Essays on Collusion: Mixed Oligopolies and Corporate Social Responsibility

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Doctoral Thesis in Economics
2020

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Biographical Note

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In 2017, she was invited by Católica Porto Business School to teach at the undergraduate degrees in Economics and Management, where she has been working as a teaching assistant in the courses of Microeconomics, Environmental Economics, International Management and Multidisciplinary Project. Since 2019, she has also worked as a consultant at CEGEA (Research Centre in Management and Applied Economics of the Católica Porto Business School), where she develops applied research in different fields.

Her research on collusion has been presented in multiple conferences, such as the 12th Annual Meeting of the Portuguese Economic Journal (Lisbon, Portugal, 2018), the 45th European Association for Research in Industrial Economics (EARIE) Conference (Athens, Greece, 2018) and the 28th Eurasia Business and Economics Society (EBES) Conference (Coventry, UK, 2019). One of her papers is published in the Journal of Industry, Competition and Trade.
I would like to express my deepest gratitude to Professor João Correia da Silva and Professor Joana Pinho, my research supervisors, for their guidance, assistance and advice throughout these years. To my co-author and friend Mariana Cunha, my gratitude for her support, encouragement and friendship over the past few years. I extend my deepest thanks to Professor Duarte Brito, Professor Joana Resende, Professor Odd Rune Straume and Professor Yassine Lefouili for very useful comments and suggestions, which allowed me to improve this thesis. My grateful thanks extend to Helder Vasconcelos, Valanta Miliou, all the participants at the conferences attended and two anonymous referees of the Journal of Industry, Competition and Trade. I gratefully acknowledge the financial support from FCT through the PhD scholarship SFRH/BD/100022/2014, the European Regional Development Fund through COMPETE 2020 - Programa Operacional Competitividade e Internacionalização (POCI) and Portuguese public funds through FCT in the framework of the project POCI-01-0145-FEDER- 006890.

To Professors António Brandão, Maria do Rosário Moreira, M. Alper Çenesiz and Pedro Mazeda Gil, I would like to devote a word of gratitude and appreciation for their advice and guidance through the adversities faced in my ten-year journey as a FEP student. To my PhD colleagues and friends, in particular Ana Isabel, Ana Sá, António, Bernardo, Carlos, Cláudia, Diogo, Francisco, Göksu, Isabel, Joana, José, Lia, Mafalda, Rodrigo, Sandra and Sara, my sincere thanks for their companionship, support and motivation. I extend my thanks to all my colleagues at Católica Porto Business School.

Finally, a special word of deep gratitude goes to my husband, André, for his unconditional support and encouragement throughout this journey and every day of my life. To my father, sister, grandma, cousin, aunt, in-laws and friends for their infinite support.
Abstract

This thesis comprises three essays on collusion, whose conclusions draw on the importance of firms’ objective functions. Two essays are included in the literature on mixed oligopolies and the other in the literature on corporate social responsibility.

In the first essay, we provide a discussion on the conditions for collusion to be sustainable in a mixed oligopoly. By allowing the public firm to be part of the collusive agreement, we show that the objective function of the public firm affects the sustainability of collusion. While the motivation to engage in the collusive agreement differs among the two types of firms, there are possible gains from colluding to both. Our conclusions show that collusion is most sustainable when firms are symmetric, i.e., when both maximize profits, which corroborates the traditional results in the literature on collusion. Finally, we show that collusion may improve consumer’s welfare, for intermediate levels of the weight the public firm attaches to consumer surplus, and total welfare for sufficiently high levels of this concern for consumers.

In the second essay, we consider a private duopoly where firms may have some degree of corporate social responsibility. In line with the existing literature, we model corporate social responsibility as a weight attached to consumer surplus in firms’ objective functions. Considering a symmetric case, where firms have the same level of social concern, and an asymmetric case, where firms have different levels of social concern, our results confirm that assigning a positive weight to consumer surplus makes collusion harder to sustain, as shown in the literature. However, extending the existing literature on corporate social responsibility and collusion by considering product differentiation, we also show that the degree of product differentiation affects collusion sustainability. For a sufficiently high level of social responsibility, collusion sustainability is actually increasing in the degree of product substitutability.
when firms are symmetric in the weight they attach to consumer surplus. If, instead, firms have different levels of social concern, collusion sustainability is increasing in the degree of product complementarity. Finally, we conclude that collusion may be welfare-improving when firms adopt a socially responsible behavior if goods are sufficiently differentiated or if the level of social concern is sufficiently high.

In the third essay, we propose to bring a new perspective to the existing literature on collusion in mixed markets by considering a vertically related market. In our setting, an upstream monopolist sells an essential good to the downstream retailers. The downstream market constitutes a mixed duopoly, where a public retailer values both profits and consumer surplus and a private retailer, with a typical behavior, maximizes profits. Assuming that the upstream monopolist may face either linear or quadratic costs in the production of the essential input it supplies to the downstream firms, we conclude that downstream collusion is sustainable as long as firms are sufficiently patient, but it is never beneficial to the upstream monopolist or consumers. This provides an important result to competition authorities as, in opposition to our previous conclusions, collusion is shown to be welfare-damaging to consumers as posed by most of the existing literature.

Our results contribute with some new insights to the literature on collusion and to competition law enforcement, in particular against cartels.


Keywords: Collusion, Corporate Social Responsibility, Firm’s Objectives, Mixed Markets, Product Differentiation.
Resumo

Esta tese compreende três ensaios sobre colusão, cujas conclusões se baseiam na importância das funções objetivo das empresas. Dois ensaios estão incluídos na literatura sobre oligopólios mistos e o outro na literatura sobre responsabilidade social corporativa.

No primeiro ensaio, contribuímos para a discussão sobre as condições para que a colusão seja sustentável num oligopólio misto. Ao permitir que a empresa pública faça parte do acordo colusivo, mostramos que a função objetivo da empresa pública afeta a sustentabilidade do acordo. Embora a motivação para cooperar difira entre os dois tipos de empresas, existem ganhos potenciais da colusão para ambas. As nossas conclusões mostram que a colusão é mais sustentável quando as empresas são simétricas, ou seja, quando ambas maximizam os lucros, o que corrobora os resultados tradicionais da literatura. Por último, mostramos que a colusão pode melhorar o bem-estar dos consumidores, para níveis intermédios do peso que a empresa pública atribui ao excedente do consumidor, e o bem-estar total, para níveis suficientemente elevados desta preocupação com os consumidores.

No segundo ensaio, consideramos um duopólio privado cujas empresas podem ter algum grau de responsabilidade social corporativa. Em consonância com a literatura existente, modelamos a responsabilidade social das empresas como um peso associado ao excedente do consumidor nas funções objetivo das empresas. Considerando um caso simétrico, no qual as empresas têm o mesmo nível de preocupação social, e um caso assimétrico, no qual as empresas têm diferentes níveis de preocupação social, os nossos resultados confirmam que a atribuição de um peso positivo ao excedente do consumidor dificulta a sustentabilidade do acordo colusivo, como mostra a literatura. No entanto, alargando a literatura existente acerca de responsabilidade social corporativa e colusão considerando a diferenciação de produto,
também mostramos que o grau de diferenciação afeta a sustentabilidade do acordo. Para um nível de responsabilidade social suficientemente elevado, a sustentabilidade da colusão é crescente com o grau de substitutibilidade dos produtos quando as empresas são simétricas no peso que atribuem ao excedente do consumidor. Se, pelo contrário, as empresas tiverem diferentes níveis de preocupação social, a sustentabilidade da colusão aumenta com o grau de complementaridade dos produtos. Por último, concluímos que o cartel pode melhorar o bem-estar dos consumidores quando as empresas adotam um comportamento socialmente responsável se os bens forem suficientemente diferenciados ou se o nível de preocupação social for suficientemente elevado.

No terceiro ensaio, propomos uma nova perspetiva à literatura existente sobre colusão em mercados mistos, considerando um mercado verticalmente relacionado. No nosso modelo, um monopolista a montante vende um bem essencial aos retalhistas a jusante. O mercado a jusante constitui um duopólio misto, em que um retalhista público considera tanto os lucros como o excedente do consumidor e um retalhista privado, com um comportamento típico, maximiza os lucros. Assumindo que o monopólio a montante pode enfrentar custos lineares ou quadráticos na produção do bem essencial que fornece às empresas a jusante, concluímos que o conluio a jusante é sustentável se as empresas forem suficientemente pacientes, mas nunca é benéfico para o monopolista a montante ou para os consumidores. Este é um resultado importante para as autoridades da concorrência, uma vez que, em oposição às nossas conclusões anteriores, se mostra que a colusão é prejudicial para o bem-estar dos consumidores, tal como demonstra a maior parte da literatura existente.

Os nossos resultados constituem novos contributos para a literatura de colusão e para as atividades de regulação, em particular na luta contra os cartéis.

**Códigos de Classificação JEL:** D42, D43, H44, L13, L21, L41, L51, M14.

**Palavras-chave:** Colusão, Diferenciação do Produto, Mercados Mistos, Objetivos das Empresas, Responsabilidade Social Corporativa.
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Chapter 1

Introduction

Antitrust enforcement requires a deep understanding of the welfare impacts of firms’ interaction and, thus, rigorous economic analysis. In this context, theoretical models provide important results to competition authorities and constitute a strategic tool at their disposal. Having this in mind, the aim of this thesis is to provide new insights to competition authorities regarding cartel enforcement. The battle against hard core cartels is at the core of antitrust enforcement worldwide. Cartels are prohibited according to Article 101 TFEU in the European Union and Section 1 of the Sherman Act in the United States. However, even without explicit collusion, it is possible that firms collude tacitly. Friedman’s (1971) Folk Theorem states that any payoff above the static Nash payoffs can be sustained as a subgame perfect equilibrium of the infinitely repeated game if the players are sufficiently patient, i.e., repeated interaction between firms may allow for coordinated behavior to occur and be sustained.1 In the EU, Court decisions have frequently handled tacit collusion under the notion of collective dominance, which corresponds to the notion of “coordinated effects” used in the US (Ivaldi et al., 2003).

This thesis comprises three essays on collusion, which are, to some extent, related to each other. It is organized in five chapters. This first chapter provides an overview of its structure

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1Fudenberg and Maskin’s (1986) Folk Theorem is an even stronger result.
and introduces the reader to the research topics. Each research topic is approached in its own chapter as an independent essay, as follows.

Chapter 2 presents the first essay of the thesis. In this essay, we provide a discussion on the conditions for collusion to be sustainable in a mixed oligopoly. Mixed oligopolies are receiving increasing attention in the literature since the role of state-owned enterprises in the markets is growing worldwide (OECD, 2016). Many sectors are characterized by having state-owned firms competing with private firms (for instance, the airlines industry). Naturally, strategic interactions between these two types of firms may arise and bring new implications for competition authorities to address. Empirical evidence shows that cartel cases have been found involving not only private but also public firms. But while the literature on collusion has been developed for decades now, there is still scope to analyze the effects of coordinated practices between private and public (or semi-public) firms. Addressing this topic, our novelty is in considering that the state-owned firm may be a member of the cartel. We study how the objective function of the state-owned firm affects its willingness to tacitly collude, and the sustainability of the cooperative outcome. Not surprisingly, we conclude that private collusion is easiest sustained than mixed collusion, i.e., collusion is easier to sustain when firms are symmetric and maximize profits. Even though the private firm’s incentives to collude are increasing for sufficiently high levels of concern for consumers by the public firm, the public firm’s incentives to collude behave contrarily. The higher is the level of concern for consumers, the higher are the incentives for the public firm to deviate by producing a larger output. Finally, we provide an important insight to competition authorities as we find that collusion may be welfare-improving to consumers, in particular, and to the society in general.

Chapter 3 presents the second essay. This essay is related to the previous one in the sense of analyzing collusion when firms’ objective functions are broader than their profit functions. However, differently from the previous, this paper belongs to the literature on Corporate Social Responsibility since the firms we model are private firms. We analyze the coordinated effects of Corporate Social Responsibility in a setting where firms include a component of consumer’s
welfare in addition to profits in their objective function, produce differentiated goods and compete in quantities. We consider a symmetric case, where firms have the same level of social responsibility, and an asymmetric case, where firms have different levels of social responsibility. Our results confirm that assigning a positive weight to consumer surplus makes collusion harder to sustain, as previously shown in the literature. However, for a sufficiently high level of social responsibility, collusion sustainability is actually increasing in the degree of product substitutability when firms are symmetric regarding the level of social concern. If firms are, instead, asymmetric in this component, then collusion sustainability is increasing in the degree of product complementarity. Further, as in the first essay, we show that collusion may be welfare-improving when firms adopt a socially responsible behavior, which provides an interesting background to competition authorities when analyzing cartel cases involving socially responsible firms.

In the first two essays we conclude that collusion may be welfare-improving. This result, we should stress, derives naturally from our modelling assumptions. Considering a vertically related market structure where the downstream market is a mixed duopoly, as in Chapter 2, and the upstream market is a monopoly, our results regarding collusion between a private and a public (or semi-public) firm are different from the previous. This leads us to our third essay, which is presented in Chapter 4.

In this essay, we consider a vertically related market constituted by an upstream monopoly and a downstream mixed duopoly. To the best of our knowledge, this is the first contribution to the literature on mixed oligopolies to analyze collusion in a vertically related market. In our model, the upstream manufacturer produces an essential input to the production of the final good by the downstream retailers. The upstream monopolist is a profit-maximizer as is one of the downstream retailers, while the other retailer is a public firm. While the private firms maximize profits, the public firm maximizes a weighted sum of profits and consumer surplus. We assume that the upstream monopolist has either linear or quadratic production costs and the downstream firms face a linear demand function for the final good. We conclude that the
collusive agreement is sustainable if firms are sufficiently patient, and that downstream firms’
incentives to collude depend on the existence of economies or diseconomies of scale and on the
weight the public firm attaches to consumer surplus in its objective function. If the upstream
monopolist faces increasing or constant marginal costs, the private firm has more incentives
to deviate from the agreement than the public firm; by contrast, the opposite result holds if
the upstream marginal costs are decreasing. Finally, in this model, downstream collusion
is always detrimental to final consumers and to the upstream firm, since output is contracted
under collusion. This is a relevant result to competition authorities as, in opposition to what we
have concluded so far, in the previous essays, even if one cartel member cares about consumer
welfare, that is not sufficient to make collusion welfare-improving.

Chapter 5 concludes by summarizing our research questions, methods and main results.
Some suggestions for future research are also presented in this final chapter.
Chapter 2

Welfare-Improving Mixed Collusion

2.1 Introduction

The role of state-owned enterprises in the markets is growing worldwide, in contrast to the (opposite) trend in the years before the financial crisis (OECD, 2016). In many markets, state-owned enterprises compete with private firms. The interaction between these distinct types of firms gives rise to new concerns and challenges to competition law enforcement, in general, and to cartel prosecution, in particular.

There are some sectors that, for their strategic nature or degree of (de)regulation, are more prone to cooperation between firms. The airlines industry is a typical example of such markets, specially when one of the players is stated-owned. Some cartel cases have been found in

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1This is a joint work with João Correia da Silva (FEP and CEF.UP) and Joana Pinho (Católica Porto Business School and CEGE). We thank Duarte Brito, Joana Resende and seminar participants at the 12th Annual Meeting of the Portuguese Economic Journal (Lisbon, 2018), the 45th EARIE Conference (Athens, 2018) and 28th EBES Conference (Coventry, 2019) for their valuable comments and suggestions.

2Due to the substantial demand expansion since the 1980s (as a result of the global reach of airline networks, as well as decreasing prices and growing income), both passengers and cargo air traffic has grown. According to OECD (2016), the volume of world trade shipments carried by air amounts to 35% of the total in terms of its value. The global extent of this industry has been giving place to the emergence of multilateral alliances. These alliances, typically used to reduce costs and/or to share risks, may potentially have detrimental impacts on competition and, ultimately, on consumers. As pointed out by Reitzes and Moss (2008), the harm of such anti-competitive actions in the airlines industry may have serious effects.
recent years in the airline industry, some of them involving state-owned firms. For instance, in 2010, the European Commission imposed fines on eleven air cargo carriers for the participation in a price fixing cartel, three of which were partially state-owned firms. However, the airline industry is not the only one where joint illegal agreements between private and state-owned firms have been found. Very recently, in 2019, the Portuguese Competition Authority (AdC) imposed a fine of 225 million euros to 14 banks, including the Portuguese public bank CGD, for the concerted practice of exchanging sensitive commercial data, between 2002 and 2013.

Other examples of cartel cases can be traced back to 1997 up to recent years. In 1997, the Spanish Competition Authority fined a price-fixing cartel formed by industrial dairy firms and led by a state-owned firm. In 2004-2005, the OECD Competition Directorate recommended penalties to be imposed on all Tunisian banks, including state-owned banks, which were found to be on a price-fixing cartel (OECD, 2013). In 2008, the Egyptian National Cement Company was found to be part of a price-fixing cartel in the cement market (OECD, 2013). The Egyptian Competition Authority referred the case to courts, and the managers of the cartelized firms (both from the private companies and the state-owned company) were fined. Also in 2008, the Hungarian Competition Authority (GVH) imposed a fine of approximately 4 million euros on the parties involved in a cartel on the market of rail freight transport services. The GVH established that the undertakings under investigation were conducting restrictive market practices on the liberalized market by: i) applying uniform prices for railway freight forwarding; and ii) market sharing. In 2015, the European Commission fined three big logistics providers for operating a price-fixing cartel on the provision of rail cargo transport services in connection with blocktrains, lasting from 2004 to 2012. Also in 2015, the French Competition Authority

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3 The companies involved were Air Canada, Air France-KLM, British Airways, Cargolux, Cathay Pacific Airways, Japan Airlines, LAN Chile, Martinair, Qantas, SAS and Singapore Airlines, and Lufthansa and its subsidiary Swiss international Airlines, to whom full immunity was given under the Commission’s 2006 Leniency Notice. Of the previous, Air France-KLM, SAS and Singapore Airlines were partially state-owned firms.

4 Case PRC/2012/9. The banks fined were BBVA, BIC (for practices of the then BPN), BPI, BCP, BES, BANIF, Barclays, CGD, Caixa de Crédito Agrícola, Montepio, Santander (for its own actions and those of Banco Popular), Deutsche Bank and UCI.

5 Case 352/94, Industrias Lácteas.


7 Case AT.40098 - Blocktrains. ÖBB Group, Austria’s largest mobility services provider and state-owned, and two of its
fined twenty logistics firms and their trade association for a price-fixing cartel that lasted from 2004 to 2010.\textsuperscript{8,9,10}

Even when firms are not state-owned, we can find evidence of cartel formation enforced by state intervention. At least two examples can be provided: one involving a raisin cartel with the support of the California state government - worried about a crisis of oversupply of California raisins;\textsuperscript{11} and the 2003 case in which Italy was required by EU law to no longer apply its law introducing quotas for the sale of matches in Italy.\textsuperscript{12}

Despite all this evidence of cartels involving public and private firms, there are still very few contributions to the economic literature that analyze the impacts of this type of cooperation. Thus, we aim at contributing to fill this gap, by considering that the state-owned firm may be a member of the cartel. In particular, we study the impact of public ownership on the conditions for collusion sustainability. We allow for different objective functions of the state-owned firm, that differ on the weight that this firm attaches to consumer surplus vis-à-vis its own profit. The weight the state-owned firm attaches to consumer surplus can enlighten us on how the objective of this firm affects its willingness to join the collusive agreement, and the sustainability of the agreement itself. Henceforth, we will use the term public firm to denote the state-owned firm. In our model, we assume a mixed duopoly where a public and a private firm play an infinitely repeated game, make simultaneous decisions regarding the quantities to produce and may compete or collude in those decisions. We assume these firms are asymmetric regarding their objective functions, but symmetric regarding everything else. Hence, while the public firm maximizes a weighted sum of consumer surplus and profits, the private firm maximizes profits. Both firms face a similar cost function, with increasing marginal costs, which, as we

\textsuperscript{8}Décision n. 15-D-19 du 15 décembre 2015.
\textsuperscript{9}Geodis, the logistics arm of France’s state-owned railway SNCF, faced the biggest fine. Royal Mail, one of the firms involved, was public at the time and privatized less than a year after the potential fine was first announced in July 2014.
\textsuperscript{10}Regarding state-owned cartels, Martyniszyn (2010) studies the problem of applicability of competition laws to a foreign state, describing repetitive unsuccessful efforts, by the US, to sue OPEC and export cartels of Chinese state-owned enterprises.
\textsuperscript{12}Case C-198/01, Consorzio Industrie Fiammiferi v. Autorità Garante della Concorrenza e del Mercato (Italian matches), 2003 E.C.R. I-8055.
will show, plays a very important role in our results.

The public firm uses the collusive agreement to give part of its production to the private firm, allowing it to produce less and, therefore, to decrease production costs. The private firm accepts to increase production as long as the gain in market share generates higher profits. Then, both firms can be better off under collusion than under competition. Regarding the sustainability of the collusive agreement, while the private firm’s incentives to collude are increasing for sufficiently high levels of the concern for consumers of the public firm, the incentives of the public firm to collude behave in the opposite way. The intuition is that the higher is the weight attached to consumer surplus, the higher the quantity the public firm desires to produce. Thus, it has incentives to deviate from the collusive agreement by producing a larger output. In turn, when deviating from the agreement, the private firm restricts output, which is natural since it is a profit-maximizer. We conclude that collusion is, therefore, easiest sustained when firms are symmetric, i.e., when both firms maximize profits. This result is not surprising, being in line with the standard literature on collusion.

Furthermore, we find that collusion is welfare-improving to consumers for intermediate levels of the weight the public firm attaches to consumer surplus. Total surplus is also higher under collusion, but for sufficiently high levels of concern for consumers by the public firm. This is so because collusion increases the payoffs of both firms, the profits of the private firm and the payoff function of the public firm through the component attached to consumer surplus, but it may also increase the profits of the public firm. In the light of these results, competition authorities should be careful in the analysis of cartel cases in mixed oligopolies, since collusion may either increase or decrease both consumer and social surplus.
Related Literature

Since the pioneer contributions of Merrill and Schneider (1966) and Sertel (1988), some authors have assumed that private firms may behave cooperatively in mixed oligopolies. Recently, Colombo (2016) and Correia-da-Silva and Pinho (2018) contributed to this literature by analyzing how the presence of a public firm in the market affects the sustainability of collusion among private firms. Colombo (2016) concludes that an increasing public ownership of the non-colluding firm may help collusion between the private firms, as the public firm tends to expand output, making the punishment harsher and, thus, decreasing the incentives to deviate. Correia-da-Silva and Pinho (2018) find that privatization always makes collusion among private firms easier to sustain. These two contributions consider, however, that the public firm is outside the cartel, while we assume that it is a cartel member.

