Impact of Modulation Formats and Fibre Non-linearities on Optical Systems
Impact of Modulation Formats and Fibre Non-linearities on Optical Fibre Systems

Thesis submitted to the University of Porto for the degree of Ph.D. in Electrical and Computing Engineering

Dep. de Engenharia Electrotécnica e de Computadores
Faculdade de Engenharia da Universidade do Porto
2001
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“What we call the beginning is often the end
And to make an end is to make a beginning
The end is where we start from”

T. S. Eliot

To Luis and Adelina with all my love
Acknowledgements

Firstly, I would like to thank Professor Artur Pimenta Alves for the continuous attention and incentives during the years in guiding my scientific activity, for the determination shown in establishing the international contacts that were fundamental to the realisation of this work, as well as for his unconditional support during the completion of this Thesis.

Sincere thanks to Professor John O’Reilly to whom I am indebted for taking me under his supervision and providing me with an exceptional environment during my stays at his TRG group at University College London. Professor O’Reilly availability and valuable insights, as well as his positive and friendly personality, were instrumental to making this Thesis possible.

I wish to thank INESC Porto for supporting, in several forms, this PhD programme. In particular, I would like to thank my colleagues and staff at the Optoelectronics and Electronics Systems Unit for providing a nice working environment and for their help and support.

I would like to thank my family for their enormous support throughout the years, for enduring my prolonged stays abroad, and their remarkable patience and generous understanding towards my erratic and constant mood alterations.

Finally, I gratefully acknowledge the partial support given to this work by the PRAXIS Programme PRAXIS/P/EEI/12275/1998 of the Portuguese Ministry of Science.
Abstract

This Thesis investigates the potential of duobinary, modified duobinary and dicode signalling schemes operating at 10 Gbit/s for application in optical fibre communication systems. In particular, the impact of optical fibre non-linearities on these modulation formats is investigated. The premise for exploring these signals was that, because some of them allow spectral compression and carrier-suppression, an enhanced immunity to non-linearities as well as to chromatic dispersion should be achieved. All signalling methods are investigated through theoretical considerations and computer simulations.

Of all of the classes of partial response signalling (PRS), it was previously known that the above mentioned electrical three-level PRS signals were suitable for transformation into optical signal format for transmission along the optical fibre. Regarding fibre dispersion, it is found that in spite of the fact that duobinary and modified duobinary reduce the optical bandwidth of near rectangular pulses, they offer no advantage over conventional binary modulation. An improvement in dispersion immunity is achieved only by properly filtering of the electrical signals prior to be injected into the fibre—these improvements are modulation format dependent. An alternative duobinary signalling scheme allows the dispersion-limited distance to be extended to \(~150\) km, an improvement of more than two times over conventional binary format. However, this increasing in dispersion immunity comes with a price: strict symmetry requirements in the PRS transmitters compared to conventional binary transmitter.

The sustained increasing demands for capacity have pushed for higher channel bit rates, high optical powers per channel and narrower spacing between channels to allow increased channel count. These factors exacerbate non-linear cross talk between the channels due to the non-linear properties of the optical fibre. Thus, the impact of fibre non-linearities on the various modulation formats under investigation is analysed. The main effects due to Kerr non-linearity (self-phase modulation, cross-phase modulation and four-wave mixing) and Raman scattering are assessed. Each of the non-linear effects is analysed separately.
Resumo

A investigação do potencial de formatos de modulação “partial-response signalling”, tais como duo-binário, duo-binário modificado e “dicode”, a débitos de 10 Gbit/s e aplicados em sistemas de comunicação por fibra óptica, foi o objectivo primordial desta Tese. Mais especificamente, o impacto das não-linearidades da fibra óptica sobre estes formatos de modulação foi objecto de análise. A premissa, na utilização deste tipo de sinais, deveu-se ao facto de, alguns deles, poderem permitir, devido a possibilitarem compressão espectral, uma imunidade acrescida à dispersão cromática da fibra bem como às suas não-linearidades.

De todas as classes de sinais PRS, era já conhecido que apenas os acima-mencionados são, na verdade, adequados para uso em sistemas por fibra óptica. No tocante à dispersão, verificou-se que, apesar de reduzirem a largura de banda óptica da ligação, não apresentavam, mesmo assim, vantagem alguma sobre modulação binária convencional. Tal só foi alcançado através de limitação da banda, por meio de filtragem dos sinais eléctricos; todavia, tais melhorias eram dependentes do formato em questão. Uma técnica alternativa de implementação de duo-binário permitiu estender a distância, limitada pela dispersão, a cerca de 150 km, uma melhoria quase duas vezes superior quando comparada com sinais binários. Todavia, esta acrescida imunidade à dispersão teve o reverso da medalha: uma simetria rígida dos parâmetros de funcionamento dos transmissores ópticos PRS relativamente ao caso binário.

A necessidade, sustentada, de cada vez maior capacidade dos sistemas, conduziu a maiores débitos, maiores potências ópticas injectadas e menor espaçamento entre canais, o que por sua vez permitiu aumentar o número de canais do sistema. No entanto, todos estes factores exacerbaram a diafonia cruzada não-linear entre canais, a qual é devida aos efeitos não-lineares da fibra. Por conseguinte, avaliar o impacto das não-linearidades sobre os vários formatos de modulação estudados foi objecto de análise. As principais não-linearidades devidas ao efeito de Kerr (“self-phase modulation”, “cross-phase modulation” e “four-wave mixing”) assim como o efeito de Raman foram avaliados individualmente.
Résumé

L’investigation du potentiel de formats de modulation signalisation à réponse partielle PRS (en anglais « partial-reponse signalling »), tel que duo-binaire, duo-binaire modifié e dicode, à débits de 10 Gbit/s, pour application en systèmes de communication par fibre optique a été le principal objectif de ce Thèse. En particulier, l’impact des non-linéarités de la fibre sur ces formats de modulation a été analysée. La prémisse pour l’utilisation de ces signaux s’est due au fait de quelques-uns permettent la compression spectrale et donc peut rendre possible une immunité augmentée aux non-linéarités, bien comme à la dispersion chromatique de la fibre. Tous les formats de modulation ont été investigués par des considérations théoriques et de simulations par ordinateur.

De toutes les classes de formats PRS, il été déjà connue que seulement les mentionnés ci-dessus sont, réellement, appropriés par l’utilisation en liaisons par fibre optique. En ce qui concerne à la dispersion, malgré les formats duo-binaire et duo-binaire modifié permettent la réduction de la largeur de bande optique du système, ils ne présent aucune avantage sur la modulation binaire conventionnelle. Ce fait a seulement été obtenu quand les filtrages des signaux, dans le domaine électrique, ont été effectués – ces améliorissements ont dépendu du format de modulation utilisé. Une technique alternative d’implémentation de duo-binaire a permit étendre la distance, limitée para la dispersion chromatique, à environ 150 kms, un améliorissement supérieur à deux fois environ que pour le cas d’un signal binaire. Cependant, l’augmentation de l’immunité de la dispersion à le revers de la médaille : l’opération des émetteurs PRS impose une symétrie stricte de leur fonctionnement en comparaison avec l’émetteur binaire conventionnelle.

La nécessité sustentée d’une croissance de la capacité a conduit à un débit chaque fois plus élevée, à des intensités optiques plus hautes, à une plus étroite séparation entre les canaux et une augmentation de son numéro. Ces facteurs ont exagéré élevé la diaphonie non-linéaire entre les canaux à cause de la non-linéarité de la fibre. À cause de cela, l’impact des non-linéarités de la fibre sur les formats PRS a été investigué. Les principaux effets à cause de la non-linéarité de Kerr (auto-modulation de la phase, SPM, modulation croisée de la phase, XPM, et la mélange à quatre ondes, FWM) et l’effet de Raman ont été évaluées.
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Chapter 1: Introduction

In the last 20 years a myriad of new telecommunication services, namely the introduction of multimedia and wireless services, the advances in cable television and the explosive growth of INTERNET have assumed particular importance. In the present fast changing environment, new and improved telecommunication infrastructures are required. Key issues to be addressed include, among others: high-speed connections, scalability, services integration, reliability, integrated operation, administration, maintenance and provision (OAM&P). In creating this new broadband telecommunication networks, optical fibre have had a major role. Optical fibre systems offer reliable transmission rates of tens of Gigabits per second (Gb/s) over a single fibre, with the capacity to carry several hundreds of thousands of simultaneous voice signals. If dense wavelength division multiplexing (DWDM) technologies are used, the fibre’s carrying capacity can be increased further by at least two orders of magnitude, where transmission capacity well over 1 Tb/s is possible [1]. The large capacity of optical fibre systems is derived from the nature of the transmitted signal and the fundamental characteristics of optical fibre. The optical carrier frequency of about 200 THz is much higher than for other communication systems by several orders of magnitude, which translates in a significant larger information-carrying capacity.

This introductory chapter starts with a discussion about some of the major limitations in a high-speed optical fibre system.

1.1 Limitations in a high-speed optical fibre communication systems

The performance of an optical fibre communication system can be degraded due to many attributes of the system itself: spectral broadening of light sources due to chirping; mode partition noise; pulse broadening from chromatic dispersion and/or polarisation dispersion within the fibre; attenuation from inherent losses in the optical fibre or the insertion loss due to the use of other optical devices; noise from spontaneous emission (ASE) in optical amplifiers; non-linear behaviour of optical fibre, opto-electronic devices and electrical components; non-ideal frequency response of components. All of these attributes will affect system performance in some way. A brief description of some of these limiting factors is given.

Information bandwidth: The information bandwidth occupied by the transmitted signal is a fundamental and almost impossible to reduce mechanism responsible for channel broadening. Its impact is of particular importance in WDM systems. Any modulated signal is spectrally
broadened due to the inherent information capacity of that channel. For intensity-modulated channels, the 3 dB full width half-maximum (FWHM) bandwidth is roughly equal to the modulation bit rate, with a 1 Gb/s signal having a 1 GHz bandwidth. For very fast modulated signals at 20 Gb/s, the information bandwidth is \(~0.15\) nm and will affect the required width of the optical filter pass band that must be used to recovery the entire transmitted signal [2].

Light sources: Due to phase fluctuations in a laser cavity and the quantum-mechanical energy-level fluctuations of the stimulated emission process, the spectral line width of the optical output from semiconductor lasers has a non-zero value [3]; even for the case of continuous-wave operation (CW) these fluctuations produce a broadening of the output spectra. Typical DFB lasers have line widths of about 10 MHz. In WDM systems with direct-detection (DD) systems, the broadening of the signal spectrum due to laser line width requires a wider optical filter bandwidth to select the desired channel. However, in today’s systems, operating at bit rates greater than hundreds of Mb/s, the laser line width is usually considered negligible compared to signal bandwidth [3]. When the injected current into the laser is directly modulated, both laser output power and laser frequency change. The latter phenomenon is known as chirp. In high-speed intensity-modulated (IM) systems, chirp broadens the laser output spectrum and, therefore, distorts transmitted signal via fibre dispersion [4]. To overcome this limitation, most high-speed transmission schemes use some form of external modulation, which increases transmitter complexity and cost.

Attenuation: Optical fibre losses, optical fibre connectors, fibre splicing and the insertion of optical and opto-electronic devices in the system are all responsible for attenuation of the light signal. Nevertheless, the current availability of optical amplifiers provides a means of overcoming attenuation limitations.

Fibre chromatic dispersion: Much of the fibre that has been deployed in existing infrastructures has been standard (or conventional) single-mode fibre with the zero in chromatic dispersion situated at a wavelength near 1300 nm. If a system using this fibre is upgraded to use erbium-doped fibre amplifiers (EDFAs), there is good reason to design the new system around their operating wavelength near 1550 nm (lower attenuation). However, the system chromatic dispersion will increase, not only because of longer span distances but also because the wavelength shift in operation results in significant dispersion (~17 ps/nm/km). Even in systems where the fibre can be chosen from the onset to have any desired dispersion, techniques to
compensate dispersion may be required. This is why much effort has gone into devising and implementing techniques to reduce the dispersion penalty inherent at the longer 1550 nm wavelength of operation [5]. In the next chapter some of the more successful dispersion compensating methods are discussed.

Fibre non-linearities: The widespread use of EDFAs compensates only for attenuation, allowing other effects such as dispersion and non-linearities to accumulate along the fibre link. These effects will cause severe limitations especially in long-haul un-regenerated spans and WDM transmission systems. The optical non-linearities in optical fibre are due to stimulated scattering processes (stimulated Brillouin scattering, SBS, and stimulated Raman scattering, SRS) and changes in the refractive index with optical power [6, 7]. Stimulated scattering is manifested as an intensity-dependent gain or loss while the power dependence of the refractive index is responsible for the Kerr effect. In scattering phenomena, such as SBS or SRS, part of the energy of the propagating optical field is transferred to local phonons. In particular, in SBS acoustic phonons are involved, while in SRS optical phonons are generated. At high-power signal levels, both of these processes can induce stimulated effects, and the intensity of the scattered light will grow exponentially once a threshold value of the incident power is exceeded. In general, SBS does not couple channels in a WDM system; on the other hand, SRS could be responsible for interaction of different WDM channels, thus given rise to cross talk [8]. Depending on the input signal shape, the Kerr non-linearity gives rise to different effects such as self-phase modulation (SPM), cross-phase modulation (CPM or XPM) and four-wave mixing (FWM) (also known as four-photon mixing - FPM) [6-8]: in SPM fluctuations in the optical signal intensity give rise to modulation of the signal phase and lead to broadening of the spectrum; in CPM (or XPM) the intensity fluctuations in one channel propagating along the fibre modulate the phase of all other co-propagating channels; and FWM (or FPM), where the mix between two (or three or more) channels will produce optical power at their sum and difference beat frequencies.

Polarisation dispersion: Fibre chromatic dispersion is not the only dispersion mechanism responsible for pulse broadening in practical single-mode fibre. In single-mode fibre there are actually two independent, degenerate propagation modes [9]. These modes are very similar, but their polarisation planes are orthogonal. In general the electric field of the light propagating along the fibre is a linear superposition of these two polarisation modes. In ideal fibres, with perfect rotational symmetry, the two modes are degenerate with equal propagation constants and any polarisation state injected into the fibre will propagate unchanged. In a real fibre there are
always some imperfections [10] (core deformities, stress or bending), which break the circular symmetry of the ideal fibre and lift the degeneracy of the two modes. The modes propagate with different phase and group velocities, and the difference between their effective refractive indices is called modal birefringence. Tsubokawa and Sasaki [11] demonstrated a pulse broadening of 17 ps for a fibre 100 km long due to polarisation dispersion. Their results indicate that 10 Gb/s transmission rates may be limited to about 150 km for some standard single-mode fibres. C. D. Poole and J. Nagel [9] demonstrated a significant temperature sensitivity of polarisation dispersion.

**Component frequency response:** The frequency response of components used in high-speed communication systems is generally non-ideal. For example, external modulators and photodetectors can have a frequency response that fluctuates considerably within the pass band or in the roll-off region; filters can exhibit ripple, especially in the roll-off region. The cumulative effect of the non-ideal frequency response of all components in the system may result in a substantial power penalty, which in some cases leads to malfunctioning of the system.

### 1.2 Scope of the Thesis

Two developments in optical fibre systems are re-shaping the design of high-capacity transmission systems. The advent of erbium-doped fibre amplifiers (EDFAs) has increased the link distance as limited by fibre loss. Wavelength-division multiplexing (WDM), on the other hand, has been the most promising technology to transmit simultaneously a large number of channels.

Much of the fibre that has been deployed in existing infrastructures has been the so-called standard or conventional single-mode fibre (SMF), with the zero in chromatic dispersion around 1.3 μm. If a system using this fibre is upgraded to use EDFAs, this forces the use of wavelengths near 1.55 μm. This translates in higher chromatic dispersion, not only because of longer distances between regenerators, but also because of the shift in operation to a wavelength with significant dispersion (~17 ps/nm/km). Optical duobinary modulation, a particular partial-response signalling (PRS) scheme, has been shown to reduce fibre chromatic dispersion impact.

EDFAs also allow higher signal power levels and transparency to wavelength-division multiplexing (WDM) of channels. These advantages allow fibre non-linear effects to play a role.
The research focus of this Thesis is to understand and assess the impact of fibre non-linearities in PRS modulation schemes. Specifically, the following objectives have been established:

- To analyse which PRS schemes can be considered for use in optical fibre systems.
- To evaluate their advantages and limitations regarding fibre chromatic dispersion.
- To investigate and assess the transmission impairments resulting from fibre non-linearities (self-phase modulation, cross-phase modulation, four-wave mixing and Raman scattering).
- To set guidelines for the design and optimisation of optical systems using PRS modulation formats.

The objectives set for this Thesis will lead inevitably to investigate theoretical and numerical models. Extensive simulations will be carried out for assessing several link parameters, such as optical signal power, frequency chirp, signal extinction ratio, optical signal polarisation, electrical and optical filtering, and limitations for each non-linear process and their combination.

Most of the existing fibre infrastructure includes standard single-mode fibre. Thus the optimisation of systems operating on SMF is a key issue for a first upgrading of transport and access networks in which systems adopting electrical repeaters are substituted by repeater less systems. For this reason, SMF will be the preferred type of fibre used in the simulated models. Dispersion-shifted Fibres (DSFs) and non-zero-dispersion fibres (NZDFs) will also be considered where and when deemed appropriate. Finally, repeater less systems (without in-line optical amplifiers) mean fibre transmission spans of about 200 km, which is sufficient for a large fraction of the world’s terrestrial links.

1.3 Organisation of the Thesis

The following chapters are intended as an in-depth analysis of the applicability of partial response signalling (or correlative level coding) schemes to optical communication systems. Their impact on system performance will be evaluated; some guidelines will be set up taking into account their limitations.

In chapter 2, a basic overview of some techniques and methods to increase capacity in high-speed optical fibre systems will be briefly described and analysed. They range from multiplexing (either in time, wavelength, or polarisation) to techniques to combat critical performance
degrading effects limiting the system capacity. PRS techniques will be introduced in this context.

The next chapter will discuss the theoretical framework regarding PRS signalling schemes. Discussion will be focussed to three-level PRS techniques, namely, duobinary, modified duobinary and dicode. As it will be shown, only these schemes (or suitable variants of these formats) are deemed adequate for optical communication systems due to a clever use of external modulators at the optical transmitter.

Chapter 4 will give a brief and general overview of the simulator to be used in this work. A generic system model will be established. An in-depth description of the major system components will be carried out and how they can be modelled. In particular, the cw Lorentzian laser, the single mode fibre and the LiNbO₃ amplitude dual-arm Mach-Zender modulator will be analysed.

Simulation results will be presented in chapter 5 based on the impact of fibre chromatic dispersion on system performance. Although propagation under the effect of group-velocity dispersion (GVD) for binary NRZ, and AM-PSK duobinary have been investigated, very little work has been reported concerning the other analysed PRS formats. Guidelines on how to choose optimal transmitting filters in repeater less PRS systems will be established. On the other hand, the impact of electrical filters parameters such as roll-off, their amplitude or phase response, or how optimal filtering varies with modulation formats, will be addressed. Finally, a detailed study of the influence of several key transmitter parameters will be conducted. Parameters to be considered are laser line width impact, modulator operating point, finite extinction ratio, non-ideal electronics, and unwanted modulator frequency chirp.

Chapter 6 is the core of this Thesis. It will deal with the impact of optical fibre non-linearities. The optical non-linearities are due to stimulated scattering processes (Brillouin and Raman scattering) and changes in the refractive index with optical power (the Kerr effect). Depending on the shape of the input signal, the Kerr non-linearity manifests by different effects, such as self-phase modulation (SPM), cross-phase modulation (XPM or CPM), and four-wave mixing (FWM or FPM). Along this chapter, the main effects due to Kerr non-linearity and Raman scattering will be assessed. Their impact on the several modulation formats under consideration will also be investigated.
1.4 Principal contributions of this Thesis

It was previously known that electrical three-level PRS signals were suitable for transformation into optical signal format for transmission along the fibre by a clever use of external modulators at the optical transmitter. Building on this, here arrange of single bit data formats, such as duobinary, modified duobinary and dicode schemes are investigated. In particular, a unified and integrated approach to analyse the potential of partial-response signalling to fibre transmission systems was developed.

Propagation under the effect of chromatic dispersion for binary and AM-PSK has been extensively studied and reported in the literature, but very little work has been reported concerning the other formats explored here. It is shown that band-limiting electrically anyone of the PRS signals was effective in improving dispersion immunity. Therefore, a systematic study regarding the impact of electrical filter parameters was carried out. The impact of filter parameters such as roll-off, amplitude and/or phase response, or how its bandwidth varies with modulation formats, is addressed for the first time for optical duobinary, modified duobinary and dicode signalling methods. Guidelines are established on how to choose optimal transmitter filters for optical repeater less PRS systems. Moreover, the impact of various transmitter parameters is studied, such as the impact of laser line width, modulator bias, modulator extinction ratio and chirp. It is found that dispersion tolerance of all optical PRS systems rely on strict symmetry in the signal and in operation of the external modulator. A known result, that binary NRZ signals are more robust to large deviations of the parameters breaking this symmetry, is confirmed and an explanation provided for why optical AM-PSK duobinary signals showed such an improved performance concerning fibre chromatic dispersion. It this, two widely accepted but apparently mutually exclusive interpretations are reconciled by drawing on theoretical principles particularised to the optical communications regime.

The sustained increasing demands for capacity have pushed for higher channel bit rates, high optical powers per channel and narrower spacing between channels to allow increased channel count. These factors exacerbate non-linear cross talk between the channels due to the non-linear properties of the optical fibre. Thus, the impact of fibre non-linearities on the various modulation formats under investigation is analysed. The main effects due to Kerr non-linearity (self-phase modulation, cross-phase modulation and four-wave mixing) and Raman scattering are assessed. Each of the non-linear effects is analysed separately. Suitable simulation set-ups were developed
to isolate and clarify the impact of each individually. It is shown that self-phase modulation induces spectral broadening and it is dispersion-dependent. Nevertheless, in the anomalous dispersion regime, cooperation between group-velocity dispersion and self-phase modulation can, in fact, narrow the spectrum. This effect has been known for some time for the conventional binary format; it is called non-linear assisted transmission. Here it is shown, for the first time, that this effect also occurs in almost all PRS signalling schemes. The single exception is AM-PSK duobinary, where, due to the occurrence of phase shifts exactly in the middle of each space, chirp polarity alternations do not allow SPM-induced chirp to compensate the dispersion-induced chirp. It is shown that SPM depends on pulse shape and bandwidth, fibre type and propagated distance. Regarding cross-phase modulation effects, it is demonstrated that they are approximately inversely proportional to the channel spacing. It is also shown that for powers less than 8-10 dBm, XPM induced interference scales almost linearly, while above this threshold, it is found that SPM also plays a role, enhancing the XPM effects in an exponential-like way. It was also shown that XPM is polarisation-dependent. Thus, even though in conventional SMF polarised light quickly and randomly changes its state of polarisation, it is observed that the impact of XPM-induced phase fluctuations is strongly reduced when light waves have orthogonal polarisations between each other. This is an indirect proof that XPM takes place in the first part of the fibre link and over a short distance. When narrow channel spacing is considered, XPM-induced interference it is much more severe. Regarding the impact of four-wave mixing (FWM), it was demonstrated that it is the most important effect on dispersion-shifted fibre. If channels carrying information are uniformly spaced, these FWM products are generated within the system optical bandwidth, and adversely affect system performance. However, it is shown that a moderate amount of dispersion is sufficient to induce a significant suppression of the FWM generated components. As in the XPM case, narrow channel separation substantially enhances FWM impact. It is established, for the first time, that PRS signalling schemes allow a greater level of suppression of FWM products than conventional binary. This interesting result is a consequence of the signalling methods investigated here are carrier-suppressed formats. Furthermore, it is also noted that the optical signal phase, induced by these schemes, appears to have an important, but subtle role requiring further detailed study. Power per channel is also an important parameter affecting the magnitude of FWM products, and for all the modulation formats investigated limiting the power per channel to a few milliwatts should be kept in mind. It is shown that for light waves with orthogonal polarisations, FWM generated tones are almost annihilated, even for the case of DSF and higher powers per channel. Finally, it
is shown that cross talk due to the Raman effect can strongly degrade system performance, even for a moderate number of channels.

The overarching finding of this Thesis is that there is a benefit to be gained in terms of system performance and tolerance to improvements in adopting a more sophisticated modulation format rather than conventional binary intensity modulation. In its, the selection of parameters for a particular system must be carefully effectuated taking account of a wide range of factors, drawing on the detailed findings and limitations established here.
References:

Chapter 2: Increasing capacity of a high-speed optical fibre system

2.1 Introduction

More than 10 million kilometres of optical fibre transmission links has been installed worldwide [1]. Since fibre cable installation is a large part of system cost, one of the major challenges of today’s system designers is how to upgrade and increase the total throughput of these links as more sophisticated optical and opto-electronic components, and equipment are being devised. Increasing the capacity involves increasing the bit rate and/or increasing the transmission distance.

To fully utilise the optical fibre capacity with efficiency, it should be possible to share that capacity. A generic term for such sharing is multiplexing. Several types of multiplexing techniques are possible. The simplest way of multiplexing is to use in parallel several optical fibres to support transmission of identical signals: it is called space-division multiplexing (SDM), which is an uneconomical way of increasing capacity for distances over hundreds of metres. However, concerning the fabrication of optical switching elements, SDM is the technology by choice and fairly mature. Another type is wavelength-division multiplexing (WDM): a number of signals can be carried simultaneously if each signal is modulated onto a different wavelength, and the optical carriers are sufficient separated to prevent channel interference. When multiple signals can be carried on a single channel by interleaving portions of each signal in time, time-division multiplexing (TDM) is the technique. Finally, polarisation-division multiplexing (PDM), where the two orthogonally polarised modes supported by single-mode fibre are used to double the information throughput. Several other techniques are then described that maximise the span length and/or bit rate by compensating for critical performance degrading effects. In particular, solitons, which are a special kind of waves that can propagate undistorted over long distances, are briefly described in subsection 2.1.4. Next, two subsections are devoted to improve system performance by compensating chromatic dispersion. First, optical phase conjugation (also called mid-span spectral inversion – MSSI) in the middle of the optical link is considered, to obtain dispersion and Kerr effect compensation. Next, the adoption of passive dispersion compensating devices, such as dispersion-compensating fibres (DCFs) or grating filters as fibre Bragg gratings (FBGs) are briefly dealt with. These components, inserted at appropriate locations along the link,
introduce a frequency-dependent phase shift that compensates fibre dispersion. All these methods deal with optical signals so they were grouped as optical-based techniques\(^1\).

Pre-chirping is essentially a pre-distortion technique used to compensate the chromatic dispersion. It is implemented at the transmitter side by modifying the spectrum of the data stream. One of the most successful transmitter techniques for dispersion compensation is referred as dispersion-supported transmission (DST). In this scheme, the optical transmitter generates a frequency-modulated signal that is converted to an amplitude-modulated signal by propagation along a dispersive fibre, i.e., chromatic dispersion is used as a way to decode information. It can be considered a particular case of the pre-chirping technique. Another method is partial-response signalling (PRS) formats, which can be used to reshape the spectrum of the signal in order to be more compatible with the transmission channel while generating controlled inter-symbol interference (ISI). As an example, the duobinary format allows a two-to-one spectral compression, which can reduce the influence of chromatic dispersion. Finally, multi-level signalling may serve as a means for increasing the bit rate. For example, if two 2 Gb/s binary signals are mapped into a four-level signal, then 2B Gb/s transmission is possible at a symbol rate of B GHz using 4-ary amplitude-shift keying (ASK). All these methods that allow some spectral shaping at the transmitter were grouped as transmission format techniques\(^1\).

