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Fuzzy Spatial Load Forecasting

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FUZZY SPATIAL LOAD FORECASTING

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Spatial Forecasting is about what the world will look like and not about what we imagine it should look like.
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Abstract

This thesis presents a computational intelligence system for forecasting electricity demand, with special emphasis in the geographical behavior of the demand growth. This new approach, called Fuzzy Spatial Load Forecasting, uses fuzzy systems and cellular automata as mathematical techniques and is completely integrated into a GIS spatial analysis support. The system simulates the geographical electricity demand as a function of geographical influence factors mapped for the region.

The innovation of the system is its ability for learning the spatial behavior of consumptions, based on the combination of information from historical maps and judgmental information from experts. The simulation of the geographical load growth results from the application of the knowledge base to new geographical and economical scenarios, specific for a region. The ability to forecast with high geographical reality is another attractive feature of the system, provided by the capabilities of the GIS spatial analysis support.

The thesis formulates the spatial load forecasting problem; presents a detailed description of the mathematical formulation of the system, including test and validation; studies the performance of the system in the modeling and propagation of uncertainties, with special emphasis in the modeling of spatial uncertainties; and finally presents several possible usages of the methodologies and their potential for future research developments.

The objective of this thesis is to explore a set of methodologies to open doors for a new generation of automated electricity distribution planning. It is our objective to develop intelligent automation, which allows the planners an intuitive observation and understanding of the geographical phenomena that influence the demand behavior. In another perspective, the planning tools should use large quantities of information with better geographical representation of the real world and covering other related planning areas. In this thesis it is our goal to reach these objectives by exploring computational intelligent techniques integrated in geographical information systems and implemented in spatial analysis.
Esta tese apresenta um sistema computacional inteligente para previsão de consumos de electricidade, sendo de especial interesse o comportamento geográfico do crescimento de novos consumos. Esta nova abordagem, denominada Previsão Geográfica de Consumos com Sistemas Difusos, utiliza os Sistemas Difusos e Autômatos Celulares como técnicas matemáticas e foi implementada fazendo uso das capacidades de análise espacial dos Sistemas de Informação Geográfica (SIG).

O sistema tem como atractivo a capacidade de aprendizagem automática, a partir de mapas históricos e a partir de informação qualitativa fornecida por especialistas. A simulação do fenómeno geográfico de crescimento resulta da aplicação da Base de Conhecimento a cenários geográficos e económicos específicos para a região. A capacidade de simulação com elevado realismo geográfico é outro dos atractivos do sistema, proporcionado pelas capacidades de análise geográfica do SIG.

A tese estuda o problema de previsão geográfica de consumos; apresenta uma descrição detalhada da formulação matemática do sistema proposto, incluindo testes de validação; estuda o comportamento do sistema na modelação e propagação de incertezas, com especial atenção para as incertezas espaciais; por fim apresenta um conjunto de possibilidade de exploração desta metodologia em aplicações e investigações futuras.

Esta tese tem como objectivo explorar um conjunto de metodologias de forma a proporcionar a base para uma nova geração de ferramentas de planeamento. Pretendemos com estas metodologias desenvolver uma automatização inteligente, que permita aos agentes de planeamento uma intuitiva observação e compreensão dos fenómenos geográficos que afectam o crescimento de consumos. Num outra perspetiva pretendemos que as ferramentas de planeamento utilizem maior quantidade de informação com maior realismo geográfico e cobrindo outras áreas de planeamento relacionadas. Nesta tese pretendemos atingir estes objectivos explorando metodologias de inteligência computacional integradas em sistemas de informação geográfica e implementadas com base em análise espacial.
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Chapter 1  INTRODUCTION

The electricity demand is characterized by magnitude, variation in time and geographical location. These three components are essential information for Electricity Distribution Planning. Spatial Load Forecasting is the methodology designed to forecast simultaneously these three characteristics of the future electricity demand.

Spatial Load Forecasting is the information basis for the Electricity Distribution Planning, especially if the Expansion Planning includes a geographical perspective. The concept of Spatial Load Forecasting is not new and from the earlier 80ies several models have been developed and some implemented in commercial software. However the use of these methodologies has been unattractive due to several factors:

- Difficulties in acquiring and managing geographical information.
- Difficulties in capturing and updating the knowledge base that describes the spatial behavior of the demand.
- Difficulty in interacting with the decision maker leading to a deficient definition of scenarios.
- Excessive simplification in the abstraction of the real world leading to an insufficient information system and consequently a distorted image about the problem.

We believe that the technological development in GIS and the efforts in interoperability at the level of information, data structures and methodologies will provide a favorable environment to revive the interest in these tools. To develop an adequate approach to implement a new generation of Spatial Load Forecasting is the deep purpose of this thesis.
1.1. PURPOSE OF THE THESIS

The core of this thesis presents a computational intelligence system for Spatial Load Forecasting designed to capture and simulate the spatial behavior of electricity demand. The Fuzzy Spatial Load Forecasting consists in Fuzzy Systems and Cellular Automata implemented in GIS Spatial Analysis. The innovations of this system are:

- the capability to capture the historical behavior of the phenomena by emulating the cause-effect relations implicit in geographical maps of past development and corresponding geographical influence factors;

- the capability to improve the historical knowledge base with judgmental knowledge, defined by human experts, about changes on the future behaviors;

- the storage of the knowledge base in a set of fuzzy rules interpretable by human experts;

- the provision of an easy tool to the electricity distribution planner to define geographic scenarios and to evaluate uncertainties in the spatial pattern of the electricity demand.

1.2. THE SOFTWARE TOOLS

The thesis is focused on the spatial analysis methodologies; consequently the GIS capabilities are used for the model implementation. More than a simple tool, the spatial analysis is defended in this thesis as a promising research area that could significantly improve the electricity distribution planning. The main motivations to use this resource are the following:

- Spatial analysis provides an efficient representation and management of geographical information that interacts with the electricity distribution-planning problem. Most of the information used in Electric Distribution Planning is geographical. The variables taking part in the planning problem, costs, uncertainties and restrictions, are highly dependent on geographical aspects.
- Algorithms could be implemented to automate the information extraction from real world representation to the information abstraction used by the distribution planning tools.

- An improvement in the interaction of the electricity planning process with other multidisciplinary planning activities (land-use planning, urban planning, environment planning) may be achieved. The developments in GIS interoperability, in information, data structures and processes allow the interaction between the several planning areas.

- Spatial analysis allows an efficient computational implementation of simulation and optimization algorithms traditionally applied in Electric Distribution Planning (load forecasting, substation location, feeder routing).

- Excellent capabilities for result visualization and analysis improve the interaction with the planner.

1.3. BACKGROUND OF THE WORK

The concept of Spatial Load Forecasting was developed in the 80ies and has been implemented and commercialized in a variety of software. The most complete description of these methodologies is compiled by Willis in his book “Spatial Load Forecasting” [1]. In the perspective of the application in Electric Load Forecasting this work uses the work of Willis as starting point. However, the subject of study is the methodology of Spatial Forecasting covering the study of applications in other multidisciplinary areas (land-use planning, urban planning, environment planning) and crossing these specialized experiences with the general concepts and principles of the Forecasting Science. The closeness to the Geographical Information Science in general and Spatial Analysis in particular is other strong background component in this work.

1.4. THESIS STRUCTURE

The thesis is composed by 7 chapters, including introduction and conclusion, and one Annex about Fuzzy Systems.
Chapter 2 states the problem of Spatial Load Forecasting describing the main ideas of the methodologies and the state of knowledge. The chapter begins by defining the scope of the spatial load forecasting in the global set of methodologies used in Forecasting Science, followed by a characterization of the demand growth behavior problem in the perspective of spatial analysis. To close the chapter the interaction between forecasting and planning is discussed.

Chapter 3 presents a detailed description of the mathematical model used in the intelligent system. This system, based on the modern understanding of computational intelligence, uses Fuzzy Systems and Cellular Automata as mathematical techniques. The chapter describes the structure of the model, detailing the fuzzy system and the cellular automata formalization and implementation. Several aspects related with the process of knowledge base construction are studied. The mathematical formulation presented in this chapter is the basis for the implementation of the Fuzzy Spatial Load Forecasting model described in Chapter 4.

Chapter 4 is the core chapter with a complete description of the Fuzzy Spatial Load Forecasting model, presenting the conceptual formulation, the structure, the implementation and the validation. The description of the model is complemented with examples for better explanation of the functionalities and model validation.

Chapter 5 is dedicated to the study of the uncertainties that affect the problem of Spatial Load Forecasting. The chapter begins by discussing particularities of spatial uncertainties and their modeling. In this chapter one explains the capabilities of the Fuzzy Spatial Load Forecasting model to define scenarios and one studies the aspects related with the propagation of uncertainties through the mathematical model. The study is based on examples of applications and on the evaluation of results, contributing to the awareness of the possible consequences of these uncertainties in Distribution Planning.

The discussion in Chapter 6 is a clarification about future work. The spatial models developed on the kernel of the thesis open excellent opportunities for research in methods for distribution planning. Some of these potential ideas and work, already done by the author, are presented in this chapter as future and present research directions.
1.5. ORIGINALITY AND CONTRIBUTIONS

The thesis is written over two general innovative concepts. The first is the adoption of the spatial analysis computational techniques to link the real world representation with the information system used in distribution planning. The second concept is the expansion of the scope of electricity distribution planning outside the utility world by developing methodologies oriented to the interoperability with other planning sectors.

These two concepts are applied to the design of the Fuzzy Spatial Load Forecasting model, which is the core contribution of the author in this thesis. The Spatial Load Forecasting is the traditional module of distribution planning that works as interface between the real world and the electric distribution planning. This interface characteristic motivates the author to explore the Spatial Load Forecasting as a starting point to implement general new planning concepts.

In the perspective of mathematical techniques the use of fuzzy systems to capture spatial behavior is one of the main innovative aspects of this thesis. Some previous works used fuzzy systems to represent spatial behaviors but none capture the behavior directly from historical maps. Other innovation is the use of fuzzy reasoning techniques to combine old and future spatial behaviors by merging historical and judgmental information.

The use of Cellular Automata in this kind of problems is not an innovation of this thesis. Cellular Automata have been used in urban planning to describe the spatial behavior. However, in this thesis the Cellular Automata are not used to represent the spatial behavior: their function here is only as a complement of the Fuzzy System functionality by providing the capability of discretization in the simulation process. These innovations have been reported to the Power Systems scientific community in the following references [2]-[7] (see Annexes).

We believe that forecasting results are only truly useful if their uncertainty in results is known. This motivated us to write a chapter (Chapter 5) that discusses the capability of the Fuzzy Spatial Load Forecasting tool to model, represent and propagate uncertainties. This chapter is based on studies presented on the following references [8],[9] (see Annexes).
As referred above, before the Chapter 6 discusses ideas related with Spatial Load forecasting but outside the scope of the thesis. This chapter contains several innovative methodologies and tools that could be useful to understand the research direction actually followed by the author.

1.6. DATA SETS, STUDY REGION AND EXAMPLES

Because the methodology developed in the thesis is based on GIS several geographical data sets are used and presented throughout the thesis. These data sets consist in a variety of thematic coverages relevant for the study (digital terrain models, road coverage, land-use, demographic coverage, electricity network, etc). The datasets were managed by commercial GIS software, also used to implement the mathematical models (ArcView). The resolutions used for the spatial analysis and for the information representation vary from 50 to 250m.

Most of the geographical information presented in the thesis corresponds to the island of Santiago in Cabo Verde. The island has a very complex orography (steep, rugged, rocky). The forecasting studies were focused on the city of Praia (capital of the country). The city concentrates more than 70% of the population of the island as a consequence of a fast urban expansion in the last 30 years. The information used in the thesis is from several sources, having been collected and digitized by the author in projects developed at INESC Porto and finished previously to this thesis work.

The forecasting examples presented are not the result of contractual or research projects. The studies have been developed specifically to test and validate the methodology developed in the thesis. This specific region was selected because of to the complete set of information available to the author. The fast demographic development and the consequent electricity demand growth was another other important reason for this selection. An effort has been done in order to formulate the forecasting scenarios in accordance with the regional reality. Due to the lack of better information in some cases it was necessary to create hypothetical scenarios. Consequently, it is important to underline that the results presented in the examples of this thesis should be considered as hypothetical scenarios that could not be used as conclusive or in intermediate regional studies.
1.7. REFERENCES


Chapter 2  THE SCOPE OF SPATIAL LOAD FORECASTING

Spatial Load Forecast (SLF) predicts where, when and how much load growth will occur in a utility service area. This information is traditionally used for expansion planning purposes to ensure that the system will be able to supply the load. Earlier work from the 80ies provides extensive documentation on different spatial load forecasting methods available to modern utility distribution planning [10]-[24]. The foundations of Spatial Load Forecasting methods are well documented by Willis [25], which was the promoter of this kind of technique to model electric consumption. In recent years some research work has been done in SLF [26]-[31]. However in our opinion, the development in SLF didn’t follow the developments in geographical information science. Consequently there are attractive research opportunities for new developments in SLF.

Forecasting the electric consumption growth is related to other research areas such as urban planning, social sciences and forecasting science. In urban planning, a large work has been done; part of this work appears as the basis of Spatial Load Forecasting methodologies. The research in this area is being developed using GIS as the implementation tool. On the other hand new geocomputational modeling methodologies and techniques [32] (Neural Networks, Cellular Automata and Fuzzy Systems) are used to simulate the growth behavior ([33]-[44]). The Social Sciences have been the basis for global long-term electricity forecasting studies. These sciences are more dedicated to study the electricity end-use and its relation with the social behavior of the consumers not using spatial approaches. Independently of the research area, all the methodologies related with forecasting must follow the principles of forecasting used and stated by the Forecasting Science [45].

The present trends of interoperability, in geographic data and GIS technologies, indicate that in the future the multiple sectors of planning (urban, environment, electricity, telecommunications, and other network facilities) will use common data, interchanging their planning results and reusing them to improve forecasts. This appears as a motivation for the research of new SLF methodologies, flexible and adaptable, working as an information interface between the electric utility information systems and the other information systems of the external world.
In this thesis we research the methods and techniques used in several forecasting areas in order to improve the functionality and the framework of Spatial Load Forecasting. This chapter presents the scope of the spatial load forecasting. The first part of this chapter is a global view about several forecasting methods and corresponding knowledge sources. Spatial structures and resolutions are an important aspect for GIS implementation discussed in the second part. In the third part we address the time dynamic aspects of load growth behavior. The fourth part introduces the concept of Geographical Influence Factors, which are basic inputs of the spatial load forecasting. The last part of the chapter describes the interaction between electric spatial forecasting models and electric distribution planning.

2.1. FORECASTING METHODS AND THEIR KNOWLEDGE SOURCES

Spatial Load Forecasting is or may be a mix of several forecasting methods. In fact, most of the forecasting methods could be used to implement SLF, but applied in a geographic framework using spatial data structures. In this section the several aspects related with the scope of the SLF are studied.

There are different types of forecasting methods and most of then able to be applied to SLF. The discussion about the adequacy of the forecasting approach depends essentially on the type of knowledge source. Figure 2.1 shows the process of selection from several types of forecasting methods based on the type of knowledge source.

Some comments are necessary to clarify the meaning of this figure:

- The forecasting methods are based in one of two types of knowledge, the judgmental and the statistical. For instance, in our problem the statistical information is the measure of spatial electrical growth along the past years and the judgmental information derives from the expert experience and from a prediction of future behavior, based on the human understanding of the development phenomena.

- The judgmental methods may be classified as the ones that predict one's own behavior (label “self”) and those that predict how others will behave (label “others”). Some SLF methods are only based on knowledge constructed from utility inside information, forecasting being based on utility planning directives.
This scenario planning could be based on the analysis of the role played by and actions among the key decision-makers (label “role”). The identification of scenarios could be done by predicting the behavior of decision-makers relatively to utility strategies. When no role exists for a specific situation the behavior may be predicted by studying how feature situations, created by the “selves”, affect intentions. The conjoint analysis forecasts the behavior based on the reaction of the consumer. Forecasting the effects of Demand Side Management is an example of intention and conjoint analysis methods.

- The tendencies for the interoperability in the planning process, in interaction with other planning areas, motivate for the use of forecasting methods based on the understanding of the interaction between the electric utility and the external world knowledge base (label “other”). The interoperability of planning (collaborative planning) is one of the ideas defended in this thesis and the forecasting models proposed should follow this branch of forecasting methods.

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**Figure 2.1** – Forecasting methods and their knowledge source (inspired in [45]). Each method type is adequate for specific knowledge source characteristics. The method proposed in this thesis is a mix of characteristics of several forecasting types.

- The methods downstream the label “others” (Figure 2.1) are used for expert knowledge environments with which people and organizations interact. This expert knowledge could be from a diversity of planning areas. The expert-opinion is the usual method to build a knowledge base. Some of these methods use analogies with past situations. It is possible to infer expert rules using regression
analysis; the approach called “judgement bootstrapping” [46] is a type of expert
system limited to inference models based in the information that experts use to
forecast.

- The forecasting models based on statistical data may be univariate or multivariate.
The univariate models are simple extrapolation models that may have the support
of the expert opinion based on analog situations. More complex univariate models
may integrate domain statistical knowledge with expert knowledge as a set of rules
(rule-based forecasting). The multivariate methods can be split in those derived
from theory such as the econometric models, and those derived from statistical
data. The econometric models are structured from theory and the parameters are
estimated from statistical data.

- The multivariate models strictly based on statistical data could be used to
construct a set of rules. This knowledge base could be complemented with expert
knowledge in order to build a rule-base forecasting method. Also the econometric
models provide an excellent way to integrate expert and statistical knowledge
sources. The forecasters could select one of these methods or could combine the
best characteristics of some of them in an Expert System.

How to classify, then, the Spatial Load Forecasting method proposed in this
dissertation? One may say that it is based on an expert system (Fuzzy Spatial Model
(FSM)) that combines statistical multivariate and expert knowledge base into a rule-based
forecasting model. The proposed spatial forecasting model should combine judgmental
information, from inside the utility and from external interacting planning sectors, with
statistical multivariate data from historical behaviors. In fact the presented model
combines characteristics of many of these forecasting branches in order to maximize the
use of the existing information.
2.2. **Spatial Structures and Resolution**

The selection of the data structures and their resolution is one of the most important issues in implementing the Spatial Load Forecasting (SLF) into the Geographic Information Systems (GIS). The data structures are composed by organized area features; these area features are elementary units containing information about the shape and other non-graphic information linked to the identifier of the elementary unit. The differences between the data structures are related with the shape of the elements, and the resolution is related with the size of these elements. In SLF one may adopt three kind of geographic data structures (see Figure 2.2).

- **Equipment areas** – The elementary units are irregular shapes defining the service areas of several electric equipment (substations, feeders, secondary substations, transformers). The advantage of this approach is the ideal match between the utility information database and the spatial analysis structure. Another advantage is that the resolution (here characterized by size and shape) matches exactly the available information. However, there are several disadvantages. The first is that the growth often occurs where no equipment exists and no shape structure is defined. The second is that the shape of the service areas may change through the lifetime of the equipment. Third, the structure is non-appropriate to the cross analysis between geographic layers from different sources. Fourth, this kind of structures must be represented as vectorial formats, which are computationally heavy for spatial analysis and intensive computation modules.

- **Uniform grid** – It is based on raster structures where the element units are regular shapes (rectangles) ordered in rows and columns in a matrix organization. The position of the elementary unit in the matrix defines the geographic position of the cell center. The greatest advantage of this type of data structure is its natural suitability for spatial analysis, the ability to cross, join and transform information from multiple data sources. As disadvantage it presents the non-existence of exact match with the available shape and size of the information. This problem may be solved by increasing the resolution, but this may have as consequence large data sets which difficult the storage and decrease computation speed.

- **Heterogeneous areas** – Each developable area may have an arbitrary size, but shapes must be regular and must have matching bounds. The basic idea is to use
the spatial analysis capabilities of the raster structure while using heterogeneous sizes for cells. The advantage of this structure is a better match between the data structure and the density of information and results. The disadvantage is greater complexity for spatial analysis and the unavailability of this kind of structures in most of the commercial GIS. Progressive resolution analysis with multiple levels of uniform structures could be used to reach the same effect of heterogeneous resolution (see Figure 2.3).

One of the ideas defended is this thesis is the interoperability of the utility information system by improving the interaction between the distribution planning and the external information world. This task is only possible using common data structures, interactive analysis methods and interoperable GIS software ([47],[48],[49]). In this perspective, the use of Uniform Grid structures is in fact most adequate and this will be the data structure used in this dissertation. Furthermore, the structures "equipment areas" and "heterogeneous area" may be implemented in a uniform grid approach, however with some loss in efficiency.

![Figure 2.2 - Three possible data structure approaches for SLF spatial analysis](image)

The selection of the analysis resolution depends on three factors: the planning horizon, the type of equipment to be planned and the spatial accuracy and density of the geographic data. In the case of SLF models the selection of resolution has an extra difficulty because the forecast must be done for different planning horizons (short, medium and long range planning), for different voltage levels (sub-transmission, distribution, etc) and including multiple sources of data to describe very heterogeneous regions (urban, rural, etc).

The planning horizon is related with the planning status of the network. For instance, in some regions where there exists a network, consumers can be mapped with relatively good accuracy. In these regions or in their surroundings some short range planning is needed. This will influence the medium and long range planning. In short range planning,
an engineer must decide on equipment size and settings and the precise location of the equipment; this kind of planning requires high forecasting resolution. Long range planning does not lead to immediate decisions on equipment and site acquisition. Rather, its purpose is to set the direction for the short range planning decisions, guiding where substations and capacities should be added. Because, by nature, long range planning presents implicitly a smaller forecasting accuracy, the resolution used in long range may be lower (larger cell size). For locations requiring shorter range planning the resolution could be higher because higher density of information with high accuracy is available.

The voltage levels influence the selection of resolution because they require different time horizons. For instance, the planning horizon for a HV/MV substation is larger than for MV/LV secondary substations. The acceptability of the lower resolution depends also on the size of the equipment service area. Fortunately, the equipment that needs higher planning horizon is also associated to larger service areas. That is to say that equipment allowing low planning resolutions has larger service areas for which it is acceptable to use these low resolution scales. Some scale indicators about the resolutions to be used in each kind of equipment are presented in Table 2.1 [1].

The accuracy of the data available is another restriction conditioning the resolution selection. Obviously it is possible to use high resolution analysis based on low accuracy input data, but the results will be illusory and the consequence may be bad for planning decisions. For this reason the forecast resolution must be proportional to the accuracy it represents. Our recommendation to deal with this problem is to include in SLF uncertainty analysis evaluating the accuracy of the forecasting outputs for correspondent uncertainties in the set of inputs.

<table>
<thead>
<tr>
<th>Planning purpose</th>
<th>Planning Horizon (years ahead)</th>
<th>Service Area (km or km²)</th>
<th>Forecasting resolution (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub-transmission network</td>
<td>5 - 25</td>
<td>10 km</td>
<td>250</td>
</tr>
<tr>
<td>Substation</td>
<td>5 - 25</td>
<td>25 km²</td>
<td>250</td>
</tr>
<tr>
<td>Medium voltage network</td>
<td>2 - 20</td>
<td>3 km</td>
<td>50</td>
</tr>
<tr>
<td>Secondary substations</td>
<td>1 - 15</td>
<td>0.5 km²</td>
<td>50</td>
</tr>
<tr>
<td>Low voltage network</td>
<td>1 - 10</td>
<td>300 m</td>
<td>5</td>
</tr>
</tbody>
</table>

Different types of regions have different density of information. For instance, the quantity of information per square km in urban regions is higher than in rural regions. Because higher density of information rewards the use of higher resolution, the resolution
necessary for regions with lower information density (e.g. rural) could be lower than the resolution necessary for high information density (e.g. urban).

A key mechanism of the SLF is based on the spatial spreading of a global value. This concept could be generalized by admitting a zooming process with sequential forecast levels (Figure 2.3). The forecast of a following level is restricted to zones that require detailed analysis: the resolution is higher constituting a regular partitioning of the previous forecast level. The global value used in the following forecast is the previous forecast result for the corresponding cell location.

![Figure 2.3 - Illustration the zooming process where global forecast is progressively distributed by smaller area subdivisions.](image)

This process is used for implementing SLF with heterogeneous structure analysis. Note that the global forecast is the non-spatial forecast covering all the geographical extent of the spatial analysis. The difficult aspect of this multi level forecast is the different behavior associated with each resolution level: each forecast level requires a specific knowledge base.
2.3. LOAD GROWTH BEHAVIOR

By “load” one could be referring to energy, load curves, peak power or installed capacity, but in general the magnitude used for distribution planning is the peak power.

There are three causes for load change inside a cell location. The first is the change in the number of consumers (customer-count or land-use). The second is the change in the use of electricity per consumer (end-use). This subdivision of growth causes is important in the SLF modeling because it corresponds to the decomposition in two different modules. All the two load change causes depend on spatial influence factors, and the corresponding modules should be implemented, at least partially, by spatial models.

The customer-count (land-use) module forecasts model the growth in the number of consumers, predicting the land suitability and projecting growth over the most suitable locations. The suitability for a specific type of consumer depends on several geographic influence factors considered relevant for the development process.

The change in the use of electricity is related to the number of appliances and to the efficiency of the electricity usage. The per-capita consumption is influenced by several factors like the emergence of new consumption appliances, the electricity price, and economic prosperity. Some of these influences change from region to region, which means they have a spatial behavior that should be modeled in a spatial analysis framework. Another important aspect that may be modeled in the end-use module is the competition between electricity and other end-use forms (natural gas, solar thermal, etc.); the availability of such end-use alternative forms reduces the use of electric appliances, and affects the electric consumption growth.

The growth behavior is partially described by the so-called diffusion of innovation also called technological forecasting. The diffusion of innovation describes a behavior for the diffusion of products, technologies, or ideas, based on the communicative interaction between the social communities. This concept is applicable to the acquisition of suitable locations for customer development (customer-count forecast) and acquisition of electric appliances (end-use forecast).

The forecasting of the behavior of diffusion of innovation follows an $S$ curve, also called growth or saturation curve ([50]-[56]). The $S$ curves are effective for modeling load growth. Thus, the $S$ curve is characterized by a slow initial load growth (dormant period),
followed by a period of rapid growth during the main development phase (growth period), and finally a slow period of growth when the area reaches saturation (saturated period). The S curve characterizes the timing for development and is essential to describe the dynamics of the growth behavior. The S curve represents the level of saturation and the derivative of the S curve is proportional to the instantaneous development.

![Sigmoid Growth Curve](image)

Figure 2.4 – The S curves shape represents a growth behavior typical for the innovation forecast. The s curves have three periods: dormant, growth and saturation.

There are several types of representation patterns for the S curve, each one with a specific designation and with an equation or model associated (e.g. Rogers's approach, Gompertz curve, logistic curve, Bass model, and other exponential patterns).

The S curves are based on the following assumptions:

- The forecasting model knows the general behavior of the growth curve. In some models the S curve is represented by well-known equations, on others growth behavior is stored as a piece of information associated with the points or parts of the curve. In both situations the general behavior for the entire curve must be known.

- The past function is indicative of the future. This means that, by following the growth behavior for the curve, the past and present saturation levels indicate the growth potential for the near future. The instant to begin the development is particularly important to the S curve.

- The maximum saturation value for the curve is known. This assumption is very important because the shape of the second half part of the curve depends on the distance to the saturation level. For instance the urban development in a specific
cell depends on the vacant area available and is estimated based on a maximum urban density allowed for the cell.

The methodology developed in this thesis (Fuzzy Spatial Model - FSM) follows the three assumptions, but implemented in a specific way. In FSM the shape of the curve is defined by a set of points represented by a fuzzy rule base (see more in Chapter 3). With this methodology the S curves results directly from the piecewise simulation process, avoiding a predefined curve shape. As demonstrated in chapter 4 the shape of the S curves varies from site to site depending on local and neighborhood effects. One advantage of non-predefined shape is the capability to model non-usual curve shapes like consumption decrease or urban redevelopment.
2.4. **Geographic Influence Factors**

The GIS spatial models, which compose the SLF, are basically a set of operations for combining spatial elements to predict and explain spatial phenomena. The spatial models adopted in SLF use a set of explanatory geographic variables, designated as spatial influence factors. The spatial influence factors can be classified in three types: structural, location factors and neighborhood factor. The structural factors are variables that affect the site of each geographical unit without influence from outside the unit (e.g. terrain slope, domestic saturation). Contrarily, the location factors are consequences of the absolute and relative location of other features outside the geographic unit (distance to roads, distance to urban center). Finally the neighborhood factors represent the influence of entities or features in an adjacent area (number of commerce, landscape visibility).

When building spatial models, working with a large number of variables usually requires a large computational effort and long time consuming. To reduce the number of variables, the most obvious approach is to select only the variables with relevance to the process. This can be done by using formal methods like significance tests or sensitivity analysis. Another approach to reduce the number of variables is to divide a region in sub-regions with uniform behavior. Thus it is possible to eliminate variables without spatial significance in the sub-region. For instance the terrain slope may have influence on a heterogeneous region, however this variable could be excluded in a flat sub-region. Also important is the formulation of the models; some times, the number of variables could be reduced by compacting several variables into only one. For instance, instead of the distance to urban centers A, B and C (3 spatial variables) use the distance to the closest urban center (1 variable). The pre-processing of some variables requires complex spatial analysis; this pre-processing is presently easy to implement by using commercial GIS functions (e.g. distance functions, reclassification functions, surface analysis functions, zonal functions, focal functions, etc.).

The influence factors to be used in SLF vary from region to region and must be selected carefully by the planner when constructing the SLF knowledge base. This is one of the reasons why the SLF must be very flexible and easy to adapt to new knowledge bases. The variables must be selected in a way that the extracted knowledge base should be transportable to other space and time with similar behavior. For instance, the use of the variable “distance to locations with high saturation” is preferable to use the variable
"distance to Central Square"; the first option is a relative measure, dynamic on time and on space, and the second is absolute location. The distance to a saturated area could be applied in other regions and times because saturation is a general concept influencing different times and spaces in the same way.

Figure 2.5 – Examples of influence factors (terrain slope a) and distance to roads b)

In the fuzzy SLF model presented in this dissertation, as in other SLF models, the planner must select the variables. However several variables are mandatory; in customer-count models the saturation level is a mandatory variable used to control the dynamics of the process. Other variables are not mandatory but are generally necessary for all regions (e.g. distance to urban centers and distance to roads). The Fuzzy Spatial Model is a model prepared to simulate the dynamics along several time stages, and in this perspective the influence factors could be divided in three different types:

- Static influence factors – these are factors that influence the process but are unchangeable with time (e.g. terrain slope, altitude, etc). For these influence factors the spatial model use the same data sets on all the stages.

- External dynamic influence factors – these are variables that change with time, but their change is indicated externally to the spatial model by human experts or by other spatial models (e.g. distance to roads, environmental indicators, etc). This kind of variables allows the interaction with the external knowledge along the dynamic process. The modeling of uncertainties by scenario identification is done by means of this type of variables.

- Internal dynamic influence factors – the data sets of these variables also change with time, but in this case the change is computed internally by the spatial models based on the results of the previous stages (e.g. saturation level, distance to
saturated areas). These variables allow the spatial model to keep control of the phenomena dynamics.

Before being used in spatial models, a spatial influence factor should be transformed into a set of geographical information with predefined data structures. This information sets should be raster (bitmap grids) coverages well georeferenced with appropriate resolution and covering a common geographical extent. Some spatial influence factors require GIS preprocessing by using spatial functions. For instance, a distance influence factor requires distance spatial functions, which could be simple Euclidian distance functions or complex minimum cost-path functions. This preprocessing could be automatic, based on a primary set of information like the coverage of roads, or could be done manually by the user preprocessing the information differently in each simulation. The spatial influence factors could be also the result of very complex modules like urban or environmental planning.

The spatial influence factors are the connection between the forecasting model and the external simulation environment. The flexibility of the forecasting model using these explanatory variables defines its interface capability.

2.5. INTERACTION BETWEEN FORECASTING AND PLANNING

Forecasting is often mistaken with planning. Planning is concerned with the idea of what we imagine for the world, while forecasting is concerned with the idea we have about what the world will be. There are two kinds of interaction between forecasting and planning:

- Planners use forecasting methods to produce information necessary for the planning process. In our problem, knowing the geographical pattern of consumption is essential for electricity distribution planning.

- Planners can use forecasting methods to predict outcomes of an alternative plan. In our problem the implementation of electricity distribution plans influences the consumption because the consumption is restricted by the capacities of the electric distribution infrastructures.
Sec. 2.5 - Interaction Between Forecasting and Planning

The Figure 2.6 summarizes the relationship between electricity distribution planning and spatial load forecasting. The spatial load forecasting uses diverse spatial influence factors. Some influence factors represent the real world environment and other result from urban or environmental planning. But other important influence factors are information about the electricity distribution planning. The costs associated with the plans and the outcomes associated with the demand allow the evaluation of outcomes. If the outcomes are not satisfactory they can implicate the revision of the electricity distribution plans. If the outcomes are satisfactory the plans are implemented and the outcomes are monitored supplying new environment for forecasting.

There is a loop of interaction between planning and forecasting. The electricity distribution planning uses the load forecasting result and the load forecasting is dependent of the electric distribution plans. An iterative simulation process between the forecasting and the planning models should be used to perform this loop interaction.

The extraction of spatial load forecasting results to use in distribution planning models is discussed in chapter 6 and 7 and consists of the association of small area load information to graph structures (points and lines) representing electric network equipment. This load information associated to the corresponding network equipment is essential for electricity distribution network planning.

Considering electricity distribution plans in spatial load forecasting is a question of problem formulation, and could be done by including the equipment capacity constraints
as an external dynamic influence factor. The measures of closeness to the limiting capacity of equipment could be converted to small area surfaces and used as one of the inputs to the forecasting model. The knowledge base of the forecasting model should have the behavior of load growth as a function of the local capacity constraints. The behavior is captured from historical responses to similar situations and from knowledge of human experts.

Repressed demand is expected in areas with capacity factors near the limit. In a case of network expansion in a region not previously covered by the electric network, the load development could be highly affected; however stand-alone electricity technologies could be intermediary solutions triggering the consumption growth.

The spatial load forecasting model presented in this thesis is prepared to model the interaction with the electricity distribution planning by automatically capturing the characteristics of load growth behavior under capacity constraints. However, the study of the dynamic behavior under capacity constraints is a complex problem that needs deeper studies that are out of the scope of this thesis.

2.6. CONCLUSIONS AND REMARKS

This chapter introduces Spatial Load Forecasting by addressing the most important aspects of the problem. This introductory view reflects the author perspective about the scope of SLF, positioning SLF relatively to general forecasting, identifying SLF problem, and orienting the discussion to the implementation in spatial analysis framework.

Stressing the importance of SLF, it must be underlined that distribution planning exists to design good solutions to supply load. Thus, because the load forecasting and its uncertainty are the basis for all distribution planning, Spatial Load Forecasting models require the highest attention in distribution planning methodologies.

In the forecasting science there is a vast set of forecasting methods with different applications. These methods are based in two types of knowledge sources: the statistical information, extracted from historical behavior, and the judgmental information, extracted from human expert knowledge. SLF could be implemented by several types of classical forecasting models, but in a spatial analysis framework. Our proposal of SLF models is
designed to extract statistical and judgmental knowledge, and store it in an computationally efficient and human interpretable knowledge base.

To describe the scope of SLF, we discussed aspects related with the possible spatial structures, the choice of the analysis resolution, the load growth behavior, and the spatial influence factors. The spatial structures may be based on equipment areas, uniform grid partition or heterogeneous grid partition. The adequate structure depends on the structure of the information base and on the computing efficiency of the spatial analysis for the spatial structure. The selection of the appropriate resolution depends on the planning purposes (equipment voltage level, time horizon, and urban morphology), on the uncertainty for input data, and on the accuracy required for forecasting outputs.

The general load behavior is characterized by growth curves with sigmoid shape, and this shape is influenced by the spatial resolution. We also introduced that load growth behavior depends on three components (customer-count, per-capita consumption, and distributed generation), all with a specific spatial behavior.

The SLF model represents for distribution planning the interface between the utility planning information and the multidisciplinary external influence factors. External and internal influence factors constitute the inputs of the SLF. There are different types of spatial influence factors, modeling different effects, and playing different roles in the spatial behavior dynamics. The number of influence factors defining a phenomenon should be as minimum as possible. Influence factors should be selected carefully in order to allow the portability of the knowledge base to different time and space environment. Recent developments in spatial analysis science allow efficient and complex pre-processing analysis for SLF inputs. The influence factors are the link between different planning sectors, allowing collaborative methodologies and allowing to the planners the scenario specification.

Closing the chapter, we discussed the framework of interaction between the forecasting and the planning models. This interaction is an iterative process because planning models use load forecast results and there are influenced by electricity distribution plans. The spatial load forecast could model the influence of distribution plans by using measures of capacity constraints as influence factors. This influence of predefined plans could be automatically captured by spatial load forecasting models but this issue deserves deeper studies not addressed in this thesis.
2.7. REFERENCES


Chapter 3  **Fuzzy Spatial Model**

Spatial Load Forecasting (SLF) could be defined as a spatial model to explain and simulate spatial behaviors related with electricity consumption. These spatial models are used to model many kinds of spatial phenomena and consist of a set of GIS operations combining spatial information elements to predict and explain the behavior of the spatial phenomena. The author proposes a GIS spatial model called Fuzzy Spatial Model (FSM) to models this spatial behavior. The Fuzzy Spatial Model is an inference model able to capture the knowledge rules of the spatial phenomena, and simulate them in different space and time environments. The scope of applications of this model could be very large, nevertheless in our approach it is used to model and simulate the spatial behavior of electricity consumption. Several modules including a fuzzy system and a cellular automaton compose the Fuzzy Spatial Model.

This chapter describes the Fuzzy Spatial Model. In the first part the general structure of the SLF is presented. The second part discusses several aspects of the Fuzzy Inference Systems module, including its structure, functionality and training. Finally, the third part describes the Cellular Automaton module and the coordination between modules.

### 3.1. Structure of the Fuzzy Spatial Model

Two main modules compose the Fuzzy Spatial Model. The first one is the Fuzzy Inference System (FIS) that estimates the suitability, which is an indicator of the potential for development. The second is the Cellular Automaton (CA) that spreads the global trending over all the region based on the preferences indicated by the suitability maps. The results of the CA module are the effective geographic distribution of the development.

The model is fed with global forecasting values. The global forecasting is external and could be implemented by many forecasting models (trending, econometric models, diffusion of innovation). The methods used for global forecasting are out of the scope of this thesis. From now we admit that a list of global forecasting values, for each time stage, is supplied by external information sources.
The Scenario Coordinator (SC) links the FSM with the forecasting environment and coordinates the dynamics of the simulation. The SC coordinates the inputs of the Fuzzy Inference System and Cellular Automaton along the several time stages. The coordination of the internal influence factors is the main responsible for dynamic behavior of the FSM. The coordination of the external influence factors defines the link of the FSM with the external spatial environment and also the link with the decision-makers that identify the scenarios. Preprocessing the inputs is also one of the functions of the scenario coordinator. Some of the preprocessing functions could have the form of very complex spatial models, defined specifically for the application and independent of the general FSM kernel.

![Diagram](image_url)

**Figure 3.1 – Structure of the Fuzzy Spatial Model**

The Fuzzy Inference System stores the spatial knowledge base, as a set of comprehensive rules, used to simulate spatial phenomena. These rules may be generated automatically from the historical observation of the spatial influence of geographical factors, or directly generate by the planner in order to simulate future behaviors of the phenomena. The Fuzzy Inference System generates a continuous map of suitability for development. Different maps are generated for each time stage as a function of the correspondent spatial inputs (geographical coverages) stated by the scenario coordinator. The FIS dictates the dynamic behavior along the time stages based on the knowledge base.

The results of the FIS (suitability maps) are used by Cellular Automata to produce discontinuous maps of events (develop or not develop), following the spatial distribution pattern of the suitability maps and respecting the event-count restrictions imposed by the global forecasting. In other words, the CA spreads units of development, oriented by the
suitability maps, until the global forecasting values are reached. The behavior of the CA is controlled by four tuning parameters, and it doesn’t use any knowledge base. The CA only controls the dynamics behavior within each time stage.

3.2. FUZZY INFERENCE SYSTEM

The research on fuzzy systems, started by Zadeh in 1965 [57], has been intensive in the last 30 years, and their theory and applications are too vast [58]-[75] to be presented in the body of this thesis. Annex A1 was written to cover minimally the theoretical background on fuzzy systems complementing the fuzzy system concepts used in this thesis.

For the purpose of this thesis, we define a Fuzzy Inference System as a set of rules representing the knowledge base that reproduces the behavior of phenomena, by observing the system input data and producing a set of inferred output data. Fuzzy Systems attempt to obtain more flexibility and more effective capability to handle and process uncertainties in complicated and ill-defined systems by modeling human thinking through a linguistic approach. The linguistic approach of system modeling can be formulated in three distinguishing features:

- **The use of linguistic variables associated or in addition to numerical variables.**
  Linguistic concepts like “close to the road”, “location with high environmental protection” or “medium urban development” are examples of linguistic representation of variables.

- **The characterization of simple relations between variables by IF-THEN fuzzy rules.** Relation between causes and consequence could be represented by a simple rule like “IF road is close THEN urban development is high”, where an antecedent part (cause) implicates the consequent (effect).

- **The formulation of complex relations by fuzzy reasoning algorithms.** The rule proposition could be composed by multiple antecedent connectives like “IF road is close AND urban center is close AND ...”. The system could be defined by a large set of rules with different grades of activation. The aggregated result represents the real system behavior.
Therefore, the main characteristic of Fuzzy Inference Systems is that they are based on the concept of fuzzy partitioning of information. The fuzzy partitioning allows for more elaborated considerations of the boundaries between the regions\(^1\) based on a weighted combination of the outputs for neighboring regions. This effectively allows a more gradual transition from one output region to another.

3.2.1. Fuzzy system structure

A variety of structures could be used to model the fuzzy systems depending on:

a) the type of fuzzy rules (Sugeno or Mamdani);

b) the operators used in the inference system (composition, conjunction, implication, aggregation, ...);

c) the characteristics of membership functions used in antecedent premises and in consequents (shape parameters, overlapping, hyperspace partitioning, ...);

d) the type of fuzzyfication;

e) the type of defuzzyfication (mean of maxima, center of gravity, ...);

f) the techniques used for modeling and training fuzzy inference systems.

Details about these aspects are discussed in Annex A. Apparently a variety of structures could be used to implement the Fuzzy Spatial Model, and the field is open for other new implementations, research and experiments. However, in this thesis, the characteristics of the problem that we aim to solve give us an orientation for selecting adequate structures. For this analysis several characteristics of the problem are identified and used to orient the definition of the fuzzy system structure:

- The conjugation of statistical with judgmental information is one of the important conditions for the proposed fuzzy systems. This demands high interpretability, by humans, of the antecedent and consequent membership functions.

- The FSM problem is characterized by very large set of geographical cells (near a million cells per map); and the number of significant variables is quite limited (about 5 variables). These facts were the motivation for an implementation based on the GIS spatial analysis functions instead of a GIS coupling with external fuzzy

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\(^1\) The term "regions" means here regions on the fuzzy system hyperspace. However, due to the characteristics of the geographical application, these regions could be imagined on a geographical space, but with other dimensions associated.
system modules. This approach is more oriented to the optimization of the geographical analysis than to the optimization of the FIS.

- On GIS implementation, stacks of maps represent the membership functions of the fuzzy system. This is a motivation to the use of memberships with the minimum number of parameters possible.

- To minimize the fuzzy inference operations the number overlapping input membership functions should be kept as minimum as possible. This is the reason for the use of normal fuzzy partitions, which allows in practice to consider only the overlapping of two memberships.

- The historical data appear as input/output sets of maps. The training requires sequential adaptation along several historical stages. The training technique requires adaptive map-by-map adaptation (not point by point) processes.

- Forecasting (the future) requires that the statistical historical knowledge must be adjusted and complemented with human judgmental knowledge. This imposes that the system should learn from input/output data first and on a second phase the rule-base should be adjusted, complemented by human knowledge about future behavior.

- We have no guarantees that both the historical data and the human knowledge cover all the rule hyperspace, representing behaviors that may occur in the future. This fact requires the use of techniques for rule interpolation in order to cover the blank spots related with the non-completeness of the rule base.

- Because the human judgmental knowledge is very important in this forecasting problem, the number of rules should be as limited as possible and highly interpretable. Limiting the number of rules is very difficult when the number of variables and the complexity of the real phenomena increases. For this reason, it is necessary to implement fuzzy reasoning mechanisms to translate the meaning of sets of rules to better interpretable forms for humans.

Based on these perspectives of the problem the following picture of fuzzy inference systems arises. In order to maintain the interpretability of the input and output
memberships the space partitioning is determined by the user and, for antecedent memberships, it will not change with training. This means that only consequent parameters have to be identified in the training process. A normal partition is recommendable in order to minimize in practice (to two) the number of membership functions overlapping. To decrease the number of map layers, used in the FIS implementation, the membership functions should be described by only three parameters (triangular fuzzy numbers). Normal fuzzy partitions with triangular fuzzy numbers are the most appropriate to minimize the number of parameters to store and to allow the use of a partition by range (the Figure 3.2 presents a partition defined by 4 ranges). With these options the $n$ input memberships for a variable $x$ are defined by only $n-1$ parameters. Another advantage of this configuration is the use of GIS reclassification functions to attribute the membership labels. For coding advantages, numerical labels instead of natural language will be used for membership labels.

![Figure 3.2 – Example of a membership partitioning. The normal partition is composed by 5 triangular fuzzy numbers characterized by 4 parameters. The user defines the shape of the membership, with higher concentration on ranges that require higher detail. The memberships are labeled with integer numbers.](image)

When choosing the type of rules, between Sugeno type [76] and Mamdani type [77] (more information in Annex A), it is necessary to weight the interpretability and the complexity of implementation. Two types of structure seem adequate for the modeling: the Mamdani type, with consequents represented by normal partitions with triangular fuzzy numbers, and the zero-order Sugeno type. In relation to the interpretability, the two models have similar level for inputs and outputs, but Mamdani type allows better rule interpretation based on linguistic labels. In relation to the complexity in implementation, the zero-order Sugeno it is quite simple especially in defuzzification and training. The computational effort motivates the choice of the zero-order Sugeno model.