Escrihuela-Villar (2008) shows that collusion is much easier to sustain between similar firms. In fact, symmetry may explain why many papers analyzing collusion in mixed markets assume that the public firm (or semi-public) firm does not participate in the cartel. To the best of our knowledge, Wen and Sasaki (2001) were the first to consider strategic interactions between a welfare-maximizing public firm and a profit-maximizing private firm. The authors consider that the public firm can hold excess of capacity as a strategic punishment to sustain a welfare-improving agreement. They conclude that, facing symmetric marginal costs, the welfare-maximizing equilibrium is only efficient when the public firm is less cost-efficient in capacity investment than the private and the discount factor is sufficiently high. The very recent contribution of Haraguchi and Matsumura (2018) is the closest to ours in purpose: to discuss welfare-improving collusion in a mixed oligopoly. However, there are important differences between their work and ours. Firstly, this being the main difference between the two contributions, while Haraguchi and Matsumura (2018) assume that the public firm proposes a (welfare-improving) collusive agreement to the private firm (that may or may not accept it), we assume that firms choose quantities simultaneously, and, still, welfare-improving collusion may occur and be sustainable. Secondly, we consider homogeneous goods and Cournot com-
petition, while they assume differentiated goods and linear demand à la Dixit (1980). Finally, they consider constant marginal costs, while we consider increasing marginal costs. Since, as in our model, social welfare is naturally increasing due to the assumptions made, Haraguchi and Matsumura (2018) focus on the comparison between Cournot and Bertrand competition. They find that Cournot competition is more welfare-improving when the discount factor is sufficiently large, whereas Bertrand competition yields greater welfare when the discount factor is small.

Our contribution also relates to another branch of the literature that studies the sustainability of collusion among heterogeneous firms. Most of the existing contributions assume cost asymmetries (Donsimoni, 1985; Bae, 1987; Harrington, 1991; Verboven, 1997; Rothschild, 1999; Vasconcelos, 2005; and Ganslandt et al., 2012; Correia-da-Silva and Pinho; 2016), but also heterogeneity in firms’ capacities (Compte et al., 2002; Bos and Harrington, 2010) or in the discount factors (Harrington, 1989; Gerson, 1989). The source of heterogeneity across firms in our model is the objective function. Finally, in the analysis of our model, we borrow some techniques used by Harrington (1991),13 in the determination of the Nash bargaining solution, and by Harrington (1991) and Verboven (1997), in the determination of the Pareto-optimal equilibria.

The remainder of this paper is organized as follows. Section 2.2 introduces the model. Section 2.3 provides the model analysis, including the study of different strategic behaviors, and the conditions for collusion to be sustainable in equilibrium. Section 2.4 analyzes the results. Section 2.5 concludes.

13Note that the use of the Nash Bargaining solution can be traced back to Harsanyi and Selten (1972), and to Myerson (1979). However, the context in which we use it is closest to Harrington (1991).
2.2 Setup

Consider an industry with one public firm, $g$, and one private firm, $p$, producing homogeneous goods, competing in quantities and interacting for an infinite number of periods. In each period, firms set their quantities, $q_g$ and $q_p$, simultaneously. The inverse demand is linear in quantities and it is given by $p = 1 - Q$, where $Q = q_p + q_g$ is the total output.\(^\text{14}\)\(^\text{15}\) Following De Fraja and Delbono (1989) and Correia-da-Silva and Pinho (2018), we assume that the total cost of producing $q$ units of the good is the same for the public and the private firm, and it is given by $C(q) = \frac{q^2}{2}$. Thus, the profit function of firm $i = \{p, g\}$ is given by:

$$\pi_i(q_g, q_p) = (1 - q_g - q_p)q_i - \frac{q_i^2}{2}.$$ \hfill (2.1)

Firms are symmetric in every dimension but one: their objective functions. The private firm aims at maximizing its individual profit, while the public firm aims at maximizing a weighted sum of the consumer surplus (CS) and its individual profit, as follows:

$$\Omega(q_g, q_p) = \mu \text{CS} + (1 - \mu)\pi_g = \mu \frac{(q_g + q_p)^2}{2} + (1 - \mu) \left[ (1 - q_g - q_p)q_g - \frac{q_g^2}{2} \right],$$ \hfill (2.2)

where parameter $\mu \in \left(0, \frac{2}{3}\right)$ measures the weight the public firm attaches to consumer surplus.\(^\text{16}\) Parameter $\mu$ can also translate the fact that firm $g$ is actually a semi-public firm, in which case it has to consider both consumer surplus, to please the state, and profits, to please shareholders (Omran et al., 2002).\(^\text{17}\) When $\mu = 0$, the firm is purely a profit-maximizer and,

---

\(^{14}\)This is a particular case of the demand function in Haraguchi and Matsumura (2018) when $\gamma = 1$.

\(^{15}\)We assume demand to be stationary. Additional challenges could arise by assuming demand growth and entry. These challenges were studied in the literature by Vasconcelos (2008) and Correia da Silva et al. (2015, 2016).

\(^{16}\)As will be clearer later on, we assume that $\mu < \frac{2}{3}$ to guarantee that the private firm produces a strictly positive output under competition.

\(^{17}\)Among the different arguments for the preference of a consumer surplus standard over a total surplus standard in competition policy debates, one may consider that part of the profits may go to foreign firms and, therefore, to foreign consumers. In that sense, assuming a preference for domestic agents is a way to justify the use of this standard (Motta, 2004). Also, if the competition enforcement agency follows total welfare as its standard, the outcome of an assessment process is
the higher is the value for $\mu$, the more firm $g$ cares about consumers.

Total surplus ($TS$) is given by the sum of consumer surplus and the profits of the two firms:

$$TS = CS + \pi_g + \pi_p = \frac{(q_g + q_p)^2}{2} + (1 - q_g - q_p)q_g - \frac{q_g^2}{2} + (1 - q_g - q_p)q_p - \frac{q_p^2}{2} =$$

$$= \frac{Q^2}{2} + (1 - Q)Q - \frac{q_g^2 + q_p^2}{2}.$$

2.2.1 Nash equilibrium

If firms choose quantities simultaneously and independently, in each stage, the private firm chooses the quantity $q_p^N$ that solves:

$$\max_{q_p} \pi_p(q_g, q_p) = (1 - q_g - q_p)q_p - \frac{q_p^2}{2},$$

while the public firms chooses the quantity, $q_g^N$, that solves:\footnote{For simplicity, we are not imposing the public firm to make a non-negative profit. Let us assume that the public firm may have others sources of funding, allowing this firm to support a negative profit, but not a negative objective function.}

$$\max_{q_g} \Omega(q_g, q_p).$$

The corresponding first-order condition (FOC) for the private firm gives that:\footnote{It is straightforward to see that the second-order conditions are satisfied.}

$$\frac{\partial \pi_p}{\partial q_p} = 0 \iff q_p = \frac{1 - q_g}{3},$$

(2.3)

which implies that the higher is the output level of the public firm, the lower is the output level of the private firm. This is a standard result in the literature on quantity competition, since the private firm only cares about its own profit.

likely to be significantly biased in favor of producer surplus rather than total welfare (Albaek, 2013).
Solving the FOC for the public firm, we obtain:

\[
\frac{\partial \Omega}{\partial q_g} = 0 \iff q_g = \frac{1 - \mu - (1 - 2\mu)q_p}{3 - 4\mu}.
\] (2.4)

Notice that, if \( \mu < \frac{1}{2} \), the greater is the output level of the private firm, the lower is the output of the public firm (as in the standard private Cournot duopoly model). However, for \( \frac{1}{2} < \mu < \frac{3}{4} \), the higher is the private output, the greater is the public output. This results from the fact that, for \( \mu > \frac{1}{2} \), the public firm weights more the consumer surplus than its own profit.

Combining the FOCs (2.3) and (2.4), we obtain:

\[
q_p^N = \frac{2 - 3\mu}{8 - 10\mu} \quad \text{and} \quad q_g^N = \frac{\mu - 2}{10\mu - 8}.
\] (2.5)

Notice that, the more the public firm cares about consumers, the higher is its output level and the lower is the output level of the private firm. However, as \( q_g^N \) increases more than \( q_p^N \) decreases in \( \mu \), we conclude that consumers are the better the higher is \( \mu \).

Furthermore, the competitive profits are:

\[
\pi_p^N = \frac{3(2 - 3\mu)^2}{8(4 - 5\mu)^2} \quad \text{and} \quad \pi_g^N = \frac{(\mu - 2)(11\mu - 6)}{8(4 - 5\mu)^2}.
\] (2.6)

As \( \mu \) increases, the profit of both firms decrease. As we are assuming that \( \mu < \frac{2}{3} \), the private firm always gets a positive profit. However, the same is not true for the public firm. More precisely, for \( \mu > \frac{6}{11} \), the public firm has losses under competition (Figure 2.1). Interestingly, we conclude that, by attaching a positive weight to CS, the public firm is able to profit more than the private rival for \( \mu < \frac{1}{2} \).

Finally, replacing expressions (2.5) in (2.2), we obtain the equilibrium payoff for the public

---

20 This relates to the literature on strategic delegation according to which owners will give incentives to his managers in order to affect the outcome of the competition between their firms and their competitors (see, for instance, Fershtman and Judd, 1987). In our model, to attach a positive weight to consumer surplus increases the output of the public firm, leading the private firm to produce less and, eventually, having lower profits.
firm under competition:

$$\Omega^N = \frac{(1 - \mu)(12 - 12\mu - 5\mu^2)}{8(4 - 5\mu)^2},$$

(2.7)

which is increasing in $\mu$.

![Figure 2.1](a) Impact on quantities. (b) Impact on profits]

**Figure 2.1.** Impact of the weight the public firm attaches to CS, $\mu$, on the Nash equilibrium of the stage game.

### 2.2.2 Perfect collusion

Assume now that the two firms aim at (tacitly) sustaining the most beneficial collusive agreement. As firms are asymmetric, the collusive agreement they establish is not straightforward. Following Harrington (1991), we assume that firms use Nash bargaining to divide the gains from collusion. For simplicity, we assume that firms have the same bargaining power, which is translated into symmetric bargaining power parameters. Thus, if firms agree to cooperate,
they produce the quantities that solve:

$$\max_{(q_g,q_p)} \left[ \pi_p(q_g,q_p) - \pi_p^N \right] \left[ \Omega(q_g,q_p) - \Omega^N \right] \equiv NP(q_g,q_p), \quad (2.8)$$

where the expressions for the disagreement payoff of the private firm, $\pi_p^N$, and public firm, $\Omega^N$, are respectively given by (2.6) and (2.7).

The FOC corresponding to the maximization of $NP$ w.r.t. $q_g$ gives:

$$\frac{\partial NP}{\partial q_g} = 0 \iff [\mu(4q_g + 2q_p - 1) - 3q_g - q_p + 1] \left[ \pi_p(q_g,q_p) - \pi_p^N(\mu) \right] = q_p \left[ \Omega(q_g,q_p) - \Omega^N(\mu) \right], \quad (2.9)$$

while the FOC corresponding to the maximization w.r.t. $q_p$ gives:

$$\frac{\partial NP}{\partial q_p} = 0 \iff [\mu q_p - (1 - 2\mu)q_g] \left[ \pi_p(q_g,q_p) - \pi_p^N \right] = (q_g + 3q_p - 1) \left[ \Omega(q_g,q_p) - \Omega^N \right]. \quad (2.10)$$

To obtain the collusive quantities, $q_g^C$ and $q_p^C$, we should combine (2.9) and (2.10). However, as, for a generic $\mu$, both equations involve the finding of the roots of third-order polynomial in $q_p$ and $q_g$, we were not able to find the closed-form solutions for the collusive output levels. Thus, we proceed the analysis numerically. In Table 2.1, in the appendix, we present the values for all market variables for different values of $\mu$. Figure 2.2 shows the evolution of the output levels as functions of the weight the public firm attaches to consumer surplus.

As we have shown previously, the output level of the private firm is strictly decreasing in $\mu$, while the output level of the public firm is strictly increasing in $\mu$, as is total output. Under collusion, total output is also increasing in $\mu$, but it is only higher than the competitive output for intermediate levels of $\mu$. When $\mu = \frac{1}{2}$, total output is actually the same under collusion and under competition. Regarding individual output levels, we can see that the output of the private firm is decreasing when the output of the public firm is increasing. This trend verifies
Figure 2.2. Output levels as functions of the weight the public firm attaches to CS, $\mu$, under collusion (solid line) and under competition (dashed line).

up to a jump in the output level of the private firm around $\mu = 0.39$ (corresponding to a fall in the output level of the public firm). Then, output of the private firm increases for some levels of $\mu$ while output of the public firm decreases and then there is an inversion of this trend when $\mu = \frac{1}{2}$. The intuition behind this result is the following. Due to our assumption of increasing marginal costs, under collusion, the public firm is willing to give part of its production to the private firm, which it accepts as long as its profit increases. At some point, the private firm is not willing to increase its output level due to the impact on profits and starts decreasing its production and, in response, the public firm starts producing more for high levels of $\mu$.

Let us explain why there is a “jump” in the collusive quantities when $\mu$ is around 0.39. For firms to be willing to collude, their payoff must be higher than under competition. Graphically, in Figure 2.3, this means that the public firm is only willing to collude for pairs of collusive quantities, $(q^C_g, q^C_p)$, in the light grey areas, while the private firm only accepts to collude for pairs $(q^C_g, q^C_p)$ in the dark grey area. Thus, the agreements $(q^C_g, q^C_p)$ that are individual rational (IR) for both firms are those that belong to the intersection of the shaded regions, as identified in the figures. The point marked in each figure is the solution of the Nash Bargaining problem (2.8). As we can see in Figure 2.3, around $\mu = 0.39$, there is a jump in the collusive quantities.

From the analysis of Figure 2.2c, we conclude that, depending on the value of $\mu$, collusion can benefit or damage consumers. More precisely, for sufficiently low or high values of $\mu$, recall that, as firms sell homogeneous goods, the greater is total output, the greater is consumer surplus.

\[21\]
consumers are better off if firms compete than if they collude. In contrast, for intermediate values of $\mu$, collusion benefits consumers, which is a surprising result. The intuition for this result is the following. For low levels of $\mu$, the public firm is more concerned about profits than consumer surplus, which corresponds to a lower pressure on output. With the increase in the level of concern for consumers, $\mu$, the public firm desires to achieve a higher output level, being willing to share its production with the private firm, as we have already explained. Hence, after the threshold at which the private firm starts decreasing its production is reached, even though the public firm increases its, total output becomes above the level under competition, leaving consumers worse off.

As guaranteed by the formulation of the Nash bargaining problem, both firms get a higher payoff under collusion than under competition (see Figure 2.4 a and c above). In addition,
Figure 2.4b shows that the profit of the public firm, despite being decreasing for sufficiently high values of $\mu$, is never negative as it was under Nash competition (for $\mu > \frac{6}{11}$). However, it is important to call the attention that, for certain intermediate levels of $\mu$, the public firm may get a lower profit under collusion than under competition. This firm is only willing to collude because it also values the consumer surplus, and the resulting gain in consumer surplus from collusion more than compensates its profit loss.

2.2.3 Deviation from the collusive agreement

Firms may have incentives to unilaterally deviate from the collusive agreement, i.e., to produce a different output to increase their own payoff. The deviation quantities are calculated as typically in the literature, by finding the quantity that maximizes individual payoff, assuming that the other firm sticks to the agreement. More precisely, if the private firm deviates from the agreement, it assumes that the public firm produces the cooperative output, $q_c^p$, and chooses the quantity that solves:

$$\max_{q_p} \pi_p(q_g^C, q_p) = (1 - q_g^C - q_p) q_p - \frac{q_p^2}{2}. \tag{2.11}$$

Solving this maximization problem, we would obtain the deviation quantities plotted in Figure 2.5a.

If the public firm deviates from the agreement, it assumes that the private firm produces the cooperative output, $q_c^p$, and chooses the quantity that solves:

$$\max_{q_g} \Omega(q_g, q_c^p) = \mu \left( \frac{q_g + q_c^p}{2} \right)^2 + (1 - \mu) \left[ (1 - q_g - q_c^p) q_g - \frac{q_g^2}{2} \right]. \tag{2.11}$$

Solving this maximization problem, we would obtain the deviation quantities plotted in
From Figure 2.5, it is clear that, while the private firm deviates from the collusive agreement by reducing its output level \( q_{pD} < q_{pC} \), the public firm’s motivation to deviate is to increase its output level \( q_{gD} > q_{gC} \).

### 2.2.4 Sustainability of collusion

Following Friedman (1971), we assume that firms use grim trigger strategies to punish deviations from the collusive agreement, meaning that they permanently revert to Nash competition after detecting a deviation. Thus, the private firm is willing to comply with the collusive agreement if and only if the following incentive compatibility constraint (ICC) is satisfied:

\[
\sum_{s=t}^{\infty} \delta^{s-t} \pi_{pC} \geq \pi_{pD} + \sum_{s=t+1}^{\infty} \delta^{s-t} \pi_{pN} \iff \delta \geq \frac{\pi_{pD} - \pi_{pC}}{\pi_{pD} - \pi_{pN}} \equiv \delta^* \quad (2.12)
\]
where $\delta^*_p$ is the critical discount factor of the private firm. Likewise, the public firm will abide by the agreement if and only if:

$$\delta \geq \frac{\Omega^D_g - \Omega^C_g}{\Omega^D_g - \Omega^N_g} \equiv \delta^*_g,$$

(2.13)

where $\delta^*_g$ is the critical discount factor of the public firm. Naturally, for the agreement to be sustainable, neither of the firms must have incentives to deviate from the agreement. Thus, the critical discount factor for collusion sustainability is:

$$\delta^* = \max \{ \delta^*_p, \delta^*_g \}.$$

Figure 2.6 presents our results regarding the impact of parameter $\mu$ on the critical discount factors. We discuss these results in the next section.

**Figure 2.6.** Impact of the weight the public firm attaches to CS, $\mu$, on the critical discount factor.
2.3 Discussion of the results and welfare implications

Let us now discuss the results regarding the sustainability of the collusive agreement. We devote a final part of this section to the discussion of some welfare implications.

As we can conclude from the analysis of Figure 2.6, collusion is easiest sustained when the two firms are symmetric (i.e., $\mu = 0$). This result is standard in the literature on collusion. In fact, when $\mu = 0$, the public maximizes its own profit with disregard for consumer surplus.\(^{22}\) As the asymmetry between the public and the private firm in our model lies on their objective functions, in this scenario, firms are symmetric. Hence, we expect a similar behavior in all competition regimes. As firms are symmetric, we assume that, under collusion, they produce quantities that maximize their joint profit and divide it in equal shares. Both public and private firm restrict their outputs under collusion to increase profits and, in case of a unilateral deviation, the deviating firm restricts output even more. As firms are symmetric, the critical discount factor is the same for the two firms, and it is equal to $\delta^* \approx 0.516$.\(^{23}\)

When deviating from the collusive agreement, as the public firm also considers consumer welfare in its objective function, it produces more than under collusion (Figure 2.5b). The private firm, on the contrary, reduces its output when deviating from the collusive agreement (Figure 2.5a), which is an expected behavior of a profit-maximizer. In this scenario, the public firm has more incentives to deviate than the private firm, i.e., $\delta^*_g > \delta^*_p$ (Figure 2.6).

The incentives for the public firm to collude are non-monotonic in the weight attached to consumer surplus in the firm’s objective function (Figure 2.6). In fact, for sufficiently high levels of $\mu$, the critical discount factor of the public firm is not only increasing in this parameter, but it is also binding (i.e., higher than the critical discount factor of the private firm). This happens because the higher is the weight attached to consumer surplus, the higher is the

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\(^{22}\)See Appendix for further detail.

\(^{23}\)This is a particular case of equation (7) in Rothschild (1997, p.724), with $\alpha_1 = 1, A = 3, B = 5$. 

21
output desired by the public firm. Then, since under collusion the public firm gives part of its production to the private firm, and the private firm has a ceiling on the output it is willing to produce, after that threshold is reached, the public has more incentives to deviate from the collusive agreement by producing a larger output in an attempt to increase consumer surplus.

Likewise, the critical discount factor of the private firm is also non-monotonic in $\mu$. The assumption of increasing marginal costs plays a key role in these results. As we have shown, with increasing marginal costs, the private firm is willing to accept the increase (or maintenance) of total output at the public firm’s expense, that transfers part of its production to the private firm. This result contrasts those under collusion among private firms, under which firms aim at restricting total output so as to increase market price. However, our result only holds as long as the profit of the private firm is increasing. As we have seen in Figure 2.4, the collusive output level of the private firm is decreasing in the weight the public firm attaches to consumer surplus for sufficiently high levels of this parameter.

An interesting case to highlight is that of $\mu = \frac{1}{2}$. Notice that when the public firm weights equally its own profit and the consumer surplus, total output is the same under competition and collusion despite individual outputs being different (Figure 2.2). This, once again, results from the fact that, under collusion, the public firm gives up part of its production to the private firm, which is welfare-improving because marginal costs are increasing in the output level. Hence, the private firm produces more and the public firm produces less under collusion than under competition. As total output remains unchanged, the consumer surplus will be unaffected by collusion. However, the industry costs will be lower under collusion. Notice that, as consumer surplus is not affected by collusion, the public firm’s willingness to collude is driven by the increase on its profit, which is an interesting result.

