Seismic analysis by macroelements of circular earth constructions: the Fujian Tulou

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Abstract

The overall earthquake response of Tulous, traditional earth buildings of the Fujian Province (China) and listed among the UNESCO World Heritage buildings, is investigated through non-linear static analysis (pushover) by using the equivalent frame approach. Although this approach implies some rough approximations, it is suitable to model complex masonry structures, like Tulous, because non-linear analysis implemented with more rigorous methods (i.e. by finite elements or by discrete elements) would lead to complex models hard to converge and with high computation costs. The earthquake analysis of a Tulou prototype was hence performed, and failure modes and overall seismic response of such complex earth structures were investigated. Since this is one of the first studies on the overall earthquake response of Tulous, further research is necessary in order to better understand their seismic behaviour.

1. Introduction

Tulous are collective fortresses located in the Fujian Province where traditionally Hakka clans live. These constructions are made of a continuous rammed earth wall, with circular or rectangular plan, and by an inner 3D wooden frame system connected to the wall along its perimeter at storeys and roof levels. They are characterized by large dimensions (diameter up to 50m and height up to 20m), with small openings mainly concentrated in the highest part of the structure (**Figure 1**).

Figure 1. *Tulou in the Fujian Province.*

Due to their heritage value and outstanding relevance, they are also inscribed in the UNESCO World Heritage list. But, limited studies have been developed in terms of their structural behaviour and performance. The need to deepen the studies in this field is evident considering that the province of Fujian is an area with a significant seismic hazard. Also, the evidence that many Tulou structures have resisted to different earthquakes occurred during their long life motivates this study.

While there are only a few specific studies available on Tulous, in recent years the interest on earthen constructions has considerably increased. As a matter of fact, the earthen material was used in many

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historical buildings, and nowadays it is recognised that due to its characteristics of eco-compatibility and energy performance there is an enormous potential for its reintroduction in ecological and sustainable architecture [1,2]. In fact, earth is still widely used in buildings, as evidenced by the one billion people living in earth constructions [2].

New studies have been recently developed to increase the background knowledge in this field. Regarding the structural behaviour, a significant work have been developed on the mechanical characterization of earthen samples and adobe bricks (as can be find, for example in [3–7]), as well as on adobe wall panels (for example in [8–10]).

Other authors studied the seismic behaviour of earth structures [11]. In this respect, an important starting point is the investigation on the damage caused by earthquakes [12–14]. Blondet *et al.* [15] highlighted the seismic vulnerability of the adobe structures associated to the combination of a low tensile strength of the material with its fragile behaviour, especially in fibres' absence. Evidences of this vulnerability were observed in recent earthquakes, as in El Salvador (2001), Iran (2003), Peru (1970, 1996, 2001 and 2007), Pakistan (2005) and China (2008 and 2009) [11].

Another important tool for investigating the seismic behaviour of earthen structures is provided by experimental shaking table tests, conducted both on reduced masonry section walls [16–18] and on scale models of entire buildings [19,20]. Although these tests provide qualitatively valid results, the models' construction time and costs are undoubtedly two hard obstacles that make this approach usually impractical.

From this point of view, it is certainly more advantageous to study earthen structures through the numerical modelling techniques that have been affirmed in recent years, such as the Finite Element Modelling (FEM), the macro-element method with the Equivalent Frame Modelling (EFM), and the Distinct Element Modelling (DEM). These modelling techniques have been applied with good results to different types of masonry, including historical and earth walls. The modern FEM techniques can provide accurate results, but non-linear FE analyses of complex earth structures appears to be not applicable due to convergence difficulties hard to solve and high computational cost involved. The same applies to DEM method. On the other hand, considerable advantages are obtained if a macro-element approach is used through the EFM. Despite introducing some approximations, they allow to reduce the computational costs without compromising the validity of the results. In macro-element modelling, the masonry is schematized as frames made up of non-deformable rigid nodes connected to each other by deformable elements, corresponding to masonry piers and spandrels. Among the different macro-element models, considerable diffusion have the codes RAN [21–24], SAM [25,26] and TREMURI [27–29]. In particular, in this study, the macro-elements modelling combined with non-linear static analysis (pushover) have been performed using TREMURI code.