2.2 Optical methods

2.2.1 Time-division multiplexing (TDM)

Optical time-division multiplexing (OTDM) is an extension of time-division multiplexing in the electrical domain to the optical domain. Initially, its implementation was difficult because the required technology was not available. With the development of opto-electronic devices on LiNbO\(_3\), the availability of optical switches/modulators, multiplexers and integrated optics was made possible.

Depending on the channel bit rate and the number of users, it is easy to realise that optical components do not exist that can accommodate a large number of users if each channel operates at moderate speeds. The fastest and most complicated TDM switches are limited to ~100 GHz [2]. However, practical OTDM systems only require the electronics to operate at the transmission rate of the individual tributaries rather than at the rate of the multiplexed signals.

\(^{1}\) This classification is not meant to be strictly rigorous: some of the techniques described could also be included in
The crucial multiplexing and demultiplexing functions can be performed in the electrical or the optical domain, as illustrated in Fig. 2.1. Electrical (de)multiplexing essentially (de)multiplexes several lower-speed channels into one high-speed channel, which is then used to modulate a single optical signal. It is just the direct replacement of other transmission media by a new transmission medium, the optical fibre. Performing (de)multiplexing in the optical domain is at the forefront of research: several lower-speed optical signals are time (de)multiplexed by a fast optical switch. The importance of this technology is that the individual tributaries can originate and terminate at locations far way from the switch, which is exactly the opposite when considering electrical TDM. According to [2] there are two major advantages of OTDM: (i) there is no output-port contention problem, and (ii) the implementation for low-speed photonic networks is quite simple and similar to electronic networks. On the disadvantages side: (i) issues related to network synchronisation, clock distribution and clock recovery, (ii) generation of high-speed optical pulses, (iii) ultra high-speed optical switches, and (iv) network control, stability and electronic processing become difficult and expensive at high-speeds.

![Diagram](image_url)

**Figure 2.1** Time-division multiplexing performed (A) electrically or (B) optically.

Some of the earlier OTDM work was done by R. Tucker *et al.* [3] in 1987, where they reported 16 Gb/s OTDM transmission over 8 km of fibre with a bit error rate (BER) less than $10^{-9}$. Several lasers were used as optical sources for the different channels. In 1989, Eisenstein *et al.* [4] used a different approach, where a single laser was used as the optical source for all channels. They were able to transmit an 8 Gb/s RZ (4 Gb/s x 2) signal over 57 km of single-
mode fibre. A third possibility for OTDM was presented in 1988, where a CW laser was used as the source and an optical switch driven at half the bit rate divided the light into two pulse streams. Two separate modulators with NRZ format modulated these two bit streams, and finally a second switch combined the two streams into one [5].

Because many successful OTDM demonstrations have taken place to date, it is likely to play a significant role in future optical communication systems. It is expected that the number of channels and aggregate bit rate will increase as OTDM technologies are maturing and improving. For example, OTDM transmission of 30 tributaries channels each at 40 Gbit/s over 85 km was reported in [6].

2.2.2 Wavelength-division multiplexing (WDM)

Since 1995, WDM technology has grown rapidly from tens of millions of dollars per year into an annual multibillion-dollar business with worldwide installations in both terrestrial and undersea environments [1].

In this technique several baseband-modulated channels are transmitted along a single fibre but with each channel at a different wavelength [7, 8]. It is by far the most active research area in optical communication systems today. The achievements in WDM have progressed at a phenomenal rate. For example, in 1995, a 160 Gb/s transmission was achieved by transmitting eight 20 Gb/s channels over 232 of standard single-mode fibre [9]. A few months later, the first WDM experiments to reach 1 Tb/s were reported at OFC’96, where 1 Tb/s (50 x 20 Gb/s) was demonstrated over 50 km [10] and 1.1 Tb/s (55 x 20 Gb/s) over 150 km [11]. Recently, transmission at 6.4 Tb/s (160 x 40 Gb/s) over 186 km of fibre was demonstrated [12].

After several years of research, the enabling technologies for the cost-effective implementation of WDM networks are coming of age. Several critical components needed to combine, distribute, isolate, and amplify optical signals at different wavelengths were created and perfected. It includes various passive and active optical devices, ranging from tunable optical filters [13-19], tunable sources [20-23], optical amplifiers [24-29], and WDM multiplexers [30-32].

Figure 2.2 depicts the basic implementation of such components in a typical WDM link [1]. At the transmitting side there are several independently modulated light sources, each emitting at a
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unique wavelength. It is followed by a multiplexer to combine all these optical signals into a single spectrum and couple them onto a single fibre. At the receiving end, the reverse operation is realised by a demultiplexer, which separate the optical signals for detection by appropriate receivers. In this design, many parameters must be considered carefully. These included transmitter source characteristics, optical amplifier performance, multiplexer-demultiplexer requirements, and receiver design (which may include optical filters).

![Diagram of an optical WDM link](image)

**Figure 2.2 - Diagram of a typical optical WDM link (after [1]).**

At the transmitter, the source wavelengths must lie within the useful gain of the amplifiers, and they must be stabilised to maintain channel spacing with acceptable accuracy. Regarding optical amplifiers their gain bandwidth must be expanded as well as its flatness improved for reducing their wavelength dependence. At the receiving end, demultiplexing is required. The critical parameters are centre frequency and adjacent channel rejection [33]. The centre frequency of the receiving filter and the source frequency must be aligned and remained aligned over the lifetime of the system. Once devices that operate at fixed frequencies and are stable under operating conditions, then passive filters can be built in order to allow WDM networks operation at standardised frequencies. The WDM standards developed by ITU specify channel spacing in terms of frequency (since WDM is essentially a frequency-division multiplexing at optical carrier frequencies). The ITU-T Rec. G.962 specifies selecting the channels from a grid of frequencies referenced to 193.100 THz (1552.524 nm in glass fibre) and spacing them 100 GHz (0.8 nm) apart. Suggested alternative spacing includes 50 GHz (0.4 nm) and 200 GHz (1.6 nm) [34].
### 2.2.3 Polarisation-division multiplexing

What is commonly called a “single-mode fibre” in reality has a split of the fundamental mode into degenerate modes orthogonal polarised, constituting the x- and y-components of the E field vector. These orthogonal modes can be used to simultaneously carry two independent channels, thus doubling the information throughput. The reasoning is that when a monochromatic radiation having a certain polarisation has imposed on it a change of SOP (state of polarisation) to the orthogonal SOP (antipodal position on the Poincaré sphere), this orthogonality survives the propagation process, and the received states of polarisation remain orthogonal.

There are two possible ways of carrying out polarisation-division multiplexing (PDM): the first is to employ a co-channel arrangement, in which two channels having the same carrier frequency are orthogonally polarised [10]; and the second is to employ polarisation interleave multiplexing, in which each channel is orthogonally polarised to the neighbouring channels [35]. P.Hill et al. [36, 37] was one of the earliest to demonstrate a polarisation-division multiplexing system.

An identical method to PDM is polarisation shift-keying direct-detection (PolSK-DD). In binary PolSK, one SOP represents a “0” and the orthogonal state a “1”. Each arriving SOP will be completely different from the state in which it was transmitted, but the two will be orthogonal to each other upon arrival at the receiver. Fukuchi et al. [38] demonstrated a single and dual channel 5 Gb/s PolSK-DD system over 1000 km of dispersion-shifted fibre (DSF) using nine erbium-doped fibre amplifiers (EDFAs). As a matter of fact, in [39] a WDM demonstration at 1.28 Tb/s using polarisation interleave multiplexing to improve system performance was reported.

### 2.2.4 Solitons

Soliton pulses in optical fibre were studied for the first time in 1973 [40] and were experimentally observed in 1980 [41]. These pulses have the peculiar behaviour of preserving their shape during propagation, assuming loss less dispersive fibres. Thus, solitons may be useful for long-haul transmission.

Soliton systems rely on the exact compensation of dispersion by fibre non-linearities [42-44]; therefore, they must maintain a fixed pulse energy throughout the link. Thus, optical amplifiers are absolute necessary for soliton systems. Solitons do not solve the problem of optical fibre losses, but they do overcome the limits set by fibre dispersion.
A major limitation in soliton systems is the so-called Gordon-Haus jitter [45]: a temporal jitter due to the random shifts in soliton frequency resulting from the non-linear interaction between noise and signal. Passive frequency filters [46, 47] were proposed to extend the limit set by the Gordon-Haus effect, but result in an exponential increase in noise around the signal wavelength. The noise accumulation, however, can be greatly reduced by sliding (slightly shifting) the filter’s centre frequency at each amplifier stage along the link [48, 49]: the soliton is able to follow the sliding; the linear amplifier noise is not. Others limitations do exist that should be taken into consideration: soliton interaction [43, 50], which determines the bit rate of a soliton system, and frequency chirp [43, 51], which is detrimental because it superimposes on the non-linearity-induced chirp and disturbs the exact balance between chromatic dispersion and the non-linearities necessary for solitons. Nevertheless, several techniques were developed to reduce these adverse phenomena, allowing the transmission of solitons over long distances [52, 53]. Although short spacing between optical amplifiers was once a concern for soliton systems, nowadays some demonstrations indicate that this is no longer a limitation [54]. For example, in [55] a 320 Gb/s soliton WDM system was demonstrated with 100 km amplifier spacing over 1100 km of standard SMF with dispersion compensation. Indeed, the potential capacity of soliton WDM systems is impressive. For example, 100 Gb/s (5 x 20 Gb/s) soliton WDM transmission has been demonstrated [56] over 10 000 km, which is a capacity of 1000 Tb/s.km.

### 2.2.5 Non-linear optical phase conjugation (OFC)

The broadening of optical pulses propagating in a fibre due to group velocity dispersion is a limiting factor in high-speed optical communication. A method for pulse narrowing following chromatic dispersion in a fibre is non-linear optical phase conjugation (OFC), also known as mid-span spectral inversion [57, 58]. It is commonly implemented by four-wave mixing (FWM) in a non-linear medium. Although the concept of OPC was proposed in 1979 [57], it was demonstrated in optical fibre transmission systems only in 1993 [59].

Figure 2.3 sketches the principle. The signal propagating down the first section of fibre of length L₁ suffers temporal pulse broadening. The phase conjugator performs complex conjugation on the (complex) individual frequencies amplitudes making up the pulse by spectral inversion about the centre optical frequency, which is that of a strong optical field used to “pump” the conjugation. The signal with a conjugate phase is then extracted by an optical filter and amplified, being propagated along a second span of fibre length L₂. As a result, fibre chromatic
dispersion, acting on the inverted spectrum signal, reverses the pulse broadening imparted by the first half, thereby restoring the original shape of the pulse.

![Spectral inversion](image)

**Figure 2.3 - Principle of spectral inversion by optical phase conjugation (after [60]).**

The two most pursued candidates as non-linear media for optical phase conjugation are DSF [59, 61] or semiconductor amplifiers [62, 63]. OPC was used to compensate 560 of standard single-mode fibre in a 2-channel WDM system with a bitrate of 10 Gb/s [64], and 40 Gb/s transmission over 202 km [65].

### 2.2.6 Dispersion compensating fibre and fibre Bragg gratings

Dispersion-compensating fibres (DCFs) are one of the most practical means of achieving dispersion compensation at present [66, 67]. The basic principle is that by joining fibres with chromatic dispersion of opposite sign and suitable lengths, an average dispersion close to zero can be obtained [68]. The negative dispersion regarding SMF is achieved by specially designed refractive index profiles. To get the desired dispersion coefficient requires an increase in the percentage of germanium oxide in the core and/or doping the fibre cladding with fluorine. As a result, DCFs are quite lossy, typically 0.32 to 0.5 dB/km. At present, dispersion of –80 to –100 ps/nm/km is typical for commercial DCFs. So the required lengths of compensating fibre are a significant fraction of the span length and loss. As an example, a 300-km span might require 50 km of compensating fibre with a loss of 18 dB; therefore, additional amplifier may be required. Nevertheless, although the DCF can be several kilometres long, it can be compacted in a reel. The reel can then be inserted at the transmitter, at the receiver, or at any point along the link. In addition, the core diameters of DCFs are typically less than 4 µm, so the non-linear coefficient
tends to be larger than in SMFs, which make them susceptible to non-linear effects. Despite these disadvantages and, additionally, the high cost of DCFs, compensating fibres are commercially available because they are simple passive devices that can be used over a very broad bandwidth. In [69] a 16-channel, 10 Gb/s, 10-nm wide system was reported in which 1000 km distance was spanned using DCF.

However, in standard DCFs (single-cladding structure), the slope in the wavelength dependence of the dispersion has the same sign (positive) as that of SMFs; thus only second-order dispersion can be compensated. By adding more cladding layers, it is possible to get DCFs with negative slope, which is particularly useful in WDM systems. A DCF of this type was used in a WDM experiment that achieved the transmission of 8 x 20 Gb/s channels over 232 km of conventional fibre [9].

A figure of merit (FOM) for DCFs is the ratio of dispersion to attenuation loss, expressed as an absolute value and in units of ps/nm/dB. Figures of merit up to 300 ps/nm/dB with dispersion near −100 ps/nm/km have been achieved [70, 71]. Commercial DCFs have FOMs in the range of 150-250 ps/nm/dB.

An alternative interferometric device that is currently attracting much attention is the chirped fibre Bragg grating (FBG) [72-74]. Gratings are written down the length of an optical fibre by periodically changing the refractive index of the fibre. Figure 2.4 depicts the principle of operation of a linearly chirped fibre Bragg grating.

![Figure 2.4 - Principle of operation of linearly chirped fibre Bragg grating ( after [60])](image)

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*Figure 2.4 - Principle of operation of linearly chirped fibre Bragg grating ( after [60]).*
The grating period is reduced linearly down the length of the device – the grating is “chirped” – so the resonance frequency depends linearly on the position along the grating. Therefore, different frequency components of the broadened pulse can be reflected at different points, accumulating a delay that varies linearly with the frequency. This delay is a key factor that determines the length of the device (typically a few centimetres). Most FBGs have the disadvantage that the compensated signal is retro-reflected, so an optical circulator is required to separate the input from the output. Nevertheless, these gratings offer low loss to non-resonant light passing through, so they are fitted for WDM systems by cascading several gratings down the fibre, each centred on a different wavelength. Dispersion compensation was demonstrated at two different wavelengths using 12-cm long chirped sampled Bragg gratings in a 240 km link a 10 Gb/s externally modulated transmitter [75]. In another experiment [76], using a 10-cm chirped fibre Bragg grating, a duobinary transmitter and EDFAs amplifiers, 10 Gb/s transmission up to distances of 700 km were demonstrated. The grating was inserted near the middle of the link span. It was also reported [77] an 11-cm un-chirped grating operating in transmission mode, thus avoiding the need for an optical circulator or coupler. Results showed 10 Gb/s transmission for 72 km of fibre with no power penalty and a distance of 106 km with a 2 dB power penalty. The suitability of chirped fibre Bragg gratings is advancing rapidly; at present, they are already commercially available.

2.3 Transmission format methods

2.3.1 Signal pre-chirping

One of the simplest techniques to compensate chromatic dispersion is signal pre-chirping [78]. In an ideal intensity-modulated system, the signal temporally broadens due to dispersion when propagating along a dispersive fibre in the anomalous region: the high-frequency components of the pulse, travelling faster, are pushed toward the leading edge of the pulse, while the low-frequency ones are pushed towards the trailing edge. For example, in a 10 Gb/s system at 1550 nm, with fibre dispersion of 17 ps/nm/km and regenerator span of 200 km, the net dispersion is 3400 ps/nm. For a chirp-free transmitter, an isolated mark requires ~0.1 nm of optical bandwidth and it will broaden by 340 ps during propagation along the regenerator span. Thus, the pulse will occupy 3 or 4 bit slots at the end of the span.

The basic idea of the pre-chirping technique is to transmit a pre-chirped pulse stream with a suitable phase modulation in order to compensate the above-described effect. If a “pre-chirped”
pulse, with lower-frequency components in the leading edge and higher-frequency components in the trailing edge, is transmitted in the fibre anomalous region, pulse width compression occurs in a first section of the link before pulse broadening starts to dominate. This behaviour allows longer distances to be bridged without regeneration.

Various techniques have been used to pre-chirp the transmitted signal. One simple method for generating a chirped bit stream is to add phase modulation through the use of an unbalanced Mach-Zender external amplitude modulator [79-81]. Another method is to frequency modulate the laser bias current, which is synchronised with the driving current of the external modulator using the same clock [82]. A third technique takes advantage of the fibre non-linearity [83]. The main advantage of pre-chirping resides in its conceptual simplicity, while the disadvantages are due to requirement of an external modulator coupled with a narrow line width laser, and a complete link redesign when some link parameters are changed, e.g., to up-grade the bit rate.

2.3.2 Dispersion-supported transmission (DST)

Another way to extend the dispersion-limited distance is to take advantage of the fibre chromatic dispersion. Dispersion-supported transmission (DST) [84, 85] is based on the frequency modulation of the optical signal that is converted into an amplitude modulation through the transfer function of the dispersive fibre, a photodiode (for direct detection) and a simple first-order low-pass filter [86] or a special decoding circuit with two threshold detection and one bit memory [87].

Fig. 2.5 summarises the principle of operation of the DST technique: pulse shapes of the transmitter driving signal $I$, the optical frequency $\nu$ and the optical power $P_{opt}$ are sketched in part (a). Optical power is assumed constant. The signal frequency can assume two different values: $\nu$ and $\nu + \Delta \nu$. Due to fibre dispersion, the different signal frequency components are delayed differently- see Fig. 5(b): as a result interference occurs, and the frequency modulation is converted into amplitude modulation of the optical signal, as sketched in Fig. 5(b). At the fibre end, $P_{opt}$ is no longer constant and can assume three different levels: an intermediate one corresponding to the input level, a positive pulse due to constructive interference, and a negative pulse due to due to the absence of the signal.
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Figure 2.5- Principle of dispersion-supported transmission: (a) transmitter signals, (b) receiver signals after dispersive fibre of length L (after [85]).

In the ideal case, the residual intensity modulation at the input of the link is kept low. Thus, the impact of non-linear Kerr effect remains low in the first fibre span. More than 180 km of standard SMF have been demonstrated without in-line optical amplifiers [88] and larger distances can be bridged by adding in-line amplifiers. The noise in the optical receiver is reduced due to the DST low-pass filter. However, the eye-opening of DST signals is lower than with other transmission formats. Thus, power sensitivities are not low, and DST is more susceptible to optical signal-to-noise degradation in amplified systems. Nevertheless, very good performance has been recorded during field experiments with successful transmission over 160 km with one in-line amplifier and even 240 km with two amplifiers [89]. Recently, a generalisation of DST was proposed and demonstrated at 10 Gb/s over 305 km of SMF, using optical EDFAs as booster, in-line amplifiers (two) and a receiver with optical pre-amplifier [90]. The technique allows a multiple bit shift, potentially leading to an extension of the dispersion limited link length.

Dispersion-supported transmission is a linear technique but fibre non-linearity and residual amplitude modulation at the transmitter can be effective in further increasing the transmission distance as investigated in [91], leading to the so-called dispersion-mediated non-linearity-enhanced transmission.
This technique avoids the use of an external modulator and potentially reduces the system cost. The main limitation is that the frequency modulation depth must be tuned to match the span length.

### 2.3.3 Partial-response signalling (PRS)

Up until the 90's, spectral efficiency was not an issue in optical fibre communications. As bit rates and the number of channels increase in optical fibre systems, transmission capacity started to be limited by the EDFA gain bandwidth (~35 nm, or, equivalently, 4.4 THz). To pack Tb/s into this gain bandwidth, spectrum efficiency has to be improved to more than 0.23 b/s/Hz.

Ideas from communication theory developed originally for microwaves and electrical signals are often applied to optical signals. Format schemes such as *partial-response signalling (PRS)* [92] were a way of achieving more efficient use of the channel bandwidth. PRS (also known as *correlative-level coding* [93, 94]) allows a controlled amount of inter-symbol interference that can be used to shape the system spectrum. This is done by introducing correlation over a span of a few bits in the original sequence into the transmitted symbol.

A. Lender [95] first introduced duobinary format as a data transmission method: it achieved the symbol rate of 2B symbols/s in a bandwidth of B Hz. Thus, the correlative schemes conceived by Lender, and later generalised by Kretzmer [96] to other PRS schemes, can be regarded as a practical means of achieving the theoretical maximum symbol rate of 2 symbols/s/Hz postulated by Nyquist, using realisable and perturbation tolerant filters [94]. Fig. 2.6 shows a method of generating the overall transfer function $H(f)$ of a PRS system [92].

![Figure 2.6- General PRS system model.](image)
The system consists of a tapped delay line with coefficients $\{x_k\}$ in cascade with a filter with frequency response $G(f)$. To maximise the bit rate in the available bandwidth, $G(f)$ must correspond to the Nyquist filter [92]. Note the separation of the PRS system into two parts: $X(f)$, which is periodic, forces the desired sample values; $G(f)$ may be used to band-limit the system transfer function, while preserving the sampled values\(^2\). The transfer function $X(f)$ of a correlative system has the following general form [92]:

$$X(f) = \sum_{k=0}^{N-1} x_k e^{-j2\pi ft}$$ (2.1)

where $T$ is the symbol spacing. By choosing judiciously various combinations of integer values for the $\{x_k\}$, different useful shapes of $H(f)$ can be obtained. For example, if $f_0 = f_1 = 1$ are the only two non-zero coefficients, the digital filter adds the previous symbol to the present symbol. This scheme is known as duobinary or class 1-partial response. If, on the other hand, the only coefficients are $f_0 = 1$ and $f_2 = -1$, the digital filter subtracts the second previous symbol from the present one. In this case, the scheme is called modified duobinary or class 4-partial response.

Another consideration when implementing a practical PRS system is the number of output levels: a compromise must be struck between the circuit complexity and the required data rate. A PRS system with $M$ non-zero pulse samples will have $m^M$ output levels for $m$-ary input unless there are special relationships between the sample values [92]. For binary input, the systems usually have either 3 or 5 output levels.

Several approaches for using PRS formats in optical fibre systems have been demonstrated. For example, regarding duobinary format (the most known PRS scheme applied to optical signals) they range from using a duobinary receiver in conjunction with a binary transmitter [97, 98], duobinary intensity modulation at the transmitter [99, 100], and optical AM-PSK duobinary signalling [101-103]. Much of the research presented in this Thesis evaluates the potential of PRS systems using optical fibre. In particular, only systems with three electrical levels will be considered since these are the most suitable for optical communications, as will be explained in the next chapter.

2.3.4 Multilevel signalling format

Multilevel techniques employ $M$ discrete signalling levels and represent $\log_2 M$ binary channels. Although the concept of multilevel signalling is not new, it has not received much attention

\(^2\)This separation is artificial; in actual implementations it may be organised quite differently.
concerning its application in optical fibre systems. The main reason is that multilevel signalling decreases the eye opening. For a fixed power, a \( M \)-level system has approximately \( 20\log_{10}(M-1) \) dB penalty relative to binary; however, at the same time decreases the symbol rate by a factor of \( \log_2 M \). As a result, for a given symbol error rate, multilevel signalling introduces an optical power penalty relative to binary of \( 10\log_{10}[(M-1)^2/\log_2 M] \) dB in direct detection systems. For example, a four-level signal requires three times the transmitted optical power of a binary signal (on-off) to obtain the same separation between received levels.

Practical implementation difficulties of multilevel signalling transmitters and receivers are also an important disadvantage in considering these systems. Few experimental demonstrations were reported [104-106], and all of them just used 4-level signals due to the excessively large optical power required at the receiver. An in-depth investigation about multilevel signalling at 10 Gb/s is reported [106]. It is stated that 4-ary ASK signals may be suited for distances between 200 and 350 km; however, the signal requires about 8 dB more power and amplifier spacing of no more than 80 km, as well source line width less than 1 MHz. At present, and for the foreseeable future, multilevel signalling systems are not a viable option for optical fibre communication systems.

2.4 Summary

This chapter gave a basic overview of some techniques and methods to increase capacity in high-speed optical fibre systems. They range from multiplexing (either in time, wavelength, or polarisation) to techniques to combat critical performance degrading effects limiting the system capacity. They exhibit a variety of different and often complimentary properties. Therefore, combination of several techniques is likely to be encountered in practical systems. Nevertheless, those carried out on the electrical side of the optical link are desirable, since they offer potentially a larger degree of freedom and flexibility regarding system upgrades. Among these, PRS systems may become very important because they are easily implemented and allow fibre transmission spans of about 200 km, which is sufficient for a large fraction of the world’s terrestrial links. Thus, next chapter will thoroughly investigate partial-response signalling in optical fibre systems.
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Chapter 3: Optical modulation formats

In this chapter the theoretical foundations regarding PRS signalling schemes are laid down. Discussion is focussed to three-level PRS techniques, namely, duobinary, modified duobinary and dicode. As will be shown later on in the chapter, and was previously known, only these schemes (or suitable variants of these formats) are deemed adequate for optical communication systems due to a clever use of external modulators at the optical transmitter (see chapter 4). It will also explore the fundamental differences among them, including power spectral density (PSD), precoding and decoding, and potential advantages and/or limitations.

Before delving into the subject, a brief note regarding the concepts of coding and modulation is required. Coding and modulation are operations performed at the transmitter to achieve efficient and reliable information transmission. Coding can be defined as an "act of transforming or translating information into signs or symbols". Coding can be described as a symbol-processing operation, i.e., a logical operation. On the other hand, modulation is a signal-processing operation, i.e., mapping the symbol information sequence into signal waveforms adequate to the physical medium. It involves two waveforms: a modulating signal that represents the information, and a carrier that suits the communication channel. Therefore, a particular code can be associated with several types of modulation (also called formats). For example, when one speaks of "a NRZ binary signal" that means coding is binary and modulation is non-return-to-zero (NRZ) in amplitude. Thus, signals are the result of information coding and modulation of some physical parameter (e.g., electrical current or intensity of light).

3.1 Theoretical foundations

3.1.1 Introduction

Perhaps the most important function of modulation is to transmit a base band signal $a(t)$ over a band pass channel centred at $f_c$. For the purpose of this discussion, it is assumed that the optical carrier $c(t)$ is of the form

$$c(t) = A \cos[2\pi f_c t + \theta(t)]$$

(3.1)

where $A$ is the amplitude, $f_c$ is the carrier frequency, and $\theta(t)$ is the arbitrary phase, uniformly distributed in $[-\pi, \pi]$. Unless otherwise stated, this random phase offset is arbitrarily set to zero. One can translate the spectrum of $a(t)$ by multiplying it with the carrier $c(t)$,
\[ s(t) = c(t)a(t) = A a(t) \cos(2\pi f_c t) \] (3.2)

It is assumed that \( s(t) \) can represent all the modulated signals to be considered in this chapter. It can be shown [1] that the signal \( s(t) \) has a power spectral density of

\[ S_s(f) = \frac{A^2}{4} \left[ S_a(f - f_c) + S_a(f + f_c) \right] \] (3.3)

Eq. (3.3) shows that \( s(t) \) is a band pass signal and its spectrum is simply that of \( a(t) \) shifted up and down in frequency by an amount equal to \( f_c \). Therefore, it is sufficient to evaluate the PSD of the base band signal to know the PSD of the modulated carrier \( s(t) \).