As we have showed in a previous section, for intermediate levels of $\mu$, consumer surplus is higher under collusion than under competition. Interestingly, notice that the level of $\mu$ for which collusion is harder to sustain belongs to that interval. Hence, consumers may be better off or worse off under collusion depending on the level that public firm attaches to their
surplus. Regarding total welfare, it is higher under competition for sufficiently low levels of \( \mu \), and under collusion for sufficiently high levels of \( \mu \) (see Figure 2.7). Recall that for low levels of the weight attached to consumers surplus by the public firm, both firms are concerned with profits. Even though the profit of the public firm may be lower under collusion than under competition, collusion generates higher profits to the private firm (otherwise, it would not accept the agreement) and to the public firm through the component attached to consumer surplus. The latter is naturally increasing the weight the public firm attaches to it. Then, the fact that total surplus is higher under collusion than under competition for high levels of \( \mu \) does not seem surprising.

![Figure 2.7](image.png)

**Figure 2.7.** Total surplus as function of the weight the public firm attaches to CS, \( \mu \), under collusion (solid line) and under competition (dashed line)

Finally, the policy implications of our paper sum up to the following. Even though collusion is harder to sustain between a private and a public (or semi-public) firm, and the discount factor for which collusion is most difficult to sustain belongs to the interval where consumer surplus is higher under collusion than under competition, we conclude that collusion may be welfare-improving. Hence, competition authorities should be aware that if a public firm is part of the collusive agreement that *per se* is not sufficient to ensure that collusion is not harmful, but it may actually improve both consumer’s and total welfare.
2.4 Conclusion

Motivated by the number of cartel cases involving public firms, we studied the conditions for collusion to be sustainable in a mixed duopoly when the public firm is a member of the cartel. We allowed for different objective functions of the public firm, that differ in the weight that this firm attaches to consumer surplus.

We concluded that the incentives for both firms to collude are non-monotonic in the weight the public firm attaches to consumer surplus. Due to the assumption of increasing marginal costs of production, under collusion, the public firm transfers part of its production to the private firm. The private firm is willing to accept the increase (or maintenance) of total output as long as its profit increases through the increase in its market share. This result is interesting since it contrasts the traditional one under which firms collude to restrict total output so as to increase market price. Nevertheless, we show that, when deviating, the private firm does so by restricting output and, on the contrary, the public firm deviates by increasing its production. Regarding collusion sustainability, although the critical discount factor of the private firm is decreasing in the weight the public firm attaches to consumer surplus for sufficiently high levels of this parameter, collusion is never as easy to sustain as when both firms maximize profits (private collusion). This corroborates the result in the literature that collusion is easier to sustain between symmetric firms. In our model, this results from the critical discount factor of the public firm becoming binding for sufficiently high levels of concern for consumers. Indeed, since the higher is the weight attached to consumer surplus, the higher is the output desired by the public firm, its incentives to deviate by producing a larger output increase. In addition, our results showed that collusion may be welfare-improving to consumers when the weight the public firm attaches to consumer surplus is not too high or too low. Total surplus is also higher under collusion for high levels of the weight attached to consumers surplus.

Finally, our paper provides two important policy implications to competition authorities.
One the one hand, collusion is harder to sustain between asymmetric firms, in particular, asymmetry regarding their objective functions. One the other hand, collusion between a public and a private firm may be welfare-improving. Future research could include other sources of asymmetry between the two firms. For instance, to assume that one firm is more cost-efficient than the other. In addition, we could assume that firms have different bargaining powers.
Appendix

Public firm maximizes individual profit ($\mu = 0$)

When $\mu = 0$, the public maximizes its own profit with disregard for consumer surplus. As the asymmetry between the public and the private firm in our model lies on their objective functions, in this scenario, firms are symmetric. Hence, we expect similar results regarding the quantities produced in all competitive regimes.

As firms are symmetric, we assume that, under collusion, they produce quantities that maximize their joint profit and divide it in equal shares. Thus, the total output is the solution of:

$$
\max_Q \{\pi_g + \pi_p\} = (1 - Q)Q - \frac{Q^2}{4}.
$$

Solving the corresponding FOC, we obtain $Q^C = \frac{2}{5}$ and, therefore, $q_p^C = q_g^C = \frac{1}{5}$.

The deviation quantities are calculated as typically in the literature of homogeneous collusion, assuming that the other firm sticks to the agreement. As firms are symmetric in this case, the deviation quantities are equal. Also, we expect similar results whether it is the public firm or the private firm that deviates from the collusive agreement. Solving, for instance, the profit maximization problem of the private firm, the private firm assumes that the public firm produces the cooperative quantity, $q_g^C$, and solves:

$$
\max_{q_p} \pi_p(q_g^C, q_p) = (1 - q_g^C - q_p)q_p - \frac{q_p^2}{2}.
$$
Hence, we find that the deviation quantity is:

\[ \frac{\partial \pi_p}{\partial q_p}(q_g, q_p^C) = 0 \Leftrightarrow q_p^D = \frac{4}{15} \]

and the deviation profit is \( \pi_p^D = \frac{8}{15} \).

To find the punishment profits, we just need to replace \( \mu = 0 \) in (2.6). As, when \( \mu = 0 \), firms are symmetric, the critical discount factor is the same for the two firms, and collusion is sustainable if and only if:

\[ \delta_p^* = \delta_g^* = \frac{16}{31} \]

**Public firm equally weights CS and individual profit (\( \mu = \frac{1}{2} \))**

When \( \mu = \frac{1}{2} \), the public firm’s problem consists in maximizing the sum of consumer surplus and profit. The FOC for its maximization problem is as follows:

\[ \frac{\partial \Omega}{\partial q_g}(q_g, q_p) = 0 \Leftrightarrow q_g = \frac{1}{2} \]

The private firm maximizes its own profit and the corresponding FOC boils down to:

\[ q_p(q_g) = \frac{1 - q_g}{3} \]

Combining the two FOCs, we obtain:

\[ q_p^N = \frac{1}{6} \quad \text{and} \quad q_g^N = \frac{1}{2} \]
Substituting these expressions in the firms’ objective functions, we find:

\[ \pi_N^p = \frac{1}{24}, \quad \pi_N^g = \frac{1}{24} \quad \text{and} \quad \Omega_N = \frac{19}{144}. \]

The cooperative quantities are the solutions of the following maximization problem:

\[
\max_{(q_g, q_p)} \left[ \frac{1}{2} \left( \frac{(q_g + q_p)^2}{2} + (1 - q_g - q_p)q_g - \frac{q_g^2}{2} - \frac{19}{144} \right) \right] \left[ (1 - q_g - q_p)q_g - \frac{q_g^2}{2} - \frac{1}{24} \right]
\]

Solving the Nash bargaining problem, we find:

\[ q_C^p = \frac{1}{3} \quad \text{and} \quad q_C^g = \frac{1}{3} \]

Replacing these expressions in the firms’ objective functions, we find:

\[ \pi_C^p = \frac{1}{18}, \quad \pi_C^g = \frac{1}{18} \quad \text{and} \quad \Omega_C = \frac{5}{36}. \]

When deviating, the private firm assumes that the public firm produces the cooperative quantity, \( q_C^g \), and solves:

\[ \max_{q_p} \pi_p(q_g^C, q_p) = (1 - q_g^C - q_p)q_p - \frac{q_p^2}{2} \]

We find that the deviation quantity is \( q_p^D = \frac{2}{9} \), and the corresponding profit is: \( \pi_p^D = \frac{2}{27} \).

When it is the public firm that deviates, it assumes that the private firm produces the cooperative quantity, \( q_C^p \), and decides to deviate from the collusive agreement by solving:

\[ \max_{q_g} \Omega(q_g, q_p^C) = \frac{(q_g + q_p^C)^2}{2} + (1 - q_g - q_p^C)q_g - \frac{q_g^2}{2} \]

Solving the maximization problem above, we find that the public produces \( q_g^D = \frac{1}{2} \) and gets a payoff of: \( \Omega^D = \frac{11}{72} \).
Replacing the obtained expressions for payoffs in the different regimes in the ICC of the private firm, we obtain:

\[ \delta_p^* \approx 0.571, \]

while the critical discount factor of the public firm is:

\[ \delta_s^* \approx 0.667. \]
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Table 2.1: Market outcomes for different values of \( \mu \).
Chapter 3

Coordinated Effects of Corporate Social Responsibility

3.1 Introduction

Over the past decades, firms have been voluntarily adopting social and environmental concerns in their business operations and in their interactions with stakeholders (European Commission, 2011). Corporate Social Responsibility (henceforth CSR) has become a major concern for a large number of firms in several industries, working as a source of market competitiveness. According to the KPMG Survey of Corporate Responsibility Reporting (2017), more than 75% of the world’s 250 largest firms now include some “non-financial” information in their annual reports, since they believe CSR data is relevant to their investors, and this trend continues to grow (KPMG, 2017). In the 2011 Communication on Corporate Social Responsibility, the European Commission defined CSR as “the responsibility of enterprises towards their impact..."
on society” (EC, 2011, p. 6). Scholars have been devoting their attention to this topic as well (e.g. Baron (2001), Goering (2007), Bénabou and Tirole (2010), Lambertini and Tampieri (2010), Kopel and Brand (2012), Fanti and Bucella (2017a, 2017b, 2017c, 2018), among others).

Pressures to engage in CSR arise from different stakeholders (customers, employees, governments and society as a whole), as noted by Baron (2001) and McWilliams and Siegel (2001). Baron (2001) explains that even if firms are completely selfish, they may strategically engage in social actions to avoid bad propaganda or, putting it differently, to be entitled to some kind of endorsement. Indeed, firms may be willing to mix corporate do-gooding with profit maximization, so as to respond to such demands. Following a moral or altruistic behavior instead, Bénabou and Tirole (2010) explain that firms are willing to sacrifice profits in the social interest. Nevertheless, these authors add to their reasoning that firms often believe that by voluntarily taking actions in the name of CSR, they may be rewarded in the marketplace by increasing demand and, therefore, profits; or they may achieve a positive image among regulators and the public opinion. The latter may be strategically used to avoid strict supervision in the future (Garriga and Melé, 2004; Bénabou and Tirole, 2010).2 According to some authors, CSR may also be used as a mechanism to deter entry, since promoting investments on the behalf of environmental or social causes would increase rivals’ costs (Baron, 2001; Garriga and Melé, 2004; Bénabou and Tirole, 2010; Fanti and Bucella, 2017b). Additionally, firms may engage in CSR because it may serve as a commitment device for their strategic decisions in oligopolistic markets (Planer-Friedrich and Sahm, 2019) and may help to promote a coordinated behavior in the market by means of decreasing market competition (Tzavara, 2008; Lambertini and Tampieri, 2012).

CSR has been covered in the literature in two major perspectives. One perspective, related to the demand side, considers CRS as a form of vertical differentiation, which is valued by consumers (e.g., Bagnoli and Watts, 2003; Alves and Santos-Pinto, 2008; Baron, 2009;

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2 For further detail on the different interpretations of CSR see, for instance, Garriga and Melé (2004).
García-Gallego and Georgantzís, 2009; Manasakis et al., 2013, 2014). A second perspective, related to the supply side, relies on the assumption that CSR is incorporated in firms’ objective function along with their individual profit (e.g., Goering (2007, 2012, 2014), Lambertini and Tampieri (2012, 2015), Lambertini et al. (2016), Fanti and Buccella (2017a, 2017b, 2017c), among others). All of these contributions consider that CSR is exogenously given. In this paper, we follow the second perspective. In our model the level of CSR is defined as a positive weight on consumer surplus in firm’s objective function. Our aim is to study how this type of objective function affects firms’ ability to coordinate their decisions. Additionally, we are also concerned about assessing whether collusion with CSR firms may be welfare-enhancing to consumers following existing contributions in the literature.

It is generally considered in the literature that collusion is detrimental to consumers (Farrell and Shapiro, 1990). This results from the standard conclusion that firms restrict output while colluding. Since we model CSR by attaching a positive component to consumer surplus in firms’ objective functions, we expect CSR to increase output and, therefore, consumer surplus. Given this, we set out to investigate whether the result of output restriction under collusion holds in the presence of socially responsible firms, which will determine if collusion is harmful to consumers or not.

To the best of our knowledge, there are only two papers that study the coordinated effects of CSR, those of Tzavara (2008) and Lambertini and Tampieri (2012). Tzavara (2008) studies whether CSR promotes a collusive behavior among firms, considering a duopoly with homogeneous products and quantity competition. In her model, CSR is incorporated in the demand function, being price-augmenting for consumers and entailing a cost for firms. Lambertini and Tampieri (2012) consider a model where firms have CSR, produce homogeneous goods and compete in quantities, as in Tzavara (2008). In their model, CSR is also incorporated as a

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3There are other studies that model CSR endogenously (Kopel and Brand, 2012; Kopel, 2015; Planer-Friedrich and Sahm, 2018, 2019).

4Some authors have shown that collusion may be welfare-enhancing, for instance, Mukherjee (2010), Deltas et al. (2012) and Mukherjee and Sinha (2019).
weight that firms assign to consumer surplus in their objective functions. Both contributions find that CSR makes collusion harder to sustain. Our paper contributes to this literature being closely related to Lambertini and Tampieri’s (2012) paper as regards the modelling of CSR. However, our setting is more general as we consider product differentiation à la Singh and Vives (1984), which captures the case of homogeneous goods as a particular one. Additionally, as in Lambertini and Tampieri (2012), firms are cost-symmetric, however, in our setting, firms may have asymmetric levels of CSR, which are exogenously given.

Our contribution also relates to the literature that studies collusion with product differentiation. Several papers in the literature have studied the effects of product differentiation on firms’ ability to collude, for instance Deneckere (1983, 1984), Ross (1992) and Rothschild (1992). These contributions have focused, mainly, on the nature of the competitive market, and on how the type of competition makes collusion easier or more difficult to sustain. The effect of horizontal differentiation in collusion sustainability is ambiguous (Ivaldi et al., 2003). On the one side, it reduces the short-term gains from undercutting rivals, since it makes it more difficult to attract customers who value a specific variety of the good. On the other side, it limits the severity of punishments. Deneckere (1983, 1984) find that the stability of collusion for substitute goods is monotonically decreasing with product homogeneity. Ross (1992) finds a negative monotonic relationship between homogeneity and cartel stability, i.e., the more homogeneous the products are, the less likely the cartel is to be stable. In addition, Ross (1992) shows that cartels of very homogeneous products are more stable than cartels of heterogeneous products. We obtain the latter result when both firms have the same level of corporate social responsibility.

The key findings of the paper are as follows. If firms have the same level of CSR (symmetric case), attaching a positive weight to consumer surplus hinders collusion. With symmetric CSR, firms’ payoffs increase under competition, collusion and deviation. Although CSR increases the difference between the collusive and Cournot payoffs, it also increases the difference between deviation and collusive payoffs. Since the second difference is higher than
the first one, firms’ incentives to deviate from the agreement are higher, making collusion less sustainable. This result is close to the ones obtained by Lambertini and Tampieri (2012) and Tzavara (2008). Nevertheless, product differentiation plays a crucial role. When firms are socially responsible, collusion sustainability increases in the degree of substitutability if the level of CSR is sufficiently high. Indeed, for a sufficiently high level of CSR, our findings confirm Rothschild’s (1992): a high degree of substitutability fosters collusion in quantities, whereas a high degree of complementarity makes it more difficult to sustain.\(^5\) We obtain the latter result for any level of CSR when both firms have the same level of CSR.

If firms are asymmetric regarding the levels of CSR (asymmetric case), conclusions about collusion sustainability may depend on the level of product differentiation, on the level of CSR and on to which firm belongs the binding critical discount factor (above which collusion is possible). When products are complements, collusion is easier to sustain the higher is the degree of differentiation. This result contrasts the one obtained in the symmetric case. Moreover, the higher is the level of CSR, the more difficult it is to sustain collusion among the two firms. As products become more substitutes, conclusions regarding collusion sustainability are not straightforward. In fact, collusion is more sustainable for a sufficiently low level of CSR or some particular combinations of the level of CSR and the degree of product differentiation.

Interestingly, under collusion, welfare may actually improve when compared to the competitive scenario when products are sufficiently differentiated or when the level of social responsibility is sufficiently high. Therefore, we believe our paper also contributes to the discussion of whether some cartel agreements may be exempted from cartel law if they promote sustainable gains as a public interest purpose.\(^6\) Nevertheless, we strongly advise competition authorities to be careful when investigating the strategies followed by these type of firms.

\(^5\)Notice that Rothschild’s (1992) results were amended in Albæk and Lambertini (1998), who show that his results were in fact similar to the ones in Deneckere (1983, 1984).

\(^6\)Schinkel and Spiegel (2017) and Treuren and Schinkel (2018) and Hashimzade and Myles (2018) follow this line of the literature by studying the effects of collusion when consumers value sustainable goods and firms must choose their sustainability levels. Differently from our paper, in these papers, sustainability is perceived by consumers who are willing to pay more for those goods and firms have additional costs to produce them. The authors find that the utility loss (from higher collusive prices) outweighs the benefits from environmental contributions, therefore the public interest is not defended.
The remainder of the paper is organized as follows. Sections 3.2 and 3.3 present the baseline assumptions of the model. In Section 3.4, we analyze the results of collusion with CSR in both cases (symmetric and asymmetric CSR levels) and in Section 3.5 we discuss their implications in terms of consumers’ welfare. Finally, Section 3.6 offers some concluding comments with a brief discussion of the main results.

3.2 Setup

We consider an economy with two firms that produce differentiated goods and compete in quantities. The demand function of each good is derived from the preferences of a representative consumer, whose quasi-linear utility function is quadratic à la Singh and Vives (1984) and is given by:

$$U(q_i, q_j) = \sum_{i=1}^{2} q_i - \frac{1}{2} \left( \sum_{i=1}^{2} q_i^2 + 2\gamma \sum_{i \neq j} q_i q_j \right) + m,$$

(3.1)

where $i = 1, 2$, $j = 1, 2$, $i \neq j$, $q_i$ and $q_j$ are, respectively, the quantities of good $i$ and good $j$; $m$ is the income spent on all other goods; and $\gamma \in [-1, 1]$ measures the degree of product differentiation. A smaller $\gamma$ indicates a larger degree of product differentiation. If $\gamma < 0$, goods are complements; if $\gamma = 0$, goods are independent and if $\gamma > 0$, goods are substitutes. When $\gamma = 1$ goods are perfect substitutes and when $\gamma = -1$ goods are complements but they may not be perfect complements.\(^7\)

\(^7\)While we begin with the a priori possible range $\gamma \in [-1, 1]$, for the case of complements, further restrictions for the consumer’s optimization problem are needed. In fact, the case of perfect complementarity corresponds to a Leontief utility function (which is concave but not strictly concave as our utility function). As discussed by Varian (1992) (chapter 10, pp. 164-165), for a quasi-linear utility function, the available income should be large enough in order to allow for demand functions to be independent of the income. This assumption can be mild for substitutes but, as pointed out by Amir et al. (2017) (see Proposition 14), it is, indeed, not compatible with perfect complements. For these goods, all the available income is spent or, in the absence of a constraint on income, consumption would go to infinity. Also, for a given income level, if products are sufficiently complements, then the consumer’s demand will not be a linear demand function. That is, to analyze a market where perfect complements are sold, one should first model how consumers allocate their income between these
Maximizing the representative consumer’s utility function (3.1) subject to a budget constraint, \( p_iq_i + p_jq_j = m \), where \( p_i \) and \( p_j \) are the prices of the two goods, yields the following linear inverse demand function of good \( i \):

\[
p_i(q_i, q_j) = 1 - q_i - \gamma q_j
\]

Moreover, we consider that firms are cost-symmetric and have increasing marginal costs of production.\(^8\) Given the model specifications, profits and consumer surplus are, respectively, given by:

\[
\pi_i = (1 - q_i - \gamma q_j)q_i - \frac{1}{2}q_i^2, \quad \text{for} \quad i = 1, 2
\]

and

\[
CS = U(q_i, q_j) - p_iq_i - p_jq_j
\]

Following Goering (2007), Lambertini and Tampieri (2010, 2012), Fanti and Buccella (2017a, 2017b, 2017c) and Fanti and Buccella (2018), we consider that CSR firms are interested in consumer’s welfare in addition to their own profits.\(^9,10\) Thus, when a firm engages in CSR or has an altruistic concern, it places a weight (given by \( \theta_i \)) on consumer surplus in its objective function, which is analogous to assume that the firm places a higher weight on particular goods and the other goods they consume.

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\(^8\)The assumption of increasing marginal costs is common in the literature of mixed oligopolies (for a detailed explanation, see De Fraja and Delbono, 1990). Our results have been checked with a linear cost function \( C(q_i) = cq_i \) and remained unchanged.

\(^9\)Following Goering (2008) and Kopel and Brand (2012), we weight the surplus of all consumers rather than each firm’s own consumers in the objective function.

\(^{10}\)Maximizing a weighted sum of profits and another component is not exclusive to this literature, being widely used in the literature of strategic delegation (Vickers, 1985; Sklivas, 1987; Fershtman and Judd, 1987; Lambertini and Trombetta, 2002) and mixed oligopolies (De Fraja and Delbono, 1990; Colombo, 2016; Correia-da-Silva and Pinho, 2018). Nevertheless, in the literature of mixed oligopolies, when modelling the objective function of a public (or semi-public) firm, it is common to assume weights on profits and consumer surplus, where the sum of the weights equals one. In the literature on CSR, however, firms are profit-maximizers (they are private firms) and may, in addition to their own profits, place a weight on consumer surplus in their objective functions (measured by \( \theta \) in our model).
output.\textsuperscript{11} This assumption seems reasonable if we recall the idea that pressures to engage in CSR arise from different stakeholders, whose participation in firm’s governance explains their influence in the market decisions. That is, in response to such a “social pressure”, firms end up complying with an exogenously settled level of social concern.\textsuperscript{12}

The objective function of firm $i$ that follows CSR rules can be specified as a parametrized combination of profits and consumer surplus. The CSR objective function ($W$) of firm $i$ is, then, given by:

$$W_i = \pi_i + \theta_i CS = p_i q_i - \frac{1}{2} q_i^2 + \theta_i \left[ (q_i + q_j) - \frac{(q_i^2 + q_j^2)}{2} + 2\gamma q_i q_j \right] - p_i q_i - p_j q_j$$

(3.2)

where $i = 1, 2, j = 1, 2, i \neq j$ and $\theta_i \in [0, 1]$ measures the level of CSR of firm $i$, that is, the social concern for consumers in the market.

