FORMING LIMIT DIAGRAMS.
DEFINITION OF PLASTIC INSTABILITY CRITERIA
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Dissertation presented to Engineering Faculty of Porto University for the accomplishment of Doctor of Philosophy in Mechanical Engineering degree requirements, realized under scientific supervision of Prof. Augusto Duarte Campos Barata da Rocha, Associate Professor of Engineering Faculty of Porto University and Prof. Jóse Joaquim de Almeida Grácio, Full Professor of Mechanical Department of Aveiro University.

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Abstract

A more general model for predicting the sheet forming limits under linear and complex strain paths is proposed. The model utilizes the Theory of Plasticity and the Marciniak-Kuczinsky analysis. A modular and user-friendly software is developed, which allows the implementation and combination of different constitutive equations, changing the corresponding subroutine, while keeping intact the main part of the program. The phenomenological approach as well as the physical one, are utilized for modelling the anisotropic behaviour of sheet metals. In order to demonstrate the generality and validity of the new model, several phenomenological constitutive equations such as, Swift hardening power law and Voce saturation hardening law, the isotropic Von Mises yield criterion, the quadratic Hill yield criterion (Hill’48), the non-quadratic Hill yield criterion (Hill’79) and the Yld96 Barlat yield criterion are implemented. Finally, a physics - based constitutive model accounting for the texture and strain path induced anisotropy is considered. This advanced model is based on the Van Houtte’s anisotropic texture plastic potential expressed in a strain rate space coupled with the Teodosiu and Hu microstructural hardening model. A detailed strain and stress based forming limits analysis is performed to assess the potentiality and efficiency of the new developed model on the Forming Limit Diagrams and Forming Limit Stress Diagrams predictions. It is also of particular interest its aptitude in the selection of the best combination of constitutive equations for an accurate description of the material behaviour. This quality of the material model was shown to be vital for good predictions on plastic flow localization from the Marciniak-Kuczinsky theory and consequently for a correct analysis on the material formability.
Resumo

Um modelo mais geral para previsão dos limites de embutidura duma chapa metálica, em trajectórias lineares e complexas é proposto. O modelo utiliza a Teoria da Plasticidade e a análise do Marciniak-Kuczinsky. Um programa modular e simples é desenvolvido, qual permite a implementação e combinação das diferentes equações constitutivas através da modificação da subrotina correspondente, mantendo intacta a parte principal do programa. A abordagem fenomenológica assim como uma física são utilizadas para a modelação do comportamento plástico anisotrópico das chapas metálicas. Para demonstrar a generalidade e a validade do novo modelo, são implementadas varias equações constitutivas como, por exemplo, as leis de encrumento de Swift e Voce, o critério de cedência isotrópico de Von Mises, o critério de cedência quadrático de Hill (Hill’48), o critério de cedência não-quadrático de Hill (Hill’79) e o critério de cedência Yld96 de Barlat. Em fin, um modelo constitutivo físico, contando com a anisotropia induzida por textura e a anisotropia induzida por trajectória de deformação, é também considerado. Este modelo avançado se baseia no potencial plástico anisotrópico de Van Houtte expresso em velocidades de deformações, combinado com o modelo microstructural de encrumento de Teodosiu e Hu. Uma análise detalhada das deformações limites de embutidura em deformações e tensões é desenvolvida para salientar o potencial e a eficácia do novo modelo, na previsão das Curvas Limites de Embutidura e das Curvas Limites de Embutidura expressas em Tensões. É também de particular interesse a sua aptidão em selecção da melhor combinação das equações constitutivas para uma descrição correcta do comportamento do material. Esta qualidade do modelo do material se comprovou a ser vital para uma boa previsão da localização do escorregamento plástico através da teoria do Marciniak-Kuczinsky e consequente, para uma análise correcta da formabilidade do material.
Résumé

Une modélisation plus générale pour les limites de formage des tôles métalliques utilisées en emboutissage, qu’elles soient déterminées en trajets de déformation linéaires ou complexes est proposée. La modélisation utilise la théorie de la plasticité et le modèle de Marciniak et Kuczinsky. Un logiciel modulaire et convivial est réalisé, qui permet de combiner librement la surface de plasticité et le modèle d’écrouissage choisi, en modifiant les routines correspondantes et sans toucher au cœur du programme principal. L’approche phénoménologique ainsi qu’une approche plus physique sont utilisées pour la modélisation du comportement plastique anisotrope des tôles. Afin de démontrer la généralité et la validité du nouveau modèle, plusieurs équations constitutives phénoménologiques sont implémentées: la loi de Swift et la loi saturante de Voce pour l’écrouissage, couplées au critère de plasticité isotrope de Von Mises, au critère anisotrope quadratique de Hill (Hill'48), au critère anisotrope non-quadratique de Hill (Hill'79) et au critère de plasticité anisotrope Yld96 de Barlat. Enfin, un modèle constitutif à bases plus physiques est considéré, qui rend compte de l’anisotropie de texture et de l’anisotropie induite par le trajet de déformation. Ce modèle performant est basé sur le potentiel plastique anisotrope de Van Houtte, exprimé en vitesses de déformation, couplé au modèle d’écrouissage microstructural de Teodosiu et Hu. Une analyse détaillée de la déformabilité est faite pour évaluer le potentiel prédicatif et l’efficacité du nouveau modèle sur la simulation des Courbes Limites de Formage en déformations et des Courbes Limites de Formage en contraintes. On s’est plus particulièrement intéressé à son utilité dans le choix de la meilleure combinaison d’équations constitutives pour une description précise du comportement du matériau. Cette qualité du modèle de comportement s’est avérée essentielle pour la bonne prédiction de la localisation de l’écoulement plastique par la théorie de Marciniak-Kuczinsky et donc pour une analyse correcte de le formabilité du matériau.
Symbols and abbreviations

Phenomenological approach

a, b, c, d, e, f, g  coefficients of BBC2000 yield function
a, c, h, p  material parameters in Barlat and Lian yield function
a, k  integer exponents used by yield criteria
ak, bk  coefficients of Cazacu-Barlat’2002 yield criterion
A, B, C  material constants on Voce hardening law
B, C, D, H  coefficients of Chu yield function
c, p, q  coefficients of Hill 1993 yield function
c1, c2, c3  independent material parameters of Yld96 Barlat yield function
CD  material constant in Drucker yield function
e  sheet thickness
f0  initial geometrical imperfection factor of M-K analysis
f, g, h, a, b, c  coefficients of Hill 1979 yield function
g(θ)  function used to define the Budiansky yield criterion
g(θ,α)  function used to define the Ferron yield criterion
hi  anisotropy coefficients in the Von Mises yield function
I2, I3  second and third invariants of stress tensor
J1, J2, J3  invariants of the stress deviator tensor
k  material constant on Swift hardening law
K1, K2  invariants of the stress tensor
L  linear transformation fourth-order tensor used by yield criteria
m  coefficient of strain rate sensibility
M  exponent used by yield criteria
n  work hardening coefficient
p  transformation matrix between the principal direction of s and the principal axes of anisotropy
r0, r45, r90  anisotropy coefficients at 0, 45 and 90 degrees from rolling direction
R0, R45, R90  anisotropy coefficients at 0, 45 and 90 degrees from rolling direction
R, r  average anisotropic parameter
R, S, T  shear yield stress in the principal anisotropic directions(Hill 1948)
s, S  isotropic plastic equivalent (IPE) transformed stress tensor
s1, s2, s3  principal values of s
S1, S2, S3  principal values of S
X, Y, Z  tensile yield stress in the principal anisotropic directions(Hill 1948)
Y  yield stress
α  strain ratio
α1, αy, αz  quantities related to the anisotropy of the materials (Yld94, Yld96)
α1, α2, γ1, γ2, γ3, c  parameters defining the material anisotropy (Karafillis –Boyce criterion)
a1, α2, α3  coefficients of Yld94 and Yld96 Barlat yield functions
\[ \beta_1, \beta_2, \beta_3 \]
angles between the principal directions of \( s (1, 2, 3) \) and the anisotropic axes

\[ \Gamma, \psi \]
functions of the second and third invariants of deviatoric stress tensor equivalent plastic strain

\[ \epsilon_0 \]
materical constant on Swift hardening law

\[ \tilde{e} \]
effective strain rate

\[ \dot{\tilde{e}} \]
plastic strain rates

\[ d\tilde{e} \]
increment of equivalent plastic strain

\[ [d\epsilon]_{yz} \]
strain increment tensor in the orthotropic referential frame of anisotropy

\[ [d\epsilon]_{mr} \]
strain increment tensor in the in the groove reference frame

\[ d\epsilon_{ij} \]
components of strain increment tensor

\[ \epsilon_1, \epsilon_2, \epsilon_3 \]
principal strains

\[ \phi \]
yield function

\[ \varphi \]
function of the second and third invariants of the deviatoric stress tensor in Lián and Chen yield function

\[ \lambda(\rho) \]
ratio of effective strain to major true strain

\[ d\lambda \]
plastic multiplier in the flow rule

\[ \rho \]
stress ratio

\[ \tilde{\sigma} \]
equivalent stress

\[ \sigma \]
Cauchy stress tensor.

\[ \sigma_1, \sigma_2, \sigma_3 \]
principal stresses

\[ [\sigma]_{yz} \]
strain tensor in the orthotropic referential frame of anisotropy

\[ \sigma_x, \sigma_y, \sigma_{xy} \]
planar components of stress tensor in the orthotropic referential frame

\[ [\sigma]_{mr} \]
strain tensor in the in the groove reference frame

\[ \sigma_{mr}, \sigma_{nr}, \sigma_{nm} \]
components of stress tensor in the groove reference frame

\[ \bar{\sigma}^0 \]
stress direction

\[ \sigma_0, \sigma_{45}, \sigma_{90} \]
uniaxial yield stress at 0, 45 and 90 degrees from rolling direction

\[ \sigma_b \]
balanced biaxial yield stress

\[ \sigma_e \]
equivalent stress

\[ \sigma_u \]
uniaxial yield stress

\[ \sigma_{uts} \]
ultimate tensile strength

\[ \xi(\alpha) \]
ratio of effective stress to major true stress

\[ \psi_0, \psi \]
initial and current angle between the M-K geometrical imperfection and direction \( y \)

\[ \psi^\prime \]
critical orientation of the M-K geometrical imperfection

**Physical approach**

The plastic strain potential and all constitutive and evolution equations are supposed form-invariant with respect to the Jaumann frame. In this work, bold letters represent the involved tensors whose components in the Jaumann frame will be labelled by a superposed hat.
The so-called Einstein summation convention, stipulating that whenever a letter index appears twice in a product, the sum is to be taken over this index, is also adopted.

\( A \)  
plastic strain rate mode

\( A_{ij} \)  
plastic strain rate mode component (i,j = 1 \ldots 3)

\( A_K \)  
plastic strain rate mode component (K = 1 \ldots 5)

\( A_{65} \)  
five-dimensional to six-dimensional conversion matrix

\( B_{56} \)  
six-dimensional to five-dimensional conversion matrix

\( C_R \)  
isogetic hardening rate parameter

\( C_P \)  
polarization rate parameter

\( C_X \)  
evolution rate parameter of backstress

\( C_{SD} \)  
evolution rate parameter for the hardening due to active dislocations

\( C_{SL} \)  
evolution rate parameter for the hardening due to latent dislocations

\( D^p \)  
plastic strain rate

\( D^p \)  
norm of \( D^p \)

\( D_{\text{vM}} \)  
von Mises equivalent plastic strain rate

\( D_{11}, D_{22} \)  
principal values of \( D^p \)

\( f(g) \)  
orientation distribution function (ODF)

\( g \)  
crystallographic orientation

\( G(A) \)  
sixth-order series expansion of strain rate mode

\( H^u \)  
hardening matrix on slip system \( s \) induced by slip system \( u \)

\( H \)  
work-hardening rate

\( I \)  
unit matrix

\( m \)  
Schmid tensor

\( m \)  
contribution ratio of persistent dislocation structures to isotropic hardening

\( M \)  
Taylor factor

\( \bar{M} \)  
average Taylor factor for polycrystal

\( n_p \)  
dissolution exponent of the currently active part of persistent dislocation structures under reversed straining

\( n_L \)  
dissolution exponent of the latent part of persistent dislocation structures

\( P \)  
polarity of persistent dislocation structures

\( q \)  
latent contribution to \( \bar{X}_{\text{sat}} \)

\( R \)  
rotation matrix

\( R \)  
isotropic hardening caused by statistically accumulated dislocations

\( R_{\text{sat}} \)  
saturation value of \( R \)

\( S \)  
directional strength of planar persistent dislocation structures

\( S_{SD} \)  
strength of the currently active slip systems of persistent dislocation structures

\( S_L \)  
strength of the latent part of persistent dislocation structures

\( S_{\text{sat}} \)  
saturation value of the directional strength of persistent dislocation structures

\( U \)  
normalized deviatoric yield stress

\( \dot{w}_f \)  
frictional dissipation

\( W \)  
rigid body rotation rate

\( \dot{W} \)  
rate of plastic power per unit volume

\( X \)  
backstress
$X_0$  
initial value of $\overline{X}_{sw}$  

$\overline{X}_{sw}$  
saturation value of $|X|$  

$\beta$  
Schmitt parameter characterizing two-stage strain-path  

$\delta_{ij}$  
Kronecker symbol, $\delta_{ij} = 1$ if $i = j$, else $\delta_{ij} = 0$  

$\Delta t$  
time increment  

$\dot{\gamma}$  
total slip rate over slip systems  

$\dot{\gamma}_s$  
slip rate on slip system $s$  

$\dot{\gamma}$  
average polycrystal slip rate  

$\theta^b$  
angle expressing the orientation of the orthotropic axes of anisotropy of the heterogeneous zone with respect to the global reference frame ($x'^o y'^o z'^o$)  

$\dot{\sigma}'$  
deviatoric stress tensor  

$\tau$  
critical resolved shear stress  

$\tau_0$  
initial value of $\tau$  

Generalities  

FLD  
forming limit diagram  

FLSD  
forming limit stress diagram  

HL  
hardening law  

YF  
yield function  

$\partial$  
symbol of partial derivatives  

$J$  
Jacobian matrix  

$J^{-1}$  
inverse of the matrix $J$  

BCC  
body-centred cubic  

FCC  
face-centred cubic  

RD  
rolling direction  

UT-BS  
uniaxial tension followed by biaxial stretching  

BS-UT  
biaxial stretching followed by uniaxial tension  

UT-PS  
uniaxial tension followed by plane strain stretching  

BS-PS  
biaxial stretching followed by plane strain stretching  

FLDcode  
ame of the new developed code for FLD prediction  

TexMic  
ame of the combined model of texture and strain path anisotropy  

BBC2000  
Banabic et al. 2000 yield function  

Hill'48  
quadratic yield function of Hill  

Hill'79  
non-quadratic Hill 1979 yield function  

Yld91  
Barlat 1991 yield function  

Yld94  
Barlat 1994 yield function  

Yld96  
Barlat 1996 yield function  

Yld2000-2d  
Barlat 2000 yield function
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1. INTRODUCTION

1.1 State of the art and objectives

The optimisation of sheet metal forming processes through the use of numerical simulation has become a key factor to a continuously increasing requirement for time and cost efficiency, for quality improvement and materials saving, in many manufacturing areas such as automotive, aerospace, building, packaging and electronic industries.

An extensive effort has been devoted to the development of reliable analytical tools and mathematical models capable of simulating the sheet metal forming processes which can be of a great value in reducing the tool try-out works, that is a time consuming process involving occupation of production equipment for costly prototype experiments.

The environmental problems associated with the various steps in the life cycles of resources, materials and energy recovery determine a need to re-evaluate the ways in which natural resources are managed. Concerning the automotive industry, part of the overall strategy is lower weight vehicles that mean increased performance, reduce fuel consumption and consequently less gas emissions to the environment. Therefore, the actual important issues as weight saving, improvement of passenger safety and better corrosion resistance has raised the application of high-strength low-alloy-carbon steel sheets and aluminium and its alloy sheets in the construction of vehicles in the last years. The formability problems encountered in press-forming steel sheets of enhanced strength and reduced thickness together with higher standards of corrosion protection impose significant constraints on manufacturing high quality products and process reliability. In this context, the aluminium alloy sheets and other steel sheets, like dual-phase steels and bake-hardening grades may extend their challenge to commercial deep drawing steels in vehicle manufacture. Moreover, new advanced structural materials such as the high-strength aluminium alloys, dual-phase and TRIP steels were recently developed to support the highest priorities of the sheet metal manufacturing industry and their use can be strongly promoted by proper numerical simulation.

The sheet metal formability is a measure of its ability to deform plastically during a forming process in order to produce a part with definite requirements on mechanics, dimension and appearance, being mainly limited by the occurrence of flow localization or instability. Its strong dependence on both the intrinsic constitutive properties of the sheet metal and the extrinsic factors involved in a practical forming operation turned the correct choice of these parameters to be one of the main aims in modern industry, their experimental study being a difficult and sometimes unfeasible task. A good understanding of the deformation processes, of the plastic flow localization and of the factors limiting the forming of sheet metal is of key importance in monitoring the formability issue.

The Forming Limit Diagram (FLD), introduced by Keeler (Keeler, 1964) and Goodwin (Goodwin, 1968), represents an useful concept for characterizing the formability of sheet metal and a very important safety tool in sheet metal forming simulation. Forming Limit Diagrams under complex strain paths help us to understand the behaviour of the material in complex loading, to estimate the severity of the strain paths imposed on the workpiece and to optimise the
shape of the dies to avoid the necking occurrence. Experimental studies have shown that the maximum admissible limiting strains strongly depend on the deformation mode, loading history and plastic anisotropy induced by cold rolling. The experimental determination of these parameters requires a vast and expensive effort.

The theoretical analysis of plastic instability is therefore of major importance in order to predict the forming limits, to examine the influence of each parameter on the necking occurrence and to improve the press performance. Industrial complex parts are manufactured by multistage forming operation. The strain ratio evolution is generally quite complex and abrupt changes can take place. Taking into account the strain path changes in FLD calculations is really important, because the number of potentially significant changes is too great to be thoroughly covered by experiments and because theoretical calculation allow general trends to be explored over a large range of variables.

Marciniak and Kuczynski (1967) have proposed the first realistic mathematical model for theoretical determination of FLD’s that suppose an infinite sheet metal to contain a region of local imperfection where heterogeneous plastic flow develops and localizes.

Numerous authors have tested the implementation of different yield criteria in the M-K model. A general result is that the predicted limit strains tend to strongly depend on the constitutive law incorporated in the analysis. In the last years better material models describing new generations of sheet metals have been developed. The use of an appropriate yield function that describes analytically the plastic behaviour of orthotropic metals allows to a better prediction of limit strains, therefore a better shape and position of FLDs. To achieve an efficient development, a high level of accuracy is desired in the simulation.

This doctoral research develops a more general model for predicting the sheet forming limits under linear and complex strain paths, based on the Marciniak-Kuczynski (1967) analysis and Theory of Plasticity, concluded in a modular and user-friendly software. Its high priority is to allow the implementation of any constitutive equations easily and without changes in the main part of the program being also adaptable for future developments in the advanced material models.

Several phenomenological constitutive equations as well as a physics -based constitutive model accounting for the texture and strain path induced anisotropy are implemented in the new model to prove firstly its efficiency and generality and secondly its validity for both of approaches of the Theory of Plasticity, namely the Phenomenological one and the Physical one.

For a practical validation of the new developed model in accuracy as well as in its performances, various advanced structural materials are considered in a meticulous experimental and theoretical study on Forming Limit Diagrams under linear and complex strain paths.

With the purpose of a better understanding of the stress-based forming limit concept, proposed by Kleemola and Pelkkikangas(1977), and Arrieux, Bedrin and Boivin(1982), as a solution for the multi-stage forming processes analysis, being apparently independent of the strain path changes, a detailed analysis of it by simultaneously comparing with the strain based forming limit concept through the use of the new model is also performed.
1.2 Outline of the thesis

The present work is structured on 7 chapters as follows:

Chapter 2 reviews the Forming Limit Diagram concept as well as the most used experimental techniques of its determination and the major progresses on the theoretical analysis of plastic flow localization under linear and complex strain path. A particular attention will be focused on the Marciniak-Kuczynsky method. Its sensitivity on several factors involved in a forming operation will be analysed and discussed.

A succinct review of the main phenomenological constitutive equations proposed in the literature including the recent developments is presented in Chapter 3. In addition, the more relevant polycrystal models describing the single crystal plasticity and the polycrystal plasticity will be also briefly reviewed in this section.

In Chapter 4 the computational method applied for the development of the new model of Forming Limit Diagram prediction for the phenomenological and physical approach of theory of plasticity will be presented. Next, the selected phenomenological constitutive equations will be reviewed and their particular implementation on the new model will be presented. After that, the physics-based plasticity model will be described in a detailed review and its application on the developed FLD code including the analytical derivatives of such constitutive model required for its numerical integration will be also presented. Furthermore, a comparison of the theoretical treatments of localized necking between the phenomenological and physical approach of theory of plasticity will be discussed.

In Chapter 5, experimental and theoretical approaches linking the material’s properties with the formability performance will be presented for a realistic validation of the new model. A series of particular studies are selected to point out important aspects of the sheet metal forming analysis.

Chapter 6 presents a study on Forming Limit Stress Diagrams (FLSD) during linear and complex strain paths. The influence of the applied constitutive law on the experimental and predicted stress limit will be analysed. Experimental and theoretical results for proportional deformations and two stage strain path changes will demonstrate the independence of the stress based forming limits on the strain path changes. Next, the two concepts of forming limits, based on strain and stress analysis respectively, will be compared and analysed.

Finally, the general conclusions of the present work are summarized in Chapter 7.

Some new topics for a future research on forming limits are also proposed in Chapter 8.
2. FORMING LIMIT DIAGRAMS

2.1 Introduction

In sheet metal forming operations, the amount of useful deformation is limited by the occurrence of unstable deformation, which mainly takes the form of localized necking or wrinkling. Failure by wrinkling occurs when the dominant stresses are compressive, tending to cause material thickening. Localized necking occurs when the stress state leads to an increase in the surface area of the sheet at the cost of a reduction in the thickness and is a very important phenomenon in determining the amount of useful deformation that can be imposed on a workpiece.

The forming limit diagram is a constructive concept for characterizing the formability of sheet metal. It was proved to be an essential tool for material selection, design and try out of the tools for deep drawing operations. Since the experimental determination of FLDs requires a wide range of sheet forming tests, consequently a large variety of expensive equipment and tremendous experimental effort, many attempts have been made to predict the FLDs, taking into account the theory of plasticity, material parameters and instability conditions.

The objective of this chapter is to review the Forming Limit Diagram concept, its experimental determination and the pertinent theoretical studies related to the analysis of forming limit diagrams (FLDs). Moreover, the theories advanced toward the understanding and predicting such strains under complex loading conditions will be described. Of particular interest is the Marciniak - Kuczynsky method (M-K) and its sensitivity on different factors.

2.2 Forming Limit Diagram concept

Stretch forming is a significant component in many sheet metal manufacturing processes. Simultaneously with the progressively thinning of a sheet, two modes of plastic instability are possible, a diffuse necking followed by a localized necking.

A study of failure in biaxially stretched sheets by Keeler and Backofen (1963) showed the existence of what is known as forming limit diagrams (FLDs). The main discovery was that the largest principal strain before any localized thinning in a sheet increased as the degree of biaxiality increased. They had tested several materials including steel, copper, brass and aluminum sheets by stretching them over solid punches. Later, Keeler (1965) found the material properties have great effect on the strain distribution in biaxial stretching of sheet metal. He pointed out that for higher exponent of the material work hardening, n, the strain distribution will be relatively homogeneous. On the contrary, materials having lower n values develop sharp strain gradients and the deformation concentrates in a very small region, then causing premature failure. A map in principal strain space (\(\varepsilon_1, \varepsilon_2\)), separates safe strain states that a material could provide, from the more severe states, which would lead to failure. By definition, \(\varepsilon_1\) is the major principal strain, and \(\varepsilon_2\) is the minor principal strain. Therefore, FLDs show the combination of major and minor in-plane principal strains beyond which failure occurs. With further development of the experimental techniques by Goodwin (1968), a FLD for mild steel was obtained serving as a criterion for most stamping processes and often being referred as Keeler-Goodwin diagram (Figure 1) due of the contribution of both authors to the understanding of material formability.
The FLDs cover strain states from uniaxial tension through plane strain to balanced biaxial tension. A typical FLD has a forming limit curve (FLC) as shown in Fig. 2.
During a forming process, the strain generally increases towards the FLC. When the strain reaches the FLC, a necking or fracture is supposed to occur. By comparing the strains measured in the formed part to the FLC, the severity and nature of the deformation can be assessed and process parameters such as lubricants and draw beads can be re-designed accordingly to assist the forming operation.

2.3 Experimental determination of the FLDs

Strains in deformed specimens are generally measured by means of the circle method. In this method, circle grids are electrochemically applied on the initial blank. As the result of deformation, the circles are distorted into ellipses. The major and minor principal strains are determined from the axes of the ellipses.

Although the concept of FLD is simple, its experimental determination is not trivial, requiring a wide range of sheet forming tests, between uniaxial tension and equibiaxial stretching. The most used tests for experimental measurement of the forming limit strains are the following:

- **Uniaxial tension** test is performed with a conventional tension machine. It provides constitutive properties of material, which are important factors influencing the formability in more complex loading conditions and reveals the yield point phenomenon. Moreover, varying the geometry of the specimens it is possible to obtain different strain paths and the corresponding limit strains.

- Test **Jovignot** (1930) consists in forming a blank sheet under a hydraulic pressure. Function of the type of the die geometry, which can be circular, elliptical and rectangular different strain paths are obtained. The main advantage of this method is that there is no direct contact between punch and sheet consequently no problems of friction exist.

- **Nakazima** test (1968) uses a hemispherical punch and a circular die to form rectangular blanks. Function of the blank width and the lubrication conditions is possible to obtain all range of the strain paths between the uniaxial tension and equibiaxial stretching.

- **Fukui** test (Fukui, 1939) consists in forming a circular metal blank through the clearance between a hemispheric punch and a conical die.

- **Swift** test (Swift, 1952) consists in forming a circular metal blank with a cylindrical flat-bottom punch. This test is particularly interesting for experimental determination of FLDs under complex strain paths.

- **Marciniak** test or in-plane stretching (I.P.S.) (Marciniak, 1973), to avoid the friction problems, deforms a blank until rupture through the use of a device composed by a flat bottom punch. In order to obtain different strain paths, punches with different geometries such as squared, elliptical or circular are used.

More details concerning these basic tests for experimental determination of FLDs can be found in several works (Jalinier, 1981; Arrieux, 1981; Barata da Rocha, 1985; Nguyen Nhat Thang, 1994).
Over the years, many researchers have been carried out in search for an easier FLD construction. Hecker (1975), proposed an experimental technique that allows drawing the entire diagram with less stretching tests by using a hemispherical punch with different widths and lubricants. Although Hecker's model was a significant improvement to FLD construction, the employed methodology was of great complexity. Tadros and Mellor (1978), Gronostajski and Dolny (1980) and Raghavan (1995), on the other hand, developed techniques not only to eliminate the tool/material friction effects, but also to keep the blank surface plane, a disadvantage of the previous models. Sing and Rao (1993) treated a method to determine the FLD through the parameters obtained by a conventional tensile test, which seems to be very helpful and less expensive than the previous ones. Since 1980, new measuring devices like CCD cameras together with more powerful computer facilities allowed to an easier and faster strains measurement. The first systems have been developed to measure one ellipse and automatically calculate principal strains. After 1985, thanks for the progress in calculation power, the square grid was exploited, automatically being possible the simultaneous measurement of many squares. So, in 1992, Tan et al. (1992) proposed an image processing technique to facilitate measuring operations. Albretskens (1994) analysed the best use to be done with Marciniak method of deformation. Mguil et al. (1997) introduced a new technique named “image correlation” which allows to replace the grids by a “mouchetis” of randomly produced strains. In 1999, Vacher et al. (1999) analysis the evolution of the thickness variation, on the successive images recorded by a digital camera during the straining of the blanks.

In the 1968, Yoshida (1968) and Keeler (1968) put on the scene one of the greatest problem involved on the FLD’s determination: “Must the FLD to be defined in terms of fracture or localised necking?”

As Coll (2000) mentioned in its interesting paper, fracture is a well-defined criterion, easy to detect, without ambiguity, but when it occurs, it is too late and the measurement of the strains is very difficult, if not impossible. So, what would be interesting to predict is the onset of localized necking, which in fact, decides correctness of the work-piece. For quality representatives, a part with a local necking is to be rejected because its aspect is not acceptable for external parts, its stiffness can be affected, exists a serious risk for fatigue initiation and also can be reduced the corrosion protection. Over the years were analysed two more failure criteria namely the pronounced necking and the onset of diffuse necking. Even that the first criterion is clearly visible, the strain measurement is difficult as for fracture, and the state of pronounced necking can extend over a non-negligible range of strain. The state of onset of diffuse necking corresponds to the load instability being very well defined but too sever to be accepted. In industrial practice, quite a lot of diffuse necking is tolerated before a part is considered unacceptable, even in plane strain. Nowadays both these criterions are no more used.

The methods for determination of FLD from the attained strains in deformed specimens by the tests presented before are based on the detection of the initiation of a strong strain gradient characteristic of the localized necking. The proposed methods for determination of the strains corresponding to the localized necking are:

- Veerman method (Veerman, 1971, 1972) that compares the deformation of the three adjacent circles and reports the evolution of the central circle deformation as a function of the average deformation of the adjacent circles. A disadvantage of this technique is that it does not take into account the strain gradients existing in the necking vicinity.
Also the method is very much time consuming because many measurements at many steps of the forming process have to be made.

- **Bragard** method, (Bragard, 1972) based on the strain distribution at fracture and using a mathematical method of parabolic regression calculation, determines the point corresponding to localized necking. In this method it is also necessary to eliminate the highest points of the strain distribution profile in order to calculate the parabolic regression. Therefore, the forming limit strains determination is strongly dependent on the choice of the points representing the strain distribution profile. Due of enormous influence of the way in which the extrapolation is made, upon the results, great care was taken by Bragard to fix severe conditions for choosing circles and so on. The advantage of this method is the relatively small account of work required, but its drawback undoubtedly results from the artificial character of the extrapolation, which in fact still to be discussion even today, thirty years later. Anyway, to assure reproducibility such extrapolation can be automatically proceeded under computer control, allowing to precisely fix the smoothing law.

- **“Double profile”** method, (D’Haeyer et Bragard, 1974) consists on the determination of two strain distribution profiles. The first one corresponds to the localized necking and is obtained by forming a sample through an adjusted press to a depth of pass giving the neighbouring strains of the initiation of localized necking. The second profile corresponds to the fracture and is obtained by forming the sample until fracture. The strain distribution profiles are represented in rapport with the circles on which have been done the strains measurement. By plotting both strain profiles on the same graphic, its last two common points are selected to define the forming limits at the localized necking.

- **Hecker** (1972) method consists on the measurement of the strains of three types of circles on the broken sample:
  - the broken circles
  - the circles that show a visible localized necking
  - the circles that don’t show a visible necking that are not broken

On the diagram of the principal strains, the points at the fracture, the points showing the localized necking and the points without necking but in vicinity of the fractured circles or those with necking are plotted. The forming limit curve is obtained by plotting the border between these different types of circles. The precision of this method depends of the number of measured circles. The clear advantages of this method, which, even today, is often considered as a reference method for FLD’s, are that there is normally only one or two broken parts to be measured for each mode of deformation and the used criterion is the industrial one. Its disadvantages come from the somewhat intuitive way of judgement and from the exhaustive and time consuming involved work. The modern techniques of electronic measuring could give a renewal of interest to the Hecker technique.

- **Kobayashi** method (Kobayashi et al., 1972) proposed to determine the necking localization as the moment when an abrupt augmentation of the surface roughness
occurs. Thus, this method requires the measurement of the roughness profile on the critical zone, for different deformation rates. Its evolution as a function of the strain shows an abrupt change of the slope, corresponding to the necking localization.

— Method n° 5 of Zürich (Parniere and Sanz, 1976). In 1973 the International Deep Drawing Research Group (IDDRG), proposed a normalization of the interpolation of the Bragard method. The strains measurement is realized on an interlocking circles grid. Considering a broken circle and three adjacent circles on both sides of the rupture, the method recommends using for interpolation only the measures corresponds to differences of the strains between two neighbouring circles less than 5%. Figure 3 presents several of such recommendations.

![Diagram](image)

If:
\[ \varepsilon_1(\text{I}) - \varepsilon_1(\text{II}) < 0.05 \]

The circles I, II and III are interpolated.

If:
\[ \varepsilon_1(\text{I}) - \varepsilon_1(\text{II}) > 0.05 \]
\[ \varepsilon_1(\text{II}) - \varepsilon_1(\text{III}) < 0.05 \]

The circles II and III are interpolated.

If:
\[ \varepsilon_1(\text{I}) - \varepsilon_1(\text{II}) < 0.05 \]
\[ \varepsilon_1(\text{I'}) - \varepsilon_1(\text{II'}) > 0.05 \]

The circles I, II, III, II' and III' are interpolated

The interpolation is impossible when:
\[ \varepsilon_1(\text{I'}) - \varepsilon_1(\text{I}) > 0.05 \]
even if,
\[ \varepsilon_1(\text{I}) - \varepsilon_1(\text{II}) < 0.05 \]
\[ \varepsilon_1(\text{I'}) - \varepsilon_1(\text{II'}) > 0.05 \]

Figure 3. Method n° 5 of Zürich (Parniere and Sanz, 1976)

— The French Society of Metalurgy, (SFM, 1974), by comparing all different tests and methods used for experimental determination of forming limit curve (FLC) showed that the FLC is strongly dependent on the type of the test that is used, especially on the plane strain region. Consequently, depending on the strain gradient nature, two types of tests can be considered. First type includes the uniaxial test on the large samples, the Nakazima test and the tests Jovignot and Marciniak, corresponding to a weak strain gradient. The second type contains the tests Fukui and Swift and those that utilize the uniaxial tests on the notched samples allowing an inferior forming limits than those attained by the first type tests. Another important conclusion is that for one type of test, the FLC doesn't depend significantly on the method of the experimental determination of the forming limits strains.
Must be noticed that in 2003, through the employment of a strain-to-stress space mapping procedure, Smith et al. (2003) proposed a new and interesting sheet metal formability model that takes into account the through-thickness normal stress, founding a good agreement with a limited set of experimental data and extending thus, the strain space forming limit diagram concept from a plane stress assumption to a full stress assumption, which in fact characterizes many sheet metal forming operations.

2.4 Theoretical methods of FLDs prediction

Forming limits of sheet metals are influenced by several physical factors of which the most important ones are material work-hardening, strain rate sensitivity, plastic anisotropy, the development of structural damage and strain path. It is difficult to experimentally assess the influence of each parameter individually since it is virtually impossible to change only one at a time. The experimental determination of the forming limits in all sheet metal forming processes is not only tedious and expensive but also nearly impossible, since the strain paths of material points are quite non-linear and distinguished from each other. The theoretical analysis of plastic instability and flow localization may supply relevant information to prevent the failure on the sheet metal forming process, to examine the influence of each parameter on the necking occurrence and to improve the press performance. Therefore, an extensive effort has been devoted to the development mathematical models capable of accurately predicting the plastic flow localization of sheet metal forming processes.

A large number of different theoretical approaches have been proposed to explain the localized necking in biaxial tensile fields and can be differentiated in two broad theoretical frameworks.

The first one, which is a linear method, is based on the plastic instability of homogeneous sheet metals and describes the initiation of localized band of straining in an otherwise uniform sheet, in order to obtain an explicit expression for predicting the limit strains (Hill, 1952; Stören and Rice, 1975; etc.).

The second one, which is a non-linear method, is based on the plastic instability of heterogeneous sheet metals. It is assumed an initial weakness, imperfection or inhomogeneity in the sheet, which gradually develops into a neck as straining proceeds, (Marciniak and Kuczynski, 1967).

In the following, a review of the most important theoretical models developed for FLDs predictions are presented.

2.4.1 Linear analysis

Giving explicit solutions for predicting limit strains, the linear analysis is easy to be applied to the actual press shop. Since the 1950's, many studies on linear stability or bifurcation analysis have been performed showing useful results through the use of linear analytical methods.

i) Swift’s diffuse necking -

In 1952, based on the Considere’s (1885) analysis, Swift (1952) developed a criterion for predicting the onset of diffuse necking with the assumption that plastic instability occurs at a
maximum load for proportional loading, calculating the critical major strain for diffuse necking as:

\[ \varepsilon_1^* = \frac{2n(1 + \rho + \rho^3)}{(\rho+1)(2\rho^2 - \rho + 2)} \]  

(2.1)

where \( \rho \) is the strain ratio, expressed by:

\[ \rho = \frac{d \varepsilon_2}{d \varepsilon_1} \]  

(2.2)

Figure 4 shows the forming limit curve obtained through the Swift criterion, being known as so-called Swift curve or diffuse necking curve. It is interesting to observe that the diffuse necking occurs in uniaxial tension, plane strain and biaxial stretching when the critical major strain reaches the same value, specifically equals to the work hardening coefficient when a power hardening law is used.

![Forming Limit Curve](image)

Figure 4. The Swift forming limit curve predicting the diffuse necking

Swift's analysis can be applied to the whole range or deformation, i.e. from uniaxial tension to balanced-equibiaxial stretching but the predicted limit strains are much lower than the experimental data when the strain ratio is negative. Anyway, in real parts, the maximum admissible strains are typically limited by localized necking and Swift's equations have only limited applicability.
ii) Bifurcation method with flow theory (Hill)

With the assumption that the onset of failure, or discontinuity of stress and velocity, leads to localized necking, Hill (1952) described the restrictions on the flow stress and rate of work hardening for the growth of localized necking. Hill observed that, during uniform deformation of a sheet, a localized sheet zone can develop along the zero-extension direction, specifically when the plastic work increment within the zone becomes less than that for uniform deformation. The criterion for the occurrence of instability is given by

\[
\frac{d\sigma}{d\varepsilon} = \frac{\sigma}{1 + R}
\]

(2.3)

where \( R \) is the plastic anisotropy parameter.

From Hill's analysis, the localized band angle \( \theta \), which is measured between the normal to the band and the major strain direction, is expressed by:

\[
\theta = \arctan(-\rho)^{1/2},
\]

(2.4)

where \( \rho \) is the strain ratio.

Thus, it is obviously that the angle has a real value only if the strain ratio is negative. In consequence, Hill's analysis predicts that the localized necking cannot form under stretching conditions when the strain ratio is positive. However, localized necking can be observed experimentally in biaxial stretched sheets and several methods have been proposed to compute the entire FLD.

Using a power law stress-strain relation,

\[
\sigma = k\varepsilon^n,
\]

(2.5)

the critical condition for localized necking for negative strain ratio becomes:

\[
\varepsilon_1^* = \frac{n}{1 + \rho},
\]

(2.6)

\( \rho \) meaning the strain ration.

Consequently, the Hill's predicted maximum principal strain \( \varepsilon_1^* \) prior to localized necking has a magnitude of \( \varepsilon_1^* = n \) under a plane strain condition and increases to a value of \( \varepsilon_1^* = 2n \) in uniaxial tension. In Figure 5 are presented the forming limit diagram predicted by Hill theory, beside of that predicted by the Swift theory.

Later, Lee and Kobayashi (1975) and Korhonen (1978) combined the Swift's instability criterion with the Hill's criterion for localized necking, applying the first one for positive strain increment ratios and the second one for negative strain increment ratios. In these approaches, it is
assumed that plastic instability depends on the prior deformation history through accumulated effective strain and on the instantaneous value of strain increment ratio. They obtained that a change towards equibiaxial tension generally increases the forming limits, a change towards plane strain having the opposite effect. The theoretical limit strains are influenced by the assumptions made concerning work hardening and anisotropy. By using a similar method, Hillier (1966) and Negroni et al. (1968) assumed that instability occurs when the forces transmitted by the sheet reach a maximum. They also found that the instability is strain path dependent.

![Formation Limit Diagrams](image)

Figure 5. Forming Limit Diagrams obtained by use of Swift theory or Hill theory respectively.

**iii) Bifurcations with vertex theory (Stören and Rice)**

The physical theory of plasticity, based on simple crystallographic slip models (Lin, 1971), predicts the development of a sharp vertex at the loading point on the yield surface of a polycrystalline aggregate. The existence of such phenomena is supported by physical theories of plasticity and also by detailed experimental investigations (Hecker, 1976).

**Stören and Rice (S-R) (1975)** incorporated the J2 deformation theory of plasticity, which is a simplified model of the corner theory, into the classical bifurcation analysis to predict the localized necking over the entire range of the FLD. They postulated that localized necking is due to the development of a corner on the yield surface. Thus there is no theoretical restriction to localized necking. They found that the angle of the localized band is not in the zero extension direction when the strain ratio is negative, and the band always coincides with the minor strain direction for positive strain ratios. This method predicts reasonable limit strains for some strain rate insensitive materials when the strain ratio is positive and underestimates forming limits when the strain ratio is negative. Accordingly, its application is limited.
Hutchinson and Neale (1978a) improved the theory of Stören and Rice and by applying the deformation theory on the finite deformation domain allowed to simulate the FLD under complex strain paths. Needleman and Tvergaard (1984) developed the S-R theory by introducing in the model of the strain rate sensitivity.

Chu (1982) took the results obtained by Stören and Rice (1975), and extended it in order to predict the forming limit by considering the influence of loading - unloading on the FLD. By using an original yield condition expressed as a polynomial function of fourth degree, the Gotoh (1985) contribution is relevant on the development of the Stören and Rice method. In 1986 Triandafyllidis (1986) introduced the bending effect in the equilibrium conditions of his mathematical model of FLD prediction. Later, Chen and Gerdeen (1989) extend this model for anisotropic materials, by using the Budiansky yield criterion.

Recently, Zhu, Weinmann and Chandra (2001) proposed a unified bifurcation analysis of sheet metal forming limits, by including the moment equilibrium in their study in addition to the force equilibrium condition. They observed that the shear terms due to the perturbation are found to vanish inside the localized neck of a region of deformation, thus, simplifying the two-dimensional problem to a one-dimensional problem.

On the base of this simplified vertex theory, Chow (2003) developed a generalized method to predict forming limits of sheet metals, by considering Hosford's high-order yield criterion (1979), Hill's quadratic yield criterion and the von Mises yield criterion. Whatever form of a yield criterion is adopted, the left hand side of the FLD always coincides with that given by Hill's zero-extension criterion. However, at the right hand side of FLD, the forming limit depends largely on the order of a chosen yield function. He concluded that typically, a higher order yield function leads to a lower limit strain.

iv) Instability analysis (Dudzinsky-Molinari)

An alternative approach is based on perturbation analysis. The sheet is assumed to be initially homogeneous. At any stage of the postulated homogeneous deformation process, a perturbation is superimposed on the basic homogeneous flow. The flow instability or stability is characterised by the fact that the perturbation is increasing or decreasing. It has been quite successfully developed in fluid mechanics and later it was used in solids mechanics for shear band analysis (Fressengeas and Molinari, 1985; Zbib, 1988). Dudzinski and Molinari (1991) first used the concept of effective instability, and then successfully adopted the instability method to prediction of forming limit diagrams. They found that the onset of instability appears to be of a little significance if the growth of the unstable modes is very slow. The effective instability analysis gives basically the same trends as the M-K method. The dependence of the limit strains on the value of the instability intensity parameter presents the same tendencies as the dependence of the limit strains on the amplitude of the initial defect in the M-K analysis. In 1996, Toth, Hirsh and Van Houtte (1996) improved the previous method by using viscoplastic crystallographic slips and Taylor approach. Considering plastic potentials and their development during large plastic deformation of textured aluminium sheets, the forming limit diagrams are obtained. In 1999, Zimmniak (1999) developed the perturbation theory in combination with a new stress-strain relation and six component Barlat '87 yield surface and obtained good prediction of the onset of necking.
The most important advantage of the instability method over M-K method is that an explicit expression for the critical strain can be obtained. However, it is found that the instability analysis can only obtain reasonable forming limit strains for some strain rate sensitive materials, while it underestimates limit strains for steels.

