good firms' net
worth

class Consumption_good_firms :
```

```
def __init__ (self, j):
    self.S         =   np.zeros(T)          # sales
    self.PC        =   np.zeros(T)          # total production cost
    self.Deb       =   np.zeros(T)          # stock of debt
    self.Q         =   np.zeros(T)          # quantity produced
    self.Qd        =   np.zeros(T)          # desired quantity to be produced
    self.cI        =   np.zeros(T)          # internal funds allocated production and investment
    self.K         =   np.zeros(T)          # capital
    self.Kd        =   np.zeros(T)          # desired capital
    self.EId       =   np.zeros(T)          # desired replacement investment
    self.Nd        =   np.zeros(T)          # desired inventories
    self.N         =   np.zeros(T)          # stock
    self.f         =   np.zeros(T)          # market share of each firm
    self.mu        =   np.zeros(T)          # variable markup over the cost
    self.E         =   np.zeros(T)          # competitiveness
    self.D         =   np.zeros(T)          # demand
    self.De        =   np.zeros(T)          # demand expected
    self.pi        =   np.zeros(T)          # average productivity
    self.c         =   np.zeros(T)          # total unit cost of production
    self.P         =   np.zeros(T)          # price
    self.RS        =   [[] for t in range(T)]   # machines that need to be replaced
    self.CD        =   np.zeros(T)          # credit demand
    self.CO        =   np.zeros(T)          # credit obtained in period t
    self.NW        =   np.zeros(T)          # liquid assets in the period t
    self.Pi        =   np.zeros(T)          # total profit of period t
    self.Idj       =   np.zeros(T)          # total investment desired by firm j
    self.Iota      =   np.zeros(T)          # unfilled demand
    self.Ld_j      =   np.zeros(T)          # labor demand for each firm
    self.best_buy =    None                 # best machine to be bought
    self.catalog  =    [[] for t in range(T)]   # catalog of machines
    self.machines =    [[] for t in range(T)]   # inventary of machines: list of (Atau, Btau, time
of purchase)
    self.instcap  =    np.zeros(T)          # maximum capacity of production installed
    self.ltsr      =   np.zeros(T)          # liquidity-to-sales ratio
    self.broked    =   False                # condition of the firm
    self.brand     =   j                    # brand
```

```
        self.taxj    =   np.zeros(T)                    # firm's tax
        self.equity  =   np.zeros(T)

        self.NW[0]    =   NW0minj
        self.mu[0]    =   mu_0
        self.Iota[0]  =   0

    def reset (self, other, t):
        self.Deb[t]   =   0
#       self.S[t]     =   other.S[t]
        self.f[t-1:t+1]    =   other.f[t-1:t+1]
        self.NW[t]    =   NW0minj
        self.machines[t]    =   other.machines[t]
#       self.mu[t]    =   other.mu[t]
        self.De[t]    =   other.De[t]
        self.E[t]     =   other.E[t]
        self.N[t]     =   0
        self.Iota[t]  =   other.Iota[t]


    def tech_choice(self,t):
        return min(self.catalog[t], key = lambda prod:  prod[0] + b * prod[1] )

    def update(self,t,econ):
        self.machines[t]=   [m for m in self.machines[t-1] if t-m[3] < eta]
        self.instcap[t]=   sum(m[2] * m[4] for m in self.machines[t-1])
        self.K[t]    =   sum(m[4] for m in self.machines[t-1])
        if self.K[t] == 0: print(self.brand)
        self.pi[t]   =   self.instcap[t] / self.K[t]

        self.De[t]   =   self.D[t-1]
        self.Nd[t]   =   self.De[t] * iota
        self.Qd[t]   =   self.De[t] + self.Nd[t] - self.N[t-1]
        self.c[t]    =   econ.W[t] / self.pi[t]
        self.mu[t]    =    self.mu[t-1] * (1 + upsilon * ((self.f[t-1] - self.f[t-2]) / self.f[t-2])) if t>1 and
self.f[t-2]>0 else self.mu[0]
```