### 3.3 Collusion with CSR

We consider that firms engage in an infinitely repeated game. Let $\delta_i \in [0, 1]$ denote the discount factor between periods for firm $i$. In a repeated game, collusion is sustainable only when none of its members has an incentive to deviate from the agreement. We focus the analysis on perfect collusion, i.e., we assume that firms aim at sustaining the most beneficial collusive agreement. Following Friedman (1971), we will assume that firms adopt grim trigger

\textsuperscript{11}One can also interpret this concern as having an empire-building motive. According to the literature on managerial empire building, managers, when not monitored by shareholders, may be tempted to overinvest, aggressively growing the firm, which reduces profitability and does not necessarily reflect the best interest of shareholders (e.g., Kanniainen (2000) and Hope and Thomas (2008)).

\textsuperscript{12}Empirical results from Spitzeck and Hansen (2010) show that stakeholders participate in operational, managerial, as well as in strategic decisions.
strategies. Then, the incentive compatibility constraint (ICC) is given by:

\[
\frac{W_i^C}{1 - \delta_i} \geq W_i^D + \frac{\delta_i}{1 - \delta_i} W_i^N \Leftrightarrow \delta_i \geq \frac{W_i^D - W_i^C}{W_i^D - W_i^N} \equiv \delta_i^* \tag{3.3}
\]

where \( W_i^D \) is the deviation payoff, \( W_i^C \) is the collusive payoff, \( W_i^N \) is the punishment (Cournot-Nash) payoff and \( \delta_i^* \) is the critical discount factor of firm \( i \). For the agreement to be sustainable, neither of the firms must have incentives to deviate. Therefore, the critical discount factor for collusion sustainability is:

\[
\delta^* = \max\{\delta_i^*, \delta_j^*\}.
\]

First, we examine the equilibrium where firms choose non-cooperatively and simultaneously their output levels, \( q_1 \) and \( q_2 \), that maximize the respective payoff function (3.2). Solving the corresponding first-order conditions, the resulting Cournot-Nash equilibrium output under CSR of firm \( i \) is:

\[
q_i^N = \frac{\gamma(1 - \theta_i) + \theta_j - 3}{\gamma^2(1 - \theta_i)(1 - \theta_j) - (\theta_i - 3)(\theta_j - 3)}. \tag{3.4}
\]

Substituting (3.4) into the payoff function yields:

\[
W_i^N = \frac{\gamma^2 \left\{ \theta_i ((5 - \theta_i)\theta_i + \theta_j - 2)\theta_j - 6 + 3 \right\} + (\theta_i - 3) [2\gamma(3 - 2\theta_i - \theta_j) + (\theta_i - 3)\theta_i - (\theta_j - 3)^2]}{2 \left[ \gamma^2(1 - \theta_i)(1 - \theta_j) - (\theta_i - 3)(\theta_j - 3) \right]^2}. \tag{3.5}
\]

If firms collude, they will choose the output level that maximizes their joint payoff, solving
the following problem:

\[
\max_{q_1,q_2} \quad W = W_1 + W_2.
\]

The resulting collusive equilibrium output and payoff of firm \(i\) are given by:

\[
q_i^C = \frac{1}{1 - (\gamma + 1)(\theta_i + \theta_j - 2)}, \quad (3.6)
\]

\[
W_i^C = \frac{2(1 + \gamma)(1 - \theta_j) + 1}{2[1 - (\gamma + 1)(\theta_i + \theta_j - 2)]^2}. \quad (3.7)
\]

As firms are symmetric, let us consider that firm \(i\) deviates from the collusive agreement and chooses the quantity \(q_i^D\) that maximizes its payoff function (3.8), taking into account that firm \(j\) produces the collusive quantity (3.6):

\[
W_i^D = p_iq_i^D - \frac{1}{2} (q_i^D)^2 + \theta_i \left[ (q_i^D + q_j^C) - \left( \frac{(q_i^D)^2 + (q_j^C)^2}{2} + 2\gamma q_i^D q_j^C \right) - p_iq_i^D - p_jq_j^C \right]. \quad (3.8)
\]

The resulting equilibrium output and payoff of the deviating firm \(i\) are given by:

\[
q_i^D = \frac{\theta_i - (\gamma + 1)(1 - \theta_j) - 2}{(3 - \theta_i)[(\gamma + 1)(\theta_i + \theta_j - 2) - 1]}
\]

\[
W_i^D = \frac{(\theta_j - 3)^2 + \gamma^2(\theta_j - 1)^2 + 2\gamma(\theta_j - 1)(\theta_i + \theta_j - 3) + \theta_i(2\theta_j - 3)}{2(3 - \theta_i)[(\gamma + 1)(\theta_i + \theta_j - 2) - 1]^2}. \quad (3.9)
\]
**Assumption 1:** Assume that all firms are active in the market in the three scenarios. That is, $\gamma$, $\theta_i$ and $\theta_j$ are such that:

- when $\gamma \in [-1, -\frac{1}{2}]$ for all $\theta_i, \theta_j \in [0, 1]$;
- when $\gamma \in (-\frac{1}{2}, \frac{1}{2}]$ if $\theta_j \in [0, \frac{1}{2(\gamma+1)}]$ and $\theta_i \in [0, 1]$ or if $\theta_j \in (\frac{1}{2(\gamma+1)}, 1]$ and $\theta_i \in [0, \bar{\theta}]$;
- when $\gamma \in (\frac{1}{2}, \frac{3}{2}]$ if $\theta_j \in [0, \frac{2-3\gamma}{\gamma+1}]$ and $\theta_i \in [0, 1]$ or if $\theta_j \in (\frac{2-3\gamma}{\gamma+1}, \frac{\gamma}{\gamma+1})$ and $\theta_i \in [0, \bar{\theta}]$ or if $\theta_j \in [\frac{\gamma}{\gamma+1}, 1]$ and $\theta_i \in [0, \bar{\theta}]$;
- when $\gamma = \frac{2}{3}$ if $\theta_j = 0$ and $\theta_i \in [0, 1]$ or if $\theta_j \in (0, \frac{2}{3})$ and $\theta_i \in [0, \bar{\theta}]$ or if $\theta_j \in [\frac{2}{3}, 1]$ and $\theta_i \in [0, \frac{13-10\theta_j}{10}]$;
- when $\gamma \in (\frac{2}{3}, 1]$ if $\theta_j \in [0, \frac{\gamma}{\gamma+1}]$ and $\theta_i \in [0, \bar{\theta}]$ or if $\theta_j \in [\frac{\gamma}{\gamma+1}, 1]$ and $\theta_i \in [0, \bar{\theta}]$,

with $\bar{\theta} = \frac{2(2-\theta_j)(\gamma+1)+3}{2(\gamma+1)} \theta_i = \frac{-2\theta_i(\gamma+1)\theta_j+8\gamma+9}{4(\gamma+1)} - \frac{1}{4} \sqrt{4(\gamma+1)\theta_i[(\gamma+1)\theta_j-2\gamma-3]+8(5\gamma+6)+9} / (\gamma+1)^2 \hat{\theta} = \frac{43-10\theta_j - \sqrt{20\theta_j(5\theta_j-13)+529}}{20}$.

This condition is imposed in order to guarantee that firms’ profits are positive in the three scenarios, that is, CSR firms keep their private nature for any level of product differentiation. Under this assumption, firms do not behave as Non-Profit Organizations (NPOs), by bearing negative profits in face of a very high concern for consumers’ welfare.\footnote{For instance, firms that compete in sectors that provide public goods, such as water utility, rail track maintenance, private air-traffic control (Bennett et al., 2003), or universities may be examples of NPOs.} Notice that as products become less differentiated, there are more restrictions on the CSR levels in order to ensure that both firms are private in the three scenarios. For instance, if firm 2 has a sufficiently low level of CSR, firm 1 can have any level of CSR. However, when $\gamma \geq \frac{2}{3}$, in order to rule out negative profits, the CSR levels cannot be too high for both firms simultaneously.

The critical discount factor for collusion sustainability, $\delta^*_i (\gamma, \theta_i, \theta_j)$, is obtained by substituting (3.5), (3.7) and (3.9) into (3.3), and is given by:

$$\delta \geq \frac{\gamma (\theta_j - 1) + \theta_j}{\left[ (\theta_i - 3)(\theta_j - 3) - \gamma^2 (\theta_j - 1) (\theta_i - 1) - (\theta_j - 3) (\theta_i - 3) \right]^2 \left[ (3 - \theta_i) [(1 + \gamma) (\theta_i + \theta_j - 2) - 1]^2 \right] \times X},$$

(3.10)
with \( Z = (\theta_j - 3)^2 + \gamma^2(\theta_j - 1)^2 + 2\gamma(\theta_j - 1)(\theta_i + \theta_j - 3) + \theta_i(2\theta_j - 3) \) and \( X = \gamma^2[\theta_i((5 - \theta_i)\theta_i + (\theta_j - 2)\theta_j - 6) + 3] - 2\gamma(\theta_i - 3)(2\theta_i + \theta_j - 3) + (\theta_i - 3)[(\theta_i - 3)\theta_i - (\theta_j - 3)^2]. \)

### 3.4 Model analysis

We consider two cases: (i) symmetric case, where both firms have the same level of CSR and (ii) asymmetric case, where one firm is a CSR firm and the other is a pure profit maximizer. In the Appendix we also provide an analysis of the asymmetric case where both firms have positive but different levels of CSR.

#### 3.4.1 Symmetric case \((\theta_1 = \theta_2)\)

Let us assume that the level of social concern is the same for the two firms: \(\theta_1 = \theta_2 = \theta\). In order to ensure that firms are active in the three scenarios, Assumption 1 is now reduced to \(\theta \in [0, \frac{2\gamma+3}{4(\gamma+1)}]\) when \(\gamma \in (-\frac{1}{2}, 1]\) and \(\theta \in [0, 1]\) when \(\gamma \in [-1, -\frac{1}{2}]\).

Substituting \(\theta_i\) and \(\theta_j\) by \(\theta\) into (3.10) and after some straightforward algebra, we have:

\[
\delta \geq \frac{[\gamma(\theta - 1) + \theta - 3]^2}{\gamma^2(\theta - 1)^2 + 4\gamma(\theta - 3)(\theta - 1) + 3(\theta - 3)(\theta - 2)} \equiv \delta_{sym}^{*}(\gamma, \theta) \tag{3.11}
\]

Proposition 3.1 summarizes our results as regards the effects of CSR on the sustainability of collusion with symmetric levels of social concern.
Proposition 3.1. (Lambertini and Tampieri, 2012) Under collusion, increasing the weight attached to consumer surplus ($\theta$) makes collusion harder to sustain.

Proof. Differentiating (3.11) w.r.t. $\theta$ yields

$$
\frac{\partial \delta^*_{\text{sym}}(\gamma, \theta)}{\partial \theta} = \frac{3 - \gamma(\theta - 1) - \theta}{\gamma^2(\theta - 1)^2 + 4\gamma(\theta - 3)(\theta - 1) + 3(\theta - 3)(\theta - 2)^2} \geq 0,
$$

for all $\theta \in [0, 1]$ and $\gamma \in [-1, 1]$.

In the three scenarios, CSR increases the output produced by each firm, raising production costs, but also reduces prices. As a consequence, firms’ profits are decreasing in the level of CSR. However, since firms are socially responsible, they are willing to sacrifice profits in the interest of consumers (Bénabou and Tirole, 2010). Consequently, firms’ payoffs increase in the three scenarios, since the positive impact of CSR on consumer surplus outweighs the negative effect on profits.

As firms become more socially responsible (i.e., as $\theta$ increases), collusion sustainability decreases. Although, with CSR, firms have higher payoffs in the three scenarios, the difference between collusive and Cournot payoffs increases less than the difference between deviation and collusive payoffs. Therefore, firms incentives to deviate from the agreement are higher, making collusion less sustainable. This result confirms those obtained by Lambertini and Tampieri (2012) and Tzavara (2008) in the case of homogeneous goods.

We have already concluded that CSR hinders collusion (see Proposition 3.1), as shown in the literature. However, our results also depend on the degree of product differentiation. Proposition 3.2 and Figure 3.1 summarize our results as regards the effects of different degrees of product differentiation on the sustainability of collusion with symmetric levels of CSR. Notice that, in Figure 3.1, we consider that there is no collusion when Assumption 1 is not satisfied.
Proposition 3.2. With symmetric levels of CSR, collusion sustainability is:

- higher when products are independent for a sufficiently low level of CSR \((\theta < \frac{\gamma}{1+\gamma})\);
- higher when products are substitutes for a sufficiently high level of CSR \((\theta > \frac{\gamma}{1+\gamma})\);
- unaffected by the degree of product differentiation for \(\theta = 1\) and for \(\theta = \frac{\gamma}{1+\gamma}\).

Proof. Assuming that Assumption 1 holds. Differentiating (3.11) w.r.t. \(\gamma\) yields

\[
\frac{\partial \delta_{sym}^* (\gamma, \theta)}{\partial \gamma} = \frac{2(\theta - 3)(\theta - 1)[\gamma(\theta - 1) + \theta - 3][\gamma(\theta - 1) + \theta]}{[\gamma^2(\theta - 1)^2 + 4\gamma(\theta - 3)(\theta - 1) + 3(\theta - 3)(\theta - 2)]^2} \leq 0,
\]

when \(\gamma \in [-1, -\frac{1}{2}]\) for all \(\theta \in [0, 1]\) and when \(\gamma \in (-\frac{1}{2}, 1]\), for \(\theta \in \left(\frac{\gamma}{1+\gamma}, \frac{2\gamma+3}{4(\gamma+1)}\right]\). □

![Figure 3.1](image.png)

Figure 3.1. Effect of the degree of product differentiation on the sustainability of collusion.
Figure 3.2 below shows how the critical discount factor changes with the level of CSR and the degree of product differentiation.\textsuperscript{14}

![Graph showing critical discount factor](image_url)

Figure 3.2. Critical discount factor for different degrees of product differentiation and different levels of CSR.

It is important to stress that the lowest curve presented in Figure 3.2 captures the framework presented by Deneckere (1983), apart from the fact that, differently from this author, we consider quadratic costs.\textsuperscript{15}

When there is no CSR ($\theta = 0$), collusion is more stable if products are more independent (less substitutes or less complements), i.e., when $\gamma \to 0$. Furthermore, the critical discount factor is increasing and concave in the degree of product differentiation.\textsuperscript{16} When products are complements or when products are substitutes, collusion is less stable since there are higher incentives for firms to deviate from the agreement. However, it is easier for firms to collude when products are more substitutes than when products are more complements. These results are in line with Ross’s (1992) and partially with Deneckere’s (1983, 1984), who conclude that Cournot competition makes collusion more difficult to sustain for very close substitutes.

\begin{itemize}
\item \textsuperscript{14}Notice that the dashed lines represent the values of the critical discount factor that do not satisfy Assumption 1.
\item \textsuperscript{15}Notice that when $\theta = 0$ the critical discount factor in (3.10) simplifies to $\delta_{\text{NoCSR}}(\gamma) \equiv \frac{(\gamma+3)^2}{\gamma(\gamma+12)+18}$.
\item \textsuperscript{16}By differentiating $\delta_{\text{NoCSR}}(\gamma)$ w.r.t. $\gamma$, yields $\frac{\partial \delta_{\text{NoCSR}}(\gamma)}{\partial \gamma} = \frac{6\gamma(\gamma+3)}{(12\gamma+\gamma^2+18)^2} > 0$ if $|\gamma| > 0$ and $\frac{\partial^2 \delta_{\text{NoCSR}}(\gamma)}{\partial \gamma^2} = \frac{6(24-\gamma^2(2\gamma+9))}{(12\gamma+\gamma^2+18)^3} > 0$, for $\forall \gamma \in [-1, 1]$.
\end{itemize}
Introducing CSR, we find that when products are complements, deviation is more tempting. As products are more complements, the existing differences between the collusive and Cournot payoffs and between deviation and collusive payoffs are higher than when products are more homogeneous. Firms prefer to compete in a separated market rather than to collude with the other firm since each firm is already a monopolist in its market and benefits from the other firm’s production due to complementarity. Additionally, by colluding, firms’ production increases, which contrasts the standard result in the literature. However, by producing more, their costs increase as well, decreasing their payoffs and making collusion less attractive.

As products become more independent, firms’ payoffs in the three scenarios start to decrease. The deviation payoff decreases more than the payoffs in the other scenarios, making deviation less tempting. Moreover, the difference between deviation and collusive payoffs decreases more than the difference between collusive and Cournot payoffs, increasing collusion sustainability.

As products become close substitutes, the results depend on the weight firms attach to consumer surplus. On the one hand, for low levels of CSR ($\theta < \frac{\gamma}{1 + \gamma}$), collusion is harder to sustain with more homogeneous products as shown in the literature (see, for instance, Ross, 1992). On the other hand, for sufficiently high levels of CSR (except for $\theta = 1$, when the critical discount factor is independent of the degree of product differentiation\(^\text{17}\)), collusion sustainability is actually increasing in the degree of product substitutability. Although the payoffs decrease in the three scenarios, the incentives to deviate are now lower since the difference between deviation and collusive payoffs is very small. This finding extends the existing literature on collusion with product differentiation and provides new insights to the literature of collusion with CSR.

\(^{17}\)Notice that when $\theta = 1$, $\frac{\partial I_{min}(\gamma, \theta)}{\partial \gamma} = 0.$
3.4.2 Asymmetric case ($\theta_1 \neq \theta_2$)

Let us consider the case where firm $i$ has a positive level of social responsibility (with $\theta_i = \theta > 0$) and the other behaves as a pure profit-maximizing firm (with $\theta_j = 0$). Also, Assumption 3.1 holds, guaranteeing firms’ private nature.\footnote{In this case, Assumption 3.1 holds when $\gamma \in [-1, \frac{3}{4}]$, for all $\theta \in [0,1]$ and when $\gamma \in (\frac{3}{4}, 1]$, for all $\theta \in \left[0, \frac{8\gamma + 9}{4(\gamma + 1)^2} - \frac{1}{4}\sqrt{\frac{40\gamma + 48\gamma + 72}{(\gamma + 1)^2}}\right]$.} Substituting $\theta_i = \theta$ and $\theta_j = 0$ into (3.10), we have the discount factor of the CSR firm:

$$\delta_{CSR}^{asym}(\gamma, \theta) = \frac{\gamma^2 [3(\theta - 3) - \gamma^2(\theta - 1)]^2}{[\theta(4\gamma + 3) + \gamma(\gamma - 3) - \theta^2(\gamma + 1)]\Omega},$$

with $\Omega = \gamma^4(\theta - 1)^2 + \gamma^3(\theta^2 - 4\theta + 3)^2 + \gamma^2(\theta - 3)\{\theta((\theta - 4)(\theta - 2) + 6) - (\theta - 3)^2(\theta - 6)\{\theta - \gamma(1 - \theta)\}$.

Computing the discount factor of the No-CSR firm we get:

$$\delta_{No-CSR}^{asym}(\gamma, \theta) = \frac{[\gamma^2(\theta - 1) - 3(\theta - 3)]^2[\gamma(\theta - 1) + \theta]^2}{\gamma^2[\gamma(\theta - 1)^2 + \theta(\theta - 1) - 3]\Phi},$$

with $\Phi = \gamma^3(\theta - 1)^2 + \gamma^2\{(\theta - 7)\theta + 9\} - 6\gamma(\theta - 3)(\theta - 1) - 6(\theta - 3)^2$.

The critical discount factor depends on the range of possible levels of CSR but also on the degree of product differentiation. As we can see in Figure 3.3, in some regions of $(\gamma, \theta)$ the binding critical discount factor belongs to the firm that has no CSR ($\delta_i < \delta_j$) and in other regions to the CSR firm ($\delta_i > \delta_j$). Also, in some regions of $(\gamma, \theta)$ there is no collusion since Assumption 3.1 does not hold and/or the critical discount factor does not belong to $[0, 1]$.\footnote{In this case, Assumption 3.1 holds when $\gamma \in [-1, \frac{3}{4}]$, for all $\theta \in [0,1]$ and when $\gamma \in (\frac{3}{4}, 1]$, for all $\theta \in \left[0, \frac{8\gamma + 9}{4(\gamma + 1)^2} - \frac{1}{4}\sqrt{\frac{40\gamma + 48\gamma + 72}{(\gamma + 1)^2}}\right]$.}
When products are complements (as $\gamma \to -1$) and collusion is possible, the binding critical discount factor always belongs to the firm that has no CSR ($\delta_i < \delta_j$), except for the extreme case when $\gamma = -1$.\footnote{Actually, the critical discount factor of firm $j$, when $\gamma = -1$, does not change with $\theta$ ($\delta_j = 0.571$).} In fact, as products become complements, the quantity produced by each firm under collusion increases. Both the payoff of the CSR firm and the profit of the No-CSR firm will increase. However, the profit of the No-CSR firm will increase less making collusion less attractive. This effect is amplified the higher is the level of CSR. Hence, since the No-CSR firm is more prone to deviate from the collusive agreement, its critical discount factor is the binding one.

When products are substitutes (as $\gamma \to 1$) and collusion is feasible, the critical discount factor belongs to the socially responsible firm for sufficiently low levels of CSR ($\delta_i > \delta_j$). For high levels of CSR, the results are not straightforward. In this case, the binding critical discount factor will depend on the combination of $(\gamma, \theta)$. As products become substitutes, quantities produced decrease with $\gamma$. Firms have fewer incentives to produce more since if one firm increases its quantity, the other would react by reducing its quantity or increasing it less. This effect is enlarged when the level of CSR increases. The payoff of the CSR firm and the profit of the No-CSR firm are decreasing in $\gamma$. However, the payoff of the CSR firm is increasing with $\theta$ and the profit of the No-CSR firm is decreasing with $\theta$. Therefore, for low
levels of CSR, the impact of $\gamma$ tends to weigh more than the impact of $\theta$, making deviation more tempting to the CSR firm. In addition, for larger levels of CSR, a larger $\gamma$ for the same level of corporate social responsibility also gives this firm more incentives to deviate from the agreement, making its critical discount factor binding in these regions of $(\gamma, \theta)$.

