A thesis submitted to the University of Lancaster for the degree of Doctor of Philosophy.

SEPTEMBER, 1981.
To my parents,
my wife
and my children
NATIONAL PLANNING OF COMMODITY IMPORT OPERATIONS:

THE CASE OF PORTUGAL'S FEED GRAIN IMPORTS

by


ABSTRACT

The thesis is concerned with the overall decision making process involved in the importing of Portugal's maize and sorghum requirements by a Government agency. Although based on this particular problem, the research provides a general framework for formulating and modelling problems associated with the planning and control of commodity import operations at a national level.

The emphasis is on the design of an integrated set of models in which

(i) the decision problem facing the Government agency is adequately set in the context of a wider Government planning process;

(ii) the solutions of the different aspects of the problem are combined to form a consistent overall decision policy.

The models described in the thesis cover all the most significant aspects of the planning and control problem facing the Government agency, namely:

(i) the specification of the amounts of maize and sorghum imports required, according to their prices on the international markets;
(ii) the definition of a delivery-inventory policy, stating
the size of the feed grain shipments and the timing of
their planned arrivals at the Portuguese ports; and
(iii) the derivation of a purchasing policy stating how
and when those shipments should be bought.

Following the analysis of the demand for and supply of feed grains in
the country, the animal feed industry is singled out as the major feed
grain user. The process of substitution between maize, sorghum and the
other raw materials used by that industry is initially analysed using a
'static' linear programming model in which all prices of the imported
raw materials are assumed to be independent of the amounts imported.
Economies of scale in the delivery-inventory operations of maize and
sorghum are later incorporated in the static import mix model. This
makes the model become non-linear. It is solved by a piecewise linear
approximation. Two further extensions of the import mix model are
considered:

(i) the model is made adaptive to the dynamics of the planning
situation; and
(ii) a measure of the risks associated with the raw materials'
unknown future price changes is incorporated in the model
(a more general quadratic programming formulation is
considered, but is found unnecessary in the context of
the particular problem analysed).

The delivery-inventory operations of maize and sorghum are modelled
jointly with those of other raw materials which share common unloading
and storage facilities at the Portuguese port terminals. The modelling of
these operations is preceded by an analysis of two important aspects of the sea transport operations: the uncertainty of vessels' arrivals and the shipping cost structure. The results of this analysis are incorporated into a simulation-optimization model which yields near-optimal delivery-inventory policies for all raw materials. By combining simulation (used as a means to evaluate policies) with a direct search technique (used to select the best policy) the model succeeds in representing adequately a highly complex problem, whilst keeping the computational effort within acceptable limits.

The final aspect of the problem analysed in the thesis concerns the feed grains purchasing operations. Firstly a purchasing operating doctrine is selected, where the purchasing of maize 'futures' contracts is shown to be the central issue. Short and medium-term forecasting models are then reviewed and applied to the maize 'futures' prices. This is followed by an analysis of the buying decisions: new models are proposed to derive both short and medium-term buying policies.
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CHAPTER 9

GENERAL INTRODUCTION

1. The Problem Considered. Objectives of the Research

As with most Operational Research work, the research described in this thesis was developed to assist an organization in the management of its operations.

The organization, Empresa Pública de Abastecimento de Cereais (EPAC), acting as an agency of the Portuguese Government, is responsible for the whole range of operations involved in the implementation of the Government's intervention programmes in the internal grain markets and holds the monopoly of all grain imports into the country.

The research covers an analysis of the decision making process concerning the feed grains import operations. Although focused on this particular problem, one of the objectives is to infer from it, in terms as general as possible, an approach for formulating and modelling the whole decision process involved in planning and managing commodity import operations at a national level.

A number of other authors have modelled several aspects of the problem. The central contribution was given by Kingsman, who identified and modelled basic components of the 'commodity purchasing problem' and analysed important structural relations between them. As will be shown later, when reviewing his work in detail, the nature of the relations within and between those components is such that complex systems, such as the one analysed
in this thesis, cannot be represented adequately by a single overall model. Under these circumstances, the success of the modelling process becomes as crucially dependent on the correct balance and integration between the submodels used to represent the various components of the system as on the appropriateness of each of those submodels.

The problem facing EPAC, although falling into the broad category of the 'commodity purchasing problems', has special features associated with the scope and the scale of the operations involved.

Some of them are associated with the nature of the organization, whose operations have to be regarded as forming part of a wider Government planning process. The understanding of the role played by the organization in this process, the definition of appropriate boundaries for the decision problem and the setting of consistent objectives that should guide the organization's actions become critical components of the overall approach required to solve the problem adequately.

Other particular features result from the interactions between the decisions concerning the imports of feed grains and other cereals by EPAC as well as other commodities' imports by separate organizations. An example of such interactions, mainly attributable to the scale of the operations, arises from the utilisation of common port terminals to unload and store maize, sorghum and other imported agricultural products.

The difficulties associated with the scope and the scale of the operations dictated, to a considerable extent, the type of approach adopted. A distinguishing characteristic of the approach is the
emphasis on the design of an integrated set of models in which

(i) the decision problem facing the Government agency
    is adequately set in the context of a wider Government
    planning process; and

(ii) the solutions of the different aspects of the problem
    are combined to form a consistent overall decision
    policy.

The formulation and modelling strategy required to achieve such a
design is derived from basic concepts of the OR method.
It involves a number of inter-related steps covering the
identification of the aspects of the problem which deserve to be
included in the analysis and the definition of a model building
procedure that takes such aspects properly into account.

Within the framework of this global strategy, the thesis aims
to provide answers to questions arising in the main areas of the
problem facing EPAC, namely:

(i) the quantification of the import requirements of maize
    and sorghum: taking into account the substitutability
    between maize, sorghum and other home produced and
    imported raw materials of the animal feed industry,
    the feed grains import requirements are adapted
    dynamically as import price changes are detected or
    predicted, in an attempt to optimize Government policy
    objectives;

(ii) the definition of a delivery policy stating the size
    of the shipments of maize and sorghum and the timing
    of their planned deliveries: the policy is based on
the analysis of relevant aspects of the shipping operations, namely the uncertainty of the vessels' arrivals and the shipping cost structure, and takes into account the interactions between the feed grains imports and the imports of other agricultural products arising from the common utilisation of port terminals in the unloading and storing operations;

(iii) the analysis of alternative maize and sorghum purchasing policies, based on short and medium term price forecasts, the objective is to minimize the expected purchasing costs, within given constraints reflecting the purchaser's attitude towards the risks associated with the variability of prices.

2. **Layout of the Thesis**

Following this brief introduction, some general background to the feed grains import planning problem is given in Chapter 2. The demand and supply of feed grains in Portugal are analysed and the animal feed industry is singled out as the major feed grain user. The decision process involved in the planning of feed grain imports is set in its overall economic and organizational context, and major inadequacies of the current planning system are identified.

In Chapter 3, the scope of the research is defined and the general method of approach is characterized. Firstly, the process of government intervention in an economy is analysed and its basic decision levels are identified. The distinction between these levels provides a useful frame of reference to define the scope of the research. The problem to be considered is then identified as a 'commodity purchasing problem'. Whilst
reviewing previous research in this area, the main components of the problem and the interactions between them are identified. The method proposed to formulate, model and solve the problem places the emphasis on the integration of the solutions of the different sub-problems to form a consistent overall policy.

Chapter 4 includes an analysis of the structure of the animal feed production in Portugal, shown to be notably stable. A static linear programme (LP) is used to study the effect of import prices on the optimal import mix (defined, for a given level of animal feed production, as the mix that minimizes the country's foreign currency expenditure). The analysis of the solutions reveals important properties of the optimal import mix, which are exploited later in the development of the overall feed grain imports planning and control system.

The following chapter focuses on two important aspects of the feed grains' shipping operations: the uncertainty of vessels' arrivals and the shipping cost structure. The results derived are used in Chapter 6, in the planning of the delivery-inventory operations of feed grains and other raw materials handled at the Portuguese grain terminals. A simulation-optimization model is developed and yields near-optimal delivery-inventory policies for all raw materials. The model proposed succeeds in representing adequately a highly complex problem, whilst keeping the computational effort within acceptable limits.

In Chapter 7, the substitution between maize, sorghum and the other raw materials of the animal feed industry is revisited. Economies of scale in the delivery-inventory operations of maize and sorghum - modelled in the previous chapter - are incorporated in the 'static' import mix model. The model becomes non-linear, being solved by a piecewise linear approximation.
Chapter 8 deals with the modelling of the feed grain purchasing operations. Firstly a purchasing operating doctrine is selected: the purchasing of maize 'futures' contracts is seen to be the central issue in the whole process. Short and medium term price forecasting models are then reviewed and applied to the maize 'futures' prices. This is followed by an analysis of the most significant aspects of the buying decisions. New models are proposed in this chapter to derive short and medium-term buying policies.

