Maria João Martins dos Santos

U. PORTO
FEUP FACULDADE DE ENGENHARIA
UNIVERSIDADE DO PORTO
DEPARTAMENTO DE ENGENHARIA E GESTÃO INDUSTRIAL

Towards solving a robust and sustainable Vehicle Routing Problem with Backhauls

Submitted to Faculdade de Engenharia da Universidade do Porto in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Industrial Engineering and Management, supervised by Pedro Amorim, Assistant Professor of Faculdade de Engenharia da Universidade do Porto, Eduardo Curcio, researcher at INESC TEC and invited professor at Faculdade de Engenharia da Universidade do Porto, and Alexandra Marques, researcher at INESC TEC

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*"Research is to see what everybody else has seen,
and to think what nobody else has thought."*

\- Albert Szent-Gyorgyi

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Abstract

Transportation is a key process in all supply chains as it constitutes the link to execute all order fulfillment activities. Nowadays, the challenges imposed by a greener circular economy require more attention to the environmental impact of transportation and logistics. An opportunity to improve the sustainability of transport operations is to plan simultaneously outbound routes (deliveries to customers) and inbound routes (pickups from suppliers) in order to reduce empty running of vehicles and increase their efficiency in terms of used capacity. Under the domain of Operations Research, this problem is framed as the Vehicle Routing Problem with Backhauls (VRPB) and it is used as a base problem throughout this thesis. The scope of this thesis is the integrated transportation planning within a sustainable and uncertainty context, addressing several challenges that are currently presented in the literature. Overall, this thesis aims to develop mathematical models and solution methods for the VRPB to provide more robust plans, understand the value of inter-firm collaboration, and help decision-makers to perform comparative analysis among plans, considering economic and environmental aspects. This research was grounded in the analysis of two case studies, namely in the wood-based panel industry and in the food supply chain. The results provide valuable managerial insights suitable to be implemented in the case studies, but also to be extended to other industries and supply chains.

Resumo

O transporte é uma atividade chave em todas as cadeias de abastecimento, uma vez que constitui o elo necessário para todas as atividades relacionadas com o cumprimento de encomendas. Hoje em dia, os desafios impostos por uma economia circular mais verde exigem mais atenção ao impacto ambiental do transporte e da logística. Uma oportunidade para melhorar a sustentabilidade das operações de transporte é planear simultaneamente rotas de saída (entregas aos clientes) e rotas de entrada (recolhas dos fornecedores), a fim de reduzir a circulação de veículos em vazio e aumentar a sua eficiência em termos de capacidade utilizada. No domínio da Investigação Operacional, este problema é enquadrado como o Problema de Roteamento de Veículos com Retorno (PRVR) e é utilizado como o problema de base ao longo de toda esta tese. O âmbito desta tese é o planeamento integrado do transporte dentro de um contexto sustentável e de incerteza, abordando vários desafios que são actualmente apresentados na literatura. De uma forma geral, esta tese pretende desenvolver modelos matemáticos e métodos de solução para o PVRP que permitam definir planos de transporte mais robustos, perceber o valor da colaboração entre empresas, e ajudar os decisores a realizar análises comparativas entre planos, considerando diferentes aspetos económicos e ambientais. Esta investigação foi fundamentada na análise de dois estudos de caso, nomeadamente na indústria de painéis derivados de madeira e na cadeia de abastecimento alimentar. Os resultados proporcionaram conhecimentos valiosos de gestão adequados para a implementação nos casos de estudo, mas também para serem aplicados a outras indústrias e cadeias de abastecimento.

Contents

Chapter 1

Motivation and overview

Transportation is a crucial element in any supply chain, since it establishes the connections between the different stages comprising an entire chain. In addition, transportation represents a significant portion of the total logistics costs (Crainic and Laporte, 1997) and accounts for about 24% of total greenhouse gases (GHG) emissions in the European Union, where road transportation alone comprises about 17% (Demir et al., 2014). Nowadays, promoting smart, green and integrated transportation solutions are challenges that drive many ongoing research projects in Europe (see `ec.europa.eu/programmes/horizon2020`). Given its importance for the supply chain and the high impact on the economic and environmental costs, transportation hides a huge potential to promote efficient and sustainable supply chains. Among several green initiatives adopted by logistics carriers, optimizing transportation planning and reducing empty trips of vehicles are becoming more popular (Lin et al., 2014; Evangelista et al., 2017).

Optimization emerges as a powerful approach to find optimal routing plans for a fleet of vehicles, considering a set of constraints. This problem is generally known as the Vehicle Routing Problem (VRP) and was firstly introduced by Dantzig and Ramser (1959) to solve a real-world logistics problem where a fleet of trucks departing from a bulk terminal had to visit a series of service stations. The VRP is defined as an optimization problem that aims to plan routes for a homogeneous fleet, in order to satisfy all customers demands in a single visit and without exceeding the capacity of the vehicle. Each vehicle departs loaded from a common depot and returns back empty to the same depot. The most common objective in the VRP is the minimization of the total routing costs (usually given by the total traveled distance). Since its first application, several new variants have been created, attempting to capture real-life aspects of transportation problems, such as characteristics of vehicles (e.g., size, capacity, weight), drivers (e.g., licenses, resting period, maximum working hours), customers (e.g., time windows) and products (e.g., compatibility between products and vehicles), among other aspects (Mancini, 2016).

Empty trips can constitute a significant part of the overall travelled distance and contribute substantially to the total routing costs (Ubeda et al., 2011). A strategy to reduce empty trips is called *backhauling*, which takes advantage of the capacity of the vehicle, when it runs empty, to pick up some load in the return trip to the depot. Under the domain of Operations Research, this problem is known as the Vehicle Routing Problem with Backhauls (VRPB) (Deif and Bodin, 1984) and it has been successfully applied in real logistics problems (Toth and Vigo, 1999; Koç and Laporte, 2018). Including backhauling in transportation planning may also be combined to optimize the selection of suppliers. When several suppliers that share similar raw-materials, with different availability and quality levels, the selection of a supplier should be related with both travelled distance (cost savings) and quantity/quality of raw-materials obtained (revenue) (Yu et al., 2016). Thus, the selection of a supplier will consider if the obtained revenue collected is enough to compensate the additional increase in the total distance traveled by the fleet of vehicles. As an extension of the classic VRPB, this problem is designated the Vehicle Routing Problem with Selective Backhauls (VRPSB) (Baldacci et al., 2010). Despite the growing importance of transport optimization and integrated planning, the literature still lacks models and methods that deal efficiently with the transportation planning with backhauling while including sustainability issues (Bektaş et al., 2019; Allaoui et al., 2019; Colicchia et al., 2013). In particular, the VRPB is not well studied in the literature, although it has been advocated that backhauling provides a suitable alternative for inducing more friendly-environmental transport operations.

Moreover, most of the optimization tools used in transportation planning have a deterministic nature, disregarding uncertainties that may turn out to be very costly. Although the uncertainty itself may not be seen as a sustainable issue, providing robust transportation plans that are immune to uncertain events can indirectly impact the sustainability of transportation. For example, a routing plan that accounts for possible delays and changes on demand would allow the vehicles to improve their efficiency of use (e.g., travel lower distances), instead of reacting to the uncertainty after the routing plan has been established. The literature shows that there are two main approaches to deal with uncertainty - stochastic programming and robust optimization (Adulyasak and Jaillet, 2016). Stochastic programming is a well-known approach that allows to represent the uncertainty as a probability distribution function and thus creating diverse scenarios with a given probability. Examples of stochastic programming include the two-stage stochastic optimization and chance-constrained optimization. Robust optimization is a fairly recent approach to deal with uncertainty when its probability distribution is not known or when only rough information may be available (Grossmann et al., 2016). In this case, the uncertainty is represented by a bounded uncertainty set and a robust counterpart of the deterministic problem is derived. Although robust optimization models may generate more conservative solutions than stochastic programming, one of its main advantages is the tractability of the robust counterpart, which, depending on the uncertainty set, may be similar to its deterministic version. So far, no study has yet been carried regarding the VRPB under uncertainty (Koç and Laporte, 2018). Moreover, very few works on VRPs have investigated uncertainties other than demand and time, possibly because these parameters have a direct impact on the service level and customer satisfaction. However, uncertainty in revenues, which are crucial parameters to consider in the selection of suppliers, is not usually addressed in the transportation planning.

Another setting in which backhauling can be effectively developed is on collaborative transportation. Collaboration between companies allows to optimize their entire network, increasing the efficiency of the vehicles and reducing empty trips (Audy et al., 2012). The literature shows that most of the works on collaborative vehicle routing investigate collaborations between carriers or between shippers (Gansterer and Hartl, 2018). This type of collaborative networks is known as horizontal collaboration, where the entities are at the same level of the supply chain and perform similar transportation services. In opposition, vertical collaboration refers to a collaborative network formed by companies at different levels in a supply chain. Lateral collaboration allows to combine horizontal and vertical collaboration, and involves decisions on similar and complementary transportation services. In the context of collaborative vehicle routing, research on vertical and lateral collaborations have not received much attention in the literature as the horizontal collaboration, despite its practical relevance in real world contexts (Gansterer and Hartl, 2018). The impact of collaboration is usually measured in terms of gains obtained by comparing the cost of the collaborative solution with the cost of the non-collaborative solution, and the percentage difference is known as synergy value. Afterwards, these gains should be fairly distributed among the participants in the collaboration, which implies solving a pricing problem. The fairness criteria are usually based on properties derived from game theory, such as the individual rationality, which ensures that each participant is allocated a cost that is lower than its individual cost (i.e., when the participant does enter in the collaboration). In this context, there is a vast literature on allocation methods to distribute collaborative gains, ranging from simple proportional allocation methods to more complex methods that require solving linear programs (Guajardo and Rönnqvist, 2016). Despite the growing research on collaborative vehicle routing, the majority of the published works either focus on solving the routing problem or on solving the profit sharing problem, but usually not both simultaneously (Gansterer and Hartl, 2018).

In summary, the focus of this thesis is on proposing transportation planning models, enhancing the potential of backhauling while coping with uncertainty and sustainable objectives, and promoting inter-firm collaboration. The models developed are applied in two case-studies, allowing to demonstrate the real benefits of different backhauling strategies and to derive a set of managerial insights that may support logistics decisions in different supply chains.

1.1. Research objectives and methodology

This research aims to investigate mathematical programming models for the VRPB that are capable of handling sustainability issues and uncertainty, by combining or extending features of existing approaches.

To handle uncertainty, the concepts and modeling aspects of robust optimization will be borrowed and adjusted to the problem under research. Robust optimization is an emergent approach in VRP literature that allows to incorporate uncertainty in optimization models without previous knowledge on the uncertainty profile, which happens in many real logistics planning problems. The existing research relating VRP and robust optimization will be analyzed, in order to identify how different uncertainties are being represented and modeled and which decisions are required for different robust optimization approaches. Moreover, the uncertainty in revenues and how these affect the routing problems and supplier selection will be examined.

To handle sustainability issues, current literature on both green VRP and collaborative VRP will be analyzed in order to identify the parameters that influence sustainable aspects of transportation, as well as the requirements to form efficient collaborative networks. In addition, different modelling approaches to tackle environmental concerns and collaboration in transportation will be examined, with the goal of adapting such approaches to

the VRPB. Several backhauling strategies are analysed and tested with instances from the literature and instances from case-studies.
The VRPB is a NP-hard combinatorial optimization problem, which means that the problem cannot be solved in polynomial time. Different solutions methods are investigated in order to efficiently solve the different VRPB models that include uncertainty and/or sustainability issues, namely exact methods (e.g., Branch-and-Cut method), metaheuristics (e.g., Adaptive Large neighborhood Search algorithm) and matheuristics (e.g., fix-and-optimize algorithm). Very few exact methods were developed so far for the VRPB, because they are not usually efficient to deal with the hard combinatorial nature of the VRPB. Metaheuristics, on the other hand, are the state-of-art methods for the VRPB, but the solutions obtained should be associated to a lower bound, which is usually obtained with an exact method. A matheuristic, a hybrid solution method that combines exact algorithms and metaheuristics, is also build in this thesis using a fix-and-optimize approach.
The scope of the research questions and their relationship is presented in Figure 1.1. Research question 0 (RQ0) provides drivers for all the other research questions. Research question 1 (RQ1) is related with research question 2 (RQ2), since they both explore the revenues in the settings of the VRPSB. Research question 3 (RQ3) is related with RQ2 since they both investigate new exact algorithms to solve variants of the VRPB. Research question 4 (RQ4) is related to RQ3, as they both examine collaborative networks and apply properties from game theory to solve the profit sharing problem. Finally, RQ4 and RQ1 relate to each other since they are investigated in real case studies.

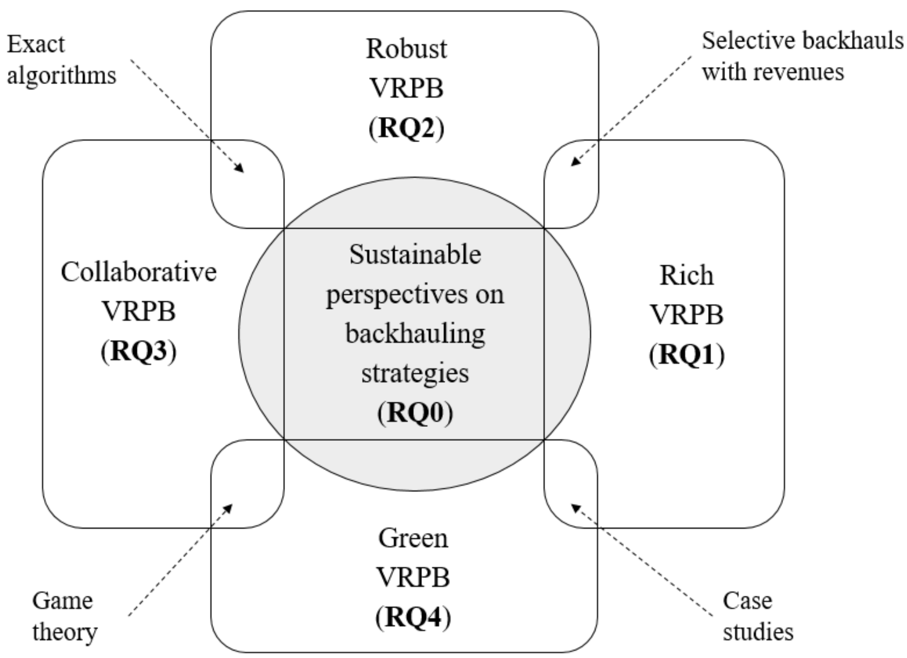

Figure 1.1 – Research questions framework.

Research Question 0

What is the role of the Vehicle Routing Problem with Backhauls in terms of sustainability?

This research question is motivated by the potential of the VRPB to be applied in real logistics problems and to cope with economic, environmental and social concerns, despite that traditional optimization approaches are mostly driven by economic goals only.
To answer this research question, an extensive review on the VRPB literature with particular focus on sustainability issues is performed. The main trends in the overall transportation planning problems are identified and compared with the current VRPB research, in order to determine interesting research lines for this type of problems, which are further tackled in this thesis.

Research Question 1

How can transportation with backhauling be enriched for real world contexts?

This research question is investigated with the goal of evaluating new strategies to address the transportation planning with backhauling other than the typical VRPB, while copping with real-life operational constraints. In order to provide an efficient solution method that is able to solve these rich versions of the VRPB in reasonable computation time, a fix-and-optimize algorithm is developed, taking advantage of the hybridization of exact methods and metaheuristics.
The answer to this research question is the application of a case-study in the forest supply chain, which assesses the impact of different backhauling strategies, as well as the impact of different inputs of the problem.

Research Question 2

How to address uncertainty in the Vehicle Routing Problem with Backhauls through robust optimization?

This research question is motivated by, on the one hand, the nonexistence of any VRPB study under uncertainty, and, on the other hand, by the increasing popularity of robust optimization among optimization problems for which the probability distribution of uncertain parameters is not available. Moreover, uncertainty in revenues is recognized as a major factor to consider in the supplier selection, but few works in the literature have considered this source of uncertainty.
The outcome of this research question will be a well defined model to represent the robust VRPB and different solution methods to solve it. The methods to be developed include exact methods and metaheuristics. The trade-off between robustness and impact on the objective function of the nominal problem will be examined. The results will allow to draw conclusions about robust optimization and its impact on the robustness, efficiency and cost of the transportation planning problem with backhauling under uncertainty.

Research Question 3

How to efficiently model and solve a collaborative Vehicle Routing Problem with Back-

hauls?

This research question is motivated by the increasing awareness of the potential of the VRPB to cope with sustainable supply chains and collaborative networks. The VRPB can reduce empty trips, which represent a large portion of the total routing costs, and allows a better use of the vehicle's capacity, which impacts considerably on the energy consumption and efficiency.
Answering this question will result in a newly mathematical formulation based on bilevel optimization that tackles the vertical collaboration between a shipper and a carrier, while ensuring the hierarchical decisions and the conflicting objectives among different stakeholders. Analysis and discussion of the results will allow to compare the new approach with typical collaborative models and evaluate the synergy effects between the two players.

Research Question 4
How to address the challenges of a practical sustainable collaborative Vehicle Routing Problem with Backhauls?

This research question is motivated by three main research gaps in the literature. The first regards the lack of studies on lateral collaboration for the collaborative vehicle routing, which allows to combine horizontal and vertical decisions. The second regards the lack of collaborative networks that incorporate other than economic concerns, such as fuel consumption or GHG emissions. The third regards the lack of works that handle both the optimization of the routing problem and the fair distribution of collaborative gains among the players.
The final output of this research question is the report of a case-study in a food supply chain evaluating the potential gains from collaboration with respect to different objectives (economic an environmental related) and the analysis of different methods to allocate the gains among the players. The results of this case-study will allow the definition of a set of managerial insights to implement collaboration in practice.

1.2. Thesis structure and synopsis

This thesis is composed of a collection of papers, and each chapter (or paper) is aligned with each research question previously presented. This section provides an overview of the chapters that compose this thesis.

Chapter 2 provides an extensive literature review on the VRPB works, with particular focus on the sustainability issues of transportation planning, aiming to answer to Research Question 0. Since its first introduction in the literature, the VRPB has been commonly used to plan routes under an umbrella of economic objectives, such as minimizing distances, costs, time and number of vehicles. With the growing interest by the scientific community and companies on developing and implementing sustainable strategies for transportation, new aspects derived from practical requirements, and environmental and social concerns have been driving the emergence of alternative modelling approaches and practices for more sustainable transport operations. In this chapter, we classify all reviewed papers according to a common taxonomy for routing problems and contextualize the VRPB literature under the domains of Rich VRP, sustainable goals, collaborative networks and reverse logistics. In this chapter, we also demonstrate that are still very different avenues to explore with the goal of bringing the VRPB closer to the theme of sustainable transportation in supply chains. Therefore, we provide a set of guidelines for future research in this context.

Chapter 3 examines and compares three different approaches to deal with the transportation planning of inbound and outbound routes in real contexts, addressing Research Question 1. The first approach applies the traditional decoupled planning, where the inbound and outbound routes are optimized separately. The second approach plans the inbound and outbound routes simultaneously, allowing the integration of inbound trips after outbound trips. The third approach is an opportunistic variant of the second where the primary concern is the optimization of outbound routes and only if it is cost-effective, inbound routes may be integrated. The underlying problem considers practical aspects from real operations in the wood-based industry, namely heterogeneous fleet of vehicles, multiple depots and open routes, split deliveries and limitation on maximum distances. Moreover, the problem considers the variable quality degree of raw-materials available at the suppliers, which are translated into different revenues. Hence, the problem is modelled as a Rich VRPB minimizing fixed and variable transportation costs and maximizing the total revenues collected. A fix-and-optimize algorithm is developed to solve the problem, combining features of exact and approximate methods. This work is applied to a case-study in the wood-based industry, on which a set of mills aims to plan optimal routes to deliver wood panels to their customers and collect raw-materials from suppliers. A set of managerial insights with application to the wood-based industry, but possible to extent to other supply chains (e.g., retailing), are derived based on a series of sensitivity analysis to different parameters of the problem and from the comparison of the different planning approaches.

Chapter 4 investigates a robust optimization approach to address the VRPB under uncertainty, and thus answers Research Question 2. In chapter 3, the revenues collected at backhaul customers are deterministic. In this chapter, the revenues are considered uncertain, and no knowledge on the probability distribution is available. The VRPB with uncertain revenues is modelled following a robust optimization approach in the literature, where the space of uncertain revenues is defined by an uncertainty set and a parameter, known as budget of uncertainty, controls the size of that space. The problem consists on planning inbound-outbound routes such that all linehaul customers are visited and a minimum amount of total revenue is collected at backhaul customers. The goal is to minimize the total travelling costs minus the sum of expected revenues. Two different solutions methods are proposed to solve the problem: a Branch-and-Cut algorithm and an Adaptive Large Neighborhood Search (ALNS) metaheuristic. Robust solutions are evaluated in terms of probability of constraint violation and price of robustness. The probability of constraint violation refers to the probability of a generated solution failing to attend the minimum revenue expected, which can be related to the budget of uncertainty. Three different methods from existing literature and one novel method proposed in this work are used and compared to estimate the probability of constraint violation. The price of robustness refers to the percentage cost difference between a robust solution and a deterministic solution. In this context, the solutions of the robust optimization model are compared with solutions obtained with a chance-constraint model. The performance, advantages and limitations of the models and methods used in this work are presented.

Chapter 5 presents a novel formulation to model a collaborative VRPB, and thus addresses Research Question 3. The collaborative VRPB considers a vertical collaboration between a shipper and a carrier, where hierarchical decisions are taken by both entities but their goals are different. The new formulation of the problem consists of a bilevel optimization model where the upper level covers the problem of the shipper and the lower level covers the problem of the carrier. In the upper level, the shipper decides the minimum cost delivery routes and the incentives to offer to the carrier to perform pickups at different backhaul customers. At the lower level, the carrier decides the incentives to accept and on which delivery routes are the backhaul customers inserted. In practice, the shipper first decides the minimum cost routes and then offers a side payment to the carrier aiming to compensate the additional travelling costs to perform additional backhauling. In this context, the main novelty of the bilevel model is that it allows to solve simultaneously the routing and the pricing (incentives) problems, instead of solving them sequentially. The bilevel problem is solved with an exact reformulation method and this approach is compared with two different approaches based on side payments. The impact of collaboration is measured through the synergy value, and the advantages and limitations of the bilevel formulation are presented.

Chapter 6 addresses the collaborative VRPB from an environmental perspective, tackling Research Question 4. The transportation network considers different entities (a retailer, a 3PL and several suppliers), which can perform different transportation related activities, such as pickups and deliveries in backhaul routes and cross-docking in intermediary facilities. Due to the existence of diverse entities and strategies, the problem addresses the case of lateral collaboration, which is not commonly investigated in the context of collaborative vehicle routing. The collaborative problem is formulated with three different objective functions: a pure economic (minimize operational costs), a pure environmental (minimize fuel consumption) and a holistic function (minimize operational and CO_2 emission costs). The impact of collaboration is then measured by the synergy value and the solutions obtained with the three formulations are compared. Afterwards, three profit

sharing mechanisms are tested using concepts from proportional allocation methods, and compared in terms of some fairness criteria. The problem is applied to a case-study in the food supply chain and managerial insights are derived from the results.
Finally, chapter 7 summarizes the main contributions of this thesis, providing answers to the research questions raised and suggestions for future work.

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Chapter 2

Vehicle Routing Problem with Backhauls

The vehicle routing problem with backhauls towards a sustainability perspective - a review

Maria João Santos *† · **Pedro Amorim***† · **Alexandra marques**† · **Ana Carvalho**‡ · **Ana Póvoa**‡

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Abstract The Vehicle Routing Problem with Backhauls (VRPB) allows to integrate inbound and outbound routes, which is an efficient strategy to reduce routing costs and also to reduce the environmental and social impacts of transportation. In this paper, we analyze the VRPB literature with a sustainability perspective, which covers environmental and social objectives, collaborative networks and reverse logistics. First, in order to better understand and analyze the VRPB literature, all related works are characterized according to a common taxonomy provided for routing problems. This taxonomy is extended to differentiate between economic, environmental and social objectives. After identification of all VRPB papers that include sustainability issues, these are analyzed and discussed in more detail. The analysis reveals that research on VRPBs with sustainability concerns is recent and relatively scarce and the most popular aspects investigated are the minimization of fuel consumption and CO_2 emissions. Future research lines driven by sustainability concerns are suggested for the VRPB as a promoter of green logistics.

Keywords vehicle routing problem · backhauling · sustainability · collaboration · reverse logistics

2.1. Introduction

In the European Union, transportation is the sector with the fastest growing energy consumption and greenhouse gas (GHG) emissions (Oberhofer and Dieplinger, 2014). Road

*Faculdade de Engenharia, Universidade do Porto, Porto, Portugal
†INESC TEC - Instituto de Engenharia de Sistemas e Computadores, Tecnologia e Ciência, Porto, Portugal
‡Centro de Estudos de Gestão (CEG-IST), Instituto Superior Técnico (IST), Lisboa, Portugal

transportation still accounts for about 18% of GHG emissions and yields 25% of empty returns (Juan et al., 2014). For these reasons, transportation can be seen as one of the most promising sectors regarding the development and the implementation of environmental strategies capable of mitigating the environmental impact and of reducing costs, enforcing therefore the sustainability of supply chains.

In this context, an optimal transportation planning should be used to design one or more routes that allow meeting all customers' needs, covering the entire network, using the full capacity of the vehicles and avoiding empty return trips, thus reducing costs and mitigating the environmental impact related with the transportation operations. The classical Vehicle Routing Problem (VRP) is one of the most famous transportation planning problem and deals with the distribution of goods from a single depot to a set of customers using vehicles with limited capacity. The problem assumes that a customer's demand must be satisfied in a single visit, that the fleet of vehicles is homogeneous (with similar capacities) and that each route must start and end at the depot. The goal is to determine optimal routes, minimizing the total cost (usually in terms of distance travelled) (Solomon, 1987).

In an attempt to introduce real-life aspects into transportation problems, several new variants of the VRP have been created (see Toth and Vigo (2002b); Golden et al. (2008); Toth and Vigo (2014)). These add significant complexity to the resulting problems. Such real-life aspects concern vehicle characteristics (e.g., size, capacity, weight), drivers (e.g., licenses, resting period, maximum working hours), customers (e.g., time windows) and products (e.g., compatibility between products and vehicles), among other aspects (Mancini, 2016). Eksioglu et al. (2009) proposed a methodology to classify a VRP based on the characteristics introduced in the problem, which was latter updated by Braekers et al. (2016). This taxonomy is used to classify the routing problems reviewed in this paper and extended to classify the objective functions in three dimensions, namely economic, environmental and social.

The Vehicle Routing Problem with Backhauls (VRPB) is a variant of the VRP that focus on the efficiency of the vehicle temporal utilization. The problem considers two different sets of customers: the set of linehaul customers (locations that receive loads from a depot) and the set of backhaul customers (locations where loads are collected and sent to a depot). The goal of the VRPB is to optimize cost-effective routing plans that integrate linehaul and backhaul customers. Although the goal is still minimizing costs, the VRPB can contribute substantially to reduce empty running of vehicles and fuel consumption, which in turn contributes to reduce the environmental impact of transportation (Pradenas et al., 2013).

The first review on the VRPB literature was carried out by Toth and Vigo (2002a), which presents an overview of the exact and heuristics methods to solve the classic VRPB until the beginning of the century. The authors describe in detail the mathematical formulations, relaxations, and heuristics algorithms that are the foundation for many solutions methods used today. A second review can be found in Parragh et al. (2008a), which presents a common classification of the main variants of the problem and reviews the research carried until 2008. Papers considering reverse logistics are cited, but outside the scope of that review. The authors generally agree that the future direction of VRPB research will entail the incorporation of more detailed real-life aspects into the problem, as well as the effects of dynamism and uncertainty. Later on, Toth and Vigo (2014) have revived the attention to several variants of the VRP, namely the VRPB. The authors followed up the previous reviews and discussed the new contributions on this topic, namely in terms of new solutions methods and extensions of the problem. In their conclusions, they highlighted that exact methods for the VRPB should be updated to the current state-of-art algorithms. More recently, Koç and Laporte (2018) has updated the literature on VRPB research. The authors provide a comprehensive review covering models, solutions methods, new variants and applications of the problem. It is concluded that the research towards the VRPB has been increasing but there is room for improvement, particularly focused on modelling aspects (e.g., continuous approximations, stochastic parameters) and solution methods (e.g., hybrid algorithms combining population-based and local search metaheuristics).

None of these reviews, however, provide any suggestions on how to enhance the benefits of the backhauling strategy in the VRPB, nor they provide examples of the VRPB under more sustainable contexts, such as reverse logistics or collaborative networks. Reverse logistics relates to the VRPB since the vehicles used to transport goods to customers are also used to carry returned products back to the depot (e.g. damaged or non-conform products, used items to recycle). For instance, the retailer Tesco uses backhauling for the returned products collected at their stores. The returned products are collected in a secure cage in each store and when they are full, they are transported by the delivery vehicles that supply that store to the distribution centre (DS Smith, 2017). Collaborative networks can be related to the VRPB whenever a carrier uses its delivery vehicles to perform additional services in their backhaul routes, reducing thus the empty trips. In fact, collaboration in routing problems aims to increase the efficiency of vehicles, by reducing empty backhauls and road congestion, and by enhancing capacities (Audy et al., 2012; Gansterer and Hartl, 2018). For example, Nestlé and United Biscuits, competitors in the food market, have arranged a collaboration system to improve their logistics operations. This system involves coordinating the collection of loads of each other company after delivering its own requests. This backhauling strategy allowed to reduce empty running from 22% to 13% in four years (Catherine Early, 2011). In addition, some relevant works that model a VRPB with environmental concerns are missing from the review of Koç and Laporte (2018), such as the one in Ubeda et al. (2011) who first propose a VRPB with such features.

This paper aims to present the research on VRPBs with sustainable concerns, by analyzing papers that introduce environmental and/or social objectives, and papers that consider the benefits of backhauling in different contexts, such as in collaborative networks or reverse logistics. This paper complements the review of Koç and Laporte (2018), allowing to show in detail and to a further extension the developments, scope and future directions for the VRPB. Nonetheless, this paper has distinct contributions from that of Koç and Laporte (2018). Particularly, the focus of this review is not on the solutions methods or models that have been developed for the VRPB, but rather on how the sustainable impacts of the VRPB have been analyzed and evaluated. In addition, a future research agenda driven by sustainability concerns is proposed. The three main contributions of this paper are:

- Present a clear classification and comparison of all VRPB works carried up to the moment, based on a common taxonomy for routing problems (Braekers et al., 2016) that is extended to differentiate between economic, environmental and social objectives;

- Review relevant works that investigate the impact of the VRPB in sustainable contexts, such as reverse logistics and collaboration;

- Provide insights on how to leverage the sustainable potential of the VRPB.

The paper is organized as follows. Section 2.2 describes the VRPB and presents its main variants. Section 2.3 defines the methodology applied in this research. Section 2.4 focuses on the VRPB literature overview, which provides a classification and a comparison of papers according to a common taxonomy. Section 2.5 analyses the relevant VRPB literature that focuses on sustainable goals, reverse logistics and collaborative networks. Section 2.6 suggests research lines for the future development of more sustainable VRPB works, based on the findings of previous sections. Section 2.7 concludes this paper.

2.2. The VRPB and its variants

The VRPB was firstly introduced in literature by Deif and Bodin (1984) as an extension of the VRP that includes two types of customers: linehaul customers are those who receive goods from a depot (forward flow - outbound) and backhaul customers are those who send goods back to the depot (backward flow - inbound). The goal is to create the most cost-effective routes that simultaneously satisfy the demand of linehaul customers and collect the pickup orders at the backhaul customers. The constraints of the VRPB are the following:

1. Each vehicle travels exactly one route;

2. Each vehicle starts and ends at the depot;

3. Total demand of linehaul and backhaul customers, considered separately, cannot exceed the total vehicle capacity;

4. Each customer (either linehaul or backhaul) is visited exactly once;

5. All linehaul customers are visited before backhaul customers (precedence constraint);

6. No routes with only backhaul customers are allowed, but routes can contain only linehaul customers.

The precedence constraint of the VRPB is a route condition that brings several benefits from an economic and practical perspective, particularly for the manufacturing industry. Firstly, the vehicles are often rear-loaded and the load is arranged in such a way that reflects the sequence of deliveries to linehaul customers. Therefore, the precedence constraint avoids problems that emerge from rearranging the vehicle loads at the delivery points (Toth and Vigo, 1996, 2002a). Secondly, linehaul customers have higher service priority than backhaul customers, which is in line with the constraint that allows designing routes with linehaul costumers only but not routes with backhaul costumers only (Toth and Vigo, 1996, 2002a). It is also worth mentioning that, for many industries where the supply needs are usually satisfied with a single vehicle trip, i.e. the load to be supplied matches the full capacity of the vehicle, the VRPB seems to be the most suitable variant to employ in such instances.

The VRPB has also been called Vehicle Routing Problem with Clustered Backhauls (VR-PCB) by Parragh et al. (2008a). Following the classification of Parragh et al. (2008a), the main variants of the VRPB are: the VRP with mixed backhauls (VRPMB), the VRP with divisible delivery and pickup (VRPDDP) and the VRP with simultaneous pickup and delivery (VRPSPD). The VRPB and the VRPBM establish that customers must be either linehaul or backhaul, never both, while the VRPDDP and the VRPSPD assume that each customer requires a delivery and a pickup. These variants are briefly described below.

In the VRPMB, the precedence constraint is not considered; instead, mixing visits between linehaul and backhaul customers are explicitly allowed. Consequently, the vehicle load may decrease or increase during the route, depending on whether the load is delivered or picked up. This load fluctuation requires checking the vehicle capacity regularly, which increases the complexity of the problem (Mosheiov, 1994). Early works on the VRPMB include those of Casco et al. (1988), Mosheiov (1998) and Salhi and Nagy (1999). The VRPMB can be efficiently applied to situations where multiple loads can fit on the truck, as is the case in food companies (Oesterle and Bauernhansl, 2016).

In the VRPSPD, each customer is associated to both a load to deliver and a load to pick up. The delivery and pickup occur simultaneously in a single visit. This procedure may decrease customer expenses and inconvenience, but can also increase the route duration (Ropke and Pisinger, 2006). The first work addressing the VRPSPD is presented in Min (1989). Also, the VRPSPD is highly linked to reverse logistics. For instance, in the soft drink industry, the vehicle has to deliver the product to the customers and, at the same time, pick up the empty bottles to recycle and transform them into new products (Privé et al., 2006).

In the VRPDDP, costumers also require both a delivery and a pickup, but vehicles are allowed to visit a customer more than once. In fact, two visits are required, one for load delivery and another for load pickup. One of the first works addressing this variant belongs Gribkovskaia et al. (2001). When a customer faces two visits, they are referred to as a "split" customer (Nagy et al., 2013).

A variant not specifically classified by Parragh et al. (2008a) assumes that backhauls are optionally visited based on a revenue they generate. In opposition to the other variants, the VRPSB aims to maximize the total revenues (or total profits, after discounting the costs). This problem is referred in literature as the VRP with deliveries and selective pickups (Gribkovskaia et al., 2008), the VRP with unrestricted backhauls (Süral and Bookbinder, 2003) or the VRP with Selective Backhauls (Baldacci et al., 2010).

2.3. Methodology

The methodology applied in this work follows a methodological approach similar to the literature review followed by content analysis described in Seuring et al. (2005). This is based on the following four steps: material collection, descriptive analytics, category selection and material evaluation.

Material collection Setting clear boundaries for a literature review is necessary in order to limit the research (Seuring et al., 2005). For the present VRPB review, the following considerations apply:

- All the works reviewed in Koç and Laporte (2018) are considered in this paper. The published VRPB papers, as well as their variants, presented in Parragh et al. (2008a) and in Toth and Vigo (2014) are also considered. Moreover, an additional search is carried but limited to scientific papers published in international journals since 2000.
- The additional search was limited to the keywords "Vehicle Routing Problem" and "Backhaul". Papers that were found with those keywords, but investigate Pickup and Delivery Problems (PDPs) were not collected. In the PDP, the linehaul customers are supplied by the backhaul customers instead of the depot (Parragh et al., 2008b).
- Three library databases were used to search for VRPB papers, namely ScienceDirect (www.sciencedirect.com), Scopus (www.scopus.com) and Google scholar (www.scholar.google.com).

Overall, 107 papers were selected - 84 papers already cited in the previous reviews and 23 papers from the additional searching process. We anticipate that most of these papers does not consider sustainability issues, for which these papers are only included for the purpose of classification of the VRPB literature (Section 2.4). The papers that consider sustainability issues are analyzed and discussed in further sections.

Descriptive analysis A preliminary analysis of all papers was undertaken, according to publication year, journal and number of citations. The aim of this descriptive analysis is to evaluate the interest of the scientific community in VRPB-related research.

Category selection The categories are selected in order to classify each work and facilitate the comparison of the VRPB literature. This set of categories are based on the taxonomy proposed by Braekers et al. (2016), with the following adaptations: i) some categories were not used, as they were adding no value to the analysis when considering the purpose of this work; ii) the classification of backhauls and node/arc covering constraints were included in the same criterion, and iii) the objective function distinguishes between economic, environmental and social dimensions. Table 2.1 provides the categories selected from Braekers et al. (2016), marked in bold.
The reasons that motivated the investigation of these specific categories are as follows.

- The trade-off between solution quality and computing time determines the use of exact or approximate solution methods. In addition, the purpose of the research, for example, comparing backhauling strategies or solving a large-scale problem, also influences the selection of the solution method. Thus, it is crucial to analyze the diversity of methods applied to the VRPB.

- The characterization of the problem's scenario and physical network is important for describing the real context of the problem and for analyzing the accuracy provided by the model regarding the problem representation (Caceres-Cruz et al., 2014).

- Transportation activities are often subjected to uncertain events, e.g., traffic and weather conditions, vehicle failure and newly assigned orders, which can jeopardize the success of a predefined routing plan. Since research devoted to the VRP under uncertainty has received increasing attention recently (Grossmann et al., 2016), it is interesting to analyze the current position of the VRPB in the context of uncertainty.

- The increasing interest on promoting more sustainable transportation has been creating opportunity to design new VRP models that enable to tackle environmental and social concerns (Lin et al., 2014). The dimensions covered by the objective(s) of a VRPB are, in fact, the most important criteria to analyze in this work.

Material evaluation All papers selected for the review were classified according to the set of categories defined above. Note that the focus of the present paper is not an exhaustive literature review, since this work was already carried in previous reviews, but rather on the analysis of VRPB works that address sustainability concerns. In this context, we designate by "sustainable VRPBs" all the works that include: i) environmental and/or social objective functions, ii) collaboration, and iii) reverse logistics. The reasons that motivated the analysis of sustainable VRPBs are as follows.

- Measuring the externalities of transportation (impact of transport operations on the environment (e.g., GHG emissions) and on the society (e.g., noise pollution) is becoming more and more important in the context of sustainable supply chains (Demir et al., 2015). In addition, green variants of the VRP have been developed in literature in order to model energy minimization (e.g., Kara et al. (2007)), pollution minimization (e.g., Bektaş and Laporte (2011)) or alternative fuel vehicles (e.g., Erdoğan and Miller-Hooks (2012)).

- Collaboration is a sustainable strategy currently used by many companies in the logistics sector that aim to increase the efficiency of vehicle use and reduce costs (Evangelista et al., 2017). Collaboration enables the participants in the transportation network to detect backhauling opportunities that otherwise are not possible. For this reason, collaborative networks can be modelled as VRPBs.

- Reverse logistics is also a hot topic in the sustainable supply chains that aims to improve the management, planning and control of all backward flows of products (Lin et al., 2014). Seen as a closed-loop routing problem, the VRPB can be analogous to a reverse logistics problem, if the same vehicles are used in the forward and backward flows.

Table 2.1 – Categories selected from Braekers et al. (2016) for the VRPB classification

1. Type of Study	1.1 Theory **1.2 Applied methods** 1.3 Implementation documented 1.4 Survey, review or meta-research
2. Scenario Characteristics	2.1 Number of stops on route **2.2 Load splitting constraint** 2.3 Customer service demand quantity 2.4 Request times of new customers **2.5 Onsite service/waiting times** **2.6 Time window structure** **2.7 Time horizon** **2.8 Backhauls** **2.9 Node/Arc covering constraints**
3. Problem Physical Characteristics	3.1 Transportation network design 3.2 Location of addresses (customers) **3.3 Number of points of origin** **3.4 Number of depot points** 3.5 Time window type 3.6 Number of vehicles 3.7 Capacity consideration **3.8 Vehicle homogeneity (Capacity)** 3.9 Travel time **3.10 Objective**
4. Information Characteristics	4.1 Evolution of information **4.2 Quality of information** 4.3 Availability of information 4.4 Processing of information
5. Data Characteristics	

2.4. Literature overview

In this section, an overview of the VRPB literature is presented, covering a general descriptive analysis of the published papers and the respective classification of all 107 works, according to the taxonomy provided in Table 2.1. The section concludes with the main remarks of the literature and the identification of the sustainable VRPB works selected for the analysis in the next section.

2.4.1 Descriptive analysis

A first glimpse at the research and application potential of the VRPB can be described by the distribution of the body of literature published in the analyzed period. In Figure 2.1, this distribution is shown and it can be observed that VRPB research has been effectively growing, particularly since the beginning of the century. Note also that the two papers found for the year 2019 only consider the first month of the year.

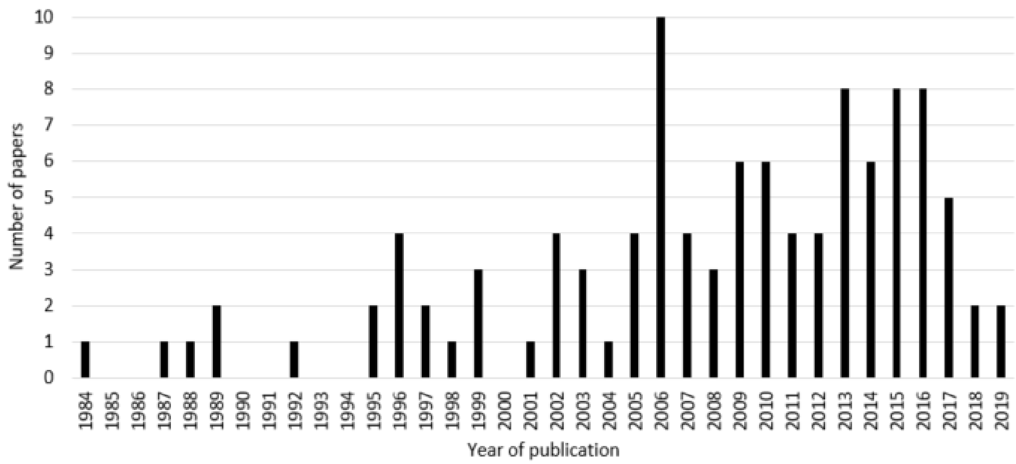


Figure 2.1 – Distribution of published papers per year.

It can be seen in Figure 2.2 that the journal where more papers were published was the European Journal of Operational Research, followed by the Expert Systems with Applications. The first journal aims to contribute to the development of operational research methodologies and best practices for decision making in order to solve real-world problems, while the second gives more emphasis to the design, testing and implementation of intelligent systems, as well as to the practical guidelines and management of such systems. Nevertheless, several papers were published in journals that focus on the theoretical and methodological aspects of the studies, such as the Computers & Operations Research, the Journal of Operational Research Society, or the Transportation Science. The category "Others" groups all journals which appear only once or twice.

To perform a ranking of VRPB works cited in the literature, Google Scholar Citations was used to collect the number of citations of each VRPB paper. From Figure 2.3, it can be seen that the most cited paper belongs to Nagy and Salhi (2005), which is one of the first

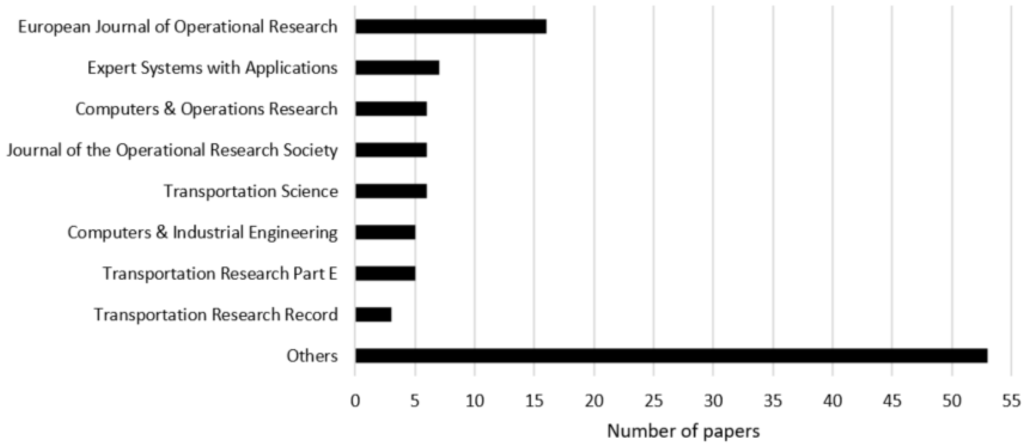


Figure 2.2 – Distribution of published papers per journal.

works extending the VRPMB and VRPSPD to the multi-depot scenario. The following most cited papers, namely Dethloff (2001), Min (1989) and Montané and Galvão (2006) propose well-know instances benchmarks for the VRPSPD. Concerning the classic VRPB, the most cited paper belongs to Ropke and Pisinger (2006), which develops one of the most efficient and flexible solution methods to solve a VRPB, an Adaptive Large Neighborhood Search (ALNS). The second most cited work is that from Ubeda et al. (2011), which is the first to investigate the potential of the VRPB to reduce CO_2 emissions. The third and final most cited VRPB paper is the one from Goetschalckx and Jacobs-Blecha (1989), which defines one of the most used benchmark instances for the standard problem.

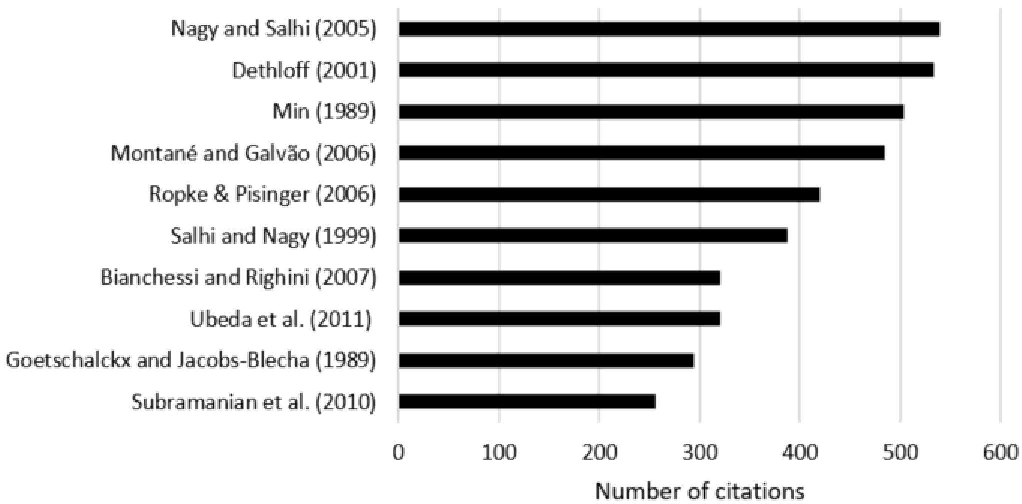


Figure 2.3 – Top 10 most cited VRPB papers.

2.4.2 Classification of VRPB works

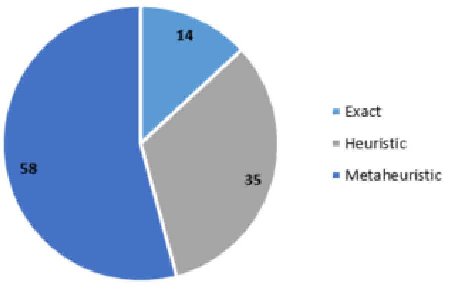
Figure 2.4 presents the number of VRPB works classified according to the categories depicted in Table 2.1. A detailed classification of each work is reported in Appendix 2.A. The first category classifies the solution method in exact algorithm, heuristic or metaheuristic (2.4a). The second category classifies the backhauling strategy according to the variant investigated (2.4b). Note that this category combines the categories 2.8 Backhauls and 2.9 Arc/node covering constraints reported in Table 2.1. The third category classifies the scenario characteristics accordingly if the problem includes load split, service/waiting time, time-windows and multi-periodic issues (2.4c). The fourth category classifies the physical characteristics accordingly if it includes multi-depot network and heterogeneous fleet (2.4d). The fifth category classifies the type of objective function of the VRPB according to the dimension it covers: economic, environmental and social (2.4e). This classification differs from the one in Braekers et al. (2016) because they do not provide such separation of the sustainable dimensions. The sixth and final category classifies the quality of information in a single aspect - if uncertainty is addressed in the VRPB (2.4f).

The classification of the VRPB works shows clear evidences of the following: i) exact methods are the least investigated solution methods for the VRPB, whereas metaheuristics are the most popular ones; ii) the majority of the problems is formulated as the classic VRPB with precedence constraint; iii) time windows, with or without consideration of service times, is the most used scenario characteristic (after the classic VRPB); iv) heterogeneous fleet and multi-depot VRPBs have been smoothly increasing in the last years; v) the vast majority of the VRPB literature only considers economic objectives; and vi) no work has yet addressed a VRPB under uncertainty. Complementary information of the literature review can be found in Appendices 2.B and 2.C, which report the main benchmark instances used in the VRPB literature and case studies and applications of the VRPB and its variants, respectively.

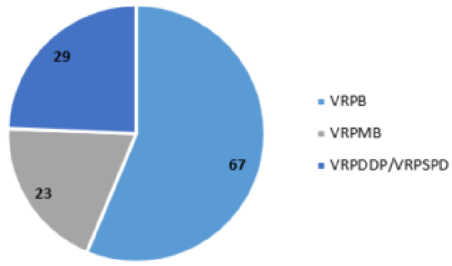
Since the previous literature reviews already present a description of 84 out of the 107 VRPB works selected for this study, only the remaining 23 works are presented below. From these, only those with pure economic objectives are presented here, since a thorough analysis of sustainable VRPB works is carried out in Section 2.5.

2.4.2.1 Exact methods

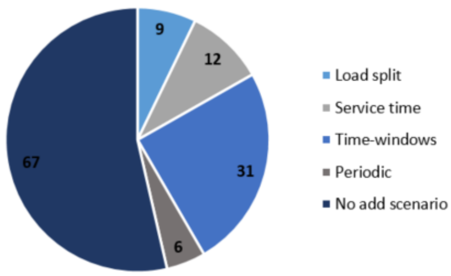
A branch-and-price algorithm is applied in Gutiérrez-Jarpa et al. (2010) to solve a VRPSB with time windows where the goal is to minimize the routing costs minus the revenue collected with pickups. The solution method developed in this work can be seen as an extension of the exact algorithm of Dell'Amico et al. (2006), although a first branch-and-price algorithm is described in Angelelli and Mansini (2002) to solve a VRPSPD with time-windows. The problem is formulated as an integer linear programming with minimization of total net costs (routing costs minus revenues collected with pickups). The instances used were based on the work of Solomon (1987), but only instances with 50 customers or less were optimally solved.



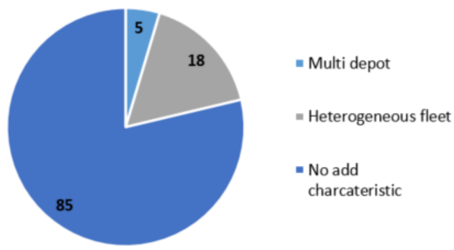
(a) Solution method



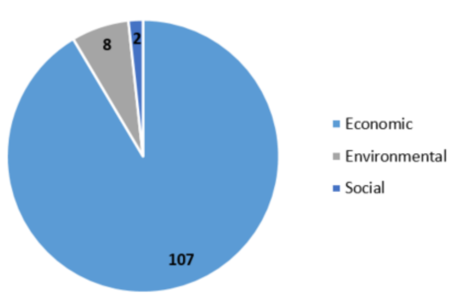
(b) Type of variant



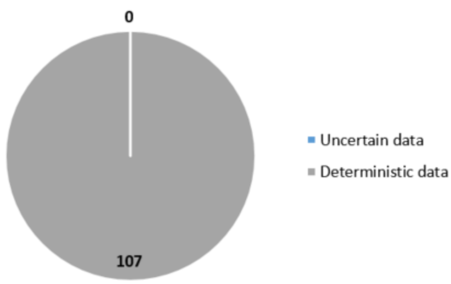
(c) Scenario characteristics



(d) Physical characteristics



(e) Objective function



(f) Quality of information

Figure 2.4 – Number of VRPB papers classified according to each category.

Davis et al. (2014) developed a two phase exact solution method to solve a weekly transportation schedule that involved collecting food donations from local sources and to deliver them to a food bank and then to deliver the food from the bank to charitable agencies. In the first phase, the problem is formulated as a capacitated set covering problem (CSCP), solving the assignment of food banks to local sources. In the second phase, the VRPB is formulated as an extension of the Miller-Tucker-Zemlin formulation, solving the routing and scheduling problem for each workday.
Recently, Granada-Echeverri et al. (2019) proposes a mixed integer programming formulation for a VRPB, which is build based on the characteristics of an Open VRP. The linehaul and backhaul routes are two subproblems of the VRPB solved as independent Open VRPs and tie-arcs connecting these routes are used to generate a solution for the VRPB. This compact formulation is demonstrated to be competitive with the two state-of-art exact solution methods proposed in Toth and Vigo (1997) and in Mingozzi et al. (1999), providing 12 new best known solutions for the VRPB instances of Goetschalckx and Jacobs-Blecha (1989) and 8 new best known solutions for the instances of Toth and Vigo (1997).

2.4.2.2 Heuristics

A cluster-first, route-second heuristic is proposed in Kumar et al. (2011) to solve a VRPB with tree networks. A tree network refers to a network covering a main highway used by the vehicles and customers locations outside the highway. The heuristic first creates different clusters of nodes (locations) and then determines a near-optimal route in each cluster. Although a cluster-first, route-second was firstly proposed by Toth and Vigo (1996) to solve a VRPB, the problem in this work takes the advantages of the tree structure, as introduced by Labbé et al. (1991).
Another two-phase heuristic is applied to solve the VRPB with heterogeneous fleet and split deliveries, while minimizing routing and handling costs in Lai et al. (2013). The first phase determines a feasible solution, using the savings algorithm proposed in Clarke and Wright (1964), and the second phase improves the solution through local search heuristics, using two different neighbourhoods for the search space. The algorithm was applied to five real instances with nearly 40 customers and two trucks with different capacities. The results were compared with the current policy of the carrier, revealing significant reductions in the total distance travelled, ranging from 2.89% to 12.61%.
A heuristic solution procedure based on Lagrangian relaxation was presented by Zhu et al. (2010) for a capacitated plant location problem with customer and supplier matching (CLCSM), where the VRPB is incorporated to reduce empty distances. The problem is formulated as an integer linear programming aiming to minimize the total opening costs of plants and the total cost of distance and number of vehicles. The heuristic first formulates a lower bound based on the Lagrangian relaxation of the problem and decomposes it into sub-problems that are as many as the number of plants. Based on the optimal relaxed solution, a feasible solution is then created for the CLCSM. The algorithm is tested for randomly generated instances and the results suggest it is suitable for producing efficient and robust solutions, even for large instances with 400 customers, 60 suppliers and 40 plants.
Ghaziri and Osman (2003) presents a heuristic based on Kononen's self-organizing feature

map (SOFM) to solve a single-vehicle VRPB The heuristic is based on a neural network architecture that consists of two separate chains of neurons (one for linehauls and one for backhauls). The interaction between the depot and these chains and the interaction between neurons in the same chain lead to create a route, which is further improved with a 2-opt procedure. The authors compare the performance of the heuristic with the branch-and-bound algorithm developed by Fischetti and Toth (1992). The results provide that the exact method is competitive for small instances (up to 50 customers), while the SOFM is better suited for small to large-sized instances (e.g., with 1000 customers).

2.4.2.3 Metaheuristics

Metaheuristics can be divided into local search and population search metaheuristics (Hertz and Widmer, 2003). A general difference between these two types of classes is that the former finds solutions by modifying and improve a single solution, while the latter performs a similar task using multiple solutions.

Local search metaheuristics Among local search metaheuristics, Tabu Search (TS) algorithms, or methods combining TS with other heuristics, are among the most popular solution methods for the VRPB, since its first introduction by Duhamel et al. (1997). For instance, Nguyen et al. (2016) proposed a TS algorithm that integrates multiple neighbourhoods, in order to solve the VRPB, with and without time windows, while considering the minimization of both the total travel distance and the number of vehicles. The algorithm was tested on VRPB instances from Goetschalckx and Jacobs-Blecha (1989) and Toth and Vigo (1997), and on VRPBTW instances from Gélinas et al. (1995). The algorithm presents good performance and can be competitive with the state-of-art metaheuristics, such as the ALNS of Ropke and Pisinger (2006), the unified hybrid genetic algorithm of Vidal et al. (2014) or the TS of Brandão (2006), among others. Lai et al. (2015), on the other hand, proposes an adaptive guidance (AG) metaheuristic that combines TS with the savings algorithm of Clarke and Wright (1964) to solve a VRPB with split deliveries. First, the VRPB is decomposed into different VRPs, one for linehaul customers and one for backhaul customers. Each problem is solved with TS and then the savings algorithm is used to merge the sub-problems and create a global solution for the VRPB. The iterative process is build upon an adaptive guidance mechanism that defines simple rules to assess the quality of a current solution and detects potential improvements. The method applied in this problem enables a reduction of about 25% in the total travel distance, compared with the current solution of the carrier. A hybrid metaheuristic that combines TS and simulated annealing (SA) is proposed by Küçükoğlu and Öztürk (2015) to solve a VRPB with heterogeneous fleet. First, a nearest-neighbour heuristic is used to create an initial solution. Then, the TS is used to generate neighborhoods and finally the SA is used to select a new solution from the neighborhood, based on an acceptance criterion. The hybrid metaheuristic is tested with Gélinas et al. (1995) instances with up to 100 nodes, for which 11 new best known solutions are created. Recently, Reil et al. (2018) makes use of TS to study all four main variants of the VRPB with time windows and three dimensional loading constraints. The solution method follows a two-phase approach that solves first the packing problem and then the routing

problem. The packing problem is solved with a TS and the routing problem is solved sequentially by a multi-start evolutionary algorithm that minimizes the number of vehicles and then by a TS to minimize the total travel distance. Several benchmark instances, either from literature and generated at random, are used to evaluate the performance of the metaheuristic. Results from this work indicate that the VRPMB can slightly increase the number of vehicles required compared with the VRPB, when both time windows and three dimensional loading constraints are considered.
A reconstruction algorithm is designed in Nikolakopoulos (2015) to solve a bi-level VRPBTW, based on a theoretic game approach of a leader aiming to minimize the number of vehicles and a follower aiming to minimize the total duration of the routes. The bilevel problem is reduced to a single level by aggregating the two objective functions, such that the one from the leader is prioritized over the one of the follower. An initial solution is created by a construction heuristic and then six different heuristics are used to partially destroy the current solutions. The improvement phase consists of a variation of the Threshold Accepting method proposed by Dueck and Scheuer (1990) and the search intensification uses local search heuristics. The reconstruction algorithm is shown to be competitive with state-of-art metaheuristics for the minimization of number of vehicles, but does not compete for the minimization of total duration of routes.
To solve the VRPBTW and the VRPMBTW, both with minimization of the number of vehicles and the distance travelled, Tarantilis et al. (2013) propose the Adaptive Path Relinking metaheuristic. A set of feasible solutions is generated and subsets of intermediate solutions are subsequently created based on a path generation method. This method uses a procedure to guide the intermediate solutions according to the recurrence of some of their attributes. Optimal local solutions are selected from the intermediary solutions and improved by local search algorithm. Tests on several VRPBTW and VRPMBTW instances are performed and the metaheuristic was able to provide several new best solutions compared with state-of-art methods. Further tests suggest that allowing mixed pickups and deliveries leads to less costly routing plans than those obtaining by forcing pickups after deliveries.
Zachariadis et al. (2015) presented a local search algorithm to optimize both, VRPSPD and VRPDDP, with two-dimensional loading constraints and different configurations of loading, namely with fixed orientation, with allowed rotation and with or without LIFO (last-in-first-out) constraints. The algorithm initiates with a feasible solution and a local search procedure is used further to solve the routing problem. In turn, the routing algorithm calls another algorithm to check loading feasibility. Tests on generated instances with up to 150 customers are performed with all models and the results suggest that allowing simultaneous pickup and delivery allow for significant reduction in the total costs, and this benefit increases with the increasing of the contribution of backhaul customers to the problem size.

Population-based metaheuristics Among population-based metaheuristics, evolutionary algorithms are the most popular ones to solve the VRPB. Ganesh and Narendran (2007) uses a genetic algorithm (GA) to intensify the search for best solutions, which are obtained using a three-phase constructive process. The construction of an initial solution starts by clustering the nodes according to a proximity criteria and then using a shrink-wrap algorithm (Sofge et al., 2002) to orient the nodes along a path, and thus by finding a route within each cluster. Finally, each route is allocated to a vehicle by solving a generalized assignment problem using a heuristic similar to Fisher and Jaikumar (1981). The GA uses a two-point crossover operator and an insertion operator for mutation in the intensified search. The metaheuristic is tested for VRPB instances of Goetschalckx and Jacobs-Blecha (1989) and Toth and Vigo (1996), as well as for VRP benchmark instances in literature. The solutions are obtained with reasonable time and with average gaps of, respectively, 0.06% and 0.42% from the best known solutions for the VRPB instances.