As we can see from Figure 3.4, collusion sustainability depends on the effects of the level of CSR and the degree of product differentiation. As $\theta$ increases, collusion is easier to sustain when the products are substitutes and the level of CSR is sufficiently high. The reverse occurs if the level of CSR is sufficiently low and products are substitutes or when products are complements for any level of CSR. The conclusion that CSR makes collusion more difficult to sustain is already known from the symmetric case. However, when goods are substitutes, the critical discount factor is decreasing with CSR for specific combinations of $(\gamma, \theta)$ and not only for sufficient high levels of CSR.

![Figure 3.4. Impact of $\theta$ and $\gamma$ on the critical discount factor.](image)

When products are complements, the critical discount factor is increasing in $\gamma$, i.e., decreasing in the degree of product differentiation, except for the extreme case of $\gamma = -1$ and for sufficiently low levels of CSR if $\gamma < -0.5$. Notice that product differentiation increases the quantities produced and, as a result, both the payoff of the CSR firm and the profit of the No-CSR firm increase, promoting the collusive behavior.

When products are substitutes, the critical discount factor is decreasing in $\gamma$ for sufficiently
low levels of CSR, but increasing for $\gamma > 0.5$. The intuition for this result is the following. As products become substitutes, quantities produced decrease since product differentiation is reduced ($\gamma$ effect). Nevertheless, a higher level of CSR pressures the socially responsible firm to produce more ($\theta$ effect). As a result, the payoff of the CSR firm increases and the profit of the No-CSR firm decreases. For low levels of CSR, however, even the socially responsible firm is more concerned about profits than consumers and, therefore, its incentives to deviate are higher. For larger levels of CSR, the critical discount factor may either increase or decrease with $\gamma$, depending on the combination of ($\gamma, \theta$). In this case, a lower degree of product differentiation, holding the level of CSR constant, tends to amplify the $\gamma$ effect over the $\theta$ effect, making deviation more tempting to the CSR firm.

In the Appendix, we provide the analysis of the asymmetric case where both firms are socially responsible but have asymmetric levels of CSR ($\theta_i, \theta_j > 0$ and $\theta_i \neq \theta_j > 0$) and conclude that the results are similar to the ones described in this section.

### 3.5 Welfare implications

The aim of this section is to analyse the effects of collusion with CSR on welfare.\(^{20}\)

The consumer surplus (CS) under collusion with CSR is given by:

\[
CS^C(\theta_i, \theta_j, \gamma) = \frac{\gamma + 1}{[1 - (\gamma + 1)(\theta_i + \theta_j - 2)]^2} \tag{3.14}
\]

Proposition 3.3 and Figure 3.5 summarize the welfare effects.

\(^{20}\)Given that we model CSR as a component of firms’ objective function, it would not make sense to analyze how firms’ decisions affect total surplus (TS). In fact, analyzing the effects on TS would mean including the component of consumer surplus (CS) at least twice on that measure: $TS = W_1 + W_2 + CS$, since $W_1$ and $W_2$ already include some portion of CS. Moreover, for the purpose of providing some policy implications, we follow Lyons (2002) who argues that many competition authorities operate under legislation and guidelines that aim at protecting consumers. Hence, our welfare analysis is based on the effects of firms’ decisions on CS.
Proposition 3.3. Under collusion, when firms are socially responsible, consumer surplus:

- is strictly increasing in the level of CSR;
- is decreasing in the degree of product substitutability for \( \gamma \in (-\frac{1}{2}, 1] \) when \( \theta_i + \theta_j \in \left[0, \frac{2\gamma+1}{\gamma+1}\right) \).

Proof. Differentiating (3.14) w.r.t. \( \theta_i, \theta_j \) and \( \theta_i + \theta_j \) yields:

\[
\frac{\partial CS^C}{\partial \theta_i} = \frac{\partial CS^C}{\partial \theta_j} = \frac{\partial CS^C}{\partial (\theta_i + \theta_j)} = \frac{2(\gamma + 1)^2}{[1 - (\gamma + 1)(\theta_i + \theta_j - 2)]^3} \geq 0,
\]

for all \( \theta_i, \theta_j \in [0, 1], \theta_i + \theta_j \in [0, 2] \) and \( \gamma \in [-1, 1] \).

Note that the numerator is always positive. Also, \( \theta_i + \theta_j - 2 \leq 0 \) and \( \gamma + 1 > 0 \) since \( \gamma \in (-1, 1] \). Hence, the denominator is also positive. When \( \gamma = -1 \) the effect of \( \theta \) on CS is 0.

Differentiating (3.14) w.r.t. \( \gamma \) we have:

\[
\frac{\partial CS^C}{\partial \gamma} = \frac{1 + (\gamma + 1)(\theta_i + \theta_j - 2)}{[1 - (\gamma + 1)(\theta_i + \theta_j - 2)]^3} < 0,
\]

when \( \theta_j \in [0, \frac{1}{2}) \) and \( \theta_i \in [0, 1] \) for \( \gamma \in \left[\frac{1-\theta_i-\theta_j}{\theta_i+\theta_j-2}, 1\right] \) or when \( \theta_j \in \left[\frac{1}{2}, 1\right] \) and \( \theta_i \in \left[0, \frac{3-2\theta_j}{2}\right) \) for \( \gamma \in \left(\frac{1-\theta_i-\theta_j}{\theta_i+\theta_j-2}, 1\right] \). This is also true when \( \theta_i + \theta_j \in \left[0, \frac{2\gamma+1}{\gamma+1}\right) \) for \( \gamma \in (-\frac{1}{2}, 1] \). \qed

Figure 3.5. Impact of \( \gamma \) on Consumer Surplus under collusion.
When we compare the symmetric with the asymmetric case, we find that in both cases, under collusion, CS increases with the degree of product differentiation if products are sufficiently differentiated \((\gamma < -0.5)\) for any level of CSR. Notice that a higher level of CSR increases quantities and, therefore, we can expect a higher consumer surplus. Also, there are two effects on consumer surplus from the change in the level of product differentiation \((\gamma)\): (i) a direct effect, as products become more differentiated, the representative consumer’s utility increases and, as a result, consumer surplus will also increase even if the quantity levels do not change; and (ii) a strategic effect, by increasing the level of product differentiation, collusive quantities increase and, therefore, consumer’s utility and surplus also increase. If products are less differentiated or substitutes, an increase in the degree of product differentiation increases CS for sufficiently high levels of CSR, but not for lower levels. Indeed, note that the direct and the strategic effects from differentiation do not hold in this case. As \(\gamma\) increases, the degree of product differentiation reduces and quantities decrease. As quantities decrease, consumer surplus is negatively affected. However, if the level of CSR is sufficiently high, the weight attached to consumer surplus gives firms incentives to produce more, compensating for the decrease in quantities caused by a change in \(\gamma\). These two opposite effects explain why consumer surplus may either decrease or increase in \(\gamma\) for \(\gamma > -0.5\).

We are also able to compare consumer welfare under collusion and under competition for the symmetric and asymmetric cases. Our results are summarized in Proposition 3.4 and Figure 3.6.

**Proposition 3.4. With CSR, consumer surplus is higher under collusion than under competition:**

- when products are sufficiently differentiated \((\gamma \leq 0)\), for any level of CSR;
- when products are homogeneous \((\gamma > 0)\) if the level of CSR is sufficiently high.
We conclude that collusion with CSR improves consumer surplus (in comparison to competition) when goods are more differentiated or the level of CSR is sufficiently high, as shown in Figure 3.6. This is an interesting result as collusion when firms are socially responsible may be, in fact, welfare-improving when compared to the case where there is no CSR or where firms compete with CSR. This extends the literature that finds that collusion hinders consumers.

Figure 3.6. Sign of the difference between CS under collusion and under competition for different degrees of product differentiation and levels of CSR.

When products are complements ($\gamma < 0$) the collusive output is always higher than the competitive output in both the symmetric and the asymmetric case. Therefore, collusion improves consumer surplus for complements. However, when products are substitutes, the collusive quantity is only higher than the competitive quantity for a sufficiently high level of CSR. In both competition scenarios, quantities decrease as products become more substitutes. However, as the level of CSR increases, the weight attached to consumer surplus in firms’ objective functions plays an important role. The higher is the level of CSR, the higher is the quantity produced by the CSR firms. Therefore, for sufficiently high levels of CSR, the effect of $\theta$ outweighs the effect of $\gamma$ on the quantity produced. By producing more, firms increase consumer surplus.
We are aware that our results follow our assumptions regarding the modelling of CSR. Also, if simultaneously products are sufficiently homogeneous and the level of CSR is low, we obtain that collusion damages consumers (when compared to the competitive scenario). This is an expected result since in this case firms’ altruistic concern is reduced and, therefore, their behavior is very similar to that of profit-maximizer firms producing closely homogeneous goods. This should be taken into account by competition authorities when analyzing markets with socially responsible firms since collusion may either be welfare-improving or welfare-detrimental. As we have shown, competition authorities should consider both the degree of product differentiation and the level of social engagement to design appropriate intervention tools. Given that the presence of socially concerned firms facilitates the attainment of higher welfare levels, even under collusion, it remains to the competition authorities to assess, on a case-to-case basis, whether these agreements work or not in the public interest.

3.6 Conclusion

In this paper we investigate the coordinated effects of corporate social responsibility in a duopoly market where firms sell differentiated goods and the level of CSR, exogenously given, is modelled as the weight that firms attribute to consumer surplus in their objective function. We consider two cases: a symmetric case, in which both firms have the same level of social responsibility; and an asymmetric case, in which one firm is socially responsible and the other behaves as a pure profit-maximizer.

We find that firms willing to mix corporate do-gooding with profit maximization face two opposite effects when they are coordinating their strategies. On the one hand, CSR increases both deviation and competition payoffs, fostering firms’ incentives to deviate. On the other hand, CSR also increases payoffs under collusion. Since the difference between collusive and competition payoffs increases less than the difference between deviation and collusive pay-
offs, the net effect is that CSR hinders the sustainability of collusion. This result corroborates previous findings in the literature on collusion with socially responsible firms producing homogeneous goods. Nevertheless, our contribution provides new insights to the literature as we show that product differentiation plays a crucial role. Our analysis captures the case of homogeneous goods as a particular one.

When firms have the same level of social responsibility, collusion is more stable when products are independent for low levels of CSR and when products are substitutes for high levels of CSR. The finding that collusion sustainability increases in the degree of substitutability contradicts a branch of the literature that studies collusion with product differentiation without CSR that concludes that Cournot competition makes collusion more difficult to sustain for very close substitutes (Deneckere, 1983, 1984; Ross, 1992). The intuition for this result is that for sufficiently high levels of CSR, the weight attached to consumer surplus in the firms’ objective function, which pressures the firms to produce more, compensates the decrease in quantities caused by competition with substitute goods. When goods are substitutes, incentives to deviate are, therefore, higher for low levels of CSR since firms are more concerned about profits.

If firms are asymmetric respecting CSR, our conclusions regarding collusion sustainability depend on the level of product differentiation, on the level of CSR and on the binding critical discount factor, which may belong to one firm or the other. When there is only one socially responsible firm, collusion sustainability increases with product differentiation, i.e., collusion is easier to sustain when firms produce complements (except for very low levels of CSR). This result contrasts the one obtained when firms have the same level of social concern. The reasoning for this result is the following. Product differentiation makes collusive quantities to increase. As a result, both the payoff of the CSR firm and the profit of the No-CSR firm are increasing with product differentiation, facilitating collusion. Nevertheless, when products are complements, the critical discount factor belongs to the No-CSR firm. This is a natural result since the quantity produced under collusion is larger than the quantity produced under competition, giving the profit-maximizing firm more incentives to deviate from the collusive
agreement as this firm only cares about profits.

As products become more substitutes, conclusions regarding collusion sustainability are not unequivocal. Given the effect of product differentiation, the quantities produced decrease with product’s homogeneity. In this case, firms have less incentives to produce more since it triggers an undercut by the rival firm. This effect is enlarged when the level of CSR increases. In fact, both the payoff of the CSR firm and the profit of the No-CSR firm are decreasing in the degree of product differentiation. However, the payoff of the CSR firm is also increasing in the level of CSR, while the profit of the No-CSR firm is decreasing. As a consequence, for low levels of CSR, the impact of the degree of product differentiation tends to weigh more than the impact of the level of CSR, expanding the incentives to deviate from the agreement of the CSR firm. For high levels of CSR, the impact that prevails depends on the combination of the degree of product differentiation and the level of CSR.

When comparing the welfare effects under competition and under collusion, our results suggest that collusion may improve consumer surplus. Specifically, consumer surplus is larger under collusion than under competition when products are complements or when firms are very concerned about consumers. Once again, the degree of product differentiation plays a role in this result. The reasoning for this result is twofold. On the one hand, product differentiation generates a direct effect as the representative consumer’s utility increases with a larger variety of goods available in the market; one the other hand, product differentiation generates a strategic effect as collusive quantities increase and, therefore, consumer’s utility and surplus also increase. Note that firms producing complements react directly to the quantity produced by the rival by increasing output in response to an increase in the output level of the rival. On the contrary, when the degree of product differentiation vanishes, i.e., when goods are more substitutes, quantities decrease. In this case, two opposite effects have to be considered: the effect of the degree of product differentiation and the effect of the level of CSR. If larger similarities between the products decrease the quantities produced, a higher level of CSR, through the component attached to consumer surplus, increases the quantities produced by socially re-
sponsible firms. As a result, for high levels of CSR, the latter outweighs the former, making collusion beneficial to consumers. On the contrary, if the level of CSR is not sufficiently high, collusion decreases output and, consequently, consumer surplus. This captures the common result in the literature on collusion that states that collusion is harmful to consumers (Farrell and Shapiro, 1990).

Since collusion may be welfare-improving, we recommend competition authorities to be meticulous when analyzing cartel cases involving socially responsible firms producing differentiated goods in oligopoly markets.
Appendix

Asymmetric case \((\theta_i \neq \theta_j > 0 \text{ and } \theta_i, \theta_j > 0)\)

Let us consider the case where both firms are socially responsible but have asymmetric levels of CSR \((\theta_i, \theta_j > 0 \text{ and } \theta_i \neq \theta_j > 0)\). When Assumption 3.1 holds, in this scenario, the critical discount factor is given by (3.10). Tables 3.1 to 3.5 (below) summarize the values of the critical discount factor obtained for different levels of CSR and product differentiation.

In this scenario, as products become substitutes (as \(\gamma \to 1\)), the conclusion regarding the effects of the CSR level and the degree of homogeneity on collusion sustainability is not unequivocal (see Tables 3.1 to 3.3 below). In fact, a narrower Assumption 3.1 makes the range of possible levels of CSR smaller. As products become close substitutes, firms incentives to increase their levels of CSR decrease. Higher levels of CSR would increase quantities and, as a result, reduce prices. However, this would also increase the production costs. Since costs are quadratic, the second effect would dominate; therefore, profits would also decrease. Once again, when one firm has a higher CSR level than the other, the binding critical discount factor is the one of the firm with lower level of CSR (if \(\theta_i > \theta_j\) then \(\delta_j > \delta_i\)), except for the extreme case of perfect substitutability \((\gamma = 1)\). When products are perfect substitutes \((\gamma = 1)\), collusion is only sustainable when the aggregate level of CSR is one \((\theta_i + \theta_j = 1)\).
Table 3.1. Critical discount factor for $\gamma = 1$ and $\theta_i \neq \theta_j > 0$.

Table 3.2. Critical discount factor for $\gamma = 0.5$ and $\theta_i \neq \theta_j > 0$.

Table 3.3. Critical discount factor for $\gamma = 0$ and $\theta_i \neq \theta_j > 0$.

As products become complements (as $\gamma \rightarrow -1$), the range of possible levels of CSR ($\theta_i$...
and $\theta_j$) that satisfies Assumption 3.1 and ensures collusion sustainability becomes wider\footnote{This is due to the fact that Assumption 3.1 becomes less restrictive for a high degree of product differentiation.} (see Table 3.4 and Table 3.5). Additionally, for the same aggregate level of CSR, collusion is easier to sustain the higher is the degree of product differentiation. One should notice that the binding critical discount factor always belongs to the firm that has a lower level of CSR (for instance, if $\theta_i > \theta_j$ then $\delta_j > \delta_i$), except for the extreme case of $\gamma = -1$, where the critical discount belongs to the firm that has a higher level of CSR. Moreover, when firms have different levels of CSR, the larger is this difference, the more difficult it is to hold the collusive agreement among them.

\begin{table}[ht]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$\gamma = -0.5$ & $\theta_j$ & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \\
\hline
0.1 & $\delta_i = 0.614$ & $\delta_i = 0.715$ & $\delta_i = 0.824$ & $\delta_i = 0.942$ & & & & & & & \\
0.2 & $\delta_i = 0.619$ & $\delta_i = 0.711$ & $\delta_i = 0.810$ & $\delta_i = 0.916$ & & & & & & & \\
0.3 & $\delta_i = 0.715$ & $\delta_i = 0.619$ & $\delta_i = 0.711$ & $\delta_i = 0.802$ & $\delta_i = 0.899$ & & & & & & \\
0.4 & $\delta_i = 0.824$ & $\delta_i = 0.711$ & $\delta_i = 0.626$ & $\delta_i = 0.635$ & $\delta_i = 0.714$ & $\delta_i = 0.798$ & $\delta_i = 0.887$ & $\delta_i = 0.981$ & & & \\
0.5 & $\delta_i = 0.942$ & $\delta_i = 0.810$ & $\delta_i = 0.711$ & $\delta_i = 0.635$ & $\delta_i = 0.645$ & $\delta_i = 0.719$ & $\delta_i = 0.798$ & $\delta_i = 0.880$ & $\delta_i = 0.967$ & & \\
0.6 & $\delta_i = 0.916$ & $\delta_i = 0.802$ & $\delta_i = 0.714$ & $\delta_i = 0.645$ & $\delta_i = 0.657$ & $\delta_i = 0.727$ & $\delta_i = 0.801$ & $\delta_i = 0.878$ & & & \\
0.7 & $\delta_i = 0.899$ & $\delta_i = 0.798$ & $\delta_i = 0.719$ & $\delta_i = 0.657$ & $\delta_i = 0.671$ & $\delta_i = 0.737$ & $\delta_i = 0.807$ & & & & \\
0.8 & $\delta_i = 0.887$ & $\delta_i = 0.798$ & $\delta_i = 0.727$ & $\delta_i = 0.671$ & $\delta_i = 0.686$ & $\delta_i = 0.750$ & $\delta_i = 0.704$ & & & & \\
0.9 & $\delta_i = 0.981$ & $\delta_i = 0.880$ & $\delta_i = 0.801$ & $\delta_i = 0.737$ & $\delta_i = 0.686$ & $\delta_i = 0.750$ & $\delta_i = 0.704$ & & & & \\
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\end{tabular}
\caption{Critical discount factor for $\gamma = -0.5$ and $\theta_i \neq \theta_j > 0$.}
\end{table}

\begin{table}[ht]
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\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$\gamma = -1$ & $\theta_j$ & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \\
\hline
0.1 & $\delta_i = 0.588$ & $\delta_i = 0.597$ & $\delta_i = 0.606$ & $\delta_i = 0.615$ & $\delta_i = 0.623$ & $\delta_i = 0.635$ & $\delta_i = 0.645$ & $\delta_i = 0.656$ & $\delta_i = 0.667$ & & \\
0.2 & $\delta_i = 0.597$ & $\delta_i = 0.597$ & $\delta_i = 0.606$ & $\delta_i = 0.615$ & $\delta_i = 0.623$ & $\delta_i = 0.635$ & $\delta_i = 0.645$ & $\delta_i = 0.656$ & $\delta_i = 0.667$ & & \\
0.3 & $\delta_i = 0.597$ & $\delta_i = 0.597$ & $\delta_i = 0.606$ & $\delta_i = 0.615$ & $\delta_i = 0.623$ & $\delta_i = 0.635$ & $\delta_i = 0.645$ & $\delta_i = 0.656$ & $\delta_i = 0.667$ & & \\
0.4 & $\delta_i = 0.606$ & $\delta_i = 0.606$ & $\delta_i = 0.606$ & $\delta_i = 0.615$ & $\delta_i = 0.625$ & $\delta_i = 0.635$ & $\delta_i = 0.645$ & $\delta_i = 0.656$ & $\delta_i = 0.667$ & & \\
0.5 & $\delta_i = 0.615$ & $\delta_i = 0.615$ & $\delta_i = 0.615$ & $\delta_i = 0.615$ & $\delta_i = 0.625$ & $\delta_i = 0.635$ & $\delta_i = 0.645$ & $\delta_i = 0.656$ & $\delta_i = 0.667$ & & \\
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1 & $\delta_i = 0.667$ & $\delta_i = 0.667$ & $\delta_i = 0.667$ & $\delta_i = 0.667$ & $\delta_i = 0.667$ & $\delta_i = 0.667$ & $\delta_i = 0.667$ & $\delta_i = 0.667$ & $\delta_i = 0.667$ & & \\
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\end{tabular}
\caption{Critical discount factor for $\gamma = -1$ and $\theta_i \neq \theta_j > 0$.}
\end{table}
Using numerical simulation, we are also able to compare consumer welfare under collusion and under competition. Our results are summarized in Table 3.6 (below).

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Table 3.6. Sign of the difference between CS under collusion and under competition for different levels of product differentiation.
Chapter 4

Can Collusion be Sustainable in Downstream Mixed Oligopolies?

4.1 Introduction

In 1997, the Spanish Competition Authority has convicted FENIL (Federación Nacional de Industrias Lácteas) and 49 producers of dairy products for coordinating through minimum price fixing.\(^1\) One of the 49 cartel members was a public firm (La Lactaria Española) and was the market leader in the sector (with 6 factories across the country and a 17% share in buying volume at that time). Dairy industries (downstream) fixed the minimum price at which the milk was to be bought from producers (upstream), as well as potential premia or discounts

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\(^1\)Expte. 352/94 Industrias Lácteas.
according to the quality standards of the product (milk).²,³,⁴ Our motivation follows from this cartel case involving a public firm, but also in view of a contribution to enlarge the literature on collusion in mixed oligopolies. To the best of our knowledge, vertical competition and downstream collusion has not been analyzed in a mixed oligopoly setting.