In Chapter 9, the 'static' import mix model is extended in two areas. Firstly it is made adaptive to the dynamics of the planning situation; secondly the risks associated with the raw materials' price changes are incorporated in the model (a more general quadratic programming formulation is considered, but is found unnecessary in the context of the particular problem analysed). Chapter 9 also includes a discussion of the interactions between the several models developed in this thesis.

Chapter 10 concludes the thesis. A summary of the contributions of this research in each of the specific areas tackled is presented. Furthermore, it is argued that the global approach taken can be generalized to other commodity import planning situations. Finally, areas of further research are indicated.
CHAPTER 2

GENERAL BACKGROUND TO THE PROBLEM

1. Introduction

As stated in the introductory chapter, the research described in this thesis focuses on the decision making process related to the import of feed grains into Portugal. This chapter aims at setting this central concern in its overall economic and organizational context.

In line with such a purpose, in section 2, the recent trends in production, imports and utilization of feed grains in Portugal are reviewed.

The subsequent section presents the global strategy of the Central Government concerning the production and consumption of feed grains, in the context of the recent evolution of the Portuguese economy.

Section 4 describes the role of EPAC as the Government agency which controls the feed grains importing process.

The chapter is rounded off with a brief look at the way the feed grains imports are currently planned and controlled, identifying some of the major inadequacies in the present system.

2. Recent Trends in the Production, Imports and Consumption of Feed Grain

Like most of the agricultural sector in Portugal, feed grain production has remained nearly stagnant in the recent past, at extremely low levels of productivity.
Table 1 shows the productions and yields of the feed grains grown in Portugal—maize, barley and oats—over the decade 1967-1976. To illustrate the backwardness and the stagnation of the home production of feed grains, the average European yields for those crops were also included in the table. It is noticeable that the gap between them, already substantial in 1967, increased over the whole decade, particularly for maize.

| Year | Maize | | | Barley | | | | | | | Oats | | | | | | | Total | Feed | Grains |
|------|-------|---|---|-------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1967 | 577  | 1,320 | 2,800 | 73  | 0.690 | 3,040 | 121 | 0.190 | 2,360 | 761 | | | | | | | | |
| 1968 | 540  | 1,250 | 2,920 | 94  | 0.990 | 2,970 | 129 | 0.550 | 2,370 | 771 | | | | | | | | |
| 1969 | 553  | 1,290 | 3,500 | 54  | 0.460 | 2,990 | 79  | 0.360 | 2,390 | 686 | | | | | | | | |
| 1970 | 591  | 1,460 | 3,310 | 54  | 0.510 | 2,690 | 72  | 0.380 | 2,300 | 717 | | | | | | | | |
| 1971 | 576  | 1,550 | 5,527 | 84  | 0.917 | 5,131 | 125 | 0.745 | 2,705 | 735 | | | | | | | | |
| 1972 | 529  | 1,552 | 3,707 | 62  | 0.697 | 3,738 | 85  | 0.505 | 2,601 | 666 | | | | | | | | |
| 1973 | 550  | 1,496 | 3,795 | 55  | 0.640 | 3,234 | 76  | 0.480 | 2,621 | 662 | | | | | | | | |
| 1974 | 486  | 1,350 | 3,687 | 75  | 0.797 | 3,390 | 99  | 0.581 | 2,854 | 660 | | | | | | | | |
| 1975 | 506  | 1,290 | 3,932 | 86  | 0.860 | 3,083 | 121 | 0.564 | 2,571 | 713 | | | | | | | | |
| 1976 | 429  | 1,166 | 3,846 | 117 | 0.818 | 2,998 | 127 | 0.589 | 2,555 | 673 | | | | | | | | |

Source: FAO Production Yearbooks.

In a study carried out by the Centro de Estudos Agronómicos da CUF (*), the basic reasons given for the low productivity of the Portuguese grain crops were the low quality of the soils, an inadequate use of fertilizers and the failure to correct the soils' acidity, the utilisation of low quality seeds and the wrong allocation of crops to the available land.

Assuming the adoption of measures involving
(i) the increase and the reallocation of the area devoted to the production of feed grains (extending considerably

(*) Centre for Agricultural Studies of CUF; CUF, the biggest industrial group in Portugal, is involved in the production of fertilizers.
the areas allocated to barley and oats),

(ii) better fertilization and correction of the acidity of the soils, and

(iii) the usage of good quality seeds,

that centre estimates the potential production of feed grains as given in Table 2.

| Crop | Average 1967-1976 | Potential | Potential increase in production relative to the 67/76 av.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Area [1000 ha]</td>
<td>Yield [m.t./ha]</td>
<td>Production [1000 m.t.]</td>
</tr>
<tr>
<td>Maize</td>
<td>397</td>
<td>1.329</td>
<td>526</td>
</tr>
<tr>
<td>Barley</td>
<td>104</td>
<td>0.728</td>
<td>75</td>
</tr>
<tr>
<td>Oats</td>
<td>196</td>
<td>0.526</td>
<td>102</td>
</tr>
<tr>
<td>Total</td>
<td>697</td>
<td>-</td>
<td>703</td>
</tr>
</tbody>
</table>

Sources: Centre de Estudos Agronómicos da CUF - Potencialidades do País para a Produção de cereais (Jan.1976)

FAO Production Yearbooks

In spite of the low levels of production of feed grains, the animal feed industry - by far the major consumer of those grains - has expanded sharply. As shown in Table 3, in the decade 1967-1976 the production of animal feeds grew exponentially at an average rate of 19.2% per year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Production [1000 m.t.]</th>
<th>Prod.Index [1967:100]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>456</td>
<td>100</td>
</tr>
<tr>
<td>1968</td>
<td>690</td>
<td>151</td>
</tr>
<tr>
<td>1969</td>
<td>796</td>
<td>175</td>
</tr>
<tr>
<td>1970</td>
<td>955</td>
<td>209</td>
</tr>
<tr>
<td>1971</td>
<td>1 195</td>
<td>262</td>
</tr>
<tr>
<td>1972</td>
<td>1 380</td>
<td>303</td>
</tr>
<tr>
<td>1973</td>
<td>1 527</td>
<td>335</td>
</tr>
<tr>
<td>1974</td>
<td>1 793</td>
<td>393</td>
</tr>
<tr>
<td>1975</td>
<td>1 851</td>
<td>402</td>
</tr>
<tr>
<td>1976</td>
<td>2 218</td>
<td>486</td>
</tr>
</tbody>
</table>

Average annual rate of growth: 19.2%

Source: IACA - Situação do Sector em 1976 (Sep.1977)
This growth of the animal feed industry is attributable to a number of different reasons, namely

(i) the increase in protein intake per capita associated with a rise in the level of income of the population (during the sixties and the early seventies Portugal was among the fastest growing economies in Europe);

(ii) the inability of the fisheries sector to respond to the increasing demand for protein diets; the decline of the sector is clearly shown by comparing fish landings in Portugal in 1967 and 1976: 400.5 and 273.3 thousand m.t., respectively (Source: OECD Economic Surveys - Portugal, July 1974 and Dec. 1977);

(iii) the rapid change in poultry and livestock feeding patterns; traditional feeding practices were replaced by 'industrial-intensive growing' techniques.

Since it was unable to satisfy internally the demand for feed grains, the country had to rely progressively more on imports. Table 4 illustrates

<table>
<thead>
<tr>
<th>Year</th>
<th>Maize (1)</th>
<th>Sorghum (1)</th>
<th>Barley (2)</th>
<th>Total [1000 m.t.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>211</td>
<td>31</td>
<td>9</td>
<td>251</td>
</tr>
<tr>
<td>1968</td>
<td>431</td>
<td>-</td>
<td>4</td>
<td>435</td>
</tr>
<tr>
<td>1969</td>
<td>417</td>
<td>-</td>
<td>10</td>
<td>427</td>
</tr>
<tr>
<td>1970</td>
<td>356</td>
<td>-</td>
<td>70</td>
<td>406</td>
</tr>
<tr>
<td>1971</td>
<td>.517</td>
<td>-</td>
<td>117</td>
<td>634</td>
</tr>
<tr>
<td>1972</td>
<td>821</td>
<td>-</td>
<td>121</td>
<td>942</td>
</tr>
<tr>
<td>1973</td>
<td>786</td>
<td>234</td>
<td>12</td>
<td>1 032</td>
</tr>
<tr>
<td>1974</td>
<td>1 006</td>
<td>378</td>
<td>27</td>
<td>1 411</td>
</tr>
<tr>
<td>1975</td>
<td>1 221</td>
<td>93</td>
<td>9</td>
<td>1 323</td>
</tr>
<tr>
<td>1976</td>
<td>1 272</td>
<td>425</td>
<td>n.a.</td>
<td>1 697</td>
</tr>
</tbody>
</table>

(1) includes a small proportion of white maize
(2) includes grain for malting and brewing

Sources: INE - Estatísticas Agrícolas (from 1967 to 1975)
EPAC records (for 1976)
the sharp rise in the imports of feed grains over the period 1967-1976. Feed barley was imported in significant quantities only during 1970, 1971 and 1972. However these imports were halted because of problems arising in the grinding operation for many animal feed producers, due to the hardness of the grain. Since then, only yellow maize and yellow sorghum have been imported for feed. Most of these imports come from the United States (US) - over 80% in 1976, according to EPAC records.