The performance of a memetic algorithm (MA) under different combinations of algorithm components and parameter values is investigated in Saremi et al. (2007) to study a VRPB with heterogeneous fleet. The MA combines EA to find global solutions and local search algorithms to find local optimal solutions. All components of the MA are evaluated, namely crossover operator and crossover rate, mutation operator, local search improvement method and selection method. The MA is tested for VRPB instances from Toth and Vigo (1999), distributed by three sets of increasing number of nodes and vehicles. For the first and second sets, the component that affects most the solution quality is the mutation operator, followed by the local search method, while the crossover operator is the most important for the third set of experiments. However, the local search method is the most significant component affecting the computational time. A differential evolution algorithm (DEA) is used by Küçükoğlu and Öztürk (2014) to solve a VRPBTW with minimization of total distance. The main distinction between the DEA and the GA is the type of operator used to build better solutions, the former uses mutation and the latter uses crossover. The population size, crossover ratio and mutation constant are evaluated for different parameter settings and the effect of each parameter, as well as their interactions, are also examined. The DEA is applied to five VRPBTW data sets from Gélinas et al. (1995), which found 23 new best solutions out of 45. Applied to a real-problem of a catering firm, the DEA allowed to reduce the total distance in 22.32%.

In Paraphantakul et al. (2012), ant colony optimization (ACO) is used to study a real-world problem with time restrictions, where the objective is to minimize the total travel time. The particularity of this problem compared with the classic VRPB is that routes composed with only backhauls are allowed. The metaheuristic is adapted from Reimann et al. (2002) and the results demonstrate that improvements in the range of 9.1% to 12.3% on the average route duration can be obtained with this metaheuristic, compared with the current method of the company. Recently, Lu and Yang (2019) applied a ACO metaheuristic to solve a multi-depot VRPSPD that minimizes total costs, covering drivers wages and fuel costs. The authors investigate the performance of different heuristics for both, creating an initial solution and improving current solutions, and compare the strategies of mixed and sequential deliveries and pickups. The authors report an improvement rate of 15% in the total costs for the case of mixed deliveries and pickups.

2.4.3 Final remarks on the VRPB

The classification of the 107 VRPB papers allows to conclude that there is great agreement with the findings of Koç and Laporte (2018), namely in what respect the solution methods used and the main goals pursued in the problems. Notwithstanding, three important aspects must be highlighted from our review.

The first aspect concerns the backhauling strategy adopted in the problem. Although the majority of the research is devoted to the classic VRPB, some works present a comparison between the classic and the mixed bakhauling strategies. When these two strategies are investigated, the focus of the study is either on the development of solutions methods capable of solving efficiently different VRPB variants (e.g., Ropke and Pisinger (2006); Tarantilis et al. (2013); Reil et al. (2018)) and/or on the evaluation and comparison of the impact of different backhauling strategies (e.g., Tarantilis et al. (2013); Nagy et al. (2013); Turkensteen and Hasle (2017); Reil et al. (2018); Lu and Yang (2019)). However, which backhauling strategy is better to cope with the goals of the decision-maker cannot be generalized to all problems. For instance, the mixing strategy is shown to be a better strategy in the work of Tarantilis et al. (2013), but it is not an efficient option in the work of Reil et al. (2018).

The second aspect concerns the nomenclature of a VRPB with several scenario and physical characteristics. A widespread trend for any variant of the VRP is the inclusion of more practical aspects in the problem, aiming to bring the problem closer to reality of companies. This has motivated the emergence of a new class of problems denominated Rich VRP. Caceres-Cruz et al. (2014) proposed a classification for a Rich VRP in three levels, according to the degree of realism associated to the model. The first level is attributed to classic VRP variants that borrow aspects of other variants (e.g., a VRPB with TW and HF). The second level is attributed to classical-advanced VRPs, such integrated problems (e.g., inventory-routing problems (IRP)) or multi-objective models (e.g., minimizing routing costs, GHG emissions and accident rate). The third level is attributed to VRPs that are able to provide more accurate data through the incorporation of uncertainty and/or dynamism, for instance. For third level Rich VRP, hybrid methods such as matheuristics are usually applied. Hereupon, it can be recognized that the majority of VRPB papers falls into the first and second categories of the Rich VRP classification.

Finally, the third aspect, which drives the main purpose of the present paper, concerns the goals of VRPBs. From Table 2.2, it becomes evident that some effort has been carried in order to incorporate objectives other than economic in VRPBs, particularly in the last years. As mentioned throughout this paper, the main goal of the present review is to analyze sustainable VRPBs. Thus, the first group of VRPB papers selected for the analysis is those that include, apart from the economic concerns, environmental and/or social aspects in the objective function, such as in Ubeda et al. (2011), Eguia et al. (2013), Chávez et al. (2016) and Turkensteen and Hasle (2017). The next group of papers addressing VRPB with sustainability concerns are those that also address collaboration, for which three works are identified: Bailey et al. (2011), Pradenas et al. (2013) and Juan et al. (2014). The last group includes the VRPB works that are studied in the context of reverse logistics, for which six papers are identified: Privé et al. (2006), Gribkovskaia et al. (2008), Rahimi et al. (2016)

Hoff et al. (2009), Nagy et al. (2013) and Soleimani et al. (2018). Overall, 13 sustainable VRPB works are analyzed in the next section, as summarized in Figure 2.5.

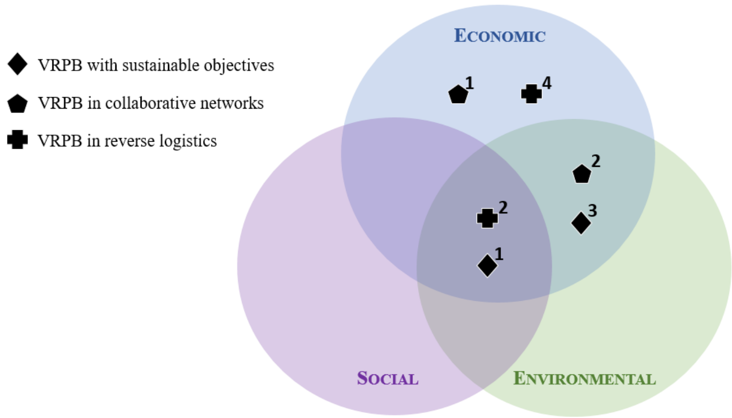


Figure 2.5 – Number of VRPB works covering different dimensions of sustainability.

2.5. Analysis of VRPBs with sustainability concerns

The VRPB works that focus on sustainable aspects, namely by considering environmental and/or social objectives, collaboration or reverse logistics, are described and discussed in this section. We designate the problems tackled in such works as VRPBs with sustainability concerns.

2.5.1 VRPBs with sustainable objectives

For the majority of VRPBs described in literature, the formulation of the problem still focus on minimizing costs (usually the distance). Accordingly, as GHG emissions and fuel consumption are proportional to the distance traveled, minimizing the distance indirectly reduce both emissions and fuel. Nevertheless, most of the studies and practical applications of the VRPB focus on the costs savings by performing inbound and outbound trips together and neglect the quantitative assessment of the impact of transportation on the environment and society.

The first study to address the environmental impact of a VRPB is described in Ubeda et al. (2011). The authors also investigate the impact of different transport strategies, namely re-scheduling deliveries, backhauling and green optimization, on the economic and environmental costs and on the operation profitability. The first strategy relies on formulating a VRP minimizing the total distance, the second strategy involves formulating a VRPB minimizing also total distance and the third strategy involves formulating the VRP with minimization of total CO_2 emissions. In all three strategies, the emissions are estimated based on the distance and distance-based emission factors. The Mole and Jameson's method is used to solve the VRPs and the Nearest Neighbor Insertion algorithm is used to solve the VRPB. From the three strategies studied, backhauling provides the highest reduction of the total distance traveled: more than 15%. The results of this case study establish that i) backhauling increases the truck's fill-rate and, consequently, the vehicles efficiency, and ii) increasing the backhauling rate leads to an increase in both economic and ecological benefits.

Chávez et al. (2016) formulate a multi-depot VRPB with minimization of the travel distance, the travel time and the total energy consumption. The GHG emissions and energy consumption are computed based on the Pollution-Routing Problem (PRP) introduced by Bektaş and Laporte (2011). In the PRP, the energy consumption is as a function of distance, load and vehicle speed. Vehicle speed, in turns, is determined according to a series of road and vehicle factors, such as road angle and friction coefficient, among others. The authors propose a Pareto ant colony optimization (PACO) algorithm and test the algorithm on 33 adapted instances of Salhi and Nagy (1999) in order to find ordered Pareto solutions. The resulting Pareto fronts suggest that energy and time have low correlation between them, whereas distance travelled affects both, time and energy consumption.

A method to assess the carbon emission effects when using a classic VRP, a VRPB and a VRPMB is presented in Turkensteen and Hasle (2017). The carbon emissions are a function of distance driven and average load factor. The results point out that, comparing with a classic VRP, the total distance is reduced in about 10-20% for the VRPB and in about 20-40% for the VRPMB. The average load in the VRPB is very similar to that of the VRP but it fluctuates considerably in the VRPMB. Moreover, savings in carbon emissions up to 25% and up to 42% can also be reached for the VRPB and the VRPMB, respectively. Although the VRPMB can induce higher benefits (lower costs and emissions), the time to rearrange the load in the vehicle during mixed visits are not considered in this study. Nevertheless, this extra time may lead to a drastic increase in the total time required to complete a route.

Pradenas et al. (2013) and Juan et al. (2014) studied a VRPB with environmental objectives in the context of collaborative networks. These works are analyzed in subsection 2.5.2.

The first work to address social objectives in a VRPB is presented in Eguia et al. (2013). In fact, the problem considers all dimensions of sustainability but translates all parameters in costs. Thus, the problem aims to minimize internal and external costs. Internal costs refer to the total costs with fuel, drivers, vehicles and tolls. External costs comprise both environmental (climate change and air pollution) and social impacts (noise and accident rate). The problem considers different type of fuels and is solved with a Clarke and Wright savings heuristic extended by the authors to include the ability to perform with a heterogeneous fleet. With a case study, the authors show that including backhauling is always a better strategy than perform individual inbound and outbound routes. They also conclude that considering social and environmental impacts results in choosing the less pollutant vehicles, if an heterogeneous fleet is considered in the routing problem. Up to the moment, only Rahimi et al. (2016) has also covered the social dimension of a VRPB, in the context of reverse logistics. This work is analyzed in subsection 2.5.3.

2.5.2 VRPBs in collaborative networks

Collaborative vehicle routing can be defined as all forms of cooperation between the participants in the network that aim to improve the efficiency of transport operations (Gansterer and Hartl, 2018). Typical participants in a transportation network are the shippers (owners of the load to be carried) and the carriers (owners of the vehicles). If the shipper owns self-support vehicles, shipper and carrier are the same entity. Otherwise, the shipper outsources a carrier to perform the transport operations.

Bailey et al. (2011) investigates the impact of a collaborative VRPB, when a carrier seeks for collaborative shipments to reduce the empty distances and maximize savings. The collaboration can occur with a shipper which offers a pickup-delivery task close to the backhaul routes of interest of the carrier. The collaboration can also occur with another carrier that does not have sufficient capacity to fulfill all its tasks. The authors developed two models and two solution methods to solve a real-world freight network problem. The first model allows at most one collaborative shipping per truck and is solved with a greedy heuristic. The second model allows multiples collaborative shipments in each route and is solved using a tabu search algorithm. The collaboration provides cost savings between 13% and 28% in backhaul routes compared with a non-collaborative network.

Besides reducing costs, the collaborative VRPB can also bring benefits from an environmental perspective, as demonstrated by Juan et al. (2014) for collaborative shippers and by Pradenas et al. (2013) for collaborative carriers.

In Juan et al. (2014), each shipper has a private fleet of vehicles, a unique depot and a unique set of customers to serve. The authors compare a non-cooperative scenario, where each shipper serves only its customers, and a cooperative scenario where a vehicle of one shipper can perform a pickup and delivery tasks for another shipper, after serving its own customers. The problem is modelled as a multi-depot VRP and solved with an Iterated Local Search algorithm. With a numerical experiment, they conclude that a reduction of about 24% in GHG emissions can be achieved with a collaborative network. Furthermore, they determined that cost savings are highly influenced by the geographical dispersion of the customers in the network but, at least, a 5% cost reduction can be achieved with collaboration.

Pradenas et al. (2013) investigated the effect of three different collaboration strategies on the costs and CO_2 emissions of a transportation system. The first strategy - total competition, considers that each company performs in own deliveries (i.e. there is no collaboration). The second strategy - total cooperation, assume that information and all resources are shared among companies. The third strategy - mixed system, assume that information may be exchanged but not the resources. They also propose a method to determine the profits sharing between participants in the collaborative network, based on the popular Shapley value of game theory (Shapley, 1953). The authors conclude that a decrease in 2% in GHG emissions results in a cost increase between 2% and 8%. Nevertheless, a cooperative network is able to bring economic profits for all companies, approximately 30% cost reduction in the total transportation costs.

2.5.3 VRPBs in reverse logistics

Reverse logistics concerns the backward flows of products (e.g., returned products, end-of-life products, waste) from its last use location to a final destination (e.g., manufacturer, recycling company, waste treatment location).

When different vehicles must be used to perform independently the delivery and the collection routes, the VRPRL cannot be seen as a VRPB or any of its variants. This occur when the material to collect is unfitted to the delivery vehicle, such as in the case of waste management (Ramos et al., 2014). On the other hand, when the same vehicle is used for the delivery and the collection tasks, the VRPRL is analogous to a VRPSPD, if the delivery and pickup occurs simultaneously (e.g., Privé et al. (2006)), or to a VRPDDP, if the tasks occur in separate routes (e.g., Nagy et al. (2013)). Although most of the literature reviewed in this paper concerning these VRPB variants can be linked to reverse logistics, this section is devoted to those papers that specifically draw managerial insights for the application of reverse logistics in practice. Consequently, papers focused on the development of solutions methods are not described.

Privé et al. (2006) study a reverse logistics problem arising in the drink industry, where the vehicles perform the deliveries of soft-drinks from a distribution center to retailers and the collections of containers with empty cans and bottles to be returned to the distribution center. Each recyclable container has an associated revenue and the delivery and collection operations occur simultaneously. Three construction heuristics are developed to solve the problem and the solutions are compared in terms of total costs, total revenues, percentage use of vehicle capacity and number of vehicles required. Compared with the current routing plan of the studied company, the proposed model and solutions methods are able to reduce the total distance up to 23%. Gribkovskaia et al. (2008) investigate a similar problem in reverse logistics, but considering that a revenue can only be collected if the delivery and pickup occurs simultaneously. Otherwise, if two visits are necessary, no revenue is collected. Note that both of the works presented can also be analogous to the VRPSB, considering that the collection of recyclable material/containers is optional, based on the revenue generated.

Hoff et al. (2009) investigate lasso solutions for the routing problem, which can be defined as intermediate solutions between the VRPSPD and the VRPDDP. More precisely, a lasso solution creates a route where a vehicle firstly delivers to one (or few) customers, then performs a simultaneous delivery and pickup at the remaining customers, and after that, it performs the pickups at the firstly visited customers while returning to the depot. A pioneer work concerning lasso solutions can be found in Gribkovskaia et al. (2001). A load control parameter β is introduced to set the free space of a vehicle before starting to collect empty bottles and tested within the range [0.0, 1.0]. The decision-maker sets this parameter but Casco et al. (1988) suggests a free space of 20%. Thus, with $\beta = 0.0$, the problem becomes analogous to a VRPSPD; $\beta = 1.0$, the problem becomes analogous to a VRPDDP. They applied a tabu search metaheuristic to solve several generated instances and demonstrated the significant impact of the load control parameter on the solution costs. Although they found out that, theoretically, solutions for the VRPSPD are less costly, lasso solutions with a low value of β are preferable from a practical perspective.

The VRPDDP is investigated and compared with the VRPSPD in the study of Nagy et al. (2013). The authors concluded that serving the customers twice instead of simultaneously can reduce costs and the number of vehicles required, and such benefits are more evident when there are considerable differences between delivery and collection quantities. Another important observation is that splitting delivery and pickup seems more advantageous for customers located near the depot or when these have a higher pickup or delivery demand.

Although the reverse logistics is itself a sustainable strategy, because is concerned with reducing waste, recycling and circular economy, the optimization of the routing problem still targets the cost reduction, as the works presented above. Notwithstanding, the literature seems to move towards more broader issues, namely by including environmental (e.g., Soleimani et al. (2018)) or social concerns (e.g., Rahimi et al. (2016)). Soleimani et al. (2018) describe the problem of a newspaper company that uses a fleet of vehicles to deliver newspapers to newsstands, while picking up unsold ones to return back to the distribution center. The newsstands can also order an amount of old newspapers to the distribution center. The problem is modelled as a non-linear multi-objective model, which is further linearized and validated with a fuzzy approach. The objectives of the problem include the minimization of total routing costs and total energy consumed. The CO_2 emissions are a function of the rate of fuel and considered as a constraint of the problem. Compared with a separate planning of routes to deliver newspapers and routes to collect unsold ones, the backhauling strategy allows a reduction of both costs (10.51%) and environmental impacts (12.5%). Rahimi et al. (2016) describe an inventory-routing problem (IRP) for the distribution of perishable products to a set of retailers and the collection of expired products from those retailers. They formulate a bi-objective mathematical model considering both economic and social concerns. The economic objective aims to maximize the expected profits and is determined by the total revenues obtained with the delivery of products minus the total routing and inventory costs. The social objective concerns the minimization of accident rate and quantity of expired products. The accident rate is modelled as a function of vehicle speed and the quantity of expired products is modelled as a function of the expiration date of each product. The control of vehicle noise and limitation on GHG emissions are introduced in the model as constraints. The authors analyze the conflicting nature of the objectives in the problem, concluding with their experiments that increasing the relative importance of the social objective over the economic one (e.g., 27% increase), improves the social issues in the inventory-routing plan in 23% and reduces the total profits by about 17%. This is because increasing the number of vehicles used at a lower speed, reduces the accident rate but increases the total routing costs.

2.5.4 Final remarks on the VRPBs with sustainability concerns

In this section, the literature on the VRPBs with sustainability concerns is firstly cross-checked with the survey of Green VRP (GVRP), carried out by Lin et al. (2014), and then with the survey of collaborative vehicle routing, carried out by Gansterer and Hartl (2018). The survey of Lin et al. (2014) proposes a classification for the GVRP into three different categories: i) the Green-VRP, ii) the Pollution Routing Problem, and iii) the VRP in

Reverse Logistics.
The first category covers the routing problems concerned with the energy consumption of vehicles, and the primary goal is the minimization of fuel consumption, translated in a cost function. The main factors that influence fuel consumption are the load, the distance traveled and the speed of the vehicle. This category also includes the alternative-fuel powered vehicles (AFV) and, as such, the risk of running out of fuel and the possibility of recharging AFV *en route* are additional factors that must also be considered in this type of problems. From the analysis of the VRPBs with sustainable concerns, the work of Eguia et al. (2013) can be placed in the category of Green-VRP since it considers the energy consumption of a heterogeneous fleet of vehicles.
The second category in the survey of Lin et al. (2014) covers the routing problem concerned with the minimization of pollution emissions, in particular, CO_2 emissions. Although a decrease in fuel consumption has an intimate relationship with a decrease in CO_2 emissions, additional factors must be considered in order to measure accurately the emissions, such as those related to the engine of the vehicle and traffic conditions. The objectives of such problems can also include the minimization of noise and congestion, among others. From the analysis of the VRPBs with sustainable concerns, most of the works described in Sections 2.5.1 and 2.5.2 can be placed in the category of Pollution-Routing Problems.
Finally, the last category in the survey of Lin et al. (2014) covers the routing problems applied in the context of reverse logistics operations. These can be further divided in i) Selective Pickups with Pricing, when only the profitable collection sites are visited, ii) Waste Collection, when the problem applies to the refuse collection services and waste recycling, iii) End-of-life Goods Collection, for the pickups of components that may be re-manufactured, and iv) Simultaneous Distribution and Collection, if the problem is analogous to a VRPSDP. From the analysis of the VRPBs with sustainable concerns, all the works described in Section 2.5.3 can be placed in the category of VRP in Reverse Logistics.
The survey of Gansterer and Hartl (2018) proposes a classification of collaborative VRPs into i) centralized collaborative planning, ii) decentralized planning without auctions, and iii) auction-based decentralized planning.
The centralized planning considers the joint optimization of the routing problem of all participants. It assumes that all participants share all their information and, as such, the problem reflects full collaboration between participants. The works of Juan et al. (2014) and Pradenas et al. (2013) can be placed in this category.
The decentralized planning considers that only part of their information is shared among participants. This information asymmetry may also reflect a hierarchy between participants in the collaborative network. The work of Bailey et al. (2011) can be placed in this category, since the studied carrier collaborates with other participants for its backhaul routes, but its predefined delivery routes are not allowed to change. In other words, the carrier is only willing to display information about its potential backhaul routes but does not share information nor requests of the delivery routes to other participants (its delivery customers are not considered in the collaborative network).
The auction-based decentralized planning assumes the existence of a bidding system, where the participants submit their requests to a common pool, which are grouped into bundles,

and then offered to participants. The participants place their bids for the bundles, which are then allocated to the best bid. From the review carried in the present paper, no VRPB work falls in this category.

2.6. Future directions for sustainable VRPB

When planning routes according to a VRPB, an integrated inbound-outbound logistics problem is considered. In the same route, the vehicle successively unloads all the requested orders at the customers (outbound logistics) and, in the return trip to the depot, the vehicle visits suppliers and load raw-materials until it reaches full capacity. As a result, it is possible to achieve significant savings by reducing the number of empty trips of vehicles, the number of required vehicles and the total travel distance. This integrated inbound-outbound perspective also highlights the sustainability side of the VRPB. Possible reductions in the number of vehicles and in the distance they travel lead to consume less fuel, generate less GHG and air pollutants and reduce noise pollution. This confirms the strong relation between the VRPB and sustainability.
When evaluating the environmental impact in the VRPB, only the CO_2 emissions have been usually considered. This is due to the correlation of the CO_2 emissions and fuel consumption, as modelled in the PRP (Bektaş and Laporte, 2011). However, the emissions of other pollutants, such as nitrogen oxides (NO_x), may depend on other parameters, such as combustion dynamics or type of emission control, among others (Demir et al., 2014). For instance, Naderipour and Alinaghian (2016) used the MEET model to estimate the emissions of CO, CO_2 and NO_x for an Open VRP. The MEET model was developed under an European project with the objective of providing a common procedure to evaluate the impacts of transportation in the environment, covering methods, emissions factors and functions to estimate the emissions of air pollutants. Therefore, in order to develop more sustainable models for the VRPB, a possible direction may be to include specialized functions in the mathematical model that estimate the individual emissions of different pollutants. In this way, it is possible to evaluate and compare the effects of transportation on local emissions (air pollutants, e.g., NO_x, CO) and on global emissions (GHG, e.g., CO_2).
The social concerns addressed in the VRPB literature are accident rate and noise, which have direct impact on the well-being of the drivers. An interesting social aspect that could be investigated for the VRPB is the equity in the working hours among drivers. Considering that the wage is proportional to the working hours, routes with different duration, may be seen unfair and, consequently, demotivate drivers. This in turns, may increase the accident rate. For instance, Ramos et al. (2014) analyze the impact of balancing working hours among drivers for a multi-objective multi-depot periodic VRP. Their results demonstrate that minimizing the maximum working hours among drivers leads to increase both, the costs and the CO_2 emissions. Nevertheless, the authors propose a compromise solution between economic, social and environmental objectives, and proof in a case study that it has potential to save distance and fuel consumption and, simultaneously, promote equity in routes duration.
The use of AFV in the VRP has been introduced in literature by Erdoğan and Miller-

Hooks (2012), which investigates the possibility of refueling a set of vehicles while *en route* in order to avoid running out of fuel. From the literature review carried out in the present paper, and also as highlighted by the review of Koç and Laporte (2018), no study has yet investigated the use of AFV for a VRPB or its variants. The closest work may be the one in Eguia et al. (2013), which considers different types of vehicles fueled by diesel. Therefore, an important direction for the future work of sustainable VRPBs could include the formulation of the VRPB with AFV. Additional research could cover the impact of using different types of fuel (Ashtineh and Pishvaee, 2019) or the limited capacity of alternative fueling stations (Bruglieri et al., 2019), as also suggested by Lin et al. (2014).
Given the importance of backhauling in collaborative networks, another future direction for the VRPB may be to investigate in more detail the aspects of collaborative VRPB. An example could be related to modelling a collaborative VRPB for which the goal pass beyond cost reduction, such as fairness in profit sharing between collaborators or balanced distribution of drivers' working hours. Another relevant study could be to investigate the collaboration between a shipper and a carrier and at what extent is the VRPB profitable for the entire network when compared with two different VRPs - one for the delivery routes and one for the pickup routes. Furthermore, the different strategies of clustered, mixed or optional backhauls could also be evaluated and compared for the collaborative VRPB. A comparison between centralized and decentralized approaches for the collaborative VRPB is also worthy of investigation. In this case, an analysis of the value of information sharing could determine different collaborative strategies to be applied in real applications of VRPB and its variants. Finally, departing from the work of Pradenas et al. (2013), which determines the coalition savings in terms of costs and GHG emissions, the research on collaborative VRPBs could be extended to determine coalition savings based on additional environmental and social indicators.
The literature review on this paper shows that reverse logistics problems can be modelled as a VRPSPD or a VRPDDP if the same vehicle is used to deliver products and pickup returned items. Nevertheless, it would be very interesting to study a more broader VRPB combined with reverse logistics. Particularly, the problem would consider a set of linehaul customers and two sets of backhaul customers - one set of delivery customers that have items to be returned (reverse logistics) and one set of suppliers of the depot (VRPB). In this way, all possible routes can be effectively planned together and the company can leverage from better efficiency of inbound and outbound transportation needs, particularly if the vehicles are self-supported. As mentioned in the previous section, several routing problems tackling reverse logistics cannot be modeled as a VRPB or one of its variants, such as in waste management. That occurs because the waste to be collected is not compatible with the delivery items, or because the vehicles used for the collections have requirements not supported by typical delivery vehicles. One possible way to overcome this drawback could be to investigate multi-compartment vehicles in order to avoid cross-contamination of different products. Addressing multi-compartment vehicles and two- and three-dimensional loading constraints for the waste collection problem is also suggested as a future research direction in the review of Lin et al. (2014).
A major evidence from this review is that uncertainty was never addressed in a VRPB. Chardy and Klopfenstein (2012) studied the impact of uncertainty inclusion in a VRP and

concluded that, while using the average scenario, which is the most used scenario in decision making, more than 15% of the tasks may not be concluded as previously estimated, while when considering uncertainty this value falls to a maximum of 2.5%. Therefore, another direction to promote more sustainable VRPBs could be to study the impact of uncertainty inclusion and the development of adequate models. In fact, we highlight that considerable attention should be given to this topic, since there are two different types of customers in the VRPB, each one with distinct concerns and uncertainties. For example, while the demand and time windows are major concerns for the success of delivery routes, the success of backhaul routes may depend on the product availability or product quality at suppliers, which may result in unmet demand for the depot or poor estimates of revenues collected with pickup loads. The lack of stochastic routing problems in the literature is highlighted in all reviews (Koç and Laporte, 2018; Lin et al., 2014; Gansterer and Hartl, 2018) and pointed as a future research direction to address more realistic cases of applications.

Stochastic programming and robust optimization are two popular approaches to deal with uncertainty in VRPs (Averbakh, 2001). The main difference between them is that the former incorporates the uncertain parameter in the VRP formulation described by its probability distribution, while the latter represents the uncertain parameters as a bounded uncertainty set, taking as input the worst realization of the uncertainty. For instance, the VRP with uncertain travel times is modelled through stochastic programming in Li et al. (2010), through robust optimization in Lee et al. (2012) and through a combined robust-scenario approach in Han et al. (2014). In addition, comparing the performance of different modelling approaches could be another line of research for the VRPB, since stochastic programming may produce less costly solutions (Adulyasak and Jaillet, 2016) but robust optimization may to provide less computational effort (Chen et al., 2016).

An additional suggestion for the development of more reliable and accurate models for the VRPB derives from the survey of Ritzinger et al. (2016), which focuses on dynamic and stochastic VRPs. In the last years, operations research has witnessed an increased interest in dynamic routing problems due to, on the one hand, the increasing importance of e-commerce and online transactions, and on the other hand, due to the exponential technological development of telematics systems that allows obtaining real-time data. Besides, these telematics systems enable the collection of a large amount of data that could be further processed to get useful statistics about stochastic information. Therefore, in order to keep the VRPB research updated, the use of dynamic approaches and systems to collect real-time information should be incorporated in the face of the traditional VRPB approaches. This would lead not only to enrich VRPB models but also to increase the potential of solutions methods to solve the related problems.

The future directions for the VRPB research and application are summarized in Figure 2.6.

2.7. Conclusions

The present analysis of literature demonstrated that backhauling is becoming an ever more interesting option for solving real-world transportation problems and that it has the poten-

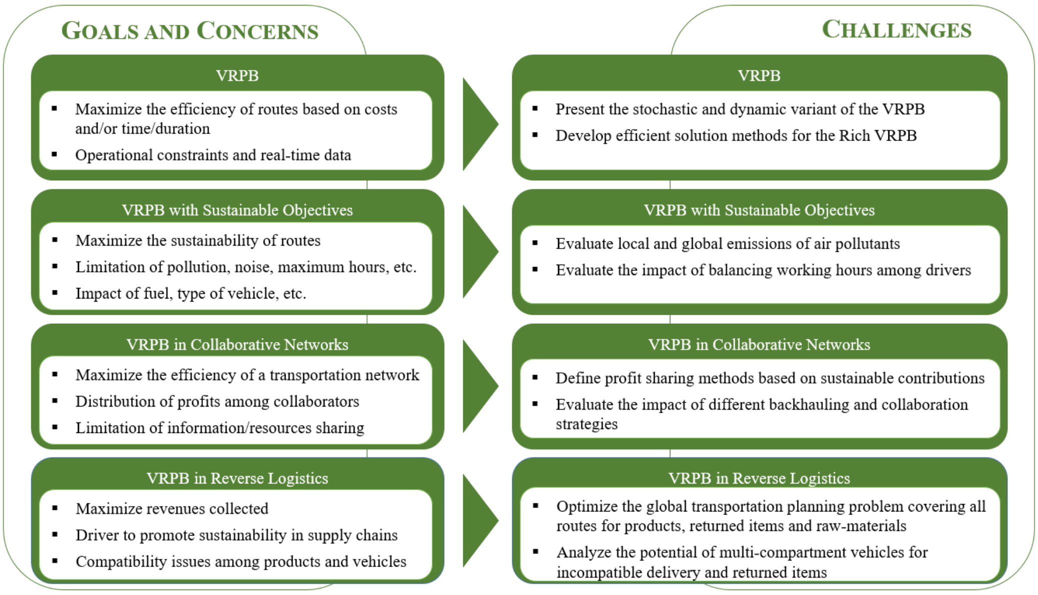


Figure 2.6 – Framework of future directions for the VRPB research.

tial to provide significant reductions in the total routing costs and increase sustainability of transportation. Although the VRPB is commonly modelled as a cost minimization problem, some literature already extents the problem to include environmental and social objectives. Environmental objectives often include the minimization of CO_2 emissions and energy consumption. Social objectives include minimization of accident rate and control of noise pollution. The analysis also reveal that the VRPB can well suit routing problems in the context of reverse logistics and collaborative networks.

As a promoter of green logistics, possible future directions for the VRPB are: i) formulation of sustainable VRPB models considering multi-objective functions with economic, environmental and social issues; ii) investigation of additional and broader aspects in collaborative VRPBs, such as contracts and profits sharing among collaborators; iii) explore the potential of different backhauling strategies when possible, such as trade-offs between precedence and mixed backhauls or also between optional and restrictive backhauls; iv) develop integrated models combining VRPB and reverse logistics, covering deliveries and pickups at customers and pickups at suppliers; v) develop efficient models to incorporate uncertainty and dynamism in VRPB models and measure the robustness of solutions.

This paper complements the reviews of Koç and Laporte (2018) and Parragh et al. (2008a) on the models, solution methods, and applications of the VRPB, but particularly it extends the literature to broader contexts, such as collaboration and reverse logistics. One innovative aspect of this paper is the focus on the sustainable perspective and impacts of the VRPB and, accordingly, the proposal of a future research agenda driven by sustainability concerns.

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Appendix 2.A Classification of VRPB works

Table 2.2 – Classification of VRPB works (1984-2004)

	Solution methods			Backhauls			Scenario characteristics				Physical characteristics		Objective function			Information
Reference	E	H	MH	Pr	Mx	PD	LS	SWT	TW	MP	MD	HF	Ecn	Env	Scl	Un
Deif and Bodin (1984)[1,2,3]		•		•									•			
Yano et al. (1987)[2,3]	•			•				•					•			
Casco et al. (1988))[1,2,3]		•			•								•			
Goetschalckx and Jacobs-Blecha (1989)[1,2,3]		•		•									•			
Min (1989)[1,2]		•				•							•			
Derigs and Metz (1992)[1]		•		•			•		•			•	•			
Gélinas et al. (1995)[1]	•			•					•				•			
Kontoravdis and Bard (1995)[2]			•			•			•				•			
Anily (1996)[1,2]		•		•									•			
Potvin et al. (1996)[1]			•	•					•				•			
Thangiah et al. (1996)[1,2,3]		•		•					•				•			
Toth and Vigo (1996)[1,2,3]		•		•									•			
Duhamel et al. (1997)[1,2,3]			•	•					•				•			
Toth and Vigo (1997)[1,2,3]	•			•									•			
Mosheiov (1998))[1,2]		•			•								•			
Mingozzi et al. (1999)[1,2,3]	•			•									•			
Salhi and Nagy (1999)[1,2,3]		•			•	•					•		•			
Toth and Vigo (1999)[1,2,3]		•		•									•			
Dethloff (2001)[1,2]		•				•							•			
Angelelli and Mansini (2002)[1,2]	•					•			•				•			
Dethloff (2002)[1]		•			•								•			
Osman and Wassan (2002)[1,2,3]			•	•									•			
Wade and Salhi (2002)[1,2,3]		•			•								•			
Cheung and Hang (2003)[3]		•			•				•			•	•			
Ghaziri and Osman (2003)		•		•									•			
Süral and Bookbinder (2003)[1,3]	•				•								•			
Wade and Salhi (2004)[1,2]			•		•								•			

Paper reviewed in: [1] Parragh et al. (2008a), [2] Toth and Vigo (2014), [3] Koç and Laporte (2018)
Legend: E - Exact, H - Heuristic, MH - Metaheuristic, Pr - Precedence, Mx - Mixed, PD - Pickup and delivery, LS - load split, SWT - Service and waiting time, TW - Time-windows, MP - multi period, MD - Multi-depot, HF - Heterogeneous fleet, Ecn - Economic, Env - Environmental, Scl - Social, Un - Uncertainty

Table 2.2 (cont.) – Classification of VRPB works (2005-2009)

	Solution methods			Backhauls			Scenario characteristics				Physical characteristics		Objective function			Information
Reference	E	H	MH	Pr	Mx	PD	LS	SWT	TW	MP	MD	HF	Ecn	Env	Scl	Un
Cho and Wang (2005)[3]		•		•					•				•			
Crispim and Brandão (2005)[1,2,3]			•		•	•							•			
Nagy and Salhi (2005)[1,2,3]		•			•	•					•		•			
Zhong and Cole (2005)[1,2,3]		•		•	•				•				•			
Brandão (2006)[1,2,3]			•	•									•			
Chen and Wu (2006)[1,2]			•			•							•			
Dell'Amico et al. (2006)[1,2]	•					•							•			
Ghaziri and Osman (2006)[1,3]		•		•									•			
Katoh and Yano (2006)[2]		•				•	•						•			
Montané and Galvão (2006)[1,2]			•			•							•			
Privé et al. (2006)[2]		•			•	•			•			•	•			
Reimann and Ulrich (2006)[1,3]			•	•	•				•				•			
Ropke and Pisinger (2006)[1,2,3]			•	•	•	•		•	•		•		•			
Tavakkoli-Moghaddam et al. (2006)[2,3]			•	•								•	•			
Bianchessi and Righini (2007)[1,2]			•			•							•			
Ganesh and Narendran (2007)[1]			•	•									•			
Saremi et al. (2007)			•	•								•	•			
Wassan (2007)[2,3]			•	•									•			
Gribkovskaia et al. (2008)[2]		•				•							•			
Wassan et al. (2008a)[2]			•			•							•			
Wassan et al. (2008b)[2,3]			•			•							•			
Gajpal and Abad (2009)[2,3]			•	•									•			
Hoff et al. (2009)[2,3]			•			•							•			
Liu and Chung (2009)[3]			•	•									•			
Wang and Wang (2009)[2,3]		•	•	•				•		•			•			
Tütüncü et al. (2009)[3]			•	•	•								•			
Zachariadis et al. (2009)[2]			•			•							•			

Paper reviewed in: [1] Parragh et al. (2008a), [2] Toth and Vigo (2014), [3] Koç and Laporte (2018)
Legend: E - Exact, H - Heuristic, MH - Metaheuristic, Pr - Precedence, Mx - Mixed, PD - Pickup and delivery, LS - load split, SWT - Service and waiting time, TW - Time-windows, MP - multi period, MD - Multi-depot, HF - Heterogeneous fleet, Ecn - Economic, Env - Environmental, Scl - Social, Un - Uncertainty

Table 2.2 (cont.) – Classification of VRPB works (2010-2014)

	Solution methods			Backhauls			Scenario characteristics				Physical characteristics		Objective function			Information
Reference	**E**	**H**	**MH**	**Pr**	**Mx**	**PD**	**LS**	**SWT**	**TW**	**MP**	**MD**	**HF**	**Ecn**	**Env**	**Scl**	**Un**
Çatay (2010)[2]			•			•							•			
Gutiérrez-Jarpa et al. (2010)	•			•			•	•	•				•			
Subramanian et al. (2010)[2]			•			•							•			
Tütüncü (2010)[2,3]			•	•								•	•			
Zachariadis et al. (2010)[2]			•			•							•			
Zhu et al. (2010)		•		•			•						•			
Bailey et al. (2011)		•		•									•			
Kumar et al. (2011)		•		•									•			
Subramanian et al. (2011)[2]	•					•							•			
Ubeda et al. (2011)		•		•									•	•		
Anbuudayasankar et al. (2012)[2]			•										•			
Paraphantakul et al. (2012)			•	•			•	•	•	•			•			
Subramanian et al. (2012)[2]	•					•							•			
Zachariadis and Kiranoudis (2012)[2,3]			•	•									•			
Belmecheri et al. (2013)[3]			•		•				•			•	•			
Eguia et al. (2013)[3]		•		•					•			•	•	•	•	
Lai et al. (2013)			•	•			•					•	•			
Nagy et al. (2013)[3]			•			•							•			
Pradenas et al. (2013)[3]			•	•				•	•				•	•		
Salhi et al. (2013)[3]		•		•								•	•			
Subramanian et al. (2013)[2]			•			•							•			
Tarantilis et al. (2013)[2]			•	•				•	•				•			
Cuervo et al. (2014)[3]			•	•									•			
Davis et al. (2014)	•			•				•		•			•			
Juan et al. (2014)			•	•									•	•		
Küçükoğlu and Öztürk (2014)			•	•				•	•			•	•			
Vidal et al. (2014)[3]			•	•				•	•				•			
Yu and Qi (2014)[3]			•			•			•				•			

Paper reviewed in: [1] Parragh et al. (2008a), [2] Toth and Vigo (2014), [3] Koç and Laporte (2018)
Legend: E - Exact, H - Heuristic, MH - Metaheuristic, Pr - Precedence, Mx - Mixed, PD - Pickup and delivery, LS - load split, SWT - Service and waiting time, TW - Time-windows, MP - multi period, MD - Multi-depot, HF - Heterogeneous fleet, Ecn - Economic, Env - Environmental, Scl - Social, Un - Uncertainty

Table 2.2 (cont.) – Classification of VRPB works (2015-2019)

	Solution methods			Backhauls			Scenario characteristics				Physical characteristics		Objective function			Information
Reference	**E**	**H**	**MH**	**Pr**	**Mx**	**PD**	**LS**	**SWT**	**TW**	**MP**	**MD**	**HF**	**Ecn**	**Env**	**Scl**	**Un**
Belloso et al. (2015)[3]		•		•									•			
Bortfeldt et al. (2015)[3]			•	•									•			
García-Nájera et al. (2015)[3]			•	•									•			
Küçükoğlu and Öztürk (2015)			•	•				•	•			•	•			
Lai et al. (2015)			•	•			•						•			
Nikolakopoulos (2015)			•	•				•	•				•			
Yalcın and Erginel (2015)[3]		•		•									•			
Zachariadis et al. (2015)			•	•									•			
Berghida and Boukra (2016)[3]			•		•				•			•	•			
Brandão (2016)[3]			•	•									•			
Chávez et al. (2016)[3]			•	•							•		•	•		
Dominguez et al. (2016)[3]			•	•						•			•			
Nguyen et al. (2016)			•	•			•		•	•			•			
Oesterle and Bauernhansl (2016)[3]		•			•				•			•	•			
Rahimi et al. (2016)	•					•			•	•			•	•	•	
Wu et al. (2016)[3]			•		•				•			•	•			
Belloso et al. (2017a)[3]		•		•								•	•			
Belloso et al. (2017b)[3]			•	•	•								•			
Lin et al. (2017)[3]			•	•					•			•	•			
Turkensteen and Hasle (2017)				•	•								•	•		
Wassan et al. (2017)[3]			•	•			•						•			
Reil et al. (2018)		•		•	•				•				•			
Soleimani et al. (2018)	•					•					•	•	•	•		
Granada-Echeverri et al. (2019)	•			•									•			
Lu and Yang (2019)			•	•	•								•			

Paper reviewed in: [1] Parragh et al. (2008a), [2] Toth and Vigo (2014), [3] Koç and Laporte (2018)

Legend: E - Exact, H - Heuristic, MH - Metaheuristic, Pr - Precedence, Mx - Mixed, PD - Pickup and delivery, LS - load split, SWT - Service and waiting time, TW - Time-windows, MP - multi period, MD - Multi-depot, HF - Heterogeneous fleet, Ecn - Economic, Env - Environmental, Scl - Social, Un - Uncertainty

Appendix 2.B Benchmark instances

Table 2.6 – Main benchmark instances used in VRPB literature

Instance	Variant	References
Goetschalckx and Jacobs-Blecha (1989)	VRPB	Toth and Vigo (1996, 1997, 1999); Mingozzi et al. (1999); Wade and Salhi (2002, 2004); Osman and Wassan (2002); Brandão (2006, 2016); Tavakkoli-Moghaddam et al. (2006); Ropke and Pisinger (2006); Ganesh and Narendran (2007); Wassan (2007); Wassan et al. (2008a, 2017); Gajpal and Abad (2009); Tütüncü et al. (2009); Anbuudayasankar et al. (2012); Zachariadis and Kiranoudis (2012); Cuervo et al. (2014); Vidal et al. (2014); Belloso et al. (2015, 2017b); Bortfeldt et al. (2015); Yalcın and Erginel (2015); García-Nájera et al. (2015); Nguyen et al. (2016); Granada-Echeverri et al. (2019)
Gélinas et al. (1995)	VRPB	Potvin et al. (1996); Thangiah et al. (1996); Duhamel et al. (1997); Zhong and Cole (2005); Reimann and Ulrich (2006); Ropke and Pisinger (2006); Ganesh and Narendran (2007); Pradenas et al. (2013); Tarantilis et al. (2013); Küçükoğlu and Öztürk (2014, 2015); Vidal et al. (2014); Nikolakopoulos (2015); Nguyen et al. (2016)
Toth and Vigo (1996)	VRPB	Toth and Vigo (1997, 1999); Mingozzi et al. (1999); Dethloff (2001); Wade and Salhi (2002); Ghaziri and Osman (2006); Ropke and Pisinger (2006); Ganesh and Narendran (2007); Wassan (2007)
Thangiah et al. (1996)	VRPB	Ropke and Pisinger (2006); Tarantilis et al. (2013)
Toth and Vigo (1997, 1999)	VRPB	Dell'Amico et al. (2006); Brandão (2006, 2016); Tavakkoli-Moghaddam et al. (2006); Saremi et al. (2007); Wassan (2007); Wassan et al. (2008a, 2017); Gajpal and Abad (2009); Tütüncü et al. (2009); Tütüncü (2010); Anbuudayasankar et al. (2012); Salhi et al. (2013); Cuervo et al. (2014); Bortfeldt et al. (2015); Dominguez et al. (2016); Nguyen et al. (2016); Belloso et al. (2017a,b); Granada-Echeverri et al. (2019)
Kontoravdis and Bard (1995)	VRPMB	Zhong and Cole (2005); Tarantilis et al. (2013)
Salhi and Nagy (1999)	VRPMB/ VRPSDP	Dethloff (2001, 2002); Nagy and Salhi (2005); Crispim and Brandão (2005); Chen and Wu (2006); Montané and Galvão (2006); Wassan et al. (2008a,b); Zachariadis et al. (2009, 2010); Çatay (2010); Subramanian et al. (2010, 2011, 2012, 2013); Nagy et al. (2013); García-Nájera et al. (2015); Chávez et al. (2016)
Min (1989)	VRPSDP	Dethloff (2001); Montané and Galvão (2006); Çatay (2010)
Dethloff (2001)	VRPSDP	Montané and Galvão (2006); Zachariadis et al. (2009, 2010); Çatay (2010); Subramanian et al. (2010, 2011, 2012)
Montané and Galvão (2006)	VRPSDP	Zachariadis et al. (2009, 2010); Subramanian et al. (2010, 2011, 2012, 2013)

Appendix 2.C Cases studies and applications of VRPB and variants

Table 2.7 – Case studies and applications of VRPB

Reference	Case study
Yano et al. (1987); Cheung and Hang (2003)	A retail company delivers goods to stores and picks up goods from suppliers
Casco et al. (1988); Eguia et al. (2013)	A supermarket chain delivers goods to customers and picks up goods from suppliers
Min (1989)	A public library sends material (e.g., books) to branch libraries and picks up material from them
Privé et al. (2006); Hoff et al. (2009)	A beverage company sends full bottles to customers and collects the empty ones
Ubeda et al. (2011)	A food retailer sends the requests to customers and picks up goods from suppliers
Anbuudayasankar et al. (2012)	A bank needs to collect and replenish automated teller machines (ATM)
Paraphantakul et al. (2012)	A cement producer sends bagged cement to its customers and picks up lignite from mines
Yu and Qi (2014)	An express delivery company receives packages at the hub, where are sorted and send to transfer stations
Davis et al. (2014)	A food bank collects donations from local sources and delivers food to charitable agencies
Lai et al. (2015)	From a port, a carrier provides door-to-door freight to import customers and collects containers at export customers
Yalcın and Erginel (2015)	A ceramics company sends requests to customers and picks up raw-materials at suppliers
Oesterle and Bauernhansl (2016)	A food industry delivers perishable goods to customers and pickups raw-materials from suppliers
Wu et al. (2016)	A mail distribution centre delivers packages to post offices and collects others
Dominguez et al. (2016)	A company delivers industrial equipment to customers and collects items at suppliers
Rahimi et al. (2016)	A food company sends perishable goods to customers and collects the expired ones
Lin et al. (2017)	A grocery chain sends requests to customers and picks up goods from suppliers
Soleimani et al. (2018)	A newspaper vendor delivers newspapers to customers and collects the unsold ones

Chapter 3

A Rich Vehicle Routing Problem with Backhauls

Integrated planning of inbound and outbound logistics with a Rich Vehicle Routing Problem with Backhauls

Alexandra Marques* · **Ricardo Soares** *† · **Maria João Santos** *† · **Pedro Amorim** *†

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Abstract This paper addresses the integration of the planning decisions concerning inbound logistics in an industrial setting (from the suppliers to the mill) and outbound logistics (from the mill to customers). The goal is to find the minimum cost routing plan, which includes the cost-effective outbound and inbound daily routes (OIRs), consisting of a sequence of deliveries of customer orders, pickup of a full truck-load at a supplier, and its delivery to the mill. This study distinguishes between three planning strategies: opportunistic backhauling planning (OBP), integrated inbound and outbound planning (IIOP) and decoupled planning (DIOP), the latter being the commonly used, particularly in the case of the wood-based panel industry under study. From the point of view of process integration, OBP can be considered as an intermediate stage from DIOP to IIOP. The problem is modelled as a Vehicle Routing Problem with Backhauls, enriched with case-specific rules for visiting the backhaul, split deliveries to customers and the use of a heterogeneous fleet. A new fix-and-optimise matheuristic is proposed for this problem, seeking to obtain good quality solutions within a reasonable computational time. The results from its application to the wood-based panel industry in Portugal show that IIOP can help to reduce total costs in about 2.7%, when compared with DIOP, due to better use of the delivery truck and a reduction of the number of dedicated inbound routes. Regarding OBP, fostering the use of OIRs does not necessarily lead to better routing plans than DIOP, as it depends upon a favourable geographical configuration of the set of customers to be visited in a day, specifically, the relative distance between a linehaul that can be visited last in a route, a neighboring backhaul, and a mill. The paper further provides valuable managerial insights on how the routing plan is impacted by the values of business-related model parameters which are set by the planner with some degree of uncertainty. Results suggest

*INESC TEC - Instituto de Engenharia de Sistemas e Computadores, Tecnologia e Ciência, Porto, Portugal
†Faculdade de Engenharia, Universidade do Porto, Porto, Portugal

that increasing the maximum length of the route will likely have the largest impact in reducing transportation costs. Moreover, increasing the value of a reward paid for visiting a backhaul can foster the percentage of OIR in the optimal routing plan.

Keywords logistics planning · vehicle routing with backhauls · rich vehicle routing · forest industry

3.1. Introduction

The optimisation of the logistics processes has a whopping effect on improving the cost-efficiency of supply chains. Specifically, in forest-based supply chains, the inbound logistics bringing the wood from the forest to the mill can represent up to 30% of the total costs (Audy et al., 2010), while the outbound logistics bringing the wood-based products from the mill to the consumers can be equally high.

Despite recent studies showing that integrated planning of supply chain operations can lead to better results than decoupled planning (e.g., Amorim et al., 2012), inbound and outbound logistics planning are still dealt separately in most forest industries, as well as in other sectors. The complexity of the logistics operations, specificities of the transportation fleet and customer service levels are frequent justifications for this fact. In the wood-panel based industry, outbound logistics planning establishes the minimum-cost daily routes, henceforth called outbound routes (ORs), for delivering the ordered amounts of finished products to customers. This process accrues from the mill's production plan and impacts on the customer order lead time. Inbound logistics establishes the inbound routes (IRs), usually of a dedicated log-truck, consisting of a sequence of full truck-load trips between a wood sourcing location and the mill. The process is affected by wood procurement planning, ultimately impacting on the upstream forest harvest scheduling decisions. Similar transportation planning settings appear in the retail industry. Namely, in cases in which the retailer has the option to pick-up products at suppliers besides just simply distributing to stores (Yano et al., 1987).

This paper studies the integration of inbound and outbound logistics in the context of the wood-based panel industry. The case study is driven from a real-life industrial application that operates on a multi-mill setting. The production strategy of the wood-based panels at each mill is Make-to-Order. The finished products are shipped to the customers in the day after its production. The stock of raw materials should be at least one week to overcome fluctuations in wood supply. The outbound logistics are planned locally, in the transportation department of each mill, while the inbound logistics are planned centrally, considering the bulk demand for all the mills. The goal here is to find daily minimum-cost outbound and inbound routes (OIRs) where the vehicle departing from each mill firstly performs a sequence of deliveries of the amounts ordered by the customers, and secondly, whenever is cost-effective, picks up a full truck-load of raw materials at a nearby supplier, and delivers it at the closest company's mill. OIRs allow better use of the delivery truck, when compared with ORs and further avoid dedicated IRs. This is possible because the driver can easily adapt the same truck that transported the wood boards with reinforcements in its structure so it can transport a full truck-load of wood chips. For wood-based supply chains, it is common that the inbound transport is carried in full truck-loads (e.g., Carlsson and Rönnqvist, 2007; Hirsch, 2011; Derigs et al., 2012).

In this paper, the problem of finding OIRs is modelled as a Vehicle Routing Problem with Backhauls (VRPB). The VRPB is a variant of the well-known Vehicle Routing Problem (VRP) where the route visits several customers, in some performing deliveries (referred as linehauls) and in others pickups (the backhauls), but all deliveries must be made before any pickups (Goetschalckx and Jacobs-Blecha, 1989). In this study, we use the VRPB as

a mean to tackle Integrated Vehicle Routing Problems, as outlined by Bektaş et al. (2015), since the routing decisions related with the process of inbound logistics and those of the outbound logistics are dealt jointly. Moreover, there are essential business-related rules arising from our application to the wood-based panel industry that determine route feasibility, which are not yet fully covered in the VRPB literature and justify the formulation of a new variant of a rich VRPB, in line with the taxonomy proposed by Lahyani et al. (2015). The first set of business-related rules addressed in this study relate to the conditions determining the visit to a backhaul: i) the backhaul can only be visited after all deliveries are performed, here called a precedence constraint, because the reinforcement of the truck for transporting the wood chips can only occur after the last delivery of the wood-based panels; ii) there is at most one backhaul visited per route because the amount picked up is always a full truck-load since there are no wood availability constraints at suppliers; iii) if there is a pickup at a backhaul it is mandatory that the same route includes its delivery at a mill. This is another type of precedence constraint ensuring that a mill is visited after a backhaul. However, operational practice indicates that the unloading mill may or may not be the mill of origin, because the company owns several mills geographically dispersed, and the truck can end the route in any of these mills, as long as the compatibility requirements between the types of raw materials available at the backhaul and accepted at the mill are accounted for; iv) a backhaul may or may not be visited, which is known in the literature as selective backhauling; v) routes without a backhaul are also feasible, and in this case, the route ends in the last linehaul visited, similarly to what occurs in an Open VRP (see Figure 3.1). There are other studies on VRPB that work with precedence constraints and selectiveness. However, the possibility to optimise the decisions about visiting or not a backhaul and further choosing the delivering mill in order to minimise total logistics costs are new and important features of the problem under study. Another important case-specific rule determines that each customer may be visited more than once by different vehicles, known in the VRP literature as *split deliveries*. The bundle of panels to be delivered at the linehaul customer is of variable size and weight. Therefore, several smaller bundles can be transported by the same truck, but larger bundles may need multiple trucks serving the same customer. Lastly, the available fleet is composed of trucks which are *heterogeneous* in terms of the transportation capacity. The transport is entirely outsourced to third-party carriers and paid based on a fixed daily use cost and a variable cost depending on the travelling distances of the 'for-hire' vehicles. We further emphasize that these business rules are also applicable in other industries besides the wood-panel one, such as in grocery retail.
The complexity of this real-world problem motivates a study about the main strengths and shortcomings of different inbound and outbound planning strategies, with greater or fewer degrees of integration. Furthermore, given the considerable size that these problems can achieve, it becomes relevant to envisage a scalable solution method, able to cope with the operational reality.
This research builds on a literature review on VRPB and other rich VRP variants with similarities to our problem. The first contribution of this paper is to develop a mathematical formulation to address a rich VRP that is primarily used to solve different planning strategies for obtaining OIRs. We apply it to a case study in the wood-based panel industry in Portugal and draw conclusions by comparing the routing plans obtained with those

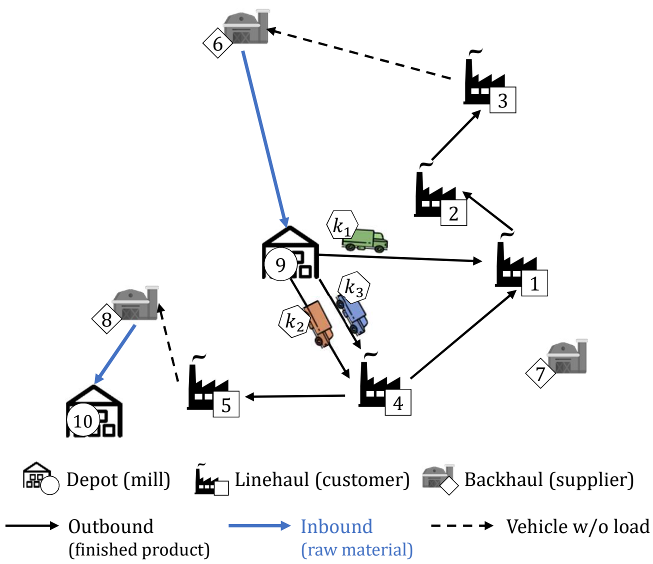


Figure 3.1 – Problem representation

alternative planning strategies. Another contribution is to provide valuable managerial insights for planners about the impact of business-related model parameters over the optimal routing plan. Another contribution is to adapt the fix-and-optimise matheuristic presented by Sahling et al. (2009) for obtaining good quality solutions for larger instances of this problem within a reasonable computational time.

The remainder of this paper is organised as follows. Section 3.2 provides a critical review of the literature regarding integrated transportation planning with a particular connection to the VRPB. This review covers extensions of VRPBs and solution methods developed to solve both artificial and real instances, and allows us to place our work in context. Section 3.3 presents the mathematical formulation of the three logistics planning strategies investigated in this work, namely the opportunistic backhauling, the integrated and the decoupled inbound-outbound transportation planning. Section 3.4 describes the solution approach developed, which is based on a fix-and-optimise algorithm. Section 3.5 presents the computational experiments performed with close-to-reality instances from a wood-based industry in Portugal. The routing plans obtained for the three planning strategies are compared, and relevant managerial insights are envisaged. The main conclusions are presented in Section 3.6.

3.2. Critical review of the state of the art

In the literature on logistics and transportation, the term integrated planning is broadly used to refer to situations where the routing decisions are tackled jointly with other decisions (Speranza, 2018). In some situations, the integration is between transportation decisions of different planning levels, for example, strategic decisions concerning the design of the transportation network and the tactical decisions related with the routes and assignment of the transport vehicles (e.g., Bouchard et al., 2017). In other situations, the integration is between the routing decisions and the decisions concerning other processes of the supply chain. The special issue by Bektaş et al. (2015) on the integrated VRP shows examples of cases where vehicle routing is interlinked with decisions related to loading, production (or inventory), location, and speed optimisation. As an example, production-routing problems integrate production, products delivery (i.e., outbound logistics), and usually also inventory decisions (e.g., Adulyasak et al., 2015). There are several examples in the forest literature where wood transportation to the mill (i.e., inbound logistics) and the upstream process of forest harvesting are planned jointly (e.g., Marques et al., 2014).

As indicated by Speranza (2018), a common feature of the studies on integrated transportation planning is that dealing with those decisions separately or hierarchically by solving the problems independently, leads to a sub-optimal solution for the integrated problem. In fact, integrated planning potentiates global efficiency gains, usually translated into cost savings. As an example, Archetti and Speranza (2015) present significant savings of around 9.5% in terms of total cost and 9.0% in terms of the number of vehicles used when using a heuristic solution for an inventory-routing problem, in comparison with the solution obtained by sequentially and optimally solving the inventory management and the routing problems.

The main particularity of our study, not yet fully covered in the literature, is that the integration is between two processes of the supply chain – inbound and outbound logistics – wherein both processes the relevant decisions are related with the optimal vehicle routes. In fact, in our problem, it is the same vehicle that may perform both processes. There are significant differences in respect to the modelling approach because, in the other cases of integrated VRPs, such as production-routing, there are at least two types of decision variables, one for each process, and the correspondent linking constraints. While in ours, there are only the decision variables related to routing. The linkage between the two processes accrues from the way the routes are built.

The problem class that mostly resembles our problem is the VRPB, firstly introduced by Deif and Bodin (1984). Since then, there are several VRPB variants being studied in the framework of practical applications, as shown in the recent review of Koç and Laporte (2018). In general terms, the VRPB consists in finding the minimum cost routes, which start and end at the depot and visit a set of customers partitioned into linehauls (customers who require deliveries), and backhauls (customers who require pickups), all must be visited contiguously (e.g., Wade and Salhi, 2002).

The VRPB is not usually considered as an example of integrated vehicle routing planning. In fact, many of the industrial applications of the VRPB focus on the outbound logistics process, for example, in retail companies (e.g., Goetschalckx and Jacobs-Blecha, 1989; Eguia et al., 2013). In these cases, the route prioritises first all the products deliveries, and only afterwards the pickups, in order to attain a high vehicle utilisation. The customers are all of the same type (e.g., stores), but with different requirements (i.e., pickup or delivery) and the picked up material can be of a different type that cannot be mixed with the delivered products, such as empty boxes, damaged products or post-consumption material in reverse logistics. In other applications, such as the distribution of equipment to rentals (e.g., Dominguez et al., 2016), or package delivery over a distribution network (e.g., Yu and Qi, 2014), the inbound and outbound material is the same, and it is all planned together as a unique logistic distribution process.