The application of these numerical modelling techniques to earth structures has not properly been explored yet. A first attempt in this field was made by Tarque [30], who compared the results obtained from the application of different modelling strategies to the Finite Element Method (FEM) results. Bettini [31] applied the same FEM method to portions of earth walls stiffened by wood and steel frames.

As part of a research project funded by the Region of Sardinia, an Italian region where a long tradition of earthen constructions still exists, the macro-elements method was applied for the first time to the earthen structures [32,33]. The validity of the results obtained was first assessed by comparison with the results of simple tuff buildings. In a second moment the results of the macro-elements numerical model were compared with those of a series of experimental tests carried out on small scale models of adobe buildings, subjected to dynamic tests on a shaking table within the Getty Seismic Adobe Project (GSAP) [20,34].

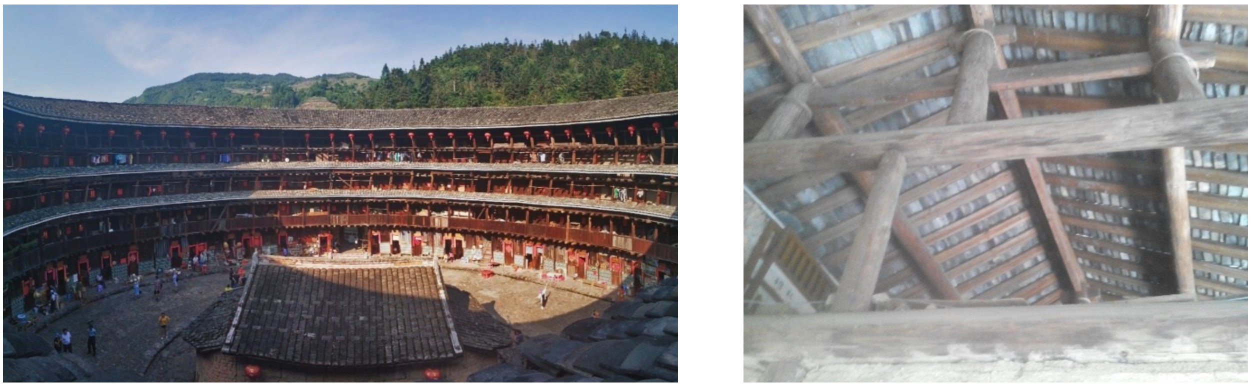
In this article, the EFM analysis has been applied to the Tulou rammed earth structure. The circular wall, typical of these buildings, has been discretized and shaped into a 24-sided polygon. The geometric characteristics and the mechanical properties of the material have been extrapolated from some studies on Huanji Tulou [35–37]. The results obtained provide a first significant contribution regarding the seismic behaviour of Tulous.

Figure 2. View of a Tulou cluster.

2. Building technology and structure of a Fujian Tulou

The Tulous are located in the mountainous area of west-south of the Fujian Province (China), and distributed in small villages developing near the waterways (**Figure 2**) [38]. Typically, they are circular buildings of about 50 m in diameter and 20 m in height. The outer wall, made of earth and built according to the rammed earth technique, has a considerable thickness of about 2 m, which gradually reduces with height with a tapered section. The access to the internal courtyard is guaranteed by a single door. (**Figure 3a**). Some small windows are located in the highest part of the building, to witness the defensive function for which these buildings were originally built. Inside, a frame of wooden pillars and beams supports the floors that host at different levels housing and commercial functions. On the last level, there is a two-pitch roof supported by a very particular truss system supported by the wooden frame and by the wall (**Figure 3b**).

Unlike this architectural features, the knowledge on the mechanical properties of the materials and on the structural features of the construction elements is much more limited. A few technical information is available about the Huanji Tulou, largely studied by some authors to investigate the effects of a violent earthquake occurred in 1918 in Fujian[35–37]. In fact, the Huanji Tulou presented as single damage a fracture that, although extensive and pronounced, did not compromise the stability of the building. After an investigation campaign for the characterization of the materials, the same authors developed a FEM model to analyse the deformations of this crack. Based on the information reported in these studies on Huanji Tulou, in the present research a numerical model of a Tulou prototype has been implemented and is shown below.