Figure 1 illustrates how the supplies of maize (home production and imports) were utilised in 1976. The relative sizes of the boxes and channels are approximately proportional to the supplies, usages and flows of this grain.

Figure 1 - Supply and utilization of maize in Portugal (1976)

Imported yellow maize
US no. 3
(1 242 000 m.t.)

Home produced maize
(487 000 m.t.)

Animal feed industry
(1 003 000 m.t.)

Human food + maize directly fed to animals
(690 000 m.t.)

(1) Imported white maize (30 000 m.t.)
(2) Weighted average of 1975 ($w_1 = 0.75$) and 1976 ($w_2 = 0.25$)
(3) Industry: oil, starch

Sources: EPAC records
IACA - Relatório e Contas, Exercício de 1976
The vast proportion of imported maize is yellow, US grade no. 3. This is the only maize supplied to the animal feed industry as well as other maize dependent industries such as oil and starch extraction. The home produced maize is also yellow and is of better quality than imported maize. It is grown in the northern areas of the country on numerous small farms (the 'minifundia'). This maize is consumed directly on the farms or traded in small local markets, and is used as a human food (flour, bread) or fed to animals. The home supply of maize for human use is supplemented by small amounts of imported yellow and white maize.

The fact that none of the home production of maize is used by the animal feed, oil and starch industries results from the price intervention programme adopted by the Government in the sector. The imported grain is sold by EPAC to those industries at prices that are fixed at levels based on the import prices. Meanwhile, the prices of the home produced maize and the maize imported to supply the flour and bread industries are maintained at higher levels through a Government support programme. This programme, which is aimed at encouraging home production, includes:

(i) the prohibition of sales of maize by the animal feed, oil and starch industries;

(ii) a guaranteed minimum price paid by EPAC to any producer willing to deliver maize at the EPAC silos and warehouses;

(iii) a constraint that EPAC may not sell this maize below the minimum guaranteed price, except to the animal feed, oil and starch industries (where the selling price is the same as that of the imported yellow maize).
The failure of home production to meet the consumption in human food and direct animal feeding has meant that no significant deliveries from producers to EPAC were ever made (in 1976, according to EPAC records, less than 30 thousand m.t. were delivered, and were resold at the price paid to the producers).

As far as the other feed grains are concerned, the supply-disappearance processes are simpler than for maize. Sorghum is not produced in the country and all the imports (US grade no. 2) are consumed by the animal feed industry. Feed barley and oats are almost entirely used inside the farms. The quantities of these grains used by the animal feed industry are negligible: in 1976 they were respectively 14 and 19 thousand m.t. (source: IACA - Relatório e Contas (Exercício de 1976)).

3. Recent Developments in the Portuguese Economy. Government Strategy in the Feed Grain Sector

The intervention of the Portuguese Central Government in the feed grain sector cannot be isolated from the overall evolution of the economy in the last few years. For this reason, the presentation of the broad objectives of such intervention is preceded by a brief review of the major social, political and economic developments taking place in the country in recent times. (*)

Until April 1974, Portugal was ruled for fifty years under a dictatorial regime centred around the concept of a 'corporate state'.

similar to that of Mussolini's fascism.

Within this institutional framework and as a result of a considerable industrial development, the Portuguese economy achieved a high rate of growth during the sixties and the early seventies. Between 1969 and 1974 the GDP average annual growth was among the highest in Europe: 6.4%. However, this growth concealed enormous distortions: modern industries, based on capital-intensive technology transferred from the West, coexisting with an extremely backward agricultural sector, with highly distorted income distributions.

The agricultural/rural sector was largely ignored in this development process: still employing 37% of the population, this sector, in 1970, accounted for only 12% of the gross domestic product. (Source: Hill, B. et al, 'An Economic Analysis of Agriculture', Heinemann, London, 1977).

Until 1974, the phenomenon of emigration was central in the evolution of the Portuguese economy. During the sixties and the early seventies about 1.5 million emigrants left the country to the then booming economies of the industrialised nations of the West (mainly France). During this period the total population decreased, reaching 8.7 million by the end of 1974. As shown in Table 5, the emigrants' remittances (under 'Transfers', in the table) together with the revenue from the tourism industry, more than offset the chronic trade deficit, leading to a comfortable surplus of the balance of payments.

The revolution of April 1974 radically changed the basis of the country's social, political and economic organization. The major structural changes brought under successive governments included:
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Exports</td>
<td>1353</td>
<td>1855</td>
<td>2288</td>
<td>1936</td>
<td>1824</td>
<td>2037</td>
<td>2433</td>
<td>3459</td>
</tr>
<tr>
<td>Imports</td>
<td>1947</td>
<td>2763</td>
<td>4277</td>
<td>3606</td>
<td>3925</td>
<td>4553</td>
<td>4748</td>
<td>5975</td>
</tr>
<tr>
<td>TRADE BALANCE</td>
<td>-594</td>
<td>-908</td>
<td>-1989</td>
<td>-1670</td>
<td>-2101</td>
<td>-2506</td>
<td>-2315</td>
<td>-2515</td>
</tr>
<tr>
<td>Services</td>
<td>204</td>
<td>153</td>
<td>55</td>
<td>-184</td>
<td>-111</td>
<td>-127</td>
<td>-96</td>
<td>n.a.</td>
</tr>
<tr>
<td>of which: tourism</td>
<td>(261)</td>
<td>(322)</td>
<td>(258)</td>
<td>(101)</td>
<td>(182)</td>
<td>(268)</td>
<td>(431)</td>
<td>(698)</td>
</tr>
<tr>
<td>Transfers</td>
<td>872</td>
<td>1104</td>
<td>1110</td>
<td>1037</td>
<td>963</td>
<td>1134</td>
<td>1635</td>
<td>2406</td>
</tr>
<tr>
<td>CURRENT BALANCE</td>
<td>482</td>
<td>349</td>
<td>-823</td>
<td>-617</td>
<td>-1249</td>
<td>-1499</td>
<td>-776</td>
<td>54</td>
</tr>
<tr>
<td>M/1.term capital mov.</td>
<td>-129</td>
<td>-142</td>
<td>272</td>
<td>-107</td>
<td>26</td>
<td>95</td>
<td>758</td>
<td>n.a.</td>
</tr>
<tr>
<td>BASIC BALANCE</td>
<td>353</td>
<td>207</td>
<td>-551</td>
<td>-924</td>
<td>-1223</td>
<td>-1404</td>
<td>-18</td>
<td>n.a.</td>
</tr>
<tr>
<td>S.ter m capital mov.</td>
<td>n.a.</td>
<td>136</td>
<td>-82</td>
<td>-89</td>
<td>98</td>
<td>-33</td>
<td>175</td>
<td>n.a.</td>
</tr>
<tr>
<td>Banks foreign posit.</td>
<td>n.a.</td>
<td>-14</td>
<td>64</td>
<td>-10</td>
<td>153</td>
<td>574</td>
<td>-198</td>
<td>n.a.</td>
</tr>
<tr>
<td>BALANCE ON OFF.SETTLEM.</td>
<td>n.a.</td>
<td>328</td>
<td>-569</td>
<td>-1023</td>
<td>-972</td>
<td>-863</td>
<td>-41</td>
<td>n.a.</td>
</tr>
<tr>
<td>Monetary movements</td>
<td>n.a.</td>
<td>-</td>
<td>336</td>
<td>844</td>
<td>504</td>
<td>250</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>CHANGES IN RESERVES</td>
<td>n.a.</td>
<td>-226</td>
<td>569</td>
<td>607</td>
<td>128</td>
<td>359</td>
<td>-103</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

Sources: OECD Economic Surveys – Portugal

(i) the institution of a democratic political system;

(ii) an extensive programme of nationalisation of basic sectors of the economy;

(iii) the replacement of former 'guild-like' unions by independent unions and the profound transformation of industrial relations;

(iv) a land reform programme involving the expropriation of the country's 'latifundia' (large estates, in the South) and the introduction of systems of collective management by the workers;

(v) the raising of low incomes and the extension of social welfare benefits;
(vi) decolonisation.

Under so extensive structural changes, taking place at a time of oil price increases and when the industrialised nations could not provide an outlet for the increasing labour force, the Portuguese economy was severely hit.