Contrarily, we argue that our case study can be considered integrated transportation planning because the inbound and outbound logistics are two separate processes that nowadays are planned independently, involving different types of customers – i.e., suppliers of raw materials vs. consumers of finished products – sharing in common the depot/mill. Yano et al. (1987) study a case resembling ours, in a retail chain with one centralized distribution centre, 40 stores and nearby vendors, where the route includes the delivery of goods to stores and the pickup of goods in nearby vendors. Planning includes dedicated routes for the vendors whenever it is not cost-efficient to include them in the delivery routes. The results of this work allowed savings in the order of a half-million dollars. With a similar strategy, Paraphantakul et al. (2012) report a case-study in a cement industry, where cement customers are linehaul customers, and lignite mines are backhaul customers. The problem was solved using an ant colony optimisation method, and the company was able to save about 12% in the average tour duration.

The literature review on VRPB reveals examples of mathematical models, exact and heuristic methods for solving distinct problem variants. A general integer linear programming formulation and set partitioning formulation for the VRPB are presented in Koç and Laporte (2018). Among the most common extensions of VRPB found in the literature are the incorporation of time windows (Ropke and Pisinger, 2006; Gutiérrez-Jarpa et al., 2010; Küçükoğlu and Öztürk, 2013; Nguyen et al., 2016), multi-periods (Davis et al., 2014; Nguyen et al., 2016), multi-depots (Chávez et al., 2015), heterogeneous fleet (Salhi et al., 2013; Lai et al., 2013) and split deliveries (Gutiérrez-Jarpa et al., 2010; Lai et al., 2015; Nguyen et al., 2016; Wassan et al., 2017). There are also variants on the nature of the backhauling, such as the mixed VRPB that also allows deliveries to linehauls after pickups in backhauls (e.g., Yazgitutuncu et al., 2009).

As the research on transportation planning advances more and more towards its practical application, several extensions of VRPs that consider real-life aspects of the logistics problems have emerged in the literature. The VRPs that cover such aspects, namely the integration of different logistics operations (e.g., inbound and outbound transport), the consideration of uncertainty or dynamism, or the inclusion of real constraints (e.g., time windows and multi-periodicity), fall into the vast class of Rich VRPs (Lahyani et al., 2015; Caceres-Cruz et al., 2014). As our problem concerns a VRP with selective backhauls, heterogeneous fleet, and split deliveries, we can classify it as a rich VRPB. Table 3.1 presents a description of other VRPBs found in the literature that relate to our work, including the real-life aspects addressed in the problem and the respective types of solution methods used to solve the VRPB.

From Table 3.1, it is possible to observe that metaheuristics are the most popular methods used to solve VRPBs. This results from the fact that the VRPB is an NP-hard problem and, as such, very few exact methods are known to be efficient for large scale problems. Yano et al. (1987) describe the problem using a set-covering formulation and then solve it using a procedure based on a Branch-and-Bound that starts from an initial solution obtained with simple heuristics. Gutiérrez-Jarpa et al. (2009) introduce a Branch-and-Cut algorithm to solve a VRPB with split deliveries and test it in new problem instances adapted from the VRP instances with up to 100 customers, but only those instances with 50 customers or less can be solved to optimality. Davis et al. (2014) use a commercial solver to find optimal

Table 3.1 – Characteristics of the Rich VRPB under study and related works in the literature

Reference	VRPB features							Solution method		
	TW	HF	SD	MD	MP	SB	MB	Exact	Metaheuristic	Matheuristic
Yano et al. (1987)						•		•		
Ropke and Pisinger (2006)	•								•	
Gribkovskaia et al. (2008)						•			•	
Gutiérrez-Jarpa et al. (2009)	•		•			•		•		
Paraphantakul et al. (2012)	•		•						•	
Küçükoğlu and Öztürk (2013)	•	•							•	
Salhi et al. (2013)		•							•	
Lai et al. (2013)		•	•						•	
Davis et al. (2014)					•			•		
Chávez et al. (2015)				•					•	
Nguyen et al. (2016)	•		•		•				•	
Oesterle and Bauernhansl (2016)	•	•					•	•		
Wassan et al. (2017)			•						•	
Our problem		•	•			•				•

Legend: TW (time-windows), HF (heterogeneous fleet), SD (split deliveries), MD (multi-depot), MP (multi-periodic), SB (selective backhauls), MB (mixed backhauls)

transportation schedules that allow food banks to collect food donations from local sources and to deliver food to charitable agencies, through food delivery points. The problem is solved in two phases: first, the problem is formulated as a set-covering model to assign charitable agencies to food delivery points, and then, the problem is formulated as a VRPB enriched with constraints related to food safety, operator workday and collection frequency, also using the optimal solution of the first phase as an input. Oesterle and Bauernhansl (2016) also study a logistic problem of a food company but considering a mixed VRPB with time windows, heterogeneous fleet, manufacturing capacity and driving time limits. The problem is formulated as a mixed integer programming model and also solved with a commercial solver in two phases. The first phase creates clusters of customers to visit, and at the second phase, the routes in each cluster are optimised.

With respect to metaheuristics, both local search and population-based methods have proved to be very efficient to deal with VRPB and its extensions. Examples of local search metaheuristics include tabu search (Gribkovskaia et al., 2008; Nguyen et al., 2016), adaptive large neighborhood search (Ropke and Pisinger, 2006), and variable neighborhood search (Wassan et al., 2017). Examples of population-based metaheuristics developed for the VRPB include ant colony optimisation (Paraphantakul et al., 2012; Chávez et al., 2015) and evolutionary algorithms (Küçükoğlu and Öztürk, 2013). Moreover, two-phase heuristics are also investigated in the works of Salhi et al. (2013) and Lai et al. (2013).

Regarding matheuristic approaches, no references related to its adaptation to the VRPB were found. However, the literature accounts for several matheuristic approaches for various solving VRP variants. For example, the fix-and-optimise approach was initially proposed by Sahling et al. (2009) for a lot-sizing problem, but it has been gaining recent in-

terest in the literature for solving several rich routing problems with real-life aspects (e.g., Neves-Moreira et al., 2019). This matheuristic consists in iteratively fixing different sets of binary variables from a mathematical model, thus allowing a commercial solver to only solve smaller parts of the global problem. Depending on the problem, the selection of the variables to be fixed or released needs to be carefully designed. Most references frame this approach in a variable neighbourhood decomposition search (Hansen et al., 2001), where the number of variables to be released is progressively increased as a way to increase the neighbourhood sizes being explored (e.g., Darvish et al., 2019; Soares et al., 2019). Other research works use distinct heuristic concepts, such as tabu search (e.g., Rieck et al., 2014) by using a tabu list for the variables being fixed.
Our work is distinct from the ones revisited in this section. It contributes to the literature because it not only describes a new formulation for a rich VRPB that can be used to address different transportation planning strategies but also investigates a fix-and-optimise method to solve the problem, which was not yet addressed in VRPB literature.

3.3. Problem formulation

This section outlines the main planning strategies for the integration of inbound and outbound logistics processes, which will be addressed in this paper. For each one of these planning strategies, mathematical formulations will be provided, which will be the basis for the sections that follow.

3.3.1 Logistics planning strategies

The integration of inbound and outbound logistics by finding the optimal OIRs can be staged in two distinct planning strategies, in opposition to a simpler strategy of decoupled planning, similar to what is used today by the company:

- Opportunistic backhauling planning (OBP): In this strategy, the primary process to be considered is the outbound logistics. The outbound transportation plan encompasses ORs and cost-effective OIRs, but another plan exists for IRs. There is an underlying idea that OIRs can provide only a residual amount of the raw materials demanded and IRs assure the vast majority of the demand.

- Integrated Inbound and Outbound Planning (IIOP): In this strategy, both processes of inbound and outbound logistics are planned jointly. The transportation plan encompasses all types of routes – ORs, OIRs and IRs.

- Decoupled Inbound and Outbound Planning (DIOP): This strategy implies that both processes of inbound and outbound logistics are planned independently. The outbound transportation plan (or delivery plan) encompasses the ORs, while the inbound plan (or supply plan) encompasses IRs, there are no OIRs. In the current situation of the case study, logistics planning occurs in separate company departments. IRs are planned centrally and ORs are planned in a department at each mill.

From the point of view of process integration, OBP can be considered an "intermediate" stage, from DIOP towards IIOP, as well as from the point of view of the level of organisational changes needed for its adoption. In fact, OBP impacts mostly on the planners of the outbound logistics in each mill and on the truck drivers while IIOP implies a major restructuring from merging (and possibly centralising) the inbound and outbound logistics planning departments. From a modelling point of view, the mathematical formulation for OBP and IIOP are similar. For the purpose of simplification, this section focuses on OBP, making the necessary adjustments to IIOP afterwards. The section ends with the description of DIOP.

3.3.2 Opportunistic backhauling planning (OBP)

OBP can be modelled as a rich, capacitated Vehicle Routing Problem with selective backhauls and split deliveries. Considering a set of mills M, a set of linehaul customers L whose demand needs to be fulfilled, and a set of suppliers backhauls B with raw materials available for the mills that may or may not be visited. The problem consists in finding the optimal daily minimum-cost routes for a set of trucks K, starting at the mill, encompassing one or many deliveries to linehauls, and including at the most one pickup of a full truckload of a given type of raw materials at a backhaul, which is selected based on the best fit with one of the possible destination mills. The set of types of raw materials to be collected at a backhaul is represented by set P. Hence, the problem components include:

- the fleet of $|K|$ trucks, where each truck $k \in K$ has a given capacity (Q_k) and can perform both deliveries and pickups. There is a fixed cost for the daily usage of a vehicle $\left(f^k\right)$ and a variable cost $\left(c_{ij}^k\right)$ proportional to the travelled distances;
- the $|M|$ mills owned by the company that are geographically dispersed. Each mill $m \in M$ receives wood chips and produces wood-based panels on a make-to-order basis. The fleet is assigned to a specific mill or origin (or depot), from where the routes start. According to operational practice, in case of a route with a backhaul, the truck can unload the raw materials in any of the company's mills, which may or may not be the mill of origin. There is a minimum amount of raw materials to be backhauled to all mills (β);
- the $|L|$ linehaul customers that are characterized by a given demand of a finished product, which must be fulfilled (q_l) at each linehaul $l \in L$. Split deliveries can occur, meaning that each customer may be visited more than once (each visit consisting in at least a ψ amount), but each truck may visit a customer at most once;
- the $|B|$ backhaul suppliers that are also geographically dispersed. Also, according to the operational practice, it is assumed that all have unlimited availability, hence pickups correspond to full-truck loads. The type of raw materials that are available may also vary amongst them;
- the $|P|$ types of raw materials consisting of wood chips of variable size and moisture content, sawdust and recycled wood. Some types of raw materials are more desirable

to the mills than others. There are also compatibility issues with respect to the types of raw materials available and demanded at the different locations.

Contrarily to other VRPs found in the literature, the time window constraints related to the earliest or latest time to arrive at each location are not of importance. However, the maximum distance travelled in a route is limited by a parameter α. It is noteworthy that the route length can be constrained in terms of travelling time, to account for driving time regulations stating maximum driving or working times. However, in this case, the value of the maximum distance travelled was set with the planner as an average of the actual routes length, already implicitly considering all the necessary stops, hence simplifying problem modelling. In summary, the characteristics of the feasible routes are: i) start at a home depot with the truck loaded up to its maximum capacity, with the products ordered by the linehaul customers; ii) perform a sequence of deliveries to the linehauls; iii) if it is cost-effective and doable during the maximum route length, the vehicle travels empty to a nearby backhaul supplier to pick up a full truck-load of raw materials to be delivered at any of the company's mill, where the route ends (specific to OIRs); and iv) if a backhaul is not visited, the route is ended when the truck is empty after visiting the last linehaul of the route (specific to ORs), as the company does not pay for trips where the truck does not transport merchandise.

3.3.2.1 Modelling approach

The rVRPB under study is modelled as a graph $G = (V, A)$ where V is the set of all vertices, $V = \{0\} \cup L \cup B \cup M$ and A is the set of all possible arcs. We adopt a standard flow VRP formulation with 3-index decision variables x_{ij}^k equal to 1 if vehicle $k \in K$ travels from customer $i \in V$ to $j \in V$ and zero otherwise. Like in the standard VRPB formulation proposed by Parragh et al. (2008), we distinguish the vertices in linehauls and backhauls, in order to model the precedence constraints.
However, the typical VRPB constraints assuring that each vertex is visited exactly once do not apply, due to the possibility of selective backhauls (i.e., backhauls may or may not be visited) and the split deliveries at the linehauls (i.e., linehauls are visited more than once).
To avoid the complexity of a multi-depot and open VRP, we propose a *2-echelon backhauls network*, starting and ending at the same fictitious depot 0. In fact, when the route starts, the fictitious depot corresponds to the mill of origin from where the customers' orders will be delivered. Since there is a fleet dedicated to each mill when the route starts, routing planning for each mill can be done separately as a single depot. When the route ends, the fictitious depot corresponds to a fictitious location whose distance from the last vertex visited in the route is equal to zero. Hence, the 2-echelon backhauls network is composed by the first echelon of backhauls corresponding to the suppliers and the second echelon of backhauls corresponding to the mills to be supplied by the backhauled amounts. Additional constraints are needed to assure that a mill can only be visited after a backhaul (see Figure 3.2).
The decisions whether a backhaul is visited in a route or not, and if so, to which mill to go next, are based on a new parameter related with the *reward paid for visiting that backhaul and a mill next* (δ_{bm}). Like in previous studies of VRP with selective pickups (e.g.,

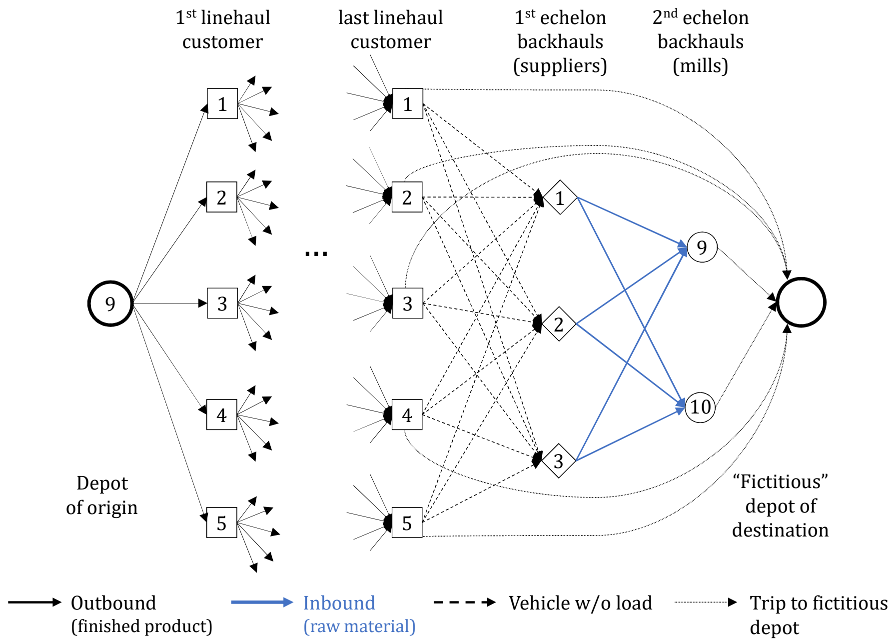
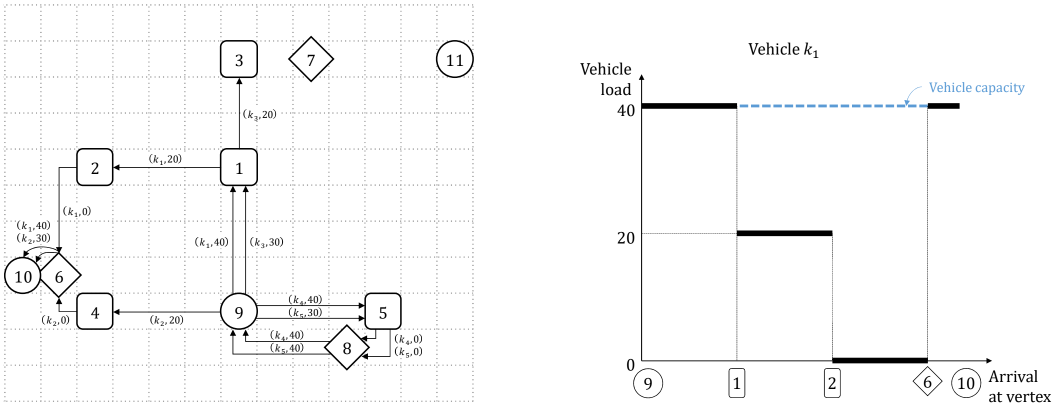


Figure 3.2 – Network representation of the problem

Gribkovskaia et al., 2008) and other formulations of VRP with profits (e.g., Aras et al., 2011), the reward is used to make an arc linehaul to backhaul more or less attractive. The reward corresponds to a payment per each ton of raw materials picked-up in a backhaul and delivered in a neighbouring mill. If the route ends after visiting the last linehaul, then there is no positive reward associated with that route. Hence, the reward parameter is used in the objective function, which trades-off between the sum of the travelling costs for visiting the backhaul after the last linehaul and moving from there to a mill, and the reward gained for visiting that backhaul. The reward parameter is also used to address compatibility issues related to the type of raw material p to be transported from a given backhaul b to a given mill m. In fact, if p is not available in b or not accepted in m then $\delta_{bm} = 0$. On the contrary, if there are several types of raw materials that can be transported from b to m, the value of δ_{bm} corresponds to the value of the most profitable material because there are no other aspects determining the choice between them. Consequently, the set P does not need to be considered in this model. However, in other real-life applications where the availability at the backhauls and or demand at the mills is limited and varies per type of product, the set P should be properly incorporated in the model, leading to a four-index decision variable x.

A new decision variable is needed to assure that, despite the possibility of *splitting the deliveries* to a linehaul, each delivery cannot exceed the truck capacity and that the total amount delivered in the several routes that visit it meets the expected demand. Previous studies used continuous variables w_i^k representing the quantity transported by vehicle $k \in K$ to/from customer $i \in V$ for a similar purpose (e.g., Nikolakopoulos, 2014). However, in the rVRPB under study, without time windows, these variables are insufficient for sub-tour elimination. In this context, a new set of continuous variables u_{ij}^k represent the load of ve-

hicle $k \in K$ when traversing arc $(i, j) \in A$. Variables u_{ij}^k are a natural adaptation of variables u_i (Bektaş et al., 2015; Toth and Vigo, 2014) to a multi-route and split delivery situation. Additional constraints are needed to account for the routes with backhauls. In this case, the truck-load is higher before visiting the first linehaul, then progressively decreases until reaching zero after visiting the last linehaul. If a backhaul is visited, the pickup corresponds to a full truck-load. As an example, for a given route k, encompassing $\{0, i, i', i'', j, 0\}$, where $i, i', i'' \in L$ and $j \in B$, then the following rules apply: $u_{0i}^k \leq u_{ii'}^k \leq u_{i'i''}^k, u_{i''j}^k = 0, u_{j'0}^k = Q_k$.
Figure 3.3 exemplifies a feasible solution for the OBP starting in the node 9, in a network composed by 5 linehauls (numbered 1 to 5), 3 backhauls (numbered 6 to 8) and 3 mills (numbered 9 to 11). For simplification purposes, only the arcs used in the solution are represented in Figure 3.3a. The demand (in ton) at the linehauls is $q_1 = 30, q_2 = 20, q_3 = 20, q_4 = 20, q_5 = 70$. The reward for visiting a backhaul is 0.1€/ton in all cases. The available fleet is composed of 5 trucks, with capacity (in ton) $Q_1 = 40, Q_2 = 30, Q_3 = 30, Q_4 = 40, Q_5 = 40$. The linear distances between vertices (d_{ij}) are computed in reference to the background grid with 1km by 1 km, for example, $d_{13} = 2$ km. The fixed cost for using a vehicle is zero, and the variable cost is 1 €/km.



(a) feasible solution for vehicles k_1, k_2, k_3, k_4, k_5

(b) variation of the load of vehicle k_1 along the route

Figure 3.3 – Example of a feasible solution for a rVRPB

The routing plan foresees the use of all five vehicles: k_1, k_2, k_4 and k_5 are OIRs while k_3 is an OR ending after visiting linehaul 3. There are split deliveries in linehauls 1 and 5. Total costs are 29€ and total revenues are 15€. The values of u_{ij}^k for truck 1 are shown in Figure 3.3b.
This example is instrumental in showing the impact of the reward value over the final routing solution. In fact, the route visiting linehaul 4 will always visit backhaul 6, and then mill 10, because the extra cost for visiting this pair backhaul-mill is 1€ $\left(d_{4,6} = 1e, d_{6,10} = 0 \implies c_{4,10}^k = 1\right)$ and the minimum revenue is 3€ ($\delta_{bm} = 0.1$€/ton, $\min\{Q_k\} = 30$ ton $\implies u_{6,10}^k \geq 30, \forall k \in K, \delta_{6,10} = 0.1$. Applying a similar logic, it is expected that the route visiting linehaul 3 will visit backhaul 7 if $\delta_{7,11} \geq 0.4$, since the extra cost for visiting the backhaul and mill is 4€ and $u_{7,11}^k \geq 30, \forall k \in K$.

3.3.2.2 Mathematical formulation

For the sake of convenience, before presenting the mathematical formulation, we resume the necessary decision variables, sets and parameters.
Decision variables:

x_{ij}^k $\begin{cases} 1 & \text{if vehicle } k \text{ travels from location } i \text{ to } j; \\ 0 & \text{otherwise.} \end{cases}$

u_{ij}^k load of vehicle $k \in K$ when traversing arc $(i, j) \in A$

Sets:

L set of linehauls (customers where finished products are delivered)

B set of backhauls (suppliers where raw materials can be picked up)

M set of mills (where raw materials are delivered if a backhaul is visited)

V set of vertices; $V = \{0\} \cup L \cup B \cup M$

K set of vehicles

Parameters:

q_i quantity to be delivered to customer $i \in L$ (ton)

c_{ij}^k cost of transportation with vehicle $k \in K$ from $i \in V$ to $j \in V$ (€)

f^k fixed cost of using vehicle $k \in K$ in a daily route (€)

Q_k transportation capacity of vehicle $k \in K$ (ton)

d_{ij} travelling distance from $i \in V$ to $j \in V$ (km)

α maximum distance travelled in a route (km)

β minimum amount of raw materials to be backhauled (ton)

δ_{bm} reward for picking up one unit of raw material at backhaul $b \in B$ and delivering it to mill $m \in M$ (€)

ψ minimum amount of order delivered to a linehaul (ton)

Model [P0]

$$\min \sum_{k\in K} \sum_{j\in V\setminus\{0\}} f^k x_{0j}^k + \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} c_{ij}^k x_{ij}^k - \sum_{k\in K} \sum_{i\in B} \sum_{j\in M} \delta_{ij} u_{ij}^k \tag{3.1}$$

subjected to:

$$\sum_{i\in V} x_{ij}^k \leq 1 \qquad \forall j \in L \cup B, \forall k \in K \tag{3.2}$$

$$\sum_{i\in B} \sum_{j\in B} \sum_{k\in K} x_{ij}^k = 0 \tag{3.3}$$

$$\sum_{i\in V\setminus B}\sum_{j\in M}\sum_{k\in K}x_{ij}^k = 0 \tag{3.4}$$

$$\sum_{i\in M}\sum_{j\in V\setminus\{0\}}\sum_{k\in K}x_{ij}^k = 0 \tag{3.5}$$

$$\sum_{i\in B}\sum_{j\in L\cup\{0\}}\sum_{k\in K}x_{ij}^k = 0 \tag{3.6}$$

$$\sum_{j\in B}\sum_{k\in K}x_{0j}^k = 0 \tag{3.7}$$

$$\sum_{j\in L}x_{0j}^k = \sum_{i\in L\cup M}x_{i0}^k b \qquad \forall k\in K \tag{3.8}$$

$$\sum_{i\in V}x_{ij}^k = \sum_{i\in V}x_{ji}^k \qquad \forall j\in V, \forall k\in K \tag{3.9}$$

$$u_{ij}^k \le Q_k x_{ij}^k \qquad \forall (i,j)\in A, \forall k\in K \tag{3.10}$$

$$\sum_{i\in V}u_{ij}^k - \sum_{i\in V}u_{ji}^k \ge \psi\sum_{i\in V}x_{ij}^k \qquad \forall j\in L, \forall k\in K \tag{3.11}$$

$$\sum_{i\in L}\sum_{j\in B\cup\{0\}}\sum_{k\in K}u_{ij}^k = 0 \tag{3.12}$$

$$\sum_{i\in V}\sum_{k\in K}\left(u_{ij}^k - u_{ji}^k\right) = q_j \qquad \forall j\in L \tag{3.13}$$

$$\sum_{i\in V}\sum_{j\in V}d_{ij}x_{ji}^k \le \alpha \qquad \forall k\in K \tag{3.14}$$

$$\sum_{i\in B}\sum_{j\in M}\sum_{k\in K}u_{ij}^k \ge \beta \tag{3.15}$$

$$x_{ij}^k \in \{0,1\}, u_{ij}^k \ge 0 \qquad \forall (i,j)\in A, \forall k\in K \tag{3.16}$$

The objective function (3.1) minimizes the total costs, decomposed into fixed costs (proportional to the number of vehicles used) and the variable costs (proportional to the total travelled distance), decreased by the revenue obtained for visiting backhauls and mills in the course of the OIR. Constraints (3.2) assure that any location can be visited at most once by each truck. Regardless, each linehaul and backhaul can be visited by several routes. Constraints (3.3)–(3.7) deal with route precedence rules, resulting from the specificities of this rVRPB for the wood-based panel industry. Specifically, constraints (3.3) state that the transport from a backhaul to another backhaul is not possible. Constraints (3.4) assure that the mill can only be visited after a backhaul. Constraints (3.5) assure that after visiting a mill, the only possibility is to go to the ending depot. Constraints (3.6) state that after visiting a backhaul, the next visit cannot be to a linehaul nor to the depot. Constraints (3.7) assure that the route cannot visit a backhaul after the depot. Constraints (3.8) and (3.9) are the typical VRP flow conservation constraints, at the depot and at each vertex, respectively. Constraints (3.10) are linking constraints, assuring that there is only a given amount transported to/from the customer if the customer is visited. Constraints (3.11) to (3.13) assure the elimination of sub-tours. Specifically, constraints (3.11) assure that the load of trucks progressively decreases as it visits the linehauls, and the amount delivered should be higher than a minimum amount. By considering the lower bound of the minimum amount, the model avoids undesirable solutions where $x^k_{ij} = 1$ and $u^k_{ji} - u^k_{ij} = 0$, which may occur for example if a linehaul (i') is visited in the course of a route from i to j, i.e., $x^k_{ii'} = x^k_{i'j} = 1$ (instead of $x^k_{ij} = 1$) but the amount delivered in i' is zero $\left(u^k_{i'i} - u^k_{ii'} = 0\right)$ due to the fact that the distance matrix does not obey to the triangular inequality (i.e., $\exists\, d_{ij} : d_{ij} > d_{ii'} + d_{i'j}$). Constraints (3.12) state that the truck leaves empty after visiting the last linehaul and constraints (3.13) assure that the demand at the linehauls is completely fulfilled. Constraints (3.13) together with constraints (3.2) account for the possibility of split deliveries at the linehauls. Constraints (3.14) assure that the maximum allowable distance of the daily route cannot be exceeded. It is noteworthy that if the maximum route length is constrained by the time travelled, then, this would require another type of auxiliary variables to count the route duration and consequent changes in these constraints, with similarities with other VRPs with time windows (e.g., Toth and Vigo, 2014). Constraints (3.15) set a minimum amount of raw materials to be backhauled to mills. Finally, constraints (3.16) determine the domain of the decision variables.

3.3.2.3 Special situation in which the rVRPB is simplified to a rich capacitated VRP

A problem variant of the rVRPB consists in removing constraints (3.14) and (3.15). In this situation, where there is no limitation to the route length and there is no minimum backhauling amount, a backhaul will be visited whenever it is cost-effective, according to the trade-off between the extra transportation cost (from travelling from the last linehaul, to that backhaul and to its closest mill) and the revenue (associated with delivering the load from the backhaul to the closest mill). From a modelling perspective, this means that, knowing which is the last visited linehaul in a route, it is possible to compute beforehand if and which backhaul and mill should be visited to minimize total costs. Consequently, the mathematical model can be simplified to a Rich Capacitated VRP (rCVRP) with split deliveries. This problem will only consist in sequencing the linehauls to be visited in each route, thus determining which linehaul will be last in each route.

This adaptation relies on a data pre-processing procedure (described in Algorithm 1) which consists in computing the minimum cost of having a given linehaul visited last in a vehicle route. If the cost of visiting a backhaul at the end of the route is lower than finishing the route at the depot (line 5), the cost associated with the arc heading to the depot is updated to the summed costs of pickup at the backhaul, delivering to the mill and returning to the depot, subtracted by the corresponding reward for performing the delivery to that mill (line 6). All combinations of vehicles, linehauls, backhauls, and mills are tested in this pre-processing stage, therefore ensuring that the vehicle arcs heading to the depot account for the minimum possible cost, which either corresponds to performing backhauling at the most advantageous locations or finishing its route after visiting the last linehaul. Finally, the sets of backhauls and mills are removed from the problem.

Algorithm 1 Data pre-processing for adapting the rVRPB to a rCVRP

for each vehicle k **in** K **do**
  for each linehaul customer j **in** L **do**
    for each backhaul customer i **in** B **do**
      for each mill customer m **in** M **do**
        if $c^k_{ji} + c^k_{im} + c^k_{m0} - \delta_{im} \cdot Q_k < c^k_{j0}$ **then**
          $c^k_{j0} := c^k_{ji} + c^k_{im} + c^k_{m0} - \delta_{im} \cdot Q_k$
$V := V \setminus (B \cup M)$; $B := \varnothing$; $M := \varnothing$

Afterwards, the new model for the rCVRP can be built upon [P0] by changing the objective function and removing constraints related with the sets of backhauls and mills, as shown in model [P1].

Model [P1]

$$\min \sum_{k\in K} \sum_{j\in V\setminus\{0\}} f^k x^k_{0j} + \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} c^k_{ij} x^k_{ij} \tag{3.1b}$$

subjected to (3.2), (3.8)–(3.13) and (3.16) of model [P0]

3.3.3 Integrated Inbound and Outbound Planning (IIOP)

As stated before, the IIOP strategy consists in jointly planning all types of routes, including OIRs, ORs only for delivery of finished products and IRs for pickup of raw materials. Model [P2] for IIOP can be built upon adaptations of [P0], that account for the IRs, as follows. Constraints (3.7) are removed to allow dedicated routes from the depot to a backhaul. A new parameter δ^D_{bm} represents the reward for picking up one unit of raw material at backhaul $b \in B$ and delivering it to mill $m \in M$ (€) in the course of the dedicated route. A new set of auxiliary continuous variables y^k_{ij} is needed to represent the amount picked up in $b \in B$ and delivered in mill $m \in M$ by vehicle $k \in K$ in a direct route. The objective function (3.1c) is adapted accordingly. A new set of constraints (3.17) defines variables y^k_{ij} and constraints (3.18) set its bounds. Considering an arc $(i, j), i \in B, j \in M$, with $x^k_{ij} = 1$, if $x^k_{0i} = 1, i \in B$, then k is in a dedicated route, and according to the conjugation of constraints (3.17) and (3.18), $y^k_{ij} = u^k_{ij}$. If $x^k_{0i} = 0, i \in B$, then k is in an OIR, and $y^k_{ij} = 0$.

Model [P2]

$$\min \sum_{k\in K} \sum_{j\in V\setminus\{0\}} f^k x^k_{0j} + \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} c^k_{ij} x^k_{ij} - \sum_{k\in K} \sum_{i\in B} \sum_{j\in M} \delta^D_{ij} y^k_{ij} - \sum_{k\in K} \sum_{i\in B} \sum_{j\in M} \delta_{ij} (u^k_{ij} - y^k_{ij}) \tag{3.1c}$$

subjected to (3.2)-(3.6), (3.8)–(3.16) of model [P0] and

$$y^k_{ij} \leq Q_k x^k_{ij} \qquad \forall i \in B, \forall j \in M, \forall k \in K \tag{3.17}$$

$$y_{ij}^k \leq u_{ij}^k - (1 - x_{0i}^k)Q_k \qquad \forall i \in B, \forall j \in M, \forall k \in K \tag{3.18}$$

$$y_{ij}^k \geq 0 \qquad \forall i \in B, \forall j \in M, \forall k \in K \tag{3.19}$$

3.3.4 Decoupled Inbound and Outbound Planning (DIOP)

As stated before, DIOP corresponds to the planning strategy currently used, where the ORs and IRs are planned independently and there are no OIRs. For the outbound logistics planning, the optimal ORs can be obtained by solving model [P3] that is an adaptation of model [P0], considering the nonexistence of backhauls and mills. For the inbound planning, the optimal IRs can be obtained by solving a model [P4], also an adaptation of model [P0], acknowledging only the routes from the depot/mill of origin to the backhauls.

Model [P3]

$$\min \sum_{k \in K} \sum_{j \in V \setminus \{0\}} f^k x_{0j}^k + \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{ij}^k x_{ij}^k \tag{3.1d}$$

subjected to (3.2), (3.8)–(3.11), (3.13)–(3.14) and (3.16) of model [P0]

Model [P4]

$$\min \sum_{k \in K} \sum_{j \in V \setminus \{0\}} f^k x_{0j}^k + \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{ij}^k x_{ij}^k - \sum_{k \in K} \sum_{i \in B} \sum_{j \in M} \delta_{ij}^D u_{ij}^k \tag{3.1e}$$

subjected to (3.2)–(3.6), (3.8)–(3.10) and (3.14)–(3.16) of model [P0]

3.4. Solution approach

3.4.1 Fix-and-optimise approach

As stated in the literature review, the complexity of the VRP problems in real-life applications justifies the use of matheuristics. In this study, all the different models presented before are solved with a fix-and-optimise (F&O) approach in case of the large instances (i.e., more than 30 customers). This solution method was firstly presented by Sahling et al. (2009) for lot-sizing problems, but has been successively used for solving complex routing problems with promising results (e.g., Neves-Moreira et al., 2016; Larrain et al., 2017).
The F&O matheuristic approach consists in iteratively solving several smaller mixed integer programming (MIP) sub-problems of the original model. The design of each sub-problem is problem-dependent and the obtained results highly depend on its adequate design. In this approach, we define a sub-problem as a set of decision variables to be either released or fixed in the original MIP model. Fixing a variable consists in setting its lower and upper bounds to the current solution value, thus precluding it from being changed in a solver iteration. On the other hand, releasing a variable consists in restoring a fixed variable to its original lower and upper bound values. For the problem at hand, two distinct sub-problem types were conceived, named `RouteRelease` and `LocationRelease`.

The `RouteRelease` sub-problem releases all decision variables associated with a given set of routes in the incumbent solution, based on proximity criteria of these routes. Route proximity is defined by the centroids of each route, which are computed as the non-weighted averages of the location coordinates that are visited. The outline of the `RouteRelease` sub-problem construction procedure is illustrated in Algorithm 2. The procedure starts by computing the centroid of each route in the incumbent solution (lines 5–7). For unused vehicles, the route's centroid is given by the depot's coordinates. A pivot route is selected at random from the incumbent solution (line 8), after which all other routes are ordered by its centroid's distance to the pivot route (line 9). The n routes with the lowest distance to centroid of the pivot route are then released in the sub-problem (lines 10–15).

Algorithm 2 Route Release sub-problem construction

```
1:  Input: vars (MIP model routing decision variables)
2:         sol (incumbent solution)
3:         n (number of routes to be released in the subproblem)
4:  released_routes = ∅; centroid_list = ∅
5:  for each route ∈ sol do
6:      compute centroid of route
7:      append centroid of route to centroid_list
8:  rt ← random route; cnt ← centroid of rt
9:  order centroid_list by descending order of their distance to cnt
10: released_routes ← n first routes ∈ centroid_list
11: for each var ∈ vars do
12:     if var is associated with a vehicle ∈ released_routes then
13:         release var
14:     else
15:         fix var
```

The `LocationRelease` sub-problem consists in releasing a given set of linehaul locations based on its geographical proximity. The procedure is described in Algorithm 3, and it starts by selecting a pivot linehaul (line 4), after which we retrieve all routes in the incumbent solution where the pivot linehaul is visited. Afterwards, we retrieve all the additional linehauls that are visited in these routes (lines 7–8). Finally, the n closest linehauls to the pivot linehaul that were previously selected are released (lines 11–18).

The overall structure of the matheuristic is shown in Algorithm 4.
The solution method requires an initial solution s_0 with objective function f_0, which is obtained through a greedy nearest neighbour heuristic (line 7): we select a random vehicle and construct its route by visiting the nearest unsatisfied linehaul until vehicle capacity is exhausted. The process is repeated until all linehaul demand is satisfied. No routes to backhauls are considered in the constructive phase.
After obtaining an initial solution, the matheuristic is then started. To that effect, sub-problem construction is initiated, whose size is controlled through the general principles of

Algorithm 3 Location Release sub problem construction

```
Input: vars (MIP model routing decision variables)
       sol (incumbent solution)
       n (number of locations to be released in the subproblem)
released_locations = ∅; candidates = ∅; loc ← random linehaul
for each route ∈ sol do
    if route transverses loc then
        for each linehaul ∈ route do
            append linehaul to candidates
order candidates by descending order of their distance to loc
released_locations ← n first locations ∈ candidates
for each var ∈ vars do
    if var is associated with a linehaul ∈ released_locations then
        release var
    else
        if var is associated with a mill then
            release var
        else
            fix var
```

a Variable Neighbourhood Decomposition Search (VNDS), similar to what is presented in Hansen et al. (2001). Sub-problems are constructed in line 13, after which the MIP model is fed the incumbent solution s_{cur} and the sub-problem is solved by a MIP solver (lines 14–15).

After each solver iteration, the obtained solution s_{solve} is evaluated against the incumbent solution (lines 16–21). If the obtained solution did not yield an improvement of at least imp (line 16), we consider this a non-improvement iteration and increment the non-improvement counter. Nevertheless, we will accept the obtained solution even if it is not significantly better than the previous one (line 21). After a given number of consecutive non-improvements, the VNDS framework takes place either by increasing sub-problem size or switching the sub-problem type, if the current sub-problem size has been maxed out (lines 22–32). In the occurrence of a significant improvement of the problem's objective function, sub-problem type is re-set to `RouteRelease` and its initial size (line 17).

The matheuristic approach always initializes with the `RouteRelease` sub-problem type and the `LocationRelease` sub-problem is used after a significant number of non-improvements of the `RouteRelease` sub-problem. This algorithmic structure was conceived by bearing in mind that `RouteRelease` would be used as a more disruptive sub-problem, which would explore more disperse sections of the solution space, while the `LocationRelease` sub-problem focuses more on intensification.

3.4.2 Data pre-processing

Pre-processing the instance data related to the network generation is a common procedure in VRPs (e.g., Parragh et al., 2008; Soares et al., 2019) to simplify the mathematical for-

Algorithm 4 Matheuristic outline

Input: MIPmodel (mixed integer programming model)
        P (list of possible sub-problems to be used)
        N_p (initial neighbourhood size for sub-problem p)
        $I_{p,n}$ (limit of consecutive non-improvement iterations for sub-problem p of size n)
        $TL_{p,n}$ (time limit for solver iterations of sub-problem p of size n)
        imp (minimum solution improvement to reset the no-improvement counter i)
s_0, f_0 = nearest_neighbour()
$s_{cur} = s_0$; $f_{cur} = f_0$; i = 0
p = "RouteRelease"
while termination criteria not met **do**
    n = np
    while $n \leq N_p$ **do**
        construct sub-problem of type p with size n
        feed MIPmodel with initial solution s_{cur}
        s_{solve}, f_{solve} = MIPsolve (MIPmodel, $TL_{p,n}$)
        if $f_{solve} < f_{cur} - imp$ **then**
            $s_{cur} = s_{solve}$; $f_{cur} = f_{solve}$; i = 0; p = "RouteRelease"
            break
        else
            if $f_{solve} < f_{cur}$ **then**
                $s_{cur} = s_{solve}$; $f_{cur} = f_{solve}$
            i = i + 1
            if $i > I_{p,n}$ **then**
                i = 0
                if $n = N_p$ **then**
                    if p = "LocationRelease" **then**
                        p = "RouteRelease"
                    else
                        p = "LocationRelease"
                    break
                else
                    increase n
return s_{cur}, f_{cur}

mulation and achieve better performance in the optimisation solver. The pre-processing procedure used prior to solving the models is threefold. The sub-set of arcs to be considered is presented in Table 3.2.

Table 3.2 – Pre-processing the problem network

Origin \ Destination	{0} depot or mill of origin	Linehaul l	Linehaul $l' \neq l$	Backhaul b	Mill m	{0} *fictitious depot*
{0} depot or mill of origin		•	•	(*)		
Linehaul l			•	(**)		•
Linehaul $l' \neq l$		•		(**)		•
Backhaul b					(+)	
Mill m						•
{0} *fictitious depot*						

Legend:
(*) is generated in IIOP but not in OBP;
(**) for each linehaul, generate the arcs to all the backhauls that lead to a cost-effective solution (i.e. satisfy line 5 of Algorithm 1);
(+) for each backhaul, only generate the arc to the minimum cost mill.

First, we remove all the arcs that lead to an unfeasible route, i.e., arcs that violate the precedence constraints (3.3) to (3.7). Second, we eliminate all arcs from linehauls to backhauls where its visit is not economically worthwhile, according to the given reward for visiting a backhaul. These arcs are only generated if they respect the condition exhibited in line 5 of Algorithm 1.

Third, arcs from backhauls are only generated to its closest mill, as delivering merchandise to more distant mills will only induce an increase of the problem's objective function.

It should be noted that this data pre-processing procedure does not cut off optimal solutions only if we do not impose a minimum inbound quantity to be collected from backhauls via constraints (3.15). If this is not the case, this procedure may induce sub-optimality or even turn the model infeasible because there are no cost-effective backhauls to visit. Therefore, in these situations, a trade-off between optimality and simplicity must be taken into account.

3.5. Computational experiments

The proposed approach was applied in a case study in a wood-based panel company in Portugal. The mathematical model was implemented in Gurobi 7.5 commercial solver. The solution method was developed in Python 3.6. The mathematical models were subject to the data pre-processing procedure described earlier and used to compare the gains of the IIOP strategy with the DIOP one, which is currently done by the company. A set of experiments were also done to provide valuable managerial insights for planners. Lastly, the performance of the proposed solution method was compared with a commercial MIP solver for problem instances of increasing size, which were based on real routing plans executed by the company.

3.5.1 Case study

This study was motivated by a real-life application in the wood-based panel company, firstly presented in Amorim et al. (2014). The focal company owns several mills, each producing a specific portfolio of wood-based panels mainly for furniture, construction and decoration. The case study is at one of the mills in Portugal that produces around 1.2 thousand tons of wood-based panels per day, in a make-to-order basis, and assures its delivery to an average of 30 customers distributed over the entire Iberian Peninsula. The average daily consumption of raw materials is 1,750 . The study uses real data regarding the customers' orders in two of the most representative operational days. There are 30 customers to be visited, whose ordered amounts are in average 35.5 ton/customer, varying between 0.05 and 399 ton. The values of the model parameters are an approximation of those provided by the planners. The distances between locations were computed by resorting to the Google Maps routing engine.

Nowadays, the outbound routes are planned to start in the morning of the next day at the opening hour of the mill of origin. It is assumed that all routes can start at the same time, and there are no time windows conditioning the time of arrival to customers, suppliers or mills. The responsible for outbound logistics determines the exact number of trucks needed for the next day and groups the customers to be visited in each route according to empirical rules that rely on the customers' geographical location. Then, the routes are assigned to the third-party logistics operators (3PL) with whom there are valid outsourced contracts. The generic contractual conditions are a fixed cost of 70 per truck used and a variable cost of 1.7 per km travelled. The fleet available at the mill of origin in each day encompassed 100 trucks, of which 20 trucks have capacity up to 10 tons, 40 trucks have a capacity of 20 tons, and 40 trucks have a capacity of 40 tons, summing up a total transportation capacity of 2,600 ton. Each vehicle must deliver at least 0.5 ton to each customer visited (i.e., $\psi = 0.5$), except when its demand is lower than this parameter. All trucks are prepared to do IRs, if needed.

Overall, the current logistics process results in a low rate of inbound-outbound flow integration, and the logistics planner has very little visibility about the arrival time to the customers and the time and characteristics of the inbound loads.

The 3PL assigns a truck driver to each route. Then, the driver is responsible for establishing the sequence for visiting all outbound customers, the path and schedules, which may or may not be optimal. The decision of either to visit a supplier (backhaul) or not is often taken by the driver, based on the extra cost for visiting a known supplier in the vicinity of the last costumer (linehaul) visited in the route, in case it is doable within the route maximum duration length set by the 3PL business rules and conditioned by transportation legislation. There are 75 possible suppliers of wood-chips for IRs. The raw materials may be delivered back to the mill of origin or alternatively to any of the other three mills owned by the company in the Iberian Peninsula. Currently, there is no minimum amount of backhauling required. According to the experience of the planners, the reward for each backhauling can go up to 10 €/ton.

The computational results were obtained with two major groups of instances (A and B) built upon the previous case study description, each one of them corresponding to a representative operational day. Instances A differ from instances B with respect to the average distance between the linehauls. The linehauls in instances A are more geographically dispersed, with an average distance between linehauls and depot of 461 km, while in instances B it is 197 km. Baseline instances A30 and B30 correspond to the situation described, with 30 linehauls, a total demand of 1,853 ton, 75 backhauls, 4 mills and 100 available trucks. Instances among the same instance group differ in the number of linehauls to visit (10, 30 or 50 linehauls) and the number of possible backhauls (0, 25, 50, 75 or 100 backhauls). The instances were generated in a cumulative manner, i.e., the largest instances contain all locations considered in smaller instances. The selection of the locations to be included/removed in the instances was performed randomly from the dataset of the case study.

3.5.2 Comparison among distinct planning strategies

Instances A10 and B10 were used in these experiments to compare and quantify the benefits of adopting an OBP or IIOP strategy versus DIOP because it is possible to solve the model quickly to optimality while larger instances require the proposed matheuristic whose gaps to optimality could bias the results. Furthermore, the resulting routing plan can be easily visualized.

To perform this comparison, two different reward values were considered (1€/ton and 7€/ton). In order to avoid results biased by different reward values for IRs and OIRs, the backhauling reward was set regardless of the type of route (whether it was a dedicated backhaul route or an opportunistic one) and is generically called reward instead of backhauling reward. The inbound quantity to be satisfied was set to 160 ton of raw materials, which corresponds to approximately twice the outbound quantities of finished products in these instances, taking into account the mills' productive efficiency. The remaining parameters remained unchanged throughout the instances, with $\alpha = 1,200$ km and $\psi = 0.5$ ton.

For the IIOP strategy, model [P2] was solved to optimality and a given backhauling amount was set. The DIOP models [P3] and [P4] were also solved to optimality with this same backhauling amount to allow a fair comparison. In respect to the OBP strategy, the rationale to allow its comparison with the remaining strategies consisted in: (i) solving the OBP model [P0] to optimality, replacing constraints (3.15) by a similar set of constraints where a maximum (instead of minimum) backhauling amount of 160 ton is set; (ii) solving model [P4] to obtain the IRs for the differential amount between 160 ton and the already backhauled amount via OBP; (iii) computing the total costs for these two models.

The obtained results are presented in Table 3.3. In these instances, the matheuristic was not required, since the computational time for proving optimality in the solver was very short (less than 5 min on average). In these experiments, the number of binary decision variables ranged from 10,000 to 17,000.

The analysis of these results shows that the logistics planning strategy leading to the lowest cost is IIOP in all the experiments. In some cases, the strategy OBP performs better than DIOP, as intuitively expected, but in others, it does not. This is because the OBP model is

Table 3.3 – Comparison between alternative Inbound and Outbound Planning strategies

Reward (€/ton)	Instance	Planning strategy	Objective Function	Costs (€)			No. routes				Backhauled amount (ton)	No. trucks used	Runtime (s)
				Total	Fixed	Transport	Total	OIR	OR	IR			
7.00	A10	Integrated	2,536	3,656	350	3,306	7	0	3	4	160	5	821
		Opportunistic	2,714	3,834	420	3,414	7	1	3	3	160	6	45
		Decoupled	2,536	3,656	350	3,306	7	0	3	4	160	5	37
	B10	Integrated	768	1,888	280	1,608	4	3	0	1	160	4	36
		Opportunistic	771	1,891	280	1,611	4	4	0	0	160	4	32
		Decoupled	896	2,016	350	1,666	7	0	3	4	160	5	517
1.00	A10	Integrated	3,496	3,656	350	3,306	7	0	3	4	160	5	60
		Opportunistic	3,496	3,656	350	3,306	7	0	3	4	160	5	47
		Decoupled	3,496	3,656	350	3,306	7	0	3	4	160	5	34
	B10	Integrated	1,802	1,962	350	1,612	5	4	1	0	160	5	21
		Opportunistic	1,811	1,971	350	1,621	6	1	2	3	160	5	332
		Decoupled	1,856	2,016	350	1,666	7	0	3	4	160	5	243

myopic in the sense that it includes all OIRs that are cost-effective for a given backhauling reward value, but does not trade-off between OIRs and IRs as it happens with the IIOP model.

Instance B10 with a reward value of 7€/ton exemplifies a case where the IIOP strategy is better than OBP and better than DIOP strategies. The total costs of the resulting logistics plans are 1,888€, 1,891€, and 2,016€ respectively. The optimal IIOP routing plan consists of three OIRs (for trucks k_1, k_2 and k_5) and one IR (for truck k_4) (Figure 3.4a). While, the optimal plan for OBP encompasses three OIRs identical to the later plan, and one extra OIR (k_3) (Figure 3.4b). The OIR k_3 is still cost-effective for that reward value, but it is costlier than doing the alternative IR k_4 as in the IIOP plan. No IRs are foreseen in the OBP strategy because the routing plan obtained by solving model [P0] already fulfils the whole demanded backhauled amount; therefore there is no stimulus for finding IRs with model [P4] afterwards.

The decoupled planning strategy for instance B10 leads to a 6.8% increase of the total costs when compared with the previous, due to the increase of the transportation costs and also the use of five vehicles instead of four (Figure 3.4c). The overall routing plan encompasses four IRs (obtained with model [P4]), plus three ORs (obtained with model [P3]). The IRs are similar to the ones of vehicle k_4 in the IIOP strategy but the ORs are not. This is because the linehauls are re-distributed in the routes in a different way when the visit to backhauls is not considered in the same model. For example, linehaul 7 was split deliveries according to the IIOP and OBP plans due to its geographical proximity to the backhaul 13. This no longer happens in the decoupled planning, and this linehaul is visited only once in the course of a longer route that extends up to linehaul 10.

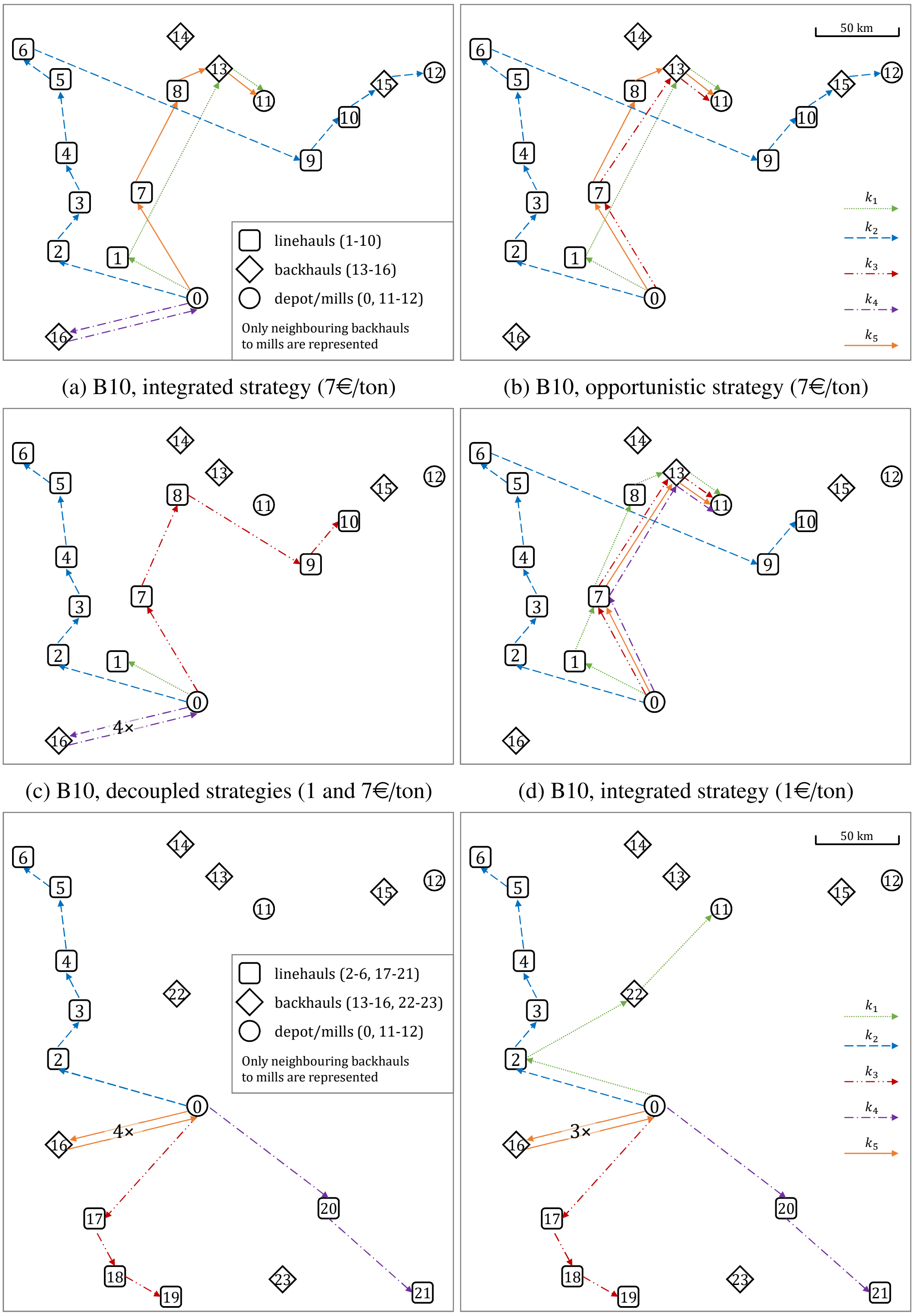
Conversely, instance A10 with a reward value of 7€/ton, exemplifies a case where the performance of the IIOP strategy is the same as the DIOP strategy (3,656€), and the OBP performs worse than the other planning strategies (3,834€, 4.9% worse). The optimal IIOP routing solution consists of three ORs and four IRs (Figure 3.4e). In this setting, with the linehauls more geographically dispersed and farther from the suppliers and neighbouring mills, it is cheaper to visit several times supplier 16 in dedicated IRs than considering OIRs. However, the solution of the OBP model, which is myopic with respect to this possibility,

encompasses one OIR that visits the cost-effective backhaul 22 (Figure 3.4f).
The analysis of these results also shows that the backhauling reward value has a significant impact on the routing plans and can lead to different conclusions with respect to the comparison between the alternative planning strategies. For example, when the reward value is lowered to 1€/ton, the results for instance B10 show that the visit to backhauls 15 and 16 are no longer cost-effective in the IIOP strategy. Hence, the routing plan consists of four OIRs, all visiting backhaul 13 (Figure 3.4d) and 1 OR. The total costs are 3.8% higher than in the experiment with a reward of 7€/ton, due to an increase in the total transportation distance and in the use of five vehicles instead of four.
It is noteworthy that lowering the value of the reward for visiting the backhauls has a negative effect on revenue and consequently, increasing the value of the objective function (134% higher than with previous experiment with 7€). For this case, the IIOP strategy still performs better than the OBP and DIOP. However, the total cost savings are reduced to 2.7% and 2.2%, respectively. This is due to the fact that with a lower reward value, the use of OIRs is less attractive, and the inbound demand must, therefore, be satisfied with dedicated backhaul routes.
In instance A10, when the reward value is lowered to 1€ per ton, there is no visit to a backhaul that is cost-effective. Hence the optimal plan for the OBP strategy does not consider any OIR, and it is identical to the IIOP and DIOP strategies described before.
These findings suggest that IIOP is the strategy that allows the optimisation of the combination between backhauling and inbound routing, but under specific circumstances that favour the supply of raw materials through cost-effective OIRs instead of direct IRs, OBP can perform better than DIOP. As shown in these experiments, these circumstances depend on the backhauling reward value for visiting a backhaul in an OIR and on the geographical configuration of the logistics network of the planning day, especially the relative distance between a linehaul that can be visited last in a route, and a neighbouring backhaul and mill.
As stated before, the opportunistic planning strategy can be considered an "intermediate" stage from DIOP towards IIOP. The transition from DIOP to opportunistic planning is smoother since it is restricted to organisational changes within the local outbound logistics offices in each mill, while the IIOP also impacts in the central office currently responsible for inbound logistics planning. During this intermediate stage using the OBP strategy, the planners need to compare the optimal routing plan with the outcome of the DIOP strategy in order to establish if backhauling is favourable for the set of customers visited in each day.

3.5.3 Managerial insights

Focusing on OBP, which is the strategy likelier to be adopted by the planners in this case study, additional experiments were designed for instances with 10 linehauls (A10, B10), to provide managerial insights on how the values of key parameters of model [P0] set by the planners with some degree of uncertainty, may actually impact on the routing plan. The parameters under study are:

- backhauling reward (δ), i.e., incentive for picking up one unit of raw material in a backhaul $b \in B$ and delivering it to a mill $m \in M$. For simplification purposes,



(a) B10, integrated strategy (7€/ton)

(b) B10, opportunistic strategy (7€/ton)

(c) B10, decoupled strategies (1 and 7€/ton)

(d) B10, integrated strategy (1€/ton)

(e) A10, integrated, decoupled (1 and 7€/ton) and opportunistic strategies (1€/ton)

(f) A10, opportunistic (7€/ton)

Figure 3.4 – Graphical representation of the planning strategies for instances A10 and B10

it is assumed that the reward is the same for all backhauls and mills that accept compatible types of raw materials. Based on experts opinions, the reward can vary in a range from 1 to 10 euros per ton;

- minimum backhauled amount (β), i.e., amount of raw materials to be backhauled in OIR. If parameter $\beta = 0$ then no visits to backhauls are required; if $\beta \geq 1$ ton, then, at least, one backhaul must be included; and if $\beta \geq 41$ ton, then, at least, two backhauls must be included), since the maximum truck capacity is 40 ton;
- maximum length of the route (α), i.e., maximum distance travelled in a route. The values tested are $1,200$ km (corresponding to the longest distance from the mill to a linehaul in instance A10) and $1,500$ km;
- minimum delivery (ψ), i.e., the minimum allowable amount of order delivered to a linehaul, conditioning the possibility of splitting the order of a linehaul into more than one deliveries done by different trucks. The values tested were $\psi = 0.5$ ton, meaning that many split deliveries are allowed, $\psi = 5$ ton and $\psi = 10$ ton (corresponding to the capacity of the smallest truck), meaning that split deliveries are more restricted.

Let us state the baseline conditions for this analysis $\delta = 6$ €/ton for instance A10 and $\delta = 1$ €/ton for instance B10, $\alpha = 1200$ km for A10 and $\alpha = 400$ km for B10, $\beta = 0$ and $\psi = 0.5$ ton. The results of the experiments presented in Table 3.4 and in the Appendix are the basis for managerial insights that can be valuable for route planning.

Impact of the variation of the value of reward for visiting a backhaul (δ)

Results generically confirm a positive effect in the objective function of increasing the value of δ, because more OIRs are performed, often with the same number of trucks. The total transportation costs increase, due to the increase in the total distances travelled, but these are compensated with a higher total reward collected. The first managerial insight that can be formulated is that planners wishing to foster an increase of OIRs should set the reward value at least equal to the extra travelling costs for visiting the most cost-effective backhaul (i.e. the costs for travelling from the last linehaul to the backhaul and from there to the closest mill).

For instance A10 the minimum δ should be 7€/ton. Below that value, there is no backhaul that is cost-effective, hence, no OIRs are included in the optimal routing plan. The number of trucks needed increases for four to five. Increasing δ to 8 €/ton improve the value of the objective function but do not change the costs, because the number of trucks and the routing plan remains the same. However, very high values of δ are not beneficial as it leads to the use of a large truck fleet. Hence, the percentual increase of total costs is much higher than the gains in the value of the objective function, and the resulting routing plan is hardly adopted in practice. For example, a δ equal to 10€/ton in instance A10 leads to costs 243% higher than in the baseline, corresponding to the highest number of 34 OIRs out of the 36 routes that compose the optimal routing plan.

Regarding instance B10, the linehauls are less geographically dispersed than in A10; thus, a slight increase in the δ leads to significant changes in the number of OIRs and the improvement in the objective function value. In fact, the baseline experiment with a δ equal to 1€/ton already leads to 3 OIRs and one for each of the trucks used. For δ equal or higher than 5€/ton the routing plan changes drastically to 39 OIRs requiring 39 extra trucks.

Impact of the variation of the required backhauled amount (β)

Experiments suggest that increasing β has a negative impact on the value of the objective function because it increases the transportation costs for the mandatory visit to backhaul. However, in some instances, such as A10, it leads to an increase in OIRs, while in others, such as B10, it leads to an increase of the number of IRs. A second managerial insight for planners relates to the fact that the geographical dispersion between the linehauls, backhauls and mills is the determining factor for finding the optimal routing plan, as discussed in Section 5.2. It is also noteworthy that, under some circumstances (e.g. for $\delta \leq 2$ and $\beta > 0$ for A10), the solution turns infeasible because the pre-processing algorithm guarantees that only cost-efficient integrated routes can be created.