```
        self.P[t]    =   (1 + self.mu[t]) * self.c[t]
        self.E[t]    =   - omega1 * self.P[t] / econ.avgP[t] - omega2 * self.Iota[t-1]

        self.best_buy=   self.tech_choice(t)
        self.RS[t]   =   [m for m in self.machines[t] if self.best_buy[0] <= b * (m[1] - self.best_buy[1])]
###
        core_machines=   [m for m in self.machines[t] if self.best_buy[0] >  b * (m[1] - self.best_buy[1])]
        core_capacity=   sum(m[2] * m[4] for m in core_machines)
        self.Kd[t]   =   max(0, np.ceil((self.Qd[t] - core_capacity) / self.best_buy[2] ))          ## number
of desired new machines
        self.Idj[t]  =   self.best_buy[0] * self.Kd[t]

        self.CD[t]   =   max(0, self.Idj[t] + self.PC[t] - self.NW[t-1])
        if self.CD[t]>   lambda1 * self.S[t-1]:
            if DEBUG:
                print ("error: credit constraint surpassed: {}>{}".format(self.CD[t],lambda1 * self.S[t-1]))
        self.f[t]    =   max(self.f[t-1] * (1 + chi * ((self.E[t-1] - econ.avgE[t-1]) / econ.avgE[t-1])) , 0)
        self.D[t]    =   self.f[t] * econ.C[t-1] / self.P[t]
        #print(self.f[t], econ.C)
        if self.instcap[t] + self.N[t-1] >= self.D[t]:
            self.Q[t]    =   max(0, min(self.D[t] + self.Nd[t] - self.N[t-1], self.instcap[t]))
            self.N[t]    =   self.Q[t] - self.D[t] + self.N[t-1]
            self.S[t]    =   self.D[t] * self.P[t]
            self.Iota[t]=   0
        else:
            self.Q[t]    =   self.instcap[t]
            self.N[t]    =   0
            self.S[t]    =   (self.Q[t] + self.N[t-1]) * self.P[t]
            self.Iota[t]=   1 - (self.Q[t] + self.N[t-1]) / self.D[t]
        self.PC[t]   =   self.Q[t] * self.c[t]
        ### constraints:
        ### - credit request should be positive
        ### - credit request should be smaller than a percentage of previous sales

        self.Ld_j[t] =   self.Qd[t] / self.pi[t]
        self.ltsr[t] =   self.NW[t-1] / self.S[t] if self.S[t] > 0 else 0    ###
```

```
    def update2(self,t,econ):
        if self.CO[t] == self.CD[t]:
            i = self.best_buy[3]
            if self.Kd[t] > 0:
                econ.cap[i].Q[t] += self.Kd[t]
                econ.cap[i].client_hist.append(self.brand)
                new_machine = (*tuple(self.best_buy[:3]), t, self.Kd[t])
                self.machines[t] =  [m for m in self.machines[t] if self.best_buy[0] >  b * (m[1] -
self.best_buy[1])]
                self.machines[t].append(new_machine)

        self.Deb[t] =  self.Deb[t-1] + self.CO[t]
        self.cI[t]  =  self.Idj[t] - self.CD[t]
        if self.S[t] - self.PC[t] - econ.bank[0].rL * self.Deb[t] > 0:
            self.Pi[t]     =  (1-tr)*(self.S[t] - self.PC[t] - econ.bank[0].rL * self.Deb[t])
            self.taxj[t]   =  tr*(self.S[t] - self.PC[t] - econ.bank[0].rL * self.Deb[t])
        else:
            self.Pi[t]     =  self.S[t] - self.PC[t] - econ.bank[0].rL * self.Deb[t]      # Capital-good firms'
profit
            self.taxj[t]   =  0