Unemployment, very limited before 1974 due to the emigration phenomenon, rose sharply: the rate of unemployment was under 0.5% in December 1974 and went up to 13.9% in December of 1978. The rise in the number of jobs after the events of 1974 fell short of the huge increase in the civilian labour force (from 3.3 million at the end of 1974 to 4.2 million at the end of 1978). This increase was largely associated with the decolonisation process: part of it was attributable to the demobilisation that followed the ending of the colonial war (160 thousand men, between 1974 and 1978); the biggest part of the increase was associated with the influx of repatriates from the former colonies (estimated at around 900 thousand, i.e. increasing the total population by about 10%).

A second major problem of the economy concerned the general rise in prices - between 1974 and 1978 the average rate of inflation was 20.6%. The increase in the price of energy and of imported raw materials, the progressive devaluation of the Escudo and the sharp increase in the internal demand for goods and services, as a consequence of the substantial increase in the total population and of the rise in the purchasing power of the lower income groups, were major causes of the inflationary pressures on the economy.

The most obvious effect of the crisis on the economy, after the 1974 events, concerned the country's balance of payments. As shown
in Table 5, the current balance showed a deficit for the first time in 1974; the situation became progressively worse until 1977, due to a combination of different reasons:

(i) a decrease in exports (attributable to the unrest in major exporting industries and to the absorption of the production by an increasing domestic demand);

(ii) a considerable increase of the imports (a consequence of the increases in prices of energy and imported raw materials and, above all, the result of the substantial increase in the domestic demand);

(iii) a decrease in the inflow of invisibles (namely emigrants' remittances and tourism revenue), at a time marked by social unrest and political uncertainty.

To make up the deficits in the current balance, successive governments were forced to draw on the strong reserves left by the previous regime and to resort to external credit, negotiated through the IMF.

Partly as a condition imposed by the IMF, starting in 1977 the monetary policy was shifted progressively to a more restrictive stance and steps were taken to try to improve the country's financial situation: the devaluation of the Escudo, the imposition of new import restrictions, measures seeking to attract greater remittances by the emigrants are examples of the instruments used to reverse the dangerous trend of the balance of payments. The results of the policy pursued started to show in 1978. In this year, the deficit of the current balance of payments was nearly halved, as shown in Table 5. Provisional figures released by the Portuguese Government (in Grandes Opções do Plano - 1980) point to a remarkable recovery of the current balance: from -776 million US $
in 1978 to 454 million US$ in 1979. According to those figures, the recovery was due to a considerable increase of the 'invisibles' (mainly emigrants' remittances and tourism) that largely offset the moderate increase in the trade deficit.

The contribution of the feed grains imports to this deficit has been considerable. During 1976, the imports of maize and sorghum alone amounted to approximately 206 million US dollars. (Source: EPAC records), i.e. equivalent to nearly 10% of the total trade deficit. It is therefore not surprising that feed grains became a target in the government's plans to control such a deficit.

Under the broad objective of reducing feed grain imports, the Government adopted a number of measures both to encourage expansion of home production and to curb the upward trend in the consumption of feed grains.

Within the first group, measures such as raising the minimum support prices and granting better credit facilities to the farmers were taken. However, their impact appears to have been negligible. In fact, the total feed grain production decreased from 673,000 m.t. in 1976 to 591,000 m.t. in 1977 and 541,000 m.t. in 1978, mainly due to the failures of the barley and oats crops. (Source: FAO Production Yearbook - 1978).

It is difficult to predict how the feed grain production will evolve in the future. However, due to the nature of some of the causes underlying the current low productivity of these crops - e.g. the low educational level of the rural population or the difficulties associated with the implementation of the land reform programme - any process of raising the present levels of production to the potential ones will be
necessarily slow.

Since 1977, the Government has imposed quotas on feed grains imports (expressed in quantity, not in value) in order to keep the growth in their consumption under control. This measure, supplemented by the control of maximum prices of animal feeds and food products (meat, milk) has been preferred to a control of the consumption based exclusively on price rises resulting from the taxation of the imported feed grains. The two basic reasons for this preference are concerned with the fight against inflation (recognised as a major problem of the economy) and the fact that food price rises would affect mostly the lower income groups of the population.

Figure 2 shows the evolution of the feed grain imports until 1978. At first sight the diagram would suggest that the quotas imposed by the Government in 1977 and 1978 did not correspond to any real restriction to the growth of the imports. In fact, during those years, the imports appeared to be in line with the evolution observed in previous years. However, the correct interpretation of the figures requires the consideration of the 10% increase in the country's population due to the decolonisation process. The corresponding increase in the demand for feed grains, felt mostly in 1977 and 1978, was likely to exceed 10% since the living standards and protein diets of the repatriates were, on average, higher than the Portuguese ones.

According to data originating from EPAC and published in 1978 in a study of the new grain terminals at the port of Lisbon, the imports, under direct government control, are expected to stabilise in the 1980's around a target of 2.4 million metric tons. However, approximate this
figure may be it shows clearly that feed grains imports will remain, in the foreseeable future, an important feature of the Portuguese economy.

4. The Role of EPAC in the Feed Grains' Importing Process

The origins of EPAC can be traced back to the old 'corporate state' organization and more recently to the Instituto dos Cereais (IC). EPAC was created in 1977 when IC was given the status of 'Empresa Pública' (a state owned enterprise). Acting as a Government agency, EPAC is not a profit oriented organization. Its corporate objective can be defined as the efficient provision of a whole range of services concerning the implementation of Government intervention programmes in the internal
grain markets. In order to fulfil its role EPAC was granted by the Government the monopoly of all grain imports into the country.

In the context of the Government intervention plans aimed at encouraging the home production of feed grains, EPAC is involved in a number of different activities. Some of them concern the price support programme for home produced feed grains. This programme, briefly outlined in section 2 for maize, covers barley and oats as well. Other actions aimed at supporting home production include importing and distributing better quality feed grain seeds.

The role played by EPAC in the feed grains importing process is essentially the same, whatever the destination of the imported grains. It will be illustrated taking the case of the animal feed industry, singled out earlier as the one that consumes most of the imported feed grains.

Figure 3 identifies the roles played by the Government, EPAC and the Industry in the supply, distribution and utilization of feed grains for that purpose.

The Government intervenes in four distinct areas:

(i) fixing the global import quotas: at the end of each calendar year, in consultation with EPAC and IACA (the association of animal feed manufacturers) the Government defines the annual import quotas for maize and sorghum (expressing the amounts that are to be imported);

(ii) fixing the internal prices at which the feed grains are sold by EPAC: these prices are readjusted from time to time, sometimes remaining at the same level over periods
of two years or more; the differences between the (fluctuating) import prices and the (fixed) internal prices are absorbed by a large Government fund designated 'Fundo de Abastecimento';

(iii) fixing standards for animal feeds (including their classification and designation and the imposition of maximum or minimum contents of several nutrients);

(iv) fixing the maximum selling prices of most of the animal feeds (about 82% of the whole production, during 1976 according to IACA - Situação do Sector em 1976 (Sept. 1977)).

Figure 3 - Roles played by the Government, EPAC and the Industry in the supply, distribution and utilization of feed grains for the production of animal feeds.

Diagram:

- FEED GRAIN EXPORTERS
  - Maize
  - Sorghum
  - Raw materials other than feed grains (O)

- EPAC
  - (purchasing, storing and selling)

- GOVERNMENT
  - Global quotas
  - Internal prices
  - Indiv. quotas

- MANUFACT. "A"
  - (mixing raw materials into animal feeds)

- MANUFACT. "B"
  - (...

- ANIMAL FEED MANUFACTURERS
  - Feeds' standards
  - Feeds' max. prices

- LIVESTOCK GROWERS

- Animal feeds

- IACA

Diagram represents the flow of feed grains and animal feeds through various entities.
Once the global import quotas are fixed by the Government, IACA defines for each of its members (78 in 1976, according to IACA-Situação do Sector em 1976 (Sept. 1977)) individual annual quotas of maize and sorghum. Each manufacturer is entitled to buy each month, from EPAC, one twelfth of his annual quota of each grain.

In the process described in Figure 3, EPAC is responsible for:

(i) purchasing the feed grains in the open international market (mostly, the US export market, as said earlier);

(ii) timing the feed grains deliveries at the Portuguese ports of Lisbon and Oporto and controlling the stock levels at its port silos or warehouses so as to maintain efficiently the supply of feed grains to the internal market;

(iii) selling the raw materials to the individual manufacturers (at the prices fixed by the Government), making sure that each manufacturer does not exceed, in each month, his individual quota.

5. The Current System for the Planning and Management of the Feed Grains' Imports: Identification of its Major Inadequacies

The basic inadequacies of the system currently adopted to plan and manage the feed grain imports can be identified considering separately

(i) the definition of the feed grains' import quotas (in which EPAC plays a role together with the Government, other Government Agencies and organizations representing the industries dependent on feed grain imports);
(ii) the purchasing, shipping and inventory of feed grains
(all within EPAC’s sphere of responsibilities).