(a) (b)

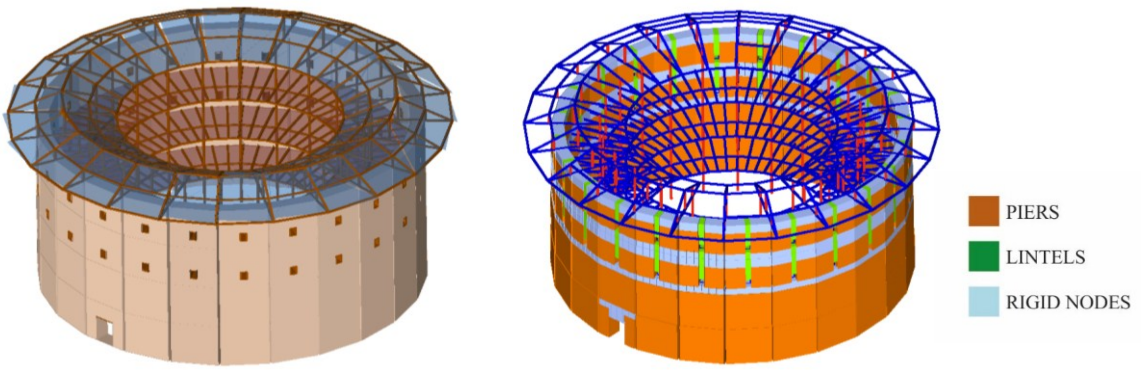
Figure 3. Interior view of a Tulou (a). Detail of the roof truss system (b).

3. Numerical model of a Tulou prototype

Among the different numerical modelling techniques available for the analysis of masonry structures as well as of earthen buildings, in the present study the prototype of a Tulou was analysed through the Equivalent Frame Method (EFM), based on macro-elements modelling. For this aim TREMURI code was used.

Contrary to techniques such as Finite Element Modelling (FEM), this modelling strategy involves rough approximations, but has proved to be valid and particularly efficient to model masonry structures [23,28,39,40]. Asprone *et al.* [32] have also applied the same analyses method to earth constructions, assessing its capability to capture the structure's response.

In accordance with what emerged from the evaluation of post-earthquake damage and dynamic test data on physical models, the EFM approach models each wall as an equivalent frame made of deformable macro-elements with non-linear behaviour (piers and spandrels) connected by non-deformable rigid nodes. The nodes' dimension depend on the openings' position, thus in turn defining also the dimension of piers and spandrels. The horizontal floors are modelled as membrane elements, whose stiffness depends on the floor type. Stiffness and geometry of walls and diaphragms allow to achieve box-behaviour and structural efficiency of the masonry structure [27–29].



(a) (b)

Figure 4. *Tulou numerical model: 3D view (a); detail of the Equivalent Frame (b).*

TREMURI code model piers and spandrels as deformable elements. In detail, deformable elements are modelled as linear elements in which three parts can be distinguished: in two thin end ones flexure deformations are addressed through a non-linear constitutive law, while shear deformations are addressed in the remaining part of the macro-element, almost coinciding with the whole masonry panel except for the two thin end parts [41]. While the connection of the rigid nodes with pier and spandrel form a 2D frame modelling a single wall, the connection between the rigid nodes of different frames allows the generation of a 3D frame, even if complex, simulating the behaviour of real masonry structures [27–29].

The application of the EFM to the Tulou's case requires some preliminary considerations about the structure and the building technology of Tulous. In detail, they are made of an external wall, usually of circular shape, and by a 3D circular wooden frame that helps increase the out-of-plane stiffness of the earth wall. The wooden floor system is supported by the radial beams of the wooden frame and is likely to be only partially rigid, because the planks are not firmly connected to the rafters supported by the radial beams. Similar considerations can be done for the two-pitch roof, supported by an A-frame truss system which is in turn supported by the wall and the wooden frame system.