*vi) Jones-Gillis (JG) theory*

In 1984, Jones and Gillis developed a new theory (J-G) for prediction of localized necking by assuming that the biaxial stretching of the sheet occurs in three steps:

1) an homogeneous deformation up to the maximum load
2) a strain concentration under constant load;
3) a localized necking due to a rapidly load decrease.

Initially, J-G theory was applied on the prediction of the right side of the forming limit diagram ($\varepsilon_2 > 0$), but Choi (1989) extended it for the negative strain ratios ($\varepsilon_2 < 0$), allowing the prediction of the entire FLD. However, the J-G method is not so user-friendly as the M-K method.

*vi) Strain gradient theory*

For finite deformations, the effect of material inhomogeneity becomes pronounced and has a dominant influence on the constitutive and instability behaviour of polycrystalline solids. In order to account for this effect, a gradient theory of plasticity was developed by Aifantis (1984, 1987, 1992, 1994) and co-workers (e.g., Zbib and Aifantis, 1989, Muhlhaus and Aifantis, 1991, Vardoulakis and Aifantis, 1991) by introducing a gradient-dependent expression for the flow stress. For a comprehensive understanding of the nature of the gradient coefficient, certain microscopically based arguments have already been advanced for its theoretical estimation. These estimations depend on the scale of interest and the particular deformation mechanisms involved (Muhlhaus and Aifantis, 1991, Vardoulakis and Aifantis, 1989, 1991).

2.4.2 Marciniak-Kuczynski method

Marciniak and Kuczynski (M-K) (1967) have developed a theory based on the assumption that necking develops from local regions of initial heterogeneity. They introduced thickness imperfections in the sheet, perpendicular to the principal stress and strain direction as a groove simulating preexisting defects in the material. Necking was considered to occur when the ratio of the thickness in the groove to the nominal thickness is below a critical value. Practically, this type of inhomogeneity could be a local thickness variation, which may originate from surface roughness or prior processing. This problem has been simplified to a one-dimensional one when the imperfection is a geometrical thickness variation or a material property variation that is a function of only the coordinate perpendicular to the infinitely long band. The plastic properties of the sheet materials were based on the model of anisotropy put forward by Hill (1950). In this analysis, they only covered the region where both principal strain components were positive. However, this idea led to major developments in the prediction of both regions of the FLDs. Azrin and Backofen (1970) discovered in their study that to obtain agreement between the M-K analysis predictions and the experiments an imperfection ratio of about 0.97 or less is required. Sowerby and Duncan (1971) presented a clear interpretation of the model and they also found a
large dependence of the predicted limit strains on the anisotropy coefficient value. Gosh (1978) showed that the material strain rate sensitivity becomes important after the maximum of the axial stress versus axial strain curve has been reached. Tadros and Mellor (1975) proposed the initiation of the localized necking at the Swift instability, not at the beginning of stretching. In 1978, Hutchinson and Neale (1978 a, b, c) extended M-K model to the entire range of plane stress state, using the deformation theory of plasticity and their predictions were in better agreement with experiments. Their work represents an important contribution to gaining insight into the roles of constitutive equations and plasticity theories on FLDs. After the pioneering work presented above many others researchers adopted the M-K method. Analysing and identifying the sources of disagreement between the predicted and experimental FLDs, refined models were developed, allowing to reasonable quantitative correlation of analytical and experimental limit strains.

2.5 Sensitivity of Marciniak- Kuczynski analysis

Research has shown that the calculated forming limit strains using M-K analysis depend sensitively on several factors, such as the material anisotropy, the material hardening, the material texture and microstructure and the strain paths. Based on this fact, numerous authors tried to increase the performance of M-K method. An overview of these studies will be presented as follows.

2.5.1 The effect of material anisotropy and yield functions

Sheet metals exhibit a highly anisotropic material behaviour due to cold rolling. Thus, the description of the yield criterion is of major importance to the accuracy of forming limit prediction by M-K model where localized necking occurs when the strain path has transformed from biaxial stretching to plane strain. Sowerby and Duncan (1971) pointed out that the yield surface shape determines the strain path transformation in the numerical simulation. In 1989, Chan (1989) showed the ability of the yield locus to change the strain ratio and consequently the level of the forming limit curve. Recently, Friedman and Pan (1998, 2000) introduced an angle parameter based on the point on the yield surface defined by the initial strain path and that of plane strain. Since this parameter denotes the extent of deformation change from a particular loading path to plane strain, it can be used to predict the effects of yield surface on limit strains. Different yield functions have been introduced by various researchers in the M-K analysis to assess the influence of the yield surface on the FLD.

The original M-K analysis was based on Hill's 1948 yield criterion (Hill, 1948). By comparing the experiments and predicted results (Painter and Pearce, 1974) it was observed that this analysis overestimates the limit strains in the equibiaxial strain region and underestimates the limit strains in the plane strain region, particularly for materials with R values less than unity. Moreover, the predicted limit strains on the right hand side of the FLDs are very sensitive to the material anisotropy, a phenomenon that has not been experimentally observed. Sowerby and Duncan (1971) by using Hill'48 yield criterion showed the strong effect of the normal anisotropic R value on the FLDs, explaining it on the effect of R value on the yield locus shape. In 1978, Parmar and Mellor (1978) investigated the dependence of limit strains on the material anisotropy based on the M-K method and non-quadratic Hill's 1979-yield function. Barata da Rocha et al. (1984) analysed in detail the influence of the anisotropic behaviour of the sheet using Hill's theory of plastic anisotropy (1948) for both linear and non-linear strain paths in
terms of the anisotropic coefficient measured during uniaxial tensile stretching in three different directions referred to the rolling direction \( (r_0, r_{45}, \text{ and } r_{90}) \). Lian et al. (1989a) have thoroughly studied all four cases of Hill'79 yield criterion, and their limitations, founding reasonable predictions of the forming limits in the positive strain ratios region, for all cases of the yield function, excepting one. Also, Lian et al. (1989b) incorporated the non-quadratic anisotropic yield criterion proposed by Barlat and Lian (1989) and found reasonable results. A diagram called yield surface shape hardening (YSSHD) is proposed which is able to represent more precisely the influence of the yield surface shape on stretchability of sheet metals.

In 1982, Rasmussen (1982) adopted an entirely different concept, using a simple model of a material, which develops a rounded vertex at the loading point, and studying the effect of rounded vertices on strain localization in sheets under balanced biaxial tension. It was shown that the presence of rounded vertices drastically destabilizes plastic flow.

Graf and Hosford (1993a, 1994) calculated the right hand side of the FLD using M-K analysis with the non-quadratic Hosford yield criterion. The predicted results, although still higher, were in better agreement with the experimental data compared to those obtained by using the Hill'48 yield criterion.

Barlat et al. (1997c) computed the FLDs, using the Yld94 yield function. The predictions match the experimental FLD reasonably well.

Xu and Weinmann (1998) proposed an M-K approach to predict forming limits by using the Hill's 1993 yield criterion under the assumption of isotropic hardening showing fairly good predictions of FLDs for both steel and aluminium. In a recent work, Banabic and Dannenmann (2001), used Hill's 1993 yield criterion associated to Swift's instability condition with diffuse necking into the M-K method and the results seem to be encouraging. However, the model validation is still in progress.


Using M-K analysis combined with the plane stress yield criterion proposed by Ferron et al. (1994) Mesrar et al. (1998) predicted the necking failure of planar-isotropic viscoplastic materials subjecting to linear, bilinear and curvilinear strain paths. Instead of using the plot of limit principal strains, the author plotted the forming limit using the effective strain as a function of the strain rate ratio. The obtained limit curve, which moreover, can be fitted by an empirical expression for both positive and negative strain ratios values, is considered having good accuracy as an intrinsic curve that defines the forming limit independent of the strain paths.

Cao et al. (2000) obtained accurate predictions of the FLD under linear and nonlinear strain paths, by applying the general anisotropic yield criterion developed by Karafillis and Boyce (K-B), (1993) on the M-K analysis. The small discrepancy is mainly due to the surface anisotropy in the experiments, pre-existence and development of the surface ridging, which is not considered in the calculation. The improvement of the accuracy of the FLD prediction is achieved through the high stress exponents in the K-B yield criterion, which have the effect of lowering the forming limit prediction to the desired level.
2.5.2 The effect of material hardening

Besides the plastic anisotropy, the material hardening behaviour also plays an important role on the prediction of theoretical forming limits.

In the original M-K model an empirical strain hardening law was adopted, as follows:

$$\sigma = k(\varepsilon_0 + \bar{\varepsilon})^n$$  \hspace{1cm} (2.7)

where $\sigma$ is the effective stress, $\bar{\varepsilon}$ the effective strain, $\varepsilon_0$ the prestrain, $n$ the strain hardening exponent and $k$ is the strength constant.

Strain rate hardening is the intrinsic resistance of a material to strain localization through the accommodation of strain rate change. Laukonis and Ghosh (Laukonis and Ghosh, 1978) found that strain rate effect is very important for steel, especially for the deformation mode near biaxial stretching, while aluminium is insensitive to strain rate. Marciniak, Kuczynski, and Pokora (1973) explained the great importance of even small positive strain-rate sensitivity. They used a simplified relation for the flow stress and strain, including also the strain rate:

$$\sigma = k(\varepsilon_0 + \dot{\varepsilon})^n \dot{\varepsilon}^m$$  \hspace{1cm} (2.8)

where $\dot{\varepsilon}$ denotes the effective strain rate and $m$ is the strain-rate sensitivity exponent.

Calculations considering the rate effect have brought closer agreement with experiment (Ghosh and Hecker, 1975). It is found that even a small $m$ can drastically improve the material formability (Ghosh, 1977; Hutchinson and Neale, 1978a; Barata da Rocha, 1989; and Zhao et al, 1996). Graf and Hosford (1990) revealed that the level of the FLD is raised by increasing the material's strain hardening exponent, $n$ and the strain-rate sensitivity, $m$. Neale and Chater (1980) and Lee and Zaveri (1982) examined the combined effects of material strain-rate sensitivity and anisotropy on sheet necking. Material strain-rate dependence is noticed to have a substantial effect.

In problems of plastic stability, an isotropic work-hardening may be an inopportune assumption since work hardening causes the yield surface curvature to grow, becoming unrealistically large, which greatly stabilizes plastic flow and leads to unrealistic predictions of the onset of the instability. Barata da Rocha (1989) showed that the anisotropic hardening and transient work hardening are important influential factors in predicting FLD and he concluded that more accurate constitutive laws should be used. It is well known that the models with anisotropic hardening give a better description of phenomenon, e.g., the Bauschinger effect. One of the simplest anisotropic hardening rules is kinematic hardening. The basic kinematic hardening rule also requires only a single parameter, i.e. the current origin of the yield surface.

Tvergaard (1978) studied sheet metal necking using a kinematic hardening model. This model was found to accelerate strain localization in the same way as the corner models, although to a lower extent. Tvergaard pointed out that the kinematic hardening model could be interpreted as a model of a material with an expanding, smooth yield surface that develops a rounded vertex at the loading point with a local curvature equal to that of the initial yield surface. He found that the
forming limit curves predicted by kinematic hardening are in far better agreement with experimental results than the similar curves predicted by standard flow theory with isotropic hardening. Kinematic hardening may give a reasonable description of the actual material behaviour. This effect of kinematic hardening is particularly strong for a high strength material, for which a considerable translation of the yield surface occurs in stress space.

Using the \textit{M-K} method to compare the effect of isotropic hardening and of the \textit{Prager-Ziegler} kinematic hardening on the FLDs, \textit{Lu and Lee} (1987) also found that the kinematic hardening model predicted forming limits that were closer to experimental data for steel sheets under proportional loading. On the other hand, the isotropic hardening model gives better predictions of the forming limits with uniaxial prestrain followed by equibiaxial straining. The overall shapes of the limit strains predicted by both isotropic and kinematic models are similar for most loading histories. However, the kinematic model can predict the lower forming limits under subsequent loading with positive strain ratios.

\textit{Hiwatashi et al.} (1998) using the \textit{M-K} analysis and an anisotropic hardening model, based on texture and dislocation structure, provided forming limit predictions that reflected the experimental tendencies not predicted by isotropic hardening model.

Recently, in order to improve the accuracy of FLD prediction under nonlinear strain paths, \textit{Yao and Cao} (2002) proposed a methodology to determine the evolution of yield surface in a large plastic deformation process. The evolution is expressed in terms of changes in back stress and yield surface curvature, which are assumed to be proportional to the accumulated plastic strain. The calculated FLDs using the proposed approach demonstrated a great improvement of the predicted FLDs under various loading conditions for the studied aluminium alloys.

A detailed theoretical analysis of necking failure in thin sheets must predict the maximum admissible strains, providing also a full description of neck initiation and growth behaviour. This is due in part to the fact that the formability of sheet metals must be assessed not only from the limiting strains but also from the way in which the strain gradients develop. \textit{Gosh} (1977) analysed the development of strain gradients during deformation of a sheet tensile specimen. The use of a "many-slice" approach permitted the determination of strain distributions in uniaxial tension. \textit{Lee and Zaverl} (1982) have given a detailed analysis of neck growth in sheet metals based on the idea of initial nonuniformity and on a "many-slice" model. To determine the left side of the FLD, plane strain condition was imposed in the neck. \textit{Barata da Rocha et. al.} (1985) studied the development of nonuniform plastic flow in sheet metals based on the idea of initial nonuniformity to start the neck growth process. They also used a "many-slice" approach to simulate the development of strain gradients and neck growth. The obtained results are consistent with experimentally observed features of flow localization and instability conditions.

To analyse the deformation thoroughly, the constitutive relation for effective or flow stress should be given to represent the material mechanical properties, which can account for both the homogeneous and nonhomogeneous deformation. In conventional methods, the stress-strain relation is obtained in homogeneous deformation. There is no strain gradient effect in this deformation, so the flow stress is only related to the local effective strain and/or strain rate. To account for the strain gradient effect corresponding to the initiation of the strain localization, a gradient dependent flow stress has been proposed (\textit{Aifantis}, 1984). Based on Aifantis's strain gradient theory, \textit{Wang et al.} (1996) incorporated the strain gradient theory into the \textit{M-K} method.
to analyse the deformation localization, and to predict the corresponding FLDs. The higher order strain gradient (second order strain gradient) was used in the power law hardening equation to describe the different material properties inside and outside the localized region. According to the studies of Zbib and Aifantis (1988), it is seen that the orientation of the localized band is insensitive to the strain gradients. Wang et al. (1996) followed Hill's criterion (1952) and simplified this problem by assuming that the localized necking initiates along the zero extension direction for the left hand side of the FLD. The localized band was assumed to initiate along the minor strain direction for positive strain ratios as proposed by Marciniak and Kuczynski (1967). This approach introduces an internal length scale into the constitutive equations and takes into account the effects of deformation inhomogeneity. It overcomes the imperfection sensitivity encountered in the conventional M-K method. Shi and Gerdeen (1991) used Barlat and Lian's nonquadratic anisotropic yield criterion (Barlat and Lian, 1989) and strain gradient theory in the constitutive equation to account for the effect of the curvature of the punch on localized necking in anisotropic sheets. The effect of the strain gradient resulting from deforming a flat sheet into a curved sheet is considered in the prediction of FLDs based on the M-K method by introducing first order strain gradient term in the constitutive equation. They found that, for the left hand side of the FLD, the stress ratios are constant throughout the deforming sheet including the homogeneous and non-homogeneous region if Hill's zero-extension direction is assumed along the direction of the initial imperfection.

2.5.3 The effects of microstructural parameters

Since a formability of a sheet metal reflects the material's response to the external forming conditions, should have a close relation with the microstructural characteristics and the dynamic variation of microstructure during forming. Numerous experiments published in the literature have shown that the forming limit strongly depends on microstructural features of materials. In order to get accurate FLD predictions, it is necessary to account for the maximum number of microstructural parameters in computation, but due the complexity of the plastic flow localization process, it is very important to determine the essential microstructural features responsible for such phenomena. Over the years different microstructural aspects on the plastic flow localization process were analysed, and a special attention was attributed to the influence of the damage and crystallographic texture on forming limits, whose review are next presented.

i) Voids or damage influence

During the sheet metal forming operation, internal damage occurs as a result of nucleation growth and coalescence of cavities around particles. This phenomenon limits the strains, which can be achieved before the appearance of localized necking. In order to assess the influence of damage on sheet necking, basing on the concept of imperfection, several models (Needelmann and Triandafyllidis, 1978; Chu and Needleman, 1980; Jaliniere and Schmitt, 1982), which can be divided in two categories, have been proposed. In the first one, the material behaviour is represented by the Gurson constitutive model for porous materials (1977), while, in the second one, the material is assumed to behave like a classical incompressible plastic material. In both cases, the void volume fraction in the imperfection is chosen more or less arbitrarily.

A model to calculate the void volume fraction in the imperfection is more rigorously developed by Barlat (1989). The internal damage is modelled by initially equi-axed cavities. Barata da Rocha (1989) assumed that the material contains an initial internal damage due to the presence
of voids. It is shown that the necking process strongly depends on this phenomenon. Lee et al. (1985), Tai (1988), and Lee et al. (1997) analysed the application of the concept of damage mechanics to anisotropic materials on the prediction of forming limit diagrams for different materials. Tjotte (1992) implemented a damage model for void growth during plastic deformation in finite element model to study the uniaxial tension and plane strain tension. Huang et al. (2000) adopted a macroscopic yield criterion for anisotropic porous sheet metal to develop a failure prediction methodology. The M-K analysis approach was employed to predict failure by assuming a higher void volume fraction inside the randomly oriented imperfection band. Based on the theory of damage mechanics, Chow et al. (2001) developed a viscoplastic constitutive model of anisotropic damage for the prediction of forming limit curve (FLC), taking into account the effect of rotation of the principal damage coordinates on the deformation and damage behaviour, obtaining good agreements between theoretical and experimental results.

The physical basis for the M-K analysis was well presented in McCarron et al. (1988). In their study, imperfections in the form of grooves were planted in samples used in equal biaxial stretching. It was found that no reductions in forming limit strain were obtained with shallow grooves for which the imperfection indices, which were defined as the thickness ratio of the groove to the nominal area, were greater than 0.990 and 0.992 for two different steels.

Tang and Tai (2000), analysed the evolution of preferred grain orientation and its effect on plastic deformation and mechanical properties based on crystallographic theory and the continuum mechanics of textured polycrystals technique and proposed a method to predict the forming limit strains using the M-K model with modification to take into account the damage due to both texture evolution and the presence of microdefects. The results show that the proposed method can significantly reduce the conventional over-estimation of limit strains in the biaxial stretching state.

ii) The effect of texture

Most of the previous studies considered the initial anisotropy to be conserved during the forming operation. Only recently, the role of the varying plastic anisotropy during biaxial deformation of sheet metals has been considered. Experimental data from Hu (1974, 1975) and Ghosh (1978) has shown that the material parameters, such as material anisotropy R and work hardening exponent n, all change during deformation. Therefore, it is very important to study the effect of the variation of these parameters on the forming limit diagram. Bate (1984), Barlat (1987) and Barlat and Richmond (1987) have shown the importance of the crystal structure and the crystallographic texture on the yield surface shape and on the onset of plastic flow localization in thin sheets. Therefore, the information concerning the crystal structure and crystallographic texture must be somehow incorporated in the constitutive equation. Applying a rate-sensitive crystal plasticity model directly in connection with the M-K method, Zhou and Neale (1995) have been able to predict forming limit diagrams for annealed FCC sheet metals with various initial textures. They found that the localized neck occurs when the total accumulated shear over all slip systems in the band of localized deformation reaches a critical value, and the critical value depends only on the slip-induced hardening law. Different crystalline orientations result in different slip distributions for a specific deformation state; the slip distribution also affects the forming limit. On the other hand, different initial textures lead to different yield surface shapes before and during deformation, and this greatly affects the forming limit diagrams. The evolution of texture, and consequently of yield surface shape, during deformation may have a great
influence on the forming limits. Wu, et al. (1998) confirmed this results in their investigation of the formability of aluminium.

Toth et al. (1996) examined the differences in the predicted limit strains, which may be caused by the development of anisotropy during straining within the framework of the M-K theory. They found that both the initial textures and their subsequent development influence the limit strains.

Hivatashi et al. (1998) and Hoferlin et al. (1998) applied an anisotropic constitutive model based on texture through the Van Houtte et al. (1995), plastic potential and dislocation structure through the Teodosiu and Hu (1995) hardening model, on the M-K analysis to predict FLDs. The crystallographic texture or the blank orientation itself can influence considerably the stretchability parameter $P$ defined as the ratio of the balanced biaxial stress to the plane strain stress. It has been shown that the stretchability in the biaxial stretching domain is improved when the $P$ value is high. Thus, they observed that limit strains near equi-biaxial stretching are significantly influenced by the slip systems, the initial texture and the back stress. Also, their study with texture-based yield locus shows that a texture, which provides a high r-value, does not necessarily lead to a low limit strain under equi-biaxial stretching. The anisotropic hardening model based on the dislocation structure and the initial texture is essential for predicting macroscopic behaviour under strain path changes. In addition, they concluded that for further accuracy, other factors such as texture evolution, damage and strain rate sensitivity should be taken into account.

In 2000, Qian et al. (2000) proposed a physically based constitutive framework for precision metal forming processes. Considering the texture evolution playing a key role for both fully plastic anisotropy and the trigger of deformation bifurcation, they assumed it as a bridge to link Rice's localization analysis with the M-K analysis for FLD prediction.

2.5.4 The effect of strain path

During an actual forming operation, a material element may undergo considerably large changes in strain path, and these changes can significantly alter the forming limits.

Nakazima et al. (1968) and Kikuma and Nakazima (1971) presented forming limit curves for several two-stage strain paths, even today remaining as an important reference on the experimental data. They have shown that the maximum curve is observed in uniaxial tension followed by equibiaxial stretching, while the minimum one is obtained for the opposite sequence (Figure 6).

Numerous other experimental works (Hecker, 1973; Kleemola and Pelkkikangas, 1977; Gronostazski et al., 1982; Chu, 1983 and Graf and Hosford, 1993a) emphasized the strong effect of the strain path and of the loading history on the forming limit diagrams. Figure 8 shows the forming limit diagrams in proportional loading and after a prestrain in uniaxial tension or biaxial stretching, respectively, followed by different strain paths obtained by Gronostazski et al. (1982). The influence of the strain path on the attained forming limits is evident and significant.
Modifying the M-K analysis, Lee and Kobayashi (1975) initiated a method for non-proportional deformation using a formulation in which the orientation of the incipient necking band is always perpendicular to the direction of major principal stress. This specific orientation of the band, however, leads to an overestimation of formability in those cases where the strain path includes negative strain increments. Rasmussen (1981) has given a formulation of M-K analysis, which allows for any strain path to be simulated and for the most critical orientation of necking bands to be determined. In this method, the forming limit is obtained for the groove orientation, which leads to the minimum calculated limiting strain. He found that the initial band orientation was independent of the prestrain when the last deformation is biaxial stretching. However, for negative strain ratios the limiting strains are achieved with groove orientations dependent on the strain path. Later Rasmussen (1982) extended this method to analyse the effect of curvilinear strain paths.

Barata da Rocha and co-workers (1984, 1985 and 1989) have studied the effect of strain path changes on the theoretical FLDs of isotropic and anisotropic rate-sensitive materials computed with M-K analysis. They have pointed out that the influence of the strain path on the formability of the material can be explained in terms of the strain rate of the necking region and through the concept of metastable plastic states in which the stability of plastic flow depends on the direction of the strain increment vector. It has been shown that uniaxial prestrain followed by balanced biaxial stretching considerably increases the limiting strains at the onset of necking. Conversely, premature instabilities were observed for strain paths consisting of prior balanced biaxial stretching followed by uniaxial tension. In addition, a change in strain path from balanced stretching towards plane strain deformation resulted in a significant loss of plastic stability. These results are in perfect agreement with the experimental behaviour of the material.
Figure 8. The influence of strain path on the forming limits for a steel
(Gronostazski et al., 1982)

Kinematic hardening and rate sensitivity of flow stress have also been analysed by Lu and Lee (1987) and Nie and Lee (1991). Graf and Hosford (1993) investigated the effect of changing strain paths on forming limits using a high-exponent yield criterion with the Marciniak and Kuczinsky analysis. Calculations incorporating abrupt path changes agreed with the general trends found experimentally. If the first stage of strain is under biaxial tension, the subsequent FLD shifts to the right and down with respect to the original FLD, whereas it shifts to the left and up when the first stage of strain is in uniaxial tension. Calculations introducing gradual strain-path changes, characteristic of stretching over a hemispherical dome, predict that the minimum of the FLD shifts to the right. Zhao et al. (1996) also considered the effect of strain path on the shape and magnitude of the FLD by considering material anisotropy, strain hardening and strain rate sensitivity. From these analyses, it is found that prestrain in balanced biaxial tension generally lowers the entire FLD, whereas prestrain in uniaxial tension raises the limits on the right hand side of the FLD without much effect on the left hand side, when the direction of the largest principal strain does not change. If the directions of the principal strains are rotated, prestrain in uniaxial or plane strain tension lowers the forming limits for most of the FLD range. Hiwatashi et al. (1998) studied the effect of strain path for proportional and two-stage straining, through the use of the M-K analysis and the texture (Van Houtte, 1995) and microstructure based-anisotropic hardening model (Teodosiu and Hu, 1995). It was shown that forming limit for the case of two-stage straining depends on the amount of the first straining and the combination of first and second strain rate modes. Also the model reproduced the same lowest curve of the FLD as the experimental one obtained by Nakazima et al. (1968), fact that is impossible to obtain by using any phenomenological constitutive equations.
2.6 Conclusions

Formability, in general terms, represents the result of complex interactions between intrinsic/constitutive properties of the sheet metal and the extrinsic influencing factors involved in a forming operation, being limited by the occurrence of a localized necking.

This review examined the Forming Limit Diagram concept, paying a especial attention on the theoretical prediction of the plastic instability. From these studies, it is seen that the predicted forming limit diagrams are affected by many different complex factors, such as strain hardening and strain rate sensitivity, the shape of the yield surface, the void volume fraction, the texture and most importantly, the strain path.

Even though the linear perturbation (or bifurcation) analysis can give an explicit expression of the critical strain, much less researches have been conducted by using it.

The M-K method has been widely used to predict forming limit diagrams, the original version of it undergoing tremendous improvement. With more and more realistic models, the predicted values can match the experimental data very well for most sheet metals. Up today, the M-K method has become one of the most important tools in predicting the sheet metal formability. The combination of the Marciniak- Kuczynski theory with accurate and advanced constitutive models may be a fascinating and fruitful device in the understanding of material behaviour and in predicting the necking occurrence.

Due of the amazing progress of forming simulation codes, as Coll (2000) affirms, “It is not exaggeration to say that, in future, the only reason to determine experimental diagrams will be to ascertain the validity of predictive models”.

3. CONSTITUTIVE MODELS FOR SHEET METALS

Since a more accurate description of the sheet behaviour is essential in order to predict the real behaviour in the press shop and consequently the correct forming limits, it seems important to review the different constitutive models for sheet metals developed over the years. Two opposite ways are being developed: a phenomenological and a polycrystalline approach.

3.1. Phenomenological approach

The sheet metals crystallographic structure induces a significant anisotropy of the mechanical properties, which plays a vital role on the material flow during forming processes and on the final product characteristics.

Macroscopically, the material is assumed to be a continuum, neither its microstructure nor the physical mechanisms of the deformation process being taken in account. For a multiaxial stress space, plastic deformation is well described with a yield surface, a flow rule and a hardening law. Plastic anisotropy is the result of the distortion of the yield surface shape due to the microstructural state.

3.1.1. Yield criteria

A yield surface is generally described by an implicit equation of the form

$$\Phi(\overline{\sigma}, Y) = \overline{\sigma} - Y = 0$$  \hspace{1cm} (3.1)

where $\overline{\sigma}$ is the equivalent stress which is a scalar function of the deviatoric stress tensor and $Y$ is the yield stress obtained from a simple test like tension, compression or shearing. In mechanical terms yield surface refers to the locus of stresses at which the elastic limit is exceeded and plastic deformation initiates. The mathematical form of a yield surface is the yield function describing the correlation among stresses, which in general sense, include six components (three normal and three shear).

Zyczkowski (1981) concludes that the yield function may be defined by considering the appearance of the plastic state when some physical quantity (energy, stress etc.) reaches a critical value, or by approximating experimental data while an analytical function is defined on the basis of a physical model (Hosford criterion), noticing that these ones are also purely phenomenological functions, being not obtained from a calculus based on the crystallographic texture of the material.

During the last two decades, numerous yield functions were introduced in order to improve the fitting with the experimental results. Recently, Banabic (2000) presents a detailed and interesting review on the most significant of them. Following this study, a brief resume of the yield criterion history will be presented.
i) Yield criteria for isotropic materials

Observing that the plastic strains usually appear as a consequence of the sliding of the crystal lattice due to shear stressing, Tresca (1864) proposed the first yield criterion in 1864 and expressed that yielding occurs when the maximum stress reaches a critical values. Its general form is:

$$\max \left\{ |\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1| \right\} = \sigma_0$$  \hspace{1cm} (3.2)

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the principal stresses.

In this case, the yield surface is represented by a polygon in the plane of the principal stress $\sigma_1$ and $\sigma_2$ as can be observed in Figure 1.

![Von Mises and Tresca yield surfaces](image)

Figure 1. The graphical representation of the Tresca and Huber-Mises-Hencky yield criteria

The next criterion proposed independently by Huber (1904), Von Mises (1913) and Henky (1924), based on the observation that hydrostatic pressure doesn't cause plastic straining of the material, assumes that when the elastic energy of distortion reaches a critical value, independent of the stress state type, the transition from an elastic to a plastic state appears.

The Huber-Mises-Hencky yield criterion can be written as:

$$\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 = 2 \cdot \sigma_0$$  \hspace{1cm} (3.3)

Its yield locus represents an ellipsis in the plane of the principal stresses $\sigma_1$ and $\sigma_2$, circumscribing the polygon associated to the Tresca yield criterion (Figure 1).

In order to represent the experimental data located between the Tresca and Huber-Mises-Hencky yield surfaces, Drucker (1949) proposed the following criterion:

$$J_2^1 - C_P J_3^2 = k^2$$  \hspace{1cm} (3.4)
where $J_2$ and $J_3$ are the second and third invariants of the stress tensor, respectively, $C_n$ is a constant and $k^2 = 18 \left( \frac{Y}{3} \right)^6$, $Y$ being the uniaxial yield limit.

Based on self-consistent polycrystal calculations, Hershey (1954) introduced a criterion later used by Hosford (1972) and representing a generalisation of the Huber-Mises-Hencky yield criterion:

$$\left( \sigma_1 - \sigma_2 \right)^n + \left( \sigma_2 - \sigma_3 \right)^n + \left( \sigma_3 - \sigma_1 \right)^n = 2 \cdot Y^m \quad (3.5)$$

where $Y$ is the uniaxial yield stress and $m$ is such that $1 < m < \infty$. If $1 < m < 2$ or $4 < m < \infty$ the corresponding yield locus is located between the Tresca and Huber-Mises-Hencky loci.

ii) Yield criteria for anisotropic materials

a) The quadratic yield criteria

Von Mises proposed in 1928 the first yield criterion for anisotropic materials in the form of a quadratic function. Although it was initially used to describe the plastic behaviour of an anisotropic single crystal, afterwards it was also used for polycrystals.

The expression of this criterion is:

$$\phi = h_{11} \sigma_x^2 + h_{22} \sigma_y^2 + h_{33} \sigma_z^2 + h_{44} \tau_{xy}^2 + h_{45} \tau_{xz}^2 + h_{55} \tau_{yz}^2 + 2h_{12} \sigma_x \sigma_y + 2h_{13} \sigma_x \sigma_z + 2h_{45} \sigma_x \tau_{xy} + 2h_{15} \sigma_x \tau_{xz} + 2h_{16} \sigma_x \tau_{yz} + 2h_{23} \sigma_y \sigma_z + 2h_{25} \sigma_y \tau_{xz} + 2h_{26} \sigma_y \tau_{yz} + 2h_{35} \sigma_z \tau_{xz} + 2h_{36} \sigma_z \tau_{yz} + 2h_{45} \tau_{xy} \tau_{xz} + 2h_{46} \tau_{xy} \tau_{yz} + 2h_{56} \tau_{xz} \tau_{yz} \quad (3.6)$$

where $h_{ij} (i,j = 1-6)$ are coefficients of anisotropy whose identification based on simple mechanical tests is very difficult.

In 1948, supposing that the material has an anisotropy with three orthogonal symmetry planes, Hill (1948) proposed an anisotropic yield criterion as an extension of the Huber-Mises-Hencky criterion.

It is expressed by a quadratic function of the following shape:

$$2f(\sigma_q) = F(\sigma_x - \sigma_z)^2 + G(\sigma_z - \sigma_y)^2 + H(\sigma_x - \sigma_y)^2 + 2L \tau_{yz}^2 + 2M \tau_{zx}^2 + 2P \tau_{xy}^2 = 1 \quad (3.7)$$

where $f$ is the yield function, $F$, $G$, $H$, $L$, $M$ and $P$ are constants specific to the principal anisotropic state, and $x$, $y$, $z$ are the principal anisotropic axes that usually coincide with the rolling, transverse and normal directions of the sheet.

Denoting by $X$, $Y$ and $Z$ the tensile yield stress and by $R$, $S$ and $T$ the shear yield stress associated to the principal anisotropic directions, the Hill’48 yield function coefficients can be expressed as dependencies of these yield stresses as follows:

$$2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}; \quad 2G = \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}; \quad 2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}$$

28
Thus, to give a complete description of the anisotropy state of the material, six independent yield stresses as well as the orientation of the principal anisotropic axes need to be known.

In simulation of the sheet metal forming process, the anisotropy coefficients are denoted as $r_0$, $r_{45}$, and $r_{90}$ and the yield stress in the directions of the principal anisotropic axes as $X = \sigma_0$, $Y = \sigma_{90}$.

The relation between the anisotropy coefficients and the coefficients introduced by Hill, easily obtained by the associated flow rule, are:

$$
\begin{align*}
    r_0 &= \frac{H}{G} ; \quad r_{90} = \frac{H}{F} ; \quad r_{45} = \frac{H}{F+G} - \frac{1}{2} 
\end{align*}
$$

Banabic (2000) in his study pointed out some remarks concerning the Hill 1948 yield criterion, mentioning the main advantages and disadvantages of this criterion. A better rigor of its basic assumptions, easily to understand, as much as a small number of mechanical parameters necessary for writing the yield function explain its wide use in practice.

Pearce (1968) and Woodthorpe and Pearce (1970) showed that for materials exhibiting $r$ values less than 1, the balanced biaxial yield stress $\sigma_b$ measured with Bulge testing was at least equal to the uniaxial yield stress $\sigma_u$, whereas Hill’s 1948 yield function predicts the contrary. This has been called “anomalous behaviour” of some materials and is due to the fact that Hill’48 criterion predicts:

$$
\sigma_b = \sigma_u \sqrt{\frac{1+r}{2}}
$$

Thus, it is obvious that if $r>1$, then $\sigma_b > \sigma_u$ and if $r<1$, then $\sigma_b < \sigma_u$.

The relation between the yield stress and the anisotropy coefficients, which is:

$$
\frac{\sigma_0}{\sigma_{90}} = \frac{r_0 (1 + r_{90})}{r_{90} (1 + r_0)},
$$

implies the second drawback of this criterion by its impossibility to represent the “second order anomalous” behaviour: $\frac{r_0}{r_{90}} > 1$ and $\frac{\sigma_0}{\sigma_{90}} < 1$ (or inverse)

Another disadvantage is that that it can be applied to materials forming only two or four “ears” in the axisymmetric deep-drawing processes and the accuracy to predict the dependence of the yield stress on the direction of tension is poor.

Olszak (1956) generalizes the function proposed by Von Mises (1928) for nonhomogeneous anisotropic materials. For an orthotropic material it can be reduced to a quadratic function with only six terms, the same such as the one proposed by Hill in 1948.
b) The non-quadratic yield criteria

Several researchers independently concluded that only non-quadratic functions are suitable to model the so-called “anomalous behaviour” of some materials such as aluminium alloys, which, according to Woodthorpe and Pearce, although have the anisotropy coefficient less than one, show an experimental yield surface located outside the surface predicted by the Von Mises yield criterion ($\sigma_a > \sigma_e$).

In order to account for this behaviour, Hill (1979) proposed a new type of yield criterion in the form of a non-quadratic function. Supposing that the directions of the principal stresses are superimposed with the anisotropy axes, the general form of this criterion is:

$$f|\sigma_2 - \sigma_3|^M + g|\sigma_3 - \sigma_1|^M + h|\sigma_1 - \sigma_2|^M + a|2\sigma_1 - \sigma_2 - \sigma_3|^M + b|2\sigma_2 - \sigma_1 - \sigma_3|^M + c|2\sigma_3 - \sigma_1 - \sigma_2|^M = \sigma_e^M$$

(3.12)

where $f$, $g$, $h$, $a$, $b$ and $c$ are coefficients of material anisotropy, $\sigma_e$ is the effective stress and $M$ is an exponent greater than one required to assure the convexity condition. Calculation of $M$ value is made by fitting of work hardening curves for equibiaxial stretching calculated on the base of experimentally determined work hardening curves in tensile test to experimentally determined work hardening curves in equibiaxial stretching.

Four special cases are derived from the general form:

1°) $a=b=h=0, f=g$

$$c|\sigma_1 + \sigma_2|^M + f(|\sigma_1|^M + |\sigma_2|^M) = \sigma_e^M$$

(3.13)

2°) $a=b, c=f=g=0$

$$a(|2\sigma_1 - \sigma_2|^M + |2\sigma_2 - \sigma_1|^M) + h|\sigma_1 - \sigma_2|^M = \sigma_e^M$$

(3.14)

3°) $a=b, f=g, c=h=0$

$$a(|2\sigma_1 - \sigma_2|^M + |2\sigma_2 - \sigma_1|^M) + f(|\sigma_1|^M + |\sigma_2|^M) = \sigma_e^M$$

(3.15)

3°) $a=b=f=g=0$

$$c|\sigma_1 + \sigma_2|^M + h|\sigma_1 - \sigma_2|^M = \sigma_e^M$$

(3.16)

Lian, Zhou and Baudeau (1989a) proved that the four forms of the Hill 1979 yield criterion could be expressed as functions of only two coefficients depending on the $r$ and $m$ parameters as follows:

Case 1°) $c = \frac{1 + 2r}{1 + r}, \quad f = \frac{r}{1 + r}$

(3.17)
3. Constitutive models for sheet metals

Case 2°) \[ c = \frac{1}{(1+r)(2^{M-1} - 1)}, \quad h = 1 - \frac{2^M + 1}{(1+r)(2^{M-1} - 1)} \] (3.18)

Case 3°) \[ a = \frac{r}{(1+r)(2^{M-2} + 2)}, \quad f = 1 - \frac{r2^{M+1}}{(1+r)(2^{M-2} + 2)} \] (3.19)

Case 4°) \[ c = \frac{r}{2(1+r)}, \quad h = \frac{1+2r}{2(1+r)} \] (3.20)

The most widely used expression of this yield criterion is the case 4°, which applies to materials exhibiting planar isotropy (same properties in any direction of the plane of the sheet). Using the coefficient of normal anisotropy (r), it can be rewritten:

\[ 2(1+r)\bar{\sigma}_w^M = |\sigma_1 + \sigma_2|^M + (1+2r)|\sigma_1 - \sigma_2|^M \] (3.21)

Although the Hill 1979 captures the Woodthorpe-Pearce “anomalous” behaviour of some materials its main disadvantage is that it is expressed using only principal stress and the predicted yield surfaces are sometimes far from the experimental surfaces predicted by the Bishop-Hill theory.

In 1991 Monheillet et al. (1991) proposed a generalization of this criterion for planar anisotropy.

Bassani (1977) demonstrated that a family of yield functions depending on four parameters, is able to approximate a relatively broad range of planar isotropic yield surfaces predicted with the Bishop-Hill (1951) polycrystal model.

\[ \phi = \left| \frac{\sigma_1 + \sigma_2}{2\sigma_b} \right|^m + \left| \frac{\sigma_1 - \sigma_2}{2\tau} \right|^{m'}, -1 \] (3.22)

where \( \sigma_b \) is the biaxial yield stress, \( \tau \) is the pure shear yield stress, whilst \( m \) and \( m' \) are two constants greater than one.

It is observed that when \( m = m' \), the criterion is reduced to the case 4° proposed by Hill in 1979, containing a difference in a definition of the coefficients.

Hosford (1979), independently from Hill, generalizes his own isotropic criterion (Hosford 1972) by developing the following non-quadratic function:

\[ \phi = F|\sigma_y - \sigma_z|^a + G|\sigma_z - \sigma_x|^a + H|\sigma_x - \sigma_y|^a = \bar{\sigma}^a \] (3.23)

This criterion is a particular expression (a=b=c=0 and f=g) of the case 4° of Hill’s 1979 yield criterion, with a essential distinction on the identification of the exponent a. Hosford established a on the yield surface for different crystallographic structures, concluding that the best
approximation were attained by $a=6$ for FCC materials and $a=8$ for BCC materials (Logan and Hosford, 1980).

The main advantage of this criterion consists in a good approximation of the yield locus computed using the polycrystal Bishop-Hill (1951) model as well as the experimental data by suitable setting the value of the exponent $a$.

An important drawback of this criterion similar to Hill’s 1979 criterion is their lack of shear stress, unable to predict the variation of plastic properties in the plane of the sheet metal. In order to analyse this variation, Hosford (1985) tried to include a shear stress component into his formulation without success.

Barlat and Richmond (1987) proposed a more general form of Hosford’s criterion for isotropic materials, by expressing it in an $x$, $y$, $z$, co-ordinate system (the so-called “tricomponent plane stress yield surface”) including also the shear stress.

$$\phi = |k_1 + k_2|^M + |k_1 - k_2|^M + 2|k_2|^M = 2\sigma_e^M \quad (3.24)$$

where, $k_1$ and $k_2$ are invariants of the stress tensor, while the exponent $M$ is an integer having the same significance as the exponent $a$ used by Hosford.

$k_1$ and $k_2$ are defined as:

$$k_1 = \frac{\sigma_x + \sigma_y}{2}$$

$$k_2 = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}. \quad (3.25)$$

The extension of this yield function to the case of normal anisotropy is:

$$\phi = a|k_1 + k_2|^M + b|k_1 - k_2|^M + c|2k_2|^M = 2\sigma_e^M \quad (3.26)$$

where $a$, $b$ and $c$ are magnitudes depending on the anisotropy coefficients, while $k_1$ and $k_2$ are computed with:

$$a = b = 2 - c = \frac{2}{1+r} \quad (3.27)$$

In 1989, Barlat and Lian (1989) generalized the previous criterion for the case of materials exhibiting planar anisotropy, introducing the next yield function:

$$\phi = a|k_1 + k_2|^M + a|k_1 - k_2|^M + c|2k_2|^M = 2\sigma_e^M \quad (3.28)$$

where:
\[ k_1 = \frac{\sigma_x + h\sigma_y}{2} \]
\[ k_2 = \sqrt{\left(\frac{\sigma_x - h\sigma_y}{2}\right)^2 + p^2\tau_{xy}^2} \]

(3.29)

with \(a\), \(c\), \(h\) and \(p\) material parameters identified by:

\[ a = 2 - c = \frac{2\left(\frac{\sigma_x}{\tau_{xy}}\right)^M - 2\left(1 + \frac{\sigma_x}{\sigma_{90}}\right)^M}{1 + \left(\frac{\sigma_x}{\sigma_{90}}\right)^M - \left(1 + \frac{\sigma_x}{\sigma_{90}}\right)^M} ; \quad h = \frac{\sigma_x}{\sigma_{90}} ; \quad p = \frac{\sigma_x}{\tau_{xy}} \left(\frac{2}{2a + 2^M c}\right)^{\frac{1}{M}} \]

(3.30)

\(\sigma_{90}\) is the uniaxial yield stress at 90° from rolling direction, whereas \(\tau_{s1}\) and \(\tau_{s2}\) are shear yield stresses for two types of tests: \(\tau_{xy} = \tau_{s1}\) for \(\sigma_x = \sigma_y = 0\); \(\tau_{xy} = 0\) for \(\tau_{xy} = 0\) for \(\sigma_y = -\sigma_x = \tau_{s2}\)

The second method for determining \(a\), \(c\), \(h\) and \(p\) is to use \(r\) values obtained from uniaxial tension tests in three different directions, for instance \(r_0\), \(r_{45}\) and \(r_{90}\). From associated flow rule and \(r\) value calculations, it was shown that \(a\), \(c\) and \(h\) depended on \(r_0\) and \(r_{90}\) only.

\[ a = 2 - c = 2 - 2\sqrt{\frac{r_0}{1 + r_0} \frac{r_{90}}{1 + r_{90}}} ; \quad h = \sqrt{\frac{r_0}{1 + r_0} \frac{1 + r_{90}}{r_{90}}} \]

(3.31)

The calculation of \(p\) requires a numerical procedure or can be used Eq. (3.30).