Our paper relates to the literature on (i) downstream collusion in vertical supply chains⁵ and (ii) vertical relations in mixed oligopolies. The first branch of the literature studies how the relative bargaining positions of upstream and downstream firms may affect supply contracts and firms’ ability to collude.⁶ Piccolo and Miklós-Thal (2012) showed that by having a strong bargaining position in input supply negotiations, firms ability to sustain collusion on output prices is greater if the firms also reach an implicit agreement on their input supply contracts. They consider vertical chains under exclusivity contracts and show that collusion on supply contracts, consisting in above marginal cost pricing plus a negative fee (slotting allowances) can increase collusion. Biancini and Ettinger (2017) also address downstream collusion in vertically related industries but, differently from the previous authors, they assume that downstream firms are not linked to a unique upstream supplier and they introduce the possibility of

²This cartel is not unique in involving industries in which firms obtain key inputs through bilateral supply contracts. In 2007, the Greek Competition Commission fined the five largest dairy product companies in Greece for horizontal price fixing and supply-sharing agreements in the acquisition of milk from the suppliers (Decision 369/V/2007). A dairy cartel among retailers (supermarkets) was also found in the UK (CA98/03/2011): https://www.gov.uk/cma-cases/dairy-products-investigation-into-retail-pricing-practices.

³Farmers are typically small enterprises, facing adverse bargaining conditions when selling their products to processing companies. Also, their bargaining power has deteriorated over the years following the increasing concentration of food retail. In the Netherlands dairy processors have pursued mergers in order to strengthen their bargaining power against retailers (Bijman, 2018). For a large urban area in the United States, Villas-Boas (2007) found that retailers’ pricing power over other players in the supply-chain on the yogurt market was consistent with a high level of bargaining power by retailers. As explained by Bonanno et al. (2018), modern food markets, such as the dairy industry, “have become characterized by high levels of concentration, complex vertical relationships and sophisticated forms of coordination” (p.6).

⁴For a complete survey on buyer power, with a particular application to retailing, see Inderst and Mazzarotto (2008). The effects of collusion on innovation and product quality are outside the scope of this paper. See, respectively, Jullien and Lefouili (2019) and Brekke et al. (2017).

⁵Horizontal cartels with vertical dimension have been studied by Jullien and Rey (2007); Piccolo and Reisinger (2011); Piccolo and Miklós-Thal (2012). The literature on the impact of downstream mergers is also relevant, since, in a large class of environments, the impact of a merger is the same as the impact of collusion. Lommerud et al. (2005) analyze the impact of a downstream merger in a vertical supply chain against a benchmark case in which input prices are exogenous.

⁶For instance, Bhaskarabhatla et al. (2016) examine the use of buyer power coupled with vertical restraints by a downstream cartel to impose targeted asymmetric punishments in the Indian retail pharmaceutical industry.
a vertical merger.\textsuperscript{7} If suppliers have some market power, damages from downstream collusion may be partially controlled. Huang (2017) showed that a supplier with market power can use nonlinear pricing (e.g. two-part tariff) in supply contracts to limit profit loss from downstream collusion by adjusting pricing in a way that downstream firms end up colluding at a large quantity.

All these contributions, nevertheless, only consider profit-maximizing firms. We are not the first to address vertical relations in mixed oligopolies (Matsumura and Matsushima, 2012; Chang and Ryu; 2015; Wu \textit{et al.}, 2016). Matsumura and Matsushima (2012) and Chang and Ryu (2015) study the privatization of public firms operating in upstream mixed duopolies, considering downstream competition between private firms.\textsuperscript{8} Wu \textit{et al.} (2016) also study the welfare implications of privatization in a mixed oligopoly with vertically related markets, but in their model there is a downstream mixed duopoly and an upstream foreign monopolist that sells an essential input to both downstream firms.

Apart from the literature on mixed oligopolies, there are some contributions analyzing vertical relations in the literature on corporate social responsibility. Brand and Grothe (2015) analyze a linear bilateral monopoly to assess the effects of firms’ social concern. The consideration of an extended objective function relates to our modelling of the objective function of the public firm. Their results show that to attach a level of social concern in firms’ objective functions increases profits and also softens the classical double marginalization problem. In addition, Goering (2012) analyzes a bilateral monopoly where one of the firms, either the upstream manufacturer or the downstream retailer, is socially responsible. This author models social responsibility through firms’ objective functions by placing a weight on consumers’ welfare.

In our model, we consider an upstream private monopoly and a downstream mixed duopoly.

\textsuperscript{7}Mendi (2009) also considers the effect of vertical integration on collusive outcomes in a downstream duopoly with asymmetric marginal costs, but considering a competitive upstream industry and a downstream duopoly while Biancini and Ettinger (2017) consider upstream and downstream oligopolies.

\textsuperscript{8}Chang and Ryu (2015) consider that foreign private firms compete with domestic firms both in the upstream and in the downstream markets.
The two downstream firms, a public firm and a private firm, only differ in their objective functions. While the private firm maximizes profits, the public firm maximizes a weighted sum of profits and consumer surplus. The three firms repeatedly play a per period game with three stages. In the first stage, the producer sets a non-discriminatory uniform wholesale price. In the second stage, the retailers may cooperatively agree on the quantities to sell in the downstream market. In the third stage, the retailers choose quantities to order and sell (if an agreement was reached in the second stage, they can honor it or deviate). The single input of the downstream firms is the good produced by the upstream monopolist. The upstream monopolist has either linear or quadratic production costs and the downstream firms face linear demand function for the final good. We focus on equilibria with trigger strategies: after a deviation, downstream firms permanently revert to Cournot-Nash behavior.

Our results are the following. If upstream costs are linear, downstream collusion is detrimental to final consumers and to the upstream firm, since both downstream firms contract their output and the input price is the same under downstream competition and collusion. Under this assumption, the collusive agreement is sustainable if firms are sufficiently patient, but the public firm has stronger incentives to abide by the collusive agreement than the private firm, i.e., the critical discount factor belongs to the private firm. If the upstream monopolist faces quadratic costs instead, collusion is also detrimental to consumers as total output is lower under collusion than under competition. As regards downstream firms’ incentives to collude, the results now depend (i) on the existence of economies or diseconomies of scale and (ii) on the weight the public firm attaches to consumer surplus in its objective function. We find that if the upstream marginal cost is increasing, the private firm has more incentives to deviate from the agreement than the public firm. By contrast, if the upstream marginal cost is decreasing, the public firm is the one more tempted to deviate. Finally, under our assumptions, we conclude that downstream collusion is never beneficial to consumers.
4.2 Setup

An upstream monopolist ($u$) produces an intermediate good and sells it to two downstream firms by setting a uniform and non-discriminatory wholesale price, $w$.\(^9\) One of the downstream firms is private ($p$) while the other is public ($g$). The output of the upstream firm is the single input of downstream firms. The technology of downstream firms exhibits constant returns to scale, and we normalize units so that they transform each unit of input into one unit of a homogeneous final good. Firms interact along an infinite number of periods, discounting future payoffs at the same constant rate $\delta \in (0, 1)$. In each period $t \in \mathbb{N}_0$, the timing of the per-period game is the following:

1. The upstream monopolist sets the uniform wholesale price, $w$.
2. Downstream firms observe $w$ and may cooperatively agree on quantities to sell, $(\hat{q}_p, \hat{q}_g)$.
3. Downstream firms simultaneously choose the quantities to sell, $q_p$ and $q_g$.

Downstream firms use trigger strategies to punish deviations from collusion. After a deviation, they permanently become unwilling to cooperate and revert to Cournot-Nash behavior. Firm $i \in \{p, g\}$ deviates from collusion whenever an agreement $(\hat{q}_p, \hat{q}_g)$ is reached in stage 2, but the firm sets $q_i \neq \hat{q}_i$ in stage 3. If firms do not reach an agreement in stage 2, then there is no agreement to deviate from and firms remain willing to cooperate in future periods.

Inverse demand in the downstream market is constant over time and given by:\(^{10}\)

\[
P = a - q_p - q_g,
\]

\(^9\)If the upstream firm sets a two-part tariff, it can fully appropriate downstream profits, regardless of whether the downstream firms are colluding or competing. It would not be interesting to study the incentives for downstream collusion in such an environment (see Armstrong, 2008; Goering, 2012; Pinopoulos, 2019). It would also be interesting to study an environment where the upstream firm is able to set discriminatory wholesale prices, but this is left for future work. Brito et al. (2019) investigated the effect of input price discrimination across vertically differentiated downstream retailers.

\(^{10}\)As demand is stationary, we will omit the time index and solve the model for an arbitrary period.
where $P$ is the market price of the final good, $a > 0$ is the choke price, and $q_i$ is the output level of firm $i \in \{p, g\}$. Total output is denoted by $Q \equiv q_p + q_g$, and corresponds to the demand of firm $u$. The cost supported by the upstream firm to produce $Q$ units is $C(Q)$, with $C'(\cdot) > 0$. The structure of the market is represented in Figure 4.1.

![Figure 4.1. Market structure.](image)

For simplicity, we assume that downstream firms have no additional costs besides the input price. Thus, the per-period profit of the downstream firm $i \in \{p, g\}$ is given by:

$$\pi_i = (a - q_p - q_g - w)q_i.$$  \hspace{1cm} (4.1)

The public and the private firm only differ in their objective functions. The private firm maximizes its own profit, whereas the public firm maximizes a weighted sum of its own profit and consumer surplus:

$$W_g = \pi_g + \mu CS = (a - q_p - q_g - w)q_g + \mu \frac{(q_p + q_g)^2}{2},$$  \hspace{1cm} (4.2)

where $\mu \geq 0$ denotes the weight attached to consumer surplus. We make the following assumption to ensure that both downstream firms are active even in the non-cooperative scenario:
Assumption 4.1. $\mu < 1$.

In the first-stage of each period, the upstream firm chooses $w$ that maximizes its profit:

$$\pi_u = wQ - C(Q),$$  \quad (4.3)

where $Q$ naturally depends on $w$. Thus, the upstream firm may charge a different input price depending on whether it expects downstream firms to cooperate or not.

It is well known that a repeated game has a plethora of equilibria. We focus on equilibria in Markovian strategies with two phases: an initial collusive phase, which lasts until some firm deviates; and a non-cooperative phase, which is irreversibly triggered by a deviation. During the collusive phase, the upstream firm expects downstream firms to choose their quantities cooperatively. In the non-cooperative phase (off-the-equilibrium-path), the upstream firm expects Cournot-Nash behavior by the downstream firms. Consistently, each downstream firm expects its rival to cooperate during the collusive phase and to set the Cournot-Nash individual output in the non-cooperative phase.

### 4.3 Analysis: Downstream market

We solve the per-period game by backward induction. First, taking the input price as given, we find the quantities that downstream firms choose non-cooperatively (Cournot-Nash equilibrium) and the quantities that they choose cooperatively through a Nash bargaining protocol (cooperative output). Then, we find the input price that maximizes the profit of the upstream monopolist given the continuation equilibrium of the per-period game.
4.3.1 Cournot-Nash equilibrium

If the downstream firms do not reach an agreement in the second-stage, they choose quantities that are mutual best-responses to the quantities chosen by the rival. For a given input price $w_N$, we can obtain the individual quantities by combining the corresponding first-order conditions, $\frac{\partial \pi_p}{\partial q_p} = 0$ and $\frac{\partial W_g}{\partial q_g} = 0$.$^{11}$

Result 4.1. For a given input price $w$, if the downstream firms choose output levels non-cooperatively, the output levels and payoffs are the following:

$$q^N_p(w) = \frac{1 - \mu}{3 - \mu} (a - w), \quad q^N_g(w) = \frac{1 + \mu}{3 - \mu} (a - w), \quad Q^N(w) = \frac{2}{3 - \mu} (a - w), \quad (4.4)$$

$$\pi^N_p(w) = \frac{(1 - \mu)^2}{(3 - \mu)^2} (a - w)^2, \quad W^N_g(w) = \frac{1 + 2\mu - \mu^2}{(3 - \mu)^2} (a - w)^2. \quad (4.5)$$

Not surprisingly, the higher is the weight the public firm attaches to consumer surplus, the greater is its output. As firms’ decisions are strategic substitutes, this expansion of public output leads to a contraction of private output.

4.3.2 Collusive agreement

Suppose downstream firms reach an agreement in stage 2 of the per-period game regarding the quantities to produce. Following Harrington (1991), we consider this collusive agreement is the result of a negotiation made through a Nash bargaining protocol. If firms do not reach an agreement, the continuation equilibrium yields the Cournot-Nash payoffs, given in (4.5).

$^{11}$Note that $\frac{\partial^2 \pi_p}{\partial q_p^2} = -2$ and $\frac{\partial^2 W_g}{\partial q_g^2} = -2 + \mu$, which means that, under Assumption 4.1, the second-order conditions of the individual maximization problems are satisfied.
We will start by characterizing the set of solutions of the Nash bargaining problem when side-payments are feasible. Then, we will show that there exists a solution of that problem with a null side-payment. Thus, the Nash bargaining problem with side-payments can be used as a technical device to find the Nash bargaining solution without side-payments.

If side-payments are possible, for a given input price $w$, downstream firms may agree on a triple $(q_p, q_g, S)$, where $S$ denotes a side-payment from the public to the private firm (if $S$ is negative, the side-payment is in the opposite direction). The Nash bargaining solution is the triple $(q_p, q_g, S)$ that maximizes the product of individual gains from collusion (subject to both gains being non-negative):

$$NP(q_p, q_g, S) = \left[ (a - q_p - q_g - w) q_p + S - \tilde{\pi}_p \right] \times \left[ (a - q_p - q_g - w) q_g + \mu CS(q_p + q_g) - S - \tilde{W}_g \right],$$

(4.6)

where $\tilde{\pi}_p = \pi_p^N(w)$ and $\tilde{W}_g = W_g^N(w)$ are the payoffs of downstream firms if they do not reach an agreement.

The Nash bargaining solution always yields a Pareto-efficient agreement. There does not exist an alternative agreement that is preferred by both firms because greater payoffs for both firms would trivially imply a greater Nash product. With side-payments (i.e., transferable utility), efficient agreements are those that maximize total payoff, which depends on total output but not on its allocation between firms: $\pi_p + \pi_g + \mu CS = (a - Q - w)Q + \mu \frac{Q^2}{2}$. The Nash bargaining solution is the agreement that maximizes total payoff and distributes in equal shares the total gain from the agreement. Total output and individual payoffs after the side-payment are uniquely determined, but not individual outputs and the side-payment.

**Lemma 4.1.** For a given input price $w$, if side-payments are feasible, the Nash bargaining solution prescribes a total output equal to:

$$Q^C(w) = \frac{a - w}{2 - \mu}.$$  

(4.7)
Proof. See Appendix.

Since marginal costs are constant and symmetric across downstream firms, the transfer of output from one firm to the other has no impact on their joint payoff. Transferring output from one firm to the other is payoff-equivalent to transferring money. Firms thus have two “instruments” to share the total profit: the allocation of output and the side-payment. This is the origin of the indeterminacy of individual output levels.

Among all possible output allocations, \( \{(q_p, q_g) \in \mathbb{R}_+^2 \mid q_p + q_g = Q^c(w)\} \), there exists one that dispenses with the side-payment (the one which distributes the collusive gain in equal shares). This particular solution of the Nash bargaining problem with side-payments is also the solution of the Nash bargaining problem in an environment where firms cannot make side-payments (a maximizer in a choice set is also a maximizer in a subset of that choice set). Therefore, we can see the Nash bargaining problem with side-payments as a technical device to find the Nash bargaining solution without side-payments.

Lemma 4.2. For a given input price \( w \), there is a unique solution to the Nash bargaining problem without side-payments. Individual output levels are:

\[
q^c_p(w) = \frac{(1 - \mu)(9 - 4\mu)}{4(3 - \mu)^2}(a - w) \quad \text{and} \quad q^c_g(w) = \frac{4\mu^3 - 17\mu^2 + 11\mu + 18}{4(3 - \mu)^2(2 - \mu)}(a - w),
\]

and the resulting payoffs are:

\[
\pi^c_p(w) = \frac{(9 - 4\mu)(1 - \mu)^2}{4(2 - \mu)(3 - \mu)^2}(a - w)^2 \quad \text{and} \quad W^c_g(w) = \frac{4\mu^3 - 15\mu^2 + 10\mu + 9}{4(3 - \mu)^2(2 - \mu)}(a - w)^2.
\]
4.3.3 Unilateral deviations from the collusive agreement

For a given input price \( w \) and an agreement \((q^c_c, q^c_g)\) given by (4.8), if the private firm deviates from the agreement, it produces the output that maximizes its individual profit presuming that the public firm produces the collusive quantity \( q^c_g(w) \):

\[
\pi_p = \left[a - q_p - q^c_g(w) - w \right] q_p. \tag{4.10}
\]

The deviation output is:

\[
q^d_p(w) = \frac{(1 - \mu) (8\mu^2 - 41\mu + 54)}{8(2 - \mu)(3 - \mu)^2} (a - w), \tag{4.11}
\]

and the corresponding profit is:

\[
\pi^d_p(w) = \left[q^d_p(w) \right]^2. \tag{4.12}
\]

If the public firm deviates from the agreement, it produces the quantity that maximizes (4.2), presuming that the private firm produces \( q^c_p(w) \):

\[
W_g = \left[a - q^c_p(w) - q_g - w \right] q_g + \mu \frac{[q^c_p(w) + q_g]^2}{2}. \]

The deviation output is:

\[
q^d_g(w) = \frac{4\mu^3 - 13\mu^2 - 2\mu + 27}{4(3 - \mu)^2(2 - \mu)} (a - w), \tag{4.13}
\]

---

12 The second-order condition is satisfied because \( \frac{\partial^2 \pi_p}{\partial q_p^2} = -2 < 0 \).

13 Under Assumption 4.1, the SOC is satisfied as: \( \frac{\partial^2 W}{\partial q_g^2} = -2 + \mu < 0 \).
and the corresponding payoff is:

\[
W_g^d(w) = \frac{32\mu^5 - 296\mu^4 + 984\mu^3 - 1247\mu^2 + 54\mu + 729}{32(3 - \mu)^4(2 - \mu)}(a - w)^2.
\] (4.14)

### 4.3.4 Critical discount factors

The collusive agreement is sustainable if and only if the continuation payoff of both downstream firms is greater if they abide by the collusive agreement than if they unilaterally deviate. As firms use trigger strategies to punish deviations, the private firm does not deviate if and only if the following incentive compatibility constraint is satisfied:

\[
\sum_{t=0}^{+\infty} \delta^t \pi_p^c(w^c) + \sum_{t=1}^{+\infty} \delta^t \pi_p^N(w^N) \Leftrightarrow \delta \geq \frac{\pi_p^d(w^c) - \pi_p^c(w^c)}{\pi_p^d(w^c) - \pi_p^N(w^N)} \equiv \delta_p(w^c, w^N),
\] (4.15)

where: \( w^c \) is the input price set by the upstream firm in the collusive phase (expecting collusive behavior by the downstream firms); and \( w^N \) is the input price set by the upstream firm in the punishment phase (expecting Cournot-Nash behavior by the downstream firms).

Similarly, the critical discount factor above which the public firm is willing to collude is:

\[
\delta_g(w^c, w^N) \equiv \frac{W_g^d(w^c) - W_g^c(w^c)}{W_g^d(w^c) - W_g^N(w^N)}.
\] (4.16)

Collusion is sustainable if and only if neither firm has incentives to deviate, i.e., if and only if \( \delta \geq \max \{ \delta_p(w^c, w^N), \delta_g(w^c, w^N) \} \).

**Result 4.2.** The higher is the input price under downstream competition, the more incentives firms have to collude, i.e., \( \frac{\partial \delta_j}{\partial w^N} < 0 \), with \( j \in \{ p, g \} \). By contrast, the higher is the input price under downstream collusion, the less incentives firms have to collude, i.e., \( \frac{\partial \delta_j}{\partial w^c} > 0 \).

**Proof.** See Appendix.
4.4 Analysis: Upstream market

In the first-stage of each period, the upstream firm chooses the price, $w$, that maximizes its individual profit, anticipating the demand by the downstream firms. Thus, it sets the price that satisfies the following first-order condition:\(^\text{14}\)

$$\frac{d\pi_u}{dw} = 0 \iff Q(w) + w \frac{\partial Q(w)}{\partial w} - C'(Q(w)) \frac{\partial Q(w)}{\partial w} = 0 \iff Q(w) = \left[ C'(Q(w)) - w \right] \frac{\partial Q(w)}{\partial w},$$

(4.17)

where: $Q(w) = Q^N(w)$, given by (4.4), if downstream firms competed or deviated in the previous period; and $Q(w) = Q^c(w)$, given by (4.7), if firms colluded in the previous period.

Replacing (4.7) in (4.17), we find that the input price in the collusive phase satisfies:

$$2w^c - C'\left(\frac{a - w^c}{2 - \mu}\right) = a,$$

(4.18)

while, replacing (4.4) in (4.17), we find that the input price in the punishment phase satisfies:

$$2w^N - C'\left(\frac{2a - 2w^N}{3 - \mu}\right) = a.$$

(4.19)

Let us proceed by focusing on two particular functional forms for the cost function of the upstream firm: linear cost and quadratic cost.

\(^{14}\)The corresponding second-order condition is: $\frac{d^2\pi_u}{dw^2} = 2 \frac{\partial Q}{\partial w} + [w - C'(Q)] \frac{\partial^2 Q}{\partial w^2} - C''(Q) \left(\frac{\partial Q}{\partial w}\right)^2 < 0.$
4.4.1 Linear upstream cost

Suppose that the upstream firm has linear production costs:

\[ C(Q) = c_1 Q \quad \text{with} \quad 0 < c_1 < a. \]

**Lemma 4.3.** If upstream production costs are linear, the input price is the same under downstream competition and collusion, and equal to the integrated monopoly price:

\[ w^c = w^N = \frac{a + c_1}{2}. \quad (4.20) \]

**Proof.** Replace \( C'(Q) \) by \( c_1 \) in (4.19) and (4.18). Assumption 4.1 guarantees that the second-order conditions are satisfied: \( \frac{d^2 \pi_u}{dw^2} = -\frac{4}{3-\mu} < 0 \) (competition) and \( \frac{d^2 \pi_u}{dw^2} = -\frac{2}{3-\mu} < 0 \) (collusion).