### 3.1.2 Power Spectral Density

Consider a digital baseband modulating signal \( a(t) \) represented in the general form as a superposition of identically-shaped, randomly-weighted pulses

\[ a(t) = \sum_{k=-\infty}^{\infty} a_k g(t-kT) \] (3.4)

where \( T \) is the time between pulses (symbol period), \( g(t) \) is the pulse shape, and \( \{a_k\} \) represents the information sequence of discrete values. Bennett [2] has demonstrated that the PSD of such a signal is composed of a continuous component and a discrete component, consisting of a set of line spectral components:

\[ S_a(f) = S_c(f) + S_d(f) \]

\[ = \frac{1}{T} |G(f)|^2 \sum_{k=-\infty}^{\infty} R_a(k)e^{-j2\pi kfT} + \frac{1}{T^2} m_a^2 \sum_{k=-\infty}^{\infty} \left[ G\left( \frac{k}{T} \right) \right]^2 \delta \left( f - \frac{k}{T} \right) \] (3.5)

where \( m_a \) is the mean, \( G(f) \) is the Fourier transform of \( g(t) \), and \( R_a(k) \) is the ensemble auto-correlation function of the symbols \( \{a_k-m_a\} \). \( R_a(k) \) is given by

\[ R_a(k) = E\left[ (a_n-m_a)(a_{n+k}-m_a) \right] = E\left[ a_n a_{n+k} \right] - m_a^2 \] (3.6)

where \( E[ \cdot ] \) denotes the expected value. If the mean \( m_a \) is null or \( G(k/T)=0, \forall k \), then \( S_d(f) \) is null. Notice that the line spectral terms occur at harmonics of the signalling frequency and have a power that is proportional to \( |G(f)|^2 \) evaluated at \( f=k/T \).

Regarding the continuous component, Eq. (3.5) illustrates the dependence of the PSD of \( a(t) \) on the spectral characteristics of the pulse \( g(t) \) and on the correlation characteristics of the information sequence \( \{a_k\} \). The rationale of PRS formats is to change \( \sum R_a(k)e^{-j2\pi kfT} \) in order to shape the power spectrum of \( a(t) \) and match it to the optical fibre frequency response.
To bring out this potential for statistical spectral shaping, the continuous component of Eq. (3.5) can be rewritten in the more familiar form

\[ S_c(f) = \frac{1}{T} |G(f)|^2 \left[ R_a(0) + 2 \sum_{k=1}^{\infty} R_a(k) \cos(2\pi kfT) \right] \]  

having drawn upon the property \( R_a(-k) = R_a(k) \).

### 3.2 PRS signalling formats

The PRS signalling schemes presented in this chapter are three-level electrical formats that still represent the input two binary states without any economy in the symbol time. For this reason they are part of a broad category of three-level formats better known as pseudo-ternary code formats [3]. They are symbol-by-symbol formats.

![Diagram of PRS signalling waveforms](image)

**Figure 3.1 - Example of PRS signalling waveforms.**

In Figure 3.1 the waveforms of the three PRS formats to be analysed are introduced. Binary NRZ is used as reference. It can be noticed from Figure 3.1 that three additional waveforms are sketched, identified with the appended words “...w/ precoding”. An inherent weakness of these signalling techniques lies on their principle of operation: because correlation is introduced among symbols in the original binary sequence, it would seem logical at the receiver that recovering a binary symbol would require an observation of several detected symbols. This also suggests that an error in a decoded symbol could propagate, and produce additional errors in the decoded sequence. This problem has been solved by precoding the input binary stream, as will be explained later [4].

#### 3.2.1 Duobinary

The duobinary modulation format is intended to increase the data transmission efficiency by means of coding. According to A. Lender [4], who discovered the concept of correlative
transmission techniques, duo means doubling the binary capacity by bandwidth compression. This format, in its coding process, takes advantage of all three levels. The duobinary format is achieved according to the following mathematical equation,

$$ b_n = a_n + a_{n-1} $$

(3.8)

where $a_n$ is the data source bit and $b_n$ is the three-level coded bit. The magnitude of the past adjacent source bit $a_{n-1}$ is algebraically added to the magnitude of the present source bit $a_n$. The resulting bit stream is normalised to ±1 and 0, as shown in Figure 3.2.

![Figure 3.2- Construction of a duobinary signal from a binary signal.](image)

The duobinary with precoding is related to the duobinary coding circuit. It is constructed by a precoder circuit, which acts on the source data, followed by the duobinary circuit of Figure 3.2. It takes advantage of all the three possible amplitude levels: the mark state is represented by the zero (or centre) level; the space state is represented either by a positive (or top) or a negative (or bottom) level. This scheme assures that the only permitted transitions in successive bits are between any two adjacent symbols. These patterns follow the unique rule [5]: the polarities of two successive bits at the extreme levels are opposite if the number of in-between bits at the centre level is odd; otherwise, no level transition occurs (see Figure 3.1). The result of duobinary formatting is the redistribution of the spectral density of the original binary data into a highly concentrated energy density at low frequencies in the base band process, or at the carrier frequency when considering a band pass signal.

**3.2.2 Modified Duobinary**

Modified duobinary is another PRS scheme that involves a correlation span of two digits. Mathematically, the modified duobinary is defined by the following equation

$$ b_n = a_n - a_{n-2} = a_n + a_{n-2} $$

(3.9)

where again $a_n$ is the data source bit and $b_n$ is the coded bit. Implementing this format can be achieved by delaying the data source stream by two time periods. The past bit $a_{n-2}$ is subtracted from the present bit $a_n$, and then the coded bit has levels ±1 and 0, according to Figure 3.3. Instead of the subtraction operation it is possible to employ the addition operation by simply
complementing the delay bit state (\(a_{n-2}\)). This is allowed since the data source bits have only two states.

![Figure 3.3- Coding circuit of the modified duobinary format.](image)

The modified duobinary with precoding is constructed by the binary source data, the precoder circuit, and the modified duobinary circuit of Figure 3.3. The principle of operation is the following: the pulse train is divided into odd and even bits. Both odd and even pulse trains follow the same rule [5]: two successive bits at the extreme levels always have opposite polarity (see Figure 3.1). Contrary to the duobinary format, transition from the top level to the bottom one or vice versa can occur. As a consequence, the width of the eye-pattern diagram is narrower, which indicates an increase in inter-symbol interference (ISI) and somewhat reduces the margin to impairments. As well as duobinary, modified duobinary allows a 2:1 bandwidth compression. In the base band form, the zero-frequency component is eliminated and the energy is centred at a frequency of \(f = 1/T\) Hz, where \(T\) is the symbol period. When considering the band pass process, energy density is concentrated at \(f_c \pm B/2\).

### 3.2.3 Dicode

Dicode, also known as twin-binary, is another modulation scheme that involves a correlation span of one digit. The dicode format is implemented according to the following mathematical equation,

\[
b_n = a_n - a_{n-1} = a_n + a_{n-1}
\]  

(3.10)

It can be seen that dicode format is similar to the modified duobinary format already discussed, except that here the delay concerns only one time period. Figure 3.4 shows the implementation of the dicode format.

![Figure 3.4- Construction of dicode format from binary data.](image)

It is a DC-free format where a positive level is transmitted whenever a state change from space to mark occurs, whereas a negative level occurs when there is a state change from mark to space.
From Figure 3.1, in the last waveform, corresponding to the case of dicode with precoding, the binary symbol 0 is represented by no signal and 1 is represented alternately by positive and negative pulses. But that is the known alternate mark inversion (AMI) code [6]. It is a DC-free format, with more level transitions than the dicode scheme, and contains little low frequency components. It occupies the same bandwidth as binary NRZ signals.

### 3.2.4 Precoding

Assuming symbol-by-symbol detection, for duobinary signalling the samples at the output of the receiver demodulator have the form

\[ r_n = B_n = A_n + A_{n+1} \]  \hspace{1cm} (3.11)

where \( \{A_n\} \) is the transmitted sequence of pulses. NRZ signalling is used for logical-to-waveform mapping. The mapping is given by

\[
\begin{align*}
0 & \rightarrow Ap(t) \\
1 & \rightarrow -Ap(t)
\end{align*}
\]

where \( \{A_n\} \) is assumed equal to the source data sequence \( \{a_n\} \). The assumed pulse is the unit rectangular pulse

\[
p_r(t) = \begin{cases} 
1 & 0 \leq t \leq T \\
0 & \text{elsewhere}
\end{cases}
\]

Considering binary input stream, \( \{a_n\} = 0, 1 \), with equal probability, then \( \{b_n\} \) takes on one of the three possible values, namely, \( b_n = 0, 1, 2 \) with corresponding probabilities \( \frac{1}{4}, \frac{1}{2}, \) and \( \frac{1}{4}, \) respectively. If \( a_{n-1} \) is the detected symbol from the \( (n-1)th \) signalling interval, then its effect on \( b_n \), the present received symbol, can be eliminated by subtraction, thus allowing to recover \( a_n \), the original data sequence

\[ a_n = b_n - a_{n-1} \]  \hspace{1cm} (3.12)

This procedure can be used recursively for every received symbol. The major problem with this process is that errors, arising from incorrect received symbols due to noise, tend to propagate. For instance, if \( a_{n-1} \) is in error, its effect on \( b_n \) is not eliminated but, in fact, it is reinforced by the incorrect subtraction. Therefore, additional errors can be produced.

**Precoding** is a technique to eliminate the error-propagation problem of symbol-by-symbol decoding. It was first introduced by Linder [4, 5]. The precoder eliminates the effect of previous symbols at the source where they are known precisely. Decisions at the output decoder are then made independently of one another. The precoding operation is performed on the binary data
source sequence. From the data sequence \( \{a_n\} \) of 1s and 0s, a new sequence \( \{p_n\} \) also of 1s and 0s, called the *precoded sequence*, is generated. For duobinary signalling, the precoded sequence is defined as

\[
p_n = a_n \odot p_{n-1}
\]  

(3.13)

where \( \odot \) denotes modulo-2 subtraction\(^1\). But since \( a_n = p_n \oplus p_{n-1} \) and from Eq. (3.11), it follows that the data sequence \( \{a_n\} \) is obtained from \( \{b_n\} \) using the relation

\[
b_n = a_n \pmod{2}
\]  

(3.14)

provided \( b_n \) is interpreted in a modulo-2 manner, i.e., level 2 = 0 modulo-2. Consequently, if \( \{b_n\} = 0,2 \) then \( \{a_n\} = 0 \), whereas \( \{b_n\} = 1 \) represents \( \{a_n\} = 1 \). Thus, the recovered data sequence is just the complement of the original binary data sequence. The most important feature to retain from precoding is that every bit in \( \{b_n\} \) is decode into \( \{a_n\} \) without resorting to a prior bit or bits. Therefore, there is no possibility of error propagation.

Considering modified duobinary format, Eqs. (3.11) and (3.13) should be rewritten as

\[
b_n = a_n - a_{n-2}
\]  

(3.15)

with \( b_n = -1, 0, +1 \), and

\[
a_n = p_n \oplus p_{n-2}
\]  

(3.16)

Again, \( b_n \equiv a_n \), provided \( a_n \) is interpreted as modulo-2. It is a simple exercise to get the precoding scheme for dicode signalling.

Due to the use of a precoder and an PRS signalling scheme in the transmitter, an important and interesting situation occurs when the coded signal assumes anyone of the three possible values \( \{-1, 0, +1\} \). In this particular case, decoding at the receiver is very simple,

\[
a_n = |b_n|
\]  

(3.17)

Therefore in the case of optical communication systems, at the receiver side, direct-detection (square-law) photo-detector receivers for binary signalling may be used without any modification. This is just the case for the three PRS signalling formats studied in this Thesis\(^2\).

---

\(^1\) For binary numbers, modulo-2 subtraction is equivalent to modulo-2 addition, which is equivalent to an Exclusive-OR operation.

\(^2\) It should be mentioned that for duobinary signalling a proper biasing at the transmitter is required to get the signal levels \( \{-1, 0, +1\} \).
3.2.5 Power Spectral Densities and eye-diagrams of PRS formats

3.2.5.1 NRZ Binary Format

In NRZ binary signalling, the elements of the information sequence \( \{a_k\} \) are either 0 or 1. Consider that the information symbols in the sequence are mutually uncorrelated, occur with equal probability, and the mean \( m_a \neq 0 \). Then, the autocorrelation is given by

\[
R_a(k) = \begin{cases} 
\sigma_a^2 + m_a^2 & k = 0 \\
m_a^2 & k \neq 0 
\end{cases}
\]

(3.18)

where \( \sigma_a^2 \) denotes the variance of an information symbol. Applying Eq. (3.5) yields the desired result for the power spectral density

\[
S_{NRZ\ Binary}(f) = \frac{1}{T} \sigma_a^2 |G(f)|^2 + \frac{1}{T^2} m_a^2 \sum_{k=-\infty}^{\infty} G\left(\frac{k}{T}\right)^2 \delta \left( f - \frac{k}{T} \right)
\]

(3.19)

If the pulse shape \( g(t) \) is assumed to be a rectangular pulse with amplitude equal to \( A \) during the interval \( T \), the Fourier transform of \( g(t) \) is

\[
G(f) = AT \frac{\sin(\pi fT)}{\pi fT} = AT \text{sinc}(fT)
\]

(3.20)

The amplitude spectrum shows that it contains zeros at multiples of \( I/T \) in frequency. As a consequence of the spectral zeros in \( G(f) \), all but one of the discrete components in Eq. (3.19) vanish. Thus, upon substitution for \( |G(f)|^2 \) in Eq. (3.19) the PSD of a NRZ binary format, assuming rectangular pulses, is

\[
S_{NRZ\ Binary} = \frac{A^2 T}{4} \text{sinc}^2(fT) + \frac{A^2}{4} \delta(f) = \frac{I}{2} \left( T \text{sinc}^2(fT) + \frac{A^2}{4} \delta(f) \right)
\]

(3.21)

where the pulse amplitude is related to the total average power by \( A = \sqrt{2I} \). Half of the power is in the carrier frequency (represented by the term \( \delta(f) \)).

Both the analytical and simulated spectra are shown in Figure 3.5. The simulated spectra was generated from a pseudo-random binary sequence (PRBS) of length \( 2^7-1 \) with 32 samples per bit for a total of 1024 bits.

Note that the first nulls occur at \( \pm 1/T = \pm B \) where \( B \) is the bit rate, and \(-90\%\) of the power that makes up the continuous part of the spectrum is contained in the main lobe. Thus, signal transmission requires a bandwidth of roughly \( \pm B \), i.e., \(-2B\), about the carrier.
3.2.5.2 DUOBINARY FORMAT

A duobinary signal is the sum of a binary signal with a delayed version of itself, according to Eq. (3.8). With no loss of generality, assume that $\{a_k\}$ is a sequence of uncorrelated random variables that takes the binary values (-1,1), and occur with equal probability. Also suppose that $g(t)$ is a rectangular pulse whose amplitude spectrum is $|G(f)|^2 = (AT)^2 \text{sinc}^2(fT)$. Thus, the three-level sequence $\{b_k\}$ can assume the values (-2,0,2) with probabilities $P(b_k=2)=P(b_k=-2)=1/4$ and $P(b_k=1)/2$. Then, the autocorrelation of a zero mean duobinary signal is

$$R_a(k) = E[b_nb_{n+k}] = E[(a_n+a_{n+1})(a_{n+k}+a_{n+k+1})]$$

$$= 2\delta(k) + \delta(k+1) + \delta(k-1)$$

having drawn upon that $E[a_na_{n+k}] = \delta(k)$. Therefore,

$$R_a(k) = \begin{cases} 
2 & k = 0 \\
1 & k = \pm 1 \\
0 & |k| \geq 2 
\end{cases}$$

By using Eq. (3.7), the power spectral density is

$$S_{NRZ \text{ Duobinary}} = 2A^2T(1+\cos2\pi fT)\text{sinc}(fT)$$

$$= 4A^2T\cos^2(\pi fT)\text{sinc}(fT)$$

(3.24)

where the trigonometric identity $\cos^2x = 1/2 + 1/2\cos2x$ was used. Both the analytical and simulated spectra for NRZ Duobinary signals are shown in Figure 3.6.
Figure 3.6- Analytical (dashed) and simulated (jagged lines) of duobinary signalling at 10 Gbit/s for rectangular pulses.

Notice the 2:1 spectrum compression compared to the NRZ binary format, as well as the absence of discrete component at the carrier frequency. As with NRZ binary signal, ~77% of the power is inside the main lobe; therefore, there is a high concentration of energy density at the carrier frequency. Nevertheless, the duobinary signal cannot be transmitted in a bandwidth bounded by the nulls of the main lobe, as was the case with a binary signal. This can be explained when considering the duobinary signal in the time domain for which the symbol period is equal to $T$, regardless of the three levels (a characteristic of pseudo-ternary code formats). The pulse requirements of the transmission channel to obtain no intersymbol interference (ISI) must still be satisfied, i.e., Nyquist’s criterion for zero ISI must hold, which relates the required transmission bandwidth to the symbol period, not the number of levels in the signal.

**3.2.5.3 Modified Duobinary Signalling**

Modified duobinary involves a correlation span of two digits; mathematically, it is defined by Eq. (3.9). Retaining all the assumptions made in the case of duobinary signalling, the autocorrelation of a modified duobinary signal is

$$R_n(k) = E[b_nb_{n+k}] = E[(a_n-a_{n+2})(a_{n+k}-a_{n+k+2})]$$

$$= 2\delta(k) - \delta(k+2) - \delta(k-2) \quad (3.25)$$

Thus,

$$R_n(k) = \begin{cases} 2 & k = 0 \\ -1 & k = \pm 2 \\ 0 & \text{otherwise} \end{cases} \quad (3.26)$$
Using Eq.(3.7), an expression for the spectrum of the NRZ Modified Duobinary signal can be obtained:

\[
S_{NRZ \text{ Modified Duobinary}} = 2A^2T \left( 1 - \cos 4\pi fT \right) \text{sinc}(fT) \\
= 4A^2T \sin^2(2\pi fT) \text{sinc}(fT)
\]  

having drawn upon the trigonometric identity \( \sin^2 x = 1/2 - 1/2 \cos 2x \). Notice that there is no \( \delta(f) \) term which means the discrete component vanishes at the carrier frequency. This is due to the fact that the coded information symbols have zero mean. Both the analytical and simulated spectra for NRZ Modified Duobinary signals are shown in Figure 3.7.

Again, as with duobinary signal, there is a 2:1 spectrum compression. There is also a redistribution of energy density, which is almost eliminated at the carrier frequency.

![Figure 3.7- Analytical and simulated spectra for NRZ modified duobinary signals.](image)

### 3.2.5.4 DICODE FORMAT

The generation of the dicode signal is similar to modified duobinary, except that it is created by correlating adjacent bits of a binary signal. The result is a signal that has levels (-1,0,+1). Eq. (3.10) conveys this creation process. Considering the assumptions made for the PRS formats already analysed, the autocorrelation of a dicode signal can be expressed as
\[ R_a(k) = E[b_n b_{n+k}] = E[(a_n - a_{n+1})(a_{n+k} - a_{n+k+1})] \\
= 2\delta(k) - \delta(k+1) - \delta(k-1) \\
= \begin{cases} 
2 & k = 0 \\
-1 & k = \pm 1 \\
0 & |k| \geq 2 
\end{cases} \quad (3.28) \]

Applying Eq. (3.7) to obtain the power spectral density

\[ S_{\text{NRZ Dicode}} = 2A^2T \left(1 - \cos 2\pi f T\right) \text{sinc}(f T) \]
\[ = 4A^2T \sin^2(\pi f T) \text{sinc}(f T) \quad (3.29) \]

Both the analytical and simulated spectra for NRZ dicode signals are shown in Figure 3.8.

Notice the absence of discrete component at the carrier frequency, as well as no spectrum compression.

![Figure 3.8](image)

**Figure 3.8**- Analytical and simulated spectra for NRZ dicode.

### 3.2.5.5 **Eye-diagrams**

Eye diagrams are used to determine the quality of a system’s response to a digital signal. An eye-pattern for a random input pulse stream is formed by superimposing the received pulse stream over a 1-symbol interval. This is done experimentally by displaying the received electrical signals on an oscilloscope synchronised externally with a clock set at \( I/T \) Hz, where \( T \) is the symbol rate. Eye-diagrams results will often be used to assess system performance throughout this Thesis.

Figure 3.9 shows eye-diagrams for the investigated PRS signals in the absence of noise. It also displays the eye-diagram for NRZ binary signals, used as reference. It assumes an ideal NRZ
pulse that has been filtered by a 5\textsuperscript{th} order Bessel filter with 3-dB points according to signal spectrum bandwidth: ~\(B\) for NRZ, and dicode signals; ~\(B/2\) for duobinary, and modified duobinary. The filter was included to improve readability of the displayed eye-diagrams, and somewhat to simulate band limiting as in real communication systems due to frequency response of various components.

It can be observed from Figure 3.9 that the PRS signals show clearly three-electrical levels: -5, 0, and +5 (arbitrary units). Furthermore, the permitted transitions for the different signals are also evident, according to their coding rules. For example, in duobinary format, no direct transition from the upper to the bottom level is allowed, i.e., the only permitted transitions in successive bits are between any two adjacent symbols. Also notice that the width of the eye-pattern diagram is narrower for modified than duobinary, which indicates an increase in intersymbol interference (ISI).

![Eye-diagrams](image)

Figure 3.9- Examples of eye-diagrams for NRZ binary, duobinary, modified duobinary, and dicode.
3.3 Summary

This chapter gave a basic description of the signalling schemes explored in this work. These include conventional binary NRZ, duobinary, modified duobinary, and dicode as PRS formats suitable for optical fibre systems. Calculating the PSD for these signals was explained. The motive for exploring these techniques is that the energy density in the transmitted signal is re-distributed around the optical carrier. It is this spectral shaping that brings some interesting properties regarding the optical field propagation along the fibre.

As a first step (and sometimes the only affordable one), simulations must be carried out to assess performance of the various signalling schemes. Thus, for modelling the optical system, a simulation tool must be used, incorporating the necessary sophisticated algorithms and complex models. Therefore, the next topic concerns system simulation. A brief description of the simulator, as well as the required mathematical models of key components used in an optical fibre system are the subject of the next chapter.
References:

Chapter 4: System simulation

4.1 Introduction

The simulation of binary or PRS signals in an optical fibre transmission system involves modelling of the generation, propagation and reception of the transmitted signal. The trade-off of any simulation is between accuracy and time. Usually it requires a large amount of time and resources, both financial as well as human, to research, develop, test and improve the complex mathematical models required to implement an optical system simulator. Due to the availability of commercial optical system simulators, with sophisticated simulation algorithms, easy-to-use graphical user interfaces, and reasonable prices, it was decided to use one of such packages. It was chosen OptSim\(^1\) from ARTIS Software Corp. Several reasons have concurred for this option: a unique approach to the Split-Step Fourier (SSF) method, which is widely used to solve numerically the non-linear partial differential equation that describes the propagation of an optical pulse in single-mode fibres, and that is the core of the simulator; a stand-alone simulation tool; easy-to-use graphical user interface with a Windows\(^2\)-like appearance; extensive component library, with possibility of creating custom components; good references from other research group with some experience using this tool; and last, but not least, an interesting price for academic institutions.

This chapter will give a brief overview of the simulator, and a description of the models used for important systems components including laser, external modulator, the single-mode fibre, and optical detector. It ends by discussing a generic model used in the simulations, and topics related to system performance assessing.

4.2 The OptSim\(^\text{®}\) simulator

OptSim is a high-end optical system simulator for professional engineering and advanced research of WDM, CATV, and other emerging optical systems. It is a stand-alone tool that includes a Windows-like user interface. The OptSim architecture is composed of the following three main parts [1]:

- A graphical editor that is the principal interface with the user.
- A software engine that performs all the numerical computation:

\(^1\) OptSim and ARTIS are trademarks of ARTIS Software Corporation.
\(^2\) Windows is a trademark of Microsoft Corporation.
• A data post-processing and display system that allows the manipulation and display of the simulation results.

Figure 4. 1 shows the OptSim editor window (A) and numerical results displayed either as a small text window or as a graphical measurement window (B).

![Figure 4. 1- The OptSim editor window (A) and the simulation results either as a text or graphical window (B).](image)

At the core of the OptSim simulator is the fibre simulation based on the Time Domain Split-Step (TDSS) method that takes into account all linear and non-linear effects. It will be detailed in section 4.3. In order to optimise both the accuracy and the computational effort, OptSim supports an incremental simulation approach with two different techniques:

- The spectral propagation technique (SPT) where only the power spectrum is propagated and linear effects are considered. Only applies to optical components.
- The variable bandwidth simulation technique (VBS) where full vector signal is propagated, thus taking into account all linear and non-linear effects and all system components.

### 4.3 Component modelling and related topics

In this section a brief description is given of the models used for important system components including laser, optical fibre, external modulator, and detector. A key component is the optical fibre itself: the coupled non-linear Schrödinger equations, taking in account all linear and non-linear effects (with the exception of the stimulated Brillouin scattering effect) will be explained. Another key component is the amplitude dual-arm Mach-Zender modulator. It is critical for the generation of optical PRS signals. It will also be discussed extensively. Finally, a short description of the optical detector will be given.
4.3.1 The optical source: CW Lorentzian Laser

Since the use of external modulators at the optical transmitter is mandatory, an optical continuous wave (CW) source is required. The laser model implements a simplified continuous wave (CW) laser considering only phase noise [2]. Laser phase noise is taken into account by generating a Lorentzian emission line shape whose FWHM (Full Width Half Maximum) is specified by parameters chosen by the user. In this model the instantaneous phase, by definition the integral of the instantaneous frequency, has a power spectrum given by:

$$\Phi_{in}(f) = \frac{k}{f^2} \quad (4.1)$$

where \(k\) is a constant depending on the 3-dB bandwidth of the laser itself. At the output of a fibre, the power spectrum of the amplitude fluctuation due to the laser phase modulation is [3]

$$\frac{\Delta S}{2\langle S \rangle} = \frac{k}{f^2 \sin^2 \left[ \frac{B_2 L}{2} \left( 2\pi f \right)^2 \right]} \quad (4.2)$$

assuming a small modulation signal. \(\langle S \rangle\) is the time averaged signal intensity. The CW Lorentzian Laser model presents the additional feature of easy-to-use since it requires just a few parameters in its specification. Two other models based on rate equations are also available. They were not considered since the trade-off among accuracy, complexity, and simulation time was not favourable.

4.3.2 Single-mode optical fibre

The implemented fibre model fully accounts for the linear and non-linear phenomena influencing pulse propagation, as well as polarisation related effects. In particular, polarisation mode dispersion (PMD) and birefringence are simulated considering their interaction with non-linearities. Non-linear Kerr effect is simulated considering its “instantaneous” (self-phase modulation-SPM, cross-phase modulation-CPM, four-wave-mixing-FWM, and parametric gain-PG) and “delayed” response (Raman cross talk and amplification). The Raman pump can be specified as either co- or counter-propagating with respect to signal propagation.