The structure of the zero-order fuzzy system is the presented in the Figure 3.3. The matching on fuzzy proposition "$x'_i$ is $A_{ij}$", is given by (3.1), where $x'_i$ is the numerical input for variable $x_i$ and $A_{ij}$ is the membership labeled $j$ on this variable.
\[ \alpha_{i,j} = \mu_{A_i}(x'_i) \quad (3.1) \]

The support value for rule \( r_k \) is given by:

\[ \beta_k = \prod_i \mu_{A_i}(x'_i) \quad (3.2) \]

Meaning that the T-norm "product" is adopted to represent the intersection of fuzzy sets. The final output \( y' \) is the weight sum (3.3), where \( b_i \) is the zero-order function coefficient or weight for rule \( r_i \) and \( N_i \) is the number of rules.

\[ y' = \frac{\sum_{i=1}^{N_i} \beta_i \cdot b_i}{\sum_{i=1}^{N_i} \beta_i} \quad (3.3) \]

The implementation of this fuzzy inference system in GIS spatial functions is particularly interesting because the operations proposed by the equations (3.1)-(3.4) are applied simultaneously on all geographical cells. The implementation requires: maps with activated membership labels (two for each variable); maps with matching values \( \alpha_{i,j} \) (two for each variable); maps with coding for activated rule (\( 2^N \) maps, where \( N_i \) is the number of variables); maps with the support values \( \beta_i \) (as many as the number of maps for coding activated rules); maps with lookup values \( b_i \) associated with support values \( \beta_i \) (as many as the maps with the support values).

![Figure 3.3 - Graphic representation showing the zero-order Takagi-Sugeno inference method.](image)

Rule coding is important, for implementation in GIS, to identify the rules and access the lookup tables containing the rule database. The rule code \( cod_k \) is a unique integer value
representing the rule $k$. Once knowing the rule code it is easy to identify the membership labels of the rule antecedents. The coding of the rule $r_i$ is done by:

$$\text{cod}_i = \sum_{l=0}^{N_j^2} L_{ij} \cdot (N_{l_{max}})$$

(3.4)

where $L_{ij}$ is the membership label for label $j$ on variable $X_i$, the $N_{l_{max}}$ is the maximum number of membership labels for variables $X_i$, and $N_j$ is the number of variables.

![Figure 3.4](image)

Figure 3.4 – Simple example with only two variables, showing: a) one of the maps with rule coding; b) the corresponding maps of support value $\beta_i$; c) the output value $y'$. The two input variables correspond to “Distance to urban center” and “Distance to roads”, partitioned in 5 linguistic labels. The output value is the potential for domestic development [0,1] (suitability map).

Even with this simplified Takagi-Sugeno system the implementation in GIS still computationally heavy. For instance, 5 variables with 2 mandatory membership activation per variable need $2^5 = 32$ maps of rule codes and the same number of maps of activation values $a_{ij}$. These maps must be stored in memory in order to parallel-compute the correspondent 32 maps of base values $\beta_i$ and finally the $y'$ is computed by using look-up functions to find maps of $b_i$ in the rule database. In total this simple example needs to store and operate simultaneously more than 128 maps. This process takes about 3 hours (in a 200MHz Pentium III computer) even for relatively small maps (30000 cells). To overcome this problem we can use only the highest activation values, which allow the reduction of 80% of memory and computing time with imperceptible changes on results.
3.2.2. Fuzzy system training with statistical information

There are several approaches for rule generation [78]-[82], some based on diverse kinds of fuzzy inference [83]-[88], using genetic algorithms [89]-[95], or using neuro-fuzzy approaches [75], [96], [97]. In this thesis we use a fuzzy-inference method based on local learning processes [98].

Because the input memberships are fixed, the training of the zero-order Sugeno fuzzy system consists only in the identification of the zero-order coefficient \( b_i \) for each rule \( r_i \). There are two kinds of training: the local learning and global learning [98]. For global learning algorithms the parameters of the model are identified using the whole training data set in a single algorithm operation. Local learning uses only the data set in the neighborhood of each rule for locally fitting the rule. Each point in the neighborhood is weighted according to its distance to the centroid of the rule. Global learning is appropriated when the system has important global behavior that should not be influenced by local discrepancies. Local learning is in general based in simpler models, decomposed in multiple sub-models, and with better efficiency on locally well-behaved problems. The typical local characteristics of our problem, and also the modeling of inputs by normal partitions, restricting the system interaction to neighboring rules, motivate us to use local learning.

Local learning computes the parameters of the model by minimizing the objective function (3.5), where \( y_r \) is the target output, (desired output or verified output for real system) and \( b_i \) is the weight value for rule \( r_i \), which is the rule to be created.

\[
E = \sum_{p=1}^{N_r} \beta_i \cdot (y_p - b_i)^2
\]  

(3.5)

Note that the error sum \( E \) is done for all the training set \( N_r \), however the support value \( \beta_i \) assumes null values when the rule \( r_i \) is not activated, which limits the sum to the local points (local learning). Contrarily to the global learning, for local learning the estimation of \( b_i \) doesn't depend on the zero-order parameters of the other rules. The application of an ordinary least squares approach the zero-order coefficient \( b_i \) is determined by the simple weighted sum (3.6).
\[ b_i = \frac{\sum_{p=1}^{N} (\beta_i \cdot y_p)}{\sum_{p=1}^{N} \beta_i} \] (3.6)

In the GIS implementation we first use maps of outputs \( y_p \) to compute the several maps \( (\beta_i \cdot y_p) \); in the next step we compute zonal-statistics maps by summing the \( (\beta_i \cdot y_p) \) in regions (zones) defined by the rule-coding maps. The \( b_i \) value is computed at the end of geographical zonal-sums. The results of \( b_i \), the sums \( \sum_{p=1}^{N} (\beta_i \cdot y_p) \) and \( \sum_{p=1}^{N} \beta_i \) are stored as rule parameters in a rule lookup table. The sums stored in lookup table could be used for parameter adjustment if new input/output data are added to the training set. In this case the adaptation is not done point by point but map-by-map.

### 3.2.3. Fuzzy rule interpolation

When learning from input/output data, only the rules implicitly represented in the learning set are generated. This leads to the incompleteness of the rule set, and when we run the fuzzy system on new hyperspace regions the so called blank-spots may appear, which reflects the lack of rules to represent these new hyperspace regions. Interpolating and extrapolating the existing rule base may solve this problem. We propose the following method to interpolate rules.

If we admit that the rule base is continuous, requiring that rules having adjacent antecedents also have adjacent consequent, the proximity of rules on antecedents (inputs) implicates a proximity in consequent (outputs).

![Figure 3.5](image_url)  

**Figure 3.5** – The figure shows an illustrative example, the interpolation of a new rule \( r_{new}(A_{new}, b_{new}) \) for a fuzzy system with only one variable \( x_i \). The \( b_{new} \) value is the rule zero-order Sugeno parameter estimated based on the parameters \( b_i \) for other existing...
rules \( r_i \). The interpolation is weighted by the distance \( d_{new, k} \) between the center of maxima of the new rule and the existent rules.

Admit that we wish to generate a new rule \( r_{new}(A_{i,1}, \ldots, A_{i,h}, \ldots, A_{N,1}, \ldots, b_{new}) \), where \( A_{i,h} \) are the known input memberships for variable \( x_i \) with correspondent label \( f_i \), and \( b_{new} \) is the output value (zero-order parameter for Sugeno function) to be estimated for rule \( r_{new} \). The input distance \( d_{new, k} \) between rule \( r_{new} \) and other rules \( r_i \) could be obtained by the sum, for all input variables, of the square difference between the center of maxima \( CM(A) \) of the two rules. The distance should be normalized for the several inputs by dividing by the range of the variable \( R_i \).

\[
d_{new, k} = \sum_{i=1}^{N} \left( \frac{CM(A_{i,new}) - CM(A_{i,k})}{R_i} \right)^2
\]  

(3.7)

There is a set of points \( d_{new, k} \), one for each existing rule, that can be used to estimate the \( b_{new} \). For each rule the proximity may be estimated by function (3.8), based on weighted least squares, weighted by the inverse of the distance.

\[
F_{new, k}(d_{new}) = c_{new,0} + c_{new,1} \cdot d_{new} + c_{new,2} \cdot d_{new}^2 + \cdots + c_{new,n} \cdot d_{new}^n
\]  

(3.8)

where \( c_{new,n} \) are the coefficients of the polynomial function \( F_{new, k}(d_{new}) \) for rule \( r_i \), and \( d_{new} \) is the distance variable. The error function to be minimized is (3.9), weighted by the inverse of the distance, and using a set of \( N_x \) points \((x, y) = (d_{new, k}, b_i)\), one for each existing rule.

\[
E = \sum_{i=1}^{N_x} \frac{1}{d_{new, k}} \left( F_{new, k}(d_{new, k}) - b_i \right)^2
\]  

(3.9)

As a result of the minimization, by using ordinary least squares or other numeric methods, we obtain the polynomial coefficients \( [c_{new,0}, c_{new,1}, c_{new,2}, \ldots, c_{new,n}] \) for the new rule \( r_{new} \). The value of \( b_{new} \) will be the value obtained from the distance function when the distance tends to zero \( b_{new} = F_{new, k}(0) \).
\[(x, y) = (d_{new,k}, b)\]

\[F_{new,k}(d_{new}) = 1.045 + 2.407 \cdot d_{new} - 1.088 \cdot d_{new}^2 - 0.918 \cdot d_{new}^3 + 0.047 \cdot d_{new}^4\]

![Graph](image)

Figure 3.6 – Example of the generation of a new rule by interpolation. The value obtained for rule output is \(b_{new} = 35.045\).

This method allows not only the generation of a rule by interpolation but also by extrapolation. In fact, the method should be considered as a trending method for fuzzy rules. This ability is particularly useful for the Fuzzy Spatial Model (FSM) because most of the blank-spots correspond to rules that represent future forecasting behavior in the extreme of the hyperspace.

### 3.2.4. Fuzzy system tuning with judgmental information

As mentioned before the spatial behavior is learned based on input/output historical data and on human judgment knowledge. The last source of information is particularly important in spatial load forecasting, where the future behavior is not only a continuity of the past behavior but is also dependent on new future behavior that can be foreseen by human experts. Therefore the rule base resulting from input/output training should be adjusted and complemented by human knowledge. We propose the following reasoning mechanism to create and tune system rules based on judgmental information described by human experts.

Human experts define the judgmental information in the form of rules (judgmental rules). For a judgmental rule \(r\), the input and output variables \((X, Y)\) are the same as the ones used in the fuzzy system rules. However, the inputs \(x\) are fuzzy memberships \(A_i\) that could be different from the linguistic labels used in the fuzzy system \(A_{ij}\). The consequent \(b_i\) must be the crisp value proposed by the human expert. This means that for the fuzzy system relation \(R\), the following judgmental rule must hold:
\( r_s : \text{if } \text{AND}(A'_i \text{ is } A_{is} \text{ and } A'_j \text{ is } A_{is}) \text{ then } y \text{ is } b'_s \)

This judgmental rule is represented by the compositional rule of inference. The results \( B'_s \) for judgmental information should be inferred from the composition between the judgmental fuzzy memberships \( A'_i \) and the fuzzy system relation \( R_i \):

\[
B'_s = A'_i \circ R_i 
\]  
(3.10)

The process of tuning new rules into the system rule-base has the following steps:

- Interpolate all system rules that could be matched by judgment rule precedents. This procedure creates and initializes the non-existent rules, but only the ones that will be tuned.

- Compute the fuzzy system output \( y_s \) for the fuzzy inputs \( A'_i \). This is the system output for judgmental rule inputs before tuning.

- Adjust the system rule base (values \( b'_s \)) to fit the system output to the value \( b'_s \) imposed by judgmental rule.

The differences between fuzzy and crisp inputs are only in the process of computing the matching value \( \alpha_{i,j} \). When fuzzy inputs are used the matching values are computed by:

\[
\alpha_{i,j} = \sup \min(\mu_{x_i}(x_i), \mu_{A_{is}}(x_i)) 
\]  
(3.11)

The graphical representation of this operator is presented in Figure 3.7. Several possibilities are open for fuzzy input matching. For instance, if the \( A'_i \) covers all the input partition, this represents the label “don't care”, for which the result of the fuzzy system is independent of variable \( x_i \), accepting all values in the variable range. Other interesting situation is when \( A'_i = A_{is} \) for each variable \( x_i \), which means that the antecedents of the judgment rule \( r_s \) are exactly the same as the antecedents of one of the system rules \( r_i \). Note that in this situation not only the input membership \( A_{is} \) is activated (with value \( \alpha_{i,j} = 1 \)) but also the adjacent memberships are activated \( (\alpha_{i,j-1} = \alpha_{i,j+1} = 0.5) \).
Figure 3.7 – Graphical representation for the antecedent-matching operator, on variable $x_i$, with uncertainty input $A_i$. In this example the fuzzy input activates four linguistic labels. Note that the number of system rules activated depends on the activation through all variables, and not only one as shown in the figure.

The base value for each activated rule is computed by:

$$\beta_i = \prod \alpha_{i,s}$$

(3.12)

The system rules are adjusted, for each judgment rule, by an adaptative training heuristic using the equation (3.13). The adjustment $\Delta b_i$ is a function of the deviation between the parameter value $b_i$ for system rule $r_i$ and the output value $b'_i$ proposed by judgmental rule $r_i$. The adjustment for each system rule is weighted by the correspondent base value $\beta_i$. The weight of the judgmental rule $r_i$ on the adjustment is regulated by the learning rate $\delta_i$ in range $[0,1]$.

$$\Delta b_i = \frac{\beta_i}{\sum_{i=1}^{\infty} \beta_i} \cdot (b'_i - b_i) \cdot \delta_i$$

(3.13)

For the coordination of the adaptive training it is preferable to begin from judgment rules with large support on the antecedent memberships, and continue through increasing rule specificity. More specific judgmental rules affect less number of system rules and consequently reduce the risk of inconsistency with the previous adjustments done by other judgmental rules.

### 3.3. Cellular Automata

In the fuzzy spatial model the cellular automaton (CA) is the module responsible for the spreading of the global forecasted value over all the geographical region, following the suitability pattern computed by the fuzzy system.

John Von Newman first introduced the CA theory in the 40th century and gained considerable popularity two decades later through the work of John Conway in the game of life. Some
of the most important references on CA are [106]-[109]. The simulation of spatial behavior is one of the most promising applications of CA [34],[39], [42], [43], [44]. Some of these CA applications have been implemented in GIS spatial structures [110]-[115].

A Cellular Automaton is a discrete dynamic system. Space, time, and the states of the system are discrete. Each point in a regular spatial lattice, called a cell, can have any one of a number of possible states. The state of each cell in the lattice is updated according to a local rule. The state of a cell at a given time depends only on its own state in the previous time step, and on the state of its neighbors. In our FSM problem the cells are the geographical grid cells, the time stages are simply algorithm iterations of the CA, the states of the cell are the possible development states, and the local heuristic rules are represented by appropriated functions.

In our formulation, at any specific moment in time \( t \), the \( CA' \) automaton is a collection of binary states \( e'_{i,j} \) in cell location \((i,j)\), with value 1 if a new development is added to the site and 0 if no development is added.

\[
CA' = \{ e'_{i,j} \}_{0 \leq i \leq r, \ 0 \leq j \leq c, \ \forall e'_{i,j} \in E}
\]  

(3.14)

where \( E \) is the finite set of states, \( r \) and \( c \) are the number of rows and columns of the map grid. The sum of all development along all the algorithm time stages gives the possible development states for \( CA' \).

A local rule specifies the iterative process of the CA and triggers the state transition from non-developed \( e'_{i,j} = 0 \) to developed \( e'_{i,j} = 1 \). The state transition is triggered according to the following rule:

\[
\text{if } P'_{i,j} > P_t \text{ then } e'_{i,j} = 1 \text{ else } e'_{i,j} = 0
\]  

(3.15)

where \( P'_{i,j} \) is the suitability (or potential for development) value for cell \((i,j)\) on time stage \( t \). \( P_t \) is a selective value specified from for CA parameters.

A selective parameter \( \varphi \) fixes the level of potential requirement for development. The selective value \( P_s \) is computed as a function of the selective parameter \( \varphi \) (value in range

---

2 The time stages in the CA theory are not the scenario time stages that we discuss in this thesis. In the FSM the time stages are algorithm cycles that occur inside one scenario stage, and may not have relation with the time evolution of the spatial phenomena.
[0,1]) and the maximum and minimum value for suitability grid computed by equation (3.16). Thus the selective parameter $\varphi$ represents the level of selectivity in the range of the existent suitability grid values (higher values represent higher selectivity of potential necessary for development):

$$
P'_s = \varphi \cdot \max_{ij} \{P'_{ij} \} + (1 - \varphi) \cdot \min_{ij} \{P'_{ij} \}
$$

(3.16)

For each iteration the development $D'_{ij}$ is computed based on the development from the previous iteration $D^{i-1}_{ij}$ (initialized with $D^0_{ij} = 0$) and on the development increase for the present time stage. The development increase is quantified in discrete steps $D_{\text{sep}}$ (in general one uses $D_{\text{sep}} = 1$).

$$
D'_{ij} = D^{i-1}_{ij} + e'_{ij} \cdot D_{\text{sep}}
$$

(3.17)

If $P^0_{ij}$ represents the potential development expected for the cell, then the value $(P^0_{ij} - D'_{ij})$ represents the adjustment of the development allowed on the cell for the next iteration. $P^0_{ij}$ is the initial value of the suitability obtained from the fuzzy system. The suitability for the next iteration is recalculated using the following formula:

$$
P'^{n+1}_{ij} = \alpha \cdot \left( P^0_{ij} - D'_{ij} \right) + \beta \cdot \frac{1}{8} \sum_{s} \left( P^0_{ij} - D'_{ij} \right) + \lambda \cdot e'^{n+1}_{ij} \cdot (P^0_{ij} - D'_{ij})
$$

(3.18)

The formula includes three components:

- a) positive feedback from the cell in the previous iteration, weighted by $\alpha$.
- b) neighborhood effect based on 8 adjacent neighbor cells $\Omega_{ij}$, weighted by $\beta$.
- c) innovation factor modeled as random noise $e'^{n+1}_{ij}$ in the range [0,1], weighted by $\lambda$.

The $\alpha$, $\beta$ and $\lambda$ are the weights for each component, with values in the range [0-1] and verifying the following condition

$$
\alpha + \beta + \lambda = 1.
$$

(3.19)

These values are parameters of the cellular automata and should be specified in accordance with the effect expected from the CA dynamics. The positive feedback, the
most important component, is simply related with the suitability in the previous stage and the range of values for $\alpha$ is $[0.3, 1]$. The neighborhood effect allows the modeling of the neighborhood influence, with higher values for $\beta$ implicating enhanced influence from adjacent cells, and the range of values used for $\beta$ is $[0, 0.5]$. If many different behaviors exist in neighboring cells the use of low values for $\beta$ is recommendable. The innovation factor represents a random behavior deviation, dependent on the deviation we tolerate; usually the values of $\lambda$ are quite low $[0, 0.2]$.

The CA algorithm iterates along the time stages until the sum of all cell development $D_{ij}$ reaches a target value with an error tolerance $ert$. The target value is the forecasted value $G$, to be spread over the region. When the forecasted value overflows the global forecast value $G$ the adjustment is done by repeating the iteration with higher value for $\varphi$.

\[
\text{IF } \left( \sum_{i} \sum_{j} D_{ij} < G - \sqrt{ert} \right) \text{ THEN new iteration } t + 1
\]

\[
\text{ELSE IF } \left( \sum_{i} \sum_{j} D_{ij} - G \right)^2 < ert \text{ THEN stop CA}
\]

\[
\text{ELSE repeat iteration } t; \text{ set } \varphi = \frac{\varphi + 1}{2}
\]

In the Fuzzy Spatial Model the CA it may be seen as the module that defines the spreading pattern, transforming a continuous map of suitability in a discrete map of effective development, and adjusting the development to the global forecast. Figure 3.8 illustrates the function of the CA in the Fuzzy Spatial Model.

![Figure 3.8 - Example of the inputs and outputs of the CA. a) represents the continuous suitability map resulting from the fuzzy system, b) the red spots represent the discontinuous map resulting from the CA. Darker red spots represent higher development values.](image)
Contrarily to other implementations in spatial models ([39], [42], [43], [110]-[115]), the dynamics and the morphology of the Fuzzy Spatial Model are not only modeled by the CA heuristic rules above. In fact, the dynamic behavior of the FSM, representing the pattern change in time, is mostly modeled by the fuzzy system knowledge base, where the internal dynamic influence factors have the main role in the modeling of the dynamic behavior. However, the pattern transformation from suitability maps into development maps, for a specific stage, is affected by the parameters of the CA.

The neighborhood effect, parameterized by weight $\beta$, has an effect of continuous spatial diffusion, with higher diffusion for higher $\beta$. The innovation factor, parameterized by $\lambda$, simulates a random deviation from the standard pattern behavior. Higher weight $\lambda$ originates higher discontinuity in the development patterns. Obviously, the innovation factor is a strong source of uncertainty, introduced by the model. The positive feedback is given by $\alpha = 1 - \beta - \lambda$ and represents the proximity between the development pattern and the suitability pattern, suggested by the fuzzy system knowledge base.

The selective value $\varphi$ is an important parameter of the CA. The value of this parameter controls the way the CA selects the locations for development and controls the number of developments per cell. Higher values for the selective parameter implicate more selective criteria, concentrating the development on cells with high suitability. When the $\varphi$ is low the development is widespread to lower suitable cells and the number of developments for the most suitable cells decrease. In general, the values for the selective parameters vary in the range $[0.8, 0.999]$, but an appropriated value depends on the combination of several characteristics of the problem (magnitude of the global value, variance of the suitability map, resolution, and pattern morphology). Higher global values and higher variance of the suitability require lower values for $\varphi$.

The tuning of the CA parameters $\beta, \lambda, \varphi$ is done by using numerical methods that minimize the error function $E$, measuring the sum of the difference between the CA result map $\{D^*_t(\beta, \lambda, \varphi)\}$ and the historical development maps $\{H^*_t\}$, for scenario time stage $h$.

$$E = \sum_t \sum_i (D^*_t(\beta, \lambda, \varphi) - H^*_t)^2$$ (3.20)

The positive feedback parameter is determined by the equation $\alpha = 1 - \beta - \lambda$. 
3.4. CONCLUSIONS AND REMARKS

This chapter presents the mathematical formulation and describes the GIS implementation of the Fuzzy Spatial Model (FSM). The FSM is an innovative spatial model with inference and simulation capabilities appropriated for the implementation of the Spatial Load Forecasting kernel.

This spatial model is based on the coupling between a Fuzzy System and a Cellular Automaton. The Fuzzy System stores the knowledge base that describes the phenomena behavior, and uses the spatial influence factors to compute the suitability for development. The Cellular Automaton spreads the effective development, given by a global forecasting value for the entire region, in accordance to the potential indicated by the suitability maps.

The fuzzy system has been designed to combine statistical and judgmental information. This is done by using inference models that build the fuzzy rule-base, using sets of historical maps representing the observed developments caused by the several influence factors. To complete the set of rules, an innovative rule interpolation method was developed, allowing the fuzzy rule trending for hyperspace extremes. To tune the rule base and to integrate the human expert knowledge fuzzy reasoning techniques were used, allowing an interpretable and functional interface between human expert rules and system rules.

The CA was inspired in recent spatial methodologies used in urban planning. However, contrarily to previous methodologies, in this model the CA only controls the phenomena dynamics inside each stage. The Fuzzy System controls the phenomena behavior and the dynamics between time stages. This structure is perceived as a great improvement avoiding the inaccurate performance of the CA in describing behaviors and hedging the model from the less understandable and interpretable effect of the CA parameters.
3.5. REFERENCES


This chapter reports the implementation and test of the Fuzzy Spatial Load Forecasting model. This Fuzzy Spatial Load Forecasting is an innovative Spatial Load Forecasting method, based on the Fuzzy Spatial Model described on Chapter 3. The text uses the mathematical formulation presented in Chapter 3 to solve the load forecasting problem discussed in Chapter 2.

The first part of this chapter describes the modules that compose the Fuzzy Spatial Load Forecasting (FSLF) system. The second part is dedicated to the implementation of the FSLF using the mathematical methodology presented on Chapter 3 (Fuzzy Spatial Model). The third part presents an example of simulation obtained with the model. The fourth part presents another example to illustrate the capacity of the model in combining statistical information from historical observations with judgmental information from human experts. The last part presents the accuracy measures used to validate the model, obtained for several processes.

4.1. STRUCTURE OF FUZZY SPATIAL LOAD FORECASTING

In the perspective of this thesis, load forecasting has three level of analysis, decoupling the problem in: global quantification of the number of consumers (global forecasting model); geographical location the development of consumers (land-use model); and temporal characteristics of per-capita consumption (end-use model).

This approach decouples the detailed load forecasting in three different forecasting problems. The global forecasting is the basic forecasting ignoring spatial or temporal consumption behavior. The land-use model simulates the spatial behavior, identifying where the development will occur. The end-use model describes the change in per capita consumption.

The Spatial Load Forecasting model must follow sequentially those three forecasting modules. The global forecasting supplies the global forecast values for land-use and the end-use module completes the consumption information for consumer development.
maps. Afterwards, the forecasted information is geographically aggregated providing a detailed characterization of consumption by equipment service area.

![Diagram](image_url)

Figure 9 – Scheme describing the sequence of the three modules necessary for Spatial Load forecasting. Forecasting the global number of consumers is the first forecasting module. The spatial behavior is modeled by the land-use model. The end-use module models the temporal behavior for per-capita consumption. The aggregation of the forecasting results gives a complete characterization of the loads.

The main scope of this work is the spatial perspective of the problem, thus our discussion is centered on land-use model with less focus on global forecasting and on end-use model. However, some discussion about the two non-spatial models, the global forecasting and the end-use, is required.

4.1.1. Global forecasting

The land-use model uses the results of a global forecasting module. This global forecasting module states the total number of consumers for the entire study region and the land-use module spreads the new customers over the region. For global forecasting, non-spatial forecasting models are used. In this thesis we assume that an external module supplies the global forecast. This module of global forecasting could be implemented in a variety of methods, covering all the forecasting science. For global forecasting, three types of methods could be used: univariate (time-series), multivariate methods and qualitative forecasting methods. A brief description of some models used in global energy forecasting follows.

The time-series methods determine the overall trend in historical data, and develop the forecast by extrapolating the historical trend. These are univariate forecasting methods. Univariate methods include smoothing, exponential smoothing, decomposition, linear trending, and non-linear growth models, among others. The purpose of these methods, in global forecasting, is to model the patterns of past number of consumers to project them into the future. These methods however do not recognize structural changes and are vulnerable to errors.
The multivariated forecasting methods, also known as causal methods, are other kind of methods that could be used for global forecasting. These methods make projections for the future by modeling the relationship between the forecasted series and other series. The external variables are called predictor or independent variables. For example, forecasting the number of consumers may be based on known economic and industrial indicators. Multivariate methods include simple-regression and multi-regression, econometric, multi-equation econometric, multivariate time-series, and a few other advanced techniques. In general multivariate methods are more complex and costly than univariate methods. The additional cost and difficulty in application results from the use of more external data series.

Qualitative forecasting methods are based on the judgment and opinions of others concerning future trends and technological changes. Qualitative methods include market research, panel consensus, scenario analysis, and historical analogy methods of predicting the future. Qualitative methods are useful when there is little data to support quantitative methods. In the public sector, these methods are useful to predict changes in policies and demand for services and specially to project long-run technological changes (technological forecasting). Formal qualitative methods are not usual, however these subjective methods are often used in informal and intuitive ways.

The choice of the best method for global forecasting depends mostly on the data we have available and also on the classes of consumers. If good sets of other external variables (predictors) highly correlated with the number of consumers are available, the multivariate methods are the most appropriate. If we have a good and well behaved set of data about the number of consumers the univariate methods could be a good solution. Finally, for behaviors highly dependent on policies or scenarios, for which we only have subjective data, the qualitative methods could be a solution.

4.1.2. The end-use model

For efficient interaction between modules a decoupling of the causes of load change is necessary. To improve the interaction between global forecasting, land-use and end-use modules the consumers are separated in classes (industrial, domestic, commercial). All the modules are applied by customer class, obtaining a decoupled consumption per class. As an example for a domestic consumer class: 1st we forecast the global number of domestic consumers; 2nd we use the spatial forecasting to estimate the geographical distribution of
consumers; 3rd we estimate the per-capita consumption characteristics for domestic consumers class. Obviously, there are dependencies among the development behaviors of consumer classes. These dependencies are modeled in the land-use module. For instance, domestic consumer development is usually higher near commercial areas and lower near the industrial consumer areas.

The basis of the end-use model is precisely the identification and the decoupling of the causes for per-capita load change. End-use models require a deep decoupling in sub-classes of consumers; this sub-classification is related with the characteristics of the electric appliance acquired by the consumers. The sub-classification of consumer classes by appliance load groups allows in first place the forecasting of installed capacity (kW) and in second place facilitates the forecasting of energy consumption by identifying direct causes for the behavior of appliance usage.

The decomposition of each customer class in sub-classes is mainly based on human expert classification. If possible, the classification should be done as a spatial classification of the fraction of each sub-class, estimating in each location the fraction of each sub-class consumer number. This sub-division must be done only if there is enough information to discriminate the load behavior and the models that represent this behavior. The urban plan directives are excellent spatial information bases for this classification. A variety of procedures may be found in [25], [117]-[125] for load decomposition on end-use models.

Table 4-1 – Example of load decomposition on end-use module.

<table>
<thead>
<tr>
<th>Customer Class</th>
<th>Seasons sub-type</th>
<th>Sub-class</th>
<th>Appliance load group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial</td>
<td>winter, summer</td>
<td>(Industrial type)</td>
<td>Light, heater, electrochemical, motor, electric arc, etc</td>
</tr>
<tr>
<td></td>
<td>mar., aug, dec.</td>
<td>Light industry, heavy industry, chemical industry</td>
<td></td>
</tr>
<tr>
<td>Domestic</td>
<td>weekday, holiday</td>
<td>(house type)</td>
<td>Light, refrigerator, A/C and heater, cooking, etc</td>
</tr>
<tr>
<td>Commercial</td>
<td>flat, single residential, old houses</td>
<td>(commercial-type)</td>
<td>light, computer, motors, A/C, etc</td>
</tr>
<tr>
<td></td>
<td>retail, offices, schools, etc.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Another decoupling level used in end-use models is the classification in seasonal sub-types (winter, summer, January, august, weekday, holiday). The decomposition in seasonal sub-types is mainly based on the time differentiability of load curves. For a specific consumer, differences in periodic cycles of load curves should be clustered in different classes in order to model separately the factors that cause these differences. Contrarily to the sub-class decomposition, which is related to the acquisition of electric appliances, the seasonal sub-type decomposition is associated with the usage of the electric appliances.
We may explain the functionality of the end-use model by decoupling it in two modules: the end-use capacity forecasting related with the number and size of electric appliances; and the end-use consumption related with usage of each electric appliance group.

For the end-use capacity sub-module, one may use qualitative forecasting methods (technological forecasting) to forecast the electric appliance markets and the evolution and diffusion of innovative technologies with different energy consumption characteristics. The number and age of consumers for a specific class is obtained from the results of the land-use model. The age of consumers is very important in developing countries, because consumption increases as long as people get used to electricity. In fact the end-use-capacity sub-module may include spatial variables extracted from external spatial models. For instance, the acquisition of new electric appliances is depends on electric distribution plans; if there is repressed demand, appliance acquisition will be low. Another aspect is the spatial competition with other energy resources like gas network: if there is access to a gas networks some electric appliances will not be acquired due to the gas alternative.

<table>
<thead>
<tr>
<th>Input Variables</th>
<th>Forecasting Modules</th>
<th>Customer classification</th>
<th>Forecasting Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global Forecasting; Spatial influence factors</td>
<td>Land-use Spatial Model</td>
<td>Customer class</td>
<td>Number of consumers</td>
</tr>
<tr>
<td>Electric appliance market</td>
<td>End-use capacity per customer</td>
<td>Decomposition</td>
<td>installed capacity (KW)</td>
</tr>
<tr>
<td>Age of consumer</td>
<td>Economic variables</td>
<td>Decomposition</td>
<td>(number and size of appliances)</td>
</tr>
<tr>
<td>Electric distribution plans</td>
<td>Climatic variables; Typical load curves</td>
<td>Seasonal sub-type</td>
<td>Consumption (KWh)</td>
</tr>
<tr>
<td>Competitiveness with other energies</td>
<td>Demand side management</td>
<td>(usage of appliances)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 10 – Scheme describing end-use model and related aspects. The end-use models use the information from the land-use model and other external variables. End-use could be divided in two parts: the capacity forecasting related with the number and size of electric appliances and the consumption forecasting related with the usage of electric appliances. The end-use model is based in several levels of customer classification.

With the end-use consumption sub-module we obtain the usage of each appliance group. This usage is different for each seasonal sub-type classification; this is why one
needs a seasonal class decomposition and the identification of typical load curves for each
class decomposition. For an end-use consumption sub-module one could use multivariate
forecasting methods. For this kind of forecasting, economic and climatic variables become
important, as well as other predictors like energy cost policies as a consequence of
demand side management.

The results from all these modules are finally aggregated in order to obtain a complete
characterization of the spatial and temporal behavior of load growth. First one aggregates
the consumption of appliance groups for each consumer class then a geographical
aggregation of the consumption for the spatial pattern of consumers is obtained from
land-use model. If one knows the location of the electric distribution equipment, the
aggregation by feeders and substation service areas is also possible.
4.2. **LAND-USE MODEL IMPLEMENTATION**

This thesis is focused on the spatial perspective of the electricity distribution problem. Thus, for spatial load forecasting we concentrate on the land-use module, which supports the gross of the spatial modeling of the consumption behavior.

The kernel of the land-use module is implemented by the Fuzzy Spatial Model (FSM), simulating the spatial behavior phenomena for customer development. The FSM is suitable to simulate complex spatial phenomena, difficult to be modeled by a statistical approach or by theory formulations. The great advantage of this model is its application to simulate phenomena explainable by judgmental information captured from human knowledge. Unfortunately, the model construction requires as basis a large set of input/output data in the form of maps, which in some situations is difficult to gather.

The FSM has a basic structure described in Chapter 3. However, each spatial phenomenon requires a specific modeling specified by a planner. The modeling depends essentially on the knowledge that exists about the phenomena. To model an adequate FSM it is necessary to specify the following aspects:

- **Defining the structure:** by deciding which spatial influence factors (variables) are relevant for the problem, how to model the time dynamics, how to model the spatial interaction, and how to model multiple outputs.

- **Model parameterization:** in this step the system knowledge base is built and the functional parameters of the model are specified. For the knowledge base construction it is necessary to define which knowledge should be learned from statistical data and which knowledge should be adjusted and updated by human knowledge. Some of the tuning parameters of the Cellular Automaton (CA) should be selected in accordance with the best simulation performance for the problem in question.

- **Interaction and coordination:** a FSM simulates a specific phenomenon that interacts with other spatial phenomena. This interaction must be coordinated by specifying the dynamics of data interchange and the specifying the interfaces between modules.
In this section these aspects related with the implementation of the Land-use model are specified. The land-use module is a spatial modeling used to forecast the number of consumers (customer-count). The customer-count spatial behavior is different for each customer class, and must be simulated by different spatial model. The forecast is based on the preference maps for the consumer class development, and based on the global forecasting for the entire region in analysis. These spatial preference maps are estimated based on spatial knowledge bases constructed with statistical and judgmental data sources.

The customer-count forecasting is the most representative application of fuzzy spatial model. The knowledge necessary to build the model is mostly from urban planning. The urban forms are defined by a mix of the influence of infrastructures, human behavior and sustainability policies. The objective of the customer-count model is to obtain the spatial distribution of the development number for several classes of customers.

### 4.2.1. The structure

The structure of the customer-count module is basically the structure of the FSM (see Figure 11), without significant complements.

![Diagram](image-url)

**Figure 11** – General structure for the customer-count module.

The fuzzy system uses a set of geographical influence factors to compute the suitability maps that drive the customer development for each customer class. The cellular automaton uses the suitability maps to spread the global forecasting value through the geographical area, computing the development maps for the customer class in analysis. The dynamics of the module is controlled by saturation level in each location, recalculated for each stage based on the new developments of all the customer classes.

### 4.2.2. The customer classes

The customer classes should be chosen by separating classes with different spatial development behavior. Examples of customer classes with different behavior are the
domestic, industrial, or commercial consumers. Certainly each of this customer type could be divided on other subclasses, however the sub-division is only useful if the influence factors affect ways in different the subclasses. Other important aspect in the selection of the customer classes is the availability of separable data to training the rule base. For instance, if there are only spatial data describing the spatial evolution of general industrial type, the partition in industrial subtypes is useless because the behavior of these separated subclasses is not known. Each customer class has a specific set of influence factors, and it is not necessary that all the different classes use the same input variables. The knowledge base for each class is modeled by a different fuzzy system and consequently by different fuzzy rule bases and different input variables.

4.2.3. The geographical influence factors

The adequate selection of the influence factors is one of the most important aspects of this forecast. There are different kinds of spatial variables. Examples of variable types are the local variables, the relative distance variables, and the neighborhood count variables. The local variables are related with the geographic cell unit by itself; for instance, the type of land-use recommendation imposed by regional administrative directives, or the terrain slope. The relative location variables are in general distance measures to features, such as the distance to the closest feature (e.g. distance to roads), the mean distance measured to several features (e.g. mean distance to local commerce facilities), or the travel distance (e.g. travel hours by car). To allow the transfer of the rule base to another geographic space it is important that the distance variables represent relative positions and not absolute positions. The neighborhood count variables are variables that give importance to number instead of distance (e.g. number of retail commerce, green areas in the neighborhood).

We have already mentioned the existence of static influence factors and dynamic influence factors. The static influence factors can be specified only once because they consist in the same data for all time stages (e.g. terrain slope). On the other hand the dynamic influence factors must be updated in each stage. For instance, the road coverage changes along time, and the new roads must be designed externally to the fuzzy spatial models. The changes in external dynamic influence factors are done by interaction with the planner. The planner observes the evolution of the customer development and adds new roads and in the next stage the fuzzy spatial model will consider these updates when building the forecast. In a more elaborated scheme the interaction between several planning sectors could be done by collaborative spatial models where several planning
models, supervised or not, can run in parallel sharing information (updating variables) and negotiating solution plans.

4.2.4. The saturation level

The dynamics of the growth behavior, in the customer-count model is modeled by introducing the saturation level of the previous stage as input variable. The saturation curve $S_{ij}^*$, in cell $(i,j)$ and for customer class $c$, represents the ratio between the occupation $O_{ij}^*$, by the customer class $c$, and the total number of consumers that can be settled in cell.

$$S_{ij}^* = \frac{O_{ij}^*}{O_{ij}^* + V_{ij} \cdot \rho_{ij}^*} \quad (4.1)$$

$V_{ij}$ is the vacant area (in km$^2$), and $\rho_{ij}^*$ is the density of customers $c$ acceptable for the cell (in customers/km$^2$). The vacant area is computed in each stage by:

$$V_{ij} = A_{ij} - \sum_{c} N_c \frac{O_{ij}^*}{\rho_{ij}^*} \quad (4.2)$$

where $A_{ij}$ is the total area of the cell (in km$^2$) and $N_c$ is the number of customer classes. Note that when the occupied area increases for one of the customer classes $c$, the general vacant area decreases (4.2) and consequently the saturation level increases (4.1) for all the classes, but with high increase for customer class $c$.

The maximum number of consumers that can be installed in a specific cell depends on restrictions imposed by urban plans. Based on these urban plan directives it is possible to define a geographic coverage where in each location the maximum density of customers $\rho_{ij}^*$ admissible for each customer-class is defined. The process allows the definition of the upper bound for the saturation level. Depending on urban directive plans the upper bound for the saturation may change significantly from site to site, conditioning the slope rate of the saturation curves estimated with the Customer-Count module. With this modeling it is possible to simulate the growth for several classes of consumers in the same space and time, and also the interaction among them.

The inclusion of the saturation level as internal dynamic variable allows the modeling of the saturation curve, not explicitly, but by a set of rules, each centered on a specific point of the saturation curve. The derivative of the saturation curve represents the
potential for development ($P/D$), which in general is proportional to the fuzzy system output. The use of multiple variables, representing the saturation level of the several customer classes, allows the modeling of competition and interaction between classes, modeling the behavior of repulsion or attraction between customer classes.

Figure 12 – The saturation curve is described by a set of fuzzy rules where some of the input variables are the saturation level for each customer class. The saturation levels are represented on fuzzy systems as linguistic labels (low saturation, medium, high). With this formulation the curve is built dynamically through the time stage evolution. The derivative of the curve is the potential for development and is proportional to the fuzzy system output.

Figure 12 illustrates how one of the saturation level variables is related with the saturation curve. However, this relation is extremely complex when we imagine the complete rule. The saturation curve may have a very different shape from site to site. A simple example of a rule that models only one of the saturation curve points has the following aspect:

$$\textbf{IF} \ (\text{distance to road is CLOSE}) \ \textbf{AND}$$
$$\ (\text{distance to urban center is CLOSE}) \ \textbf{AND}$$
$$\ (\text{terrain slope is MODERATE}) \ \textbf{AND}$$
$$\ (\text{domestic saturation is MEDIUM}) \ \textbf{AND}$$
$$\ (\text{industrial saturation is LOW})$$

$$\textbf{THEN} \ \text{Domestic PFD is 20 consumers per stage per km}^2$$

4.2.5. The Fuzzy System

The formal representation of the fuzzy system function for customer-count is:

$$\hat{S}_{ij} = f(S_{a,ij}^{t-1}, ..., S_{a,ij}^{t+n}, \ i_{ij}^{t-1}, ..., i_{ij}^{t+n}) \ (4.3)$$

where $\hat{S}_{ij}$ represents the output of the fuzzy system for stage $t$ and customer-class $cc$,

$S_{a,ij}^{t+n}$ represents one of the $N_a$ saturation level, for customer-class $cc$ and stage $t-1$, and

$i_{ij}^{t+n}$ represents one of the $N_i$ influence factors for stage $t-1$. 
The rule base is constructed independently for each customer class. In a first step the rules are generated based on the methodology explained in Chapter 3. The input variables and the output should be stored in the same spatial data structures and with the same resolution. The external influence factors \( l \) are computed based on GIS spatial operations. For instance, the vectorial road coverage is converted to a raster structure, which is used to compute the distance from each location to the closer road. The saturation level \( S^c \) for each class \( c \) is computed (using eq. (4.1) and (4.2)) with the historical maps, storing in each cell \((i, j)\) the number of consumers \( O^c_{ij} \) for class \( c \), and with the maps of maximal tolerable consumer density \( \rho^c \) for class \( c \). The customer density maps could be acquired from regional urban planning directives. All the data, input and outputs, must correspond to the same time stage.

At the second phase of the training, the planner experts update and adjust the rule-base. In practice, this process allows to correct old planning practices and allows also the definition of new urban practices for the future. For instance, suppose that in the past the development in dangerous sloping terrain was not restricted, but new planning practices prohibit the development on this kind of terrain. For this kind of situation the planner adjusts the rule-base by defining his own fuzzy reasoning rule:

\[
\text{IF (slope is HIGH) AND (other variables DON'T CARE) THEN Domestic PdD is 0 consumers per stage per km}^2
\]

In this example the linguistic label “don't care” states that the whole range of values is allowed for other variables different than “slope”. The adjustment process must follow an order of increasing detail, with the more detailed reasoning rules at the end of the adjustment process. The new planning situations are translated to the rule base by automatically adding new rules and adjusting inference parameters. For instance, suppose that new emplacements for high development are expected farther from urban centers:

\[
\text{IF (distance to road is (Mildereate U CLOSE)) AND (distance to urban center is (Mildereate U CLOSE)) AND (terrain slope is (not HIGH)) AND (domestic saturation is MEDIUM) AND (industrial saturation is LOW) THEN Domestic PdD is 20 consumers per stage per km}^2
\]

In this example the reasoning rule matches a set of rules in the fuzzy system rule-base. If some of these rules don’t exist, because the correspondent behavior didn’t happen in
the past, then the new rules are generated by trending (see Chapter 3). Finally all the rules are adjusted in order to output a potential-for-development (PfD) equal to 20 consumers per stage per km².

4.2.6. The Cellular Automaton

The global number of consumers at each time stage and for each consumer class must be specified as input to the fuzzy spatial models. Many kinds of methods could be used to forecast these values [129], namely regression models, econometric models, and technological methods. Because this thesis is focused on spatial analysis this kind of global forecasting will be not discussed, and we will admit that the global trending values are known input values.

As described before, the CA parameters have an effect on the development morphologies, and must be chosen in accordance with the FSM application. As explained in Chapter 3, there are four empirical parameters that adjust the CA dynamics: the positive feedback \( \alpha \), the neighborhood parameter \( \beta \), the innovation parameter \( \lambda \), and the selective parameter \( \phi \).

The neighborhood parameter controls the influence of the neighborhood on the development. In practice higher values for the parameter result in higher geographical smoothing in the development.

The innovation parameter represents the behavior randomness of the development pattern relatively to the pattern of potential for development proposed by the Fuzzy System module. Higher values for the innovation parameter results on more discontinuous development patterns. The parameter is important to trigger the development in green land (non-developed areas), but it is also a source of uncertainty for the model.

The positive feedback is computed as consequence on the specified values for innovation and neighborhood parameters, by the following formula \( \alpha = 1 - \beta - \lambda \).

The selective parameter depends on the dynamics of customer sitting. If the customers are very selective, with high preference for the highest suitable sites, the \( \phi \) parameter should be high. This high parameter value is applicable to high competitive consumer classes with small number of global forecasting. For domestic consumers the
competitiveness is not very high and the selective parameter values are in general low. On the other hand, for industrial consumers there is a higher selectivity in geographical regions with higher potential. For this type of consumers the selective parameter has higher values.
4.3. LAND-USE APPLICATION EXAMPLE

The following example illustrates the application of the FSM on customer-count. In this example we forecast the spatial development of domestic consumption in the island of Santiago, (Cabo Verde) [126], [127]. We aim to forecast the domestic development along one scenario of seven stages, which were obtained for illustration purposes and cannot be seen as reflecting the actual situation in the region. The geographical influence factors are characterized by the following spatial variables:

- V1 - Distance to main urban centers (4 linguistic labels)
- V2 - Distance to secondary urban centers (4 linguistic labels)
- V3 - Domestic saturation level (6 linguistic labels)
- V4 - Distance to roads (5 linguistic label)
- V5 - Distance to the Coast (3 linguistic label)
- V6 - Terrain slope (4 linguistic labels)

The spatial variables V1, V2 and V3 are internal dynamic influence factors, recalculated for each iteration with the development result of the previous stage. The saturation is recalculated in each iteration as a function of the development on the previous iteration. We define urban center as the areas with saturation level higher than 0.8. This allows, for each iteration, the redefinition of the urban center areas implicating new distance maps for variables V1 and V2. The use of two kinds of urban center was necessary because the development has different behavior for each of these urban types.

The Distance to roads is an external dynamic influence factor because the planner, along the time stages, updates the road coverage. In this scenario we admit that on stage 2 a new ring road on the main urban center will be built and on stage 6 a new coast road will be constructed on the south coast near the city.