This lemma relates to a well-established result in the literature that, under Cournot competition in the downstream market, the price charged by the monopolistic supplier does not depend on the fierceness of downstream competition (Greenhut and Ohta, 1976).

Replacing \( w^N = \frac{a + c_1}{2} \) in (4.4), we obtain the individual output levels under competition:

\[ q_p^N = \frac{1 - \mu}{2(3 - \mu)}(a - c_1), \quad q_g^N = \frac{1 + \mu}{2(3 - \mu)}(a - c_1), \quad Q^N = \frac{a - c_1}{3 - \mu}, \]

As the input price does not depend on the weight the public firm attaches to consumer surplus, the impact of \( \mu \) on equilibrium quantities and profits is driven by the usual working forces of Cournot competition. In particular, if \( \mu \) increases, the output of the public firm increases and the output of the private firm decreases (due to strategic substitutability). As the slope of the best-response function is smaller than one in absolute value, total output is increasing in \( \mu \). Hence, consumers and the upstream firm benefit from an increase in \( \mu \).
Replacing (4.20) in (4.7), we obtain the total output under collusion:

\[ Q^c = \frac{a - c_1}{4 - 2\mu} < Q^N. \]  

(4.21)

As a result, the profit of the upstream firm is lower when downstream firms collude (input price is the same but total output is lower). In addition, as consumer surplus only depends on total output, consumers are worse off under collusion.

**Result 4.3.** *If upstream costs are linear, downstream collusion:*

(i) leads both firms to contract output, but the public firm contracts more;

(ii) is detrimental to final consumers and to the upstream firm.

**Proof.** As:

\[ q_p^N - q_p^c = \frac{3(1 - \mu)}{8(3 - \mu)^2} (a - c_1) > 0 \]

and

\[ q_g^N - q_g^c = \frac{(6 - \mu)(1 - \mu)}{8(3 - \mu)^2(2 - \mu)} (a - c_1) > 0, \]

we conclude that both firms contract output under collusion. Furthermore, as:

\[ \frac{q_p^N - q_p^c}{q_g^N - q_g^c} = \frac{3(2 - \mu)}{6 - \mu} \leq 1, \]

the public firm contracts output more than the private firm. \(\square\)

Replacing \(w^N = w^c = \frac{a + c_1}{2}\) in (4.15) and (4.16), we obtain the critical discount factor of the private firm:

\[ \delta^*_p = 1 - \frac{16(2 - \mu)(3 - \mu)^2}{(6 - \mu)(16\mu^2 - 81\mu + 102)} \]

15Recall that we are considering \(\mu < 1\) (Assumption 4.1).
and for the public firm:
\[ \delta_g^* = 1 - \frac{8(3 - \mu)^2}{3(8\mu^2 - 40\mu + 51)}. \]

These critical discount factors are plotted in Figure 4.2. The greater is the weight the public firm attaches to consumer surplus: the less incentives the private firm has to collude (\(\delta_p^*\) is increasing in \(\mu\)); and the more incentives the public firm has to collude (\(\delta_g^*\) is decreasing in \(\mu\)). While \(\delta_p^*\) increases from \(\frac{9}{17} \approx 0.529\) (for \(\mu = 0\)) to \(\frac{121}{185} \approx 0.654\) (for \(\mu = 1\)), \(\delta_g^*\) decreases from \(\frac{9}{17} \approx 0.529\) (for \(\mu = 0\)) to \(\frac{25}{57} \approx 0.439\) (for \(\mu = 1\)). If firms are sufficiently patient, collusion is sustainable, with the private firm having stronger incentives to deviate (i.e., \(\delta^* = \delta_p^* < 1\)).

\[ \begin{align*}
\delta_p^* & \quad \text{for } \mu = 0 \\
\frac{9}{17} & \approx 0.529 \\
\delta_g^* & \quad \text{for } \mu = 0 \\
\frac{9}{17} & \approx 0.529 \\
0.654 & \quad \text{for } \mu = 1 \\
\frac{121}{185} & \approx 0.654 \\
0.439 & \quad \text{for } \mu = 1 \\
\frac{25}{57} & \approx 0.439 \\
0.654 & \quad \text{for } \mu = 0 \\
\frac{121}{185} & \approx 0.654 \\
0.439 & \quad \text{for } \mu = 1 \\
\frac{25}{57} & \approx 0.439
\end{align*} \]

**Figure 4.2.** Critical discount factors of the private firm (solid line) and of the public firm (dashed line) when upstream costs are linear.

**Result 4.4.** If upstream costs are linear, the public firm has stronger incentives to abide by the collusive agreement. The agreement is sustainable if and only if \(\delta > \delta_p^*\).
4.4.2 Quadratic upstream cost

Suppose now that the cost function of the upstream firm is quadratic and given by:

\[ C(Q) = c_1 Q + \frac{c_2}{2} Q^2, \quad \text{with} \quad 0 < c_1 < a. \]  

(4.22)

If \( c_2 > 0 \), marginal production cost is increasing, i.e., there are diseconomies of scale in upstream production; while, if \( c_2 < 0 \), marginal production cost is decreasing, i.e., there are economies of scale in upstream production. To avoid marginal cost from becoming negative for large output levels, which would be unrealistic, we modify the cost function in (4.22) whenever \( c_2 < 0 \) by setting \( C(Q) = C(\frac{c_1}{|c_2|}) \) for all \( Q > \frac{c_1}{|c_2|} \). As a result, marginal cost is zero instead of negative for \( Q > \frac{c_1}{|c_2|} \). Marginal cost and marginal revenue are plotted in Figure 4.3.

![Figure 4.3. Marginal cost (MC) and marginal revenue (MR) of the upstream monopolist.](image)

For marginal cost and marginal revenue to cross in the region where marginal cost is strictly positive \( (Q < \frac{c_1}{|c_2|}) \), we make the following assumption (plotted in Figure 4.4).

**Assumption 4.2.** \( c_2 > c_2 \equiv -\frac{(2-\mu)c_1}{a} \).
Replacing (4.22) in (4.19), we obtain the input price in the punishment phase:

$$w^N = \frac{(3 - \mu)(a + c_1) + 2ac_2}{2(c_2 + 3 - \mu)}. \quad (4.23)$$

Replacing this expression in (4.4), we obtain the output levels in the punishment phase:

$$q^N_p = \frac{1 - \mu}{2(c_2 + 3 - \mu)}(a - c_1); \quad q^N_g = \frac{1 + \mu}{2(c_2 + 3 - \mu)}(a - c_1); \quad \text{and} \quad Q^N = \frac{a - c_1}{c_2 + 3 - \mu}.$$

The corresponding payoffs are:

$$\pi^N_p = \frac{(1 - \mu)^2}{4(c_2 + 3 - \mu)^2}(a - c_1)^2; \quad W^N_g = \frac{1 + 2\mu - \mu^2}{4(c_2 + 3 - \mu)^2}(a - c_1)^2; \quad \pi^N_u = \frac{(a - c_1)^2}{2(c_2 - \mu + 3)}.$$

Similarly, replacing (4.22) in (4.18), we obtain the input price in the collusive phase:

$$w^c = \frac{(2 - \mu)(a + c_1) + ac_2}{c_2 + 2(2 - \mu)}. \quad (4.26)$$

**Result 4.5. The input price is higher (resp. lower) under downstream collusion than under**

---

16 The second-order condition of the maximization problem of the upstream firm (see Footnote 14) in the punishment phase is $-\frac{4(c_2 + 3 - \mu)}{(3 - \mu)^2} < 0$, which is satisfied because Assumption 4.2 implies that $c_2 > -2$.

17 The second-order condition of the maximization problem of the upstream firm in the collusive phase, $-\frac{c_2 + 4 - 2\mu}{(2 - \mu)^2} < 0$, is also satisfied under Assumption 4.2.
downstream competition if \( c_2 < 0 \) (resp. \( c_2 > 0 \)).

**Proof.** The difference between the input price in the collusive phase and in the punishment phase is:

\[
 w^c - w^N = -\frac{(1 - \mu)c_2}{2(c_2 + 4 - 2\mu)(c_2 + 3 - \mu)(a - c_1)},
\]

Under Assumption 4.2, \( c_2 > -2 \). This allows us to conclude that \( w^c < w^N \iff c_2 > 0 \). □

Inspection of Figure 4.5 suggests that, depending the concavity/convexity of the upstream firm’s cost function, the input price may be increasing or decreasing in the weight the public firm attaches to consumer surplus. The higher is \( \mu \), the more the public firm produces and the greater is total output. If the marginal cost of the upstream firm is increasing (\( c_2 > 0 \)), a higher \( \mu \) implies a higher marginal cost, which is passed-through to downstream firms in the form of a higher input price. The opposite occurs if the marginal cost of the upstream firm is decreasing (\( c_2 < 0 \)): a higher \( \mu \) implies a lower marginal cost, which is passed-through in the form of a lower input price.

![Figure 4.5](image)

Figure 4.5. Input price under downstream competition (dashed line) and downstream collusion (solid line), for \( a = 1 \) and \( c_1 = 0.5 \).

Replacing expression (4.26) in (4.8), we obtain the individual collusive output levels:

\[
 q_p^c = \frac{(2 - \mu)(1 - \mu)(9 - 4\mu)}{4(3 - \mu)^2(c_2 + 4 - 2\mu)}(a - c_1) \quad \text{and} \quad q_g^c = \frac{4\mu^3 - 17\mu^2 + 11\mu + 18}{4(3 - \mu)^2(c_2 + 4 - 2\mu)}(a - c_1),
\]

(4.27)
and total output:

\[ Q^c = q_p^c + q_g^c = \frac{a - c_1}{c_2 + 4 - 2\mu} \]  

(4.28)

If \( c_2 > 0 \), it is immediate that total output is lower under collusion than under competition. The usual output contraction associated with collusion is reinforced by the fact that the input price is lower under collusion than under competition (Figure 4.5b). By contrast, if \( c_2 < 0 \), the input price is higher under collusion and thus the two effects act in opposite directions.

**Result 4.6.** Total output is greater if downstream firms compete than if they collude.

**Proof.** Comparing the output level under collusion, in (4.28), with the industry output under competition, given by (4.24), we obtain:

\[ Q^N - Q^c = \frac{1 - \mu}{(c_2 + 4 - 2\mu)(c_2 + 3 - \mu)}(a - c_1) > 0. \]

This means that the ‘downstream’ effect (the benefit of contracting output associated with charging a higher price to final consumers) outweighs the ‘upstream’ effect (the resulting increase of the input price). As consumer surplus only depends on total output, we conclude that downstream collusion is detrimental to consumers.

When \( c_2 > 0 \), downstream collusion is detrimental to the upstream firm, as it charges a lower price and sells a lower quantity under collusion than under competition. By contrast, when \( c_2 < 0 \), the impact of downstream collusion on the upstream firm’s profit is not straightforward because the input price is higher but output is lower.

**Result 4.7.** The upstream firm is damaged by downstream collusion.

**Proof.** \( \pi^c_u - \pi^N_u = -\frac{1 - \mu}{2(c_2 + 4 - 2\mu)(c_2 + 3 - \mu)}(a - c_1)^2 < 0. \)
Let us now analyze the incentives for the downstream firms to cooperate. Replacing expression (4.26) in (4.9), we obtain the individual collusive payoffs:

$$\pi_c = \frac{(2 - \mu)(9 - 4\mu)(1 - \mu)^2(a - c_1)^2}{4(3 - \mu)^2(c_2 + 4 - 2\mu)^2}$$

and

$$W_g = \frac{(2 - \mu)(4\mu^3 - 15\mu^2 + 10\mu + 9)(a - c_1)^2}{4(3 - \mu)^2(c_2 + 4 - 2\mu)^2}.$$

(4.29)

A necessary condition for a firm to be willing to collude is that its per-period payoff is greater under collusion than under competition. This condition is not necessarily satisfied if $c_2 < 0$ because the input price is higher under collusion than under competition. Thus, from the point of view of the downstream firms, the gain from restricting output in the downstream market may not compensate the payment of a higher input price.

**Result 4.8.** The per-period payoff of downstream firms is higher under collusion than under competition if and only if $c_2 > \tilde{c}_2$, with:

$$\tilde{c}_2 = -\frac{3 - \mu}{3\mu^3 - 12\mu^2 + 8\mu + 9} \left[ (2 - \mu)(3 + 3\mu - 2\mu^2) - \sqrt{(2 - \mu)(1 + 2\mu - \mu^2)(4\mu^3 - 15\mu^2 + 10\mu + 9)} \right] > c_2.$$

(4.30)

**Proof.** See Appendix.

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![Figure 4.6](image_url)

**Figure 4.6.** Parameter region where downstream firms have higher payoff under collusion than under competition (dark grey area), for $a = 1$ and $c_1 = 0.5$.

In the light grey area of Figure 4.6, Assumption 4.2 is satisfied but there is no collusion because the public firm has lower payoff in the collusive phase than in the punishment phase.
If \( c_2 < 0 \), the greater is the weight the public firm attaches to consumer surplus, the weaker need to be the economies of scale in upstream production for collusion to benefit both firms. Henceforth, we restrict the analysis to the parameter region where collusion benefits downstream firms.

**Assumption 4.3.** \( c_2 > \tilde{c}_2 \).

![Figure 4.7](image)

**Figure 4.7.** Output levels of the private firm under collusion (solid line), deviation (dotted line), and punishment (dashed line), for \( a = 1 \) and \( c_1 = 0.5 \).

![Figure 4.8](image)

**Figure 4.8.** Output levels of the public firm under collusion (solid line), deviation (dotted line), and punishment (dashed line), for \( a = 1 \) and \( c_1 = 0.5 \).

The individual outputs in the three regimes (collusion, deviation and punishment) are plotted in Figures 4.7 and 4.8. When \( c_2 > 0 \), two forces lead the output level of the private firm to decrease in \( \mu \): (i) there is strategic substitutability and the public firm expands production as \( \mu \) increases (because it cares more about consumer surplus); and (ii) the input price is increasing in \( \mu \) (Figure 4.5b). When \( c_2 < 0 \), the input price is decreasing in \( \mu \), which could lead the
private firm to expand production as $\mu$ increases. However, this effect is outweighed by the increase in the output of the public firm.

**Result 4.9.** When there are diseconomies of scale in input production:

(i) the private firm produces the highest output level under deviation and the lowest under collusion: $q^d_p > q^N_p > q^c_p$.

(ii) if $c_2 < \frac{18-9\mu+\mu^2}{\mu(5-2\mu)}$, the public firm produces the highest output level under deviation and the lowest under collusion: $q^d_g > q^N_g > q^c_g$.

(iii) if $c_2 > \frac{18-9\mu+\mu^2}{\mu(5-2\mu)}$, the public firm produces the highest output level under deviation and the lowest under competition: $q^d_g > q^c_g > q^N_g$.

**Proof.** See Appendix.

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**Figure 4.9.** Comparison of the public firm’s output in the three market regimes, when $a = 1$ and $c_1 = 0.5$.

**Figure 4.10.** Downstream firms’ payoffs under competition (dashed line), collusion (solid line) and deviation (dotted line), for $a = 1$, $c_1 = c_2 = 0.5$. 

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Finally, as we can see in Figure 4.11, the results regarding the critical discount factors depend on whether the upstream marginal costs are increasing or decreasing.

\[
\begin{align*}
(a) & : c_2 = -0.5 \\
(b) & : c_2 = -0.1 \\
(c) & : c_2 = -0.01 \\
(d) & : c_2 = 0.5
\end{align*}
\]

Figure 4.11. Critical discount factor of the private (dashed line) and of the public firm (solid line), for \( a = 1, c_1 = 0.5 \).

**Result 4.10.** When input production costs are quadratic and the public firm attaches a sufficiently low weight to consumer surplus (i.e., \( \mu \) is sufficiently low):

(i) If \( \tilde{c}_2 < c_2 < -0.256945 \), the critical discount factor of the private firm is decreasing in \( \mu \) while the critical discount factor of the public firm is increasing in \( \mu \).

(ii) If \( -0.256945 < c_2 < -0.046003 \), both firms’ critical discount factors are increasing in \( \mu \).

(iii) If \( c_2 > -0.046003 \), the critical discount factor of the private firm is increasing in \( \mu \), while the critical discount factor of the public firm is decreasing in \( \mu \).

**Proof.** See Appendix.

**Corollary 4.1.** When input production costs are quadratic and the public firm attaches a sufficiently low weight to consumer surplus (i.e., \( \mu \) is sufficiently low):

\[
\delta^* = \begin{cases} 
\delta_g^* & \text{if } \tilde{c}_2 < c_2 < -0.0958273 \\ 
\delta_p^* & \text{if } c_2 > -0.0958273 
\end{cases}
\]

(4.31)

Analyzing Figure 4.12, we conclude that: if the upstream marginal costs are increasing, the private firm is the one with more incentives to deviate. By contrast, if the upstream marginal
costs are decreasing, the public firm is more likely to deviate.

Figure 4.12. Comparison between the critical discount factors.

4.5 Conclusion

In this paper, we studied a vertically related market where an upstream monopolist supplies an essential input to two downstream retailers. The downstream market is a mixed duopoly where a public and private retailer operate. These retailers only differ in their objective functions. While the private firm maximizes profits, the public firm maximizes a weighted sum of consumer surplus and profits. In order to assess the impact and the sustainability of collusion in the downstream market, we compared market outcomes under competition and collusion assuming that the upstream monopolist, facing either linear or quadratic production costs, sets a uniform wholesale price. By developing this study, we shed some light on the literature on collusion in mixed oligopolies since, to the best of our knowledge, collusion in vertically related mixed markets had not yet been analysed. After presenting our conclusions, we believe there is scope for a deeper study on collusion in vertically related markets involving public and private firms.

An important result of this study, albeit not the most surprising, is the one that corroborates most of the literature on collusion: collusion is detrimental to consumers. Under our
assumptions, output under collusion is always lower than under competition, thereby reducing consumer welfare. With this result, we show that even if one cartel member cares about consumer welfare (in our model, the public firm), that is not sufficient to make collusion welfare-improving, which constitutes a relevant policy implication.

Moreover, since collusion generates a contraction of the upstream firm’s demand, the upstream profit is always lower under collusion. Despite the fact that collusion increases the input price when the upstream monopolist faces decreasing marginal costs (decreasing it under every other assumption regarding the upstream firm’s cost function considered), this positive impact on the input price does not outweigh the decrease in output. Hence, downstream collusion is detrimental to the upstream firm in every case analyzed.

Regarding the sustainability of the collusive agreement, we found different results whether upstream costs are linear or quadratic. If upstream costs are linear, the private firm has more incentives to deviate than the public. If upstream costs are, instead, quadratic, downstream firms’ incentives to abide by the collusive agreement were found to depend on the existence of economies/diseconomies of scale and on the weight that the public firm attaches to consumer surplus. As in the case of constant marginal costs, the private firm has more incentives to deviate if the upstream marginal cost is increasing, but this result is reversed if the upstream marginal cost is sufficiently decreasing.

Finally, we can outline some avenues for future research. We would like to extend our current work by considering a setting where the upstream firm is able to set discriminatory wholesale prices. To provide more insights to the study of vertically related mixed oligopolies, it would also be interesting to analyze a less concentrated upstream market and to introduce some source of buyer power. In light of the evidence provided by cartel cases involving private and public firms in downstream markets, our intuition is that the assumption of buyer power may have a substantial impact on collusive outcomes.
Appendix

Proof of Lemma 4.1

Note that:

\[
\max_{(q_p, q_g, S)} NP(q_p, q_g, S) = \max_{(q_p, q_g)} \left\{ \max_S NP(q_p, q_g, S) \right\}
\]

(4.32)

and:

\[
\max_S NP(q_p, q_g, S) = \max_S \left[ \pi_p(q_p, q_g) - \tilde{\pi}_p + S \right] \left[ \pi_g(q_p, q_g) + \mu CS(q_p, q_g) - \tilde{W} - S \right]^{2}
\]

\[
= \frac{1}{4} \left[ \pi_p(q_p, q_g) - \tilde{\pi}_p + \pi_g(q_p, q_g) + \mu CS(q_p, q_g) - \tilde{W} \right]^{2}
\]

\[
= \frac{1}{4} \left[ \Delta(q_p, q_g) \right]^{2},
\]

(4.33)

where \( \Delta \) is the joint gain from collusion for the downstream firms. Hence, for a given input price, \( w \), the output pair that solves the maximization problem (4.6) is the one that maximizes \( \Delta(q_p, q_g) \) or, equivalently, the one that maximizes joint payoff:

\[
\max_{(q_p, q_g)} \left\{ \pi_p(q_p, q_g) + \pi_g(q_p, q_g) + \mu CS(q_p, q_g) \right\}
\]

\[
= \max_{(q_p, q_g)} \left\{ (a - q_p - q_g - w)q_p + (a - q_p - q_g - w)q_g + \mu \frac{(q_p + q_g)^2}{2} \right\}
\]

\[
= \max_Q \left\{ (a - Q - w)Q + \frac{\mu}{2} Q^2 \right\}.
\]

Solving the corresponding first-order condition, we can find the solution to the latter problem:

\[
Q = \frac{a - w}{2 - \mu}.
\]

18Note that \( \max_S (A + S)(B - S) = (A + \frac{B - A}{2})(B - \frac{B - A}{2}) = \frac{(A + B)^2}{4} \).