The most important advantage of the yield criterion proposed by Barlat and Lian consists in the inclusion of all the planar stress components as well as the planar anisotropy. By correctly choosing the M exponent, the prediction of the yield locus and the distribution of the \(r\)-coefficient and uniaxial yield stress in the plane of sheet metal it is performed with a very good accuracy compared with those given by Bishop-Hill theory.

On the basis of the approach proposed by Barlat, Chu (1995) introduced a generalization of the case 4° of Hill 1979 criterion, the material constants being expressed as dependence’s of the anisotropy coefficients.

\[ C|\sigma_x + B\sigma_y|^m + H(\sigma_x - B\sigma_y)^2 + (2D\tau_{xy})^{m/2} = \sigma_b^m \]

(3.32)

Another way to include the shear stress component was proposed by Hill (1990) in a new yield function, which is a generalisation of his previous criterion (Hill 1979) expressed in a general coordinate system.
\[
\phi = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_{xy}^2}{4t^2} \left( \sigma_x - \sigma_y \right)^2 + \frac{\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2}{m/2-1}.
\]

(3.33)

\[
\left\{ -2a(\sigma_x^2 - \sigma_y^2) + b \left( \sigma_x - \sigma_y \right)^2 \right\} \left( 2\sigma_b \right)^m = 2(\sigma_x^4 + \sigma_y^4)
\]

(3.34)

where \( \sigma_b \) is the equi-biaxial yield stress, \( \tau \) is the pure shear \( (\sigma_1 = -\sigma_2) \) yield stress, \( a \) and \( b \) are material constants, while the exponent \( m \) is calculated from:

\[
\left( \frac{2\sigma_b}{\sigma_{45}} \right)^m = 2(1 + r_{45})
\]

The parameters \( a \) and \( b \) can be identified through the yield stresses or by use of the anisotropy coefficients. More details about the computation of the Hill'90 coefficients can be found in Lin and Ding's work (1996) that also concluded that the first identification procedure ensured better results. Five material parameters are necessary for defining the yield function such as \( \sigma_0 \), \( \sigma_{45} \), \( \sigma_{90} \), \( \sigma_b \) and \( r_{45} \) or \( \sigma_{45} \), \( \sigma_b \), \( r_0 \), \( r_{45} \) and \( r_{90} \), obtained from three uniaxial tensile tests at 0°, 45° and 90° from rolling direction and a biaxial tensile test.

Lin and Ding (1996) proposed a more general expression of Hill 1990 yield criterion by the addition of non-quadratic terms which allow better results when the yield function coefficients are identified on the basis of the anisotropy coefficients, contrary to the original Hill 1990 yield criterion situation.

In 1993, in a very comprehensive analyse, Hill (1993) pointed out some situations, which cannot be reproduced entirely by any of the criteria described above. He shows that in the cases of all previous criteria the condition \( \sigma_0 = \sigma_{90} \) enforces \( r_0 = r_{90} \), whereas experimentally it was observed that some materials in especial the aluminium alloys and brass, although having almost equal yield stress in rolling and transverse directions, the anisotropy coefficients in these directions are different. Banabic (1998, 1999) called this behaviour as “anomalous behaviour of second order” to distinguish it from the “anomalous behaviour” of aluminium alloys described by Woodthorpe and Pearce. The new yield criterion proposed by Hill (1993) is a polynomial function of third degree:

\[
\frac{\sigma_{1}^2}{\sigma_0^2} - \frac{c\sigma_1 \sigma_2}{\sigma_0 \sigma_{90}} \sigma_1 \sigma_2 + \frac{\sigma_2^2}{\sigma_{90}^2} + \left\{ (p+q) - \frac{(p\sigma_1 + q\sigma_2)}{\sigma_b} \right\} \frac{\sigma_1 \sigma_2}{\sigma_0 \sigma_{90}} = \sigma_u^2
\]

(3.35)

with

\[
\frac{c}{\sigma_0 \sigma_{90}} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_{90}^2} - \frac{1}{\sigma_b^2}
\]

(3.36)

The coefficients \( p \) and \( q \) are calculated from the normality condition of the strain rate tensor to the yield surface. Five mechanical parameters from two uniaxial tensile tests and one equibiaxial tensile test are necessary to define the yield function coefficients.

The most significant advantages of the Hill 1993 yield criterion are that it captures the “first order anomalous behaviour” and the “second order anomalous behaviour”, it has a relatively
simple and friendly expression and it has a great flexibility due to the five mechanical parameters considered in identification of the yield function coefficients.

The main disadvantages of this model are the non-homogeneity of the yield function with respect to $\sigma_1$ and $\sigma_2$, not allowing an explicit expression of the strain increments; it is expressed using principal stresses being used only in the case when the principal stresses directions are superimposed with the orthotropic axes; the yield surface predicted by this function is far from the one computed using the polycrystal theories (Taylor or Bishop-Hill).

Barlat et al. (1991) proposed a six-component yield criterion (denoted Yld91) that extends the isotropic Hershey-Hosford criterion to anisotropic (orthotropic) cases by replacing the stress tensor with the principal values of the stress deviator modified with weighting coefficients. This weighting procedure is equivalent with the application of a fourth order linear transformation operator on the stress tensor.

$$\tilde{S} = \tilde{L} \cdot \tilde{\sigma}$$

The idea of introducing a fourth order linear tensor to describe anisotropy was first reported by Sobodka (1969).

Therefore, the isotropic Hosford criterion is rewritten in a form containing the deviator principal stresses $S_1$, $S_2$ and $S_3$:

$$\phi = |S_1 - S_2|^m + |S_2 - S_3|^m + |S_3 - S_1|^m = 2 \cdot \sigma_e^n$$

After a complex number transformation and the Bishop-Hill notation,

$$A = \sigma_y - \sigma_z, \quad B = \sigma_z - \sigma_x, \quad C = \sigma_x - \sigma_y; \quad F = \tau_{yz}, \quad G = \tau_{zx}, \quad H = \tau_{xy},$$

Barlat obtained the following expression of the isotropic Hosford yield criterion:

$$\phi = (3 I_2)^{n/2} \cdot \left(2 \cos \left(\frac{2 \theta + \pi}{6}\right) + 2 \cos \left(\frac{2 \theta + 3 \pi}{6}\right) - 2 \cos \left(\frac{2 \theta + 5 \pi}{6}\right) \right)^m = 2 \sigma_e^n,$$

where

$$\phi = \arccos \left(\frac{I_3}{I_2^{3/2}}\right)$$

$I_2$ and $I_3$ are the second and third invariants of the stress determinant that were modified by multiplying the stress components with some weighting coefficients which are identified using three uniaxial tensile tests in the orthotropic axes directions and three pure shearing tests.

The main advantages of the Barlat yield criterion are its generality and flexibility, its good accuracy on the yield surfaces prediction by comparing with those predicted by polycrystal
Theories, the easy implementation on the finite-element codes and the realistic estimation on the distribution of the uniaxial yield stress and r-coefficient in the sheet plane.

The essential disadvantage consists in the relatively complicated expression of the associated flow rule which leads to a difficult manipulation and use in analytical computations.

Lian and Chen (1991) generalized the Hill 1979 yield criterion to the three-dimensional stress state using a similar methodology. They proposed a six-component expression of the Hill 1979 criterion:

$$
2^n J_2^{m/2} \left\{ f \left| \sin \left( \frac{\varphi}{3} \right) \right|^m + g \left| \cos \left( \frac{\varphi + \pi}{6} \right) \right|^m + h \left| \cos \left( \frac{\varphi - \pi}{6} \right) \right|^m \right\} + \\
+ 3^{m/2} \left[ a \left| \cos \left( \frac{\varphi}{3} \right) \right|^m + b \left| \cos \left( \frac{\varphi + \pi}{3} \right) \right|^m + c \left| \cos \left( \frac{\varphi - \pi}{3} \right) \right|^m \right] = \sigma_e^m
$$

(3.42)

where $\varphi$ is a function of the second and third invariants of the deviatoric stress tensor. This methodology is very general and can be applied to any yield criterion written by using principal stresses in order to obtain a six-component expression.

In 1993, Karafillis and Boyce (1993) proposed an original and very general yield criterion by expressing it as a weighted combination between the von Mises and Tresca criteria and using a linear transformation to pass from the isotropic to the anisotropic case. The proposed yield function is:

$$
\phi = (1-c)\phi_1 - c\phi_2,
$$

(3.43)

where,

$$
\phi_1 = |S_1 - S_2|^2 + |S_2 - S_3|^2 + |S_3 - S_1|^2 = 2\sigma_e^{2k}
$$

(3.44)

and

$$
\phi_1 = |S_1|^{2k} + |S_2|^{2k} + |S_3|^{2k} = \frac{2^{2k} + 2}{3^{2k}} \sigma_e^{2k}
$$

(3.45)

$S_1$, $S_2$ and $S_3$ are the principal deviatoric stresses, $c$ is a weighting coefficient and $2k$ is an exponent with the same significance as the exponent $a$ in the Hosford's criterion. To achieve the anisotropy, Karafillis and Boyce used the above-described linear transformation:

$$
\tilde{S} = \tilde{L} \cdot \tilde{\sigma},
$$

(3.46)

where $\tilde{S}$ is a deviatoric stress tensor associated to an “Isotropic Plastic Equivalent” (IPE) state, $\tilde{\sigma}$ is the actual anisotropic stress tensor and $\tilde{L}$ is a linear operator associated to the material defined as:
3. Constitutive models for sheet metals

\[
L = \begin{bmatrix}
1 & \beta_1 & \beta_2 & 0 & 0 & 0 \\
\beta_1 & \alpha_1 & \beta_3 & 0 & 0 & 0 \\
\beta_2 & \beta_3 & \alpha_2 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_1 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_2 & 0 \\
0 & 0 & 0 & 0 & 0 & \gamma_3 \\
\end{bmatrix}
\]  \quad (3.47)

where,

\[
\beta_1 = \frac{\alpha_2 - \alpha_1 - 1}{2} \\
\beta_2 = \frac{\alpha_1 - \alpha_2 - 1}{2} \\
\beta_3 = \frac{1 - \alpha_1 - \alpha_2}{2}
\]  \quad (3.48)

\(\alpha_1, \alpha_2, \gamma_1, \gamma_2, \gamma_3\) and \(c\) are parameters defining the anisotropy of metallic material.

Karafillis–Boyce criterion predicts the yield surfaces and the variation of the uniaxial yield stress and anisotropy coefficients in the plane of the sheet, in a very good agreement with the experimental data as well as with the prediction of the Bishop-Hill (1951) theory. Another advantage is that only uniaxial tensile tests are used to identify the material parameters. The main disadvantage of the criterion is the relative complexity of the tensor operator \(\bar{L}\) identification procedure which requires a numerical solution.

In order to increase the potential of the theoretical models to simulate the complex plastic behaviour of those alloys, in especial the aluminium ones, that is very difficult to model with the criteria above, Barlat and his coworkers (1997a) proposed in 1994 the so-called Yld94, a more general expression of the yield function introduced by himself in 1991. The generalization consisted in weighting the terms of the yield criterion with the coefficients \(\alpha_i\), \(\alpha_2\) and \(\alpha_3\):

\[
\phi = \alpha_1 |S_2 - S_3| + \alpha_2 |S_3 - S_1| + \alpha_3 |S_1 - S_2| = 2\bar{\sigma}_n
\]  \quad (3.49)

The coefficients \(\alpha_i\), \(\alpha_2\), \(\alpha_3\), are computed using the transformation:

\[
\alpha_k = \alpha_1 p_{1k}^4 + \alpha_2 p_{2k}^2 + \alpha_3 p_{3k}^2
\]  \quad (3.50)

where \(P\) is the transformation matrix between the principal direction of \(S\) and the principal axes of anisotropy while \(\alpha_1\), \(\alpha_2\) and \(\alpha_3\) are quantities related to the anisotropy of the materials.

For plane stress state, six independent coefficients are available, and can be identified from three uniaxial tensile tests for \(\sigma_0\), \(\sigma_{45}\), \(\sigma_{90}\) and \(r_{90}\) and one biaxial tensile test for \(\sigma_b\).
The yield surface predicted by this criterion is in good agreement with the Bishop Hill theory and the experimental ones, but the finite-element simulation based on this criterion, has shown some inaccuracies in representing the blank earring.

In order to improve the performance of this criterion, Barlat et al. (1997b) introduce in the so-called Yld96 yield criterion, the quantities $\alpha_x$, $\alpha_y$, and $\alpha_z$ depending on the angles $\beta_1$, $\beta_2$, $\beta_3$ between the principal directions of the stress tensor and the anisotropic axes, as follows:

$$\alpha_x = \alpha_{x0} \cos^2 2 \beta_1 + \alpha_{x1} \sin^2 2 \beta_1$$
$$\alpha_y = \alpha_{y0} \cos^2 2 \beta_2 + \alpha_{y1} \sin^2 2 \beta_2$$
$$\alpha_z = \alpha_{z0} \cos^2 2 \beta_3 + \alpha_{z1} \sin^2 2 \beta_3$$

$$\cos^2 \beta_i = \begin{cases} 
1.1, & \text{if } |s_i| \geq |s_j| \\
3.1, & \text{if } |s_i| < |s_j|
\end{cases}$$

(3.52)

Under plane stress condition, the eight parameters, such as $c_1, c_2, c_3, c_6, \alpha_x, \alpha_y, \alpha_z$ and the exponent which is established in accordance with the material crystallographic structure, are necessary to completely identify the yield function. A good flexibility of the criterion is achieved due the great number of the criterion parameters that are computed from eight mechanical parameters obtained from three uniaxial tests and one biaxial testing stretch.

Using Yld96 yield criterion, a very good agreement of the blank earring prediction and the experimental data is obtained for cylindrical cup deep drawing (Yoon et al., 2000) Also, a very good accuracy on the prediction of yield surfaces as well as on the distribution of the uniaxial yield stress and anisotropy coefficients in the sheet plane has been found.

As disadvantages, there are three problems associated with Yld96 with respect to FE simulations such as: there is no proof of convexity, which is an important requirement in numerical simulations to ensure the uniqueness of a solution; the derivatives are difficult to obtain; the plane stress implementation in FE codes does not provide any particular problem and leads to good results but for full stress states some numerical problems have been encountered (Barlat et al, 2003).

Banabic et al. (2000b) proposed a new yield criterion called BBC2000 for orthotropic sheet metals under plane-stress conditions, derived from the one proposed by Barlat and Lian in 1989 (1998) through the addition of two coefficients, namely $b$ and $c$.

The equivalent stress is defined by the following expression:

$$\bar{\sigma} = \left[ a(b \Gamma + c \Psi)^{2k} + a(b \Gamma - c \Psi)^{2k} + (1-a)(2c \Psi)^{2k} \right]^{\frac{1}{2k}}$$

(3.53)

where $a$, $b$, $c$, and $k$ are material parameters, while $\Gamma$ and $\Psi$ are functions of the second and third invariants of a fictitious deviatoric stress tensor $\sigma^t$, which is related to the actual stress tensor $\sigma$ by the Karafillis-Boyce linear transformation (1993):

$\Gamma$ and $\Psi$ can be expressed as explicit dependencies of the actual stress components:
\[ \Gamma = (d + e) \sigma_{11} + (e + f) \sigma_{22} \]

\[ \Psi = \sqrt{\left[ \frac{1}{2} (d-e) \sigma_{11} + \frac{1}{2} (e-f) \sigma_{22} \right]^2 + g^2 \sigma_{xy}^2} \]  \hspace{1cm} (3.54)

where \( d, e, f, g \) are anisotropy coefficients of material and \( k \) value is set in accordance with the crystallographic structure of the material (Hosford, 1972): \( k = 3 \) for BCC alloys, and \( k = 4 \) for FCC alloys.

A more effective strategy of the identification of the seven yield function coefficients from the seven mechanical parameters such as \( \sigma_0, \sigma_{45}, \sigma_{90}, \sigma_b, r_0, r_{45} \) and \( r_{90} \), is to impose the minimization of an error function by adopting the downhill simplex method proposed by Nelder and Mead (Press, 1992) for the numerical minimization, not needing the evaluation of the gradients.

**BBC2000** yield criterion has an increased flexibility due to the seven coefficients used to describe the yield surface. Also, the predicted yield surfaces and the distribution of the anisotropy coefficients and uniaxial yield stress are in very good agreement with the experimental data.

Recently, Paraianu et al. (2003) proposed an improvement of this criterion, in order to account for an additional mechanical parameter, namely, the biaxial anisotropy coefficient, introduced independently by Barlat et al. (2003) and Pöhlandt et al. (2002). Aretz (2003) introduced a more flexible yield function in the BBC formulation.

A recent proposal of Barlat et al. (2003) is a new plane stress yield function, called **Yld2000-2d**, that well describes the anisotropic behaviour of sheet metals, in particular aluminium alloy sheets. Its expression is given by:

\[ \phi = \phi' (X') + \phi''(X'') = 2\bar{\sigma}^m, \]  \hspace{1cm} (3.55)

where \( \phi' \) and \( \phi'' \) are two isotropic functions as follows:

\[ \phi'(s) = |s_1 - s_2|^m \]  \hspace{1cm} (3.56)

\[ \phi''(s) = |2s_2 + s_1|^m + |2s_1 + s_2|^m \]  \hspace{1cm} (3.57)

while \( X' \) and \( X'' \) are the linearly transformed stress tensor:

\[ X' = C' \sigma = C'T\sigma = L'\sigma \]  \hspace{1cm} (3.58)

\[ X'' = C'' \sigma = C'T\sigma = L''\sigma \]  \hspace{1cm} (3.59)

\( C' \) and \( C'' \) or \( L' \) and \( L'' \) represent the linear transformation and \( T \) a matrix relating the deviatoric to the Cauchy stresses.
**Yld2000-2d** criterion solve the two problems associated with **Yld96** with respect to FE simulation namely no proof of convexity and difficult analytical calculation of the derivatives, providing a simpler formulation than **Yld96** with at least the same accuracy and attesting its convexity and its straightforward implementation into FEM. A great flexibility of the function is assured by the eight yield function coefficients, which are computed using as input the experimental values of the stresses and r values in tension along three directions and the balanced biaxial flow stress as well as the balanced biaxial anisotropy coefficient. It was shown an excellent ability of the **Yld2000-2d** to reproduce the experimental as well as polycrystal results.

**Cazacu-Barlat'2002** (2002) yield criterion is a very interesting generalization to anisotropic conditions of the stress deviator invariants in the framework of the theory of representation of tensor functions, e.g. **Boehler** (1978), **Liu** (1982). Assuming that yielding is insensitive to hydrostatic pressure, for an isotropic material, the yield function depends on stress through $J_2 = trs^2 / 2$ and $J_3 = trs^3 / 3$, the second and third invariants of the stress deviator s, respectively. To introduce orthotropy in the expression of an isotropic criterion, **Cazacu and Barlat** (2002) proposed generalizations ($J_2^o$ and $J_3^o$) of the stress deviator invariants. $J_2^o$ and $J_3^o$ are homogeneous functions of degree two and three in stresses, respectively, are insensitive to pressure and are invariant to any transformation belonging to the symmetry group of the material. Relative to the orthotropic reference frame $(x, y, z)$, they are expressed as follows:

\[
J_3^o = \frac{1}{27} (b_1 + b_2) \sigma_x^3 + \frac{1}{27} (b_3 + b_4) \sigma_y^3 + \frac{2}{27} b_{11} \sigma_{xy} \sigma_{xz} \sigma_{yz} + \frac{1}{27} [2(b_1 + b_4) - b_2 - b_3] \sigma_z^3 + \\
\frac{2}{9} (b_1 + b_4) \sigma_x \sigma_z \sigma_y - \frac{1}{9} (b_1 \sigma_y + b_2 \sigma_z) \sigma_y^2 + \frac{1}{9} (b_3 \sigma_z + b_4 \sigma_x) \sigma_y^2 \\
-\frac{1}{9} [(b_1 - b_2 + b_4) \sigma_x + (b_1 - b_3 + b_4) \sigma_y] \sigma_x^2 - \frac{\sigma_{yz}^2}{3} [2b_6 \sigma_y - b_4 \sigma_z - (2b_9 - b_8) \sigma_x] \\
- \frac{\sigma_{zy}^2}{3} [2b_{10} \sigma_z - b_5 \sigma_y - (2b_{10} - b_5) \sigma_x] - \frac{\sigma_{zx}^2}{3} [b_6 + b_7] \sigma_x - b_6 \sigma_y - b_5 \sigma_z \\
J_2^o = \frac{a_1}{6} (\sigma_x - \sigma_y)^2 + \frac{a_2}{6} (\sigma_y - \sigma_z)^2 + \frac{a_3}{6} (\sigma_z - \sigma_x)^2 + a_4 \sigma_{xy}^2 + a_5 \sigma_{xz}^2 + a_6 \sigma_{yz}^2 (3.60)
\]

where all the coefficients $a_k$ and $b_k$ ($k = 1\ldots6$) reduce to one in the isotropic case.

Using these generalized invariants any isotropic yield criterion can be extended to describe orthotropy. In **Cazacu and Barlat** (2003), this approach was used to extend **Drucker's** (1949) isotropic yield criterion. Hence, the proposed orthotropic criterion is:

\[
\phi = \left( J_2^o \right)^3 - c \left( J_3^o \right)^2 = k^2 (3.62)
\]

For a full stress state, the criterion involves 18 material parameters. For plane stress conditions, the 10 anisotropy coefficients and the value of c can be determined using different methods, for
instance from the measured uniaxial yield stresses $\sigma_b$ and strain ratios $r_b$ in five different orientations $\theta$, namely at 0°, 30°, 45°, 75° and 90°, and the value of the balanced biaxial stress, $\sigma_b$ (see more details in Cazacu and Barlat, 2003).

Excellent results were shown on the generalization of Drucker's yield criterion to orthotropy by the use of this method. Beside of the all-previus presented criteria, several other non-quadratic yield criteria with a restrained use have been developed. The Gotoh (1977) proposed a yield criterion, which in order to remove the disadvantages of the Hill criterion is expressed as a polynomial yield function of fourth degree instead of the quadratic one, for orthotropic rolled sheet metals. The main disadvantage of this criterion is its complex form and the large number of mechanical tests needed for coefficients identification. In 1990, Zhou (1990) generalized the Hosford yield criterion for the case when the principal directions are not coincident with the orthotropic axes. After four years, Zhou (1994) proposed a generalization of the case 4° of Hill 1979 yield criterion by including the shear stress component. Monteillet (1991) has developed another generalization of case 4° of Hill 1979 yield function. In 1991 Banabic and Balan (1999a, b) proposed a new yield criterion for stress states where the orthotropy directions are also principal directions of the stress tensor that achieved a good agreement between the simulated yield surface and the experimental one.

Other criteria have been proposed on the basis of different principles. Vetter (1991) has proposed the representation of the yield function with the help of Bezier's interpolation of experimental data. In order to describe planar anisotropy Vetter's criterion contains 17 parameters, which ensured its flexibility, the most important advantage of the criterion. As disadvantages can be mentioned the unfriendly form of the yield function making it improper for analytical computation and the large number of mechanical tests required for identifying the parameters (four tests: uniaxial tension, biaxial tension, plane strain and pure shearing).

iii) Yield criteria in polar coordinates

Based on the fact that a function of the stress points ($\sigma_2 + \sigma_1$) and ($\sigma_2 - \sigma_1$) can express any two dimensional yield criterion for planar isotropy, Budiansky (1984) proposed a general yield criterion written as a parametric dependence in polar coordinates:

$$x = \frac{\sigma_2 + \sigma_1}{2\sigma_{bl}} = g(\theta)\cos\theta, \quad y = \frac{\sigma_2 - \sigma_1}{2\sigma_s} = g(\theta)\sin\theta$$

where $g(\theta) > 0$ is the radial coordinate of a point located on the yield surface, $\theta$ is the associated polar angle, $\sigma_s$ is the yield stress in pure shearing and $\sigma_{bl}$ is the yield stress in equi-biaxial tension.

Ferron (Ferron et al., 1994) extended this criterion of Budiansky for planar anisotropy cases by considering the radial coordinate of point located on the yield surface $g(\theta, \alpha)$ as a function of the associated polar angle $\theta$ and of the angle characterizing the principal stress directions with respect to the orthotropy axes $\alpha$. Using a similar variable substitution as Budiansky:
where $\sigma$ is the equivalent stress usually taken as the equibiaxial yield stress $\sigma_y$, the yield surface is described by the equation:

$$\psi(x_1, x_2, \alpha) = 1$$  \hspace{1cm} (3.65)

with $x_1$ and $x_2$ two parametric functions written as:

$$x_1 = x_1(\theta, \alpha) = g(\theta, \alpha)\cos \theta$$

$$x_2 = x_2(\theta, \alpha) = g(\theta, \alpha)\sin \theta$$  \hspace{1cm} (3.66)

A good agreement with the experimental data was obtained on the simulation of the yield surfaces as well as on the distribution of the uniaxial yield stress and anisotropy coefficients in the sheet plane (Tourki et al., 1994).

### 3.1.2 Work hardening

Work hardening, or strain hardening, is an intrinsic ability of material to strengthen or harden with increasing strain level and is one of the most important properties influencing the formability of sheet metals. During plastic deformation, a region undergoing thinning can resist further deformation by virtue of strain hardening and can spread deformation to its neighbouring regions, thus promoting more uniform thinning. There are generally two parameters for the description of the strain hardening behaviour. One is the strain-hardening rate, $\theta$ defined as the slope of true stress true strain $(\bar{\sigma} - \bar{\varepsilon})$, i.e. $\theta = \partial \bar{\sigma} / \partial \bar{\varepsilon} | \dot{\varepsilon}$ (Estrin and Kubin, 1986, 1991; Ling and McCormick, 1990). Another is the strain-hardening index, $n$, defined as $n = \partial \ln \bar{\sigma} / \partial \ln \bar{\varepsilon}$

The phenomenological hardening models describe the work hardening as the evolution of the equivalent stress on the relation with the equivalent strain. Generally can be written as:

$$\bar{\sigma} = f_{HR}(\bar{\varepsilon})$$  \hspace{1cm} (3.67)

where $f_{HR}$ is a function of the equivalent plastic strain $\bar{\varepsilon}$ which is defined by the time integral of the equivalent plastic strain rate:

$$\bar{\varepsilon} = \int \dot{\varepsilon} dt$$  \hspace{1cm} (3.68)

Material parameters of the function $f_{HR}(\bar{\varepsilon})$ are usually identified by fitting to a stress-strain curve obtained by a uniaxial tensile test or recently by a biaxial stretching test.

The typical formulation of the hardening law is the Holomon power law expressed as:

$$\bar{\sigma}(\bar{\varepsilon}) = k \bar{\varepsilon}^n$$  \hspace{1cm} (3.69)

In order to improve the accuracy, numerous hardening laws were used to characterize the hardening behaviour of the material under plastic deformations such as:
1. **Swift equation:**
\[ \bar{\sigma}(\bar{\varepsilon}) = k(\varepsilon_0 + \bar{\varepsilon})^n \]  
(3.70)

2. **Ludwick equation:**
\[ \bar{\sigma}(\bar{\varepsilon}) = \sigma_0 + k\bar{\varepsilon}^n \]  
(3.71)

3. **Hartley law:**
\[ \bar{\sigma}(\bar{\varepsilon}) = \sigma_0 \exp\left[\frac{(\varepsilon_0 + \bar{\varepsilon})}{\varepsilon^*}\right] \]  
(3.72)

4. **Voce equation**
\[ \bar{\sigma}(\bar{\varepsilon}) = A - B \exp(-C\bar{\varepsilon}) \]  
(3.73)

5. **Marciniak equation**
\[ \bar{\sigma}(\bar{\varepsilon}) = \frac{c(\varepsilon_0 + \bar{\varepsilon})}{1 + c(\varepsilon_0 + \bar{\varepsilon})} \sigma_e \]  
(3.74)

where \( \bar{\sigma} \) and \( \bar{\varepsilon} \) are the effective stress and strain, \( k, \varepsilon_0, \sigma_0, \varepsilon^*, \sigma_e, A, B, C \) are material constants specifics of the respective laws.

Beside of the deformation history, the strain rate and temperature also influence the work hardening behaviour but its effects are usually neglected in sheet metal forming. Nevertheless there are many hardening laws that take in account the effects of the strain rate and temperature. (see Banabic 1992).

Figure 2 shows the three types of hardening, which can be distinguished on the base of the yield locus expansion:

1. The isotropic hardening corresponds to an expansion of the yield surface without distorsion.

2. The so-called kinematic hardening is defined by a translation of the yield surface, which is expressed by.
\[ \bar{\sigma}(\sigma - X) = \sigma_0 \]  
(3.75)

where "the back stress" \( X \) is a second-order tensor with the dimensions of a stress which gives the position of the centre of the yield locus and varies during the plastic deformation while \( \sigma_0 \) is constant.

3. The mixed hardening model combines the isotropic and kinematic hardening assuming that an expansion and a shift of the yield locus occur simultaneously during plastic deformation. It is generally expressed in a similar form as Eq. 3.75, but \( \sigma_0 \) is a variable as well as \( X \). The key to the mixed-hardening model is the determination of the evolution of \( X \) and \( \sigma_0 \).
3.2 Physical approach

Whereas the phenomenological approach counts with the mechanical reaction of the material for a given deformation, the understanding of the physical mechanisms of the deformation process is the goal of the physical approach of the theory of plasticity.

One of the major causes of anisotropy in most metallic engineering materials is their polycrystalline nature. The metals used for cold working applications are polycrystalline materials and plastic deformation is caused by crystallographic slip on preferred atomic planes in
preferred directions. The slip systems have particular orientations relative to the unit cell of the crystal lattice, and consequently the plastic behaviour of each crystallite is highly anisotropic. Also, the macroscopically plastic behaviour of the polycrystal strongly depends on the orientations of the crystal lattices of the individual crystallites. The anisotropy of the individual crystallites will be cancelled out on the macroscopic level if the crystal orientations are randomly distributed. The crystallographic texture that are usually encountered in industrial materials for cold working processes often exhibit pronounced preferred crystal orientations, thereby causing a pronounced macroscopic anisotropy.

Over the years several polycrystal models were proposed, which differ in the single crystal constitutive model and the particular procedure used to average the mechanical behaviour over a representative number of grains (Kocks, Tomé and Wenk, 1998).

### 3.2.1 Single crystal plasticity

The dominant deformation mechanism, encountered in plastic forming of polycrystal metals, is multiple crystallographic slip. According to Schmid (1924), whose law describes the plastic behaviour of a single crystal in terms of the activation of crystallographic slip systems, a single crystal yields on a particular slip system if the resolved shear stress reaches a critical value, the so-called critical resolved shear stress (CRSS) \( \tau^s \):

\[
m^s : \sigma^s = \tau^s \quad \text{(one system s)}
\]

(3.76)

The inner product \( m^s : \sigma^s \) is the projection of the applied stress onto the straining direction of the particular slip system.

In general, there is more than one slip system available: each is represented by a plane perpendicular to its \( m^s \). Thus, equation Eq 2.72 must be generalized as:

\[
m^s : \sigma^s \leq \tau^s \quad \text{(for all system s)}
\]

(3.77)

The expressed inequality is necessary in order for all inactive system to have a resolved stress less than their CRSS.

An important consequence of the 'yield condition' (Eq. 3.77) is that not all arbitrary combinations of slip modes can be simultaneously activated.

The Schmid law allows to determine the slip system(s) that will be activated as well as the yield stress level when a given macroscopic stress-loading mode is applied on a single crystal. Bishop and Hill (1951a, 1951b) constructed the single crystal yield locus on the basis of the Schmid law and applied the maximum work criterion to it, showing that the crystallographic slips achieved from their approach are the same as those obtained by Taylor's (1938) earlier hypothesis of minimal work dissipation by the slips.

To simplify the crystal approach, it is possible to use a description of the single grain behaviour that is more suitable for numerical applications (Leque et al., 1987a,b; Arminjon, 1991; Gambin, 1991; Darrieulat and Piot, 1996; Maniatty and Yu, 1996; Gambin and Barlat, 1997). In this case, the description can represent not only one grain, but also a grain distribution
around a particular orientation and the polycrystal can be described by a few specific orientations only.

3.2.2 Polycrystal plasticity

The single-crystal plasticity models can be applied to a simulation of the polycrystal in several ways. An overview of existing models has been given by Lefers et al. (1988), Van Houtte (1994) and by Aerdoudt et al. (1993).

Considering the deformation kinematics and taking the lattice rotation into account Asaro (1983) and Teodosiu (1992) proposed the direct application of the single-crystal plasticity model to the finite-element simulation of the polycrystalline materials in which each grain of the polycrystal consists of many finite elements. This method seems to be the most accurate solution but requires a vast computational effort, which is hardly accepted in industry.

More practical methods are obtained by use of the averaging procedure over the polycrystal before the finite-element simulation. The central problem of polycrystal averaging is the interaction of grains across their interfaces: interactions that require both equilibrium and compatibility condition to be met. Based on the assumptions adopted about the distribution of stresses and strains in the polycrystal, there are three essential approaches, in which can be classified the most of the averaging methods: the Taylor theory, the Sachs theory and the self-consistent models. The Taylor theory (Taylor, 1938) or Taylor-Bishop-Hill theory (Bishop and Hill, 1951a, b; Van Houtte, 1988) assumes a homogeneous strain over the polycrystal and ignores the stress equilibrium at the grain boundaries. Contrary, the Sachs theory (Sachs 1928) assumes a homogeneous stress over the polycrystal and does not satisfy the compatibility condition at the grain boundaries. Such restricted condition imposed by these theories may be severe assumptions in some situations, the obtained results being not in a good agreement with experiment data. To solve this problem, Van Houtte (1982) extends the Taylor theory to the relaxed constraint model (RC), which imposes a mixture of local compatibility and local equilibrium, based on grain shape considerations. In 1994, Van Bael (1994) developed appropriate polynomial series expansions that provide an analytical continuous representation of the average Taylor factor, in order to overcome the difficulties due to the discrete nature of the Taylor results. Self-consistent polycrystal models aim at deducing the overall response of the aggregate from the known properties of the constituent grains and an assumption concerning the interaction of each grain with its environment, fulfilling compatibility and stress equilibrium in an average sense. Some authors applied the self-consistent approach (Eshelby, 1957) to consider the stress equilibrium and strain compatibility at the boundaries physically (Kröner 1961, Hihi et al. 1985, Molinari et al. 1987). In 1993, Lebensohn and Tomé (1993) developed a general averaging scheme that relates the behaviour of a grain (crystal) to macroscopic properties. Their approach is based on the assumption that each grain can be treated as an inhomogeneity embedded in the homogeneous effective medium represented by the polycrystal. Such a formulation leads to an interaction equation that linearly relates the stress and strain rate in the grain with the overall stress and strain rate of the effective medium. The condition that the average of stress and strain rate over all the grains has to be consistent with the equivalent macroscopic magnitudes, makes for the self-consistent resolution of the problem. In 1995, Van Houtte (1995) solved the inclusion problem of the self-consistent model using the upper-bound theorem, instead of the inappropriate assumption of linearity in plastic response or instead of the
time consuming finite-element method, and the result was fitted to an analytical function, which allows a rapid computation, by the relaxed Taylor model with an appropriate relaxation.

A possible simplification to the crystal approach for numerical simulations is to describe the behaviour of the polycrystal as a whole and link the associated parameters to the main coefficients of the grain orientation distribution function of the material (Toth et al., 1991; Arminjon and Imbault, 1996; Van Houtte, 1994)

3.2.3 Work hardening

Plastic deformation of crystalline materials leads to the formation of three-dimensional distributions of dislocations, which are characteristic of crystal structure of the material being deformed, temperature of deformation, strain and strain rate. Additionally, grain boundaries, precipitates and stacking fault energy affect the distribution of the dislocations. The hardening of crystals during plastic deformation is due to the increase in dislocation density and the mutual interaction between dislocations.

The work hardening of the slip system can be expressed as an evolution equation of the critical resolved shear stress in the form of:

\[ \dot{\tau}_c = H^{u} |\dot{\gamma}_s|, \]  
(3.78)

where \( H^{u} \) is the hardening matrix whose components correspond to the hardening on slip system \( s \) induced by slip system \( u \), evolving as deformation proceeds. Teodosiu (1992) and Gil Sevillano (1993) presented a review of the several hardening models of this type which were proposed during the years.

The simplest hardening model at the single crystal level provided by the Taylor's isotropic assumption, consider that all slip systems have the same value of the critical resolved shear stress (\( \tau_c \)) and the evolution rate is proportional to the total of the slip rates in all slip systems. The evolution equation is written as

\[ \dot{\tau}_c = H\dot{\gamma} \]  
(3.79)

with the total slip rate

\[ \dot{\gamma} = \sum_s |\dot{\gamma}_s| \]  
(3.80)

The work-hardening rate \( H \) can phenomenological be described by the differential form of a power type equation:

\[ H = nK(\gamma_0 + \gamma)^{n-1} \]  
with \( K, \gamma_0, n \) material parameters,  
(3.81)

or of a Voce type equation:
\[ H = H_0 (\tau_{sat} - \tau_c) \] (3.82)

where \( H_0 \) and \( \tau_{sat} \) are material parameters which denote the evolution rate and saturation value of \( \tau_c \), respectively. Kocks (1976) interpreted this saturation as the balance between the hardening due to the accumulation of dislocations and the softening due to the annihilation by dynamic recovery.

Based on the dislocation theory and the microstructural observations, in 1995, Teodosiu and Hu developed an anisotropic hardening model, introducing three internal variables to describe the anisotropic strength of the dislocation structures, the polarity and the back stress.

Muller et al. (1994) describes the internal stress due to the dislocation structures on the base of the self-consistent approach both at grain scale and at subgrain scale. This model cannot describe the cross hardening and subsequent softening due to the microband formation and requires more computational effort than the Teodosiu-Hu model.

In 1997, Winther and Jensen (1997) showed that the critical resolved shear stress on a slip system increases with the dislocation boundaries depending on the orientation of the slip system. Studying the effect of the texture evolution on the strain-induced plasticity, Strauven and Aernoudt (1987) showed an insignificant influence of it on the softening under reverse deformation whereas other works reported by Teodosiu (1992) support the predominance of the dislocation structures at moderate strains.

Assuming that the individual crystals have different orientations and the applied resolved shear stress for slip varies from grain to grain, there are important differences between plastic deformation in single-crystal and polycrystalline materials. A few grains yield first, followed progressively by the others. The grain boundaries act as strong barriers to dislocation motion, so that unless grain size is large, the stage I easy-glide exhibited by single crystals does not occurs in polycrystals. The stress strain curve is therefore not simply a single crystal curve averaged over random orientations. The macroscopic yield stress at which all grains yield depends on grain size. A grain in a polycrystal is not free to deform plastically as though it were a single crystal, for it must remain in contact with, and accommodate the shape changes of its neighbours (Hull and Bacon, 1984).

3.3 Conclusions

In order to assess the progress made in the field of the sheet metal constitutive models, over the last decades, a succinct review, with resource on the macroscopic and microscopic approaches, was presented in this chapter. Future improvements are expected to result from more advanced material models that combine the advantages of phenomenological material description with the most important physics-based aspects of plasticity. A viable model of polycrystal plasticity must be based on physical insight and must be tractable. Plasticity is a too complex phenomenon that all its aspects could be captured in a single model (Kocks, 1998). Since qualitatively different length scales are involved, the task is to judge which aspects to emphasize and which to downplay in a particular effort (Kocks, 1991).
4. A MORE GENERAL MODEL FOR FORMING LIMIT DIAGRAMS PREDICTION

4.1 Introduction

The complexity of the plastic deformation has lead to the development of numerous techniques to predict or evaluate the formability of the raw materials. In the last few years, important developments have been achieved, based on new theoretical models and experimental validation. It is well known that after the introduction of the FLD’s concept the research in this field of sheet-metal formability has focused mainly on the development of some mathematical models for the theoretical determination of FLD’s, the Marciniak and Kuczynski (1967) theory being the first realistic such mathematical model.

Bearing in mind that the predicted limit strains strongly depend on the constitutive law incorporated in the analysis and that many and better material models describing new generations of sheet metals like car body sheet steels, aluminium sheets have been developed in the last years and will be continuously developed in the future, the present chapter introduces a more general code for FLDs prediction applied for phenomenological and physical approach of plasticity theory. Treating the Marciniak – Kuczynsky (M-K) theory by a new approach, the code consists of a main part and several subroutines, which allow the implementation of any hardening law, yield function or constitutive equation, by merely changing the respective subroutine. The Newton-Raphson numerical method is used to solve the theoretical treatments of localized necking taking into account linear and complex strain paths.

The chapter is divided into four sections. Section 4.2 presents the general structure of the program and its individual subroutines including the advantage of the code as a user-friendly post processor for a FE code. Section 4.3 contains a detailed description of the new model for phenomenological and physical approach of theory of plasticity. Then, an overview of the considered phenomenological constitutive equations and their implementation on the FLD code are presented. Starting with the implementation of the Von Mises yield function for the isotropic materials, this new strategy allowed the implementation of the quadratic Hill criterion (Hill’48), non-quadratic Hill criterion (Hill’79) and Yld96 Barlat yield function. After that, the new model was extended to a physical approach of the plastic theory, by considering the most advanced physical model at the present time, which is the combined model of texture and strain-path-induced anisotropy, based on the Van Houtte anisotropic model (Van Houtte et al., 1995b) and the Teodosiu and Hu hardening model (Teodosiu and Hu, 1995). Finally, the conclusions are pointed out in the Section 4.4.

4.2 Structure of the program

The general structure of the FLD code is shown in Figure 1. The model consists of several subroutines, such as Hardening Law subroutine (1), Yield Function subroutine (2) and Flow Rule subroutine (3). Moreover, the computations of homogeneous and heterogeneous zone are considered independently, being their connection realized through the MK conditions: force equilibrium and geometrical compatibility.
General structure of the program

\textit{M-K analysis} \hspace{2cm} \textit{Theory of Plasticity}

![Diagram of the program structure](image)

Figure 1. Structure of the FLDcode

Figure 2 shows the well-defined input and output data for a specific subroutine. It can be noticed that the employment of different hardening laws or yield functions is possible to realize changing the corresponding subroutine, without having to modify the main part of the program. This fact gives a wonderful flexibility of the model, which beside of its ability on the implementation of different constitutive equations, allows to several combinations between hardening models and anisotropic plastic potentials for the best description of the material behaviour.