As far as the first decision is concerned, the current system contains two major deficiencies:

(i) the system is not adaptive to changes in the import prices of the feed grains;

(ii) it ignores the substitutability between maize and sorghum, on the one hand, and between those feed grains and other raw materials used in the animal feed industry, on the other.

The first can be illustrated by taking the definition of import quotas for the maize starch extraction industry. In this industry there is no other raw material that can replace maize in the production process.

For a given maize import price, the imposition of an import quota (expressed as a quantity of maize) is the measure used by the Government to limit the foreign currency expenditure incurred by that industry. In adopting a given value for the import quota, the Government does it in the belief that that value corresponds to the best compromise (or something acceptably close to it) between the several goals to be accomplished: economic, social and political ones. It is obvious that
a considerable departure from the circumstances prevailing at the time of fixing a quota can convert this quota into an unacceptable compromise. The fluctuating nature of commodity prices, frequently subject to significant and unpredictable changes, calls for an adaptive system for planning the import quotas. The present system of fixing the quotas annually does not give the planner the flexibility required to respond rationally to changes in the prices of the imported commodity.

The inappropriateness of the current system is even more obvious when it comes to defining the import quotas for maize, sorghum and other substitutable raw materials of the animal feed industry, whose relative prices fluctuate considerably over time. The system, based on the establishment of separate annual quotas for each raw material, irrespective of the evolution of their relative prices, does not enable the planner to adapt the import mix to significant price changes, in the attempt to save foreign currency.

In addition to being non-adaptive, the definition of the import quotas is based on too simplistic a criterion, essentially involving extrapolations from previous import figures, regardless of changes in the prices of raw materials or in the country's needs.

This approach - not based on an explicit attempt to satisfy most economically the requirements corresponding to pre-established animal feed production targets - entails an unnecessary wastage of foreign currency for the country and the unfeasibility for the Industry to produce adequate animal feeds with the available raw materials. This is illustrated in a letter dated 4th April, 1977 and sent by IACA to
the Government. Making the point about the inconsistency of the import quotas in force at that time (apparently too stringent for oil seed meals) IACA described the Industry's possible courses of action in the following terms: 'either to reduce even more the production of animal feeds or, in order to make full use of the available feed grains, to reduce drastically the incorporation of oil seed meals (...)'.

Considering now the management of the import operations carried out by EPAC, it can be said to follow traditional methods, without the aid of any significant quantitative analysis. As far as the purchasing operations are concerned this has meant the adoption of a 'hand-to-mouth' type policy: in the absence of reliable price forecasts, the purchaser buys the commodity just when it is needed for consumption. Under the adoption of this policy the purchasing price will tend, in the long run, to the average market price.

Figure 4 shows the evolution of the purchases and deliveries of yellow maize over 1976. The purchases were made on a 'c&f' Lisbon/Oporto basis (cost and freight, delivered at the main Portuguese ports), the insurance being made under a different contract, with a Portuguese insurance company. From the analysis of Figure 4, it is clear that the flow of maize took place at a steady rate over the year. This was a result of the very limited storage capacity at the grain terminals. Additionally, Figure 4 shows how close the purchasing policy was to a 'hand-to-mouth' policy. The time lag between the line 'Purchase commitments' and the line 'Deliveries - specified latest date' varied between one and two months, which is the normal time required by exporters to deliver grain shipments to Portugal, after their orders have been placed.
As to the shipping and storage operations, EPAC management has a complex task to perform without the help of such quantitative analysis. Under severe storage capacity constraints and facing considerable uncertainty in the dates of the vessels' arrivals, a difficult trade-off must be made between the risk of shortage, on the one hand, and the operating costs associated with transporting, unloading and storing the imported grains, on the other. The shortage risk can be decreased in two ways: by increasing the grains' safety stocks in port (at the expense of an increase in the length of stay of the vessels in port) or, in the event of an imminent shortage, by making an attempt (not always successful) to buy 'afloats' (grain already in
a vessel near the Portuguese coast, without an assigned destination, that for a high price can be diverted at short notice to the port where the shortage is about to occur.

EPAC's difficulties in defining a sound policy for the delivery of grain to the ports were evident in the events of 1976. Even having vessels chartered with very low nominal rates of unloading (normally 1500 m.t./work day, according to EPAC records) and in spite of the purchase of six 'afloats' at excessively high prices (Figure 5), EPAC management could still not avoid shortages of maize. A major shortage occurred in October at both ports simultaneously and there were three other occurrences involving only one port.

Figure 5 - Yellow maize purchasing prices, c&f - Lisbon/Oporto (1976)

After 1976 some major changes affecting the feed grain import operations took place:

(i) a new grain terminal in Oporto (Leixões) replaced the warehouses previously used in the unloading/dispatching
operations;

(ii) the development of a new grain terminal in Lisbon was initiated and is under way;

(iii) the purchases of feed grains are made, since March 1977, on a 'fob' basis (free on board, at the exporting port); the sea transport of the grains since then has been made under a contract between EPAC and the two major Portuguese ship charterers and shipowners;

(iv) there has been a progressive shift from purchasing directly in 'physical' (i.e. purchases of grain made directly to the exporters) to purchasing through the Chicago Commodity Exchange (first in 'futures', then converted to physical).

Although these developments can contribute to the improvement of the efficiency of the feed grains' import operations, their full potential will not be realized unless fundamental changes in the attitude of the management of EPAC and other organizations take place. These changes include

(i) the integration of the decisions emanating from different organizations, or from different departments within each organization, into a common decision process;

(ii) the replacement of improvised decision-making by a planning system supported by quantitative analysis.

It is hoped that the method and the results of this research will provide a basis and a stimulus for such changes.
CHAPTER 3

SCOPE AND METHOD OF THE RESEARCH

1. Introduction

In the previous chapter it was shown that the planning and control of the feed grain imports into Portugal is of considerable importance and that it has not been approached adequately by the Government and EPAC. These two facts clearly point to the need for an analysis of the problem. However, the breadth of the decision problem is such that it is possible to employ different types of analyses, with distinct (and perhaps equally relevant) emphases. The objective of this chapter is to define the scope and the method of the research described in this thesis.

As a first step to accomplish that objective, in section 2, the process of government intervention in an economy is analysed and its basic decision levels are identified and characterised. The distinction between these decision levels provides a useful frame of reference to define the scope of the analysis.

Having defined the scope of the research in section 3, the decision problem under study is then identified as a 'commodity purchasing problem' in section 4. Whilst reviewing previous research in this area, the main components of the problem are identified and the interactions between them are analysed.

The final section describes the method of approach adopted in the formulation and modelling of the problem. Derived from basic concepts of the OR method, the emphasis of the approach is on the
integration of the solutions of the several problem components into a consistent overall decision policy.

2. The Process of Government Intervention in the Economy

The process of government intervention in the economy can be regarded as a two-stage process involving

(i) what is commonly known as 'government economic planning' or simply 'government planning', and
(ii) the implementation of agreed plans.

The first of these stages has been defined in different ways, varying in precision and emphasising different relevant aspects of the government planning process. For Dahl et al. (p.20) government planning is 'an attempt at rationally calculated action (by government) to achieve a goal'. Sirkin stresses the level at which the government intervention is made when he states (p.45) that government planning involves 'the attempt, by centralizing the management of allocation of resources sufficiently, to take into account social costs and social benefits which would be irrelevant to the calculations of the decentralized decision maker'. Other authors emphasise the pragmatic nature of the economic planning process. In a paper on the theory and practice of economic planning, Hilhorst states (p.1.1) that 'Policy-making (...) is concerned with more than setting priorities or designing a new society. It is very much concerned with developing concerted government action that is feasible and that leads to desired results.'
Economic planning is part of the art of deciding to do the 'feasible'.

In the United Nations report entitled 'Economic Planning in Europe', government planning is defined in a more specific way, as a process involving three separate but related stages:

(i) the derivation of major policy goals and the ordering of their relative priorities;

(ii) the translation of these goals into a set of explicit, consistent and quantified economic targets for a specified period of time by an assessment of resources, analysis of relevant interdependencies between economic variables and a survey of possible effects of acceptable policy alternatives;

(iii) the selection of policy measures designed to achieve those targets.

In the planning process, even at a sectional level, the number of complexity of the goals to be accomplished are usually quite considerable. Since many of those goals are frequently in direct conflict, government planners are asked to make trade-offs between them in the attempt to reach the best possible compromise. This compromise expresses the way of best satisfying globally the original set of goals, within the economic, social and political constraints imposed by the planning environment. The set of economic targets defined in the second stage of the planning process can be interpreted as expressing that best compromise, or something acceptably close to it, in terms that are sufficiently pragmatic, so as to make possible the definition of concrete policy measures.
The proliferation of measures required to reach the agreed targets reflects both the wide range of goals sought through their implementation and the need for each goal to be approached through a combination of measures. In spite of the need for an integrated implementation of such measures, governments face the need to split the implementation process among different specialized agencies, given the size and complexity of the tasks involved. Although specialization and coordination are not incompatible in principle, the dynamic nature of the planning process, carried out by successive governments frequently with different views on how to intervene in the economy, can lead to the severing of essential links among inter-related agencies or between these and the central government. A permanent organizational effort is therefore required to maintain an overall harmony in the government intervention process. The recognition of this need is the reason for the inclusion, by Halcrow, of a fourth stage in the planning process:

(iv) the selection of appropriate agencies to implement policy measures and the definition of the degree of constraint to be exercised in limiting their activities.