Impact of the variation of the delivery amount at a linehaul (ψ)

Experiments indicate that increasing ψ has a slightly negative effect on the value of the objective function. Although this may imply the use of fewer vehicles, this also decreases the possibility of creating integrated routes and, as such, the possibility of collecting a higher total reward.

There is a complementary relation between the key parameters ψ and δ in fostering the number of OIRs in the optimal routing plan. In practice, if ψ is low, means that more visits to the linehauls are allowed, and so, there is more flexibility in the routing plan to include OIR, especially if the reward for visiting a backhaul is high. In fact, the number of OIRs is maximized (34 out of 35 routes) if ψ is very low (e.g., 0.5 ton) and δ is very high (e.g., 10€). However, these high number of integrated routes (e.g. 34 out of 36 routes) can hardly represent the common practice (Figure 3.5). Hence, another managerial insight for planners relates to the importance of properly addressing the trade-off between the offered reward and the maximum number of visits allowed to a linehaul, which is specific for each case.

Impact of the variation of the maximum length of the route (α)

Experiments show that increasing α tends to improve the objective function, due to the decrease in the number of required vehicles and the possibility to visit a larger number of linehauls is the same route. However, without a direct impact on the number of OIR. As an example for instance A10, increasing α from $1,200$ km to $1,500$ km, all other parameters remaining the same as in the baseline scenario, lead to a decrease of 32% in the value of the objective function, related with the use of 3 vehicles instead of 4. In instance B10, the increase from 400 km to 800 km, leads to a decrease of 22% in the value of the objective function due to longer routes, using the same fleet of 3 trucks.

In summary, results show that there are several trade-offs that need to be analysed by planners to balance the increase of OIRs and the increase in transportation costs. In particular, results suggest that α is the parameter that impacts the most in improving the value of the objective function and costs (improvements of 32% in instance A10 e 22% in B10, because it enables to use fewer vehicles, and fewer distances travelled, however, do not necessarily foster OIRs.

Moreover, the main parameter to be taken into account for planners willing to improve OIRs is δ. As discussed before, OIRs tend to be included in the routing plan when the reward value is above a threshold, corresponding to visiting the first cost-effective backhaul. The value of this threshold depends on the geographical dispersion of nodes in the transportation network and particularly the distance between the last visited linehaul, the closest backhaul and its neighbouring mill.

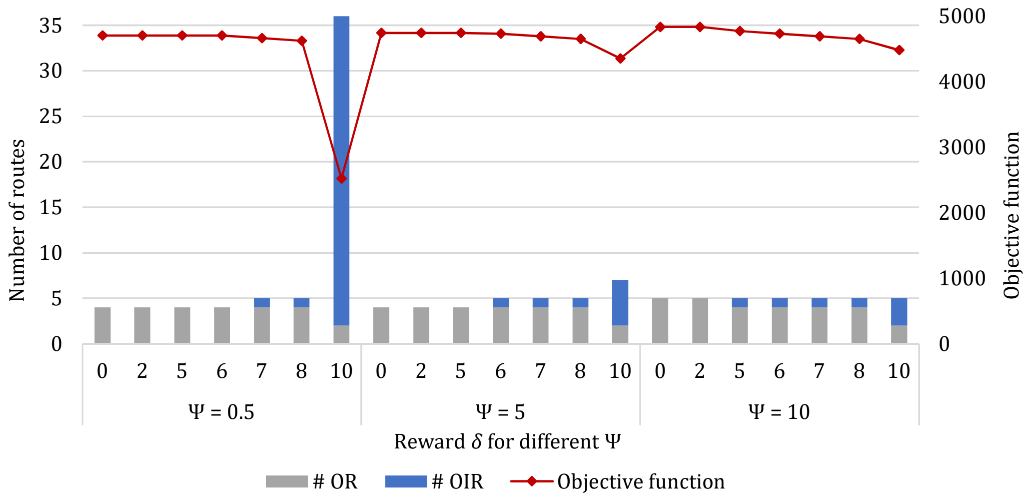


Figure 3.5 – Impact of the variation of the reward (δ) and minimum delivery (ψ) ($\alpha = 1,200, \beta = 0$)

3.5.4 Performance of the solution approach

Despite the fact that the solver is able to obtain optimal solutions within a few minutes for problem instances of 10 linehauls, this is not the case for larger instances. In these cases, the use of the matheuristic is justified in order to obtain good quality solutions in a shorter computational time. A set of computational experiments was envisaged to validate the proposed solution approach. Instances of group A and B were solved using the standalone MIP solver approach and the fix-and-optimise matheuristic. These experiments were performed in an Intel Xeon E5-2450 @ 2.10GHz CPU with capacity for 16 simultaneous processing threads.

Both approaches were run for 3,600s, with $\alpha = 1,200$, $\beta = 0$ and $\psi = 0.5$. The MIP solver was executed once for each instance using Gurobi's default parameters and the fix-and-

Table 3.4 – Summary of the experiments on the impact of the values of model parameters

Inst.	Parameter				Objective Function	Costs (€)			Routes			Runtime (s)	MIP Gap
	α	β	ψ	δ		Total	Fixed	Transport	Total	OIR	OR		
A10	**1,200**	**0**	**0.5**	**6**	**4,703**	**4,703**	**280**	**4,423**	**4**	**0**	**4**	**19**	**1.8%**
	–	–	–	7	-1%	+5%	+25%	+4%	+1	+1	0	105	0.6%
	–	–	–	8	-2%	+5%	+25%	+4%	+1	+1	0	112	0.3%
	–	–	–	10	-46%	+243%	+800%	+208%	+32	+34	-2	272	1.9%
	–	1	–	–	0%	+5%	+25%	+4%	+1	+1	0	19	1.0%
	–	41	–	–	+2%	+12%	+50%	+10%	+2	+2	0	190	0.0%
	1,500	–	–	–	-32%	-32%	-25%	-32%	-1	0	-1	26	1.6%
	–	–	5	–	+1%	+6%	+25%	+4%	+1	+1	0	28	2.0%
	–	–	5	10	-7%	+35%	+75%	+33%	+3	+5	-2	304	0.4%
	–	–	10	–	+1%	+6%	+25%	+4%	+1	+1	0	130	1.9%
	–	–	10	10	-5%	+21%	+25%	+21%	+1	+3	-2	308	0.7%
B10	**400**	**0**	**0.5**	**1**	**2,002**	**2,002**	**210**	**1,792**	**3**	**3**	**0**	**16**	**0.0%**
	–	–	–	2	0%	+4%	+33%	+1%	+1	-2	+3	28	0.0%
	–	–	–	5	-105%	+284%	+1,300%	+165%	+39	+36	+3	32	0.0%
	–	1	–	–	+2%	+4%	+33%	+1%	+1	-2	+3	39	0.0%
	–	41	–	–	+7%	+11%	+67%	+4%	+2	-1	+3	41	0.0%
	800	–	–	–	-22%	-20%	0%	-22%	0	-2	+2	72	0.0%
	–	–	5	–	0%	0%	0%	0%	0	-3	+3	25	0.0%
	–	–	5	5	-23%	+57%	+267%	+32%	+8	+5	+3	20	0.0%
	–	–	10	–	0%	0%	0%	0%	0	-3	+3	29	0.0%
	–	–	10	5	-15%	+25%	+133%	+13%	+4	+1	+3	18	0.0%

Legend: α: maximum length of the route (km); β: minimum backhauled amount (ton); ψ: minimum delivery amount (ton); δ: backhauling reward (€/ton); Runtime: computational time after which no better solution was obtained (seconds); MIP Gap: percentual difference obtained by Gurobi between the upper and lower bound of the branch-and-bound method. All models were run with a maximum time limit of 3,600s. The first row of results for each instance (highlighted in bold) contains the baseline values, and the rows that follow exhibit either the absolute or the percentual variation compared with the baseline values (except for Runtime and MIP Gap values).

optimise approach was run 10 times for each instance, using the parameters described in Table 3.5.

Table 3.5 – Used parameters for the matheuristic approach

Parameters		Value
Termination criteria	Time limit	3,600s
No-improvement criterion	Improvement between consecutive iterations lower than	100
`RouteRelease` sub-problem	Subproblem sizes	4, 6, 8, 16 routes
	No-improvement limit to change subproblem size	2 iterations
	MIP solver iteration time limit	Multiples of 15s (according to sub-problem size)
`LocationRelease` sub-problem	Subproblem sizes	2, 4, 6, 8 linehauls
	No-improvement limit to change subproblem size	2 iterations
	MIP solver iteration time limit	Multiples of 15s (according to sub-problem size)

Table 3.6 summarizes the computational results of the MIP solver and matheuristic approaches for the 30 problem instances.

The results demonstrate that both the MIP solver and the matheuristic are adequate for solving instances up to 10 linehauls (groups A10 and B10), as the solver is able to prove optimality for most instances and the matheuristic easily reaches the same solution as the MIP solver. For larger instances, the MIP solver yields optimality gaps up to 32% for instances of groups A30, A50, B30 and B50. Specifically to instances of group A, it is possible to observe the increase in the number of routes that perform backhauling as the number of backhaul locations progressively increases. In instances of group B30, a single OIR is used when backhauling is allowed, and in group B50 no opportunistic backhauling is performed. However, the obtained solutions by the MIP solver when the number of backhauls increases do not necessarily improve, contrarily to what would be expected. Furthermore, for instances of group B50, the solver is unable to find a single feasible solution within the 1-hour limit for 4 out of the 5 instances. This fact is probably due to the increase in model size and complexity when more backhaul locations are being considered, thus requiring more time for Gurobi to reach identical solutions when exploring the branch-and-bound tree.

The matheuristic approach exhibits small standard deviation values for the 10 repetitions performed for each instance, thus suggesting that the obtained results are robust. The negative percentual difference values between the solver and the matheuristic suggest that the matheuristic is able to converge correctly to better solutions, as opposed to the solver, which exhibits very high optimality gaps. This negative percentual difference tends to be increasingly more expressive with the increase in instance size. Furthermore, results also suggest that the matheuristic also takes better advantage of the increase in the number of backhaul locations, as the objective function values generally decrease when the number of backhaul locations increases.

Table 3.6 – Computational results of the MIP and matheuristic approaches for 30 problem instances

Problem instance						**MIP Solver**					**Fix-and-optimise**					**% diff.**
											Objective Function					
Group	$\|L\|$	$\|B\|$	$\|K\|$	$\sum q_i$	$\sum Q_k$	OF	MIP Gap	Runtime	No. routes	No. OIRs	Average	Standard Deviation	Runtime	No. routes	No. OIRs	
A10	10	0	100	82	2,600	3,215	0.0%	50	3	0	3,215	0	31	3	0	0.0%
	10	25	100	82	2,600	3,215	0.0%	205	3	0	3,215	0	37	3	0	0.0%
	10	50	100	82	2,600	3,215	0.0%	60	3	0	3,215	0	28	3	0	0.0%
	10	75	100	82	2,600	3,215	0.0%	55	3	0	3,215	0	30	3	0	0.0%
	10	100	100	82	2,600	3,215	0.0%	31	3	0	3,215	0	33	3	0	0.0%
A30	30	0	100	1,853	2,600	13,745	18.4%	3,110	53	0	13,627	19	2,957	53	0	-0.9%
	30	25	100	1,853	2,600	13,696	19.6%	3,444	54	26	13,404	20	2,885	53	25	-2.1%
	30	50	100	1,853	2,600	13,701	20.1%	2,758	54	32	13,374	21	2,879	53	31	-2.4%
	30	75	100	1,853	2,600	13,833	20.9%	3,187	55	33	13,366	19	2,576	53	31	-3.4%
	30	100	100	1,853	2,600	13,702	20.1%	1,877	56	34	13,369	20	2,731	53	31	-2.4%
A50	50	0	100	2,061	2,600	20,986	29.9%	2,968	65	0	19,465	363	3,463	64	0	-7.2%
	50	25	100	2,061	2,600	19,717	26.5%	1,902	64	30	19,019	194	3,527	63	26	-3.5%
	50	50	100	2,061	2,600	19,907	27.3%	3,469	65	37	19,079	313	3,397	64	31	-4.2%
	50	75	100	2,061	2,600	20,829	30.5%	3,019	65	36	19,167	423	3,464	64	32	-8.0%
	50	100	100	2,061	2,600	21,365	32.3%	2,653	66	33	19,116	327	3,513	65	36	-10.5%
B10	10	0	100	82	2,600	1,575	1.9%	129	3	0	1,575	0	39	3	0	0.0%
	10	25	100	82	2,600	1,575	2.7%	117	3	0	1,575	0	39	3	0	0.0%
	10	50	100	82	2,600	1,575	0.0%	53	3	0	1,575	0	39	3	0	0.0%
	10	75	100	82	2,600	1,575	0.0%	51	3	0	1,575	0	44	3	0	0.0%
	10	100	100	82	2,600	1,575	0.0%	51	3	0	1,575	0	61	3	0	0.0%
B30	30	0	100	818	2,600	8,695	20.1%	3,032	23	0	8,210	9	2,267	22	0	-5.6%
	30	25	100	818	2,600	8,626	19.4%	3,354	23	1	8,212	15	2,730	21	1	-4.8%
	30	50	100	818	2,600	9,162	30.5%	2,473	23	0	8,206	12	2,292	22	1	-10.4%
	30	75	100	818	2,600	8,350	16.8%	3,262	21	1	8,206	10	2,876	21	1	-1.7%
	30	100	100	818	2,600	9,003	32.1%	441	23	1	8,198	2	2,474	22	1	-8.9%
B50	50	0	100	2,054	2,600	–	–	–	–	–	45,393	956	3,419	68	0	–
	50	25	100	2,054	2,600	–	–	–	–	–	45,325	793	3,420	69	0	–
	50	50	100	2,054	2,600	–	–	–	–	–	45,759	526	3,388	64	0	–
	50	75	100	2,054	2,600	46,991	13.8%	2,830	69	0	45,592	655	3,359	65	0	-3.0%
	50	100	100	2,054	2,600	–	–	–	–	–	45,768	646	3,412	67	0	–
															Average	**-3.0%**

Legend: $|L|$: number of linehauls to be visited; $|B|$: number of possible backhauls; $|K|$ total number of vehicles available; $\sum q_i$: total quantity to be delivered to linehauls (ton); $\sum Q_k$: total vehicle transportation capacity (ton); OF: Final value of the objective function (€); Runtime: average computational time after which no better solution was obtained (seconds); MIP Gap: Percentual difference obtained by Gurobi between the upper and lower bounds of the branch-and-bound method; No. routes: total number of routes; No. OIR: number of routes that include visit to a backhaul; % diff.: percentual difference of the fix-and-optimise average objective function towards Gurobi's objective function (a negative difference favours the matheuristic). For the fix-and-optimise approach, the route indicators correspond to the repetition whose objective function value was closest to the obtained average.

From these results, we can say that the proposed matheuristic approach is adequate for solving the problem at hand. For instances of considerable size, the MIP solver starts to struggle in finding feasible solutions in an acceptable time limit, and apparently also has greater difficulties taking advantage of backhauling, while the matheuristic is able to decrease the overall logistics costs with an increase in the number of backhaul locations, therefore yielding more consistent results.

3.6. Conclusions and future work

Integrating planning processes requires a thorough assessment of both quantitative benefits pertaining to the expected decrease in the related costs and qualitative impacts related to the usual need of breaking functional silos. This work explored, mainly, the quantitative aspect of integrating outbound and inbound logistics routes. We used as a background a case-study from the wood-based panel industry, but the results and approaches developed are generalisable for other settings in which this integration can be modelled as an rVRPB (e.g., grocery retail, cement distribution). Besides modelling three possible planning strategies (i.e., OBP, IIOP, and DIOP), we have also developed a matheuristic to tackle real-world instances of this problem.

Three key conclusions emerge from our computational study. Firstly, the intuitive idea that intermediate levels of integration would always result in better planning outcomes was not verified. DIOP outperforms OBP in certain geographical contexts where the distribution network is more dispersed. In our studies, this happened in instance A10 when the average distance between linehaul customers and the depot of origin is 197 km. In this case, it was actually cheaper to assure the supply of raw material through dedicated inbound routes (i.e. going to and from the nearest supplier) than including a visit to a supplier at the end of the outbound route (i.e. after visiting all customers). The IIOP model does this trade-off, but the OBP model is myopic to the possibility of doing direct inbound routes, hence, leading to worse results than DIOP.

Secondly, we confirm that there are important parameters dealt by the planners with some degree of uncertainty that actually can have a great influence on the total costs of the routing plan. This study analysed four of these parameters - backhauling reward, minimum backhauled amount, maximum length of the route and minimum delivery amount allowed. Results suggest that increasing the maximum length of the route leads to the largest impact in the performance of the routing plan but including a quantitative reward for each supplier visited will likelier increase the proportion of integrated inbound and outbound routes in the overall routing plan. In fact, the total reward (€/ton) should be equal or higher than the extra transportation costs for the most cost-effective supplier. Meaning that the extra distance travelled empty from the last customer to the nearest supplier and then full from there to the neighbouring mill is minimized.

Finally, the developed matheuristic proved to be a suitable approach to tackle this problem and this fact reiterated the interest of fix-and-optimise to solve routing problems.

Future work could be devoted to merging the qualitative and quantitative assessments related to the integration of planning processes. In particular, the study of integrated inbound and outbound routes is of interest due to its potential in improving the ever-relevant sustainability dimension.

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Appendix 3.A Supplementary material

Table 3.7 – Experiments on the impact of the values of model parameters

Inst.	Parameter				Objective Function	Costs (€)			Routes			Runtime (s)	MIP Gap
	α	β	ψ	δ		Total	Fixed	Transport	Total	OIR	OR		
A10	1200	0	0.5	6	4703	4703	280	4423	4	0	4	19	1.8%
A10	1200	0	0.5	7	4665	4945	350	4595	5	1	4	105	0.6%
A10	1200	0	0.5	8	4625	4945	350	4595	5	1	4	112	0.3%
A10	1200	0	0.5	10	2523	16123	2520	13603	36	34	2	272	1.9%
A10	1200	0	5	6	4732	4972	350	4622	5	1	4	28	2.0%
A10	1200	0	5	7	4692	4972	350	4622	5	1	4	219	0.6%
A10	1200	0	5	8	4652	4972	350	4622	5	1	4	123	0.3%
A10	1200	0	5	10	4356	6356	490	5866	7	5	2	304	0.4%
A10	1200	0	10	6	4732	4972	350	4622	5	1	4	130	1.9%
A10	1200	0	10	7	4692	4972	350	4622	5	1	4	175	1.1%
A10	1200	0	10	8	4652	4972	350	4622	5	1	4	237	0.5%
A10	1200	0	10	10	4482	5682	350	5332	5	3	2	308	0.7%
A10	1200	1	0.5	6	4705	4945	350	4595	5	1	4	19	1.0%
A10	1200	1	0.5	7	4665	4945	350	4595	5	1	4	109	0.8%
A10	1200	1	0.5	8	4625	4945	350	4595	5	1	4	153	0.2%
A10	1200	1	0.5	10	2523	16123	2520	13603	36	34	2	2523	0.7%
A10	1200	1	5	6	4732	4972	350	4622	5	1	4	137	0.5%
A10	1200	1	5	7	4692	4972	350	4622	5	1	4	357	0.7%
A10	1200	1	5	8	4652	4972	350	4622	5	1	4	241	0.6%
A10	1200	1	5	10	4356	6356	490	5866	7	5	2	165	0.2%
A10	1200	1	10	6	4732	4972	350	4622	5	1	4	96	0.2%
A10	1200	1	10	7	4692	4972	350	4622	5	1	4	57	0.4%
A10	1200	1	10	8	4652	4972	350	4622	5	1	4	200	0.1%
A10	1200	1	10	10	4482	5682	350	5332	5	3	2	94	0.5%
A10	1200	41	0.5	6	4802	5282	420	4862	6	2	4	190	0.0%
A10	1200	41	0.5	7	4722	5282	420	4862	6	2	4	226	0.1%
A10	1200	41	0.5	8	4642	5282	420	4862	6	2	4	126	0.2%
A10	1200	41	0.5	10	2523	16123	2520	13603	36	34	2	185	0.8%
A10	1200	41	5	6	4828	5308	420	4888	6	2	4	234	0.5%
A10	1200	41	5	7	4748	5308	420	4888	6	2	4	155	0.5%
A10	1200	41	5	8	4668	5308	420	4888	6	2	4	180	0.6%
A10	1200	41	5	10	4356	6356	490	5866	7	5	2	107	0.6%
A10	1200	41	10	6	4830	5310	350	4960	5	2	3	166	0.3%
A10	1200	41	10	7	4750	5310	350	4960	5	2	3	57	0.4%
A10	1200	41	10	8	4670	5310	350	4960	5	2	3	89	0.7%
A10	1200	41	10	10	4482	5682	350	5332	5	3	2	67	0.1%
A10	1500	0	0.5	6	3215	3215	210	3005	3	0	3	26	1.6%
A10	1500	0	0.5	7	3177	3457	280	3177	4	1	3	357	1.0%
A10	1500	0	0.5	8	3137	3457	280	3177	4	1	3	123	2.3%
A10	1500	0	0.5	10	1034	14634	2450	12184	35	34	1	504	0.3%

Legend: α: maximum length of the route (km); β: minimum backhauled amount (ton); ψ: minimum delivery amount (ton); δ: backhauling reward (€/ton); Runtime: computational time after which no better solution was obtained (seconds); MIP Gap: percentual difference obtained by Gurobi between the upper and lower bound of the branch-and-bound method. All models were run with a maximum time limit of 3,600s.

Table 3.7 (cont.) – Experiments on the impact of the values of model parameters

Inst.	Parameter				Objective Function	Costs (€)			Routes			Runtime (s)	MIP Gap
	α	β	ψ	δ		Total	Fixed	Transport	Total	OIR	OR		
B10	400	0	0.5	1	2002	2002	210	1792	3	0	3	16	0.0%
B10	400	0	5	1	2002	2002	210	1792	3	0	3	25	0.0%
B10	400	0	10	1	2002	2002	210	1792	3	0	3	29	0.0%
B10	400	0	0.5	2	2001	2081	280	1801	4	1	3	28	0.0%
B10	400	0	5	2	2001	2081	280	1801	4	1	3	21	0.0%
B10	400	0	10	2	2002	2002	210	1792	3	0	3	18	0.0%
B10	400	0	0.5	5	-108	7692	2940	4752	42	39	3	32	0.0%
B10	400	0	5	5	1534	3134	770	2364	11	8	3	20	0.0%
B10	400	0	10	5	1710	2510	490	2020	7	4	3	18	0.0%
B10	400	1	0.5	1	2041	2081	280	1801	4	1	3	39	0.0%
B10	400	1	5	1	2041	2081	280	1801	4	1	3	37	0.0%
B10	400	1	10	1	2059	2099	280	1819	4	1	3	21	0.0%
B10	400	1	0.5	2	2001	2081	280	1801	4	1	3	33	0.0%
B10	400	1	5	2	2001	2081	280	1801	4	1	3	36	0.0%
B10	400	1	10	2	2019	2099	280	1819	4	1	3	30	0.0%
B10	400	1	0.5	5	-108	7692	2940	4752	42	39	3	27	0.0%
B10	400	1	5	5	1534	3134	770	2364	11	8	3	19	0.0%
B10	400	1	10	5	1710	2510	490	2020	7	4	3	16	0.0%
B10	400	41	0.5	1	2133	2213	350	1863	5	2	3	41	0.0%
B10	400	41	5	1	2133	2213	350	1863	5	2	3	32	0.0%
B10	400	41	10	1	2133	2213	350	1863	5	2	3	20	0.0%
B10	400	41	0.5	2	2053	2213	350	1863	5	2	3	27	0.0%
B10	400	41	5	2	2053	2213	350	1863	5	2	3	20	0.0%
B10	400	41	10	2	2053	2213	350	1863	5	2	3	10	0.0%
B10	400	41	0.5	5	-108	7692	2940	4752	42	39	3	38	0.0%
B10	400	41	5	5	1534	3134	770	2364	11	8	3	40	0.0%
B10	400	41	10	5	1710	2510	490	2020	7	4	3	14	0.0%
B10	800	0	0.5	1	1566	1606	210	1396	3	1	2	72	0.0%
B10	800	0	0.5	2	1503	1743	210	1533	3	3	0	23	0.0%
B10	800	0	0.5	5	-778	7222	2800	4422	40	40	0	54	0.0%

Legend: α: maximum length of the route (km); β: minimum backhauled amount (ton); ψ: minimum delivery amount (ton); δ: backhauling reward (€/ton); Runtime: computational time after which no better solution was obtained (seconds); MIP Gap: percentual difference obtained by Gurobi between the upper and lower bound of the branch-and-bound method. All models were run with a maximum time limit of 3,600s.

Chapter 4

A Robust Vehicle Routing Problem with Backhauls

A robust optimization approach for the vehicle routing problem with selective backhauls

Maria João Santos [*†] · **Eduardo Curcio**[*] · **Mauro Henrique Mulati**[‡] · **Pedro Amorim**[*†] · **Flávio Keidi Miyazawa**[‡]

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Abstract The Vehicle Routing Problem with Selective Backhauls (VRPSB) aims to minimize the total routing costs minus the total revenue collected at backhaul customers. We explore a VRPSB under uncertain revenues. A deterministic VRPSB is formulated as a mixed-integer programming problem and two robust counterparts are derived. A novel method to estimate the probabilistic bounds of constraint violation is designed. A robust metaheuristic is developed, requiring little time to obtain feasible solutions with average gap of 1.26%. The robust approach studied demonstrates high potential to tackle the problem, requiring similar computing effort and maintaining the same tractability as the deterministic modeling.

Keywords vehicle routing problem with selective backhauls · robust optimization · branch-and-cut · adaptive large neighborhood search

4.1. Introduction

The Vehicle Routing Problem with Backhauls (VRPB) is an extension of the classic Vehicle Routing Problem (VRP), where two different types of customer are considered in the network: linehaul customers, those who receive goods from a depot (outbound logistics), and backhaul customers, those who send goods back to the depot (inbound logistics). Integrated inbound-outbound logistics planning can reduce the distance of empty trips, which are responsible for significant transportation costs (Liu and Chung, 2008), thus increasing

[*]Faculdade de Engenharia, Universidade do Porto, Porto, Portugal

[†]INESC TEC - Instituto de Engenharia de Sistemas e Computadores, Tecnologia e Ciência, Porto, Portugal

[‡]Instituto de Computação - Universidade Estadual de Campinas (Unicamp) and Departamento de Ciência da Computação – Universidade Estadual do Centro-Oeste (Unicentro), Brazil

the efficiency of both vehicles and routes, and reducing fuel consumption and emissions of air pollutants (Pradenas et al., 2013).

One approach whereby the VRPB includes an optional selection of backhauls is the Vehicle Routing Problem with Selective Backhauls (VRPSB) (Baldacci et al., 2010). In this type of problems, the backhauls are selected according to the revenues they provide. The first application of the VRPSB is described in Yano et al. (1987) to solve a real transportation problem of a company operating in a retail chain. The company owned a fleet of vehicles and had to plan delivery routes from the distribution center to some stores and pickup routes from vendors to the distribution center. Since the private fleet could not cover all the vendors, due to capacity limitations, the company outsourced carriers to pickup the material from the remaining ones. Therefore, one of the decisions of the problem was to select which backhauls to visit. By optimizing the VRPSB, the company was able to save a considerable amount of money.

Most of the exact approaches used to solve VRP rely on cut and column generation algorithms (Poggi and Uchoa, 2014; Baldacci et al., 2012). Therefore, these are also the majority of exact solution methods used to solve both VRPB and VRPSB. Gutiérrez-Jarpa et al. (2009) formulate a VRPSB as a mixed integer linear programming model and use a branch-and-price algorithm to solve adapted instances from Solomon (1987). Only those instances with 50 customers or less could be optimally solved. Very recently, Granada-Echeverri et al. (2019) model a VRPB with a mixed integer linear programming formulation based on characteristics of an Open VRP, allowing to obtain several new best known solutions for the instances of Goetschalckx and Jacobs-Blecha (1989) with up to 113 customers and of Toth and Vigo (1997) with up to 100 customers.

Due to high computational performance, heuristics are the most used algorithms to solve the majority of VRPB and its variants. For the VRPSB, García-Nájera et al. (2015) proposes a similarity-based selection multi-objective evolutionary algorithm (SSMOEA) to solve a multi-objective problem considering minimization of number of vehicles, travel distance and uncollected revenue. The metaheuristic was tested for adapted instances of Goetschalckx and Jacobs-Blecha (1989), providing solutions for instances with up to 150 customers in the range of 1000 to 8000 seconds.

Notwithstanding the practical application of the VRPSB to real-world problems (e.g., parcel services), few works have been described in the literature that use this variant of the VRP (Gutiérrez-Jarpa et al., 2009). This statement is also valid for other similar problems such as the VRP with Deliveries and Selective Pickups (VRPDSP). The VRPDSP often arises in reverse logistics, where a vehicle delivers a request to a customer and may collect an amount of material in the same location, resulting in a profit (Süral and Bookbinder, 2003; Privé et al., 2006; Bruck et al., 2012; Coelho et al., 2012). Thus, in the VRPDSP, customers requiring deliveries are also those requiring pickups.

In the VRPSB, selecting one backhaul rather than another depends highly on the revenue it provides and its impact in mitigating routing costs. For instance, the revenue may be related to the quality of products to pick up or the suppliers may have several products available with different quality attributes (Zanjani et al., 2010; Alvarez and Vera, 2014; Andersson et al., 2016). However, to the best of our knowledge, only Allahviranloo et al. (2014) reported the development of a selective VRP model with uncertainty in revenues, showing evidence of the advantages of addressing uncertainty in humanitarian logistics. More surprisingly, a recent literature review on the VRPB research shows that no work has yet investigated this problem under uncertainty, which is usually present in practice (Koç and Laporte, 2018).

To deal with uncertainty in VRPs, two main approaches stand out in the literature: stochastic programming and robust optimization (Averbakh, 2001; Bertsimas and Sim, 2003). Stochastic programming is a well-known technique used when the probability distribution of the uncertain parameters are known, and the goal is to optimize the expected value of a solution while maintaining its feasibility for the scenarios considered (Birge and Louveaux, 2011). When no knowledge on the probability distribution of the uncertainty is available, robust optimization emerges as a suitable alternative (Chardy and Klopfenstein, 2012; Grossmann et al., 2016). In this case, the goal is to obtain a solution that is robust (i.e. feasible) against all possible uncertainty realizations. Robust Optimization (RO) is, in fact, the most recent trend to deal with uncertainty in optimization problems (Grossmann et al., 2016).

The present paper analyses the robustness of routing plans obtained under revenue uncertainty for a robust VRPSB, where it is required to achieve a minimum revenue from collected raw-materials. The interest on studying the uncertain revenues derive from two main aspects: (i) a deviation in the quality of raw-materials can impact the efficiency of the productive processes (at the depot) and, consequently, the production costs of a company, and (ii) for large distances, and particularly for daily routes, the deviation in the expected revenue may turn the predefined routes very costly for consecutive days, which may cause significant financial damage to the company. Particularly for the wood-based supply chain, the quality of raw-materials is becoming more and more important, since, in one hand, it allows to differentiate suppliers (Andersson et al., 2016) and, on the other hand, it impacts the efficiency of subsequent manufacturing processes (Zanjani et al., 2010; Alvarez and Vera, 2014). In addition, although several characteristics of raw-materials can be easily obtained, such as diameter and length, others can only be roughly estimated, such as moisture.

An interesting finding from the literature review on RO is that, in opposition to exact methods, few metaheuristics have been proposed to solve the VRP under uncertainty. In fact, Solano-Charris et al. (2015) argue that, up to the date of their work, only four metaheuristics were proposed in the literature: a particle swarm optimization (PSO) algorithm (Moghaddam et al., 2012), a differential evolution algorithm (DEA) (Cao et al., 2014), and two Ant Colony Optimization (ACO) algorithms (Toklu et al., 2013; Toklu et al., 2014). More recently, some research has been driven by improving or adapting well-known metaheuristics to handle the robust VRP. Gounaris et al. (2016) present an Adaptive Memory Programming (AMP) metaheuristic to solve the VRP under demand uncertainty, which considers two different cases of polyhedral uncertainty sets - budget support and factor-model support. The AMP metaheuristic is enhanced with a new mechanism to identify and select promising solution components and a new augmented objective function. The LANTIME metaheuristic, based on the Tabu Search (TS) algorithm, is adapted by Groß et al. (2019) to solve a robust VRP with uncertain travel times, which are represented by interval scenarios. The adapted version of the metaheuristic includes a new procedure to evaluate the cost of a solution, which compares a candidate solution in a given scenario and in worst-case scenario. Hu et al. (2018) study a robust VRP where the uncertain travel time and uncertain demand are assumed to belong to a new route-dependent budgeted uncertainty set. To solve the robust problem, an Adaptive Variable Neighborhood Search (AVNS) is modified in order to include two new steps in the standard algorithm: an adaptive shaking and a local search. The adaptive shaking step uses two exchange operators to explore neighborhoods, whereas the local search step uses a randomized variable neighborhood descent (RVNS) heuristic to improve the solutions. An enhanced Large Neighborhood Search (LNS) is proposed in Wu et al. (2017) and in Pelletier et al. (2019) to solve large scale VRP. The former represents the uncertain travel time as discrete scenarios and proposes a new robust criterion to evaluate the robustness of solutions. The LNS is applied to explore the solution space and it is hybridized with a series of local moves to improve the solutions. The latter investigates several uncertainty sets for the robust Electric-VRP with uncertain energy consumption, namely box, polyhedral (budget and route-dependent budget), and ellipsoidal sets. The LNS used to solve the problem is combined with a Set Partitioning (SP) formulation in a two-phase method. First, the metaheuristic generates a set of candidate routes and then a complete robust solution is achieved by solving the SP problem. Furthermore, the LNS proposed by Pelletier et al. (2019) uses all the components of the Adaptive LNS (ALNS), except the adaptive procedure on the basis that previous results have shown that no solution improvement was obtained for the particular problem. Nonetheless, the ALNS has been successfully applied for the robust VRP under uncertainty in travel/service times. Braaten et al. (2017) uses the ALNS in a modular fashion, allowing to investigate different versions of the metaheuristic, namely by excluding a preprocessing phase, excluding the local search heuristics, and replacing a new acceptance rule. This new acceptance rule only accepts a new solution that is not worse than a previous one, instead of measuring it against a given threshold, as in the original ALNS proposed by Ropke and Pisinger (2006). Eshtehadi et al. (2020) assumes that the uncertain service times belong to an interval, but the robust counterpart derived is based on the very conservative worst-case approach. The ALNS used to solve the problem departs from the enhanced method proposed by Demir et al. (2012) and introduces new components, such as new removal operators. Finally, metaheuristics have also been enhanced to solve problems beyond the scope of robust VRP, such as the location-routing problem (LRP) with uncertain spatial customer distribution, demand and service time (Schiffer and Walther, 2018), the inventory-routing problem (IRP) with uncertain demand (Fardi et al., 2019), the intermodal freight transportation problem (IFTP) with uncertain transportation costs and uncertain capacities of terminals (Abbassi et al., 2019), and the Capacitated Arc Routing Problem (CARP) with uncertain demand (Tirkolaee et al., 2018).

We explore the aspects of RO and its respective budget of uncertainty, using exact and metaheuristics solution methods. We define a function to assess the robustness of solutions embedded in a state-of-art metaheuristic for a large spectrum of VRPB variants, namely the ALNS. In addition, we compare a RO model with a chance-constrained (CC) model and study four different methods to derive bounds for the probability of constraint violation. Moreover, a factor model support describing the uncertain revenues is proposed as an alternative to the budget of uncertainty. The main contributions of this paper are presented as follows, by decreasing order of significance.

1. A novel method to estimate tighter bounds for the probability of constraint violation for the robust optimization under polyhedral uncertainty sets;

2. Two different formulations for the VRPSB under revenue uncertainty, contributing simultaneously for two gaps in literature (lack of works dealing with uncertainty in VRPBs and uncertain revenues in VRPs);

3. A simple and efficient procedure to check the feasibility of robust solutions embedded in the state-of-art metaheuristic ALNS.

The remainder of this paper is organized as follows. The deterministic formulation, as well as the RO models for the VRPSB are detailed in Section 4.2. The methods to derive probabilistic bounds and an alternative uncertainty support are also defined. In Section 4.3, both solution methods used in this work are presented, which are respectively a Branch-and-Cut (B&C) algorithm and an ALNS metaheuristic. The main results are presented and discussed in Section 4.4, particularly regarding the robustness of the models under uncertainty, the structure of the solutions and computational performance of both solution methods. Finally, conclusions and insights of this work are stated in Section 4.5.

4.2. Problem description and formulations

In this section, a mixed integer programming (MIP) formulation for the deterministic VRPSB is presented first. The deterministic model is then reformulated with the proper robust counterpart in order to incorporate uncertainty in revenues. Afterwards, four different methods of estimating probabilistic bounds are designed to estimate the probability of constraint violation. Furthermore, a new support for the uncertainty set, namely the factor model support, is presented and illustrated for the VRPSB under revenue uncertainty.

4.2.1 Deterministic VRPSB

The problem can be defined by an undirected graph $\mathcal{G} = (\mathcal{V},\mathcal{E})$ and a fleet of K vehicles. Set $\mathcal{L} = \{1,\dots,n\}$ comprises the linehaul customers with demands $q_i > 0$ for all $i \in \mathcal{L}$. Set $\mathcal{B} = \{n+1,\dots,n+m\}$ comprises the backhaul customers with revenues $p_i > 0$ for all $i \in \mathcal{B}$. The depot is partitioned into two vertices: an origin depot 0 and a destination depot $n+m+1$. We anticipate that a split depot allows a better adaptation of the problem to be solved in the B&C, as discussed in Section 4.3.1. Finally, the set of all vertices in the network is $\mathcal{V} = \{0,n+m+1\} \cup \mathcal{L} \cup \mathcal{B}$. The edges in $\mathcal{E}$ are given by $\{(0, j)\colon j \in \mathcal{L}\} \cup \{(i, j)\colon i \in \mathcal{L} \cup \{n+m+1\},\ j \in \mathcal{L} \cup \{n+m+1\},\ i < j\} \cup \{(i, j)\colon i \in \mathcal{L},\ j \in \mathcal{B}\} \cup \{(i,n+m+1)\colon i \in \mathcal{B}\}$ with costs $c_{ij} \geq 0$ for all edges $(i, j) \in \mathcal{E}$. Note that, for the sake of a clean notation we are using the notation (i, j) to represent an (undirected) edge given by $\{i, j\}$. Set $\mathcal{K} = \{1,\dots,K\}$ represents the fleet of identical vehicles, each one with a capacity $C > 0$, all of them initially located at the depot. A route is a sequence of vertices given by $r = (i_0,i_1,i_2,\dots,i_s,i_{s+1})$ with $i_0 = 0$ and $i_{s+1} = n+m+1$ and with $\{i_1,i_2,\dots,i_s\} \subseteq \mathcal{L} \cup \mathcal{B}$. The total cost of a route is given by the sum of its costs minus its revenue. The objective is to find a set of at most $|\mathcal{K}|$ routes (tours) minimizing the total cost in such a way that: (i) all linehaul customers are visited, each one in exactly one route; (ii) all the linehaul customers of a route must be visited before the backhaul customer (if any); (iii) there is no route with just one customer if it is a backhaul; (iv) the sum of the demands of the linehaul customers in a route is at most C; and (v) a minimum revenue obtained from the raw-materials picked up at backhaul customers is attained. Four additional features of real world transportation problems are assumed for the VRPSB: (vi) the revenue associated to each backhaul customer is a function of external factors not related to route planning; (vii) each route visits at most one backhaul customer; (viii) backhaul customers have enough raw-material available to supply the depot with multiple vehicles; and (ix) backhaul customers can only be visited after all deliveries are performed (precedence constraint). These assumptions derive from the fact that many inbound routes are supported by full truckload vehicles that, in a single visit to a backhaul customer, collect an amount of load that equal their capacity. Example of such cases are found in the manufacturing industry (e.g., forests with large supply for mills) and in retail (e.g., supplier inbound operations to the retailers' distribution centers). In particular, for wood-based supply chains the inbound transport is often carried in full truckloads (e.g., Carlsson and Rönnqvist (2007); Hirsch (2011); Derigs et al. (2012)). It is also worth mentioning that woods parks do not usually present capacity problems to accommodate several types of raw-materials. Finally, the precedence constraint is an important aspect in practice because: (i) vehicles are often rear-loaded and this precedence constraint avoids the rearrangement of the loads at linehaul customers (Dominguez et al., 2016; Kumar et al., 2011) and (ii) linehaul customers have higher service priority than backhaul customers (Ropke and Pisinger, 2006; Tütüncü et al., 2009) because they have time-windows necessary to inspect the received loads.

To better illustrate the network of the VRPSB, an example of a feasible solution is given in Figure 4.1. In this example, the network is composed of seven linehaul customers (white circles), five backhaul customers (grey circles) and a depot (partitioned into two squares). Four different routes are created. Two vehicles visit backhaul customers after visiting the linehaul customers (dashed and double lines) and two other vehicles only visit linehaul customers, returning then back empty to the depot (full and dotted lines).

The deterministic VRPSB is presented as a two-index vehicle flow formulation, using two decision variables: x_{ij}, that represents the number of times the edge (i, j) is used in a tour, and κ, that denotes the number of distinct paths that begin at the origin depot 0 and end at the destination depot $n+m+1$. The following notations are also used to formulate the problem: $\delta(\mathcal{X},\mathcal{Y}) = \{(i, j) \in \mathcal{E} : i \in \mathcal{X},\ j \in \mathcal{Y}\}$, $\delta(\mathcal{S}) = \{(i, j) \in \mathcal{E} : i \in \mathcal{S},\ j \notin \mathcal{S}\}$, $\delta(\mathcal{X}, i)$ corresponds to $\delta(\mathcal{X}, \{i\})$, and $\delta(i)$ is equivalent to $\delta(\{i\})$. The deterministic VRPSB reads as follows:

$$min \sum_{(i,j)\in\mathcal{E}} c_{ij}x_{ij} - \sum_{j\in\mathcal{B}} p_j x_{j,n+m+1} \tag{4.1}$$

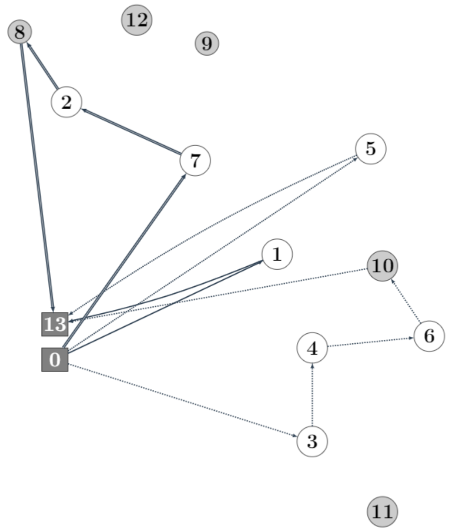


Figure 4.1 – Example of a feasible solution for a VRPSB instance with $\mathcal{V} = \{0, 13\} \cup \mathcal{L} \cup \mathcal{B}$, where $\mathcal{L} = \{1,2,3,4,5,6,7\}$ and $\mathcal{B} = \{8,9,10,11,12\}$. Omitting costs, demands and revenues. The depot are the squares and $\mathcal{K} = \{1,2,3,4\}$. Each type of line (full, dotted, dashed and double) represent the total route of each vehicle.

subjected to

$$\sum_{(i,j)\in\delta(0)} x_{ij} = \kappa, \tag{4.2}$$

$$\sum_{(i,j)\in\delta(n+m+1)} x_{ij} = \kappa, \tag{4.3}$$

$$\sum_{(i,j)\in\delta(i)} x_{ij} = 2 \qquad \forall i \in \mathcal{L}, \tag{4.4}$$

$$\sum_{(i,j)\in\delta(i)} x_{ij} = 2x_{i,n+m+1} \qquad \forall i \in \mathcal{B}, \tag{4.5}$$

$$\sum_{(i,j)\in\delta(\mathcal{S})} x_{ij} \geq \kappa \qquad \forall S \subseteq \mathcal{V},\ \mathcal{S} \in 0,\ \mathcal{S} \notin n+m+1, \tag{4.6}$$

$$\sum_{j\in\mathcal{B}} p_j x_{j,n+m+1} \geq \beta, \tag{4.7}$$

$$\sum_{(i,j)\in\delta(\mathcal{S})} x_{ij} \geq 2r(\mathcal{S}) \qquad \forall \mathcal{S} \subseteq \mathcal{L},\ |\mathcal{S}| \geq 2, \tag{4.8}$$

$$x_{ij} \in \{0,1,2,\ldots,K\} \qquad \forall (i,j) \in \delta(\mathcal{B}, n+m+1), \tag{4.9}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{E} \setminus \delta(\mathcal{B}, n+m+1), \tag{4.10}$$

$$\kappa \in \{1,\ldots,K\}. \tag{4.11}$$

Equation (4.1) is the objective function, which aims at minimizing the total costs, i.e., total travel costs minus total revenues obtained from visiting backhaul customers. Equations

(4.2) and (4.3) impose that exactly κ paths are connected to the origin depot 0 and to the destination depot $n+m+1$. Equation (4.4) ensures that each linehaul customer is visited exactly once by a vehicle by forcing the connection of exactly two edges – one input edge and one output edge. By the definition of the problem any route is allowed to visit a backhaul customer at most once, and a backhaul can be visited by several distinct routes. In this way, Equation (4.5) makes the number of routes that enters in each backhaul equals the number of routes that connects that backhaul to the destination depot $n+m+1$, taking advantage that there are no edges among the backhauls neither between them and the origin depot. Equation (4.6) ensures the existence of κ routes that begin at the origin depot 0 and end at the destination depot $n+m+1$, accomplishing this by delimiting the number of selected edges to the routes in each possible cut S separating 0 and $n+m+1$. The equations (4.5) and (4.6) together guarantee that whether a backhaul is chosen, there is only one in a route and it is connected to the destination depot, i.e., both equations are necessary to assure the precedence constraints among the linehauls and backhauls. Furthermore, these equations assure that there are no routes with two extreme points in 0 nor two extreme points in $n+m+1$. We anticipate that these constraints can be relaxed if the problem allows for multiple backhaul visits per route. For that case, we demonstrate the modelling changes in 4.B.

Equation (4.7) ensures that a minimum amount of revenue β is achieved with a given routing plan. We emphasize that, in practice, the wood industry is concerned with the different characteristics of raw-materials, which have great impact on the manufacturing processes. The work of Andersson et al. (2016) distinguishes between "hard" constraints that reflect the logs requirements in the wood industry, and the "soft" constraints that reflect desirable logs properties. The authors also note that both type of constraints are often uncertain.

Equation (4.8) denotes, simultaneously, the well-known capacity inequalities (CI) and the subtour elimination constraints (SEC) for the part of the routes made only by linehaul customers. The value of $r(S)$ represents the minimum number of routes (vehicles) necessary to attend the demand of the linehaul customers in $S \subseteq \mathcal{L}$. The optimal value of $r(S)$ can be calculated by the Bin Packing Problem (Martello and Toth, 1990), which is NP-hard. A widely used lower bound to $r(S)$ is given by $k(S) = \lceil q(S)/C \rceil$, where $q(S) = \sum_{i \in S} q_i$. These constraints work as follows: (i) if S has demand $q(S) > C$, at least $r(S)$ routes must connect S with $\mathcal{V} \setminus S$, and, since there is a factor of 2 on the right hand side, this ensures that a vehicle must enter and exit the cut S; and (ii) given a route of vertices in S that is a subtour, it is known that connections between S and $\mathcal{V} \setminus S$ are such that $\sum_{(i,j)\in\delta(S)} x_{ij} < 2r(S)$, thus eliminating route S. In this paper, we substitute $r(S)$ by $k(S)$ in Equation (4.8), giving rise to the well known rounded capacity inequalities (RCIs). We emphasize that this substitution maintains the correctness of the formulation (Irnich et al., 2014). Equations (4.9) to (4.11) define the domain of the decision variables.

Remark If $\beta = 0$ then the deterministic VRPSB can be converted into an asymmetric capacitated vehicle routing problem (ACVRP). To do that, it is necessary to execute the steps described in Algorithm 1.

Algorithm 1 Pseudo-code to eliminate backhauls from the deterministic VRPSB ($\beta = 0$)

for each $j \in \mathcal{L}$ **do**
  Calculate the new traveling costs $c'_{j,n+m+1}$:
  $c'_{j,n+m+1} = \min\{\min_{i \in \mathcal{B}}\{c_{ji} + c_{i,n+m+1} - p_i\}, c_{j,n+m+1}\}$
Formulate the model (4.1)–(4.11) with $\mathcal{B} = \emptyset$ and new traveling costs $c'_{j,n+m+1} \forall j \in \mathcal{L}$

Algorithm 1 calculates new minimal costs for the arcs from the linehaul customers to the depot and then removes the selective backhaul customers from the problem. The algorithm can be performed in $O(|\mathcal{L}| \cdot (|\mathcal{B}| + 1))$ and its output results in an ACVRP.

4.2.2 Robust VRPSB

Robust optimization is a fairly recent research field (Gorissen et al., 2015) that opened a plethora of possibilities to deal with uncertainty in many optimization problems. It is a suitable approach to address uncertainty when its probability distribution is not known, or when infeasibility cannot be tolerated (Li et al., 2011; Gorissen et al., 2015).

Robust optimization assumes that uncertain parameters belong to a bounded uncertainty set, and the main aim is to provide an optimal solution that is feasible for this entirely uncertainty set (Ordóñez, 2010). Defining the uncertainty set is, therefore, a major decision that will greatly influence the design of the model and the formulation of the problem. One of the main advantages of robust optimization is precisely its computational tractability for many types of uncertainty sets (Gorissen et al., 2015).

In this work, we consider that revenue in backhauls is uncertain. As mentioned in Section 4.1, this is a very common issue in wood-based supply chains, where a minimum amount of revenue related to quantity and quality of raw materials has to be collected everyday and it is not possible to know the quality of wood beforehand. Revenue uncertainty is only considered in Equation (4.7), meaning that a minimum revenue requirement of β must be satisfied considering the deviations controlled by the uncertainty set. This allows for a trade-off between the nominal profit obtained in the objective function and the feasibility of Equation (4.7). In addition, it will be possible to assign probabilistic bounds of violating Equation (4.7), as it will be detailed in Section 4.2.3. Finally, it is noteworthy that this approach is very intuitive, meaning that the decision maker aims to optimize the nominal objective function, but at the same time must meet a minimum revenue requirement for the all cases considered in the uncertainty set.

The first robust optimization approach used in this work follows the *budget of uncertainty* Γ of Bertsimas and Sim (2004) to develop the robust version of the VRPSB under study. The uncertain revenues are assumed to belong to a space represented by a polyhedral set $\mathcal{U}(\Gamma)$. The choice of the polyhedral set comes with two explanations. Firstly, the resulting model is more computationally tractable than others with non-linear sets, which is desirable for a problem that is already NP-hard. Secondly, by considering a Γ polyhedral set, it is possible to derive tight probabilistic bounds, as shall be further demonstrated in the next subsection.

The uncertain revenue $\tilde{p}_j$ of each backhaul $j \in \mathcal{B}$ is modeled as a symmetric and independent variable, bounded by the interval $[\bar{p}_j - \dot{p}_j, \bar{p}_j + \dot{p}_j]$, where $\bar{p}_j$ is the nominal revenue of backhaul j and $\dot{p}_j$ is the deviation in the revenue. It is thus possible to define a scale deviation $\varepsilon_j = (\tilde{p}_j - \bar{p}_j)/\dot{p}_j$ varying between $[-1,1]$, such that the revenue of each backhaul is given by:

$$\tilde{p}_j = \bar{p}_j + \dot{p}_j \varepsilon_j. \tag{4.12}$$

The uncertainty set $\mathcal{U}(\Gamma)$ is represented as follows:

$$\mathcal{U}(\Gamma) = \{\varepsilon : |\varepsilon_j| \leq 1 \ \forall j \in \mathcal{B}, \sum_{j \in \mathcal{B}} |\varepsilon_j| \leq \Gamma\}, \tag{4.13}$$

where $\varepsilon \in \mathbb{R}^{|\mathcal{B}|}$ is the vector containing the deviation ε_j for each $j \in \mathcal{B}$. As observed from Equation (4.13), the *budget of uncertainty* Γ is introduced in order to control the number of times the worst realizations of revenues are allowed to occur. In the VRPSB, the *budget of uncertainty* falls in the range $[0, |\mathcal{B}|]$, but $\Gamma = \min\{|\mathcal{B}|, |\mathcal{K}|\}$ is already the worst scenario for the problem, since the maximum number of deviations is limited by either the number of backhauls or the number of vehicles (number of routes). Therefore, when $\Gamma = 0$, the model becomes the deterministic version of the problem – no deviations are considered for any revenue; when $\Gamma = \min\{|\mathcal{B}|, |\mathcal{K}|\}$, the model becomes its worst-case deterministic version – all revenues suffer deviation, which is equivalent to the Soyster (1973) approach. Thus, Γ is used to adjust the robustness of the method against the level of conservatism of the solution (Bertsimas and Sim, 2004), or in other words, Γ is used to control the size of the uncertainty set.
Based on the uncertainty set $\mathcal{U}(\Gamma)$, the uncertain revenues in Equation (4.7) are reformulated in the problem, as described by the following semi-infinite and non-linear constraint:

$$\sum_{j \in \mathcal{B}} \bar{p}_j x_{j,n+m+1} + \dot{p}_j x_{j,n+m+1} \varepsilon_j \geq \beta \quad \forall \varepsilon \in \mathcal{U}(\Gamma). \tag{4.14}$$

Following the first theorem from Bertsimas and Sim (2004), the dual formulation is incorporated into the original model, thus obtaining its tractable robust counterpart, where λ and μ_j represent the dual variables. Therefore, the tractable robust counterpart derived from the dual is as follows:

$$min \sum_{(i,j) \in \mathcal{E}} c_{ij} x_{ij} - \sum_{j \in \mathcal{B}} \bar{p}_j x_{j,n+m+1}$$

subjected to

$$(4.2) - (4.6)$$

$$(4.8) - (4.11)$$

$$\sum_{j \in \mathcal{B}} \bar{p}_j x_{j,n+m+1} - \lambda \Gamma - \sum_{j \in B} \mu_j \geq \beta, \tag{4.15}$$

$$\lambda + \mu_j \geq \dot{p}_j x_{j,n+m+1}, \qquad \forall j \in \mathcal{B}, \tag{4.16}$$

$$\lambda, \mu_j \geq 0, \qquad \forall j \in \mathcal{B}. \tag{4.17}$$

Considering the above reformulation, the robust VRPSB model presents $1+|\mathcal{B}|$ more variables than the deterministic version, as a result from including the dual variables λ and μ_j. In addition, the robust model also includes $2 \cdot |\mathcal{B}| + 1$ more constraints, as a result of including Constraints (4.15), (4.16) and (4.17).

Note that the uncertainty only appears in the constraints but not in the objective function. This is aligned with basic assumptions in robust optimization, namely that the objective is certain and the uncertain constrains are hard, in the sense that cannot be violated (Gorissen et al., 2015). As such, the decision-maker is optimizing the expected revenue but concerned that, in the worst-case, the minimum revenue required is met. In this work, optimizing over the expected value instead of the worst-case value is valid because this operational routing problem is performed every day and worst-cases are not usually catastrophic and can be compensated by better scenarios in the next day. Still, we include the constraint of minimum required revenue, because a minimum quality/revenue of raw-material in the worst-case scenario may be desirable for the sustainability of the production process. According to Mulvey et al. (1995), a solution is said to be *solution robust* if it remains close to the optimal for all uncertainty realizations, and a solution is said to be *model robust* if it remains feasible for all uncertainty realizations. Therefore, we can contextualize our solutions as *model robust*, because the main concern is on the feasibility and not on the quality of a solution. A similar approach can be found in Alvarez and Vera (2014).

4.2.3 Estimates of probabilistic bounds

In robust optimization, it is not necessary to acknowledge the probability distribution of the uncertainty sources. Nevertheless, it is necessary to consider how many revenues will deviate from their nominal value, represented by Γ. The budget of uncertainty determines the size of the uncertainty set and can be defined using historical values and the opinion of experts. Γ can also be defined based on probabilistic bounds of constraint violation, as shown in Bertsimas and Sim (2004).

In this section, we describe how Γ can be defined using different probabilistic bounds. We first start with a case where no probability distribution is assumed for the uncertainty sources. Then we describe *a priori* and *a posteriori* probabilistic bounds that can be obtained assuming that the uncertainty sources follow an uniform distribution. We also describe a novel method that provides tighter probabilistic bounds for $\mathcal{U}(\Gamma)$ when random parameters are independent, identically and uniformly distributed.

The choice for an uniform distribution comes from the fact that, since the random parameters are not known from historical data, one could assume that all of them can occur with the same probability. It is important to note that it is still a conservative assumption, given that the extreme values have the same probability to occur than the mean value. However, this assumption allows to derive much tighter probabilistic bounds for the problem. Obviously, if it is not possible to assume any distribution, standard probabilistic bounds (Bertsimas and Sim, 2004; Guzman et al., 2016) can still be assumed.

4.2.3.1 Probabilistic bound based on Bertsimas and Sim (2004)

Based on Bertsimas and Sim (2004), the budget of uncertainty can be calculated according to probabilistic bounds of constraint violation without assuming any distribution for the uncertain parameters. Assuming that $\varepsilon_j \in J$ are independent, symmetrically distributed have the expected value 0, the bound of violating constraint (4.15) can be calculated as follows:

$$Pr\left(\sum_{j\in\mathcal{B}} \tilde{p}_j x_{(j,n+m+1)} < \beta\right) \approx 1-\phi\left(\frac{\Gamma-1}{\sqrt{|\mathcal{B}|}}\right), \tag{4.18}$$

where $\phi(\theta)$ is a standard Gaussian cumulative distribution function. According to Equation (4.18), the probability of constraint violation is expected to decrease as the budget of uncertainty increases. Therefore, this bound can be used to estimate the budget of uncertainty based on confidence levels and avoid solving the problem several times to obtain an adequate budget of uncertainty.

4.2.3.2 *A priori* probabilistic bound based on Guzman et al. (2017a) and Kang et al. (2015)

If the probability distribution function of the uncertainty parameters is assumed to be known, it is possible to derive tighter probabilistic bounds, as presented by Guzman et al. (2017a) and Kang et al. (2015). Assuming that each uncertain parameter ε_j is independent, has the expected value equal to 0, and has a known cumulant generating function given by $\Lambda_j(\theta)$, the following bound holds:

$$Pr\left(\sum_{j\in\mathcal{B}} \tilde{p}_j x_{(j,n+m+1)} < \beta\right) \le \exp\left(\min_{\theta>0}\left\{-\theta\Gamma + \sum_{j\in\mathcal{B}} \Lambda_j(\theta)\right\}\right). \tag{4.19}$$

Here, we assume that each parameter ε_j follows an uniform distribution supported in $[-1,1]$, which gives $\Lambda_j(\theta) = \ln(\frac{\sinh\theta}{\theta})$. Finally, Γ in Equation (4.19) is obtained by means of Algorithm 1 from Guzman et al. (2017a), given a desired probability of constraint violation.