4.3.2.1 Field vector propagation equations

In this subsection the physical phenomena imparting light propagation along single-mode optical fibres will be briefly described. They are incorporated in a suitable form of the field propagation equations, which fully account for linear dispersion, non-linear susceptibility and non-linear polarisation effects. The assumptions, accuracy, validity and derivation to get the approximated
non-linear (vector) propagation equation, known as coupled non-linear Schrödinger equations, are the subject of several excellent books (e.g., see [4-6]). It can be written as [7]:

\[
\frac{\partial \mathbf{A}}{\partial z} + \left\{ j \beta_0 + \frac{\beta_1}{\partial t} - \frac{i}{2} \beta_2 \frac{\partial^2}{\partial t^2} - \frac{1}{6} \beta_3 \frac{\partial^3}{\partial t^3} \right\} \mathbf{A} + \alpha \mathbf{A} = -j \gamma \left\{ \left( 3 |A_x|^2 + 2 |A_y|^2 \right) A_x + A_x^* A_y^2 \right\}
\]

(4.3)

where

\[
\beta_i(\omega) = \frac{\partial^{(i)} \beta}{\partial \omega^i} \bigg|_{\omega=0} \quad i = 1, 2, 3
\]

is the propagation constant matrix, \( \mathbf{A} \) is the mode amplitude vector, \( \alpha \) is the attenuation constant, and \( \gamma \) the non-linearity coefficient. It is assumed that all parameters are normalised so the total power in Watts is given by: \( P_{\text{tot}} = |A_x|^2 + |A_y|^2 \). Typically, \( z \) is expressed as \( km \) and \( \gamma \) is \( \text{Watt}^{-1} \cdot \text{km}^{-1} \). In deriving the above non-linear field propagation equation some simplifying assumptions were made [4]: \( P_{\text{NL}} \), the non-linear component of the induced polarisation vector \( P \), is treated as a small perturbation to \( P_L \), the linear part of the polarisation, i.e., \( P_{\text{NL}} \ll P_L \); pulse width greater than 1 ps and optical power lower than 1 Watt\(^3\) such that \( \omega_0 T_0 \gg 1 \) and \( T_\theta/T_0 \ll 1 \). \( T_0 \) is the pulse width, \( \omega_0 \) is the centre wavelength, and \( T_R \) is related to the slope of the Raman gain.

### 4.3.2.2 Numerical Solution of the Propagation Equation

Eq. (4.3) is a non-linear partial differential equation that in general does not possess closed-form solutions. A numerical approach is, therefore, necessary to deal with the propagation equation. To solve Eq. (4.3) two different classes of numerical methods are considered [4]: finite differences, and pseudospectral methods. In general, these latter methods are faster by up to an order of magnitude to achieve the same accuracy. Of these pseudospectral methods, the one that has been widely used is the split-step method. In order to understand the split-step method it is useful to rewrite Eq. (4.3) in the form

\[
\frac{\partial \hat{U}(T, z)}{\partial z} = \left[ \hat{L} + \hat{N} \right] \hat{U}(T, z)
\]

(4.4)

\[
= \left[ \hat{L}_N(T) + \hat{L}_S(T) + \hat{N}(z) \right] \hat{U}(T, z)
\]

\(^3\)For powers lower than 1 Watt, stimulated Raman scattering becomes negligible. Furthermore, if the system under analysis is single wavelength, most of the time SRS can be modelled as just extra loss because the amplified Stokes wave is well out-of-band.
where $\hat{L}$ is a linear operator and $\hat{N}$ is a non-linear operator. To simplify the coupled equations the following transformations were used:

$$
A = U e^{-j\beta_2 z} \\
T = t - \beta_1 z
$$

The operators in the right-hand side of Eq.(4.4) have the following definitions:

- $\hat{L}_A(T)$ is a linear operator that induces differential group delay (DGD); it does not depend on $z$.
- $\hat{L}_D(T)$ is a scalar operator responsible for chromatic dispersion that does not depend on $z$.
- $\hat{N}(z)$ is the non-linear operator that accounts for all phenomena that do not depend on $T$, i.e., attenuation, birefringence and fibre non-linearity.

The philosophy behind the split-step method resides in the longitudinal coordinate $z$ being discretised in $\Delta z$ steps (over this length all the terms of $\beta_i(w)$ are considered as constants\textsuperscript{4}). It is also assumed that the operators act one at a time over the elementary step $\Delta z$. Therefore, the solution to (4.4) can be written as a sequence of new operators resulting from the independent spatial integration of the original operators [7]:

$$
U(T, z + \Delta z) = L^{1/2}_D L^{1/2}_z N L^{1/2}_z L^{1/2}_A U(T, z)
$$

(4.5)

where

$$
L^{1/2}_z = \exp \left\{ L_z \frac{\Delta z}{2} \right\}
$$

(4.6)

$$
L^{1/2}_A = \exp \left\{ L_A \frac{\Delta z}{2} \right\}
$$

(4.7)

$$
NU(T, z) = \exp \left\{ \int_{z}^{z+\Delta z} N(\xi) U(T, \xi) d\xi \right\}
$$

(4.8)

and

$$
U'(T, z) = L^{1/2}_D L^{1/2}_z U(T, z)
$$

(4.9)

\textsuperscript{4} They may vary from spatial step to spatial step, though.
In Eq. (4.5) a procedure called *symmetrised split-step method* [4] was used. The main difference is that the effect of non-linearity is included in the middle of the segment rather than at the segment boundary. This is illustrated in Figure 4.2. It should be remarked that Eq. (4.5) is not an exact solution to Eq. (4.4) since \( L_1 \), \( L_2 \), and \( N \) are non-commuting operators. Thus, the "\( \approx \)" symbol in Eq. (4.5). In the right-hand of (4.5) it is assumed that the operators act independently of one another, neglecting their mutual, spaced-distributed, interaction.

![Figure 4.2- Schematic illustration of the symmetrised split-step method.](image)

Note that the ordering of the operators in Eq. (4.5) is arbitrary. A critical decision is related to the selection of the step interval \( \Delta z \) to ensure convergence. The problem is compounded by the fact that operators describe different phenomena, whose characteristic lengths (i.e., the spatial length over which their effect is noticeable) are different.

The usual method of implementing operator splitting is by the so-called *split-step Fourier (SSF)* method: the non-linear operator is calculated in the time domain; the linear operator is evaluated in the frequency domain, by using the FFT (Fast-Fourier Transform) and converted back to time domain again with an inverse-FFT. Being a linear operator, \( L \) is fully characterised by its impulse responses \( h(t) \), and the correct way of computing its effect on \( A(t,z) \) is via a convolution product in time. Mathematically,

\[
SSF \Rightarrow A'_t[n] = A[n] \otimes h[n] = FFT^{-1} \left( FFT \left( A[n] \right) \times FFT \left( h[n] \right) \right)
\] (4.10)

where \( A[n] \) is the signal sampled in time. It is known [8] that SSF as calculated above implements a *circular convolution* – indicated with the \( \otimes \) symbol. The circular convolution creates a signal fold-over error effect – or *aliasing* – in the resulting array \( A'_t[n] \), which may contain less samples than needed to compute the actual convolution product \( A_t[n] \). Thus, there is
an intrinsic error to the SSF method and there is no way the exact result $A_L[n]$ can be derived from $A'_L[n]$.

It is in the way the linear operator is calculated that resides the uniqueness of OptSim. The split-step method used in the tool is called Time Domain Split-Step (TDSS) [9], in which $L$ is computed in the time domain by calculating the convolution product in sampled time as:

$$TDSS \Rightarrow A_L[n] = A[n] * h[n] = \sum_{k=-\infty}^{\infty} A[k] h[n-k]$$  \hspace{1cm} (4.11)

The effective implementation of this method is more difficult to achieve. It implements the linear operator $L$ by means of Finite Impulse Response (FIR) filters. It is claimed that this method present several significant advantages over SSFs methods [7, 9].

There are several limitations inherent in the use of the non-linear Schrödinger equations (4.3) for pulse propagation in optical fibres. Firstly, neglecting the second-order derivative of $A(z,t)$ with respect to the propagation direction $z$, the so-called slowly-varying envelope approximation. Secondly, that backward propagating waves are totally ignored. Thirdly, the effects induced by SRS are not modelled. SRS is a delayed non-linear response of the optical fibre. However, it should be mentioned that, in the current version of OptSim, SRS effects as well as Raman amplifier models are incorporated.

**4.3.2.3 NON-LINEAR EFFECTS**

The response of any dielectric medium to light becomes non-linear for intense electromagnetic fields, and optical fibres are no exception. The optical non-linearities in optical fibre are due to stimulated scattering processes (Brillouin and Raman scattering) and changes in the refractive index with optical power [10, 11]. Stimulated scattering is manifested as an intensity-dependent gain or loss. In scattering phenomena, such as SBS (Stimulated Brillouin Scattering) or SRS (Stimulated Raman Scattering), part of the energy of the propagating optical field is transferred to local phonons. In particular, in SBS acoustic phonons are involved, while in SRS optical phonons are generated. The power dependence of the refractive index is responsible for the Kerr effect. Depending on the shape of the input signal, the Kerr nonlinearity manifests by different effects, such as self-phase modulation (SPM), cross-phase modulation (XPM or CPM), and four-wave mixing (FWM or FPM).

The general equation that describes light propagation in optical fibres is the wave equation [4]
\[ \nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\mu_0 \frac{\partial^2 P}{\partial t^2} \quad (4.12) \]

where the induced polarisation \( P \) characterises the medium and it is a function of the electrical field \( E \). In the case of weak non-linear behaviour of the medium, as in optical fibres, \( P \) satisfies the general relation [12]

\[ P = \varepsilon_0 \left[ \chi^{(1)} \cdot E + \chi^{(2)} : EE + \chi^{(3)} : EEE \right] \quad (4.13) \]

where \( \varepsilon_0 \) is the vacuum permittivity, \( \chi^{(1)} \) is the linear susceptibility (represents the dominant contribution to \( P \)), and the second- and third-order susceptibility tensors \( \chi^{(2)} \) and \( \chi^{(3)} \) are responsible for the non-linear behaviour. In optical fibres, the term \( \chi^{(2)} \) vanishes due to silica (SiO\(_2\)) symmetric molecules. As a consequence, the non-linear effects in optical fibres originate from the third-order susceptibility. In particular, the real part of \( \chi^{(3)} \) is responsible for the Kerr effect, while the imaginary part for the Raman effect. In the Kerr effect, due to the intensity dependence of the refractive index resulting from the contribution of \( \chi^{(3)} \), the refractive index of fibre becomes [13]

\[ n = n_0 + n_2 |E|^2 \quad (4.14) \]

where \( n_0 \) is the linear part, \( |E|^2 \) is the optical intensity inside the fibre, and \( n_2 \) is the non-linear index coefficient related to \( \chi^{(3)} \).

**Self-Phase Modulation – SPM**

Due to \( n_2 \), the non-linear contribution to the refractive index results in a phase change for light propagating along the fibre of [14, 15]

\[ \Phi^{NL} = \frac{2\pi}{\lambda} n_2 |E|^2 z_{eff} = \frac{\omega}{c} n_2 |E|^2 z_{eff} \quad (4.15) \]

with

\[ z_{eff} = \frac{[1 - \exp(-\alpha z)]}{\alpha} \]

\( z_{eff} \) is the effective non-linear fibre distance, that is smaller than the actual \( z \) because of fibre loss. It is this non-linear phase shift that gives rise to self-phase modulation. In the case of an optical signal carrying information, the time-dependent non-linear phase shift causes a chirp in the optical transmitted field. In the absence of group-velocity dispersion (GVD), this chirp causes a non-linear broadening of the signal spectrum, which is dependent on the bandwidth and on the shape of the injected signal. When chromatic dispersion is considered, the interaction between GVD and SPM depends on the sign of
the dispersion $\beta_2$: in the normal dispersion region, the linear chirp (generated by GVD) and non-linear chirp (due to SPM) have the same sign, while in the anomalous region they have opposite signs. Therefore, in the first case the pulse is broadened by the combined effects of SPM and dispersion; in the latter case, the pulse is narrowed.

**Cross-Phase Modulation - XPM**

When several signals having different carrier frequencies propagate through a fibre, the intensity-dependent non-linear phase of the signal at frequency $\omega_j$ depends also on the intensity of the signals at frequencies different from $\omega_j$ according to the expression [16]

$$\Phi_{i}^{\text{NC}} = \frac{2\pi n_2 z}{\lambda_i} \left[ |E_i|^2 + 2 \sum_{i \neq j} |E_j|^2 \right] = \frac{\omega_j n_2 z}{c} \left[ |E_i|^2 + 2 \sum_{i \neq j} |E_j|^2 \right]$$

(4.16)

The first term in the square brackets is responsible for SPM. The second term results from phase modulation of one wave by the co-propagating waves and is responsible for XPM. From Eq. (4.16) it is evident that XPM is twice as effective as SPM for the same intensity. XPM causes a further non-linear chirp, thus interacting with GVD as in the case of SPM. However, from Eq. (4.16) it can be deduced that XPM is only effective as long as the interacting signals are superimposed in time. Thus, the presence of chromatic dispersion means that pulses in interfering channels will not, in general, remain superimposed on the pulses in the channel of interest. Therefore, increasing GVD decreases XPM efficiency.

**Four-Wave Mixing – FWM**

When signals at different frequencies propagate along the fibre, besides XPM, another important effect occurs: four-wave mixing (FWM). It is a parametric interaction among waves satisfying a particular phase relationship called phase matching. The origin of parametric processes lies in the non-linear response of bound electrons of a medium to an applied optical field [17].

Assuming three co-propagating and co-polarised waves of frequencies $f_i, f_j, \text{ and } f_k (k \neq i, \ j)$, they generate through XPM the frequency [18]

$$f_{jk} = f_i + f_j - f_k$$

(4.17)

Thus, three waves give rise, by FWM, to nine new optical waves [19]. Besides this configuration, so-called non-degenerate, in which the interacting waves have different
frequencies, FWM can occur in a degenerate configuration when two of the interacting frequencies are the same.

The power evolution along the fibre of an FWM-generated wave can be obtained by solving the coupled propagation equations of the four interacting waves. In the case where the power exchange among the interacting waves is very low (not depleted pump condition), these equations can be analytically solved. Assuming three waves at fibre input, the peak power of the mixing product is given by [19]

$$P_{ijk}(z) = \eta \left( \frac{D_{ijk}}{3} \gamma z_{eff} \right)^2 P_i P_j P_k$$  \hspace{1cm} (4.18)

where $\gamma$ is the non-linear coefficient, $z_{eff}$ is the effective interaction length (see Eq. (4.15)), and $D_{ijk}$ is the degeneracy factor (equal to 3 if degenerate FWM is considered, 6 otherwise). The factor $\eta$ is the FWM efficiency, and is given by [19]

$$\eta = \frac{\alpha^2}{\alpha^2 + \Delta \beta^2} \left[ 1 + \frac{4e^{-az} \sin^2 (\Delta \beta z/2)}{(1-e^{-az})^2} \right]$$  \hspace{1cm} (4.19)

The quantity $\Delta \beta$ is the difference of the propagation constants of the various waves, due to dispersion, and is given by [20, 21]

$$\Delta \beta = \beta_i + \beta_j - \beta_k - \beta_{ijk} = \frac{2\pi \lambda^2}{c} (f_i - f_k)(f_j - f_k) \left[ D(\lambda) - \frac{\lambda^2}{c} \left( \frac{f_i + f_j - f}{2} \right) \frac{dD(\lambda)}{d\lambda} \right]$$  \hspace{1cm} (4.20)

where the dispersion $D$ and its slope are computed at a generic wavelength $\lambda$. The propagation constant $\beta$ was expanded in a Taylor series and terms up to the third order were retained. Significant FWM occurs only if $\Delta \beta = 0$, a requirement that is referred to as phase matching. Thus, the use of low dispersion fibres enhances the efficiency of generation of four-wave mixing waves by reducing the phase mismatch naturally provided by the fibre dispersion. For this reason, in long-haul WDM systems using dispersion-shifted fibre (DSF), cross talk due to FWM is the dominant non-linear effect [22]. Assuming $\Delta \beta = 0$ and $\eta = 1$, and, in addition, that all channels have the same input peak power, the ratio of the generated power $P_{ijk}$ to the transmitted power of the channel at the receiver $P_{out}$ can be expressed as [21]

$$\frac{P_{ijk}}{P_{out}} = \left( \frac{D_{ijk}}{3} \right)^2 (\gamma z_{eff} P_{in})^2$$  \hspace{1cm} (4.21)
Stimulated Brillouin Scattering – SBS

Stimulated Brillouin scattering is a non-linear process that can be described as a parametric interaction among the pump (or signal) wave, the Stokes wave, and an acoustic wave [23]. The pump wave generates acoustic waves by electrostriction, which in turn induce a travelling refractive index gradient in an optical fibre. This induced grating scatters the pump light through Bragg diffraction. Since it is a moving grating with acoustic velocity \( v_A \), the scattered light is downshifted in frequency due to the Doppler effect. Since the three waves must satisfy the energy and momentum conservation law, the following two relationships among the frequencies and wave vectors must hold [23]

\[
\omega_A = \omega_p - \omega_S \tag{4.22}
\]

\[
k_A = k_p - k_S \tag{4.23}
\]

where \( \omega_A, \omega_p, \) and \( \omega_S \) are the frequencies, and \( k_A, k_p, \) and \( k_S \) are the wave vectors of the acoustic, pump, and Stokes waves, respectively. A diagram of the three interacting waves is shown in figure Figure 4.3.

![Figure 4.3- Schematic illustration of stimulated Brillouin scattering.](image)

SBS in optical fibres occur only in the backward direction, with the Brillouin shift given by [23]

\[
v_B = \frac{\omega_A}{2\pi} = \frac{2n v_A}{\lambda_p} \tag{4.24}
\]

where \( n \) is the refractive index, and \( \lambda_p \) the pump wavelength. Assuming \( v_A = 5960 \text{ m/s} \) and \( n = 1.45 \) for silica fibres, than \( v_B \equiv 11.1 \text{ GHz} \) at \( \lambda_p = 1.55 \mu\text{m} \). The growth of the Stokes wave is characterised by the Brillouin-gain coefficient \( g_B(v) \). Assuming steady-state conditions and CW or quasi-CW pump (pulse width \( T_0 \gg T_B \) and spectral width \( \Delta v_p \ll \Delta v_B \), where the last term is the Brillouin-gain line width), it can be shown that the Brillouin gain has a Lorentzian spectral profile given by [23]

\[
g_B(v) = \frac{(\Delta v_B)^2}{(v - v_B)^2 + (\Delta v_B/2)^2} g_B(v_B) \tag{4.25}
\]
where \( g_B(n_B) \) is peak value of the Brillouin-gain coefficient, occurring at \( \nu = n_B \). The Brillouin-gain line width \( \Delta n_B \) is very small, typically \( \sim 10 \) MHz.

To analyse SBS, a coupled equations system composed of Eq. (4.12) and by the propagation equation of the acoustic wave have to be solved\(^5\). However, a simplified analysis can be carried out if the pump depletion is neglected. Under this assumption, it is possible to estimate the \textit{Brillouin threshold}: above this threshold value the Stokes wave intensity is found to grow exponentially in the backward direction. The Brillouin threshold is found to occur at a critical pump power \( P_{P}^{th} \) obtained by using the relation \cite{23}

\[
\frac{g_B P_{P}^{th} n_{eff}}{A_{eff}} = 21 \tag{4.26}
\]

where \( A_{eff} \) is the effective fibre core area.

In optical communications systems, the spectral width \( \Delta n_P \) increases considerably depending on the bit rate \( B \) at which the input signal is modulated. For \textit{On-Off Keying (OOK)} formats (the most common one in optical fibre systems), it can be shown that the Brillouin threshold is substantially increased \cite{15}. In \cite{15} several techniques to increase the SBS threshold are briefly described. An interesting one is the suppression of the carrier component of the modulated signal, which is achieved using PRS signalling schemes described in chapter 3. Experiments have shown that for duobinary signals at 10 Gbit/s the SBS threshold was more than 20 dBm for single-channel links \cite{24, 25}. It was also shown that this threshold increases linearly with the bit rate, making this method of SBS suppression very attractive in high-bit rate systems. As a consequence, in optical communications, SBS is not considered a system limitation, in general. Therefore, it will not be analysed in the rest of this Thesis.

\( \square \) \textit{Stimulated Raman Scattering – SRS}

SRS is a non-linear parametric interaction between light and molecular vibrations. Incident light is partially scattered by molecules and experiences a downshift in optical frequency. Incident light acts as pump for generating the frequency-shifted radiation

\(^5\) Thus, SBS is very difficult to incorporate into an expression of \( \chi^{(3)} \) as it is done with other non-linear effects, namely the Kerr and Raman effect.
called the Stokes wave. SRS is similar to SBS, but several differences exist due to a fundamental change [11, 15, 26]: acoustical phonons participate in SBS whereas optical phonons participate in SRS. Thus, SRS can occur in either the forward or the backward propagation direction (generally backward SRS is not observed in optical fibres [26]). Also the Raman gain coefficient $g_R$ is about three orders of magnitude smaller than the Brillouin gain coefficient, so the SRS threshold is about three orders of magnitude larger than the Brillouin gain coefficient for single-channel systems. It is of the order of 1 W [11, 26]. However, the most significant feature of the Raman gain is that it extends over a large frequency range (up to 40 THz) with a broad peak near 13 THz. In the case of pure silica, $g_R$ is maximum for the frequency component that is downshifted from the pump frequency by about 13.2 THZ [26]. As in the Brillouin case, a Raman threshold power $P_{p}^{th}$ is defined as the pump power at which a pump depletion of 3 dB occurs. It is given by [11, 26]

$$\frac{g_R P_{p}^{th} z_{eff}}{b A_{eff}} = 16$$  \hspace{1cm} (4.27)

where $b$ is 1 if the pump and Stokes waves are co-polarised, and 2 if the polarisations are scrambled as occurs in a real fibre. Above this threshold value, the Stokes wave intensity builds up almost exponentially.

It is clear that SRS is negligible for a single modulated channel. However, in WDM systems, due to the broadband gain bandwidth, SRS can couple different channels and give rise to cross talk. In this case, the Raman effect, at the expense of those at shorter wavelengths, amplifies the signals at longer wavelengths. Degradation of the shorter wavelength signal can occur, even when the overall optical power is smaller than the Raman threshold.

In OptSim, the SRS-induced cross talk is simulated using the following non-linear operator of the generalised non-linear Schrödinger equation [2]:

$$\frac{\partial A(z,t)}{\partial z} = - j \gamma \left\{ (1 - f_R) A(z,t)^2 + f_R A(z,t) \int_{-\infty}^{t} h_R(t-\tau) A(z,\tau)^2 d\tau \right\}$$  \hspace{1cm} (4.28)

where $f_R$ is the reference frequency for the Raman gain profile, and the impulse response $h_R(t)$, obtained from the inverse Fourier transform of the Raman gain spectral-shape, can be expressed as
4.3.3 Amplitude dual-arm Mach-Zender Modulator

External modulators offer a way to reduce or eliminate chirp because the laser source operates in a steady state, CW mode. The CW laser has the narrowest line width possible. Furthermore, operating the laser in the CW state also simplifies the laser-driving circuitry and extends its lifetime. The external modulator is typically either a LiNbO₃ modulator or an electroabsorptive modulator. Only LiNbO₃ modulators will be considered, since they present high bandwidths, chirp less operation, high linearity, dual-drive operation, and cost advantages, making them a common choice in today’s high speed optical systems.

An intensity (amplitude) Mach-Zender modulator (MZ) is an electro-optic device based on the use of a MZ interferometer designed to modulate the optical signal. It consists of a substrate with an embedded optical wave-guide region in which light inputting one end of the wave-guide is split into equal components, propagates over the two arms of the interferometer and then recombines to give the output signal. The two arms of the interferometer are sufficiently separated to forbid evanescent coupling between them. The resulting output signal depends on the relative phase of the combining fields, which is determined by the optical path length of each arm.

4.3.3.1 Transfer Characteristic

The basic working principle of a dual-drive MZ modulator is shown in Figure 4.4. The inputting optical field, $E_{in}$, undergoes a phase change $\phi$ in each of the arms. In LiNbO₃ devices, the linear electro-optic Pockels effect causes a change in the refractive index of refraction of the wave-guides proportional to an applied electric field. If electrodes are placed over or alongside the wave-guides, a voltage $v$ applied to them creates an internal field that vary linearly with the applied voltage [27]. Thus, the respective phase change in each arm is due to the applied voltage acting on that arm. The resulting exit field from the modulator is given by [28]

$$E_{out}(t) = \frac{E_{in}(t)}{2} \exp\left[ j\phi_1(t) \right] + \frac{E_{in}(t)}{2} \exp\left[ j\phi_2(t) \right]$$

$$= E_{in}(t) \cos \left[ \frac{\phi_2(t) - \phi_1(t)}{2} \right] \exp \left\{ j \left[ \frac{\phi_2(t) + \phi_1(t)}{2} \right] \right\}$$ (4.30)
where use of the trigonometric identity $2\cos(x) = \exp(jx) + \exp(-jx)$ was made in the last equality. Assuming that the interaction lengths of each arm are equal, the phase change in each waveguide varies linearly with the applied voltage as

$$\phi_1(t) = \pi \frac{v_1(t)}{V_\pi}, \quad \phi_2(t) = \pi \frac{v_2(t)}{V_\pi}$$

(4.31)

where $V_\pi$ is the switching voltage required for a phase shift of $\pi$ radians. Substituting in Eq. (4.30), the electrical field at the output of the device can be written as

$$E_{out}(t) = E_{in}(t) \cos \left\{ \frac{\pi}{2V_\pi} [v_2(t) - v_1(t)] \right\} \exp \left\{ j \frac{\pi}{2V_\pi} [v_2(t) + v_1(t)] \right\}$$

(4.32)

Dividing Eq. (4.32) by $E_{in}(t)$ gives the transfer characteristic $\chi_E(t)$ in the electric field domain for the MZ modulator

$$\chi_E(t) = \cos \left\{ \frac{\pi}{2V_\pi} [v_2(t) - v_1(t)] \right\} \exp \left\{ j \frac{\pi}{2V_\pi} [v_2(t) + v_1(t)] \right\}$$

(4.33)

The transfer characteristic in the power domain is obtained by squaring the magnitude of Eq. (4.33) to get

$$\chi_p(t) = \cos^2 \left\{ \frac{\pi}{2V_\pi} [v_2(t) - v_1(t)] \right\}$$

(4.34)

4.3.3.2 Chirp Parameter

As it is often done with directly modulated lasers, it is convenient to express the amount of frequency chirping in an external MZ modulator in terms of a parameter $\alpha$ called the chirp parameter. By definition, the chirp $\alpha$ of a modulator is the ratio of the phase modulation to the amplitude modulation [28]
\[ \alpha = 2S \left( \frac{d\phi}{dt} \right) \left( \frac{dS}{dt} \right) \]  

(4.35)

where the phase is the average phase change in each arm

\[ \phi = \frac{\phi_1(t) + \phi_2(t)}{2} = \frac{\pi}{2V_\pi} \left[ v_2(t) + v_1(t) \right] \]  

(4.36)

and the intensity \( S \) is the square of the electrical field

\[ S = E^2 = E_0^2 \cos^2 \left[ \frac{\phi_2(t) - \phi_1(t)}{2} \right] = E_0^2 \cos^2 \left[ \frac{\pi}{2V_\pi} [v_2(t) - v_1(t)] \right] \]  

(4.37)

where \( E_0=constant \) assumes that the input signal is CW light, i.e., the chirp parameter is due entirely to the modulator. Substituting the values for intensity and phase in Eq. (4.35) and taking differentiation with respect to time leads to

\[ \alpha = \frac{\left( \frac{dv_1(t)}{dt} \right) + \left( \frac{dv_2(t)}{dt} \right)}{\left( \frac{dv_1(t)}{dt} \right) - \left( \frac{dv_2(t)}{dt} \right)} \cot \left[ \frac{\pi}{2V_\pi} [v_2(t) - v_1(t)] \right] \]  

(4.38)

Some useful expressions can be obtained when the voltages \( v_1(t) \) and \( v_2(t) \) have mathematically similar waveforms [29]. For example, consider the case where the device is driven with similar waveforms and that one of the arms will have a certain dc bias voltage

\[ v_1(t) = V_b + V_1s(t), \quad v_2(t) = V_2s(t) \]  

(4.39)

where \( s(t) \) is an arbitrary ac waveform normalised such that its peak-to-peak value is unity. \( V_1 \) and \( V_2 \) are constants and equal the peak-to-peak values of \( v_1(t) \) and \( v_2(t) \), respectively. Assuming operating condition for the modulator to be at the half power point, i.e., where the bias places the modulator at the midpoint of the optical response curve \( (V_b=V_\pi / 2) \), and considering the small signal limit \( (V_1<< V_\pi \) and \( V_2<< V_\pi) \) then \( \cot(\cdot) \) in Eq. (4.38) is approximately equal to -I. Thus, the chirp is given as

\[ \alpha = \frac{V_2 + V_1}{V_2 - V_1} \]  

(4.40)

Therefore, for small signal drive conditions using mathematically similar waveforms and suitable bias, the chirp parameter is a constant and given uniquely in terms of the peak-to-peak applied voltages \( V_1 \) and \( V_2 \). Notice that by setting \( V_1 = - V_2 \), i.e., when one electrical signal is an inverted replica of the other, chirp free operation is possible. In this particular case, the dual-drive MZ modulator operates in a push-pull format. One speaks of push-pull because when one
arm “pushes” a phase change, the other arm “pulls” it at the same time. Another advantage of using a dual-drive MZ modulator in a push-pull configuration is related to the driving voltage. For example, if a modulator requires an 8 V peak-to-peak swing, a dual-drive push-pull format requires only 4 V swings on each input (the factor 2 in the cosine term in Eq. (4.33)). Lower voltage swings ease the requirements on driving circuitry. It is the push-pull operation of the MZ modulator that will allow generation of optical PRS signals, as explained in next subsection. A remarking note regarding the phase of the output field: it is not equal to the argument of the exponential in Eq. (4.32) since the co-sinus function can change from positive to negative values. This will be shown in the next subsection.