![Diagram of hardening law subroutine](image)

\textbf{a) Hardening Law Subroutine}
4. A more general Model for Forming Limit Diagrams Prediction

Yield Function Subroutine (2)

Input
\[ [\sigma]_{xyz} \]

Stress tensor in orthotropic axes of anisotropy

Output
\[ \bar{\sigma}_{YF} \]

Effective Stress

b) Yield function Subroutine

Flow Rule Subroutine (3)

Input
\[ [\sigma]_{xyz} \]
\[ d\bar{\varepsilon} \]

- Stress tensor in orthotropic axes of anisotropy
- Increment of equivalent strain

Output
\[ [d\varepsilon]_{xyz} \]

Increments of plastic strain in orthotropic axes of anisotropy

c) Flow Rule Subroutine

Figure 2. The structure of program subroutines

Another advantage of the FLDCode consists in its use as a postprocessing for Finite Element (FE) simulations, its structure allowing a more flexible interface between the necking criterion and FE programs as can be observed in Figure 3. In this case, FE code provides all information related to the homogeneous region, specifically the stress and strain state in orthotropic reference frame, which subsequently will be used as the input data of the FLDCode. Next, FLDCode computes the imperfection region from Marciniak-Kuczynski analysis, defining the
corresponding stress and strain state in orthotropic reference frame and finding the plastic instability moment when the failure condition is achieved.

\[
\begin{align*}
\text{FLDcode} & \quad \text{Input data} \\
\rightarrow & \\
\text{FE code} & \quad [\sigma]_{xyz}^a, [d\varepsilon]_{xyz}^a \\
\rightarrow & \\
\text{FLD code} & \quad [\sigma]_{xyz}^b, [d\varepsilon]_{xyz}^b \\
\rightarrow & \\
\text{M-K Necking Criterion} & \quad \text{NO} \\
\rightarrow & \\
\text{Necking localization} & \quad \text{YES} \\
\end{align*}
\]

Figure 3. The interface between necking criterion and FE programs

The version of the FLDcode as postprocessing for FE simulation is schematically presented in Figure 4. Due to the fact that the homogeneous zone is completely defined from the FE code as was previously mentioned, the adapted FLDcode consists only in the computation of the imperfection region keeping the origin structure of the program. Also the connection between the two zones of the material, actually between the FE code and FLD code still to be realized through the M-K conditions: force equilibrium and geometrical compatibility.
4. A more general Model for Forming Limit Diagrams Prediction

Figure 4. FLDcode structure as a postprocessing for FE simulation
4.3 Description of the model

4.3.1 Phenomenological approach of Theory of Plasticity

The simulation of plastic instability is carried out in the framework of heterogeneous materials using the Marciniak-Kuczinsky (M-K) analysis coupled with the theory of plasticity. The rigid plasticity, the plane stress condition and isotropic work hardening of the material is assumed.

![Figure 5. Initial geometrical imperfection of the M-K analysis](image)

The M-K analysis (Marciniak and Kuczinsky, 1967) is schematically illustrated in Figure 5. The model, as discussed in Chapter 2, is based on the growth of an initial defect in the form of a narrow band inclined at an angle \( \psi_0 \) with respect to the principal axis. The initial value of the geometrical defect is characterized by the ratio \( e_0^b/e_0^a \) where \( e_0^a \) and \( e_0^b \) are the initial thickness in the homogeneous region and in the groove respectively. The \( x, y, z \)-axes correspond to rolling, transverse and normal directions of the sheet, whereas \( 1 \) and \( 2 \) represent the principal stress and strain directions in the homogeneous region. The set of axis bound to the groove is represented by \( n, t, z \)-axes where ‘\( t \)’ is the longitudinal one. This two-zone material is subjected to plastic deformation applying a constant incremental stretching of the homogeneous part. The plastic flow occurs in both regions, but the evolution of strain rates is different in the two zones. When the flow localization occurs in the groove at a critical strain in homogeneous region, the limiting strain of the sheet is reached. The model assumes that the orthotropic axes and the principal stress directions are superimposed in region A and the major strain occurs along the \( x \)-axis. The homogeneous zone and the imperfection are conventionally denoted with the superscript ‘\( a \)’ and ‘\( b \)’ respectively.
The main equations of the M-K analysis are related to equilibrium and compatibility requirements.

The Equilibrium condition that indicates the same force perpendicular to the necking band in region A and B conforms to:

\[
\begin{align*}
\sigma_{nn} e^a &= \sigma_{nn}^b e^b \\
\sigma_{nn} e^b &= \sigma_{nn}^b e^b
\end{align*}
\]  
(4.1)

where \( \sigma_{nn} \) and \( \sigma_{nn}^b \) are components of stress tensor in the groove reference frame, while \( e^a \) and \( e^b \) are the sheet thickness outside and inside the groove respectively, given by

\[ e^a = e_0^a \exp(\varepsilon^a_n) \quad \text{and} \quad e^b = e_0^b \exp(\varepsilon^n_b) \]  
(4.2)

The Compatibility requirement assumes that the elongation in the direction of the necking band is identical in both regions:

\[ de_n^a = de_n^b \]  
(4.3)

The original M-K analysis assumed the pre-existing thickness imperfection in the form of a groove perpendicular to the principal strain direction. Hutchinson and Neale (1978 a, b) found a definite imperfection orientation angle which gives the minimum limiting strain for each proportional strain path on the left hand side of FLD. Moreover, in the general case of anisotropic materials, Barata da Rocha (1985, 1989) pointed out that the limiting strains are sensitive to the groove orientation, even on the right hand side of the FLD.

Thus, the band rotation is described by

\[ \tan(\psi + d\psi) = \tan(\psi)(1 + \varepsilon^a_n)/(1 + \varepsilon^a_n) \]  
(4.4)

'Theory of Plasticity' is the name given to the mathematical study of stress and strain in plastically deformed solids, especially metals (Hill, 1950). Hence, a general definition of the components of this theory namely of the hardening rule, the yield function and the associated flow rule is next summarized.

The Hardening Rule representing the relationship between the equivalent stress and equivalent strain is:

\[ \bar{\sigma} = f_{HR}(\bar{\varepsilon}) \]  
(4.5)

where \( f_{HR} \) is a function of the equivalent strain \( \bar{\varepsilon} \).

The Yield Function describing the yield surface and representing the relationship between the equivalent stress and stress tensor in the orthotropy referential frame of anisotropy is expressed as:
\[ \bar{\sigma}_{\text{YF}} = f_{\text{YF}}(\sigma_{xx}, \sigma_{yy}, \sigma_{xy}) \]  

(4.6)

where \( f_{\text{YF}} \) is a function of the stress tensor \([\sigma]_{\text{yz}}\).

The associated Flow Rule results by the normality of \( d\bar{\varepsilon} \) to the yield surface:

\[ d\varepsilon_y = d\lambda \frac{\partial f_{\text{YF}}}{\partial \sigma_y}, \]  

(4.7)

where \( d\lambda \) is a proportionality factor by the conservation of the plastic work and the Euler theorem on homogeneous functions.

In the model, for a given yield function, it is useful to know the correlation between the stress ratio \( \rho \) and the strain ratio \( \rho \). Thus, considering that the principal anisotropy axes of orthotropic symmetry are coincident with the principal axes of stress in the homogeneous region these relation could be calculated in the follow way:

\[ \rho = \frac{d\varepsilon_{12}}{d\varepsilon_{11}} = \frac{d\lambda \frac{\partial f_{\text{YF}}}{\partial \sigma_{12}}}{d\lambda \frac{\partial f_{\text{YF}}}{\partial \sigma_{11}}} = \frac{\partial f_{\text{YF}}}{\partial \sigma_{12}} = f_\rho(\alpha), \]  

(4.8)

where \( f_\rho(\alpha) \) is a function of stress ratio.

Similarly \( \alpha = f_\rho(\rho) \), where \( f_\rho(\rho) \) is a function of strain ratio.

**i) Theoretical treatments of localized necking**

**i.1. Computation of stress and strain state in homogeneous region ‘zone A’**

Small increments of equivalent strain \( d\bar{\varepsilon}^a \) are imposed in the homogeneous region (zone A). The strain path is characterized by the stress ratio \( \alpha = \frac{\sigma_{12}}{\sigma_{11}} \), which coupled with the yield function gives the stress tensor in the orthotropic referential frame of anisotropy \([\sigma^a]_{\text{yz}}\).

\[
\begin{align*}
\sigma_{xx} &= f_{\text{YF-a}}(\bar{\sigma}, \alpha) \\
\sigma_{yy} &= \alpha \sigma_{xx}
\end{align*}
\]  

(4.9)

where \( f_{\text{YF-a}} \) is a function of the equivalent stress \( \bar{\sigma} \) and the stress ratio \( \alpha \).

The stress tensor correspondent to the homogeneous region can be also obtained through another method, more efficient (Barlat, 2000), by imposing the stress direction and applying then the yield function and Euler theorem of homogeneous functions (see Appendix A1).
4. A more general Model for Forming Limit Diagrams Prediction

Through the flow rule, the strain increment matrix \([d\varepsilon^a]_{yz}\) in the principal axis of anisotropy is calculated. By an axes change, the stress and strain states in the groove reference frame are also known.

The structure of M-K analysis computation presented in Figure 6 clearly illustrates that only four results of homogeneous zone are necessary in the computation of heterogeneous zone. When the ratio \(d\bar{\varepsilon}^b / d\bar{\varepsilon}^a\) is greater or equal to ten, the code considers necking is occurring.

i.2. Computation of stress and strain state in imperfection region ‘zone B’

In order to compute the equivalent increment strain \(d\bar{\varepsilon}^b\) and the stress value in longitudinal direction of the groove \(\sigma_{nz}^a\), the Newton-Raphson method is applied.

The current imperfection value \(f\) is characterized by the ratio of the sheet thickness in regions A and B and is expressed as a function of the initial defect \(f_0\).

\[
f = f_0 \exp(\varepsilon_3^b - \varepsilon_3^a)
\]  \hspace{1cm} (4.16)

The condition of force equilibrium between zones A and B allows to calculate the flow stress value in the normal direction of the groove and the flow shear stress in the groove, whereas through the yield function the equivalent flow stress \(\bar{\sigma}_{yz}^b\) is evaluated.

The flow stress is calculated by the hardening rule \(\bar{\sigma}^b = f_{HR}(\bar{\varepsilon}^b)\) and through the flow rule the strain matrix increment in the imperfection region \([d\varepsilon^b]_{yz}\) in the orthotropic referential frame of anisotropy is determined.
The following two nonlinear equations in $d\bar{e}^b$ and $\sigma_u^b$ can be written.

\begin{align}
G_1(d\bar{e}^b, \sigma_u^b) &= \bar{\sigma}^b - \bar{\sigma}_{YF}^b = 0 \\
G_2(d\bar{e}^b, \sigma_u^b) &= d\varepsilon_u^a - d\varepsilon_u^b = 0
\end{align}

(4.17) and (4.18)

$G1$ and $G2$ are two polynomial functions respectively in $d\bar{e}^b$ and $\sigma_u^b$. $G1$ represents the Yield Criterion whereas $G2$ represents the deformation Compatibility requirement in longitudinal direction of the necking band.

From equations (4.17) and (4.18), the iterative formula for Newton–Raphson’s method is described as follows:

\begin{equation}
\begin{bmatrix}
\frac{d\bar{e}^b_{i+1}}{} \\
\frac{\sigma_u^b_{i+1}}{}
\end{bmatrix}
= \begin{bmatrix}
\frac{d\bar{e}^b_i}{} \\
\frac{\sigma_u^b_i}{}
\end{bmatrix}
- J^{-1} \begin{bmatrix}
G1\left(d\bar{e}^b_i, \sigma_u^b_i\right) \\
G2\left(d\bar{e}^b_i, \sigma_u^b_i\right)
\end{bmatrix}
\end{equation}

(4.19)

The Jacobian matrix $J$ represents the derivatives of functions $G1$ and $G2$ reported by $d\bar{e}^b$ and $\sigma_u^b$ and $J^{-1}$ the inverse of the matrix $J$.

\begin{equation}
J = \begin{bmatrix}
\frac{\partial G1}{\partial d\bar{e}^b} & \frac{\partial G1}{\partial \sigma_u^b} \\
\frac{\partial G2}{\partial d\bar{e}^b} & \frac{\partial G2}{\partial \sigma_u^b}
\end{bmatrix}
\end{equation}

(4.20)

Applying the chain rules, the derivatives $\frac{\partial G1}{\partial d\bar{e}^b}$ and $\frac{\partial G1}{\partial \sigma_u^b}$ are obtained from the Yield Function and Hardening Rule derivation whereas, $\frac{\partial G2}{\partial d\bar{e}^b}$ and $\frac{\partial G2}{\partial \sigma_u^b}$ from the Flow Rule derivation by $d\bar{e}^b$ and $\sigma_u^b$.

Applying the chain rules, the Yield Function derivatives are obtained, i.e.

\begin{align}
\frac{\partial \bar{\sigma}^b}{\partial d\bar{e}^b} &= f_{YF} \left( \frac{\partial \sigma_x^b}{\partial d\bar{e}^b}, \frac{\partial \sigma_y^b}{\partial d\bar{e}^b}, \frac{\partial \sigma_{xy}^b}{\partial d\bar{e}^b} \right) \\
\frac{\partial \bar{\sigma}^b}{\partial \sigma_u^b} &= f_{YF} \left( \frac{\partial \sigma_x^b}{\partial \sigma_u^b}, \frac{\partial \sigma_y^b}{\partial \sigma_u^b}, \frac{\partial \sigma_{xy}^b}{\partial \sigma_u^b} \right)
\end{align}

(4.21) and (4.22)

where,
\[
\frac{\partial \sigma^b_{x1}}{\partial \sigma^b_{x1}} = \frac{\partial \sigma^b_{x1}}{\partial \sigma^b_{x1}} = \sin \psi^2 \\
\frac{\partial \sigma^b_{y1}}{\partial \sigma^b_{x1}} = \frac{\partial \sigma^b_{y1}}{\partial \sigma^b_{x1}} = \cos \psi^2 \\
\frac{\partial \sigma^b_{y1}}{\partial \sigma^b_{x1}} = \frac{\partial \sigma^b_{y1}}{\partial \sigma^b_{x1}} = -\cos \psi \sin \psi
\]

By applying the partial derivatives on the equation of referential framework rotation, the next quantities are obtained:

\[
\frac{\partial \sigma^b_{x1}}{\partial \sigma^b_{x1}} = \frac{\partial \sigma^b_{x1}}{\partial \sigma^b_{x1}} = \sin \psi^2 \\
\frac{\partial \sigma^b_{y1}}{\partial \sigma^b_{x1}} = \frac{\partial \sigma^b_{y1}}{\partial \sigma^b_{x1}} = \cos \psi^2 \\
\frac{\partial \sigma^b_{y1}}{\partial \sigma^b_{x1}} = \frac{\partial \sigma^b_{y1}}{\partial \sigma^b_{x1}} = -\cos \psi \sin \psi
\]
when the principal strain in the groove area was 10 times that in the nominal area. Cao et al. (2000), defined the deformation severity indices as:

\[
\begin{align*}
    f_{nn} &= \frac{d\varepsilon_{nn}^b}{d\varepsilon_{nn}^a} \\
    f_{nl} &= \frac{d\varepsilon_{nl}^b}{d\varepsilon_{nl}^a}
\end{align*}
\]  

(4.29)

and they proposed that necking occurs when either \( f_{nn} \) or \( f_{nl} \) is greater than 10.

Following Barata da Rocha et al. (1984) in the present work it is assumed that the plastic flow localization occurs when the equivalent strain increment in imperfection region \( (d\bar{\varepsilon}^b) \) is ten times equals or greater than in homogeneous zone \( (d\bar{\varepsilon}^a) \). When the necking criterion is reached the computation is stopped and the corresponding strains \( (\varepsilon_{nn}^a, \varepsilon_{nl}^a) \) accumulated at that moment in the homogeneous zone are the limit strains. The analysis is repeated for different values of \( \psi_0 \) (between 0 and 90 degrees) and the limit point on the FLD is obtained through the minimization of the curve \( \varepsilon_{nn}^a \) versus \( \psi_0 \). After the evaluation of several strain paths between uniaxial tension and biaxial stretching, the theoretical forming diagram will be plotted.

### i.3. Complex strain paths

Numerous experimental and theoretical works (Section 2.4.4) have shown that the maximum admissible limiting strains strongly depend on the deformation mode, loading history and plastic anisotropy induced by cold rolling. Consequently, the forming limit diagram for complex strain paths is very useful to understand the behaviour of the material in complex loading, to estimate the severity of the strain paths imposed on the workpiece and to optimise the shape of the dies to avoid the necking occurrence.

In consequence, following the method proposed by Barata da Rocha (1985) the FLDcode is extended to take into account the complex strain paths whose simulation involves a prestrain of the homogeneous zone followed by a drastic change in strain path:

\[
\begin{align*}
    \rho &= \rho_1 \quad \text{if} \quad \varepsilon_1 < \varepsilon^* \\
    \rho &= \rho_2 \quad \text{if} \quad \varepsilon_1 > \varepsilon^*
\end{align*}
\]  

(4.30)

(4.31)

where \( \rho_1 \) and \( \rho_2 \) characterized the two strain paths that are applied and \( \varepsilon^* \) is the prestrain value.

Barata da Rocha showed in a previous work (Barata da Rocha, 1985) that the band orientation, which minimizes the critical strains, depends on the level of prestrain and of the sequence of strain paths. The minimization of the critical strain is achieved as a function of the initial band orientation. In the present model, the initial band orientation at the beginning of the second strain path is equal to the final band orientation at the end of the first stage.

FLDcode is able to simulate any two stage strain paths but due to previous experimental and theoretical interest, expressed for certain complex strain paths which demonstrated a pronounced
and interesting effect on the FLD level, the following ones were particularly studied: uniaxial tension followed by biaxial stretching (UT-BS), biaxial stretching followed by uniaxial tension (BS-UT), uniaxial tension followed by plane strain stretching (UT-PS) and biaxial stretching followed by plane strain stretching (BS-PS). Moreover, a deformation history involving a prestrain in a certain strain path, i.e., uniaxial tension, plane strain or biaxial stretching following by several strain paths in all range between uniaxial tension and biaxial stretching was considered.

4. A more general Model for Forming Limit Diagrams Prediction

i.4 Some particular conditions for FLD at 90° from RD

In order to predict the FLD at 90° from rolling direction three particular conditions must be taken into account:

1. The stress direction becomes:

\[ \tilde{\sigma}^0 = \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix}, \]  

(4.32)

where \( \alpha \) represent the stress ratio.

2. The geometrical defect must be rotated with \( \frac{\pi}{2} \) radians thus meaning that the initial orientation of the groove is between \( \frac{\pi}{2} \) and \( \pi \) radians.

3. The band rotation is expressed by

\[ \tan(\psi + \psi') = \tan\psi'(1 + d\varepsilon_\gamma)/(1 + d\varepsilon^\alpha) \]  

(4.33)

ii) Application of the FLD code on different constitutive equations

In order to verify the efficiency of the new model, several constitutive equations were considered and successfully implemented in the code, such as, the isotropic Von Mises yield criterion, the quadratic Hill yield criterion (Hill'48), non-quadratic Hill yield criterion (Hill'79), Yld’96 Barlat yield function (Barlat et al. 1997), one of the most accurate anisotropic plastic potential, the Swift hardening power law and Voce saturation hardening law. In the following, a brief description of the implementation of each of these theoretical models considered to describe the material behaviour is presented.

ii.1 Hardening law description

Each material is completely defined macroscopically by its yield surface and its work-hardening law \( \tilde{\sigma}(\tilde{e}) \), which in the present work takes two forms:

1. Swift equation:

\[ \tilde{\sigma}(\tilde{e}) = k(\tilde{e}_0 + \tilde{e})^\alpha \tilde{e}^m \]  

(4.34)
where $\bar{\sigma}$ and $\bar{\varepsilon}$ are the effective stress and strain, $k, n, m, \varepsilon_0$ are material constants.

The derivatives of equivalent stress reported to equivalent strain increment and to the stress value on the longitudinal axes of the groove, required on the Newton Raphson computation, are:

$$
\frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}} = \bar{\sigma} \left( \frac{n}{\bar{\varepsilon}_0 + \bar{\varepsilon}} + \frac{m}{\bar{\varepsilon} \frac{d}{dt}} \right),
$$

(4.35)

with $dt$ the time increment and

$$
\frac{\partial \bar{\sigma}}{\partial \sigma_n} = 0
$$

(4.36)

2. Voce equation:

$$
\bar{\sigma}(\bar{\varepsilon}) = A - B \exp(-C \bar{\varepsilon})
$$

(4.37)

where $A, B, C$ are the material constants. Each constant is calculated by fitting experimental stress/strain data.

$$
\frac{\partial \bar{\sigma}}{\partial \bar{\varepsilon}} = BC \exp(- C \bar{\varepsilon})
$$

(4.38)

The flow stress being independent of the $\sigma_n$, its derivative $\frac{\partial \bar{\sigma}}{\partial \sigma_n}$ is equal to zero as in Swift case (Eq. 4.36).

ii.2 Yield function description

a) Von Mises yield function

For ideal case of isotropic materials, Von Mises equation is given by:

$$
\sigma_{yp}^2 = \sigma_{xx}^2 - \sigma_{xx} \sigma_{yy} + \sigma_{yy}^2 + 3\sigma_{xy}^2
$$

(4.39)

where $\sigma_{yp}$ represents the equivalent stress and $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ are the stress tensor components.

The relation between the stress ratio $\alpha$ and the strain path $\rho$ corresponding by Von Mises criterion is:

$$
\alpha = \frac{2\rho + 1}{\rho + 2}
$$

(4.40)

The derivatives of yield functions useful in Newton-Raphson method are:
4. A more general Model for Forming Limit Diagrams Prediction

\[
\frac{\partial \bar{\sigma}_{yf}}{\partial \bar{\sigma}_{ef}} = 2\sigma_{xx} \frac{\partial \sigma_{xx}}{\partial \bar{\sigma}_{ef}} + 2\sigma_{yy} \frac{\partial \sigma_{yy}}{\partial \bar{\sigma}_{ef}} - \left( \sigma_{yy} \frac{\partial \sigma_{xx}}{\partial \bar{\sigma}_{ef}} + \sigma_{xx} \frac{\partial \sigma_{yy}}{\partial \bar{\sigma}_{ef}} \right) + 6\sigma_{xy} \frac{\partial \sigma_{xy}}{\partial \bar{\sigma}_{ef}} \quad (4.41)
\]

\[
\frac{\partial \bar{\sigma}_{bf}}{\partial \sigma_{bf}} = 2\sigma_{xx} \frac{\partial \sigma_{xx}}{\partial \sigma_{bf}} + 2\sigma_{yy} \frac{\partial \sigma_{yy}}{\partial \sigma_{bf}} - \left( \sigma_{yy} \frac{\partial \sigma_{xx}}{\partial \sigma_{bf}} + \sigma_{xx} \frac{\partial \sigma_{yy}}{\partial \sigma_{bf}} \right) + 6\sigma_{xy} \frac{\partial \sigma_{xy}}{\partial \sigma_{bf}} \quad (4.42)
\]

Equations 4.25 and 4.26 and the derivatives of the flow rules allows the calculation of \(\frac{\partial \bar{\sigma}_{ef}}{\partial \bar{\sigma}_{ef}}\) and \(\frac{\partial \bar{\sigma}_{bf}}{\partial \sigma_{bf}}\).

\[
\frac{\partial \bar{\sigma}_{ef}}{\partial \bar{\sigma}_{ef}} = \frac{d\bar{\sigma}_{ef}}{d\bar{\sigma}_{ef}} + d\varepsilon_{xx} \left( -\frac{\partial \bar{\sigma}_{ef}}{\partial \bar{\sigma}_{ef}} \right) + \frac{d\bar{\varepsilon}_{ef}}{\bar{\sigma}} \left( \frac{\partial \sigma_{xx}}{\partial \bar{\sigma}_{ef}} - 0.5 \frac{\partial \sigma_{xy}}{\partial \bar{\sigma}_{ef}} \right) \quad (4.43)
\]

\[
\frac{\partial \bar{\sigma}_{ef}}{\partial \bar{\sigma}_{xy}} = \frac{d\bar{\sigma}_{ef}}{d\bar{\sigma}_{xy}} + d\varepsilon_{xy} \left( -\frac{\partial \bar{\sigma}_{ef}}{\partial \bar{\sigma}_{xy}} \right) + \frac{d\bar{\varepsilon}_{ef}}{\bar{\sigma}} \left( \frac{\partial \sigma_{yy}}{\partial \bar{\sigma}_{ef}} - 0.5 \frac{\partial \sigma_{xy}}{\partial \bar{\sigma}_{ef}} \right) \quad (4.44)
\]

\[
\frac{\partial \bar{\sigma}_{ef}}{\partial \bar{\sigma}_{xy}} = \frac{d\bar{\sigma}_{ef}}{d\bar{\sigma}_{xy}} - d\varepsilon_{xy} \left( -\frac{\partial \bar{\sigma}_{ef}}{\partial \bar{\sigma}_{xy}} \right) + 1.5 \frac{d\bar{\varepsilon}_{ef}}{\bar{\sigma}} \frac{\partial \sigma_{xy}}{\partial \bar{\sigma}_{ef}} \quad (4.45)
\]

\[
\frac{\partial \bar{\sigma}_{ef}}{\partial \sigma_{bf}} = \frac{d\bar{\sigma}_{ef}}{d\sigma_{bf}} \left( \frac{\partial \sigma_{bf}}{\partial \sigma_{bf}} - 0.5 \frac{\partial \sigma_{yy}}{\partial \sigma_{bf}} \right) \quad (4.46)
\]

\[
\frac{\partial \bar{\sigma}_{ef}}{\partial \sigma_{bf}} = \frac{d\bar{\sigma}_{ef}}{d\sigma_{bf}} \left( \frac{\partial \sigma_{bf}}{\partial \sigma_{bf}} - 0.5 \frac{\partial \sigma_{xx}}{\partial \sigma_{bf}} \right) \quad (4.47)
\]

\[
\frac{\partial \bar{\sigma}_{ef}}{\partial \sigma_{bf}} = 1.5 \frac{d\bar{\sigma}_{ef}}{\bar{\sigma}} \frac{\partial \sigma_{yy}}{\partial \sigma_{bf}} \quad (4.48)
\]

where \(\frac{\partial \bar{\sigma}}{\partial \bar{\sigma}_{ef}}\) results by Hardening Law derivation.

b) Quadratic (Hill 1948) Hill’s criterion

To take into account anisotropy the Hill'48 yield function (Hill, 1948) was employed, which is certainly the most popular equation describing the behaviour of orthotropic materials such as rolled sheets. Hill’48 yield function is a generalization of the Von Mises criterion:

\[
\bar{\sigma}_{yf}^2 = \sigma_{xx}^2 - 2H\sigma_{xx}\sigma_{yy} + (F + H)\sigma_{yy}^2 + 2P\sigma_{xy}^2 \quad (4.49)
\]

where F, H, P can be calculated using the anisotropy coefficients at 0, 45 and 90 degrees from rolling direction \(R_0\), \(R_{45}\), and \(R_{90}\).
4. A more general Model for Forming Limit Diagrams Prediction

\[ F = R_0 \left[ R_{90}(1 + R_0) \right] \]
\[ H = R_0 / (1 + R_0) \]
\[ P = (R_0 + R_{90})(2R_{45} + 1)/[2R_{90}(1 + R_0)] \]  \hspace{1cm} (4.50)

The flow rule of Hill'48 allows to write:

\[ d\varepsilon_{xx} = d\bar{\varepsilon}(\sigma_{xx} - H\sigma_{yy})/\bar{\sigma} \]
\[ d\varepsilon_{yy} = d\bar{\varepsilon}[F + H)\sigma_{yy} - H\sigma_{xx}] / \bar{\sigma} \]  \hspace{1cm} (4.51)
\[ d\varepsilon_{xx} = d\bar{\varepsilon}[(H - 1)\sigma_{xx} - F\sigma_{yy}] / \bar{\sigma} \]
\[ d\varepsilon_{yy} = Pd\bar{\varepsilon}\sigma_{xy} / \bar{\sigma} \]

Hill 1948 yield function presents the following relation between the stress ratio \( \alpha \) and the strain path \( \rho \):

\[ \alpha = \frac{\rho + H}{F + H(1 + \rho)} \]  \hspace{1cm} (4.52)

The derivative of yield stress corresponding to Hill'48 criterion results from the derivation of this yield function and from equations (4.23 and 4.24):

\[ \frac{\partial \bar{\sigma}_{xx}}{\partial \varepsilon_{xx}} = 2\sigma_{xx} \frac{\partial \sigma_{xx}}{\partial \varepsilon_{xx}} + 2(F + H)\sigma_{yy} \frac{\partial \sigma_{yy}}{\partial \varepsilon_{xx}} - 2H \left( \sigma_{yy} \frac{\partial \sigma_{xx}}{\partial \varepsilon_{xx}} + \sigma_{xx} \frac{\partial \sigma_{yy}}{\partial \varepsilon_{xx}} \right) + 4P \sigma_{xy} \frac{\partial \sigma_{xy}}{\partial \varepsilon_{xx}} \]  \hspace{1cm} (4.53)
\[ \frac{\partial \bar{\sigma}_{yy}}{\partial \sigma_{xx}} = 2\sigma_{xx} \frac{\partial \sigma_{xx}}{\partial \sigma_{xx}} + 2(F + H)\sigma_{yy} \frac{\partial \sigma_{yy}}{\partial \sigma_{xx}} - 2H \left( \sigma_{yy} \frac{\partial \sigma_{xx}}{\partial \sigma_{xx}} + \sigma_{xx} \frac{\partial \sigma_{yy}}{\partial \sigma_{xx}} \right) + 4P \sigma_{xy} \frac{\partial \sigma_{xy}}{\partial \sigma_{xx}} \]  \hspace{1cm} (4.54)

Equations 4.25 and 4.26 and the derivatives \( \frac{\partial \varepsilon_{xx}}{\partial \varepsilon_{xx}}, \frac{\partial \varepsilon_{yy}}{\partial \varepsilon_{xx}}, \frac{\partial \varepsilon_{xx}}{\partial \sigma_{xx}}, \frac{\partial \varepsilon_{yy}}{\partial \sigma_{xx}} \) resulted by derivation of the flow rule, allow to calculate \( \frac{\partial \varepsilon_{xx}}{\partial \sigma_{xx}} \) and \( \frac{\partial \varepsilon_{yy}}{\partial \sigma_{xx}} \):

\[ \frac{\partial \varepsilon_{xx}}{\partial \sigma_{xx}} = \frac{d\varepsilon_{xx}}{d\sigma_{xx}} + d\varepsilon_{xx} \left( \frac{- \partial \sigma_{xx}}{\partial \varepsilon_{xx}} \right) + \frac{d\varepsilon_{xx}}{\bar{\sigma}} \left( \frac{\partial \sigma_{xx}}{\partial \varepsilon_{xx}} - H \frac{\partial \sigma_{yy}}{\partial \varepsilon_{xx}} \right) \]  \hspace{1cm} (4.55)
\[ \frac{\partial \varepsilon_{yy}}{\partial \sigma_{xx}} = \frac{d\varepsilon_{yy}}{d\sigma_{xx}} + d\varepsilon_{xx} \left( \frac{- \partial \sigma_{xx}}{\partial \varepsilon_{xx}} \right) + \frac{d\varepsilon_{xx}}{\bar{\sigma}} \left( (F + H) \frac{\partial \sigma_{yy}}{\partial \varepsilon_{xx}} - H \frac{\partial \sigma_{xx}}{\partial \varepsilon_{xx}} \right) \]  \hspace{1cm} (4.56)
4. A more general Model for Forming Limit Diagrams Prediction

\[ \frac{\partial \varepsilon_{xy}^b}{\partial \varepsilon_x^b} = \frac{d\varepsilon_{xy}^b}{d\varepsilon_x^b} - \varepsilon_{xy}^b \left( \frac{\partial \sigma}{\partial \varepsilon_x^b} \right) + P \frac{d\varepsilon_{xy}^b}{\sigma} \frac{\partial \sigma_{xy}^b}{\partial \varepsilon_x^b} \] (4.57)

\[ \frac{\partial \varepsilon_{xx}^b}{\partial \sigma_x^b} = \frac{d\varepsilon_{xx}^b}{\sigma} \left( \frac{\partial \sigma_{xx}^b}{\partial \sigma_x^b} - H \frac{\partial \sigma_{xy}^b}{\partial \sigma_x^b} \right) \] (4.58)

\[ \frac{\partial \varepsilon_{yy}^b}{\partial \sigma_x^b} = \frac{d\varepsilon_{yy}^b}{\sigma} \left( (F + H) \frac{\partial \sigma_{xy}^b}{\partial \sigma_x^b} - H \frac{\partial \sigma_{xx}^b}{\partial \sigma_x^b} \right) \] (4.59)

\[ \frac{\partial \varepsilon_{xy}^b}{\partial \sigma_x^b} = P \frac{d\varepsilon_{xy}^b}{\sigma} \frac{\partial \sigma_{xy}^b}{\partial \sigma_x^b} \] (4.60)

where \( \frac{\partial \sigma}{\partial \varepsilon_x^b} \) results by Hardening Law derivation, (Eq. 4.35 or Eq. 4.38).

The Hill’ 48 criterion has been widely used for many years. As was specified in the previous chapter, a satisfactory agreement with experiments was obtained for steel but this description was not fully suitable for aluminium alloys being not able to reproduce the “anomalous behaviour” of aluminium alloys. Thus, Hill proposed his 1979 criterion in order to account for this behaviour.

c) Non-quadratic (Hill 1979) Hill’s criterion

In the present work, the Hill 1979 yield criterion for planar isotropy, case 4 in Hill’s paper (1979) was used. It supposes that principal stress and material symmetry axes are superimposed, and it presents the following equation:

\[ 2(1 + R)\bar{\sigma}^M = |\sigma_1 + \sigma_2|^M + (1 + 2R)|\sigma_1 - \sigma_2|^M \] (4.61)

where \( M \) is an exponent experimental identified (see Chapter 3) and \( R \) is an average anisotropic parameter determined from uniaxial tests at 0, 45 and 90 degrees to the rolling direction:

\[ R = \frac{R_0 + 2R_{45} + R_{90}}{4} \] (4.62)

According to the flow theory of plasticity, the associated Flow Rules are:

\[ d\varepsilon_1 = \frac{d\varepsilon}{2M (1 + R) \bar{\sigma}^{(M-1)}} \left( M |\sigma_1 + \sigma_2|^{(M-1)} + M(1 + 2R)|\sigma_1 - \sigma_2|^{(M-1)} \right) \] (4.63)

\[ d\varepsilon_2 = \frac{d\varepsilon}{2M (1 + R) \bar{\sigma}^{(M-1)}} \left( M |\sigma_1 + \sigma_2|^{(M-1)} - M(1 + 2R)|\sigma_1 - \sigma_2|^{(M-1)} \right) \]
For Hill 1979 yield function, the relation between the stress ratio \( \alpha \) and the strain path \( \rho \) is:

\[
\alpha = \frac{(1 + \rho)(1 + 2R)}{(1 - \rho)(1 + 2R)} - 1
\]

\[
\frac{1}{(1 - \rho)(1 + 2R)} + 1
\]

(4.64)

Derivative of yield stress corresponding to Hill 1979 yield criterion reported by \( d\bar{e}^b \) respectively \( \sigma^b_u \) are:

If \( \sigma_1 \) is not equal to \( \sigma_2 \) and \( f_{YF}(\sigma_1, \sigma_2) = \bar{\sigma}_{YF} \) then:

\[
T_1 = M \text{ sign}\left| \sigma_1 + \sigma_2 \right| \left| \sigma_1 + \sigma_2 \right|^{(M-1)}
\]

(4.65)

\[
T_2 = (1 + 2R) M \text{ sign}\left| \sigma_1 - \sigma_2 \right| \left| \sigma_1 - \sigma_2 \right|^{(M-1)}
\]

(4.66)

\[
\frac{\partial \bar{\sigma}_{YF}}{\partial \bar{e}} = \left[ T_1 \left( \frac{\partial \sigma_1}{\partial \bar{e}} + \frac{\partial \sigma_2}{\partial \bar{e}} \right) + T_2 \left( \frac{\partial \sigma_1}{\partial \bar{e}} - \frac{\partial \sigma_2}{\partial \bar{e}} \right) \right] \frac{1}{2(1 + R)}
\]

(4.67)

\[
\frac{\partial \bar{\sigma}_{YF}}{\partial \sigma_u^b} = \left[ T_1 \left( \frac{\partial \sigma_1}{\partial \sigma_u^b} + \frac{\partial \sigma_2}{\partial \sigma_u^b} \right) + T_2 \left( \frac{\partial \sigma_1}{\partial \sigma_u^b} - \frac{\partial \sigma_2}{\partial \sigma_u^b} \right) \right] \frac{1}{2(1 + R)}
\]

(4.68)

Else when \( \sigma_1 \) equals with \( \sigma_2 \) we have:

\[
\frac{\partial \bar{\sigma}_{YF}}{\partial \bar{e}} = T_1 \left( \frac{\partial \sigma_1}{\partial \bar{e}} + \frac{\partial \sigma_2}{\partial \bar{e}} \right) \frac{1}{2(1 + R)}
\]

(4.69)

\[
\frac{\partial \bar{\sigma}_{YF}}{\partial \sigma_u^b} = T_1 \left( \frac{\partial \sigma_1}{\partial \sigma_u^b} + \frac{\partial \sigma_2}{\partial \sigma_u^b} \right) \frac{1}{2(1 + R)}
\]

(4.70)

It is well known that the Hill’79 can model the Woodthorpe-Pearce “anomalous” behaviour of some materials but the main disadvantage is that it is expressed using only principal stress and the predicted yield surfaces are sometimes far from the experimental surfaces predicted by the Bishop-Hill theory.

d) Yld96 yield criterion

Barlat Yld96 yield function (Barlat et al., 1997) was shown to be consistent with the polycrystal-based yield surfaces and the advantage of this phenomenological model is that it takes into account all factors (grain structure, precipitates, heterogeneous microstructure) that contribute to anisotropy, by using mechanical test results. Based on experimental and polycrystal
results, this yield function appears able to describe the flow surface for aluminium alloy sheets and possible for other FCC and even for BCC materials. The Yld96 yield function has the form:

\[ \phi = \alpha_1 |s_2 - s_3|^\alpha + \alpha_2 |s_3 - s_1|^\alpha + \alpha_3 |s_1 - s_2|^\alpha = 2\sigma^a \]  

(4.71)

In equation (4.71) \( \alpha \) is a material parameter equal to 6 for BCC and 8 for FCC materials and \( s_1, s_2, s_3 \) are the principal values of the so-called isotropic plastic equivalent (IPE) transformed stress tensor \( s \), which is given by:

\[ s = L : \sigma \]  

(4.72)

where \( L \) is a fourth-order symmetric and deviatoric tensor that depends, in case of an orthotropic material, on three independent parameters \( c_1, c_2 \) and \( c_3 \), while \( \sigma \) is the Cauchy stress tensor.

\[
L = \begin{bmatrix}
\frac{(c_2 + c_3)}{3} & -\frac{c_3}{3} & -\frac{c_2}{3} & 0 & 0 & 0 \\
-\frac{c_3}{3} & \frac{(c_3 + c_1)}{3} & -\frac{c_1}{3} & 0 & 0 & 0 \\
-\frac{c_2}{3} & -\frac{c_1}{3} & \frac{(c_2 + c_1)}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & c_4 & 0 & 0 \\
0 & 0 & 0 & 0 & c_5 & 0 \\
0 & 0 & 0 & 0 & 0 & c_6
\end{bmatrix}
\]  

(4.73)

The coefficients \( \alpha_1, \alpha_2, \alpha_3 \) are computed using the transformation:

\[ \alpha_k = \alpha_k p_{1k}^2 + \alpha_p p_{2k}^2 + \alpha_s p_{3k}^2 \]  

(4.74)

\( p \) is the transformation matrix between the principal direction of \( s \) and the principal axes of anisotropy; \( \alpha_x, \alpha_y, \) and \( \alpha_z \) are quantities related to the anisotropy of the materials and depend on the angles \( \beta_1, \beta_2, \beta_3 \) between the principal directions of \( s \) (1,2,3) and the anisotropic axes, respectively given by:

\[ \alpha_x = \alpha_{x0} \cos^2 2\beta_1 + \alpha_{x1} \sin^2 2\beta_1 \]  

\[ \alpha_y = \alpha_{y0} \cos^2 2\beta_2 + \alpha_{y1} \sin^2 2\beta_2 \]  

\[ \alpha_z = \alpha_{z0} \cos^2 2\beta_3 + \alpha_{z1} \sin^2 2\beta_3 \]  

(4.75)

\[ \cos^2 \beta_1 = \begin{cases} 
1, & \text{if } |s_1| \geq |s_3| \\
3, & \text{if } |s_1| < |s_3|
\end{cases} \]  

\[ \cos^2 \beta_2 = \begin{cases} 
1, & \text{if } |s_1| \geq |s_3| \\
3, & \text{if } |s_1| < |s_3|
\end{cases} \]  

(4.76)

\[ \cos^2 \beta_3 = \begin{cases} 
1, & \text{if } |s_1| \geq |s_3| \\
3, & \text{if } |s_1| < |s_3|
\end{cases} \]
with \( s_1 \geq s_2 \geq s_3 \) \hspace{1cm} (4.77)

As the present work is considered under plane-stress condition, the isotropic plastic equivalent transformed stress tensor \( s \) becomes:

\[
\begin{bmatrix}
    s_{xx} & s_{xy} & 0 \\
    s_{xy} & s_{yy} & 0 \\
    0 & 0 & s_{zz}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
    c_2 (s_{xx} - s_{yy}) + c_1 s_{xy} & c_3 s_{xy} & 0 \\
    c_3 s_{xy} & c_1 s_{yy} - c_3 (s_{xx} - s_{yy}) & 0 \\
    0 & 0 & -s_{xx} - s_{yy}
\end{bmatrix} \hspace{1cm} (4.78)
\]

and

\[
s_{1,2} = \frac{s_{xx} + s_{yy}}{2} \pm \sqrt{\left(\frac{s_{xx} - s_{yy}}{2}\right)^2 + s_{xy}^2} \hspace{1cm} (4.79)
\]

while \( s_3 (= s_{zz}) = -s_1 - s_2 = -s_{xx} - s_{yy} \) due to the deviatoric nature of \( s \).

Furthermore, for the plane stress condition, \( p, \alpha_x, \alpha_y, \alpha_z, \beta_1, \beta_2 \), and \( \beta_3 \) reduce to:

\[
p = \begin{bmatrix}
    \cos \beta & -\sin \beta & 0 \\
    \sin \beta & \cos \beta & 0 \\
    0 & 0 & 1
\end{bmatrix} \hspace{1cm} (4.80)
\]

\[
\alpha_x = \alpha_{x0} = \alpha_{x1} \\
\alpha_y = \alpha_{y0} = \alpha_{y1} \\
\alpha_z = \alpha_{z0} \cos^2 2\beta + \alpha_{z1} \sin^2 2\beta \hspace{1cm} (4.81)
\]

Then,

\[
\alpha_1 = \alpha_x \cos^2 \beta + \alpha_y \sin^2 \beta \\
\alpha_2 = \alpha_x \sin^2 \beta + \alpha_y \cos^2 \beta \\
\alpha_3 = \alpha_{z0} \cos^2 2\beta + \alpha_{z1} \sin^2 2\beta \hspace{1cm} (4.82)
\]

where \( \alpha_{z0} = 1 \) and \( \theta = \arctg \left( \frac{s_1 - s_{xx}}{s_{xy}} \right) \hspace{1cm} (4.83) \)

Using the normality rule, the associated flow rule of Yld96 is:

\[
d\epsilon_y = d\lambda \frac{\partial \bar{\sigma}}{\partial \sigma_y} \hspace{1cm} (4.84)
\]

where \( d\lambda \) is a scalar function.
The computation of $\frac{\partial \bar{\sigma}}{\partial \sigma_y}$ is lengthy but straightforward as is summarized as follows.