Table I. Mechanical properties of earth material

f_c [MPa]	f_{v0} [MPa]	E [MPa]	w [kN/m^3]
1.00	0.10	1000	12

In order to obtain an Equivalent Frame Model of a circular Tulou (**Figure 4**), the earth wall has been approximated to a 24-side polygonal wall. Assuming a regular distribution of the openings (constant horizontal spacing and vertical alignment), a small window has been positioned at the two upper levels in each wall side. Furthermore, to take into account the low stiffness of wooden floors and roof sections, the horizontal diaphragms were appropriately modelled accounting for their low rigidity and orthotropic behaviour. **Figure 4b** shows the Equivalent Frame implemented to analyse the Tulou through the EFM.

The geometry and dimensions of the structural elements have been extrapolated from the data available in the studies on the Huanji Tulou carried out by Stanislawski [35] and Liang et al. [36]. Also, the mechanical properties of the construction materials have been defined by referring to literature-based data [37]. **Table I** shows the mechanical properties of the earth material assumed in the model of the Tulou prototype under consideration.

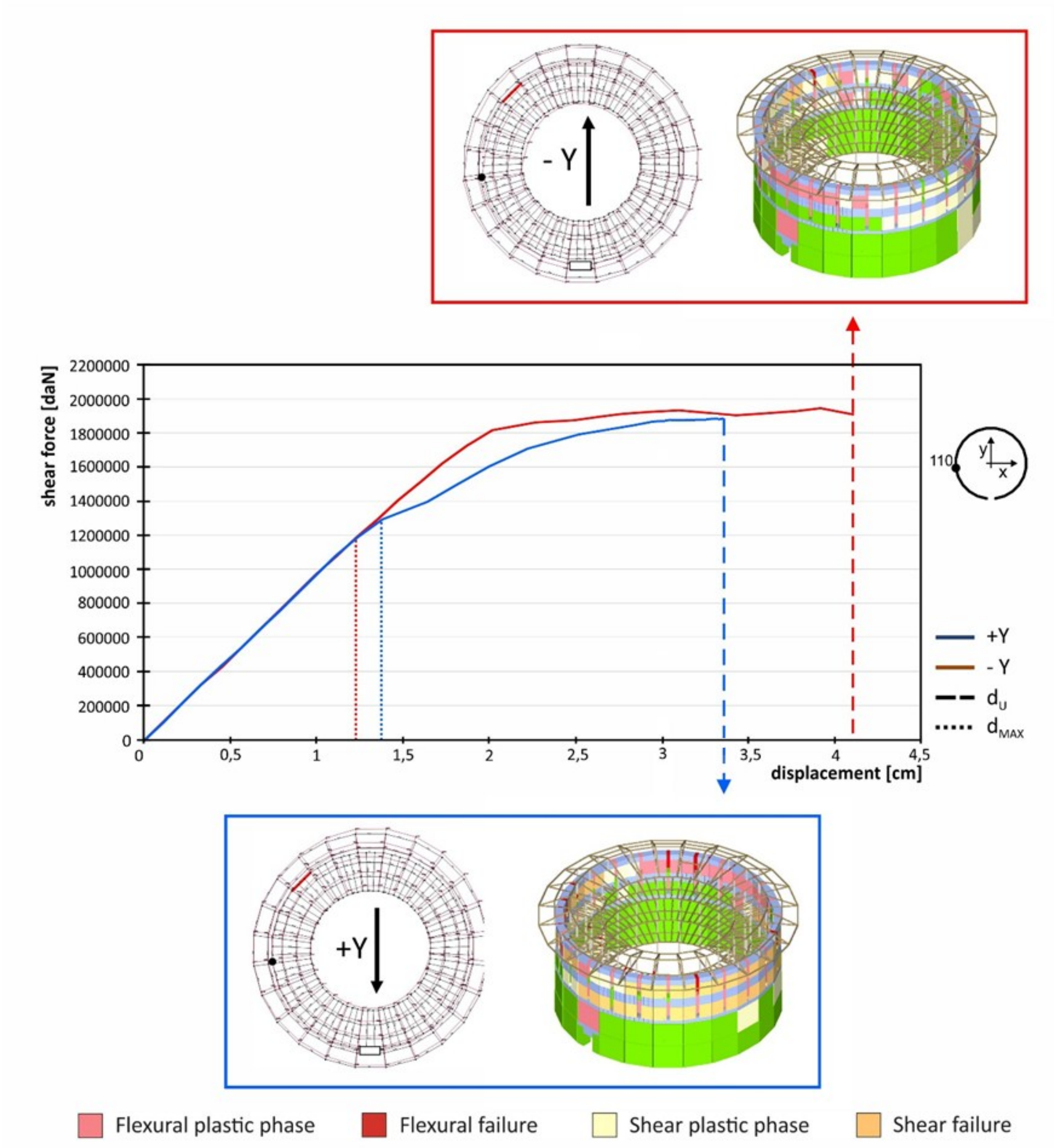


Figure 5. *Pushover analyses of the Tulou numerical model for Y loading direction of the seismic action.*

4. Seismic analysis of a Tulou prototype modelled through the EFM

Regarding the analysis methods for the seismic assessment, in the last decades, the nonlinear static analyses (pushover) have become more and more popular due to the performance-based earthquake engineering concepts [29,38] [29,42]. In this paper a pushover analysis was carried out on Tulou model implemented in TREMURI code by assigning mass proportional horizontal forces and monitoring the displacements of an appropriately chosen control node. The capacity curve obtained from the pushover non-linear static analysis was compared with the displacement demand, thus allowing to evaluate the Tulou safety under the Fujian seismic risk. The displacement demand has been obtained from the ADRS spectrum at the Life Safety Limit State, considered as an Ultimate limit State, and corresponding to 475 year return period. The analysis was carried out assuming an acceleration value a_g = 1.6g, which corresponds to the peak ground acceleration (PGA) provided by different studies on the seismic risk in the Fujian Province. The ADRS spectrum was drawn according to the Eurocode 8 rules, and therefore assuming F_0 = 2.45 and T_C* = 0.32 for a_g = 1.6*g*. Since the Tulou are constructed in the mountain area of the Fujian Province, and therefore probably on a rocky subsoil, in this first study on the Tulou seismic response, the effects related to the stratigraphic and topographic amplification have been neglected.