        self.NW[t]  =  self.NW[t-1] + self.Pi[t] - self.cI[t]
        self.equity[t] =  self.K[t] + self.N[t] + self.NW[t]-self.Deb[t]
        self.broked =  self.NW[t] < 0 or self.f[t] < f_threshold or sum(m[4] for m in self.machines[t])
== 0
        if self.broked:
            econ.broked_firms[t].append(self.brand)
```

6. ODD protocol:

The ODD (Overview, Design and Details) protocol permits to the scientific community to have a more homogeneous account of the different Agent-Based Models and a standardized description that facilitates comparison and the utilization and replication for others. Facilitates the reading and emphasizes the most important aspects that are common to Agent-Based Models. We follow the second update of Grimm et al. (2020)(see also Grimm et al. (2006)).

6.1. Overview

Purpose and patterns

The reason why this model was constructed, as the first predecessor of the family K+S models, Dosi, Fagiolo, & Roventini (2006), states, to incorporate endogenous growth and business cycles. The Dosi et al. (2013) model offers those characteristics as well with a simple credit system that fuels the real sector and is particularly good for studying the impact of policies regarding labor and credit markets in the overall variables of interest to an economy. We could simply start with original K+S model (Giovanni Dosi et al., 2010) but as it lacks the ability to study monetary policies we chose this one.

Entities, state variables and scales

The model has three types of agents: capital-good firms (F1), consumption-good firms (F2), a bank and a labor supply (Ls) that represents both consumers and workers even if they do not make any actual decisions (they do not qualify as agents). Implicitly also has a state who can set a unemployment benefit and collect taxes and an unmodeled central bank who sets the policy rate and the minimum of rate of reserve required for banks to hold (in this case, only one bank).

In the baseline model the:

Capital-good sector has 50 firms;

Consumption-good has 200 firms;

And a monopolistic bank.

Each one of the three sectors characterized by different state variables that define every particular agent in each period *t*. For the capital-goods sector, in each period *t*, every company, *i*, is characterized by its:

Variable	Meaning
S_i	Sales
Q_i	Quantity produced
RD_i	Amount spent on research and development
c_i	Unit cost of production
$c(A_i^\tau, t)$	Labor unit cost in consumption good industry
IN_i	Funds for innovation
IM_i	Funds for imitating other firms' technology
x_i^A	Independent draw of a beta distribution for productivity A
x_i^B	Independent draw of a beta distribution for productivity B
p_i	Price
A_i^τ	Productivity of the machine currently produced by a F1 in F2
B_i^τ	Current productivity in a capital-good firm
A_i^{in}	Productivity of a new innovative machine
B_i^{in}	Productivity of the new technology in producing machines
A_i^{im}	Productivity of a imitated machine
B_i^{im}	Productivity of technology imitated from other firm
θ_i^{in}	Draw from a Bernoulli distribution in the innovation process
θ_i^{im}	Draw from a Bernoulli distribution in the imitation process
MIN	Choosing machine function
NW_i	Liquid assets in the period t
Π_i	Total profit of period t

$client\_list_i$	List of clients
$client\_hist_i$	List of the historical clients
PC_i	Total production costs
Ld_i	Labor demand

Table 2 - State variables of capital-goods sector (own elaboration)

For the consumption-good sector, in each period *t*, every company, *j*, is characterized by its:

Variable	Meaning
S_j	Sales
PC_j	Total production cost
Deb_j	Stock of debt
Q_j	Quantity produced
Q_j^d	Desired quantity to be produced
cI_j	Internal funds allocated production and investment
K_j	Physical capital (number of machine tools)
K_j^d	Desired additional capital
N_j^d	Desired inventories
N_j	Actual inventaries
f_j	Market share of each firm
μ_j	Variable markup over the cost
E_j	Competitiveness
D_j	Demand
D_j^e	Demand expected
π_j	Average productivity

c_j	Total unit cost of production
p_j	Price of the good produced
RS_j	Machines that need to be replaced
CD_j	Credit demand
CO_j	Credit obtained in period t
NW_j	Liquid assets in the period t
Π_j	Total profit of period t
I_j^d	Total investment desired by firm j
I_j	Unfilled demand
Ld_j	Labor demand for each firm
$best\_buy_j$	Best machine to be bought
$catalog_j$	Catalog of machines
$machines_j$	Stock of machines: list of price, labor cost, productivity, period of purchase, number of machines with the same technology
$instcap_j$	Maximum capacity of production installed
$ltsr_j$	Liquidity-to-sales ratio
$broked$	Condition of the firm: True or False

Table 3 - State variables of consumption-goods sector (own elaboration)