First, the procedure used to compute the quantity $\bar{\sigma}(\sigma)$ is presented. By defining the stress components $\sigma_{xx}, \sigma_{yy}$ and $\sigma_{xy}$ as

$$[\bar{\sigma}] = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix},$$

the symmetric matrix $s$ is

$$[\bar{T}_k] = \begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{bmatrix} = \begin{bmatrix} \frac{c_3(\bar{\sigma}_1 - \bar{\sigma}_2) + c_2 \bar{\sigma}_1}{3} \\ \frac{c_1 \bar{\sigma}_2 - c_3(\bar{\sigma}_1 - \bar{\sigma}_2)}{3} \\ c_6 \bar{\sigma}_3 \end{bmatrix}. \quad (4.86)$$

The principal values of $s$ are expressed as:

$$[\bar{\tilde{\sigma}}] = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \frac{\bar{L}_1 + \bar{L}_2}{2} + \sqrt{\left(\frac{\bar{L}_1 - \bar{L}_2}{2}\right)^2 + L_3^2} \\ \frac{\bar{L}_1 + \bar{L}_2}{2} - \sqrt{\left(\frac{\bar{L}_1 - \bar{L}_2}{2}\right)^2 + L_3^2} \\ -(s_1 + s_2) \\ \tan^{-1}\left(\frac{s_1 - \bar{L}_1}{L_3}\right) \end{bmatrix}. \quad (4.87)$$

Therefore, it results $\bar{\sigma}(\sigma)$:

$$\bar{\sigma} = \left\{ \frac{1}{2} \left\langle \frac{1}{\phi} \right\rangle \right\} = \left\{ \frac{1}{2} \left( \alpha_1 |s_2 - s_3|^a + \alpha_2 |s_3 - s_1|^a + \alpha_3 |s_1 - s_2|^a \right) \right\}^{\frac{1}{2}} \quad (4.88)$$

where
\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix} =
\begin{bmatrix}
\alpha_x \cos^2 \beta + \alpha_y \sin^2 \beta \\
\alpha_x \sin^2 \beta + \alpha_y \cos^2 \beta \\
\alpha_{z0} \cos^2 2\beta + \alpha_{z1} \sin^2 2\beta
\end{bmatrix}
\] (4.89)

The quantity \( \frac{\partial \bar{\sigma}}{\partial \sigma} \) is obtained by applying the chain rules, i.e.

\[
\frac{\partial \bar{\sigma}}{\partial \bar{\sigma}_k} = (2m \bar{\sigma}^{(m-1)})^{-1} \frac{\partial \phi}{\partial \bar{\sigma}_k} = (2m \bar{\sigma}^{(m-1)})^{-1} \sum_p \sum_q \frac{\partial \bar{\eta}_p}{\partial \bar{\sigma}} \frac{\partial \bar{L}_q}{\partial \bar{\sigma}_k} \quad \text{(for k=1-3)}
\] (4.90)

In consequence, it is necessary to compute the following derivatives:

\[
\begin{bmatrix}
\frac{\partial \phi}{\partial \bar{\eta}_i} \\
\frac{\partial \phi}{\partial \bar{L}_j}
\end{bmatrix} =
\begin{bmatrix}
\alpha_x \left( s_1 - s_2 \right) s_1 - s_2 + \alpha_y \left( s_3 - s_1 \right) s_3 - s_1 \left( s_3 - s_1 \right)^{a-2} \\
\alpha_x \left( s_1 - s_2 \right) s_1 - s_2 + \alpha_y \left( s_2 - s_3 \right) s_2 - s_3 \left( s_2 - s_3 \right)^{a-2} \\
\alpha_x \left( s_2 - s_3 \right) s_2 - s_3 + \alpha_y \left( s_3 - s_1 \right) s_3 - s_1 \left( s_3 - s_1 \right)^{a-2}
\end{bmatrix} \left( \alpha_x - \alpha_y \right) \sin 2\theta \left( \left| s_2 - s_3 \right|^{a} + \left| s_3 - s_1 \right|^{a} \right) - 2 \left( \alpha_{z0} - \alpha_{z1} \right) \sin 4\theta \left| s_1 - s_2 \right|^{a}
\] (4.91)

\[
\begin{bmatrix}
\frac{\partial \bar{\eta}_i}{\partial \bar{L}_j} \\
\frac{\partial \bar{L}_i}{\partial \bar{L}_j}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{2} + \frac{\left( \bar{L}_1 - \bar{L}_2 \right)}{4r} & \frac{1}{2} \frac{\left( \bar{L}_1 - \bar{L}_2 \right)}{4r} & \bar{L}_2 \\
\frac{1}{2} \frac{\left( \bar{L}_1 - \bar{L}_2 \right)}{4r} & \frac{1}{2} \frac{\left( \bar{L}_1 - \bar{L}_2 \right)}{4r} & \bar{L}_3 \\
-1 & -1 & 0
\end{bmatrix}
\] (4.92)

where \( r = \sqrt{\left( \frac{\bar{L}_1 - \bar{L}_2}{2} \right)^2 + \bar{L}_3^2} \).

Note that \( \frac{\partial \eta_i}{\partial \bar{L}_j} = \frac{\partial \beta}{\partial \bar{L}_j} \) is obtained from the relation of

\[
\tan \beta = \frac{\eta_i \left( \bar{L}_1 - \bar{L}_2 - \bar{L}_3 \right) - \bar{L}_i}{\bar{L}_3}
\] (4.94)

which gives

\[
\frac{\partial \beta}{\partial \bar{L}_j} = \frac{1}{1 + \tan^2 \beta} \left( \frac{\partial \tan \beta}{\partial \bar{\eta}_i} \frac{\partial \bar{\eta}_i}{\partial \bar{L}_j} + \frac{\partial \tan \beta}{\partial \bar{L}_j} \right) \quad \text{(for j=1-3)}
\] (4.95)

70
and 
\[
\left[ \frac{\partial \tilde{L}_j}{\partial \tilde{\sigma}_i} \right] = \begin{bmatrix} c_1 + c_2 & -c_3 & 0 \\ \frac{3}{3} & \frac{3}{c_1 + c_3} & 0 \\ 0 & 0 & c_6 \end{bmatrix} \tag{4.96}
\]

Finally, \( \frac{\partial^2 \tilde{\sigma}}{\partial \tilde{\sigma}_i \partial \tilde{\sigma}_j} \) is obtained using the following chain rule:

\[
\frac{\partial^2 \tilde{\sigma}}{\partial \tilde{\sigma}_i \partial \tilde{\sigma}_j} = \frac{\sigma^{(i,j)}}{2a} \frac{\partial^2 \phi}{\partial \tilde{\sigma}_i \partial \tilde{\sigma}_j} - a - 1 \frac{\partial \tilde{\sigma}}{\partial \tilde{\sigma}_i} \frac{\partial \tilde{\sigma}}{\partial \tilde{\sigma}_j} \quad \text{(for } i, j = 1-3) \tag{4.97}
\]

Now,

\[
\frac{\partial^2 \phi}{\partial \tilde{\sigma}_i \partial \tilde{\sigma}_j} = \frac{4}{a} \sum_{r,s} \sum_{b} \sum_{d} \frac{\partial^2 \phi}{\partial \tilde{n}_r \partial \tilde{n}_b} \left( \frac{\partial \tilde{L}_r}{\partial \tilde{\sigma}_i} \frac{\partial \tilde{L}_b}{\partial \tilde{\sigma}_j} \right) + \frac{4}{a} \sum_{b} \sum_{d} \frac{\partial \phi}{\partial \tilde{n}_b} \left( \frac{\partial \tilde{L}_b}{\partial \tilde{\sigma}_i} \right) \left( \frac{\partial \tilde{L}_d}{\partial \tilde{\sigma}_j} \right) + \frac{4}{a} \sum_{b} \sum_{d} \frac{\partial \phi}{\partial \tilde{n}_b} \left( \frac{\partial^2 \tilde{L}_b}{\partial \tilde{\sigma}_i \partial \tilde{\sigma}_j} \right) \tag{4.98}
\]

Note that \( \left( \frac{\partial^2 \tilde{L}_i}{\partial \tilde{\sigma}_j \partial \tilde{\sigma}_k} \right) \) vanishes.

\[
\left[ \frac{\partial^2 \phi}{\partial \tilde{n}_i \partial \tilde{n}_j} \right] = \frac{a}{a-1} \begin{bmatrix} \alpha_1 |s_1 - s_2|^{a-2} + \alpha_2 |s_3 - s_1|^{a-2} & -\alpha_3 |s_1 - s_2|^{a-2} \\ -\alpha_3 |s_1 - s_2|^{a-2} & \alpha_1 |s_2 - s_3|^{a-2} + \alpha_4 |s_1 - s_2|^{a-2} \\ -\alpha_2 |s_3 - s_1|^{a-2} & \alpha_1 |s_2 - s_3|^{a-2} \\ \frac{g_1}{a/(a-1)} & \frac{g_2}{a/(a-1)} \\ -\frac{g_1}{a/(a-1)} & 0 \\ -\frac{g_2}{a/(a-1)} & 0 \\ \alpha_1 |s_3 - s_1|^{a-2} + \alpha_2 |s_3 - s_1|^{a-2} & 0 \\ \frac{g_3}{a/(a-1)} & \frac{g_4}{a/(a-1)} \end{bmatrix} \tag{4.99}
\]

where

\[
g_1 = (\alpha_x - \alpha_y) \sin 2\theta \begin{bmatrix} -a(s_3 - s_1)|s_3 - s_1|^{a-2} \begin{bmatrix} -2a(\alpha_{x0} - \alpha_{x1})(s_1 - s_2)|s_1 - s_2|^{a-2} \sin 4\theta 
\end{bmatrix} 
\end{bmatrix} 
\]

\[
g_2 = (\alpha_x - \alpha_y) \sin 2\theta \begin{bmatrix} -a(s_2 - s_3)|s_2 - s_3|^{a-2} \begin{bmatrix} +2(\alpha_{x0} - \alpha_{x1})(s_1 - s_2)|s_1 - s_2|^{a-2} \sin 4\theta 
\end{bmatrix} 
\end{bmatrix} \tag{4.100}
\]

\[
g_3 = \begin{bmatrix} (\alpha_x - \alpha_y) \sin 2\theta \begin{bmatrix} a(s_2 - s_3)|s_2 - s_3|^{a-2} \begin{bmatrix} +a(s_3 - s_1)|s_3 - s_1|^{a-2} \begin{bmatrix} -\alpha_2 |s_3 - s_1|^{a-2} 
\end{bmatrix} 
\end{bmatrix} 
\end{bmatrix} 
\end{bmatrix} 
\]

\[
g_4 = 2(\alpha_x - \alpha_y) \cos 2\theta \begin{bmatrix} -|s_2 - s_3| |s_2 - s_3|^{a-2} + |s_3 - s_1|^{a-2} \begin{bmatrix} -8(\alpha_{x0} - \alpha_{x1}) \cos 4\theta |s_1 - s_2|^{a-2} 
\end{bmatrix} 
\end{bmatrix} 
\]
Also,

\[
\frac{\partial^2 \bar{\eta}_1}{\partial L_i \partial L_j} = \begin{bmatrix}
\frac{1}{4r} \frac{(\bar{L}_1 - \bar{L}_2)^2}{16r^3} & \frac{1}{4r} \frac{(\bar{L}_1 - \bar{L}_2)^2}{16r^3} & -\frac{(\bar{L}_1 - \bar{L}_2)\bar{L}_3}{4r^3} \\
-\frac{1}{4r} \frac{(\bar{L}_1 - \bar{L}_2)^2}{16r^3} & \frac{1}{4r} \frac{(\bar{L}_1 - \bar{L}_2)^2}{16r^3} & -\frac{(\bar{L}_1 - \bar{L}_2)\bar{L}_3}{4r^3} \\
-\frac{1}{4r} \frac{(\bar{L}_1 - \bar{L}_2)\bar{L}_3}{4r^3} & -\frac{1}{4r} \frac{(\bar{L}_1 - \bar{L}_2)\bar{L}_3}{4r^3} & \frac{1}{r} \frac{\bar{L}_3^2}{r^3}
\end{bmatrix}
\]  

(4.101)

where \( r = \sqrt{\left(\frac{\bar{L}_1 - \bar{L}_2}{2}\right)^2 + \bar{L}_3^2} \) (Eq. 4.93).

\[
\frac{\partial^2 \bar{\eta}_i}{\partial L_i \partial L_j} = 0
\]  

(4.102)

and

\[
\frac{\partial^2 \bar{\eta}_s}{\partial L_i \partial L_j} = 0
\]  

(4.103)

while

\[
\frac{\partial^2 \bar{\eta}_h}{\partial L_i \partial L_j} = -2\tan \beta \frac{\partial \beta}{\partial L_i} \frac{\partial \beta}{\partial L_j} + \frac{1}{(1 + \tan^2 \beta)} \frac{d^2 (\tan \beta)}{dL_i dL_j} \quad \text{(for } i, j = 1-3) \]  

(4.104)

with

\[
\frac{d^2 (\tan \beta)}{dL_i dL_j} = \begin{bmatrix}
\frac{\partial^2 \bar{\eta}_1}{L_3 \partial^2 \bar{L}_1} & \frac{\partial^2 \bar{\eta}_1}{L_3 \partial \bar{L}_1 \partial \bar{L}_2} & -\frac{\partial \bar{\eta}_1}{L_3 \partial \bar{L}_1} + \frac{\partial^2 \bar{\eta}_1}{L_3 \partial \bar{L}_2 \partial \bar{L}_3} + \frac{1}{L_3} \\
\frac{\partial^2 \bar{\eta}_1}{L_3 \partial \bar{L}_1 \partial \bar{L}_2} & \frac{\partial^2 \bar{\eta}_1}{L_3 \partial^2 \bar{L}_2} & -\frac{\partial \bar{\eta}_1}{L_3 \partial \bar{L}_2} + \frac{\partial^2 \bar{\eta}_1}{L_3 \partial \bar{L}_2 \partial \bar{L}_3} \\
-\frac{\partial \bar{\eta}_1}{L_3 \partial \bar{L}_1} + \frac{\partial^2 \bar{\eta}_1}{L_3 \partial \bar{L}_1 \partial \bar{L}_3} + \frac{1}{L_3} & -\frac{\partial \bar{\eta}_1}{L_3 \partial \bar{L}_2} + \frac{\partial^2 \bar{\eta}_1}{L_3 \partial \bar{L}_2 \partial \bar{L}_3} & -\frac{2\partial \bar{\eta}_1}{L_3 \partial \bar{L}_3} + \frac{\partial^2 \bar{\eta}_1}{L_3 \partial \bar{L}_3 \partial \bar{L}_3} + \frac{2(\bar{\eta}_1 - \bar{L}_3)}{L_3^2}
\end{bmatrix}
\]  

(4.105)

The yield function coefficients can be calculated from four mechanical tests such as uniaxial tension at 0°, 45° and 90° from rolling direction (RD) and a biaxial stretching test, which
provides seven test results: the anisotropy factors and uniaxial yield stress at 0°, 45° and 90° from RD and balanced biaxial yield stress. The computation of the Yld96 coefficients \((c_1, c_2, c_3, c_6, \alpha_x, \alpha_y, \alpha_{z1})\) was realized by a de-coupling scheme where the four variables \((c_1, c_2, c_3, c_6)\) and the three variables \((\alpha_x, \alpha_y, \alpha_{z1})\) are treated separately and the Newton Raphson numerical method was used to solve the respectively systems. A detailed description of the computation of Yld96 coefficients can be found in (Barlat et al., 1997) and (Yoon et al., 2000).

4.3.2 Physical approach of Theory of Plasticity

Sheet metal forming processes often impose very intense forming sequences, leading to severe strain-path changes and accumulated strains up to several hundreds per cent. The main advantage of the phenomenological models is the computational efficiency, whereas the disadvantages have been experienced under such multi-axial and non-proportional loading conditions or on the difficult identification of the parameters in the case of the advanced ones. In contrast, the physical approach generally improves the accuracy and identification facilities but an expensive computation effort is often needed.

During the plastic deformation, various and complex physical phenomena are taking place simultaneously, being impossible their inclusion into a unique efficient physical model. Thus, in order to develop an accurate and computational practical model, from the complicated evolution of the material structures, the most important factors must be extracted into a physical model with a certain degree of simplification.

Among various physical models, the most advanced and accurate one seems to be the combined model of texture and strain-path-induced anisotropy based on the Van Houtte's anisotropic texture model (Van Houtte et al., 1995) and Teodosiu and Hu microstructural hardening model (1995). It takes into account the initial anisotropy and the induced anisotropy, which are the most important macroscopic properties for non-proportional loading at large strains. Moreover, this model allows a rapid computation since it is constructed in the framework of continuum mechanics.

Hoferlin (2001), after a interesting and detailed personal interpretation of the model, pointed out the three crucial advantages by opting for a texture-based yield locus: Firstly, the fast and systematic identification of the parameters defining the texture-based yield locus compared by the identification of the advanced phenomenological yield loci in stress space which require complex mechanical tests, repetition of tests for statistical accuracy and time consuming manufacturing of ad hoc test samples. Furthermore, the texture-based approach allows to account for the through-thickness heterogeneity of the sheet properties and the subsequent mechanical properties such as side-bulging during tensile test (Van Bael, Hoferlin et al., 1998). Secondly, it provides a direct link between thermo-mechanical processing of the sheet and its mechanical behaviour. Thirdly, the distortional hardening of the yield locus can easily be accounted for sheet forming simulation.

He also emphasized the three essential advantages of the use of the advanced microstructure-based hardening model, such as: it reproduces a large variety of strain-path change effects with a reasonable amount of material parameters (Teodosiu, 1995); it accurately predicts stresses as the hardening rate upon strain path change (Teodosiu, 1992), which is capital for the prediction of
strain localisation; it precisely reproduces the mechanical behaviour under cyclic deformation. Furthermore, the formulation of the model remains phenomenological, making its implementation rather classical.

Therefore, the new general code for FLD prediction was extended to the Physical approach of Theory of Plasticity, by applying this combined model of texture and strain path anisotropy. Its review and theoretical treatments of localized necking required on the implementation in the FLDcode is presented in the next sections.

i) The Combined Plasticity Model of Texture and Strain-Path induced Anisotropy description.

In the physically based plasticity model based on texture and microstructure, called TexMic (Hiwatashi et al., 1997), the shape of the yield locus is generated with a potential in strain rate space, developed by Van Houtte et al. (1995), derived from experimental texture measurements. The position and the size of the yield locus are computed by a mixed hardening model based on microstructural observations and on the theory of dislocations developed by Teodosiu and Hu (1995).

i.1 Macroscopic plastic potential derived from polycrystalline plasticity

In the mixed hardening model, the rate of plastic power per unit volume \( \dot{W} \) presents a component \( \dot{W}_h \) affected by kinematic hardening (backstress) and a component \( \dot{W}_i \) which is a dissipation associated with the non-directional resistance to glide and is affected by isotropic hardening:

\[
\dot{W} = \dot{W}_h + \dot{W}_i \tag{4.106}
\]

If \( X \) represents the backstress and \( D^p \) defines the plastic strain rate, the \( \dot{W}_h \) term can be obtained as:

\[
\dot{W}_h = X : D^p \tag{4.107}
\]

The frictional dissipation in a grain with a crystallographic orientation \( g \) can be written as

\[
\dot{\omega}_f = \tau \sum_s |\dot{\gamma}_s| \tag{4.108}
\]

where \( \tau \) is a frictional stress identical assumed in every slip system and \( \dot{\gamma}_s \) is the slip rate in each slip system \( s \). The active slip systems and the slip rates are calculated in such a way that they achieve the imposed plastic strain rate \( D^p \) while \( \dot{\omega}_f \) is minimal. The Taylor factor is defined as:

\[
M = \frac{\dot{\omega}_f}{\tau D^p_{\text{eq}}} \tag{4.109}
\]
where $\dot{\omega}_f^\ast$ is the minimum of $\dot{\omega}_f$, and $D_{\text{vt}}$ is the von Mises equivalent plastic strain rate defined as:

$$D_{\text{vt}} = \sqrt[2]{\frac{3}{2}} D^p,$$  \hspace{1cm} (4.110)

with $D^p$ the norm of $D^p$,

$$D^p = \|D^p\| = \sqrt{D^p : D^p}$$  \hspace{1cm} (4.111)

The plastic strain rate mode $A$ is defined as

$$A = \frac{D^p}{\|D^p\|}$$  \hspace{1cm} (4.112)

The macroscopic dissipation $\dot{W}_i$ is obtained by averaging over the polycrystal the minimum plastic power $\dot{w}_f^\ast$ dissipated in each grain of crystallographic orientation $g$, by using the orientation distribution function (ODF) $f(g)$ as weighting factor,

$$\dot{W}_i = \int f(g) \dot{w}_f^\ast d g$$  \hspace{1cm} (4.113)

Being only $M$ a function of $g$ results:

$$\dot{W}_i = \tau D_{\text{vt}} \bar{M}$$  \hspace{1cm} (4.114)

where $\bar{M}(A)$ is the average Taylor factor and can be calculated as a function of the direction of the plastic strain rate $A$, when the ODF has been determined by means of an X-ray measurement.

$$\bar{M}(A) = \int f(g) \bar{M}(g, A) dg.$$  \hspace{1cm} (4.115)

In order to define $\dot{W}_i$ analytically a sixth-order series expansion, $G(A)$ is used, where $A$ is a given plastic strain rate mode. This function $G(A)$ satisfies the following relationship:

$$G(A) = \sqrt[2]{\frac{3}{2}} \bar{M}(A), \forall A$$  \hspace{1cm} (4.116)

$G(A)$ is then fitted to a set of discrete values of $\bar{M}(A)$ that have been obtained by means of the Taylor theory (Van Houtte et al., 1995). Therefore $\dot{W}_i$ is obtained as

$$\dot{W}_i = \tau D^p G(A)$$  \hspace{1cm} (4.117)
The critical resolved shear stress \( \tau \) and \( X \) are assumed to be independent of \( D^p \) and the deviatoric stress \( \sigma' \) is given by

\[
\sigma' = X + \tau U
\]  
(4.118)

where the normalized deviatoric yield stress \( U \) is a second order tensor with Cartesian components

\[
U_{ij} = \frac{\partial (D^p G)}{\partial D^p_j}
\]  
(4.119)

Deriving analytically the normalized dissipative yield stress corresponding to any imposed plastic strain-rate using the vector representation of a deviatoric tensor, (Van Bael, 1994; Teodosiu, 1999) it results:

\[
U_k = GA_k + \frac{\partial G}{\partial A_L} (\delta_{KL} - A_k A_L)
\]  
(4.120)

\( X \) represents the centre of the yield locus in deviatoric stress space, \( \tau \) gives its size and \( U \) as a function of \( A \) corresponds to its shape. These macroscopic variables should change as the texture and microstructure evolve during the deformation.

In this model, it is assumed that the texture is constant with respect to a frame that rotates with the total spin \( \mathbf{w} \). Consequently, the plastic strain potential and all constitutive and evolution equations are supposed form-invariant with respect to this Jaumann frame. In the following, the components of all tensor-valued state variables in the Jaumann frame will be labelled by a superposed hat.

i.2 Microstructural hardening model

One of the most important features of the microstructural organization at the large strains is the formation and evolution of persistent dislocation structures. Whenever a sufficient amount of monotonic deformation is allowed along the same deformation path, persistent dislocation structures gradually form, which are more or less parallel to the main slip plane. Dislocation sheets introduce a directional hardening of the material, and hence a plastic anisotropy. In addition to this similar effect but not identical to the texture anisotropy, dislocation sheets display a certain polarity, which arises from the fact that on each side of the sheet there is an excess of dislocations of the same sign and this sign is different on the opposite sides of the dislocation sheet (Kocks et al., 1980)

The macroscopic transients accompanying the complex strain-path changes that are involved on the metal forming processes, are in general the phenomenological counterpart of the modification or dissolution of preformed structures and the formation of new ones that correspond to the last deformation mode.

Two-stage strain path tests (see overviews, e.g., by Aernoudt et al. 1987 and Teodosiu 1992) have been widely used to investigate the influence of strain-path changes on the work-hardening
transients. To characterize a two-stage strain-path, a scalar parameter $\beta$, defined as the double-contracted tensor product between the directions of the strain rate tensors during the first and second deformation stages, was proposed by Schmitt et al. (1985). From its definition it is obvious that $\beta$ varies from -1 (Bauschinger test) to 1 (monotonic test). When $\beta = 0$, the deformation sequence is called orthogonal. According to Schmitt et al. (1985), the parameter $\beta$ is suitable to characterize the yield loci of severally prestrained mild steel. The ratio between the flow stress before unloading and the yield stress after reloading equals one at $\beta = 1$, is smaller than one at $\beta = -1$, and reaches a maximum value at $\beta = 0$. Bacroix, Genevois and Teodosiu (1994) confirm that $\beta$ is an adequate parameter to characterize strain path changes showing a slightly difference but similar tendency on the experimental data obtained from different strain path changes with the same $\beta$ value.

The simplest strain-path change is the stress reversal. According to microscopic observation, the first stage of reversed deformation representing a microplastic regime of rapid work hardening, corresponds to the coarsening of preformed dislocation cell structures, the extended plateau in the stress strain curve, signifying the work-hardening stagnation, to the partial disintegration of the preformed structures, and the subsequent resumption of work-hardening to the formation of new cell structures (Teodosiu, 1995).

The orthogonal sequence is particularly interesting, since it involves new slip systems that were latent during the predeformation. During a subsequent orthogonal deformation, the flow stress is always higher than during the monotonic deformation, at the same accumulated strain. For a small prestrain, the work-hardening rate is always positive, whereas for a large prestrain, a rapid hardening due to microyielding is followed by a work softening and then by a resumption of work-hardening. Microscopic analysis shows that the strain rate is mainly carried by parallel microbands along the newly active slip planes, and that a microstructural softening may occur when the microbands succeed to shear the preformed dislocation sheets (Rauch and Schmitt, 1989).

Based on this microstructural evidence and on the fact that the plastic behaviour of metals depends not only on the current state of deformation, but also on the deformation history, Teodosiu and Hu describe the hardening of the material by four internal state variables, denoted by $(R, X, S, P)$. $R$, $S$ and $X$ have the dimension of stress, $P$ has no dimension. $R$ is a scalar, $S$ is a fourth-order tensor and $P$ and $X$ are second-order tensors. For a well-annealed material, all their initial values are zero.

The scalar variable $R$ describes the isotropic hardening produced beyond the yield limit by the statistically accumulated dislocations.

The tensor variable $X$, called the backstress, is intended to describe the rapid changes in stress under strain path changes. Additionally, it will be used to describe effects that are typical of the large-amplitude cyclic deformation, including cyclic hardening or softening.

The tensor variable $S$ describes the directional strength of planar persistent dislocation structures, being the most important descriptor of the microstructural evolution in the present model. The choice of its order is due to the necessity to describe the anisotropic contribution of persistent dislocation structures to the flow stress.
The tensor variable $P$ is associated with the polarity of persistent dislocation structures, which is due to the excess of dislocations of the same sign on each side of the dislocation sheet. When a microstructure is not at all polarized, $P = 0$. For a microstructure completely polarized after a monotonic deformation, $P = A$, where $A$ is the direction of the plastic strain rate tensor during the predeformation. Therefore, without loss of generality it is possible to assume that $0 \leq |P| \leq 1$.

The evolution laws of the internal variables are written in a work-hardening-recovery format, which takes into account the mechanisms of formation and dissolution of dislocation structures.

The evolution of $R$ is given by the equation:

$$\dot{R} = C_R (R_{sat} - R)^{\gamma}$$  \hspace{1cm} (4.121)

where $C_R$ characterizes the rate of the isotropic hardening produced by the statistically accumulated dislocations and $R_{sat}$ is the saturation value of $R$. The average polycrystal slip rate $\bar{\dot{r}}$ is an average of the total slip rate over the polycrystal. It is defined by:

$$\bar{\dot{r}} = D^\rho G (A)$$  \hspace{1cm} (4.122)

while its integral over time step is approximated as follows:

$$\Delta \bar{r} = \int_{t_f}^{t} \bar{\dot{r}} (\hat{D}^\rho) d\tau = \bar{\dot{r}} (\hat{D}^\rho) (t_f - t_i) = \bar{\dot{r}} (\hat{D}^\rho_f) \Delta t = D^\rho G_f (A) \Delta t$$  \hspace{1cm} (4.123)

If $R_i$ denotes the initial value of $R$, its updated value at the end of the deformation increment is:

$$R_f = R_{sat} + (R_i - R_{sat}) e^{-C_R \Delta \bar{r}}$$  \hspace{1cm} (4.124)

The evolution of the polarity tensor $P$ is described by the equation

$$\dot{P} = C_p (\hat{A} - \hat{P}) \bar{\dot{r}}$$  \hspace{1cm} (4.125)

whose update is:

$$\dot{P}^{(f)}_k = \hat{A}^{(f)}_k (1 - e^{-C_p \Delta \bar{r}}) + e^{-C_p \Delta \bar{r}} \dot{P}^{(i)}_k$$  \hspace{1cm} (4.126)

where $C_p$ characterizes the polarization rate of the persistent dislocation structure and the superscript $i$ and $f$ denoting values at the start respectively at the end of the deformation increment.

The tensor variable $X$, called the backstress, is intended to describe the rapid changes in stress under strain path changes. The evolution of $X$ is governed by the equation

$$\dot{X} = C_X (\hat{X}_{sat} - \hat{X}) \bar{\dot{r}}$$  \hspace{1cm} (4.127)

and its update:
\[
\dot{X}_k = \overline{X}_{\text{sat}} \dot{U}_k (1 - e^{-C_x \Delta t}) + \dot{X}_k e^{-C_x \Delta t}
\]

(4.128)

where \( C_x \) characterizes the saturation rate of \( X \) and \( \overline{X}_{\text{sat}} \) is a material parameter characterizing the saturation value of \( |X| \). The dependence of \( X \) on the persistent dislocation structure is included in the scalar function \( \overline{X}_{\text{sat}} (S, A) \). It is assumed that

\[
\overline{X}_{\text{sat}} = X_0 + (1 - m) \sqrt{S_D^2 + q |\dot{S}_L|^2}
\]

(4.129)

where \( X_0 \) is the initial value of \( \overline{X}_{\text{sat}} \), \( q \) and \( m \) are material parameters.

**Hypothesis:** The variables \( P \) and \( S \) are supposed constant over the increment and equal to their values \( P_i \) and \( S_i \) at the beginning of the increment. On the other hand, \( \dot{A} \) and the other strain rate potential variables are supposed constant, but equal to their values at the end of the increment. Whenever the internal variables are projected on \( \dot{A} \), the value of \( \dot{A}' \) is used too. Finally, the direction of \( \dot{S}_L \) is supposed constant during the strain increment, so that only its norm has to be updated.

In order to describe such evolution processes relating that dislocation structures associated with the current direction of the strain rate evolve quite differently from the rest of the persistent dislocation structures, the fourth tensor \( S \) is decomposed in two parts, namely \( S_D \) which represents the yield strength associated with dislocations of to the currently active slip systems and \( S_L \) which is associated with the latent part of the persistent dislocation structures.

\[
S = S_D A \otimes A + S_L
\]

(4.130)

with \( S_D = A : S : A \).

The evolution of \( S_D \) is governed by the equation

\[
\dot{S}_D = C_{SD} \left[ h_p (S_{\text{sat}} - S_D) - h_X S_D \right] \dot{P}
\]

(4.131)

where \( C_{SD} \) and \( S_{\text{sat}} \) are material parameters.

The polarity tensor \( P \) appears in the expressions of \( S_D \) and \( \overline{X}_{\text{sat}} \) through the functions \( h_X \) and \( h_p \), which are defined by the relations:

\[
h_X = \frac{1}{2} \left( 1 - \frac{\dot{X}}{\overline{X}_{\text{sat}} G} \right)
\]

(4.132)

\[
h_p = \begin{cases} 
\frac{1 - C_p}{C_{SD} + C_p} \frac{S_D}{S_{\text{sat}}} - \dot{P} : \dot{A} \neq 0, & \text{if } \dot{P} : \dot{A} \geq 0 \\
\left( 1 + \dot{P} : \dot{A} \right)^p \left( 1 - \frac{C_p}{C_{SD} + C_p} \frac{S_D}{S_{\text{sat}}} \right), & \text{otherwise.}
\end{cases}
\]

(4.133)
The evolution of $\dot{\mathbf{S}}_L$, results from the interaction between microbands and the preformed microstructures. The rate equation for the tensor $\dot{\mathbf{S}}_L$ is slightly different, as the strength of the latent dislocation structures decreases with progressing deformation:

$$
\dot{\mathbf{S}}_L = -C_{SL} \left( \frac{\dot{\mathbf{S}}_L}{S_{sat}} \right)^n \dot{\mathbf{r}}
$$

(4.134)

The corresponding update equation of $\dot{\mathbf{S}}_D$ and $\dot{\mathbf{S}}_L$ is described as:

$$
\dot{S}_D' = C_2 + (S_D - C_2) e^{-C_3 \Delta\Gamma}
$$

(4.135)

and

$$
\dot{S}_L' = \left( (S_L')^{n_k} + C_1 \Delta\Gamma \right)^{1/n_k}
$$

(4.136)

with

$$
C_1 = \frac{n_k C_{SL}}{(S_{sat})^n}
$$

(4.137)

$$
C_2 = \frac{h_p S_{sat}}{h_p + h_y}
$$

(4.138)

and

$$
C_3 = C_{SD} (h_p + h_y)
$$

(4.139)

The size of the yield locus is proposed as:

$$
\tau = \tau_0 + R + mS
$$

(4.140)

where $\tau_0$ is the initial value of $\tau$, $m$ is a material parameter and $S$ is the norm of $\mathbf{S}$, i.e.

$$
S = |\mathbf{S}| = \sqrt{S_L^2 + S_D^2}
$$

(4.141)

Finally, $\dot{\mathbf{S}}'$ is updated with:

$$
\dot{\mathbf{S}}'_{kj} = \frac{S_{L}'}{S_{L}'} \dot{\mathbf{S}}'_{kj} + \left( S_D' - S_D' \frac{S_{L}'}{S_{L}'} \right) \hat{A}_k \hat{A}_j
$$

(4.142)

The model involves 13 material parameters:

$\tau_0$, $X_0$, $S_{sat}$, $R_{sat}$, $C_P$, $C_{SL}$, $C_{SD}$, $C_X$, $C_R$, $n_P$, $n_L$, $m$ and $q$. 

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80
which are identified by means of mechanical tests for three strain paths (monotonic test, stress reversal and orthogonal sequence) and by first assuming planar isotropy, thus retaining only the plastic anisotropy induced by the microstructural evolution. To this end, the experimental results obtained in monotonic uniaxial tension and monotonic simple shear at various angles \( \omega \) with respect to the rolling direction have been averaged with respect to \( \omega \) before the model identification. The Bauschinger sequences of simple shear, involved different amount of forward shear, while the orthogonal strain-path changes consisted by first imposing a true tensile strain in the rolling direction followed by a shearing in the same direction.

### i.3 Partial derivatives of the hardening stress

Expressing the increment of the plastic strain as:

\[
\Delta \varepsilon^p = \Delta \hat{\varepsilon}_p = D^p \Delta \hat{\varepsilon} = (\Delta \varepsilon^p) \hat{\varepsilon}^p,
\]

with \( \Delta \varepsilon^p = D^p \Delta t \) \hspace{1cm} (4.143)

and reading the five-dimensional vector representation of the hardening stress as:

\[
\hat{\sigma}_k' = \hat{X}_k + \tau \hat{U}_k,
\]

with K=1-5, the variation of the hardening stress caused by a variation of the plastic strain is obtained by chaining the partial derivatives of different variables of the model.

\[
\frac{\partial \hat{\sigma}_k'}{\partial \Delta \hat{\varepsilon}_M^p} = \frac{\partial \hat{X}_k}{\partial \Delta \hat{\varepsilon}_M^p} + \frac{\partial \tau}{\partial \Delta \hat{\varepsilon}_M^p} \hat{U}_k + \tau \frac{\partial \hat{U}_k}{\partial \Delta \hat{\varepsilon}_M^p}
\]

Following Teodosiu (1999) and according to the evolution law of \( \mathbf{X} \), the variation of the back-stress is computed as follows:

\[
\frac{\partial \hat{X}_M^f}{\partial \Delta \hat{\varepsilon}_M^p} = X_{sat} \frac{\partial \hat{U}_M}{\partial \Delta \hat{\varepsilon}_M^p} (1 - e^{-c_x \Delta \varepsilon^p}) + C_X (X_{sat} \hat{U}_M - \hat{X}_M^i) e^{-c_x \Delta \varepsilon^p} \hat{U}_M
\]

By the derivation of the equation (Eq. 4.140) the variation of the critical resolved shear stress is obtained:

\[
\frac{\partial \tau}{\partial \Delta \hat{\varepsilon}_M^p} = -C_R (R_i - R_{sat}) e^{-c_x \Delta \varepsilon^p} \hat{U}_M + m \frac{S'}{S} \left( S_D' \frac{\partial S_L^f}{\partial \Delta \hat{\varepsilon}_M^p} + S_L' \frac{\partial S_D^f}{\partial \Delta \hat{\varepsilon}_M^p} \right)
\]

with

\[
\frac{\partial S_D^f}{\partial \Delta \hat{\varepsilon}_M^p} = C_2 C_3 e^{-c_y \Delta \varepsilon^p} \hat{U}_M
\]

\[
\frac{\partial S_L^f}{\partial \Delta \hat{\varepsilon}_M^p} = C_2 C_3 e^{-c_y \Delta \varepsilon^p} \hat{U}_M
\]
\[ \frac{\partial S'_I}{\partial \delta M} = \frac{C_1}{n_L} (S'_L)^{(1+n_L)} \epsilon'_M \] (4.150)

Finally, the variation of the normalized yield stress is expressed by:

\[ \frac{\partial \epsilon'_K}{\partial \delta M} = \frac{1}{\Delta G} \left[ \delta_{MN} - \hat{A}'_M \hat{A}'_N \left( \delta_{MN} \left( G - \hat{A}'_L \frac{\partial G}{\partial \hat{A}'_L} \right) + \delta_{KL} - \hat{A}'_K \hat{A}'_L \right) \frac{\partial^2 G}{\partial \hat{A}'_K \partial \hat{A}'_L} \right] \] (4.151)

**ii) Application of the FLD code on the Combined Plasticity Model of Texture and Strain-Path induced Anisotropy. Theoretical treatments of localized necking.**

In this section is presented a detailed structure of the physical development of the FLDmodel extended to a large plastic strains analysis. Figure 7 shows the structure of the M-K computation, which keeps the same form as in macroscopic approach of theory of plasticity, still requiring only four results, actually the same ones, from homogeneous zone in the computation of heterogeneous zone.

![Figure 7. The structure of M-K analysis computation](image)

In addition, few particular assumptions are considered:

The first one is that instead of strain increment framework, the strain rate framework is adopted being more properly for the large plastic strain context.

The second one is that it is considered that the orthotropic axes of anisotropy in homogeneous zone are fixed and superimposed by the principal axes of anisotropy whereas in the groove they are rotated with the Jaumann spin:

\[ \dot{R} R = W \] (4.152)

where \( R \) is the rotation matrix and \( W \) the rigid body rotation rate.
Thirdly, as the Marciniack – Kuckzinski necking criterion can be defined in several ways (see Section 4.3.1-i.2.), in the present analysis, the plastic flow localization is supposed to occur when the maximum principal value $D_{11}^b$ of $\dot{D}^b$ is larger than 1000 $D_{11}^*$ (Hiwatashi, 1998).

The rigid plasticity behaviour and plane stress condition are assumed too.

Figure 8 illustrates the two-zone model used in the M-K analysis and the corresponding coordinate systems. If $\psi$ is the angle which characterize the groove orientation, then it is assumed that $\theta^b$ is the angle which expresses the orientation of the orthotropic axes of anisotropy of the heterogeneous zone with respect to the global reference frame (xoyoz), which in fact, in the present study coincides with the orthotropic axes of anisotropy in homogeneous zone (xyz). Therefore can be defined the corresponding rotation matrix can be defined as follows:

$$R_\psi = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix} \quad \text{(4.153)}$$

and

$$R_{\theta^b} = \begin{bmatrix} \cos \theta^b & -\sin \theta^b \\ \sin \theta^b & \cos \theta^b \end{bmatrix} \quad \text{(4.154)}$$

Figure 8. Initial geometrical imperfection of the M-K analysis
ii.1 Computation of stress and strain state in homogeneous region 'zone A'

On the homogeneous region A small increments of deformations are imposed. The strain rate tensor \( \dot{D}^a \) is assumed to be constant during an increment and it is represented as

\[
\dot{D}^a = \begin{bmatrix}
\dot{D}_{11}^a & 0 & 0 \\
0 & \dot{D}_{22}^a & 0 \\
0 & 0 & -\dot{D}_{11}^a - \dot{D}_{22}^a
\end{bmatrix}
\]  

(4.155)

with \( \dot{D}_{11}^a > 0 \) and \( \dot{D}_{11}^a \approx \dot{D}_{22}^a \)  

(4.156)

Since the homogeneous zone orthotropic axes of anisotropy are fixed, the rigid body rotation rate is equal to zero:

\[
W^a = 0
\]  

(4.157)

The strain path is expressed by the strain rate mode that is characterized by the ratio:

\[
\rho = \frac{D_{22}^a}{D_{11}^a}
\]  

(4.158)

The material model leads to the deviatoric stress in region A, and the plane stress condition then gives the total stress \( \dot{\sigma}^a \) at the end of the increment.

The general algorithm of the computation of homogeneous zone A is presented as follows:

- Rotation of matrix \( \dot{D}^a \) in the groove reference frame in order to obtain \( D_{nt}^a \).

\[
D_{nt}^a = R_v^T \dot{D}^a R_v
\]  

(4.159)

- Calculation of the actual plastic strain rate direction \( \dot{A}^a \):

\[
\dot{A}^a = \frac{\dot{D}^a}{\sqrt{\dot{D}^a : \dot{D}^a}}
\]  

(4.160)

- Conversion of \( \dot{A}^a \) from 3 x 3 component notation in a five-dimensional vector notation \( \dot{A}_t^a \) (see Appendix A2)

- Computation of the function \( G(\dot{A}_t^a) \) through the texture model

- Computation of the normalized effective deviatoric stress \( \dot{U}_t^a \)

- Computation of \( \Delta \Gamma^a \)
• Computation of the internal variables $S^a_f, P^a_f, X^a_f, R^a_f$ evolution and the yield locus $\tau^a_f$.

• Computation of the deviatoric stress $\dot{\sigma}^a_I$ by using the flow rule, according to the dual plastic potential theory

• Conversion of deviatoric stress $\dot{\sigma}^a_I$ from five-dimensional vector notation in $3 \times 3$ component notation $\dot{\sigma}^a_I$

• Computation of stress matrix $\dot{\sigma}^a_I$

$$\dot{\sigma}^a_I = \dot{\sigma}^{a\prime}_I - \dot{\sigma}^{33}_I I$$  \hspace{1cm} (4.161)

where $I$ represent the unit matrix.

• Computation of the stress matrix in the groove reference frame by rotation with $R_\psi$

$$\sigma^a_m = R_\psi^T \dot{\sigma}^a R_\psi$$  \hspace{1cm} (4.162)

The geometrical compatibility conditions at large strains are:

$$D^a_n = D^b_n$$
$$W^a_n + D^a_m = W^b_n + D^b_m$$

hence

$$W^b_n = D^a_n - D^a_m$$
$$D^b_{3n} = D^b_{13} = 0$$
$$W^b_{13} = W^a_{13} = 0$$
$$W^b_{3n} = W^a_{3n} = 0$$  \hspace{1cm} (4.163)

ii.2 Computation of stress and strain state in heterogeneous region ‘zone B’

The algorithm of the computation of the imperfection region is:

• Initialisation $D^{b}_{mn}, D^{b}_{nt}$

• Compatibility requirement: $D^b_n = D^a_n$

• Rotation of matrix $D^b_{mn}$ in orthotropic axes of anisotropy.

$$\hat{D}^b_i = R^T_{\phi_e} R_\psi D^b_{mn} R^T_\psi R_{\phi_e}$$  \hspace{1cm} (4.165)

• Calculation of the direction of actual plastic strain rate $\hat{D}^b_f$: 
\[ \hat{A}_i^b = \frac{\hat{D}_i^b}{\sqrt{\hat{D}_i^b : \hat{D}_i^b}} \]  

(4.166)

- Conversion of \( \hat{A}_i^b \) from 3 x 3 component notation in a five-dimensional vector notation \( \hat{A}_i^b \) (see Appendix A2)

- Texture model computes the function \( G(\hat{A}_i^b) \) and the derivatives of \( G \), namely \( \frac{\partial G}{\partial \hat{A}_i} \) and \( \frac{\partial^2 G}{\partial \hat{A}_i \partial \hat{A}_j} \) respectively.

- Computation of the normalized effective deviatoric stress \( \hat{U}_i^b \) and its derivatives \( \frac{\partial \hat{U}_K}{\partial \hat{D}_M} \) by using Eq. 4.151.

- Computation of the evolution of the internal variables \( S_j^b, P_j^b, X_j^b, R_j^b \) and their corresponding derivatives by use of Eq. 4.147, Eq. 4.149 and Eq. 4.150.

- Computation of the yield locus size through the critical resolved shear stress \( \tau_i^b \) and calculation of its respective derivative \( \frac{\partial \tau_i^b}{\partial \hat{D}_M} \) with Eq. 4.146.

- The flow rule, according to the dual plastic potential theory, gives the deviatoric stress \( \hat{\sigma}_i^b \).