4.3.3.3 GENERATING OPTICAL PRS SIGNALS

Generation of optical PRS signals can be accomplished by biasing a dual-drive MZ external modulator at maximum extinction and applying a baseband PRS signal, either duobinary, modified duobinary, or dicode, as shown in Figure 4.5. Also shown in Figure 4.5 is the conventional way of biasing a MZ modulator to produce a binary, intensity-modulated signal.

![Binary Signals and PRS Signals](image)

**Figure 4.5**- Mach-Zender drive condition to get an intensity OOK(left) and an optical PRS signal.

As the electrical signal passes through the point of maximum extinction, the phase of the optical carrier is reversed, providing a kind of phase-shift keying. Therefore, the optical PRS signals can also be appropriately called Optical AM-PSK Duobinary, or Optical AM-PSK Modified Duobinary, or Optical AM-PSK Dicode. The baseband electrical signals are produced according
to the rules outlined in the previous chapter. Thus, a binary-like intensity-modulated signal is produced; however, the PSD of the optical field is that of the PRS scheme.

In the case of a duobinary signal, besides the delay-and-add circuit used to implement the ideal duobinary signal (see Figure 3.2, chapter 3), there is another way of producing an electrical duobinary signal [30, 31]. Considering the Nyquist filter (see subsection 2.3.3, chapter 2), just the first arch of the cosine filter for duobinary is required. Thus, the quarter-cycle cosine filter can be easily approximated in practice by analogue filters: Bessel filters, due to their linear phase response, are good candidates. Figure 4. 6 shows the Bode diagram of the modulus of the transfer function of the ideal duobinary filter (cosine) and its approximation by a 5th order low-pass Bessel filter (3-dB cut off at ~1/4 the bit rate) for a 10 Gbit/s NRZ signal – top diagram. It can be seen that the first arch of the cosine is well approximated by the Bessel filter. In the bottom diagram, a comparison between the phases of the same two filters is made. The x-axis is displayed in a linear scale to meet in evidence the phase linearity of the Bessel filter. Above 4.5 GHz, the phase is no longer linear but it is already well above the cut off frequency.

![Bode diagram for ideal duobinary filter and Bessel filter](image)

**Figure 4. 6-** Comparison between the Bode diagram for the ideal duobinary filter (cosine) and the 5th order Bessel filter with a 2.5 GHz pass band (top – magnitude; bottom - phase).
The narrow \textit{LPF (Low-Pass Filter)} act as an analogue converter, which simultaneously converts signals from binary to duobinary and trims the high frequency components in the duobinary signal spectrum. As will be seen in chapter 5, these signals are also produced by modulating both the amplitude and phase. Thus, some authors called them optical AM-PSK duobinary signals [32, 33] – from now on this definition will also be adopted in this Thesis.

4.4 A generic system model

To analyse the performance of optical fibre transmission systems, a generic model of the transmission system can often be useful. Different performance evaluation parameters could then be used to assess the impact of the various transmission impairments. A generic model of the transmission system is depicted in Figure 4. 7.

![Figure 4. 7- A generic system model.](image)

The transmitter section is composed of four components: a \textit{digital data source}, an \textit{electrical driver}, a \textit{laser}, and an \textit{external amplitude modulator}. The digital data source simulates a pseudo-random logical signal generator. Baud rate, sequence length, and logical signal level (number of bits per symbol) are all user-defined parameters. The electrical driver converts the logical input signal, a binary sequence of "0s" and "1s", into an electrical signal. Several mapping laws are possible; of those available, only NRZ and NRZ raised cosine will be used. The electrical levels of the output signal are user-defined. The optical source is a CW Lorentzian laser (described in subsection 4.3.1): the output state polarisation is aligned with the geometrical \textit{x-axis}. Light exiting the laser is coupled to an external amplitude dual-arm Mach-Zender modulator (see subsection 4.3.3), which impresses the information upon the optical carrier.
Notice that in the electrical-driving circuit part, the delay-and-add configuration for the generation of duobinary signals is depicted.

The optical link part includes the optical fibre, and, eventually, optical couplers and splitters, optical filters, optical polarisers, and optical amplifiers. The optical fibre model is the most fundamental and complex of the OptSim tool (for an in-depth description see subsection 4.3.2). In general, SMF will be the preferred type of fibre used in the simulated models.

The optical receiver section implements a simplified, noiseless single channel receiver. It is composed by an ideal PIN photodiode, an ideal electrical amplifier, and an electrical 5\textsuperscript{th} order Bessel filter, with a cut off of ~0.7 bit rate. Eventually, an optical raised cosine filter can be incorporated. This component is useful when one wants to simulate a simple receiver whose only relevant characteristics are the electrical (and optical) filter bandwidth; details of the receiver in terms of noise level are irrelevant. Sometimes a built-in receiver, called sensitivity optical receiver, will be used. It is a complete receiver, and provides an easy way of estimating receiver sensitivity, considering certain parameters from datasheets or from experimental measurements. Moreover, this component includes a semi-analytical technique that allows estimates of the receiver BER versus the received optical power, either in electrical noise limited case or in the ASE noise limited one.

The lay out of the optical link is done. However, one still needs to determine what kind of measurements should be done. Also shown in Figure 4. 7 are two measurements components: the optical probe, and the electrical scope. The optical probe produces diagrams such as power, phase, instantaneous frequency, Stokes parameters, and spectrum of the optical signal. The electrical scope allows diagrams such as signal amplitude, eye diagram, histogram at the optimum sampling instant, and spectrum of the electrical signal.

The eye-diagram technique will be the method used to determine the system performance throughout this Thesis. To measure system performance with the eye-diagram method, generation of random data should be provided. This assures generation of marks and spaces at a uniform rate but in a random manner, which is the characteristic of data streams found in practice. A convenient approach is to use pseudo-random streams. The word pseudo-random means that the generated combination or sequence of marks and spaces will eventually repeat but that it is sufficiently random for test purposes. If the sequence is \textit{N-bit long}, one speaks of a
sequence length of \( N \) bits. A \( N \)-bit long sequence comprises \( 2^N - 1 \) different \( N \)-bit long combinations; after this limit has been generated, the data sequence will repeat. Eye-diagrams are obtained by slicing the received bit stream into segments of \( T \) seconds, and overlaying the slices. These slices have been delayed by an integer number of bit periods. A great deal of system performance information can be derived by simple inspection of the eye-diagram. According to [34], the following information concerning the signal amplitude distortion, timing jitter, and system non-linearities can be deduced (see Figure 4.8):

- The width of the eye-opening defines the sampling time interval. The best time instant to sample the received signal is when the height of the eye-opening is largest or, equivalently, the inter-symbol interference is minimised;
- The height of the eye-opening is reduced as a result of amplitude distortion in the data signal. The greater the eye-closure the more difficult to make a correct decision on the received data stream;
- The height of the eye-opening at the particular sampling time instant indicates the noise margin or immunity to noise of the signal.
- The rate at which the eyes closes as the sampling instant is swept, i.e., the slope of the eye-diagram sides determines system sensitivity to timing errors;
- Timing jitter (or phase distortion) is due to noise in the receiver and pulse distortion in the optical fibre. The amount of distortion \( \Delta T \) at the threshold level is a measure of the amount of jitter;
- Any non-linearities of the fibre will give rise to an asymmetry in the eye-diagram. If a purely random data stream propagates along a “truly” linear system, all the eye-openings will be identical and symmetrical.

![Figure 4.8- Simplified eye-diagram and its interpretation (after [34]).](image-url)
The eye-diagram is a useful tool to visualise the effect of inter-symbol interference. ISI has two detrimental effects on the eye: first, it reduces the height of the eye-opening, which brings the two amplitude levels, corresponding to the space and the mark bit, close and decreases the guard interval against noise; in case of severe signal distortion, the trace of a particular symbol may occur in the wrong side of the decision threshold; second, it can narrow the width of the eye-opening, thus reducing the possible range of sampling instants, and increasing the sensitivity to time jitter.

Experimentally, eye-diagrams are obtained by displaying the received electrical signals on an oscilloscope synchronised externally with a clock set at $1/T$ Hz, where $T$ is the symbol rate.

In this Thesis, eye-opening penalty (EOP) is used as the criterion of system performance. The eye-opening penalty is defined as [24]

$$EOP(dB) = -10 \log_{10} \left( \frac{B \times P_{\text{avg}}}{A \times P_0} \right)$$  \hspace{1cm} (4.41)

where $A$ is the eye-opening of the reference eye-diagram, and $B$ is that of the eye-diagram distorted by optical fibre and band-limiting filter at the transmitter (which will be the case for some PRS signalling systems). The reference eye-diagram is for binary NRZ signal in a back-to-back situation, i.e., transmitter and receiver are directly connected. If the band-limiting filter is implemented, the average power of the modulated optical signal may be changed, so it is necessary to calibrate the eye-opening using the normalised average optical power. This normalised average optical power is $P_{\text{avg}}/P_0$ in expression (4.41), where $P_0$ is the average optical power without the filter, and $P_{\text{avg}}$ is the average optical power with it; thus $P_{\text{avg}}/P_0$ equals one when no band-limiting filter is inserted.

4.5 Summary

This chapter gave a brief and general overview of the simulator used in this work. A generic system model was established. An in-depth description of the major system components was given and how they can be modelled. In particular, the cw Lorentzian laser, the single mode fibre and the LiNbO$_3$ amplitude dual-arm Mach-Zender modulator were analysed.
Regarding optical fibre, the numerical split-step method for solving the coupled non-linear Schrödinger equations, which represent the field propagation along an optical fibre, was extensively discussed. A description was also given of optical non-linearities in optical fibres, and their impact on signal propagation. Stimulated scattering processes, such as SBS and SRS, and different effects due to Kerr non-linearity, such as SPM, XPM, and FWM, were described.

The electric-field domain transfer function for a LiNbO₃ MZ external modulator was derived. Frequency chirping and chirp parameter were also discussed. It was also shown that when an amplitude dual-arm MZ modulator is operated in a push-pull configuration, i.e. the two arms are driven with mathematical similar waveforms, chirp free operation of the transmitter is possible. The use of a dual-arm MZ modulator, operated in a push-pull mode, for generating optical PRS signals was explained. An alternative approach for the generation of optical duobinary signals was also considered.

Lastly, the use of eye-diagram for assessing system performance was described. The eye-opening penalty was defined. This will be the key parameter used to evaluate system performance throughout this Thesis.

Up to this point, the various pieces of the this Thesis puzzle, ranging from the theoretical framework to simulator and system model, have been described and put into their place. Now, it is time for starting analysing system performance for different PRS modulation formats conditioned to transmission impairments mainly due to optical fibre. One such impairment is fibre chromatic dispersion. It causes pulse broadening and induces inter-symbol interference. It will be dealt with in the next chapter.
Appendix 4A – Pseudo-Random Duobinary Sequences

System performance can be assessed either by experimental testing or by numerical simulation with “realistic” sequences. By “realistic” sequences one means those that somehow assures generation of marks and spaces at a uniform rate but in a random manner, which is the characteristic of data streams found in practice. A convenient approach is to use pseudo-random streams. The word pseudo-random means that the generated combination or sequence of marks and spaces will eventually repeat but that it is sufficiently random for test purposes. If the sequence is N-bit long, one speaks of a sequence length of N bits. A N-bit long sequence comprises $2^N - 1$ different N-bit long combinations. The sequence length determines how the spectrum low frequencies are simulated: the longer the sequence, the better approximated is the low-portion of the spectrum. However, longer sequences result into larger simulation running times; thus, more computational power and/or effort are required, which translates into more system cost. As a compromise between accuracy and simulation time and computational power, 7-bit long sequences will be used as default throughout this Thesis.

One can define a duobinary pseudo-random sequence as that whose power spectral density is a square cosine and that respects the duobinary coding rule. If the symbol $\Delta$ represents in a practical sense one delay unit ($D = e^{-j2\pi f t}$), duobinary coding can be described as the application of the “operator” $(1 + \Delta)$ to data coming from the source. Regarding precoding, this mathematical operation can be represented by the “operator” $(1 + \Delta)^\dagger \mod 2$. Thus, duobinary with precoding can be considered as the application of the “operator” $(1 + \Delta)^\dagger \mod 2$ to binary data. When binary pseudo-random sequences are used as input data, duobinary with precoding does not give a duobinary pseudo-random sequence. Effectively, the precoding operator, as opposed to the duobinary one, is a non-linear operator that breaks the randomness of data as well as the periodicity of pseudo-random sequences. Let’s clear this point by the following example of a 15-bit binary pseudo-random sequence:

$$Duobinary: \quad (1 + \Delta) \overline{[110011001111010]} = \overline{210110012221110}$$

$$Duobinary \text{ w/ precoding:} \quad (1 + \Delta)^\dagger \mod 2 \overline{[110011001111010]} = \overline{222211211210100}$$

where PRS stands for pseudo-random sequence.
For a random signal, the number of symbols “0” and “2” should be the same, and their total should be approximately equal to the number of “1s”. In the above example, that is not the case when the duobinary with precoding “operator” is applied to the binary sequence. Furthermore, this latter example also shows that sequence periodicity is no longer possible. As a matter of fact, if the duobinary sequence where to repeat itself, one arrives to an impossibility: the rule that states that the only permitted transitions in successive bits are between any two adjacent symbols is violated.

Therefore, precoding is useless when the input data stream is a binary pseudo-random sequence. Moreover, it is also misleading since the duobinary sequence is no longer pseudo-random which can adversely affect the simulations. It can be shown that this fact can be extended to the other PRS signalling schemes. Thus, throughout the rest of this Thesis, binary pseudo-random sequences will be used as input data, and no precoding operation will ever be done on this input data before their transformation in PRS signals.
References:


Chapter 5: Dispersion immunity of modulation formats

5.1 Introduction

Before considering non-linearities, it is important to consider the effect of chromatic dispersion alone on the pulse evolution along the fibre path. Although propagation under the effect of group-velocity dispersion (GVD) for binary NRZ [1], and AM-PSK duobinary [2-5] was investigated, very little work has been reported concerning the other analysed PRS formats [6]. Up until the present Thesis, guidelines on how to choose optimal transmitting filters in repeaterless PRS systems has not been given. Several observations have been made that filtering at approximately half the bit rate of duobinary signals leads to improved propagation distances [5]. This result is somewhat intuitive by simple inspection of the spectrum of duobinary signals. On the other hand, the impact of electrical filters parameters such as roll-off, their amplitude or phase response, or how optimal filtering varies with modulation formats, has never been addressed either for optical duobinary or modified or dicode signalling schemes. Finally, a detailed study of the influence of several key transmitter parameters was conducted. Parameters to be considered are laser line width impact, modulator operating point, finite extinction ratio, non-ideal electronics, and unwanted modulator frequency chirp. The methodology employed in analysing the impact of these parameters is based on that reported in [7] for the case of optical AM-PSK duobinary signals.

5.2 System model and parameters

System model is based on the generic model described in the previous chapter. It is assumed that systems are repeaterless (no EDFA along the link) and performance is limited by electrical receiver noise and signal distortion.

The transmitter consists of a dual arm Mach-Zender modulator, operated in a push-pull configuration. The optical PRS signals were generated by applying the different baseband (three-level) electrical signals to both arms of the LiNbO₃ modulator biased at maximum extinction, i.e., at $V_\pi$ (see chapter 4). The electrical signals before driving the modulator can be passed through a band-limiting filter, resulting in finite rise and fall times between levels. All the three basic PRS signalling formats were considered, plus some other variants due to applying band-limiting filters. Binary NRZ is used as reference against which all other results are compared.
All the simulations were carried out using the OptSim tool, v. 3.0. Data was generated by a 10 Gb/s $2^7$-1 pseudo-random binary sequence (PRBS). It was assumed that fibre was standard SMF: loss coefficient of 0.2 dB/Km, chromatic dispersion of 16 ps/um/km and dispersion slope of 0.06 ps/nm²/Km; fibre non-linearities were not considered. The receiver was a built-in component of the programme, called Sensitivity Optical Receiver. It simulates a complete optical receiver: PIN photodetector, amplifier and a 5th order, 7.5 GHz Bessel post-detection filter. An ideal optical pre-amplifier was also included before the optical transducer, whose variable gain was calibrated to exactly compensate fibre loss.

The different transmitter model implementations were based on the technique illustrated in chapter 4 for the generation of optical PRS signals. Electrical PRS signals were obtained by a delay-and-add circuit for duobinary and a delay-and-subtract for modified duobinary and dicode (see Figure 5.1a). Furthermore, in the case of duobinary an alternative implementation was considered: the duobinary filter was approximated by a single low-pass Bessel filter with a 3-dB cut-off at $\sim 1/4$ the bit rate (see Figure 5.1b).

![Diagram](image)

**Figure 5.1** - Generation of electrical PRS signals: (a) delay-and-add or delay-and-subtract circuit followed by an optional band-limiting filter, and (b) approximation of duobinary filter (cosine) by a single low-pass filter.

### 5.3 Simulation results and discussion

#### 5.3.1 Optimal filtering

As shown in Figure 5.1 it is important to determine how to choose optimal transmitting filters. Very little has been reported on this subject [3, 5, 8]. Several observations regarding filtering are
drawn upon using the optical spectrum bandwidth as a first guideline [5], as explained in chapter 3. However, due to the fact that practical optical fibre system are a non-linear systems, makes it difficult to offer clear statements about optimal filters. Nevertheless, simulations regarding electrical filters structures and parameters (e.g., filter order, amplitude and/or phase response, filter bandwidth) were carried out to gain some insight.

Butterworth and Bessel filters of various orders were examined. These filters represent a wide range of properties. The Butterworth filter is known for its magnitude response that is maximally flat in the passband and monotonic overall; on the other hand, it presents a poor-time delay performance, giving rise to effects such as overshoot and ringing, when driven by pulse signals. The Bessel filter is known for its maximally flat group delay response; the price paid for almost constant time delay is signal attenuation and less steepness in the transition region between passband and stopband [9]. In short, Butterworth filters present good frequency-domain performance preserving signal amplitude. On the other hand, Bessel filters show excellent time-domain performance that minimises waveform distortion. The simulations presented here assume ideal, but realisable, low-pass filters to band-limit the signal at the transmitter. Some intuitive guidelines can be followed a priori. For example, duobinary and modified duobinary filtering should be done at approximately half the bit rate since their first nulls in their spectra are located at half their bit rate. NRZ and dicode signals, on the other hand, have their first nulls in their spectra at more or less their bit rate, a good indication for filter bandwidth. Eye diagrams were used to assess filter performance over runs of 2048 bits and a 3-bit pattern was considered to get more accurate system performance for highly distorted systems.

Figure 5. 2 condense results regarding the impact of band-limiting filters on optical systems in the linear regime, i.e., only considering fibre dispersion. The link distance was 70 km, which is an accepted 1-dB penalty distance limit, observed in both practice and computer simulation at 10 Gbit/s [10]. For Butterworth filters, the optimal filtering bandwidth depends on the filter roll-off (directly related to the filter order). Filters with fast roll-offs tend to have wider bandwidths. As shown in Figure 5. 2 (left side), the optimal filtering bandwidth increases with filter order (from $1B$ to $1.2B$); this effect is more pronounced in binary and dicode formats. Furthermore, low-orders (slow roll-offs) present better performance than high-order filters. This is apparent from $I$-$A$ to $I$-$D$ in Figure 5. 2.
Figure 5.2: Eye-opening penalty for different filter structures against filter order and filter bandwidth: I- Butterworth filters; II- Bessel filters, and different signalling formats: A- binary; B- duobinary C- modified duobinary; D- dicode.

For Bessel filters, the optimal filtering bandwidth dependence on filter order is not as much evident as in the case of Butterworth filters. In fact, some uniformity can be observed, particularly for duobinary and modified formats. Regarding dispersion penalty, although high-
order filters do present slightly better results, the difference is indeed small (less than 0.2 dB for all the signalling schemes). This is an expected result, since Bessel filters trade constant delay for poor attenuation characteristics. Therefore, they have slow and identical roll-offs, with small differences from 4th to 7th order filters in the region around the optimal filtering bandwidth.

Overall, simulations have shown that Bessel filters have a slight better performance than Butterworth filters. However, this result should be considered carefully. It should be remembered that 70 km is a relatively short distance, and Bessel filters have superior characteristics regarding signal distortion. Thus, their impact on eye-diagram distortion is less severe than Butterworth filters. These latter filters, on the other hand, allow signals to span longer distances due to their superior attenuation characteristics. Furthermore, additional care should be exercised when comparing Butterworth and Bessel filters taking into account phase and attenuation response. According to [11] when one considers frequency normalisation, with the cut off defined as the −3 dB point, the Bessel filter seems to be better than the Butterworth. However, when discrimination is taking into account, the opposite is true.

Nevertheless, a brief summary of the results is: (i) for duobinary and modified duobinary, the best bandwidths are approximately 0.6-0.7 B, where B is the bit rate; for binary and dicode, they are around 1.0-1.1 B; (ii) for Butterworth filters, the best filtering is obtained for low-order, slow roll-off filters; (iii) for Bessel filters, best performance is achieved by high-order filters. In practical implementations, trade-offs among filter complexity, cost and performance must be weighted according to system specifications.

Recall from Figure 5.1 that an alternative way of generating an optical AM-PSK duobinary is possible by using a single low-pass Bessel filter with a 3-dB cut-off at ~1/4 the bit rate. Eye-opening penalty against electrical filter bandwidth and filter order is plotted in Figure 5.4.

A good compromise is obtained by using 5th order Bessel filter with a cut-off frequency of 2.8 GHz. As explained in chapter 4, the ideal duobinry filter (cosine) is well approximated by this filter. As described in next section, this optical AM-PSK duobinary signal presents unique properties.
5.3.2 Dispersion limit estimation

Although not explicitly stated, previous results indicated that band limiting could actually improve system performance. In fact, in this section it is shown that limiting the spectral occupancy of optical PRS signals gives an improvement in dispersion immunity. Band limiting can be achieved by an appropriate choice of electrical transmitter filter response. Results agree with published experimental work [12-14].

In this Thesis (through numerical simulations) and in other reported work, duobinary signalling has been demonstrated to improve the dispersion immunity of 10 Gbits/s optical systems using standard SMF and operating in the third window (1550 nm). Various approaches have been experimentally verified, ranging from using a duobinary receiver in conjunction with a binary transmitter [15], duobinary intensity modulation at the transmitter [16, 17], and optical AM-PSK duobinary signalling [18-20]. Simulation and experimental results in [2] demonstrated that no improvement in dispersion immunity over conventional binary NRZ system is observed when “ideal” duobinary signalling is used. By “ideal” duobinary it is meant that the electrical signal that drives the MZ modulator is a perfect rectangular NRZ duobinary (three-level) signal, i.e., zero rise and fall times. In this present Thesis, the analysis is extended to the other signalling schemes under consideration.
In the following dispersion immunity is evaluated for the various signalling formats. These simulations assume zero chirp and an infinite intrinsic\(^1\) extinction ratio for the external modulator.

Figure 5.4 shows simulations of receiver eye-opening penalty, EOP, versus distance for optical systems using different filter structures for both conventional and PRS signalling schemes. The PRS signals, as mentioned before, are generated either by delay-and-add filter (duobinary) or by delay-and-subtract (modified duobinary and dicode) filters, and then band limited by various Butterworth and Bessel filters with the indicated filter order and 3 dB cut off frequency, prior to driving the modulator. Additionally, passing the electrical binary NRZ signal by a 2.8 GHz, 5\(^{th}\) order Bessel filter before driving the modulator, also generates an electrical duobinary signal. Note that all systems in Figure 5.4 use the same receiver filter, and an ideal optical pre-amplifier, whose gain is matched to exactly compensate fibre loss.

![Graph showing receiver eye-opening penalty (EOP) versus distance for different filter structures.](image)

*Figure 5.4- The effect of various electrical filter structures on receiver performance versus distance for 10 Gbit/s binary, duobinary, modified duobinary and dicode systems.*

It was found that band limiting was effective in extending the dispersion limit. The AM-PSK duobinary with 2.8 GHz Bessel filter (also called by some authors PSBT- Phase-Shaped Binary Transmission [4]) yields the best tolerance to chromatic dispersion among those studied here. If

\(^1\) By intrinsic, it is meant the extinction ratio that is inherent to the device, deriving from asymmetry in the Y-branches or unequal loss through the MZ arms.
the 1-dB penalty limit criterion is used, only the AM-PSK duobinary with 2.8 GHz Bessel filter is able to bridge distances up to 120 km.

However, up to 60-70 kms, the common accepted distance for a 1-dB penalty limit, only those schemes where band limiting was implemented present better results than binary NRZ - of the order of 0.5 dB. Therefore, it is highly questionable if this small gain is enough to warrant the added complexity and cost of the transmitter. In the author's opinion, the format of choice for distances up to 70 km is, no doubt, the conventional binary NRZ format.

As shown in Figure 5.5, the spectra of the duobinary and modified duobinary are reduced by a factor of two as compared to the binary and dicode signal. But, after 90 km, the eye patterns of the ideal duobinary and modified are nearly closed; no benefit is brought by spectral compression. However, when a low-pass filter is included an improvement in tolerance to dispersion is observed in dicode and duobinary with 2.8 GHZ Bessel filter (the best eye-opening). Again, notice that up to 50-70 km there is no significant difference among the various schemes under study.