The financial sector has only one bank that is defined by the following variables in each period, *t*:

Variable	Meaning
MTC	Maximum total credit available
$Loans$	Total loans conceded to the economy
$Deps$	Aggregated deposits of the firms

$\boldsymbol{Res}$	Reserves
$\boldsymbol{\Pi_b}$	Profit of the bank
$\boldsymbol{NW_b}$	Net worth of the bank

Table 4 - State variables of the banking sector (own elaboration)

Process overview and scheduling

In any given time period, *t*, the following microeconomic decisions take place in the following sequential order:

1. Policy variables are determined (interest rates; tax rate; unemployment subsidy);
2. Capital-good firms perform R&D to innovate or to imitate other firms, to get the best pair of technologies A_i^τ, B_i^τ that minimizes equation 3.10 and advertise their machines to consumption-good firms;
3. Consumption-good firms compute their desired quantity to produce and the machines necessary to produce that quantity. In addition, firms verify the obsolescence of their machines and count the machines that need replacement. The model distinguishes both types of investment, the substitution investment (RId) and the expansion investment desired (EId). As external funds are expensive and conditioned, firms will spend first their funds and if necessary will borrow from banks. Firms that need credit, calculate their credit demand (CD) summing their investment and production necessities since both labor and the machine-tools (or machines) are payed in advance.
4. The total amount of disposable credit in the economy (equation 3.31) is distributed by the bank to each firm, satisfying firstly the firms with highest liquidity-to-sales ratio which means that if the sum of the all credit demands exceeds the maximum credit available, firms can be rationed;
5. In both sectors firms hire workers according to their plans or the funds that they have specially in what respects to consumption-good firms;
6. The market for consumption goods "opens". The demand for each firm is determined by the market share (*f*) of the firm which in turn depends on price competitiveness;

7. Firms in both sectors compute their profits and net liquid assets after paying back their due loans to the bank to the extent that they have liquidity to do that and deposit their savings, if any;

8. In both sectors, firms with near-zero market shares and/or negative net liquid assets are expelled (Schumpeterian part) from their industry and replaced by new and identical firms (number of firms is maintained fixed);

9. Machines that were ordered at the beginning of the period are delivered in the end of the same period but only become part of the capital stock in t+1.

10. At the end of each time step, aggregate variables are computed, summing over the corresponding microeconomic quantities.

6.2. Design

Basic principles – As in all K+S models, we are dealing with a model whose main driver is technological change to create endogenous growth and business cycles together with the reflections of Keynesian dynamics in the relations between aggregate variables. This one in particular, has a simple credit system from which we can derive some monetary policy implications related to interest rates. In this model, we pretend to study questions of inequality in income distribution in individuals as in companies earnings and how monetary policy impacts that distribution.

Emergence – the system develops itself first from the machine-tool sector and the heterogeneity of productivities or technologies is the main force behind its dynamics and tend to create concentration of sales both in capital and in consumption-good sector. Both industries seem to concentrate their activity around a few companies after a good number of periods.

Adaptation – firms in the capital-good sector chose which machine-tool to produce from a maximum of tree technologies sets, (A_i, B_i): one resulted from innovation process (A_i^{in}, B_i^{in}), if succeeded; another from an imitation process, (A_i^{im}, B_i^{im}) if succeeded; and one technology pair inherited from the previous period, (A_i^{τ}, B_i^{τ}). The firm predicts the technology that fits best its clients minimizing equation 3.10. Consumption firms who receive the brochure of company, *i,* compare the price and productivity, A_i, according with equation 3.10, with all the stock of machine-tools and decide if productivity compensates the price and how many machines-tools to order. Firms also discard machines other than η periods.

Objectives – in the two adaptive behaviors listed above: we describe the way capital-good firms choose which machine to produce from the technologies resulted from previous or from the period's R&D; and how consumption-good firms choose which machines to order, and therefore their supplier. It happens that capital-good firms anticipate how consumption firms choose and predict the same criterion demonstrated in equation 3.10.