The variables V5 and V6 are static influence factors, because they don’t change along the seven stages. The proximity to the sea is significant to model the preference for the coast line and to model if the cell is on sea or on land. The slope influence factor is a significant variable that influences the development.
Figure 13 – Evolution of the number of consumers along seven stages. On stage 2 a ring road was constructed (white) and on stage 6 a coast road was constructed (black). The image shows an evolution gradually growing out of the historical city center. The development is high near the roads. We observe also high development for locations with lower slope and close to the coastline.

The study region has 2400 km² including one main urban center and three secondary centers. The resolution on GIS spatial analysis was 250m, which represents maps with 38400 cells. The historical growth is based on the geographical building growth along the last 30 years. As training results we obtained 2512 rules completed with 266 rules defined by expert knowledge. The expert knowledge was applied to adjust and generate rules that describe new behaviors for high saturation levels, new behaviors for development on slopes, and generating rules that restrict development on inland.

The results follow the expected pattern with high development near the city, near the roads, in regions with smaller slopes, and close to the coastline. Along the time stages the central area saturates and the development is forced out. It is interesting to observe that small urban concentrations are developed apart from the city center, becoming sub-urban centers. The development of new roads affects the development pattern, by increasing the suitability of the closest pixels.
As discussed before, for the FSM, the shape of the saturation curves is a result of the dynamic behavior implicit in the fuzzy rule base. Figure 14 shows the saturation level evolution along seven stages for seven different locations. As observed, the shape of the saturation curve may be different from site to site but in general it follows an S shape with three different phases (dormant, development and saturation). As expected the development is faster on preferable sites. In Figure 14 we may observe faster development near the urban centers.

Figure 14: Saturation curves for seven different locations build based on the Fuzzy Spatial Model results of seven development stages.

In Chapter 2, when we discussed aspects related with the load growth behavior, we referred that the cell size and the maximal density determine the shape of the S curve in each location. In Figure 15 we observe the effect of different resolutions (1000, 2000 and 4000 m) on three different points (P1, P2 and P3). From the results obtained we observe that lower analysis resolution (bigger cell size) implicates soft development rates. By comparing the development for the three points we conclude that this relation between development and resolution is a complex phenomena with significant different characteristics from site to site.
Figure 15—The S curves for different cell sizes (1000, 2000 and 4000 m) are presented for three different locations (P1, P2 and P3).

The above example shows that the Fuzzy Spatial Model could be successfully applied to model very complex spatial behaviors for the development of consumers. In this model the shape of spatial load behavior is not predefined. The knowledge base effectively represents rules of relative concepts in space and time, allowing the translation of the knowledge base to other regions with similar behavior and to other time, in the future.
4.4. MERGING STATISTICAL AND JUDGMENTAL INFORMATION

The Fuzzy Spatial Model (FSM) is based on two types of knowledge sources, the statistical information, extracted from historical behavior and the judgmental information, extracted from human expert knowledge. It is essential, when training the FSM, the merging of statistical and judgmental information. Chapter 3 describes in detail the aspects related with the training of the FSM, and the methodology used to merge the two types of information in a unique knowledge base [130].

To exemplify the capability of the FSM to merge statistical and judgmental information we present in this section an example comparing results of two different rule bases. The first simulation (A) uses the rule base constructed for the example presented in 4.3. The second simulation (B) uses the same rule base, but updated with judgmental information defined by one "master rule". To merge the judgmental information the expert defines several master rules; however for the sake of exemplification we will consider only the following master rule:

\[
\text{IF (distance to road is (MODERATE U HIGH U VERY HIGH)) AND (other variables is (DON'T CARE)) AND THEN Domestic PdD decrease 20\%}
\]

The rule base was tuned with the additional judgmental information defined by expert knowledge. This process created 146 additional rules by interpolation and changed 1560 rules in the original rule-base.

With the initial rule base we obtained, as a result of the simulation the development A (Figure 13) and with the upgraded rule base we obtained the simulation B which has visualization maps for development similar to simulation A. Due to the visual similarity of the results we compare the results by computing the maps of relative change in development \((A-B)/A\).
Figure 16—Perceptual change on development, for the 5th time stage, due to the upgrade with the additional judgmental information. a) dark zones represent decrease in development. b) dark zones represent increase on development.

The master rule stated by the expert to adapt the rule base says that the development "not close" to the road infrastructure will be 20% lower. In figure 8 a) we could observe that for some areas "not close" to the roads the development decreases significantly (dark zones). This decrease in development for some regions implicates increase in others. In Figure 16 b) we may see that this increase will happen near the roads (dark zones) but, because "very near the roads" and "near the urban center" the area is saturated, the development is pushed out; however, it will happen always near roads as required by the "master rule".
4.5. VALIDATING THE LAND-USE MODEL

The Spatial Load Forecasting, developed in this thesis, has particular characteristics, relatively to traditional forecasting methods [128], [129], that must be evaluated with appropriated processes. These characteristics are the spatial behavior, the capability to model temporal behavior, and the capacity to merge historical and judgmental information. To evaluate these three characteristics of the model we will formulate decoupled processes to independently validate these aspects [131]. These processes are:

- the spatial validation, measuring the performance of the model when predicting the spatial development with a knowledge base acquired from the same time period behavior.

- the temporal forecasting validation, measuring the performance of the model with a knowledge base acquired from previous time period.

- the temporal backcasting validation, measuring the performance of the model when applied to predict spatial development at a time period preceding the time periods used to build the knowledge base.

- the validation of the merging judgmental and historical information, measuring the performance of the model when a new judgmental behavior is added to the knowledge base by a simple human expert rule.

In order to validate the forecasting method we use two different measures of accuracy: the Coefficient of Variation (CV) to measure the accuracy on level; and the Turning Point (TP) to measure the accuracy on changes. These two measures of accuracy are required because for Electricity Distribution Planning it is necessary to assess the levels of load development (levels) and the changes from green-field to new load developed area (changes). Changes are particularly important for Electricity Distribution Planning because they condition network expansion (associated with fixed costs). On the other hand, the level of development is equally important but conditions mainly the sizing of the network (associated with variable costs depending on sizing).

The Coefficient of Variation (CV) relates the Root Mean Square Error (RMSE) to the average value of the actual data. The CV is used because it is a free unit measure of
accuracy and doesn’t depend on the scaling of the units. It is a relative measure (like the Mean Absolute Percentage Error (MAPE)) for which errors in high or low scale levels have the same importance (this is not true for MAPE), but larger errors have more importance (in RMSE).

\[ CV = \frac{RMSE}{\sum_{i=1}^{h} V_i / h} \]  

(4.4)

where the \( \sum_{i=1}^{h} V_i / h \) represents the mean value of the validation sample and RMSE is the Root Mean Square Error given by:

\[ RMSE = \sqrt{\frac{\sum_{i=1}^{h} (V_i - F_i)^2}{h}} \]  

(4.5)

where \( i \) is the index of the forecasted output (one index represent one geographical cell in a specific time period), and \( h \) is the number of forecasted points (number of cells in the geographical coverage times the number of periods), and \( V_i \) is the actual result for point \( i \). \( F_i \) is the forecasted value for point \( i \).

As explained before, the Turning Point (TP) measures the accuracy of change from green-field to developed area. The four possible turning point situations are the following:

<table>
<thead>
<tr>
<th>Did the change occur?</th>
<th>Was a change predicted?</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>TP_1=a/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>TP_2=b/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TP_3=c/o</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( a \) represents successful predictions when change occurs; the \( c \) represents errors when change occurs and was not predicted; \( b \) represent errors when change was predicted but did not occur, \( o \) represents the number of change occurrences \( (a+c) \), and \( (a+b) \) represents the number of change predictions. The number of change predictions is not necessarily equal to change occurrences \( o \).

4.5.1. Spatial Validation

To validate the spatial behavior, a cross validation procedure is used. A spatial random selection is used to separate a calibration sample \( [C] \) from a validation sample \( [V_i] \), where \( i \neq j \). The calibration sample is used to train the system and build the fuzzy rule knowledge
base. Using this knowledge base the forecasting method is applied producing the forecast sample [F] in the point correspondent to [V].

The rule knowledge base is purely constructed from historical data to decouple the spatial validation from the effect of merging judgmental information. In our example the historical data consist in the development observed along 3 time periods and correspond to 3 geographical coverages of domestic development in Santiago island in Cabo Verde. To decouple the spatial behavior from the temporal behavior the selection of the calibration and validation samples was done for the three temporal stages as shown in Figure 17.

The data for each time period are stored in one development map covering the entire island. From this geographical coverage we randomly selected half of the points for the calibration sample \([C]\) (near 15000 points per period) and the other half for the validation sample \([V]\).

The system was trained with a calibration set \([C]\) of 45000 points, using six variables as influence factors generating approximately 2500 fuzzy rules (1750 for Test B). Considering the points that activate each rule, an average of 40 significant points has been used to calibrate each rule. The input variables used for this training were those used on previous examples:

- V1 - Distance to main urban centers (4 linguistic labels)
- V2 - Distance to secondary urban centers (4 linguistic labels)
- V3 - Domestic saturation level (6 linguistic labels)
- V4 - Distance to roads (5 linguistic label)
- V5 - Distance to the Coast (3 linguistic label)
- V6 - Terrain slope (4 linguistic labels)

The number of linguistic labels used for each variable is an important accuracy factor that is the object of our study. To test this aspect we idealized two different scenarios (Test A), the first using the variables and linguistic labels previously described and the second (Test B) using the same variables but adopting only 3 linguistic labels for variable V4 (Distance to roads).
After the training step, using only historical information, we proceed to forecasting using the same input variables and labels and using a given value of global forecast. The global forecast (number of new consumers for all the coverage) was previously obtained from the development maps for each period \( (p_1=1000; p_2=1500; p_3=2000) \). The three maps of forecasting values cover the entire region for the three periods. Using the forecasting results \([F]\) and the validation sample \([V]\) for the validation points (points not used for calibration), we estimated the errors for "Test A" and "Test B".

As previously explained we use the coefficient of variation (CV) to measure the accuracy on the forecasting level (deviation on the magnitude of forecasting). Table 4-2 are presents the CV and the Root Mean Square Error (RMSE), used to compute the CV, and the mean value for the validation sample. The CV values are a percentual value relative to the mean value for the validation sample. The several columns represent the global results and the results for each period. "Test A" and "Test B" are used to compare the accuracies for different fuzzy partition on input variables. "Test A" uses an input structure with 5 linguistic labels for variable V4 (distance to roads). "Test B" corresponds to a structure with the same variables but using only 3 linguistic labels for variable V4.

Table 4-2 – Spatial Validation results to measure the accuracy on the forecasting level, including the Root Mean Square Error, and the coefficient of variation for Test A (5 linguistic labels for variable V4) and Test B (5 linguistic labels for variable V4).

<table>
<thead>
<tr>
<th></th>
<th>Period P1</th>
<th>Period P2</th>
<th>Period P3</th>
<th>Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum_{i=1}^{h} V_i / h )</td>
<td>1.502</td>
<td>1.511</td>
<td>1.520</td>
<td>1.512</td>
</tr>
<tr>
<td>Test A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.163</td>
<td>0.156</td>
<td>0.171</td>
<td>0.163</td>
</tr>
<tr>
<td>CV</td>
<td>10.9%</td>
<td>10.3%</td>
<td>11.35%</td>
<td>10.8%</td>
</tr>
<tr>
<td>Test B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.174</td>
<td>0.168</td>
<td>0.184</td>
<td>0.175</td>
</tr>
<tr>
<td>CV</td>
<td>11.6%</td>
<td>11.1%</td>
<td>12.1%</td>
<td>11.6%</td>
</tr>
</tbody>
</table>

The results show that the spatial mean error (CV) rounds the 11%. Comparing the values for the three periods we observe that Period P2 has lower error because the three
periods where used for calibration and P2 is the intermediary period that probably benefits of better fitting.

Comparing results for “Test A” with results for “Test B” we observe lower accuracy when using only 3 linguistic labels instead of 5. When more linguistic labels were used the knowledge base is fragmented in a larger number of smaller regions and consequently more rules are used in the knowledge base but each rule describes the behavior in a smaller region of the space.

![Coefficient of Variation (CV)](image)

Figure 18 – variation of accuracy measure (CV) with the saturation level.

The error is not the same for different saturation levels. The figure shows the variation of the error for different levels of saturation. The error increases for the increasing part of the saturation curve (S curve). For a saturation level between 40% and 70% the area develops very fast and forecasting is more difficult. When the area approximates saturation the development becomes slow and restricted by a maximum value and consequently forecasting becomes more accurate.

As explained before, measuring the error level is not enough for Spatial Load Forecasting. It is also very important to have accuracy in predicting changes from greenfield to developed area. This accuracy is evaluated with Turning Point (TP) measures (TP1, TP2 and TP3), described previously. TP2 and TP3 are measures of unsuccessful predictions and TP1 is a measure of successful predictions.
Table 4-3 – Spatial Validation results to measure the accuracy in forecasting change. TP₁ represents successful predictions when change occurs; TP₂ represents errors when change was predicted but did not occur, and TP₃ represents errors when change occurs that was not predicted.

<table>
<thead>
<tr>
<th>Test</th>
<th>Period P1</th>
<th>Period P2</th>
<th>Period P3</th>
<th>Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>48</td>
<td>73</td>
<td>108</td>
<td>229</td>
</tr>
<tr>
<td></td>
<td>47</td>
<td>65</td>
<td>102</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>88.8%</td>
<td>85.9%</td>
<td>84.1%</td>
<td>85.7%</td>
</tr>
<tr>
<td></td>
<td>9.1%</td>
<td>3.1%</td>
<td>10.3%</td>
<td>7.8%</td>
</tr>
<tr>
<td></td>
<td>11.2%</td>
<td>14.1%</td>
<td>15.9%</td>
<td>14.3%</td>
</tr>
<tr>
<td>B</td>
<td>45</td>
<td>68</td>
<td>94</td>
<td>207</td>
</tr>
<tr>
<td></td>
<td>85.7%</td>
<td>83.9%</td>
<td>81.9%</td>
<td>83.3%</td>
</tr>
<tr>
<td></td>
<td>8.1%</td>
<td>9.3%</td>
<td>5.1%</td>
<td>7.1%</td>
</tr>
<tr>
<td></td>
<td>14.3%</td>
<td>16.1%</td>
<td>18.1%</td>
<td>16.7%</td>
</tr>
</tbody>
</table>

The number of observed changes and number of predicted changes are quite approximate with an error between 7% and 10%. For “Test B” the difference between the total number of predicted and occurred changes is higher due to a less detailed partition of knowledge base space.

Observing the Turning Point accuracy measure TP₁ we observe approximately 85.7% for “Test A” and 83.3% for “Test B”. This accuracy on forecasting changes is lower than the accuracy on forecasting level (Table 4-2). However, we must recognize that forecasting behaviors of “spatial pioneer” consumers is extremely difficult because usually these consumers display unusual behavior.

It is interesting to observe that in general we obtain better accuracy for TP₂ (change predicted but did not occur) than for TP₁ (change occurs but was not predicted). This shows that the real changes that occurred are more wildly behaved than the prediction from the model. This could be improved by better calibration of the Cellular Automata parameters; increasing the “innovation factor” will increase the wildness of predicted behavior. This calibration will decrease the TP₃ but increase TP₂.

We note that the change in accuracy along the three periods decreases. For latest periods, more innovative behaviors at development borders are activated and the changes occur mostly in these regions, and consequently are more exposed to error.
4.5.2. Temporal Validation - forecast

To validate the temporal behavior we used two different procedures: the forecast validation and the backcast validation. The forecast validation consists in using the forecasting models, calibrated with the historical information excluding the latest period, and measuring their success in forecasting the latest known data sample. This test is almost as good as the real forecast; however, this test only uses historical behavior for calibration and is unable to model forward changes on behavior.

As Shown in Figure 19, for forecasting validation we use the data samples from periods $P_1$ and $P_2$ to calibrate the forecasting model. The knowledge base generated with this historical data is used to validate the latest data sample available $P_3$. In this test the forecasting model doesn’t know any information about the behavior of the latest period $P_3$.

![Figure 19 - Scheme identifying, for forecast temporal validation process, the data sets used for calibration validation and forecasting.](image)

The $[C_i]$ data sample (periods $P_1$ and $P_2$), used to calibrate the knowledge base totalize approximately 60000 points. The influence factors, used as inputs, are the same used for validating spatial behavior (six variables). Similarly to the spatial validation process we used the two test scenarios “Test A” and “Test B” to study the influence of fuzzy input partitions in the forecasting accuracy.

After the training step, we used the model to forecast the period $P_3$ and we obtained the forecasting map with forecasting values for approximately 30000 points. With the forecasting map $[F]$ and with validation sample $[V]$ in period $P_3$ we measured the accuracy. We used the Coefficient of Variation (CV) to measure the forecasting level accuracy and the Turning Point (TP) to measure the accuracy in forecasting changes.
Table 4.4 – Forecasting temporal validation results to measure the accuracy on the forecasting level. Accuracy measures (RMSE and CV) for period P3. Test A corresponds to 5 linguistic labels for variable V4 and Test B to 5 linguistic labels.

<table>
<thead>
<tr>
<th></th>
<th>Test A</th>
<th>Test B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{i=1}^{h} V_i / h$</td>
<td></td>
<td>1.520</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.228</td>
<td>0.240</td>
</tr>
<tr>
<td>CV</td>
<td>15.0%</td>
<td>15.8%</td>
</tr>
</tbody>
</table>

In the Table 4.4 we observe that the accuracy is lower than the one observed for spatial validation: the CV measures increase from 11% to 15%. This increase is expected because, contrarily to the spatial validation process, the forecasting knowledge base has no information about the behavior of the forecasted period. Other reason for this decrease in the forecasting accuracy is the possible changes in behaviors resulting for multiple factors. Obviously, these changes in behaviors could not be captured from historical information. This accuracy of the model could be significantly improved by merging judgmental information.

Comparing the CV values for “Test” A and “Test B” we observe that a more refined partition (Test A) improves the accuracy. However, the improvement is relatively small when compared with the results obtained for the spatial validation tests. This means that spatial overfitting by increasing the detail of the partition space does not necessarily implicate better accuracy in forecasting.

To test the accuracy on changes we used the several Turning Point (TP) measures. The measures use the forecasting sample [F] and the validation sample for period P3.

Table 4.5 – Forecast temporal validation results to measure the accuracy for forecasting change. The $\mathbf{TP}_1$ represent successful predictions when change occurs; $\mathbf{TP}_2$ represent errors when change was predicted but did not occur; and $\mathbf{TP}_3$ represent errors when change occurs that was not predicted.

<table>
<thead>
<tr>
<th></th>
<th>Test A</th>
<th>Test B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed changes</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>Predicted changes</td>
<td>98</td>
<td>96</td>
</tr>
<tr>
<td>$\mathbf{TP}_1$</td>
<td>82.2%</td>
<td>79.3%</td>
</tr>
<tr>
<td>$\mathbf{TP}_2$</td>
<td>8.5%</td>
<td>9.6%</td>
</tr>
<tr>
<td>$\mathbf{TP}_3$</td>
<td>17.8%</td>
<td>20.7%</td>
</tr>
</tbody>
</table>

The accuracy in the number of predicted changes is approximately 90%, with lower number of predicted than occurred changes.
As on spatial validation tests we continue observing lower values of error for TP₂ (changes predicted but did not occur) than for TP₃ error (changes occurs but was not predicts). This continues to happen because the real development is more wildly behaved than the behavior simulated by the forecasting model.

Comparing the values obtained for temporal validation (Table 4-5) with values obtained for spatial validations (Table 4-3) for period P₃, we observe that the accuracy of the successful predictions TP₁ is lower (82.2% vs. 84.1% for “Test A”, and 79.3% vs. 81.9% for “Test B”). This happens because for temporal validation the behavior for period P₃ is completely unknown. Contrarily, for spatial validation the behavior for period P₃ is partially known.

4.5.3. Temporal Validation - backcast

One of the interesting aspects of the Fuzzy Spatial model is its capability to capture and store the knowledge base and the possibility of applying this rule base to other regions with region with similar behavior but shifted in time. This is only possible if the model has good backcasting validity. With the backcast validity we test if the model still predicts efficiently earlier behavior based on a knowledge base constructed with recent data sets. Obviously, this approach may suffer from contamination because the knowledge base is influenced by what happens recently, but it is this kind of contamination that we wish to evaluate.

Contrarily to forecasting validation, for backcasting validation we use the data samples from period P₂ and P₃ to calibrate the model knowledge base. The model calibrated with this historical data is used to validate the earlier data sample corresponding to period P₁.

<table>
<thead>
<tr>
<th>Backcast Validation</th>
<th>Calibration</th>
<th>Calibration</th>
<th>Forecast Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>[V₁], [F₁]</td>
<td>[C₁]</td>
<td>[C₂]</td>
<td>[F₂]</td>
</tr>
</tbody>
</table>

Start of historical data | P₁ | P₂ | P₃ | Current time | End of forecast horizon

Figure 20– Scheme identifying, for backcast temporal validation process, the data sets used for calibration validation and forecasting.

As on previous validation test six variables were used as influence factors. The calibration data sample (periods P₂ and P₃) totalize approximately 60000 points, and the
validation data sample (period P₁) contains approximately 30000 points. Similarly to the
previous validation processes we used the two test scenarios “Test A” and “Test B” to
study the influence of fuzzy input partitions in the prediction accuracy.

For period P₁, using the forecasting map [F] and the validation sample [V] we
measure the level accuracy with the measure CV and we measure the change accuracy
with TP₁, TP₂ and TP₃.

<table>
<thead>
<tr>
<th>Vₙ / h</th>
<th>Test A</th>
<th>Test B</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.177</td>
<td>0.183</td>
</tr>
<tr>
<td>CV</td>
<td>11.8%</td>
<td>12.2%</td>
</tr>
</tbody>
</table>

Table 4-6 – Backcasting temporal validation results to measure the accuracy on the
forecasting level. Accuracy measures (RMSE and CV) for period P₃. Test A
corresponds to 5 linguistic labels for variable V₄ and Test B to 5 linguistic labels.

In Table 4-6 we observe and error of 11.8% for “Test A”, with marginally higher
values for “Test B” 12.2%. As expected these values are higher than values observed for
spatial validation (Table 4-2) because, contrarily to spatial validation using cross validation,
in this backcast validation the period P₁ is completely unknown to the knowledge base.

The accuracy of the backcasting (12%) is considerably better than the forecasting
(15%). This shows that the future behavior continues storing the past behavior and this
one could be efficiently captured. The forecasting predictions of are worse than the
backcasting predictions because future data samples contain the past and future behaviors
but past data samples don’t contain a complete characterization of the future behaviors.

We continue to use the Turning Point (TP) measures to evaluate the accuracy on
changes. The table contains the values obtained for TP measures.

<table>
<thead>
<tr>
<th>Observed changes</th>
<th>Test A</th>
<th>Test B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted changes</td>
<td>46</td>
<td>45</td>
</tr>
<tr>
<td>TP₁</td>
<td>87.4%</td>
<td>84.3%</td>
</tr>
<tr>
<td>TP₂</td>
<td>8.4%</td>
<td>9.5%</td>
</tr>
<tr>
<td>TP₃</td>
<td>12.6%</td>
<td>15.7%</td>
</tr>
</tbody>
</table>

Table 4-7 – Backcast temporal validation results to measure the accuracy for forecasting
change. TP₁ represents successful predictions when change occurs; TP₂ represents
errors when change was predicted but did not occur, and TP₃ represents errors when
change occurs that was not predicted.
The accuracy TP, obtained for backcast validation is 87.4% for “Test A” and 84.3% for “Test B”. These values are significantly higher than those obtained for forecast validation (82.2% for Test A and 79.3% for Test B). This occurs because in backcast we don’t have the additional difficulty of the innovative behaviors because these behaviors could also be captured from the future information samples. On other hand the accuracy obtained for backcast validation TP1 still slightly lower than values obtained for the spatial validation process (88.8% for Test A and 85.7% for Test B). The difference observed between accuracy values in backcasting tests and spatial validation tests are a consequence of a better knowledge of the P1 behavior for spatial validation tests due to the cross validation process.

4.5.4. Validating the merging of judgmental information

Validating the accuracy of the model relatively to historical information is quite easy if the historical information exists. However, evaluating the accuracy of model based on judgmental information is a very different problem. Usually the quality of this kind of judgmental models is controlled in input and not in outputs. Controlling the quality of the judgmental information and its sources allows a subjective evaluation about the forecasting accuracy.

We are particularly interested in studying the accuracy of our model in merging the judgmental information. One possible process to do that is to observe the performance of the model when we merge judgmental information described by a well defined macro rule. The accuracy could be measured by comparing the forecast results of the calibrated model with the real development that includes this judgmental information. Unfortunately we couldn’t separate judgmental and historical data in an historical data sample. To overcome this problem we idealize a validation process illustrated in the figure.
The data from periods P₁, P₂ and P₃ are used to calibrate the system using only the historical data. The data sample from period P₃ is modified manually and is used as validation sample. The modification of the P3 data sample is done by manual updating in accordance with a macro rule that represents the judgmental information. The macro rule used for the validation test is the following:

**IF** (distance to road is (MODERATE U HIGH U VERY HIGH)) **AND** 
(other variables is (DON'T CARE)) **AND** 
**THEN** Domestic P&D decrease 20% 

For this simple judgmental rule is quite easy to update the map containing the validation sample, for it is only necessary to select the regions in the map where distance is moderate, high or very high. This corresponds to selecting the regions where distance to roads is greater than 200 m as shown in the Figure 22.

After selecting the geographical region matched by the macro-rule, we apply the consequent of the rule to this region reducing in 20% the development for period P₃. With this process we recreate manually the merging of judgmental information with historical information. This data sample allows the validation of the merging process by comparing results of manual and automatic adjustment. We admit that testing the model for a simple macro rule validates the model for any kind of complex macro-rules.
In summary, the validation process for merging judgmental information follows the following steps:

1. Calibrate the Fuzzy Spatial Model with the calibration sample \([C]\) using the historical data from periods \(P_1\), \(P_2\) and \(P_3\), capturing the historical behavior inclusive for period \(P_3\).

2. Manually update the data sample of \(P_3\), changing the historical behavior by merging the judgmental information from the macro-rule. This generates the validation sample \([V]\). Count the global value of development (1840 consumers).

3. Use the model's automatic process to merge the judgmental information, updating the knowledge base generated from historical information.

4. With the updated knowledge base, forecast the period \(P_3\) generating the forecasting \([F]\). Use the global value of development as global forecasting value.

5. With \([V]\) and \([F]\), measure accuracy.

As in previous validation processes we use measures of level accuracy (CV) and measures of change accuracy. These measures were only possible for period \(P_3\) and for "Test A" because the 5 logistic labels were used.

The measure of error obtained for \(P_3\) is low when compared with the error obtained for the other validation test. One of the reasons for this low error is the use of complete data sets for the three periods for validation. Comparing the accuracy measures, with automatic adaptation "Merge" with the expected results obtained for manual adaptation "No-Merge" (both using the three periods for calibration), we conclude that these values are very close which shows that the accuracy was not very affected by the impact of the change on the knowledge base. Note that the impact of the knowledge base update is 20% (decrease of the PFD for the new behavior) for 30% of the validation points (percentage of locations distancing more than 200 m from roads). If the initial knowledge
base was erroneously applied to forecast the new behavior the error will be approximately 6% higher due to the misadjusted knowledge base (20% misadjusted in 30% of the points). Instead of the 6% error increase we observe a very small increase (from 10.1% to 10.2%) which proves that the process used to merge judgmental information works properly by a correct adaptation of the knowledge base.

Table 4-8 – Validating the merge of judgmental and historical information. Results to measure the accuracy on the forecasting level. Accuracy measures (RMSE and CV) for period P3. “Merge” represent the measures of error after the adaptation of the knowledge base with the judgmental information. “No-Merge” represented the expected error of the system admitting an ideal adaptation of the knowledge base.

<table>
<thead>
<tr>
<th></th>
<th>Merge</th>
<th>No-Merge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{i=1}^{h} V_i / h$</td>
<td>1.518</td>
<td>1.520</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.155</td>
<td>0.154</td>
</tr>
<tr>
<td>CV</td>
<td>10.2%</td>
<td>10.1%</td>
</tr>
</tbody>
</table>

To clarify how the merging process influences the error we scatter the error with the distance to the roads and we compare this scatter for the two situations: with and without merging.

![Figure 23 – Variation of accuracy measure (CV) with the distance to roads. The two series represent the accuracy with and without merging judgmental information.](image)

In Figure 23 we observe that the error is lower close to the roads, which is explainable by higher saturation levels and consequently lower error (see Figure 18). Between 200 and 500 meters we observe the higher errors consequence of the characteristic of these regions with high rates of development mixed with innovative behaviors. For distances larger then 500 m the error decreases because the non-change areas (with no level error) also increases non-allowing a correct observation of the error. Comparing the two data series, “merge” and no-merge, we observe that the two accuracy measures are very close, which proves that the merging method is working properly. The difference between the
two series observed between 100 and 200 meters is consequence of the crisp selection (selecting regions >200m and reducing 20% the development); the fuzzy reasoning method performs a soft and gradual adaptation instead of crisp. This difference could not be considered a problem with the method because it is simply a deficiency in the validation process. The differences observed for distances larger the 400m are a consequence of the decrease in development and could not be interpreted as decrease of error due to the merging methodology.

To measure the accuracy in change we use the turning point measures (TP₁, TP₂ and TP₃), where TP₂ and TP₃ represent the errors and TP₁ the accuracy. In the Table 4-9 we may see that the change accuracy is relatively high when compared with validation tests, totalizing approximately 89.4%.

Table 4-9 - Validating the merge of judgmental and historical information measuring accuracy in changes. The TP₁ represent successful predictions when change occurs; TP₂ represent errors when change was predicted but did not occur; and TP₃ represent errors when change occurs that was not predicted.

<table>
<thead>
<tr>
<th></th>
<th>Merge</th>
<th>No-Merge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed changes</td>
<td>98</td>
<td>108</td>
</tr>
<tr>
<td>Predicted changes</td>
<td>86</td>
<td>96</td>
</tr>
<tr>
<td>TP₁</td>
<td>89.4%</td>
<td>89.2%</td>
</tr>
<tr>
<td>TP₂</td>
<td>5.5%</td>
<td>4.3%</td>
</tr>
<tr>
<td>TP₃</td>
<td>10.6%</td>
<td>10.8%</td>
</tr>
</tbody>
</table>

Comparing the situations of merge and no-merge we conclude that the accuracy doesn't decrease when we use the merging methodology. On the contrary, we observe an increase in the accuracy (the measures of successful predictions TP₁ increase and the measures of errors TP₂ and TP₃ decrease).

With this validation process we proved that the methodology for merging judgmental information works properly changing in the knowledge base what was expected to change, in the correct proportion.
4.6. CONCLUSIONS AND REMARKS

The structure of the Spatial Load forecasting model presented in this thesis is composed of three main components: the Global Forecasting model, the Land-use model and the End-Use model. This Global Forecasting module states the number of consumers for the entire study region. The End-Use module models the temporal behavior for per-capita consumption. The Land-Use model is the most important in the perspective of this thesis and simulates the spatial behavior of the forecasting.

For Global Forecasting thee types of methods could be used: univariate (time-series), multivariate methods and qualitative forecasting methods. Choosing the best method for global forecasting depends mostly on the data we have available and also on the classes of consumers.

The End-Use model is based on the identification and the decoupling of causes of per-capita load change, decomposing the problem by customer classes, by seasonal sub-types of consumption and by sub-classes associated with electric appliance load groups. The End-Use module has two levels of forecasting: the forecasting of the capacity per-consumer and the energy consumption load curve characterization.

The Global Forecasting module and the End-Use module are coupled to the land-use module in order to obtain a complete characterization of the spatial and temporal behavior of load growth.

The Land-Use module implementation is the central issue of this chapter. The kernel of the land-use module is the Fuzzy Spatial Model (FSM), simulating the spatial behavior phenomena for customer development. Aspects of the implementation are discussed, namely: the structure; the selection of customer classes; the selection of geographical influence factors; the saturation level used to control the dynamics of the system; the interpretability of the rule base; The judgmental macro rules; the parameters used in the fuzzy system; and the parameters used in the Cellular automata.

One example is presented for forecasting the development of domestic consumers in the island of Santiago (Cabo Verde). This example illustrates the training and the application of the Land-Use model. Six spatial variables are used to simulate the behavior of the development along 7 stages. The example also explains how to formulate scenarios and how the simulation process interacts with the forecasting environment. Aspect related
with computing time and information storage are also referred. The results of the example show the spatial evolution of the number of consumers and show aspects related with the evolution of the saturation curves in different locations.

To observe the capabilities of the process to merge judgmental and statistical information and example is presented. The comparisons of simulation results with and without the merging some judgmental information show the effects in the spatial development pattern.

To validate the Spatial Load Forecasting model four different validation test are performed: spatial validation; forecast temporal validation; backcast temporal validation; and validation of the judgmental/historical information merge methodology. Two different accuracy measures were used: the measure of the accuracy in forecasting level and the accuracy measures in change from green-field to developed area.

The accuracy observed for the forecasting level varies from 80% to 90%, which seems reasonably good for the tested example and for distribution planning purpose. The accuracy observed for changes are higher, ranging from 70% to 85%, admitting different types of error (predicted/non-occurred and non-predicted/occurred). These change accuracy values, important for expansion planning, are a consequence of the difficult predictive characteristics of the “spatial pioneer” consumers. Apparently these accuracy values are in the range of other Spatial load Forecasting models, however the comparison with other models is only possible when tested in the same forecasting problem and in the same conditions, which unfortunately was not possible in this thesis. The model performs well for spatial and temporal forecasting and temporal backcasting. Due to innovative behaviors, the accuracy in temporal forecasting is lower. When the knowledge base of the model is misadjusted is practical impossible the improvement of the forecasting performance by using only statistical information, because the new behavior is not in the historical information. The ability of the forecasting method to merge judgmental information provides to the model the capability to improve the accuracy in temporal forecasting.
4.7. REFERENCES


Chapter 5  UNCERTAINTIES IN SPATIAL LOAD FORECASTING

The Spatial Load Forecasting (SLF) proposed in this thesis presents original ability to cross information from multiple sources, formats and accuracy. Unfortunately this input information has associated uncertainties, which are crossed and propagated to the forecasting results.

In this chapter we will study several kinds of uncertainties affecting the spatial load forecasting and we will focus on the modeling and propagation of these uncertainties through the SLF model. The first part of the chapter presents aspects related with the types of uncertainties and their modeling, always in the perspective of electricity distribution planning. Motivated by the spatial characteristics of the problem studied in this thesis we will give special attention to the spatial uncertainties. The second part of the chapter studies the capabilities and performance of the SLF in modeling uncertainties by using scenarios. The third part of the chapter studies the propagation of multiple kinds of uncertainties through the model. The second and the third part of the chapters include examples about the studies of uncertainty modeling and results in the SLF model.

5.1. UNCERTAINTIES IN SPATIAL LOAD FORECASTING

Uncertainties in Spatial Load Forecasting have a major influence on distribution planning. Uncertainties are not under control of the planners, however their knowledge and modeling allow a better evaluation of risk and allow to identify the robustness of solutions in decision models. In distribution planning, the following are important: the evaluation of uncertainties in load growth, costs and reliability [132]-[134]. In this thesis we are mainly interested in load growth uncertainties, but the cost and reliability uncertainties are also influenced by the spatial characteristics of the load growth.

In automated planning tools (which includes Spatial Load Forecasting) we have uncertainties in the data inputs, uncertainties related with the computation model, and uncertainties related with the decision process. We are worried with the uncertainties
related with the computation models (how do the uncertainties propagate inside SLF model?), but we must first model the input uncertainties, and because SLF is linked to other multi-disciplinary planning models (urban and environment planning) the inputs are influenced by uncertainties in related decision process. All the aspects will be studied in the second and third sections of this chapter.

In the perspective of the type of uncertainty, we can find: uncertainties with stochastic behavior (short-term load forecasting, equipment failure), well represented by known probability functions; or uncertainties that exhibit a chaotic behavior with non-repetitive nature (facility construction, policy resolutions). This distinction of uncertainties orientates the construction of the uncertainty models. These can be scenarios, intervals, probabilistic models or fuzzy models. A mix of these approaches will be used to study SLF uncertainties.

Another classification for uncertainty is based on the dimensional perspective: we can have magnitude uncertainties (how much?), time uncertainties (when?) and spatial uncertainties (where?). These classifications are the ones that better fit the uncertainty in outputs of spatial load forecasting, and this is the uncertain information we are interested in supplying to the automated planning models that use load forecast maps.

Due to the use of Geographic Information Systems (GIS) as a support system for Spatial Load Forecasting we find the study of issues related with spatial uncertainty interesting. In Spatial Load Forecasting we deal with various types and origins of spatial data, such as vector GIS data, raster data, statistical tables, etc. They can also be classified as positional data, thematic data and temporal data. The spatial data accuracy is a definition of how closely the spatial data represent the real world.

5.1.1. Accuracy in data

The first source of uncertainty in spatial data is the inaccuracy in data. The inaccuracy is related with several causes [135] [136].

- **Completeness** - What features have been omitted?
- **Consistency** (logical uncertainty) - What non-existent features are represented? Are the relations between geographic features consistent?
- **Thematic Accuracy** (thematic uncertainty) - How correct is their classification?
Attribute Accuracy (attribute uncertainty) - How correct are the values associated with the attributes of the geographical features?

Temporal Accuracy (temporal uncertainty) - How correct is the data in the represented time-stage?

Positional Accuracy (positional uncertainty) - How far away is a map feature from its actual location in the world?

There are many sources of error: the age of data, the map scale, the density of observations, the format and GIS data structures, errors in data acquisition and preprocessing, errors in data analysis and data conversion classification and generalization. The decision-makers that use spatial distribution planning must recognize these errors. To formalize error assessment, it is recommendable to follow the data quality standards, actually emerging from the ISO TC/211 activity.

The lack of accuracy, referred to in the above cause components, can be interpreted as divided in four types of spatial uncertainty (positional, logical, thematic and temporal). The modeling of these uncertainties will be discussed later in this section. The attribute accuracy can’t be considered as a spatial uncertainty measure, but it constitutes an excellent way to store spatial uncertainties in a uniform support structure.

5.1.2. Amount of data

The uncertainty is not only related with errors on data, but also with the amount of information available for a scenario. One way to reduce uncertainty is to provide more descriptive information about the data; increasing the following aspects can achieve this.

Data detail - corresponds to how much information and attributes are stored for each geographical feature (electric line, substation, etc). For instance, the data detail needed for short-range is higher than the data detail needed for long-range Distribution Planning.

Data density - this is a measure of how many features per area are stored. For instance, the data density needed for urban planning is higher than the data density of rural planning. This density can be measured in the same theme or in multiple overlapping thematic data.
Resolution – it is the degree to which closely related entities can be discriminated. For vectorial data, the resolution may imply a minimum feature size (smaller features imply higher resolutions and larger number of features). For raster data (data stored in adjacent pixels) the resolution is determined by the size of the pixels, because it is impossible to store anything that falls between the pixels.

5.1.3. Choosing accuracy and resolution

The main concern that we have about SLF uncertainty in data is its potential impact upon distribution planning decisions. Choosing the accuracy and resolution aims at specifying minimum required data whose uncertainty causes acceptable consequences on the planning process.

The basic procedure consists in evaluating the uncertainty on model outputs for a specific input data set. If the output uncertainty is unacceptable the initial reaction is to reduce uncertainty in the input data. However other options exist such as accepting stronger consequences or reducing the vulnerability of the decision to uncertainties [137].

Reducing the uncertainty in data implies the use of more detailed data, more densified, or higher resolution data. The three hypotheses must be weighted in order to find a balanced set of data within an acceptable cost. Due to the error propagation in spatial analysis, decreasing uncertainty only in some thematic data may be useless, because the uncertainties in the other inaccurate thematic data still propagate in the spatial analysis. When opting for reducing the uncertainty we must try to reduce uncertainties in input data that have more impact on the results although not forgetting the equilibrium among the several thematic data.

Accepting stronger consequences in the decisions may be a solution when the trade-off for decreasing uncertainty is high. For instance, in long-range planning this hypothesis can be tolerable, if the risk is not too high, because it is known that in the future more accurate data will be available resulting in more accurate plans.

The third option of reducing the vulnerability of the decision involves establishing an association between reduction of the influence of data on a decision and the resulting reduction in the consequences of its uncertainty. This objective can be reached for SLF by evaluating multiple forecasts in a scope of scenario modeling of uncertainties. And it
could be adopted by automated planning models that use the SLF results by using edging and robust optimization.

5.1.4. Modeling Spatial Uncertainties

As referred to before, the uncertainty associated with spatial data may include positional uncertainty, thematic uncertainty, logical uncertainty, and temporal uncertainty [135]. In the following paragraphs we present some models for these kinds of uncertainties.

5.1.4.1. Positional uncertainty

The positional uncertainty derives from the inaccuracy of the observation and data gathering or from the conversion between different data structures. For example, the conversion from vector to raster formats may incur in a substantial amount of error, depending upon the cell size and the algorithms used in the conversion [139].

There are two types of positional uncertainty. The first is the absolute positional uncertainty referred to how close the location associated to the data representation is to its real location on the land. The second is the relative positional uncertainty referred to how similar a shape on data representation is to the shape of the object in real world. For Spatial Load Forecasting we are worried with relative positional uncertainty because in our modeling all input influence factor must be modeled relatively (for instance, distance to geographical features).

Various approaches have been used to measure positional uncertainty [140]. The majority of these approaches are based on the resolution of the source of data and resolution losses on data conversions. By introducing the confidence regions in a geographic feature (represented by point, line or polygon), we can quantify the uncertainty caused by the elementary components of this feature. An easy way to represent positional uncertainty is using spatial membership representation (fuzzy set theory). In this kind of representation we can use distance and reclassification spatial functions in order to map a membership value. For instance, the location of a road (a line feature) could be represented by a fuzzy set, whose geographical representation corresponds to multiple polygons each representing an \( \alpha \)-cut of the fuzzy set. The distance to the road is a fuzzy set mapped from the spatial calculation of the distance for each \( \alpha \)-cut.
5.1.4.2. Thematic uncertainty

The thematic uncertainty refers to the uncertainties in the classification of a specific feature in the correct thematic class (e.g. classifying a site as land or water). The thematic uncertainties, related with classification, have their origin in data sources (remote sensing misclassification, use data classifications from inadequate framework, etc).

There are several models and techniques to model thematic uncertainties [142] [143]. If there is enough repetitive information (e.g. multiple satellite imagery) the probabilistic representation can be appropriated. For instance, for classifying land use we can assign to each class a probability value (P[urban]=0.55; P[rural]=0.40; P[water]=0.05), representing the probability of each class in different geographic coverages or table fields. Using fuzzy set theory we can classify the thematic data by attributing a grade of membership to each class, and these values may be computed by a variety fuzzy classification methods.

The thematic uncertainties can be important for Spatial Distribution Planning because thematic coverages, mapping classes, are used to assess lookup tables of input values. For instance, if there are uncertainties in land use directives (development for domestic or for industries), these uncertainties will have strong impact on the spatial load forecast for domestic or industrial consumer development.

5.1.4.3. Logical uncertainties

The logical uncertainty, also referred as topological uncertainty, refers to the uncertainty associated with the relationship among geographic entities. This type of uncertainty is especially significant in network design because the topological relationship among electric components is important for the analysis. Little research has been undertaken in this area [144]. Because the possible relations among components lead to a large number of combinations, this kind of uncertainties is in general ignored in the spatial analysis. In Spatial Load Forecasting (SLF) the logical uncertainty is not important because the inputs for this kind of problem are not based on network topologies or on relationships among geographical features. However, the spatial forecast results are extracted to build possible electric network topologies and consequently their output uncertainties generate logical uncertainties. The positional uncertainty can lead to logical uncertainty, especially when one uses conversion functions between different GIS data structures. For instance, when converting a raster data structure to vector network structures, this is done when extracting the possible network topologies from load
forecasting results. More difficult than extracting the topology is the transfer of the positional uncertainty measures to logical uncertainty. The topological uncertainty can be modeled as a measure of existence of each geographical component. For instance, two electric lines may be (or not) connected if a third line exists between them. If we associate a probability measure of existence to lines or interconnected components we can transfer the SLF output uncertainty to a topological uncertainty measure for the network optimization models used in distribution planning.

5.1.4.4. Temporal uncertainties

Temporal uncertainty occurs when the data set is not synchronized with the time stage that it represents. One case is when the input data are not updated (old data) for the current representation. Another case is when we have some data from the past but we have uncertainty about the exact date that they represent. Last, and the most important for planning methodologies, we have the case of future data representation, as spatial load forecasting results [145]. The uncertainties in future spatial data depend on decisions and events, and some of them can be forecasted but always with associated uncertainties about the timing for their occurrence. As expected, due to the crossing and accumulation of uncertainty sources, the uncertainties in data increase with the forecasting time horizon [146].

The temporal uncertainty is about the lack of information on the timing for the appearance of geographical features or a customer. The modeling of time uncertainties in inputs is done by scenarios if we are dealing with singular features (for instance, building a new road on stage 2 or on stage 5). If the features are countable (for instance, number of customers) we can use probabilistic models or fuzzy models for the number of consumers that appear in each stage. The occurrence of events must be modeled for the global time horizon. For instance, the evolution of the global forecasting (total number of consumers) could follow different time series, but the total number of consumers for the global forecasting horizon should be the same.

Extracting the temporal uncertainties from the Spatial Load Forecasting results is a difficult problem because we only have information about the counting of consumers or about the load of aggregated consumer groups and we don't have information about the timing for appearance of each individual consumer. The way to extract the temporal uncertainty from Spatial Load Forecasting results is using scenario uncertainty models
allowing the output separation in multiple layers. When constructing the multiple possibilities of network topology, based on the several layers of spatial load forecasting results, the uncertainty based on subjective probabilities of each scenario could be extracted to the network components.

An important aspect to focus is the relation between time uncertainty and resolution. For long time horizon the uncertainty on data increases as a consequence of the lower data detail, the decreasing number of sources of data, and incomplete data. Consequently, each map unit (cell) contains lower descriptive information and the resolution to be used to describe the real world will be lower. For instance, if we are planning for a short time horizon we expect a high-resolution path for an underground cable (2 meters resolution). However, if we are planning for a long time horizon, for which most of the urban structures are unknown, it is still possible to plan the path for an underground cable, but spatial planning results with high resolution are not credible. In Spatial Distribution Planning there are geographical zones with heterogeneous detail of information, zones with well-known infrastructures mixed with areas to be developed several year later. These issues suggest that in Spatial Distribution Planning we must use multiple levels of analysis details (multiple resolution) directly related with the time uncertainty of each region.

5.1.5. Uncertainty Propagation on Models

Previously, we discussed methods for identifying and modeling uncertainty. In the following paragraphs we discuss issues related with the propagation of error through the spatial analysis process. The general computation process associated with Automated Planning Tools is a data flow including several interacting modules (spatial load forecasting, substation siting, service area optimization, line routing; network design, etc), each one composed of complex sets of GIS spatial functions. The spatial analysis modules, based on GIS functions, have exceptional capabilities to cross data from several sources. Unfortunately, this capability is also a disadvantage due to the propagation and cascading of the error through the spatial computation model, producing results with specific levels of uncertainty [148] [149].