![Figure 5.5- Spectra and eye-diagrams evolution of various formats for several distances.](image)

Why restriction of the spectral occupancy gives such an improvement against chromatic dispersion? In particular, why AM-PSK duobinary, generated by a 2.8 GHz Bessel filter, excels all the other schemes in dispersion immunity? Two explanations have been given to answer these fundamental questions. In [5] a frequency domain explanation based on communications
theoretical principles was put forward. It is explained why it is mandatory to band limit the spectrum of optical duobinary field to $f_0 \pm B/2$, where $f_0$ is the optical carrier frequency and $B$ is the bit rate. The required band limiting can be accomplished either in the optical domain or with an appropriate choice of electrical transmitter filter response. It is stated: "this filtering suppresses spectral components beyond one-half the bit rate away from the carrier that would otherwise undergo severe phase distortion which, in turn, would increase the non linear, intensity envelope distortion that falls within the bandwidth of the receiver". Another interpretation in the time-domain was proposed in [4]. It is based on an interference phenomenon between marks and spaces: since there is a phase shift occurring exactly in the middle of each space, its impact is to maintain the energy of the mark symbol in their time-slot by maintaining the adjacent space to a low level. It is stated:"the whole spectrum, including its phase and not only the “width” of its intensity, has to be taken into consideration when accounting for the effect of group-velocity dispersion. We showed that some phase shift occurring in each “0” neighbouring a “1” allows to keep the energy of the pulse over a longer distance in the “1” bit-time". Some controversy was set off between the authors of these two different interpretations [21]. Neither one was convinced by the other’s comments nor further light was shed on why such a good performance is observed for certain AM-PSK duobinary signals regarding chromatic dispersion tolerance.

Nevertheless, an indisputable fact remains. Duobinary signals generated by a single low-pass filter with a 3-dB cut-off at $\sim 1/4$ the bit rate do present some striking differences regarding all other signals. These differences can be observed either in the electrical signal, or the optical signal or even in the spectrum.

Figure 5. 7 shows the optical waveform representing a duobinary signal generated by a 2.8 GHz 5th order Bessel filter. Regarding the conventional binary NRZ signal, this duobinary signal presents some special features: $\pi$ phase shifts exactly in the middle of each space, and intensity rebounds when spaces are also transmitted. These rebounds result from secondary lobes suppression due to the duobinary filter (see spectrum in top of Figure 5. 6). In fact, it is the only signalling format where such suppression is so evident. Furthermore, note the smooth transitions between marks and spaces, and the oscillatory behaviour of the rebounds.
Figure 5.6- Optical signal correspondent to a duobinary signal generated by 2.8 GHz 5\textsuperscript{th} order Bessel filter: intensity (solid line) and phase (dotted line).

Compare it with an ideal NRZ duobinary signal - Figure 5.7. Notice that although there are also $\pi$ phase shifts, they are now dislocated since the phase of a space is undefined. Also, there is not any intensity rebounds. Thus, above immunity dispersion results are clearly linked to the characteristics of the signal itself.

Figure 5.7- Optical signal intensity (solid line) and phase (dotted line) for a NRZ duobinary signal.
It is the author's belief that neither the frequency domain nor the time domain interpretation accurately and completely describes the improvement to dispersion immunity observed for the AM-PSK duobinary with a single low-pass filter. The answer may lie in-between these two explanations. Another interesting and more convincing explanation, from the author's point of view, is reported in [22]. It conciliates both the frequency as well as the time domain interpretation.

The key point relates to what is the lowest bandwidth pulse shape with the minimum ISI. That question has been answered in classical communications [23]. The pulse requirements of the transmission channel to obtain no intersymbol interference (ISI) must be satisfied, i.e., Nyquist's criterion for zero ISI must hold. However, Nyquist's "brick-wall" response filter is unrealisable. In that respect, PRS schemes can be regarded as a practical means of achieving the theoretical maximum symbol rate of 2 symbols/s/Hz postulated by Nyquist, using realisable and perturbation tolerant filters. The pulse shape of duobinary signal generated by low-pass filtering has some properties of the Nyquist pulses. The duobinary cosine filter is well approximated by a filter with a cut off at $\sim 1/4 B$ (see Figure 4. 6). Furthermore, the optical signal with oscillatory rebounds do somewhat resembles the oscillations observed with pulses obeying the Nyquist criterion (the $\text{sinc}$ pulse): the field is approximately zero at the centre of each space thereby minimising the effect of ISI. Besides, due to the phase shifts, isolated spaces$^2$ are surrounded by "1" and "-1", and the signal is anti-symmetric around the centre of the space. This means that the electric field (and thus the intensity) is zero in the centre of the isolated space, and the bottom of the eye remains open. This aspect corresponds to the time-domain explanation. On the other hand, when the same NRZ pulses shapes are used, the frequency content of a binary and PRS signals are similar. The high-frequency components in the optical spectrum are due to the sharp transitions associated with the edges of the transmitted signals. The relatively slow modulation of the phase (at the bit rate) cannot affect them substantially. It is only when band limiting is implemented that the transmitted pulse shape enables a significant narrowing of the optical spectrum, thus obtaining high spectral efficiency. This is striking evident for duobinary generated by single-low pass filter, as can be observed in the displayed spectra in Figure 5. 6. This represents the frequency-domain interpretation. Of all the signalling schemes analysed, only the optical AM-PSK duobinary signal, generated by a single low-pass filter with a cut off at

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$^2$ Isolated spaces for binary modulation are the symbols most impaired by ISI, with energy form surrounding marks spreading into the space time-slot, thus raising the bottom of the eye.
\(-1/4\) \(B\), represents approximately a minimum bandwidth system. Therefore, the improved immunity to chromatic dispersion stems from this fact.

### 5.3.3 Influence of transmitters parameters in PRS systems

As bit rates increase, it is more and more difficult to implement optical transmitters. Hence, non-ideal transmitters characteristics must be considered in overall system performance evaluation. Therefore, a detailed study of the influence of several key transmitter parameters is required. Parameters to be considered are laser line width impact, modulator operating point, finite extinction ratio, non-ideal electronics, and unwanted modulator frequency chirp. The methodology employed in analysing the impact of these parameters is based on that reported in [7].

In the rest of this chapter binary and PRS signalling schemes are investigated at distances that correspond to an \(EOP\) penalty of approximately 1.5 dB. These are:

- 74 km for binary NRZ signals;
- 74 km for ideal duobinary signals;
- 118 km for duobinary signals filtered by a 4\(^{th}\) order Butterworth filter, 6 GHz cut off;
- 135 km for duobinary signals by a low-pass 5\(^{th}\) order Bessel filter, 2.8 GHz cut off;
- 73 km for ideal modified duobinary signals;
- 82 km for modified duobinary signals filtered by a 5\(^{th}\) order Bessel filter, 7 GHz cut off;
- 82 km for ideal dicode signals;
- 92 km for dicode signals filtered by a 5\(^{th}\) order Bessel filter, 11 GHz cut off.

These systems were selected according to results displayed in Figure 5.2. Other possibilities could also be considered. This choice pretends to limit the evaluated schemes to a manageable number. Some arbitrariness is implicit in this choice.

#### 5.3.3.1 Laser line width

When light from a laser with a finite line width is transmitted over a dispersive medium, such as optical fibre, another source noise occurs. It is a kind of relative intensity noise (RIN). For this noise, fibre dispersion induces a conversion of phase fluctuations to intensity modulation. This process is called \(PM\)-to-\(AM\) (phase modulation to amplitude modulation) [24]. The extent of this
PM-AM conversion depends on the modulation frequency, modulation depth, and the amount of dispersion, i.e., the length of the fibre.

From Figure 5.8, the finite linewidth of the laser source can have a great impact on system performance. To keep the total penalty below, for example, 2 dB means that a 30% variation in the normalised EOP is permissible. Thus, linewidths of up to 20 MHz are allowed which mandates the use of external modulators at the transmitter. Another interesting observation derives from the fact that filtered signals with Bessel filters are more prone to increasing laser line width. In general, PRS systems with band limiting are more sensitive to source line width increases. This observation can be explained by the fact that the total RIN from PM-to-AM conversion inversely scales with the square of $f$, the instantaneous frequency (see CW Lorentzian laser, chapter 4).

![Figure 5.8- Penalty vs. laser linewidth for various signalling schemes. The EOP penalty is normalised to the eye-opening of an ideal, monochromatic laser source.](image)

**5.3.3.2 MODULATOR OPERATING POINT**

For binary NRZ signals, the modulator is biased about the midpoint of its transfer characteristic, with the peak-to-peak driving voltage set equal to the switching voltage (see Figure 4.5). On the case of PRS signals, the driving peak-to-peak voltage is set equal to two times the switching voltage. The modulator is biased such that maximum excursions in the electrical drive cause the
optical signal to assume its peak intensity. This happens with either a 0 or \( \pi \) phase shift, corresponding to "I" or "-I", respectively (see Figure 4.5).

The effects of a change in the mean value, or bias, of the modulator are shown in Figure 5.9. From the graph, it can be observed that a mere 20% offset in the bias voltage can have a drastic impact on PRS signals. Of these, the filtered ones show a slight improvement concerning the "ideal" signals. On the other hand, binary NRZ signals are very insensitive to variations in the operating point. Note that because PRS signals have three levels, which should be symmetric on the modulator characteristics, both positive and negative offset causes a penalty. Although the bias voltage variation is more restrictive for PRS systems, in actual implementations this should not be of great concern since it is relatively easy to maintain the bias voltage under strict control.

![Figure 5.9: System penalty versus modulator bias. The bias is normalised to the mean-to-peak bias voltage. Penalty is normalised to the eye-opening for mean bias voltage.](image)

### 5.3.3.3 Impact of a Finite Optical Extinction Ratio

The extinction ratio (ER) is defined as the ratio between the output optical power corresponding to the maximum transmission value \( P_I \) and the one corresponding to the minimum transmission value \( P_0 \)

\[
ER = \frac{P_I}{P_0}, \quad 1 \leq ER \leq \infty \quad (5.1)
\]

Thus, the optical power falling onto the photodetector is no longer \( P_I/2 \). Instead, it is equal to
\[ P_{av} = P_1 + P_0 - \frac{P_1}{2} \left( \frac{1 + ER}{ER} \right) \]  

(5.2)

This extinction ratio definition assumes an optical waveform made of near rectangular and identical pulses. Following [25], for OOK/DD (On-Off Keying/Direct Detection), assuming for simplicity that the thermal noise is dominant, the penalty due to extinction ratio, \( P_{ER} \), is given by

\[ P_{ER} = \frac{P_{av}(ER)}{P_{av}(ER = \infty)} \geq \frac{ER + 1}{ER - 1} \]  

(5.3)

The penalty \( P_{ER} \) in decibels is plotted in Figure 5.10 as function of \( ER \) (also in decibels).

![Power penalty due to extinction ratio versus ER, extinction ratio.](image)

When the extinction ratio is 12.5 dB or more, the penalty is less than 0.5 dB, and fast decreasing towards zero. Note that an \( \infty \) value corresponds to an ideal extinction ratio, i.e., there is no optical power in the space time-slot.

Figure 5.11 shows the simulation results for the signalling systems under study. Note the 1.5 dB penalty when the ideal case is considered, i.e., the extinction ratio is \( \infty \).

Several remarks can be made from inspection of Figure 5.11. First, power penalties are more severe than expected when Eq. (5.3) is considered – for an extinction ratio of 6 dB a power penalty of \( \sim 2.3 \) dB should be expected, which translates in a EOP of around 6 dB. Notice that only in one case this happens – duobinary generated by a single low-pass Bessel filter. That should come as no surprise, since to get the expression relating power penalty and extinction ratio some simplistic assumptions were made [25]. Second, in the case of binary NRZ signals, to lower the optical extinction ratio actually improves system dispersion immunity, provided that the associated penalty can be tolerated. This is a “surprising” result. Intuitively, a finite extinction ratio should be associated with system performance degradation. Quite the opposite,
as it is evident from Figure 5.12, where the eye-diagram versus extinction ratio is displayed, at different distances.

![Figure 5.11- Extinction ratio penalty in decibels vs. extinction ratio (dB).](image)

The explanation for this “strange” behaviour is, indeed, simple [26]: in on-off keying, the optical signal is unipolar; as the extinction ratio is reduced, the signal starts to resemble a bipolar signal; thus, the effect of dispersion on the ones and zeros becomes more symmetrical as light from one can coherently interact with light from the other.

![Figure 5.12- Eye-diagram evolution vs. extinction ratio for binary NRZ signals at several distances.](image)
In PRS signals, due to the $\pi$ phase shifts, there is the break-up of this coherent interaction, which results in a strong degradation in performance. As a rule of thumb, for this signals, the modulator requires an extinction ratio of 20 dB to get a penalty of less than 0.5 dB. Notice that in the case of duobinary signals, generated by a single low-pass Bessel filter, it is not possible to give a clear definiton of extinction ratio due to the intensity rebounds in spaces (see Figure 5. 7). Nevertheless, one can speak of an equivalent extinction ratio which would give the same penalty considering a perfectly rectangular NRZ signal. This also applies when binary NRZ signals are considered after propagation along an optical fibre, where dispersion and ISI distort pulses that are not anymore near rectangular, nor identical, and nor can be superpositioned at the receiving side.

5.3.3.4 IMPACT OF A DELAY DIFFERENCE IN MODULATOR SIGNAL ARMS

The addition of a delay to one of the modulator signals arms was investigated. Such delay, in real systems, is expected due to nonideal driver electronics Therefore, its impact is an important parameter to be considered.

The delay introduction breaks the symmetry under push-pull operation. Thus, transitions from 0s to 1s take place at slightly different times in the two modulator electrodes. The modulator is no longer in the ideal push-pull state. This destroys the ideal $\pi$ phase shifts between $+1$ and $-1$, which are responsible for the generation of PRS optical signals. In the binary case, the requirement should be much more relaxed. Indeed, this can be observed in Figure 5. 14.

The results indicate that a 0.5 dB penalty for a 10-ps delay with PRS transmission; the exception is modified duobinary transmission, where a higher tolerance is achieved; for binary transmission more than 20 ps can be tolerated.

5.3.3.5 INFLUENCE OF CHIRP

It is known that controlled frequency chirping of MZ modulators can be used to extend the system reach of binary digital lightwave systems operating at 1550 nm over SMF [27, 28]. This is accomplished by operating a dual-drive MZ modulator in a push-pull state in which the signal applied to one arm is an exact inverted replica of that applied to the other arm. The idea is to
adjust the weighting of the applied signals to obtain a desired amount of frequency chirping. For a definition of the chirp factor see chapter 4.

![Graph](image)

Figure 5. 13- System penalty vs. delay difference in modulator signal arms.

Note that a non-ideal extinction ratio induces an implicit chirping that it is not take into account by the chirp factor. This is called *residual chirp*, and can have a negative impact unless the intrinsic extinction ratio of the modulator is no worse than 20 dB [29]. Optsim tool, in the used version, does not allow to assess the impact of this residual chirp as mentioned above. Nevertheless, as seen in a previous section, for PRS signals the extinction ratio should be kept higher than 20 dB, which according to [29] limits the impact of the residual chirp. Thus, residual chirp should be of no concern in PRS systems using MZ modulators with an extinction ratio equal or better than 20 dB.

The effect of added chirp is shown in Figure 5. 14. The only formats that depend on the sign of \( \alpha \) are binary NRZ and dicode electrically filtered signals. An improvement is observed for binary transmission with positive chirp. Filtered PRS signals are more susceptible to chirp. In fact, the impact of added chirp is much more important for these schemes. In particular, filtered duo-binary signals for high values of chirp do present a very strong degradation, with highly distorted and almost closed eye-diagrams (results not shown). As a final remark, note that the \( \alpha \) factor refers to the definition of phase that depends on power, according to [30]. This definition differs from the chirp parameter \( C \) that relates the phase to a function of time, as defined in [1]. Note that the meaning of the two definitions is completely different; therefore, the impact on
system performance is also different. Thus, care should be exercised when comparing results from different studies.

Figure 5.14- Eye-opening penalty versus modulator chirp parameter.

5.4 Summary

Simulation results were presented in this chapter that are based on the impact of fibre chromatic dispersion on system performance. Results have shown that band-limiting PRS signals as well as binary can be effective in improving dispersion immunity of all signalling schemes investigated. An explanation was provided for why optical AM-PSK duobinary signals present such an improved performance regarding fibre chromatic dispersion. Finally, the impact of several transmitter parameters were discussed, such as laser line width, modulator bias, modulator extinction ratio and chirp factor. It was found that dispersion tolerance of optical PRS systems rely on the symmetry in the signal and the MZ modulator. In this respect, binary NRZ signals do present a larger tolerance on parameters breaking this symmetry.

The main results can be summarised as follows:

- Bandlimiting was effective in extending the dispersion limit;
For duobinary and modified duobinary, the best bandwidths of electrical filters are approximately $0.6-0.7 \, B$, where $B$ is the bit rate; for binary and dicode, they are around $1.0-1.1 \, B$;

- For Butterworth filters, the best filtering is obtained for low-order, slow roll-off filters; for Bessel filters, best performance is achieved by high-order filters;

- AM-PSK duobinary with 2.8 GHz Bessel filter yields the best tolerance to chromatic dispersion among those studied.

- Binary NRZ is the format of choice for distances up to 70 km.

- Laser line widths of up to 20 MHz are required, which mandates the use of external modulators at the transmitter; PRS systems with band limiting are more sensitive to source linewidth increasing;

- Offset in the bias voltage can have a drastic impact on PRS signals. A maximum fluctuation of 20% regarding the ideal operating point should be a design target. On the other hand, binary NRZ signals are very insensitive to variations in the operating point;

- In PRS signals, a finite extinction ratio can result in a strong degradation in performance. As a rule of thumb, for this signals, the modulator requires an extinction ratio of 20 dB to get a penalty of less than 0.5 dB. In the case of binary NRZ signals, to lower the optical extinction ratio to a certain point actually improves system dispersion immunity;

- Delay difference in the two electrical signal arms feeding the modulator is critical in PRS systems; in general, it should be kept below 10 ps, while more than 20 ps can be tolerated in binary systems;

Results presented in this chapter were obtained in the so-called linear regime. Thus, besides chromatic dispersion, the influence of fibre non-linearities should also be considered. Therefore, PRS signals transmission in the non-linear regime will be the subject of next chapter. The impact of non-linearities, such as self-phase modulation, cross-phase modulatio, four-wave mixing, and Raman scattering, will be investigated.
References:

Chapter 6: Non-linearities impact on modulation formats

6.1 Introduction

The response of any dielectric medium to light becomes non-linear for intense electromagnetic fields, and optical fibres are no exception. The optical non-linearities in optical fibre are due to stimulated scattering processes (Brillouin and Raman scattering) and changes in the refractive index with optical power [1, 2]. The non-linear effects in optical fibres originate from the third-order susceptibility. In particular, the real part of $\chi^{(3)}$ is responsible for the Kerr effect, while the imaginary part for the Raman effect.

The Kerr non-linearity gives rise to different effects, depending on the shape of the injected field into the fibre. In the following, the main effects due to Kerr non-linearity and Raman scattering are assessed. Their impact on the several modulation formats under consideration is also investigated. Each of the non-linear effects is analysed separately. Suitable simulation set-ups are developed to isolate and clarify the impact of each individually.

6.2 Impact of self-phase modulation (SPM)

Self-phase modulation occurs when intensity modulated signal propagates along an optical fibre. Due to the non-linear index coefficient, SPM gives rise to a non-linear phase shift, which increases in magnitude with the propagated distance. If a complex signal is injected into the fibre, the time-dependence of the non-linear phase shift causes a chirp in the transmitted field. This results in SPM-generated frequency components that broad the spectrum over its initial width. This broadening may result in penalties due to filter bandwidth or pulse distortion arising from chromatic dispersion. The spectrum broadening depends on the bandwidth and on the shape of the injected signal.

When the combined effects of GVD and SPM on pulse evolution along the fibre are considered, the net effect depends on the sign of the dispersion $\beta_2$. If the fibre dispersion coefficient is positive (normal dispersion regime) SPM-induced non-linear chirp and GVD linear chirp are of the same sign, while in the opposite case (anomalous dispersion regime) they have opposite sign. Therefore, it is expected that in the first case pulse broadening is enhanced by SPM, while in the latter case it is reduced. This pulse narrowing suggests that SPM can be used to compensate dispersion in the anomalous dispersion regime. The ultimate example of this is
soliton transmission, where complete compensation can be attained if a particular pulse shape is selected (the soliton).

### 6.2.1 System model and parameters

The system model used is the same described in chapter 5. The only difference concerns SMF: while in chapter 5 fibre non-linearity index was not considered, here this parameter is activated, i.e., the Kerr effect is taken into account; fibre non-linearity coefficient is $\gamma = 1.2 \text{ W}^1.\text{Km}^{-1}$.

In a second and later stage, the optical 10 Gb/s PRS signals, as well as binary NRZ, are sent over a SMF fibre span of 100 km. The power at the input is varied from 7.5 to 17.5 dBm using an optical booster amplifier. EDFA noise has been turned off in order to simplify the analysis. This slightly different configuration is also used to study SPM impact on the studied PRS signals when two types of fibres other than SMF are considered: dispersion-shifted fibre (DSF) and non-zero dispersion fibre (NZDF).

### 6.2.2 Simulation results and discussion

#### 6.2.2.1 Standard single-mode fibre

Figure 6. 1 shows simulations on the performance of different 10 Gbit/s systems in conventional fibre for 8 and 32 mW average powers, as well as the linear case (obtained by setting the non-linearity coefficient switch to the off position).

The linear case shows a 1-dB penalty at roughly 60 km for binary NRZ format (see Figure 6. 1A). When non-linearity is taken into account, the 1-dB penalty is extended to near 100 km at 32 mW average power. Note that the curves for 2 and 8 mW are almost identical. This may indicate that up to these powers dispersion is the dominant effect. As length is increased, all powers show the same steep degradation in performance.

Regarding duobinary signals, the same dispersion compensation trend by non-linearity can be observed from Figure 6. 1B. However, unlike the binary case, this effect is clearly power dependent. When filtering is implemented, as in Figure 6. 1C, duobinary signal curves present some changes as compared with the ideal duobinary case. Penalty decreases becoming almost zero at 32 mW average power at a distance of about 60 km when the non-linearity is considered.
Also note that the curve for 32 mW crosses that for 8 mW at around 120 km. It is seen that high powers exhibit a more drastic degradation in performance as distance is increased.

Figure 6.1- Simulation of PRS systems transmission performance as a function of distance using SMF fibre. Results are shown for linear transmission (filled diamond and bold line), 2 mW (open diamonds and dash-two dots line), 8 mW (open squares and dotted line) and 32 mW average power (open triangles and dashed line).
For AM-PSK duobinary signals, generated by a Bessel filter with a cut off of 2.8 GHz, Figure 6. 1D shows the simulations results. At high powers higher penalties are encountered when non-linearity is taken into account. This substantial degradation indicates that in this case non-linearity seems to play the opposite role regarding the previous signalling schemes: instead of compensating dispersion, it appears to enhance it. A possible explanation may lie down in the signal itself. As seen in the previous chapter, there are $\pi$ phase shifts exactly in the middle of each space. Furthermore, the polarity of these phase shifts alternates as shown in Figure 5. 6. This may imply that due to phase polarity alternations, the SPM-induced chirp will not partial compensate the chromatic dispersion. Note that for distances above 120 km, the linear, 2 mW and 8 mW curves tend to converge. This distance represents more or less the accepted dispersion limit for a 1-dB penalty.

Modified duobinary signals (Figure 6. 1E and F) present an interesting behaviour. For high powers, the system performance actually improves instead of degrading. That means the linear chirp induced by GVD and the non-linear one due to SPM have opposite sign. As a consequence, net chirp is reduced, leading to pulse narrowing; thus, partial compensation occurs. This phenomenon is well known and particularly efficient when the transmitted pulse has the particular shape of a hyperbolic secant (the soliton). Nevertheless, partial compensation does occur for other pulse shapes, as seen here. The method of non-linear assisted transmission is based on this effect [3]. Note that the effect is more evident for the case when filtering is present. Also in this case, note that the curve for 32 mW crosses that for 8 mW at 55 km, with a steeper rise in penalty than the other curves.

Regarding dicode signalling scheme the same pattern arises as in the case of modified duobinary format. Here again, the use of electrical filtering allows greater distances to be bridged, as well the effect of power dependence is very clear.

Overall, almost all of the analysed signalling formats present partial compensation, in a more or less higher degree. The exception concerns AM-PSK duobinary. In fact, this format seems to be more sensitive to degradation in performance when non-linearity is taken into account. Another interesting fact is that partial compensation takes place in the range of 20 to 50 km. This is an interesting result when one considers the effective non-linear length $z_{eff}$ of the fibre, already defined in chapter 4. When $\alpha L >> 1$ then $z_{eff} \approx 1/\alpha$. For SMF, this length is about 20 km.
Moreover, a last remark concerning modified duobinary and dicode signals is in order. For distances up to 70 km they present a “gain” (negative EOP), i.e., a significant improvement in performance can be expected due to partial compensation of chromatic dispersion by self-phase modulation. This interesting result needs to be validated by further detailed study and experimental data in order to have a real assessment of its potential. Does this improvement warrant the added complexity at the transmitter? Finally, why in general electrical filtered signals show better performance than unfiltered signals? As already stated, SPM-induced chirp is dependent of the bandwidth and shape of the injected signal. Thus, when filtering is implemented, the resulting signal shape and bandwidth increases the efficiency of this effect, i.e., a higher partial compensation is achieved.

In order to better understand the impact of SPM on the different modulation formats, the eye-opening penalties against different launched powers for the different signals propagating along 100 km of SMF are shown in Figure 6.2.

![Eye-opening penalty versus injected power for various signals propagating along SMF fibre.](image)

**Figure 6.2-** Eye-opening penalty versus injected power for various signals propagating along SMF fibre.

It is observed that eye-opening penalty increases for very strong transmitted power, highlighting the SPM-induced *phase modulation* (PM) to *intensity modulation* (IM) conversion through fibre dispersion [4]. By increasing the power, SPM grows and depletes the signal. Thus, power increases up to the point where non-linearity dominates over chromatic dispersion implies that signal degradation starts to occur (for an average optical power higher than 15 dBm). On the
other hand, for lower input powers it can also be observed an improvement in the system performance. It is a direct consequence of GVD induced chirp and SPM induced chirp are of opposite polarities in the anomalous regime of SMF fibre. This effect is power as well as modulation format dependent. The only exception concerns AM-PSK, which appears to be insensitive to any improvement (as expected from previous results). Another interesting observation can be made regarding the impact of electrical filtering. Only in the case of duobinary signals filtered by a 4th order Butterworth filter, with a 6 GHz cut off frequency, do they present better performance than unfiltered ones.

Lastly, it was also observed that when the non-linear SPM effect started to grow the signal spectrum began to widen and the pulse energy remained more confine within the bit-slot, as exemplified in Figure 6. 3 for duobinary signalling. All other formats present the same behaviour. Note that for high power noise-like behaviour appears in the relative bit pattern.

![Figure 6.3- Spectra (top) and relative bit pattern (bottom) of duobinary signals for different injected optical powers.](image)

6.2.2.2 Dispersion-shifted fibre (DSF)

This section deals with the impact SPM can have on the same modulation formats when dispersion shifted fibre is used as the transmission medium. The main transmission parameters are: loss coefficient of 0.2 dB/Km, chromatic dispersion of 0.4 ps/nm/km and dispersion slope of 0.05 ps/nm².km, and fibre non-linearity coefficient of 1.2 W⁻¹km⁻¹.

The same system model used in the case of SMF was used: optical 10 Gb/s PRS signals, as well as a NRZ, were sent over a DSF fibre span of 100 km. The power at the input was varied from
7.5 to 17.5 dBm using an optical booster amplifier. EDFA noise has been turned off in order to simplify the analysis.

Figure 6.4 shows EOP versus injected power for the several PRS signals under study. Note an improvement in performance (negative EOP) for almost all of the signalling schemes; the exception, are duobinary filtered signals. Moreover, the gain depends on the modulation format, being more pronounced in the case of dicode signals filtered by a 5\textsuperscript{th} Bessel filter.