Error! Reference source not found. shows the capacity curves obtained assuming the loading direction of the seismic action and control node as reported in the legend. Under ideal conditions, the control node should coincide with the centre of mass. However, for the Tulou geometry, with all the masses concentrated close to the earth wall, the centre of mass is located in the centre of the Tulou court. Considering that in the centre of the Tulou there are no structural elements, therefore no control node is available close to the centre of mass. Some preliminary analyses have allowed to understand the sensitivity to the choice of the control node.

The two pushover capacity curves have been obtained reversing the direction of the seismic action. The displacement capacity corresponds to the final displacement obtained at ultimate from the pushover curve. At this point, failure is considered attained for the sudden loss of overall shear resistance at the wall base, equal to at least 20% of the maximum value of the base shear recorded during the analysis. **Error! Reference source not found.** shows the pushover curve and the displacement demand obtained from the ADRS response spectrum. Since the displacement demand is far lower than the displacement capacity for the design seismic action defined through the ADRS response spectrum, the structure appears able to resist the Fujian earthquakes without suffering any damages that could compromise its stability.

A more detailed picture of the Tulou seismic response can be provided by observing the damage sequence. As an example, **Figure** reports the pushover curve for +X loading direction of the seismic action and the related damage sequence of the Tulou wall.

The status at four points along the curve of **Figure a** is analysed. At the first point (**Figure 6b**) flexural yielding occurs in the wall above the Tulou entrance. The loading direction is parallel to the wall. A few spandrels also yield in the upper level. In Tremuri, yielding of the macro-elements entails that the macro-element has reached its flexural capacity but failure is not attained because there is still some ductility available in the element. At the second point (**Figure 6c**), the other macro-elements above the opening yield in shear, while flexural-compression yielding occurs in both piers next to the Tulou entrance. Again, these are not failure points, because some small ductility is provided not only in flexure but also in shear. At the third point (**Figure 6d**) many piers of the first level close to the Tulou entrance yields in shear, together with some other elements especially at the opposite side of the Tulou, but the overall curve does not drop. Finally, at the fourth point (**Figure 6e**), many piers of the first level close to the Tulou entrance fail in shear, together with other piers at the upper level and at the opposite side of the circular wall, thus causing a sudden drop of the shear overall load-bearing capacity higher than 20%. During the loading sequence there is also an increasing number of upper lintels that yield or even fail. Since the role of the lintels is to connect the vertical load bearing walls, that are the main lateral load carrying elements, lintel failure do not induce overall failure of the Tulou, and the pushover curve retains its shape without being practically affected by lintel yielding.