The planning process just outlined corresponds to what Anthony, in 'Planning and Control Systems - A Framework for Analysis', defines as 'strategic planning' (p.16): 'the process of deciding on objectives of the organization, on changes in these objectives and on the policies that are to govern the acquisition, use and disposition of resources'.
The second stage of the government intervention process -
early described as the implementation of agreed plans - concerns
the planning and control of ongoing operations, in the attempt of
achieving efficiently the pre-established economic targets, within
the guidelines and constraints imposed at the strategic planning
level. In this implementation process, Anthony⁵ makes a
distinction between two different sub-processes that he defines
as follows (pp.17-18):

(i) 'Management control is the process by which managers
assure that resources are obtained and used effectively
and efficiently in the accomplishment of the organization's
objectives';

(ii) 'Operational control is the process of assuring that
specific tasks are carried out effectively and efficiently'.

The distinction between these two processes is unclear from the
definitions, requiring further clarification. The following quotation from
Anthony ⁵ (p.92) is helpful in this respect: 'Two basic
characteristics distinguish operational control from management
control. First, operational control focuses on specific tasks or
transactions, whereas management control focusses on the whole
stream of ongoing operations.

Second, operational control is essentially objective, whereas
management control is essentially subjective. Operational control
is objective in the sense that it has to do primarily with
activities for which the correct decisions can be objectively
determined. At least conceptually, and often practically, a valid
decision rule can be stated mathematically and programmed into a
computer. Management control is essentially subjective in that
decisions in this process inherently involve management judgement,
and there is no objective or "scientific" way of determining the
best course of action in a given set of circumstances'.

As is often the case in attempts to define basic concepts,
the divisions between the processes identified earlier are somewhat
fuzzy. This is inevitable in any classification of processes associated
with decision problems ranging 'continuously' from highly complex
and ill-defined (therefore of consequences extremely difficult
to appraise) to simple and well defined (where the results associated
to the decisions can be interpreted 'objectively'). Situations can
be found that do not fit clearly in a single category or when the
fitting into one or another category is to some extent, a matter of
interpretation. Nevertheless, however controversial this classification
may be, the distinction between the defined processes - in the type
of analysis that can be usefully carried out, in the type of
information required and in the type of results that can be achieved
- is clear enough to provide a useful frame of reference for the
definition of the scope of the research.

3. Scope of the Research

In the introductory chapter, the objective of the research was
defined as 'the formulation and modelling of essential aspects of
the decision process involved in the management of commodity import
operations'. It is now easier to define more precisely what is
meant by this objective:
(i) the focus of the research is on the decision processes occurring at the management control and operational control levels (the expression 'management of import operations' was chosen to reflect this intent);

(ii) the emphasis of the study is on the quantitative analysis of the decision problems posed at those levels; using Anthony's terminology, the study is an attempt (a) to identify the aspects of the problem that should be a matter of operational control and (b) to define an integrated set of models to help the management to derive operational control rules.

One can now analyse the implications of this definition of the research objectives, considering specifically the planning and control of feed grains imports into Portugal.

Strategic planning decisions are not the primary concern of the research. According to the classification criterion defined earlier, these decisions correspond to what was described as the 'government economic planning' process: deriving policy goals, translating them into explicit economic targets, selecting policy measures to achieve those targets and setting up the organizational structure capable of implementing these measures.

One point must be made concerning the strategic choice of economic targets. In an industry dependent on an imported raw material that has no substitutes (the maize starch extraction industry is an example) the establishment of a production target implies automatically a decision on an import target. However, the
situation is rather different in the case of the animal feed industry, which depends on imports of several substitutable raw materials. In this case the strategic decision is made when fixing the industry's production targets (corresponding to what was previously designated the best economical, social and political compromise, in the government planner's view). The decision of translating those production targets into import targets of each of the raw materials concerned should be regarded as part of the management of import operations. In other words, that conversion of production targets into import targets should be interpreted as part of the job of assuring that the available resources 'are used effectively and efficiently'. And as such, it will be covered in this research.

The differences between those decisions - establishing production targets for the animal feed industry and converting them into import targets - provides an excellent illustration of the convenience of adopting the division between strategic and non-strategic decisions as a boundary for the scope of the analysis. As a matter of fact, it is difficult to find two decision problems as different in complexity, breadth, type of analysis involved in their 'solution' and type of data required, as these. The first problem - setting appropriate production targets - is extremely ill-defined and involves the consideration of decision factors as diverse as the effects of different production levels on unemployment, balance of payments, growth of the economy, nutritional standards of the population or even on the reaction of the industrialists and the political forces representing their interests. In making a decision, the government has a wide choice of alternative courses
of action, combining different levels of restriction of the animal feed production with measures aimed at replacing meat diets by fish diets, replacing home production of livestock by direct imports of milk, eggs or meat or encouraging a better use of natural pastures, among many others. By contrast, the second decision - converting the production targets into import targets - poses a more restricted and relatively well defined problem. The objective is to establish the import mix that meets the requirements imposed by the production targets using the country's resources most efficiently.

Another implication of the definition given earlier for the scope of the research is that the discussion of the types of measures adopted by the government to achieve the desired economic targets or of decisions regarding the organization structure and the division of responsibilities in the process of implementing those measures is not a primary concern of the research. The adoption of import quotas to control the feed grain imports or the procedure used in converting the global import quotas into quotas for the individual manufacturers of animal feed are examples of strategic choices that are unquestioned in the study. However, this does not mean that the analysis carried out does not have significant implications for strategic planning decisions. As a result of the study it was possible to identify, for example, the potential benefits associated with making the import quotas responsive to price changes or those resulting from the coordination of decisions currently taken by different bodies without proper consultation. Certainly these are important factors in organizing the overall structure of the import planning and control system. However, it has to be recognised that the impact of the results of the analysis is only one of several factors that has to
be taken into account in deciding what is the best set of policy measures or what is the best overall organizational structure for the purpose of attaining the desired goals.

The second part of the definition of the objectives for the research further restricts its scope. As implied there, central concern of the research is the quantitative analysis of the most important aspects of the decision process associated with the management of the feed grains import operations. The main aspects covered in the study are:

(i) the conversion of animal feed production targets into import quotas for each feed grain;
(ii) the planning of the sea transport, discharge and storage operations;
(iii) the planning of the purchasing operations.

They can be regarded as basic components of a global 'commodity purchasing problem'. The next section presents an overview of the problem and stresses the sources of interaction between the decisions concerning each of those components.

4. The Commodity Purchasing Problem: an Overview

4.1 General

Feed grains, like many other raw materials referred to as commodities, are traded on large international exchanges. Buyers and sellers of a commodity, represented by members of the exchange, trade 'cash' and 'futures' contracts, for immediate or deferred delivery of specified amounts of the commodity. All the transactions
take place under rules set by each exchange. The purpose of these rules is to simplify the trade and to guarantee a fair deal for all the market participants. In addition to the role of regulators and standardizers of the trading procedures, commodity exchanges act as conduits for information reflecting what happens around the world to each commodity that may affect its price. In this role, large commodity exchanges extend their influence to transactions taking place outside their control. The prices of these transactions follow closely the evolution of the prices quoted at the major commodity exchanges.

Reacting to ever changing estimates of production, consumption, stocks, etc., prices on the commodity exchanges fluctuate considerably. It is not unusual for a commodity price to change by 5% from one day to the next or to double, or to halve, in less than six months. This price volatility is a major source of difficulty in the purchasing decision process and it is the prime reason for singling out the 'commodity purchasing problem' from other purchasing problems.

Although many authors made relevant contributions to the analysis of specific areas of that problem, the central contribution towards its overall definition - i.e. the identification of its major components and the interactions between them - is due to Kingsman. The commodity purchasing problem, in what can be regarded as its simplest configuration, involves a sequence of three basic stages the buyer has to go through when deriving an efficient purchasing policy. Kingsman (p.25) describes them as follows:

' (a) first determine his requirements day-by-day or week-by-week into the future and update his current stocks and
any forward contracts to derive his present stock
cover and convert his forward consumption requirements
into order quantities specifying delivery dates,
(b) collate the available information on the state of
the market and the past prices to derive forecasts of
the future price movements,
(c) use the results from the previous stages with a purchasing
policy to arrive at the most appropriate buying decisions
for the present price offer'.