There are two techniques of error propagation control, the formal error propagation and the simulation modeling. These two techniques can be applied in three theoretic frameworks, Bayesian probability theory, mathematical theory of evidence, and fuzzy set theory. Bayesian theory is based on fundamental probability theory and is appropriate for
dealing with uncertainty about randomness or variability facts. Opposed to Bayes theory, the theory of evidence, developed by Dempster and Safer [150], allows a certain degree of doubt or ignorance (measure function that does not need to sum one). The fuzzy set theory [151] allows a variable degree of membership to a value or class, modeling the ambiguity associated with the identification of alternatives, entities or facts.

5.1.5.1. Formal error propagation methods

Formal methods for error propagation are mathematical functions describing how uncertainties are modified through the GIS spatial analysis operations. These methods are a function of the nature of input uncertainties and are specific for each GIS operation. The problem with formal error propagation is that they are only available for a small number of spatial operations [152].

When uncertainties are described by probability measures, it is possible to use Bayesian networks to model the conditional probability. For example, given that one condition is true (e.g. substation located on site X), how does this affect the next decision to be made (e.g. network design). When the mathematical spatial operation is unknown it is possible to derive a formal mathematical expression using the simulation modeling.

When uncertainties are represented as fuzzy numbers, large sets of operations can be modeled as fuzzy sets operations (theoretic operations, arithmetic operations, ranking operations, etc). The computation complexity of the operation depends on the representation used for the membership functions. If the operation is too complex it is possible to use the Extension Principle. This principle is one of the most basic ideas of fuzzy set theory providing a general method for extending non fuzzy mathematical concepts in order to deal with fuzzy quantities. The great advantage of a formal method in spatial analysis is the lower computational load. The automated planning tools used in distribution planning are composed by complex modules that are too computationally heavy to be implement by simulation methods or by the extension principle of fuzzy theory.

5.1.5.2. Simulation methods

Simulation modeling starts with the definition of a stochastic error model allowing the production a set of perturbed version of the original data set $V$.

$$E = f(V_a + U_a(\varepsilon_a), \ldots, V_n + U_n(\varepsilon_n))$$

(5.1)
Based on the stochastic models of each kind of uncertainty (positional, thematic, topological or temporal) it is possible to generate random fields $U$, which avoid the application of traditional inversion methods. If the variables are independent the random field can be defined by using a probability function, with mean 0 and standard deviation describing the uncertainty on the input by a random noise $\varepsilon$ in $[0-1]$. This approach defines an output value that is the realization value $E$ which is the mean function value contaminated by the uncertainty, as a random outcome with particular probability density function. Repeating the simulation multiple times one obtains the probability function for the global uncertainty.

When there are several uncertainties, represented by corresponding random fields, it is necessary to consider the correlation $\rho$ between the several uncertainty sources. This definition of the correlation factor can be very difficult, especially when the number of variables to be correlated is higher than two. The random field must be generated as

$$E = f(V_1 + U_1(\varepsilon_1), ..., V_n + U_n(\varepsilon_n), \rho(V_i + U_i(\varepsilon_i)))$$ (5.1)

Another process to simulate multiple sources of uncertainty is the sequentially simulation. In this method, the random field inclusions $U_i$ are generated sequential, simulating one uncertainty class at a time. The sequential simulation also allows the control of the auto-correlation for each category and the adjustment of the relative size of the inclusion.

Repeating the simulation process a large number of times will indicate how uncertainty is propagated through the analysis. If the outcome of the analysis is a geographical coverage, simulation results describe multiple results allowing the visualization of uncertainties.
5.2. SCENARIO PLANNING ON SPATIAL LOAD FORECASTING

In the previous section we discussed several ways to model uncertainty, and scenario planning is one of these uncertainty modification techniques. Scenario planning could be used to model all kinds of uncertainties discussed before, but it is more appropriate to model uncertainties resulting from dissimilar decisions and policies. The forecast for a specific scenario is a strong statement about what will happen in the future. In the next section we study the uncertainty propagation, an approach more appropriated to model small deviations from this statement.

Scenario planning consists in the study of multiple possibilities about what future should look like. The scenarios represent several contrasting futures, identifying economic, technological, or event possibilities for the future. The scenario planning requires the participation and responsibility of the Decision-Maker in the construction and evaluation of the scenarios. The Decision-Maker must structure the uncertainties by scenario identification processes. The scenario identification process helps decision-makers to structure uncertainties identifying the events leading to different plans for the forecasting environment. Once these events identified, it is necessary to verify the correlation among them. This will eliminate many unrealistic scenarios resulting from the spatial and temporal combination of the events. Because of their nature, simple forecasting models are unable to model uncertainties by exploring the logic of how events occur. Scenario planning allows the decision-maker to define alternative environments for which decisions are taken.

A scenario identification process escapes the rules of a systematic processing. The planner must identify the scenario, by providing a consistent and coherent alternative for the possible futures. Because the number of possible scenarios is huge, and because the interrelation between events is very complex, the scenario identification must be a human process that selects a limited set of relevant scenarios. So, the planner should follow the following three steps for scenario identification:

- Identify the uncertainty factors and events that influence the decision environment and analyze the logic relation between events. For instance, identify the different plans for development of new roads and identify the timing, and the
planning relation for the construction of the several roads (e.g. if road A exists don’t build road C).

- Develop scenarios that are consistent visions of the future. The alternative futures must be credibly coordinated events, with very different outcomes. Identify a minimum number of very probable combinations of events. Try to diversify the scenarios, use only representative and contrasting combination of events.

- Ensure that events with identified uncertainty are critical for the decision. Due to the multiplicity of scenarios resulting from the spatial and temporal combination of events, it is important to limit our analysis only to the combinations of events that critically affect decisions and their performance.

Is not necessary for the Spatial load Forecasting (SLF) to associate a probability to a scenario because each scenario is forecasted independently. However, some decision models will use the forecasting results, probabilities or weighting process and require a stochastic or subjective probability value to characterize the scenario. The planner, for each scenario, must define these values, and the forecast result maps will be associated with the same probability defined for the scenario. In general, subjective probabilities are used to characterize the probability of the solution space for which the scenario is representative. Note that if we specify the probability of each scenario, based on the probability of each combination of events, all possible scenarios should be considered, which in general is impossible due to the large combination of possible events.

The Fuzzy SLF has a Scenario Coordinator module, which allows the planner to identify scenarios and to interact with the system along the several stages. The planner identifies the scenario by identifying the geographical input coverage that the models use in each stage. The geographical input coverage contains the modifications indicated by the planner. The variables to be modified by the planner, in order to identify the scenarios, are the previous designed external dynamic influence factors, and the global forecasting values.

Table 5-1 – Example of the scenario coordination table, specifying one scenario with 7 stages.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Road coverage</th>
<th>Global forecasting</th>
<th>Other inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td>road coverage I - actual road</td>
<td>1000 new customer</td>
<td>...</td>
</tr>
<tr>
<td>Stage 2</td>
<td>road coverage II - add a new ring</td>
<td>1500 new customers</td>
<td>...</td>
</tr>
<tr>
<td>Stage 3</td>
<td>road coverage II - same as on stage 2</td>
<td>2000 new customers</td>
<td>...</td>
</tr>
</tbody>
</table>
5.2.1. Example

To illustrate the functionality of the scenario coordinator and the effects of the scenario planning, three different scenarios are here compared for the load forecasting example presented in Chapter 4.

![Figure 24 - Representation of the Ring Road and the Coast Road to be built on stages 2 and 6.](image)

To identify the scenarios, we first identify the factors that may affect the number of customer development. In our example, there are only two influencing factors that could be changed externally by the user: the road coverage and coverage of secondary urban centers. Because we don’t have credible information for the emergence of new secondary urban centers, we will use only the road coverage. For road development, two possible new events were identified: a) the construction of a new ring road surrounding the main urban center, b) the construction of a coast road on the south part of the main urban center. These roads are digitized on road coverage, but this design for the road is only a guess, the road may follow very different paths (positional uncertainties not considered). Notice that these two road designs aim to be two contrasting events that are representative for many variations of road designs. The next step consists in identifying the logical relation between the timing for road construction (temporal uncertainties for singular events). We identified that, due to the limited resources, it is impossible to construct the two roads at the same time. Thus we specified that a road should be
constructed separately on stage 2 and 6, but without information about which one will be constructed first.

Another dynamic factor that affects the development in the number of customer is the global forecasting. The global forecasting depends on the global economic and social behavior. Different assumptions or different forecasting methods may produce different growth curves. In general the uncertainties in global forecasting are continuous variations, but the uncertainties could be too high, and a good procedure is to separate them in several contrasting growth curves. In this example we identify two contrasting curves (temporal uncertainties for countable variables): a) the first with gradual growth; b) the second with sigmoid growth shape.

![Figure 25 – Two global trending curves for scenario identification (linear growth – thick line; sigmoid growth – dashed line).](image)

The next step is the combination of the events of the several variables in the final scenario specification, having in consideration the correlation between events. In the road coverage there are two combinations and from global forecasting we have other two, resulting in 4 combinations. However, the sigmoid global forecast curve has been forecasted assuming the construction of the ring road on stage 2. On the other hand the gradual growth forecast was assumed as independent of the road constructions. So three possible final scenarios remain:

**Scenario 1** – global forecasting with gradual growth; coast road constructed on stage 2; ring road constructed on stage 6.

**Scenario 2** - global forecasting with gradual growth; coast road constructed on stage 6; ring road constructed on stage 2.
Scenario 3 - global forecasting with sigmoid growth; coast road constructed on stage 6; ring road constructed on stage 2.

5.2.1.1. Uncertainty modeling of singular events

The SLF results obtained for each scenario have differences in spatial distribution and on per cell growth behavior. The maps of the evolution of the saturation maps are presented on Figure 26 (scenario 1) and Figure 27 (scenario 2). Observing the two simulation results, it is difficult to perceive the differences, but they exist and reach approximately 10% difference in some locations. These differences are more evident close to the new roads.

Figure 26 – Spatial Load forecast results for scenario 1, showing the location and magnitude of load development for 4 of the 7 simulation time stages (dark zones represent higher load development). The scenario 1 is characterized by a global forecasting with gradual growth; a construction of a coast road on stage 2 and a ring road constructed on stage 6.
Figure 27 – Spatial Load forecast results for scenario 2, showing the location and magnitude of load development for 4 of the 7 simulation time stages (dark zones represent higher load development). The scenario is characterized by a global forecast with gradual growth, coast road constructed on stage 6, and ring road constructed on stage 2.

For better understanding and analysis of the differences obtained for the two scenario results, we present in Figure 28 the difference between the result maps for scenario 1, 2.

The legend class label with “scenario 1>scenario 2” shows areas where we noted high development due to the attraction caused by the construction of the coast road. The effect of attraction is clearly observable near the coast road, especially in areas that were non-attractive before the construction of the road. In contrast, the legend class labeled with “scenario 1<scenario 2” shows, near the ring road, the areas with higher development attraction due to the construction of the ring road. Far from both roads we also could observe differences, some labeled as “scenario 1>scenario 2” and other labeled as “scenario 1<scenario 2”. These differences appear on the poor development areas for which the development decreases due to the displacement of consumers to areas near the new roads. This happens because the global number of consumers is still the same, and consequently if in some areas the development increases in other the development must decrease.

These examples evidenciate the effect of location and time uncertainties for singular events. This kind of uncertainties results in significant location uncertainties for spatial
load development. The consequences of these uncertainties are very important for electric distribution planning because the optimal substation locations and cable routing may change significantly.

![Maps](image)

Figure 28 - Comparison maps showing the difference between simulation results on scenario 2 and scenario 3. The maps show differences on 4 different time stages. The legend (scenario 1 > scenario 2) shows areas where we noted high development due to the attraction caused by the construction of the coast road (effect near the coast road). The legend (scenario 1 < scenario 2) shows areas where we noted high development due to the attraction caused by the construction of the ring road (effect near the ring road).

5.2.1.2. Uncertainty modeling of countable events

In a second part of the scenario planning example we will study the uncertainty modeling of countable events, by studying the influence of time uncertainties in the global forecasting. For that we compare the results on scenario 2 and scenario 3. These two scenarios differ in the shape of global forecasting growth, as show the Figure 25. The total number of consumers to develop in the two stages is the same but this development is different in each stage. This scenario formulation is a way to model time uncertainties in countable events (number of new consumers to be appear).

On Figure 29 we can observe three development maps for the corresponding time stages. The evolution observed in the maps reflects the push to development outside the urban center, mainly caused by the saturation on the central areas. The results obtained for scenario 2 and 3 are very similar without perceptible difference on development maps.
However when the differences as observed in the evolution of the saturation level for 4 different locations (A, B, C and D) we perceive an evolution pattern that reflects the evolution pattern of the global forecasting. From the chart it is evident the earlier development and saturation of locations near the urban center. From the chart we also observe higher differences for zones with high development, reflecting an effect of the global trending uncertainties on the development magnitude of load growth. On the saturated locations differences are very small.

![Maps showing stages 2, 3, and 5 with graphs illustrating the evolution of the load development](image)

Figure 29 – Spatial load forecast result showing the evolution of the load development (development for time stage 2, 3 and 5 for scenario 2). The chart shows the evolution of the saturation level and the difference between the saturation on scenario 2 and scenario 3. The chart shows the influence of global forecasting curves in the time uncertainty.

This example shows that time uncertainty in global forecasting causes impact on the magnitude and timing for load growth. Time uncertainties in global forecast directly influence the time uncertainty in spatial outputs, as show the chart in Figure 29. This example shows how the model could estimate the time uncertainties in small area basis based on the time uncertainties of the global area. Note that uncertainties in global forecasting affect the timing for development instead of development spatial pattern.
5.3. UNCERTAINTY PROPAGATION ON SPATIAL LOAD FORECASTING

The scenario identification states the large uncertainties, and the smaller deviations from these strong statements are modeled by the uncertainty propagation models. Without further analysis the uncertainty propagation models could seem secondary and less important in uncertainty evaluation. However, this is not true. The distribution expansion planning has always important fixed cost components, for which the important is not the magnitude of the development, but the load existence. If there is a development, even if very small, the network expansion for the cell is required involving significant fixed costs. Thus, in our spatial problem it is essential to model the uncertainties resulting from small deviations from forecasting statements. Anyway, the uncertainty propagation is an important issue that has been a focus of research for the geographical information science in the last years. Results presented on nice maps could be very informative and conclusive for decision-makers; however, the real value of this information is only available when the spatial uncertainty of the map is known. Inaccurate information could be useful if its uncertainty is known, however ignoring uncertainty in supposedly accurate information could lead to very bad decisions.

The Fuzzy Spatial Model propagates and crosses information from input spatial variables and from its parameters. As in other spatial models the uncertainties are modeled in three main steps as follows:

- The first step consists of identifying, for each input variable and parameter, the kind of uncertainty and how it can be extracted from the data sources.

- The second step consists of the definition of the conceptual model for the uncertainty; this depends on the kind of uncertainty identified in the first step.

- Finally the third step consists of the definition of an appropriated model for the uncertainty propagation.

Concerning the uncertainty identification, the most important uncertainty source of FSM is the spatial uncertainty related with the spatial influence factors. There are several types of spatial uncertainties discussed previously in chapter 2 (positional, thematic, temporal or topological). The extraction of these uncertainties could be based on statistics, symmetry considerations, or qualitative description. On the other hand, there are
uncertainties in Cellular Automata parameters, extracted by qualitative description. There are also uncertainties in global forecasting that are propagated to the SLF.

To implement the second step we have the following conceptual models of uncertainty: intervals, probabilistic models or fuzzy models. The interval modeling could be used to model uncertainties described qualitatively, by using stochastic modeling. The probabilistic models are the most appropriated when uncertainties are probabilistic, by using probability distributions. Fuzzy models could be used to model uncertainties in the Fuzzy System producing fuzzy maps of potential for development. However in the Fuzzy Spatial Model (FSM) the Fuzzy System is coupled to the Cellular Automata which is an iterative algorithm non-modeled by fuzzy operations. Thus, the fuzzy models are not appropriate to model the uncertainty propagation in the FSM.

To implement the third step one could use two types of models for uncertainty propagation: simulation methods (Monte Carlo method [153]) and formal methods (Taylor methods [154] or Rosenblueth’s method [155]). Formal uncertainty propagation methods use explicit mathematical models describing the uncertainty propagation on spatial operations. If the mathematic modeling is possible, the formal approaches are interesting due to their easy applicability. Unfortunately, the majority of the functions on spatial analysis are too complex for using formal models. The simulation methods are the alternative for methods like FSM where the formal methods are not applicable. The main advantage of the simulation techniques is their applicability independently of the complexity of operations involved in the module. Motivated by the complexity of the spatial analysis used in Fuzzy Spatial Load Forecasting we will adopt simulation methods to model uncertainty propagation on our model.

5.3.1. Example

To illustrate the modeling process and the effects of the uncertainty propagation we present in this section one example for the scenario 1 of the example presented in the previous section. Remembering, this scenario consists of a domestic load forecasted along seven stages. The influence factors and the set of global forecasting values are the following:

V1 - Distance to main urban centers
V2 - Distance to secondary urban centers
V3 - Domestic saturation level
V4 - Distance to roads
Constructing a coast road on stage 2
Constructing a ring road on stage 6
V5 - Distance to the Coast
V6 - Terrain slope

Global forecasting with gradual growth - [1000; 1500; 2000; 2500; 3000; 3500; 4000]

The Table 5-2 summarizes the uncertainty modeling for each uncertainty source and presents the implementation formulas used in the simulation. The general function \( F(\mu, \sigma) \) is the normal probability function, where \( \mu \) is the mean value and \( \sigma \) is the standard deviation. The general \( F^{-1}(\mu, \sigma, \epsilon) \) is the inverse of the probability function, where \( \epsilon \) is a random value in \([0,1]\) representing the probability value. The \( \epsilon_{ij} \) represent a random value in \([0,1]\), for cell \((i,j)\).

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Uncertainty Modelization</th>
<th>Simulation Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to centers</td>
<td>saturated center ( S_{center} \in [75; 85] ) resolution error ([-250; 250])</td>
<td>( DC_{ij}^* = \text{MinDist}<em>{\text{cells} \leq (S</em>{ij} &gt; 75 + \epsilon_{ij} \cdot 10)} + (\epsilon_{ij} \cdot 500 - 250) )</td>
</tr>
<tr>
<td>Saturation Level</td>
<td>initial saturation ( F(\mu(S_{ij}^c), 0.2 \cdot \mu(S_{ij}^c), \sigma(S_{ij}^c)) ) development for stage ( t ) ( F(\mu(D_{ij}^t), \sigma(D_{ij}^t)) )</td>
<td>( S_{ij}^* = F^{-1}(\mu(S_{ij}^c), \sigma(S_{ij}^c)) + \sum_{t=1}^{\infty} F^{-1}(\mu(D_{ij}^t), \sigma(D_{ij}^t)) \cdot \epsilon_{ij} )</td>
</tr>
<tr>
<td>Distance to roads</td>
<td>position of existent roads ( F(0.25) ) position of new roads ( F(0.1000) )</td>
<td>( DR_{ij}^* = F^{-1}(\mu(DR_{ij}^c), \sigma(DR_{ij}^c)) \cdot \epsilon_{ij} )</td>
</tr>
<tr>
<td>Terrain Slope</td>
<td>terrain elevation ( F(\mu(Z_{ij}^c), \sigma(Z_{ij}^c)) )</td>
<td>( TS_{ij}^* = \text{Slope} \left( F^{-1}(\mu(Z_{ij}^c), \sigma(Z_{ij}^c)) \right) \cdot i, j \in \Omega_{ij} )</td>
</tr>
<tr>
<td>Global Forecasting</td>
<td>from global forecasting module ( F(\mu(G^c), \sigma(G^c)) )</td>
<td>( GF_{ij}^* = F^{-1}(\mu(G^c), \sigma(G^c)) + \epsilon )</td>
</tr>
<tr>
<td>CA parameters</td>
<td>neighborhood weight ( \beta \in [0.15; 0.25] ) innovation weight ( \lambda \in [0.05; 0.15] ) selective weight ( \varphi \in [0.8; 0.9] )</td>
<td>( \beta_{ij}^* = 0.15 + \epsilon \cdot 0.1 ) ( \lambda_{ij}^* = 0.05 + \epsilon \cdot 0.1 ) ( \varphi_{ij}^* = 0.8 + \epsilon \cdot 0.1 )</td>
</tr>
</tbody>
</table>

All the distance influence factors have a minimum uncertainty that is affected by the resolution. For distance influence factors the uncertainty has deviation value in the interval \([-250m; 250m]\). In this example the distance to centers are computed as the distance to cells with saturation higher than \( S_{center} \); this value is a source of uncertainty specified qualitatively by an interval value \([75\%; 85\%]\). The distance to new roads (coast
and ring roads) are affected by larger positional uncertainties with \( \sigma = 1000m \); the existing roads have lower uncertainty extracted from the digitalization result with \( \sigma = 25m \). The modeling of uncertainty for distance to roads is computed based on stochastic models; first, the probability of existing a road on each location is computed and then the discrete probability function for the distance in each cell location. The uncertainty in terrain slope is computed by stochastic methods based on digital elevation model uncertainties. For the global growth, we assumed a standard deviation 20% of the global forecasting value. For the uncertainty in initial saturation level, a standard deviation of 20% was assumed. From stage 2 to stage 7, the uncertainty in saturation level is computed by stochastic methods including the uncertainty for all previous developments. The uncertainties in Cellular Automata parameters are described qualitatively by specification of deviation intervals.

The simulation process uses 30 simulation runs (index \( s \)), where each uncertainty source is affected by a random noise modeled in the simulation formulas of each input uncertainty (see Table 5-2). On spatial inputs the random noise is computed for each cell location \( e_{ij} \) and for each simulation run; on the CA inputs the random noise \( \varepsilon \) is modeled as one value for each simulation run. The simulation formulas depend on uncertainty modelization, previously discussed. One important aspect in this example is the recursivity of the uncertainty propagation. Excluding the initial stage, the saturation level is computed based on the development results of the previous stages, thus the uncertainty on the development is a source of uncertainty for the next stage.

The result of the simulation is a stack of 30 development maps for each stage (given by equation (5.1)). The simulation is completed for a stage \( t \), next are computed the statistics for the 30 simulations, and the process continues repeating for the next stage \( t+1 \).

\[
D_{ij}^t = f(DC_{ij}^t, S_{ij}^t, DR_{ij}^t, TS_{ij}^t, GF_{ij}^t, \beta^t, \kappa^t, \varphi^t) \tag{5.1}
\]

On table of Figure 30 we may observe the stochastic results of the simulation, the mean value \( \mu \) and the standard deviation \( \sigma \). By comparing the \( \sigma \) for the several points we can confirm that uncertainty varies from site to site, but in general higher uncertainties are associated with locations with faster development. The faster development areas are located closer to the urban centers (faster development follows the order \( A>B>C>D>E>F \)).
To analyze the uncertainty evolution through the saturation curve, we classified the results by saturation level and for each saturation interval we computed the standard deviation (Figure 31).
This example is presented to understand the uncertainty propagation through the FSM models. This particular application of the FSM uses recursive uncertainties, with uncertainty inputs estimated as outputs of the previous stages. This structure raises the question, if the uncertainty is amplified or attenuated along the time stages. Observing the evolution of the standard deviation along the seven stages we can see (in Figure 32) that the uncertainties decrease when we forecast for larger number of stages. This fact is a consequence of the sum of the stochastic modeling of the development in several stages (see simulation function for saturation level, on Table 5-2). The sum of several stochastic independent variables (development in each stage) is the convolution of their density functions; in this case, the result of the convolution with the stochastic variable of a new development stage leads to a distribution function with lower variance. This effect is similar to what happens when aggregated simultaneous loads in the network: larger number of simultaneous consumers implicate lower variance on aggregated load.

![Figure 32 – Evolution of the uncertainty, over all location cells, trough the evolution of the time stages.](image)

In practice if the same uncertainty per stage is used in inputs (independent of the stage period), the model forecasts with lower uncertainty when forecasting with more number of stages. In fact, if we use shorter periods for stages the uncertainty per stage in global forecasting becomes higher (it is easier to forecast the emergence of new consumers in the next year than in the next month). Then, when we use more time stages in the same forecasting horizon we should use larger uncertainties in the per-stage global forecasting uncertainties.

The spatial pattern of the uncertainties could be observed on the Figure 33. The evolution of the higher uncertainties along the time stages shows higher values of uncertainties for locations with more intensive development. The locations with higher terrain complexity (higher density of contour lines) have higher uncertainty consequence
of the slope uncertainty. Higher spatial uncertainty is also observable near the new roads. This uncertainty is caused by high positional uncertainties resulting from the guessing of the road path.

![Maps showing stages 2, 3, 5, and 7 with annotations](image)

Figure 33 – Evolution of the spatial uncertainty on the saturation level. The dark zone represents higher standard deviation for the saturation level.

Along the stages the uncertainty in saturation growth is proportional to the development, decreasing when the saturation reaches maximum value (observe the zones A and D). Another important observation is that uncertainties are higher for higher slopes; this may be clearly observed at zones B and C. We observed that all the indicated zones (A, B, C, D) reach saturation levels very close to the maximum for time stage 7. However in some of these zones (B, C) the uncertainties are still high even when the location is saturated. These maps of uncertainty (Figure 33) show clearly that the outputs of Fuzzy Spatial Load Forecast could represent the characteristics of location uncertainties and time uncertainties.

The results of Spatial Load Forecasting are used by other automated planning tools (e.g. substation sitting, line routing, network topology design). The information for these automated planning tools extracts and uses the load forecasts and the uncertainty associated. These planning modules use the raster load forecast maps to compute vectorial results such as the location of loads and substations as points and electric lines as sets of
line segments. The stochastic modeling may be extracted from the raster forecast maps and stored in a vector database (points and lines); this way, each load point will have as attributes the mean value and the standard deviation. These parameters may be propagated over the network design modules in order to obtain stochastic modeling for load and capacity on lines and substations.

In some situations, uncertainty in load affects the topology of the network design. When vacant areas are developed, new lines must be built to supply this load. If we admit erroneously that some load exists (type II error) this may implicate high consequences in planning because building new lines for vacant areas has in general high costs. If we admit erroneously that load doesn't exist (type I error) the consequences may be smaller because investment may be delayed but appear in the next time stages.

![Figure 34](image)

Figure 34 – a) Mean value of development for time stage 5; b) developed areas for 3 different confidence levels 90.0%, 99.0%, and 99.9%.

To hedge for type II errors, it is recommendable to increase the confidence in load development. To illustrate this, we present in Figure 34a) the mean value for development for stage 5 (the number of new consumers per 250m pixel varies between 0 and 6). We may observe that development may happen at a large number of locations, which is very important if the network happens to be designed to supply locations which higher development uncertainty, like vacant areas. In Figure 34b) we mapped the developed locations for 3 different confidence levels 90.0%, 99.0%, and 99.9%. At higher confidence levels the developed area shrinks, restricting the number of developed areas. This shrinking effect is more relevant on areas with higher uncertainty.

Similarly to the way we compute the confidence level one may compute the probability of existence of a predefined level of load. These probabilities may be
propagated to the lines and substations leading to topological uncertainties in the results of the network design. This exemplifies the extraction of SLF uncertainties to topological uncertainties in the electric planned components.

5.4. CONCLUSIONS AND REMARKS

The SLF model produces load maps, which are the basis for Distribution Planning. In this chapter we have shown that there are many sources of uncertainty, most of them spatial uncertainties, requiring special modeling. The uncertainties affecting the SLF may be propagated to the forecasted load maps and consequently to the Distribution Planning process.

The modeling of uncertainty is essential in SLF. The relevance of a forecasting map is only solid when its uncertainty is understood. The Fuzzy Spatial Model allows two different ways of evaluating output uncertainties: the scenario planning and the uncertainty propagation. The Scenario Planning produces the forecast statements imposed by contrasting futures resulting from different decisions, politics or other events. The Uncertainty Propagation models small deviations from the forecasting statements originated by uncertainties on input deviations.

The FSM was designed to guarantee a good interaction with the planer allowing the scenario identification by using the Scenario Coordinator module. The spatial influence factors affect the pattern of the spatial development, implicating consequences on substation sitting and network design, and on the other hand the non-spatial input uncertainties affect the magnitude and timing for development and planning.

For Uncertainty Propagation, one requires the identification and modeling uncertainties on inputs and parameters. The complexity of Fuzzy SLF models requires the use of simulation methods to model the uncertainty propagation. In this chapter, simulation methods have been used to evaluate the error propagation. Different kinds of uncertainties have been modeled and the uncertainties in several outputs have been analyzed.

The small deviations modeled by the Uncertainty Propagation are particularly important because they allow the modeling of initial developments that impose fix costs, which are extremely important for distribution planning.
5.5. REFERENCES


Chapter 6  FURTHER SPATIAL PLANNING TOOLS

This thesis is about Fuzzy Spatial Load Forecasting, and the core ideas of the innovative techniques developed have been discussed in the previous chapters.

The central concept in the forecasting exercise has been the use of computational intelligence tools. However, this use cannot be separated from the technological basis for representing data and knowledge and interfacing with the planner – the GIS. GIS Spatial analysis tools have a promising future in Electric Distribution Planning. These tools establish a link from automated planning concepts and real world modeling.

This thesis is not about Distribution Planning, only on forecasting. However, the availability of credible spatial load forecast opens a large door and offers big hope into a sort of evolution in the final system design methods that a planner can make use of.

In order to achieve a more complete work with this thesis, this chapter discusses complementary aspects to the forecasting exercise, in the use of GIS tools for the following purposes:

- Spatial Distributed Generation Forecasting.
- Feeder routing
- Substation siting and service area optimization

In each topic, besides a discussion of the state of the art, the text presents and discusses innovative ideas and work already done by the author. These discussions must be understood as a classification of future work, in the follow up of the thesis on spatial forecasting. However, these is more than just vague hints on work to be done in the future and it was felt that it would be adequate to dedicate to these ideas more than just a few paragraphs in the traditional chapter devoted to future research.
6.1. AUTOMATED PLANNING

The automated planning tools need information from the real world provided by expert planners or, as defended in this thesis, by GIS spatial analysis preprocessing tools. This information could be grouped as follows:

- Information about timing, location, size, characteristics and uncertainties of loads. This information is the one obtained from Spatial Load Forecasting, which was studied in detail in the previous chapters.

- Information about timing, location, size, characteristics and uncertainties related with Disperse Generation. Nowadays, distribution planning depends not only on consumptions but also on generations. Disperse Generation has an impact in distribution planning with an increased importance. The geographical characteristics of this impact are similar to the consumption distribution. In this perspective, one will require a Spatial DG Forecasting (DG siting). The first section of this Chapter will be dedicated to this subject, discussing the spatial analysis tools to DG energy resource potential and presenting some ideas for a spatial DG forecast model.

- Information on the location of electric network components and on topologies. Substation Siting, Line Routing, and Network Design are old automated planning tools that could be strongly improved with the use of GIS spatial and network analysis. These tools will use as information basis the load and DG forecast and also detailed geographical information about cost and restrictions. The second section of the chapter is dedicated to open some ideas about future research in this area.

6.2. SPATIAL DG FORECASTING

The centralized traditional power system faces new challenges due to the increase in Distributed Generation [156]-[160]. Distributed Generation is an alternative that is explored and the utility planners must have it in consideration. The DG has impact in distribution planning because it could delay, and in some cases avoid, new investment in power delivery infrastructures. On the other hand, it may require local additional investments.
In the spatial perspective, the DG characteristics are very similar to load characteristics: it is spatially distributed; in most of the cases, the growth behavior is not controllable by the utility; and the development follows a typical technology innovation growth pattern. We have different types of DG technologies as well as consumers; there are spatial influence factors that influence the development; in general, a global value of development, for the entire region, could be forecasted easily (easier than in spatial forecasting) or in some cases could be stated by policies. Then, to forecast the spatial pattern of DG development we could use processes similar to the ones used to forecast spatial load development.

In spatial load forecasting we have several kinds of customer classes with different characteristics and different consumption scales that must be modeled in independent knowledge bases but interacting in the forecasting process. In a similar way, there are several types of DG technologies with different growth behaviors, different influence factors, and different production scales.

In the perspective of energy resources we differentiate two classes of DG technologies. On one side the ones with geographically dependent energy resources (solar, wind, hydro), and on other side the ones that use transportable energy resources (generation systems driven by internal combustion, biomass, gas turbines, and the fuel cells).

Some DG technologies are more appropriate to supply local consumption, and their development is highly dependent on the location, type and density of consumers. Other DG consist in larger systems oriented to produce energy to the electric network; their development location is more dependent of the geographical location of the electric distribution network.

These differentiations are important to define the influence factors that are more relevant to simulate the development behavior. For instance, the spatial distribution of an energy resource is the most important influence factor for solar, wind or hydro DG systems. The DG system must produce on the location where the energy resource exists. The spatial distribution of consumers is an important influence factor for consumption oriented DG types. The distance to electric network infrastructures is an important influence factor for generation oriented to produce energy to the network. Distance to gas infrastructures and costs of fuel transportation are important influence factors for non-
renewable generation. The reliability of the existing energy system (or in extreme cases the avoidance of an energy network) is another factor that could influence the DG development.

The spatial analysis systems based on GIS are the adequate tools for technical-economical evaluation of high potential regions to develop DG systems [161]-[164]. These tools together with the methodology used in the Fuzzy Spatial Models (see Chapter 3) could be used to build Spatial DG Forecasting. The Spatial DG Forecasting is a novel idea, presented in this chapter on account of its importance to Distribution Planning. Also there is a strong relation with the core research of this thesis.

![Diagram of Spatial DG Forecasting Structure](image)

Figure 35 – Structure of the Spatial DG forecasting

A general framework for Spatial DG Forecasting is presented in Figure 35. The general geographical influence factors are the energy resources, the costs, the externalities, the existent reliability and the load characterization. The Fuzzy Spatial Model (FSM) could simulate the development behavior for each DG technology. As on Spatial Load Forecasting the FSM for Spatial DG forecasting must use a global forecasting value. The spatial behavior of the phenomena is learned, captured, and stored, on the knowledge base of the FSM, based on historical and judgmental information. This knowledge base could be learned from past experience in a different geographical region, and could be complemented with judgmental information from human expert, which becomes extremely relevant in the case of DG forecasting.

The energy resources are spatial assessments of the energy available for each technology, which could be: maps of mean wind speed, maps of mean monthly global radiation, maps with mean flow in rivers, or maps with the characterization of collectable biomass resources. The information about the energy resource combined with the
characteristics of the energy conversion system allows the calculation of the annual energy that could be produced by the system.

The Figure 36 shows two examples of energy resource assessment that could be used as influence factors for Spatial DG Forecasting. The methodologies used to this assessment are based on spatial analysis methodologies. These methodologies are out of the scope of the thesis, but some these developments done by the author are explained in [161]-[164].

![Wind Energy Resources][1] ![Solar Energy Resources][2]

Figure 36 – Example of wind energy resources. Wind and solar energy resources for the region of La Rioja (Spain). Assessment done in the scope of the project ENERGIS.

We call "levelised costs" to an economic value that includes all the costs related with the implementation of the DG system, transformed in annual $/kWh. They depend on a large set of components (DG equipment, installation, fuel transportation, operation and maintenance, new infrastructure, etc.). Most of these costs depend on the geographical location where the DG system is installed. Each technology has a specific set of cost components that must be evaluated. In some regions the cost components are more important than on other. For instance, in the Amazonian region the cost of transportation is a cost component very important that could be decisive for the development of a certain DG technology.

The energy resources and all the costs related with the DG system could be encapsulated in only one influence factor. This influence factor is the LEC (Levelized Electric Cost) that represents the energy production cost in each location ($/kWh). The lower value of LEC is a good indicator of economic preference for the development of a specific DG technology. The Figure 37 represents the LEC for 4 different systems in a Marajó study region (Amazonia, Brazil).
The economical influence factors (based on LEC maps) could be processed in order to do a spatial filtering the areas that are technically and politically acceptable. To do this geographical filtering we could use simple spatial analysis (crossing information from different geographical coverages) or we could use very advanced spatial analysis systems. The author developed a spatial decision and negotiation tool to help the definition of permitted areas for development DG facilities [164].

Another influence factor to be considered is externalities. The externality-evaluation assesses non-direct benefits (or collateral impacts) of the technology penetration. For instance, the DG technologies with higher environmental impact will be less acceptable on more environmental sensitive regions; this information should be captured in the knowledge base of the Spatial DG Forecasting. Figure 38 shows an example of map that could be used as influence factor representing environment externalities.

Cost and reliability are always the important aspects that drive the development of DG systems. A geographical coverage with indexes of reliability is an important factor that influences the development of DG technologies. In the geographical perspective a DG system has greater incentives for development if the reliability of the existing electric system is low. This means that the spatial evaluation of the existing reliability is an
important influence factor. Rural regions with low reliability indices for the existing energy systems have larger potential for a development of DG technologies; this is especially true in developing areas without an energy network or grid.

Figure 38 – Example map, in La Rioja (Spain), about environmental externalities, the red areas represent less environmentally tolerable areas for wind farm development.

The development of some DG technologies is highly correlated with the spatial Load Forecasting. The distributed generation dedicated to supply local consumption depends on the magnitude of the local loads and also depends in the coincidence behavior between load curves and DG generation curves. For these DG technologies the load forecasting results are one of the most important influence factors and define also the saturation level for DG development (the saturation level of DG is bounded by the maximum load). The use a separated spatial load forecast for different customer classes is also important because some DG technologies have different correlation for different customer classes. For instance, the development of gas turbines and fuel cell systems are highly correlated with industrial consumers, but on the other hand photovoltaics or small Diesel systems may be more correlated with domestic consumption.

The global forecasting is a module based on the trending and on market studies providing a technology growth value for an overall region. The global growth is forecasted based on macroeconomic evaluation of industry and on government policy for distributed generation technologies. For instance, police directives can state that in the next 5 years,
200 MW of wind DG will be developed. Based on this global forecast information, the Spatial DG Forecasting will find the spatial pattern for this development (see Figure 39).

![Figure 39 - Example map, for La Rioja, showing the preferential spatial development for wind farms (blue areas) admitting a global development of 200 MW of capacity.](image)

Based on the proposed spatial influence factors and on the global forecasting for specific technologies it is possible to built a Fuzzy Spatial Model similar to that one build for spatial load forecasting. The fuzzy system knowledge base could be built with historical development behavior captured from previous development experience or could be built based on judgmental information of experts. With this knowledge base, the Spatial DG Forecasting could simulate the development along the forecasting horizon using as inputs the changes in spatial influence factors and the global forecast defined by policies or market projection for the technologies.

The saturation level controls the dynamics of the forecasting process. This definition of the saturation depends on the type of DG technology. For instance, for wind farms the saturation depends on the occupation area of the wind turbines; for small photovoltaic systems the saturation depends on the loads and on the number of consumers.

This section suggests that a feasible Spatial DG Forecasting may be built around the methodologies developed in the previous chapters aggregated with other models used in energy planning. They provide the necessary material to develop this kind of forecasting tools. Nowadays the Spatial DG Forecasting is, for distribution planning, almost as
important as the Spatial Load Forecasting, and consequently some work in this direction will be useful for utilities and energy planning entities.

6.3. FEEDER ROUTING

The Spatial Load Forecasting (SLF) and the Spatial DG Forecasting (SDGF) are the basic information for modern electric distribution planning. When we link this information with the data structures of the automated planning tools, the question emerges about how feeder routing is generated. The usual process is the individual definition, by a planner with experience, of multiple possible line and cable routing, each one associated with several possibilities of capacity and line types. The GIS could be used to automate and perform these tasks.

The automated feeder routing is one of the most promising GIS applications in distribution planning, and is one of the most interesting research fields for people involved in the application of spatial analysis in distribution planning. This reason and also the strong relation between feeder routing and SLF or SDGF motivates the author to a brief introduction of new ideas in this research field. There are some research works related with feeder routing [166]-[174]. However, in the opinion of the author, this research and tools are not profiting from the development of GIS spatial and network analysis capabilities. In GIS Science a large research has been done in routing algorithms. These GIS research works [175]-[180] focus mainly in the geographical aspects of hydrologic networks and road networks. In the area of operational research the routing and network optimization is a broad research area [181]-[183]; however, most of this research is oriented to optimization of graph networks structures and not to spatial network structures.

In the perspective of this work, we are interested in network extraction with GIS and this may be reached with the so-called shortest path algorithms [184]-[186]. The number of shortest path algorithms is very numerous due to its interest in many network applications (electricity, gas, water, telecommunications, roads, etc).

The shortest path algorithms have their basis on graph theory and network analysis. Traditionally, vector GIS is used to support shortest path and network analysis, an approach in the domain of graph theory. This approach is effective when the network path (nodes and edges) is predefined. However, our objective is to extract a network path
from a continuous surface. In this perspective, the use of raster GIS structures (spatial analysis) has advantages because we are able to extract feeder routing directly from information about terrain characteristics. The difficulties in modeling network analysis in raster structures are related with the large networks necessary to the artificial modeling of the surface (depending on the analysis resolution) and consequently the large computational time. Other difficulties when using raster structures are in the modeling of no-planar networks (over crossing lines) and flow directions.

In order to understand network analysis in raster structure we must present the basis of network modeling in raster structures. Its basis consists in modeling the surface as a network structure. In surfaces, the information is location oriented instead feature oriented and this information modeling varies continuously over the geographical regions. The surface network consists in modeling the center of each elementary cell as a node linked to the 8 neighborhood cells by hedges. Each node representing the cell center is associated with the cost of traversing this particular cell. Starting from source point, it is possible to calculate the accumulated cost of traversing cells obtaining multiple path and costs connecting sources to destinations. The minimization of this accumulated costs gives the least-cost path (also called the shortest path or fastest path).

![Example of path from cell-to-cell in raster network modelization.](image.png)

The cost of traversing the cell could be unique in all directions (depending only on location) or could be dependent on the traversing direction [178], [187]. This distinction is important in cases that routing depends on geographic neighborhood aspects, for instance the dependence of terrain slope.

In order to avoid the zigzagging outlines some models use a larger cluster of neighborhood cells [187] instead the 8-neighborhood structure presented in Figure 40.
Interpolating the line path over the cells is another method used to avoid the zigzagging [188].

In the raster network, the information about flow direction could be modeled by storing this direction in the raster structure: each cell stores one or several flow directions defining the upstream and/or downstream flow direction. This kind of structure can only model planar networks. Over-crossing lines should be modeled in different coverages. Analysis with structures based on ranking of preferential direction flows is an interesting future research that could be very promising to model uncertainties in routing and corridor modeling. Corridor analysis is a variant of surface analysis that finds a connection between a source and destination by continuous and elongated geographical areas.

The cost surfaces that store the cost of traversing each location are built based on spatial cost functions that could result from crossing information from a variety of thematic coverages. For instance, the costs for underground electric distribution cable are dependent on the terrain slope, the terrain type, the existence of obstacles, land usage, accessibility of the location, terrain cost, etc. Each kind of line or cable has a specific cost coverage, and this will influence the path of the feeder routing. For instance, the path for a typical routing of aerial electric lines is straighter than the underground cable routing because aerial lines are less dependent on terrain geographical characteristics.

The cost path algorithms implemented in raster structures, and available in some commercial GIS, could compute paths from several sources to all destinations in the surface. The sources could be points, lines or areas. This means that we could find the least-cost path from each load location in the surface to an existent network or to an interconnection point in the network. By using multiple sources the result of cost-path optimization provide not only the path but also the service area associated to each source.

Due to the specificity of the raster structure, the network problem reaches easily a very large number of nodes. For instance, for a small region with 10x10km using a resolution of 50m, the network structure representing the surface area reaches approximately 40000 nodes and 199204 edges. This is the main reason why for raster GIS we only hear of implementations of algorithms that optimize simple problems. The optimization algorithms available in GIS are simple point to point least-cost path. The shortest path problem is a subtype of the minimum cost flow problem where the flow $y_{ij}$ is 1 or 0
depending if the edge belongs to the path \( P \) or not. The \( a_{ij} \) is the edge cost representing the cost to traverse the two half cells \( i \) to \( j \). The formal formulation for optimizing the path \( P \) into a surface \( A \) is:

\[
\text{minimize } \sum_{(i,j) \in A} a_{ij} y_{ij}
\]

Subject to the divergence restriction that represents the difference between the total flow departing and the total flow arriving at node \( i \):

\[
\sum_{(i,j) \in A} y_{ij} - \sum_{(j,i) \in A} y_{ij} = \begin{cases} 
1 & \text{if } i = \text{source} \\
-1 & \text{if } i = \text{destination} \\
0 & \text{otherwise}
\end{cases}
\]

where, path \( P \) is stated by:

\[
y_{ij} = \begin{cases} 
1 & \text{if } (i,j) \in P \\
0 & \text{otherwise}
\end{cases}
\]

Large number of algorithms could be used to solve this discrete optimization problem [181]. The implementation on raster GIS is based on spatial analysis operators (local, focal and zonal function) using the aspects of the raster structures to faster the algorithms [189], [190].

---

Figure 41 – Example of raster GIS Least cost path. Up-left matrix is the raster representation of source (1 represents one line source, 2 represents a point source; D are the destinations and P represent prohibitive areas). Up-right matrix is the cost to traversing each cell. Down-left matrix present the results of the least-cost path, the least
cost to reach each cell. Down-right matrix represent the allocation of cells to the sources and the direction of paths.

Tree-optimization models are also possible by using the variety of spanning tree optimization methods [192]. However, optimizing these problems becomes very complex for the typical size of raster structures and these kind of optimization functions and operators are not available in commercial GIS. There is some research and processes implemented in GIS that reach near optimal solutions in acceptable computation time. One of those processes consists in combining neighborhood destinations in order to share the costs in part of the path [191].

![Figure 42 - Comparison of simple point-to-point optimization to spanning tree optimization. From McIlhaga [191]](image)

Other processes consist in computing the path by steps, sequentially calculating the paths between closest destinations [193].