- Conversion of deviatoric stress \( \hat{\sigma}_i^b \) from five-dimensional vector notation in 3 x 3 component notation \( \hat{\sigma}_i^b \) (see Appendix A2)

- Computation of stress matrix \( \hat{\sigma}_y^b \)

\[ \hat{\sigma}_y^b = \hat{\sigma}_y^b - \hat{\sigma}_y^{33} \mathbf{I}, \]  

(4.167)

where \( \mathbf{I} \) represent the unit matrix.

- Computation of the stress matrix in the groove reference frame

\[ \sigma_{nt}^b = R_y^T R_y \hat{\sigma}_y^b R_y^T R_y \]  

(4.168)

- Computation of the actual value of the geometrical imperfection:

\[ f = f_1 \exp \left[ \left( D_{m}^{hf} - D_{n}^{hf} \right) \Delta t \right] \cdot \exp \left[ \left( D_{11}^{hf} + D_{22}^{hf} \right) \Delta t \right] \]  

(4.169)
• Computation of $D_{mn}^b$ and $D_{nt}^b$ through the use of Newton–Raphson’s method

At the end of the increment, the following equilibrium conditions must to be satisfied:

$$
\begin{align*}
\sigma_{mn}^b &= \sigma_{nn}^a / f \\
\sigma_{nt}^b &= \sigma_{nt}^a / f
\end{align*}
$$

(4.170)

In consequence, on the base of such equilibrium conditions, it is possible to define two polynomial functions $G_1$ and $G_2$ in $D_{mn}^b$ and $D_{nt}^b$ expressed as follows:

$$
\begin{align*}
G_1(D_{mn}^b, D_{nt}^b) &= \sigma_{mn}^b - \sigma_{mn}^b = 0 \\
G_2(D_{mn}^b, D_{nt}^b) &= \sigma_{nt}^b - \sigma_{nt}^b = 0
\end{align*}
$$

(4.171)

The iterative formula for Newton–Raphson’s method performed to solve the equations system presented in (Eq.4.171), is described as follows:

$$
\begin{bmatrix}
D_{mn}^{b, i+1} \\
D_{nt}^{b, i+1}
\end{bmatrix} = 
\begin{bmatrix}
D_{mn}^{b, i} \\
D_{nt}^{b, i}
\end{bmatrix} - J^{-1} \begin{bmatrix}
G_1(D_{mn}^{b, i}, D_{nt}^{b, i}) \\
G_2(D_{mn}^{b, i}, D_{nt}^{b, i})
\end{bmatrix}
$$

(4.172)

where, $J$ is the Jacobian matrix which is expressed as:

$$
J = \begin{bmatrix}
\frac{\partial G_1}{\partial D_{mn}^b} & \frac{\partial G_1}{\partial D_{nt}^b} \\
\frac{\partial G_2}{\partial D_{mn}^b} & \frac{\partial G_2}{\partial D_{nt}^b}
\end{bmatrix}
$$

(4.173)

and matrix $J^{-1}$ represent the inverse of $J$.

• Computation of the Jacobian matrix.

$$
\begin{align*}
\frac{\partial G_1}{\partial D_{mn}^b} &= \frac{\partial \sigma_{mn}^b}{\partial D_{mn}^b} - \frac{\partial \sigma_{mn}^b}{\partial D_{mn}^b} \\
\frac{\partial G_1}{\partial D_{nt}^b} &= \frac{\partial \sigma_{nt}^b}{\partial D_{nt}^b} - \frac{\partial \sigma_{nt}^b}{\partial D_{nt}^b} \\
\frac{\partial G_2}{\partial D_{mn}^b} &= \frac{\partial \sigma_{mn}^b}{\partial D_{mn}^b} - \frac{\partial \sigma_{mn}^b}{\partial D_{mn}^b} \\
\frac{\partial G_2}{\partial D_{nt}^b} &= \frac{\partial \sigma_{nt}^b}{\partial D_{nt}^b} - \frac{\partial \sigma_{nt}^b}{\partial D_{nt}^b}
\end{align*}
$$

(4.174)
Therefore, in order to compute the Jacobian matrix, the computation of the derivatives of stress matrix in the groove reference frame reported by the strain rate tensor in the same reference frame is needed, could be performed in two steps, as follows:

Firstly, the derivatives $\frac{\partial \hat{\sigma}^b}{\partial \hat{D}^b}$ in the orthotropic axes of anisotropy reference frame are computed, which are then transposed on the groove reference frame in order to obtain the quantities $\frac{\partial \sigma_{nt}^b}{\partial D_{nt}^b}$.

By using the Eq.146 and Eq.148, the variation of the hardening stress caused by a variation of the plastic strain rate $\frac{\partial \hat{\sigma}^b}{\partial \hat{D}^b}$ in a five-dimensional vector notation is calculated, which next is transposed in a six-dimensional vector notation (see Appendix A3) with the purpose of computation of the quantities $\frac{\partial \hat{\sigma}^b}{\partial \hat{D}^b}$ through the Eq. 167:

$$\frac{\partial \hat{\sigma}^b}{\partial \hat{D}^b} = \frac{\partial \hat{\sigma}^b}{\partial \hat{D}^b} - \frac{\partial \hat{\sigma}^b}{\partial \hat{D}^b} I,$$  \hspace{1cm} (4.175)

with K, M = 1-6 and the stress and strain rate matrix in a six-dimensional vector notation reading:

$$\hat{\sigma}^b = [\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3, \hat{\sigma}_12, \hat{\sigma}_{13}, \hat{\sigma}_{23}]$$ \hspace{1cm} (4.176)

and

$$\hat{\varepsilon}^b = [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{\varepsilon}_3, 2\hat{\varepsilon}_{12}, 2\hat{\varepsilon}_{13}, 2\hat{\varepsilon}_{23}]$$ \hspace{1cm} (4.177)

respectively.

Finally, $\frac{\partial \sigma_{nt}^b}{\partial D_{nt}^b}$ is obtained as follows (see Appendix A4):

$$\frac{\partial \sigma_{nt}^b}{\partial D_{nt}^b} = R^b R^b \frac{\partial \hat{\sigma}^b}{\partial \hat{D}^b} R^{bT} R^{bT}$$ \hspace{1cm} (4.178)

with

$$R^b = R^T \Omega^b$$ \hspace{1cm} (4.179)

and
\[ R^{bT} = R_{\phi}^T R_{\psi} \]  

(4.180)

where \( R_{\phi}^T \) and \( R_{\psi}^T \) represent the transposes of \( R_{\phi} \) and \( R_{\psi} \), respectively.

Secondly, the derivatives \( \frac{\partial \sigma^{b^*}}{\partial D_{n_l}^{b}} \) are calculated by derivation of Eq. 170 and are expressed as follows:

\[
\begin{align*}
\frac{\partial \sigma_{nn}^{b^*}}{\partial D_{nn}^{b}} &= \frac{\sigma_{nn}^a}{f} \cdot \Delta t; & \frac{\partial \sigma_{nn}^{b^*}}{\partial D_{nt}^{b}} &= 0.0 \\
\frac{\partial \sigma_{nt}^{b^*}}{\partial D_{nn}^{b}} &= \frac{\sigma_{nt}^a}{f} \cdot \Delta t; & \frac{\partial \sigma_{nt}^{b^*}}{\partial D_{nt}^{b}} &= 0.0
\end{align*}
\]

(4.181)

- Rotation of matrix \( D_{n_l}^{b} \) in the groove orthotropic axes of anisotropy.

\[
\hat{D}_{l}^{b} = \hat{R}_{\phi}^{T} R_{\psi} D_{n_l}^{b} R_{\psi}^{T} \hat{R}_{\phi}
\]

(4.182)

- Computation of the principal strain rates corresponding to zone B.

\[
\begin{align*}
D_{11}^{b} &= \frac{\hat{D}_{xx}^{b} + \hat{D}_{yy}^{b}}{2} + \sqrt{\left( \frac{\hat{D}_{xx}^{b} - \hat{D}_{yy}^{b}}{2} \right)^2 + D_{xy}^2} \\
D_{22}^{b} &= \frac{\hat{D}_{xx}^{b} + \hat{D}_{yy}^{b}}{2} - \sqrt{\left( \frac{\hat{D}_{xx}^{b} - \hat{D}_{yy}^{b}}{2} \right)^2 + D_{xy}^2}
\end{align*}
\]

(4.183)

- Finding the plastic instability occurrence by checking the condition:

\[
D_{11}^{b} \geq 1000 D_{11}^{a}
\]

(4.184)

Therefore, as previously mentioned, when the maximum principal strain rate value of heterogeneous zone is greater or equal to the maximum principal strain rate value of homogeneous zone the analysis assumes that plastic flow localization occurs and the accumulated principal strains in the homogeneous zone represent the forming strain limits.

If the failure condition (Eq.4.184) is not satisfied, the computation continues after the actualisation of the internal variables and after the computation of the rotation matrix \( R_{\theta_{1,1}} \). that indicates the actual position of the groove orthotropic axes of anisotropy.

- Computation of the rotation matrix \( R_{\theta_{1,1}} \) by using the Hughes formula (Hughes, 1984) that is expressed as:
(R_{\delta_k})_{k+1} = r^b(R_{\delta_k})_k \quad (4.185)

where

r^b = 1 + \left(1 - \frac{1}{2} \Delta W^b \right)^{-1} \Delta W^b \quad (4.186)

with $1$ the unit matrix and $\Delta W^b$ the increment of the rigid body rotation rate of the heterogeneous zone represented by:

$\Delta W^b = \begin{bmatrix}
0 & \Delta W_{nl}^b & 0 \\
-\Delta W_{nl}^b & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \quad (4.187)$

where

$\Delta W_{nl}^b = W_{nl}^b \Delta t \quad (4.188)$

and $W_{nl}^b$ results from the geometrical compatibility conditions at large strains expressed by the Equations 4.163 and 4.164.

Thus, following Hughes (1984), $r^b$ can be expressed as:

$r^b = 1 + \frac{2}{1 + \frac{1}{4} (\Delta W_{nl}^b)^2} \left(\frac{1}{2} \Delta W_{nl}^b - \frac{1}{4} (\Delta W_{nl}^b)^2 \right) \quad (4.189)$

- Evaluation of the actual groove orientation through the angle $\psi_{k+1}$ expressed as:

$tg(\psi_{k+1}) = tg(\psi_0) \cdot exp(\epsilon_{11}^e - \epsilon_{22}^e) \quad (4.190)$

For each considered strain path, the analysis is repeated for different values of $\psi_0$ (between 0 and 90 degrees) and the limit point on the FLD is obtained through the minimization of the curve $\epsilon_{11}^e$ versus $\psi_0$.

### ii.3 Complex strain paths

The essential performance of the present physical model on the Forming Limit Diagram prediction consists, as previously reported, in its ability to reproduce experimental tendency under strain path changes that can't be reproduced by another phenomenological model. In consequence, FLDcode was extended to account for complex strain paths at large plastic strains analysis. In this case, such prestrain will be accumulated on the internal variables of the physical model. Thus, the initial condition of the second loading assumes that internal variables values at the beginning of the second loading are equal to their values at the end of the first loading:
\[ \hat{X}_{2i} = \hat{X}_{1f} \]
\[ \hat{S}_{2i} = \hat{S}_{1f} \]
\[ \hat{P}_{2i} = \hat{P}_{1f} \]
\[ R_{2i} = R_{1f} \]  \hspace{1cm} (4.191)

where the subscripts "f" and "i" represent the initial moment of the second loading and the end of the first loading, respectively. It is also considered that the initial band orientation at the beginning of the second strain path is equals to the final band orientation at the end of the first stage.

Among the several strain path changes, the advantage of modeling the material hardening on the basis of the microstructure evolution is relevant for a strain history involving a biaxial stretching prestrain followed by a uniaxial tension (BS-UT), which presents a work softening phase whose severity increases with the amount of prestrain. This cross effect results from the interaction between the currently active slip systems and the previously formed dislocation structure being described by the hardening model (Hiwatashi, 1998a; Hoferlin et al, 1989). Therefore, the decrease of the work hardening is responsible for the severe drop of formability, especially when the magnitude of the biaxial prestrain is high.

4.3.3 The comparison of the theoretical treatments of localized necking between the phenomenological and physical approach of theory of Plasticity

Figure 9 shows that the same method could be applied in the computation of theoretical treatments of localized necking in the Phenomenological and Physical approach of Plasticity. In both cases after the computation of homogeneous region, the Newton Raphson numerical method is used to solve a system of two equations, which proceeds from certain conditions of M-K analysis eventually coupled with certain plasticity theory condition such as the yield criterion in the phenomenological approach, in order to compute the respective unknowns necessary to define the state of stress and strain or strain rate, respectively, in the groove. Afterwards, when the assumed M-K failure criterion is achieved, the plastic instability localization is found.

4.4. Conclusion

A more flexible mathematical method was applied to Marciniack - Kuckzinsky theory to develop a general code for FLDs prediction in monotonic and two-stage strain paths. Therefore, the yield function and the work hardening law that achieve the best description of the mechanical behaviour of each material can be implemented without major difficulty.

A very easy and rapid comparison of the performance of different yield functions and work hardening laws for the same material can be also performed.

Additionally, the structure of the code allows to a more flexible development of the interfaces between necking criterion and FE programs.
Figure 9. Theoretical treatments of localized necking on both approaches of theory of plasticity: phenomenological one versus physical one
Figure 10. The scheme of the constitutive equations implemented in the FLDcode
The generality and effectiveness of the new code was definitely proved by successful implementation of several combinations of different constitutive equations, like the quadratic Hill criterion (Hill'48), non-quadratic Hill criterion (Hill'79) and Yld'96 Barlat yield function coupled with Swift or Voce equation as the hardening model.

Moreover, by applying the same method on the computation of theoretical treatments of localized necking, the new model was extended to a physical approach of the plastic theory, by considering the most advanced physical model at the present time, namely the combined model of texture and strain-path-induced anisotropy. Due of the fact that the texture based plastic potential is a potential in plastic strain rate space, the generality of the new method is absolutely completed.

Observing Figure 10, which presents all considered constitutive equations efficiently applied in the FLDcode, can be concluded that the most important advantage of the new method is its validity for the Phenomenological approach of the Plasticity Theory as much as the Physical one, at the same time, could by applied with equal degree of success for plastic potentials in stress space as well as in strain rate space.
5. APPLICATION AND VALIDATION OF THE “FLDcode” ON THE STRAIN-BASED FORMING LIMITS ANALYSIS

5.1 Introduction

This chapter aims at applying and validating the new model qualities on the Forming Limit Diagrams analysis. Due the complexity of the material behaviour and the essential importance of the constitutive equations on the localized necking prediction as well as on the entire process of sheet forming simulations, the objective of the present analysis is also to assess the efficiency of the FLDcode in the correct choice of such materials models for their best description. For this purpose, four industrial relevant metal sheets with high quality for deep drawing are considered. These are: AA6016-T4 and AA5182-0 aluminium alloy sheets and two new generation steel sheets namely a bake-hardening steel (BH steel) and a DC06 steel with two different thickness such as 0.8mm and 1.5 mm. The chapter falls in five sections. Section 5.2 contains a description of the mechanical behaviour of each considered material, including the theoretical performances of the FLDcode in the material characterization. Section 5.3 describes the experimental determination of the FLDs for all analysed materials. Section 5.4 points out the advantages of the FLDcode structures concerning the FLDs theoretical analysis. Section 5.5 is devoted to the experimental validation of the FLDcode predicted results. Section 5.6 brings a comparative analysis on strain-based forming limits, based on phenomenological and physical approach of Plasticity Theory. Finally, the most relevant conclusions are presented in Section 5.7.

5.2 Materials characterization and mechanical behaviour

The mechanical behaviour for all considered materials was determined in uniaxial tension tests along different directions, performed by an INSTRON MODEL 4507 uniaxial tension test machine equipped with a High Resolution Digital Automatic Extensometer (Biaxial Strain Gauge) INSTRON. For all tests the reference length was 50 mm and the rate of data acquisition was 5 point/sec.

The anisotropy factors values (r-values), defined by the magnitude of the ratio between the plastic width strain and the plastic thickness strain in a uniaxial tensile test, were determined using an electro-chemical grid printed on the surface of the test-pieces of circles of 5 mm diameter. The strains were calculated from measurements of deformed grids using a NC Profile Projector having an accuracy of ±1 μm. By averaging twenty values for each direction, the r-values were obtained. They can be observed from the data presented in Table 1, Table 2, Table 3 and Table 4, besides the corresponding yield stresses, the ultimate tensile strengths and the uniform elongations.

Table 1. AA6016-T4 uniaxial plastic properties at 0°, 45° and 90° from rolling direction

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Yield Stress σ_Y [MPa]</th>
<th>Ultimate tensile strength σ_ut [MPa]</th>
<th>Uniform elongation</th>
<th>r value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>127</td>
<td>280</td>
<td>22%</td>
<td>0.80</td>
</tr>
<tr>
<td>45°</td>
<td>127</td>
<td>279</td>
<td>23%</td>
<td>0.43</td>
</tr>
<tr>
<td>90°</td>
<td>114</td>
<td>243</td>
<td>16%</td>
<td>0.61</td>
</tr>
</tbody>
</table>
5. Application and validation of the "FLD code" on the strain-based forming limits analysis

Table 2. AA5182-0 uniaxial plastic properties at 0°, 45° and 90° from rolling direction

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Yield Stress $\sigma_Y$ [MPa]</th>
<th>Ultimate tensile strength $\sigma_{uts}$ [MPa]</th>
<th>Uniform elongation</th>
<th>$r$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>143</td>
<td>374</td>
<td>24%</td>
<td>0.67</td>
</tr>
<tr>
<td>45°</td>
<td>154</td>
<td>300</td>
<td>23%</td>
<td>0.66</td>
</tr>
<tr>
<td>90°</td>
<td>137</td>
<td>339</td>
<td>23%</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table 3. BH steel uniaxial plastic properties at 0°, 45° and 90° from rolling direction

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Yield Stress $\sigma_Y$ [MPa]; Upper/Lower</th>
<th>Ultimate tensile strength $\sigma_{uts}$ [MPa]</th>
<th>Uniform elongation</th>
<th>$r$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>210 / 208</td>
<td>410</td>
<td>23%</td>
<td>2.2</td>
</tr>
<tr>
<td>45°</td>
<td>224 / 218</td>
<td>417</td>
<td>21%</td>
<td>1.5</td>
</tr>
<tr>
<td>90°</td>
<td>214 / 211</td>
<td>409</td>
<td>22%</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Table 4. DC06 steel (0.8mm) uniaxial plastic properties at 0°, 45° and 90° from rolling direction

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Yield Stress $\sigma_Y$ [MPa]</th>
<th>Ultimate tensile strength $\sigma_{uts}$ [MPa]</th>
<th>Uniform elongation</th>
<th>$r$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>136</td>
<td>374</td>
<td>29%</td>
<td>2.6</td>
</tr>
<tr>
<td>45°</td>
<td>134</td>
<td>369</td>
<td>27%</td>
<td>2.1</td>
</tr>
<tr>
<td>90°</td>
<td>136</td>
<td>365</td>
<td>28%</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Table 5. DC06 steel (1.5mm) uniaxial plastic properties at 0°, 45° and 90° from rolling direction

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Yield Stress $\sigma_Y$ [MPa]</th>
<th>Ultimate tensile strength $\sigma_{uts}$ [MPa]</th>
<th>Uniform elongation</th>
<th>$r$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>150</td>
<td>361</td>
<td>29%</td>
<td>1.7</td>
</tr>
<tr>
<td>45°</td>
<td>161</td>
<td>370</td>
<td>26%</td>
<td>1.6</td>
</tr>
<tr>
<td>90°</td>
<td>158</td>
<td>364</td>
<td>28%</td>
<td>2.4</td>
</tr>
</tbody>
</table>

The material flow behaviour, in the sheet plane, is characterised on the base of the true stress-true strain curves for three orientations between the tensile axis and the rolling direction, namely at 0°, 45° and 90° from RD. They are presented in the next figures. Through these curves, certain anisotropic flow behaviour in the sheet plane for each considered material is revealed. Actually, it is remarked a rather weak anisotropic flow behaviour for all materials. Additionally, the uniform elongation at 0° is slightly greater than the uniform elongations corresponding to the other two directions, namely, 45° and 90° from RD. In consequence, in the present analysis, the assumption of considering the true stress-true strain curve at 0° from RD to identify the material parameters of the hardening model seems to be reasonable and acceptable.

It also can be noticed that the AA5182-0 and AA6016-T4 aluminium alloys don’t exhibit a strong anisotropic flow behaviour during uniaxial tension as was observed for AA1050-0 aluminium sheet.
(Lopes et al, 1999), which presented a 45° uniform elongation higher with 25% and 30% than the 90° and 0° uniform elongation, respectively.

Figure 1. True stress – true strain curves for AA6016-T4 aluminium alloy

Figure 2. True stress – true strain curves for AA5182-0 aluminium alloy
Figure 3. True stress – true strain curves for DC06 (0.8mm thickness) steel sheet

Figure 4. True stress – true strain curves for DC06 (1.5mm thickness) steel sheet
By fitting experimental true stress – true plastic strain ($\sigma$-$\varepsilon$) data of each studied material, measured in the uniaxial test along the RD, the hardening material parameters, corresponding to a Swift equation:

$$\bar{\sigma} = k(\bar{\varepsilon}_0 + \bar{\varepsilon})^m$$  \hspace{1cm} (5.1)

and to a Voce equation:

$$\bar{\sigma} = A - B \exp(-C\bar{\varepsilon})$$  \hspace{1cm} (5.2)

were identified. They are expressed in the Table 6 and Table 7, respectively.

Table 6. Material constants of Swift equation.

<table>
<thead>
<tr>
<th>Material</th>
<th>k</th>
<th>n</th>
<th>m</th>
<th>$\varepsilon_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA6016-T4</td>
<td>417.854</td>
<td>0.245</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>AA5182-0-0</td>
<td>596.123</td>
<td>0.31</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>BH steel</td>
<td>548.67</td>
<td>0.195</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>BH steel</td>
<td>579.99</td>
<td>0.195</td>
<td>0.012</td>
<td>0.01</td>
</tr>
<tr>
<td>DC06 steel (0.8mm)</td>
<td>455.55</td>
<td>0.253</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>DC06 steel (1.5mm)</td>
<td>521</td>
<td>0.25</td>
<td>0</td>
<td>0.01</td>
</tr>
</tbody>
</table>
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Table 7. Material constants of Voce equation

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA6016-T4</td>
<td>318.103</td>
<td>191.103</td>
<td>8.706</td>
</tr>
<tr>
<td>AA5182-0-0</td>
<td>416.068</td>
<td>273.068</td>
<td>8.3016</td>
</tr>
<tr>
<td>BH steel</td>
<td>421.124</td>
<td>211.124</td>
<td>11.425</td>
</tr>
<tr>
<td>DC06 steel (0.8mm)</td>
<td>388.541</td>
<td>252.541</td>
<td>10.15</td>
</tr>
<tr>
<td>DC06 steel (1.5mm)</td>
<td>447.482</td>
<td>297.482</td>
<td>6.835</td>
</tr>
</tbody>
</table>

The next tables (Tables 8-12) show, for each considered material, the Yld96 seven parameters \( (c_1, c_2, c_3, c_6, \alpha_x, \alpha_y, \alpha_z) \) obtained by a numerical identification on the base of the experimental data \( (\sigma_0, \sigma_{45}, \sigma_{90}, r_0, r_{45}, r_{90}) \).

Table 8. AA6016-T4 material constants describing the Yld96 yield surface shape

<table>
<thead>
<tr>
<th>( \sigma_0 [\text{MPa}] )</th>
<th>( a )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_6 )</th>
<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>123.44</td>
<td>8</td>
<td>1.0474</td>
<td>0.7752</td>
<td>1.0724</td>
<td>0.9288</td>
<td>2.00</td>
<td>3.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 9. AA5182-0 material constants describing the Yld96 yield surface shape

<table>
<thead>
<tr>
<th>( \sigma_0 [\text{MPa}] )</th>
<th>( a )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_6 )</th>
<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>138.225</td>
<td>8</td>
<td>1.0067</td>
<td>0.9048</td>
<td>1.0018</td>
<td>0.7791</td>
<td>1.72</td>
<td>2.05</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 10. BH steel material constants describing the Yld96 yield surface shape

<table>
<thead>
<tr>
<th>( \sigma_0 [\text{MPa}] )</th>
<th>( a )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_6 )</th>
<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>216.56</td>
<td>6</td>
<td>1.2329</td>
<td>1.2165</td>
<td>0.9808</td>
<td>1.066</td>
<td>0.21</td>
<td>0.27</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 11. DC06 steel (0.8mm) material constants describing the Yld96 yield surface shape

<table>
<thead>
<tr>
<th>( \sigma_0 [\text{MPa}] )</th>
<th>( a )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_6 )</th>
<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>142.204</td>
<td>6</td>
<td>1.3375</td>
<td>1.3020</td>
<td>1.023</td>
<td>1.2769</td>
<td>0.16</td>
<td>0.22</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 12. DC06 steel (1.5mm) material constants describing the Yld96 yield surface shape

<table>
<thead>
<tr>
<th>( \sigma_0 [\text{MPa}] )</th>
<th>( a )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_6 )</th>
<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>155.088</td>
<td>6</td>
<td>1.1604</td>
<td>1.3052</td>
<td>0.9783</td>
<td>1.0661</td>
<td>0.3</td>
<td>0.27</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figures 6, 7, 8 and 9 show the comparison between the experimental and modeled \( \sigma-\epsilon \) curves obtained by use of Swift and Voce equations, for all considered materials under uniaxial tension, respectively. As it can be seen, the three curves corresponding to each particular material, are
superimposed, but the maximum equivalent strain corresponding to localized necking achieved in uniaxial tension through the M-K model is different for Voce and Swift equations.

Fig. 6. Experimental and theoretical true stress – true plastic strain curve at 0° from RD for AA6016-T4 aluminium alloy

Fig. 7. Experimental and theoretical true stress – true plastic strain curve at 0° from RD for AA5182-O aluminium alloy
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Fig. 8. Experimental and theoretical true stress – true plastic strain curve at 0° from RD for DC06 steel sheet

Fig. 9. Experimental and theoretical true stress – true plastic strain curve at 0° from RD for BH steel sheet

It can be noticed that the saturation stress-type Voce equation reproduces with a better accuracy then a power-law-hardening model (Swift equation) the maximum experimental equivalent strain before necking, for all considered materials. Moreover, it is observed that by use of Voce equation the
maximum admissible equivalent strain is predicted almost perfect, with a slight overestimation, for such aluminium alloys, and is underestimated for the respective steel sheets, while the one predicted by use of Swift equation are always considerable overestimated. These observations bring out the question: Is the saturation stress-type Voce equation the best macroscopic hardening model to reproduce the mechanical behaviour of any material and the respective forming limit strains? In order to find the requested answer, in Figure 10 are represented the experimental and theoretical limit strains under uniaxial tension for AA6016-T4, AA5182-0-0, BH steel and DC06 steel, respectively. It is observed that in terms of forming limit strains, calculated through the M-K analysis, the Swift equation predicts with a very good accuracy the experimental data for the BH steel and DC06 steel sheets, while the Voce equation reproduces very well the experimental forming limits for AA6016-T4 and AA5182-0 aluminium alloys sheets. So, even that the predicted maximum admissible equivalent strain obtained by using Swift equation for the respective steel sheets is overestimated, the corresponding forming limit strains are in perfect agreement with the experimental ones. More details concerning the difference that appears in approximation of the work hardening curve by a power-law-hardening model (Swift equation) and a saturation stress-type Voce equation, up to the maximum equivalent strain will be presented forward in one of the next subsections.

![Figure 10. Experimental and theoretical limit strains during uniaxial tension for AA6016-T4, AA5182-0-0, BH steel and DC06 steel, respectively.](image-url)
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Another important observation is that the initial values of the M-K geometrical defect, which were used in simulation in order to obtain the best accuracy, are equal to 0.998 for AA6016-T4, to 0.996 for AA5182-0, to 0.992 for DC06 steel and to 0.993 for BH steel. It must be noticed that according to previous work by Barlat, based in the microstructural analysis (Barlat, 1989), the correct initial defect value is in the range 0.996-0.998. So, for each particular material, using a unique value of the M-K geometrical defect, which is the closest of such correct imperfection value and more appropriate in terms of accuracy, the simulated results obtained by Swift and Voce equation were compared. Certainly, concerning the aluminium alloys, a better agreement between the simulated results by Swift equation and the experimental data during uniaxial tension can be obtained, if another defect value is used, maybe around 0.98 but, at the same time, problems in the reproducibility of the right part of the diagram appear, as Figure 10* shows. The same situation exists for the steel sheets, when Voce hardening law is used in simulation, a higher value of the imperfection factor leading to better simulated result during uniaxial tension, simultaneously with a considerable over evaluation of the limit strain in the stretching zone.

![Graphs showing formability limit predictions for AA6016-T4 and BH steel](https://via.placeholder.com/150)

*a) AA6016-T4 Forming Limit Diagram

*b) BH steel Forming Limit Diagram

Figure 10*. The influence of the initial defect value on the predicted uniaxial tension limit strain

The material plastic anisotropy is mainly pointed out through the anisotropy coefficients values. Thus, it is very important, in terms of numerical simulation, the correct prediction of the evolution of the R-value with the angle to the rolling direction. Undoubtedly the adopted yield function plays a vital role in the anisotropy factors prediction. Hence, the next Figures present, for each distinct material, the predicted distribution of the anisotropy factor as a function of the tensile loading axis between 0° and 90° degrees by the different yield functions implemented in the FLDcode. It is well known that the Von Mises yield function considers the anisotropy factors equal to one while Hill 1979 yield function is a function of an average value of anisotropy factors. So, related to these two yield functions, no R evolution occurs as can be seen in the respective Figures. It is remarked that the predicted anisotropy factors at 0°, 45° and 90° from RD, by Hill 1948 and Yld96 are in perfect agreement with the experimental ones for all materials. Moreover, they are able to simulate very well the entire distribution of the anisotropy factor as a function of the tensile loading axis between 0° and 90° degrees at each material, except for the AA5182-0 aluminium alloy, for which only Yld96 R-prediction perfectly reproduces the experimental data.
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Figure 11. Distribution of anisotropy coefficients for AA6016-T4 aluminium alloy

Figure 12. Distribution of anisotropy coefficients for AA5182-0 aluminium alloy
Figure 13. Distribution of anisotropy coefficients for DC06 steel sheet

Figure 14. Distribution of anisotropy coefficients for BH steel sheet
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Figure 15. Normalized yield stress as a function of the tensile loading axis for AA6016-T4 aluminium alloy

Figure 16. Normalized yield stress as a function of the tensile loading axis for AA5182-0 aluminium alloy
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Figure 17. Normalized yield stress as a function of the tensile loading axis for DC06 steel sheet

Figure 18. Normalized yield stress as a function of the tensile loading axis for BH steel sheet
Figure 19. Experimental and theoretical yield surface for AA6016-T4 aluminium alloy

Figure 20. Experimental and theoretical yield surface for AA5182-0 aluminium alloy
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

![Graph](image1)

Figure 21. Experimental and theoretical yield surface for DC06 steel sheet

![Graph](image2)

Figure 22. Experimental and theoretical yield surface for BH steel sheet
The plastic anisotropy is also revealed by the evolution of the yield stresses values with the different tensile loading orientations from rolling direction. The predicted distribution of the normalized yield stress with respect to the angle with the rolling direction are shown in Figures 15, 16, 17 and 18, respectively. As was expected, the Von Mises and Hill 1979 yield functions predict no evolution of normalized yield stress for the tensile loading axis between 0 and 90 degrees. Therefore, their predicted normalized yield stress is the unit value. Also, the Hill 1948 yield function tends to reproduce the shape of normalized stress distribution but the predicted values are rather far from the experimental data for each present analysed case. Simultaneously, it can be observed that the match between the experimental and Yld96 predicted curve is excellent because the yield stresses values at 0°, 45° and 90° orientations were imposed in the computation of yield function coefficients.

The predicted plane stress yield surfaces and the experimental ones for AL6016–T4, AA5182-0, DC06 steel and BH steel sheets are presented in Figures 19, 20, 21 and 22, respectively. It is remarkable the very good agreement between the experimental and predicted curves obtained by using Yld96 yield function, for all studied materials, excepting the case of DC06 steel sheet where it is observed some difference on the predicted curve and the experimental one. Concerning the aluminium alloys of the present analysis, the Von Mises and Hill 1948 yield functions cannot reproduce the shape of yield locus in balanced biaxial tension stress states whereas Hill 1979 yield locus is not in agreement with the experimental shape of the yield surface only in transverse uniaxial tension. Regarding the analysed steel sheets, it is observed that the predicted Hill 1948 and Hill 1979 yield surfaces are hugely overestimated in the balanced biaxial tension stress state, whereas the corresponding Von Mises predicted curves are slightly underestimated in the respective zone.

5.3 Experimental Forming Limit Diagrams

The experimental FLDs for each distinct material of the present analysis, were determined for linear strain paths (LSP) namely, uniaxial tension, plane strain and biaxial stretching with circular and elliptical die rings, and for complex strain paths namely, uniaxial tension followed by biaxial stretching (UT-BS) and biaxial stretching followed by uniaxial tension (BS-UT). Pre-straining in uniaxial tension at certain values of deformation followed by stretching with different elliptical die rings (UT-X) was also considered.

An electro-chemical method was used to print on the surface of the test-pieces a grid of circles of 2.5mm and 5mm diameter pattern respective to uniaxial tension test and 5mm respective to bulge test. The strains were calculated from measurements of deformed grids using a NC Profile Projector having an accuracy of ±1 μm. All strains were measured on the top surface in true sense:

\[ \varepsilon_1 = \ln \left( \frac{l_1}{l_0} \right) = \text{true major principal strain} \]  \hspace{1cm} (5.3)

\[ \varepsilon_2 = \ln \left( \frac{l_2}{l_0} \right) = \text{true minor principal strain} \]  \hspace{1cm} (5.4)
The onset of localized necking was determined from strain distribution profiles near the necking region through the use of the Bragard method (Bragard, 1972) and following the calculation procedure N° 5 of Zürich recommended by the International Deep Drawing Research Group (IDDRG) (Parniere and Sanz, 1976) in Zürich in 1973.

A brief description of the experimental determination of the limit strains is next presented. On the base of the uniaxial tensile test and the bulge test and different types of specimens which can be observed from Figure 23, all range of strain paths between uniaxial tensile and equibiaxial stretching is covered.

![Figure 23. The types of specimens used on the experimental determination of Forming Limit Diagram](image)

The elliptical die rings are characterized by the quantities “b” and “a”, which in the present analysis take values equal to 70, 90, 110 and 130mm, for all materials, being firstly considered that “b” is along the rolling direction (RD) and secondly, that “a” is along the RD.

Figure 24 shows the strains attained near the necking region for AA5182-0 and BH steel during the bulge tests with two different elliptical die rings, namely characterized by “b”=70mm and “b”=90mm respectively. It is noticed that for a same elliptical die ring, different strain path is
obtained for each material, due to the corresponding anisotropy. Moreover, from this figure, it can be observed the influence of the geometry sample in the strain path identification.

Figure 24. The strains near the necking region for AA5182-0 and BH steel during the bulge test with two different elliptical die rings.

Thus, through the same experimental test by varying the geometry of the sample, diverse strain paths are obtained. Figure 25 shows that the left side of the FLD diagram is assured by the uniaxial tensile test with samples of varies geometry, while right one is guaranteed by the bulge test with elliptical and cylindrical die rings.

The bulge test (Figure 26) equipment consists of a blank holder and die, an hydraulic power supply, an electronic console, a measuring unit (LVDT) and data acquisition equipment interfaced to a personal computer. The bulging pressure, the height of the pole and the curvature of the bulge specimen are recorded through the interface, for data treatment after the test has been performed. The bulge sample is a disc with a diameter of 250 mm while the hydraulic Bulge pressure is imposed at a rate of 1 bar/sec. The clamping force is established to perform the test without sliding of the Bulge sample and it is set to 300 kN for the aluminium sheets and 400 kN for the steel sheets.

The major advantage of this test is that it eliminates friction effects and simulates loading and strain conditions founded in many forming processes.
5. Application and validation of the "FLDcode" on the strain-based forming limits analysis

Figure 25. The tests used on the experimental determination of Forming Limit Diagram

Figure 26. Bulge test
For the experimental determination of the Forming Limit Diagrams under complex strain paths, two types of strain path changes were performed. The first one, schematically represented in Figure 27, involves a prestrain in uniaxial tension followed by a bulge test with cylindrical and elliptical dies respectively. Moreover, Figure 27 shows the specific samples for each involved strain path. It can be observed that the bulge sample is cut from the prestrained uniaxial tension large sample. In the present work two different uniaxial tension prestrains, up to 7% strain and 14% strain respectively, were considered.

![Figure 27. The complex strain path UT-BS: Prestrain in uniaxial tension – Reloading in biaxial stretching](image)

The second one, involving a prestrain in equibiaxial stretching followed by a reloading in uniaxial tension is sketched in Figure 28, that also shows the corresponding specimens necessary at each particular test. Thus, it is observed that the reloaded uniaxial tensile samples have been cute from
the pole of the bulge specimens, which are prestrained up to 5% strain in this analysis, the tensile axis being parallel to the rolling direction.

Figure 28. The complex strain path BS-UT:
Prestrain in biaxial stretching– Reloading in uniaxial tension

For each performed test, two strain distributions, namely along the rolling direction and along the transverse direction were determined. Hence, Figure 29 presents the respective strain distribution obtained in uniaxial tension at 0° from RD, for a DC06 (0.8mm) steel sheet. The point corresponding to localized necking is determined by applying a mathematical method of parabolic regression calculation, according to Bragard method. Notice that, the highest points of the strain distribution profile are eliminated and only the measures correspondent to differences of the strains between two neighboring circles less than 5% (method N° 5 of Zürich) are used for interpolation.
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![Figure 29. Strain distribution of DC06 steel sheet during uniaxial tension at 0° from RD](image)

The Figures 30, 31, 32 and 33 show the results of the experimental FLDs for the investigated linear and complex strain paths of each particular material. It is remarked that under linear strain paths all regions between uniaxial tensile along rolling direction and uniaxial tension along the transverse one were covered. Moreover, besides the previously mentioned complex strain paths, for AA6016-T4 additional strain histories were performed, involving prestrains up to 7% and to 14% in uniaxial tension at 90° from RD followed by equibiaxial stretching and by stretching with elliptical matrix.

In all cases, an evident formability reduction is observed for plane strain conditions, thus corroborating previous results obtained for several metals (Barata da Rocha, 1984). Moreover, it is showed, experimentally as well as theoretically, that the lowest forming limit, in terms of proportional loadings, is obtained for the plane strain deformation state. Barlat (1989), through the M-K analysis, explains this effect due to the fact that in this deformation state, the materials cannot take advantage of the "yield surface shape hardening" (Lian et al., 1979) and only work-hardening is available to counterbalance the thickness imperfection growth.

It is also noticed a lower forming limit, in the biaxial stretching zone, for aluminium alloys than for steel sheets, confirming that BCC materials have a better stretchability than FCC materials (Barlat, 1987). Additionally, it is interesting to note a higher limiting strain in uniaxial tension in the transverse direction than in the rolling direction for the BH steel, while for the other materials, almost the same uniaxial tension limit is observed for both directions. It can be remarked the strong dependence of the limit strains on the strain path history, corroborating previous works (see Section 2.5.4). A considerable increase of limit strains in uniaxial prestrain followed by equibiaxial stretching as well as a premature instability for equibiaxial prestrain followed by uniaxial tension were observed, being in agreement with Barata da Rocha's results (1984).
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Figure 30. Experimental FLDs under linear and complex strain paths for AA5182-0-0

Figure 31. Experimental FLDs under linear and complex strain paths for AA6016-T4
Figure 32. Experimental FLDs under linear and complex strain paths for DC06 steel sheet

Figure 33. Experimental FLDs under linear and complex strain paths for BH steel sheet
5.4 Theoretical Forming Limit Diagrams

On the base of its structure, FLDeCode allows to a detailed theoretical study on Forming Limit Diagrams, which are presented in the following.

5.4.1 Influence of the Constitutive Equations in Forming Limit Diagram prediction

Through the use of several combinations of constitutive equations, which can be seen in Figure 34, to predict the plastic flow localization with M-K method, some interesting comparisons have been performed, in order to assess the effects of constitutive equations on Forming Limit Diagrams during linear and complex strain paths.

Figure 34. The theoretical combination of constitutive equations used in FLDs prediction

i) Hardening law influence in forming limit prediction

The influence of hardening law is pointed out in Figure 35 by the use of two different hardening laws, namely Swift hardening model and Voce equation, whereas the yield shape is described by the same yield function. Concerning the FLDeCode use, in this case only the “Hardening Law Subroutine” was changed as can be observed in Figure 34.

a) FLDs obtained by Von Mises yield function  b) FLDs obtained by Hill 1948 yield function
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Figure 35. Hardening law influence on AA6016-T4 FLDs during linear strain paths

Remembering that a unique defect value, specific for each material, was assumed in simulations, these results show that the Forming Limit Diagrams obtained using Swift equation are always higher than those obtained using Voce equation, whatever yield function is used. The most studied strain path changes are uniaxial tension followed by biaxial stretching (UT-BS), which considerably increase the forming limit strains, and biaxial stretching followed by uniaxial tension (BS-UT) that implies premature instabilities. Therefore, the influence of hardening law was pointed out for these two strain path changes and is presented in Figure 36. It can be seen that during complex strain paths the computed forming limit curves obtained using Swift hardening model are always higher than the simulated curves obtained by use of Voce equation. Obviously, this conclusion is a consequence of the hardening law influence in forming limits prediction during linear strain paths.

Figure 36. Hardening law influence on AA6016-T4 FLDs during complex strain paths

a) FLDs obtained by Von Mises yield function coupled with Swift or Voce equation
b) FLDs obtained by Hill 1948 yield function coupled with Swift or Voce equation
c) FLDs obtained by Yld96 yield function coupled with Swift or Voce equation

Figure 36. Hardening law influence on AA6016-T4 FLDs during uniaxial tension followed by biaxial stretching (UT-BS) and biaxial stretching followed by uniaxial tension (BS-UT).

Since the hardening law was identified in uniaxial tension, the considerable difference observed between the predicted forming limit strains under uniaxial tension, obtained by using different hardening laws, is attributed to the geometrical defect value, which is assumed in the simulation (see Figure 10*).

Barata da Rocha (1984a) has shown that the angle, which minimizes the forming limit strains, depends on the strain paths and for a given material, to every linear strain path corresponds a critical orientation of the geometrical defect.

a) Yld 96
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Figure 37. Evolution of limit strain with initial band orientation during uniaxial tension for AA6016-T4

It is interesting to see if there is some influence of the considered hardening law on such critical angle. For this purpose, Figure 37 presents the influence of the initial orientation of the M-K geometrical defect on the strain limits corresponding to the flow localization after uniaxial tension. A strong influence of the considered hardening law on the critical orientation of the groove is observed. In the present case of AA6016-T4, the initial orientation, which minimizes the computed plastic strains, is 0.3 radians by use of Swift equation and 0.4 radians by use of Voce equation, independently of the applied yield function. In consequence, the critical orientation of the M-K narrow band is dependent by the hardening law considered to describe the material work hardening. Moreover, must be noticed that if the same critical orientation of the geometrical defect, for both hardening models is assumed, different values on the corresponding strains are also found. Such difference is probably due to the hardening model evolution around the necking localization moment. Similar results were obtained for a BH steel sheet and will be afterwards presented in the next chapter.

On continuity of this analysis, Figure 38 shows the evolution of the limit strains with initial band orientation during equibiaxial stretching and the respective hardening law influence. It can be observed a significant difference between the attained limit strain values by use of the Swift law and Voce equation respectively, for each considered yield function. At the same time, no influence of the hardening law on the critical orientation of the M-K defect is found, this one being achieved at 0.0 radians. Additionally, it is remarked the influence of the yield function on the shape of such evolution of major limit strain with initial band orientation and also on its level.

In conclusion, beside of the strain paths, the hardening model used in the M-K analysis also influences the angle, which minimizes the forming limit strains while no effect of the yield function on it was found.
Figure 38. Evolution of limit strain with initial band orientation during equibiaxial stretching

**ii) Effect of the yield surface shape in FLDs**

The influence of the Yield Function used in the simulation (Figure 8) on the FLD prediction is illustrated in the Figures 39, 40, 41 and 42.