19The second-order condition, equal to \(-2 + \mu\), is trivially satisfied under Assumption 4.1.
We can then write the joint collusive gain, (4.33), as follows:

$$\Delta = \frac{(1 - \mu)^2}{2(2 - \mu)(3 - \mu)^2}(a - w)^2.$$  \hfill (4.34)

As individual gains from collusion are symmetric (Footnote 18), we obtain the individual payoffs under collusion after the side-payment:

$$\left(\pi^c_p + S\right) - \bar{\pi} = \frac{\Delta}{2} \iff \pi^c_p + S = \bar{\pi} + \frac{\Delta}{2},$$  \hfill (4.35)

$$\left(\pi^c_g - S + \mu CS^c\right) - \bar{W} = \frac{\Delta}{2} \iff W^c_g - S = \bar{W} + \frac{\Delta}{2}.$$  \hfill (4.36)

Proof of Lemma 4.2

For a given input price, $w$, replacing $S = 0$, $\bar{\pi} = \pi^N_p(w)$ and (4.34) in (4.35), we obtain the collusive profit of the private firm (when side-payments are not feasible):

$$\pi^c_p(w) = \frac{(1 - \mu)^2}{(3 - \mu)^2}(a - w)^2 + \frac{(1 - \mu)^2}{4(2 - \mu)(3 - \mu)^2}(a - w)^2 = \frac{(9 - 4\mu)(1 - \mu)^2}{4(2 - \mu)(3 - \mu)^2}(a - w)^2.$$

Using (4.1) and (7), we can obtain the output level of the private firm under collusion:

$$q^c_p(w) = \frac{\pi^c_p(w)}{a - Q^c(w) - w} = \frac{(1 - \mu)(9 - 4\mu)}{4(3 - \mu)^2}(a - w).$$

Similarly, we can obtain the collusive payoff of the public firm (in the absence of side-payments):

$$W^c_g(w) = \frac{4\mu^3 - 15\mu^2 + 10\mu + 9}{4(3 - \mu)^2(2 - \mu)}(a - w)^2.$$

The collusive output level of the public firm is:

$$q^c_g(w) = Q^c(w) - q^c_p(w) = \frac{4\mu^3 - 17\mu^2 + 11\mu + 18}{4(3 - \mu)^2(2 - \mu)}(a - w).$$


Proof of Result 4.2

Replacing the expressions for the profits of the private firm in the different regimes, given by (4.5), (4.9) and (4.12), we obtain the threshold on the discount factor above which the private firm is willing to collude:

\[
\delta_p(w^c, w^N) = \frac{(18 - 7\mu)^2(a - w^c)^2}{a^2(6 - \mu)(16\mu^2 - 81\mu + 102) - (8\mu^2 - 41\mu + 54)^2 w^c(2a - w^c) + 64(2 - \mu)^2(3 - \mu)^2 w^N(2a - w^N)}.
\]

(4.36)

Differentiating this expression with respect to \(w^N\), we obtain:

\[
\frac{\partial \delta_p}{\partial w^N} = -\frac{128(18 - 7\mu)^2(3 - \mu)^2(2 - \mu)^2(a - w^c)^2(a - w^N)}{[a^2(6 - \mu)(16\mu^2 - 81\mu + 102) - (8\mu^2 - 41\mu + 54)^2 w^c(2a - w^c) + 64(2 - \mu)^2(3 - \mu)^2 w^N(2a - w^N)]^2} < 0,
\]

while, differentiating \(\delta_p\) with respect to \(w^c\), we obtain:

\[
\frac{\partial \delta_p}{\partial w^c} = \frac{128(18 - 7\mu)^2(3 - \mu)^2(2 - \mu)^2(a - w^c)(a - w^N)^2}{[a^2(6 - \mu)(16\mu^2 - 81\mu + 102) - (8\mu^2 - 41\mu + 54)^2 w^c(2a - w^c) + 64(2 - \mu)^2(3 - \mu)^2 w^N(2a - w^N)]^2} > 0.
\]

Replacing the expressions (4.5), (4.9) and (4.14) for the payoffs of the public firm in (4.16), we obtain:

\[
\delta_g(w^c, w^N) = \frac{(4\mu^2 - 13\mu + 9)^2(a - w^c)^2}{D}
\]

where:

\[
D = 3a^2(8\mu^2 - 40\mu + 51)(1 - \mu)^2 - (32\mu^5 - 296\mu^4 + 984\mu^3 - 1247\mu^2 + 54\mu + 729) w^c(2a - w^c)
\]

\[
+ 32(2 - \mu)(3 - \mu)^2 (\mu^2 + 2\mu + 1) w^N(2a - w^N).
\]

Differentiating \(\delta_g\) with respect to \(w^N\), we obtain:

\[
\frac{\partial \delta_g}{\partial w^N} = -\frac{64(3 - \mu)^2(2 - \mu)(1 + 2\mu - \mu^2)(4\mu^2 - 13\mu + 9)^2(a - w^c)^2(a - w^N)}{D^2} < 0,
\]
while differentiating it with respect to $w^e$, we obtain:

$$\frac{\partial \delta_g}{\partial w^c} = \frac{64(2 - \mu)(1 + 2\mu - \mu^2)(3 - \mu)^2 (4\mu^2 - 13\mu + 9)^2 (a - w^c)(a - w^N)^2}{D^2} > 0.$$ 

Proof of Result 4.8

1. Public firm’s payoff

Comparing expressions (4.25) and (4.29), we have that the per-period payoff of the public firm is higher under collusion than under competition if and only if:

$$W^c_g > W^N_g \iff \frac{g(c_2)(a - c_1)^2}{4(3 - \mu)^2(c_2 + 4 - 2\mu)^2(c_2 + 3 - \mu)^2} > 0 \iff g(c_2) > 0,$$

where:

$$g(c_2) = (1 - \mu) \left(9 + 8\mu - 12\mu^2 + 3\mu^3\right) c_2^2 + 2(1 - \mu)(2 - \mu)(3 - \mu) \left(3 + 3\mu - 2\mu^2\right) c_2$$

$$+ (1 - \mu)^2(2 - \mu)(3 - \mu)^2.$$

If $c_2 \geq 0$, the condition $g(c_2) > 0$ is surely satisfied, as $9 + 8\mu - 12\mu^2 + 3\mu^3 > 0$ for $\mu \in [0, 1)$. Therefore, the public firm surely gets a higher per-period payoff under collusion than under competition.

Consider now that $c_2 < 0$. We have that $g(c_2) = 0 \iff c_2 = c_2^\pm$, with:

$$c_2^\pm = -\frac{3 - \mu}{9 + 8\mu - 12\mu^2 + 3\mu^3} \left[(2 - \mu)(3 + 3\mu - 2\mu^2) \pm \sqrt{(2 - \mu)(3 + 3\mu - 2\mu^2)^2 - (2 - \mu)(1 - \mu)(9 + 8\mu - 12\mu^2 + 3\mu^3)}\right].$$

It is straightforward to see that $c_2^+ < 0$ and $c_2^- < c_2^-$. Thus, $g(c_2) > 0 \iff c_2 < c_2^+ \lor c_2 > c_2^-$. However, combining Assumption 4.2 and $c_1 < a$, we obtain that $c_2 > \mu - 2$. As $c_2^+ < \mu - 2$
(Figure 4.13), we conclude that $W^c_g > W^N_g \iff c_2 > \hat{c}_2$, with:

$$
\hat{c}_2 \equiv c_2 = -\frac{3 - \mu}{3\mu^3 - 12\mu^2 + 8\mu + 9} \left[ (2 - \mu)(3 + 3\mu - 2\mu^2) - \sqrt{(2 - \mu)(1 + 2\mu - \mu^2)(4\mu^3 - 15\mu^2 + 10\mu + 9)} \right].
$$

(4.37)

![Figure 4.13. Representation of $c^\pm_2$ as function of $\mu$.](image)

2. Private firm’s payoff

The per-period profit of the private firm is higher under collusion than under competition if and only if:

$$
\pi^c_p > \pi^N_p \iff \frac{(1 - \mu)^2(a - c_1)^2p(c_2)}{4(3 - \mu)^2[c_2 + 2(2 - \mu)]^2(c_2 + 3 - \mu)^2} > 0 \iff p(c_2) > 0,
$$

where:

$$
p(c_2) = c_2^2 \left( 9 - 11\mu + 3\mu^2 \right) + 2c_2(2 - \mu)(3 - \mu)(3 - 2\mu) + (2 - \mu)(3 - \mu)^2.
$$

Again, if $c_2 \geq 0$, condition $p(c_2) > 0$ is trivially satisfied, as $9 - 11\mu + 3\mu^2 > 0$ for $\mu \in [0, 1)$.

Let us now analyze the case where $c_2 < 0$. Start by noticing that $p(c_2) = 0 \iff c_2 = \hat{c}_2^\pm$, with:

$$
\hat{c}_2^\pm \equiv -\frac{3 - \mu}{9 - 11\mu + 3\mu^2} \left[ (2 - \mu)(3 - 2\mu) \pm \sqrt{(2 - \mu)^2(3 - 2\mu)^2 - (2 - \mu)(3\mu^2 - 11\mu + 9)} \right].
$$

As $9 - 11\mu + 3\mu^2 > 0$ for $\mu \in [0, 1)$, it follows that $\hat{c}_2^\pm < 0$. Thus, $p(c_2) > 0 \iff c_2 < \hat{c}_2^\pm$. 

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\( \hat{c}_2^+ \lor c_2 > \hat{c}_2^- \). Again, combining Assumption 4.2 and \( c_1 < a \), we obtain that \( c_2 > \mu - 2 \). As \( \hat{c}_2^+ < \mu - 2 \) (Figure 4.14), we conclude that \( p(c_2) > 0 \iff c_2 > \hat{c}_2^- \).

![Figure 4.14. Representation of \( \hat{c}_2^\pm \) as function of \( \mu \).](image)

In sum, the public firm gets a higher per-period payoff under collusion than competition than under if \( c_2 > c_2^- \); while the same occurs to the private firm when \( c_2 > \hat{c}_2^- \). In Figure 4.15, we represent the pairs \((\mu, c_2)\) for which both firms have a higher payoff under collusion than under competition (dark grey region), and the pairs where the private firm gets a higher payoff under collusion but the public firm does not. In particular, we conclude that if \( c_2 > \hat{c}_2^- = \tilde{c}_2^- \), both firms get a higher per-period payoff under collusion than under competition.

![Figure 4.15. Combinations of the parameters \( \mu \) and \( c_2 \) where payoffs are greater under collusion than under competition, for \( a = 1 \) and \( c_1 = 0.5 \).](image)

**Proof of Result 4.9**

(i) Comparing the private collusive output level, given in (4.27), to the Nash output level, given
in (4.24):

\[ q_p^N - q_p^c = \frac{(1 - \mu)[\mu(5 - 2\mu)c_2 + 3(2 - \mu)(3 - \mu)]}{4(3 - \mu)^2[c_2 + 2(2 - \mu)][(c_2 + 3 - \mu)]} (a - c_1) \]

As Assumption 4.3 implies \( c_2 > -1 \), we have that:

\[ \mu(5 - 2\mu)c_2 + 3(2 - \mu)(3 - \mu) > -\mu(5 - 2\mu) + 3(2 - \mu)(3 - \mu) = 5\mu^2 - 20\mu + 18 > 0, \forall \mu < 1, \]

which implies that: \( q_p^N > q_p^c \).

Replacing expression (4.26) in (4.11), we obtain the deviation output level of the private firm. Comparing it with the Nash output level, we obtain:

\[ q_p^d - q_p^N = \frac{(1 - \mu)[c_2(2 - \mu)(9 - 4\mu) + (3 - \mu)(6 - \mu)]}{8(3 - \mu)^2[c_2 + 2(2 - \mu)][(c_2 + 3 - \mu)]}(a - c_1). \]

Using again that \( c_2 > -1 \), we have that \( c_2(2 - \mu)(9 - 4\mu) + (3 - \mu)(6 - \mu) > (8 - 3\mu)\mu > 0 \), which implies that \( q_p^d > q_p^N \).

\[(ii)\] Replacing (4.26) in (4.13), we obtain the deviation output level of the public firm. Using the expression for the public output level under collusion, (4.27), and Nash com-

\[ q_g^d - q_g^N = \frac{(1 - \mu)[c_2(1 - \mu) + (3 - \mu)(3 + \mu)] + 3(1 - \mu)(3 - \mu)}{4(3 - \mu)^2[c_2 + 2(2 - \mu)][(c_2 + 3 - \mu)]}(a - c_1). \]

Thus, as \( c_2 > -1 \) (Assumption 4.3):

\[ q_g^d > q_g^N \iff c_2 > -\frac{3(1 - \mu)(3 - \mu)}{(1 - \mu)\mu + (3 - \mu)(3 + \mu)} \equiv \hat{c}_2. \]

As illustrated in Figure 4.16, the previous inequality is always satisfied under Assumption 4.3.

Now, using the expressions for the public output level under collusion, (4.27), and Nash com-
petition, (4.24), we obtain:
\[ q_N^g - q_g^c = \frac{(1 - \mu) [18 - 9\mu + \mu^2 - \mu(5 - 2\mu)c_2]}{4(3 - \mu)^2(c_2 + 2(2 - \mu))(c_2 + 3 - \mu)} (a - c_1) > 0 \iff c_2 < \frac{18 - 9\mu + \mu^2}{\mu(5 - 2\mu)} \]

Thus, if \( c_2 < \frac{18 - 9\mu + \mu^2}{\mu(5 - 2\mu)} \), by transivity, we have that \( q_d^g > q_N^g > q_g^c \). By contrast, if \( c_2 > \frac{18 - 9\mu + \mu^2}{\mu(5 - 2\mu)} \), we still need to compare \( q_d^g \) and \( q_g^c \). As:
\[ q_d^g - q_g^c = \frac{(1 - \mu)(9 - 4\mu)}{4(3 - \mu)^2(c_2 + 2(2 - \mu))} (a - c_1) > 0, \]
we conclude that the public firm produces a higher output under deviation than under collusion.

\[ \square \]

**Proof of Result 4.10**

Replacing (4.27) in (4.15) and (4.16), we can obtain the thresholds for the discount factor above which the private and the public firms are willing to collude, respectively:
\[ \delta_p^*(c_2, \mu) = \frac{(18 - 7\mu)^2(c_2 - \mu + 3)^2}{[c_2(2 - \mu)(9 - 4\mu) + (3 - \mu)(6 - \mu)][c_2(12\mu^2 - 65\mu + 90) + (3 - \mu)(16\mu^2 - 81\mu + 102)]} \]
and:
\[ \delta_g^*(c_2, \mu) = \frac{(2 - \mu)(1 - \mu)(9 - 4\mu)^2(c_2 + 3 - \mu)^2}{D}, \]
with:

\[ D = c_2^2 \left( 24\mu^5 - 224\mu^4 + 736\mu^3 - 847\mu^2 - 243\mu + 810 \right) \\
+ 2c_2(2 - \mu)(3 - \mu) \left( -16\mu^4 + 104\mu^3 - 176\mu^2 - 81\mu + 297 \right) \\
+ 3(1 - \mu)(2 - \mu)(3 - \mu)^2 \left( 8\mu^2 - 40\mu + 51 \right). \]

Differentiating (4.38) with respect to $\mu$ and evaluating it at $\mu = 0$, we get:

\[ \frac{\partial \delta^*_p}{\partial \mu}(c_2, 0) = 4(c_2 + 3) \left( 10c_2^3 + 62c_2^2 + 97c_2 + 21 \right) \left( 9(c_2 + 1)^2(5c_2 + 17)^2 \right). \]

As:

\[ 10c_2^3 + 62c_2^2 + 97c_2 + 21 = 0 \iff c_2 \approx -3.782 \lor c_2 \approx -2.161 \lor c_2 \approx -0.257, \]

and Assumption 4.3 implies that $c_2 > -1$, we conclude that:

\[ \frac{\partial \delta^*_p}{\partial \mu}(c_2, 0) > 0 \iff c_2 > -0.257. \]

Analogously, differentiating (4.39) with respect to $\mu$ and evaluating it at $\mu = 0$, we get:

\[ \frac{\partial \delta^*_g}{\partial \mu}(c_2, 0) = -2(c_2 + 3) \left( 47c_2^3 + 313c_2^2 + 536c_2 + 24 \right) \left( 9(c_2 + 1)^2(5c_2 + 17)^2 \right). \]

As \[ 47c_2^3 + 313c_2^2 + 536c_2 + 24 = 0 \iff c_2 \approx -0.046, \]
we conclude that:

\[ \frac{\partial \delta^*_g}{\partial \mu}(c_2, 0) < 0 \iff c_2 > -0.046. \]

Using $\delta^*_p(c_2, 0) = \delta^*_g(c_2, 0)$ and the continuity of $\delta^*_p$ and $\delta^*_g$, we conclude that, \( \exists \epsilon > 0 \) such that:

1. If $c_2 \in (\tilde{c}_2, -0.257)$, for $\mu \in (0, \epsilon)$, the critical discount factor of the private firm is decreasing in $\mu$, while the critical discount factor of the public firm is increasing in $\mu$. 

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Thus: $\delta^*_p(c_2, \mu) < \delta^*_g(c_2, \mu)$.

2. If $c_2 \in (-0.257, -0.046)$, for $\mu \in (0, \epsilon)$, the critical discount factor of both firms is increasing in $\mu$, and:

$$
\left( \frac{\partial \delta^*_g}{\partial \mu} - \frac{\partial \delta^*_p}{\partial \mu} \right) (c_2, 0) = \frac{-2(c_2 + 3)^2 (67c_2^2 + 236c_2 + 22)}{9(c_2 + 1)^2(5c_2 + 17)^2} > 0 \iff -3.427 < c_2 < -0.096.
$$

Thus:

2.1. If $c_2 \in (-0.257, -0.096)$, we have that $\frac{\partial \delta^*_p}{\partial \mu} (c_2, 0) > \frac{\partial \delta^*_g}{\partial \mu} (c_2, 0) > 0$ and, therefore,

$$
\delta^*_p(c_2, \mu) < \delta^*_g(c_2, \mu), \text{ for } \mu \in (0, \epsilon).
$$

2.2. If $c_2 \in (-0.096, -0.046)$, we have that $\frac{\partial \delta^*_p}{\partial \mu} (c_2, 0) > \frac{\partial \delta^*_g}{\partial \mu} (c_2, 0) > 0$ and, therefore,

$$
\delta^*_p(c_2, \mu) > \delta^*_g(c_2, \mu), \text{ for } \mu \in (0, \epsilon).
$$

3. If $c_2 > -0.046$, for $\mu \in (0, \epsilon)$, the critical discount factor of the private firm is increasing in $\mu$, while the critical discount factor of the public firm is decreasing in $\mu$. Thus:

$$
\delta^*_p(c_2, \mu) > \delta^*_g(c_2, \mu).
$$

---

20Recall that both derivatives are positive.
Chapter 5

Conclusion

This three-essay thesis aimed at providing some new insights to the literature on collusion and some policy implications to competition authorities while handling their battle against cartels.

In Chapter 2, we presented our first essay providing a discussion on the conditions for collusion to be sustainable in a mixed oligopoly. In this essay, we studied a mixed cartel, i.e., a cartel between a private and a public firm. Our purpose was to study how the objective function of the public firm affects its willingness to tacitly collude and the sustainability of the collusive agreement. Attaching different weights to consumer surplus to the public firm’s objective function, we concluded that collusion is easier to sustain when both firms maximize profits. This captures the standard result in the literature that collusion is easier to sustain among symmetric firms. Particularly, we found that the private firm has more incentives to abide by the collusive agreement as the weight attached to consumer surplus increases, for sufficiently high levels of concern for consumers, but the public firm, on the contrary, has more incentives to deviate. Finally, we showed that consumer surplus is higher under collusion than under competition for intermediate levels of the weight the public firm attaches to consumer surplus, and total surplus is larger under collusion for sufficient high levels. Therefore, we conclude collusion between a public and a private firm may be welfare-improving.
In the second essay, presented in Chapter 3, we analyzed the coordinated effects of Corporate Social Responsibility in a setting where firms are willing to take into account consumer’s surplus in addition to profits in their objective function. In our setting, firms produce differentiated goods, compete in quantities and may be symmetric or asymmetric regarding their concern for consumers. Our results showed, in line with the existing literature on collusion and corporate social responsibility, that assigning a positive weight to consumer surplus makes collusion harder to sustain. However, the degree of product differentiation also affects collusion sustainability for different levels of social concern. On the one hand, for a sufficiently high level of social responsibility, we found that collusion sustainability is increasing in the degree of product substitutability when firms are symmetric regarding the level of social concern. On the other hand, if firms are, instead, asymmetric regarding their levels of social concern, collusion sustainability is increasing in the degree of product complementarity. Finally, we concluded that, under certain conditions, collusion may be welfare-improving when firms adopt a socially responsible behavior.

Finally, in Chapter 4 we presented our third essay. In this essay, we considered a vertically related market constituted by an upstream monopoly and a downstream mixed duopoly. By doing this, we provided some new insights to the literature on mixed oligopolies. Our aim was to study if collusion could be sustained in a downstream mixed duopoly supplied by a unique upstream monopolist and its implications. To answer our research question, we compared market outcomes under a competitive and a collusive scenario assuming that the upstream monopolist faced either linear or quadratic production costs. While downstream firm’s incentives to collude were found to depend on the existence of (dis)economies of scale and the level of concern for consumers, collusion is sustainable if firms are sufficiently patient. A relevant result of this study is that collusion is detrimental to consumers and to the upstream monopolist since total output is contracted. This brings an important policy implication as it shows, in addition to our previous conclusion that collusion may be welfare-improving, that it is not sufficient to have some concern for consumers to make collusion beneficial to them. By
concluding this, this paper corroborates most of the literature on collusion.

Future research could extend the current line of research. Regarding the first essay, it would be interesting to assume other sources of asymmetries between the two firms, e.g. cost asymmetries, or different bargaining powers. We would also like to further develop our study on the coordinated effects of corporate social responsibility by considering a market with more than two firms. This would allow us to analyze partial collusion, i.e., leaving some firms out of the collusive agreement. To expand the study of vertically related mixed oligopolies, it would be interesting to assume, as an addition to our third essay, that the upstream firm is able to set discriminatory wholesale prices and to analyze a less concentrated upstream market. Finally, in the three essays that constitute this thesis, collusion was designed to achieve the most profitable (or beneficial) agreement. We are aware that there could be other profitable agreements, eventually easier to sustain, when firms fail to maximize joint payoffs (imperfect collusion). Therefore, a final suggestion would be to extend these papers to allow imperfect collusion.
References


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