![Graph of EOP versus injected power for various signals](image)

**Figure 6.4-** Eye-opening penalty versus injected power for various signals propagating along DSF fibre.

All the modulation formats are almost insensitive to SPM induced chirp along the 100 km of DSF fibre. Only for very strong signals some degradation in performance can be observed. In the graph, such effect is only clearly visible for the case of dicode signals electrically filtered by a Butterworth filter (cut off frequency of 6 GHz). A very slight degradation also occurs when AM-PSK duobinary scheme is considered. These results should be expected since the fibre dispersion is almost zero, i.e., the dispersion length $L_D$ [5] is much larger than both fibre length $L$ and the non-linear length $L_{NL}$. Thus, SPM dominates. Therefore, spectral broadening due to the SPM-generated frequency components is expected. The extent of this spectral broadening also depends on the pulse shape as well as on the propagated distance.

When the effects of GVD can be neglected, as with DSF fibre, the impact of SPM on the various signalling schemes is rather feeble for short-to-medium distances of fibre. Figure 6.5 shows the
eye-diagrams for the several launched powers in case of dicode signals; no degradation or distortion is visible. The same pattern occurs for the other signalling schemes. Its impact can adversely affect performance only for long-haul systems and/or with in-line amplification.

![Eye-diagram evolution versus injected power for dicode signals and DSF fibre.](image)

**6.2.2.3 NON-ZERO DISPERSION FIBRE (NZDF)**

In recent years an impressive demand for high-bit services has been required. New optical fibres have been introduced on the market to meet the requirements posed by WDM systems that need huge bandwidths. When many high power channels are injected in the fibre, non-linear effects could arise and affect transmission efficiency. One way to overcome non-linear effects was to deploy a new category of fibre called *non-zero dispersion fibre (NZDF)*. Its main characteristic is its very low (but not zero) chromatic dispersion value in the window between 1500 and 1600 nm [6]. The main transmission parameters are: loss coefficient of 0.25 dB/Km, chromatic dispersion of 4.3 ps/nm/km and dispersion slope of 0.05 ps/nm².km, and fibre non-linearity coefficient of 1.8 W⁻¹km⁻¹.

The same system model used in the two previous cases was used: optical 10 Gb/s PRS signals, as well as a NRZ, were sent over a NZDF fibre span of 100 km. The power at the input was varied from 7.5 to 17.5 dBm using an optical booster amplifier. EDFA noise has been turned off in order to simplify the analysis. Results are depicted in Figure 6. 6.

This is the intermediate case between SMF and DSF. GVD and SPM cooperate wit each other to maintain a reduced net chirp. Although small in magnitude, improvements in performance are noticeable. Note that all the analysed modulation formats present some penalty reduction even in the case of AM-PSK duobinary- compare it against those for SMF and DSF fibres. Also evident from the graph is the significant improvement in performance for modified duobinary and dicode signals as compared to the two previous cases, where SMF and DSF fibres were used.
Figure 6. 6- Eye-opening penalty versus injected power for various signals propagating along NZDF fibre.

Moreover, system degradation at high powers is more gradual – Figure 6. 7 exemplifies this effect for the case of modified duobinary signals. Note eye-distortion evolution is minimal as injected power is varied. Signal shape impact is also clear from the results, as expected.

Figure 6. 7- Eye-diagram evolution versus injected power for modified duobinary signals and NZDF fibre.

Overall, the impact of the spectral broadening introduced by SPM is very limited if there is no chromatic dispersion to transform this spectral broadening into pulse broadening. In the anomalous dispersion regime, cooperation between GVD and SPM can, in fact, narrow the spectrum. This non-linear assisted transmission depends on pulse shape and bandwidth as well as propagated distance. The only investigated format that seems immune to SPM-induced PM-IM conversion through GVD is AM-PSK duobinary. No partial compensation is realised for this signalling scheme. For the 100 km fibre distance, NZDF presents the best compromise for all the studied signalling schemes. Thus, a very moderate amount of dispersion at 1550 nm is beneficial
in either all the three possible cases: i) GVD dominates; ii) SPM dominates; iii) neither GVD nor SPM dominates.

The stimulated Brillouin scattering threshold and the overcompensation of chromatic dispersion by the SPM effect can essentially limit the power increase. In the case of binary NRZ with a bit rate far larger than the SBS line width, the threshold is almost bit rate independent, and it is around 10 dBm [7]. So, a SBS suppression technique should be adopted [8]. For duobinary modulation format, due to the absence of the optical carrier, a SBS threshold of 23 dBm at 10 Gbit/s was demonstrated [7]. It was also shown that the threshold increases linearly with the bit rate, as the spectrum broadens proportionally. Thus, SBS is not a limitation for duobinary modulation.

It is the author’s firm belief that for the others PRS formats, where the absence of the optical carrier in the spectrum (see chapter 3) also occurs, that SBS suppression is also verified as in the case of duobinary modulation. However, to the best of the author’s knowledge, there is no experimental data or even numerical results that support this statement.

6.3 Impact of cross-phase modulation (XPM or CPM)

Increasing fibre capacity demands require increasing WDM channel count and channel data rates. This can lead to decreased transmission performance because of non-linear effects due to fibre propagation [1, 8]. One particular effect that has attracted increased attention is cross-phase modulation (XPM or CPM) [9-15].

Cross-phase modulation occurs because the effective refractive index of a wave depends not only on the intensity of that wave but also on the intensity of other co-propagating waves [16]. It is a non-linear phenomenon specific to WDM transmission systems [2]. It gives rise to a non-linear phase modulation of each channel in a WDM system, which depends on the overall power in all the other channels. Because of chromatic dispersion of optical fibres, such phase modulation can be converted into intensity modulation in away that a noise-like effect is generated and, thus, can degrade system performance [17-20].

6.3.1 System model and parameters

The model used was based in [19, 21]: two channels are used, one, referred as a probe, is a CW signal, and the other carries binary data and is referred to as the interfering or pump channel.
The signal powers injected into the fibre are 10 dBm for the pump signal and approximately 0 dBm for the probe signal. The probe's intensity fluctuations at the fibre output can then be used as a measure for the interference caused by XPM. The transmitter consists of a chirp-free dual arm LiNbO$_3$ Mach-Zender modulator, operated in a push-pull configuration. The fibre was 80 Km of standard SMF: 0.2 dB/Km, chromatic dispersion of 16 ps/nm/Km and dispersion slope of 0.06 ps/nm$^2$/Km; non-linearity coefficient of 1.2 W$^{-1}$km$^{-1}$. An ideal PIN photodetector and a 5th order 7.5 GHz cut-off Bessel filter represented the optical receiver. The default optical filter bandwidth was 2R, where R is the channel bit rate. Data was generated by a 10 GB/s 2$^7$-1 pseudo-random sequence. Schematic of the implemented model is shown in Figure 6. 8.

This model is based on the coupled non-linear Schrödinger equations (see chapter 4, Eq. 4.3). Neglecting the interaction that arises among polarisation components, the propagation equations can be written as [16]:

\[
\frac{\partial A_1}{\partial z} + \beta_1 \frac{\partial A_1}{\partial t} + \frac{j}{2} \beta_{21} \frac{\partial^2 A_1}{\partial t^2} + \frac{\alpha_1}{2} A_1 = j\gamma \left[ |A_1|^2 + 2|A_2|^2 \right] A_1
\]

\[
\frac{\partial A_2}{\partial z} + \beta_2 \frac{\partial A_2}{\partial t} + \frac{j}{2} \beta_{22} \frac{\partial^2 A_2}{\partial t^2} + \frac{\alpha_2}{2} A_2 = j\gamma \left[ |A_2|^2 + 2|A_1|^2 \right] A_2
\]

The first right-hand term is responsible for SPM, already discussed in the previous section. The second right-hand term is responsible for XPM. Assuming that $A_2$ represents the probe and $A_1$ the pump, one have $|A_2|^2<<|A_1|^2$. Thus, one can neglect the terms containing $|A_2|^2$ in Eq. (6.1). Evolution of the pump, governed by the upper equation, is then unaffected by the probe channel. However, evolution of the probe signal is affected considerably by the presence of the pump pulse because of XPM. As such, the bottom equation of Eq. (6.1) represents the combined
effects of XPM and GVD on the shape and the spectrum of the probe signal. Therefore, this pump-probe configuration is useful to isolate the XPM effect. Moreover, the validity of this model is reinforced by the fact that the main cause of penalty in systems employing NRZ signal format is vertical eye closure, due to distortion of the “I” level, rather than timing jitter [22]. Thus, the distortion of the “I” level of a modulated channel can be estimated by the distortion of a continuous-wave (cw) probe channel.

6.3.2 Simulation results and discussion

6.3.2.1 Impact of optical filter parameters

Cross-phase modulation appears only in multi-wavelength systems. Thus, key optical components in WDM systems are those performing the function of combining (multiplexing) different wavelength channels and splitting (demultiplexing) them. Combining different wavelengths is a relatively simple task and can be achieved with a component such as a star coupler or an arrayed waveguide grating [23]. Demultiplexing requires optical filters with sharp wavelength cut-offs; it is a much more challenging problem when practical systems are considered [24].

In this section a very brief analysis regarding the impact of optical filters was carried out. The implemented model is a raised cosine transfer function filter. This optical raised cosine filter is useful when the exact shape of the filters is not know in details, and one just wants to describe filters simply by their central frequency and passband. The filter parameters considered are filter centre frequency, and its bandwidth (defined as the full-width half maximum, FWHM, bandwidth).

The following table displays the eye-opening penalty for a fibre distance of 80 km against optical filter bandwidth (in GHz).

<table>
<thead>
<tr>
<th>Binary</th>
<th>EOP (dB)</th>
<th>10</th>
<th>12.5</th>
<th>15</th>
<th>17.5</th>
<th>20</th>
<th>22.5</th>
<th>25</th>
<th>27.5</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>W/filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOP (dB)</td>
<td>2.28539</td>
<td>1.61409</td>
<td>1.46254</td>
<td>1.45198</td>
<td>1.49221</td>
<td>1.44107</td>
<td>1.45255</td>
<td>1.42225</td>
<td>1.42228</td>
<td>1.40049</td>
<td>1.37707</td>
<td>1.5308</td>
<td></td>
</tr>
<tr>
<td>EOP (dB)</td>
<td>1.66012</td>
<td>1.33906</td>
<td>1.29751</td>
<td>1.16632</td>
<td>1.08524</td>
<td>1.0536</td>
<td>0.99635</td>
<td>0.94741</td>
<td>0.8845</td>
<td>0.89183</td>
<td>0.89546</td>
<td>0.91736</td>
<td></td>
</tr>
<tr>
<td>EOP (dB)</td>
<td>0.78543</td>
<td>0.6571</td>
<td>0.53845</td>
<td>0.48944</td>
<td>0.46568</td>
<td>0.44319</td>
<td>0.43544</td>
<td>0.43382</td>
<td>0.43039</td>
<td>0.42264</td>
<td>0.42552</td>
<td>0.45627</td>
<td></td>
</tr>
<tr>
<td>EOP (dB)</td>
<td>0.61333</td>
<td>0.57168</td>
<td>0.55312</td>
<td>0.49899</td>
<td>0.47221</td>
<td>0.46349</td>
<td>0.46549</td>
<td>0.43587</td>
<td>0.4379</td>
<td>0.44272</td>
<td>0.44108</td>
<td>0.47677</td>
<td></td>
</tr>
<tr>
<td>EOP (dB)</td>
<td>2.30863</td>
<td>2.00666</td>
<td>1.82178</td>
<td>1.80475</td>
<td>1.7065</td>
<td>1.66869</td>
<td>1.56407</td>
<td>1.50478</td>
<td>1.47278</td>
<td>1.42633</td>
<td>1.40592</td>
<td>1.48545</td>
<td></td>
</tr>
<tr>
<td>EOP (dB)</td>
<td>2.45714</td>
<td>1.4114</td>
<td>1.30739</td>
<td>1.17882</td>
<td>0.99362</td>
<td>0.83404</td>
<td>0.73088</td>
<td>0.69838</td>
<td>0.69668</td>
<td>0.71243</td>
<td>0.80991</td>
<td>0.80991</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1- Eye-opening penalty vs. optical filter bandwidth.
Since an external modulator is used, the optical bandwidth of an NRZ signal in frequency units can be approximately 1.2 times the bit rate, $B$ [25]. Thus, keeping this rule in mind, and analysing the above results in Table 6.1, it was decided as a good compromise to choose an optical filter bandwidth of 20 GHz, i.e., $\sim 2B$ in frequency units (nevertheless, some subjectivity is inherent in this choice). When channel separation was less or equal than 0.3 nm a filter bandwidth of 15 GHz was chosen.

6.3.2.2 XPM-INDUCED INTERFERENCE AS FUNCTION OF CHANNEL WAVELENGTH SEPARATION

6.3.2.3.1 Standard single-mode fibre (SMF)

The standard deviation, $Std$, of the probe output intensity, normalised to the average probe intensity at the receiver, is plotted in Figure 6.9 as a function of the wavelength spacing between the two signals.

Three different cases are depicted in Figure 6.9. They are distinctive of one another due to the type of input electrical signals before generation of the PRS electrical signals: $A$- ideal electrical NRZ signals, with instantaneous rise and fall time (roll-off parameter equals zero); $B$- NRZ raised cosine pulses with a roll-off parameter of 0.5; and, $C$ - NRZ raised cosine pulses with a roll-off parameter of 0.8. The different pulses versus roll-off parameters are shown in Figure 6.10. The reason behind is to approximate the simulated electrical signals as possible to "real" signals with finite rise and fall times. A roll-off factor of 0.5 represents more or less a symmetrical rise and fall time of 20% of the bit period (~20 picoseconds).

It is clearly seen from Figure 6.9 that the interference is almost proportional to $I/\Delta\lambda$ over the entire simulated wavelength range. This result is consistent with experimental data for the case of conventional binary signals [19]. However, for modified duobinary and dicode some discrepancy from linearity can be observed for channel spacings less than 0.5 nm.
Figure 6.9- XPM as function of wavelength separation with pump power of 10 dBm and SMF. A- input data: ideal NRZ pulses; B- input data: NRZ raised cosine pulses with a roll-off of 0.5; C- input data: NRZ raised cosine pulses with a roll-off of 0.8. Note the $1/\Delta \lambda$ scaling of the horizontal axis.

The reduction of signals distortions by the walk-off effect was an expected one. It plays an important role in non-linear phenomena involving two or more overlapping optical pulses. Due to group-velocity mismatch, pulses at different wavelengths propagate at different speeds along the fibre. XPM ceases to occur when the faster moving pulse has completely walked through the
slower moving pulse. Separation between two channels is governed by the walk-off parameter defined as [26]

\[ d_{12} = \beta_1(\lambda_1) - \beta_1(\lambda_2) = \frac{1}{v_g(\lambda_1)} - \frac{1}{v_g(\lambda_2)} = D \Delta \lambda_{12} \]  

where \( \lambda_1 \) and \( \lambda_2 \) are the centre wavelengths of the two pulses, \( \beta_1 \) is specified at these wavelengths, and \( \Delta \lambda_{12} = \lambda_1 - \lambda_2 \) is the wavelength separation between channels 1 and 2. The approximation in Eq. (6.2) is valid in a non-zero dispersion region. For pulses of width \( T_0 \), a walk-off length \( L_w \) can be defined by the relation [26]

\[ L_w = \frac{T_0}{d_{12}} \]  

(6.3)

Therefore, increasing channel separation reduces the walk-off length, thus reducing interference for all modulation formats, as depicted in Figure 6.9.

![Figure 6.10- Input electrical signal shapes according to the roll-off parameter.](image)

Notice the significant decrease observed for NRZ, duobinary, dicode and modified duobinary when a finite small rise and fall time was considered. PRS signals generated or transformed by filtering, as in the case of AM-PSK and duobinary with Butterworth filter, were unaffected by signal shaping. When considering raised-cosine signals with a roll-off parameter of 0.8, results are very uniform for all the modulation formats: variation regarding maximum and minimum values is always less than 0.5% for the considered channel separation value. Thus, signal shape
is an important parameter regarding the XPM impact. A possible explanation is that assuming finite rise and fall times is equivalent to a strong attenuation of the secondary lobes of the signal spectrum. Thus, pulses with steeper leading and trailing edges broaden more rapidly with fibre propagation simply because such pulses have a wider spectrum to start with. When raised-cosine signals with different roll-off factors are used, the pulse widths as well as the steepness of their edges are also changed. Therefore, the length of fibre where interference takes place is smaller for pulse shapes with relatively broad leading and trailing edges, according to the definition of the walk-off length. As result, XPM-induced interference is reduced, consistent to the results displayed in Figure 6.9.

6.3.2.3.2 Non-zero dispersion fibre (NZDF)

Instead of using SMF, let's consider now non-zero dispersion fibre (NZDF). Its main characteristic is its very low (but not zero) chromatic dispersion value in the window between 1500 and 1600 nm [6]. The main transmission parameters are: loss coefficient of 0.25 dB/Km, chromatic dispersion of 4.3 ps/nm/km and dispersion slope of 0.05 ps/nm².km, and fibre non-linearity coefficient of 1.8 W⁻¹km⁻¹. All other parameters remain are kept unchanged. Two different cases are depicted in Figure 6.11. They are distinctive of one another due to the type of input electrical signals before generation of the PRS electrical signals: A- ideal electrical NRZ signals, with instantaneous rise and fall time (roll-off parameter equals zero); and, B- NRZ raised cosine pulses with a roll-off parameter of 0.5.

It is clearly seen from Figure 6.11 that interference is again almost proportional to $I/\Delta\lambda$ over the entire simulated wavelength range. Moreover, although an important change in the fibre dispersion value was considered, thus reducing the walk-off parameter, results show that there are not significant differences when compared with Figure 6.9, where SMF was considered. Once again, notice some discrepancy from linearity for the modified duo-binary and decode cases when channel spacings are less than 0.5 nm. Therefore, although walk-off length is different for this case, the magnitude of the XPM-induced interference is of the same order. One possible conclusion is that XPM interference is mostly generated near the beginning of the fibre - indeed, in what follows, this will be demonstrated.
Figure 6. 11- XPM as function of wavelength separation with pump power of 10 dBm and NZDF. A- input data: ideal NRZ pulses; B- input data: NRZ raised cosine pulses with a roll-off of 0.5. Note the $1/\Delta\lambda$ scaling of the horizontal axis.

6.3.2.3 XPM-INDUCED INTERFERENCE AS FUNCTION OF INTERFERING CHANNEL POWER AND SPACING

6.3.2.3.1 Standard single-mode fibre (SMF)

Figure 6. 12 plots the dependence of XPM-induced intensity interference on the pump signal power for SMF. Again, the standard deviation, Std, of the probe output intensity, normalised to the average probe intensity at the receiver, is the parameter considered as function of the signal interfering average power. The wavelength spacing considered is 0.5 nm.

From Figure 6. 12, small differences can be seen among the investigated modulation formats. Comparing the results in A against those in B and C, a significant reduction of the XPM-induced
interference can be observed. These results are consistent with the explanation put forward in the previous subsection.

![Graphs showing roll-off effects](image)

Figure 6. 12- XPM as function of interfering pump power for channel spacing of 0.5 nm and SMF. A- input data: ideal NRZ pulses; B- input data: NRZ raised cosine pulses with a roll-off of 0.5; C- input data: NRZ raised cosine pulses with a roll-off of 0.8.

Note that the dependence is almost linear for low pump powers (< 10 dBm). For sufficient high optical powers other non-linearities should be considered as SPM, SBS and possibly MI. In particular, for high powers, SPM-induced chirp is of opposite sign regarding the linear chirp.
induced by GVD. As a consequence, net chirp is reduced, leading to pump pulse narrowing. It is the effect of partial compensation – see previous section. Thus, pump pulse narrowing translates into a longer walk-off length $L_W$, the pump-probe interaction time is longer, which significantly enhances XPM impact. It was found that the interference due to SPM only and XPM only do not add up to yield the total SPM+XPM interference, since both are complex functions of the input powers [18].

Figure 6.13 depicts results when the wavelength spacing is 0.2 nm. Significant differences can be seen among the modulation formats and pulse shapes considered, as well as when comparing to the case of channel spacing of 0.5 nm (Figure 6.12). Comparing the results in A against those in B, a significant reduction of the XPM-induced interference can be observed. Nevertheless, dependence is just approximately linear for pump powers less than 6 dBm, which indicates that XPM induced interference is much more severe for very narrow channel separation: factors of ~ 1.5 - 2.5 according to specific modulation format. Also note that signal shape dependence is not so evident as in the case of 0.5 nm channel separation.

![Graph A](image1.png)

![Graph B](image2.png)

Figure 6.13 - XPM as function of interfering pump power for channel spacing of 0.2 nm and SMF. A- input data: ideal NRZ pulses; B- input data: NRZ raised cosine pulses with a roll-off of 0.5.
6.3.2.3.2 Non-zero dispersion fibre (NZDF)

Figure 6. 14 plots the dependence of XPM-induced intensity interference on the pump signal power for NZDF. Again, the standard deviation, $\text{Std}$, of the probe output intensity, normalised to the average probe intensity at the receiver, is the parameter considered as function of the signal interfering average power. The wavelength spacing considered is 0.5 nm.

![Graph A](image)

**A**

**Roll-off: 0**

![Graph B](image)

**B**

**Roll-off: 0.5**

Figure 6. 14: XPM as function of interfering pump power for channel spacing of 0.5 nm and NZDF. A- input data: ideal NRZ pulses; B- input data: NRZ raised cosine pulses with a roll-off of 0.5.

Comparing the results in A against those in B, a significant reduction of the XPM-induced interference can be observed. Note that the dependence is almost linear for low pump powers (< 10 dBm). For sufficient high optical powers other non-linearities should be considered such as SPM. Comparing to the case of SMF, there is a slight increase in the magnitude of the XPM-induced fluctuations in intensity of the probe.

Figure 6. 15 shows results when the wavelength spacing is 0.2 nm. Significant differences can be seen among the modulation formats and pulse shapes considered, as well as when comparing to the case of channel spacing of 0.5 nm (Figure 6. 14). Comparing the results in A against those in
B, a small reduction of the XPM-induced interference can be observed. On the other hand, dependence is just approximately linear for pump powers less than 6 dBm, which indicates that XPM induced interference is much more severe for very narrow channel separation. Also note that signal shape dependence is not so evident as in the case of 0.5 nm channel separation. Furthermore, for sufficient high pump powers, a steepest increase in the interference magnitude can be observed, as well as there seems to be convergence towards same range of values (not so evident from figure for modified duobinary and diode).

![Figure 6. 15- XPM as function of interfering pump power for channel spacing of 0.2 nm and NZDF. A- input data: ideal NRZ pulses; B- input data: NRZ raised cosine pulses with a roll-off of 0.5.](image)

It was also verified that XPM interference was mostly generated near the beginning of the fibre. Figure 6. 16 shows the variation of XPM interference as function of propagation distance. Channel separation is 0.5 nm; pump and probe power are 10 and 0 dBm, respectively. A loss less SMF fibre was simulated. Note that a zero variation means the final value was reached, which in the simulated conditions represents an 80 km distance. The plotted data show that a variation of less than 5% was achieved for all the considered modulation formats – even after 1 km of fibre.
For the above system parameters, the calculated walk-off length is 12.5 km (Eq. (6.3) ). Thus, since the dispersion parameter of the simulated fibre is kept constant, the intensity fluctuations measured at the end of fibre are essentially determined by the phase shift induced in the beginning of the fibre. These simulated results are in agreement with theoretical results in [10, 11].

![Graph](image)

Figure 6.16- Variation of XPM-induced interference as function of propagation length.

### 6.3.2.4 XPM Impact vs. Light Wave Polarisation

XPM-induced coupling among light waves is polarisation dependent. In Figure 6.17, for each of the investigated signalling schemes, two curves obtained by simulations are plotted: one for parallel polarisations, and the other for orthogonal polarisations. Linear polarised pump and probe waves have been assumed at the SMF fibre input.

Note the different scaling in the y-axis between the two plots. Again, this difference is related to signal shape: pulses with broad leading and trailing edges are less broadened with fibre propagation simply because such pulses have a narrower spectrum to start with. When comparing the ideal NRZ input pulses against NRZ raised-cosine ones (roll-off factor of 0.5), results show similar variations for unfiltered modulation formats either for co-polarised or orthogonal polarised pump and probe waves. The range of variation lies between 35 to 85%; modified duobinary and dicode present the highest range values. Concerning filtered signals, i.e., duobinary with a 4th Butterworth filter (cut off frequency of 6 GHz) and duobinary with a 5th Bessel filter (cut off of 2.8 GHz), their range of variation is much more narrow. Higher values are obtained when pump and probe are orthogonal polarised, with AM-PSK 2.8 GHz Bessel
format showing less than 2% of variation while duobinary 6 GHz Butterworth presents a variation of less than 13%. Nevertheless, when electrical NRZ raised-cosine pulses with a roll-off of 0.5 are used as input data, results show lower fluctuations on the probe. Therefore, the impact of XPM-induced interference is reduced for this type of signals. This reduction is marginal when electrically filtered signals are considered.

Figure 6.17 - Normalised standard deviation vs. co-polarised or orthogonal pump and probe waves. A- input data: ideal NRZ pulses; B- input data: NRZ raised cosine pulses with a roll-off of 0.5. Both waves are assumed linearly polarised at fibre input.
Figure 6.17 shows that XPM-induced fluctuations are indeed polarisation dependent. When the pump and probe waves have orthogonal polarisations the impact of XPM-induced non-linear phase shifts is lower than the case of parallel polarisations. In general, for all modulation formats, results considering normalised standard deviation as the measuring parameter show a reduction in interference amplitude of 20-30%. The only exception concerns modified duobinary where much higher values are obtained (in the range of 80-90%). These results were obtained by considering fibre birefringence. However, a special feature of the simulation tool was also enabled. It is called "recovering original polarisation switch": if this switched is turned on, the optical field polarisation, at the end of the fibre span, is rotated to recover the whole rotation induced by birefringence during propagation. Thus, this switch allows the analysis of the effects on the polarisation induced by polarisation-mode dispersion (PMD) and non-linearity only. Since the PMD switch was turned off, one is able to analyse the non-linearity effect. In fact, it is an artefact to maintain the state of polarisation constant along the fibre. Nevertheless, some simulations were run where this special switch was set to off. Although the obtained values are lower in magnitude, they indicate that relative state of polarisation between the propagated waves limits the impact of XPM (results not shown). This is an indirect proof that XPM interference is mostly generated near the beginning of the fibre. In fact, a parameter called the birefringence correlation length was set to 0.2 km. This indicates the distance over which the state of polarisation of a signal loses memory of the input state of polarisation due to PMD and birefringence [27].

Identical results were also obtained when the light waves were circularly polarised (results not shown).

6.4 Four-wave mixing (FWM)

Four-wave mixing is a third-order non-linearity in silica fibres, which is analogous to intermodulation distortion in electrical systems. When wavelength channels are located near the zero-dispersion point, three optical frequencies will mix to produce a fourth inter modulation product (see chapter 4 for the expressions). The efficiency of four-wave mixing depends on fibre dispersion and channel spacings [28-30]. Since the dispersion varies with wavelength, the signal waves and the FWM-generated waves have different group velocities. This destroys the phase matching of the interacting waves; thus the efficiency at which power is transferred to the newly
generated frequencies is lowered. The higher the group velocity mismatches and the wider the channel spacings, the lower is the impact of four-wave mixing.

Nevertheless, the influence of FWM quadruples when halving the channel spacing [31], while cross-phase modulation effects are approximately inversely proportional to the channel spacing (as demonstrated in the previous section). Thus, FWM will ultimately limit the channel density and the capacity of Dense-WDM systems [8].