Figure 39. Theoretical FLDs under linear strain paths obtained by use of Swift equation and several yield functions
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Figures 39 and 41 present the experimental and theoretical FLDs simulated when the Swift hardening model is considered and several yield functions are used to describe the yield surface during linear and complex strain paths respectively. Figures 40 and 42 present the same results with Voce equation.

As expected, in both cases the simulated FLDs present the same shape but the forming limit strain values are different. Comparing with the experimental data the best agreement was found when the shape of yield locus was described by Yld96 criterion and the hardening law represented by Voce equation. At the same time, Hill’s 1948 yield criteria overestimated the forming limit in biaxial stretching. Although Hill’s 1979 criterion leads to a satisfactory limit strain in plane strain and biaxial stretching, it does not reproduce the experimental shape of the FLD correctly. Comparing these forming limit predicted results with the predicted shape of yield locus (Figure 19) a strong influence can be observed on the right side of the diagram. Thus, the yield function has a strong effect on the shape of the yield surface and a tremendous effect on the level of the Forming Limit Diagram in the biaxial region as previous studies also have shown (Barlat, 1987; Lian et al. 1989, see more details in Section 2.5.1).

Figure 40. Theoretical FLDs under linear strain paths obtained by use of Voce equation and several yield functions

Moreover, a strong influence of the yield functions in uniaxial prestrain followed by biaxial stretching (UT-BS) can be observed. In balanced biaxial prestrain followed by uniaxial tension (BS-UT) this effect is not so pronounced. Also, no yield function influence is found in BS-UT when Voce equation is used. The explanation of these results consists in the remarkable effect of the shape of the yield surface on the stretchability, in the right part of the Forming Limit Diagram in linear strain paths and in the insignificant effect in the left part of the diagram, as can be observed from Figures 38 and 39. In Figures 40 and 41 it is shown that besides the strong influence of the yield
function in UT-BS whatever the hardening law is considered, a low effect of the yield surface shape in BS-UT is detected when Swift hardening law is used and no effect of yield functions when Voce equation is used.

Figure 41. Theoretical FLDs during complex strain paths (UT-BS and BS-UT) obtained by using Swift equation and several yield functions

Fig. 42 Theoretical FLDs during complex strain paths (UT-BS and BS-UT) obtained by using Voce equation and several yield functions
iii) Influence of balanced biaxial yield stress on FLDs

Recently, in order to attain higher performances, the balanced biaxial yield stress was introduced in the coefficients identification of the newest developments of yield functions (Barlat et al., 1997; Banabic et al., 2000b; Cazacu and Barlat, 2002; Barlat et al., 2003; Banabic et al., 2004). Through the use of Barlat Yld96 yield criterion, the effect of this mechanical parameter in the shape of yield locus and consequently on Forming Limit Diagram under proportional and non-proportional loadings can be studied. Such results can be observed in Figures 43, 44 and 45 mentioning that in all these Figures, the so called “ratio” denotes the balanced biaxial yield stress divided by the uniaxial yield stress at 0 degrees from rolling direction.

![Figure 43. Influence of balanced biaxial yield stress on the shape of yield surface](image)

As expected and as observed from Figure 43, the balanced biaxial yield stress defines the shape of the yield surface on the stretching zone. Consequently, it has a strong effect on the forming limit strains in the stretching zone as Figure 44 also shows. An increase of balanced biaxial yield stress value leads to a decrease of the biaxial stretching limit strain. Moreover, no influence is found in the left part of the diagram.

In Figure 45 it can be observed the evolution of forming limit curves during complex strain paths varying the balanced biaxial yield stress value. The conclusion is that an increase of balanced biaxial yield stress leads to a dramatic decrease of formability during a uniaxial prestrain followed by biaxial stretching. At the same time no effect is found for a biaxial stretching followed by uniaxial tension.
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Figure 44. Influence of balanced biaxial yield stress on the Forming Limit Diagram under linear strain path

Figure 45. Influence of balanced biaxial yield stress on the Forming Limit Diagram under complex strain paths (UT-BS and BS-UT)
Thus, besides the influence of the anisotropic coefficients, the work hardening coefficient and strain rate sensitivity on the FLDs, the balanced biaxial yield stress seems to be a very important parameter, which must be taken in account. This is an important result obtained theoretically, through the use of FLDCode.

5.5 Comparison between experimental and theoretical FLDs

The numerical simulation success consists in its ability to entirely reproduce the experimental data with a very good accuracy. As is known, a lot of factors influences the final results but the most important on FLD prediction through the M-K analysis still to be the applied constitutive equations. Figure 46 presents the experimental forming limits for AA6016-T4 aluminium alloy under proportional and non-proportional loading and the computed ones, when the shape of yield surface is described by the Yld96 yield function and the hardening model is expressed by Voce equation.

![Experimental and theoretical Forming Limit Diagram under linear and complex strain paths for AA6016-T4 aluminium alloy](image)

Figure 46. Experimental and theoretical Forming Limit Diagram under linear and complex strain paths for AA6016-T4 aluminium alloy
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It can be observed a high standard of accuracy on the predicted curves as much in linear strain path as in uniaxial tension followed by biaxial stretching (UT-BS) for sheet samples cut along rolling and transverse direction, covering in this way all range between uniaxial tension at 0° from RD and uniaxial tension at 90° from RD. Moreover, the forming limit curve computed for biaxial stretching followed by uniaxial tension (BS-UT) is close to the experimental point but isn’t the lowest curve on the diagram as Nakazima et al. (1968) and Kikuma & Nakazima (1971) have shown in their experimental work. As in previous theoretical studies of Barata da Rocha (1984) the lowest one still to be the curve computed in biaxial stretching followed by plane strain (BS-PS).

Figure 47 summarizes the FLDs for two distinguished cases (Figure 47*) namely when the major principal strain ε₁ is parallel and perpendicular respectively to the RD, both during uniaxial prestraining and in subsequent FLD testing which consists on the variation of the subsequent strain ratio simulating strain paths range between uniaxial tensile tests and equibiaxial stretching.

![Diagram of FLDs](image)

**Figure 47.** Experimental and theoretical FLDs under non-proportional loading (UT-X) for AA6016-T4 aluminium alloy

Experimentally, stretching with elliptical die rings after 7% and 14% uniaxial prestrain along rolling and also transverse direction were performed. It is observed in the first case, that the tensile prestrain, along the RD, shifts the whole FLD to the left, rising the strain limits in biaxial tension.
region and consequently increasing the slope of the right-hand side of the FLD. Similar results are reached for the second case of tensile prestrain along the transverse direction, with the difference that the prestraining shifts the curves to the right. In this case it is also remarkable the very good agreement between experimental data and the predicted curves. In both cases, the leftward respective rightward shift of the FLDs for small prestrains with no increase in the level of the minimum indicates a decrease in formability. As Graf and Hosford (1993) mentioned, Zandrahimi and Wilson attributed a similar decrease of formability to a rearrangement of the dislocation substructure, which produces an unstable flow \( \frac{d\sigma}{d\varepsilon} < \sigma \) during the early stages of the new strain path until the hardening rate \( \frac{d\sigma}{d\varepsilon} \) approaches that characteristic of the new strain path.

Relating to AA5182-0 aluminium alloy, from Figure 48 it can be also observed the excellent agreement between the experimental data and the predicted curve obtained by using the Yld96 yield function and Voce equation, during linear strain path as in uniaxial tension followed by biaxial stretching (UT-BS). Similar with the prior observations made for AA6016-T4 aluminium alloy, the forming limit curve computed for biaxial stretching followed by uniaxial tension (BS-UT) is very close to the experimental point and to the lowest curve computed in biaxial stretching followed by plane strain (BS-PS). Moreover, the experimental point obtained in prestrain at 7% followed by stretching with elliptical matrix is near to the corresponding predicted curve.

![Figure 48](image.png)

**Figure 48. Experimental and theoretical FLDs under linear and complex strain paths for AA5182-0 aluminium alloy**

The very good performance of Yld96 yield function in forming limit prediction for linear and two strain path changes for the selected aluminium alloys is explained by its ability to accurately describe the respective material behaviour.
For the sake of a non-tedious analysis, the results for the studied DC06 and BH steel sheets will be presented in a context of another analysis to point out some interesting aspects like influence of thickness of the sheet on the FLDs or a comparison of a phenomenological and a physical approach of Plasticity Theory on the FLD prediction.

5.5.1 Influence of the sheet thickness on the FLDs

In the next investigation the influence of the sheet thickness on the FLDs was analysed, which is a subject of discussion since a long time, many related studies presenting contradictory results. For this purpose, a DC06 steel sheet was used, with two different thickness values, namely 0.8 mm and 1.5 mm respectively. Considering that it is important and useful to observe the effect of the sheet thickness on the mechanical behaviour of the material, a study of such effect on the most relevant mechanical parameters of material is firstly performed. Consequently, in Figure 49, comparing the true stress - true strain curves obtained during uniaxial tension at 0°, 45° and 90° from RD, for both cases, no significant difference are observed. Also weak anisotropic flow behaviour in the sheet plane is detected. On the contrary, the anisotropy factors values are considerable affected by the sheet thickness. Thus, a thinner sheet is characterized by higher values of the anisotropy factors than a larger thickness sheet. In terms of normalized yield stresses an increase of thickness sheet allows an increase of normalized yield stress. Additionally, Figures 50 and 51 show a very good agreement between experimental data and the predicted anisotropy coefficients and normalized yield stress distributions obtained by using the Yld96 yield function.

![Figure 49. Thickness effect on the true stress - true strain curve for DC06 steel sheet](image)

After analysing how the sheet thickness influences the mechanical behaviour of the material, our attention is paid for what is happening in terms of formability. Thus, Figure 52 expresses the experimental and theoretical forming limit diagram during linear strain paths for both kind of sheets. It is remarked that from an experimental point of view the sheet thickness does not affect in a noteworthy mode the forming limits. Except the uniaxial tension case where the forming limit for the sheet of 1.5 mm thickness is slightly lower than the one of 0.8 mm thickness sheet, the material formability keeps a same level. Concerning the predicted curves, a clear differentiation on the stretching zone is observed. Thus, an increase of thickness sheet improves the material formability on equibiaxial stretching zone. Also, it is notable the excellent accuracy on the reproducibility of the
experimental data attained by use of the Yld96 yield function, excepting the uniaxial forming limit at 90° from rolling direction, which are overestimated.

Figure 50. Thickness effect on the distribution of anisotropy coefficients for DC06 steel sheet

Figure 51. Thickness influence on the normalized yield stress as a function of the tensile loading axis for DC06 steel sheet
Concerning the influence of sheet thickness on the material formability during complex strain paths, Figure 53 shows the experimental data and the corresponding predicted curves for certain strain path changes. It can be observed, from experimental as much as theoretical sight, a considerable effect of sheet thickness on the attained forming limit strains during a uniaxial tensile prestrain followed by an equibiaxial stretching (UT-BS). An augmentation of sheet thickness increases the material formability for this type of strain path changes. Oppositely, no significant difference on the forming limit curves is found by the variation of thickness sheet for an equibiaxial stretching prestrain followed by uniaxial tensile test. For a strain history involving 7% and 14% uniaxial tensile prestrain followed by stretching with elliptical die ring, it is experimentally remarked that the influence of the sheet thickness amplifies with the increase of prestrain value. So, for a 7% prestrain the reached forming limits for both 0.8 mm and 1.5 mm thickness sheet are almost the same, while a much higher forming limit is achieved for 14% prestrain by the larger thickness sheet compared with the thinner one. Theoretically, this effect is not captured, the same slight thickness effect being found independently of the prestrain level.
Figure 53. Influence of the thickness on the FLD predictions under complex strain paths.

Coll (2000) mentioned that concerning the reasons of the thickness effect on the material formability, many studies were made. Some works claimed that there is no real influence, another ones pointed out the effect of strain rate, while most of them attributed it to size effect.

5.6 Forming Limit Diagrams prediction on the base of a phenomenological and a physical approach of Plasticity Theory

The aim of the present work is a study on Forming Limit Diagrams prediction during linear and complex strain paths on the base of both approaches of the plasticity theory, the phenomenological one and the physical one.

For this purpose two advanced constitutive models of the plastic anisotropy are considered, namely the Yld96 Barlat phenomenological yield function and the combined model of texture and strain-path-induced anisotropy called TexMic model described in previous section. The studied material is the aforementioned bake-hardening high-strength steel sheet.

The Department of Metallurgy and Materials Engineering of Leuven University (MTM, K.U. Leuven) determined the initial crystallographic texture and the Laboratory of the Mechanical and
Thermodynamic Properties of Materials of Paris-Nord University (LPMTM - CNRS, Univ. Paris Nord) performed the identification of the microstructural model. The 13 material parameters of the TexMic model are presented in Table 13.

Table 13. Material parameters of the microstructural model

<table>
<thead>
<tr>
<th>$C_F$</th>
<th>$C_R$</th>
<th>$C_{SD}$</th>
<th>$C_{SL}$</th>
<th>$C_X$</th>
<th>$m$</th>
<th>$n_i$</th>
<th>$n_p$</th>
<th>$q$</th>
<th>$\tau_0$</th>
<th>$R_{sat}$</th>
<th>$S_{sat}$</th>
<th>$X_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.53</td>
<td>31.2</td>
<td>3.51</td>
<td>4.0</td>
<td>148.6</td>
<td>0.265</td>
<td>-0</td>
<td>70</td>
<td>1.84</td>
<td>213.8</td>
<td>88.9</td>
<td>251.6</td>
<td>2.0</td>
</tr>
</tbody>
</table>

The elevated capability of the Yld96 yield function on describing the selected material behaviour has been previously shown, by simulating the anisotropy factors and normalized stress distributions, as a function of the tensile loading axis (Figures 14 and 18, respectively). The internal variables of the Texmic model characterize the influence of the deformation history on the plastic behaviour of metals, which is also important beside the current state of deformation. For sufficiently large monotonic strains, all internal variables are supposed to approach some saturation values, which correspond to the balance between the work-hardening and the recovery rates. Figure 54 illustrates the evolution of the internal variables of the Texmic model with strain, in the case of a simple tensile test. It is observed that $X$ saturates very rapidly, while all the other internal variables have a smaller rate. This is due to the numerical values of the material parameters:

$$C_X >> C_R > C_{SD}, C_{SL}, C_F$$  \hspace{2cm} (5.5)

Another source of rapid change of the internal variables is the evolution of the strain rate direction, included in their definition.

Figure 54. Time evolution of the internal variables during monotonic straining
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Figure 55. Experimental and predicted FLDs by Yld96 yield function and Swift law

Figure 56. Experimental and theoretical FLDs obtained using the TexMic model
Figure 55 shows very good predictions of limit strains in linear strain paths obtained using Yld96 and Swift hardening law. In complex strain paths the predicted limit strains are good and the forming limit curve computed for equibiaxial stretching followed by uniaxial tension (BS-UT) is very close to the experimental point, but maintains the same problem encountered for the previous analysed materials namely to be not the lowest curve of the diagram, as Nakazima (1968) experimentally found.

The combined model of texture and strain-path-induced anisotropy predicts with a very good accuracy the limit strains between the plane strain region and the equibiaxial stretching zone, but underestimates the uniaxial tension limit strain, as can be seen in Figure 56. In addition, an excellent agreement is obtained between the experimental data and the predicted curve for the uniaxial tension followed by equibiaxial stretching (UT-BS) and for the equibiaxial stretching followed by uniaxial tension (BS-UT), when the TexMic model is used. Moreover, it must be noticed that the combined model of texture and strain-path-induced anisotropy reproduces the experimental Nakazima tendency by predicting the forming limit curve under equibiaxial stretching prestrain followed by uniaxial tension as the lowest curve on the forming limit diagram. Actually, as Hoferlin et al. (1998) and Hiwatashi et al. (1998) remarked, the uniaxial tension reloading after the biaxial prestrain presents a work softening phase whose severity increases with the amount of prestrain. Such cross effect, as Rauch and Schmitt (1989) in their experimentally work suggested, results from the interaction between currently active slip systems and the previously formed dislocation structure and is described by the microstructural hardening model (Teodosiu and Hu, 1995). Moreover, the decrease of the work hardening caused by the ulterior microband formation, is responsible for the severe drop of formability, especially when the magnitude of the biaxial prestrain is high.

Another important observation is that the initial value of the M-K geometrical defect must be equal to 0.992 when the Yld96 is used to have good predicted results and equal to 0.998 when the TexMic model is used, the latter being the correct defect value according to previous work by Barlat (1989).

Next, in order to point out the effect of microstructure on the FLDs prediction for both proportional and non-proportional loadings, Swift hardening model with and without the strain rate sensitivity, Voce law and the microstructural hardening model was applied on the numerical simulation, keeping the shape of the yield locus described by the Von Mises yield function.

Thus, relating to linear strain paths (Figure 57) it is observed that the highest curve is obtained with the Swift law when sensitivity strain rate is taken into account and the lowest one when the Voce law is used, whereas there is a small difference in biaxial stretching region between the simulated curves obtained with the microstructural model and the Swift equation without strain rate sensitivity. Figure 58 shows the influence of the hardening law for the UT-BS and BS-UT strain paths. The performance of the microstructural hardening model to correctly estimate the limit curve for a strain history involving an equibiaxial stretching followed by a uniaxial deformation is remarkable.
Figure 57. Hardening Law influence on Forming Limit Diagrams during linear strain paths

Figure 58. Hardening Law influence on Forming Limit Diagrams under complex strain paths
5. Application and validation of the "FLDcode" on the strain-based forming limits analysis

Figure 59. The effect of the texture potential in FLDs prediction during linear strain paths

Figure 60. The effect of the texture potential in FLDs prediction during complex strain paths
5. Application and validation of the "FLDcode" on the strain-based forming limits analysis

![Graph](image)

a) FLDs obtained using Yld96 yield function and Swift equation

![Graph](image)

b) FLDs obtained using TexMic Model

Figure 6.1. Computed and experimental FLDs under non-proportional loading
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

Figures 59 and 60 present the effect of the texture strain rate potential on FLDs prediction during linear and complex strain paths respectively. As previous mentioned, it has been found a dramatic effect of the shape of the yield surface on the stretchability, in the right part of the Forming Limit Diagram for linear strain paths. Additionally, the analyse performed in Section (5.3.1.ii) pointed out the strong influence of yield functions in UT-BS strain path and a weak influence in BS-UT strain path. As shown in Figures 59 and 60, the predicted limit strains obtained by using the texture potential in the right part of the Forming Limit Diagram and for the strain histories involving uniaxial deformation followed by equibiaxial stretching are in excellent agreement with the experimental data proving thus its high ability on the forming limit prediction.

Figure 61 shows FLDs determined under non-proportional loading by using a sequence of two linear strain paths, keeping constant the preliminary strain ratio and the amount of prestrain and varying the subsequent strain ratio. It can be observed a good agreement between the predicted curves and the experimental points obtained in uniaxial prestrain at 7%, respectively 14%, followed by stretching with elliptical matrix for both plastic models.

5.7. Conclusions

For a validation of the more general FLDcode developed in the present work and for proving its efficiency and utility, a meticulous study on Forming Limit Diagram prediction was carried out. Four metal sheets of deep-drawing quality were selected for such study: two aluminium alloys (Al 6016-T4 and Al 1582) and two steels (a low carbon steel DC06 and a bake hardening steel).

Due to the vital importance of the material mechanical behaviour on the numerical simulation success, an experimental and theoretical analysis on such most representatives factors expressed as work hardening behaviour, anisotropy factors, normalized yield stress evolution as a function of the tensile loading angle and plane stress yield surfaces for all considered materials were performed. From all presented results, an excellent match between the experimental data revealing the plastic anisotropy of each particular material and the corresponding Yld96 predictions is observed. This success is a consequence of considering all this mechanical parameters in the Yld96 yield function coefficients identification.

The experimental Forming Limit Diagrams for each here attended material under linear and complex strain paths were performed, a general and briefly description of their determination being also presented.

To assess the effects of constitutive equations on Forming Limit Diagrams prediction during linear and complex strain paths, a theoretical study on the plastic flow localization with M-K method, using several combinations of different hardening laws and yield functions, through the FLDcode, was carried out.

It was shown that during linear and complex strain paths the computed forming limit curves obtained using Swift hardening model are always higher than the simulated curves obtained by use of Voce equation. Moreover, it was proved that the hardening model used in the M-K analysis
5. Application and validation of the “FLDcode” on the strain-based forming limits analysis

affects the angle, which minimizes the forming limit strains while no effect of the yield function on such angle value was found.

Beside a strong effect of the shape of the yield surface on the stretchability, in the right part of the Forming Limit Diagram, a considerable influence of yield functions in uniaxial prestrain followed by biaxial stretching (UT-BS) and a weak influence in balanced biaxial prestrain followed by uniaxial tension (BS-UT) were observed. Moreover, no effect of the yield function on the FLDs is found when Voce equation is used.

An increase of balanced biaxial yield stress leads to a decrease of formability on the biaxial stretching region for proportional loadings and consequently during a uniaxial prestrain followed by a biaxial stretching. At the same time no effect is found for a biaxial stretching followed by a uniaxial tension.

Concerning the accuracy of the FLDcode through the applied constitutive equations, a successful correlation between the experimental FLDs and the computed limit strains during linear and complex strain paths for AA6016-T4 as well as for AA5182-0aluminium alloys was revealed, when the shape of yield locus was described by Yld96 yield criterion and the hardening law represented by Voce equation. Moreover, although this yield function was developed to describe the behaviour of aluminium and its alloys, it appears to be very efficient in description of BCC materials behaviour as well as in their forming limits prediction when combined with Swift equation and considering the initial defect value 0.993 and 0.992 respectively, as the results obtained for both investigated types of steel, the DC06 and BH steel sheets showed.

Experimentally and theoretically, the effect of the sheet thickness on the mechanical behaviour of the material and subsequent on the FLDs was analysed. It was observed no significant difference on the work-hardening behaviour while an increase of sheet thickness leads to an increase of normalized yield stress and a decrease of the anisotropy factors values. With respect to the material formability under proportional loading, the effect of the sheet thickness is experimentally insignificant while a clear differentiation on the stretching zone is observed on the simulated curves. At the same time, during a uniaxial tensile prestrain followed by an equibiaxial stretching an augmentation of sheet thickness increases the material formability whereas no consequence is detected for an equibiaxial stretching prestrain followed by uniaxial tensile test.

Additionally, the physical approach of the plasticity theory versus the phenomenological one are pointed out in an interesting study on Forming Limit Diagrams prediction during linear and complex strain paths, by using the combined plasticity model of texture and strain-path-induced anisotropy and the Yld96 Barlat yield function through the M-K theory.

TexMic model predicts with a very good accuracy the limit strains between the plane strain region and biaxial stretching zone but underestimates the uniaxial tension limit strain, whereas a perfect agreement between the experimental data and the predicted curve was observed for complex strain paths. It was remarked the excellent ability of the microstructural hardening model to correctly estimate the limit curve for a strain history involving an equibiaxial stretching followed by a uniaxial deformation, uniquely reproducing the Nakazima experimental tendency which cannot be
predicted, by conventional phenomenological models and the excellent agreement with experimental data of the predicted limit strains obtained by using the texture potential in the right part of the FLDs and for the strain histories involving uniaxial deformation followed by equibiaxial stretching.

Through all these results it was definitely proved the huge potential and high standard of the Fldcode on the Forming Limit Diagram predictions and its efficiency in the selection of the best combination of constitutive equations to simulate the behaviour of different materials which is vital for good predictions on plastic flow localization from the M-K theory and consequently for a correct analysis on the material formability.

Moreover, the present study brings one more proof of the precious contribution of the Marciniack - Kuckzinsky theoretical model on Forming Limit Diagrams prediction.
6. APPLICATION OF THE “FLDcode” ON THE STRESS-BASED FORMING LIMITS ANALYSIS

6.1 Introduction

Industrial stampings of complex shape often involve multistage forming operations. Assuming the strong influence of strain path changes on the Forming Limit Diagrams, a stress-based forming limit concept was proposed by Kleemola and Pelkkikangas (1977), and Arrieux, Bedrin and Boivin (1982). This concept seems to be independent of the strain path changes and it’s utility was promoted as solution to the analysis of multi-stage forming processes. In a recent study, Stoughton (2000) shows that all of the apparent path-dependent effects on the forming limit vanish when properly viewed in stress coordinates. Thus, application of the stress-based forming limit extends the validity of the forming limit criterion to applications involving non-proportional loading. Moreover, this fact not only eliminates a critical obstacle to our ability to asses formability, but by forcing us to look at stress distributions we can modify the forming process to lower the stresses below the critical levels and determine the desired processing conditions more quickly.

In this chapter a detailed study on Forming Limit Stress Diagrams (FLSD) during linear and complex strain paths is developed. The experimental calculation of stress forming limits is described. The necking phenomenon was simulated by Marciniack-Kuczinsky (M-K) model using the FLDcode. The influence of the constitutive law incorporated in the analysis on the predicted limit stress is shown by use of different yield functions such as Von Mises isotropic yield function, quadratic and non-quadratic criterion of Hill (Hill 1948 and Hill 1979) and Barlat Yld’96 yield function. The effect of the hardening model on the FLSD is analysed by using several hardening laws, namely Swift law and Voce law. An analysis of the influence of work hardening coefficient, strain rate sensitivity and the balanced biaxial yield stress on the theoretical FLSD is also presented. The influence of strain path changes on the stress-based forming limits is shown and some remarks about FLSD concept are presented. The two concepts of forming limits, namely forming limit diagram and forming limit stress diagram are compared and analysed.

6.2 Experimental Analysis of the Forming Limit Stress Diagrams

The Forming Limit Stress Diagrams curve represents the forming limit diagram expressed by using the principal stress components in-plane of the sheet. The stress states cannot be determined directly on experimental parts and this operation generally needs a plastic calculation. The experimental measurement of the stress state is a difficult and not practical task and usually is necessary a plastic calculation to solve it. Based on the experimental forming limit strains and using the plasticity theory, the forming limit stresses are computed. A briefly definition of the required equations for the computation of stress state are presented. It is assumed that the principal anisotropy axes of orthotropic symmetry are coincident with the principal axes of stress. The strain path is characterized by the strain ratio

\[ \rho = \frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1} \]  

(6.1)
and stress ratio

\[ \alpha = \frac{\sigma_2}{\sigma_1} \]  

(6.2)

Beside the components of plasticity theory described in section 3, namely yield function, hardening rule and the corresponding flow rule, it is also necessary to define the effective strain rate as a function of the strain rate tensor.

For materials with in-plan isotropy or for cases with non shear stress in a coordinate system aligned with the anisotropy, the effective stress can be expressed in terms of the principal stresses:

\[ \bar{\sigma} = \bar{\sigma}(\sigma_1, \sigma_2) \]  

(6.3)

The plastic strain rates are expressed from the associated flow rule as follows:

\[ \dot{\epsilon}_{ij} = \dot{\varepsilon} \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}} \]  

(6.4)

The relation between the stress ratio \( \alpha \) and the strain ratio \( \rho \), which is obtained by the flow rule as is shown in Eq. (4.7), is required and presented as:

\[ \rho = \rho(\alpha) \]  

(6.5)

and

\[ \alpha = \alpha(\rho) \]  

(6.6)

By the flow rule and the conservation of the plastic work the plastic strain rate potential is defined, expressing the effective strain rate as a function of the strain tensor rate components as follows:

\[ \ddot{\varepsilon} = \ddot{\varepsilon}(\dot{\varepsilon}_1, \dot{\varepsilon}_2) \]  

(6.7)

The effective strain is the time integral of the effective strain rate.

\[ \bar{\varepsilon} = \int \ddot{\varepsilon} dt \]  

(6.8)

The effective stress is computed by the hardening law, which formally can be expressed as:

\[ \bar{\sigma} = \bar{\sigma}(\bar{\varepsilon}) \]  

(6.9)

From Eq. (6.2) and (6.3) results the major and minor true stresses expressed as follows:
\[ \sigma_1 = \frac{\bar{\sigma}}{\zeta(\alpha)} \quad (6.10) \]

and

\[ \sigma_2 = \alpha \sigma_1 \quad (6.11) \]

with \( \zeta(\alpha) \) a function of material parameters.

To compute the stress state after two stage strain paths it is assumed that the material starts to yield plastically during the second loading stage only when the effective stress raises to the same level of stress attained at the end of the prestrain loading stage.

By Stoughton (2000), the principal stresses at the end of the secondary stage are given by:

\[ \sigma_1 = \frac{\bar{\sigma}(\bar{\varepsilon}(\varepsilon_{1f}, \varepsilon_{2i}) + \bar{\varepsilon}(\varepsilon_{1f} - \varepsilon_{1i}, \varepsilon_{2f} - \varepsilon_{2i}))}{\zeta(\alpha(\varepsilon_{2f} - \varepsilon_{2i})/(\varepsilon_{1f} - \varepsilon_{1i}))} \quad (6.12) \]

and

\[ \sigma_2 = \alpha \left( \frac{\varepsilon_{2f} - \varepsilon_{2i}}{\varepsilon_{1f} - \varepsilon_{1i}} \right) \sigma_1 \quad (6.13) \]

where \( (\varepsilon_{1i}, \varepsilon_{2i}) \) represents the prestrain state, \( (\varepsilon_{1f}, \varepsilon_{2f}) \) the final strain state at the secondary loading stage and \( \alpha(\rho), \bar{\varepsilon}(\varepsilon_1, \varepsilon_2), \bar{\sigma}(\varepsilon), \bar{\zeta}(\alpha) \), represents the function defined by Eq. (6.6), Eq.(6.7), Eq.(6.9), and Eq.(6.10) respectively.

Previous works (Arrieux et al., 1997) show that there is the possibility to predict the forming limit in strain space for a given prestrain from a single curve in stress space representing the material forming limit.

Knowing the principal stress tensor, the equivalent stress is computed by the use of the yield function Eq.(6.3). The effective strain results from the inverse of the hardening law, expressed as:

\[ \bar{\varepsilon} = \bar{\varepsilon}(\bar{\sigma}) \quad (6.14) \]

Combination of Eq. (6.1) and Eq.(6.7) allows to compute the major and minor strain rates:

\[ \dot{\varepsilon}_1 = \frac{\bar{\varepsilon}}{\lambda(\rho)} \quad (6.15) \]

and

\[ \dot{\varepsilon}_2 = \rho \dot{\varepsilon}_1 \quad (6.16) \]
where \( \lambda(\rho) \) is a function of the material parameters.

Applying the time integration of the principal strain rates, the forming limit strains are obtained. In complex loading case, the forming limit strains derived from a locus of forming limits in stress space \((\sigma_1, \sigma_2)\) and a prestrain of \((e_{1i}, e_{2i})\) are given by

\[
\varepsilon_{1f} = e_{1i} + \bar{\varepsilon}(\sigma_1, \sigma_2) - \bar{\varepsilon}(e_{1i}, e_{2i}) \frac{\lambda(\rho)}{\sigma_1} (6.17)
\]

and

\[
\varepsilon_{2f} = e_{2i} + (e_{1f} - e_{1i}) \rho \left( \frac{\sigma_2}{\sigma_1} \right) (6.18)
\]

where \( \rho(\sigma), \bar{\varepsilon}(\sigma), \bar{\varepsilon}(\varepsilon_1, \varepsilon_2), \lambda(\rho) \), represents the function defined by Eq. (6.5), Eq.(6.14), Eq.(6.7), and Eq.(6.15) respectively.

These equations show that the experimental forming limit stress diagram depends on the shape of the yield surface as well as the hardening law used to describe the work hardening material behaviour. Explicit definitions of these equations are given in Stoughton (2000), by applying the Hill’s quadratic and non-quadratic normal anisotropic plastic potentials, Hosford’s non-quadratic normal anisotropic plastic potential and Hill’s quadratic generally anisotropic plastic potential.

### 6.3 Theoretical Analysis of the Forming Limit Stress Diagrams

The theoretical Forming Limit Stress Diagrams presented in this work were calculated by using FLDC ode, the more general code for FLD’s prediction, whose detailed description is presented in Chapter 4. The general structure of the FLDC ode including the prediction of the stress-based forming limit is shown in Figure 1.

Marciniack – Kuczynsky necking criterion is reached when the effective strain increment in the groove \((d\bar{\varepsilon}^b)\) is ten times greater or equals to that in the homogeneous zone \((d\bar{\varepsilon}^h)\). Following the theoretical treatments for the computation of the forming limit strains (Chapter 4), the corresponding stresses \((\sigma_1^a, \sigma_2^a)\) accumulated in the homogeneous zone at the moment of the plastic flow localization are the limit stresses. The analysis is repeated for different initial orientations \((\psi_0)\) of the groove in the range between 0 and 90 degrees and the limit point on the FLSD is obtained after minimization of the curve \(\sigma_1^a\) versus \(\psi_0\).

Several constitutive equations, namely, Von Mises yield function, quadratic Hill's criterion (Hill 1948), non-quadratic Hill’s criterion (Hill 1979), Yld96 Barlat yield criterion combined with Swift hardening law or Voce hardening law, whose review can be found in the Section (4.3.1), are considered in the theoretical simulation of the FLSD.
In order to assess the influence of the strain paths changes on the stress-based forming limit, different strain histories involving prestrain in uniaxial tension, plane strain stretching and biaxial stretching respectively followed by several strain paths between uniaxial tension and biaxial stretching are simulated and analysed.

6.4 Results and discussion

6.4.1 Materials characterization

Three sheet materials were considered in our study, namely a bake-hardened steel (BH steel), an AA6016-T4 aluminium alloy and a pure aluminium (A1). Their mechanical behaviour were determined in uniaxial tension along different directions, for BH steel and AA6016-T4 aluminium sheet materials being previous presented (Chapter 5) while Table 1 presents the mechanical characteristics of aluminium A1.

In all simulations the M-K initial geometrical defect value (f0) is considered to be equal to 0.998.

Table 1. A1 uniaxial plastic properties at 0°, 45° and 90° from rolling direction

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Yield Stress $\sigma_Y$ [MPa]</th>
<th>Ultimate tensile strength $\sigma_{uts}$ [MPa]</th>
<th>Uniform elongation</th>
<th>r value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>30</td>
<td>91.5</td>
<td>24%</td>
<td>0.93</td>
</tr>
<tr>
<td>45°</td>
<td>30</td>
<td>98</td>
<td>29%</td>
<td>0.52</td>
</tr>
<tr>
<td>90°</td>
<td>30</td>
<td>90</td>
<td>25%</td>
<td>1.23</td>
</tr>
</tbody>
</table>
The Swift equation identified for Aluminium A1 is expressed as follows:

$$\bar{\sigma}(\bar{\varepsilon}) = 132.6(\bar{\varepsilon} + 0.0002)^{0.24} \quad (6.19)$$

6.4.2 Experimental Forming Limit Stress Diagrams

Due the necessity of a plastic calculation for the determination of the experimental forming limit stress diagrams a study about the influence of the constitutive equations on the experimental forming limits will be presented.

i) Influence of the hardening law on the experimental FLSD

Figures 2, 3 and 5 show the forming limit stress curves for AA6016-T4 aluminium alloy obtained on the base of experimental FLDS and by using two hardening models namely, Swift equation and Voce equation whereas the shape of yield function is described by the same yield function. A considerable influence of the hardening law in the biaxial stretching zone is noticed. Whatever the yield function is used the Swift equation contributes for a limit stress on the biaxial stretching region higher than the one obtained by use of Voce equation. On the strain path range between the uniaxial tension and plane strain no dependence of the calculated forming limit stresses on the hardening law is observed.

![Figure 2. Experimental FLSDs obtained by use of Von Mises yield function coupled with Swift or Voce equation](image-url)
Figure 3. Experimental FLSDs obtained by use of Hill'48 yield function coupled with Swift or Voce equation.

Figure 4. Experimental FLSDs obtained by use of Hill'79 yield function with Swift or Voce equation.
A notable remark is that the experimental uniaxial tensile stress limit seems to be slightly greater than the ultimate tensile strength for AA6016-T4 aluminium alloy, which is 280MPa as Table 1 from Chapter 5 shows.

Table 2. Experimental and computed uniaxial tension data of AA6016-T4 aluminium alloy

<table>
<thead>
<tr>
<th>Constitutive Equations</th>
<th>$\varepsilon_{1\text{exp}}$</th>
<th>$\varepsilon_{2\text{exp}}$</th>
<th>$\bar{\varepsilon}$</th>
<th>$\sigma_{uts}$ [MPa]</th>
<th>$\bar{\sigma}$ [MPa]</th>
<th>$\sigma_1$ [MPa]</th>
<th>$\sigma_2$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Von Mises Voce</td>
<td>0.22</td>
<td>-0.1</td>
<td>0.19</td>
<td>280</td>
<td>0.2203</td>
<td>292.369</td>
<td>300.815</td>
</tr>
<tr>
<td>Von Mises Swift</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>291.600</td>
<td>300.023</td>
<td>17.648</td>
</tr>
<tr>
<td>Hill48 Voce</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>292.3045</td>
<td>290.944</td>
<td>0</td>
</tr>
<tr>
<td>Hill48 Swift</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>291.5098</td>
<td>290.153</td>
<td>0</td>
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</tbody>
</table>

Observing the data presented in Table 2, it can be noticed that the equivalent strain $\bar{\varepsilon}$ up to the maximum load, calculated through the assumed plastic potentials, having as input the experimental forming strain limits, is greater than the experimental one $\varepsilon_{exp}$. This obviously leads to a higher equivalent stress $\bar{\sigma}$ than the experimental ultimate tensile strength $\sigma_{uts}$, and implicitly to higher stress limits. It is also observed that by using the Hill 48 plastic potential a better uniaxial tensile stress limit is obtained, compared with that obtained by use of Von Mises plastic potential. A more accurate plastic potential, able to accurately compute the equivalent strain up to the maximum load, could be solution of this problem. Additionally, it is notable that the forming strain limits corresponding to uniaxial tension test leads to different stress ratio, depending on the plastic potential applied in the plastic calculation of the stress state.

On the base of this remark, it was observed that the experimental forming stress limits presented by Stoughton (2000) for the aluminium used in Graf-Hosford study (1993), are around 350.3 MPa while the experimental ultimate tensile strength is 282.5 MPa, as is shown in Graf and Hosford (1993a). The same problem was noticed for the data presented in Figures 9, 10 and 11, which were calculated by Laboratoire de Mécanique Appliquée CESALP-Université de Savoie. Table 3 shows the respective data corresponding to uniaxial tension test at 0° and 90° from RD.

While for aluminium AL1 the situation is almost the same like in the AA6016-T4 case, a considerable discrepancy between the calculated forming limit stress values and the corresponding experimental $\sigma_{uts}$ for Titanium and especially for Soldur34, is remarked. Part of this result could be attributed to the use of the Von Mises yield function, in such plastic calculation.

Table 3. The stress forming limits compared with $\sigma_{uts}$ for different materials

<table>
<thead>
<tr>
<th>Materials</th>
<th>Degree to RD</th>
<th>$\sigma_{uts}$ [MPa]</th>
<th>$\sigma_1$ [MPa]</th>
<th>$\sigma_2$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL1</td>
<td>0°</td>
<td>90</td>
<td>~105</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>90°</td>
<td>90.5</td>
<td>~105</td>
<td>0</td>
</tr>
<tr>
<td>Soldur 34</td>
<td>0°</td>
<td>554</td>
<td>~800</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>90°</td>
<td>580</td>
<td>~700</td>
<td>0</td>
</tr>
<tr>
<td>Titanium</td>
<td>0°</td>
<td>488</td>
<td>~600</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>90°</td>
<td>458</td>
<td>~600</td>
<td>0</td>
</tr>
</tbody>
</table>
ii) Influence of the yield surface on the experimental FLSD

Figure 5 and Figure 6 present the AA6016-T4 experimental FLSDs obtained on the base of the experimental FLDS and by using three different yield functions namely Von Mises isotropic yield function, quadratic Hill 1948 yield function and nonquadratic Hill1979 yield function whereas the hardening behaviour of the material is described by the same equation, either Swift equation or Voce equation. A slightly difference in the strain path range between uniaxial tension and plane strain and a more accentuate difference on the predicted limit stresses in the biaxial stretching region are observed. On the studied case, the highest limit stress curve is predicted by Von Mises yield function and the lowest is the one predicted by Hill'48 yield function. The balanced biaxial limit stress calculated by using Hill 1979 yield function tends to the same value of the balanced biaxial limit stress calculated by using Von Mises yield function.

![Graph showing FLSD](image)

Figure 5. Experimental FLSDs obtained by Swift hardening law and several yield functions

iii) Influence of the strain path changes on the experimental FLSD

The most important characteristic of the stress- based forming limit concept is its independence of the strain path changes. This significant conclusion is a result of previous studies applied on different materials and using on the required plastic calculation different yield functions such as Von Mises yield function, quadratic (1948) and nonquadratic (1979) Hill's yield criterions and Hosford's's non-quadratic normal anisotropic plastic potential (Arrieux, 1982, Nguyen, 1994; Stoughton, 2000) combined with Swift or Voce equation.
6. Application of the “FLDcode” on the stress-based forming limits analysis

Figure 6. Experimental FLSDs obtained by Voce hardening law and several yield functions

In order to observe the limit stress state behaviour for the materials considered in our study, the experimental FLSDs after proportional and non-proportional loadings for AA6016-T4 aluminium alloy and a BH steel are presented in Figures 7 and 8 respectively. A uniaxial tension prestrain at 7% and 14% and a biaxial stretching prestrain at 5 %, respectively followed by several strain paths between uniaxial tension and biaxial stretching were considered.

For the required plastic calculation, the shape of the yield surface is approximated by either Von Mises yield function or Hill 1948 yield function and the hardening behaviour is described by the Voce equation for the aluminium alloy and Swift equation for the BH steel.

As it was expected, in all these cases, the forming limit stresses are overlaid in a single curve and no dependence on the strain path changes is observed.
6. Application of the "FLDcode" on the stress-based forming limits analysis

Figure 7. Experimental FLSDs of AA6016-T4 during linear and complex strain paths

Figure 8. Experimental FLSDs of BH steel during linear and complex strain paths
Some more experimental FLSDs that were determined by Laboratoire de Mécanique Appliquée CESALP-Université de Savoie are presented in the Figures 9, 10 and 11.

The studied materials are a pure aluminium (Al), a commercial steel SOLDUR34 and Titanium. The experimental forming stress limits are determined from the experimental forming limit diagrams measured with a Marciniak device, using a step-to-step calculation along the experimental strain paths. It is assumed that the symmetry axes of the Marciniak rectangular blanks are overlaid with the principal strain directions. For the stress matrix calculation the Hill 1948 yield function is used.

These experimental forming limit curves are determined after proportional and non-proportional loading at 0° (a) and 90° (b) degrees from rolling directions. It was considered a 10% uniaxial tension prestrain (TENSION) and a biaxial stretching prestrain (STRETCHING) at 9% for aluminium, 11% for steel and 12% for titanium respectively, followed by several strain paths covered all range between uniaxial tension and biaxial stretching.

It is observed a tight dispersion of the resulted limit stresses, which map to a narrow band whose boundaries can define a lower and an upper forming limit stress curve as suggested by Arrieux (2000).

![Flsd Diagrams](image)

\[ a) \text{0° from rolling direction} \quad b) \text{90° from rolling direction} \]

Figure 9. Experimental FLSDs of pure aluminium during linear and complex strain paths

It is observed that for a given material, at 0° as at 90° from rolling direction the stress- based forming limits have similar values, with a slightly difference on the shape of the curves.

Also, it is noticed a gradual increase of the forming limit stress from the uniaxial tension to balanced biaxial stretching for Titanium sheet whereas a smooth behaviour on the limit stresses of the Al aluminium and Soldur 34 steel sheets is achieved.
Figure 10 Experimental FLSDs of Soldur 34 during linear and complex strain paths

Figure 11 Experimental FLSDs of Titanium during linear and complex strain paths
6.4.2 Theoretical Forming Limit Stress Diagrams

In previous studies by Arrieux (1994, 1997), Nguyen (1994), Vacher (1998) theoretical study on the stress-based forming limit diagrams prediction was realized only by use of the isotropic von Mises yield criterion and quadratic Hill's yield criterion coupled with Swift equations. In the present work, several combinations of constitutive equations have been used for the prediction of plastic flow localization with M-K method.

i) Hardening law influence on FLSD prediction

Figure 10 points out the influence of the hardening law on the predicted limit stress and predicted limit strains by using two hardening laws, namely Swift hardening model and Voce law, while the yield shape is described by the same yield function.