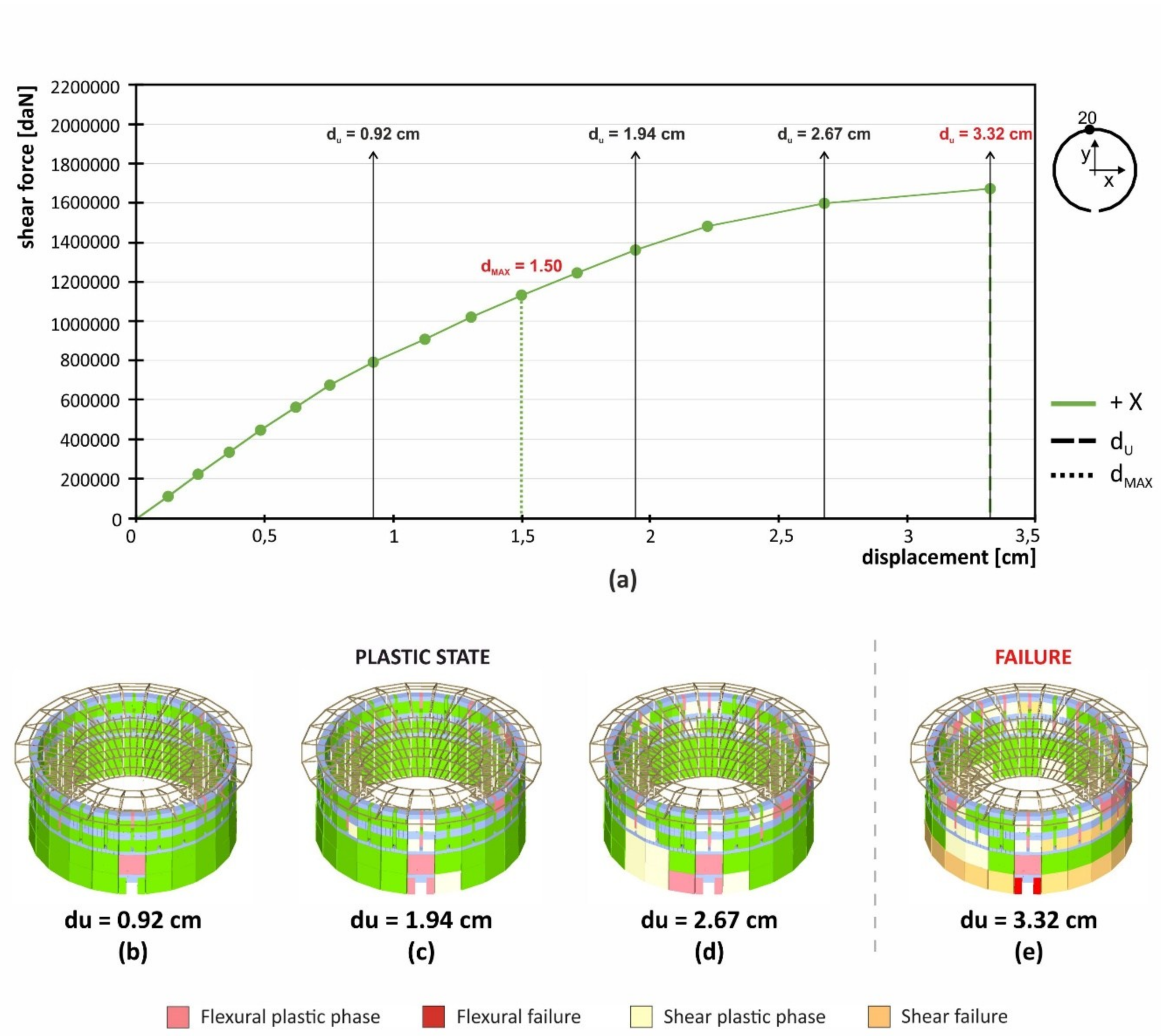


Figure 6. *Pushover curve (a) and sequence damage at different displacement levels: 0.92cm (b);1.94cm (c); 2.67 (d); 3.32 (e).*