4.2.3.3 *A priori* probabilistic bound based on the Irwin-Hall distribution

In this section, we derive a new probabilistic bound assuming that each uncertain parameter ε_j is independent and uniformly distributed in $[-1,1]$. This bound differs from Equation (4.19) since it does not rely on Markov's inequality and moment generating functions. It is inspired by the geometrical interpretation of the uncertainty set $\mathcal{U}(\Gamma)$ through the Irwin-Hall distribution. Some previous works have used the Irwin-Hall distribution in robust optimization. Chassein (2016) uses the Irwin-hall to derive the volume of the uncertainty set. Li and Morales (2017) mention the Irwin-hall to calculate the probability of the total realized uncertainty being above a specific deviation of a defined Γ, but does not provide a general framework that assigns a Γ according to the probability of constraint violation. Finally, Fliedner and Liesiö (2016) applies the Irwin-hall to define the probability of the summation of absolute deviations supported in $U[-1,1]$ being above a specified Γ in the uncertainty set, however it assumes absolute deviations and does not associate the probability with constraint violation bounds. Thereby, consider the following box uncertainty set:

$$\mathcal{U}^{HC} = \{\varepsilon \in \mathcal{R}^{|\mathcal{B}|} : -1 \leq \varepsilon_j \leq 1 \ \forall j \in \mathcal{B}\}, \tag{4.20}$$

where its feasible solution space is defined by a hypercube in $[-1,1]^{|\mathcal{B}|}$. Assuming that each variable ε_j is uniformly distributed in $[-1,1]$, $\mathcal{U}^{HC}$ will have infinite equiprobable points in $[-1,1]^{|\mathcal{B}|}$. Given that $\mathcal{U}(\Gamma) \subseteq \mathcal{U}^{HC}$ and since all points in $\mathcal{U}^{HC}$ are equiprobable, the percentage of realizations considered by $\mathcal{U}(\Gamma)$ can be calculated by its hypervolume divided by the hypervolume of $\mathcal{U}^{HC}$:

$$Pr_{scen}(\Gamma) = \frac{Vol(\mathcal{U}(\Gamma))}{Vol(\mathcal{U}^{HC})}, \tag{4.21}$$

which also can be interpreted as $Pr\left(\sum_{k=1}^{|\mathcal{B}|} U[-1,1] \leq \Gamma\right)$, with $U[-1,1]$ being an uniform distribution in $[-1,1]$. Since $Pr_{scen}(\Gamma)$ is the percentage of realizations acknowledged by the polyhedral set $\mathcal{U}(\Gamma)$, Constraint (4.14) is satisfied at least with probability $Pr_{scen}(\Gamma)$ and therefore:

$$Pr\left(\sum_{j \in \mathcal{B}} \tilde{p}_j x_{(j,n+m+1)} < \beta\right) \leq 1 - Pr_{scen}(\Gamma), \tag{4.22}$$

when each $\tilde{p}_j$ is uniformly distributed in $[\bar{p}_j - \dot{p}_j, \bar{p}_j + \dot{p}_j]$.
To calculate $Pr_{scen}(\Gamma)$ it is necessary to resort to the Irwin-Hall distribution, which is the distribution of the sum of n independent and identically distributed uniform random variables in $[0,1]$. The cumulative distribution of the Irwin–Hall is defined as:

$$F_{IH}(t,n) = Pr\left(\sum_{k=1}^{n} U[0,1] \leq t\right) = \frac{1}{n!}\sum_{j=0}^{t}(-1)^j \binom{n}{j}(t-j)^n, \tag{4.23}$$

where n is the number of independent and identically distributed uniform random variables and $t \in [0,n]$. Since our aim is to calculate $Pr\left(\sum_{k=1}^{|\mathcal{B}|} U[-1,1] \leq \Gamma\right)$, it is necessary to parametrize $F_{IH}(t)$ to acknowledge the sum of uniform distributions supported in $[-1,1]$ instead of $[0,1]$ which can be easily done by considering $n = |\mathcal{B}|$ and $t = \frac{\Gamma + |\mathcal{B}|}{2}$:

$$
\begin{aligned}
&Pr\left(\sum_{k=1}^{n} U[0,1] \leq t\right) \\
&= Pr\left(\sum_{k=1}^{|\mathcal{B}|} U[0,1] \leq \frac{\Gamma+|\mathcal{B}|}{2}\right) \\
&= Pr\left(\left(\sum_{k=1}^{|\mathcal{B}|} 2\cdot U[0,1]\right) - |\mathcal{B}| \leq \Gamma\right) \\
&= \left(\sum_{k=1}^{|\mathcal{B}|} U[-1,1] \leq \Gamma\right).
\end{aligned} \tag{4.24}
$$

Thence, $Pr_{scen}(\Gamma)$ can be calculated as follows:

$$
Pr_{scen}(\Gamma) = F_{IH}(t,n) = \frac{1}{n!}\sum_{j=0}^{t}(-1)^j\binom{n}{j}(t-j)^n, \tag{4.25}
$$

with $t = \frac{\Gamma+|\mathcal{B}|}{2}$ and $n = |\mathcal{B}|$. With that, it is easy to see that when $\Gamma = 0$, $Pr_{scen}(\Gamma)$ is 50%, in the same way, when $\Gamma = |\mathcal{B}|$, $Pr_{scen}(\Gamma)$ becomes 100%. For a geometrical derivation and further details on the Irwin-Hall distribution we refer the readers to Marengo et al. (2017). Considering that the maximum number of visited backhauls is limited by $n^* = \min\{|\mathcal{B}|,|\mathcal{K}|\}$, a tighter probabilistic bound can be achieved by assuming that the actual support of random deviations from visited backhauls is given by $U[-1,1]^{n^*}$. Hence, $Pr_{scen}(\Gamma)$ can be tighten as follows:

$$
Pr^*_{scen}(\Gamma) = F_{IH}(t^*,n^*), \tag{4.26}
$$

with $t^* = \frac{\Gamma+n^*}{2}$ and $n^* = \min\{|\mathcal{B}|,|\mathcal{K}|\}$.

4.2.3.4 *A posteriori* probabilistic bound based on Guzman et al. (2017b)

All the above methods to obtain probabilistic bounds are referred to as *a priori* bounds, which means that the probabilistic bound of violating a constraint is determined before considering the solution of the optimization model. The probabilistic bound (GMF12) in Guzman et al. (2017b) estimates the *a posteriori* probability of constraint violation. Although this method is more challenging than the previous, it provides a tighter bound because it allows to characterize one single solution to the problem (Guzman et al., 2017b), rather than every feasible solution of the uncertainty set. Considering that each $\varepsilon_j\ \forall j \in \mathcal{B}$ is independent and follows a continuous uniform distribution supported in $[-1,1]$, the exact probability of constraint violation of a solution x^* can be defined by the following equation:

$$
Pr\left(\sum_{j\in\mathcal{B}} \bar{p}_j x^*_{(j,n+m+1)} + \sum_{j\in\mathcal{B}^*} \dot{p}_j x^*_{j,n+m+1}\varepsilon_j < \beta\right) = F_{\Theta}(h(x^*)), \tag{4.27}
$$

where F_Θ is the cumulative distribution function of $\sum_{j\in\mathcal{B}} \dot{p}_j x^*_{j,n+m+1}\varepsilon_j$, the deterministic part of the constraint is given by $h(x^*) = \beta - \sum_{j\in\mathcal{B}^*} \bar{p}_j x^*_{(j,n+m+1)}$, and $B^* = \{j : \dot{p}_j x^*_{j,n+m+1} > 0\}$. Because calculating F_Θ is quite challenging, the brute-force enumeration algorithm proposed by Guzman et al. (2017b) is used to calculate it.

4.2.4 Factor model support for uncertain revenues

An alternative set for the uncertain revenues can be described by a factor model support (Gounaris et al., 2013), which enables to consider not only the deviation from the nominal revenues, as in the budget of uncertainty presented in Equation (4.13), but also the different factors that influence that deviation. Consider now the following set:

$$\mathcal{U}' = \{\tilde{p} \in \mathbb{R}_+^{|B|} : \tilde{p} = \bar{p} + \Phi\xi \text{ for some } \xi \in \Xi\}, \tag{4.28}$$

where

$$\Xi = \{\xi \in \mathbb{R}^F : \xi \in [-e, +e],\ e^T\xi \in [-\Omega F, +\Omega F]\}. \tag{4.29}$$

The parameter $F \in \mathbb{N}$ represents the number of independent factors $\xi_1, ..., \xi_F$ that affect the revenue deviation of each backhaul customer. Matrix $\Phi \in \mathbb{R}^{|B|\times F}$ represents the disturbance of each factor on the revenue of each backhaul. Parameter $\Omega \in [0,1]$ represents the percentage of surplus factors that can be above or below the point estimate 0, such that if $\Omega = 0$, the number of factors ξ_f below and above the point estimate $0 \in \mathbb{R}^F$ are the same. Finally, e is column vector of ones with dimension F. The resulting robust counterpart for the factor model support is given by:

$$min \sum_{(i,j)\in\mathcal{E}} c_{ij}x_{ij} - \sum_{j\in\mathcal{B}} \bar{p}_j x_{j,n+m+1}$$

subjected to

$$(4.2) - (4.6)$$
$$(4.8) - (4.11)$$

$$\sum_{j\in\mathcal{B}} \bar{p}_j x_{j,n+m+1} - \sum_{f=1}^{F} (v_f^+ + v_f^-) - \Omega F(\rho^+ + \rho^-) \geq \beta, \tag{4.30}$$

$$(v_f^+ - v_f^-) + (\rho^+ - \rho^-) = \sum_{j\in\mathcal{B}} \Phi_{jf} x_{j,n+m+1} \qquad \forall f = 1..F, \tag{4.31}$$

$$v_f^+, v_f^-, \rho^+, \rho^- \in \mathbb{R}_+ \qquad \forall f = 1..F, \tag{4.32}$$

with v_f^+, v_f^-, ρ^+ and ρ^- being the dual variables from $\mathcal{U}'$.

4.2.4.1 Simple illustration

Although this type of support can be of interest in optimization problems, parameters F, Ω and Φ must be assigned by the decision maker. As such, we exemplify how the uncertain revenues can be described by a factor model support. For this example, consider $|\mathcal{B}| = 10$, and that three factors ($F = 3$) have influence on the revenue provided by raw-materials of backhauls customers, namely moisture (Φ_{j1}), heterogeneity (Φ_{j2}) and ration length/thickness (Φ_{j3}):

$$\Phi = \begin{bmatrix} 50 & 121 & 71 \\ 47 & 97 & 50 \\ 79 & 40 & 120 \\ 55 & 60 & 116 \\ 54 & 115 & 60 \\ 100 & 70 & 31 \\ 10 & 108 & 117 \\ 111 & 30 & 81 \\ 20 & 115 & 94 \\ 47 & 80 & 126 \end{bmatrix}.$$

Finally, by considering $\Omega = 0$ we are assuming that the same number of factors deviate below and above the point estimate 0. On the other hand, when $\Omega = 1$, it means that all factors deviate above (or below) the point estimate. Intermediate values of Ω indicate that some factors will deviate while others remain steady. Figure 4.2 illustrates how the parameter Ω influences the solution. For this example, we have used instance B3 from Goetschalckx and Jacobs-Blecha (1989) and a minimum revenue required of $\beta = 750$.

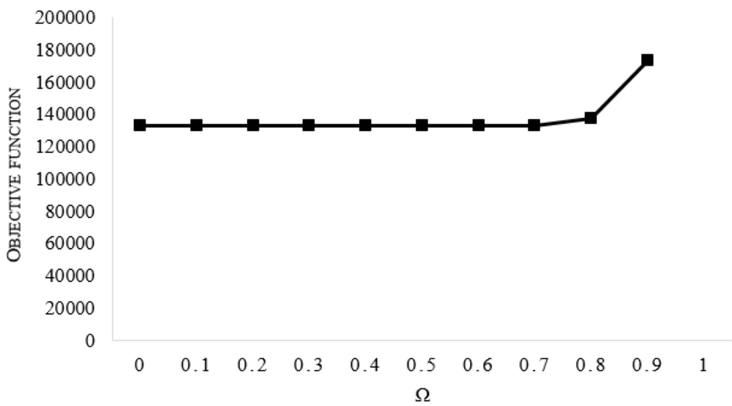


Figure 4.2 – Influence of Ω in the objective function of a robust solution.

From Figure 4.2, it can be observed that, in this example, a value of Ω within the range $[0, 0.7]$ leads to the same solution value. Increasing $\Omega =$ to 0.8 leads to a subtle increase in the objective function, whereas it is observed a drastic increase if $\Omega = 0.9$. The growth of this parameter is of utmost importance, since when it reaches $\Omega = 1.0$ no feasible solution is obtained. This extreme case can also be analogous to the worst-case solution, where all uncertain realizations occur at the same time.

4.3. Solution methods

This section first introduces the exact method, a B&C algorithm, and then the ALNS metaheuristic developed for this problem. The B&C method is designed following the work of Lysgaard et al. (2004) and is combined with CPLEX to solve the robust VRPSB. The ALNS developed is based on the work of Ropke and Pisinger (2006) and includes additional procedures to incorporate the robust nature of the problem while solving the problem. We anticipate that both of the solution methods presented in this section are used to solve small, medium and large instances adapted from literature, and afterwards compare their computational performance. For illustrative instances, where the goal is to investigate the impact of robustness parameters, probabilistic bounds and different uncertainty sets, a simple CPLEX is used.

4.3.1 Branch-and-Cut

The B&C algorithm developed is partially based on the B&C for the CVRP presented in Lysgaard et al. (2004), which is well known for its efficiency. Since that work, the methods for the CVRP itself evolved and the state-of-the-art algorithms heavy rely on the branch-and-cut-and-price framework (Poggi and Uchoa, 2014), which involves separation algorithms to identify promising cuts and pricing algorithms to generate suitable columns for the problem, as these models are based on set partitioning-like formulations (Toth and Vigo, 2014). Our exact approach is based on B&C, which can be viewed as a component to an eventual branch-and-cut-and-price based method.

The authors of Lysgaard et al. (2004) have made the source code of a software library called CVRPSEP publicly available, which implements several cut separation routines for the CVRP. In this paper, some of these separation routines are used in our implementation of the B&C to solve the robust VRPSB. More precisely, our B&C algorithm makes use of the RCI separation routine from the CVRPSEP, as well as it utilizes the max-flow routine, from the same library, to separate the κ-cuts we used in our approach. For a comprehensive material about the separation algorithms we refer to Toth and Vigo (2014). The B&C is applied to the resolution of the Robust VRPSB formulation, which is presented in Section 4.2.2, given by the following equations: (4.1), (4.2) – (4.6), (4.8) – (4.11), and (4.15) – (4.17).

The formulation presents suitable features for the B&C method we developed, namely: (i) the undirected graph, (ii) the split depot, and (iii) the κ-cuts. Using an undirected graph to represent the network in the VRPSB avoids symmetric solutions and keeps the number of variables in the formulation smaller than an hypothetically directed graph. In the VRPSB, it is mandatory to constrain a backhaul vertex to be, at most, on one extreme point of a route, and therefore, the identification of the depot-connected edges of a given route is essential.

To accomplish that we splitted up the depot into the origin depot 0 and the destination depot $n+m+1$, as already mentioned. Considering this transformation, the problem can be seen as seeking paths that connects 0 to $n+m+1$, and, if a backhaul customer is present in a path, it must be connected to the $n+m+1$. A linehaul customer can also be connected to $n+m+1$, meaning that its route has no backhaul at all. On the other hand, we must ensure that there are no routes where there are two extreme points in 0 nor two extreme points in $n+m+1$. The κ-cuts in Equations (4.6) solve this problem, as those cuts restrict the solution to have at least κ paths between 0 and $n+m+1$ in a solution.
As there is an exponential number of κ-cuts, we rely on the B&C capabilities for seeking and adding violated cuts to the model. To separate these constraints, we use the implementation of the max-flow algorithm, presented in the CVRPSEP. If the max-flow value is smaller than κ that cut is violated and added to the model. Note that Equations (4.2) and (4.3) are specialized and stronger κ-cuts.
The RCIs, at Equation (4.8), ensures that the capacity limit of each vehicles is respected. Each time the B&C has to call the RCI separation routine from the Lysgaard's CVRPSEP, it is necessary to make a conversion on the graph. This transformations occurs as follows: given a candidate solution, the vertices 0, $n+m+1$ and the vertices of $\mathcal{B}$ are all shrunk into a single supervertex that plays the role of the depot on the separation algorithm. If violated cuts are found, and considering that the edges are not incident to the depot, the cuts are added to the model and it is re-optimized.
The B&C algorithm adds the valid inequalities described above via the branch-and-cut framework of CPLEX by using its callback functions. The separation step is composed by four heuristics for the RCIs, accordingly Lysgaard et al. (2004), and a max-flow algorithm for the κ-cuts. The branching strategy is kept as the default branching of CPLEX, which typically branches on variables. An RCI is considered violated if the difference between its left and right hand sides is greater than an RCI tolerance limit. In this work, this limit is 0.2. A κ-cut is considered violated if the max-flow value is smaller than κ.
Finally, the initial model given to the B&C algorithm is composed by the objective function in Equation (4.1), the constraints in equations (4.2)–(4.5) and (4.15)–(4.16), the bounds in equations (4.9)–(4.11) and (4.17), and two additional dedicated RCI cuts in the form of Equation (4.8): one for the origin depot with $S = \{0\}$, and the other for the destination depot with $S = \{n+m+1\}$. As the model has an exponential number of κ-cuts in Equation (4.6) and also an exponential number of RCI constraints in Equation (4.8), they are added on demand in a branch and cut fashion.

4.3.2 Adaptive Large Neighborhood Search

The ALNS was firstly proposed by Ropke and Pisinger (2006) and since then, it has shown to be effective for many VRPBs (Masson et al., 2013; Li et al., 2016b; Ghilas et al., 2016), as well as other VRPs (Li et al., 2016a; Dayarian et al., 2016; Zajac, 2017).
Large neighborhoods can be explored by consecutively applying two heuristics: first, a destroy operator, which partially destroys the current solution and then a repair operator to make the solution feasible again, generating thus a new solution. The main features of ALNS are using diverse destroy and repair operators to create different neighborhoods

(Pisinger and Ropke, 2010) and the adaptive search behavior as a consequence of the performance of each pair of destroy-repair operators during the iterative process.

In this work, the general procedures of the ALNS, along with all related ALNS parameters, are conducted as in Ropke and Pisinger (2006). The pseudo-code of ALNS is outlined in Algorithm 2. First, an initial solution x is generated with a simple greedy heuristic and the weights of each destroy and repair operators are stored in ρ^- and ρ^+, starting at 1 for each operator. Then, for each iteration, a pair of destroy-repair operators are applied based on the their past performance (score). First, the destroy operator removes some vertices from the solution ($d(x)$), and next, a repair operator inserts the removed vertices into different positions ($r(d(x))$), creating a new solution x^t. If the new solution x^t satisfies the acceptance criteria, it replaces x. If the new solution x^t is better than the best solution found so far, it replaces x^b. This comparison is based on the solution cost, c. Finally, scores and weights are dynamically adjusted as the search for a new solution progresses.

Algorithm 2 Pseudo-code of ALNS framework

Input: x, ρ^-, ρ^+
Output: x^b
generate a solution x
set $x^b = x$; $\rho^- = (1, \ldots, 1)$; $\rho^+ = (1, \ldots, 1)$
while acceptance criteria is not met **do**
    select destroy operators $d \in \Omega^-$ and repair operators $r \in \Omega^+$, using weights ρ^- and ρ^+
    $x^t = r(d(x))$
    if accepted (x^t, x) **then**
        $x = x^t$
    if $c(x^t) \leq c(x^b)$ **then**
        $x^b = x^t$
    update ρ^- and ρ^+
return x^b

The destroy operators used are the *Shaw Removal*, the *Random Removal* and the *Worst Removal* heuristics. The *Shaw Removal* selects the nodes to remove according to similarity criteria, namely distance and quantity (Shaw, 1998). As the name suggests, the *Random Removal* selects at random the nodes to remove from the solution. Finally, the *Worst Removal* selects the nodes to remove according to their impact on the decrease of the objective function. The objective is to remove those customers located in positions that largely increase the total routing costs.

The repair operators used are the *Greedy Insertion* and the *k-Regret Insertion*. The former heuristic is very simple, inserting the removed nodes into positions in the routes such that the objective function increases the minimum possible. The latter applies the concept of *regret* to insert first the customers for which the insertion in a later stage would be more costly (with higher *regret*). The *regret* is determined as the sum of differences in the costs as a result of inserting customer i in the first best route and in the k^{th} best route. The values for $k = 2, 3, 4$ were used.

To develop the robust ALNS, robustness was embedded into the algorithm during the creation of new neighborhoods. Thus, instead of creating a new solution with the destroy-repair operators and afterwards evaluating its robustness, the robustness parameters are already introduced, based on a polyhedral uncertainty set, during the repair procedure, as detailed in Algorithms 3 and 4.

The adapted procedure starts by inserting all linehaul customers into a previously destroyed solution $d(x)$. After that, a solution x^1 is achieved with all linehaul customers and with the backhauls not removed by the destroy operator. The backhauls in x^1 are ordered by decreasing values of deviation in revenue multiplied by the number of times that the backhaul is present in the solution. Thus, if two different backhauls are visited the same number of times in the solution, the one with a higher deviation is the first to be addressed by the budget of uncertainty. However, if a backhaul is visited twice and another one only once, unless its deviation is very small, the former backhaul is the first to be addressed by the budget of uncertainty.

At this stage, the total revenue minus total deviation (*Revenue'*) is determined using Algorithm 4 and the feasibility of the solution is accessed by considering if this *Revenue'* is sufficient to achieve the minimum revenue required, β. If the *Revenue'* is higher or equal to β, then the expected revenue of the solution x^1 is the total revenues collected with all selected backhauls. Otherwise, the solution is infeasible and the expected revenue takes a very large negative value. The total cost of x^1 is then recalculated. The respective heuristic in use proceeds by inserting potential backhauls in available routes and creates a new solution x^2. The total expected revenue and total cost of x^2 are determined as described previously and compared with x^1. At the end, the solution with the lowest total cost is selected as the neighbor solution $r(d(x))$ which will be used in the ALNS (line 5 of Algorithm 2). An example of the procedure to obtain such solutions is illustrated in Figure 4.3.

Algorithm 3 Pseudo-code to embed robustness into ALNS

Input: $d(x)$
Output: $r(d(x))$
B = vector of backhauls in solution $d(x)$
$x^1 = d(x)$
Insert all linehaul customers in the solution with repair operator $r \in \Omega^+$
$x^1 = r(d(x))$
Revenue (x^1) = Algorithm 4 (x^1)
Total cost: c_{x^1} = Total distance - Revenue (x^1)
Add a backhaul to B
$x^2 = r(d(x))$
Revenue (x^2) = Algorithm 4 (x^2)
Total cost: c_{x^2} = Total distance - Revenue (x^2)
if $c_{x^2} < c_{x^1}$ **then**
    $r(d(x)) = x^2$
else
    $r(d(x)) = x^1$
return $r(d(x))$

Algorithm 4 Function to assess robustness

Input: x^1
Output: (x^1)
N_i = number of times backhaul i is present in the solution x^1
$\bar{B}$ = vector of backhauls in solution x^1 ordered by decreasing $\dot{p} \cdot N_i$
$Revenue'(x^1) = 0$
Revenue $(x^1) = 0$
for $i \in \bar{B}$ **do**
  Revenue $(x^1) \mathrel{+}= N_i \cdot \bar{p}_i$
  if $\Gamma \geq 1$ **then**
    $Revenue'(x^1) \mathrel{+}=$
    $N_i \cdot (\bar{p}_i - \dot{p}_i)$
    $\Gamma = \Gamma - 1$
  else
    $Revenue'\ (x^1) \mathrel{+}= N_i \cdot (\bar{p}_i - \dot{p}_i \cdot \Gamma)$
    $\Gamma = 0$
  remove backhaul i from $\bar{B}$
if $Revenue'(x^1) \leq \beta$ **then**
  Revenue $(x^1) \mathrel{+}= -999999$
return Revenue (x^1)

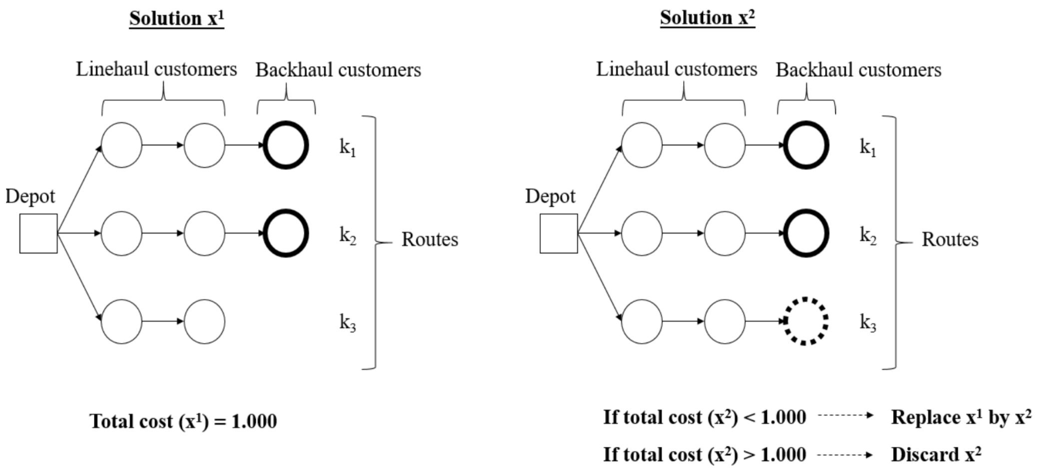


Figure 4.3 – Example of procedure to embed robustness during the creation of a feasible solution. Solution x^1 is first evaluated in terms of total cost, for a given Γ value; then a backhaul is inserted, producing solution x^2, and the total cost is again evaluated, for the same value of Γ. If the total cost of x^2 is lower than x^1, solution x^1 is discarded and the iterative process continues, otherwise, solution x^2 is discarded and the former is kept.

4.4. Computational tests

The aim of the computational tests is two-fold: (i) evaluate the performance of models and robustness of solutions, namely in terms of probabilistic bounds, feasibility and robustness; and (ii) evaluate the performance of the B&C and the ALNS to solve the robust VRPSB.
The first aim is addressed using CPLEX and the experiments are performed on small instances, due to their rapid convergence to the optimal solution. The second aim is obtained by a series of experiments on large size instances, testing the two solution methods developed. An additional set of tests are presented in 4.B that compares the solution quality and computational performance of two different cases - one allowing single backhaul visits per route and one relaxing this constraint by allowing multiple visits.
The deterministic, RO and CC models for the first aim are implemented in OPL and solved using IBM ILOG CPLEX Optimization Studio 12.6. The CC model developed to be compared with the RO model is presented in the Appendix A. It is important to note that a single-commodity formulation is used with OPL, instead of the two-index vehicle flow formulation presented in Section 4.2, because it avoids the exponential constraints (4.8). In fact, the resolution time is not a concern at this stage, since the main goal is to analyze the structure of the solutions and their feasibility for different robustness parameters. The ALNS algorithm is implemented in C++. The B&C is implemented in C++, using the C callable API of CPLEX 12.7.1. The experiments of the ALNS were performed on a computer equipped with the processor Intel Core i7 of 2.20GHz and with 16 GB of RAM, and the operating system is the Windows 10. Besides, the B&C experiments were run on a computer with processor Intel Xeon Gold 6142 of 2.60GHz with 64 on-line cores, 256 GB of RAM, and the Ubuntu 18.04.2 LTS operating system with the Linux kernel version 4.15.0-47-generic. Each B&C test was performed sequentially, although at most 32 of them were in parallel at any time in the same machine. We add that, as the B&C lower bounds are used to evaluate the quality of the ALNS solutions, we do not avoid using a powerful available machine to perform its tests.

4.4.1 Problem instances

The instances used in the experiments are adapted from the standard VRPB instances in Goetschalckx and Jacobs-Blecha (1989). The only new parameters added to these instances are the revenues and deviations, which are randomly generated for each backhaul of each instance as follows. From the original class of instances B of Goetschalckx and Jacobs-Blecha (1989), we have used their quantities as proxies for the minimum and maximum revenues. Then, 50 values are randomly simulated within this range, for each backhaul i. The average value from this simulation corresponds to the revenue, $\bar{p}$, and the standard deviation corresponds to the deviation, $\dot{p}$. Furthermore, these values are simulated for the 10 backhaul customers from class B instances and then replicated for instances with higher values, such that the revenue and corresponding deviation for backhaul customer 1 are the same for backhaul customers 11, 21, and so on. These values are reported in Table 4.1.

Table 4.1 – Values of revenue $\bar{p}$ and deviation $\dot{p}$ for each backhaul customer used for instances in the computational tests

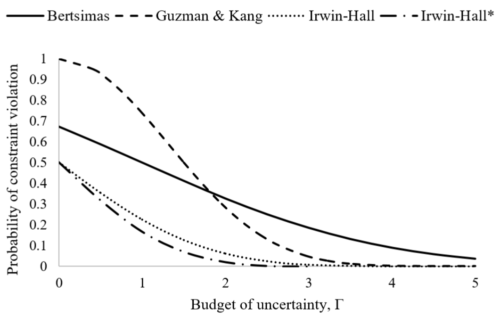
Backhaul number	$\bar{p}$	$\dot{p}$
1, 11, 21, 31, 41, 51, 61, 71	438	242
2, 12, 22, 32, 42, 52, 62, 72	404	194
3, 13, 23, 33, 43, 53, 63, 73	454	239
4, 14, 24, 34, 44, 54, 64, 74	392	231
5, 15, 25, 35, 45, 55, 65, 75	464	229
6, 16, 26, 36, 46, 56, 66	380	201
7, 17, 27, 37, 47, 57, 67	384	235
8, 18, 28, 38, 48, 58, 68	355	222
9, 19, 29, 39, 49, 59, 69	459	229
10, 20, 30, 40, 50, 60, 70	415	253

4.4.2 Evaluation of the VRPSB under uncertainty

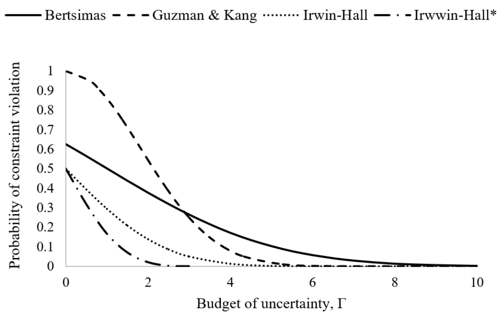
Throughout this section, several tests are conducted in order to analyze and evaluate the robust optimization model (Section 4.2.2). First, the different bounds obtained by the different methods that estimate *a posteriori* the probability of constraint violation are characterized. Next, the exact probability of constraint violation of these solutions is presented and compared with the *a posteriori* probabilistic bounds. Then, the nominal and robust solutions obtained for different values of β and Γ are analyzed and compared with the solutions obtained with a chance-constrained model. The structure of nominal and robust solutions are finally analyzed and discussed.
All the tests presented in the section are solved with CPLEX and tested with four instances and four different values of minimum revenue required to cover a wide range of instance characteristics. The standard instances tested are: A4 (20 linehaul customers, 5 backhauls and 3 vehicles), B3 (20 linehaul customers, 10 backhauls and 3 vehicles), C4 (20 linehaul customers, 20 backhauls and 4 vehicles) and F4 (30 linehaul customers, 30 backhauls and 4 vehicles). The values of minimum revenue required β are 0, 500, 750 and 1000.

4.4.2.1 Probabilistic bounds

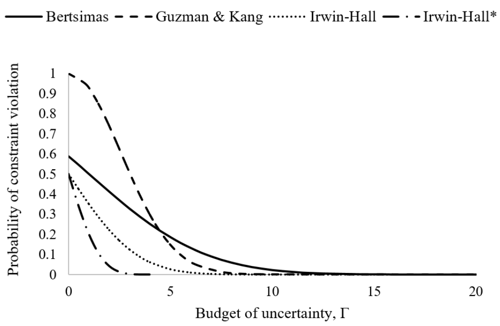
In this section, the methods that estimate the probability of violating Constraint (4.7), based on the values of the budget of uncertainty Γ, are analyzed and compared. The results are presented in Figures 4.4a, 4.4b, 4.4c and 4.4d for $|\mathcal{B}| = 5$, $|\mathcal{B}| = 10$, $|\mathcal{B}| = 20$ and $|\mathcal{B}| = 30$, respectively. It is important to note that the *a posteriori* method based on Guzman et al. (2017b) is not analyzed in this section, since it is not influenced by the robust parameters.
From these results, it can be shown that, as expected, the probability of constraint violation decreases as the budget of uncertainty increases, for all the methods investigated. Also, as more backhauls exist in an instance, the sooner the probabilistic bound reaches its minimum, i.e. the sharper is the curve provided by the bound.



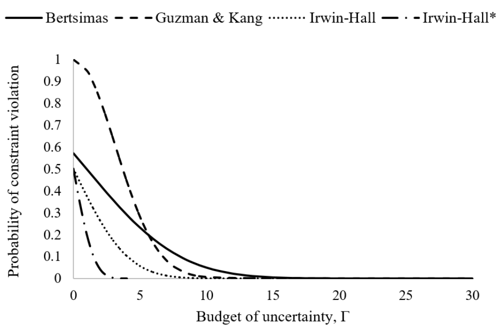
(a) Instance A ($|\mathcal{B}| = 5$)



(b) Instance B ($|\mathcal{B}| = 10$)



(c) Instance C ($|\mathcal{B}| = 20$)



(d) Instance F ($|\mathcal{B}| = 30$)

Figure 4.4 – Probability of constraint violation obtained with different methods.

The method based on Bertsimas and Sim (2004) is the most conservative among the four *a priori* probabilistic bounds. In fact, when the instance only has 5 backhauls (Figure 4.4a), the probability of constraint violation never reaches zero. When the number of backhauls increases to 10 (Figure 4.4b), a probability of zero only occurs when Γ is maximum, i.e. when $\Gamma = |\mathcal{B}|$. In the other cases (Figures 4.4c and 4.4d), a probability of zero is achieved but only latter than any of the other methods.
The method based on Guzman et al. (2017a) is very conservative for the lowest values of Γ in each instance, but the probability of constraint violation decreases drastically with increasing the degree of conservatism.
Finally, the methods based on the Irwin-Hall distribution are indisputably the ones that provide the tighter bounds, showing always the lowest values for the probability of constraint violation for any value of Γ. In fact, by considering that the worst-case occurs when $\Gamma_{max} = \min\{|\mathcal{B}|, |\mathcal{K}|\}$ (Irwin-Hall*), it is possible to obtain an even tighter bound than when $\Gamma_{max} = |\mathcal{B}|$ (Irwin-Hall). It is important to note that when the function Irwin-Hall* is used, the same probabilistic bounds are achieved for the cases illustrated in Figures 4.4a and 4.4b, since on both cases the number of vehicles is three, which is lower than the number of backhauls. Similarly, the same bounds are achieved for the cases illustrated in Figures 4.4c and 4.4d, where the number of vehicles is four.

4.4.2.2 Comparison of *a priori* and *a posteriori* probabilistic bounds

In this section, the method that estimates *a posteriori* the probability of constraint violation is compared with the best *a priori* method, as concluded in Section 4.4.2, i.e. the Irwin-Hall* distribution method.
Table 4.2 and Table 4.3 report the different values of probability of constraint violation for instances with 3 and 4 vehicles, respectively. Since the method of Irwin-Hall distribution only considers the value of budget of uncertainty and number of backhauls, the corresponding probability values are only presented once for each instance. On the other, the method based on Guzman et al. (2017b) depends on the value of β and the specific backhaul customers visited, as well as the respective number of visits, and for that the tables report the corresponding different values for each instance and β.
The results from both Tables 4.2 and 4.3 allow to conclude that much tighter bounds can be derived for any instance, for any β and for any Γ using the method that estimates probabilistic bounds *a posteriori*. An interesting observation is that it seems that the method based on Guzman et al. (2017b) estimates a lower probabilistic bound for solutions that visit the same backhaul more than once. Taking for example instance C4 with $\beta = 1000$, where it is shown that the probability of constraint violation when $\Gamma = 2$ is lower than when $\Gamma = 2.5$, for in the former case, the backhaul customer 15 is visited twice while in the later case, it is only visited once.

4.4.2.3 Performance of the models

In this section, the two different models under uncertainty, RO and CC, are analyzed and compared in terms of their performance. The performance metric used is the Price of

Table 4.2 – Probability of constraint violation for instances with three vehicles.

			Budget of uncertainty, Γ						
Instance	β	Method	0	0.5	1	1.5	2	2.5	3
A4	0	Irwin-Hall*	50.00%	31.77%	16.67%	7.03%	2.08%	0.26%	0.00%
		Guzman *post*	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	500	Guzman *post*	0.85%	0.85%	0.85%	0.85%	0.00%	0.00%	0.00%
	750	Guzman *post*	26.17%	26.17%	0.69%	0.69%	0.25%	0.14%	-
	1000	Guzman *post*	14.41%	14.41%	8.97%	-	-	-	-
B3	0	Irwin-Hall*	50.00%	31.77%	16.67%	7.03%	2.08%	0.26%	0.00%
		Guzman *post*	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	500	Guzman *post*	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	750	Guzman *post*	2.21%	2.21%	2.21%	1.42%	1.42%	0.20%	-
	1000	Guzman *post*	16.69%	16.69%	15.38%	6.40%	-	-	-

(-) No feasible solution was found.

Table 4.3 – Probability of constraint violation for instances with four vehicles.

			Budget of uncertainty, Γ								
Instance	β	Method	0	0.5	1	1.5	2	2.5	3	3.5	4
C4	0	Irwin-Hall*	50.00%	33.81%	20.05%	10.11%	4.17%	1.32%	0.26%	0.02%	0.00%
		Guzman *post*	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	500	Guzman *post*	1.07%	1.07%	1.07%	1.07%	0.00%	0.00%	0.00%	0.00%	0.00%
	750	Guzman *post*	22.95%	22.95%	0.29%	0.29%	0.29%	0.12%	0.00%	0.00%	0.00%
	1000	Guzman *post*	8.15%	8.15%	8.15%	6.40%	0.23%	0.34%	0.17%	0.01%	-
F4	0	Irwin-Hall*	50.00%	33.81%	20.05%	10.11%	4.17%	1.32%	0.26%	0.02%	0.00%
		Guzman *post*	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	500	Guzman *post*	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	750	Guzman *post*	0.38%	0.38%	0.38%	0.38%	0.38%	0.12%	0.00%	0.00%	0.00%
	1000	Guzman *post*	10.07%	10.07%	10.07%	6.26%	0.18%	0.18%	0.18%	0.02%	-

(-) No feasible solution was found.

Robustness (PoR), which measures the trade-off between constraint violation and the effect on the nominal objective function (Bertsimas and Sim, 2004), and is determined as the difference between robust and nominal solutions. Note that the metric PoR must only be used when the objective is certain, otherwise it would be comparing two different scenarios - the worst-case and the nominal case (Gorissen et al., 2015).

The results are reported for instances with three and four vehicles respectively in Tables 4.4 and 4.5. The first two columns identify respectively the instance and the model. The third column presents the value of minimum revenue β and the fourth reports the value of the nominal solution. The next columns present the value of the PoR obtained with each model, for each level of probability of constraint violation. Note that the probabilities used to test the CC model correspond to the probabilities obtained with Equation (4.26) from the Irwin-Hall distribution method. Thus, a probability of 50.00% corresponds to a $\Gamma = 0$, a probability of 0% corresponds to the maximum Γ applied, and so forth.

Table 4.4 – PoR obtained for RO and CC solutions of instances with three vehicles.

Instance	Model	β	Nominal solution	Probability of constraint violation						
				50.00%	31.77%	16.67%	7.03%	2.08%	0.26%	0.00%
A4	RO	0	136,068	0	0	0	0	0	0	0
		500	136,581	0	0	0	0	3,193	3,193	3,193
		750	136,581	0	0	3,193	3,193	4,284	6,719	-
		1000	139,774	0	0	1,091	-	-	-	-
	CC	0	136,068	0	0	0	0	0	0	0
		500	136,581	0	0	0	0	3,193	3,193	3,193
		750	136,581	0	2,762	3,193	3,193	-	-	-
		1000	139,774	0	0	-	-	-		-
B3	RO	0	132,731	0	0	0	0	0	0	0
		500	132,731	0	0	0	0	0	0	0
		750	132,731	0	0	0	770	770	1,623	-
		1000	132,731	0	0	770	9,635	-	-	-
	CC	0	132,731	0	0	0	0	0	0	0
		500	132,731	0	0	0	0	0	0	0
		750	132,731	0	0	0	5,043	-	-	-
		1000	132,731	0	0	-	-	-	-	-

(-) No feasible solution was found.

The analysis of both Tables indicate that, firstly, as expected, the PoR increases while increasing the budget of uncertainty (or decreasing the probability of constraint violation), for a given level of β. When the PoR remains the same, it means that the solution structure also remains the same, and so it does not impact the total expected revenue. These values provide the minimum value of budget of uncertainty that defines the worst-case for a given β, for each instance. Secondly, increasing the value of β tends to force the creation of a new routing plan, probably due to the need of augmenting the visits to backhaul customers. Taking for example, the results obtained with instance B3: when $\beta = 500$, the PoR = 0 and remains the same for all levels of probability; when $\beta = 750$, the PoR changes when the probability is 7.03%; and when $\beta = 1000$, the PoR changes sooner, when the probability is 16.67%. Nevertheless, these results are not always in accordance, as the tests with instance

Table 4.5 – PoR obtained for RO and CC solutions of instances with four vehicles.

Instance	Model	β	Nominal solution	Probability of constraint violation								
				50.00%	33.81%	20.05%	10.11%	4.17%	1.32%	0.26%	0.02%	0.00%
C4	RO	0	128,792	0	0	0	0	0	0	0	0	0
		500	131,464	0	0	0	0	2,117	2,117	2,117	2,117	2,117
		750	131,464	0	0	2,117	2,117	2,117	5,371	11,245	11,245	11,245
		1000	133,580	0	0	0	3,254	9,341	9,932	10,035	15,317	-
	CC	0	128,792	0	0	0	0	0	0	0	0	0
		500	131,464	0	0	0	0	369	2,116	2,116	2,116	2,116
		750	131,464	0	369	2,116	2,116	2,174	11,244	11,244	11,244	11,244
		1000	133,580	0	0	9,128	9,128	11,777	-	-	-	-
F4	RO	0	145,456	0	0	0	0	0	0	0	0	0
		500	145,456	0	0	0	0	0	0	0	0	0
		750	145,456	0	0	0	0	0	1,861	9,254	9,254	9,254
		1000	145,456	0	0	0	1,861	9,253	9,254	9,254	15,061	-
	CC	0	145,456	0	0	0	0	0	0	0	0	0
		500	145,456	0	0	0	0	0	0	0	0	0
		750	145,456	0	0	0	0	9,254	9,254	9,254	9,254	9,254
		1000	145,456	0	0	9,254	9,469	19,444	-	-	-	-

(-) No feasible solution was found.

C4 reveals that when $\beta = 750$, the PoR changes sooner than when $\beta = 1000$, although the PoR value is higher for the highest β.

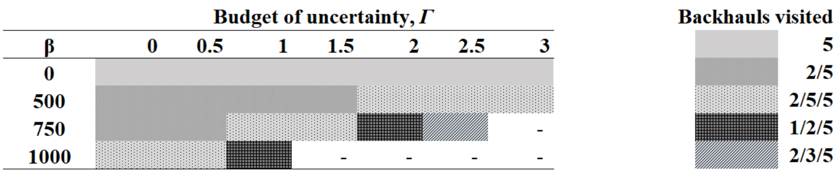
From the comparison between the two models, it is possible to conclude that the CC model is more conservative than the RO model, at least for the same probabilistic bounds obtained with the method of Irwin-Hall distribution. The results with the RO model always provide a PoR lower or, at most, equal to the CC model, for any of the instances tested, for a given value of probability. In addition, a $PoR > 0$ seems to occur sooner for the CC model than for the RO model. Finally, for all instances tested, the CC model provides more infeasible solutions than the RO model.

4.4.2.4 Solution structure

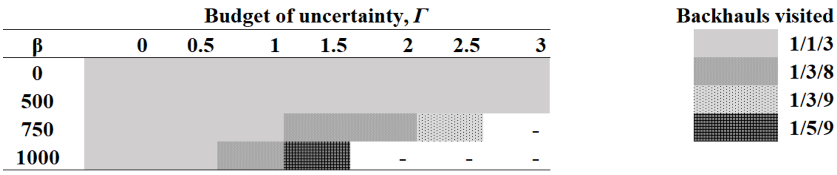
For each value of β and Γ, the solution structure (routing plan) obtained with each instance tested with the RO model is presented in Figure 4.5, specifying which backhaul customers are visited in each routing plan. All solutions depicted in Figures 4.5a and 4.5b comprise a plan with 3 routes, while all the solutions in Figures 4.5c and 4.5d comprise a plan with 4 routes.

As expected, increasing the minimum revenue required leads to increase the number of backhaul customers visited, except in the case of instance B3, as shown in Figure 4.5b. Although, in this case, the first routing plan obtained includes two visits to the same backhaul customer (1), whereas all the other plans include distinct backhaul customers. It also seems that, at least, one specific backhaul is always included in any route. For example, backhaul 5 is included in any routing plan of instance A4, backhaul 1 in instance B3, backhaul 15 in instance C4 and backhaul 9 in instance F4.

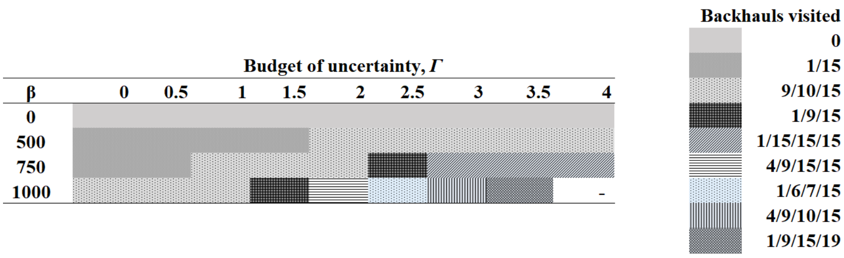
With the exception of instance C4, at least one backhaul customer is visited when there is no requirement of minimum revenue ($\beta = 0$). This means that in these cases, it is profitable to visit a backhaul customer, even if there is no need to. Also, in instances A4 and C4, all



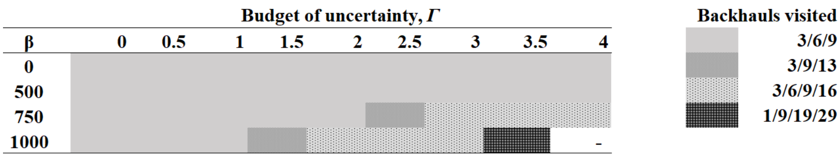
(a) Instance A4



(b) Instance B3



(c) Instance C4



(d) Instance F4

Figure 4.5 – Solution structures obtained by changing β and Γ.

routing plans obtained when there is a minimum revenue required, are different than the cases when such requirement does not exist. In fact, it seems that both instances are very sensitive to changes in any parameter, β or Γ. On the other hand, for instances B3 and F4, the same routing plan is obtained whenever $\beta = 0$ or $\beta = 500$ or even for any value of β with, at least, a $\Gamma \leq 0.5$.

4.4.3 Evaluation of solution methods

To evaluate the computational performance of the B&C and the ALNS, both methods are tested for instances with different vehicle capacities C, numbers of vehicles $|\mathcal{K}|$, linehaul customers $|\mathcal{L}|$ and backhaul customers $|\mathcal{B}|$. These instances are randomly selected from the universe of Goetschalckx and Jacobs-Blecha (1989), such that, at least, two instances from each class (from A to N) are obtained (see Table 4.6). The instances are divided into 14 classes A–N. The classes G and L have, each one, 3 instances, and the remaining classes have 2 instances each. The instances of a class are almost equal: they are different only in respect to the vehicle capacity C and the number of vehicles $|\mathcal{K}|$ while their graphs, costs, revenues, and deviations are identical. The number of backhaul customers $|\mathcal{B}|$, in relation to $|\mathcal{L}|$, corresponds to: 25% in class A; 50% in B; 100% in C; about 25% in D; 50% in E; 100% in F; about 25% in G; about 50% in H; 100% in I; about 25% in J; about 50% in K; 100% in L; 25% in N; and 50% in M.
Each instance is tested for two different values of budget of uncertainty, $\Gamma = 0$ and $\Gamma = 2$, and for two different values of minimum revenue required, $\beta = 0$ and $\beta = 750$. For the B&C, the running time is limited to one hour for each test and is worth recovering that the RCI tolerance is 0.2. For the ALNS, 30 replications of each test are performed.

Table 4.6 – Instance characteristics

Inst	C	$\lvert\mathcal{K}\rvert$	Inst	C	$\lvert\mathcal{K}\rvert$	Inst	C	$\lvert\mathcal{K}\rvert$	$\lvert\mathcal{L}\rvert$	$\lvert\mathcal{B}\rvert$
A2	2550	5	A4	4050	3				20	5
B1	1600	7	B2	2600	5				20	10
C1	1800	7	C4	4150	4				20	20
D2	1700	11	D3	2750	7				30	8
E1	2650	7	E3	5225	4				30	15
F1	3000	6	F4	5500	4				30	30
G2	4300	6	G4	5300	5	G6	8000	4	45	12
H2	5100	5	H3	6100	4				45	23
I1	3000	10	I4	5700	6				45	45
J1	4400	10	J3	8200	6				75	19
K2	6000	8	K4	6200	7				75	38
L1	4400	10	L3	5000	9	L5	6000	8	75	75
M2	5200	10	M4	8000	7				100	25
N2	5700	10	N6	8500	8				100	50

As already mentioned, the VRPSB is NP-hard and, therefore it is not expected to achieve optimality for solutions obtained with exact methods for large instances. However, the advantage of the B&C developed is to obtain an adequate lower bound for each instance and, consequently, measure the efficacy of the ALNS.

The results obtained from these tests are presented in Table 4.7. The first three columns present respectively, the instance name, the budget of uncertainty Γ and the minimum revenue required β. The next four columns refer to the main results obtained from the B&C, ordered by the lower bound (LB), solution value (z), the gap of the B&C ($G_{B\&C}$), and the time of the tests in seconds (T(s)). The gap corresponds to the percentage difference between solution value and the respective lower bound. When the solution time of the B&C reaches 3600s, it means the computing time has stopped at its limit.

The next six columns in Table 4.7 show the results from the ALNS, namely the worst, the average, and the best solution values given by z, the standard deviation (sd(z)), the total number of infeasible solutions (#i) obtained out of 30 runs, and the average solution time in seconds (AT(s)). Finally, the last two columns present the most common performance metrics used to compare the methods, namely the G_{SOL} and G_{LB}. The former metric is determined by the percentage difference between the best solution value found by the ALNS and the solution value obtained with the B&C, while the latter compares the ALNS solution with the lower bound value from the B&C. In addition, the last line in the table presents the average values of gap and solution time determined for both methods, as well as the average standard deviation of solutions obtained with ALNS.

The B&C is able to provide optimal solutions for almost all tests with classes of instances A to H (up to 45 linehaul customers), as determined by the values of $G_{B\&C}$. On the other hand, the exact method is unable to find a feasible solution in 18 out of the 90 tests. An interesting observation from the B&C results is related to the tests on the class of instances I. These have the same number of linehaul customers as the class of instances H, but almost the double number of backhauls. The difference between instances I1 and I4 are the number of vehicles, respectively 10 and 6, and vehicle capacities, respectively 3000 and 5700. Yet, the tests with instance I1 do not provide an optimal solution in one hour, while with instance I4, the optimal solution is found in no more than one minute.

The class of instances J have almost the same total number of customers (94) of class I (90), however, the number of linehauls of the J instances represents 167% of the $|\mathcal{L}|$ of the I instances. From this point forward (classes J, K, L, M, and N), no optimal guaranteed solutions are found and 18 of the 33 tests failed in returning a feasible solution. It is also noteworthy that among L5 instances the better gap (2.60%) is presented by the robust version with $\Gamma = 2$ and $\beta = 750$. Despite its usefulness in the actual solution of the problem, all the B&C tests provide lower bounds that are useful to analyze the performance of the ALNS.

The solutions obtained with ALNS can be considered reasonably stable, as indicated by the average standard deviation of 1.5%. On the other hand, infeasible solutions are found in seven instances, but only for the worst-cases, i.e. when $\Gamma = 2$.

Comparing solutions obtained with the B&C and with the ALNS, it can be observed that the metaheuristic is able to reach 31 out of 51 of the known optimal solutions proven by the B&C, given by a G_{LB} of 0.00%. It can also be demonstrated that the highest G_{LB} values are obtained for the worst-cases. Furthermore, except for two cases (instance H2, $\Gamma = 2$,

Table 4.7 – Computational results from B&C and ALNS

			B&C Results				ALNS results							
Inst	Γ	β	LB	z	$G_{B\&C}$	T(s)	Worst z	Avg z	Best z	sd(z)	#i	AT(s)	G_{SOL}	G_{LB}
A2	0	0	164,457	164,457	0.00%	0.5	164,457	164,457	164,457	0.0%	-	0.7	0.00%	0.00%
	0	750	164,457	164,457	0.00%	0.8	166,001	165,176	164,457	0.4%	-	0.9	0.00%	0.00%
	2	750	164,457	164,457	0.00%	0.3	168,480	165,909	164,457	0.5%	-	1.0	0.00%	0.00%
A4	0	0	136,068	136,068	0.00%	0.2	139,157	137,181	136,068	1.1%	-	0.5	0.00%	0.00%
	0	750	136,581	136,581	0.00%	0.2	137,092	136,617	136,581	0.1%	-	0.7	0.00%	0.00%
	2	750	140,865	140,865	0.00%	0.2	150,866	144,846	140,865	2.2%	17	0.5	0.00%	0.00%
B1	0	0	209,034	209,034	0.00%	1.5	209,089	209,047	209,034	0.0%	-	1.0	0.00%	0.00%
	0	750	209,034	209,034	0.00%	1.0	209,634	209,222	209,034	0.1%	-	1.5	0.00%	0.00%
	2	750	209,034	209,034	0.00%	2.3	211,511	209,985	209,089	0.2%	-	1.9	0.03%	0.03%
B2	0	0	162,983	162,983	0.00%	3.5	165,428	163,236	162,983	0.3%	-	0.7	0.00%	0.00%
	0	750	162,983	162,983	0.00%	1.5	168,811	163,343	162,983	0.6%	-	1.1	0.00%	0.00%
	2	750	163,431	163,431	0.00%	5.9	177,880	168,503	163,473	2.0%	2	1.0	0.03%	0.03%
C1	0	0	186,636	186,636	0.00%	2.4	191,913	187,691	186,636	0.9%	-	2.2	0.00%	0.00%
	0	750	187,655	187,655	0.00%	6.7	194,101	189,743	187,655	1.2%	-	3.7	0.00%	0.00%
	2	750	188,842	188,842	0.00%	24.3	200,405	192,907	188,842	1.9%	-	4.2	0.00%	0.00%
C4	0	0	128,792	128,792	0.00%	0.2	130,760	129,953	128,792	0.7%	-	1.6	0.00%	0.00%
	0	750	131,464	131,464	0.00%	0.5	142,010	137,440	131,464	1.9%	-	1.7	0.00%	0.00%
	2	750	133,580	133,580	0.00%	0.2	147,191	141,817	135,682	2.6%	23	1.5	1.57%	1.57%
D2	0	0	296,077	297,853	0.60%	3600.0	317,855	300,514	297,853	1.8%	-	2.3	0.00%	0.60%
	0	750	297,853	297,853	0.00%	2724.5	315,653	300,486	297,853	1.5%	-	4.6	0.00%	0.00%
	2	750	295,983	297,853	0.63%	3600.0	318,202	301,785	297,853	1.8%	-	5.5	0.00%	0.63%
D3	0	0	214,017	214,017	0.00%	292.8	215,464	214,395	214,017	0.2%	-	1.7	0.00%	0.00%
	0	750	214,017	214,017	0.00%	98.8	224,750	215,665	214,017	1.3%	-	2.5	0.00%	0.00%
	2	750	214,598	214,598	0.00%	294.0	227,802	219,152	214,598	1.5%	-	2.8	0.00%	0.00%
E1	0	0	198,239	198,239	0.00%	45.5	204,070	199,989	199,005	0.9%	-	1.4	0.39%	0.39%
	0	750	198,239	198,239	0.00%	75.0	221,881	211,357	203,036	2.2%	-	2.3	2.42%	2.42%
	2	750	198,649	198,649	0.00%	41.7	230,836	221,851	217,390	2.0%	-	3.4	9.43%	9.43%
E3	0	0	149,576	149,576	0.00%	2.3	161,201	152,294	149,576	3.1%	-	2.3	0.00%	0.00%
	0	750	151,519	151,519	0.00%	0.8	164,712	156,179	151,519	2.5%	-	5.8	0.00%	0.00%
	2	750	154,399	154,399	0.00%	2.8	179,894	162,333	154,399	3.7%	-	2.8	0.00%	0.00%
F1	0	0	183,723	183,723	0.00%	18.5	193,113	186,508	183,723	1.7%	-	4.2	0.00%	0.00%
	0	750	183,723	183,723	0.00%	20.5	209,893	188,612	183,723	3.4%	-	4.4	0.00%	0.00%
	2	750	183,723	183,723	0.00%	23.8	209,893	190,643	183,723	3.8%	-	4.6	0.00%	0.00%
F4	0	0	145,456	145,456	0.00%	0.1	149,676	149,041	145,456	0.9%	-	2.3	0.00%	0.00%
	0	750	145,456	145,456	0.00%	0.1	159,387	150,835	145,638	1.9%	-	5.8	0.13%	0.13%
	2	750	145,456	145,456	0.00%	0.1	156,699	150,631	147,027	1.8%	16	3.9	1.08%	1.08%
G2	0	0	225,110	225,110	0.00%	80.8	234,233	226,795	225,110	1.0%	-	5.7	0.00%	0.00%
	0	750	225,110	225,110	0.00%	114.8	234,333	227,184	225,663	0.9%	-	5.6	0.25%	0.25%
	2	750	225,444	225,444	0.00%	459.2	239,360	231,080	226,568	1.6%	-	6.2	0.50%	0.50%
G4	0	0	202,551	202,551	0.00%	1.6	202,961	202,725	202,551	0.1%	-	3.3	0.00%	0.00%
	0	750	202,551	202,551	0.00%	4.3	217,448	204,516	202,551	1.4%	-	3.7	0.00%	0.00%
	2	750	203,320	203,320	0.00%	22.8	221,936	209,843	204,237	2.0%	2	4.1	0.45%	0.45%
G6	0	0	180,176	180,176	0.00%	64.6	189,408	183,619	181,322	1.0%	-	2.6	0.64%	0.64%
	0	750	180,630	180,630	0.00%	60.5	196,594	187,711	180,947	2.4%	-	3.9	0.18%	0.18%
	2	750	182,081	182,081	0.00%	71.4	207,143	198,803	191,603	2.7%	13	3.7	5.23%	5.23%

Table 4.7 (cont.) – Computational results from B&C and ALNS

				B&C Results					ALNS results					
Inst	Γ	β	LB	z	$G_{B\&C}$	T(s)	Worst z	Avg z	Best z	sd(z)	#i	AT(s)	G_{SOL}	G_{LB}
H2	0	0	192,446	192,446	0.00%	149.9	200,616	196,179	192,904	1.1%	-	4.8	0.24%	0.24%
	0	750	193,092	193,092	0.00%	103.2	207,403	200,886	195,225	1.8%	-	6.9	1.10%	1.10%
	2	750	197,064	197,545	0.24%	3600.1	216,619	207,616	202,268	2.0%	-	10.7	2.39%	2.64%
H3	0	0	182,499	182,499	0.00%	11.5	195,735	186,102	182,499	2.1%	-	9.6	0.00%	0.00%
	0	750	186,316	186,316	0.00%	26.7	203,299	192,831	187,053	1.9%	-	10.9	0.40%	0.40%
	2	750	188,727	188,727	0.00%	74.9	206,504	198,095	190,855	1.7%	-	16.6	1.13%	1.13%
I1	0	0	244,435	261,403	6.94%	3600.1	267,657	259,439	256,536	1.2%	-	10.4	-1.86%	4.95%
	0	750	244,074	260,212	6.61%	3600.0	262,249	258,349	256,474	0.7%	-	12.1	-1.44%	5.08%
	2	750	244,741	262,372	7.20%	3600.0	265,021	259,621	256,717	0.9%	-	24.5	-2.16%	4.89%
I4	0	0	191,397	191,397	0.00%	49.2	201,384	193,115	191,397	1.3%	-	7.8	0.00%	0.00%
	0	750	191,397	191,397	0.00%	60.0	202,672	193,951	191,462	1.8%	-	7.4	0.03%	0.03%
	2	750	191,397	191,397	0.00%	46.3	204,411	195,577	191,619	2.2%	1	8.8	0.12%	0.12%
J1	0	0	274,601	-	-	3600.3	318,741	303,254	304,320	1.2%	-	22.6	-	10.82%
	0	750	273,684	-	-	3600.1	317,682	303,092	303,092	1.4%	-	28.1	-	10.75%
	2	750	275,849	-	-	3600.1	324,314	306,533	309,652	1.3%	-	35.5	-	12.25%
J3	0	0	216,813	245,947	13.44%	3600.2	262,717	234,677	225,100	3.9%	-	14.7	-8.48%	3.82%
	0	750	218,177	232,069	6.37%	3600.3	257,619	238,005	226,555	3.7%	-	17.5	-2.38%	3.84%
	2	750	215,679	238,870	10.75%	3600.1	253,178	238,957	228,175	2.3%	-	16.1	-4.48%	5.79%
K2	0	0	264,088	327,849	24.14%	3600.4	284,098	278,224	275,513	0.9%	-	24.7	-15.96%	4.33%
	0	750	267,466	281,093	5.10%	3600.1	288,174	279,027	275,745	1.1%	-	37.7	-1.90%	3.10%
	2	750	264,448	314,323	18.86%	3600.1	295,169	283,514	276,723	1.5%	-	40.3	-11.96%	4.64%
K4	0	0	245,828	269,840	9.77%	3600.1	266,449	259,759	257,710	0.8%	-	17.8	-4.50%	4.83%
	0	750	246,304	266,482	8.19%	3600.2	271,202	262,880	258,558	1.4%	-	24.6	-2.97%	4.98%
	2	750	247,350	269,572	8.98%	3600.1	271,429	266,705	260,791	1.1%	-	29.6	-3.26%	5.43%
L1	0	0	266,306	-	-	3600.2	307,468	294,782	291,604	1.1%	-	37.6	-	9.50%
	0	750	268,661	-	-	3600.1	306,696	294,407	291,850	0.9%	-	42.4	-	8.63%
	2	750	268,488	-	-	3600.2	300,141	294,134	291,505	0.8%	-	53.4	-	8.57%
L3	0	0	254,446	-	-	3600.8	291,922	279,328	274,465	1.1%	-	39.6	-	7.87%
	0	750	253,382	-	-	3600.4	282,923	278,122	273,860	0.8%	-	39.0	-	8.08%
	2	750	251,903	-	-	3600.3	284,272	278,334	274,561	0.9%	-	96.3	-	8.99%
L5	0	0	236,821	287,121	21.24%	3600.3	270,441	257,813	249,581	1.7%	-	34.4	-13.07%	5.39%
	0	750	235,730	268,902	14.07%	3600.2	271,250	257,389	249,666	2.4%	-	40.9	-7.15%	5.91%
	2	750	243,111	249,420	2.60%	3600.1	276,468	261,355	250,822	2.3%	2	37.5	0.56%	3.17%
M2	0	0	311,663	-	-	3600.1	356,401	348,356	342,540	1.0%	-	52.3	-	9.91%
	0	750	305,740	-	-	3600.1	365,427	349,324	341,859	1.8%	-	61.9	-	11.81%
	2	750	306,403	-	-	3600.1	383,867	354,778	342,164	2.6%	-	69.9	-	11.67%
M4	0	0	267,153	316,326	18.41%	3600.1	295,958	287,917	282,514	1.3%	-	30.8	-10.69%	5.75%
	0	750	268,303	312,923	16.63%	3600.2	294,224	287,284	282,018	1.0%	-	68.7	-9.88%	5.11%
	2	750	266,505	-	-	3600.5	306,852	293,250	284,497	1.7%	-	81.5	-	6.75%
N2	0	0	300,459	-	-	3600.3	347,618	338,144	328,669	1.3%	-	57.5	-	9.39%
	0	750	298,437	-	-	3600.1	348,852	338,435	329,922	1.4%	-	63.3	-	10.55%
	2	750	302,526	-	-	3600.4	354,570	338,931	328,484	1.4%	-	78.8	-	8.58%
N6	0	0	251,446	332,761	32.34%	3601.0	289,457	278,882	275,535	1.1%	-	29.1	-17.20%	9.58%
	0	750	252,833	-	-	3600.1	289,093	279,720	275,533	1.2%	-	36.9	-	8.98%
	2	750	247,635	-	-	3600.1	297,811	283,606	276,690	1.9%	-	45.6	-	11.73%
Avg					**3.24%**	**1616.7**				**1.5%**		**18.5**	**-1.26%**	**3.27%**

β = 750 and instance L5, Γ = 2, β = 750) the G_{SOL} is always non-positive for instances where feasible solutions – with no guarantee of optimality – were found by the B&C, which means that the ALNS can generate better solutions for these instances than the B&C, at least for one hour of computing time.