Figure 10.a. FLSD and FLD obtained by using Von Mises yield function and several hardening laws

Figure 10.b. FLSD and FLD obtained by using Hill 1948 yield function and several hardening laws
These results show that the highest curve is obtained with the Swift law and the lowest one when the Voce law is used. It is observed a stronger influence of hardening law on FLSD in equibiaxial stretching region than in uniaxial tension and plane strain region, whereas the FLD is more influenced by the hardening law in uniaxial tension region than in equibiaxial stretching region when Hill 1948 and Yld96 yield functions are used to describe the yield locus. It must be remarked that since no hardening law influence on the forming limit stress, in uniaxial tension region, was experimentally found, the hardening law influence obtained theoretically is due to the fact that a unique geometrical defect value, as was previously mentioned, was considered in simulation. Figure 10* proves this remark, by presenting the predicted forming stress limits for AA6016-T4 when different values of the defect are considered, namely equal to 0.998 for Voce
equation and equal to 0.98 for Swift equation. In this way, both hardening models achieved the best accuracy on the uniaxial tension stress limits. Hence, no influence of the hardening model is also theoretically found.

![Figure 10](image)

**Figure 10**. Influence of the hardening law on the FLSD of AA6016-T4, when different defect values are used

To a better assessment of the correlation between predicted limit stress and predicted limit strains, the major principal stress – major principal strain curve during uniaxial tension and balanced biaxial stretching obtained by using Swift hardening law or Voce hardening law respectively were plotted and presented in the Figures 11 and 12. The stronger influence of the hardening model on the stress space of forming limits during balanced biaxial stretching than in uniaxial tension is clearly illustrated in these figures 11 and 12. It is also discerned that the amount of the hardening law influence is much affected by the applied yield function during the uniaxial tension strain path.

![Figure 11a](image)

**a) $\sigma_t$-$\varepsilon_t$ curve during uniaxial tension**

![Figure 11b](image)

**b) $\sigma_t$-$\varepsilon_t$ curve during balanced biaxial stretching**

Figure 11. Major principal stress – major principal strain curve during uniaxial tension and balanced biaxial stretching obtained by using Von Mises yield function coupled with Swift hardening law or Voce hardening law respectively.
6. Application of the “FLDcode” on the stress-based forming limits analysis

Fig. 12. Major principal stress – major principal strain curve during uniaxial tension and balanced biaxial stretching obtained by using Yld96 yield function coupled with Swift or Voce hardening law respectively.

A clear view by comparing the forming limits during uniaxial tension and balanced biaxial stretching is presented on the Figures 13 and 14.

Figure 13 Comparison between $\sigma_1$-$\varepsilon_1$ curves during uniaxial tension and balanced biaxial stretching
6. Application of the “FLDcode” on the stress-based forming limits analysis

Fig. 14. Comparison between $\sigma_1$-$\varepsilon_1$ curves during uniaxial tension and balanced biaxial stretching

It can be concluded that a higher forming limit is attained during balanced biaxial stretching than the one attained during uniaxial tension in all cases, but the difference between both of them depends on the assumed hardening model. Voce equation implies a considerable difference in terms of forming limit strains and an insignificant discrepancy in terms of forming limit stresses during the specified strain paths (UT and BBS). By use of the Swift equation it is detected a well-defined difference between the forming limits as much in strains as in stresses and a great influence of the yield function on the evolution of this discrepancy. For example, the limit strains achieved during uniaxial tension is significant lower that the one attained in balanced biaxial stretching by use of Von Mises yield function whereas this difference is much reduced when Yld96 yield function is used (See Figures 13.a and 14.a).

Figure 15. Influence of the initial orientation of the M-K geometrical defect on the predicted forming strain (a) and stress (b) limits during uniaxial tension
Figure 15 presents the influence of the initial orientation of the M-K geometrical defect on the strain and stress state, respectively, corresponding to the flow localization after uniaxial tension for a BH steel and by use of Yld96 yield function. The obtained results corroborate the strong influence of the considered hardening law on the critical orientation of the groove observed in the previous chapter. In the present study, when Swift equation is used, the initial orientation, which minimizes the computed plastic strains is 0.3 radians whereas by use of Voce equation the critical groove orientation is found at 0.5 radians. The same conclusion was observed by use of Von Mises yield function and Hill'48 yield function. In addition, it is interesting to observe that when Voce hardening equation is used, the angle, which minimizes the forming limit strains, is 0.4 radians for the aluminium alloy and 0.5 radians for the steel sheet while no change is observed when Swift law is used.

**ii) Effect of the yield surface shape on FLSD prediction**

Previous works (Barlat, 1989; Lian et al. 1989) have shown the drastic effect of the shape of the yield surface on the stretchability, in the right part of the Forming Limit Diagram in linear strain paths.

![FLSD graph](image)

Figure 17. Theoretical FLSD obtained by using Swift hardening law and several yield functions

In order to analyse how the yield surface shape influences the Forming Limit Stress Diagram, in Figures 17 and 18 the simulated FLSD are presented, when several yield functions are used to describe the yield surface and the hardening behaviour is approximated either by Swift law or by
Voce law. Considering identical combination of constitutive equations like those used to predict the FLSDs presented in Figures 17 and 18, the FLDs are predicted and illustrated in Figure 19.

Figure 18. Theoretical FLSD obtained by using Voce hardening law and several yield function

Figure 19. Influence of the yield surface shape on the theoretical FLDs
It is noticed a strong effect of the yield surface shape on the equibiaxial stretching region of the FLSD but not so tremendous effect like in FLD’s case. When Swift law is applied some yield surface shape influence is found on the entire simulated FLSD while no influence in uniaxial region is found when Voce law is considered. In stress space, the highest curve is the one obtained with Hill’48 and the lowest one tends to be the Von Mises predicted curve whereas in strain space the situation is exactly inverse being the highest curve the one predicted by Von Mises yield function and the lowest one the Hill’48 predicted curve.

Figure 20. Major principal stress – major principal strain curve obtained by using Swift hardening law and different yield functions.

Figure 21. Major principal stress – major principal strain curve obtained by using Voce hardening law and different yield functions.
Figures 20 and 21 show the evolution of the major principal stress – major principal strain curves obtained by using Swift and Voce hardening law respectively, combined with different yield functions, which allows to explain the effect of the shape of the yield surface on the FLSD and FLD during linear strain paths as well as the previous remark related to the lowest and highest curve. As was expected, a small, in the Swift case, and inexistent, in the Voce case, influence of the yield function on the forming limit stresses is observed during uniaxial tension. In strain state this influence is more evident, but by comparing with the one observed during balanced biaxial stretching, is much smaller.

iii) Influence of work hardening coefficient on FLSD

Varying the work hardening coefficient (n) from Swift equation between 0.1 and 0.35, several FLSD are simulated and presented in Figure 22. It was observed that when n increases the forming limit stresses increase during uniaxial tension and balanced biaxial stretching, and decrease during plane strain. A weak influence of the work- hardening coefficient on the FLSD can be noticed, compared with its influence on the FLD, which is more pronounced, as can be observed from Figure 23.

![Figure 22. Influence of work-hardening coefficient on FLSD for BH steel](image)

The major principal stress – major principal strain curves during uniaxial tension, plane strain and balanced biaxial stretching presented in Figures 24, 25 and 26 are in agreement with the concluded results concerning the influence of the work- hardening coefficient on the stress-based forming limit and strain based forming limit respectively. Indeed, the domain of maximum
admissible deformations and stresses expands progressively as the value of n increases during uniaxial tension and balanced biaxial stretching zones. On the plane strain zone the maximum admissible stresses gradually decrease while maximum admissible strains increase with the work-hardening coefficient augmentation. In terms of the stress, the amount of this variation is slighter compared with the one attained in strains terms.

Figure 23. Influence of work-hardening coefficient on FLD for BHSteel

Figure 24. σ₁-ε₁ curve during uniaxial tension
Figure 25. $\sigma_1$-$\varepsilon_1$ curve during plane strain

Figure 26. $\sigma_1$-$\varepsilon_1$ curve balanced biaxial stretching
iv) Influence of strain rate sensitivity coefficient on FLSD

Figure 27 shows the influence of the strain rate sensitivity coefficient on the FLSDs level, presenting several curves predicted by use of different strain rate sensitivity coefficients.

![Figure 27. Influence of strain rate sensitivity coefficient on FLSD for BH steel](image)

Figure 27. Influence of strain rate sensitivity coefficient on FLSD for BH steel

![Figure 28. Influence of strain rate sensitivity coefficient on FLD for BH steel](image)

Figure 28. Influence of strain rate sensitivity coefficient on FLD for BH steel
It is observed that the level of the FLSD strongly depends on the strain rate sensitivity. When strain rate sensitivity increases the predicted forming limit stress decreases while the predicted limit strain increases as was also shown in previous works (Barata da Rocha et al., 1986). From the point of view of strain-based forming limit criterion, the augmentation of the strain rate sensitivity allows to an appreciable improvement of the forming material performance while from the point of view of stress-based forming limit criterion, this augmentation leads to a considerable reduction on the material formability. Consequently there is a delicate situation, which requires a careful attention to deal with these two forming limit concepts.

![Graph showing \( \sigma - \varepsilon_1 \) curve during balanced biaxial stretching](image)

Figure 29. \( \sigma_1 - \varepsilon_1 \) curve during balanced biaxial stretching

Figure 29 shows clearly the correlation between the influence of strain rate sensitivity on the forming limit stress diagram and forming limit diagram during balanced biaxial stretching.

v) Influence of balanced biaxial yield stress on FLSD

In the previous section the importance of the balanced biaxial yield stress on the level of the Forming limit Diagrams was pointed out. Representing an essential mechanical parameter in the coefficients identification of the newest yield functions developments (Barlat et al., 1997; Banabic et al., 2000; Barlat et al., 2002), the biaxial yield stress influence on the material forming limits through the stress-based forming limit concept is also interesting to analyse. Using the Barlat Yld’96 yield criterion and presupposing a gradually variation of the balanced biaxial yield stress, several Forming Limit Stress Diagrams were predicted and presented in Figure 30.

In these figures, like as was already specified in the Chapter (5) “ratio” means the balanced biaxial yield stress divided by the uniaxial yield stress at 0 degree from rolling direction.
6. Application of the "FLDcode" on the stress-based forming limits analysis

![Graph showing the influence of balanced biaxial yield stress on the Forming Limit Stress Diagram for AA6016-T4](image)

Figure 30 Influence of balanced biaxial yield stress on the Forming Limit Stress Diagram for AA6016-T4

Excepting the uniaxial tension limit stress, which keeps a constant value, the level of the stress-based forming limit criterion increase as an increase of the balanced biaxial yield stress. At the same time, as was shown in the previous Chapter (5), a decrease on the level of the strain based forming limit criterion occurs. Hence, there is again the controversy situation of the two concept of forming limits, which doesn't allow us to conclude how the augmentation of the balanced biaxial yield stress influences the material forming limit, because looking from stress point of view the formability of the material improves whereas from strain point of view it decreases.

vi) The influence of initial M-K geometrical defect value on FLSD

As it is very well known, the initial value of the geometrical defect plays a vital role in the M-K analysis. Thus, Figure 31 shows the dependence of the forming limit stresses with the initial value of the geometrical defect from M-K.

Conform expectation, a strong sensibility of the FLSD by the initial defect value is observed. The level of the predicted forming limit stresses rises when the initial defect value increases, presenting the same behaviour like the one achieved by the forming limit strains concerning the respective dependence as was shown in previous studies and as also can be observed in Figure 32. It is important to remark, once again, that even that the M-K initial geometrical imperfection value is considered an adjustable parameter in order to achieve the best accuracy in the predicted forming limit curves, in fact latest works by Barlat shows that it must be a constant M-K analysis parameter with values in the stretched range between 0.996-0.998, as was already mentioned in the previous section.
6. Application of the "FLDcode" on the stress-based forming limits analysis

Figure 31. Influence of the initial value of the M-K geometrical defect on FLD

Figure 32. Influence of the initial value of the M-K geometrical defect on FLD
vi) Strain path changes influence on theoretical forming limit stress diagrams

Since the no dependence of the FLSDs on the strain path changes was experimentally showed, for the present studied materials, an analysis of the predictions of the developed general FLDCode concerning stress-based forming limit criterion during complex strain paths is performed. Benefiting of the great potential of the numerical simulation model, various two-strain path changes at numerous values of prestrain were considered and simulated, in order to find the moment of necking localization.

The theoretical FLSDs under proportional and non-proportional loadings for BH steel are presented in Figures 33 and 39 whilst the ones corresponding to AA6016-T4 aluminium alloy are presented in Figures 35 and 37.

For an easier analysis of the stress-based forming limit criterion reported by strain based forming limit diagrams, the corresponding FLDS are also illustrated on the Figures 34 and 40 for BH steel and Figures 36 and 38 for AA6016-T4 aluminium alloy respectively.

The steel sheet counts with a uniaxial tension prestrain at 7%, 14% and 18%, a biaxial stretching prestrain at 5%, 7% and 10% and a plane strain prestrain at 5%, 8% and 13% respectively followed by several strain paths between uniaxial tension and biaxial stretching. The aluminium alloy sheet is pretrained at 7%, 14% and 17% in uniaxial tension, at 5%, 8% and 10% in biaxial stretching and at 5%, 9% and 14% in plane strain respectively followed also by several strain paths between uniaxial tension and biaxial stretching.

The material hardening behaviour is approximated by Swift power law for the BH steel and by Voce saturation law for the aluminium alloy.

The shape of the yield surface is described first by Hill 1948 yield function and then by the Barlat Yld96 yield function. While the Hill 1948 yield criterion was used in previous FLSDs simulations (by Arrieux et al.) there is a absolutely new opportunity to analyse the stress-based forming limits by use of the Yld96 yield function, one of the most accurate anisotropic yield function at the present time.

All these results show that for every distinct combination assumed on the simulation, the predicted forming limit stresses during the various complex strain paths are overlaid on the ones attained during linear strain path. Therefore the independence of the stress-based forming limit diagram on the strain path changes is definitely demonstrated. Consequently, no surprising results are observed, but it is amazing the manner in which all the confuse lattice of the complex forming Limit Diagrams vanishes in a single curve in stress-based forming limit criterion.

It can be seen that the shape of the forming limit in Figure 33 is slightly different from that in Figure 39 and the curves are superimposed even more when Yld96 plastic potential is used than in the case of Hill's quadratic potential. In the case of aluminium alloy the curves are perfectly overlaid for both of two plastic potential functions that are used, observing also a difference on the forming limits shape.
6. Application of the “FLDcode” on the stress-based forming limits analysis

Figure 33. Theoretical FLSDs during linear and complex strain paths for BH steel

Figure 34. Theoretical FLDs under linear and complex strain paths for BH steel
Figure 35. Theoretical FLSDs under linear and complex strain paths for AA6016-T4

Figure 36. Theoretical AA6016-T4 FLDs under linear and complex strain paths
Figure 37. Theoretical AA6016-T4 FLDs under linear and complex strain paths

Figure 38. Theoretical AA6016-T4 FLDs under linear and complex strain paths
6. Application of the "FLDcode" on the stress-based forming limits analysis

Figure 39. Theoretical FLSDs after complex strain paths for BH steel

Figure 40. Theoretical FLDs after complex strain paths for BH steel
Besides the shape of the forming limits, the attained values of the forming limit stresses are also influenced by the assumed yield function, in the present study much more for the steel case than in the case of aluminium alloy.

The most important conclusion of these results are that even using a very accurate anisotropic plastic potential like Yld96 yield function capable to describe very well the plastic behaviour of the material it was proved the independence of the stress- based forming limit diagrams on the strain path changes.

4.4 Comparison between experimental and theoretical FLSD’s

Since various pure experimental and theoretical studies were prior presented, the agreement between the experimental data and the predicted results is of high interest. Figures 41, 42, 43 and 44 show the experimental and theoretical Forming Limit Stress Diagrams under linear and complex strain paths for AA6016-T4 aluminium alloy and BH steel respectively.

![Graph showing FLSDs for AA6016-T4](image)

**Figure 41.** Experimental and theoretical FLSDs for AA6016-T4

The Von Mises yield function, Hill’s quadratic and non-quadratic function combined with the saturation law are used in the plastic calculation required to obtain the experimental and simulated aluminium alloy forming limits. It can be seen that for each of these constitutive equations combinations the predicted curves are in a very good agreement with the corresponding experimental data.
6. Application of the “FLDcode” on the stress-based forming limits analysis

Figure 42. Experimental and theoretical FLSDs for AA6016-T4

Figure 43. Experimental and theoretical FLSDs for AA6016-T4
Also, the simulated BH steel stress-based forming limits curves obtained by use of Hill's 1948 plastic potential with the power hardening law accurately reproduce the related experimental data, as Figure 44 shows.

![Figure 44. Experimental and theoretical FLSDs for BH steel](image)

All these results conclude that any yield function is able to simulate with very good accuracy the associated experimental forming limit stress diagram. As this is a controversial remark from the strain-based forming limits point of view, it will be more discussed on the following subsection.

### 6.5. Some critical remarks of the FLSD concept

#### 6.5.1. Particularity of the FLSD

The method used to determine the experimental Forming Limit Stress Diagrams leads to a strong dependence of the experimental curves on the constitutive equations applied on the plastic calculation required in the computation of the stress state. In consequence, to compare the theoretical and experimental FLSDs, the same plastic potential function and hardening law must be used in simulation as in the plastic calculation of the experimental stress-based forming limits like Stoughton (2000) also pointed out.
6. Application of the “FLDcode” on the stress-based forming limits analysis

6.5.2. Relativity of the FLSD concept compared by FLD concept

Figure 45. Forming Limit Stress Diagram for AA6016-T4 during linear strain paths

Figure 46. Forming Limit Stress Diagram for AA6016-T4 during linear strain paths
Figure 45 shows the remarkable difference between the predicted forming limit strains in equibiaxial stretching zone obtained by using quadratic Hill yield function and Yld96 yield function. At the same time, Figure 46 shows only a small difference between predicted balanced biaxial limit stresses by using of these two yield functions. Looking to the forming limit stress diagram it can be concluded that there is an insignificant difference between Hill 1948 and YLD96 yield functions, but in reality only the YLD96 is able to predict with an excellent accuracy the experimental shape of FLD whereas Hill'48 considerable overestimates the right part of the diagram.

To understand this remark Figure 47 presents the true stress-true plastic strain curve. A small stress increment is correlated with a large strain increment in especial in the necking occurrence region. Accordingly, an insignificant error in the forming limit stress prediction allows a substantial error in the forming limit strain prediction.

![Figure 47 True stress – true plastic strain curve for AA6016-T4 aluminium alloy](image)

Since Figures 41, 42, and 43 show good predictions on the Forming Limit Stress Diagrams for every considered yield function, could be concluded that Von Mises, Hill’48 and Hill’79 yield functions coupled with Voce hardening law are able to correctly predict the forming limit of the AA6016-T4 aluminium alloy from stress state sight. Hence, emerges the question if this is a right conclusion when the necking occurrence is analysed from the limit strains point of view. Observing Figure 48 that shows the corresponding strain-based forming limit diagrams, it is noticed a different and more complicated situation, the predicted forming limit curves being considerable far from the experimental data in the right part of the diagram namely between plane strain and biaxial stretching. Moreover, no one of these plastic potentials is able to accurately predict the AA6016-T4 aluminium alloy FLDs like only Yld96 potential (Figure 45) is capable to do it.
Therefore, perfect prediction on the stress-based forming limits doesn’t allow always to a correct analysis on the material formability. A solution of this problem could be the assurance of the accuracy on the forming limit prediction from the stress perspective as from the strain perspective by use of the properly plastic potential and hardening law in order to have the best characterization of the plastic behaviour of the material.

![Graph](image)

*a) FLSDs obtained by using Hill’48 potential with Voce equation*

![Graph](image)

*b) FLSDs obtained by using Von Mises potential with Voce equation*

![Graph](image)

*c) FLSDs obtained by using Hill’79 potential with Voce equation*

Figure 48. Forming Limit Diagrams for AA6016-T4 under linear and complex strain paths
6.6. Forming Limit Stress Surface

Based on the Forming Limit Stress Diagrams under off axes orthotropic loadings, Arrieux (1994), Nguyen (1994) and Vacher (1998) proposed the **Forming Limit Stress Surface** (Figure 49) represented by a three dimensional diagram ($\sigma_1, \sigma_2, \theta$) of the forming limit stresses and the angle between the principal strain and orthotropy direction, as a stop test at the time of a numerical simulation by the FEM of deep drawing operations.

![Figure 49. Forming limit stress surface for off axis solicitations, with direct path](image)

The stress surface calculated on rectilinear strain paths is used to predict the occurrence of the strain localization in a direct or broken strain path being the lowest one and in the same time
very closed by the others ones. Similarly, Vacher et al. (1998) also propose the Forming Limit Strain Surface (Figure 50), which can be used to determine the best orientation of the sheet to optimise the deep drawing operation. These strain surfaces can’t be used to detect the necking occurrence for the broken strain paths because for each broken path there is a unique strain surface.

The method using the forming limit stress surface seems to be a very interesting and useful suggestion for the prediction of the feasibility of deep drawing operations of thin plates for various directions of loading, could being applied for anisotropic and viscoplastic materials, for in and off axes straining under linear and broken strain paths.

6.7 Conclusions

For a better understanding of the Forming Limit Stress Diagram concept during linear and complex strain paths, a detailed experimental and theoretical study was performed.

Several combinations of different constitutive equations were considered on the plastic calculation of the experimental forming stress limits and also on the simulation of the necking phenomenon through the Marciniak-Kuczinsky theory that was performed by using the new developed FLDcode.

It was revealed experimentally as well as theoretically a considerable influence of the hardening law and also a strong effect of the yield surface shape on the equibiaxial stretching region of the forming limit stress diagrams. Whatever is the applied yield function, the Swift equation contributes for higher limit stresses than the ones attained by use of Voce equation. At the same time, on the strain path range between uniaxial tension and plane strain, a lower hardening law influence is observed on the predicted curves while no one is detected for the experimental ones. A slightly influence of the yield surface shape between uniaxial tension and plane strain on the stress limits was found, noticing even no influence on the uniaxial theoretical stress limits when Voce law was considered.

The difference on the forming stress limits during uniaxial tension by use of different hardening laws is related with the unique initial defect value of the M-K analysis, considered in simulation for both hardening models.

A weak influence of the work- hardening coefficient on the FLSD was noticed. When the work-hardening coefficient increases the forming limit stresses increase during uniaxial tension and balanced biaxial stretching, and decrease during plane strain. The predicted stress- based forming limits diminishes when the strain rate sensitivity increases. An augmentation on balanced biaxial yield stress leads to a growth of the balanced biaxial limit stress.

It is observed a strong sensibility of the FLSD by the initial M-K geometrical defect value, the level of the predicted FLSD rising as the initial defect value increases.

The independence of the stress- based forming limit diagram on the strain path changes is definitely demonstrated as much experimentally as theoretically, the forming limit stresses under various complex strain paths obtained for every distinct assumed combination of the plastic potential and hardening model, being overlaid on the ones attained during linear strain paths.
To compare the experimental Forming Limit Stress Diagrams with the theoretical curves, it is necessary to use identical constitutive equations in the plastic calculation required in the computation of the stress state from the experimental limit strains like in the simulation.

A very small error in the forming limit stress prediction allows a very big error in the forming limit strain prediction, which in fact is a strong limitation for industrial applications.

As it was shown previously, there is the possibility to have an excellent agreement between the predicted FLSDs and corresponding experimental data while the simulated FLDs are considerable far from the experimental data. Good predictions on the strain-based forming limits allows to a correct analyse on the material formability and good predictions on the stress-based forming limits means a perfect stop test on the applications involving non-proportional loading due its independence on the strain path changes and its more simplicity by compared to the strain based limits. In order to assure a concomitant accuracy of these two forming limit concepts appropriate constitutive equations, capable to describe with a high precision the plastic behaviour of the material, are required.

Thus, it can be concluded that a combination of the strain based forming limit criterion and stress- based forming limit criterion leads to a powerfully and safety forming limit tool on the sheet metal forming simulation.
7. GENERAL CONCLUSIONS

The theoretical analysis of plastic instability plays a significant role on the optimisation of sheet metal forming processes. Since the continuous progress on the materials science provides new advanced structural materials to satisfy the actual priorities of the sheet metal manufacturing industry, better and sophisticated material models, in order to describe the behaviour of such materials, are developed. Moreover, the selected constitutive equations keep a crucial significance on the prediction of plastic flow localization. Based on these issues, the aim of the present research is the development of a more general and user-friendly model for Forming Limit Diagram prediction. The simulation of plastic instability is carried out in the framework of heterogeneous materials using the Marciniak-Kuczynski (M-K) analysis coupled with the Theory of Plasticity. The new model (FLDcode) consists on a main part and individual subroutines for each of essential components of the theory of plasticity, describing the plastic deformation, namely the hardening law, the yield function and the associated flow rule. Hence, it allows the implementation of different constitutive equations by solely change of the respective subroutine, keeping intact the main part of the program. Simultaneously, it allows easy combinations between various yield criteria and hardening laws.

In the interest of flexibility, the homogeneous and heterogeneous zones are independently computed, their connection being realized through the M-K conditions: force equilibrium and geometrical compatibility. In this way, the conception of the interface between the necking criterion and FE programs is facilitated. Specifically, the computation of the homogeneous zone is replaced by the FE model, which provides all required data for computation of the inhomogeneous zone in order to predict the forming limits.

For a complete study of the considered topic, the mathematical description of the anisotropic material behaviour is based first, on a phenomenological approach and second, on a physical one. The rigid plastic behaviour and plane stress condition are assumed.

A general algorithm that could be applied at any kind of phenomenological models, with the plastic potential expressed in stress space, is developed. The Newton-Raphson numerical method is used to solve the nonlinear equations of localized necking, taking into account linear and complex strain paths.

The generality and effectiveness of the new model was completely proved by the successful integration in the code of several constitutive equations such as, Swift strain-hardening power law and Voce saturation strain-hardening law, the isotropic Von Mises yield criterion, the quadratic Hill yield criterion (Hill’48), non-quadratic Hill yield criterion (Hill’79) and Yld'96 Barlat yield function (Barlat et al. 1997).

Finally, a physics-based model namely the Combined Plasticity Model of Texture and Strain-Path induced Anisotropy is considered and implemented with success in the FLDcode. This advanced model is based on the Van Houtte’s anisotropic texture model (Van Houtte et al., 1995) and Teodosiu and Hu microstructural hardening model (Teodosiu and Hu, 1995). The initial anisotropy is attributed to the crystallographic texture while the strain path induced anisotropy is mainly attributed to persistent dislocation structures that develop inside each grain of the polycrystal during plastic deformations. The anisotropic work-hardening behaviour under
strain-path changes at moderately large strains is modelled by using an internal - variable approach, several intragranular deformation mechanisms being taken into account in defining the internal variables. Since the Van Houette's texture potential is expressed as a six-order strain rate potential, appropriate numerical treatments and a dedicated algorithm are developed, maintaining the fundamental structure of the new method, defined in the macroscopic approach. The Jacobian matrix required in the Newton-Raphson computation, performed for solving the theoretical treatments of localized necking, is based on an analytical derivation of the constitutive model. Additionally, in this context of large plastic deformations, it is considered the rotation of orthotropic axis of anisotropy of the imperfection region of M-K model with the Jaumann spin.

Consequently, the essential advantage of the new method is revealed by its great generality, due to the applicability for plastic potentials expressed in stress space as well as in strain rate space with resource to both approaches of the Plasticity Theory, the phenomenological one and the physical one.

In order to point out all capabilities of the new code as well as its performance in accuracy, a detailed study on Forming Limit Diagram prediction accompanying a suggestive experimental work was carried out. Four metal sheets of deep-drawing quality have been selected. They are: AA6016-T4 and AA5182-0 aluminium alloys, a low carbon steel and a bake hardened steel. Remarkable accuracies on the predicted forming limits for each distinct material were achieved when the shape of yield locus was described by Yld96 yield criterion and the hardening law represented by Voce equation for aluminium alloys and by Swift equation for steel sheets, respectively. Furthermore, the FLDCODE is able to provide a sensitive analysis for strength-hardening coefficient, anisotropy coefficient, uniaxial yield stress and yield criterion. The presented results show an excellent match between the experimental data and the corresponding Yld96 predictions of such relevant factors of material mechanical behaviour, demonstrating the high standard of this yield function. Even if the Yld96 yield function has been elaborated to describe the behaviour of aluminium and its alloys, fact that was fully accomplished, the present work showed that it is also very efficient in description of BCC materials behaviour as well as in their forming limits prediction. Notice that different initial defect values, namely 0.996-0.998 and 0.992-0.993, must be considered in the simulations for the aluminium alloys and for steel sheets, respectively. Moreover, through the use of Yld96 yield function, it was shown that the balanced biaxial yield stress is an important parameter on the forming limit prediction under linear and complex strain paths.

An experimental and theoretical analysis indicates that the sheet thickness affects in certain proportion the mechanical behaviour of the material and consequent, the FLDS under proportional as well as under non-proportional loadings. The effect of the sheet thickness on the material formability under proportional loading was experimentally observed to be insignificant, while theoretically, it was clear contoured on the stretching zone. At the same time, the material formability increases with an augmentation of sheet thickness, during a uniaxial tensile prestrain followed by an equibiaxial stretching, whereas for an inverse strain path sequence, no consequence is detected.

The FLDCODE ability to combine different constitutive equations, allows to analyse the influence of the hardening model and the yield function on the plastic flow localization with the M-K method, under linear and various two-stage strain paths. It was shown the considerable dependence of the entire forming limit level on the assumed hardening law and the strong effect
of the shape of the yield surface on the stretchability, in the right part of the Forming Limit Diagram. Subsequently, it was observed that the influence of the yield function varies with the types of the strain path changes.

Exceptionally, the FLDCode brings face to face both approaches of the Plasticity Theory, the physical one and the phenomenological one in an interesting study on forming limits prediction during linear and complex strain paths, by analysing the performances of the combined plasticity model of texture and strain-path-induced anisotropy and the Yld’96 Barlat yield function coupled with Swift hardening law, through the M-K theory. Corroborating with previous results of Hiwatashi (1998), the main advantage of modelling the material hardening on the basis of the microstructure evolution is observed on the predicted FLD under a strain history involving a biaxial prestrain followed by a uniaxial tension. Explicitly, the physics-based plasticity model is able to reproduce the Nakazima experimental tendency, which cannot be predicted by conventional phenomenological models. This effect is ascribed to the occurrence of transient hardening, possibly as a consequence of the interaction between the currently active slips and the previously formed dislocation structures. It was remarked that TexMic model predicts with a very good accuracy the BH steel forming limit strains under linear and complex strain paths, excepting the uniaxial tension limit strain, which is underestimated. More research must be done to find the origin of this unexpected result. In the presented case of BH steel sheet, another advantage of the physics based model consists on the use of the correct initial value of the M-K geometrical defect, according to previous work by Barlat, namely equal to 0.998, while for good predicted results, it must be equal to 0.992, when the phenomenological models are used.

Kleemola and Pelkkikangas(1977) and Arrieux, Bedrin and Boivin (1982) proposed the concept of stress based forming limit diagram, as a solution to the serious inconvenient of the strong dependence of the FLDS on the strain path changes in the multi-stage forming processes analysis. Very attractive due to its independence on the strain path change, a particular care was attributed to it, in a meticulous experimental and theoretical study.

Several combinations of different constitutive equations have been used on the plastic calculation of the experimental forming stress limits and also on the simulation of the necking phenomenon through the M-K analysis. It was observed a considerable influence of the hardening law and also of the yield surface shape on the equibiaxial stretching region of the forming limit stress diagrams, while a slight effect on the strain path range between uniaxial tension and plane strain was found. Moreover, it was explained that the influence of the hardening laws on the forming stress and strain limits, respectively, under uniaxial tension, is attributed to the imperfection factor, applied in simulation, which takes the same value for both hardening models. The critical orientation of the M-K narrow band was shown to be dependent on the hardening model applied in the M-K analysis. It was noticed certain sensibility of the Forming Limit Stress Diagram on the rheological parameters and on the initial M-K geometrical defect value.

Experimental and theoretical Forming Limit Stress Diagrams under various complex strain paths obtained for several combinations of the plastic potential and hardening model, was shown to be superimposed on those achieved during linear strain path. Thus, it was unequivocally proved the independence of the stress based forming limit diagram on the strain path changes.
For a correct evaluation of the accuracy, in terms of stress based forming limits, identical constitutive equations, in the plastic calculation of the experimental forming limit stresses like in the simulation, are required.

Being definitely proved the great potential and efficiency of the FLDcode on the Forming Limit Diagram and Forming Limit Stress Diagrams predictions as well as its aptitude in the best selection of the appropriate constitutive equations for the accurate description of the material behaviour, another significant advantage of the FLDcode could be its ability to be adapted for any new refined material model that could be further developed with the purpose of describing the improved performances of new generation of sheet metals.

The finally remark of this work is that for a correct analysis on the material formability and for an efficient and perfect stop test on the applications involving non-proportional loading, the combination of the strain-based forming limit criterion and stress-based forming limit criterion, in an accurate safety forming limit tool of the sheet metal forming simulation, seems to be a very promising approach, when appropriates constitutive equations, capable to describe with a high precision the plastic behaviour of the material, are used in the M-K analysis.
8. FUTURE DEVELOPMENTS

On the base of the present work, a future research related to developing a more advanced forming limit criterion by combining the strain based forming limit criterion and stress based forming limit criterion would be of relevant interest. It could account for:

- the essential necessity of an accurate description of the material behaviour for a correct analysis on the forming limits;
- the special care, which must be brought in the formulation of the continuum mechanics theory in order to describe correctly the large deformation imposed to the sheets during sheet metal forming processes;
- the significant advantage of the phenomenological approach, by the rapidity in computation, which means an industry priority on the time and cost efficiency;
- the performance of the latest developments on the phenomenological plastic potentials.

Hence, such model would aim at connecting the most advanced physically - based hardening model accounting for the evolution of the anisotropic work hardening induced by the microstructural evolution at large strains of Teodosiu and Hu (1995), with some advanced phenomenological anisotropic yield criterions that become available in the last two years, such as Yld2000 (Barlat et al. 2002), Cazacu-Barlat'2002 (Cazacu et al. 2002) and BBC2003 (Banabic et al., 2004) (see more details in Barlat et al. 2004).

The phenomenological anisotropic plastic potentials are usually expressed in a stress space. On the application of the stress based forming limit criterion, the identification of the corresponding plastic strain rate potentials is indispensable. Therefore, this question is a crucial point of the future work.

Subsequently, the extension of the forming limit model for the post-processing of three-dimensional finite element simulations would be another subject of research. Simultaneous, this issue will require the extension of the Marciniak- Kuczynski method, assumed in a plane stress approach, for the analysis of three-dimensional deformation sequences.

The consideration of new structural materials such as the high-strength aluminium alloys, dual-phase and TRIP steels would be a very valuable new topic.

Finally, the validation of the model by simulating linear and multiple-stage strain paths, specifically, two, three or even more stage strain paths, associated with corresponding experimental data, would be an interesting topic of research. Moreover, a simple shear test and an off axis uniaxial tension at large strains can be also considered. For this purpose, specific algorithms have to be elaborated in order to simulate simple shear strain paths and uniaxial tension strain paths at large strains for different loading directions, by considering the rotation of the orthotropic axes of anisotropy.

The proposed forming limit criterion could provide a considerable increase of accuracy and computational efficiency in the contemporary sheet metal forming simulation, being able for a correct forming analysis of the modern generation of sheet metals under multi-stage operations.
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Appendix

A.1 Computation of stress tensor in the homogeneous zone of the M-K model

The stress tensor is computed by imposing the stress direction and by using the yield function coupled with Euler theorem of homogeneous functions.

The stress direction is expressed as:

\[
\tilde{\sigma}_0 = \begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix}, \tag{A.1}
\]

where \(\alpha\) represent the stress ratio.

By using the yield function the quantity \(\phi_0\) is computed:

\[
\phi_0 = \phi(\tilde{\sigma}_0) \tag{A.2}
\]

Hardening rule allows to calculate the equivalent stress \(\bar{\sigma}\), while the stress tensor is expressed as:

\[
\bar{\sigma} = k \tilde{\sigma}_0, \tag{A.3}
\]

where \(k\) is a proportional factor, which is calculated as follows:

First, the yield criterion is written as:

\[
\phi(\bar{\sigma}) = 2 \bar{\sigma}^a \tag{A.4}
\]

Second, \(\phi\) is a homogeneous function of degree \(a\) and by applying the Euler theorem of homogeneous function results:

\[
\phi(\bar{\sigma}) = \phi(k \tilde{\sigma}_0) = k^a \phi(\tilde{\sigma}_0) = k^a \phi_0 \tag{A.5}
\]

Hence, the factor \(k\) is:

\[
\Rightarrow k = \left( \frac{\phi(\bar{\sigma})}{\phi_0} \right)^\frac{1}{a} = \frac{1}{\phi_0} \left( \frac{2 \bar{\sigma}}{\phi_0} \right)^\frac{1}{a} \tag{A.6}
\]

Finally, the stress tensor reads:
\[ \tilde{\sigma} = \sqrt{\frac{2}{\phi_0}} \frac{1}{\tilde{\sigma}_0} \]  

\text{(A.7)}

**A.2 Five-dimensional vector representation**

The plastic behaviour of plastically incompressible materials is due to the deviatoric part of a quantity, the hydrostatic part having no contribution in the plastic dissipation. These deviatoric tensors \( T \), contain only five independent components \( T_{ij} \) as a consequence of the properties \( T_{ij} = T_{ji} \) and \( T_{ii} = 0 \). On the base of this fact, Van Houpte et al. (1988) proposed a rigorous method to represent such tensors in a five-dimensional vector space, by conserving their norm:

\[ T_{ij} T_{ij} = T_p T_p \]  

\text{(A.8)}

with \( i,j = 1-3 \) and \( p = 1-5 \)

This conversion, which has been adopted in the present work, is defined as:

\[
\begin{align*}
T_1 &= \frac{1}{\sqrt{2}} (T_{11} - T_{22}) \\
T_2 &= \frac{3}{\sqrt{2}} (T_{11} + T_{22}) = -\sqrt{\frac{3}{2}} T_{33} \\
T_3 &= \sqrt{2} T_{23} \\
T_4 &= \sqrt{2} T_{31} \\
T_5 &= \sqrt{2} T_{12}
\end{align*}
\]  

\text{(A.9)}

The reversed operation, namely the conversion from 5-component notation in 3×3 component notation is expressed as:

\[
\begin{align*}
T_{11} &= \frac{1}{\sqrt{2}} T_1 + \frac{1}{\sqrt{6}} T_2 \\
T_{22} &= -\frac{1}{\sqrt{2}} T_1 + \frac{1}{\sqrt{6}} T_2 \\
T_{33} &= -\sqrt{\frac{2}{3}} T_2 \\
T_{23} &= \frac{1}{\sqrt{2}} T_3 \\
T_{31} &= \frac{1}{\sqrt{2}} T_4 \\
T_{12} &= \frac{1}{\sqrt{2}} T_5
\end{align*}
\]  

\text{(A.10)}
A.3 Transformation between $5 \times 5$ and $6 \times 6$ dimensional vector representations

The symmetry of the second order tensor allows only six independent components out of nine. For practical purpose, instead of $3 \times 3$ component notation, a six-dimensional vector representation following Voigt convention, is defined as:

$$T_1 = T_{11}, \ T_2 = T_{22}, \ T_3 = T_{33}, \ T_4 = T_{12}, \ T_5 = T_{13}, \ T_6 = T_{23}$$ \hspace{1cm} (A.11)

Two tensors, namely $A_{65}$ and $B_{56}$ are introduced in order to allow the following conversions:

$$T_{\alpha} = A_{\alpha k} T_K$$ \hspace{1cm} (A.12)

and

$$T_M = B_{M \beta} T_{\beta}$$ \hspace{1cm} (A.13)

where $\alpha, \beta = 1 \div 6$ and $K, M = 1 \div 5$

From Equations A.9 and A.10, the tensor $A_{65}$ is identified as follows:

$$A_{65} = \left[ A_{\alpha k} \right] = \begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 \\
0 & \sqrt{\frac{2}{3}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\
0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}$$ \hspace{1cm} (A14)

The tensor $B_{56}$, resulting from (A.8) and (A.10) and also representing the transverse of tensor $A_{65}$ is given by:
The conversion of a fourth order tensor between the $5 \times 5$-dimensional and $6 \times 6$-dimensional vector representations reads:

$$T_{\alpha \beta} = A_{\alpha K} \ T_{KM} \ B_{K\beta}$$

(A.16)

where $\alpha, \beta = 1 \div 6$ and $K, M = 1 \div 5$

A.4 The translation of the partial derivatives $\frac{\partial \dot{\sigma}^b}{\partial \dot{D}^b}$ in the M-K groove reference frame

Figure 1 represents the referential frames considered in M-K model, namely the global reference frame ($x^o y^o z^o$), the orthotropic axes of anisotropy (xyz) and the groove reference frame (ntz). The stress tensor representation in each such reference frame is defined as $\sigma^o$, $\dot{\sigma}$, $\sigma$, respectively, while the corresponding strain rate tensor is identified as $D^o$, $\dot{D}$ and $D$ respectively.

Figure 1. Representation of the considered M-K referential frames

In agreement with specifications of Chapter 4, the angle $\psi$ characterizes the M-K groove orientation, whereas the angle $\theta$ expresses the orientation of the orthotropic axes of anisotropy with respect to the global reference frame ($x^o y^o z^o$). Therefore, defining the corresponding rotation matrixes as $R_\psi$ and $R_\theta$ respectively, the stress tensor translation from orthotropic reference frame in the groove reference frame reads:
\[ \sigma = R_{\psi}^T \sigma^o R_{\psi} = R_{\psi}^T R_\theta \dot{\sigma} R_\theta^T R_{\psi} \quad \text{(A.17)} \]

If matrix \( R \) is defined as:

\[ R = R_{\psi}^T R_\theta \quad \text{(A.18)} \]

and matrix \( R^T \) as:

\[ R^T = R_\theta^T R_{\psi} \quad \text{(A.19)} \]

the Equation A.12 can be rewritten as:

\[ \sigma = R \dot{\sigma} R^T \quad \text{(A.20)} \]

The strain rate tensor in orthotropic reference frame can be expressed as a translation of the strain rate tensor in the groove reference frame as follows:

\[ \dot{D} = R^T D R \quad \text{(A.21)} \]

Thus, the partial derivatives \( \frac{\partial \sigma}{\partial D} \) read:

\[ \frac{\partial \sigma_{ij}}{\partial D_{qr}} = \frac{\partial \sigma_{ij}}{\partial \hat{\sigma}_{kl}} \frac{\partial \hat{\sigma}_{kl}}{\partial \hat{D}_{op}} \frac{\partial \hat{D}_{op}}{\partial D_{qr}} \quad \text{(A.22)} \]

\[ \frac{\partial \sigma_{ij}}{\partial \hat{\sigma}_{kl}} = R_{im} \frac{\partial \hat{\sigma}_{mn}}{\partial \hat{\sigma}_{kl}} R_{nj}^T = R_{im} \delta_{mk} \delta_{ni} R_{nj}^T = R_{ik} R_{jl} \quad \text{(A.23)} \]

\[ \frac{\partial \hat{D}_{op}}{\partial D_{qr}} = R_{os}^T \frac{\partial \hat{D}_{st}}{\partial D_{qr}} R_{rp} = R_{os}^T \delta_{sq} \delta_{tr} R_{rp} = R_{qs} R_{rp} \quad \text{(A.24)} \]

Finally, the partial derivatives of the stress tensor reported to strain rate tensor, in the groove reference frame, are obtained from the following expression:

\[ \frac{\partial \sigma_{ij}}{\partial D_{qr}} = R_{ik} R_{jl} \frac{\partial \sigma_{kl}}{\partial \hat{D}_{op}} R_{op} R_{rp} \quad \text{(A.25)} \]

with \( i, j, q, r, k, l, o, p, q, r = 1-3 \)