This description of the decision process, is applicable only
when the definition of the requirements and orders (size and delivery
dates) is independent of the evolution of prices and of the buying
decisions. It has the merit of identifying the essential aspects
of the decision process and it provides a useful basis to review previous
work on price forecasting and on the derivation of purchasing policies,
before problems of higher complexity are considered.

4.2 Price Forecasting

The models used to derive commodity price forecasts can be grouped
into two distinct categories, according to their underlying method:
statistical models and econometric models. The statistical models,
based on the analysis of time series of daily prices, attempt to
identify patterns in the past price movements and to extrapolate them
into the future. Their main limitation is identified by Kingsman
and Jex\textsuperscript{42} when they state (p.2) : 'The obvious drawback with such
methods is that a linear trend (or other type of movement) is extrapolated
continuously into the future without limit. In practice, however,
such trends do not continue indefinitely. An event such as weather
damage leading to the partial destruction of a crop (...) may occur,
thus reversing the trend of prices and bringing new factors into play. Obviously any system based on extrapolating the recent price movements into the future cannot predict in advance that a reversal of the trend will occur (...). For this reason, the use of statistical models is restricted to short-term forecasts, up to about two months into the future.

The successful use of statistical models in commodity purchasing requires that (i) price trends do exist and (ii) they are identifiable from the observation of past prices. Until recently there was no conclusive statistical evidence to show that these conditions hold. The use of statistical models (such as Brown's exponential smoothing or Trigg and Leach's adaptive exponential smoothing) to obtain commodity price forecasts was formerly justified on the evidence of positive purchasing results derived using them. However, in 1978, Taylor and Kingsman showed conclusive evidence for the existence of price trends in daily commodity price series and proposed the first formal model of price trends. This model, discussed in detail in chapter 8, was adopted to derive short-term forecasts of maize 'futures' prices.

The limitations of statistical methods - only applicable to the derivation of short-term price forecasts - led to the development, by Kingsman of models of market behaviour which enable future price changes and trend reversals to be anticipated well in advance. These 'behavioural' models can be regarded as econometric models to the extent that they relate measures of price (e.g. average 'cash' price over specified months of the year) to a few 'fundamental' economic variables (measuring the commodity's supply and demand factors). However, the approach adopted by Kingsman differs considerably from the one traditionally used by economists. The basic differences
are stated in the following quotations from that author 42 (pp.4-7):

(i) "... any market behavioural model (...) should be a dynamic model that explicitly takes account of the actual or expected changes in supply and demand factors";

(ii) "The second characteristic of the approach used, which again differs from the traditional models, is that it is the market's view of the situation at the time that matters primarily, i.e. the estimates and not the actual values that are only known in hindsight";

(iii) "Thirdly, it is assumed that future price expectations arise implicitly as a result of explicit expectations or forecasts of supply and demand changes";

(iv) "Fourthly any model should be based on how the market actually behaves, not on elegant or attractive theories of how the market ought to behave. The form of the model should be dictated by reality, not by the elegancies of particular theories nor the elegancies or difficulties of particular methods of mathematical analysis".

(v) "Finally, the model takes the form of an explicit price determination equation with supply and demand treated as exogenous variables (...). An important market characteristic for consideration in any model is that market factors are generally discussed in relative terms".

Amongst the considerable number of models developed by Kingsman using this approach, one of them, concerning maize cash prices 40 is of particular interest in the context of the problem analysed in this thesis. As described later in chapter 8, this model was adapted to produce medium-term forecasts of maize 'futures' prices.
4.3 **Purchasing**

Once the order quantities and their delivery dates are specified and after deriving forecasts of the future price movements, a difficult problem remains to be solved: that of deciding when to purchase each specified order. Kingsman \(^{41}\) characterized this problem, identifying three basic stages in the decision process:

(i) the definition of a 'potential (or strategic) buying period';
(ii) the selection of an 'active buying period';
(iii) the 'tactical' buying, within this active buying period.

Senior management of companies involved in the purchasing of commodities invariably set a limit on how far in advance of its delivery date can each order be purchased. Alternatively, management can place constraints on the total capital commitment into purchases and stocks at any time. As noted by Kingsman \(^{41}\) (p.27), 'the net result of either policy is that each scheduled delivery order will have a potential buying period during which it can be purchased'. The choice of a potential buying period places the limits on the freedom of action of buyers. Its length will depend on how far into the future good forecasts can be made and on the attitude of management towards risk and uncertainty and towards consistency and variability of the purchasing performance.

As pointed out by Kingsman \(^{41}\), in the case of agricultural commodities, the length of the potential buying period can be as long as 9 months. Referring to this case, in which the buyer has around 180 trading days in which he can purchase the commodity, that author writes (p.27): 'In making a decision a buyer does not and cannot predict a price for each of the 180 days. He forms some idea of the general trend of prices and uses this to break the potential buying period into a set of sub-buying periods. He then chooses one of these as the one he will be considering
making a purchase since it contains his lowest forecast prices. This will typically be one month in length or so. Of course, as time progresses he may modify his views and his choice of active buying period'.

In previous research no attempt has been made to treat, with minimal formality, the decision process involved in the choice of the active buying period. This important aspect of the problem confronting the purchasing will be discussed later in chapter 8. A decision rule will be proposed, to enable the derivation of the active buying period from medium-term forecasts. The rule seeks the minimization of the long-term purchasing costs, within constraints set according to the purchaser's conception of permissible risks in the purchasing operation.

Having specified an active buying period, the purchaser still faces a complex decision. Within that period (with several buying opportunities), when offered a particular price quotation, the purchaser has to decide whether to wait until the next opportunity or to buy at the present time and, in the latter case, to decide how much should be bought. This is what Kingsman \(^{38,41}\) calls the 'tactical' buying decision. This author author \(^{36,37}\) proposed a dynamic programming (DP) model to solve the problem, under the objective of minimizing the expected purchasing cost over the active buying period chosen. At each buying opportunity, the solution (i.e. the buying decision) depends on

(i) the current price offer, and

(ii) the prior distributions of future prices (normally derived from statistical forecasting models).

In order to be able to derive a solution for the DP model, the assumption of mutual independence of the prior distributions of the future prices is required. However, as pointed out by Taylor \(^{60}\), this assumption is incompatible with the type of non-stationary observed
in commodity price series. The recognition of this fact has an important implication: the buying policies derived from Kingsman's DP model can no longer be regarded as optimal. Other policies should then be considered, in an attempt to improve the purchasing results. This question will be analysed in chapter 8, where an alternative policy, derived heuristically, will be proposed for the 'tactical' purchasing of commodities through the 'futures' markets.

4.4 Sources of Additional Complexity

The structure of the commodity purchasing problem can be considerably more complex than the one implied earlier where the decision process was described as a three-stage sequence. Interactions between the several decisions involved in the overall process can occur, namely when

(i) the definition of the commodity requirements is dependent on the price of the commodity, or when

(ii) the order quantities and delivery dates cannot be defined independently from the purchasing decisions when attempting to minimise the overall operating costs (inventory, delivery and purchasing costs).

The first of these situations occurs in the 'blending problem', in which the objective is to mix several substitutable commodities in the attempt of satisfying most efficiently a number of constraints imposed on the mix. Under fixed prices, this problem is formulated as a linear programme (LP), usually under the objective of minimising the global cost of the mix. However, when prices do fluctuate, the
problem becomes more complex. As Kingsman noted, the additional difficulty in this case concerns the determination of the prices at which the raw materials should be costed in the objective function. A simple two-stage procedure

(i) determine the optimal mix with current prices and,

(ii) having fixed the requirements of each commodity, apply a purchasing rule to derive the best buying decisions,

generally will not lead to an optimal solution. In fact, it assumes that the ratios between the prices of the different commodities at an initial point in time will remain the same over all the feasible buying opportunities, which is not generally true.

Kingsman proposed a model to solve jointly (without partition) the blending and purchasing decisions, in which the DP rules mentioned earlier were imbedded into an LP. Leaving aside the question of the validity of the assumptions involved (partially discussed earlier), it was shown by that author (pp.333-353) that the full integration of the blending and purchasing decisions could only be achieved for problems with a small number of variables and constraints. For large problems – such as the one involved in the definition of the feed grains' import quotas – the computational effort required would be insuperable. However, the suboptimization introduced by partitioning the problem, i.e. treating separately the mix formulation and the purchasing decisions, can be minimized if the following procedure is adopted:

(i) re-running the LP whenever significant changes in the raw materials' price forecasts or a significant number of purchases take place;
(ii) ensuring (through the LP structure) that existing stocks and committed purchases are used most efficiently in the blend (these stocks and committed purchases need to be updated from run to run);

(iii) costing each raw material in the objective function of the LP on the basis of estimates of the purchasing prices that are expected to be paid rather than with the current prices or estimates of average market prices over the buying period.