6.4.1 System model and parameters

It is well known that WDM systems on DSF fibre with the zero dispersion wavelength within the transmission band are severely distorted by FWM [29]. To cope with this problem, on one hand, and to avoid the distortions induced by GVD, on the other hand, there has been a trend to deploy non-zero dispersion fibre (NZDF).

Therefore, the simulation set-up models this worst-case situation in order to better assess FWM impact on the modulation formats under investigation. The model used was based in [32]: two channels are used and optically combined, one is a CW signal, and the other carries a modulated optical signal. The un-modulated carrier stands as a worst-case scenario since it can be considered as a continuous string of "on" pulses. The transmitter consists of a chirp-free dual arm LiNbO$_3$ Mach-Zender modulator, operated in a push-pull configuration, which generates the optical signal (binary, duobinary, and its filtered variants, modified duobinary and dicode), as explained in previous chapters. A 50 GHz channel spacing (0.402 nm) between the two channels was used. An EDFA, used as booster, followed the coupler to increase the total launched power into the fibre to 6 dBm. It was implemented as a fixed power output, in order to keep the injected power constant. The fibre was 20 Km of DSF: 0.2 dB/Km, chromatic dispersion of 0.4 ps/nm.km and dispersion slope of 0.059 ps/nm$^2$/Km; zero-dispersion wavelength was 1550.2 nm; non-linearity coefficient of 1.8 W$^{-1}$.km$^{-1}$. Dispersion was varied from 0 to 4 ps/nm.km in increments of 1ps/nm.km. The spectrum after propagation was viewed on an optical spectrum analyser (OSA). Schematic of the implemented model is similar to the one depicted in Figure 6.8 for the case of XPM (including the receiver section).

Based on Eq. (4.17), chapter 4, for a two channel system, i.e., pump waves at $f_1$ and $f_2$ can generate FWM products at frequencies $f_{FWM1}=2f_2-f_1$ and $f_{FWM2}=2f_1-f_2$. In this case, Eq. (4.20) for the phase-mismatching can be rewritten as
\[ \Delta \beta = -\frac{\lambda^4 \pi}{c^2} \frac{dD}{d\lambda} 2 (f_i - f_k)^2 (f_i - f_0) \]  

(6.4)

Note that in Eq. (6.4) the phase-matching condition is always satisfied when \( f_i \) coincides with the zero-dispersion wavelength, i.e., \( f_i = f_0 \).

### 6.4.2 Simulation results and discussion

#### 6.4.2.1 FWM impact vs. fibre dispersion

The levels of the first order FWM frequencies were evaluated for the modulation formats under consideration. Figure 6.18 shows the spectra plots against dispersion.

![Spectra plots](image)

**Figure 6.18**- Spectra of various modulation formats for different values of dispersion. No entry means FWM products are no longer recognisable.

The numerical values of the levels of the FWM generated tones are given in Table 6.2. They are measured against the maximum level of the optical carriers (in this case it was the cw carrier). Note the asymmetry in the power of the FWM generated tones for PRS input optical signals. This happens because the two input lights have not the same optical power; furthermore, PRS signalling schemes do not have a carrier peak in the frequency domain as in the case of binary NRZ. Thus, the intensity beating of the two input lights is different for the two FWM tones.
From the Figure, it can be seen that it is hard to recognise the FWM products either for the case of dicode (just for dispersion values above 1 ps/nm.km) or binary modulation (when dispersion is higher than 2 ps/nm.km).\(^1\)

<table>
<thead>
<tr>
<th></th>
<th>Dispersion (ps/nm.km)</th>
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<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Optical Binary-FWM 1=FWM 2</td>
<td>28.1</td>
</tr>
<tr>
<td>Optical Duobinary-FWM 1</td>
<td>32.4</td>
</tr>
<tr>
<td>Optical Duobinary-FWM 2</td>
<td>28.6</td>
</tr>
<tr>
<td>Optical Du. Butterworth-FWM 1</td>
<td>31.8</td>
</tr>
<tr>
<td>Optical Du. Butterworth-FWM 2</td>
<td>28.3</td>
</tr>
<tr>
<td>Optical AM-PSK Bessel-FWM 1</td>
<td>32.5</td>
</tr>
<tr>
<td>Optical AM-PSK Bessel-FWM 2</td>
<td>27.1</td>
</tr>
<tr>
<td>Optical Modified-FWM 1</td>
<td>33.4</td>
</tr>
<tr>
<td>Optical Modified-FWM 2</td>
<td>28.7</td>
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<td>33.2</td>
</tr>
<tr>
<td>Optical Dicode-FWM 2</td>
<td>27.8</td>
</tr>
</tbody>
</table>

Table 6.2- Levels of the optical FWM signals for the various modulation formats. Signal input power is 6 dBm.

The optical power spectrum of the received signals shows that FWM products decrease with increasing dispersion. The amount of reduction is very significant for all modulation formats, including conventional binary. Notice that FWM products are strongly suppressed even for relative moderate amounts of fibre dispersion – an average 4dB for almost all the modulation formats just for a dispersion value of 1ps/nm.km. Thus, fibre dispersion is a very powerful and convenient way of controlling the impact of FWM products on system performance. In fact, non-zero dispersion fibre was developed as a way of improving system performance regarding both fibre dispersion and FWM.

In view of results, regarding the new generated frequency identified as $FWM\, 1$ it is obvious that indeed PRS signalling schemes do help into suppressing the FWM effects as compared to binary. An interesting fact is that as dispersion is increased the differences in the level of suppression by the different formats tend to converge to the same value. For zero dispersion, the range of variation between the best and the worst value as compared to the binary case was 3.7-5.3; this variation is down to 3.2-3.5 when fibre dispersion value is 2 ps/nm.km.

Concerning the other FWM product, named as $FWM\, 2$, the picture is not as clear as in the case of $FWM\, 1$. In fact, for zero dispersion the improvement is marginal (around 0.5 dB), getting a little bit higher when dispersion is introduced. Interestingly, AM-PSK duobinary presents a worst record than binary.

\(^1\) This does not necessarily mean these formats are more immune to FWM cross talk.
Analysing the results, an asymmetry in FWM products regarding the PRS signals is evident - for the binary case, FWM tones with almost the same levels were generated. Overall, modified doobinary presents the greatest level of suppression for almost all the situations.

It should be mentioned that an approximation was subjacent to the already presented results as well as to the next ones: the bandwidth of FWM products is nearly equal to that of signal light; thus, the effect of spectrum spread due to baseband modulation can be ignored. This is illustrated in Figure 6. 19, in good agreement with [29].

![Data Channel and FWM Products](image)

**Figure 6. 19-** Data signal (thick line) and FWM products FWM 1 and 2 (dotted line). Notice the different scaling for this latter situation.

### 6.4.2.2 FWM Impact vs. Signal Power

In the case where the power exchange among the interacting waves is very low, a general statement about the power evolution of the FWM generated frequencies can be made: it scales linearly with the injected signals power, according to Eq. (4.18) – chapter 4. This was confirmed through simulations. Results will be presented for the case of signals total power of 10 dBm – above this value, according to previous results, other non-linear effects should be taken into consideration, namely SPM.
Figure 6. 20 shows the spectra plots for the analysed formats when fibre dispersion is zero (worst-case situation). As can be seen from the figure, a significant increase in the levels of first order FWM products was observed. The level was so high that in some cases (NRZ, duobinary and modified) these newly generated FWM frequencies start to act as new pumps, and through interference with the nearest signal carrier, a new set of tones were created again by FWM – identified as 2\textsuperscript{nd} FWM 1 and 2\textsuperscript{nd} FWM 2, which represent second order FWM products. The numerical values are given in Table 6.3. Again notice that for the case of dicode modulation, FWM tones are not anymore recognisable when dispersion is 3 ps/nm.km.

![Spectra of various modulation formats](image)

Figure 6. 20- Spectra of various modulation formats for input signal power of 10 dBm. Zero dispersion is assumed.

As in the case of an injected power of 6 dBm, for the new generated frequency identified as FWM 1, PRS signalling schemes achieve better suppression of the FWM effects as compared to binary. From the results, it seems that higher levels are attained that in the previous case. This may be linked with the fact that this modulation schemes have no optical carrier peak in their spectra. An interesting observation can be made regarding the dicode format: for zero dispersion (or very low values at least), it performs better than any of the other formats.

Concerning FWM 2, differences to the binary case are much less marked. In fact, only for modified duobinary one gets a suppression value 1 dB better than binary in a consistent way. Again, AM-PSK duobinary presents worst results than binary.

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<tr>
<th></th>
<th>Dispersion (ps/nm.km)</th>
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<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td>Optical Binary-FWM 1=FWM 2</td>
<td>19.7</td>
</tr>
<tr>
<td>Optical Duobinary-FWM 1</td>
<td>24.8</td>
</tr>
<tr>
<td>Optical Duobinary-FWM 2</td>
<td>20.1</td>
</tr>
<tr>
<td>Optical Duo. Butterworth-FWM 1</td>
<td>24.1</td>
</tr>
<tr>
<td>Optical Duo. Butterworth-FWM 2</td>
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</tr>
<tr>
<td>Optical AM-PSK Bessel-FWM 1</td>
<td>24.8</td>
</tr>
<tr>
<td>Optical AM-PSK Bessel-FWM 2</td>
<td>19.9</td>
</tr>
<tr>
<td>Optical Modified-FWM 1</td>
<td>25.5</td>
</tr>
<tr>
<td>Optical Modified-FWM 2</td>
<td>20.7</td>
</tr>
<tr>
<td>Optical Dicode-FWM 1</td>
<td>26.9</td>
</tr>
<tr>
<td>Optical Dicode-FWM 2</td>
<td>19.9</td>
</tr>
</tbody>
</table>

Table 6.3- Levels of the optical FWM generated products. Signal input power is 10 dBm.
For comparing purposes, Figure 6. 21 depicts the results obtained for the two signal powers just considered: 6 and 10 dBm. Remark the differences regularity for all the modulation formats regarding the FWM 2 tone – on average, an 8 dB difference (rightmost graph). It should compare with the more erratic case of FWM 1, where these differences are somehow dependent of the modulation format under consideration.

![Image of graphs](image)

Figure 6. 21- Comparison of the FWM 1 and 2 for injected signal powers of 6 and 10 dBm, respectively (rememver measurements are against the maximum level of the cw signal carrier).

An important conclusion that can be drawn is that for dispersion values higher than 3 ps/nm.km the levels of the FWM products tend to saturate. The saturation point is primarily power dependent; although signal shape dependence was also observed, its impact is far less important than the input power of the light waves.

### 6.4.2.3 FWM impact vs. Light waves polarisation

The lasers representing the two channels have the same initial polarisation (along the x-axis), but polarisation for one source is rotated along the equator of the Poincaré sphere by 90°. The FWM products are maximized when the polarisations are aligned and nearly completely reduced to zero when the two polarisations are orthogonal. This is illustrated in Figure 6. 22 for the case of zero dispersion – worst-case situation (the injected signal power is 10 dBm). Compare it with Figure 6. 20. Numerical results are displayed in Table 6. 4. Note the noise-like appearance for modified and decode spectra outside the main lobes.
Figure 6.22- Spectra for various modulation formats when input waves have orthogonal polarisation. Input power of 10 dBm is assumed.

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</thead>
<tbody>
<tr>
<td>Dispersion (ps/nm.km)</td>
<td>44.2375</td>
<td>45.6659</td>
<td>42.5613</td>
<td>44.9386</td>
<td>44.2242</td>
<td>40.9593</td>
</tr>
</tbody>
</table>

Table 6.4- Levels of the optical FWM generated products when input light waves are orthogonally polarised. Signal input power is 10 dBm. No entry means FWM products are no longer detected.

Therefore, FWM efficiency becomes almost zero when injected lights are assumed to have orthogonal states of polarisation. Fibre birefringence switch was in the on state, which means that the principal axes of induced birefringence were modelled as changing randomly along the fibre length. As in the case of XPM, this is an indirect proof that FWM products are mostly generated near the beginning of the fibre. A more precise statement can be made at this point: the relationship of the states of polarisation of the light waves is the important parameter, not the states of polarisation themselves. As a matter of fact, other polarisation values were tested, ranging from linear to circular ones, and results show little variations: when the states of polarisation of the two input waves were completely orthogonal, FWM power becomes almost zero.

Overall, PRS signalling schemes showed a greater level of suppression than binary. The only exception, concerns AM-PSK duobinary format just for the generated tone frequency given by $f_{FWM2}=2f_1 f_2$, where $f_1$ is the cw light carrier and $f_2$ for modulation format data. This result seems to be a direct consequence of the absence of the peak carrier in the signal frequency domain for these signals. Furthermore, there is also some evidence pointing up to the fact that periodic phase modulation allows a reduction in the FWM products power [33]. As seen in chapter 4, generation of the optical PRS signals also introduces a phase shifts in a periodic way, which are specific to each one of the modulation formats under consideration. Thus, the better suppression
of FWM products can also be tied with the “phase modulation” inherent to the process of generating optical PRS signals. However, further detailed study is required.

6.4.2.4 FWM IMPACT VS. CHANNEL SPACING

When channel spacing between the two channels is narrowed to 25 GHz (0.201 nm), the impact of FWM is much more evident, as seen in Figure 6.23. Notice that even for moderate amounts of dispersion, FWM products have almost the same power – marginal reductions are observed for dispersion values up to 4 ps/nm.km – variations of 0.4 to 2 dB for the range of dispersion values taken into simulations, according to the modulation format under consideration.

![Figure 6.23](image)

Figure 6.23- Spectra of various modulation formats for different values of dispersion. Channel separation is 25 GHz (0.201 nm) and injected signal power of 10 dBm.

Here, again, PRS modulation formats perform better than conventional binary format in suppressing the FWM products.

Even in the case of narrow channel spacing, FWM products can be neglected when injected light waves have orthogonal states of polarisation, as can be seen from Figure 6.24 where spectra for
optical AM-PSK duobinary scheme is presented. All modulation formats show the same behaviour.

![Graphs showing optical AM-PSK duobinary scheme for D=0 ps/nm.km, D=1 ps/nm.km, and D=2 ps/nm.km](image)

Figure 6. 24- Spectra for various values of dispersion for optical AM-PSK duobinary format when input waves have orthogonal states of polarisation. Input power of 10 dBm is assumed.

### 6.5 Raman effect

Stimulated Raman scattering (SRS) severely limits the performance of WDM systems. It leads to a transfer of energy from shorter-wavelength channels to longer-wavelength channels within the Raman gain bandwidth. In silica fibres the Raman-gain coefficient extends over a large frequency range (up to 40 THz), being maximum for the frequency component that is downshifted from the pump frequency (which is the one whose power is depleted) by about 13.2 THz. This energy transfer depends on the channel spacing: it increases as the wavelength separation between two channels increases until about 100 nm [34]. Thus, the channel dependence of SRS on the channel spacing is in the opposite direction of XPM and FWM. An important feature of the Raman effect is that when the pump power exceeds a threshold value, the transfer efficiency builds up almost exponentially. It has been demonstrated that assuming a triangular Raman gain profile, equally spaced channels, and a fractional power $D$ lost by the shortest wavelength channel, there will be almost no system degradation due to SRS if the maximum power per channel does not exceed the value estimated by the following equation [35]

$$P_{ch} < \left(1 - 10^{-10}\right) 10^6 \frac{A_{eff}}{N(N-1)\Delta f L_c} \quad (6.5)$$

where $P_{ch}$ is the power per channel, $L_c$ is the fibre total effective length, $A_{eff}$ is the core effective area, $N$ is the number of channels, $\Delta f$ is the channel frequency spacing, and $X$ is the power penalty expressed as $X = -10\log_{10}(1-D)$.

#### 6.5.1 System model and parameters

A 16-channel system is launched onto a SMF ($D=16 \text{ ps/nm.km}$ and $\gamma =1.8 \text{ W}^{-1}\text{Km}^{-1}$) 20 km long. Each channel is based on the single channel system model described in chapter 5. For the
laser, a line width of 10 MHz and an average optical power of 0 dBm were considered. Regarding the optical modulator, an extinction ratio of 30 dB and an insertion loss of 3 dB were assumed. Each pseudo-random sequence, used for generating the input data, where started at random in order to assure de-correlation among the 16 channels. The centre wavelength of the shorter-wavelength channel was 1543.97 nm, and the one belonging to the longer-wavelength was 1556.03. Channel spacing was 0.804 nm (100 GHz). Finally, an optical booster amplifier was used, which injected a 20 dBm optical power into the fibre. Although the OptSim simulator uses an improved variant of the split-step Fourier method, the channel count was restricted to 16 because of long calculation times. Thus, a compromise had to be established between number of channels and simulation times. Finally, the characteristic Raman constant $\gamma R$ that controls the amount of delayed non-linearity (Raman effect) is varied from 0 to 0.5 through parametric runs.

6.5.2 Simulation results and discussion

Figure 6. 25 illustrates the cross talk induced by SRS as function of the fibre Raman constant $\gamma R$. The optical power spectrum of the received signal shows the spectral tilting induced by the SRS. The tilting increases with increasing $\gamma R$. Notice, for the eye-diagram, a progressive "closing" and distortion of the eye also with increasing values of $\gamma R$.

Overall, the spectrum tilting is less severe for PRS signalling schemes than conventional binary, although for duobinary modulation the improvement is very marginal. However, for modified duobinary there is a moderate improvement, which is considerably increased for AM-PSK duobinary. Thus, again, band limiting is a very effective technique that enables an improvement in the system performance. The spectrum tilting was measured as the difference in the peak power of the PSD (in dB) between the longer-wavelength channel and the shorter-wavelength channel.

Furthermore, if the average optical power at the end of the fibre is measured, both for the shorter- as well as for the longer-wavelength channel, AM-PSK duobinary presents the lower increment (decrement) concerning the longer (shorter) wavelength channel – see Figure 6. 26. Also note that results suggest that for this modulation format the increasing (decreasing) is almost linear and very smooth, when compared to the other modulation formats, specially duobinary and conventional binary.
Figure 6.25- Spectra and eye-diagrams for various modulations formats as function of the Raman constant $fR$.  

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Figure 6.26- Average output power at the end of fibre for various formats when the shorter wavelength channel (top figure) or the longer-wavelength channel (bottom figure) is considered.

6.6 Summary

This chapter gave an assessment of the impact of the main effects due to Kerr non-linearity (self-phase modulation, cross-phase modulation, and four-wave mixing) and stimulated Raman scattering induced cross talk. Suitable simulation set-ups were developed in order to isolate and clarify the impact of each individually. It was shown that SPM has an adverse impact for single channels; in fact, it limits the amount of injected power before penalties and signal distortions are so strong that system performance is totally degraded. XPM and FWM are two non-linear effects that introduce cross talk among the transmitted channels in a WDM system. Moreover, if the optical bandwidth is sufficiently large, even the Raman effect gives rise to channel interaction. All these effects introduce a cross talk component that is located in the same bandwidth of the channel to be selected; thus, their effect cannot be limited by increasing selectivity of the optical filter in the receiver. Much more subtle and complex approaches are required, some of which are beneficial for some effects and, at the same time, enhance the adverse effects of others non-linearities. As far as SPM, it was shown that it depends on fibre type, and its dispersion, as well as pulse shape and bandwidth, and propagated distance along the fibre. The XPM-induced chirp can be reduced by either increasing the channel spacing, or the
link dispersion or changing the state of polarisation of input waves to be orthogonal to each other. As far as FWM is concerned, its effect can be limited by increasing the frequency spacing among the channels, operating in fibre with high GVD or using input light waves whose states of polarisation are exactly orthogonal between each other. Raman-induced cross talk increases with increasing channel spacing; on its own, it sets an upper limit to the optical bandwidth that can be used. It strongly depends on the average optical power along the link. Above a certain threshold, the induced cross talk grows exponentially. These are general conclusions regarding all the analysed modulation formats. However, the impact of these effects depends on the modulation format.

Therefore, as SPM is concerned, an in-depth investigation was carried out regarding propagation distance, optical input power, and fibre type. Regarding the fibre type, SPM impact is more important for high GVD fibres and high-injected powers. Nevertheless, in the anomalous dispersion region, the cooperation between fibre dispersion and SPM has a beneficial effect, allowing longer distances to be bridged. This effect, known as non-linear assisted transmission, is particularly evident for modified and dicode formats for distances up to 70 km. The single exception concerns AM-PSK duobinary, which shows always an increasing EOP penalty. This partial compensation of dispersion is dependent of fibre type: almost non-existent in DSF (since one can neglect the effects of GVD), with a moderate expression in NZDF, and with a strong impact for SMF fibres. In this latter case, partial compensation takes place in the range of 20 to 50 km, and for powers less than 15 dBm. Above this threshold, non-linearity starts to dominate, and signal degradation occurs. Overall, for high GVD values and longer distances duobinary filtered by a 4th Butterworth filter, with a cut off of 6 GHz, showed the best results. On the other hand, for low-values of GVD and distances up to 70 km, dicode modulation format stood out as the best compromise.

Regarding cross-phase modulation effects, it was shown that it scales almost inversely proportional with channel spacings. It was also demonstrated that the magnitude of the XPM-induced interference is smaller for pulse shapes with relatively broad leading and trailing edges: for raised cosine shaped input signals, roll-off factors greater than 0.5 should be considered (in terms of rise and fall times, this means that they should not exceed 20% of the bit period, for a bit rate of 10 Gbit/s). Also, input average power should be kept under 8-10 dBm, otherwise the combined effect of XPM and SPM induce strong penalties. In all the simulated situations, the filtered versions of the duobinary signalling scheme showed better performance (which was
marginal in some cases), thus indicating that some benefits are brought up by band-limiting electrically duobinary signals. Input waves with orthogonal polarisations have shown to be an effective way to strongly reduce the impact of XPM, even when fibre birefringence is taken into account. Another method to reduce XPM is to increase fibre dispersion. Since XPM is only effective as long as the interacting signals are superimposed in time, which, incidentally, do happen in the initial part of the fibre link, higher chromatic dispersion implies that pulses in interfering channels will not remain superimposed on the pulses in the channel of interest, or that the superimposition takes place in a very short time. When channel spacing is narrower XPM-induced interference is much more severe; also, the impact of pulse shapes is not so important as compared to spacings greater or equal than 0.5nm.

Considering the impact of FWM, PRS signalling schemes showed, in general, better suppression of the new FWM generated products in all the simulated situations. FWM impact versus fibre dispersion, input average power per channel, and dependency of states of polarisation of input light waves were object of analysis. For each one of these case studies, PRS signals showed some improvements over binary. A good compromise was achieved by modified duobinary format. The method of choice to strongly reduce the impact of FWM is to increase link dispersion (even moderate amounts of dispersion is sufficient to suppress the FWM products). Another method is to polarise the input waves with orthogonal states of polarisation between each other, even for high pump powers. Power per channel should be carefully controlled, since increasing the pump power results in a significant increase in power of the FWM products. When channel spacing is narrowed FWM impact is much more severe. Nevertheless, PRS formats still presented better suppression of the FWM products. As in the case of larger channel separation, orthogonal polarisation of the input waves is a good method for reducing FWM efficiency.

Finally, it was shown that cross talk due to the Raman effect could strongly degrade system performance, even for a moderate number of channels.
References:

Chapter 7: Conclusion

In this Thesis, duobinary, modified duobinary and dicode signalling schemes were investigated. Particular emphasis has been placed on developing a unified and integrated approach to analyse the potential of these partial-response signalling formats to fibre transmission.

In chapter 2 a basic overview of some techniques and methods to increase capacity in high-speed optical fibre systems was given. They ranged from multiplexing (either in time, or wavelength or polarisation) to techniques to combat critical performance degrading effects limiting the system capacity. In this perspective, partial-response signalling formats applied to optical fibre systems were introduced.

The theoretical foundations regarding duobinary, modified duobinary and dicode were presented in chapter 3. Their properties and fundamental differences in terms of spectrum, eye-diagrams, and potential advantages and limitations were then examined. Computer simulation was then used to validate the theoretical results.

In order to assess performance of the various modulation formats investigated, simulations were carried out. In chapter 4 a brief overview of the simulator was given. The models used for the more important optical link components, including laser, the single mode fibre, and Mach-Zender external modulator, were extensively described. Then, a generic simulation model was established all along with a definition of the eye-opening penalty, the key parameter to be used in the evaluation of the optical PRS systems.

In chapter 5, simulation results were presented based on the impact of fibre chromatic dispersion on system performance. Since it was previously known that band limiting electrically duobinary signals was effective in improving dispersion immunity, a systematic study regarding the impact of electrical filter parameters was then carried out. Filter parameters such as roll-off, amplitude and/or phase response were assessed for all the modulations formats investigated. Then, guidelines were established on how to choose optical transmitter filters for the analysed PRS formats. It was shown that band limiting also had a positive impact on modified duobinary and dicode dispersion immunity. However, improved tolerance to dispersion of optical PRS systems
was shown to rely on strict symmetry in the electrical signals and the MZ modulator operation, as opposed to convention binary intensity modulation.

Finally, the impacts of fibre non-linearities on the various modulation formats under investigation were analysed in chapter 6. It was shown that there was a benefit to be gained in terms of tolerance to some of the non-linear impairments investigated.

### 7.1 Contributions

The main points and conclusions derived of the present work can be summarised as follows:

- A unified and integrated approach to analyse the potential of duobinary, modified and dicode formats was developed. The theoretical foundations and simulation framework were presented in chapters 3 and 4.

- In chapter 5, the impact of fibre chromatic dispersion on system performance was evaluated for the various modulation schemes. Significant improvements in dispersion immunity were achieved by band limiting electrically anyone of the PRS signals.

- Guidelines were established on how to choose optimal transmitter filters for optical PRS systems. The impact of electrical filter parameters such as roll-off, amplitude and/or phase response, as well as how each one of the modulation formats impacted filter bandwidth.

- Tolerance to fibre dispersion of all the optical PRS systems relies on strict symmetry in the signal and in the external modulator operation. Parameters breaking this symmetry, such as laser line width, modulator bias, modulation extinction ratio and chirp, were assessed and limits were established.

- In chapter 6, it was shown by the first time that due to cooperation between self-phase modulation and group-velocity dispersion, non-linear assisted transmission occurred for almost all the PRS signalling schemes – the single exception has been AM-PSK duobinary.
The impact of SPM on the various modulation formats investigated was dependent on pulse shape and bandwidth, fibre type and propagated distance.

Cross-phase modulation penalties were affected by channel wavelength separation, channel average power, and state of polarisation of the input light waves. Limits were established, and shown that band-limiting duobinary electrical signals did present larger tolerance to this non-linear impairment that other modulation schemes.

It was also established that optical PRS signalling schemes allowed a greater level of suppression of the four-wave mixing generated products than conventional binary. FWM penalties were dependent on power per channel, state of polarisation relationship of the input light waves, and channel spacing.

Finally, it was shown that cross talk due to the Raman effect strongly affected system performance, even for a moderate number of input channels.

### 7.2 Suggestions for future work

This work laid the ground foundations and framework for a number of possible paths that are worthy of further investigation. They are summarised as follows:

- Investigate more thoroughly the interdependence between parameters and/or effects for these modulation formats.

- Investigate higher bit rates systems, namely 20 and 40 Gbit/s.

- Investigate other formats other than NRZ, such as RZ, for use in PRS systems.

- Investigate the impact of in-line optical amplifiers, thus allowing multi-span, multi-amplifier links to be considered.

- Investigate the use of these PRS signalling schemes in long-haul systems.
7.3 Concluding remarks

In this Thesis, a unified and integrated approach to analyse the potential of duobinary, modified duobinary and dicode formats was presented. The findings and limitations established here have shown that there is a benefit to be gained in adopting a more sophisticated modulation format rather than binary intensity modulation. However, a careful selection of parameters for a particular system must be made, taken account of a wide range of factors.