![Figure 43 - Example of the stepped accumulation process. The least cost path is computed sequentially from point 1, 2 and 3. From Berry [193]](image)

The author studied and developed methodologies to solve the feeder routing problem in electric distribution [194]. The feeder routing problem is complex problem because include:

- fixed costs, associated with geographical features and with the fixed cost of the feeder,
- variable costs, associated with the flow (maximal and mean load).
fixed and variable costs associated with the connection to the sources (could be points or lines)

The methodology proposed by the author to solve the problem in radial structures consists in an iterative method that uses the point-to-point least cost path algorithms available in commercial GIS. The algorithm starts by optimizing the cost-path for all paths $P$ admitting the existence of only fixed costs (fixed cost for source points $f_s$, and fixed cost $f_y$ to traverse the two half-cell $i$ and $j$). In this first iteration the accumulated flow $x_y$ is assumed 0, and the objective function is the following:

$$\text{minimize } \left( f_s \cdot z_s + \sum_{(i,j) \in A} f_y \cdot y_{ij} \right) \quad (6.4)$$

$z_s$ is the restriction defining the site of the known sources:

$$z_s = \begin{cases} 
1 & \text{if is the source of } P \\
0 & \text{otherwise} 
\end{cases} \quad (6.5)$$

$y_{ij}$ is the restriction defining the edges that constitutes the path $P$:

$$y_{ij} = \begin{cases} 
1 & \text{if } (i,j) \in P \\
0 & \text{otherwise} 
\end{cases} \quad (6.6)$$

The result is a radial network structure reaching all the destinations in the region. Based in this radial structure the loads $d_i$ in each destination are back-accumulated obtaining the flow in each edge $x_{ij}$ (assuming a very small value if the edge doesn’t belongs to path $P$) and the accumulated flow in each source $s_s$ (assumed very small if the cell is not a source). The flow divergence implemented by the back-accumulation step is represented by:

$$\sum_{(i,j) \in A} x_{ij} - \sum_{(j,i) \in A} x_{ij} = \begin{cases} 
 s_i & \text{if } i = \text{source} \\
 -d_i & \text{if } i = \text{destination} \\
 0 & \text{otherwise} 
\end{cases} \quad (6.7)$$

With this value of flow, the variable cost associated with the cell is calculated ($v_s$ is the coefficient of variable costs on sources; $v_y$ is the coefficient of variable costs to
traverse the two half-cells $i$ and $j$. The next step consists in optimizing the cost-path admitting coverage of incremental costs (fixed and variable costs divided by the flow that traverse the edge).

$$
\text{minimize } \left( \frac{f_s + v_s \cdot s_s \cdot z_s}{s_s} + \sum_{(i,j) \in A} \frac{f_y + v_y \cdot x_y \cdot y_y}{x_y} \right) \quad (6.8)
$$

The equations (6.8) state that the optimization algorithm continues to be a point-to-point least cost path, permitting the application of least cost-path operators available on GIS. However the objective function includes network flows $x_y$ in addition to the path topology $y_y$. The accumulated flow $x_y$ and the service area accumulated flow $s_s$ (is the accumulated flow in the source point) are a common variable.

The part of the objective function (equation 6.8) that includes the source point variables $(f_s, s_s, v_s, z_s)$ represents the sources cost components. This component is important to optimize the allocation of loads between neighborhood sources. This is an important aspect for substation siting and service area optimization discussed in the next section.

The process is repeated iteratively until no changes are obtained between iterations. Then the raster structure could be extracted to a graphs format obtaining a radial network of lines and nodes.

The Figure 44 shows an example of application of the feeder routing algorithm developed by the author [194]. The problem consists in connecting 2 new wind farms (DG1 and DG2) and 3 new loads (L1, L2 and L3) to two possible points (P1 and P2). For this problem one has specified: the costs of new line connections (in P1 and P2); the fixed and variable costs for the class of line equipment to be constructed (per km cost for multiple capacities); the loads (L1, L2, L3) and capacity of the new wind farms (DG1 and DG2); and the geographical fixed costs (represented in the figure as a background grid).

The heterogeneity of the geographical cost for this problem is clearly identified by the different darkness of the background grid (see areas A1, A2 and A3). A1 is a non-expensive area, the area A2 is a very expensive due to its environmentally protected characteristics (only underground cable could be used), and A3 are complex terrain
mountains also relatively expensive. For this example we have intentionally used a low resolution (1 km) in order to understand the grid structure; however, a real analysis requires resolution 20 times higher (50m). In Figure 44 the raster structure paths observed under the lines (squares under the lines) are the results of the raster network analysis. The lines observed in red are vectorized from raster results at the end of the analysis.

![Figure 44 - Feeder routing results connecting 2 new wind farms (DG1 and DG2) and 3 new loads (L1, L2 and L3) to two possible points (P1 and P2), using the geographical cost grids (background surface grid representing highest cost with dark colors)](image)

In Figure 44 we could observe the efficiency of the algorithm in the optimization of the spanning tree (observe the tree result layout feeding the loads L1, L2 and L3). The effect of the geographical costs are also evident, notice how the line paths avoid the areas with high cost of traversing (A2) but always minimizing the path length. Notice also that two connection points (P1, P2) are available for connection; however, the algorithm found that the point P1 is the best connection solution for both loads and DG.

From equation (6.8) we observe that concentrating the flow in some paths decreases the components of the incremental cost of the fix cost ($f_i / x_{ij}$) and consequently decreases the overall minimum objective function for the spanning tree. For the initial iteration we obtain concentrated flows in non-optimal paths with highly undesired influences in the final result. To overcome this problem, after the flow accumulation step, we calculate a mean focal flow given an imprecise modelization about the flow path. This way, we don’t force the path to a line: we give more flexibility to the optimization to
change the flow to neighborhood cells with lower fixed cost. This process provides solutions approximate to the global optimum and accelerates the iterative process. Most of the problems are solved in 3 or 4 iterations.

Due to the influence of existing networks, the ideal application of the algorithm is by stepped time stages, and interacting with the planner between stages. However, to test the algorithm in large green field areas and a large time horizon we applied the algorithm to the load forecasting results we obtained in SLF, admitting the non existence of electric network and predefining locations for several substations.

Figure 45 – Example of large green-field feeder routing. In all images the green background represents geographical traversing costs based on terrain type, slopes, and proximity to roads (dark regions represent highest costs). a) Load points extracted from SLF and substation points with predefined locations. b), c) and d) represent results for three iterations of the feeder routing algorithm.

Figure 45 shows the application of the algorithm to all the load forecasted region (SLF results from the previous chapters) supposing a completely new network layout with predefined locations for 7 substations. Observing the map a) it is quite difficult to imagine how the layout of the network will be. The map b) shows the initial feeder routing solution (using only fixed costs) optimizing routing from substations to each point, but not optimizing the overall spanning tree. In the second interaction c) we observe big changes due to the inclusion of the variable cost components resulted from flow
accumulation. And finally on map d) presents the final routing. This application shows that the algorithm is not only useful to feeder routing but is also an interesting tool to define optimal service areas of substations.

6.4. **Substation Siting and Service Area Optimization**

Optimizing substation siting and optimizing its service area are important because one obtains the location of source points for distribution and the end points for transmission lines. We decided to present in this section a brief reference to this problem because service area optimization use SLF as one of the most important inputs and also because this kind of facility location problem is a an interesting spatial GIS application in electricity distribution planning.

There are several research works in the field of electric substation siting [195]-[200]. However, as on feeder routing, most of these research works use predefined candidate locations. We believe that spatial GIS tools could significantly improve the site optimization by adding spatial analysis capabilities, and by providing more realistic and complete information in the geographical perspective. The extraction and definition of high potential sites to locate the facilities is the most interesting potential of spatial GIS methodologies.

Substation site optimization includes optimizing three levels of equipment: the transmission network; the substation and the feeder routing for distribution. The site of the substation should be located at a convenient cost-distance from the transmission network in order to minimize costs of the transmission network. The location to install the substation facility should have minimum local cost. The substation must be proximal to the load in order to minimize the distribution feeder cost.

Three different optimization aspects are considered: the site, size and service area. Admitting known the spatial load forecasting, the site and the number of substations are the necessary variables for the optimization. The service area and the site for each substation are optimized simultaneously and the size depends on the load covered by the service area.

The location to install the substation has geographical costs that depend on type of soil, slopes, drainage, accessibility, terrain usage, land cost, landscaping, etc. The
exposition to lightning and adverse weather is another geographical aspect to have in consideration. Feeder getaway is another aspect to be considered, evaluating impacts in the neighborhood of the substation site.

The transmission costs are evaluated by singular line routing, evaluating the distance and costs of extending the transmission lines from the existing network to the substation locations, and also between substations. As shown in the previous section, routing from an existing network requires knowledge on the geographical cost and an estimate on substation capacity. The routing between substations requires also the knowledge of geographical cost, capacity and demands also the unknown position of both substations. These aspects must be taken in consideration but could be simplified in the site optimization process.

The feeder routing is the most important cost component. This routing could be done by the routing methodologies presented in previous sections, obtaining the radial spanning tree. These routing algorithms optimize the radial feeder tree, automatically optimizing the service areas corresponding to the predefined substations.

The feeder routing module produces as output the service area and the incremental cost to supply the loads covered by the substation service area. The Figure 46 shows that the optimal service area is very dependent of the position and number of substations. The service area for each substation defines the loads required to be covered by the substation and consequently estimates the corresponding substation capacity.

![Figure 46 - Example of two different routings for different positions and number of substations.](image)

There is a vast range of algorithms and applications for facility location [201]-[204]. However, the formulation of the substation-siting problem in GIS raster structures is
As shown in Figure 47, along the iteration process the substation site moves towards direction to the center of the service area redefining in each iteration the service area by reallocating the loads to the least cost-distance substation.

For real applications the problem is in general more easily solved than the example presented in Figure 47. In real problems we depart from existing network infrastructure and consequently there are substations with fixed locations for existing substations; the new substations have less flexibility and are highly influenced by the existent structure. An additional difficulty for real expansion planning problems is the multi-stage aspect. This aspect could be implemented by the spatial tools proposed by the author and using the Spatial Load Forecasting results for the several temporal stages; however, this requires a more elaborated formulation for the models.
6.5. CONCLUSIONS AND REMARKS

The Spatial DG Forecasting, the feeder routing, substation siting and service area optimization are very interesting research fields for the present and for near future. These are research areas the author is presently exploring and could be seen as an extension of the work (Spatial Load Forecasting) presented in this thesis. In some of these areas we propose ideas and models (Spatial DG Forecasting), actually ready to be implemented, and on other areas we present some developed and implemented models (feeder routing, substation siting, service area optimization). In all these fields the research areas are very wide with much work to be done. The reader should see this chapter as ideas with high potential for the future research.

The first section of the chapter was dedicated to the discussion of a novel idea of Spatial DG Forecasting. Spatial DG forecasting (SDGF) is a problem very similar to Spatial Load Forecasting (SLF) and the approach to follow for this forecast problem could be the same used in SLF. Then, the center of the research presented in this thesis, the Fuzzy Spatial Model, could be adapted to forecast distribution generation. In this perspective, it was discussed how to model a SDGF, analyzing the structure of the modeling, studying the geographical influence factors and other aspects related with the functionality of the model.

The feeder routing and substation siting is another research field very promising and with high potential to be implemented in spatial analysis. This research field is very wide involving yearly researches in distribution planning and involving large number of algorithms object of research in network optimization. The GIS provides computational leverage and structures that improve the performance and realism of automated tools.

The second and third sections of this chapter discuss multiple perspectives of these problems and propose methodologies that make feasible the implementation of complex optimization for very large and geographical real distribution planning problems.

The extraction of geographical information is behind all the ideas presented in this chapter. The existing automated planning tools always admit that someone (usually the planner) provides the information about the network components and topologies. The automated planning tools only optimize these alternatives. The GIS spatial analysis tools
proposed in this chapter are tools to help the planner to extract from the geographic reality the network components and their attributes. This is a family of tools that doesn’t exist yet but with the development of GIS its development is actually feasible. This chapter is certainly a necessary complement to the core ideas of the thesis, on Spatial Load Forecasting.
6.6. REFERENCES


Chapter 6 - Further Spatial Planning Tools


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When implementing the methodologies developed in this thesis we confirmed that Spatial Load Forecasting are models of difficult access due to their complexity in problem formulation and onerous burden in data acquisition and preprocessing. Concerning to data accessibility, the development in GIS interoperability is facilitating these tasks. Most of the utilities are doing efforts to construct their Geographical Information System and formulate organizational schemes to exchange geographical information. It is expected that in near future the access to information and its management will be facilitated. Concerning the complexity in problem formulation, we conclude from our work that it is possible to facilitate the process and improve the performance of Spatial Load Forecasting by approaches similar to those in the present work.

7.1. GENERAL CONTRIBUTIONS

In this work a new approach for spatial load forecasting was developed opening doors to a new generation of Electricity Distribution Planning tools and concepts. We believe that the Fuzzy Spatial Model will facilitate and motivate a better Electricity Distribution Planning with better performance in the following aspects:

- more automated in the alternative definition, more flexible in the problem statement, and more interactive in the planning activity;
- more accurate and closer to the geographical reality of the problem, by using appropriate systems specialized to deal with geographical information and experienced in facilitating the information management;
- open to interaction with other sectorial planning, providing the basis for the concept of a global geographical collaborative planning,
- oriented to use the maximum density and type of information available, by combining information sources and types (statistical and judgmental, geographical and non geographical, internal or external to the utility).
focused in uncertainty modeling and evaluation.

7.2. GENERAL CONCLUSIONS ABOUT THE RESEARCH

From the research point of view we found that Spatial Load Forecasting is not a research field exclusive to Power Systems. In fact in its essence it is a wide multidisciplinary research topic. In the particular case of this work an overview of different approaches to solve problems was a strong contribution to our research. This is a good example how a multi-disciplinary research inspection may benefit research in our specific applied science. However, we believe this work is also valuable to the other research areas; for instance: urban planning or spatial behavior modeling in general.

The Fuzzy Spatial Load Forecast is a forecasting approach objectively designed to facilitate the task of forecasting, especially to facilitate the acquisition and storage of knowledge. However, the forecasting process is much more than the method. Forecasting follows standards and practices that include: formulating the problem by defining objectives and structures; obtaining information by identifying sources, collecting and preparing data; implementing the method, which was the topic of this thesis; using the forecasts which includes the presentation of results and the analysis of consequences and interactions with the planning.

7.3. THE MATHEMATICAL APPROACHES

The mathematical formulation of the Fuzzy Spatial Model, which is the mathematical model for the Fuzzy Spatial Load Forecasting, consists of coupling between a Fuzzy System and a Cellular Automaton. The Fuzzy System stores the knowledge base and simulates the spatial behavior. However, it only simulates an indicator of potential for development. The effective development results from discrimination operations done by the Cellular Automaton. The Cellular Automaton spreads the effective development over the region based in a predefined global forecasting value for the entire region and based on the suitability for development indicated by the Fuzzy System.

We found this combination of mathematical methodologies very efficient. In works implemented in urban planning, Cellular Automata are used to store the knowledge base. In our approach we stored the knowledge base in the Fuzzy System because it was found
that: it is very difficult to find the parameters that describe the spatial behavior; and the representation of the knowledge base in a cellular automata is not interpretable. Alternatively the Fuzzy System is very efficient in capturing and simulating behavior and is excellent in storing the knowledge in an interpretable way. The Fuzzy System as designed is not adequate to represent discontinuous and countable phenomena, it only can represent a continuous image of the spatial behavior. Using the discretizing countable capabilities of the Cellular Automaton coupled to the results supplied by the Fuzzy System resulted in an excellent solution for the problem.

Fuzzy Inference Systems revealed to be the adequate methodology to capture and store spatial behavior. Using a set of historical maps to represent the influence factors and maps of effective development to represent the consequences of these influence factors, the fuzzy inference model is able to build a large set of comprehensive fuzzy rules. An interesting aspect of this rule formulation is their interpretability and it is even possible to visualize the rules geographically mapped. By using fuzzy reasoning techniques it was possible to create a methodology to merge the information known by experts with the information stored in the knowledge base. This merging of historical and judgmental information appears as one of the most interesting capacities of the model allowing a better use of all the information available.

7.4. METHODOLOGY IMPLEMENTATION

We found that the direct implementation of the Fuzzy Spatial Model in the GIS was a useful approach from the point of view of research because most of the aspects of the mathematical model could be analyzed and understood under a geographical perspective. However, for a future industrial application of this methodology the implementation as an independent module coupled to the GIS could be more computationally efficient and portable between GIS technologies.

Concerning the interface, the approach is very transparent and easy to use for the planner. To train with historical information it is only necessary to select the causal and the consequent geographical maps; the automated characteristics of the approach are able to find and store the knowledge source independently of its complexity. Natural linguistic rule statements do the updating of the knowledge base with judgmental information. These interface characteristics of the model are not an issue of software interface, they
come as a direct benefit offered by the approach and are the product of the research direction stated as an objective of this thesis.

The results obtained in the presented examples anticipate very good perspectives for the usefulness of the methodologies. When analyzing the results we clearly found very interesting aspects in the spatial behavior, ignored and unsuspected by simple observation of the inputs. The automatic capture and construction of the local load growth behavior was another surprise; the $S$ load growth curves are automatically built in each location without a previous definition of the shape characteristics. The ability to simulate consequences of geographical events (changes in infrastructures or formulation of urban planning scenarios) is another attractive feature, due to the capability to clearly show the consequences of these changes.

7.5. Methodology validation

Regarding the model validation, we studied several perspectives of the forecasting model following standards and practices of the forecasting science. Due to the specificity of the forecasting a validation process has been idealized to better understand the performance of the model. Unfortunately it was not possible to compare the results of our model with other Spatial Load Forecasting applications due to access difficulties to these models. Another kind of test that wasn’t done is the forecasting application to a real case study with complete set of historical information, about causes effects, events, and historical judgments. This kind of case study is very difficult to find, especially in Spatial Load Forecasting.

Concerning the validation tests done we conclude that the accuracy in the forecasting level (in magnitude) varies from 80% to 90% which seems satisfactory for the forecasting problem studied. The accuracy for discrete changes (error in the development events) ranges from 65% to 80%, because this forecasting problem is clearly more difficult due to behavior wild characteristics of individual consumers. We found that the accuracy of the forecast is better for more detailed description of the input variables (sample fragmentation in the modeling procedure). The spatial validation shows that the Fuzzy Spatial Load Forecasting simulates the spatial behavior satisfactory but with more difficulty to forecast in green field regions and in regions with strong load growth. From the temporal validation we conclude that the past knowledge contains a good
representation of the future behavior, but is not complete. This means that the knowledge base should be constantly updated to incorporate new particularities of the load behavior. By comparing the backcasting with the forecasting we also found that future behavior perform a better representation of the past behavior than the vice versa. This indicates that it could be interesting to use the experience of more developed regions to represent the behavior of less developed regions with similar evolution. We have also validated the process of merging judgmental information and concluded that the methodology performs well, adapting the knowledge base closer to the expected.

7.6. Modeling Uncertainties

Modeling and dealing with spatial uncertainties have very particular aspects never explored in Power Systems. However, this is a very important topic of study in Geographical Information Science. Chapter 5 explores these characteristics in the perspective of the Electric Distribution Planning, with focus in Spatial Load Forecasting. We conclude that there are two types of consequences resulting from the Spatial Load forecasting uncertainties: first, the uncertainty in the magnitude of the forecasting influences the sizing and the timing for the reinforcement of the distribution infrastructure; second, the uncertainties in discrete change (from green field to habited) influences the expansion planning. An important conclusion is that the second case is the most sensible for Distribution Planning and simultaneously is the most difficult for the Spatial Load Forecasting model. This means that the spatial performance of the Spatial Load Forecasting model is very important and deserves more attention specially in developing regions with strong spatial expansion.

Two types of modeling and propagation have been studied in the Fuzzy Spatial Load forecasting. The first is the Scenario Planning consisting in the study of forecasting statements imposed by contrasting futures. The ability of Fuzzy Spatial Load Forecasting to model these kinds of uncertainties is very attractive for decision makers because it allows a clear analysis about consequences and impacts of policies, planning schedules, and isolated events. The second is the uncertainty propagation, modeling the small deviations originated by uncertainties on input deviations. The small deviations caused by uncertainty propagation are particularly important because small changes in green field regions are the basis to evaluate the uncertainties on expansion planning.
7.7. **FURTHER DEVELOPMENTS**

These thesis studies one of the most relevant issues related with the spatial aspect of the load forecasting. However, towards an industrial application some additional work must be developed:

- implementing the methodology in an independent portable software module being able to couple to the different GIS technologies;
- studying and developing global forecasting models and implementing them in independent software modules to be coupled to GIS
- developing the end-use model.
- further research in the problem formulation for multiple consumer type by using the coupling of several Fuzzy Spatial Load Forecasting modules.
- researching in the interaction between the Spatial Load Forecasting and the Electricity Distribution Planning by studying the dependency between the two processes.

As shown in chapter 6 some of the approaches and concepts defended for Spatial Load Forecasting could be extended to other modules of distribution planning tools. A vast and promising research areas and topics are open and we hope that this thesis may be a source of motivation to explore these new capabilities.
APPENDIX A - FUZZY SYSTEM

This annex contains some basics on the fuzzy systems, theory and practice procedures that are necessary for a correct understanding of the Fuzzy Spatial Model developed on Chapter 3 of this thesis. Fuzzy systems is a very vast research field, and obviously this annex could be very limited. For a more deep study and research in this field we recommend to reading of some of the references included in the text.

The annex is divided in four parts: the first presents some basics on fuzzy set theory and relations ([210]-[224]); the second part addresses the fuzzy logic and reasoning ([58]-[68]); the third part addresses some theoretical and practical aspects of the fuzzy systems structures ([62], [64], [67], [68], [224]-[230]); finally it are presents some practical aspects related with fuzzy system modeling and identification ([231]-[245]).

FUZZY SETS AND RELATIONS

What is a fuzzy set?

A fuzzy set, introduced by Zadeh, is a set with grade membership in the real interval \( \mu_a(x) \in [0,1] \). A fuzzy set \( A \), a fuzzy subset of \( X \), is denoted by:

\[
A = \{ (\mu_A(x)/x) \mid x \in X \} \tag{0.1}
\]

where \( \mu_A(x) \) is known as the membership function, and where \( X \) is known as the universe of discourse.

Properties of fuzzy sets

The height of a fuzzy set \( A \), \( hgt(A) \), is defined by:

\[
hgt(A) = \sup_{x \in X} \mu_a(x) \tag{0.2}
\]

Fuzzy sets with height equal to 1 are called normal, and called subnormal if is less than 1.

The core of the fuzzy set is a crisp subset of \( X \):

\[
\text{core}(A) = \{ x \in X \mid \mu_a(x) = 1 \} \tag{0.3}
\]
The support of a fuzzy set is also a crisp subset of X:

$$\text{supp}(A) = \{x \in X|\mu_x(x) > 0\}$$  \hspace{1cm} (0.4)

The width of a fuzzy set is given by the difference between the supremum and infimum of the support:

$$\text{width}(A) = \sup(\text{supp}(A)) - \inf(\text{supp}(A))$$  \hspace{1cm} (0.5)

The $\alpha$-cut of a fuzzy set is defined by:

$$\text{acut}(A, \alpha) = \{x \in X|\mu_x(x) \geq \alpha\}$$  \hspace{1cm} (0.6)

A fuzzy set is characterized a convex fuzzy set when:

$$\forall x_1, x_2, x_3 \in X, x_1 \leq x_2 \leq x_3 \rightarrow \mu_x(x_1) \geq \min(\mu_x(x_1), \mu_x(x_3))$$  \hspace{1cm} (0.7)

When $N_A$ fuzzy sets $A_i$ are fuzzy subsets of universe $X$, such a tuple of fuzzy sets $(A_1, \ldots, A_n, \ldots, A_N)$ is called a fuzzy partition when:

$$\forall x \in X, \sum_{i=1}^{N} \mu_{A_i}(x) = 1$$  \hspace{1cm} (0.8)

where $A \neq \emptyset$ and $A_i \neq X$. A fuzzy partition formed by fuzzy sets which are normal and convex does not contain more than two overlapping fuzzy sets.

![Figure 0.1](image.png)  

Figure 0.1 – a) Height, core and support of a fuzzy set. b) A fuzzy partition

**Union and Interception**

The interception and union on fuzzy sets are a more general case of these operations in classical sets. Because the membership value is not limited to values 0 and 1, an infinite number of possible definitions can be chosen to implement interception and union. Triangular norms (T-norms) and triangular conorms (T-conorms or S-norms) represent general forms of operators for interception and unions. A T-norm is a binary relation from $[0,1] \times [0,1]$ to $[0,1]$ satisfying the following criteria:
T-1 \quad T(a,1) = a

T-2 \quad T(a,b) \leq T(c,d), \text{ whenever } a \leq c, \ b \leq d

T-3 \quad T(a,b) = T(b,a)

T-4 \quad T(T(a,b),c) = T(a, T(b,c))

The conditions defining a S-norm are, besides T-2, T-3 and T-4:

S-1 \quad S(a,0) = a

The following table shows some examples of T-norms and S-norms.

- proposed by Lukasiewicz: \quad T(a,b) = \max(a+b - 1,0) \quad S(a,b) = \min(a + b,1)
- proposed by Zadeh: \quad T(a,b) = \min(a,b) \quad S(a,b) = \max(a,b)
- proposed by Bandler: \quad T(a,b) = a \cdot b \quad S(a,b) = a + b - a \cdot b

**Fuzzy relations**

Fuzzy sets can be extended to have multi-dimensional membership functions. These multi-dimensional fuzzy sets are normally referred to as fuzzy relations. An n-ary fuzzy relation \( R \) in \( X_1 \times \cdots \times X_n \) is a multi-dimensional fuzzy subset of \( X_1 \times \cdots \times X_n \) and is denoted by:

\[
R = \left\{ \mu_x(x_1, \ldots, x_n) : (x_1, \ldots, x_n) \in X_1 \times \cdots \times X_n \right\}
\]

Such fuzzy relations can represent an association or correlation between elements of the product space. Fuzzy relations can be used to model linguistic associations, correlation, or correspondences.

The reduction in dimension in fuzzy relations is possible by using projection mechanisms. The projection can be done by taking the supremum of the membership function for the dimension to be eliminated. The projection of a fuzzy relation \( R \) in \( X' = X_1 \times \cdots \times X_n \), is defined by:

\[
\text{proj}(R; X') = \left\{ \sup_{i_1, \ldots, i_n \neq j} \mu_x(x_1, \ldots, x_n \mid x_i \in X_1, \ldots, x_n \in X_n) \right\}
\]

where \( R \) is a fuzzy subset of \( X' = X_1 \times \cdots \times X_n \) and \( X' \times X' = X^n \). The indices \( i_1, \ldots, i_k \) are a complementary to \( i_1, \ldots, i_k \) with respect to the indices \( 1, \ldots, n \).
It is also possible to extend the product space of a relation by using a cylindrical extension, which is defined by:

\[
\text{cext}(R;X^n) = \{ \mu_R(x_1, \ldots, x_n)/\{x_1, \ldots, x_n\}|x_i \in X_1, \ldots, x_i \in X_n \}
\] (0.11)

where R is a fuzzy relation on \(X^n\). This means that a fuzzy relation or set is extended over an enclosing Cartesian product space with the restriction that, if \(R\) is a fuzzy set on \(X^n\) and \(X^n \subset X^n\):

\[
R = \text{proj}(\text{cext}(R;X^n),X^n)
\] (0.12)

The composition of fuzzy relations is defined as follows: suppose there is a fuzzy relation \(R\) in \(X \times Y\) and \(A\) is a fuzzy set in \(X\), then fuzzy subset \(B\) of \(Y\) can be induced by \(A\), given the composition of \(R\) and \(A\). This is denoted by:

\[
B = A \circ R
\] (0.13)

and is defined by:

\[
B = \text{proj}(R \circ \text{cext}(A;X \times Y),Y)
\] (0.14)

Assuming the cylindric extension as implicit the composition of relations can be regarded as consisting of two phases: combination and projection. These two phases could be implemented by using general T-norm and S-norm for operators. One example is the use of the sup-min composition proposed by Zadeh. This approach leads to the following implementation where \(A\) is a fuzzy set with membership function \(\mu_A(x)\) and \(R\) is a fuzzy relation with membership function \(\mu_A(x,y)\):

\[
\mu_B(y) = \sup \min(\mu_A(x),\mu_A(x,y))
\] (0.15)

where the cylindrical extension of \(A\) is implicit and \(\sup\) (S-norm) and \(\min\) (T-norm) represent the projection and combination, respectively.

Fuzzy Logic and Reasoning

Fuzzy propositions

A fuzzy proposition represents statements like "\(x\) is big", where big is a linguistic label, defined by a fuzzy set on the universe of discourse of variable \(x\). Fuzzy propositions connect variables with linguistic labels defined for those variables. Fuzzy propositions can
be combined by means of logical connectives like and or. Linguistic modifiers can be used to modify the meaning of the linguistic label used in fuzzy propositions.

The and and or fuzzy connectives are implemented by T-norms and S-norms, respectively. The choice of T-norms and S-norms for the logical connectives depends on the meaning and context of the propositions and the relations among them. If the propositions are related to different universes, a logical connective will result in a fuzzy relation. For example, consider the following proposition \((p; x_1 \text{ is } A, \text{ and } x_2 \text{ is } A_2)\), where A1 and A2 have membership functions \(\mu_A(x_1)\) and \(\mu_A(x_2)\). The fuzzy relation \(P\) with membership function \(\mu_P(x_1, x_2) = T(\mu_A(x_1), \mu_A(x_2))\) can then represent the proposition \(p\), where \(T\) is a T-norm that is used to model the and-connective. Such a combination of propositions, in fact a proposition itself, can be the antecedent of a fuzzy rule.

**Fuzzy rules**

In order to reason with fuzzy logic, fuzzy rules have to be represented by an implication function. Such implication has the same function as the truth table of the classical implication in classical logic. This implication \((A \rightarrow B)\) can be seen as a representation of the statement \((\text{if } A \text{ then } B)\). In fuzzy logic this types of statements are often referred to as fuzzy if-then statements or fuzzy rules.

A fuzzy rule is an if-then statement where the antecedent and the consequent consist of fuzzy propositions. The antecedent may contain a combination of propositions connected by means of the logic connectives and or:

\[
\text{if } x_1 \text{ is } A, \text{ and } x_2 \text{ is } A_2 \text{ then } y \text{ is } B
\]

When fuzzy sets A1, A2 and B are identified by the membership functions \(\mu_A(x_1)\), \(\mu_A(x_2)\) and \(\mu_A(y)\), the following fuzzy relation \(R\), representing the fuzzy rule, can be constructed.

\[
R = I(T(A_1, A_2), B)
\]

where \(T\) is a conjunction based on a T-norm and \(I\) is a fuzzy implication function. As the \(T\) represents the and connective, the \(I\) represents the if-then connective. Hence a fuzzy rule can be represented by a fuzzy relation with the following membership function:

\[
\mu_R(x_1, x_2, y) = I(T(\mu_A(x_1), \mu_A(x_2), \mu_A(y))
\]

(0.17)
When we have a set of fuzzy rules, it is necessary to aggregate then into a unique fuzzy relation. The fuzzy rules are considered as a set of \( N_r \) parallel rules when they have a premise based on the same \( N_x \) variables:

\[
\begin{align*}
\ r_i & : \quad \text{if } x_i \text{ is } A_{1,i} \text{ and } \ldots \text{ and } x_{N_x} \text{ is } A_{N_x,i} \text{ then } y \text{ is } B_i \\
\ r_k & : \quad \text{if } x_i \text{ is } A_{1,k} \text{ and } \ldots \text{ and } x_{N_x} \text{ is } A_{N_x,k} \text{ then } y \text{ is } B_k \\
\ r_N & : \quad \text{if } x_i \text{ is } A_{1,N} \text{ and } \ldots \text{ and } x_{N_x} \text{ is } A_{N_x,N} \text{ then } y \text{ is } B_N
\end{align*}
\]

The rules have a number of properties that provide a classification for rule bases. Some of these rule base properties are consistency, continuity and completeness.

- **Continuity of a rule base** requires that rules with adjacent premises have adjacent consequents.

- **Consistency of a rule base** addresses the consistency of the knowledge represented by that rule base (no contradiction, for instance).

- **Completeness of a rule base** measure the completeness of the knowledge represented by the rule base. An incomplete rule base has so-called **blank spots**.

**Fuzzy reasoning**

A fuzzy rule can be used to infer knowledge about the consequent of the rule using data, which is a fuzzy subset of the same universe as the premise of the rule.

The inference of a single fuzzy rule is an application of the composition of fuzzy relations. The main inference scheme introduced in fuzzy logic was the composition rule of inference. However the reasoning schemes could be generalized from classical logic defining the generalized modus ponens and the generalized modus tollens.

The **compositional rule of inference** assumes that a fuzzy rule (if \( x \) is \( A \) then \( y \) is \( B \)) is represented by a fuzzy relation \( R \). A result \( B' \) can be inferred (from \( R \) by \( A' \)) through the composition of \( A' \) and \( R \):

\[
B' = A' \circ R
\]  

(0.18)

The compositional rule of inference assumes that a fuzzy relation representing the rules \( R_k \) exists and is represented by a fuzzy implication \( R_i = I(\cap(A_{1,i} \ldots A_{N_x,i}), B_i) \). When a suitable implication operator \( I \) is chosen, a correspondent composition operator should be
chosen. If the conjunction, the implication and composition operators are based on the same T-norms the inference operations may be simplified. Assume the following rule:

\[ r_k : \text{if } A_1' \text{ is } A_{1,k} \text{ and } A_2' \text{ is } A_{2,k} \text{ then } y \text{ is } B_k' \]

the inference will be represented by:

\[ B_k' = T_c(A_1', A_2') \circ T(T_c(A_{1,k}, A_{2,k}), B_k) \]  \hspace{1cm} (0.19)

where \( T_c \) is the T-norm for the conjunction operator and \( T_i \) is the T-norm for implication and composition. In the same T-norm is used the inference is simplified as follow:

\[ B_k' = T(A_1', A_2') \circ T(T(A_{1,k}, A_{2,k}), B_k) = T(T(hgt(T(A_{1,k}, A_{2,k})), hgt(T(A_{1,k}, A_{2,k}))), B_k) \] \hspace{1cm} (0.20)

The inferences that use this simplification are designed as T-implication. For T-implications, in most cases, the T-norm is chosen to be either a min operator or product operator. An example of the inference of a rule is shown schematically in figure.

Figure 0.2 – Inference of one rule when the conjunction, implication and composition are based on min operator, a), b) and c), or product operator, d) e) and f). The left columns shows the case of fuzzy data, a) and d), and the center columns shows the case of a crisp data.

The knowledge of a fuzzy system is represented by a set of parallel fuzzy rules (fuzzy rule base). The inference process for a fuzzy rule base requires aggregation procedures. Two different inference approach exist for inference the fuzzy rule base.
The local inference approach performs inference with individual rules (using $R_\delta$) and aggregates the result. This approach is normally used in conventional expert systems.

The global inference approach assumes a relation $R$ to represent the rule base, where $R$ is the aggregation of the fuzzy relations $R_\delta$ representing the individual rules. On other words, all rules are combined before and the overall result is inferred from the rule base.

Fuzzy Systems Structures

The fuzzy system is the application of the compositional rule $\circ$ inference, given a relation $R$, representing the fuzzy system and a relation $A'$, representing the fuzzy system input, a fuzzy output $B'$ can be obtained by composition of $A'$ and $R$: $B' = A' \circ R$.

However, the input $x'$ and output $y'$ of the modeled system are usually numerical values, so a translation is necessary to obtain fuzzy inputs (fuzzification) and to obtain numeric values on the output (defuzzification). The knowledge base of the fuzzy system is stored on the fuzzy rule base, which is represented by fuzzy relations using fuzzy implication functions and finally aggregated in one fuzzy relation $R$.

![Figure 0.3 - Schematic representation of a fuzzy system.](image)

**Input fuzzification**

Theoretically the fuzzification phase is the construction of a fuzzy relation $A'$, then the fuzzy input relation is the conjunction of the $N_x$ fuzzy input sets $A'_x$ where $N_x$ is the number of inputs. The complete fuzzy input relation is determined by combining the fuzzy sets for each input, where $T$ is the T-norm used to perform the conjunction in the antecedent.
\[ A' = \left\{ \left( \prod_{i=1}^{N_x} T \left( \mu_{A_i} \left( x_i \right) / \left( x_1, \cdots, x_{N_x} \right) \right) \right) | x_i \in X_1, \cdots, x_{N_x} \in X_{N_x} \right\} \quad (0.21) \]

In practice, it can be stated that the fuzzification phase consists of determining the matching between the inputs and the fuzzy sets that represent the linguistic labels for the inputs in rule premises. The matching between input and linguistic labels is usually (for T-implications) determined by: \( \alpha_{i,j} = \text{hgl}(A'_i \cap A_j) \), where \( \alpha_{i,j} \) represents the matching between the input data \( A'_i \) for input \( x_i \) and the \( j \)th fuzzy set \( A_{i,j} \) on the universe of discourse of \( X_j \). When the input is a numerical value \( x'_i \) the matching value is the correspondent membership value in the linguistic label \( \alpha_{i,j} = \mu_{A_i}(x_i) \).

The linguistic labels \( A_{i,j} \) could be found based on two distinct approaches. In the first labeled as reasoning mechanisms, which is the more usual on traditional methods of fuzzy modeling, the linguistic description is constructed subjectively on the basis of a the a priori knowledge about the system. The second direction of modeling is system identification. For this approach the membership functions are selected and tuned based on the use of input-output data processed by systematic methodologies.

**Output defuzzification**

Because the fuzzy output of the inference system \( B' \) is not appropriated for physical interpretation, in general the fuzzy output is translated to a numerical representation by using defuzzification methods. Several defuzzification methods exist, two of then referred in this anex.

Center of Gravity (COG) - this is an averaging technique calculating the center on mass where the membership value represents the mass.

\[
\text{cog}(B') = \frac{\int \mu_y(y) \cdot y \cdot dy}{\int \mu_y(y) \cdot dy} \quad (0.22)
\]

Mean of Maxima (MOM) - can be used when the output \( B' \) is the union of two or more membership functions (\( N_m \) is the number of membership functions) defined on the universe of discourse of the output variable. The technique is based on the weight mean of the central value of each membership \( y_i(B'_m) \).
Appendix A - Fuzzy System

\[
\text{mom}(B') = \frac{\sum_{k=1}^{\infty} \mu_{\kappa_k}(y_k(y_k(B'_k))) \cdot y_k(B'_k)}{\sum_{k=1}^{\infty} \mu_{\kappa_k}(y_k(B'_k))}
\]  
(0.23)

The the central value \( y_c(B'_c) \) of each output membership \( B'_c \) is given by:

\[
y_c(B'_c) = \frac{\sup (acut(B'_c, hgt(B'_c))) + \inf (acut(B'_c, hgt(B'_c)))}{2}
\]  
(0.24)

Figure 0.4 – Schematic representation of the COG a), and MOM b) defuzzification methods.

**Fuzzy inference**

The theory the fuzzy inference consists in the composition of fuzzy inputs and in fuzzy relations that describe the fuzzy rule base. This involves quite a complex calculus with multi-dimensional functions (fuzzy relations). Thus, in practice, fuzzy systems are modeled using local inference. Using local inference, the inference of a rule base is broken down to inference of individual fuzzy rules and the results are aggregate afterwards. Other practical simplification is the consideration of only numerical inputs and outputs.

Basically the inference in fuzzy systems is represented by the following steps:

1) Matching of fuzzy proposition "x is \( A_x \)" used in the premises of fuzzy rules \( r_i \), with the numerical data \( x'_i \) (fuzzy system inputs): \( \alpha_{i,x} = \mu_{A_x}(x'_i) \), where \( \alpha_{i,x} \) is a numerical value representing the matching. In the case of fuzzy inputs \( A'_x \) the matching is normally represented by \( \alpha_{i,x} = hgt(A'_x \cap A_x) \).

2) Determining the degree of fulfillment \( \beta_i \) for each rule \( r_i \): \( \beta_i = \frac{1}{\alpha_{i,x}} \), where \( T \) is the T-norm representing the and connective in the antecedents of the rules.

3) Determining the result \( B'_i \) of each individual rule \( r_i \): \( \mu_{B'_i}(y) = T(\beta_i, \mu_{x}(y)) \), where \( I \) is the implication used to model the fuzzy rules.
4) Aggregation of the overall result \( B' \) of the results \( B'_r \) of the individual fuzzy rules \( r_i \), each implication type have associated an aggregation operator, \( T \)-implications complies with conjunction aggregation \( \mu_B(y) = \bigcup_k \mu_B^k(y) \).

The four steps of the practical approach of the fuzzy inference corresponds to the four operator indicated on Figure 0.5. The composition (1) has the combination and the projection phases. The result of the composition is the matching value \( \alpha_{i,k} \) for each variable \( x_i \) in a specific rule \( r_i \). The conjunction (2) is used to compute the fulfillment \( \beta_i \) for each rule \( r_i \) by using the matching value from the several variables. The implication (3) determines the matching on the “then” part of the rule, producing as result the membership \( \mu_{k_i}(y) \) for each rule \( r_i \). Finally the aggregation combines the fuzzy relations into a single fuzzy relation.

To simplify the composition step it is acceptable and usual to admit that some input linguistic labels \( A_{i,k} \) and \( A_{j,k} \), associated with different rules \( k \) and \( k_j \), are equal and are equal to one of the fuzzy sets \( A_{i,k} \) of a fuzzy partition on the universe \( X_i \). This simplification reduces the number of input linguistic labels and allows a step 1) calculation independent from the rule index \( k \). Instead the matching value will be indexed by the input variable index \( i \) and by the partition index \( j \): \( \alpha_{i,j} \).

![Diagram](image)

Figure 0.5 – Scheme representing the structure of a fuzzy system and the four functional operators showing the correspondent inputs and outputs.

In practical implementation of fuzzy systems there are several inference methods such as “max-min” “max-prod” and “sum-prod”. The max-min inference method uses a \( \min \) operator for the conjunction in the antecedent of the rule as well as for implication
function and uses a max operator for the aggregation. The application for the compositional rule of inference results in:

$$\mu_y(y) = \max_i \min_j \left( \beta_i, \mu_{A_i}(y) \right)$$  \hspace{1cm} (0.25)

with:

$$\beta_i = \min \alpha_{i,j}$$  \hspace{1cm} (0.26)

$$\alpha_{i,j} = \sup_i \min_j \left( \mu_{A_i}(x_i), \mu_{A_j}(x_j) \right)$$  \hspace{1cm} (0.27)

Figure 0.6: Graphical max-min inference method. a) and b) represent the composition, for A1 and A2 partitions, with numerical inputs. c) and d) represent the implication using the support value resulting from the conjunction.

The max-product inference method is characterized by scaling the consequent $B_i$ of the fuzzy rule $\eta_i$ with the degree of fulfillment $B'$ of that rule and aggregating these result $B'_i$ to obtain the fuzzy system output by means of a max operator.

$$\mu_y(y) = \max_i \beta \cdot \mu_i(y)$$  \hspace{1cm} (0.28)

There are two distinguishable variations of max-product method with respect to the determination of the support value $\beta_i$. Either the min or the product operator is used for
the combination of the matching values $\alpha_{i,k}$ and thus representing the conjunction in the rule antecedent.

$$\beta_i = \frac{\min_i \alpha_{i,k}}{\prod_i \alpha_{i,k}}$$  
(0.29)

For the composition using to compute the matching value $\alpha_{i,k}$ it may be used the conjunction or the product operator. However if the inputs are numeric the result is the same.

$$\alpha_{i,k} = \frac{\text{hgl}(A'_i \cap A_{i,k})}{\text{hgl}(A'_i \ast A_{i,k})}$$  
(0.30)

Figure 0.7 - Graphical max-product inference method, using the min operator to compute the base value.

The sum-product inference method uses the product operator as implication, however the aggregation is done by summing the results $\mu_x(y)$ on all rules. Because the aggregation sum may result in values higher than 1, a bound sum to limit the upper value is used.

$$\mu_x(y) = \min \left( \sum_i \beta_i \ast \mu_x(y) \right)$$  
(0.31)
The determination of the support value $\beta_i$ for sum-product inference is based on the product operator. If the inputs are numeric values the $\beta_i$ and $\alpha_{i,i}$ could be determined as follows.

$$\beta_i = \prod_i \alpha_{i,i} = \prod_i \mu_{A_i}(x_i^i)$$  \hspace{1cm} (0.32)

![Graphical sum-product inference method, using the product operator to compute the base value.](image)

Mamdani and Sugeno fuzzy rules

In fuzzy systems it is usual to distinguish two types of rules: Mamdani rules and Sugeno rules. The Mamdani rules are the rules discussed thus far, where the antecedent and the consequent of the rules consist in fuzzy predicates. Contrarily the Sugeno rules have a fuzzy antecedent part and a functional consequent part represented by linear functions of the fuzzy system inputs. The Mamdani rules have the following general form:

$$r_i: \text{if } x_i \text{ is } A_i^i \text{ and ... and } x_N \text{ is } A_N^i \text{ then } y_i \text{ is } B_i^i \ldots, y_N \text{ is } B_N^i$$

Normally, in Mamdani rules, one uses the max-min inference method, with the min operator for conjunction and implication, and the max operator used for aggregation. Essentially the Mamdani rules use fuzzy proposition as consequent and the implication is represented by a conjunction (T-implication). In practice the Mamdani rules represent an interpolation in the hyperspace, where the interpolation points are the centroids of the rules (points where the antecedent proposition is 1 for one specific rule, and 0 for all other rules).
The general form of the Sugeno rule is as follows:

\[ r_i : \text{if } x_i \text{ is } A^i_1 \text{ and } \ldots \text{ and } x_n \text{ is } A^i_n \text{ then } y_i = f_i(x_1, \ldots, x_n) \]

The form of this rule shows that its consequent is a function \( f_i \) of the fuzzy system inputs \( x \). In general, linear functions are used in the consequent: \( f_i = b_{i1} + \sum_{i=1}^{n} b_{i,i} \cdot x_i \). The order of the polynomial function used in the consequent defines the designation order of the Sugeno fuzzy system \( f_i = b_{i1} \) is used for a zero order Sugeno fuzzy system). A simple weighted sum is used to obtain the final output as an aggregation of the multiple rule output. In the case of linear consequent functions this results in:

\[
y' = b'_i + \sum_{i=1}^{N} b'_i \cdot x_i = \frac{\sum_{i=1}^{N} \beta_i \cdot b_{i,i}}{\sum_{i=1}^{N} \beta_i} + \sum_{i=1}^{N} \frac{\sum_{i=1}^{N} \beta_i \cdot b_{i,i}}{\sum_{i=1}^{N} \beta_i} \cdot x_i \tag{0.33}
\]

where \( \beta_i \) is the support value for rule \( r_i \), and \( b_{i,i} \) are the coefficients stored on the rule base. The determination of the support value \( \beta_i \) is in general based on the product operator.

\[
\beta_i = \prod_{i} \alpha_{i,i} = \prod_{i} \mu_{a,i}(x_i) \tag{0.34}
\]

Each rule of the Sugeno fuzzy system represents by itself an approximation function in the area near the centroid of the rule. The aggregation of the several rules, by weighted sum, represents the interpolation between the functions weighted by the “distance” to the respective rule centroids. The zero-order Sugeno rules are very similar to Mamdani rules, the main difference is on the consequent predicate: Mamdani rules use a vague predicate “is” instead of the equality predicate “=” used in Sugeno rules.