Moreover, sensitivity analysis was carried out to take into account the uncertainties on the evaluation of the elastic modulus *E* of the earthen material. **Figure7** shows the pushover curves of this preliminary analyses, carried out by varying the elastic modulus of the earth material between 100MPa and 1000MPa. Control node 20 and +X direction of the seismic action were considered. The comparison between the different pushover curves obtained for different values of the elastic modulus highlights how the progressive reduction of the elastic modulus implies on the one hand an increase in ductility, and on the other hand a tendency to an elastic-fragile behaviour for very low values of *E*. Conversely, the failure modes remain unchanged, with the masonry piers of the lower levels that collapse due to shear forces. Given these results, the subsequent analyses have been conducted assuming an elastic modulus of 1000MPa, that appears to be more adherent to the nature of the earthen material used in the Tulou construction, and that in any case leads to capacity curves very close to those obtained with significantly lower values (i.e. 700 MPa) of the elastic modulus.

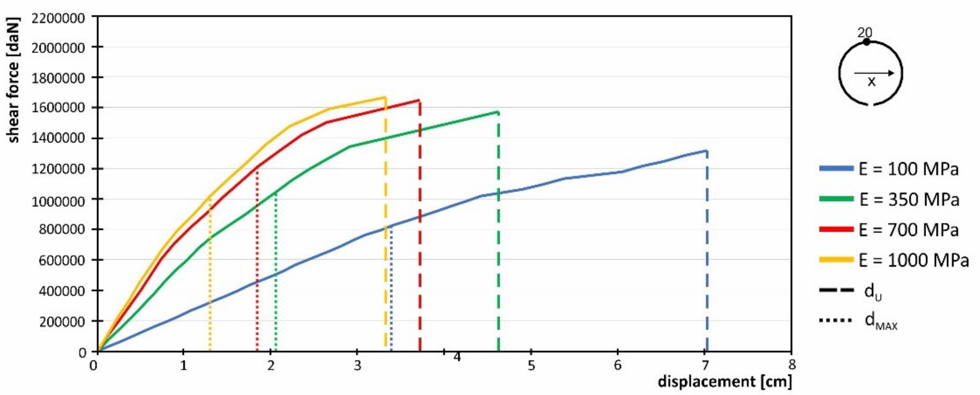


Figure 7. *Sensitivity analyses about the influence of the elastic modulus value.*

Conclusions

Tulou are earth constructions located in the Fujian Province of China, with high architectural value and included in the UNESCO World Heritage List. Despite the undoubted valence of these constructions, few studies have been conducted on them, particularly regarding their structural behaviour.

The present study attempts to deepen this aspect, analysing the seismic response of the Tulou by macro-elements through the Equivalent Frame Method (EFM), one of the numerical modelling techniques that in recent years has proved to be particularly efficient and valid for the analysis of complex masonry structures.

Starting from the data available in the scientific literature, the model of a Tulou prototype has been implemented in TREMURI code and subjected to non-linear static analysis. The circular wall has been approximated to a polygonal wall, and modelled as a series of consecutive frames, each one corresponding to a wall side of the polygonal wall. The wooden structure has been modelled through linear elements, while horizontal diaphragms with appropriate stiffness have been used to model the flexible wooden floors and roof pitches.

Based on the results obtained from the Tulou analysis by macro-elements, the EFM has proved to be a valid approach to simulate the in-plane behaviour of the Tulou. Moreover, these structures seem to have good earthquake resistance, mostly due to the circular shape of the wall that is able to channel the horizontal forces in in-plane internal forces giving the structure enough displacement capacity.

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