With respect to the solution time, the ALNS can obtain solutions very fast. In fact, even for the larger instances (L, M and N), a solution is found in less than 2 minutes, on average. From these results, we can argue that the ALNS developed is a very efficient and an effective solution method for the VRPSB under uncertainty, thus generating very good quality solutions in a very short time.

From Table 4.7, the performance of the robust optimization approach used in this work can also be demonstrated. The results do not allow to infer on the solution time of the robust optimization, since for some instances, the computing time of deterministic solutions are inferior to the computing time of robust solutions (e.g., G2), but for other instances, the opposite occur (e.g., I4). Nevertheless, it can be shown that, at least, the robust modelling has no more significant intractability than the deterministic modelling. Based on these results, it can be concluded that the robust optimization used in this work is efficient to solve the VRPSB under revenue uncertainty.

4.5. Conclusions and future work

This work presents the first study of a Vehicle Routing Problem with Backhauls (VRPB) under uncertainty. More precisely, we consider the problem where the visits to backhauls are optional, each visit has an associated revenue, which is uncertain, and a minimum revenue is required to satisfy the depot demand. Following a robust optimization approach, the revenues are considered to be bounded by a polyhedron set of uncertainty and a parameter, called budget of uncertainty, is applied to control the size of the uncertainty set. An alternative robust approach based on a factor model support is designed and illustrated in a simple example from a hypothetical case of a wood-based industry. A chance-constrained model is also designed for the VRPB under uncertain revenues and compared with the robust optimization model. Four different methods to estimate probabilistic bounds for the minimum revenue constraint violation are defined, tested and compared. Finally, two different solution methods, one exact and one metaheuristic, are developed to solve both deterministic and robust problems.

Several insights could be retrieved from this study. First, nominal solutions are very sensitive to uncertainty, since the structure of the solution changes quickly even for low values of budget of uncertainty. Second, and as expected, the cost of robust solutions is higher than the cost of nominal solutions. The solutions obtained with the chance-constrained model are much more conservative than the solutions obtained with the robust optimization model. In terms of probabilistic bounds, the method that estimates *a posteriori* the probability of constraint violation of a solution is the most rigorous one, since it produces the exact value for that probability. However, it requires the knowledge of the solution structure. A novel method based on the Irwin-Hall distribution provides much tighter bounds than the other *a priori* probabilistic methods, which allows to better characterize the bounds of robust

solutions based only on values of budget of uncertainty.
The B&C developed in this work finds optimal solutions in a reasonable time, but only for small and medium instances. However, the computing time was limited to one hour in this work. As such, increasing the time limit may result in a higher lower bound and, consequently, in a considerably lower gap. On the other hand, the ALNS shows high efficiency and efficacy to solve all instances, thus obtaining high quality solutions in a very short time (less than two minutes). It is also shown that the robust approach used in this work can create a tractable robust model, where solutions may be obtained with similar computing effort as with the deterministic model.
Different directions for future work on this subject arise. Firstly, considering that very few exact methods have been successfully applied in VRPBs, it may be interesting to evaluate more promising solution methods, such as Branch-and-Price algorithms. Thus, better lower bounds of the problem may be found, which improves the evaluation of the performance of metaheuristics, such as the ALNS developed. Another future direction for research concerns the modeling aspects of uncertainty sets. For instance, the uncertainty set may be built using a known distribution or some statistics obtained from historical information, such as the distributionally robust optimization (Gabrel et al., 2014). As such, the uncertainty set can be represented by tight bounds, which may produce better probabilistic bounds for the robust problem and decrease the conservatism of robust solutions. Finally, it would be interesting to apply the robust VRPSB in practice and gather insights from the perspective of integrated logistics.

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Appendix 4.A Chance-constrained VRPSB

In this section, a chance-constraint (CC) approach is proposed in order to benchmark it with the robust optimization method. In a CC model, the constraint of minimum revenue must be satisfied with a probability $(1-\alpha)$, as described in Equation (4.33), where α represents the probability of constraint violation. In this case, it is also assumed that the parameters $\tilde{p}_j$ are independent and follow an uniform distribution function in $[\bar{p}_j - \dot{p}_j, \bar{p}_j + \dot{p}_j]$:

$$Pr\left(\sum_{j\in\mathcal{B}} \tilde{p}_j x_{(j,n+m+1)} \geq \beta\right) \geq 1-\alpha. \tag{4.33}$$

To produce a tractable counterpart of the CC model, an approximate method is applied. The idea of the method is to approximate the resulting distribution in an uniform distribution supported in $[\sum_{j\in\mathcal{B}}(\bar{p}_j - \dot{p}_j)x_{(j,n+m+1)}, \sum_{j\in\mathcal{B}}(\bar{p}_j + \dot{p}_j)x_{(j,n+m+1)}]$, since it is extremely hard to calculate the convolution sum of the non-identical uncertain parameters $\tilde{p}_j$ and the approximate uniform distribution can be used as an over-estimator of the actual resulting distribution. The approximation is described as follows:

$$\sum_{j\in\mathcal{B}} \bar{p}_j x_{j,n+m+1} - \Upsilon^{-1}(1-\alpha)\frac{\sum_{j\in\mathcal{B}} 2\dot{p}_j x_{j,n+m+1}}{\sqrt{12}} \geq \beta, \tag{4.34}$$

where Υ^{-1} is the inverse of a cumulative distribution function of the standardized uniform distribution bounded in $[-\sqrt{3}. \sqrt{3}]$, $\sum_{j\in\mathcal{B}} \bar{p}_j x_{j,n+m+1}$ is the mean of the resulting uniform distribution and $\frac{\sum_{j\in\mathcal{B}} 2\dot{p}_j x_{j,n+m+1}}{\sqrt{12}}$ is its standard deviation. By replacing (4.7) with (4.34), the resulting CC model reads as follows:

$$min \sum_{(i,j)\in\mathcal{E}} c_{ij}x_{ij} - \sum_{j\in\mathcal{B}} \bar{p}_j x_{j,n+m+1}$$

subjected to

$$(4.2) - (4.6)$$
$$(4.8) - (4.11)$$
$$(4.34)$$

Appendix 4.B Comparison of single and multiple backhaul visits

In this section, we present and discuss the modelling and algorithmic aspects of allowing multiples backhaul visits per route in the robust VRPSB.
The robust model presented in Section 4.2.2 allows for only one backhaul visit per route.

Allowing multiple backhaul visits forces the model to include all potential arcs between one linehaul and one backhaul customers and between two backhaul customers. Thus, the set of edges must change to $\bar{\mathcal{E}}$, given by $\{(0,j)\colon j\in\mathcal{L}\}\cup\{(i,j)\colon i\in\mathcal{L}\cup\{n+m+1\},\ j\in\mathcal{L}\cup\{n+m+1\},\ i<j\}\cup\{(i,j)\colon i\in\mathcal{L},\ j\in\mathcal{B}\}\cup\{(i,j)\colon i\in\mathcal{B},\ j\in\mathcal{B}\}\cup\{(i,n+m+1)\colon i\in\mathcal{B}\}$. Therefore, the model with multiple backhaul visits is formulated as follows:

$$min \sum_{(i,j)\in\mathcal{E}} c_{ij}x_{ij} - \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{B}} \bar{p}_j x_{i,j} \tag{4.35}$$

subjected to

$$(4.2)-(4.4)$$
$$(4.6)$$
$$(4.9)-(4.11)$$
$$(4.17)$$

$$\sum_{(i,j)\in\delta(i)} x_{ij} = 2, \qquad \forall i\in\mathcal{B}, \tag{4.36}$$

$$\sum_{(i,j)\in\delta(\mathcal{S})} x_{ij} \geq 2r(\mathcal{S}), \qquad \forall\mathcal{S}\subseteq\mathcal{L}\cup\mathcal{B},\ |\mathcal{S}|\geq 2, \tag{4.37}$$

$$\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{B}} \bar{p}_j x_{i,j} - \lambda\Gamma - \sum_{j\in\mathcal{B}}\mu_j \geq \beta, \tag{4.38}$$

$$\lambda+\mu_j \geq \sum_{i\in\mathcal{V}} \dot{p}_j x_{i,j}, \qquad \forall j\in\mathcal{B}, \tag{4.39}$$

Note that, in the case of single backhaul visits, the worst-case is achieved when the budget of uncertainty is the minimum number between the number of routes and the number of backhaul customers, i.e., $\Gamma = \min\{|\mathcal{B}|,|\mathcal{K}|\}$. In the case of multiple backhaul visits, however, the number of routes do not influence the robust parameter Γ, since any number of backhauls can be visited by all routes. Thus the worst-case is achieved when $\Gamma = |\mathcal{B}|$, which leads to a higher number of possible scenarios.

To analyze the different solutions and the computational performance of both modelling cases, tests are carried with four small instances, for different values of β and Γ. The tests are performed with OPL (using a commodity flow formulation) and the time limit is set to 600 seconds. The results are reported in Table 4.9. The first three columns describe the instance, the value of β and the value of Γ, respectively. The next four columns report the objective function (z), the total distance (d), the MIP gap (Gap) and the time in seconds ($T(s)$) for the case of single visits. The last four columns report the same indicators for the case of multiple visits.

In terms of solution structure, the results show that allowing multiple backhaul visits always lead to better or equal solutions than in the case of restricting the visits to one per route. For example, with instances A4, the same value of total revenue can be achieved with shorter distances, since several backhaul customers can be visited in the same route (e.g., instead of visiting three backhauls in three routes, the same three backhauls are visited in just two routes). With instances C4, a lower value of total revenue may be achieved but this still compensates the lower total distance traveled. In fact, allowing multiple visits requires only three routes to satisfy both the linehaul customers demand and the minimum required revenue, whereas in the case of single visits, four routes must be created. This also occurs for instance F4. Moreover, it is possible to observe that allowing multiple visits always lead to feasible solutions, even for the highest values of Γ and β. For the case of single visits, however, no feasible solutions are obtained for the worst-cases if $\beta = 1000$, which can be seen for all the four instances tested.

In terms of computational performance, allowing multiple visits tends to lead to high computational times and MIP gaps, because the number of variables increases drastically and several combinations of two or more backhauls can be created for the same route. In fact, whereas the previous model that tackles single visits considers only the backhaul arc $x_{i,n+m+1} (\forall i \in \mathcal{B})$, the model that allows multiple visits considers also the arcs linking linehaul to backhaul customers and between backhaul customers, i.e. $x_{j,i} (\forall j \in \mathcal{V}/\{0\}, \forall i \in \mathcal{B})$.

Table 4.9 – Comparison of single and multiple backhaul visits.

Instance	β	Γ	Single backhaul visits z	d	Gap	T(s)	Multiple backhaul visits z	d	Gap	T(s)
A4	0	0	136,068	136,532	0.01%	7	136,068	136,532	0.01%	20
	500	0	136,581	137,449	0.01%	9	136,581	137,449	0.01%	25
	500	1	136,581	137,449	0.01%	11	136,581	137,449	0.01%	30
	500	2	139,774	141,106	0.01%	31	139,010	140,342	0.01%	53
	500	3	139,774	141,106	0.01%	27	139,010	140,342	0.01%	49
	500	4	139,774	141,106	0.00%	8	139,010	140,342	0.01%	31
	500	5	139,774	141,106	0.01%	11	139,010	140,342	0.01%	24
	1000	0	139,774	141,106	0.01%	8	139,010	140,342	0.01%	43
	1000	1	140,865	142,171	0.00%	4	139,131	140,437	0.01%	23
	1000	2	–				143,294	145,064	0.01%	92
	1000	3	–				145,928	148,128	0.01%	35
	1000	4	–				145,928	148,128	0.01%	37
	1000	5	–				145,928	148,128	0.01%	14
B3	0	0	132,731	134,061	0.01%	3	132,731	134,061	0.01%	2
	500	0	132,731	134,061	0.01%	4	132,731	134,061	0.00%	8
	500	1	132,731	134,061	0.01%	4	132,731	134,061	0.01%	5
	500	2	132,731	134,061	0.01%	2	132,731	134,061	0.01%	6
	500	3	132,731	134,061	0.00%	2	132,731	134,061	0.01%	6
	500	4	132,731	134,061	0.01%	2	132,731	134,061	0.00%	3
	500	5	132,731	134,061	0.01%	3	132,731	134,061	0.01%	8
	500	10	132,731	134,061	0.00%	2	132,731	134,061	0.01%	2
	1000	0	132,731	134,061	0.01%	1	132,731	134,061	0.00%	7
	1000	1	133,501	134,748	0.01%	1	133,501	134,748	0.01%	6
	1000	2	–				134,010	136,179	0.01%	5
	1000	3	–				134,010	136,179	0.00%	6
	1000	4	–				134,010	136,179	0.00%	4
	1000	5	–				134,010	136,179	0.00%	5
	1000	10	–				134,010	136,179	0.00%	1

(–) No feasible solution was found.

Table 4.9 (cont.) – Comparison of single and multiple backhaul visits.

			Single backhaul visits				Multiple backhaul visits			
Instance	β	Γ	z	d	Gap	T(s)	z	d	Gap	T(s)
C4	0	0	128,792	128,792	0.01%	22	128,792	128,792	0.01%	59
	500	0	131,464	132,366	0.01%	11	130,108	130,987	3.06%	600
	500	1	131,464	132,366	0.01%	54	130,108	130,987	0.01%	150
	500	2	133,580	134,918	0.01%	378	130,619	131,882	3.41%	601
	500	3	133,580	134,918	0.01%	137	130,619	131,882	2.15%	600
	500	4	133,580	134,918	0.01%	126	130,619	131,882	0.01%	191
	500	5	133,580	134,918	0.01%	157	130,619	131,882	2.13%	601
	500	10	133,580	134,918	0.01%	36	130,619	131,882	0.01%	103
	500	15	133,580	134,918	0.01%	16	130,619	131,882	0.01%	311
	500	20	133,580	134,918	0.01%	16	130,619	131,882	0.01%	76
	1000	0	133,580	134,918	0.00%	3	130,619	131,882	0.01%	144
	1000	1	133,580	134,918	0.01%	5	130,619	131,882	0.01%	235
	1000	2	142,921	144,700	0.01%	52	131,721	133,474	0.01%	421
	1000	3	143,615	145,345	0.01%	26	131,721	133,474	0.01%	462
	1000	4	–				132,232	134,369	3.02%	601
	1000	5	–				133,031	135,248	0.01%	364
	1000	10	–				133,031	135,248	0.01%	248
	1000	15	–				133,031	135,248	0.01%	289
	1000	20	–				133,031	135,248	0.01%	211
F4	0	0	145,456	146,749	0.01%	207	145,456	146,749	0.75%	600
	500	0	145,456	146,749	0.01%	60	145,456	146,749	0.85%	601
	500	1	145,456	146,749	0.01%	115	145,456	146,749	0.01%	542
	500	2	145,456	146,749	0.01%	243	145,456	146,749	0.01%	312
	500	3	145,456	146,749	0.01%	202	145,456	146,749	1.26%	601
	500	4	145,456	146,749	0.00%	97	145,456	146,749	0.01%	290
	500	5	145,456	146,749	0.00%	73	145,456	146,749	1.54%	601
	500	10	145,456	146,749	0.00%	72	145,456	146,749	0.01%	318
	500	15	145,456	146,749	0.00%	78	145,456	146,749	0.56%	600
	500	20	145,456	146,749	0.00%	63	145,456	146,749	0.19%	600
	500	25	145,456	146,749	0.00%	59	145,456	146,749	0.01%	175
	500	30	145,456	146,749	0.00%	62	145,456	146,749	0.01%	258
	1000	0	145,456	146,749	0.00%	53	145,456	146,749	0.01%	204
	1000	1	145,456	146,749	0.00%	54	145,456	146,749	0.01%	329
	1000	2	154,710	156,383	4.09%	600	150,349	152,456	4.57%	601
	1000	3	155,185	156,882	2.57%	600	157,043	158,774	7.96%	600
	1000	4	–				153,262	155,641	6.02%	600
	1000	5	–				152,521	155,043	5.47%	600
	1000	10	–				150,599	153,121	4.12%	600
	1000	15	–				148,503	150,688	1.46%	600
	1000	20	–				149,272	151,457	2.60%	600
	1000	25	–				148,728	150,918	1.26%	600
	1000	30	–				148,162	150,347	1.54%	600

(–) No feasible solution was found.

CHAPTER 5

A Collaborative Vehicle Routing Problem with Backhauls

A bilevel approach for the collaborative transportation planning problem

Maria João Santos [*†] · **Eduardo Curcio**[†] · **Pedro Amorim**[*†] · **Margarida Carvalho**[‡] · **Alexandra Marques**[†]

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Abstract The integration of the outbound and the inbound logistics of a company leads to a large transportation network, allowing to detect backhauling opportunities to increase the efficiency of the transportation. In collaborative networks, backhauling is used to find profitable services in the return trip to the depot and to reduce empty running of vehicles. This work investigates the vertical collaboration between a shipper and a carrier for the planning of integrated inbound and outbound transportation. Based on the hierarchical nature of the relation between the shipper and the carrier and their different goals, the problem is formulated as a bilevel Vehicle Routing Problem with Selective Backhauls (VRPSB). At the upper level, the shipper decides the minimum cost delivery routes and the set of incentives offered to the carrier to perform integrated routes. At the lower level, the carrier decides which incentives are accepted and on which routes the backhaul customers are visited. We devise a mathematical programming formulation for the bilevel VRPSB, where a routing and a pricing problems are optimized simultaneously, and propose an equivalent reformulation to reduce the problem to a single-level VRPSB. The impact of collaboration is evaluated against non-collaborative approaches and two different side payment schemes. The results suggest that our bilevel approach leads to solutions with higher synergy values than the approaches with side payments. Its main limitation regards the complexity of solving exactly the mathematical formulation for large instances.

Keywords bilevel optimization · vertical collaboration · vehicle routing problem with selective backhauls · exact reformulation

[*]Faculdade de Engenharia, Universidade do Porto, Porto, Portugal

[†]INESC TEC - Instituto de Engenharia de Sistemas e Computadores, Tecnologia e Ciência, Porto, Portugal

[‡]CIRRELT and Department of Computer Science and Operations Research, University of Montreal, Canada

5.1. Introduction

Among the different logistics operations, transportation comprises the major portion of the total costs and it is strongly associated with a negative impact on the environment (Wang et al., 2019). Thus, promoting sustainable initiatives for transportation is becoming a target for many companies and supply chains. Reducing empty running is one of the most popular initiatives to increase the efficiency of vehicles, which impacts directly on reducing costs, fuel consumption, and pollutant emissions (Evangelista et al., 2017). Traditionally, the vehicles travel empty when returning to their original location, and this empty distance may represent up to 25% of the total route distance (Juan et al., 2014; Turkensteen and Hasle, 2017). An efficient way to reduce empty running is to provide pickup loads for vehicles that would return empty to their depot. This is known as backhauling. For a company, backhauling allows to reduce the total costs of transportation by creating integrated outbound-inbound routes instead of dedicated delivery and dedicated pickup routes. For example, Sainsbury's uses backhauling to create integrated routes such that, after delivery all requests at stores, the vehicles collect stock at warehouses in the return trip to the distribution centre (Early, 2011). Differently, Tesco uses backhauling under the context of reverse logistics. After delivering the requests to a store, a vehicle can collect, at that store, returned products to be delivered at the distribution centre.

Backhauling is also widely applied in the context of collaborative transportation. For instance, Nestlé and United Biscuits, competitors in the food market, have arranged a collaboration to improve their logistics operations. The companies, which share a common depot, make their own delivery transport but the vehicles collect loads from each others customers in the return trip. This backhauling strategy allowed to reduce empty running from 22% to 13% in four years (Early, 2011). In the study of Juan et al. (2014) different carriers collaborate with each other through backhauling by allowing each carrier to service customers from other carrier's depot. For the overall network, the collaboration have provided average reductions of 16% on the total distance costs and of 24% on the environmental costs. Both of the above examples refer to horizontal collaboration, where the participants in the collaborative network are stakeholders at the same level in a supply chain. Vertical collaboration, on the other hand, refers to the case where participants are stakeholders at different levels in a supply chain, and usually involves an hierarchical relation between them. An example of vertical collaboration between a set of retailers and a service logistics provider (LSP) is investigated in Cruijssen et al. (2005). The retailers need to serve all their customers, either using outsourcing or collaborating with the LSP. The LSP assumes the leading of the collaboration, and offers to retailers reduced tariffs to serve their stores. Each tariff represents the cost reduction that the LSP is able to offer to a retailer, and it depends on the degree of synergy achieved with collaboration.

In the present work, we investigate a case of vertical collaboration between a shipper and a carrier, where the shipper is the leading entity of the collaboration. The shipper aims to promote the creation of integrated routes such that transportation costs are minimized, whereas the carrier aims to maximize the revenues collected during its backhaul trips. The shipper must then offer incentives to the carrier to motivate it to perform integrated routes. Based on the conflicting and hierarchical nature of the objectives of the shipper and carrier,

we propose a bilevel formulation for the collaborative transportation planning problem. A bilevel optimization model is composed of two levels: the upper level describes the problem of a leader and the lower level describes the problem of a follower. The main characteristic of the bilevel problem is that the lower level is part of the constraints of the upper level problem. Thus, this represents a sequential game where first, in the upper level, the leader (shipper) takes a decision, and afterwards, in the lower level, the follower (carrier) observes the strategy of the leader and solves its optimization problem. The collaborative problem is formulated as a Bilevel Vehicle Routing Problem with Selective Backhauls (VRPSB) and solved by reducing it to an equivalent single-level mixed integer program. The properties of the bilevel problem are analyzed, and the efficiency of the formulation is evaluated and compared against traditional modelling approaches.
The main contributions of this work are relevant for both literature and practice. First, the bilevel formulation proposed allows to explicitly model the interactions and the goals of both participants in the network, as well as it ensures the individual rationality. Second, the collaborative problem is formulated such that it can solve simultaneously the routing and pricing problems, where routing decisions are taken jointly by both participants, and pricing decisions are taken by the shipper when offering incentives to the carrier. Third, a thorough analysis on the properties of the bilevel approach, and a comparison with other alternative approaches (e.g., side payments), allow to gather several managerial insights on the potential application of the bilevel approach to form prominent collaborative networks. Fourth, the problem studied fits well real cases where several backhauling opportunities may arise. Particularly in the forestry industry, the idea of motivating carriers to use their empty vehicles to perform backhauling for shippers is widely applied (Marques et al., 2020; Audy et al., 2012).
The remainder of the paper is structured as follows. A literature review on the collaborative vehicle routing is presented in Section 5.2. Section 5.3 defines the bilevel VRPSB, describing the mathematical formulation and assumptions of the problem, as well as the properties of the model. An exact single-level reformulation developed for solving the bilevel model is proposed in Section 5.4. The computational experiments are presented and discussed in Section 5.5, covering the generation of the data sets used in this work, the managerial insights obtained with the different transportation planning strategies, and the evaluation of the computational performance of models and solution method. Section 5.6 concludes this paper, presenting the main insights and limitations of this work, as well as suggestions for future work in the research field.

5.2. Background literature

The main motivation for a player to join a collaborative network is to reduce its total costs (or increase its profits). Therefore, each player expects that its costs (profits) are lower (higher) in collaboration than in the case where they perform individually (*stand alone solution*). This is designated as the *individual rationality*. The difference between a solution for the entire collaborative network and the stand alone solution is known as the *coalition gain* (Cuervo et al., 2016), whereas the augmented percentage profit defines the *synergy*

value (Cruijssen et al., 2007).
Collaboration can be one of the three types: *i)* horizontal collaboration, if players are at the same level of the network, *ii)* vertical, if players are at different levels of the network, and *iii)* lateral, if there is a combination of both. A recent review on collaborative vehicle routing concludes that horizontal collaboration is the most investigated type in the literature (Gansterer and Hartl, 2018). Thus, collaboration can be achieved differently depending on the business context. This work focus on type *ii)* hence, next, we review the literature on different business models considering vertical collaboration, the use of cooperative game theory to incentive collaboration and the application of bilevel programming to model leadership in collaborative situations.
A case of vertical collaboration is studied in Ergun et al. (2007), which considers a shipper interested in identifying repeatable and continuous tours for carriers, in order to minimize their re-positioning needs and, consequently, the routing costs. By combining inbound and outbound routes, the shipper can negotiate discounts with the carrier and thus pay less for the overall transportation. The problem is modeled as a time-constrained lane covering problem and the results show savings between 5.5% and 13%. Bailey et al. (2011) study the problem of a carrier seeking for collaborative shipments with potential partners in a transportation network, receiving a revenue for each shipment. The collaboration can occur with a shipper, which offers a pickup-delivery task close to the backhaul routes of the carrier of interest, or with other carriers, who do not have sufficient capacity to fulfill all their tasks. In a case-study, the authors demonstrate that the carrier can reach savings between 13% and 28% compared with the stand alone solutions. A problem investigated in Xu et al. (2017), that involves a manufacturer with a private fleet and outsourcing options, shows that collaboration with a third party logistics (3PL) may allow to reduce the total costs in 10%. Cruijssen et al. (2005) propose an alternative to outsourcing, which they designate as insinking. In opposition to outsourcing, which is decided by shippers, the *insinking* allows a logistics service provider (LSP) to motivate shippers to be their customers. In this case, the LSP selects the shippers it wishes to serve in order to build strong synergies. Based on the synergy value, the carrier then determines customized tariffs to motivate the shippers to collaborate.
Collaboration may be tackled with cooperative game theoretical tools. In this context, typically, the participants problems are aggregated into one large optimization problem, and afterwards, the benefits (savings or profits) are determined and shared among the participants. This usually requires solving a pricing problem. Moreover, the collaboration should attend specific criteria, such as individual rationality (i.e., each participant cannot perform better individually than in collaboration). Several methods to allocate the profits from the collaboration have been investigated in the literature. Among them, the Shapley value (Shapley, 1953) is the most commonly used. This method distributes the profits among the players, taking into account the contribution of each player to the overall coalition gain. For example, Krajewska et al. (2008) applies the Shapley value to fairly allocate the profits in a collaborative Pickup and Delivery Problem with Time Windows (PDPTW), while Pradenas et al. (2013) use it in a collaborative Vehicle Routing Problem with Backhauls and Time Windows (VRPBTW). Cruijssen et al. (2005) have also used the Shapley value to determine customized tariffs to offer the participants in the collaborative Vehicle Rout-

ing Problem with Time Windows (VRPTW). Another approach to solve the profit sharing problem is to optimize the routes of a leading participant selfishly and then provide side payments for each of the remaining participants (Özener et al., 2011). These side payments represent a part of the profits generated for a leading entity that should be sufficient to compensate the losses of one or multiple participants. Dahl and Derigs (2011) investigated different compensation schemes to solve a collaborative problem between carriers that are allowed to share orders. These compensation schemes distinguish between orders executed by private fleet or by a partner fleet, and orders served by dedicated vehicles or inserted in already existing routes. In Liu et al. (2010), the compensation scheme cover the side payments received by a carrier if it executes orders from their partners and the penalty costs if the carrier needs to outsource. Nevertheless, using side payments is not always a guarantee that an efficient collaboration is created (Özener et al., 2011).

Recently, Defryn et al. (2019) have put in evidence that the traditional modelling of collaborative vehicle routing problems presents some fragility when it comes to consider different goals of participants in the collaboration. Due to the capabilities of bilevel optimization to explicitly consider the different optimization problems of players, it is expected that bilevel models are able to overcome this difficulty. To the best of our knowledge, only Xu et al. (2018) developed a bilevel formulation for a collaborative VRP. The bilevel formulation considers a centralized logistics platform that allocates vehicles to customers orders (upper level) and a set of vehicle owners that execute those orders (lower level). The upper level problem aims to minimize the variable and the fixed routing costs, whereas the lower level aims to minimize the total empty distances. This work highlights the benefits of balancing the different objectives of different players in a transportation network, but the coalition gain or profit sharing are not discussed.

Finally, it is worth mentioning that although bilevel optimization models are still emergent in the field of collaborative vehicle routing, they are of very much use in pricing problems (e.g., define the price of vaccines to sell in the market (Lunday and Robbins, 2019), prices of shared transportation (Qiu and Huang, 2016), storage price for outbound containers in dry ports (Qiu et al., 2015), setting revenue shares of retailers and prices of suppliers in marketplaces (de Matta et al., 2017). Thus, considering that we propose to solve simultaneously a routing and a pricing problem, a bilevel optimization model seems well suited for the purpose of this work.

Our work differs in several aspects from the above literature. First, we study a problem of vertical collaboration, where the upper level problem belongs to the shipper and the lower level problem belongs to the carrier, and where the goals of each player are different. Most of the literature on vertical collaborative transportation considers a single goal of minimizing the costs or maximizing the savings for the entire network. Only Defryn et al. (2019) have recently put in evidence that different players in collaboration can pursue different goals, but the problem they study involves the collaboration between carriers, and thus it is a case of horizontal collaboration. Second, we assume that the shipper is the leading entity of the collaboration and the incentives offered are based on the response function of the carrier. In opposition, Cruijssen et al. (2005) considers that the carrier is the leading entity and the tariffs offered to shippers are based on the Shapley value. Third, we aim to demonstrate the advantage of using a bilevel formulation to handle the collaborative problem instead of a traditional planning with side payments. We further develop an exact reformulation to solve the problem up to optimality. The work of Xu et al. (2018) provides only one single example of the capabilities of the bilevel formulation, and it describes a genetic algorithm to solve the hierarchical problem. Finally, our study contributes to the scientific literature with a thorough analysis on the benefits of the bilevel approach, as well as its limitations, against traditional modelling strategies.

5.3. Problem description

This section describes in detail the problem investigated in this work. First, the collaborative problem investigated in this work is presented, describing the perspectives of both players - the shipper and the carrier, and how their problems relate to each other. Next, the bilevel formulation for the collaborative VRPSB is presented, where the upper level is the cost minimization problem of the shipper and the lower level is the profit maximization problem of the carrier. The relevant properties of the bilevel VRPSB, which allow to propose a single level formulation, are defined afterwards. The section concludes with an illustrative example of a collaborative network formed by a shipper and a carrier.

5.3.1 The collaborative transportation planning problem

The transportation network is composed of a common depot, a set of linehaul customers (customers of the shipper) and a set of backhaul customers (suppliers of the shipper). The fleet of vehicles of the carrier are located at the common depot.

The shipper does not own a fleet of vehicles but, on the basis of a contract with a carrier, sends regular shipments to meet the demand of all its customers (outbound routes). The shipper plans the delivery routes such that (almost) all capacity of the carrier's vehicles are used when departing from the depot. The shipper has also requests to be picked up at different suppliers, for which typically it is the supplier who sends a full truck load vehicle to the depot of the shipper (inbound routes).

The shipper recognizes that integrating some inbound trips in the outbound routes of the carrier, may lead to reduce its total routing costs. For the carrier, this strategy may also bring benefits, because guaranteeing a full truck load in the return trip to the depot reduces empty backhaul distances. Thus, to motivate the carrier to collaborate and perform an integrated outbound-inbound route, the shipper must pay an additional incentive. However, the carrier, which may serve other requests to other shippers, may not be willing to collaborate. For example, the carrier can get a better incentive from another service or the distance to perform any integrated outbound-inbound route exceeds the maximum distance allowed.

A main distinct feature of this problem is the way we consider the competition between incentives. The incentives for backhauling offered by the shipper compete with the costs of pure inbound routes (for the shipper) and the external incentives (for the carrier). The competition with the former follows the rational principle that the cost of integrating a backhaul customer in a delivery route must be lower than the cost of a pure inbound route to visit this customer. The competition with the latter comes from the fact that, after deliveries, a vehicle of the carrier has a remaining distance that can be used to provide external services

while returning to the depot. We assume that the total remaining distance is used to perform a single backhauling service, either for the shipper or for an external entity.
Another feature of our problem is that outbound and inbound routes are treated differently. The first reason comes from the fact that outbound routes are the priority in the routing problem, which is based on the typical assumptions of the VRPB, and its variants. The second reason is that each delivery customer can only be visited once, whereas the backhaul customers can be visited multiple times. Therefore, inserting the same backhaul customer in different routes would lead to different routing costs. By using an incentive for each backhaul customer, independently on the route it is inserted, allows to standardize the cost of backhauling. Third, the costs of delivery routes are predefined in a contract with the carrier, i.e. it will execute them no matter what. After deliveries, the carrier seeks for backhauling services but there is no guarantee that it will find one. Thus, only after deliveries, the competition between backhauling services arises, and the carrier will select the most profitable one (hence, the need to motivate the carrier to perform a service not covered by the contract for the deliveries).
In summary, the collaborative problem not only should solve a routing problem but also a pricing problem, i.e. the shipper must define which incentives for the backhauling should offer to the carrier, considering other competing incentives. The problem should also consider the hierarchical nature of the shipper and the carrier, and the different goals that each one pursuits. The goal of the shipper is to define the minimum cost routes and the minimum incentives necessary to motivate the carrier to collaborate. The goal of the carrier is to select the most profitable services to perform with its fleet of vehicles, until returning to the depot.

5.3.2 Mathematical programming formulation

The following sets are used in the formulation. Set $V = \{0 \cup L \cup B\}$ represents all nodes in the network, where $\{0\}$ is the depot, $L = \{1,...,n\}$ is the subset of linehaul customers and $B = \{n+1,...,n+m\}$ is the subset of backhaul customers. Set $K = \{1,...,k\}$ denotes the delivery vehicles of the carrier. Each arc (i, j) in the network has an Euclidean distance d_{ij} and an associated symmetric cost c_{ij}, such that $c_{ij} = c_{ji}$ and $i \neq j$. The unitary cost of distance travelled for the shipper is c^U_{ij} and for the carrier is c^L_{ij}. The cost of a dedicated inbound vehicle is $2c^U_{i0}$, which pays the load and no-load distances between a backhaul customer i and the depot $\{0\}$. Each linehaul customer i requires a given quantity q_i to be delivered and the depot requires a minimum amount of raw-materials Q_0 to be collected at backhaul customers. All vehicles have similar capacity C. The total distance travelled by one delivery vehicle cannot exceed the maximum distance allowed D_{max}. The expected unitary profit per unit of distance of an external service outside the collaboration performed by the carrier is given by ϕ.
The routing problem is modelled using a single commodity flow formulation, since only one type of product can be carried on each arc, for delivery or for pickup load. The routing problem is modelled as a VRPSB with the exception that pure inbound routes are also allowed. Allowing the creation of all type of routes (only outbound, only inbound and integrated outbound-inbound) brings more benefits to the optimization than forcing backhaul

customers to be visited in integrated routes (Marques et al., 2020).
The profit sharing problem is combined with the routing planning through the incentives for backhauling offered by the shipper to the carrier, which leads to the Collaborative VRPSB. Finally, the collaborative VRPSB can be formulated as a mixed-integer bilevel VRPSB, where the upper level describes the problem of the shipper (Problem (5.1)-(5.9)) and the lower level describes the problem of the carrier (Problem (5.14)-(5.18)).
The upper level decision variables are:

$$x_{ij}^k := \begin{cases} 1, & \text{if vehicle } k \text{ travels on arc } (i,j) \\ 0, & \text{otherwise} \end{cases} \qquad \forall k \in K, \forall i,j \in V \setminus B$$

$Z_b :=$ incentive offered to visit backhaul customer b, $\forall b \in B$

$O_b :=$ number of visits to backhaul customer b by dedicated inbound vehicles, $\forall b \in B$

$y_{ij} :=$ load in a vehicle between customers i and j, $\forall i,j \in V \setminus B$.

The lower level decision variables are:

$$\hat{x}_{ij}^k := \begin{cases} 1, & \text{if vehicle } k \text{ travels on arc } (i,j) \\ 0, & \text{otherwise} \end{cases} \qquad \forall k \in K, \forall i \in L, \forall j \in B$$

$Z_{ext}^k :=$ external incentive offered in route k, $\forall k \in K$.

5.3.2.1 Upper level problem

The objective function of the shipper is the minimization of the total cost of the routing plan as in Equation (5.1) below. The total cost comprises three aspects: *i)* the cost associated to the total distance travelled to visit all linehaul customers, *ii)* the total incentives payed to the carrier to visit backhaul customers, and *iii)* the total cost of outsourcing dedicated inbound vehicles. Note that the upper and lower problems interact through the two variables present in the second term of the objective function, namely Z_b and $\hat{x}_{ij}^k$. Moreover, this term of the function is nonlinear, but it can be linearized as we will see later (through constraints (6.4.2)-(6.4.5)).

$$\min \sum_{i \in V \setminus B} \sum_{j \in L} \sum_{k \in K} c_{ij}^U \cdot x_{ij}^k + \sum_{i \in L} \sum_{b \in B} \sum_{k \in K} Z_b \cdot \hat{x}_{ib}^k + \sum_{b \in B} O_b \cdot 2c_{b0}^U \tag{5.1}$$

All routes start at the depot, as expressed by Constraints (5.2) and the flow constraints in delivery routes, given by Constraints (5.3), guarantee the connectivity of the locations visited in each route.

$$\sum_{i \in L} x_{0i}^k \leq 1, \qquad \forall k \in K \tag{5.2}$$

$$\sum_{i \in V \setminus B} x_{ij}^k = \sum_{i \in V \setminus B} x_{ji}^k + \sum_{b \in B} \hat{x}_{jb}^k, \qquad \forall j \in L, \forall k \in K \tag{5.3}$$

Equations (5.4) guarantee that all linehaul customers are visited exactly once by only one vehicle.

$$\sum_{i \in V \setminus B} \sum_{k \in K} x_{ij}^k = 1, \qquad \forall j \in L \tag{5.4}$$

The total load carried on each vehicle must decrease gradually as linehaul customers are visited in each route, which is ensured by Equations (5.5). These constraints are also subtour elimination constraints. Furthermore, Constraints (5.6) guarantees that the capacity of each vehicle is never exceeded on each route.

$$\sum_{i \in V \setminus B} y_{ij} = \sum_{i \in V \setminus B} y_{ji} + q_j, \qquad \forall j \in L \tag{5.5}$$

$$y_{ij} \leq x_{ij}^k \cdot C, \qquad \forall i, j \in V \setminus B, \forall k \in K \tag{5.6}$$

The demand of linehaul customers is fully satisfied with Equations (5.7) and the minimum demand of the depot is satisfied with Constraints (5.8). It is assumed that, each time a vehicle visits a backhaul customer (either on integrated or on pure inbound routes), it returns full to the depot, so that the quantity delivered at the depot matches exactly the total capacity of the vehicle. This rationale is based on the common practice of vehicles travelling in full truck load, such as in the forestry industry (Marques et al., 2020).

$$\sum_{j \in L} y_{0j} = \sum_{j \in L} q_j \tag{5.7}$$

$$\sum_{i \in L} \sum_{b \in B} \sum_{k \in K} \left(\hat{x}_{ib}^k + O_b \right) \geq \left\lceil \frac{Q_0}{C} \right\rceil \tag{5.8}$$

The domain of the upper level variables is as follows:

$$x_{ij}^k \in \{0,1\}, Z_b, y_{ij} \geq 0, O_b \in \{0, ..., \lceil \frac{Q_0}{C} \rceil\}, \forall i, j \in V \setminus B, k \in K, b \in B. \tag{5.9}$$

To linearize the objective function of the upper level, we use the McCormick constraints (McCormick, 1976). Thus, we introduce a new variable $A_b^k = Z_b \cdot \sum_{i \in L} \hat{x}_{ib}^k$ and derive the following constraints. If $\sum_{i \in L} \hat{x}_{ib}^k = 0$, then the inequality (6.4.2) ensures that A_b^k is also zero (A_b^k is higher than a negative number from inequality (6.4.4) and cannot be negative due to the Equation (6.4.5)). On the other hand, if $\sum_{i \in L} \hat{x}_{ib}^k = 1$, the inequality (6.4.2) ensures that A_b^k is lower than M (large number), and inequalities (6.4.3) and (6.4.4) guarantee that $A_b^k = Z_b$. Note that we can make $M = max\{2c_{i0}^U\}, \forall i \in B$, which is the upper bound of each inventive Z_b.

$$A_b^k \leq M \cdot \sum_{i \in L} \hat{x}_{ib}^k, \qquad \forall b \in B, \forall k \in K \tag{5.10}$$

$$A_b^k \leq Z_b, \qquad \forall b \in B, \forall k \in K \tag{5.11}$$

$$A_b^k \geq Z_b - (1 - \sum_{i \in L} \hat{x}_{ib}^k) \cdot M, \qquad \forall b \in B, \forall k \in K \tag{5.12}$$

$$A_b^k \geq 0, \qquad \forall b \in B, \forall k \in K \tag{5.13}$$

5.3.2.2 Lower level problem

The objective function of the carrier is described by Equation (5.14), which is the maximization of the total profits collected with all routes. The profit collected with integrated routes is determined as the difference between the total incentives accepted and the total travelling cost of including the backhaul customers in delivery routes. The profit collected with the external services is equivalent to the total net external incentives accepted. Finally, the profit collected with a delivery route corresponds to the difference between the total cost charged to the shipper and the total effective cost of the deliveries routes paid by the carrier. Note that this term could be removed since it is constant (it does not include decision variables of the lower level).

$$\begin{aligned} \max \sum_{i \in L} \sum_{b \in B} \sum_{k \in K} Z_b \cdot \hat{x}_{ib}^k - \sum_{i \in L} \sum_{b \in B} \sum_{k \in K} \hat{x}_{ib}^k \cdot (c_{ib}^L + c_{b0}^L) + \\ \sum_{k \in K} Z_{ext}^k \cdot (1 - \sum_{i \in L} \sum_{b \in B} \hat{x}_{ib}^k) + \sum_{i \in V \setminus B} \sum_{j \in L} \sum_{k \in K} (c_{ij}^U - c_{ij}^L) \cdot x_{ij}^k \end{aligned} \tag{5.14}$$

Constraints (5.15) forces the precedence constraint of a typical VRPSB, where backhaul customers can only be linked to a last linehaul customer in a route. First, delivery routes are the priority in the VRPSB, since these can only be performed by delivery vehicles. Second, the load to be collected at any backhaul customer can fill all the capacity of a vehicle. Thus, no mixing of delivery and pickup loads is possible.

$$\hat{x}_{bj}^k = 0, \qquad \forall b \in B, j \in L, k \in K \tag{5.15}$$

Constraints (5.16) enforce that the maximum distance D_{max} is never exceeded in any route.

$$\sum_{i,j \in V \setminus B} d_{ij} \cdot x_{ij}^k + \sum_{i \in L} \sum_{b \in B} \sum_{k \in K} \hat{x}_{ib}^k \cdot (d_{ib} + d_{b0}) \leq D_{max}, \qquad \forall k \in K \tag{5.16}$$

The external incentive, given by Equations (5.17), must be equal to the remaining distance of a delivery route multiplied by the unitary profit ϕ. This is an important aspect considered by the carrier, since the shortest are the delivery routes, the higher the distance remaining to perform additional services.

$$Z^k_{ext} = (D_{max} - \sum_{i \in V \setminus B} \sum_{j \in L} d_{ij} \cdot x^k_{ij}) \cdot \phi, \qquad \forall k \in K \tag{5.17}$$

The domain of the lower level variables is as follows:

$$\hat{x}^k_{ib} \in \{0, 1\}, \qquad \forall i \in L, \forall b \in B, \forall k \in K \tag{5.18}$$

The nonlinear terms in the objective function of the lower level are linearized through the definition of a new variable $G^k = Z^k_{ext} \cdot (1 - \sum_{i \in L} \sum_{b \in B} \hat{x}^k_{ib})$ and the respective McCormick constraints (6.4.7)-(5.22). Note that M can be set to the upper bound of the maximum external service, i.e. $M = D_{max} \cdot \phi$.

$$G^k \leq M \cdot (1 - \sum_{i \in L} \sum_{b \in B} \hat{x}^k_{ib}), \qquad \forall k \in K \tag{5.19}$$

$$G^k \leq Z^k_{ext}, \qquad \forall k \in K \tag{5.20}$$

$$G^k \geq Z^k_{ext} - \sum_{i \in L} \sum_{b \in B} \hat{x}^k_{ib} \cdot M, \qquad \forall k \in K \tag{5.21}$$

$$G^k \geq 0, \qquad \forall k \in K \tag{5.22}$$

5.3.3 Properties of the bilevel VRPSB

One particular characteristic of bilevel problems is its intrinsic hierarchical structure. The upper level is the dominant player and the first to select an action. Afterwards, the lower level observes the decisions of the upper level and optimizes its own objective function. Each strategy selected by the lower level is called a *rational response* and the set of all responses is known as the *rational set* (Colson et al., 2007; Safari et al., 2014; Sun et al., 2008). Knowing the rational set, the upper level can anticipate the response of the lower level and decide the final strategy that minimizes its costs.
The bilevel VRPSB states that, for each strategy of the shipper, i.e. for a fixed set of upper decision variables, the carrier accepts or withdraws each incentive offered by the shipper, in each route. If any incentive offered is not accepted in a route, the carrier performs an external service. The profit for the carrier of an integrated route P^k_b is determined as the difference between the incentive Z_b and the additional travelling cost to visit backhaul customer b, as follows:

$$P^k_b = Z_b - \sum_{i \in L} \hat{x}^k_{ib} \cdot (c^L_{ib} + c^L_{b0}), \qquad \forall k \in K, \forall b \in B. \tag{5.23}$$

The profit of an external service P^k_E equals the external incentive on that route, as follows:

$$P^k_E = Z^k_{ext}, \qquad \forall k \in K. \tag{5.24}$$

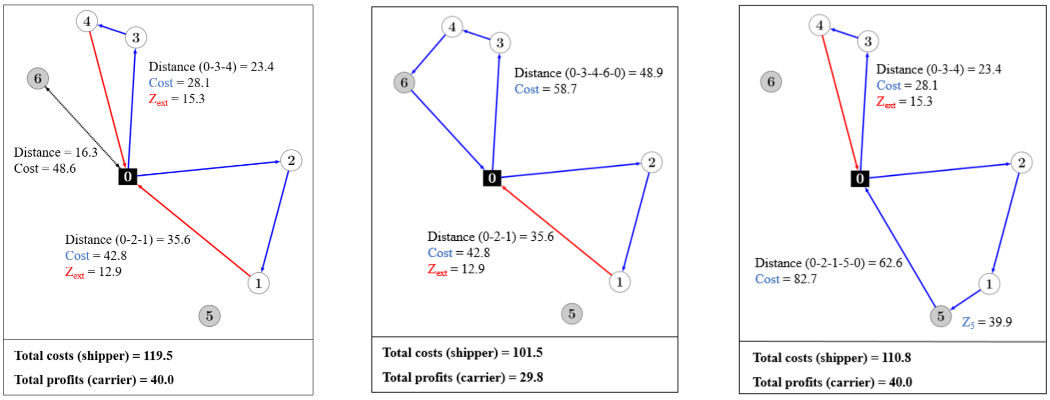
Given a fixed input of upper decision variables, the optimal response of the lower level problem, F^k, corresponds to the optimal solution of the carrier, as follows:

$$F^k = \max\{P^k_E, \max_{b\in B}\{P^k_b\}\}, \qquad \forall k \in K. \tag{5.25}$$

When the rational response is not singleton, i.e. more than one response of the lower level may be obtained for a single strategy of the upper level, two different approaches can be applied - the *optimistic* and the *pessimistic* approaches (Colson et al., 2007; Sinha et al., 2018). The optimistic approach assumes that the lower level will select the rational response that is more favourable to the upper level. In opposition, the pessimistic approach assumes that the lower level will select the least favourable response. The optimistic approach is the most investigated in literature and allows to slightly reduce the non-cooperative nature of bilevel models (Kozanidis et al., 2013; Sinha et al., 2018). For this reason, the bilevel VRPSB is build upon an optimistic approach, such that when $\max_{b\in B} P^k_b = P^k_E$, the optimal response of the carrier in route k is to accept the incentive offered by the shipper.
It is also worth mentioning that, although we use an optimistic approach, the problem of the lower level considers also its most optimistic case. In fact, as the upper level does not know exactly the problem of the lower level, it is assumed that the lower level can always achieve the highest possible profits in each route, considering that all the remaining distance can be used to provide external services. Thus, the incentives offered by the upper level are set to cover the highest external incentive in each route.

5.3.4 Numerical example

In the following numerical example, we demonstrate the rationale upon the bilevel model is build to model collaboration. Consider a shipper that needs to send requests to customers 1, 2, 3 and 4, and requires a full truck load from one of the backhaul customers 5 or 6. The carrier has two vehicles available.
Figure 5.1 illustrates the bilevel solution and two non-collaborative solutions, for the purpose. The non-collaborative solutions correspond to the separated planning (VRP, optimization of inbound and outbound routes independently) and the integrated planning (VRPSB, all routes are optimized under the perspective of the shipper only). The non-collaborative models are detailed in 5..1.
With a VRP (separated planning) (Figure 6.1c), the solution of the shipper includes the least cost routes for deliveries and an inbound route with the backhaul nearest to the depot. The carrier benefits with the external incentives from both delivery routes. This is designated as the stand alone solution.
With a VRPSB (integrated planning) (Figure 6.1d), where the shipper controls the fleet of vehicles, it would select backhaul customer 6 instead of backhaul customer 5, due to the lower backhaul distance, i.e. $d_{46} + d_{60} < d_{15} + d_{50}$. However, the carrier would lose profits in visiting a backhaul customer in a route that generates a higher external incentive.
Under a bilevel approach (Figure 6.1a), the shipper sets only an incentive for backhaul



(a) VRP (b) VRPSB (c) Bilevel VRPSB

Figure 5.1 – Routing plans obtained with each planning approach. The square is the depot, the white circles are linehaul customers and the grey circles are backhaul costumers. Blue lines represent the (part of) the routes performed by the carrier to serve the requests of the shipper. Red lines represent the part of the routes performed by the carrier to serve external services. Black line represents the pure inbound route outsourced to a supplier.

customer 5. No incentive is provided for backhaul customer 6 because it would be higher or, at least, equal to the cost of an inbound route. The carrier, in turns, would integrate the backhaul visit in its longest route, where the backhaul incentive competes with a lower external incentive, leaving the shortest route to guarantee a higher external incentive.
In this example, it was demonstrated how the bilevel VRPSB incorporates the rational response of the carrier into the problem of the shipper, by guaranteeing that the profits achieved with collaboration are not lower than its stand alone solution. In Section 5.5.3, we compare the bilevel approach with a traditional VRPSB with side payments, where the shipper plans all the delivery and integrated routes and decides on the side payment of each integrated route.

5.4. Single-level reformulation

One classic way to handle mathematically a bilevel problem is to reformulate it into a single-level optimization problem (Sinha et al., 2018) and then solve it with an exact method to find the global optimum. The reformulation based on Karush-Kuhn-Tucker (KKT) conditions is the most popular method (Zeng and An, 2014), where the lower level problem is replaced by its corresponding KKT conditions, which include primal feasibility, dual feasibility and complementary slackness conditions. A similar reformulation method is based on duality theory, where the lower level is replaced by its primal and dual constraints, enforcing strong duality. The duality-based reformulation leads to a lower number of variables than the KKT conditions, rendering a better computational performance (Garcia-Herreros et al., 2016). Nevertheless, both of these methods can only be of use in practice when the lower level is linear with continuous variables, for which the optimal solutions correspond to vertices of the feasible region.

Recently, solvers for mixed integer bilevel programming are available: MibS* and bilevel†, whose technical details are respectively discussed in Tahernejad et al. (2017) and Fischetti et al. (2017). In both solvers, only the integer upper level variables can appear in the lower level, since such assumption guarantees the existence of an optimal solution (Vicente et al., 1996). The bilevel VRPSB presents both binary and continuous decision variables, and the upper level continuous variable Z_b appears in the objective function of the lower level. Therefore, the general purpose techniques mentioned above are not suitable for the problem at hand. In order to overcome the lack of solution approaches to our problem, we will focus in its specific structure. In particular, we show how the rational set of the lower level problem can be represented by a finite set of inequalities, allowing to reduce the bilevel VRPSB to a single-level mixed integer linear programming (MILP) problem. In this way, the obtained single level problem can be solved by off-the-shelf solvers.
With the purpose of transforming the lower level objective function in a set of disjunctive constraints describing its rational set, we make use of Big-M constraints. This new set of constraints is then incorporated into the problem of the upper level, which results in a single level MILP VRPSB model. The detailed description of the reformulation technique applied in this work is presented in the following steps.

Step 1. The integer variables of the lower level problem, $\hat{x}^k_{ib}$, are replaced by the corresponding flow variables of the upper level, x^k_{ij}. By extending the entire set of locations to $\{i, j\} \in V = \{0\} \cup L \cup B$, all constraints of the lower level can be re-written using only the decision variables of the upper level, as follows.

$$x^k_{bj} = 0, \qquad \forall b \in B, j \in L, k \in K \tag{5.15'}$$

$$\sum_{i,j \in V} d_{ij} \cdot x^k_{ij} \leq D_{max}, \qquad \forall k \in K \tag{5.16'}$$

$$Z^k_{ext} = (D_{max} - \sum_{i \in V} \sum_{j \in L} d_{ij} \cdot x^k_{ij}) \cdot \phi, \qquad \forall k \in K. \tag{5.17''}$$

Step 2. The objective function of the lower level problem is to maximize the total profits of the carrier. The profits obtained with delivery routes are fixed *a priori* by the upper level, because these routes are not influenced by the decisions of the carrier. The decisions of the carrier only influence which backhaul customers are visited, if any, and on which routes. Therefore, the objective of the lower level can be translated into the maximization of profits obtained by accepting the different incentives, namely those offered by the shipper (5.26) and those offered by its competitors (5.27).

$$F^k \geq P^k_b, \qquad \forall k \in K, \forall b \in B \tag{5.26}$$

$$F^k \geq P^k_E, \qquad \forall k \in K. \tag{5.27}$$

*available at https://github.com/coin-or/MibS
†available at https://msinnl.github.io/pages/bilevel.html

Step 3. To enforce that, in each route, the carrier decides in favour for the incentive that maximizes its profits, a set of disjunctive constraints are established, using M as a large number (e.g., $M = D_{max} \cdot \phi$). Constraints (5.28) introduce a binary variable H_E^k that takes the value of 1 if the external incentive Z_{ext}^k is higher than the profit of visiting any backhaul b in route k (i.e. $P_E^k > P_b^k$) and 0 otherwise. Similarly, Constraints (5.29) introduce a binary variable H_b^k that takes the value of 1 if $P_b^k > P_E^k$ and $P_b^k > P_i^k, \forall i \in B, b \neq i$, and 0 otherwise.

$$F^k - P_E^k \leq (1 - H_E^k) \cdot M, \qquad \forall k \in K \tag{5.28}$$

$$F^k - P_b^k \leq (1 - H_b^k) \cdot M, \qquad \forall k \in K, \forall b \in B. \tag{5.29}$$

Step 4. If the external incentive is accepted for a given route ($H_E^k = 1$), all the arcs between backhauls and linehaul customers must be zero, which is enforced by Constraints (5.30). Otherwise, the incentive offered for a backhaul b is preferred ($H_b^k = 1$), for which Constraints (5.31) ensure that, for route k the arcs containing the remaining backhauls are all set to zero, since each vehicle visits only one backhaul. Constraints (5.32) guarantee that if $H_b^k = 1$, then, at least, one arc linking a linehaul to a backhaul customer must exist. Equation (5.33) ensures that only a single incentive may be accepted by the carrier in each route, while Constraints (5.34) are used to guarantee that either the incentive to visit a backhaul b is accepted or the external incentive is accepted.

$$(1 - H_E^k) \geq x_{ib}^k, \qquad \forall k \in K, \forall i \in L, \forall b \in B \tag{5.30}$$

$$(1 - H_b^k) \geq x_{ij}^k, \qquad \forall k \in K, \forall i \in L, \forall j \neq b \in B \tag{5.31}$$

$$\sum_{i \in L} x_{ib}^k \geq H_b^k, \qquad \forall k \in K, \forall b \in B \tag{5.32}$$

$$\sum_{b \in B} H_b^k + H_E^k = 1, \qquad \forall k \in K \tag{5.33}$$

$$\sum_{b \in B} H_b^k \leq 1, \qquad \forall k \in K. \tag{5.34}$$

Concluding the reformulation procedure, the resultant model is a single level VRPSB with the following (MILP) formulation:

$$min \sum_{i \in V} \sum_{j \in L} \sum_{k \in K} c_{ij}^U \cdot x_{ij}^k + \sum_{b \in B} \sum_{k \in K} A_b^k + \sum_{b \in B} O_b \cdot 2c_{b0}^U \tag{5.35}$$

subjected to

$$(5.2) - (5.9)$$

$$(5.15'), (5.16) \text{ and } (5.17'')$$

$$(5.23) - (5.24)$$

$$(5.26) - (5.34)$$

Theorem 1. *Any optimal solution of Problem* (5.35) *is also optimal to the bilevel VRPSB.*

Proof. Next, we will show the correctness of each step in the single level reformulation. Start by noticing that if a leader's optimal solution has $x^k_{ib} \neq \hat{x}^k_{ib}$, for some i, j and k, then changing x^k_{ib} to $\hat{x}^k_{ib}$ still results in an optimal solution for the leader: the modified leader's strategy is feasible, the follower's feasible region does not change, and none of the objective functions, (5.1) and (5.14), changes. Consequently, we can restrict x^k_{ij} to mimic the follower's reaction as done in **Step 1**.

Note that the follower's problem can be decompose in $|K|$ maximization problems, one for each vehicle, since there is no lower level linking constraint with the different vehicles. Hence, we can focus on each of these optimization problems, namely, on the profit F^k that can be obtained by each vehicle $k \in K$. Recalling that F^k can be modeled accordingly with Equation (5.25), **Step 2** and **Step 3** linearize it through a set of 4 constraints (Constraints (5.26) to (5.29)), and new binary variables, H^k_E and H^k_b, are added to model the type of incentive accepted (external or backhaul). With these newly introduced variables, in **Step 4**, we can ensure that the x^k_{ib} reflect the follower's reaction.

In this way, we can conclude that any optimal solution of Problem (5.35) is also optimal to the bilevel VRPSB. □

5.5. Computational experiments

The computational experiments performed in this section cover three main analysis. The first set of experiments aims to evaluate the solutions obtained with the bilevel approach for the collaborative transportation problem. Bilevel solutions are compared with stand alone solutions and the synergy values of collaboration are determined. The second set of experiments aims to analyse potential side payments schemes, and to compare them with our bilevel approach. We argue that our bilevel approach, as it incorporates the rational response of the carrier into the problem of the shipper, provides balanced solutions as well as higher synergy than side payments schemes. The third set of experiments analyses the scalability of our approach to solve the bilevel model (single level reformulation) in comparison with solving non-collaborative models.

Each set of experiments reports the results for two different scenarios: one that allows multiple visits to the same backhaul customer, and one that forbids more than one visit to the same backhaul customer. The motivation behind is to analyze the efficiency of the bilevel approach to deal with different contexts of backhauling. Most of the literature considers that backhaul customers can only be visited once, but in practice, they may have enough availability of raw-material, which allows multiple pickups (Marques et al., 2020; Santos et al., 2020). The bilevel model presented in Section 5.3.2 allows multiple backhaul visits. To forbid multiple visits to the same backhaul customer, it is sufficient to include constraint (5.36) and change the dimension of the variable to $O_b \in \{0, 1\}$. Hence, Constraint 5.36 ensures that each backhaul customer can only be visited once by all type of vehicles.

$$\sum_{i \in L} \sum_{k \in K} \hat{x}^k_{ib} + O_b \leq 1, \qquad \forall b \in B \tag{5.36}$$

All data sets used in the computational experiments are collected from Augerat et al. (1995) and adapted to the VRPSB, as described next. All models are coded in Python 3.6.3 and solved with Gurobi, on a computer equipped with the processor Intel Core i7 of 2.20GHz and 16 GB of RAM.