**Partitioning the fuzzy system hypersurface**

When building the fuzzy system, one of the first question which arises, after having chosen the inputs, is that of how many fuzzy sets are needed and how the fuzzy sets should be divided on the universe of discourse of the fuzzy system inputs. There is no standard design scheme that can be employed to choose the number and position of the fuzzy sets. Actually, the choice of number of fuzzy sets and how the fuzzy system hypersurface are partitioned requires the knowledge on the non-linearity of the system function approximation and knowledge on how the outputs are related with the inputs. In
relation with the partitioning of the universe of discourse of the outputs the fuzzy sets should be better identified in regions where the system function has more complex behavior or on regions where higher accuracy on output is required. The partitioning of the inputs depends on the influence of each input on the output and on the accuracy for the particular region of the universe of discourse of the outputs. When the process knowledge exists in the form of human expertise, then one has to use intuitive processes or knowledge acquisition processes such as used to build expert systems. When the process knowledge exists in the form of input-output data sets then one may use algorithmic or logical operations (e.g. clustering, neural-networks, genetic algorithms). When the output function of the system is characterized by well known non-linearities, then this knowledge may be used to chose the “position” of the fuzzy sets.

Overlapping the fuzzy sets

Normally the set of fuzzy sets chosen form a fuzzy partition composed by membership function convex and normal, where there are no more than two overlapping membership function on the universe of discourse. In this case only the neighboring characteristic points (centroid of the rules) affects the system output. When more than two overlapping membership functions are fired a rule is never the only one that is active and never represents exclusively the output. The interference, due to more than two overlapping fuzzy sets leads to smoothing effects on the fuzzy system behavior that are not easy to see a priori.

Shape of the fuzzy sets

Considering only normal and convex fuzzy sets forming fuzzy partitions, the influence of the shape on the results is characterized by the linearity, the core and the asymmetry. The use of a non-linear membership function introduces non-linearity and smoothing in the fuzzy output; this may be useful but also may difficult the interpretation of the results. When memberships with core interval are used may exist regions in the hyperspace core where only one rule is activated, originating uniform values for output. For Sugeno rules with order higher than zero the output on the core region is not uniform but is computed by the function of the unique fired rule. The effect of the asymmetry depends on the method used for defuzzification.
Two usual shape functions are the bell-shaped and the trapezoidal function. The bell-shaped membership function is a non-linear function, with possible core interval and possible asymmetry. The mathematical representation is given by:

\[
bell(x,a,b,c,d) = \min(\text{sigm}(x,a,b), 1 - \text{sigm}(x,c,d))
\]  
(0.35)

with:

\[
\text{sigm}(x,a,b) = \begin{cases} 
0, & \text{if } x < a \\
\frac{2}{b-a} \left( \frac{x-a}{(b-a)^{2}} \right), & \text{if } a \leq x \leq \frac{a+b}{2} \\
1-\frac{2}{(a-b)^2} \left( \frac{x-b}{(a-b)^{2}} \right), & \text{if } \frac{a+b}{2} \leq x \leq b \\
1, & \text{if } x > b
\end{cases}
\]  
(0.36)

The trapezoidal membership function is a linear function, with possible core interval and possible asymmetry. The mathematical representation is given by:

\[
\text{trap}(x,a,b,c,d) = \begin{cases} 
\frac{(x-a)}{(b-a)}, & \text{if } a < x < b \\
1, & \text{if } b \leq x \leq c \\
\frac{(x-d)}{(c-d)}, & \text{if } c < x < d \\
0, & \text{otherwise}
\end{cases}
\]  
(0.37)

Figure 0.9 – a) Bell-shaped membership function. b) Trapezoidal membership function.

**FUZZY SYSTEM MODELING**

The fuzzy system modeling consists on the design and training of the fuzzy system. In order to identify a suitable fuzzy system for a given problem, membership functions (parameters) and rule base (structure) must be specified. This can be done by interpreting prior expert knowledge, by automatically learning from input-output data or by combination of both methods.
The system identification consists in:

- **membership characterization**, consisting on the identification of the partition of the input and output spaces, defining the location, number and shape of the membership functions.

- **rule generation**, consists on the specification of input output relations in the form of IF-THEN rules.

- **parameter adjustment**, consists on the derivation of optimal inference parameters and of the tuning of the input and output membership parameters.

### Membership characterization

As discussed before, the partitioning, overlapping and shape of the membership are important aspects of the membership characterization. Selecting the characteristics of the memberships must be in accordance with the type of the fuzzy system, the source of the knowledge base and the method used for parameter identification. The characterization of the membership of the inputs is necessary for Mamdani and Sugeno systems. However, the output membership exists only in Mamdani systems. Some learning methods need differentiable membership functions and others require simple shapes to perform interpretability.

If there is no input/output data or if the human expert knowledge is important the memberships function position and the partition could be defined by human intuition in accordance to the linguistic meaning associated with the partition of the variable range. In general, the input membership functions are defined first followed by the output memberships based on a priori knowledge about the input output relation.

When the input-output data sets exist and are the basis for fuzzy system modeling, several algorithms may be used. The clustering algorithms are an intuitive approach for partitioning. Some clustering methods define, for each point, the grade of proximity to each cluster center obtaining an approximate representation of the membership. The clustering may be done for one input at each time, for all input, or for all input/output hyperspace. The second and third hypotheses are more appropriate to model the overall system. However, to obtain the membership function the projection of the clustering results for each variable is necessary.
Rule generation

As the membership characterization also the rule generation could be directly defined by human expert knowledge, based on fuzzy reasoning methods. In some cases this set of rules are used as an initial significant set and on a second step algorithms are used to complement the rule base with other rules. In other cases the human knowledge is used to complement the rule base with rules not detectable from input/output data (e.g., future behaviors).

For the cases where the input and output partitions are defined before any relation between inputs and outputs, the Fuzzy Associative Memories (FAM) may be defined a posteriori. A FAM is a table that represents the nonlinear mapping between inputs and output associating to each combination of input fuzzy sets an output fuzzy set. The same outputs fuzzy set could be the result of different combinations of fuzzy set inputs, which represent different rules. Is not necessary the existence of an output fuzzy set for all the combination of inputs; however, the non-existence of some outputs in the FAM may lead to non-completeness of the rule base due to the appearance of blank-spots on the inference process. If the number of rules is small, with small number of variables and a correspondent small partition, it is possible to construct the FAM by simple inspection using only human knowledge. When the number of rules is too large the construction of the FAM must be done by heuristics using input/output data. Defining the maximum matching values in input and output membership functions defines the rule and the correspondent cell on FAM.

Figure 0.10 – Representation of a FAM table for one example of spatial fuzzy system with two variables. Variable $x_1$ represents the distance to urban centers characterized by three linguistic classifiers represented by the corresponding membership functions ($A_{11}, A_{12}, A_{13}$). Similarly variable $x_2$ represent the distance to roads. The rules are represented by table cells, each rule having one linguistic consequent representing the level of development. The same output consequent could be used in several rules (table cells). Some table cells may be empty due to real system inconsistency (e.g., locations close to the center and far from roads don't exist).
The clustering algorithms may be used to generate the rule base by clustering the overall input/output space. This kind of partitioning creates a set of clusters that characterize a relation between inputs and outputs, each of these clusters representing a rule and also its input and output membership. In the clustering process appropriated indexes must be used to determine the optimal number of clusters and consequently the number of rules.

![Figure 0.11 - Visualization of a two dimension partitioning based on the clustering of input/output point vector. The membership functions are the projection of the proximity measure to the centroid points and the rules local region is defined by the points near each centroid. a) represents the partition of the hyperspace between three rules, b) represents the partition of the hyperspace between two rules.](image)

Automatic learning processes, using input-output data, use algorithms to insert or eliminate rules. Some learning processes begin by creating one rule for each input/output vector and proceeds by eliminating the less significant rules. Other learning processes begin from a limited set of rules and proceed by adding new rules to fit new input/output vector.

**Parameter adjustment**

Once the structure of the fuzzy system defined, it is necessary to adjust its parameters in order to approximate, as much as possible, the results of the fuzzy system to the real physical system behavior. Two kind of parameters could be adjusted, the parameters of the inference system and the parameters of the membership functions.

For inference systems composed by differentiable operators and memberships it is possible to adjust the system parameters by using analytical methods to minimize the error between the real system and the fuzzy system outputs. Particularly for Sugeno fuzzy systems, most of the existing learning algorithms choose the parameters of the model in order to minimize the objective function $E$ for a training set of $N$, vectors of real system input/output data:
\[ E = \sum_{p=1}^{N_r}(y_p - y'_p)^2 \]  

(0.38)

where \( y_p \) is the output of the real system for training vector \( p \) and \( y'_p \) is the output of the fuzzy system for inputs of the training vector \( p \). A variety of methods could be used to minimize this function, namely ordinary least squares, weighted least squares, recursive least squares or orthogonal least squares. The complexity of the models depends on the system and on type of parameters to identify. For instance, to identify the weight parameters of a zero-order Sugeno system, supposing fixed input memberships, a simple ordinary least squares could be used.

\[
\begin{bmatrix}
\beta_{i,1} \\
\beta_{i,2} \\
\vdots \\
\beta_{i,N_r}
\end{bmatrix} = \left[
\begin{array}{ccc}
\sum_{p=1}^{N_r}(\beta_{i,1} \cdot \beta_{i,2}) & \sum_{p=1}^{N_r}(\beta_{i,1} \cdot \beta_{i,3}) & \ldots & \sum_{p=1}^{N_r}(\beta_{i,1} \cdot \beta_{i,N_r}) \\
\sum_{p=1}^{N_r}(\beta_{i,2} \cdot \beta_{i,1}) & \sum_{p=1}^{N_r}(\beta_{i,2} \cdot \beta_{i,3}) & \ldots & \sum_{p=1}^{N_r}(\beta_{i,2} \cdot \beta_{i,N_r}) \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{p=1}^{N_r}(\beta_{i,N_r} \cdot \beta_{i,1}) & \sum_{p=1}^{N_r}(\beta_{i,N_r} \cdot \beta_{i,2}) & \ldots & \sum_{p=1}^{N_r}(\beta_{i,N_r} \cdot \beta_{i,N_r})
\end{array}
\right]^{-1} \begin{bmatrix}
\sum_{p=1}^{N_r}(\beta_{i,1} \cdot y'_p) \\
\sum_{p=1}^{N_r}(\beta_{i,2} \cdot y'_p) \\
\vdots \\
\sum_{p=1}^{N_r}(\beta_{i,N_r} \cdot y'_p)
\end{bmatrix} 
\]  

(0.39)

with

\[
\beta_{i,j} = \prod \alpha_{i,j} = \prod \mu_{a,j}(x'_i) 
\]  

(0.40)

where \( \beta_i \) is the zero-order coefficient for rule \( r_i \), \( \beta_{i,j} \) is the support value computed for each rule \( r_i \) based on the input values \( x'_i \), corresponding to each training vector \( p_i \), and \( y'_p \) is the outputs for the training vector \( p_i \), the training set is composed by \( N_r \) points and the rule base is composed by \( M_r \) rules. By using an orthogonal least squares method it is also possible to adjust the parameters of the input membership, and also by iterative processes the simultaneous adjustment of all parameters of the fuzzy system is possible. The coefficients of the consequent functions on the Sugeno system could be identified by using local learning or global learning. In global learning the overall training set is used; on the other hand for local learning only the training points on the rule region are used, weighting their significance by measuring the proximity to the rule centroid.

Other methods used to adjust the parameters of Sugeno systems are the gradient descent methods. These methods are only applicable to fuzzy systems with a differentiable operator in inference mechanism. In this method the error on the output is backpropagated in order to correct the parameters of the fuzzy system. Based on the backpropagation of the error the adaptation on the coefficients \( b_i \) should be
compensated by $\Delta b_i$, value computed as function of the derivative of the error in order to the parameter $b_i$. The total compensation is the sum of the error for all learning points $p$, affected by the learning rate $\eta$.

$$\Delta b_i = -\eta \sum_{i=1}^{N_p} \frac{\partial E_p}{\partial b_{i,j}}$$

(0.41)

The neuro-fuzzy learning methods are also based on error backpropagation. These methods are based on the similarity of the structures of fuzzy systems and 3-layer neural networks. Adopting the training methods used on neural-networks, the algorithms enable one to determine the system parameters in an iterative process. The backpropagation of the error may be based on heuristics instead of gradient descendental. This allows the application not only to Sugeno systems but also to Mamdani systems. The learning algorithm for fuzzy systems may be designed to insert or delete rules, and adjust the membership parameters. To adjust the membership parameters, one measures the output error which indicates, whether the degree of fulfillment of a rule has to be higher or lower. This information is used to shift the membership functions, and make their supports larger or smaller. With this heuristics it is easy to define constraints for the learning procedure in order to maintain the partition form or to obtain an interpretable rule base.

Figure 0.12 – Visualization of a possible neuro-fuzzy learning process. a) shows the initial partition, b) shows the partitioning after fuzzy set tuning. In this example exist constrains that force the liner memberships to cross on 0.5 membership value.
REFERENCES


APPENDIX B - SCIENTIFIC PAPERS ON THE SUBJECT OF THE THESIS

IEEE BUDAPEST POWER TECH'99


PES WINTER MEETING 2000


PMAPS'2000


ISAP'2001


PMAPS'2002

Vladimiro Miranda, Cláudio Monteiro, Maria Teresa Ponce de Leão, "Validation process for a Fuzzy Spatial Load Forecasting ", paper proposed for PMAPS2002 - 7th International Conference on Probabilistic Methods Applied to Power Systems, Naples - Italy, September, 2002
Fuzzy Inference Applied to Spatial Load Forecasting

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Abstract – Forecasting the future electric demand and its geographical distribution is a prerequisite to generate expansion-planning scenarios. The load growth pattern is related with the urban structure and its land-use. The model for simulating urban structures must be explicitly dynamic and must contains mechanisms for linking its macrostructure to micro behaviours. The paper presents a methodology, which uses a fuzzy inference model over a GIS support, to capture the behaviour of influence factors on the load growth pattern and map the potential for development. The load growth dynamic is simulated based on extended cellular automata in which the potential for development and demand for location in each stage drive the system into the following stage developments. By providing a series of simulation scenarios, the study unveils potential load growth maps to be used in expansion planning studies.

Keywords – Spatial Load Forecasting, Geographic Information Systems, Fuzzy Inference, Cellular Automata, Distribution Planning.

I. INTRODUCTION

The treatment of the distribution planning problem is finally reached maturity, after many years where researchers failed to recognize that there was more in it than just optimization techniques. In fact, the distribution planning problem is first of all a problem of massive data treatment in two dimensions - space and time; then, it is clearly a problem of generation of scenarios; and only in third place it is a problem of decision, where optimization techniques may play a part, perhaps not too important. Having worked intensively and with success on this part of the problem [1], and having also important work done with Geographical Information Systems (GIS) [2], we feel to be in the right position to appreciate the importance of the other steps to be taken.

The key factor in distribution planning is the load forecast - but due to its geographical dimension, one cannot adopt simple prediction models as in generation or transmission systems: one must make use of spatial load forecasting methods.

Land-use spatial load forecast simulation methods have been used to model the process of the load growth in order to predict load evolution in a spatial and temporal basis [3]. This methodology is particularly suited to high spatial resolution for long range forecasting and multi-scenario planning [4]. Several models have been used to simulate load growth and improve the performance of distribution load forecasting [5, 6]. The Geographical Information System (GIS) technology [7] and its spatial analysis capabilities [8] provides an excellent platform to implement spatial load forecasting techniques.

Spatial load forecasting (SLF) based on land-use is a modeling technique that consists on identifying and mapping areas with similar growth pattern for each customer-class. Land-use pattern recognition techniques are used to identify and model some of the causes of load growth. A land-use class has a pattern template of influences. Each template models the influences of several geographical factors on the land-use pattern development and is represented by spatial shape rules defining influence factor coefficients. Examples of those functions are, for instance, the positive influence of a radial distance to an urban center or the negative influence of the distance to a waste treatment center.

In recent years some works have enhanced the land-use methods applied to urban redevelopment, using fuzzy logic, GIS, multi-objective programming [9, 10, 11].

In urban planning, a large work is being done to model land-use conversion. This is done under the assumption that a simulation approach under the self-organization paradigm is appropriate for addressing the process of land development [12, 13, 14]. To simulate the dynamics of the process, a recent work adopts cellular automata (CA) - this approach emphasizes the way in which locally-made decisions give rise to global patterns.

In this paper we will follow the land-use approach for (SLF), expanding the traditional modeling on dynamic simulation used in urban planning. The model presented has two innovations: the first is the use of a Fuzzy Inference model to capture the geographical pattern of influence factors, and the second is the dynamic simulation modeling of new consumers based on cellular automata (CA).

The fuzzy inference module automatically models the influence of several geographical factors, i.e. distance to roads, distance to urban center, number of houses in the neighborhood, number of shops in the vicinity. To accomplish our objectives we split the problem into three levels, all implemented on a GIS platform.

- First, we identify the most important geographical influence factors, we normalize the maps and we define the membership functions for the input variables (i.e. distance to roads, distance to urban center, number of houses around, number of shops around each point).
- Second, we generate automatically the rules for the fuzzy system and we adjust the fuzzy system parameters based on historical data and on the experience of the planning experts. The output of the fuzzy system is the potential for development.
- Third, using the database of fuzzy rules and new thematic covers representing influence factors, we compute the new maps of potential for development.

The map of potential for development represents a continuous and static map indicating the actual preference for development and load growth. Applying the fuzzy rules
on other regions or other time scenario the model allows the estimation of the load in each map location based on the similarity with other time-space experience or/and based on planners expertise for future planning rules.

The load growth or land-use development change is a discontinuous and dynamic problem. The development is discontinuous because the load increase has minimum values, generally characterized by an integer number of consumers. The development is dynamic because the development for a following time stage is highly dependent of the state in previous stages.

We found that a CA model could demonstrate an ability to act under these two characteristics (discontinuous and dynamic), computing the map of development (growth of number of consumers) in short term stages. In each time step \( t \) the CA recalculate the potential growth \( P(t) \); this calculation is based on three factors, which determine potential growth [15].

- The vacant space, which is related with the saturation level, characterized by an \( S \) curve (sigmoid function).
- The interaction and feedback, which is related with the potential values on a location and in the neighborhood (calculated based on the potential in the previous iteration).
- The innovation effect, which is modeled as random noise.

II. SPATIAL LOAD FORECASTING STRUCTURE

The fuzzy system, the cellular automata, the end use model and the scenario generator compose the main model structure. The fuzzy inference model uses historical data about the influence factors (input variables), and analogue histories of development, to extract or generate fuzzy rules to be used by the fuzzy system. The planner is allowed to formulate rules that are not automatically recognized from historical data.

The set of fuzzy rules can be applied on other space-time scenario characterized by a new set of geographical data. The fuzzy system, implemented on the GIS, computes a new scenario map of potential for development. This map of potential is recalculated in each stage for each consumer class.

A global trend module is necessary to compute the growth in the number of consumers in each time stage and for each consumer class. The result is applied to the cellular automata, which dynamically and discontinuously spread the consumers over the map of potential. The cellular automata compute simultaneously for all consumer classes, taking in account saturation levels.

The process can be repeated along several stages, recalculating the potential for each consumer class and allowing the interaction with the user or with other planning events or new infrastructures.

The scenario generator module controls the scenario orientation for each time stage. The objective of this module is the generation of a tree of scenarios for load growth, which can be done in two different ways:

a) In each stage the user can redefine the geographic thematic coverage that represent the influence factors used as input (e.g. new road; new urban plan guidelines); the new geographic element can have several hypothesis leading to the ramification of the scenario tree.

b) The second hypothesis of interaction is on the global trending module; different socio-economic scenarios can be formulated leading to several growth curves for each class of consumers. The scenario generator formulates the several hypotheses in a tree of scenarios for load growth.

Finally the result - maps with the number of consumers of each class - is transferred to an end-use model that computes the load growth. The end-use model is not within the scope of this paper; its objective is to model the demand of each consumption type as function of typical load curves and other variables relevant for consumption values. In this respect, in our team we have also considerable experience, namely with Neural Network models [16].

III. FUZZY INFERENCE MODEL

To capture the regression (function approximation) between geographic influence factors and preference (development) we have developed a geographical fuzzy inference model. Some conventional SLF methods use simple linear or polynomial regressions, without geographical partitioning [3]. In some cases the user specifies the weights parameters. More advanced methods use geostatistics to compute these relations. Examples of these methods are geographical weight regression and expansion methods [17].

The fuzzy systems deal with qualitative information allowing the implementation of linguistic descriptions for influence factors (close to the road; far from urban center; many houses; few industries). Other advantage of a fuzzy approach is its capacity for generalization, allowing information aggregation and extrapolation to other space-time scenarios with less descriptive information.
Many variants and operations can be used in fuzzy-logic inference [18]. This section will describe briefly the technique we implemented on a GIS; for more detailed information on fuzzy logic see references [19, 20].

Fuzzy systems can model continuous input/output relationships; our objective is to model a function approximation (regression) between several geographic variables.

A basic component of a fuzzy system is a fuzzy rule. The rules are expressed using linguistic labels such as the rule:

IF (road is close) AND (urban center is close) THEN (development growth index is 0.8).

Fuzzy membership functions (MFs) associate linguistic labels (e.g. close) with a particular area of inputs or outputs. In the example shown above the THEN-part of each rule does not consist of a membership variable but of the crisp value 0.8. This kind of systems is called zero-order Sugeno fuzzy system. In an n-th order Sugeno fuzzy system the THEN-part of each rule consists of a polynomial of degree n in the input variables. Different shapes of the MFs can be proposed such as triangular, trapezoidal, or Gaussian. For the following discussions we assume the Gaussian shape.

\[
m_{\nu}(x) = \exp \left( -\frac{(c_{\nu} - x)^2}{2\sigma_{\nu}^2} \right)
\]

Where \(i\) denotes the index of the different MFs defined for variable \(v\) and \(x\) denotes the input for variable \(v\). The parameters \(c_{\nu}\) and \(\sigma_{\nu}\) are the center and the "width". In order to minimize the number of layers used on the GIS implementation, for each input only two membership functions have values higher than zero and their sum may be one. This can be achieved by using appropriate width and dividing each membership values \(m_{\nu}(x)\) by the sum of all membership values, indexed as \(iv\) for label \(i\) in variable \(v\), leading to normalized MFs.

\[
M_{\nu}(x) = \frac{m_{\nu}(x)}{\sum_{i} m_{i}(x)}
\]

As input variables we can have three kinds of influence factors: distance factors (e.g. distance to roads, distance to urban centers); zone-count factors (number of houses on 5 km radius); local factors (e.g. terrain slope, urban planning directives). As output we have the map of development potential for each consumer class (number of additional consumers).

After the MFs definition we can formulate the rules \(j\) in term of linguistic values. Input variables are combined in expressions using fuzzy operators such as fuzzy AND (T-Norm) or fuzzy OR (T-conorm). In the case of Gaussian MF the fuzzy AND can be performed by the arithmetic product of membership values across the input variables \(x\).

\[
G_{j}(x) = \prod_{i} M_{\nu}(x) \quad \text{(3)}
\]

For each consumer class \(c\) the output value is calculated by the OR operation and can be generated by:

\[
O_{c}(x) = \sum_{i} w_{i} G_{j}(x) \quad \text{(4)}
\]

where \(w_{i}\) is the THEN-part (or output weight) of the fuzzy rule \(j\). The output weights \(w_{i}\) can be set manually by domain experts. Alternatively a given training data set

\[
D = \{ (\xi^{k}, \xi^{*}) \}_{k=1}^{M}, \xi^{k} \in \mathbb{R}^{n}, \xi^{*} \in \mathbb{R}^{n} \quad \text{(5)}
\]

could be used to perform training, where \(M\) is the number of training points, \(n\) is the number of influence factors and \(m\) is the number of consumer classes. The goal of this training is to find the output weights that minimize the summed square error.
\[ E = \frac{1}{2} \sum_{i=1}^{N} (O_i(\xi^* - \zeta^*))^2 \]  

(6)

If the IF-part of the fuzzy rules is fixed, the determination of weights \(w_j\) can be solved by the method of least squares based on standard matrix techniques

\[
\begin{bmatrix}
\sum g_i(\xi^*)
\vdots
\sum g_i(\xi^*)
\end{bmatrix}
\begin{bmatrix}
w_1
\vdots
w_j
\end{bmatrix} =
\begin{bmatrix}
\sum g_i(\xi^*)
\vdots
\sum g_i(\xi^*)
\end{bmatrix}
\begin{bmatrix}
\sum g_i(\zeta^*)
\vdots
\sum g_i(\zeta^*)
\end{bmatrix}
\]  

(7)

When implemented on GIS the rules are coded as map regions. To map the zones in maps of rules we used geographical reclassification function. The number of rules activated in each geographical location is 2^n (the same number of maps is needed to store rule coding), where \(n\) is the number of input variables. For each rule map we compute a stack of maps with membership values \(G_i(x)\).

These membership values are functions of the geographical value for the input variables (influence factors). All the calculations associated with each rule \(j\) are computed based on zonal functions available on GIS in which the zones are the regions where rules are activated.

The method was implemented on ArcView GIS programmed on Avenue language. We needed 17.5 minutes to training 3 output classes with 3 geographical input variables using 67000 training points (area with 4000 km^2) on a Pentium II 200MHz.

The set of rules (variable labels, codification index and weights) is stored on a GIS database to be used on other time-space scenarios.

Every time changes are made on input maps (e.g. new roads, new demographics and new industries) the output maps for development potential must be recalculated using the database of rules. Such recalculations must be done in each stage of the forecasting time. This stage varies between 0.5 and 5 years depending on the urban development dynamics. The calculation of new maps for development potential takes approximately 2/3 of the training time.

A new training is necessary if the new region uses different urban planning philosophies (different rules and different variables). If the urban planning rules change through time it is also recommended the retraining of the fuzzy inference model. If no data exists to describe the urban planning behaviour the planner experts can define their own rules and insert them directly on the database.

The obtained maps of development potential are continuous on space and static in time. To solve the SLF problem, which is discontinuous and dynamic, we use a cellular automata model. There are several possible approaches. In this paper, we sketched our experiments with CA.

IV. CELLULAR AUTOMATA

The CA theory was first introduced by Jon Von Neuman [21] and is ideally applied for dynamic discrete modeling [22]. A CA is a discrete dynamical system because space and system states are discrete and this states changes sequentially over time and space. Each point in a rectangular spatial grid, called a cell, can have any one of a finite number of states. The states of the cells in the lattice are updated according to a local rule, which depends of they own state and the state of its neighbors on the previous time step. The state of the entire lattice is updated synchronously in discrete time steps.

In our formulation at any specific point of time \(t\), the automaton is a collection of states \(s_x\) characterized by the saturation levels for the several consumers classes and the cell location \(i,j\).

\[ CA = \{ s_{x} \}, \quad 0 < i \leq r; \quad 0 < j \leq c; \quad \forall x_s \in S \]  

(8)

CA is the cellular automaton, \(S\) the finite set of states, \(n\) and \(m\) are the number of rows and columns.

The possible states are the combination of possible saturation level for the several consumer classes.

For each consumer class the change must be between adjacent states. The sum of developed area must be lower than the saturation level (e.g. if the industrial development change from 2 to 3 consumers the development must decline from 20 to 10 consumers). This method allows the simulation of growth, decline and redevelopment. An appropriated classification of the consumers according to the load consumption, area utilized and time-scale for typical development is very important.

The state transition is done according to a set of heuristic rules. In this model the state transition function depends on the functions of potential-to-development and potential-to-decline obtained from the fuzzy inference model.

For each consumer class, one computes the potential for development and potential for decline using 3 component factors:

a) positive feedback of the cell on the previous iteration, weighted by \(\alpha\);

b) neighborhood effect based on the 8 adjacent neighborhoods [23], weighted by \(\beta\);

c) innovation factor modeled as random noise, weighted by \(\lambda\).

\[ P_r(t+1) = \alpha \cdot P_r(t) + \beta \cdot \frac{1}{8} \sum_{j \in N} P_j(t) + \lambda \cdot \varepsilon_r(t) \]  

(9)

\[ D_r(t+1) = \alpha \cdot D_r(t) + \beta \cdot \frac{1}{8} \sum_{j \in N} D_j(t) + \lambda \cdot \varepsilon_r(t) \]  

(10)

Where \(\alpha, \beta, \lambda\) and \(\varepsilon\) are the weights for each component, with values \([0, 1]\) and \(\alpha + \beta + \lambda = 1\). \(P_r(t+1)\) is the potential to development in time stage \((t+1)\) on site \(i\), \(D_r(t+1)\) is the potential-to-decline in time stage \((t+1)\) and site \(i\); \(\Omega\) is the set of adjacent neighbors cells.
For each consumer class, one checks if the saturation level is reached (if the sum of all consumer on cell $i$ occupies more than the global saturated level $s_D$) and if conditions to redevelop are filled $(Z_i \cap Z_J)$, where $Z_i$ is the set of cells with highest potential-to-development and $Z_J$ is the set of cells with highest potential-to-decline. If the cell $i$ is saturated the consumer class with higher potential-to-development $C$ develops (increase state $s_C$) but the consumers class with higher potential-to-decline $D$ must decline (decrease state $s_D$) in order to maintain the global saturation level.

$$\text{if } \sum_{c} s_c > s_D \text{ then } \begin{cases} s_C(t+1) = s_C(t) + 1, & i \in (Z_C \cap Z_J) \\ s_D(t+1) = s_D(t) - 1, & i \in (Z_J \cap Z_C) \end{cases}$$

If the cells are not saturated, one increases the saturation in the areas with higher potential-to-development. If the cell is not empty, one decreases the saturation level on cells with higher potential-to-decline.

$$\text{else } \begin{cases} s_C(t+1) = s_C(t), & i \not\in (Z_J) \\ s_D(t+1) = s_D(t), & i \not\in (Z_C) \end{cases}$$

This operation is repeated until all the global growth and global decline, for each consumer class in each stage, is allocated.

Trending models must estimate the global growth and global decline, for each consumer class, (e.g. the growth for year 2001 in all region is 250 industrial consumers and 5000 domestic consumers). On the end of each stage the maps of potential-to-development and potential-to-decline may be recalculated, using the fuzzy inference model and the new geographic data computed with the CA and introduced by the planner.

On figure 4 we can see the test results, a prototype demonstration for four different stages of domestic consumer development in one city (located on the island center) in an African country (Cape Verde). The influence factors to compute the potential-to-development were the distance to the roads and the distance to the urban centers.

The pattern of evolution follows the pattern of potential, more intense growth near the urban center and near roads.

V. CONCLUSIONS

The major problem of simulation methods is capturing and retaining the richness of the simulated system. The fuzzy inference approach allows the automatic identification of the chains of cause/effect in the form of rules. These rules can be transposed to other time-space environments, with similar planning philosophies. Other advantage of the fuzzy inference model is the ability to store new planning rules directly specified by the planner. The generalization ability of fuzzy systems allows the generation of continuous and stationary maps of potential-to-development and potential-to-decline.

From maps of potential, one needs to take a step into generating scenarios of development, conditioned by the set of rules establishing an environment for growth.

In the paper we have explore the possibilities offered by cellular automata in providing a massive parallel mechanism of simulating evolution.

The results are preliminary but promising. However, one recognizes that further research is necessary to understand the coupling of automata behavior with the inference model and the rules derived, and to control the characteristics of this coupling.

With this approach, we hope to have opened a way to represent the dynamics and discrete characteristics of urban and rural development, seen from the point of view of the
energy planner. Hopefully, final model wills satisfactory simulate the growth, decline and redevelopment of several classes of consumers in a territory.

VI. REFERENCES


VI. BIOGRAPHIES

Vladimiro Miranda received his Licenciado, Ph.D. and Agregado degrees from the Faculty of Engineering of the University of Porto, Portugal (FEUP) in 1977, 1982 and 1991, all in Electrical Engineering. In 1981 he joined FEUP and currently holds the position of Professor Associado Agregado. In 1989 he joined also INESC, a research and development institute, having held for many years the position of Head of Information and Decision in Energy Systems. In 1996 he was appointed President of the Executive Board of INESC-Macrual (South China) and Full Professor in the University of Macau. He is currently the President of the Scientific Board of INESC Porto and Executive Board Adviser for Power Systems. He has been a member of several Expert Committees in Power Systems. He has had responsibility over several research projects within the European Union programmes and also in cooperation with Latin America and Portuguese speaking African countries, and he is the chairman of the Luso-Afro-Brazilian Cooperation Network in Power. He has also acted as consultant for the promotion of projects with China, within the framework of the EUREKA initiative. He has authored or co-authored many papers, namely in his areas of interest, related with Power System planning and the application of fuzzy sets and other soft computing techniques to Power Systems.

Cláudio Monteiro was born in France, on March 14th, 1968. He received his Licenciado and M.Sc. degrees from FEUP in 1993 and 1996, in Electrical Engineering and Computers. In 1993 he joined INESC as a researcher in the Power System Unit. Presently is working in his Ph.D. related integrating distribution planning models and Geographical Information Systems under uncertain reasoning.
FUZZY INFERENCE IN SPATIAL LOAD FORECASTING

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Abstract - Forecasting electric demand and its geographical distribution is a prerequisite to generate expansion planning scenarios for distribution planning. This paper presents a comprehensive methodology that uses a fuzzy inference model over a GIS support, to capture the behavior of influence factors on load growth patterns and map the potential for development. The load growth is spread over maps with cellular automata. The interaction with a scenario generator inputs data into a graph generator, which will serve as a basis for more classic network planning tools.

Keywords - Spatial Load Forecasting, Geographic Information Systems, Fuzzy Inference, Cellular Automata.

1. INTRODUCTION

In distribution planning, research has been concentrated for many years in developing algorithms to optimize network design. In the last years, however, the concept of optimization has been challenged.

Two things contributed for this: the development of multiple criteria models and the emergence of the risk analysis paradigm, in the context of multiple future possible scenarios. Also evolutionary computing techniques have taken over other methods and given opportunity for the development of really comprehensive truly dynamic in time.

The first author was deeply involved in this movement of change of point of view. The work in [1] reported the first Genetic Algorithm model for Distribution Planning and established the bases for future evolutionary computing approaches, including multiple criteria e a tree of futures. This work already presented a decision choice procedure based on the minimization of decision risks. But the clear distinction between risk analysis and other decision paradigms was to be made clear in other papers such as in [2].

It is now clear that we have witnessed the completion of a phase in solving the Distribution Planning problem. It must be recognized, however, that the departing point is still the same as 20 years ago: data must include a definition of nodes (for possible injections and loads) and a graph (for possible lines to be built thus forming and expanding the network).

We know how to solve the network design problem in the space of graphs. However, how does one define such graph to begin with?

The key factor in distribution planning is load forecast - but in distribution planning we deal with vast geographical regions, not with a few nodal loads. Utilities around the world are increasingly adopting Geographic Information Systems (GIS) to serve as the background digital representation of regions and networks.

Land-use spatial load forecast (SLF) simulation methods have been used to model the process of the load growth in order to predict load evolution in a spatial and temporal basis [3]. This methodology is particularly suited to high spatial resolution for long range forecasting and multi-scenario planning [4].

The challenge that researchers face presently, in the distribution planning problem, is to derive a consistent way of establishing an acceptable correspondence between the geographical space and the space of graphs, so that the geographical model may generate an adequate input to the network design models. The reverse projection of the results of such models into the geo-referenced space is quite straightforward.

In this paper we describe an approach to generate adequate spatial load forecasts in regions and to generate graphs (with nodes and branches) representing possible investments for system development, which may serve as input for an evolutionary computing algorithm to proceed with actual system design proposals. The aim is to allow an automated (as much as possible) generation of robust solutions, that may be good regardless of the future or acceptable in a majority of credible future scenarios.

The model is based:

1. on a fuzzy inference engine (built after neuro-fuzzy concepts) which extracts knowledge from past data and produces a set of rules that condition demand growth in a region
2. on a cellular automata engine that helps in spreading to a whole map the effects of rules
3. on a scenario generator that allows building a tree of futures
4. on a grid generator that prepares graphs with potential branches and nodes to be fed into network design and optimization tools.

This allows one to generate maps representing potential for growth, from which one may build, for a succession of time stages, maps of forecasted load. From these, one may finally
derive a graph of nodal loads and of potential system branches - the departing point for final network design.

2. SPATIAL FORECASTING

Spatial Load Forecasting (SLF) refers to models used to predict load growth in a region based on the influence of several control factors, defined as “influence factors”. Examples of those factors are, for instance, the influence of a radial distance to an urban center or of the distance to a waste treatment center.

In recent years some works have enhanced the land-use methods applied to urban redevelopment, using fuzzy logic, GIS, multi-objective programming[5, 6, 7].

In urban planning, a large work is being done to model land-use conversion. This is done under the assumption that a simulation approach under the self-organization paradigm is appropriate for addressing the process of land development [8, 9]. To simulate the dynamics of the process, a recent work adopts cellular automata (CA) - this approach emphasizes the way in which locally-made decisions give rise to global patterns.

3. THE FUZZY INFERENCE MODEL

3.1 Influence factors, development and rules

The module described in this section is a spatial model that uses a set of explanatory geographical variables, designated as spatial influence factors, to predict the potential for development, for a specific consumer class.

Spatial factors to be considered in SLF are local structural factors, relative location factors, and neighborhood factors. The structural factors are variables that affect the site unit they are primarily associated with, and their effect is confined to the geographic unit boundaries (e.g. slope, altitude, land use classification).

The distance to certain geographical features defines the relative location factors; for instance, proximity to roads, proximity to urban centers.

The neighborhood effect represents the influence of entities or features in an adjacent area or in its own geographic unit. Important neighborhood factors are load saturation levels: they models the dynamics of change from stage to stage influencing the results on each following time stage.

Each point in a map is associated with saturation curves $S_i$ for each type $i$ of consumer. A saturation curve describes the number of consumers of a given type, at a certain location, as a function of time. It usually displays a S shape (figure 1).

In our approach, we do not define a priori any saturation curve. Instead, the shape of the saturation curve is built dynamically as a function of all geographical influence factors and the dynamic interaction between different consumer classes. This comes as an output of the inference process, and the rule learning procedure also allows the learning of saturation growth at each location and its weight.

The result of the module is a map for the potential for development (PFD); it is a continuous map of values between zero and a maximum value representing the maximum possible growth in one stage for a geographic unit.

The function that models the growth phenomena for each consumer class $i$ could be represented by

$$ S^i_t = f(S^i_{t-1}, ..., S^i_{t-n}, I_1, ..., I_p) $$

where $S^i$ represents the output of the model (potential-for-development for consumer class $i$) and is the differential of the saturation curve on stage $t$. The $S_i$ represent saturation levels for each consumer class at stage $t$ and the $I_i$ represent other geographic influence factors.

In our approach, the functions that relate saturation and its derivative or PFD are established by the rules in the fuzzy inference engine. These functions are represented as a set of thousands of fuzzy rules. Here's one example of a rule:

$$ IF \ (\text{distance to road is CLOSE}) \ AND \ (\text{distance to urban center is MODERATE CLOSE}) \ AND \ (\text{terrain slope is MODERATE}) \ AND \ (\text{domestic saturation is MEDIUM}) \ AND \ (\text{industrial saturation is LOW}) \ THEN \ Domestic PFD \ is \ 20 \ consumers \ per \ stage \ per \ km^2 \ AND \ Industrial PFD \ is \ 0.1 \ consumers \ per \ stage \ per \ km^2 $$

These rules are automatically generated and used by the spatial model and are easily understood by human specialists. The rules are stored in the GIS database and are used as in a lookup table in the process.

The geographical data are stored as a structure of grids of values (raster structures). In the process the system checks the rules activated in each location. If the rule is activated, an activated value $Q_i$ is computed. At the same location several rules will be activated simultaneously and the result will be a weighted sum of their activated values.

The fact that these rules can be generated and understood simultaneously by the system and by human experts is a great advantage because historic and human knowledge can be joined and interpreted.

As we will see in a further section, we employ a cellular automata module to compute the development based on

Figure 1 – The number of consumers as a function of time defines the Saturation curve at a location; its derivative is seen as the potential for development PFD. The saturation level at stage t is characterized through fuzzy descriptors.
potential for development maps. Moreover, these development maps are used to compute the new saturation levels for each consumption class.

3.2 Training or tuning the neuro-fuzzy rule model
To capture the regression (function approximation) between geographic influencing factors and preference (development) we have developed a geographical fuzzy inference model. Some conventional SLF methods use simple linear or polynomial regressions, without geographical partitioning [3]. In some cases the user specifies the weights parameters; more advanced methods use geostatistics to compute these relations. Examples of these methods are geographical weight regression and expansion methods [10].

The fuzzy systems deal with qualitative information allowing the implementation of linguistic descriptions for influence factors (close to the road; far from urban center; many houses; few industry). Other advantage of a fuzzy approach is its capacity for generalization, allowing information aggregation and extrapolation to other space-time scenarios with less descriptive information.

Many variants and operations can be used in fuzzy-logic inference [11]. This section will describe briefly the technique we implemented on a GIS; for more detailed information on fuzzy logic see references [12].

A basic component of a fuzzy inference system is a fuzzy rule. The rules are expressed using linguistic labels such as the rule: IF (road is close) AND (urban center is close) THEN (development growth index is 0.8). Fuzzy membership functions (MFs) associate linguistic labels (e.g. close) with a particular area of one of input or output variable. In our case, the THEN-part of each rule does not consist of a membership variable but of a crisp value 0.8. This is called a zero-order Sugeno fuzzy system. In an n-th order Sugeno fuzzy system the THEN-part of each rule consists of a polynomial of degree n in the input variables.

Different shapes of the MFs can be proposed such as triangular, trapezoidal, or Gaussian, for instance:

$$m_i(x_i) = \exp \left( - \frac{(c_i - x_i)^2}{2 \sigma_i^2} \right)$$  \hspace{1cm} (1)

where $i$ denotes the index of the different MFs defined for variable $v$ and $x_i$ denotes the input for variable $v$. The parameters $c_i$ and $\sigma_i$ are the center and the "width". In order to minimize the number of layers used on the GIS implementation, for each input value only two membership functions have values higher than zero and their sum may be one. This can be achieved by adopting an adequate normalizing procedure.

As input variables we can have three classes of influence factors: distance factors (e.g. distance to roads or to urban centers); zone-count factors (number of houses on a 5 km radius); local factors (e.g. terrain slope, urban planning directives). As output we get a map of development potential for each consumer class (number of additional consumers).

After the MFs definition we can formulate the rules $j$ in term of linguistic values. Input variables are combined in

![Image](image.png)

Figure 2- Illustration of the fuzzy inference process - on each map location, membership functions are activated by input values for several influence factors; several layers of rule zones are mapped; the weights of rules are applied to each case and a map of Pdf is generated for each consumer class - the darkest zones in the map on the right represent the zones with higher Pdf (near centers and near roads).
expressions using fuzzy operators such as fuzzy AND (T-Norm) or fuzzy OR (T-conorm). In the case of Gaussian MF the fuzzy AND can be performed by the arithmetic product of membership values across the input variables \( x_i \).

\[
G_i(x_i) = \prod_j M_{ij}(x_i)
\]  

(2)

For each consumer class \( c \) the output value is calculated by the OR operation and can be generated by

\[
O_c(x_c) = \sum_j w_j G_j(x_c)
\]  

(3)

where \( w_j \) is the THEN-part (or output weight) of the fuzzy rule \( j \). The output weights \( w_j \) can be set manually by domain experts. Alternatively, a given training data set

\[
D = \{ \xi^k, \zeta^k \} \text{ where } k = 1, \ldots, M, \xi^k \in \mathbb{R}^n, \zeta^k \in \mathbb{R}^m
\]  

(4)

could be used to perform training, where \( M \) is the number of training points, \( n \) is the number of influence factors and \( m \) is the number of consumer classes. This training would find the output weights that minimize the summed square error.

\[
E = \frac{1}{2} \sum_k (O_k - \zeta_k^k)^2
\]  

(5)

If the IF-part of the fuzzy rules is fixed, the determination of weights \( w_j \) can be solved by the method of least squares based on standard matrix techniques

\[
\begin{bmatrix}
\begin{bmatrix}
\sum_{i=1}^{M} G_i(e_i^r) \\
\sum_{i=1}^{M} G_i(e_i^c) \\
\sum_{i=1}^{M} G_i(e_i^k)
\end{bmatrix}
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
\sum_{i=1}^{M} G_i(e_i^r) \\
\sum_{i=1}^{M} G_i(e_i^c) \\
\sum_{i=1}^{M} G_i(e_i^k)
\end{bmatrix}
\end{bmatrix} \begin{bmatrix}
\begin{bmatrix}
G_j(e_j^r) \\
G_j(e_j^c) \\
G_j(e_j^k)
\end{bmatrix}
\end{bmatrix}
\]  

(6)

When implemented on GIS the rules are coded as map regions. The number of rules activated in each geographical location is \( 2^v \) (the same number of maps is needed to store rule coding), where \( v \) is the number of input variables. For each rule map we compute a stack of maps with membership values \( G_j(x) \).

These membership values are functions of the geographical value for the input variables (influence factors). All the calculations associated with each rule \( j \) are computed based on zonal functions available on GIS, in which the zones are the regions where rules are activated.

The set of rules (variable labels, codification index and weights) is stored on a GIS database to be used on other time-space scenarios.

The obtained maps of development potential are continuous on space and static in time. To solve the SLF problem, which is discontinuous and dynamic, we use a cellular automata model.

4. CELLULAR AUTOMATA

The CA theory was first introduced by Jon Von Neuman [13] and is ideally applied for dynamic discrete modeling [14]. A CA is a discrete dynamical system because space, time and system states are discrete and these states change sequentially over time and space. Each point in a rectangular spatial grid, called a cell, can have any one of a finite number of states.

The states of the cells in the lattice are updated according to a local rule, which depends on the cell state and the state of its neighbors at the previous time step. The state of the entire lattice is updated synchronously in discrete time steps.

In our formulation at any specific point of time \( t \), the CA automaton is a collection of binary states \( e_i^j \) in cell location \((i,j)\), with value 1 if the new consumer is added to the site and 0 if no consumer is added.

\[
CA = \{ e_i^j \} , 0 \leq i \leq r, 0 \leq j \leq c; \forall e_i^j \in E
\]  

(7)

where \( E \) is the finite set of states, \( r \) and \( c \) are the number of rows and columns of the map grid.

The CA is an iterative process computing development based on potential for development and computing new potential based on previous iteration development.

The Potential for Development (PID) is initially set by the fuzzy system. The PID is represented as a stack of continuous maps, one for each consumer type, representing the potential growth number of consumer per stage and per geographic unit (e.g. 20 domestic consumers per stage and per km²). The Development, which is the output of the CA, represents the effective number of consumer growth. A global geographical trending controls the global development, the sum of all developments in the region. The CA process finishes when the sum of all cell developments reaches the global trending value (e.g. the growth for year 2001 in the whole region tends to 250 industrial consumers and 5000 domestic consumers).

The iterative process of the CA is based on state transitions \( S_i(t) \); in our model, these will be transitions from non developed to developed. The state transition is done according to a set of rules such as

\[
\text{if } P_i(t) > P_k(t) \text{ then } S_i(t) = 1 \text{ else } S_i(t) = 0
\]

In our model a transition exists if the cell has a PID value \( P_i(t) \) higher than a specified boundary value \( P_k(t) \). The boundary value is specified by the system by ranking PID intervals.