Learning – Learning is not implemented.

Fitness – the system is evaluated with basis on the dynamics of its main aggregate variables such as the output, output growth rate, aggregate consumption, unemployment rate and investment. Also very important, is to verify the distribution of profits between sectors and inside them, if market shares are concentrated or disperse among the majority of firms and if firms who are technically broke, are not participating in the market, for example, in borrowing credit.

Prediction – agents do not foresee the consequences of their actions although, there are some built-in restrictions in the model, specially, in what concerns credit demand and consequently investment concerning consumption-good firms, as the dependence of R&D from previous periods' sales in what respects capital-good firms.

Sensing – it is assumed that capital-good firms have information about other's capital-good firms technologies in the imitation process and that they imitate the closest pair of technologies. The bank is also conscientious of the financial situation of each firm.

Interaction – capital-good firms interact first with consumption-good firms, by sending them (historical clients plus some new ones) an information brochure about the machine-tools for sale. Consumption firms react, sending their orders to capital firms and ask credit to banks according to their credit demands. Bank lends to consumption-good firms. Then aggregate consumption is distributed according to each firms' market share.

Stochasticity – there is some level of stochasticity in the model since, if from the Bernoulli draw, the innovation process is succeeded, then the pair of productivities, (A_i^{in}, B_i^{in}), is determined from two independent and random draws of a Beta distribution x_i^A and x_i^B (see equations 3.7 and 3.8). New clients (NC_i) too, are attributed randomly by the code interpreter. Firms which are the substituents of bankrupt ones, mimic some characteristics of existing firms who are chosen randomly .

Collectives – each collective group coincides with the type of agent, capital-good firms, consumption-good firms with the difference that we only have one bank (it could be a collective, but for the sake of simplicity there is only one). In this case the traits of each firm from both sectors impose the formation of these collectives, in other words, they are not an endogenous phenomenon of the model.

Observation – the kind of results that we will observe in this model will be mainly aggregate and omniscient results at the level of the economy and at the sector level.

6.3. Details

Initialization – We initialize our model, first, by setting a wage in period 0 that begins in 1 because we only care about the relative variations of wages. Then define the pair of productivities (A_i^{τ}, B_i^{τ}) in *t=0* that will allow capital-good firms to make the first sell. These productivities can begin in 1 and can vary randomly generated following a uniform distribution, within an interval to create heterogeneity between firms. To simplify, consumption-good firms only have one type of machine, in other words, machines in the same firm have all the same productivity, this only in period 0, that is, the initial time step. After that, we provide to firms from both sectors a stock of liquid assets NW, which can variate within an interval at the semblance of what we made with the pair of productivities.

Similar to W0, the consumer price index is set in 1. Then, we define a labor supply and an initial unemployment rate.

Input data – The model does not use input data to represent time-varying processes.

Submodels – There is no submodels.

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1. **Annex** *(if any)*

final

GRADEMARK REPORT

FINAL GRADE

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GENERAL COMMENTS

Instructor

PAGE 1

PAGE 2

PAGE 3

PAGE 4

PAGE 5

PAGE 6

PAGE 7

PAGE 8

PAGE 9

PAGE 10

PAGE 11

PAGE 12

PAGE 13

PAGE 14

PAGE 15

PAGE 16

PAGE 17

PAGE 18

PAGE 19

PAGE 20

PAGE 21

PAGE 22

PAGE 23

PAGE 24

PAGE 25

PAGE 26

PAGE 27

PAGE 28

PAGE 29

PAGE 30

PAGE 31

PAGE 32

PAGE 33

PAGE 34

PAGE 35

PAGE 36

PAGE 37

PAGE 38

PAGE 39

PAGE 40

PAGE 41

PAGE 42

PAGE 43

PAGE 44

PAGE 45

PAGE 46

PAGE 47

PAGE 48

PAGE 49

PAGE 50

PAGE 51

PAGE 52

PAGE 53

PAGE 54

PAGE 55

PAGE 56

PAGE 57

PAGE 58

PAGE 59

PAGE 60

PAGE 61

PAGE 62

PAGE 63

PAGE 64

PAGE 65

PAGE 66

PAGE 67

PAGE 68

PAGE 69

PAGE 70

PAGE 71

PAGE 72

PAGE 73

PAGE 74

PAGE 75