5.5.1 Data sets

The original data sets define the number of locations (depot and linehaul customers) and respective coordinates, the demand of linehaul customers, the number of vehicles and respective capacity.
The adapted data sets comprise the following modifications: *i)* the backhaul customers locations correspond to the 5 to 17 last locations in the original data set, and *ii)* the number of vehicles available is determined as the minimum number required to meet all the demand of linehaul customers without exceeding vehicle capacity, i.e. $|K| = \lceil \frac{\sum_{i \in L} q_i}{C} \rceil$ in each data set. The adapted instances used in this work are reported in Table 5.1.

Table 5.1 – Adapted instances used in this work

Original	Adapted	$\lvert L \rvert$	$\lvert B \rvert$	$\lvert K \rvert$	D_{max}
P-n16-k8	A	10	5	6	150
P-n19-k2	B	13	5	2	150
P-n20-k2	C	14	5	2	150
P-n22-k2	D	16	5	2	150
P-n22-k8	E	16	5	6	150
B-n31-k5	F	25	5	4	250
B-n31-k5	G	20	10	3	250
B-n38-k6	H	25	12	4	250
B-n41-k6	I	25	15	4	250
B-n45-k6	J	27	17	4	250
B-n45-k6	K	39	5	6	250
B-n45-k6	L	35	9	5	250
B-n50-k6	M	44	5	6	250
B-n50-k6	N	40	9	6	250

The remaining parameters of the bilevel VRPSB are set as follows. The depot demand is a multiple of the capacity of a vehicle, such that $Q_0 = [C, 2C, \ldots, |K|C]$. The maximum distance allowed is 150 for instances with less than 20 nodes, and 250 for the remaining ones. These limits are sufficient to create integrated inbound-outbound routes. The unitary cost per unit of distance is set to 1.2 € for the shipper and 1.0 € for the carrier, following the study in Yu and Dong (2013). The unitary profit obtained with an external service equals the unitary profit obtained with a delivery route, such that $\phi = 0.2$ €\unit of distance.
Throughout the experiments, 10 small instances with varying Q_0 are used, since optimal solutions are obtained in reasonable time. The motivation to use different Q_0 values is to determine if increasing the number of required visits to backhaul customers increases the performance of the collaboration. The last set of experiments uses also medium size instances. We anticipate that large instances are not tested, as the exact method developed

in this work is not efficient to solve them. However, the motivation of our work is not on the efficiency of the solution method but rather on the efficiency of the bilevel formulation to handle the collaborative problem.

5.5.2 Bilevel versus traditional planning

With the bilevel VRPSB model proposed in this work, we aim to demonstrate that it is efficient to handle the collaborative problem, while incorporating the different goals of the players, and solving the routing and the pricing problems simultaneously. The impact of collaboration is determined by two measures. The first is the network costs (NC), which is given by the difference between the total costs of the shipper and the total profits of the carrier (Equation 5.37). The second is the synergy value (SV), which provides the percentage gain that a collaborative network can reach compared with the stand alone solution (Equation 5.38).

$$\mathrm{N}C = Costs - Profits \tag{5.37}$$

$$SV = \frac{NC_{VRP} - NC_{collab}}{NC_{VRP}} \tag{5.38}$$

Tables 5.2 and 5.3 present a comparison between stand alone solutions obtained with the VRP model and collaborative solutions obtained with the bilevel VRPSB model, for the scenarios investigated. Both read as follows: total costs of the shipper (column "Costs"), total profits of the carrier (column "Profits"), costs of outsourcing inbound vehicles to suppliers (column "Out."), network costs (column "NC"), total incentives offered by the shipper and accepted by the carrier (column "Incent."), and the synergy value of the collaborative solution (column "SV").

Table 5.2 shows that seven out of ten instances do not promote collaboration, i.e. the optimal solution for the shipper is to outsource all necessary vehicles to the suppliers. Despite disappointing, this outcome is reasonable since the same backhaul customer can be visited as many times as the depot demand requires. Thus, when the cost of visiting the backhaul customer closest to the depot with an inbound vehicle is relatively low, and no backhauling incentive is lower, the shipper always tend to outsource all necessary vehicles to that closest backhaul customer. Nonetheless, when collaboration occurs, synergy values can reach about 11%.

From Table 5.3, it is possible to observe very different results compared with the previous scenario of unlimited visits to backhaul customers. When the number of visits is limited to one, the incentives proposed by the shipper compete with more diverse options than only the nearest backhaul customer. This seems to increase heavily the potential for collaboration, as shown in eight out of ten instances. Moreover, any solution of the scenario with limited visits provides equal or higher synergy values than in the scenario with unlimited backhaul visits.

We were expecting to see an increase in the synergy value with increasing Q_0, but such was not verified for all instances. Therefore, we cannot conclude on the impact of increasing

Table 5.2 – Stand alone and collaborative solutions in a scenario with unlimited visits to backhaul customers.

		VRP (stand alone) solutions				Bilevel VRPSB (collaborative) solutions					
Inst.	Q_0	Costs	Profits	Outs.	NC	Costs	Profits	Incent.	Outs.	NC	SV
A	C	275	180	68	95	265	180	58	0	85	10.0%
	2C	342	180	135	162	324	180	117	0	144	11.3%
	3C	410	180	203	230	387	180	180	0	207	10.0%
	4C	478	180	270	298	454	185	247	0	270	9.4%
	5C	545	180	338	365	521	187	314	0	334	8.6%
B	C	192	60	38	132	192	60	0	38	132	0.0%
	2C	230	60	76	170	230	60	0	76	170	0.0%
C	C	204	60	38	144	204	60	0	38	144	0.0%
	2C	242	60	76	182	242	60	0	76	182	0.0%
D	C	207	60	38	147	207	60	0	38	147	0.0%
	2C	245	60	76	185	245	60	0	76	185	0.0%
E	C	323	180	53	143	323	180	0	53	143	0.0%
	2C	376	180	106	196	376	180	0	106	196	0.0%
	3C	429	180	159	249	429	180	0	159	249	0.0%
	4C	482	180	212	302	482	180	0	212	302	0.0%
	5C	535	180	265	355	535	180	0	265	355	0.0%
F	C	499	200	113	299	478	200	88	0	278	7.0%
	2C	611	200	226	411	581	200	177	0	381	7.4%
	3C	724	200	338	524	685	200	281	0	485	7.4%
	4C	837	200	451	637	798	200	281	113	598	6.1%
G	C	429	150	91	279	424	150	86	0	274	1.7%
	2C	520	150	182	370	515	150	86	91	365	1.2%
	3C	611	150	273	461	606	150	86	182	456	1.0%
H	C	444	200	51	244	444	200	0	51	244	0.0%
	2C	495	200	101	295	495	200	0	101	295	0.0%
	3C	546	200	152	346	546	200	0	152	346	0.0%
	4C	596	200	203	396	596	200	0	203	396	0.0%
I	C	469	200	35	269	469	200	0	35	269	0.0%
	2C	504	200	70	304	504	200	0	70	304	0.0%
	3C	538	200	105	338	538	200	0	105	338	0.0%
	4C	573	200	140	373	573	200	0	140	373	0.0%
J	C	379	200	50	179	379	200	0	50	179	0.0%
	2C	429	200	100	229	429	200	0	100	229	0.0%
	3C	479	200	150	279	479	200	0	150	279	0.0%
	4C	529	200	200	329	529	200	0	200	329	0.0%
Average											2.3%

Table 5.3 – Stand alone and collaborative solutions in a scenario with limited visits to backhaul customers.

		VRP (stand alone) solutions				Bilevel VRPSB (collaborative) solutions					
Inst.	Q_0	Costs	Profits	Outs.	NC	Costs	Profits	Incent.	Outs.	NC	SV
A	C	275	180	68	95	265	180	58	0	85	10.0%
	2C	345	180	137	165	324	180	117	0	144	12.5%
	3C	416	180	209	236	387	180	180	0	207	12.4%
	4C	488	180	281	308	457	180	245	0	277	10.1%
	5C	562	180	355	382	528	180	248	68	348	8.9%
B	C	192	60	38	132	192	60	0	38	132	0.0%
	2C	264	60	110	204	246	60	53	38	186	8.5%
C	C	204	60	38	144	204	60	0	38	144	0.0%
	2C	276	60	110	216	255	60	50	38	195	9.6%
D	C	207	60	38	147	207	60	0	38	147	0.0%
	2C	270	60	101	210	253	60	45	38	193	8.3%
E	C	323	180	53	143	323	180	0	53	143	0.0%
	2C	393	180	123	213	393	180	69	53	213	0.2%
	3C	466	180	196	286	466	180	69	126	286	0.1%
	4C	542	180	272	362	542	180	69	202	362	0.1%
	5C	623	180	352	443	622	180	69	283	442	0.1%
F	C	499	200	113	299	478	200	88	0	278	7.0%
	2C	616	200	230	416	581	200	177	0	381	8.4%
	3C	733	200	348	533	685	200	281	0	485	9.1%
	4C	854	200	468	654	802	200	281	117	602	7.8%
G	C	429	150	91	279	424	150	86	0	274	1.7%
	2C	534	150	197	384	515	150	86	91	365	5.0%
	3C	647	150	310	497	609	150	179	91	459	7.6%
H	C	444	200	51	244	444	200	0	51	244	0.0%
	2C	502	200	108	302	502	200	0	108	302	0.0%
	3C	584	200	191	384	583	200	77	108	383	0.3%
	4C	672	200	278	472	671	200	83	191	471	0.2%
I	C	469	200	35	269	469	200	0	35	269	0.0%
	2C	508	200	75	308	508	200	0	75	308	0.0%
	3C	554	200	121	354	554	200	0	121	354	0.0%
	4C	601	200	168	401	601	200	0	168	401	0.0%
J	C	379	200	50	179	379	200	0	50	179	0.0%
	2C	439	200	109	239	439	200	0	109	239	0.0%
	3C	499	200	169	299	499	200	0	169	299	0.0%
	4C	561	200	232	361	561	200	0	232	361	0.0%
Average											3.7%

the demand of the depot. Overall, the results show that any solution of the bilevel model leads to costs for the shipper that are always equal or lower than the stand alone solution. Similarly, any solution of the bilevel model leads to profits for the carrier that are always equal or higher than the stand alone solution. Based on these results, we argue that our bilevel approach is efficient to deal with the collaborative problem where different goals are considered and decisions are taken hierarchically. We must also emphasise that, although the solution of the bilevel VRPSB seems to only benefit the shipper, in fact the carrier also gains using a bilevel approach, since the upper level considers the most optimistic case of the lower level problem, as implicit by the bilevel formulation (demonstrated in Section 5.3.3).

5.5.3 Bilevel versus compensation schemes

One alternative to model a collaborative transportation problem involves a leading participant optimizing selfishly the routing problem, and then compensating another participant with a side payment, so that the latter does not lose with the collaboration (Özener et al., 2011). Some works in the literature determine the side payments as a fixed value (e.g., Caballini et al. (2016), Defryn et al. (2016)), whereas other compute the side payment as a value dependent on the distance (e.g., Liu et al. (2010), Dahl and Derigs (2011), Archetti et al. (2016)).

In this work, we use the integrated problem (VRPSB) to model selfishly the routing problem, and we propose two different compensation schemes. In the first scheme, the side payment corresponds to the difference in the profits of the carrier between the stand alone solution (VRP) and the integrated solution (VRPSB). The first side payment is designated as SP_Δ and their values are presented in Table 5.4, for each VRPSB solution (before side payment). The second scheme computes the side payment as a value proportional to the backhaul distance. This is designated as $SP(sp)$, where sp stands for the proportion used, as it is computed as Equation (5.39). The first compensation scheme provides the side payment after the routing, whereas the second determines the side payment while solving the routing problem.

In this section, we compare the performance of the collaboration of our bilevel approach with compensation schemes. Tables 5.5 and 5.6 report the objective functions of the shipper and the carrier, for the different side payments, along with the synergy value of the solutions. To facilitate the comparison, the synergy values of the bilevel solutions are also reported.

$$SP(sp) = sp \cdot (d_{lb} + d_{b0}), \qquad \forall\ sp = \{0.50, 0.75\}, l \in L, b \in B \tag{5.39}$$

In the bilevel model we assume an optimistic approach such that when the profit of an external incentive equals the profit of a given backhaul visit, the carrier performs an integrated route. The same assumption applies to the case of using side payments. Therefore, as expected, a side payment SP_Δ would always motivate the carrier to collaborate because the profits gained are the same as in the stand alone solution. However, for most of the

Table 5.4 – VRPSB solutions and respective side payments determined after routing, for the two different scenarios

		Unlimited visits			Limited visits		
Inst.	Q_0	Costs	Profits	SP_Δ	Costs	Profits	SP_Δ
A	C	250	164	16	250	164	16
	2C	293	149	31	293	149	31
	3C	339	131	49	341	134	46
	4C	386	115	65	395	117	63
	5C	433	100	80	458	104	76
B	C	184	46	14	184	46	14
	2C	222	46	14	232	39	21
C	C	200	47	13	200	47	13
	2C	233	39	21	248	41	19
D	C	204	49	11	204	49	11
	2C	239	40	20	246	41	19
E	C	314	160	20	314	160	20
	2C	367	143	37	367	143	37
	3C	420	143	37	438	146	34
	4C	473	143	37	514	146	34
	5C	526	143	37	594	146	34
F	C	450	172	28	450	172	28
	2C	526	145	55	526	145	55
	3C	611	126	74	611	126	74
	4C	723	121	80	726	121	79
G	C	398	124	26	398	124	26
	2C	464	97	53	464	97	53
	3C	556	97	53	557	97	53
H	C	436	167	33	436	167	33
	2C	486	167	33	493	167	33
	3C	537	167	33	552	168	32
	4C	588	167	33	626	149	51
I	C	469	200	0	469	200	0
	2C	504	200	0	508	200	0
	3C	538	200	0	553	170	30
	4C	573	200	0	599	170	30
J	C	361	163	37	361	163	37
	2C	403	130	70	403	130	70
	3C	453	130	70	455	131	69
	4C	503	130	70	515	131	69

Table 5.5 – Comparison of synergy values from bilevel solutions and from the integrated planning with different side payment schemes, for the case of unlimited visits to backhaul customers.

			SP_{Δ}			$SP(0.50)$			$SP(0.75)$		Bilevel
Inst.	Q_0	Costs	Profits	SV	Costs	Profits	SV	Costs	Profits	SV	SV
A	C	266	180	9.3%	267	181	9.3%	275	180	0.0%	**10.0%**
	2C	324	180	11.3%	329	185	11.3%	342	180	0.0%	**11.3%**
	3C	388	180	9.5%	394	186	9.5%	410	180	0.0%	**10.0%**
	4C	451	180	9.0%	461	190	9.0%	478	180	0.0%	**9.4%**
	5C	514	180	8.6%	528	194	8.6%	545	180	0.0%	**8.6%**
B	C	198	60	-4.7%	192	60	0.0%	192	60	0.0%	**0.0%**
	2C	236	60	-3.7%	230	60	0.0%	230	60	0.0%	**0.0%**
C	C	213	60	-6.1%	204	60	0.0%	204	60	0.0%	**0.0%**
	2C	254	60	-6.6%	242	60	0.0%	242	60	0.0%	**0.0%**
D	C	215	60	-5.5%	207	60	0.0%	207	60	0.0%	**0.0%**
	2C	259	60	-7.4%	245	60	0.0%	245	60	0.0%	**0.0%**
E	C	334	180	-7.6%	323	180	0.0%	323	180	0.0%	**0.0%**
	2C	404	180	-13.9%	376	180	0.0%	376	180	0.0%	**0.0%**
	3C	457	180	-11.0%	429	180	0.0%	429	180	0.0%	**0.0%**
	4C	510	180	-9.0%	482	180	0.0%	482	180	0.0%	**0.0%**
	5C	563	180	-7.7%	535	180	0.0%	535	180	0.0%	**0.0%**
F	C	478	200	7.0%	475	197	7.0%	487	210	7.0%	**7.0%**
	2C	581	200	7.4%	576	195	7.4%	600	210	5.1%	**7.4%**
	3C	685	200	7.4%	688	198	6.5%	713	210	4.0%	**7.4%**
	4C	802	200	5.4%	801	198	5.4%	826	210	3.3%	**6.1%**
G	C	424	150	1.7%	420	146	1.6%	429	150	0.0%	**1.7%**
	2C	517	150	0.7%	511	146	1.2%	520	150	0.0%	**1.2%**
	3C	608	150	0.5%	602	146	1.0%	611	150	0.0%	**1.0%**
H	C	469	200	-10.0%	444	200	0.0%	444	200	0.0%	**0.0%**
	2C	520	200	-8.3%	495	200	0.0%	495	200	0.0%	**0.0%**
	3C	570	200	-7.1%	546	200	0.0%	546	200	0.0%	**0.0%**
	4C	621	200	-6.2%	596	200	0.0%	596	200	0.0%	**0.0%**
I	C	469	200	0.0%	469	200	0.0%	469	200	0.0%	**0.0%**
	2C	504	200	0.0%	504	200	0.0%	504	200	0.0%	**0.0%**
	3C	538	200	0.0%	538	200	0.0%	538	200	0.0%	**0.0%**
	4C	573	200	0.0%	573	200	0.0%	573	200	0.0%	**0.0%**
J	C	398	200	-10.2%	373	175	-10.2%	379	181	-10.2%	**0.0%**
	2C	473	200	-18.8%	423	175	-8.0%	429	181	-8.0%	**0.0%**
	3C	522	200	-15.5%	473	175	-6.6%	479	181	-6.6%	**0.0%**
	4C	572	200	-13.1%	523	175	-5.6%	529	181	-5.6%	**0.0%**
Average				-2.7%			1.4%			-0.3%	**2.3%**

Table 5.6 – Comparison of synergy values from bilevel solutions and from the integrated planning with different side payment schemes, for the case of limited visits to backhaul customers.

		SP_{Δ}			$SP(0.50)$			$SP(0.75)$			Bilevel
Inst.	Q_0	Costs	Profits	SV	Costs	Profits	SV	Costs	Profits	SV	SV
A	C	266	180	9.3%	267	181	9.3%	275	180	0.0%	**10.0%**
	2C	324	180	12.5%	329	185	12.5%	345	180	0.0%	**12.5%**
	3C	387	180	12.4%	396	189	12.4%	416	180	0.0%	**12.4%**
	4C	458	180	9.7%	468	189	9.6%	488	180	0.0%	**10.1%**
	5C	534	180	7.3%	542	189	7.8%	562	180	0.0%	**8.9%**
B	C	198	60	-4.7%	192	60	0.0%	192	60	0.0%	**0.0%**
	2C	253	60	5.0%	257	68	6.9%	264	60	0.0%	**8.5%**
C	C	213	60	-6.1%	204	60	0.0%	204	60	0.0%	**0.0%**
	2C	267	60	4.3%	270	66	5.9%	276	60	0.0%	**9.6%**
D	C	215	60	-5.5%	207	60	0.0%	207	60	0.0%	**0.0%**
	2C	265	60	2.2%	265	72	8.3%	270	60	0.0%	**8.3%**
E	C	334	180	-7.6%	323	180	0.0%	323	180	0.0%	**0.0%**
	2C	404	180	-5.0%	393	180	0.0%	393	180	0.0%	**0.2%**
	3C	471	180	-1.8%	466	180	0.0%	466	180	0.0%	**0.1%**
	4C	547	180	-1.4%	542	180	0.0%	542	180	0.0%	**0.1%**
	5C	628	180	-1.2%	623	180	0.0%	623	180	0.0%	**0.1%**
F	C	478	200	7.0%	475	197	7.0%	487	210	7.0%	**7.0%**
	2C	581	200	8.4%	576	195	8.4%	602	221	8.4%	**8.4%**
	3C	685	200	9.1%	693	195	6.6%	719	221	6.6%	**9.1%**
	4C	805	200	7.4%	811	195	5.8%	836	221	5.8%	**7.8%**
G	C	424	150	1.7%	420	146	1.6%	429	150	0.0%	**1.7%**
	2C	517	150	4.4%	512	144	4.3%	526	161	5.0%	**5.0%**
	3C	609	150	7.6%	607	145	7.2%	630	169	7.2%	**7.6%**
H	C	469	200	-10.0%	444	200	0.0%	444	200	0.0%	**0.0%**
	2C	526	200	-8.1%	502	200	0.0%	502	200	0.0%	**0.0%**
	3C	583	200	0.3%	570	187	0.3%	580	196	0.3%	**0.3%**
	4C	678	200	-1.3%	658	187	0.2%	667	196	0.2%	**0.2%**
I	C	469	200	0.0%	469	200	0.0%	469	200	0.0%	**0.0%**
	2C	508	200	0.0%	508	200	0.0%	508	200	0.0%	**0.0%**
	3C	583	200	-8.1%	554	200	0.0%	554	200	0.0%	**0.0%**
	4C	629	200	-6.9%	601	200	0.0%	601	200	0.0%	**0.0%**
J	C	398	200	-10.2%	373	175	-10.2%	379	181	-10.2%	**0.0%**
	2C	473	200	-14.1%	427	178	-4.5%	434	184	-4.8%	**0.0%**
	3C	524	200	-8.6%	483	158	-8.6%	494	185	-3.4%	**0.0%**
	4C	584	200	-6.3%	542	158	-6.3%	554	185	-2.2%	**0.0%**
Average				0.0%			2.4%			0.6%	**3.7%**

instances reported in Tables 5.5 and 5.6, the costs of the shipper are higher than its stand alone solution. In these cases, the network costs surpass the stand alone solution, which result in negative synergy values. These results also put in evidence the lack of efficiency of determining the side payment only after solving the routing optimization problem.

With respect to the schemes where the side payments are determined while solving the routing problem, the results show that when using the highest proportion ($sp = 0.75$) the solutions tend to be non-collaborative ($SV = 0.0\%$). On the other hand, using the lowest proportion ($sp = 0.50$), many solutions do not attend the individual rationality of the carrier, since the profits are lower than in the stand alone solution (e.g., instances F, G, J). In such cases, collaboration would not take place. Nonetheless, Tables 5.5 and 5.6 also show that, when collaboration occurs, several solutions provided by these side payments allow the carrier to collect more profits than with the bilevel approach, but this comes at higher costs for the shipper.

Generally, the results demonstrate that any solution of the bilevel approach has higher or equal synergy value than any solution obtained through the compensation schemes analyzed. Based on these results, we argue that the bilevel model proposed can more effectively capture the interactions of the different players than the compensation schemes, guaranteeing a more balanced solution with a higher synergy value.

Finally, as also observed from the results in the previous section, synergy values obtained with any side payments are higher, on average, for a scenario with limited visits than a scenario with unlimited visits to backhaul customers.

5.5.4 Computational limits of the collaborative VRPSB

The VRPB is strongly NP-hard since it generalizes the VRP (Toth and Vigo, 2002). Thus, since the VRPSB generalizes the VRP (note that if there are no bakchauls, we have a simply a VRP), VRPSB is also NP-hard. In addition, bilevel optimization problems are proven to be strongly NP-hard and a mere assessment of the optimal solution is also NP-hard, even for the simplest linear bilevel program (Jeroslow, 1985).

In this section, we aim to evaluate the practical difficulty of solving the collaborative transportation planning formulations proposed. More precisely, we aim to empirically analyze at what extent the collaborative formulation increases the practical difficulty of the problem.

Instances from F to N are tested with a computing time limit of 3600 seconds. Tables 5.7 and 5.8 provide the upper bound (UB), the computing time required to achieve the best solution and the percentage gap obtained with each model, for both scenarios (limited and unlimited backhaul visits). To avoid repetition, the UB is only displayed for instances not already covered by Tables 5.2, 5.3 and 5.4.

The results show that the bilevel formulation is effectively the main reason behind the complexity of solving the collaborative problem. The computing time to solve an instance with the bilevel model tends to be higher than with the non-collaborative models, as well as the percentage gap. Nevertheless, the complexity of the bilevel approach does not seem to be much different than for the traditional VRPSB for some instances (e.g., instances L and M). Moreover, the bilevel model solves simultaneously a routing and a pricing problem, which is expected to be more computational challenging than solving a routing problem only.

In general, the computing time to solve the bilevel problem tends to increase with increasing size of the instance. It seems that the number of linehauls customers have more influence than the number of backhaul customers, but some instances present exceptions. For example, instance N has more linehaul customers than instance L but it is solved in a much shorter time. On the other hand, the exact method seems suitable to solve bilevel instances with relatively high number of backhaul customers. These type of instances fit well real industries that have a wide range of backhauling opportunities, such as the forestry (Marques et al., 2020).

Furthermore, it seems more challenging to solve the problems in a scenario with limited visits to backhaul customers, than in a scenario with unlimited visits. On average, both the computing time and the percentage gap are higher for the scenario with limit visits. These results support that limiting the number of visits brings additional complexity the bilevel model, since the incentives for backhauling also compete with diverse options for pure inbound routes other than the least costly one.

Finally, we point out that the focus of this work is on the modelling aspects of the collaborative problem rather than on the solution methods. Using the properties of the bilevel optimization, we have demonstrated an effective way to solve a hierarchical collaborative problem. The rationale used to design the reformulation method could be applied to design a metaheuristic, and thus guarantee higher efficiency when solving the problem.

5.6. Conclusions and future research

This work investigates an innovative formulation for a collaborative transportation planning between a shipper and a carrier. The shipper offers incentives to the carrier in order to create cost-effective integrated inbound-outbound routes. These incentives compete with each other with other potential incentives offered to the carrier by external companies. The problem of the shipper is a cost minimization VRPSB and the problem of the carrier is a VRP with profits. Based on the hierarchical nature of the players and on the conflicting objectives, the collaborative problem is formulated as a bilevel optimization problem. The upper level describes the problem of the shipper and the lower level describes the problem of the carrier. To solve the bilevel problem, we convert it in an equivalent single-level mixed integer linear program by exploring problem-specific characteristics of the lower level, and then, standard linearization techniques.

This work conducts an extensive analysis on the properties of the bilevel approach to handle the collaborative problem. The bilevel model is compared with traditional non-collaborative routing problems and with different side payment schemes, in order to assess the impact of the collaboration and the approach applied. In addition, the impact of limiting the number of backhaul visits is also evaluated. Finally, the computational limits of the collaborative formulation is compared against traditional single-level routing problems.

The results of this work put in evidence the advantages of the bilevel approach to handle a collaborative transportation planning, although the computational effort tend to be higher

Table 5.7 – Computational performance of each model, considering unlimited backhaul visits.

		VRP			VRPSB			Bilevel		
Inst.	Q_0	UB	*gap*	*time*	UB	*gap*	*time*	UB	*gap*	*time*
F	C		0.0%	10		0.0%	25		0.0%	92
	2C		0.0%	11		0.0%	19		0.0%	1579
	3C		0.0%	10		0.0%	43		0.0%	145
	4C		0.0%	9		0.0%	76		0.0%	118
G	C		0.0%	14		0.0%	21		0.0%	24
	2C		0.0%	16		0.0%	39		0.0%	73
	3C		0.0%	6		0.0%	30		0.0%	61
H	C		0.0%	5		0.0%	17		0.0%	25
	2C		0.0%	5		0.0%	5		0.0%	27
	3C		0.0%	5		0.0%	10		0.0%	24
	4C		0.0%	6		0.0%	16		0.0%	27
I	C		0.0%	13		0.0%	36		0.0%	35
	2C		0.0%	24		0.0%	30		0.0%	70
	3C		0.0%	18		0.0%	25		0.0%	57
	4C		0.0%	15		0.0%	45		0.0%	45
J	C		0.0%	10		0.0%	37		0.0%	377
	2C		0.0%	11		0.0%	34		0.0%	98
	3C		0.0%	15		0.0%	19		0.0%	131
	4C		0.0%	14		0.0%	19		0.0%	457
K	C	472	0.0%	287	455	0.0%	1072	472	0.5%	3600
	2C	522	0.0%	289	499	0.0%	161	522	0.0%	650
	3C	571	0.0%	162	549	0.0%	652	571	0.0%	2568
	4C	621	0.0%	114	599	0.0%	223	621	0.0%	416
L	C	452	0.0%	169	435	0.0%	1079	452	0.0%	1224
	2C	502	0.0%	283	477	0.0%	1248	502	0.0%	1949
	3C	552	0.0%	825	527	0.5%	3600	552	0.0%	1531
	4C	601	0.0%	877	577	0.0%	1216	601	0.0%	796
M	C	542	1.9%	3600	527	2.7%	3600	543	2.1%	3600
	2C	591	1.0%	3600	567	1.1%	3600	595	2.5%	3600
	3C	640	0.0%	3545	617	1.6%	3600	646	3.3%	3600
	4C	692	1.8%	3600	666	0.0%	2215	702	7.3%	3600
N	C	474	0.0%	22	460	0.0%	11	474	0.0%	444
	2C	523	0.0%	22	500	0.0%	14	523	0.0%	985
	3C	573	0.0%	22	548	0.0%	4	573	0.0%	206
	4C	622	0.0%	21	597	0.0%	34	622	0.0%	167
Average			0.1%	504		0.2%	654		0.4%	926

Table 5.8 – Computational performance of each model, considering limited backhaul visits.

		VRP			VRPSB			Bilevel		
Inst.	Q_0	*UB*	*gap*	*time*	*UB*	*gap*	*time*	*UB*	*gap*	*time*
F	C		0.0%	9		0.0%	17		0.0%	45
	2C		0.0%	12		0.0%	15		0.0%	114
	3C		0.0%	11		0.0%	68		0.0%	242
	4C		0.0%	23		0.0%	22		0.0%	37
G	C		0.0%	12		0.0%	5		0.0%	79
	2C		0.0%	15		0.0%	32		0.0%	40
	3C		0.0%	7		0.0%	10		0.0%	121
H	C		0.0%	4		0.0%	7		0.0%	33
	2C		0.0%	12		0.0%	21		0.0%	41
	3C		0.0%	4		0.0%	11		0.0%	362
	4C		0.0%	8		0.0%	17		0.0%	509
I	C		0.0%	11		0.0%	28		0.0%	451
	2C		0.0%	12		0.0%	5		0.0%	202
	3C		0.0%	23		0.0%	12		0.0%	224
	4C		0.0%	26		0.0%	17		0.0%	146
J	C		0.0%	22		0.0%	16		0.0%	338
	2C		0.0%	14		0.0%	13		0.0%	1026
	3C		0.0%	34		0.0%	14		0.0%	587
	4C		0.0%	29		0.0%	24		0.0%	173
K	C	472	0.0%	290	455	0.0%	368	472	0.0%	873
	2C	538	0.0%	162	503	0.0%	217	538	0.0%	1800
	3C	649	0.0%	73	578	0.0%	1374	637	1.7%	3600
	4C	773	0.0%	81	654	0.8%	3600	734	1.2%	3600
L	C	452	0.0%	173	435	0.0%	633	452	4.8%	3600
	2C	512	0.0%	338	477	0.5%	3600	512	1.3%	3600
	3C	578	0.0%	422	532	0.0%	635	580	11.4%	3600
	4C	661	0.0%	324	591	0.0%	695	658	0.6%	3600
M	C	542	2.0%	3600	527	2.2%	3600	578	11.8%	3600
	2C	612	0.0%	2160	567	0.0%	2308	623	4.4%	3600
	3C	725	0.7%	3600	627	1.3%	3600	701	3.5%	3600
	4C	843	1.6%	3600	690	0.0%	1014	789	3.3%	3600
N	C	474	0.0%	22	460	0.0%	8.5	474	0.0%	1062
	2C	529	0.0%	19	500	0.0%	10	529	0.0%	799
	3C	600	0.0%	22	548	0.0%	9	600	0.0%	235
	4C	690	0.0%	20	600	0.0%	7	677	0.0%	1909
Average			0.1%	434		0.1%	629		1.3%	1356

than than the traditional non-collaborative formulations. Therefore, one main direction for future work includes the development of an efficient metaheuristic to solve the problem. Such metaheuristic could encompass a genetic algorithm or another method of the family of evolutionary algorithms, since these are the most used advanced methods to solve bilevel optimization problems (Sinha et al., 2018). The bilevel model proposed in this work is designed under an optimistic approach for both players. Because the objectives of each player are different and they collaborate in a hierarchical structure, considering the most optimistic case of the lower level can be seen as a robust strategy to achieve robust solutions for the upper level. An interesting future line of research could encompass investigating the collaborative problem under a pessimistic approach and compare it the optimistic.

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5..1 Traditional models

Traditionally, the transportation planning does not consider collaboration. In this section, two traditional formulations are presented for the transportation planning, namely a separated model (which is described by a typical VRP) and an integrated model (which is described by a VRPSB). Both models are simplifications of the bilevel VRPSB presented in Section 5.3.2 that exclude the lower level problem from the formulation. Consequently, the variable and all constraints related to the incentives are also excluded in the non-collaborative models. On the other hand, the maximum distance allowed per route is taking into account in both models, since the carrier would never accept to exceed this distance, even for a single delivery route. In addition, the objective function of the carrier is treated as an expression in both non-collaborative models. The mathematical formulation of the traditional models are presented.

Separated model

The separated model describes the problem where inbound and outbound routes are planned separately and integrated routes are not allowed. The problem is formulated as an Open VRP and the objective function in (5.40) minimizes the total routing costs of delivery vehicles, plus the constant expression given by the minimum number of vehicles necessary to satisfy the depot demand. This states that the demand of the depot can only be satisfied by dedicated inbound vehicles. The complete formulation of the separated model is presented as follows:

$$min \sum_{i \in V} \sum_{j \in L} \sum_{k \in K} c^U_{ij} \cdot x^k_{ij} + \sum_{b \in B} O_b \cdot 2c^U_{b0} \tag{5.40}$$

subjected to

$$\begin{aligned} &(5.2) - (5.4) \\ &(5.5) - (5.36) \\ &(5.16) \\ &x^k_{ij}, O_b \in \{0,1\}, y_{ij} \geq 0, \qquad \forall i,j \in V = \{0\} \cup L, b \in B, k \in K, w \in \mathbb{Z}^+_0 \end{aligned} \tag{5.41}$$

Integrated model

The integrated model describes the problem where inbound and outbound routes are planned jointly by the shipper. This model considers that the shipper assumes control over all vehicles of the carrier used in the network. As the lower level variables are not considered, the shipper no more compete with others for backhaul routes. Instead, the shipper assumes that the unitary cost to visit a backhaul customer is the same as visiting a linehaul customer. Thus, the objective function in (5.42) is to minimize the total routing costs and outsourcing of dedicated inbound vehicles. The constraints of the integrated model are the same as those from the upper level problem with the additional backhaul customers constraints of the lower level problem. Note that the variables of the lower level are substituted by variables of the upper level in the backhaul customers constraints.

$$min \sum_{i \in V} \sum_{j \in L} \sum_{k \in K} c_{ij}^U \cdot x_{ij}^k + \sum_{b \in B} O_b \cdot 2c_{b0}^U \tag{5.42}$$

subjected to

$$(5.2) - (5.36)$$

$$(5.15') \text{ and } (5.16')$$

$$x_{ij}^k, O_b \in \{0,1\}, y_{ij} \geq 0, \qquad \forall i, j \in V, b \in B, k \in K, w \in \mathbb{Z}_0^+ \tag{5.43}$$

CHAPTER 6

A Green Vehicle Routing Problem with Backhauls

A green lateral collaborative transportation problem under different collaboration strategies and profit allocation methods

Maria João Santos[*†] · **Sara Martins**[*†] · **Pedro Amorim**[*†] · **Bernardo Almada-Lobo**[*†]

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Abstract Collaboration between companies in transportation problems seeks to reduce empty running of vehicles and to increase the use of vehicles' capacity. Motivated by a case study in the food supply chain, this paper examines a lateral collaboration between a leading retailer (LR), a third party logistics provider (3PL) and different producers. Three collaborative strategies may be implemented simultaneously, namely *pickup-delivery*, *collection* and *cross-docking*. The *collaborative pickup-delivery* allows an entity to serve customers of another in the backhaul trips of the vehicles. The *collaborative collection* allows loads to be picked up at the producers in the backhauling routes of the LR and the 3PL, instead of the traditional outsourcing. The *collaborative cross-docking* allows the producers to cross-dock their cargo at the depot of another entity, which is then consolidated and shipped with other loads, either in linehaul or backhaul routes. The collaborative problem is formulated with three different objective functions: minimizing total operational costs, minimizing total fuel consumption and minimizing operational and CO_2 emissions costs. The synergy value of collaborative solutions is assessed in terms of costs and environmental impact. Three proportional allocation methods from the literature are used to distribute the collaborative gains among the entities, and their limitations and capabilities to attend fairness criteria are analyzed. Collaboration is able to reduce the global fuel consumption in 26% and the global operational costs in 28%, independently of the objective function used to model the problem. The collaborative pickup-delivery strategy outperforms the other two in the majority of instances under different objectives and parameter settings. The collaborative collection is favoured when the ordering loads from producers increase. The collaborative cross-docking tends to be implemented when the producers are located close to the depot of the 3PL.

[*]Faculdade de Engenharia, Universidade do Porto, Porto, Portugal
[†]INESC TEC - Instituto de Engenharia de Sistemas e Computadores, Tecnologia e Ciência, Porto, Portugal

Keywords Collaboration · Backhauling · Cross-docking · Profit allocation

6.1. Introduction

Transportation is a crucial activity in any supply chain, responsible for a large portion of the total logistics costs. In terms of environmental impact, transportation is responsible for about 14% of the total CO_2 emissions worldwide, from which road transportation alone represents about 75% (Liu et al., 2020). Thus, under the contemporary need of improving supply chains sustainability, adopting green initiatives for transportation activities presents opportunities to promote such goal. Among these green initiatives, collaboration is a relatively wide-spread practice among companies (Evangelista, 2014; Ramanathan et al., 2014). The goal of collaborative transportation is to identify hidden costs in logistics operations that cannot be managed individually, but can be reduced collectively (Ergun et al., 2007). Although the literature on collaborative vehicle routing has been increasing, there are still some limitations that we aim to address in this work.

The first limitation concerns the type of collaboration usually investigated in the literature. There are three types of collaboration: i) horizontal, when the participants are at the same level of the supply chain (e.g., two suppliers), ii) vertical, when the participants are at different levels (e.g., a carrier and a supplier), and iii) lateral, when there is a combination of both previous types (e.g., two suppliers and two carriers). However, within collaborative vehicle routing problems, horizontal collaboration is the most investigated type in the literature (Gansterer and Hartl, 2018), whereas less research has been devoted to vertical and lateral collaboration.

The second limitation regards the analysis of the impact of different collaborative strategies. Strategies applied in the collaborative transportation planning may include exchanging customers between companies (Dai and Chen, 2012; Fernández et al., 2016), sharing vehicles' capacity of different companies with a common depot (Cruijssen et al., 2007), consolidating cargo of different companies in single delivery routes (Ergun et al., 2007), or providing loads in backhaul routes to reduce empty trips (Juan et al., 2014). Moreover, the majority of the literature on collaborative vehicle routing focus only on economic indicators to assess the gains of collaboration. Nonetheless, collaborative transportation should also be assessed in terms of environmental impact.

Another limitation concerns the simultaneous analysis of the value of collaboration in terms of impact on the routing problem and impact on the allocation of the collaborative gains between the participants in the transportation problem. According to Gansterer and Hartl (2018), the majority of the literature on collaborative transportation focus either on solving the transportation problem or on solving the allocation problem, both usually not both.

Therefore, this work tackles the three aforementioned limitations found in the literature of collaborative vehicle routing. We address the first limitation by studying a case of lateral collaboration between different participants in a transportation network, where they may perform similar services (e.g., serving customers of each other), leading to horizontal collaboration, but also complementary transportation activities (e.g., cross-docking), leading to vertical collaboration. Studying a lateral collaboration will allow us to investigate the impact of different collaborative strategies that may be applied simultaneously, namely *pickup-delivery*, *collection* and *cross-docking*. The *collaborative pickup-delivery* refers to a strategy where customers of one participant are served by vehicles of the other participant.

The *collaborative collection* considers a strategy that allows vehicles of other participants to pickup cargo at the producers, thus avoiding empty running of vehicles when returning to the depot and, at the same time, reducing the need of outsourcing. Finally, the *collaborative cross-docking* refers to a strategy that allows loads from different locations to be consolidated in an intermediary location before arriving to the final destination. The second limitation is addressed by analyzing the collaborative model under three different objective functions, covering economic and environmental issues, allowing to measure the impact of collaboration under a sustainable context. The third limitation is addressed by solving the integrated routing and the profit sharing problems, and analyzing the effect of different collaborative solutions on the distributed profit shares determined by different allocation methods.

This approach is validated and applied to an European leading retailer (LR) and a third-party logistics provider (3PL). Several practical constraints are taken into account, such as the interdependence between different strategies, which allows to gather relevant insights to implement in real contexts. The case-study is motivated by the potential of reducing logistics costs through collaboration, enabled by the transportation network. The LR and 3PL distribution centers are strategically located in distinct geographical areas that benefit their operational activities. The LR has distribution centers located closer to dense areas of stores to be supplied, while the 3PL is centrally positioned to serve different national and international customers. With greater or lesser frequency, both serve common areas, being the 3PL located in an area with more dispersed retail stores and closer to the LR suppliers. The participants of such transportation networks are usually aware of the hidden potential of collaboration to mitigate costs and CO_2 emissions, and the main goal of this paper is to provide managerial insights on that subject. It is worth mentioning that this case study represents a common transportation problem setting in practice. Thus, the approach developed in this work can be easily extended and replicated for other supply chains. Also, the use of consolidation centers is a relatively common strategy used by carriers and shippers to reduce transportation costs, but it is rarely examined under the context of collaboration. Therefore, this work provides insights into the impact of collaboration achieved by combining strategies commonly used in collaborative networks, such as backhauling, and others that are not usually accounted for, such as cross-docking.

The remainder of this paper is structured as follows. Section 6.2 presents the literature review on collaborative vehicle routing problems relevant for this work, and frame the position of our paper within current research. Section 6.3 provides a description of the collaborative problem and strategies investigated, the mathematical formulation of the collaborative vehicle routing problem and the allocation methods used for profit sharing. Section 6.4 reports the computational experiments on randomly generated and case-study instances, discussing the impact of collaboration under economic and environmental contexts and the performance of different allocation methods. Section 6.5 concludes the paper describing the main findings and managerial insights of this work, and proposing future research as well.

6.2. Related literature

The collaborative transportation problem can be characterized according to the type of entities involved, or type of logistics services offered in the transportation network. Horizontal collaboration refers to the case where the entities involved are at the same level of network, offering the same type of service (e.g., carrier collaboration) (Audy et al., 2012). Vertical collaboration refers to the case when the entities are at different levels of the supply chain (e.g., supplier-retailer collaboration). Lateral collaboration, which combines horizontal and vertical collaborations, exploits the benefits of both types of collaboration, contributing to the overall management of transportation activities (Mason et al., 2007). The present work describes a case of lateral collaboration, involving different entities (LR, 3PL and producers) which may execute similar transportation services, leading to horizontal collaboration, as well as complementary services, leading to vertical collaboration.

It has been demonstrated that collaboration unveils hidden opportunities to reduce the logistics burden of transportation for individual companies. For example, when a carrier (or shipper) needs to deliver to long distance customers, which are conveniently located near to other carrier, the two entities may collaborate through customer sharing. In such cases, the customer of the former carrier can be supplied by the other carrier, allowing to reduce the total distances travelled by the vehicles of both entities (e.g., Fernández et al. (2016)). Another opportunity for collaboration arises when the total loads to deliver to a customer, or in a region, are relatively small compared to the full capacity of the vehicle used to carry them. In such cases, the carrier can exchange these loads with other carrier operating in the same region, or serving the same customer, increasing the average load of the vehicles in use (e.g., Paul et al. (2019)). In practice, most of the vehicles returning to the depot travel empty. Thus, backhauling emerges as a powerful opportunity for collaboration, by allowing vehicles to perform additional services in these returning trips, thus reducing the total empty running. The additional services may include pickup and deliveries between different locations (e.g., Juan et al. (2014)) or one or more pickups to supply the destination depot of the vehicles (e.g., Frisk et al. (2010)). Another possibility of collaboration relies on the use of intermediary facilities between a pickup and a delivery location, in order to reduce the total traveled distance and number of vehicles required. In such cases, the loads of different suppliers that serve the same depot can be consolidated in a intermediary location, which are then carried by a single vehicle to the depot (e.g., le Blanc et al. (2006); Neves-Moreira et al. (2016)). Although some of the above mentioned works tackle more than one collaborative strategy simultaneously, they only cover one type of collaboration, specifically horizontal collaboration. In opposition, this paper tackles different collaborative strategies that imply a combination of horizontal and vertical collaboration. Particularly, we consider strategies that are based on capacity sharing (which we designate as *collaborative pickup-delivery*), backhauling (which we designate as *collaborative collection*) and use of intermediary facilities (which we designate as *collaborative cross-docking*).

The literature on collaborative vehicle routing shows two main streams of research - the collaborative vehicle routing problem and the profit sharing problem. Recent reviews on each stream can be found in Gansterer and Hartl (2018) and Guajardo and Rönnqvist (2016), respectively. The first stream focus on optimizing the global collaborative problem (e.g., minimizing the total cost in the collaborative network), developing efficient methods to solve it and measuring the impact of the collaboration. Common metrics applied in the collaborative transportation are the *performance* and the *synergy value*. The performance provides the absolute gains (or savings) obtained from collaboration, determined as the difference between the profit (or cost) of a non-collaborative solution and a collaborative solution (Cuervo et al., 2016). The value of a collaborative solution is provided by the sum of all individual profits (or costs) of each entity before collaboration, also known as stand alone solution. The synergy value provides the percentage profit increase (or cost reduction) created by the collaboration, compared to the stand alone solution (Cruijssen et al., 2007). More recently, some works also assess the impact of collaboration in terms of environmental impacts (e.g., Juan et al. (2014); Chabot et al. (2018)). In this work, we analyse the synergy value of collaborative solutions obtained with a fuel consumption minimization model and with a model minimizing both operational and environmental costs, which we designate as the *holistic model*. We further assess the environmental impact of such solutions in comparison with the traditional economic models.

The second stream of the literature focus on developing allocation methods to distribute the gains from collaboration among the participants, according to some fairness criteria, or properties (Frisk et al., 2010). Common properties desirable in the profit sharing problem are *efficiency*, *individual rationality* and *group rationality* (Dahlberg et al., 2019). The first property ensures that all costs (profits) obtained with the collaboration are fully distributed among the participants. The second property guarantees that each participant is not allocated higher costs (lower profit) than its stand alone solution. The third property applies the same principle as the previous property for a coalition (group of participants), i.e. the participants in a coalition cannot be allocated a higher cost (lower profit) than the cost (profit) of that coalition. A stable allocation is one that ensures the properties of efficiency and group rationality. The Shapley value is one of the most frequently used methods for profit sharing in the literature, which allocates to each participant the respective average marginal cost that results from its participation in the coalition. However, this method requires solving all possible coalitions, and in some cases it cannot guarantee stability (Guajardo and Rönnqvist, 2016). Another method widely used is the nucleolus, which defines an excess vector related to the degree of acceptance of an allocated cost for each coalition, and the goal is the maximization of the minimum excess. The nucleolus method outperforms several methods proposed in the literature, in particular regarding stability, but it is relatively hard to compute, and usually requires an algorithm to solve a sequential set of linear programs (Guajardo and Rönnqvist, 2016). The proportional methods are the simplest allocation methods in the literature, because in opposition to the previous methods they do not require solving an optimization problem, and depending on the rules applied, they may provide stability. Thus, although having been subject to some criticism from the research community, the proportional methods are easy to compute and easy to communicate (Guajardo and Rönnqvist, 2016), can be more easily scalable for real life instances (Ozener, 2014), and therefore they are commonly implemented in practice (Liu, 2010). Proportional methods can also apply several rules to allocate the costs, namely based on demand, on distances or on stand alone costs (Guajardo and Rönnqvist, 2016). Currently, there is no general acceptance of one method over another in the research community, since an allocation method hardly fulfils all the properties defined in the literature. For this reason, there is a vast literature on tailored methods adapted to specific case-studies or applications (Guajardo and Rönnqvist, 2016). In this work, we investigate three proportional allocation methods following the work of Lunday and Robbins (2019), and evaluate their performance in terms efficiency and individual rationality.

One of the pioneer works tackling both the vehicle routing and the profit sharing problems is described in Krajewska et al. (2008) for carrier collaboration. The collaborative problem is modelled as a multi-depot Pickup and Delivery Problem with Time Windows (PDPTW) which merges the requests of all carriers. The problem is optimized for distance-based cost minimization and the profit sharing is solved with the Shapley value. Another collaborative PDP is investigated in Dai and Chen (2012) where both customer sharing and vehicle sharing can be adopted between carriers. Three different methods are studied for the profit sharing method, and each one requires solving a linear program. The first allocation method minimizes the difference between the value of the collaborative (global) solution and the Shapley value of each carrier. The other two methods minimize the difference between the allocation ratios of any pair of carriers, following different proportional rules. In Zibaei et al. (2016), the collaborative problem of customer exchange is formulated as a multi-depot VRP and the profit sharing is solved with different allocation methods already proposed in the literature. Collaboration between a supplier and its customers is examined in Özener et al. (2013), which describes an Inventory Routing Problem (IRP) minimizing the average transportation costs and proposes three allocation methods to solve the profit sharing problem. The first method allocates to each customer a cost proportional to its individual cost. The second allocates the cost on a per-route basis, considering that several customers can be served in the same route, thus considering synergies between customers. The third method allocates the costs to customers based on the solution of the dual problem, and considers the synergy between customers and between routes. In a latter work, Ozener (2014) formulates a collaborative VRP with minimization of travelling and CO_2 emissions costs. The authors apply similar allocation methods presented in prior work, but now distributing both the transportation costs and emissions costs among the customers. Another work that considers environmental issues on both routing and profit sharing problems is described in Pradenas et al. (2013). The authors formulate a collaborative Vehicle Routing Problem with Backhauls and Time Windows (VRPBTW) with minimization of energy consumption, and the total costs are distributed according to the Shapley value. Sanchez et al. (2015) also apply the Shapley value but the collaborative model is formulated as a VRPTW with constraints limiting the carbon footprint of transportation. Wang et al. (2018) study the collaboration of multiple centers (distribution centers (DCs), logistics centers (LCs), depots) in a two-echelon VRP, where the first echelon contains DCs and LCs, and the second echelon contains the DCs and the customers. The problem is solved considering the minimization of costs and CO_2 emissions. Afterwards, the Minimum Cost-Remaining Savings (MCRS) method distributes the costs among the collaborators. This allocation method is based on a bilevel methodology that first allocates a minimum benefit to all participants, and only after it distributes the remaining amount.

From the above literature, the most similar work to ours is the one of Wang et al. (2018).

However, while Wang et al. (2018) model the collaborative problem with a bi-objective function, our work includes two models, one with only economic goals and one with only environmental goals. Therefore, we examine how different formulations of the collaborative model impact on the synergy value. Another distinct feature of our work is that we study three different collaborative strategies that rely on *backhauling*, *capacity sharing* and *cross-docking*, whereas Wang et al. (2018) only focus on customer sharing considering that the demand of the customers of one participant can be satisfied by any other participant. Finally, we study simple proportional allocation methods which are easily adapted to real instances. Differently, Wang et al. (2018) uses an allocation method that may outperform the proportional methods from a theoretical point of view, but which are hard to implement in practice due to the need of solving an optimization for all possible coalitions formed.

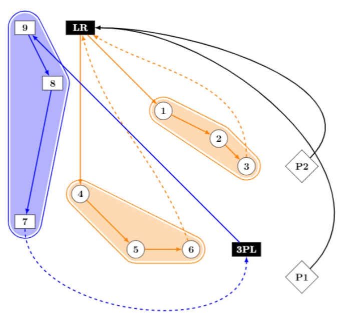
6.3. The collaborative transportation planning

In this section, we detail the problem, and the methodologies involved to solve the routing and the profit sharing problems. First, a description of each collaborative strategy and their requirements is presented. Then, the mathematical models used to formulate the collaborative problems under different contexts (minimization of costs and environmental impacts) are formally defined. Finally, the proportional allocation methods used to solve the profit sharing problem are described.

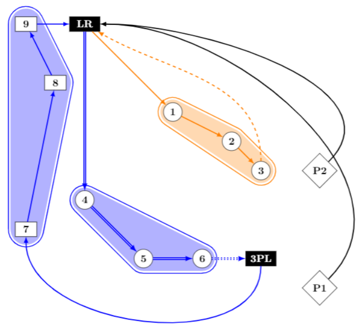
6.3.1 Illustration of collaborative strategies

Traditionally, each participant in the transportation network is responsible for serving its customers, optimizing their routes individually. Figure 6.1a illustrates the case of individual planning, which does not consider collaboration. The LR plans the delivery routes to satisfy the demand of its stores and the 3PL plans the delivery routes to satisfy the demand of its customers. After all deliveries are made, their dedicated vehicles return empty to the respective depot. The producers supply the LR through outsourced vehicles. Because the LR and the 3PL are located in different areas but operate in the same region, there are hidden opportunities to increase the efficiency of the logistics operations through collaboration. Thus, different participants may collaborate through the joint planning of transport operations, implementing one or more collaborative strategies.

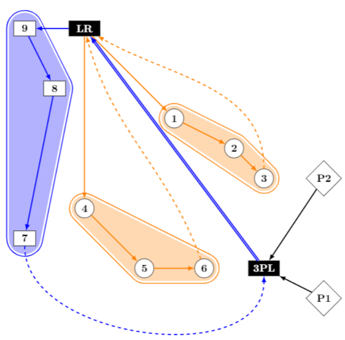
Figure 6.1b illustrates the case of *collaborative pickup-delivery*, which allows vehicles of the 3PL to serve stores of the LR. In this case, after all deliveries made to the 3PL customers, a 3PL vehicle may pickup a set of requests at the depot of the LR, delivering them to stores in the way back to its depot. This collaborative strategy does not include the possibility of vehicles of the RL serving customers of the 3PL, due to practical or market-related reasons (e.g., retailers do not serve other retailers). The collaborative pickup-delivery is particularly important when the stores are located along the backhaul trips of the 3PL, or located near to his depot. Figure 6.1c illustrates the case of *collaborative cross-docking*, which allows the producers to ship their loads to the depot of the 3PL, instead of sending directly to the depot of the RL. These loads can be consolidated in the delivery routes of the 3PL, or can be collected latter by the vehicles of the RL. The *collaborative cross-docking* emerges as a solution to lower the transportation costs of the producers. In particular, if several loads of different producers are consolidated, it might be possible to reach full-truck loads. Figure 6.1d illustrates the case of collaborative collection, which allows the dedicated vehicles of both, the RL and the 3PL, to pickup loads at the producers. This strategy provides a cost reduction for the producers, and allows to reduce empty running of dedicated vehicles.



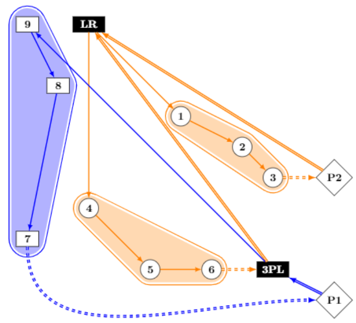
(a) No collaboration



(b) Collaborative pickup-delivery



(c) Collaborative cross-docking



(d) Collaborative collection

Legend: Circles represent stores, squares represent 3PL customers and diamonds represent producers. The blacks rectangles are the depots of LR and 3PL. Blue clusters and orange clusters represent the set of locations visited by 3PL and by LR, respectively. Blue lines and orange lines represent the routes executed by dedicated vehicles of 3PL and vehicles of LR, respectively. Black lines represent routes executed by outsourced vehicles from producers. Full and dotted lines represent loaded and empty vehicles, respectively.

6.3.2 Vehicle routing problem

The transport network is composed of a set of stores $S = \{1..., s\}$, a set of producers $P = \{1..., p\}$, a set of 3PL customers $R = \{1..., r\}$ and the depots of LR and 3PL represented by α and β, respectively. The set of all locations is given by $V = \alpha \cup \beta \cup S \cup P \cup R$. Each store $i \in S$ and 3PL customer $i \in R$ demands quantity q_i to be delivered, whereas each producer $j \in P$ has quantity q_j available to be sent to the LR. The subsets $K^{LR} = \{1, ..., k^{LR}\}$ and $K^{3PL} = \{k^{LR} + 1, ..., k^{LR} + k^{3PL}\}$ represent the limited number of dedicated vehicles of the LR and the 3PL, respectively. The vehicles are homogeneous with capacity C, and the set of all vehicles is given by $K = K^{LR} \cup K^{3PL}$. Each arc $(i, j) \in V$ is associated to an Euclidean distance $d_{ij} = d_{ji}$. The transportation cost with the dedicated fleet is given by c_{ij} for the arc

(i, j), whereas the unitary cost of outsourced vehicles is given by o. The 3PL can cross-dock loads from the producers with an associated cost cd. The decision variables of the problem are as follows:

$$x_{ij}^k := \begin{cases} 1, & \text{if vehicle } k \text{ from LR travels on arc } (i, j) \\ 0, & \text{otherwise} \end{cases} \qquad \forall i, j \in V, \forall k \in K^{LR}$$

$$z_{ij}^k := \begin{cases} 1, & \text{if vehicle } k \text{ from 3PL travels on arc } (i, j) \\ 0, & \text{otherwise} \end{cases} \qquad \forall i, j \in V, \forall k \in K^{3PL}$$

$$w_{ij} := \begin{cases} 1, & \text{if producer } i \text{ sends the load to depot } j \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in P, \forall j = \{\alpha, \beta\}$$

$$t_i^k := \begin{cases} 1, & \text{if cross-docked load from producer } i \text{ is carried by vehicle } k \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in P, \forall k \in K$$

$$y_{ij} := \text{load carried by a vehicle on arc } (i, j), \qquad \forall i, j \in V.$$

6.3.2.1 Objective functions

In this work, three different objective functions are evaluated: i) minimization of operation costs, ii) minimization of fuel consumption, and iii) simultaneous minimization of operation and CO_2 emission costs.