The development \( D_i(t) \) is recalculated in each iteration incrementing the number of consumers, by steps \( D_{step} \) (measured in number of consumers), only on cells marked as developed \( S_i(t) = 1 \).

\[
D_i(t) = D_i(t - 1) + S_i(t) \cdot D_{step}
\]  

(8)

The new potential \( P_i(t+1) \) is recalculated based in three components:

- positive feedback of the cell on the previous iteration, weighted by \( \gamma \);
- neighborhood effect based on the 8 adjacent neighborhoods [15], weighted by \( \beta \);
- innovation factor modeled as random noise, weighted by \( \lambda \);

and is given at time \( t+1 \) by
\[ P_i(t+1) = \alpha \cdot P_i(t) + \beta \cdot \frac{1}{8} \sum_{j \in n_i} P_j(t) + \lambda \cdot e_i(t) \]  \hspace{1cm} (9)  

where \( \alpha \), \( \beta \) and \( \lambda \) are the weights for each component, with values \([0,1]\) and \( \alpha + \beta + \lambda = 1 \), and \( \Omega \) is the set of adjacent neighbors cells. \( P_i(t) \) is the updated potential to develop in time stage \( t \) on site \( i \), computed based on the output of the fuzzy inference model \( P_i(0) \) and on the development computed by the CA on iteration \( t \):

\[ P_i(t) = P_i(0) - D_i(t) \]

At the end of each stage the PiD maps may be recalculated, using the fuzzy inference model and the new geographic data computed with the CA or introduced by the planner.

5. SCENARIO GENERATOR

A scenario is a sequential set of stages (time sequence) including the input data and parameters to be used in each stage. The scenario generation requires the participation of the Planners specifying some of geographical input data (e.g. road coverage, land use impositions) and parameters for each stage (global trend and parameters \( \alpha \), \( \beta \) and \( \lambda \)). Some of the input data are automatically calculated by the system or by other modules. For instance, the saturation level is recalculated used as input on stage \( t \) is recalculated with the results of development from stage \( t-1 \). Other input data, as road coverage, may come from external planning models or simply by interaction with the planner.

Multiple options on input data for a specific stage give place to a tree of scenarios that can automatically generated by a scenario generator module. For instance, two possible values for global trending at stage \( t \) and the event of building or not a bridge at stage \( t+2 \) originate 4 different scenarios.

For each scenario a sequence of the fuzzy system module and the CA module will run as many times as the number of stages. The result is a sequence of geographical maps with consumer growth along the several stages. The maps of consumption are computed based on the number of consumers and on End Use models that define the consumption behavior of each consumption class.

6. NETWORK GENERATOR

The SLF model is the data feeder for several models more familiar to network designers. The load scenarios describing maps of forecasted values along time stages are the base data for the following expansion planning modules:

- secondary network routing
- substation siting
- substation service area optimization
- routing for lines interconnecting substation

The main objective of these modules is the pre-processing of geographical data in order to obtain a set of robust feature options to be used in automated planning tools. These options are basically electric facility sizing and location (e.g. sizing and for lines and substations, routing for lines and geographical siting for substations). It is known that facility location may be highly affected by demand uncertainty and its geographical distribution.

The referred modules build a vectorial topology (based on graphs and nodes) and related databases to store attributes associated with each option. In fact, the procedure allows the extraction of information from the geo-environment and its storage as attributes of the network features.

7. EXAMPLE

In this section we illustrate the application and results of the SLF model implemented in ArcView GIS and programmed in Avenue. This study presents the forecasting value for domestic consumption in the island of Santiago in Cabo Verde (Africa). The result is one forecasted scenario with eleven stages, which were obtained for illustration purposes and cannot be seen as reflecting the actual situation in Santiago.

Figure 4 – Maps of PiD, a sample of 4 out of the 11 stages obtained with the fuzzy inference engine. Notice that at the center there is a growing saturation effect and that at later stages the potential for development concentrates mainly in peripheral zones, following roads and avoiding high slopes.

Figure 4 – Maps of forecasted number of consumers (of domestic type) after the action of the cellular automata.
REFERENCES


8. CONCLUSIONS

The research emphasis in automated distribution planning must nowadays be focused on the difficult task of establishing a correspondence between comprehensive geographical representations and the design optimization algorithms, which require data in the form of a graph with nodes and branches.

This paper demonstrates that coupling GIS tools with fuzzy inference engines may allow building the desired integrated environment for engineers, planners and decision makers.

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Scenario Identification Process in Fuzzy Spatial Load Forecasting

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Abstract: In this paper we present a Spatial Load Forecasting (SLF) model based on Fuzzy inference and Cellular Automata. The first part of the paper describes the Fuzzy SLF model, the training process, the implementation on a Geographical Information System (GIS) and the Fuzzy SLF functionality over a multiple time stages. In the second part the environment assessment for decision process, by identifying scenarios resulting from geographical non-repetitive events, will be discussed.

Keywords: Spatial Load Forecasting, Geographic Information Systems, Fuzzy Inference, Cellular Automata, and Scenario Identification.

I. INTRODUCTION

Forecasting and Planning are complementary processes that can’t be dissociated. Planners use forecasts to predict and revise the outcome of alternative plans, and plans may be used to define the environment and events that influence the forecasts. In Distribution Planning one uses consumption forecasts that are a consequence of geographic influence factors, most of them resulting from urban plans. The urban plans are the basis to set non-repetitive events (e.g. building a road, land use directives, etc.) that define scenarios. The scenarios will represent several contrasting futures identifying economic, technological, or event possibilities for the future.

There is also a relation, at the uncertainty structuring level, between forecasts and scenarios. Forecasts are strong statements about what will happen in the future when all relevant variables are taken into account. It is possible to structure uncertainties related with small deviations from a forecast statement. This kind of uncertainty modeling is presented in [1]. Contrary to the approach of structuring uncertainties into forecasts, scenario planning requires the participation and responsibility of the Decision-Maker in the construction and evaluation of the scenarios, namely by running several forecasts. Scenario planning models multiple plans that depend on the future.

In this paper we will describe an expert system, implemented into a GIS for small area spatial load forecasting. We also discuss and illustrate, with an example, the scenario planning approach for structuring uncertainties related with geographical events.

Land-use Spatial Load Forecasting (SLF) simulation methods have been largely used to model the process of the load growth in order to predict load evolution on a spatial and temporal basis [2]. The authors developed a SLF model joining the characteristics of Fuzzy Inference System and Cellular Automata [3]. The Fuzzy Inference model captures the geographical pattern of influence factors, estimating the potential for development, and cellular automata models the dynamics and spreading process over the geographical region, forecasting the development for consumer growth. The authors have further improved the model by transferring the dynamics of the forecasting model, from the Cellular Automata (CA) to the Fuzzy Inference Model [4]. In this last approach, the saturation levels (consumer/km²) of each consumer class are considered as input variables of the fuzzy system, and the CA model simulates only the consumer spreading over the geographical region. This temporal dynamics of Fuzzy SLF is controlled by a module (Scenario Coordinator) that defines for each stage the parameters and the geographical themes used in the simulation. The Scenario Coordinator allows the interaction with the planner for the identification of scenarios.

II. FUZZY SPATIAL LOAD FORECASTING

Spatial Load Forecasting (SLF) refers to models used to predict load growth in a region, based on the influence of several control factors, defined as “influence factors”. Examples of those factors are, for instance, the influence of a radial distance to an urban center or of the distance to a waste treatment center.

In recent years some works have enhanced the land-use methods applied to urban redevelopment, using fuzzy logic, GIS or multi-objective programming [5, 6, 7].

In urban planning, a large work is being done to model land-use conversion. This is done under the assumption that a simulation approach under the self-organizing paradigm is appropriate for addressing the process of land development [8, 9]. Recently, to simulate the dynamics of the process cellular automata (CA) ~has been adopted- this approach emphasizes the way in which locally made decisions give rise to global patterns.

The authors developed Fuzzy SLF models, completely implemented in a GIS support. The kernel of the Fuzzy SLF is a set of rules storing a spatial and temporal behavior of consumption development. Selecting a set of geographical influence factors, the model is able to automatically capture the historical behavior of consumption growth or to allow the direct specification of expert knowledge. The model can be applied to study other regions with a similar behavior by using the set of rules previously generated.

As shown in Figure 1, three main modules compose the Fuzzy SLF structure. The fuzzy system uses a set of geographical influence factors to compute a continuous map of Potential-for-Development (PFD). The CA uses these maps to simulate consumer growth over all the geographic region, which is the effective Development (D) at stage t. The third
module, the scenario coordinator, is responsible for the time coordination by looping the process throughout the stages of each scenario. The Scenario Coordinator is a table collecting information to be used in each scenario. Part of the data may be acquired from the results of previous stages and other part may by specified directly by the planners., for instance, the saturation levels are computed based on results from the previous stages, the influence factors may be geographical coverages specified by the planner and the parameters are constants necessary to calibrate the other modules.

![Diagram of fuzzy system](image)

Figure 1 – Three kernel modules (Fuzzy System, Cellular Automata, and Scenario Coordinator) constitute the Fuzzy SLF model. This kernel must be coupled to other modules that can be designed independently.

The Fuzzy SLF is coupled with two other modules, the Global Trending and the End Use modules. These modules can be developed independently and will be not discussed here. The Global Trending forecasts, as output for each time stage, the total number of new consumers, for each class that appears in the overall region. The End Use module describes the consumption of each consumer class, and uses the result of the Fuzzy SLF to estimate the electric power consumption growth.

### III. SPATIAL FUZZY SYSTEM

The Spatial Fuzzy System is a GIS spatial model implementing a fuzzy inference engine similar to the fuzzy systems used in control theory [10].

The module described in this section uses a set of explanatory geographical variables, designated as spatial influence factors, to compute the PFD, for each specific consumer class.

Spatial influence factors to be considered in SLF are local factors, relative location factors, and neighborhood factors. The local factors are structural factors related with the site, and their effect is confined to the geographic cell unit (e.g. slope, altitude, and land use classification).

The distance to certain geographical features defines the relative location factors. For instance, proximity to roads, proximity to urban centers proximity to prohibitive areas or undesired proximity to certain facilities.

The neighborhood effect represents the influence of entities or features in an adjacent area or in its own geographic unit. Important neighborhood factors are the load saturation levels; these influence factor model the dynamics of the development change from stage to stage, influencing the results in the following time stages.

Each point in a map is associated with saturation curves $S_i$ for each type $i$ of consumer. A saturation curve describes the number of consumers of a given type, at a certain location, as a function of time. It usually displays an S shape curve (Figure 2). The derivative of this S curves represent the PFD.

![Saturation Curve](image)

Figure 2 – The saturation curve $S$ is represented in the fuzzy rules as a set of linguistic classifiers (low, medium, and high). The curve is built dynamically throughout the time stages. One of the Fuzzy System inputs is the saturation level and the output is the PFD.

In our approach, we do not define a priori any saturation curve. Instead, the shape of the saturation curve is built dynamically following the pattern implicit on thousands of fuzzy rules that compose the Fuzzy System.

The function that models the growth phenomena for each consumer class $i$ could be represented by

$$\dot{S}_c |_t = f(S_{i-1}, ..., S_{i-t}, I_1, ..., I_p)$$  \hspace{1cm} (1)

where $\dot{S}_c$ represents the output of the model (PFD for consumer class $c$) and is the derivative of the saturation curve on stage $t$. The $S_{i-1}$ represent saturation levels for each consumer class at stage $t-1$ and the $I_i$ represent other geographic influence factors.

#### A. Implementation on GIS

Many variants and operations can be used in fuzzy-logic inference [10]. This section will briefly describe the technique implemented on a GIS - for more detailed information on fuzzy logic see references [11]. The Fuzzy SLF model is completely developed in the GIS language using the GIS spatial function to implement the fuzzy system.

A basic component of a fuzzy inference system is a fuzzy rule (see Figure 3). Rules are expressed using linguistic labels such as the rule: IF (road is close) AND (urban center is close) THEN (development growth index is 0.8). Fuzzy membership functions (MFs) associate linguistic labels (e.g. close) with a particular area of one of input or output variable.

In our case, the THEN-part of each rule does not consist of a membership variable but of a crisp value 0.8. This is called a zero-order Sugeno fuzzy system. In an nth order Sugeno fuzzy system the THEN-part of each rule consists of a polynomial of degree n in the input variables.

Different shapes of MFs can be adopted such as triangular, trapezoidal, or Gaussian.

The inputs are maps computed by the basic GIS functions (e.g. distance functions, statistical or focal neighborhood functions, surface functions...). The fuzzy system intermediate operations are specified in the set of rules stored in GIS.
lookup tables. These tables that contain thousands of rules are activated on specific regions of the maps. The activation is directly related with the linguistic label. Because several labels are activated at the same location (e.g. 0.3 close-to-road and 0.7 moderate-close), the combination of labels activates many rules in the same location, leading to computations with many maps in order to represent all the activations and the membership values.

After the definition of MF’s we can formulate rules j in term of linguistic values. Input variables are combined in expressions using fuzzy operators such as fuzzy AND (T-norm) or fuzzy OR (T-conorm). In the case of a Gaussian MF, for a zero-order Sugeno fuzzy system, the fuzzy AND can be performed by the arithmetic product of membership values across the input variables x.

$$G_j(x) = \prod_{i} M_{i_n}(x)$$

For each consumer class c the output value is calculated by the OR operation and can be generated by

$$O_c(x) = \sum_j w_j G_j(x)$$

where w_j is the THEN-part (or rule output weight) of the fuzzy rule j. Based on the multiple maps of rule activation and on the weight of each rule, stored in lookup tables, a continuous map of potential for development is computed, defined as PFD.

B. Training and tuning fuzzy system rules

The set of rules is meant to represent the function approximation $S_c$. In order to capture the regression between geographic influence factors and potential for development, one must train the fuzzy inference model. In the present fuzzy inference model training consists in finding the rule weights that better approximate the function $S_c$. The training is based on a training data set composed by input/output vectors. The inputs are the geographical influence factors defined by the user as significant variables, in this case represented by maps (e.g. distance to roads, domestic saturation level, terrain slope, ...). The output is the historical development for a specific set of inputs, also represented as maps (e.g. consumer growth per year per square kilometer). Each vector corresponds to one pixel of a map and may be formally represented in the following way:

$$D = \{x^k, y^k\}, \quad k = 1, \ldots, M, \quad x^k \in \mathbb{R}^n, \quad y^k \in \mathbb{R}^m$$

These historical data could be used to perform training, where M is the number of training points, n is the number of influence factors and m is the number of consumer classes. This training would find the output weights that minimize the summed square error.

$$E = \frac{1}{2} \sum_{k=1}^{M} \left( O_c(x^k) - \xi^k \right)^2$$

Several methods can be used to train the fuzzy system, like the fuzzy relational model [12], the fuzzy basis function based model [13], and the neural-network-based fuzzy model [14].

If the IF-part of the fuzzy rules is fixed, the determination of weights $w_j$ can be solved by the method of least squares based on standard matrix techniques.

$$\begin{bmatrix} w_0 \\ \vdots \\ w_n \end{bmatrix} = \left( \begin{bmatrix} \sum g_i(x^1) \\ \vdots \\ \sum g_i(x^M) \end{bmatrix} \right)^{-1} \left( \begin{bmatrix} \sum g_i(x^1) \xi^1 \\ \vdots \\ \sum g_i(x^M) \xi^M \end{bmatrix} \right)$$

When implemented on a GIS, the rules are coded as map regions. The number of rules activated in each geographical location is $2^v$ (the same number of maps is needed to store rule coding), where v is the number of input variables. For each rule map we compute a stack of maps with membership values $G_j(x)$. These membership values are functions of the geographical value for the input variables (influence factors). All the calculations associated with each rule j are computed based on zonal functions available on the GIS, in which the zones are the regions where rules are activated.

The set of rules (variable labels, coding index and weights) is stored in a GIS database to be used on other time-space scenarios.

The maps of development potential thus obtained are continuous in space and static in time. To solve the SLF problem, which is discontinuous and dynamic, we use a cellular automata model.

IV. SPREADING WITH CELLULAR AUTOMATA

The CA theory was first introduced by Jon Von Neumann [15] and is ideally applied for dynamic discrete modeling [16]. A CA is a discrete dynamical system because space, time and system states are discrete and these states change sequentially over time and space. Each point in a rectangular spatial grid, called a cell, can have any one of a finite number of states. The states of the cells in the lattice are updated according to a local rule, which depends on the cell state and the state of its neighbors in the previous time step. The state of the entire lattice is updated synchronously in discrete time steps.

In our formulation, at any specific point of time t, the cellular automaton is a collection of binary states $c_{ij}^t$ in cell location (i,j), with value 1 if a new consumer is added to the site and 0 if no consumer is added.
where E is the finite set of states, r and c are the number of rows and columns of the map grid.

The CA is an iterative process, computing the development based on potential-for-development and calculating new potential based on the previous iteration development.

The Potential for Development (PFD) is initially set by the fuzzy system. The PFD is represented as a stack of continuous maps, one for each consumer type, representing the potential growth in number of consumers per stage and per geographic unit (e.g. 20 domestic consumers per stage and per km²). The output of the CA is the Development and represents the effective number of consumer growth. A global geographical trend controls the global development that is the sum of all developments in the region. The CA process finishes when the sum of all cell developments reaches the global trended value (e.g. the growth for year 2001 in the whole region tends to 250 industrial consumers and 5000 domestic consumers).

The iterative process of the CA is based on state transitions S[t]; in our model, these will be transitions from non-developed to developed. The state transition is done according to a set of rules such as

\[ \text{if } P_i(t) > P_k(t) \text{ then } S_i(t) = 1 \text{ else } S_i(t) = 0 \quad (8) \]

In our model, a transition exists if the cell has a PFD value \( P_i(t) \) higher than a specified boundary value \( P_b(t) \). This value is specified by the system by ranking PFD intervals.

The development \( D_i(t) \) is recalculated in each iteration incrementing the number of consumers, by steps \( D_{\text{step}} \) (measured in number of consumers), only on cells marked as developed, with \( S_i(t) = 1 \).

\[ D_i(t) = D_i(t-1) + S_i(t) \cdot D_{\text{step}} \quad (9) \]

The new potential \( P_i(t+1) \) is recalculated based on three components:

- positive feedback of the cell on the previous iteration, weighted by \( \alpha \)
- neighborhood effect based on the 8 adjacent neighborhoods [17], weighted by \( \beta \)
- innovation factor modeled as random noise, weighted by \( \lambda \); and is given at time \( t+1 \) by

\[ P_i(t+1) = \alpha \cdot P_i(t) + \beta \cdot \frac{1}{8} \sum_{j \in B_i} P_j(t) + \lambda \cdot \xi_i(t) \quad (10) \]

where \( \alpha, \beta, \lambda \) and \( \Omega \) are the weights for each component, with values \([0,1]\) and \( \alpha + \beta + \lambda = 1 \), and \( \Omega \) is the set of adjacent neighbors. \( P_i(t) \) is the updated potential to development in time stage \( t \) on site \( i \), computed based on the output of the fuzzy inference model \( P_i(0) \) and on the development computed by the CA on iteration \( t \):

\[ P_i(t+1) = P_i(t) \cdot D_i(t) \quad (11) \]

At the end of each stage the PFD maps may be recalculated, using the fuzzy inference model and the new geographic data computed with the CA or introduced by the planner.

V. SCENARIO IDENTIFICATION PROCESS

The scenario identification process helps decision-makers to structure uncertainties identifying the events leading to different plans for the forecasting environment. Once these events identified, is necessary to verify correlations among them: this will eliminate many unrealistic scenarios resulting from the spatial and temporal combination of the events.

Because of their nature, simple forecasting models are incapable of modeling uncertainties by exploring the logic of how events occur. Scenario planning allows the decision-maker to define alternative environments for which decisions are taken.

A scenario identification process escapes to the rules of a systematic processing. The scenario must be identified by the planner, providing a consistent and coherent alternative for the possible futures. The Fuzzy SLF has a Scenario Coordinator module, which allows the planner to identify scenarios and to interact with the system along the several stages.

To help the identification of scenarios we may suggest the following three steps:

- Identify the uncertainty factors and events that influence the decision environment and analyze the logic between events.
- Develop scenarios that are consistent visions of the future. The alternative futures must be credibly co-ordinating events with very different outcomes.
- Ensure that events with identified uncertainty are critical for the decision. Due to the multiplicity of scenarios resulting from the spatial and temporal combination of events, it is important to limit our analysis only to the combinations of events that critically affect decisions and their performance.

In the Fuzzy SLF, most of the data is spatial data resulting from urban plans (road network, land use, etc.). The plans for urban development may result from the combination of the collaborative planning between several planning sectors as economic development, environment, utilities, etc.

VI. EXAMPLE

In this section the paper illustrates the application and results of the Fuzzy SLF and the scenario identification process. In this example we study the forecasting of domestic consumption in the island of Santiago in Cabo Verde (Africa). For each scenario we aim to forecast consumption growth along seven stages, which were defined for illustration purposes and cannot be seen as reflecting the actual situation in the region.

The geographical inputs (influence factors) considered are the following:

- Distance to main urban centers (4 linguistic labels)
- Distance to secondary urban centers (4 linguistic labels)
- Saturation Level (6 linguistic labels)
- Distance to roads (5 linguistic labels)
- Elevation (3 linguistic labels)
- Terrain slope (4 linguistic labels)

Linguistic labels associated with fuzzy membership functions reclassify the influence factor values (e.g., distance to roads between 0 and 2 km: VERY CLOSE; distance to roads between 1 and 3 km: MODERATE CLOSE).

The study region has 2400 km² including one main urban center and three secondary centers. The resolution on GIS spatial analysis was 250m which represents cell based maps with 38400 cells. The historical growth is based on the geographical building growth along the last 30 years. As training results we obtained 2500 rules completed with 146 rules defined by expert knowledge.

To identify the scenarios we first identified the factors that may affect the development.

We assume that the development of new urban centers is not credible; however, we admit that the areas defined as urban centers are related with saturation level. We do not consider these influence factors to identify scenarios, but in each stage we reclassify the urban centers and new distances to the centers will be recalculated.

In the same way, the saturation level is recalculated in each stage as a result of the Fuzzy SLF but this presents small differences and only a relative little influence in building multiple scenarios.

The influence factor "Distance to Roads" is an uncertainty factor especially adequate to formulate scenarios based on events. As for the construction of new roads, we admit the two possible events in this example: 1) constructing a new ring road around the main urban centre; 2) constructing a road along the south coast near the city. Once the location of the new features defined, one must define the timing of their construction. Because of insufficient economic resources only one of the events may happen in the first 6 stages.

The global trending, which is the forecasting target in each stage for the whole island, is another uncertainty factor that may be considered for identifying scenarios. Econometric analysis supplies us two possible global growth scenarios:

Figure 4 — Evolution of the number of consumers for scenario 2, along several stages. In this stage the Ring road was constructed on stage 2 and the coast road was constructed on stage 6.
1) scenario with gradual growth [1000, 1500, 2000, 2500, 3000, 3500, 4000]; 2) scenario with sigmoid growth [1000, 1200, 1800, 2500, 3200, 3800, 4000]. – see Figure 5.

Based on the referred events we identified several credible scenarios, for which we suspected of critical effect on decisions. In this work we identify, for illustration purposes, the following three scenarios:

Scenario 1
Global trending with gradual growth; coastal road built in stage 2; ring road built in stage 6

Scenario 2
Global trending with gradual growth; ring road built

in stage 2; coastal road built in stage 6

Scenario 3
Global trending with sigmoid growth; ring road built in stage 2; coastal road built in stage 6

In Figure 4 we can observe the evolution of the growth of number of consumers for scenario 2, along the several stages. We didn't present images of the other two scenarios because the map results are too similar and indistinguishable for the quality of image presentation, possible in the paper.

We observed several differences on map results among the three scenarios. In order to analyze these differences we computed comparison maps between scenario 1 and scenario 2 and between scenario 2 and scenario 3. In both comparisons, the differences range between −10% and +10%. The comparison between scenarios 1 and scenario 2, related with spatial uncertainties for road construction, displays consequences on the spatial pattern for the development. On other hand, the comparison between scenario 2 and scenario 3, related with the global trending scenario, displays consequences on the magnitude of the development.

Figure 6 – Comparison maps between developments for scenario 1 (Scn1) and scenario 2 (Scn2). Dark zones represent locations with higher development for scenario 2 and light zones represent locations with higher development for scenario 1. The new roads constructed in each scenario become one attraction factor for development, displacing consumers from other zones. The white arrows represent the consumer displacement influenced by the “ring road” and the black arrows represent the displacement influenced by the “coastal road”
A. Comparing scenario 1 and scenario 2

To study the effects of the uncertainties resulting from spatial events (constructing new roads) we compare scenario 1 and scenario 2 by computing the spatial differences between maps of development. Scenario 1 represents the influence of constructing the "coastal road" and scenario 2 represents the influence of constructing the "ring road". At stage 6, both scenarios have the two roads built, but as we will see the different timings for constructing the roads will influence significantly the spatial patterns for future developments.

In Figure 6 illustrates the influences of the events of each scenario. In stage 3 the scenario 2 indicates higher development near the ring road, constructed in stage 2, on sites quite far from the existent roads where the new road improves the accessibility (zones A, I and B). On scenario 1 the "coastal road" influences higher development in zones H and G. Note that zones C, F and D show development values higher for scenario 2 than for scenario 1. However this difference is not caused by any positive influence of the "ring road". The results are influenced by the negative effect of the coastal road, moving consumers from zones C, F and D to zones E, G and H. This effect is caused by the small elasticity of the global trending. If the development increases in a specific zone the fixed value for the global trending forces the decreasing in other zones with lower potential for development.

In stage 5 the areas near the "ring road" saturate and the adjacent areas with slower development (zones A and B) register the higher positive influence from the "ring road". It is interesting to observe that the increase in development in scenario 2 attenuates the development in zones C, N and M. For scenario 1, the coastal road continues to influence positively the development in zones G and H by displacing consumers from zones O, L and F.

On stage 7 we continue to observe the influence of the coastal road. However, due to the faster growth, near the "ring road" the positive influence is decreasing, this fact can be observed on the lower intensity of zones A and B. It is worth to notice the higher development in zone Q; the "ring road" doesn't influence directly this zone, but the higher development in the Northeast zone of the urban center, caused by the "ring road", influences positively the development in this area.

From this analysis we conclude that influence of discrete spatial and temporal events may originate very complex changes in the spatial load forecasting, and consequently the distribution planning. We also conclude that these events may affect drastically the directions for future development.

![Figure 7](image)

Figure 7 - a) map of saturation levels: comparison between scenario 2 and scenario 3, dark zones corresponding to areas where saturation in scenario 3 is higher than in scenario 2, and light zones corresponding to the inverse situation. b) chart representing, for the two scenarios, the saturation curves at 4 different points located on the map.

B. Comparing scenario 2 and scenario 3

The global trending differentiates scenario 2 and 3. In scenario 2 a uniform growth and in scenario 3 a sigmoid growth was used as illustrated on Figure 6.

The sum of the global trending along the 7 stages is the same for the two scenarios (175000 new consumers). The difference between the two scenarios is that on scenario 3 the global growth is more concentrated in the last three stages.

In order to compare the effects on the development caused by the two scenarios we analyze the differences among the saturation levels. The results from the comparison are presented on Figure 5. Due to the lack of space we only present a comparison map for the last stage. By observing the several maps we concluded that higher global trending intensify the growth on areas with higher potential.

On Figure 7 we can observe a higher development, for scenario 3, represented by the dark zones, on the areas of higher-potential corresponding to stages 5 and 6.

The chart in Figure 7 shows the evolution of the development in four different sites. The sites with faster development are sited on areas close to the urban center.
A>B>C>D. We may notice that in scenario 3 the saturation curves have a relatively higher growth in last stages influenced by similar behavior of the global trend. However the effect of different global trendings is different in different places as observed by comparing curves C and D.

VII. CONCLUSION

In the first part of this paper a Fuzzy SLF model, capable of capturing geographic behavior, expert knowledge and to store it as a set of fuzzy rules, was presented. The knowledge information stored as rules may be applied in other time-space scenarios which similar consumer growth behavior. The model was designed to deal with spatial influence factors and was completely implemented into a Geographic Information System.

In Spatial Analysis many kinds of uncertainties exist and their effects may be very difficult to model. These uncertainties may be modeled as small deviations from the forecast result or may be a planning scenario. In this paper we discussed the scenario identification process. Most of the uncertainty inputs for Spatial Load Forecasting result from non-repetitive events and urban plans, these kinds of uncertainties must be structured as scenarios requiring the active participation of the Decision-Maker.

We also explained the special ability of the Fuzzy SLF to structure these kinds of uncertainties by interaction with the Decision-Maker.

In a simple example of scenarios we observed the complexity of the SLF problem and the very different consequences resulting from quite different plans on urban planing and global demographic forecast. This proves beyond doubt how useful a process like Fuzzy SLF is, for Distribution Planning.

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Uncertainty Propagation in a Fuzzy Spatial Load Forecasting Model

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Abstract – This paper presents a framework to model uncertainties and error propagation in Spatial Load Forecasting (SLF). The proposed approach, developed by the authors, is fully discussed either in what concerns the procedure of inclusion of the uncertainties in the SLF model and their propagation through the time stage simulation. In the end relevant information can be extracted to help the planner.

Keywords – Spatial Load Forecasting, Geographic Information Systems, Fuzzy Inference, Cellular Automata, Uncertainty modeling.

I. INTRODUCTION

In the last decade, the planning engineers identified, in power system planning environment, a vast range of uncertainties, which in some cases have been incorporated in their models. The traditional approaches include uncertainties related with equipment failures, loads, costs and uncertainties related with unique events or fundamental options. The uncertainties to be considered are basically of spatial and temporal origin or from the models themselves.

The authors have worked on including high geographic realism in distribution planning models. The technologic target to reach this geographic realism is the integration of planning models with Geographic Information System (GIS). We call Spatial Distribution Planning (SDP) the GIS spatial models used as tools for Automated Distribution Planning. The departure point for Spatial Distribution Planning is Spatial Load Forecasting (SLF), modeling the geographical growth of power consumption. Land-use Spatial Load Forecasting simulation methods have been largely used to model the process of the load growth in order to predict load evolution is a spatial and temporal basis [1].

The authors developed a SLF model joining the characteristics of Fuzzy Inference System and Cellular Automata, firstly presented in [2]. The Fuzzy Inference Model captures the geographical pattern of influence factors, estimating the potential for development, and the cellular automata model the dynamics and spreading process over the geographical region, predicting consumer growth. In a further work, the authors improved the model by transferring the dynamics of the forecasting model from the Cellular Automata (CA) to the Fuzzy Inference Model [3]. In this last improvement the saturation levels (consumer/km²) of each consumer class are considered as input variables of the fuzzy system, and the CA model simulates consumer spreading over the geographical region.

In the present paper we describe a method to model uncertainties in the Fuzzy Spatial Load Forecasting model. We will begin by identifying the uncertainties in the model and their importance throughout the process. Finally the paper presents the model for uncertainty propagation over different scenario time-stages. As a result, the model outputs fuzzy maps for consumer growth. These results can be used as basis for uncertainty modeling in Spatial Distribution Planning.

II. FUZZY SPATIAL LOAD FORECASTING

The Fuzzy SLF is a complex spatial analysis model composed by several modules illustrated on Figure 1. A more detailed description of the Fuzzy SLF may be found in another paper [4]. Three main modules in that Figure 1 have been completely implemented into a GIS environment and compose the Fuzzy SLF.

![Figure 1 - Three kernel modules (Fuzzy System, Cellular Automata, and Scenario Coordinator) compose the Fuzzy SLF model. This kernel must be coupled to other modules that can be designed independently.](image)

The Fuzzy System uses spatial influence factors to compute maps of Potential-for-Development – P/D (as number of consumers). The influence factors are geographical coverage, some of them specified by the user (e.g. distance to roads, terrain slope, etc.) and other resulting from previous iterations of the Fuzzy SLF (e.g. saturation level, distance to urban centers). The result of this module is the P/D, which is a continuous map, ranking the potential for development in each location.

In a small area basis, the load growth follows an S curve pattern representing the dormant period followed by a rapid growth period rapidly reaching saturation, a period of small growth. This S curve (Figure 2) or Gompertz curve is typical of distribution and becomes sharper as one subdivides the study area into more and more small areas. So, each point of a map is associated with saturation curve $S_i$ for each type $i$ of consumers and describes the number of consumers of a given type, at a certain location and function of time. The slope at each point of the curve is the P/D.

In our approach, we do not define a priori any saturation curve. Instead, the shape of the saturation curve is built dynamically following the pattern implicit in thousands of fuzzy rules that compose the Fuzzy System.

The kernel of the Fuzzy System module is based on a set of fuzzy rules, built automatically by the fuzzy inference system or by direct specification of expert knowledge.
The Fuzzy system may be represented formally by the equation 1.

$$\hat{S}_t = f\left(S_{\text{low}}, \ldots, S_{\text{high}}, I_1, \ldots, I_n \right)$$  \hspace{1cm} (1)

where $\hat{S}$ represents the output of the model (PID for consumer class c) and is the derivative of the saturation curve on stage $t$. The $S_{i-1}$ represents the saturation levels for each consumer class at stage $t-1$, and the $I_i$ represents other geographic influence factors.

The Cellular Automata uses the map of preferential PFD and spreads a global number of consumers over all the regions. The output of this module is the effective development $D_{i-1}$ in each stage $t$. The CA module may be formally represented by the following function.

$$D_{i-1} = g\left(S_{i-1}, G_i, \alpha, \beta, \lambda \right)$$  \hspace{1cm} (2)

where $D_{i-1}$ represents the output of the model, $G_i$ is the global trending for stage $t$ and $\alpha, \beta, \lambda$ are the CA tuning parameters representing weigh factors for feedback, neighborhood and innovation effects respectively [4].

The Scenario Coordinator controls the dynamics of the model and allows the interaction with the planner by computing the values to be used as input for the next time stage. The most important input for the dynamics of the growth process is the saturation level. The Scenario Coordinator recalculates this saturation level (3) based on the development or the sum of developments obtained as outputs on all the antecedent stages.

$$S_{i-1} = S_{i-0} + \sum_{i=0}^{t-1} D_i$$  \hspace{1cm} (3)

Observing the huge amount of inputs, outputs and the process of the Fuzzy SLF, we may expect a large number of uncertainties, some of them of spatial type others resulting from non-geographical values.

The uncertainties in Fuzzy SLF may be structured in two different ways. One of them is consists on a scenario identification process, requiring the participation of the decision-maker to structure several possible and very different futures. This approach implicaes running several forecasts and is adequate to model non-repetitive events with drastically different consequences. This process is analyzed in detail in [4].

The second approach consists on modeling the uncertainty propagation inside the Fuzzy SLF. This approach is adequate to model small deviations from the forecasted values.

III. MODELING UNCERTAINTIES IN FUZZY SLF

Modeling uncertainties in spatial models follows three main steps. The first step consists in identifying for each data layer which kind of uncertainty exists and can be extracted from the data sources. The second step consists in the definition of the conceptual model for the error (this depends on the kind of error identified in the first step). Finally the third step consists in the definition of an appropriate model for the uncertainty propagation.

As explained before, the Fuzzy Spatial Load Forecasting model is composed by two main compact modules (Fuzzy System; Cellular Automata), and by one model that manages the time dynamics (Scenario Coordinator), repeating the process through the multiple time-stages.

A Sources of uncertainty in the Fuzzy Inference System

The inputs of the fuzzy system are the spatial influence factors and the output is the Potential for Development (PFD). The significant influence factors can vary depending on the particular study region environment. We will discuss some possible kind of thematic influence factors in the perspective of the different types of spatial uncertainties.

The Fuzzy system modeling is a technique that can be used to simplify the representation of complex non-linear relationships between the variables and consequently between their uncertainty. In this section we will try to understand the relationship between these uncertainties and their propagation through the Fuzzy SLF modules.

1) Uncertainties in Distance to Features

Distance to features is one of the most common influence factors; examples are: the distance to roads or the distance to urban centers. This kind of uncertainty has its source on positional uncertainties. In the point of view of distance functions, the positional uncertainties are related with the uncertainty on the position of the feature (road or urban center). Another source of uncertainty is the insufficient resolution used in the spatial analysis.

The positional uncertainty is always higher than the analysis resolution, because is impossible to store or retrieve information which falls between pixels. If we are using an analysis resolution of 250 meters, the uncertainty in distance may be represented by a uniform distribution with values between ~125m and 125m around the pixel center.

The uncertainties in position for the feature vector depend on the source error and on the inaccuracy of observation on data gathering, and at the same time are a consequence of the conversions between data structures (vector to raster). For instance, if we use maps with resolution of 1:100000 scale the error will be higher than 100m. If the conversion from vector to raster is necessary, the uncertainty will be at least equal to the resolution of the conversion. Based on maps of probabilities for the existence of the feature in a specific site, it is possible to compute the distribution function from each
location to the geographic feature [5]. However this may be a
difficult process, only possible by using simulation methods.

When we try to compute the distance to an urban center
the following question arises: "what is and where is the urban
center?". First we must define what we consider urban center;
we can admit that urban centers are areas with saturation level
higher than a specified value $s_{center}$. If a stochastic model for
the saturation level exists, it is possible to model the
uncertainties in distance-to-center. Another way to model
uncertainties in distance-to-centers is by introducing random
noise on $s_{center}$; this approach will be used, in this work, for
structuring this kind of uncertainties in a simulation method.

2) Uncertainties in Slope and Elevation

Uncertainties related with Digital Elevation Models
(DEM) exemplify a kind of uncertainty related with the
density of information and resolution. If the digital terrain
model has been built based on higher density of features per
area, the uncertainty will be lower. One way to reduce the
uncertainty in slope and elevation is the use of a higher
resolution when we build the DEM. The modeling of
uncertainties for spatial algorithms, used to interpolate data in
order to build the DEM, may be difficult depending on the
complexity of the algorithm.

The slope uncertainty can be computed as function of the
uncertainties of the DEM. The random error field may be
computed by the spatial auto-correlation of the
neighborhood height around the location. The probability
distribution function may be also obtained by simulation
methods [6].

3) Uncertainties in Saturation level

The uncertainty related with the saturation level can be
classified as temporal uncertainties. In the input of the Fuzzy
System, this uncertainty is the accumulation of uncertainties
resulting from the Fuzzy SLF obtained from the previous
time-stages. Because in our model the derivative of the
saturation curve represents the PIF, which is the result of the
Fuzzy System, the uncertainty in the saturation level will
cause different levels of uncertainty on PIF. This aspect is
particularly important when the curve changes from the
dormant state to the growing state, because this point defines
the trigger time for the development in the vacant area.

4) Uncertainties in Land Use

Land use is an example of a thematic uncertainty.
Thematic uncertainty refers to uncertainties in the
classification of a specific feature in a correct thematic class.
There are several models and techniques to model thematic
uncertainties [7, 8]. If there is enough repetitive information
(e.g. multiple satellite imagery) a probabilistic representation
can be used.

5) Sources of uncertainty in Cellular Automata

The Cellular Automata (CA) uses the PIF as input, which
results from the Fuzzy System module and the global trending
values. As referred previously, the global trending results
from an external module that forecasts the consumer growth
in the overall area. The methods used for global forecasting
may be based on extrapolation models, multivariate models,
econometric models, rule-based forecasting, or expert
systems [9]. Equation 2 expresses the relation of the CA
modules and their inputs.

6) Uncertainties in Potential for Development

As discussed previously, the uncertainties in PIF are a
consequence of the multiple uncertainties in influence factors
and uncertainties in saturation levels. The potential for
development is a continuous map used by the CA for an
evaluation and ranking of the most attractive locations for
development. When uncertainty exists, it is difficult to claim
in absolute that a site has a higher potential for development
than another does. The uncertainty in ranking PIF will lead to
uncertainty in the development computed by the CA module.

7) Uncertainties in global trending values

The global trending uncertainties are not a spatial
uncertainty, they are uncertainty values for an entire region.
The global trending value is used by the CA module to stop
the iterative process of spreading consumers over the
geographical region. The uncertainty in this value can be
modeled as a probability distribution function or as a fuzzy
number. The way the uncertainty model is built depends on
the global forecasting module, which is out of scope of this
paper. In this work we only consider a set of distribution
functions for the global growth in each stage.

8) Scenario Coordinator uncertainties

For each time stage the process is repeated and new
saturation levels are calculated associated with the
correspondent uncertainty. As expressed by Equation 3 this
map of saturation-level is the sum of the development found
for each antecedent stage added to the initial saturation level.
The uncertainty associated with the development in each
stage is progressively accumulated and summed to the
uncertainty of the actual saturation level.

IV. MODELING UNCERTAINTY PROPAGATION

In this section we will model uncertainty propagation
throughout the Fuzzy SLF process. There are two techniques
for examining uncertainty propagation, the simulation method
and formal methods.

Formal error propagation uses explicit mathematical
models describing the uncertainty propagation on GIS spatial
functions. If the mathematics modeling is possible, the formal
approaches are interesting due to their easy applicability.
Unfortunately, the majority of the functions in spatial analysis
are too complex for using formal models. The use of formal
propagation techniques in Fuzzy SLF is very difficult because
the modules are composed by a large set of spatial GIS
functions, the majority of them with unknown uncertainty
propagation mechanisms.

The simulation methods are the alternative for models like
Fuzzy SLF where the formal methods are not applicable. The
main advantage of the simulation techniques is their
applicability independently of the functions or set of
functions involved in the module. A disadvantage is the
computational load, which makes the approach unattractive.
A Case Study for Fuzzy SLF

In this work we used simulation methods to study the propagation of uncertainties in Fuzzy SLF, using as basis a hypothetical example for domestic consumer growth for Santiago Island in Cabo Verde (Africa), with an area of 4000 km² covering urban and rural areas. We will focus our analysis on the region of the main urban center.

The input variables considered in the case study are:
- distance to roads
- distance to main urban center
- distance to secondary urban centers
- slope
- saturation level.

Five linguistic labels were used to reclassify each input variable (very small, small, medium, high, very high). The analysis resolution used was 250 meters.

The fuzzy inference model was trained based on historical values, using approximately 67000 training points. As training result we obtained 2500 rules, completed with 146 rules defined by expert knowledge.

To study the uncertainty propagation we simulated uncertainty in several input variables, repeating the model 30 times per stage. For each stage we obtained the distribution function of the Saturation, PI and Development.

In this simulation we modeled uncertainties in distance to urban centers, in saturation level, in DEM and in global growth.

As explained before we assumed that the urban center is an area with saturation level higher than a specified value $S_{cenr}$. To simulate uncertainty in distance to urban center we modeled a random value for $S_{cenr}$ with uniform distribution in [75; 85]. For each simulation a new urban center area is defined by reclassifying the saturation values higher than $S_{cenr}$. In a second step, based on reclassified areas, a new map of distance to urban center is computed and used as input to the Fuzzy SLF model.

To model uncertainty in DEM a normal distribution function was used, having as mean the value at the central cell and with standard deviation the mean deviation between value on central cell and the eight neighborhood cells. The slope map and correspondent uncertainty is computed in each simulation based on the DEM generated with random noise.

The uncertainty in saturation level is modeled as the sum of developments in all previous stages with the saturation level for the initial stage. The Development map for each stage is modeled as a normal distribution function where the stochastic modeling results from the antecedent stages. For the initial saturation level, a random noise was added modeled by a normal distribution with mean value 0 and standard deviation 10% of the saturation level.

In this work we simulated seven stages with the following global growth [1000; 1500; 2000; 2500; 3000; 3500; 4000]; the uncertainties in global growth were modeled by adding random noise with mean 0 and standard deviation 20% of the global growth. The evolution throughout the stages for the mean value of the growth can be observed on Figure 4. The pattern of the evolution follows the behavior implicit in the fuzzy rules, with higher growth near roads and urban centers, close to the coastline and on sites with tolerable slope.

![Image showing the evolution of the saturation level over stages](image)

<table>
<thead>
<tr>
<th>Site</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
<th>Stage 5</th>
<th>Stage 6</th>
<th>Stage 7</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
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<td>$\mu$</td>
<td>$\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
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<tr>
<td>A</td>
<td>42.42</td>
<td>4.09</td>
<td>83.98</td>
<td>5.58</td>
<td>92.23</td>
<td>4.70</td>
<td>91.20</td>
</tr>
<tr>
<td>B</td>
<td>16.30</td>
<td>2.01</td>
<td>37.03</td>
<td>1.48</td>
<td>68.47</td>
<td>3.59</td>
<td>88.36</td>
</tr>
<tr>
<td>C</td>
<td>0.84</td>
<td>1.16</td>
<td>15.01</td>
<td>1.34</td>
<td>39.94</td>
<td>0.95</td>
<td>79.83</td>
</tr>
<tr>
<td>D</td>
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<td>0.63</td>
<td>9.21</td>
<td>1.34</td>
<td>22.92</td>
<td>1.56</td>
<td>47.21</td>
</tr>
<tr>
<td>E</td>
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<td>4.39</td>
<td>1.16</td>
<td>15.19</td>
<td>1.51</td>
<td>33.55</td>
</tr>
<tr>
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<td>0.55</td>
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<td>0.91</td>
<td>6.12</td>
<td>1.31</td>
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</tr>
<tr>
<td>G</td>
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<td>0.90</td>
<td>6.51</td>
<td>1.26</td>
<td>10.68</td>
</tr>
</tbody>
</table>

Figure 3 - Some results of the simulation method for several sites. The map shows the observed sites. The chart shows the saturation curves in each site.

The table shows the correspondent mean and standard deviation.
saturation level (90-100); for this range the growth stops and consequently the uncertainties associated decrease.

The uncertainty of the forecasting is obviously dependent on the uncertainty in input data. However, in this work we are especially interested in uncertainty propagation through the models.

![Figure 6](image)

**Figure 6** – Evolution of the standard deviation of the saturation level along the seven time-stages.

The Fuzzy SLF model is an iterative model using the results of the previous time-stage to compute the following stage development. This structure rises the question: is the uncertainty amplified or attenuated along the time stages? Observing the evolution of the standard deviation along the seven stages we can see (in Figure 6) that the uncertainties decrease when forecasting for larger number of stages. This fact is a consequence of the sum of the stochastic models of the development in several stages (eq. 3). The sum of several stochastic independent variables (development in each stage) is the convolution of their density functions; in this case the result of the convolution with the stochastic variable of a new development stage leads to a distribution function with lower variance. In practice the model forecasts with lower uncertainty when we are predicting for a higher number of stages. For instance, if we are forecasting for 2 stages the growth may happen in the first or in the second stage, which implicates higher uncertainty when forecasting one stage than when forecasting the global growth for the two stages.

As discussed before, the PID can be interpreted as the derivative of the saturation curve $S$; consequently, one expects higher PID for the development phase (saturation level between 25-75) and lower values on the dormant phase (saturation level between 0-25) or in the saturated phase (saturation level between 75-100). This expectation was confirmed and can be observed on the mean value of PID and of Development as function of the several saturation levels (see Figure 7).

![Figure 7](image)

**Figure 7** – Mean value of the PID and Development as function of the saturation level.

As expected, the Development follows the pattern of the

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**Figure 4** – Maps of mean growth for the six time-stages.

We can observe in Figure 3 the evolution of the saturation level, along the seven time stages, at different locations. The shape of the saturation curves results from the dynamic behavior implicit in the fuzzy rules. This shape may be different from site to site, but in general it follows an S shape with a dormant phase, and development phase and a saturation phase. The development is faster on preferable sites; in Figure 3 we can observe faster developments near the urban center (A>B>C>D>E>F).

On the table of Figure 3 we may observe the stochastic results of the simulation (30 simulations per stage), the mean value $\mu$ and the standard deviation $\sigma$ By comparing the $\sigma$ for the several points we can see that uncertainty varies from site to site, but in general higher uncertainties are associated with locations with faster development.

Classifying the location by saturation level (for all stages) we observed that the standard deviation of the saturation value grows proportionally to its mean saturation level. In Figure 5 we observe that standard deviation is approximately 3% of the saturation level.

![Figure 5](image)

**Figure 5** – Relation between standard deviation and mean for saturation level.

One exception is when the growth reaches the maximal
PID, but in this case with mean values lower than the PID.

In Figure 9 we may observe the standard deviation for PID and Development. The Development has higher uncertainties than the PID because the stochastic variable “global growth” is introduced directly in the CA module (see Figure 1).

Contrary to the Saturation Level, the standard deviations for PID and for Development are not proportional to their magnitude. In Figure 9 one may observe a considerable rising of uncertainty corresponding to the change from dormant phase to the development phase of the saturation curve; along the growth phase, the growth of uncertainty is quite slow; and at the saturation phase the uncertainty decreases.

![Figure 9 - Standard deviation for PID and Development as function of the saturation level.](image)

On Figure 8 we may observe the spatial pattern for the uncertainty associated with the saturation level along the several stages. Along the stages the uncertainty in saturation growth is proportional to the development, decreasing when the saturation reaches maximum value (observe the zones A and D). Another important observation is that uncertainties are higher when for higher slopes; this may be clearly observed at zones B and C. We observed that all the indicated zones (A, B, C, D) reach saturation levels very close to the maximum for stage 7. However in some of these zones (B, C) the uncertainties are still high even when the location is saturated.

V. EXTRACTING UNCERTAINTIES

The SLF model generates the basis data for several network designers used in Distribution Planning. The load scenarios, describing maps of forecasted values along time stages, are the base data for the following expansion planning modules:

- secondary network routing
- substation siting
- substation service area optimization
- routing for lines interconnecting substations

The main objective of these modules is the pre-processing of geographical data in order to obtain a set of robust feature options to be used in automated planning tools. These options are basically electric facility sizing and location (e.g. sizing for lines and substations, routing for lines and geographical siting for substations). Facility location may be highly affected by demand uncertainty and its geographical distribution.
These modules build both a vectorial topology (graphs and nodes) and related databases to store attributes associated with each option. In fact, the procedure allows the extraction of information from the geo-environment and its storage as attributes of the network features.

Figure 10 - a) Development (consumers/pixel) for stage 5. b) Developed area with level of confidence 95.0%, 99.0%, and 99.9%.

These modules are not within the scope of this paper; however, we will refer to them to explain how the uncertainty maps of SLF may be useful and how we can extract these uncertainties from models that use SLF results.

The network design modules use raster load maps to compute vectorial results such as the location of loads and substations as points and electric lines as sets of line segments. The stochastic modeling may be extracted from the raster forecast maps and stored in a vector database; this way, each load point will have as attributes the mean value and the standard deviation. These parameters may be propagated over the network design module in order to obtain stochastic modeling for power on lines and substations.

In some situations, uncertainty in load affects the topology of the network design. When vacant areas are developed, new lines must be built to supply this load. If we admit erroneously that some load exist (type II error) this may implicate high consequences on planning because building new lines for vacant areas have in general high costs. If we admit erroneously that load doesn't exist (type 1 error) the consequences may be smaller because investment may be delayed but appear in following stages.

To hedge for type II errors, it is recommendable to increase the confidence in load development. To illustrate this, we present in Figure 10a) the mean value for development for stage 5 (the number of new consumers per 250m pixel varies between 0 and 6). We may observe that development may happen at a large number of locations, which is very important if the network happens to be designed to supply locations which higher development uncertainty, like vacant areas.

In Figure 10b) we mapped the developed locations for 3 different confidence levels 90.0%, 99.0%, and 99.9%. At higher confidence levels the developed area shrinks, restricting the number of developed areas. This shrinking effect is more relevant on areas with higher uncertainty.

Figure 11 - Example of output from the network design modules. The graph represents the meshed network resulting from several radial solutions. Substations are identified as large black dots and loads are represented by small black dots.

Similarly to the way we compute the confidence level one may compute the probability of existence of a predefined level of load. These probabilities may be propagated to the lines and substations leading to topological uncertainties in the results of the network design.

VI. CONCLUSION

The SLF model produces load maps which are the basis for Distribution Planning. In this paper we have shown that there are many sources of uncertainty, most of them spatial uncertainties, requiring special modeling. The uncertainties affecting the SLF may be propagated to the forecasted load maps and consequently to the Distribution Planning process.

The complexity of Fuzzy SLF models requires the use of simulation methods to model the uncertainty propagation. In this paper, simulation methods have been used to evaluate the error propagation. Different kinds of uncertainties have been modeled and the uncertainties in several outputs have been analyzed. From the simulation result we observe that:

- The uncertainty in consumption growth varies from
site to site, but in general this uncertainty is higher for sites with faster development.

- Uncertainty in consumption growth is proportional to the saturation level, but when a site reaches the maximal saturation the uncertainty in forecasting decreases significantly.
- The uncertainty propagation in Fuzzy SLF decreases with the increase in the number of time-stages. The uncertainty in spatial influence factors influences the uncertainties in location for development while the uncertainty in global forecasting affects the magnitude of the development.
- Concerning the development in a specific stage we conclude that the uncertainties are particularly important when the saturation curve changes from the dormant phase to the development phase and from the development phase to the saturation phase.

Two ways of structuring uncertainties may be used for Fuzzy SLF, the first is the Scenario Planning for which the planner identifies several contrasting scenarios. The second approach is an internal modeling of small deviations from the forecasted value.

This paper presented a development using the last approach. It reported studies on the propagation of several spatial uncertainties through the model and on the process of projecting forecast uncertainties to network design models. We concluded that a variety of types of spatial uncertainties exist and may have important implications in forecasting. The uncertainties in loads may be simply extracted and transferred to load points vector structure, electric lines and substations. The effects of these uncertainties on the network design, especially when load uncertainties are transferred to the topological uncertainties of the network, must be dealt with carefully.

REFERENCES

MERGING JUDGMENTAL AND STATISTICAL INFORMATION TO BUILD SPATIAL LOAD FORECASTING SYSTEM KNOWLEDGE BASES

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Abstract – The forecasting methods are based on two types of knowledge sources, the statistical information, extracted from historical behavior and the judgmental information, extracted from human expert knowledge. For Spatial Load Forecasting (SLF) it is essential the merge of statistical and judgmental information. This paper presents a novel SLF model based on Fuzzy Systems (FS) and implemented in a Geographical Information System. The paper is concentrated on the training and tuning of the FS rule base. This process is done in three steps: first, build up of the FS with statistical information; second, rule interpolation to represent innovation behavior; and third, rule base tuning with judgmental information.

Keywords. – Spatial Load Forecasting, Geographic Information Systems, Fuzzy Systems, Fuzzy Inference, Fuzzy Reasoning.

I. INTRODUCTION

Spatial Load Forecasting (SLF) methods predict where, when how much load growth will occur in a utility service area. This information is traditionally been used for expansion planning purposes to ensure that the distribution system will be able to supply the load[1][2].

These simulation methods have been used to model the process of load growth in order to predict load evolution in a spatial and temporal basis. The SLF methodology is particularly suited to high spatial resolution for long range forecasting and multi-scenario planning.

In general the SLF methods are implemented in Geographical Information Systems (GIS). The GIS is ideal to support this kind of models because of its ability to manage spatial information; model and simulate the phenomena behavior; visualize data and simulation results; and establish the interaction between the planner and simulation environment.

The authors developed a Fuzzy Spatial Model (FSM), which uses a fuzzy system and cellular automata[3] to model the potential-for-development and the development respectively[4][5]. The fuzzy system simulates the spatial growth behavior and the cellular automata project a global growth value (number of consumers for overall region) into a small area basis (maps with the location of effective number of consumer at each place).

Fuzzy control or reasoning systems [6] are very adequate to model spatial growth behavior [7][8] because:

- It allows knowledge representation by linguistic concepts like “close to the road”, “location with high environment protection” or “medium saturation status for urban development”.

- It allows the knowledge representation by comprehensive rules, where cause and consequence are represented by If-Then fuzzy rules. Comprehensive rules allow better knowledge interaction between the system and the experts.

- Comprehensive knowledge bases stored as a rule base could be transferred to other space and time environments.

The first part of the paper presents the principles and structure of the Fuzzy Spatial Model, including one illustrative example of the simulation.

In the second part we will describe the process used to construct the rule base, based on statistical information.

The third part describes fuzzy-interpolation-rule methodologies used to fill in the blank spots of the knowledge base.

And finally, the fourth part describes the fuzzy system tuning process based on judgmental information, merged by fuzzy reasoning process.

II. FUZZY SPATIAL MODEL

Two main modules compose the Fuzzy Spatial Model. The first is the Fuzzy System (FS) that estimates the potential-for-development, which is an indicator of the suitability of likelihood of evolution.

The second is the Cellular Automaton (CA) that spreads an externally defined global trending over all the region9 based on the preferences indicated by the suitability maps. The results of the CA module are the effective geographic distribution of the development.

The Global Forecasting is an external module that could be implemented by many forecasting models (trending, econometric models, diffusion of innovation).

The Scenario Coordinator (SC) links the FSM with the forecasting environment and coordinates the dynamics of the simulation. The SC coordinates the inputs of the Fuzzy System and Cellular Automata along the several time stages.
The Fuzzy System stores the spatial knowledge base as a set of rules used to simulate spatial phenomena. These rules may be generated automatically from the historical observation of the spatial influence of geographical factors, or the planner may directly generate these rules in order to simulate a future behavior of the phenomena.

The Fuzzy System generates a continuous map of potential-for-development for the development of the phenomena. Different maps are generated for each time stage as a function of the spatial inputs (geographical coverage) stated by the scenario coordinator.

The FS knowledge base dictates the dynamic behavior along the time stages based on its knowledge base.

III. FUZZY SYSTEM STRUCTURE

Apparently a variety of structures could be used to implement the Fuzzy System. However the specific characteristics of the Fuzzy Spatial Model (FSM) condition the orientations for such structure.

Conjugating statistical with judgmental information is one of the important requirements for FSM. This demands high interpretability, by humans, of the membership functions of the antecedent and consequent parts of each rule. In order to maintain the interpretability of the input and output linguistic values and their associated membership functions, the space partitioning is determined by the user and, for antecedent memberships, it will not change with training. This means that only consequent parameters have to be identified with in the training process.

The FSM problem is characterized by a very large set of geographical cells (near a million cells per map); on the other hand the number of significant variables is quite limited (about 5 variables). This characteristic motivated us to an implementation based on GIS spatial analysis functions instead of a GIS coupling with external fuzzy system modules. This approach is more oriented for the optimization of the geographical analysis than for the optimization of the fuzzy system.

In GIS implementation, stacks of maps represent the membership functions of the fuzzy system. This fact motivates the use of memberships with the smallest number of parameters as possible (triangular membership functions).

The use of normal fuzzy partitions with triangular fuzzy numbers is the most appropriate because it minimizes the number of parameters to be stored and allows for the use of a partitioning by range.

When choosing the type of inference rules, between Sugeno and Mamdani models, it is necessary to weight the interpretability and the complexity of implementation. Two types of structure seem adequate for the modeling, the Mamdani type with consequent represented by normal partitions with triangular fuzzy numbers, and the zero-order Takagi-Sugeno type. In relation to interpretability, the two models have similar levels for inputs and outputs, but better rule interpretation for Mamdani type. In relation to the complexity in implementation, the zero-order Sugeno model is quite simple especially in defuzzification and training. The heavy computational effort motivates the choice of the zero-order Sugeno model.

The neuro-fuzzy structure of the zero-order fuzzy system is presented in Figure 2. The matching on fuzzy proposition \( x'_i = A_{i,j} \) is given by \( \alpha_{i,j} \), where \( x'_i \) is the numerical input for variable \( x_i \) and \( A_{i,j} \) is the membership labeled \( j \) on this variable. The support value for rule \( r_i \) is given by \( \beta_i \). The final output \( y' \) is the weight sum, where \( b_i \) is the zero-order function coefficient for rule \( r_i \) and \( N_i \) is the number of rules.

The implementation of this neuro-fuzzy system with GIS spatial functions is especially interesting because the operations are applied simultaneously at all the geographical cells. The implementation requires: maps with activated membership labels (two for each variable); maps with matching values \( \alpha_{i,j} \) (two for each variable); maps with coding for activated rule \( (2^N \) maps, where \( N \) is the number of variables); maps with the support values \( \beta_i \) (as much as the number of maps for coding activated rules); maps with lookup values \( b_i \) associated with support values \( \beta_i \) (as much as the maps with the support values). The rule coding is important to identify the rules and access the lookup tables containing the rule database. The coding of the rule \( r_i \) is done by \( Cod_i \) where \( L_{i,j} \) is the membership label for label \( j \) on variable \( X_i \), the \( N_{i,max} \) is the maximal number of membership labels for variables \( X_i \), and \( N_i \) is the number of variables.
IV. MERGING STATISTICAL INFORMATION

The historical data appear as input/output sets of maps. The inputs are the geographical coverages that represent the geographical influence factors (map with distance to roads; map with distance to urban centers; map with terrain slope; map with the saturation level, others). The outputs are the development observed in the correspondent stage (map with the number of new consumers). The training requires sequential adaptation along several historical period stages.

Because the input memberships are fixed, the training of the zero-order Sugeno fuzzy system consists only in the identification of the zero-order coefficients $b_i$ for each rule $r_i$. There are two kinds of training, local learning and global learning. For the global learning algorithms, the parameters of the model are identified using the whole training data set in a single algorithm operation. On the other hand local learning uses only the data set in the neighborhood of each rule for locally fitting the rule. Each point in the neighborhood is weighted according to its distance to the centroid of the rule.

The global learning is appropriated when the system has important global behavior that should not be influenced by local discrepancies. Local learning is in general a simple model, decomposed in multiple sub-models, and with better efficiency in locally well behaved problems. The typical local characteristics of our problem, and also the modeling of inputs by normal partitions, restricting the system interaction to neighborhood rules, motivate us to the use of local learning.

Local learning computes the parameters of the model by minimizing the objective function (1), where $y_r$ is the output for real system and $b_i$ is the centroid value for rule $r_i$, which is the rule to be created.

$$ E = \frac{1}{2} \sum r_i \cdot (y_r - b_i)^2 \quad (1) $$

Note that the error sum $E$ is done for all the training set $N_r$, however the support value $b_i$ assumes null value when the rule $r_i$ is not activated, which limits the sum to the local points (local training). Contrarily to the global learning, for local learning the estimation of $b_i$ doesn't depend on the zero-order parameters of the other rules. Based on ordinary least squares the zero-order coefficient $b_i$ is determined by a simple weighted sum

$$ b_i = \frac{\sum (\beta_i \cdot y_r)}{\sum \beta_i} \quad (2) $$

In the GIS implementation, we first use maps of outputs $y_r$ to compute the several maps $(\beta_i \cdot y_r)$; in the next step, we compute zonal-statistics maps by summing the $(\beta_i \cdot y_r)$ in regions (zones) defined by the rule-coding maps. The $b_i$ value is computed at the end of geographical zonal-sums. The results of $b_i$, the sums $\sum (\beta_i \cdot y_r)$ and $\sum \beta_i$ are stored as rule parameters in a rule lookup tables. The sums stored in a lookup table could be used for parameter adaptation if new input/output data are added to the training set. In this case the adoption is not done point by point but map by map.

V. INTERPOLATING FUZZY RULES

When learning from input/output data, only the rules matched by the learning set are generated. This leads to the incompleteness of the rule set, and when we run the fuzzy system in a new region the so called blank-spots appear, which reflects the lack of rules to represent the characteristics of this new region. Interpolating and extrapolating the original rule base could solve this problem.
If we admit that the rule base is continuous, requiring that rules having adjacent premises have also adjacent consequents, the proximity of rules in premises (inputs) implicates a proximity in consequent (outputs). Based on this assumption it is possible to trend the output value of a new rule \( b_{new} \) and its rule consequent (zero-order parameter for Sugeno functions), based on the distance \( d_{new,k} \) between the new rule antecedents and the antecedents of the neighboring rules (index \( k \)).

\[
(y_i, y_j) = (a_{i,j}, b_i)
\]

\[
F_{new}(d_{new}) = 1.045 + 2.407 \cdot d_{new} - 1.088 \cdot d_{new}^2 - 0.918 \cdot d_{new}^3 + 0.047 \cdot d_{new}^4
\]

Figure 4 - Example of the interpolation of one rule, the value obtained for rule output is \( b_{new} = 35.045 \).

For each rule one obtains a two dimension point \((x_i, y_i) = (d_{new,k}, b_i)\), where \( b_i \) is the consequent for rule \( k \). Due to the relation between rule adjacencies in antecedents and consequents the set of points presents a convergence to a specific value when distance \( d_{new} \) tends to zero. This convergence value is determined by \( b_{new} = F_{new}(0) \), where \( F_{new} \) is the polynomial trending function for the scattered set of points, obtained from the following minimization function.

\[
E = \sum_{i=1}^{N} \frac{1}{d_{new,k}} (F_{new}(d_{new,k}) - b_i)^2
\]

This method allows not only the interpolation of the rules but also suggests extrapolation. In fact, the method should be considered as a trending method for fuzzy rules. This ability is particularly useful for FSM because most of the blank-spots correspond to rules that represent future forecasting behavior in the extreme of the hyperspace defined by the data.

VI. MERGING JUDGMENTAL INFORMATION

As mentioned before, the spatial model behavior is learned based on input/output historical data and on human judgment knowledge. The last source of information is particularly important in spatial load forecasting, where the future behavior is not only a continuity of the past behavior but is also dependent on new future behavior that can be foreseen by human experts. Therefore the rule base resulting from input/output training should be adjusted and complemented by human knowledge. The authors propose a reasoning mechanism to create and tune system rules merging judgmental information, described by human experts, into the fuzzy system knowledge base.

The human experts describe the judgmental information by specifying "master rules"; these master rules use the same structure of the fuzzy system but don't necessarily use the same linguistic labels on the antecedents. The fuzzy system rule base is tuned by the adaptation of the system response to the fuzzy inputs, where these fuzzy inputs represent the linguistic labels of the master-rule antecedents.

The inference method uses fuzzy inputs instead crisp. The differences between fuzzy and crisp inputs are only on the process to compute the matching value \( \alpha_{i,k} \). With fuzzy inputs, the matching values are computed by:

\[
\alpha_{i,k} = \sup_{j} \min(\mu_{\beta}(x_i), \mu_{\beta_{j}}(x_i))
\]

The graphical representation of this operator is presented on figure 5. Several possibilities are open for fuzzy input matching. For instance, if the fuzzy input \( A'_i \) covers all the original input partition, this represents the label "don't care", for which the result of the fuzzy system is independent of variable \( x_i \), accepting all values on the variable range.

Figure 5 - Graphical representation for the premise-matching operator, on variable \( x_i \), with uncertainty input \( A'_i \). In this example the fuzzy input activates four linguistic labels. Note that the number of system rules activated depends on the activation through all variables, and not only one as show the figure.

The base value for each activated rule is computed by:

\[
\beta_i = \prod_k \alpha_{i,k}
\]

The system rules are adjusted, for each judgment rule, by an adaptive training heuristic using the equation (6). The adjustment \( \Delta b_i \) is function of the deviation between the parameter value \( b_i \) for system rule \( r_i \) and the output value \( b'_i \) proposed by judgment rule \( r_{i} \). The adjustment for each system rule is weighted by the correspondent base value \( \beta_i \). The weight of the judgment rule \( r_i \) on the adjustment is regulated by the learning rate \( \delta_i \) in range \([0,1]\).

\[
\Delta b_i = \frac{\beta_i}{\sum_{i} \beta_i} (b'_i - b_i) \cdot \delta_i
\]
VII. EXAMPLE

To illustrate the effects of merging judgmental with statistical information we present in this section an example comparing results of two different rule bases. The first simulation uses a rule base constructed from statistical information. The second simulation uses the same rule base, but updated with judgmental information defined by one "master rule".

The example consists in a hypothetical case for domestic consumer growth in Santiago Island in Cabo Verde (Africa), with a area of 4000 km2 covering urban and rural zones. We will focus our analysis on the region of the main urban center.

The input variables considered in the case study are:
- distance to roads
- distance to main urban center
- distance to secondary urban centers
- slope
- saturation level.

Five linguistic labels were used to reclassify each input variable (very small, small, medium, high, very high). The analysis resolution used was 250 meters.

The rule base was constructed based on historical values using approximately 67000 training points. As training result we obtained 2542 rules.

In order to merge judgmental information, the expert may defines several master rules; however for this simple example we will consider only the following master rule:

IF (distance to road is (MEDIUM OR HIGH OR VERY HIGH)) AND
   (other variables is (DON'T CARE))
THEN Domestic PFD decrease 20%

The rule base was tuned with the additional judgmental information defined by expert knowledge. This process created 146 additional rules by interpolation and changed 1560 rules on the original rule-base.

Figure 6 - Evolution of the number of consumers along four stages.

On Figure 6, we may observe the spatial evolution, simulated with the first rule base, of the saturation of consumers, with a more intense growth near the urban center, near roads and far from sloppy areas. The spatial pattern follows the behavior captured and stored in the fuzzy rule base. The following represents one rule example:

IF (distance to road is (CLOSE)) AND
   IF (distance to main urban center is (MODERATE CLOSE))
   AND
   IF (distance to secondary urban center is (CLOSE)) AND
   IF (terrain slope (VERY SMALL)) AND
   IF (saturation level is (MEDIUM)) AND
THEN Domestic PFD is 25 consumers per stage per km^2

In figure 7 we may observe the decrease in development far from the roads (compared with the first case), as expected due to the direct effect of the "master rule".

Figure 7 - Darker zones indicate higher decrease of development due to the merge of the "master rule". This decrease is highest for the highest distance from roads.

In figure 8 we may observe the indirect effect of the "master rule". The merged "master rule" is less permissive relatively to the development on regions without roads. Consequently the urban pressure pushes out the development originating higher development near the roads but far from urban center (as compared to the original case without the expert-defined master rule).
VIII. CONCLUSION

This paper presents the mathematical formulations and describes the GIS implementation of a Spatial load Forecasting model (the Fuzzy Spatial Model (FSM)).

This spatial model is based on the coupling between a Fuzzy System and a Cellular Automaton. The Fuzzy System stores the knowledge base that describes the phenomena behavior, and uses the spatial influence factors to compute the suitability for development. The Cellular Automaton spreads the effective development, given by a global forecasting value for the entire region, in accordance with the potential indicated by the suitability maps. The phenomena behavior and the dynamics among time stages are controlled by the Fuzzy System.

The presented fuzzy system has been designed to combine statistical and judgmental information. This is done by using inference models that build the fuzzy rule-base using sets of historical maps representing the observed developments caused by the several influence factors.

The paper presented an innovative rule interpolation method, allowing fuzzy rule trending for the extremes of the space of analysis.

The paper also describes how rule base tuning and human expert knowledge integration is done. The use of fuzzy reasoning techniques allowed an interpretable and functional interface between human expert rules and system generated rules.

In summary, the paper describes the latest developments in a novel Spatial Load Forecasting intelligent system, capable of merging historical statistical information with human judgmental information and of storing it on a highly comprehensible rule base. The results supplied by such SLF intelligent system become more understandable and subject to interpretation by planners, and do not appear as "magic numbers" out of a black box.

REFERENCES


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VALIDATION PROCESS FOR FUZZY SPATIAL LOAD FORECASTING

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Abstract – This paper reports the method used to validate a spatial load forecasting model based on Fuzzy Inference Systems (FIS) and implemented in a Geographical Information System. The validation process not only confirms the adequacy of the rule base but is necessary to define confidence intervals to the predicted spatial demand.

Keywords – Spatial Load Forecasting, Geographic Information Systems, Fuzzy Inference, Forecasting validation.

1. INTRODUCTION

Spatial Load Forecasting (SLF) aims at predicting where, when and how load growth will occur in a utility service area, maintaining a full geographical representation. This information is useful for distribution expansion planning purposes, serving as basis for establishing the evolution of the design of a distribution system in a given region.

SLF methods must be implemented on a Geographical Information System (GIS). The GIS environment provides an ability to: manage spatial information; model and simulate phenomena behavior; visualize data and simulation results; and establish interaction between planners and simulation environment.

The first and important systematic works in SLF have been conducted by Willis and are described in a remarkable series of publications - see for instance [1][2]. More recently, the authors of this paper have developed a successful approach to Spatial Load Forecasting based on Fuzzy Inference Systems (FIS) and Cellular Automata, with important results [3-8].

The classical SLF approach depended too much on the a priori definition of numerical constants, values or parameters. The recent SLF models based on FIS rely instead on capturing knowledge from past maps and building a rule base describing the interaction among influencing factors that explain the evolution of demand along time.

Two concepts are therefore fundamental in modern SLF: the extraction of knowledge under the form of rules (from past or analog cases with geographical representation) and the application of the set of rules to generate the simulation of load growth (in maps of future development).

The questions remain: how accurate are the rules? How faithful is the prediction? How important is the uncertainty associated with the spatial forecast?

This paper presents the methodology used to validate a rule base used together with a FIS to produce a SLF.

2. SPATIAL LOAD FORECASTING WITH FIS

2.1 The Fuzzy Inference System

A FIS is extremely adequate to model spatial growth behavior because:

- it allows knowledge representation by linguistic concepts such as "close to road", "location with high environment protection" or "medium saturation status for urban development";

- it allows knowledge representation by comprehensive rules, where cause and consequence are represented by if-then fuzzy rules; rules allow a better interaction between the system and human experts, because of their self-explanatory characteristics;

- a comprehensive knowledge base stored as a rule base may be applied to other space and time environments, due to its capacity of generalization.

There are some basic data whose definition is prior to the application of a FIS prediction system. One of the most important is a global growth forecast, valid for the geographical region as a whole - generated from some economic or aggregated model, external to the SLF process (trending, econometric, diffusion of innovation model).

![Figure 1 - Structure of the Fuzzy Spatial Model](image)

Figure 1 sketches the organization of the FSM concept, composed of two main models. The first is the Fuzzy System (FS); it estimates the suitability or the potential for development at each map cell. The second is the Cellular Automaton (CA); it spreads the global forecast over all the region based on the preferences indicated by the suitability maps. The results of the CA module are the effective geographic distribution of the development.

The Scenario Coordinator (SC) links the FSM with the
forecasting environment and with externally imposed conditions and coordinates the dynamics of the simulation, namely the inputs of the Fuzzy System and Cellular Automaton along the several time stages.

The Takagi-Sugeno structure of the Fuzzy System (FS) organizes its rules in a neural-like form, the inputs propagating throughout the network until an output is generated. In the Takagi-Sugeno inference model, the antecedent of a rule is fuzzy, but the consequent is crisp, and a function of the input values.

The FSM problem is characterized by a very large set of geographical cells (may reach a million cells per map). On the other hand the number of significant variables is in general limited (a typical value would be of 5 variables). This characteristic motivated us to do an implementation based on the GIS spatial analysis functions instead of a GIS coupling with external fuzzy system modules.

![Graphical representation showing the zero-order Takagi-Sugeno inference method.](image)

Figure 2 - Graphic representation showing the zero-order Takagi-Sugeno inference method.

The neural structure of the zero-order Takagi-Sugeno FS is represented in Figure 2. The matching on the fuzzy proposition "\( x_i \in A_{i,j} \)" is given by \( \alpha_{i,j} \), where \( x_i \) is the numerical input for variable \( x_i \) and \( A_{i,j} \) is the membership labeled \( j \) on this variable. The support value for rule \( r_i \) is given by \( \beta_i \). The final output \( y' \) is the weighted sum for all rules, where \( b_i \) is the zero-order function coefficient for rule \( r_i \) and \( N_i \) is the number of rules.

The implementation of this fuzzy system with GIS spatial functions is especially interesting because the operations are applied simultaneously on all the geographical cells. The implementation requires: maps with activated membership labels (two for each variable); maps with matching values \( \alpha_{i,j} \) (two for each variable); maps with coding for activated rule (2 \( N \) maps, where \( N \) is the number of variables); maps with the support values \( \beta_i \) (as much as the number of maps for coding activated rules); maps with lookup values \( b_i \) associated with support values \( \beta_i \) (as much as the maps with the support values). The rule codification is important to identify the rules and access the lookup tables containing the rule database. The codification of the rule \( r_i \) is done by \( Cod_i \), where \( l_{i,j} \) is the membership label for label \( j \) on variable \( X_i \), the \( N_{l_{max}} \) is the maximal number of membership labels for variables \( X_i \), and \( N_i \) is the number of variables.

Here's one example of a rule:

\[
\text{IF (distance to road is CLOSE) AND (distance to urban center is MODERATE CLOSE) AND (terrain slope is MODERATE) AND (domestic saturation is MEDIUM) AND (industrial saturation is LOW) THEN}
\]

DomesticPFD is 20 consumers per stage per km² AND Industrial PFD is 0.1 consumers per stage per km²

These rules are automatically generated and used by the spatial model and are easily understood by human specialists. The rules are stored in the GIS database and are used as in a lookup table in the process.

### 2.2 Cellular Automata

The application of the FIS rules leads to the production of maps of potential for development. These maps must be transformed into maps of prediction of actual development, and this is done through a Cellular Automata (CA) model.

The CA theory was first introduced by Jon Von Neuman [9] and is ideally applied for dynamic discrete modeling [10]. In our formulation, at any specific point of time \( t \) the CA is a collection of binary states \( e_{ij} \) in cell location \( (i,j) \), with value 1 if a new consumer is added to the site and 0 if no consumer is added.

\[
CA = \{ e_{ij} \} \quad 0 \leq i \leq r; \quad 0 \leq j \leq c; \quad \forall e_{ij} \in E
\]

where \( E \) is the finite set of states, \( r \) and \( c \) are the number of rows and columns of the map grid.

The CA is an iterative process computing development based on potential for development and computing new potential based on previous iteration development.

The Potential for Development (PFD) is initially set by the fuzzy system. The PFD is represented as a stack of continuous maps, one for each consumer type, representing the potential growth number of consumer per stage and per geographic unit (e.g. 20 domestic consumers per stage and per km²). The Development, which is the output of the CA, represents the effective number of consumer growth. A global geographical trend controls the global development, the sum of all developments in the region. The CA process finishes when the sum of all cell developments reaches the global trending value (e.g. the growth for year 2001 in the whole region tends to 250 industrial consumers and 5000 domestic consumers).

The iterative process of the CA is based on state transitions \( S_i(t) \); in our model, these will be transitions from non developed to developed. The state transition is done according to a set of rules such as

\[
\text{if } P_i(t) > P_b(t) \text{ then } S_i(t) = 1 \text{ else } S_i(t) = 0
\]

In our model a transition exists if the cell has a PFD value
The development $D_i(t)$ is recalculated in each iteration incrementing the number of consumers, by steps $D_{step}$ (measured in number of consumers), only on cells marked as developed $S_i(t)=1$.

$$D_i(t) = D_i(t-1) + S_i(t) \cdot D_{step} \quad (8)$$

Development maps may then be generated such as in Figure 3.

The new potential $P_i(t+1)$ is recalculated based in three components:

- positive feedback of the cell on the previous iteration, weighted by $\alpha$;
- neighborhood effect based on the 8 adjacent cells [11], weighted by $\beta$;
- innovation factor $\lambda$ modeled as random noise

and is given at time $t+1$ by

$$P_i(t+1) = \alpha \cdot P_i(t) + \beta \cdot \frac{1}{8} \sum_{j \in \Omega_i} P_j(t) + \lambda \cdot e_i(t) \quad (9)$$

where $\alpha$, $\beta$ and $\lambda$ are the weights for each component, with values $[0,1]$ and $\alpha+\beta+\lambda=1$, and $\Omega$ is the set of adjacent neighbors cells. $P_i(t)$ is the updated potential to development in time stage $t$ on site $i$, computed based on the output of the fuzzy inference model $P_i(0)$ and on the development computed by the CA on iteration $t$:

$$P_i(t) = P_i(0) - D_i(t)$$

At the end of each stage the PfD maps may be recalculated, using the fuzzy inference model and the new geographic data computed with the CA or introduced by the planner.

3. VALIDATING PROCESS

The Fuzzy Spatial Load Forecasting model has particular characteristics that require appropriated validation processes. These characteristics are the spatial behavior, the capability to model judgmental information and the temporal behavior. To evaluate these three characteristics we have formulated decoupled tests to independently validate the several aspects: spatial validation, temporal forecasting validation, temporal backcasting validation and validation of the merging of judgmental information.

Two different measures of accuracy are used: the Coefficient of Variation (CV) to measure the accuracy on level; and the Turning Point (TP) to measure the accuracy on changes. These two measures of accuracy are required because for Electricity Distribution Planning one needs to assess the level of load development (levels) and the changes from green-field to new load developed area (changes).

The Coefficient of Variation (CV) relates the Root Mean Square Error (RMSE) to the average value of the actual data.

$$CV = \frac{RMSE}{\sum_{i=1}^{h} V_i / h} \quad (1)$$

The RMSE is the Root Mean Square Error given by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{h} (V_i - F_i)^2}{h}} \quad (2)$$

where $i$ is the index of the forecasted output (representing a geographical cell in a specific time period), $h$ is the number of forecasted points (number of cell in the geographical coverage times the number of periods), $V_i$ is the actual result for point $i$ and $F_i$ is the forecasted value for point $i$.

The Turning Point (TP) measures the accuracy of prediction in the change from green-field to developed area. The four possible turning point situations are the following:

<table>
<thead>
<tr>
<th>Did the change occur?</th>
<th>Was a change predicted?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>TP$_a$=a/o</td>
</tr>
<tr>
<td>No</td>
<td>TP$_c$=c/p</td>
</tr>
<tr>
<td></td>
<td>TP$_b$=b/p</td>
</tr>
<tr>
<td></td>
<td>TP$_d$=d/p</td>
</tr>
</tbody>
</table>

The value $a$ represents successful predictions when change occurs; $d$ represents successful predictions when no change was forecasted; $c$ represents errors when change occurs and was not predicted; $b$ represents errors when change was predicted but did not occur; $o$ represents the number of change occurrences ($a+c$), and $p$ represents the number of change predictions ($a+b$). Change prediction $p$ is not necessarily equal to change occurrence $o$.

3.1 Spatial Validation - example

To validate the spatial behavior, a cross validation procedure is used. A spatial random selection is used to separate a
calibration sample \( [C_i] \) from a validation sample \( [V_i] \), where \( i \in j \). The calibration sample is used to train the system and build the fuzzy rule knowledge base. Using this knowledge base, the forecasting method is applied producing the forecast sample \( [F_i] \) in the point corresponding to \( [V_i] \).

We will describe a validation procedure applied to a given region in the island of Santiago, in Cabo Verde (Africa).

The historical data consists in the development observed along 3 time periods and corresponds to 3 geographical coverages of domestic development in Santiago. To decouple the spatial behavior from the temporal behavior, the selection of the calibration and validation samples was done for the three temporal stages.

From this geographical coverage we’ve randomly selected half of the points for a calibration sample \( [C_i] \) (near 15000 points per period) and the other half for a validation sample \( [V_i] \). The system was trained with a calibration set \( [C_i] \) of 45000 points, using six variables as influence factors and generating approximately 2500 fuzzy rules.

After the training step, using only historical information, we produced a forecast for a following time period using the same input variables and labels and adopting a given value for a global forecast. The global forecast (number of new consumers for all the coverage) was previously obtained from the development maps in each period \( (p_1=1000; p_2=1500; p_3=2000) \). We’ve produced three maps of forecasting values covering the entire region in the three periods. Using the forecasting results \( [F_i] \) and the validation sample \( [V_i] \) for the validation points (points not used for calibration), we estimated the accuracy measures CV and TP.

The results in Table 1 show that the spatial mean error (CV) reaches 11%. Comparing the values for the three periods we observe that Period P2 has lower error because the three periods where used for calibration and P2 is the intermediary period that probably benefits of better fitting.

The error is not the same for different saturation levels. Saturation is a concept illustrated in Figure 4. In our FIS model, one observes the evolution of saturation, instead of defining its curve externally.

Table 1 - Spatial Validation results to measure the accuracy on the forecasting level. Including the Root Mean Square Error (RMSE), and the coefficient of variation (CV).

<table>
<thead>
<tr>
<th>Period</th>
<th>( \sum_{i=1}^{h} V_i / h )</th>
<th>RMSE</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>1.502</td>
<td>0.163</td>
<td>10.90%</td>
</tr>
<tr>
<td>P2</td>
<td>1.511</td>
<td>0.156</td>
<td>10.30%</td>
</tr>
<tr>
<td>P3</td>
<td>1.520</td>
<td>0.171</td>
<td>11.35%</td>
</tr>
<tr>
<td>Global</td>
<td>1.512</td>
<td>0.163</td>
<td>10.80%</td>
</tr>
</tbody>
</table>

Figure 4 - The number of consumers as a function of time defines the Saturation curve at a location; its derivative is seen as the potential for development P/D. The saturation level at stage 1 is characterized through fuzzy descriptors.

Figure 5 shows the variation of the error for the different levels of saturation (of growth in a map cell). The error increases when in the growth part of the saturation curve, because for this saturation level (40% to 70%) the area develops very fast and the forecasting is more difficult. When the growth approximates saturation the development becomes slow, as if restricted by a maximum value, and consequently the forecast becomes more accurate.

Figure 5 - Variation of accuracy (CV) with the saturation level.

As explained before, measuring the level of error is not enough for Spatial Load Forecasting; it is also very important to have accuracy in predicting changes from green-field to developed area. This accuracy is evaluated with Turning Point (TP) measures \( (TP_1; TP_2; TP_3 \) and \( TP_4 \), described previously. The \( TP_2 \) and \( TP_3 \) are measures of unsuccessful predictions and \( TP_1 \) and \( TP_4 \) are measures of successful predictions.

Table 2 - Spatial Validation results to measure the accuracy for forecasting change. \( TP_1 \) represents successful predictions when change occurs; \( TP_2 \) represents errors when change was predicted but did not occur; \( TP_3 \) represents errors when change occurs and was not predicted; and \( TP_4 \) represents successful predictions when no change was forecasted.

<table>
<thead>
<tr>
<th></th>
<th>Period P1</th>
<th>Period P2</th>
<th>Period P3</th>
<th>Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changes</td>
<td>48</td>
<td>73</td>
<td>108</td>
<td>229</td>
</tr>
<tr>
<td>Predicted changes</td>
<td>47</td>
<td>65</td>
<td>102</td>
<td>214</td>
</tr>
<tr>
<td>TP1</td>
<td>88.8%</td>
<td>85.9%</td>
<td>84.1%</td>
<td>86.3%</td>
</tr>
<tr>
<td>TP2</td>
<td>9.3%</td>
<td>11.5%</td>
<td>13.8%</td>
<td>11.5%</td>
</tr>
<tr>
<td>TP3</td>
<td>11.2%</td>
<td>14.1%</td>
<td>15.9%</td>
<td>13.7%</td>
</tr>
<tr>
<td>TP4</td>
<td>98.3%</td>
<td>97.1%</td>
<td>95.8%</td>
<td>97.1%</td>
</tr>
</tbody>
</table>
The number of observed changes and of predicted changes are quite close with an error between 7% and 10%. Observing the Turning Point measures of error TP_1 and TP_2 we conclude that the error sums up to approximately 25%. These values are considerably high but we must recognize that forecasting behaviors of "spatial pioneer" consumers is extremely difficult because usually these consumers display unusual behavior. We note that the change accuracy along the three periods decreases in opposition with the accuracy in level, that has better values for the intermediate period P_2. For the latest periods, more innovative behaviors at development borders are activated and the changes occur mostly in these regions, and consequently these are more exposed to error.

### 3.2 Temporal Validation – forecast and backcast

To validate the temporal behavior we used two different procedures: the forecast validation and the backcast validation. The forecast validation consists in using the forecasting models, calibrated with the historical information excluding the latest period, and measuring their success in forecasting the latest known data sample. This test is almost as good as the real forecast. However, only uses historical behavior for calibration and is unable to model forward changes on behavior.

![Diagram](Figure 6 - Scheme identifying, for forecast temporal validation process, the data sets used for calibration validation and forecasting.)

As shown in Figure 6, for forecasting validation we use the data samples from periods P_1 and P_2 to calibrate the forecasting model. The knowledge base generated with this historical data is used to validate the latest data sample available P_3. In this test the forecasting model doesn't know any information about the behavior of the latest period P_3. The [C_j] data sample (periods P_1 and P_2), used to calibrate the knowledge base totalize approximately 60000 points. After the training step, we used the model to forecast the period P_3 and we obtained the forecasting map with forecasting values for approximately 30000 points. With the forecasting map [F_j] and with validation sample [V_j] in period P_3 we measured the accuracy. We used the Coefficient of Variation (CV) to measure the forecasting level accuracy and the Turning Point (TP) to measure the accuracy in forecasting changes.

One of the interesting aspects of the Fuzzy Spatial model is its capability to capture and store the knowledge base and the possibility of applying this rule base to other region with region with similar behavior but shifted in time. This is only possible if the model has good backcasting validity. With the backcast validity we test if the model still predict efficiently earlier behavior based on a knowledge base constructed with recent data sets. Obviously, this approach may suffer from contamination because the knowledge base is influenced by what happens recently, but it is this kind of contamination that we wish to evaluate.

![Diagram](Figure 7 - Scheme identifying, for forecast temporal validation process, the data sets used for calibration validation and forecasting.)

As shown in Figure 7, contrarily to forecasting validation, in backcasting validation we use the data samples from period P_2 and P_3 to calibrate the model knowledge base. The model calibrated with this historical data is used to validate the earlier data sample correspondent to period P_1. The calibration data sample (periods P_2 and P_3) adds up to approximately 60000 points, and the validation data sample (period P_1) contains approximately 30000 points.

**Table 3 – Temporal validation results for forecasting and backcasting.**

<table>
<thead>
<tr>
<th></th>
<th>[\sum_{i=1}^{i=h} V_i / h]</th>
<th>RMSE</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecasting P3</td>
<td>1.520</td>
<td>0.228</td>
<td>15.00%</td>
</tr>
<tr>
<td>Backcasting P1</td>
<td>1.502</td>
<td>0.177</td>
<td>11.08%</td>
</tr>
</tbody>
</table>

In Table 3 we observe that the accuracy for forward validation is lower than the one observed for backward validation: the CV measure increases from 11% to 15%. This increase is expected because, contrarily to the spatial validation process, the forecasting knowledge base has no information about the behavior of the forecasted period. Other reason for this decrease in forecasting accuracy is the possible changes in behavior resulting for multiple factors. Obviously these changes in behavior could not be captured from historical information. This accuracy of the model could be significantly improved by merging judgmental information.

For backcasting we observe and error of 11.8%. As expected, these values are higher than values observed for spatial validation (Table 1) because, contrarily to spatial validation using cross validation, in this backward validation the period P_1 is completely unknown for the knowledge base. The accuracy of the backcast (11.8%) is considerably better than of the forecast (15.0%). This shows that the future behavior continues keeping in storage the past behavior and this could
be efficiently captured. The predictions from forecasting are worse than the predictions from backcasting because future data samples contain both the past and future behavior but past data sample don't contain a complete characterization of the future behavior.

To test the accuracy on changes we used the several Turning Point (TP) measures. The measures use the forecasting sample \([F]\) and the validation sample for period \(P_1\) for forecasting validation and for period \(P_1\) for backcasting validation.

As shown in Table 4, for forecasting validation, the accuracy in the number of predicted changes is approximately 90%, with lower number of predicted than occurred changes. The total forecasting validation error in change (TP1 and TP2) totalsizes approximately 30%. Comparing the values obtained for temporal validation (Table 4) with values obtained for spatial validation (Table 2) for period P3, we observe that TP2 is lower for the temporal validation test and TP3 is considerably higher. This happens because the behavior for period P3 was affected by innovative changes not known by the model knowledge base, but the behavior from previous periods P1 and P2 is still valid and consolidated. Thus the knowledge base remains valid to predict changes following the historical behavior (failing less for predicted/non-occurred) but is unable to predict the changes corresponding to the innovative behaviors (failing more for occurred/non-predicted).

<table>
<thead>
<tr>
<th></th>
<th>Forecasting P3</th>
<th>Backcasting P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed changes</td>
<td>108</td>
<td>48</td>
</tr>
<tr>
<td>Predicted changes</td>
<td>98</td>
<td>46</td>
</tr>
<tr>
<td>TP1</td>
<td>87.4%</td>
<td>89.8%</td>
</tr>
<tr>
<td>TP2</td>
<td>12.6%</td>
<td>10.2%</td>
</tr>
<tr>
<td>TP3</td>
<td>18.3%</td>
<td>14.7%</td>
</tr>
<tr>
<td>TP4</td>
<td>90.0%</td>
<td>91.0%</td>
</tr>
</tbody>
</table>

The total error obtained for backcast validation (TP1 and TP2) is approximately 22%. These values are significantly lower than the obtained for forecast validation but still slightly higher than values obtained for the spatial validation process (21%). This occurs because in backcast we don't have the additional difficulty of the innovative behaviors as these behaviors could also be captured from the future information samples. The difference observed between accuracy values in backcasting tests and spatial validation tests are a consequence of a better knowledge of the behavior in P1 for spatial validation tests due to the cross validation process.

**4. CONCLUSION**

In this paper we discussed the validation process for a spatial forecasting model. Four different tests are performed to validate the Spatial Load Forecasting model: spatial validation, forecast, backcast and temporal validation. Two different accuracy measures were used: the measure of the accuracy in forecasting level and the measure of the accuracy in predicting changes from green-field to developed area.

Our experience teaches us that validating Spatial Load Forecasting requires more complex analysis than usual aggregated forecasting models. The accuracy observe for the forecasting level varies from 80% to 90%, which we classify as good for the tested example and for distribution planning purposes. The inaccuracy observed for changes are higher, ranging from 65% to 80%. These change accuracy values, important for expansion planning, are a consequence of the difficult predictive characteristics of the “spatial pioneer” consumers.

These results are extremely helpful in building “confidence intervals” for demand growth at each geographical location.

**REFERENCES**