The first objective function $OF(C)$ describes the minimization of the total sum of distance-based and cross-docking costs of the LR, 3PL and producers, as follows:

$$OF(C) = C_{LR} + C_{3PL} + \sum_{i \in P} C_i \tag{6.1}$$

where

$$C_{LR} = \sum_{i \in V} \sum_{j \in V} \sum_{k \in k} x_{ij}^k c_{ij} \tag{6.1.1}$$

$$C_{3PL} = \sum_{i \in V} \sum_{j \in V} \sum_{k \in k} z_{ij}^k c_{ij} + \sum_{i \in P} w_{i\beta} cd_i \tag{6.1.2}$$

$$C_i = w_{ij} q_i d_{ij} o, \qquad \forall i \in P, \forall j \in \{\alpha, \beta\} \tag{6.1.3}$$

The second objective function $OF(E)$ describes the minimization of fuel consumption, which is dependent on the distance travelled and the load carried by all vehicles. We use the Fuel Consumption Rate (FCR) introduced by Xiao et al. (2012) to determine the fuel consumption of each entity in the transportation network. The FCR is influenced by the rate of consumption of the empty vehicle (ρ), the rate of consumption of the full loaded vehicle (ρ^*), the load in the vehicle on the arc (y_{ij}) and the capacity of the vehicle (C). The objective function in (6.2) minimizes the total fuel consumption of all vehicles used by the LR, the 3PL and the producers, as follows:

$$OF(E) = \sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\left(FCR_{ij}^{LR} + FCR_{ij}^{3PL} + FCR_{ij}^{P}\right)d_{ij} \tag{6.2}$$

where

$$FCR_{ij}^{LR} = \rho x_{i,j,k} + \frac{\rho^* - \rho}{C} y_{ij}x_{ij}^k \qquad \forall i \in V, \forall j \in V, \forall k \in K^{LR} \qquad (6.2.1)$$

$$FCR_{ij}^{3PL} = \rho z_{i,j,k} + \frac{\rho^* - \rho}{C} y_{ij}z_{ij}^k \qquad \forall i \in V, \forall j \in V, \forall k \in K^{3PL} \qquad (6.2.2)$$

$$FCR_{ij}^{P} = \rho w_{ij} + \frac{\rho^* - \rho}{C} w_{ij}q_i \qquad \forall i \in P, \forall j \in \{\alpha,\beta\} \qquad (6.2.3)$$

The third objective function $OF(CE)$ in (6.3) minimizes the economic and environmental costs of the network, given by the sum of the costs of travelling, cross-docking and CO_2 emissions. The parameter e represents the unitary cost of CO_2 emissions and the parameter η is used to convert the fuel consumption into the amount of emissions.

$$OF(CE) = OF(C) + e\cdot\eta\, OF(E) \tag{6.3}$$

Note that the right hand side of equations (6.2.1) and (6.2.2) is non-linear. To linearize them, we first define the auxiliary variables $A_{ij}^k = y_{ij}x_{ij}^k$ and $B_{ij}^k = y_{ij}z_{ij}^k$, and then apply a Big-M reformulation with M as a large number, as follows.

$$A_{ij}^k \le Mx_{ij}^k, \qquad \forall i \in V, \forall j \in V, \forall k \in K \qquad (6.4.1)$$

$$A_{ij}^k \le y_{ij}, \qquad \forall i \in V, \forall j \in V, \forall k \in K \qquad (6.4.2)$$

$$A_{ij}^k \ge y_{ij} - (1 - x_{ij}^k)M, \qquad \forall i \in V, \forall j \in V, \forall k \in K \qquad (6.4.3)$$

$$A_{ij}^k \ge 0, \qquad \forall i \in V, \forall j \in V, \forall k \in K \qquad (6.4.4)$$

$$B_{ij}^k \le Mz_{ij}^k, \qquad \forall i \in V, \forall j \in V, \forall k \in K \qquad (6.4.5)$$

$$B_{ij}^k \le y_{ij}, \qquad \forall i \in V, \forall j \in V, \forall k \in K \qquad (6.4.6)$$

$$B_{ij}^k \ge y_{ij} - (1 - z_{ij}^k)M, \qquad \forall i \in V, \forall j \in V, \forall k \in K \qquad (6.4.7)$$

$$B_{ij}^k \ge 0, \qquad \forall i \in V, \forall j \in V, \forall k \in K \qquad (6.4.8)$$

6.3.2.2 Constraints

In this section, we first present the constraints associated with the stores, followed sequentially by those associated with the 3PL customers, producers, and vehicles. The last set of constraints describe precedence requirements in the transportation network, which are typical constraints of the Vehicle Routing Problem with Backhauls (VRPB).
Constraints (6.5) impose that each store is visited exactly once. The flow conservation is defined by constraints (6.6). Constraints (6.7) and (6.8) force the vehicles of the LR to depart from its depot. Constraints (6.9) ensures that the total total demand of the stores is satisfied, and constraints (6.10) impose that the load carried by each vehicle decreases along the visits to the stores.

$$\sum_{i\in V}\sum_{k\in K}\left(x_{ij}^k + z_{ij}^k\right) = 1, \qquad \forall j \in S \tag{6.5}$$

$$\sum_{i\in V} x_{ij}^k = \sum_{l\in V} x_{jl}^k, \qquad \forall j \in V, \forall k \in K \tag{6.6}$$

$$\sum_{i\in S} x_{\alpha i}^k \leq 1, \qquad \forall k \in K \tag{6.7}$$

$$\sum_{i\in S} x_{\alpha i}^k \geq x_{jl}^k, \qquad \forall j \in V, \forall l \in V, \forall k \in K \tag{6.8}$$

$$\sum_{j\in S} y_{\alpha j} = \sum_{j\in S} q_j \tag{6.9}$$

$$\sum_{i\in V} y_{ij} = \sum_{i\in V} y_{ji} + q_j, \qquad \forall j \in S \tag{6.10}$$

Constraints (6.11) and (6.12) impose that each 3PL customer is visited exactly once by vehicles of the 3PL only. The flow conservation is ensured by Constraints (6.13). Constraints (6.14) and (6.15) force the vehicles of the 3PL to depart from its depot. Constraint (6.16) ensures that vehicles departing from the depot of the 3PL carry all loads necessary to attend the demand of its customers, as well as the cross-docked loads from producers. Constraints (6.17) impose that the load carried by a vehicle decreases along the visits to 3PL customers.

$$\sum_{i\in V}\sum_{k\in K} z_{ij}^k = 1, \qquad \forall j \in R \tag{6.11}$$

$$\sum_{i\in V}\sum_{k\in K} x_{ij}^k = 0, \qquad \forall j \in R \tag{6.12}$$

$$\sum_{i\in V} z_{ij}^k = \sum_{l\in V} z_{jl}^k, \qquad \forall j \in V, \forall k \in K \tag{6.13}$$

$$\sum_{j\in R} z_{\beta j}^k \leq 1, \qquad \forall k \in K \tag{6.14}$$

$$\sum_{i\in R} z_{\beta i}^k \geq z_{jl}^k, \qquad \forall j \in V, \forall l \in V, \forall k \in K \tag{6.15}$$

$$\sum_{j \in V} y_{\beta j} = \sum_{i \in R} q_i + \sum_{i \in P} q_i w_{i\beta} + \sum_{i \in P} \sum_{k \in K} q_i x^k_{i\beta}, \tag{6.16}$$

$$\sum_{i \in V} y_{ij} \geq \sum_{i \in V} y_{ji} + q_j, \qquad \forall j \in R \tag{6.17}$$

Each producer can either send loads to the depot of the LR or to the depot of the 3PL, or wait for the backhauling of the LR or the 3PL, which is guaranteed by Constraints (6.18). Constraint (6.19) guarantees that the total load arriving at the depot of the LR corresponds, at least, to the total load sent directly by the producers and the total load collected in backhaul routes. If a producer sends loads to the depot of the 3PL, either vehicles of the LR pickup them in backhaul trips or the 3PL send them in the delivery routes, as enforced by Constraints (6.20). However, if the 3PL performs pickups at producers, the LR must collect these at the depot of the 3PL, which is enforced by Constraints (6.21). Furthermore, Constraints (6.22) ensures that vehicles of the LR only visit the depot of 3PL if there are cross-docked loads. Also, mixed visits between 3PL customers and the depot of LR are only allowed if there are cross-docked loads, as guaranteed by Constraints (6.23). Constraints (6.24) ensures that the load in the vehicle increases while collecting loads at the producers. The pair of Constraints (6.25) and (6.26) ensure that cross-docked loads are not split when consolidated in backhaul routes for the LR or in delivery routes to 3PL customers.

$$w_{i\alpha} + w_{i\beta} + \sum_{j \in V} \sum_{k \in K} \left(x^k_{ji} + z^k_{ji} \right) = 1, \qquad \forall i \in P \tag{6.18}$$

$$\sum_{i \in V} y_{i\alpha} + \sum_{i \in P} q_i w_{i\alpha} \geq \sum_{i \in P} q_i, \tag{6.19}$$

$$w_{i\beta} \leq \sum_{k \in K} \left(x^k_{\beta\alpha} + \sum_{j \in P} x^k_{\beta j} + z^k_{\beta\alpha} + \sum_{j \in R} z^k_{j\alpha} \right), \qquad \forall i \in P \tag{6.20}$$

$$\sum_{k \in K} z^k_{i\beta} \leq \sum_{k \in K} \left(x^k_{\beta\alpha} + \sum_{j \in P} x^k_{\beta j} \right), \qquad \forall i \in P \tag{6.21}$$

$$\sum_{i \in P} \sum_{j \in P} \left(z^k_{i\beta} + z^k_{ij} + w_{i\beta} \right) \geq \sum_{j \in V} x^k_{\beta j}. \qquad \forall k \in K \tag{6.22}$$

$$\sum_{i \in R} z^k_{\alpha i} \leq \sum_{i \in P} w_{i\beta}, \qquad \forall k \in K \tag{6.23}$$

$$\sum_{i \in V} y_{ji} = \sum_{i \in V} y_{ij} + q_j \sum_{i \in V} \sum_{k \in K} \left(x^k_{ij} + z^k_{ij} \right), \qquad \forall j \in P \tag{6.24}$$

$$A^k_{\beta\alpha} = \sum_{i \in P} t^k_i q_i, \qquad \forall k \in K \tag{6.25}$$

$$\sum_{j \in V} B^k_{\beta j} = \sum_{i \in R} \sum_{j \in V} z^k_{ij} q_i + \sum_{i \in P} t^k_i, \qquad \forall k \in K \tag{6.26}$$

Constraints (6.27) ensure that the same vehicle is either used by the LR or the 3PL, but not both. Constraints (6.28) ensure that the capacity of the vehicles is never exceeded in any arc of the routes.

$$\sum_{i\in V}\left(x^k_{\alpha i}+z^k_{\phi i}\right)\leq 1, \qquad \forall k\in K \qquad (6.27)$$

$$y_{ij}\leq \sum_{k\in K}\left(x^k_{ij}+z^k_{ij}\right)C, \qquad \forall i,j\in V \qquad (6.28)$$

The precedence of deliveries before pickups is ensured by Constraints (6.29) for LR vehicles, and by Constraints (6.30) for 3PL vehicles. Additional precedence constraints are necessary for the vehicles of the 3PL to forbid mixed visits between stores, 3PL customers and suppliers, ensured by Constraints (6.31) to (6.33).

$$x^k_{ji}+x^k_{\beta i}=0, \qquad \forall i\in S, j\in P, k\in K \qquad (6.29)$$

$$z^k_{ji}+z^k_{j\alpha}=0, \qquad \forall i\in S, j\in P, k\in K \qquad (6.30)$$

$$z^k_{i\alpha}+z^k_{\beta i}=0, \qquad \forall i\in S, k\in K \qquad (6.31)$$

$$z^k_{ij}+z^k_{ji}=0, \qquad \forall i\in S, j\in R, k\in K \qquad (6.32)$$

$$z^k_{ij}=0, \qquad \forall i\in P, j\in R, k\in K \qquad (6.33)$$

6.3.3 Profit sharing problem

After solving both the stand alone problem (Appendix 6.A) and the collaborative problem (described in Section 6.3.2), the global gains provided by the collaboration must be shared among the participants. Due to the simplicity of application and practical use in real contexts, we investigate proportional allocation methods to solve the profit sharing problem. Three allocation methods are collected from Lunday and Robbins (2019). The first allocation method (M1) distributes the collaborative gains equally among the participants. The second allocation method (M2) first applies the principle of M1, and then reimburses the potential losses of participants. The third allocation method (M3) follows an inverse rationale of that of M2, first reimbursing the losses and only after distributing the remaining gains equally.

The following numerical example demonstrates the principles of each proportional method. Consider that 4 participants are enrolled in the collaboration - the LR, the 3PL and two producers (P1 and P2). Table 6.1 presents the individual cost of each participant, s^0, in the stand alone solution and its difference, Δ^*, compared with the collaborative solution. It also shows the allocated share, a, of each participant obtained with each allocation method, along with the total cost after receiving the allocated gain, s^1. The total gains is given by the sum of the positive values only, i.e. $40+8+14=62$.

In this example, it is possible to observe that an equal distribution of gains is not a good approach to all participants. The total cost of the 3PL would be higher than its stand alone cost, which puts in evidence that the criteria of individual rationality is not met if the allocation method M1 applies. Using the method M2, the gains allocated to the 3PL would only be sufficient to cover the additional costs and, as such, this participant would not

Table 6.1 – Numerical example

			M1		M2		M3	
	s^0	Δ^*	a	s^1	a	s^1	a	s^1
LR	100	40	15.5	84.5	14.0	86.0	10.5	89.5
3PL	80	-20	15.5	84.5	20.0	80.0	30.5	69.5
P1	20	8	15.5	4.5	14.0	6.0	10.5	9.5
P2	14	14	15.5	-1.5	14.0	0.0	10.5	3.5

receive any benefit compared with the stand alone solution. For this reason, the 3PL would not be interested in participating in the collaboration. The method M3 allocates the gains among participants such that the individual rationality is met, ensuring that all participants are allocated with a total cost lower than its stand alone cost. For these reasons, the method M3 dominates the other two methods in terms of fairness, which is also supported by the results of Lunday and Robbins (2019).

6.4. Computational experiments

The computational experiments performed in this work have two main focus. The first is to analyse the performance of the collaboration under different objectives (economic and environmental) for various scenarios. The second research focus is to compare different proportional allocation methods to solve the profit sharing problem based on some fairness criteria. The ultimate goal of the computational experiments is to characterize the potential of collaboration between the different participants in the case-study, which will allow to draw a set of managerial insights.

The models are coded in Python 3.6.3 and solved with Gurobi on a computer equipped with the processor Intel Core i7 and 16GB of RAM. The computational experiments are applied to randomly generated instances from Solomon (1987) and to a case study instance.

6.4.1 Instances

The randomly generated instances depart from the data sets of 25 locations in Solomon (1987), respectively for a cluster (C), random (R) and random cluster (RC) network. For each type of network, 10 instances are created with 15 random locations and 2 fixed locations. The former represents the total number of stores, 3PL customers and producers, and the latter represents the depots of the LR and the 3PL. These instances are then divided in two 5-instance data sets designated as s8-r5-p2 (8 stores, 5 3PL customers and 2 producers) and s10-r3-p2 (10 stores, 3 3PL customers and 2 producers), respectively. In total, 30 different random instances are created. The demand of each store and 3PL customer are randomly generated from an uniform distribution [2,5]. The ordering load from the LR to each producer is set at 5. The number of dedicated vehicles available is 5, with a capacity of 15. In the case study instance, the network is built based on real locations and distances taken from the real context of application, which is detailed further in this section.

The remaining inputs of the models are considered in both randomly generated and case study instances. The economic parameters are taken from the real case study, as follows. The travelling cost of a dedicated vehicle is 0.8€ per unit of distance. The cost of an outsourced vehicle is 0.09€ per unit of distance and unit of load. The cross-docking cost is 1.3€ per unit of load. The parameters necessary to determine the FCR are set according to Cheng et al. (2018), namely $\rho^* = 0.296$ and $\rho = 0.39$. The parameters associated with the CO_2 emissions follow the convention that one liter of fuel generates 2.32 kg of CO_2 per liter of fuel and the cost of CO_2 emissions is 37.2€ per ton (Bektaş and Laporte, 2011).

6.4.2 Analysis of collaborative solutions

Each instance was tested with the three different objective functions, for both the collaborative and the non-collaborative models. The average results obtained for data sets s8-r5-p2 and s10-r3-p2 are presented in Tables 6.2 and 6.3, respectively. Each one reports the average operational cost and average fuel consumption of the stand alone (s^0) and the collaborative (s^1) global solutions, the standard deviations (sd) of such solutions and the synergy values (sv) achieved in terms of operational costs and fuel consumption.

The results demonstrate that the synergy value is influenced by the type of network. In particular, type RC leads to the highest synergy values both in terms of cost and fuel consumption, while type R leads to the lowest. Furthermore, these results are consistent among the different objective functions and across the different data sets. The leverage of RC networks comes from the fact that customers and producers are more distant to the depots than in the other types of network, which leads to higher total costs and fuel consumption in the stand alone solutions. However, in full collaboration, the cost and fuel reductions are usually higher for the RC networks than networks C and R, and consequently, the synergy values achieved are also higher. It is also expected that full collaboration leads to higher synergy values than the cases of limited collaboration (e.g., when only two strategies can be applied). Nonetheless, limiting the type of strategies that can be selected may reduce the leverage of RC networks, as documented in Appendix 6.B.

The holistic model provides very similar solutions than those obtained with the objective function OF(C), either for the stand alone and the collaborative solutions. This occurs because the cost of CO_2 emissions is much smaller than the operational costs. The results show that using the objective OF(CE) allows to modify the routing plan in order to reduce fuel consumption, only as long as the cost of the economic solution does not increase. Also, as expected, solutions obtained with the environmental objective OF(E) tend to prioritize fuel reduction in detriment of the costs. Nevertheless, the relative difference between costs obtained with OF(E) and OF(C) are very small.

A closer look to the collaborative solutions reported in Tables 6.2 and 6.3 allows to identify and compare the strategies that promote deeper collaboration, under different contexts. Following this reasoning, Tables 6.4 and 6.5 present the number of times each collaborative strategy was selected in each 5-instance data set in the three types of network, and for each objective function.

The collaborative pickup-delivery, which allows the 3PL to visit RL customers, seems to influence heavily the collaboration, as it is presented in almost all solutions independent on the objective function and type of network considered. The collaborative collection, which

Table 6.2 – Average results obtained with data set s8-r5-p2

		Network type C				Network type R				Network type RC			
		Cost	*sd*	Fuel	*sd*	Cost	*sd*	Fuel	*sd*	Cost	*sd*	Fuel	*sd*
$OF(C)$	s^0	243.6	26.8	108.0	11.1	320.2	28.3	141.1	11.0	401.1	13.6	178.2	5.9
	s^1	214.7	20.8	93.4	6.8	293.8	21.2	129.2	10.2	354.1	21.5	151.7	12.4
	sv	12%	3%	13%	4%	8%	2%	8%	2%	12%	3%	15%	5%
$OF(E)$	s^0	244.3	27.4	107.0	11.3	321.1	28.2	138.1	10.9	401.6	13.0	174.4	6.5
	s^1	214.9	20.7	93.3	6.8	298.0	21.7	125.7	8.6	357.9	25.7	150.7	11.5
	sv	12%	3%	12%	4%	7%	2%	9%	2%	11%	5%	14%	4%
$OF(CE)$	s^0	243.6	26.8	107.1	11.2	320.2	28.3	138.3	10.9	401.1	13.6	174.9	5.7
	s^1	214.7	20.8	93.3	6.9	293.8	21.2	128.4	10.4	354.1	21.5	151.5	12.3
	sv	12%	3%	13%	4%	8%	2%	7%	1%	12%	3%	13%	5%

Table 6.3 – Average results obtained with data set s10-r3-p2

		Network type C				Network type R				Network type RC			
		Cost	*sd*	Fuel	*sd*	Cost	*sd*	Fuel	*sd*	Cost	*sd*	Fuel	*sd*
$OF(C)$	s^0	229.4	31.6	101.4	14.4	329.7	24.4	143.6	11.2	390.8	21.5	172.1	9.6
	s^1	205.8	20.9	87.9	8.4	300.8	19.2	129.6	9.1	341.5	24.0	144.4	10.3
	sv	10%	4%	13%	5%	9%	1%	10%	1%	13%	2%	16%	2%
$OF(E)$	s^0	229.7	31.8	100.7	14.1	329.9	24.4	142.1	10.3	391.2	22.3	169.6	7.0
	s^1	207.4	21.6	87.3	8.3	305.1	21.5	127.8	8.2	344.7	28.0	142.7	9.2
	sv	9%	5%	13%	4%	7%	1%	10%	1%	12%	2%	16%	2%
$OF(CE)$	s^0	229.4	31.6	100.8	14.2	329.7	24.4	142.2	10.3	390.8	21.5	170.1	8.0
	s^1	205.8	20.9	87.8	8.4	300.8	19.2	129.4	9.1	341.5	24.0	144.3	10.3
	sv	10%	4%	12%	5%	9%	1%	9%	1%	13%	2%	15%	3%

allows dedicated vehicles to perform pickups at producers, is presented mostly in the solutions obtained with the fuel minimization model. These results indicate that collaborative collection is effective in reducing fuel consumption, but possibly at a relatively high cost. The collaborative cross-docking is never presented in any solution. A possible obstacle to implement the collaborative cross-docking may be the high cost of cross-docking compared with the travelling cost of an outsourced vehicle, for small loads, or the relatively high distance between producers and 3PL depot.
The results in Tables 6.4 and 6.5 also show that the network type RC creates the solutions with the higher total number of collaborative strategies, followed by the network type C and then type R. Based on these results, we can argue that, in general, the more strategies are implemented, the higher is the impact on collaboration, and thus the higher is the synergy value. Note that in RC networks, the number of collaborative collections is higher than in networks C and R. This shows that collaborative collections are leveraged by RC networks. However, if this strategy is not allowed, the advantage of RC networks is reduced comparing to networks C and R, as reported in Appendix 6.B.

Table 6.4 – Number of collaborative strategies used in solutions from the data set s8-r5-p2

	Network type C			Network type R			Network type RC			
OF	(C)	(E)	(CE)	(C)	(E)	(CE)	(C)	(E)	(CE)	Total
Collection	4	4	4	1	6	1	6	8	6	40
Pickup-delivery	5	5	5	5	5	5	5	5	5	45
Cross-docking	0	0	0	0	0	0	0	0	0	0
Total	9	9	9	6	11	6	11	13	11	

Table 6.5 – Number of collaborative strategies used in solutions from the data set s10-r3-p2

	Network type C			Network type R			Network type RC			
OF	(C)	(E)	(CE)	(C)	(E)	(CE)	(C)	(E)	(CE)	Total
Collection	7	9	7	4	8	4	8	10	8	65
Pickup-delivery	5	4	5	5	5	5	5	5	5	44
Cross-docking	0	0	0	0	0	0	0	0	0	0
Total	12	13	12	9	13	9	13	15	13	

6.4.3 Sensitivity analysis

As evidenced by the previous results, the collaborative pickup-delivery is adopted in almost all solutions, but the collaborative cross-docking was never selected. Thus, with the goal of exploring situations that may benefit cross-docking and determine the respective synergy value, we perform several sensitivity analysis to different parameters involved in the routing problem. These parameters comprise the geographic dispersion of the producers around the 3PL depot, the quantity ordered to producers, the cost of cross-docking and the cost of outsourcing.

The differences between the base scenario and the scenarios used in the sensitivity analysis are provided in Table 6.6. The first scenario considers that in each 5-instance data set the locations of the different producers are close to the depot of the 3PL. The second scenario considers that the LR orders to each producer a quantity that matches the capacity of the dedicated vehicles. The third scenario assumes that the cost of cross-docking is halved. The last scenario assumes that the outsourcing cost is 0.3 per unit of load and unit of distance, which is a common price in practice charged by outsourced carriers to transport small loads.

Table 6.6 – Parameter settings of each scenario

	Base scenario	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Location of producer $p \in P$	random	fixed	random	random	random
Ordering quantity (q_p)	5	5	15	5	5
Cost of cross-docking (cd)	1.3	1.3	1.3	0.65	1.3
Cost of outsourcing (o)	0.09	0.09	0.09	0.09	0.3

The results from the sensitivity analysis to each scenario are presented in Table 6.7, which provides the number of collaborative strategies implemented and the synergy values in terms of operational costs and fuel consumption. As the solutions obtained with the holistic model are the same as those obtained with the economic model, we only present the sensitivity analysis for the objectives OF(C) and OF(E). Furthermore, as similar findings are obtained for both data sets, this section only presents the results for the data set s10-r3-p2. Finally, the results obtained with the scenarios 3 and 4 are the same as the ones obtained in the base scenario, for the respective objective function. As such, Table 6.7 only reports evidences from scenarios 1 and 2.

If the economic model is used in scenario 1, the number of strategies selected, particularly collaborative collections, is reduced comparing to the base scenario. On the other hand, using the environmental model tends to promote both collaborative collections and cross-docking, but leads to lower synergy values.

In scenario 2, there is a general trend to promote collaborative collections with all producers if the economic model is used. Otherwise, the number and type of collaborative strategies implemented is the same as in the base scenario. Nonetheless, in both cases, the synergy values tend to increase, as a result of the larger reduction of costs and fuel consumption at the producers.

In order to further explore potentially beneficial situations for collaborative cross-docking, sensitivity analysis to combined scenarios are also performed, namely by combining scenario 1 with one of the other scenarios. For this purpose, only the economic model is used, since the solutions obtained with the environmental model, in terms of collaborative strategies, are not affected by the cost parameters nor the ordering quantity, as demonstrated previously. The results from the sensitivity analysis to combined scenarios is presented in Table 6.8. It can be observed that collaborative cross-docking is heavily influenced by the costs parameters when the producers are located close to the 3PL depot. When the cost of cross-docking reduces to half (combined scenarios 1 + 3), the number of collaborative cross-docking strategies increases, independently on the number of collaborative

Table 6.7 – Sensitivity analysis to different scenarios and objective functions

		Base scenario			Scenario 1			Scenario 2		
		C	R	RC	C	R	RC	C	R	RC
$OF(C)$	Collection	7	4	8	0	2	1	10	10	10
	Pickup-delivery	5	5	5	5	5	5	3	5	5
	Cross-docking	0	0	0	0	0	0	0	0	0
	sv^{cost}	10%	9%	13%	6%	8%	8%	21%	17%	25%
	sv^{fuel}	13%	10%	16%	6%	8%	8%	12%	10%	17%
$OF(E)$	Collection	9	8	10	5	5	5	9	8	10
	Pickup-delivery	4	5	5	4	4	5	4	5	5
	Cross-docking	0	0	0	5	5	5	0	0	0
	sv^{cost}	9%	7%	12%	3%	7%	6%	21%	16%	25%
	sv^{fuel}	13%	10%	16%	12%	12%	9%	12%	10%	16%

collections previously established in scenario 1, except for network type C. Increasing the outsourcing cost (combined scenario 1 + 4) leads to the highest total number of collaborative strategies adopted. In particular, cross-docking is used in the majority of the solutions obtained for all types of network. Furthermore, the synergy values of both costs and fuel consumption increase substantially comparing to all other scenarios.

Table 6.8 – Sensitivity analysis to combined scenarios

	Scenarios 1 + 2			Scenarios 1 + 3			Scenario 1 + 4		
	C	R	RC	C	R	RC	C	R	RC
Collection	10	9	7	0	4	1	5	5	6
Pickup-delivery	2	4	4	5	5	5	4	5	5
Cross-docking	0	0	0	0	4	1	5	5	4
sv^{cost}	18%	16%	11%	6%	8%	8%	24%	20%	13%
sv^{fuel}	5%	8%	8%	6%	12%	9%	13%	13%	10%

6.4.4 Performance of allocation methods

In this section, we analyze the impact of each allocation method on the cost reduction (gains) of each participant in the collaboration. In particular, we evaluate the performance of the different proportional allocation methods and compare the individual gains achieved by all entities in different collaborative solutions.

Table 6.9 presents the results obtained with each allocation method for different solutions, namely the total allocated cost of each participant after the profit distribution (s^1) and the percentage cost reduction (g) in comparison with its stand alone solution (s^0). Five different collaborative solutions obtained with data set s10-r3-p2 are selected for this analysis: a) a solution with only collaborative pickup-delivery (from base scenario, network type R), b) a solution with only collaborative collection (from base scenario, network type C), c) a solution with both collaborative pickup-delivery and collection (from base scenario, network type RC), d) a solution with both collaborative collection and cross-docking and e) a solution with all three collaborative strategies (both from combined scenarios 1 + 4, network type C).

Table 6.9 – Percentage gains of each entity obtained with each allocation method, for different collaborative solutions

			M1			M2		M3	
			s^0	s^1	g (%)	s^1	g (%)	s^1	g (%)
a)	Only pickup-delivery	LR	212	177	17%	180	15%	196	7%
		3PL	97	101	-4%	97	0%	81	16%
b)	Only collection	LR	122	125	-3%	122	0%	119	2%
		P1	8	2	70%	4	51%	5	34%
		P2	9	3	63%	5	46%	6	31%
c)	Pickup-delivery and collection	LR	214	190	11%	196	8%	201	6%
		3PL	142	160	-13%	142	0%	128	10%
		P1	15	-9	160%	-3	120%	1	90%
		P2	18	-6	134%	0	101%	4	76%
d)	Collection and cross-docking	LR	124	126	-1%	124	0%	108	13%
		3PL	68	50	26%	51	25%	52	24%
		P1	50	26	48%	27	47%	34	32%
		P2	55	30	45%	31	43%	38	30%
e)	Pickup-delivery, collection and cross-docking	LR	138	110	20%	111	20%	118	15%
		3PL	90	92	-3%	90	0%	69	23%
		P1	50	22	56%	23	55%	30	41%
		P2	55	26	52%	27	50%	34	38%

The results in Table 6.9 put in evidence that both allocation methods M1 and M2 are not suitable to benefit all participants, for any of the solutions presented. Method M1 always lead to losses for the 3PL, while method M2 only compensates these losses. Both methods would encourage the 3PL to leave the collaboration. In opposition, as expected, method M3 provides an allocated cost that fulfils the fairness properties of efficiency and individual rationality.

Another evidence of these results in the base scenario is the large percentage gains of the producers when compared with the RL and the 3PL. This occurs because the costs of the producers, even all together, represent a small portion of the total costs of the network (e.g., solutions b) and c)). However, as the costs of producers increase, the individual percentage gains seem to be more balanced between the different entities participating in the collaboration (e.g., solutions d) and e)).

6.4.5 Case-study

In this section, we solve and analyze the routing and the profit sharing problems for a case-study instance. The instance presents a transportation network for a typical day of the week for both LR and 3PL. The network is composed of 20 stores (numbered from 0 to 20), six 3PL customers (numbered from 21 to 26) and two producers (numbered 27 and 28). The demand from stores range between 2 and 10 pallets and from 3PL customers range between 1 and 18. Each dedicated vehicle has a standard capacity of 33. Each producer has a load of 10 pallets to send to the LR.

Table 6.10 presents the set of routes created with the stand alone and the collaborative solutions, for different objective functions. The operational cost and fuel consumption of each solution, as well as the synergy of collaboration, are presented in Table 6.11. One first evidence of these results is that, independently of the objective function used, only three vehicles are necessary to perform the transportation services in a collaborative network, whereas the stand alone solutions always require five vehicles (three for the deliveries to stores and the other two for the deliveries to 3PL customers). As expected, the routes created with the holistic model share many similarities with the routes created with the economic objective function. In both cases, the collaborative solution implements the three collaborative strategies.
Table 6.10 also shows that the collaborative solution obtained with the environmental objective function does not implement collections, which contrasts with the results obtained with the randomly generated instances. In the case study instance, the producers are located much closer to the depot of the 3PL than the depot of the RL and the stores, similarly to scenario 1. From the sensitivity analysis, it was determined that, in fact, solutions from scenario 1 generate lower number of collaborative collections when compared with the base scenario. However, these are compensated by an increase of cross-docking strategies, which is not verified in the case study instance.
In addition, Table 6.11 shows that high synergy values can be obtained with collaboration. In particular, collaboration can reduce the operational costs in 28% and the fuel consumption in, at least, 26%, independently of the objective function used to model the routing problem. Moreover, the results suggest that an economic objective function is sufficient to assess the collaborative potential of the network, instead of the holistic model.

Table 6.10 – Routes created in stand alone and collaborative solutions for each objective function

OF	Stand alone solution (s^0)	Collaborative solution (s^1)
(C)	LR - 1 - 18 - 19 - 16 - LR	LR - 6 - 13 - 11 - 4 - 3PL - 27 - LR
	LR - 6 - 20 - 9 - 10 - 17 - 14 - 7 - 8 - 3 - 12 - LR	
	LR - 13 - 11 - 4 - 5 - 15 - 2 - LR	
	3PL - 26 - 21 - 25 - 23 - 3PL	3PL - 26 - 21 - 23 - LR - 1 - 18 - 14 - 17 - 10 - 7 - 9 - 20 - 5 - 3PL
	3PL - 24 - 22 - 3PL	3PL - 22 - 24 - 25 - LR - 16 - 12 - 19 - 3 - 8 - 2 - 15 - 3PL
(E)	LR - 2 - 15 - 13 - 5 - 4 - 11 - LR	LR - 16 - 19 - 1 - 18 - LR
	LR - 12 - 3 - 8 - 14 - 17 - 10 - 7 - 9 - 20 - 6 - LR	
	LR - 16 - 19 - 1 - 18 - LR	
	3PL - 26 - 21 - 23 - 25 - 3PL	3PL - 26 - 21 - 23 - LR - 6 -2 - 13 - 5 - 4 - 11 - 3PL
	3PL - 24 - 22 - 3PL	3PL - 22 - 24 - 25 - LR - 12 - 3 - 8 - 14 - 17 - 10 - 7 - 9 - 20 - 15 - 3PL
(CE)	LR - 1 - 18 - 19 - 16 - LR	LR - 16 - 19 - 3 - 8- 7 -2 - 15 - 3PL - 27 - LR
	LR - 2 - 13 - 11 - 4 - 5 - 15 - LR	
	LR - 6 - 20 - 9 - 10 - 17 - 14 - 7 - 8 - 3 - 12 - LR	
	3PL - 26 - 21 - 23 - 25 - 3PL	3PL - 26 - 21 - 23 - LR - 1 - 18 - 14 - 17 - 10 - 7 - 9 - 20 - 5 - 3PL
	3PL - 24 - 22 - 3PL	3PL - 22 - 24 - 25 - LR - 12 - 6 - 13 - 11 - 4 - 3PL

The results obtained for the profit sharing problem with the three allocation methods are presented in Table 6.12. As the solution obtained with the environmental objective function only considers collaboration between the LR and the 3PL through pickup-delivery strategies, the producers do not receive any profit. Thus, since the profits are only shared between the two, the cost reduction is much more pronounced than in the case when all

Table 6.11 – Synergy values of collaborative solutions obtained with each objective function

	(C)		(E)		(CE)	
	Cost	Fuel	Cost	Fuel	Cost	Fuel
s^0	1895	780	1900	760	1895	772
s^1	1364	570	1367	562	1364	570
sv	28%	27%	28%	26%	28%	26%

entities participate in the collaboration. In fact, if the producers also participate in the collaboration, the allocated share of profits is reduced by half for the LR and the 3PL. This may raise some concerns among the participants in collaboration. On the one hand, the LR and the 3PL may not be willing to give up half of the profits, and on the other, the producers are certainly open to collaborate because it can only reduce their costs.

Therefore, one possible way to overcome this limitation may be to define a cap for the share profit of the producers based on the carried and consolidated load quantities. For example, a new rule of the allocation method could ensure that a producer would not receive a profit share higher than the share it would receive by providing full truck loads. In order to explore this setting, we perform an additional set of experiments in the case study instance considering that the loads at each supplier match the capacity of the dedicated vehicles (i.e. $q_{i \in P} = C = 33$). The results obtained with each allocation method are reported in Table 6.13. It is possible to observe that producers providing full truck loads receive a profit share such that it does not impact the profit shares of the LR and the 3PL, if the allocation method M3 is applied. In fact, the cost reduction of the LR and the 3PL is kept at 33% and 35% respectively, with or without collaborating producers, and the producers can reduce their costs in 61% and 44% respectively if they enter the collaboration.

Based on these results, we can argue that limiting the profit share of participants that have relatively low contribution to the total costs (and/or environmental impact) of the network, such as the producers, may be a good rule to promote the willingness to enter the collaboration. Thus, combining this rule with method M3 seems to be a suitable strategy to handle the limitation of overcompensating some participants while attending desirable fairness criteria.

Table 6.12 – Allocated costs and percentage gains obtained with each allocation method

			M1		M2		M3	
		s^0	s^1	g	s^1	g	s^1	g
$OF(C)$	LR	806	630	22%	630	22%	674	16%
	3PL	764	762	0%	762	0%	631	17%
	P1	136	-41	130%	-41	130%	3	98%
	P2	189	12	91%	12	93%	56	70%
$OF(E)$	LR	811	448	45%	448	45%	545	33%
	3PL	764	595	22%	595	22%	498	35%

Table 6.13 – Performance of the allocation methods for the case where producers provide full truck loads

			M1		M2		M3	
		s^0	s^1	g	s^1	g	s^1	g
$OF(C)$	LR	806	531	37%	531	37%	562	33%
	3PL	764	605	23%	605	23%	512	35%
	P1	448	144	68%	144	68%	175	61%
	P2	624	321	49%	321	49%	352	44%
$OF(E)$	LR	811	448	45%	448	45%	545	33%
	3PL	764	595	22%	595	22%	498	35%

6.5. Conclusions and future work

This work investigates a case of lateral collaboration for the transportation planning problem of a network involving a LR, a 3PL and several producers. Three different strategies can be applied in the collaborative network, namely pickup-delivery, collection and cross-docking. The collaborative vehicle routing problem is formulated based on the Vehicle Routing Problem with Backhauls (VRPB) under three different objective functions: minimization of operational costs, minimization of fuel consumption, and minimization of operational and CO_2 emissions costs. Three different proportional methods are applied to solve the profit sharing problem. Method M1 distributes the collaborative gains equally among the participants. Method M2 first distributes the gains equally and then reimburses participants for their possible losses. Method M3 first reimburses the participants with losses and the remaining gains are then distributed equally among the participants.

The results of this work indicate that the collaborative pickup-delivery is the most implemented strategy in collaboration and tends to be independent of the objective function used and type of network. The collaborative cross-docking is favoured when the producers are located closer to the depot of the 3PL than the depot of the LR, particularly when an economic objective function drives the optimization problem. On the other hand, the collaborative collection is favoured when producers are dispersed in the network (in particular when using an environmental objective function) and the ordering quantities are relatively high. The type of network RC tends to lead to higher synergy values than network types C and R because it is more prone to incorporate all types of collaborative strategies in a single solution. However, the leverage of RC may be reduced if collaborative backhauling is not allowed. Another insight of this work is that using a holistic model that minimizes both the operational and CO_2 emissions costs leads to similar solutions obtained with the economic model. Thus, it is possible to examine both the economic and environmental impact using only a traditional economic model, and thus avoiding the increased complexity of modelling the fuel consumption of vehicles in the transportation planning problem. For the case study instance, the synergy value is relatively high both in terms of costs and environmental impact. In particular, the results show synergies over 26% for both objective terms. Moreover, among the methods used in the profit sharing problem, only method M3 is able to meet both fairness properties of efficiency and individual rationality. Nonetheless, it may

be necessary to adjust the allocated shares of participants that provide a low contribution to the collaboration, such that participants with higher contributions do not perceive to be more harmed if entering the grand coalition than a sub-coalition.
This work contributes to the literature by providing a study of the collaborative vehicle routing under a sustainable context, and by exploring the impact of different collaborative strategies on both the routing and the profit sharing problems. It also contributes to practice since it provides valuable managerial insights based on the case study investigated and sensitivity analysis performed. It would be worth to integrate the routing and the profit sharing problems, generating solutions that comprise the routes to execute and the profit allocation to each participants after profit distribution. However, this suggestion for future work is challenging from a modelling and solution approach perspective because it adds the complexity of the profit sharing problem to an already complex routing problem.

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Appendix 6.A Stand alone model

The stand alone problem considers that each entity in the transportation network operates individually. Thus, the LR can only serve the stores, and afterwards the dedicated vehicles return to the depot empty. The 3PL can only serve its customers, and afterwards the dedicated vehicles return to the depot empty. Each producer may only send the available loads directly to the depot of the LR, through outsourced vehicles. Therefore, the stand alone model can be extended from the collaborative presented in Section 6.3 by including a set of constraints that forbid the vehicles to travel in specific arcs of the network. These additional constraints are as follows. Constraints (6.34) and (6.35) forbid vehicles used by the RL to visit 3PL customers and producers, respectively. Similarly, Constraints (6.36) and (6.37) forbid vehicles used by the 3PL to visit stores and producers, respectively. Constraint (6.38) ensures that vehicles of the LR and the 3PL do not visit the depots of each other, and Constraints (6.39) ensure that all loads available at the producers are sent directly to the depot of the LR.

$$x_{ij}^{k} = 0, \qquad \forall i \in V, \forall j \in R \tag{6.34}$$

$$x_{ij}^{k} = 0, \qquad \forall i \in V, \forall j \in P \tag{6.35}$$

$$z_{ij}^{k} = 0, \qquad \forall i \in V, \forall j \in S \tag{6.36}$$

$$z_{ij}^{k} = 0, \qquad \forall i \in V, \forall j \in P \tag{6.37}$$

$$x_{\alpha\beta}^{k} + x_{\beta\alpha}^{k} + z_{\alpha\beta}^{k} + z_{\beta\alpha}^{k} = 0 \tag{6.38}$$

$$w_{i\beta}^{k} = 0, \qquad \forall i \in P \tag{6.39}$$

Appendix 6.B Impact of full collaboration versus limited collaboration

The problem presented in Section 6.3 represents the case of full collaboration. This section provides a comparison between the full collaboration setting and settings of limited collaboration.

The first limited setting covers the case where only collaborative pickup-delivery and collaborative cross-docking can be selected. Thus, it is sufficient to include in the original model the two additional constraints (6.35) and (6.37), which forbid backhauling. The second limited setting covers the case where only collaborative backhauling and collaborative cross-docking can be selected. Thus, it is sufficient to include in the original model the additional constraints (6.36), which forbid vehicles of the 3PL to serve customers of the RL.

The average synergy values obtained for limited and full collaboration settings are presented in Tables 6.14 and 6.15. The results show the benefits of full collaboration comparing with both cases of limited collaboration, as the synergy values regarding costs and fuel consumption are much higher in the former case for both instances. It is also demonstrated that the type of network is susceptible to the type of collaborative strategies allowed.

Forbidding collaborative pickup-delivery reduces drastically the synergy value of collaboration, which is more pronounced for network type R. On the other hand, forbidding collaborative collection, the highest reduction of the synergy value is shown for network type RC. Moreover, even in the cases of limited collaboration, the cross-docking strategy does not provide any benefit, and thus it is never selected.

Table 6.14 – Average synergy values obtained with different collaborative strategies, for instance s8-r5-p2

		Pickup + Cross-docking			Collection + Cross-docking			Full collaboration		
		C	R	RC	C	R	RC	C	R	RC
$OF(C)$	sv^{cost}	11%	8%	7%	4%	1%	6%	12%	8%	12%
	sv^{fuel}	10%	8%	7%	8%	3%	11%	13%	8%	15%
$OF(E)$	sv^{cost}	11%	8%	7%	4%	0%	6%	12%	7%	11%
	sv^{fuel}	9%	7%	6%	8%	4%	10%	12%	9%	14%
$OF(CE)$	sv^{cost}	11%	8%	7%	4%	1%	6%	12%	8%	12%
	sv^{fuel}	9%	7%	6%	8%	3%	10%	13%	7%	13%

Table 6.15 – Average synergy values obtained with different collaborative strategies, for instance s10-r3-p2

		Pickup + Cross-docking			Collection + Cross-docking			Full collaboration		
		C	R	RC	C	R	RC	C	R	RC
$OF(C)$	sv^{cost}	7%	8%	7%	5%	1%	7%	10%	9%	13%
	sv^{fuel}	6%	7%	7%	10%	3%	12%	13%	10%	16%
$OF(E)$	sv^{cost}	7%	8%	7%	5%	0%	7%	9%	7%	12%
	sv^{fuel}	6%	7%	6%	10%	4%	11%	13%	10%	16%
$OF(CE)$	sv^{cost}	7%	8%	7%	5%	1%	7%	10%	9%	13%
	sv^{fuel}	6%	7%	6%	10%	3%	11%	12%	9%	15%

CHAPTER 7

Conclusions and future work

This thesis approaches the integrated transportation planning and the opportunities to cope with sustainable challenges. Throughout the thesis, the integrated transportation problem is addressed as a Vehicle Routing Problem with Backhauls (VRPB), which consists on the integrated planning of outbound (delivery) and inbound (pickup) routes with the goal of reducing empty running of vehicles. Even though the VRPB can reduce considerably the number of vehicles, empty and total distances in comparison with the decoupled transportation planning, the problem is recurrently driven by economic concerns only. Nonetheless, the concept of the VRPB itself matches two of the main targets of sustainable transportation, namely the efficient use of vehicles and the minimization of empty trips. Moreover, the fuel wasted on empty trips and the resulting consequences (e.g., GHG emissions) can be avoided, which remarks the green nature of VRPB.
Therefore, the first step conducted in this research encompasses a review on the VRPB literature, allowing to establish the connections between the VRPB and sustainable transportation. Chapter 2 is dedicated to this research step, which identifies the main limitations of the current VRPB literature and the challenges to promote sustainable versions of the VRPB. Some of these challenges are the drivers for the remaining chapters. Chapter 3 focuses on the analysis of different backhauling opportunities to improve the integrated planning in real life contexts, addressing thus the challenge of efficient formulations for the Rich VRPB, demonstrating the advantages of integrated planning in comparison with decoupled planning. Chapter 4 is devoted to the mathematical formulation of a robust VRPB, addressing the lack of studies on the VRPB under uncertainty. Chapter 5 focus on the efficient formulation of a collaborative VRPB, solving some limitations of the currently used approaches in practice. Chapter 6 focus on the development of green VRPB models and their impact on collaborative networks, addressing sustainable challenges identified not only in the VRPB literature but also in the literature of collaborative VRP. In this chapter, the main findings and contributions of this thesis are presented, followed by the answers to the research questions and suggestions for future work.

7.1. Contributions

The first contribution of this thesis is advanced in chapter 2 with the analysis, classification and discussion of the VRPB literature in the light of sustainable transportation. The VRPB problems are firstly classified according to a common taxonomy of VRP provided in the literature, which is extended to distinguish the objective function in economic, environmental and social. Afterwards, the context of the VRPB application is examined in order to highlight the works addressing VRPB models with environmental and/or social concerns, and VRPB models used in both collaborative vehicle routing and reverse logistics. This literature review provides statistical evidences that, out of 107 reviewed papers, eight address environmental or sustainable concerns, six examine the VRPB in the reverse logistics, three study the collaborative VRPB, and none addresses uncertainty. On the other hand, using the VRPB as an instrument to promote sustainable transportation seems to be arising the interest of the research community and practitioners, as the results also shows that most of the sustainable VRPB literature was published in the last decade. In face of these findings, chapter 2 reports the current challenges and future research directions for the VRPB research. From the aforementioned work, of which the PhD candidate was the main author, the following research paper has resulted:

- M. J. Santos, P. Amorim, A. Marques, A. Carvalho, A. Póvoa. The vehicle routing problem with backhauls towards a sustainability perspective - a review. *TOP*, 2019. https://doi.org/10.1007/s11750-019-00534-0

The main contribution of chapter 3 is a new mathematical formulation of the integrated inbound-outbound transportation planning adapted to model real transportation problems of the wood-based industry. The formulation relies on a Rich VRPB with several operational constraints (heterogeneous fleet, multi-depots, split deliveries, selective backhaul customers) aiming to minimize fixed and variable routing costs and maximize revenues. The integrated planning is compared with an opportunistic strategy, where the planning of outbound routes are prioritized over the inbound routes, and a decoupled strategy, that plans the inbound and outbound routes separately. For all the case-study instances tested, the integrated planning allowed to achieve the best results, with an average cost reduction of 2.7%. The computational experiments further analyse the influence of diverse parameters and network configurations on the integrated planning, which allows to draw some insights. The most relevant one derives from the relationship between network configuration and revenues collected at backhaul customers - the less dispersed are the locations and the higher are the revenues, the higher is the potential to create integrated routes. However, this may lead to an increase in the number of vehicles to perform non-viable integrated routes (i.e. more than necessary). Therefore, the trade-off between maximum number of vehicles and revenues should be a properly managed in the integrated planning. In this work, the main responsibilities of the PhD candidate were the literature review, conducting the experiments on sensitivity analysis and discussion of the results. The work described in chapter 3 has result in the following research paper:

- A. Marques, R. Soares, M. J. Santos, P. Amorim. Integrated planning of inbound and outbound logistics with a Rich Vehicle Routing Problem with Backhauls. *Omega*, 2019. https://doi.org/10.1016/j.omega.2019.102172

The contributions of chapter 4 are two-fold. Firstly, it provides the first study of a VRPB under uncertainty. The revenues collected at backhaul customers are considered uncertain and a robust version of the problem is formulated following a well-known robust optimization approach in the literature. The robust model considers that uncertain revenues are represented by a polyhedral uncertainty set and uses a parameter to control the size of that set, known as budget of uncertainty. The robust model is compared with a chance-constrained model and the results indicate that the former approach is less conservative. The budget of uncertainty is set by the user and reflects in some degree its risk aversion. Thus, it can be related to the probability of constraint violation. In this context, the second contribution of this chapter is an efficient method to estimate the probability of constraint violation. This new method is inspired by the geometrical derivation of the uncertainty set through the Irwin-Hall distribution proposed in the work of Marengo et al. (2017). Further, it is compared with two methods proposed in the literature, and the results clearly demonstrate that the new method outperforms them by providing the tightest probability bounds of constraint violation. The PhD candidate was the main author of the this work, leading the different tasks with the exception of the Branch-and-Cut algorithm which was developed by the third author. The second author was actively enrolled on the development of the Adaptive Large Neighborhood Search (ALNS) metaheuristic, and on the development of the methods to estimate probabilistic bounds. The following research paper result from this work:

- M. J. Santos, E. Curcio, M. H. Mulati, P. Amorim., F. K. Miyazawa. A robust optimization approach for the vehicle routing problem with selective backhauls. *Transportation Research Part E: Logistics and Transportation Review*, 2020. https://doi.org/10.1016/j.tre.2020.101888

The main contribution of chapter 5 is a new mathematical formulation of the collaborative VRPB which allows to solve simultaneously the routing problem and the profit sharing problem. The collaborative problem considers the case where a carrier, initially hired to perform deliveries at the shipper's customers, may be motivated to perform backhaul routes for the shipper under proper incentives. Chapter 5 addresses this collaborative problem, which implies solving a VRPB and defining proper incentives to induce shipper-carrier collaboration. Based on the hierarchical nature between shipper and carrier, the new mathematical formulation is based on bilevel optimization which allows to solve the problem of the shipper (upper level) while anticipating the rational response of the carrier (lower level problem). As a result, the carrier would only accept incentives that generate higher profits, or at least equal, to its stand alone solution, which puts in evidence the ability of the bilevel formulation to tackle the property of individual rationality. The computational experiments conclude that solutions obtained with the bilevel formulation tend to generate higher synergy values than solutions obtained with traditional planning with side payments. The PhD candidate was the main author of this chapter, and the second and third authors contributed to the development of the mathematical formulation and the exact reformulation technique to approach the problem solving. The work on this chapter result in the following research paper:

- M. J. Santos, E. Curcio, M. Carvalho, P. Amorim., A. Marques. A bilevel approach for the collaborative transportation planning problem. *Submitted to International Journal of Production Economics*, 2020.

The main contribution of chapter 6 is a set of efficient modelling approaches to tackle the collaborative VRPB with environmental concerns. The collaborative problem considers a set of different participants (a retailer, a 3PL and several suppliers) that operate at the same level of the transportation network but may also perform complementary transportation services (e.g., cross-docking). Moreover, two different backhauling strategies can be adopted in collaboration, namely pickup and delivery services and collections to supply a depot. Solving the routing problem considering diverse strategies at the same time is motivated by the fact that it may unveil opportunities otherwise hidden. In opposition to the hierarchical approach described in the previous chapter, the collaborative problem in this chapter follows a joint planning approach. The collaborative problem is formulated by merging the problems of all participants into one large mathematical optimization problem with an overall goal for the entire network, and not for the participants individually. Also, the routing and profit sharing problems are solved sequentially. In particular, the profit sharing problem is addressed by simple proportional allocation methods, which are easy to communicate and implement in practice. The computational experiments conducted in this research indicate that tackling environmental concerns in the objective function may reduce substantially both environmental costs and fuel consumption, with a very slight increase on the operational costs, and can create reasonably high values of synergy. Nonetheless, with a holistic function, the collaborative solutions tend to follow the solutions obtained with an economic objective function and lead to similar synergy values obtained with the environmental objective. The PhD candidate is the main author of this work, and the second author has contributed to the problem formulation. The following working paper has resulted from the work in this chapter:

- M. J. Santos, S. Martins, P. Amorim., B. Almada-Lobo. A green collaborative transportation problem - analysis of different strategies and profit allocation methods. *Working paper*, 2020.

7.2. Answers to the research questions

In this section, we provide the answers to each research question raised in this thesis.

Research Question 0

What is the role of the Vehicle Routing Problem with Backhauls in terms of sustainability?

The VRPB focuses the reduction of empty running of vehicles by complementing outbound routes with inbound routes, which leads to reducing the total distances travelled by the vehicles. As transportation is responsible for a significant part of GHG emissions, reducing total distances is expected to reduce fuel consumption and emission of pollutants. Nonetheless, other parameters besides the distance have influence on the environmental impact of transportation, such as the load carried, the vehicle speed and the characteristics of the vehicle, fuel and road. Optimizing transportation can also reduce the traffic accident rate and noise, which may also be influenced by the previously mentioned parameters. Although few works have considered the environmental and social impacts of the VRPB, the literature shows clear evidence of the increasing interest by the research community on such topics in the last decade.

The sustainable impact of the VRPB can be effectively demonstrated in real cases, considering rich contexts, collaborative networks or reverse logistics. The literature shows that the VRPB allows reductions up to 20% in the total distance and up to 25% in CO_2 emissions in comparison with the traditional VRP. These reductions can almost double if mixed deliveries and pickups are allowed. However, a mixed strategy should always be validated in real cases in order to account for the effort required to rearranged the load in the vehicle. The VRPB can be related to collaborative networks, as the main goal of collaboration is to increase the efficiency of vehicles use and reduce empty trips, and as such, they are driven by similar goals of the VRPB. The collaborative VRPB is demonstrated to be a powerful strategy for both carriers and shippers, allowing savings of up to 24% in GHG emissions and up to 30% in the total costs. The VRPB can also be linked to reverse logistics as this problem considers the backward flows of products (e.g., returned products). In reverse logistics, the linehaul and backhaul customers are the same, which implies that the classic VRPB is hardly applied. However, if the same vehicle is used to deliver and collect products, other VRPB variants can be effectively applied. Case studies show that VRPB in reverse logistics can reduce up to 23% in total distances, up to 12.5% in the environmental impact and up to 23% in the social impact. Nevertheless, this requires analysis of trade-offs between the three dimensions of sustainability.

Research Question 1

How can transportation with backhauling be enriched for real world contexts?

In order to investigate the real potential of the integrated inbound-outbound transportation planning, the VRPB should be the closest as possible as real operations, and it should be compared with the traditional decoupled planning. To bring the model closer to the real problem, the VRPB can be enriched with several business-related rules. In the case study investigated for the wood-based supply chain, the business-related rules include the compatibility between type of vehicles and loading requirements, the interdependence between delivery and pickup operations, the selection of suppliers based on revenues, the possibility to perform only deliveries, only pickups or both, and the possibility of splitting deliveries to the same customer. Furthermore, other practical aspects such as heterogeneous fleet and multi-depots are also included in this Rich VRPB.

However, as more practical constraints are considered in the VRB model, the harder is to solve exactly the problem. To overcome this difficulty, we first reduce the complexity of the model by formulating it as a two-echelon VRPB. The first echelon includes the routing decisions between linehaul and backhaul customers, and as such it also includes the selection of suppliers. The second echelon includes the routing decisions between backhaul customers and last destinations, i.e. which mills are supplied by which backhaul customers. Another suggestion to avoid the difficulty of solving exactly the problem is to solve it with a matheuristic, because it allows to reduce the computational effort of exact methods, while providing solutions as closest as possible to their optimum. Nevertheless, solutions obtained with the matheuristic should always be compared with the lower bounds obtained with exact methods, in order to measure the effectiveness of the approximate algorithm.

Research Question 2
How to address uncertainty in the Vehicle Routing Problem with Backhauls through robust optimization?

Robust optimization is an emergent technique to deal with uncertainty in routing problems when the probability distribution of the uncertainty is not known in advance. The literature shows that there are three main alternatives to represent the uncertainty space, namely the box, the ellipsoidal and the polyhedral uncertainty sets. The box uncertainty set only allows to represent the worst-case values of the uncertainty, and as such it is considered a very conservative approach. The ellipsoidal allows to reduce this conservatism but it also reduces the tractability of the model as it originates quadratic functions in the constraints that incorporate the uncertain parameters. Thus, the polyhedral uncertainty set provides a suitable trade-off between conservatism of solutions and model tractability.
In this thesis, we demonstrate two alternatives to model the VRPB with uncertain revenues using polyhedral uncertainty sets from the literature, namely the factor model support and the budget of uncertainty. In particular, the budget of uncertainty approach leads to solutions much less conservative than a chance-constrained programming model, and allows to obtain different solutions for different types of decision-makers, i.e. different degrees of risk aversion. Thus, it enables to investigate the trade-off between robustness of solutions and risk aversion of the decision-maker. Nevertheless, the polyhedral uncertainty set requires mild assumptions on the distributions and relationships of the uncertain parameters. We execute the experiments considering that each uncertain parameter is independent on the others and that their values rely on an uniform distribution interval, since this is a good approximation when there is lack of knowledge on the distribution profile of the uncertainty. Moreover, some statistical evidences from past information can be used to build less conservative uncertainty sets and, as such, provide less conservative robust solutions.

Research Question 3
How to efficiently model and solve a collaborative Vehicle Routing Problem with Backhauls?

Bilevel optimization is a well-known strategy to model a collaborative game between two (or more) entities that share common aspects of an optimization problem. In the context of collaborative vehicle routing, a bilevel optimization model can effectively represent a game between a shipper and a carrier, which have different objective functions and whose relationship is based on a hierarchical structure.
In this thesis, we demonstrate how the bilevel optimization can leverage the collaboration between a shipper and a carrier, which occurs mainly through backhauling. The problem of the shipper, which aims to minimize total costs, can be described by the upper level problem, while the problem of the carrier, which aims to maximize total profits, can be described by the lower level problem. Furthermore, the profit sharing problem can also be incorporated in the bilevel optimization model, which ensures the individual rationality of both entities.
Although the literature provides some exact solution methods that are able to solve bilevel optimization problems, in general these cannot be applied to our case, due to the existence

of continuous and integer variables in both upper and lower level problems. Thus, we develop a reformulation technique that allows to solve exactly the problem, while considering the individual rationality in the profit sharing problem. Furthermore, we show that a bilevel approach is more effective than side payments approaches, which are common strategies applied in practice.

Research Question 4
How to address the challenges of a practical sustainable collaborative Vehicle Routing Problem with Backhauls?

In practice, collaborative networks involve several entities that may perform both similar as well as complementary transportation services. However, the relationship or dependencies between these services is either neglected in collaborative models, or they rely on a set of assumptions that aim to reduce the complexity of the models.
In this thesis, we demonstrate how to mathematically formulate a collaborative model considering the dependencies of three transportation services, namely cross-docking, pickup-delivery and backhauling, and show on which situations these services may conduct to collaborative gains for the entire transportation network. We further show that independently on the objective function used, either pure economic, pure environmental or a mixed economic-environmental, significant savings in both costs and environmental impact can be achieved. This means that modelling the collaborative problem using a traditional cost minimization function is suitable to promote environmental sustainability of transportation. Nevertheless, determining the collaborative gains for the entire network does not necessarily implies that all participants would profit from collaboration. In fact, it is often the case that one or more participants may increase their total costs in comparison with their individual costs when collaboration does not occur. Hence, distributing the collaborative gains among the participants as fairest as possible is an important requirement to implement the collaboration in practice. The literature shows that there are plenty of allocation methods used to solve the profit sharing problem, but each one should be carefully considered to handle case-specific problems, since there is no "one size-fit all" method. Furthermore, to be implemented in practice, the allocation method should provide simplicity and should be easily understood by all the decision-makers. In this context, we show that using proportional rules to distribute the gains among participant is a promising approach to solve efficiently the profit sharing problem in practice.

7.3. Future work

The research conducted in this thesis allow to answer all the research questions previously established, and which were mainly motivated by the findings on chapter 2. Nonetheless, several opportunities and literature gaps related to the VRPB research still remain, and deserve further attention by the research community.
Chapter 2 shows that there is room to explore further the potential of the VRPB to handle sustainability concerns. For example, social concerns are hardly considered in the VRPB literature and only cover accident rate and noise. However, it has been recognized that

the equity of working hours among drivers has a major impact on their performance and motivation. Thus, an opportunity for research could be exploring the impact of this social concern in integrated routing plans in comparison with traditional decouple planning. Also, the environmental aspects addressed in the VRPB literature only covers CO_2 emissions, which have a global impact on the environment, and can be easily converted based on fuel consumption. However, the emission of other pollutants, such as NOx, is influenced by other vehicle parameters and impact locally on the environment. Thus, another opportunity for research entails an investigation of the impact of integrated planning considering both local and global emissions. Another interesting line of research could cover the investigation of the logistics requirements and constraints of the VRPB in practice and respective challenges. For example, it is often the case that delivery and pickup loads cannot be transported at the same time due to compatibility issues, but this challenge may be overcome with multi-compartment vehicles. Also, the use of Alternative Fuel Vehicles (AFV) has been increasing due to their efficiency on reducing pollution, but so far no VRPB study had considered this type of vehicles, which opens up another opportunity for research. Another challenge not yet addressed in the VRPB literature relates with the use of dynamic approaches to model and solve this problem.

In chapter 3, a matheuristic was developed to solve the Rich VRPB. This solution method is based on a fix-and-optimize approach which iteratively solves smaller mixed-integer programming (MIP) sub-problems of the original model, and where each sub-problem consists of a set of decision variables to fix or release. Compared with a common MIP solver, the matheuristic developed is able to provide higher solution quality and higher computing efficiency, but it still consumes a relatively high amount of time to provide a good solution in large size instances. Nonetheless, matheuristics have been successfully applied in other optimization problems, and their performance are highly dependent on the problem design. Thus, another opportunity for future work is the improvement of the matheuristic proposed in chapter 3, which may include redesign of the MIP sub-problems or testing different heuristics.

The main motivation of chapter 4 was the lack of research on VRPB under uncertainty. The approach used to model the robust VRPB was the well-known approach of budget of uncertainty, and it has succeed as demonstrated by the computational results. However, as the robust optimization is an emergent field of research, investigating new robust approaches and uncertainty sets is seen as a promising opportunity for future VRPB research. For instance, the uncertainty set of revenues may be built using a different distribution or leveraging some statistics obtained from historical information, such as the distributionally robust optimization. The uncertainty set would then present tighter bounds, leading to better estimates of the probability of constraint violation and also less conservative robust solutions.

In chapter 5, the bilevel VRPB model proposed to study the vertical collaboration problem is shown to succeed for solving both the routing and the profit sharing problems. The bilevel mathematical program is reduced to a single-level through an exact reformulation technique, and a common solver is then used to solve the problem. This strategy however requires too much effort for solving large size instances. As evolutionary algorithms are the most advanced solution methods applied in the bilevel optimization literature, a possible direction for future VRPB research may entail the investigation of these type of metaheuristics to solve the bilevel VRPB. Another opportunity for research involves the analysis of different concepts that builds the bilevel model. In chapter 5, the bilevel VRPB considers the most opportunistic case of the upper level, in face of the most optimistic scenario of the lower level. Thus, the model can be designed using other rationales (e.g., less opportunistic-pessimist scenario, less opportunistic-optimistic scenario) and the collaborative solutions can be compared, which may allow to measure in some degree the robustness of the different approaches.

Finally, in chapter 6, both the joint routing planning and the profit sharing problems are investigated and solved for the lateral collaborative problem. Although this work fills some of the gaps found in the related literature, a major challenge still remains - solving the routing and the profit sharing problems simultaneously. Chapter 5 presents a formulation that can handle this challenge, but such formulation is not suited for the problem carried in chapter 6, as there are no hierarchical decisions to be made nor conflicting objectives among the participants. Therefore, an opportunity for future research, not only concerning the VRPB literature but extended also to the collaborative VRP literature, encompasses the development of efficient formulations that allow to merge the routing and the profit sharing problems into an integrated optimization problem, or the development of efficient algorithms that can solve both problems simultaneously.