This procedure, its difficulties and ways of overcoming them will be analysed in chapter 9. The analysis includes a discussion of how different attitudes towards the risks associated with the volatility of commodity prices can affect the blending - purchasing decisions. The criterion implicitly accepted in previous work - the minimization of expected cost of the mix - will be questioned and alternative criteria will be considered.

The other major potential source of complexity is associated with interactions between the definition of order quantities and delivery dates, on the one hand, and the purchasing decisions, on the other. These interactions will generally occur when

(i) at each buying opportunity the purchaser is offered the option of buying the commodity for alternative forward delivery dates (the quoted prices varying with the date of delivery and the margins between these prices changing with time); or

(ii) there are economies of scale in the delivery process.

For a given set of price quotations (each of them corresponding to a particular delivery period), if the purchaser decides
to buy, he has several choices to take advantage of the economies of scale in the delivery process. He can add new purchases to any previously committed purchases yet to be delivered (with dates specified in previous buying opportunities) or to group them in one or more new delivery dates.

In this situation, the sequential process presented earlier—first decide on the size and date of the deliveries and then decide when to purchase them—will generally not lead to the minimization of the total inventory-delivery-purchasing costs. In fact, since each price offer is related to a delivery date, any decision on order quantities and delivery dates will potentially affect purchasing costs and, vice-versa, purchasing decisions will have implications on inventory and delivery costs. Therefore, the minimization of the total costs will generally require a joint decision on deliveries and purchases. It is not difficult to realize the potential complexity of the problem, if the following points are considered:

(i) the purchaser's freedom of choosing both the quantities and delivery dates of each buying opportunity makes the problem highly combinatorial;

(ii) for any given delivery date, the fact that purchasing decisions are based on heuristic rules makes it difficult to state explicitly, at any given buying opportunity, what is the purchasing price that is expected to be paid until the end of the buying period; for this reason accurate comparisons between expected purchasing prices associated with different delivery dates are difficult to make;
in some situations, the economies of scale achievable in the delivery process cannot be defined explicitly as a function of the amount to be delivered, since they also depend on the state of the delivery-inventory system (this situation will occur, for example, when grains that are purchased f.o.b. at any exporting country, have to be transported by sea and unloaded at a grain terminal; as it will be shown later in Chapter 6, the unit shipping cost will depend both on the size of the vessel carrying the grain and on the length of the vessel's stay in port; this length of stay, in turn, will be a function of the state of the grain terminal when the vessel arrives, depending on the number of vessels queueing or unloading at the terminal and on the available storage space in the terminal's silos).

Having identified the potential complexity of the problem, it is clear that an overall model building exercise aimed at representing the commodity purchasing problem in reasonably general terms would be bound to fail. Such a situation is not unique, as illustrated in the following quotation from Simpson 56 (pp.7-8) when he refers to problems in the production planning and control area: '(...) many problems do have special features - and often these features dominate technical effectiveness of planning and control systems. In such cases it is clearly not justifiable, even if it were practical, to build overall optimising models. Generally it will be much more efficient to identify such critical features and to design the overall system around a good, if not optimal, procedure for handling these critical
aspects. But again, of course, the other functions in the total system must be adequately covered and effective links forged between them.

It has been suggested elsewhere that it is only the exploitation of the special features that permits of their solution at all. This philosophy seems well suited to those production planning and control situations which do not fit into the standard mould (and this is most problems, perhaps) (...). Later (p.8), the same author asks: 'But how should we go about this task of identifying the aspects which it is worthwhile handling, and of devising procedures which take such aspects into account well enough?'.

Likewise, in the commodity purchasing area and, more specifically, in the planning of commodity import operations, it is more relevant (or realistic) to devise a general approach that enables the analyst to answer those questions in each particular problem than to try to identify and analyse a general problem structure that could then be adapted to particular situations. In the next section such an approach, derived from basic concepts of the OR method, will be broadly described.

5. Method of Approach
5.1 General

The words quoted earlier from Simpson mention critical aspects of the formulation and modelling of decision problems. Vollmann 71, who regards problem solving as a 'process of system design', suggests that those aspects can be described as:

(i) the 'inclusion process' (i.e. the selection of the system to be analysed, defining its boundary);
(ii) the 'structuring process' (i.e. the identification of
the structure of relations between the attributes of the
set of entities of the chosen system);

(iii) the 'model building process' (i.e. the development of a set
of models that represent adequately the relevant interaction
between the components of the system).

Although presented in a sequence, these stages are obviously inter-
related. In practice the analysis of the problem alternates between
emphasizing the inclusion, the structuring and the model building
processes. However this division provides a useful basis to describe
the broad characteristics of the overall approach adopted in the
formulation and modelling of the feed grains import operations.

5.2 The Inclusion Process. The Criteria Set

The first limitation imposed on the boundary of the system under
analysis arises from the definition of the scope of the study. In
restricting the analysis to what was described before as the 'management
of import operations', consideration was given to:

(i) the role of EPAC in the overall importing process
(largely on the 'management of import operations');

(ii) the profound difference between the complexity and the type
of analysis required to cover 'strategic planning' decisions,
on the one hand, and 'management of operations', on the other
(point raised before, in section 3).

It is clear that, by leaving aside the strategic aspects of the
problem (that are considered 'uncontrollable' or given), some
suboptimization is incurred. But this is inevitable in any analysis.
In Rivett's words53 (p.40), 'we must guard against (...
suboptimization, while remembering that the boundary must be put somewhere, and only God optimizes' (judging by the current state of this world, even God appears to have problems ...).

Having restricted the scope of the study to the quantitative analysis of the management of feed grain import operations, the boundary of the system under analysis was drawn in such a way that

(i) all the most relevant operations involved in the feed grain import operations were included in the system under analysis;

(ii) the increase in the effectiveness of the system studied is not achieved at the expense of a loss of effectiveness of related systems (i.e. the boundary does not divide 'non-separable' subsystems).

The implications of these criteria for the definition of the planning and control system for the feed grains imports were as follows:

(i) the decisions included in the system were: definition of requirements (given the strategic decisions on production targets and on maize imports for human food), purchasing, delivery (sea transport and unloading at the grain terminals), storage at the grain terminals;

(ii) because of the substitutability between feed grains and other raw materials of the animal feed industry, these raw materials were included in a joint definition of requirements;

(iii) grains and oilseeds sharing the utilisation of the grain terminals where maize and sorghum are unloaded and stored
were considered in a joint definition of a delivery-inventory policy.

It is obvious that such a planning and control system exceeds the sphere of EPAC's responsibilities. In adopting such a boundary for the system under analysis, it is assumed that it is necessary for EPAC to be embedded in a wider organization that enables decisions (ii) and (iii) to be taken jointly. This entails a change in the current organizational structure, necessary to allow efficient import operations.

The problem of establishing a boundary for the system under analysis is tied to setting the goals and objectives of the system and to agreeing by which criteria the system's performance is to be judged. In the system selected for analysis the objective emerged clearly from the recognition of the main difficulties facing the Portuguese economy (deficit of balance of payments and unemployment). Given industrial production targets and stated needs of maize for human food, the objective was defined as the minimisation of the country's foreign currency expenditure in all the operations under analysis. This objective is compatible with the attempt of maximising the incorporation of home produce raw materials in industry and human food which, in turn, is appropriate in view of the unemployment crisis the country is undergoing. Clearly, under this national objective it would be inconsistent to restrict the analysis to the feed grains neglecting the interactions between their imports and the imports of other raw materials.

The set of criteria by which the performances of different components of the overall system are judged was derived from that overall objective.
In establishing the 'criteria set', congruency between the criteria was maintained. As will be shown later, adaptations of the overall objective were required either to simplify the model building process (by using surrogate criteria) or to extend it to situations dominated by risk or uncertainty.

5.3 The Structuring Process

The structuring process is concerned with the determination of key relationships among the attributes of the entities included in the system under analysis. Obviously the inclusion and structuring processes are inseparable, in the sense that when taking the decision of including a given entity in the system, it is assumed that the attributes of that entity play a relevant role in the overall structure of the system. And, conversely, it is finally from the analysis of the structure of the overall system that conclusions can be drawn as to whether or not an entity initially included should be rejected (as irrelevant or as too complex to be handled).

In reviewing the commodity purchasing problem it was shown that its complexity is such that an attempt to build an overall optimising model will necessarily be bound to fail. The main objective during the structuring process is to try to find out

(i) the 'special features' of the system that can lead to an acceptable partition into smaller sub-systems; and

(ii) how to 'forge effective links' between these sub-systems so as to minimize suboptimization.

Clearly the establishment of these links will be unnecessary between sub-systems that are 'separable' (i.e. when the contribution
of one of them towards the effectiveness of the overall system is not significantly affected by the state of the other. It is only when dealing with 'non-separable' systems that those links become crucial. When optimizing a non-separable part of the overall system, if suboptimization is to be avoided then one of the following two propositions, transcribed from Daellenbach et al.\(^{19}\) (p.14), has to hold:

(i) 'the states of the ignored parts are already optimal with respect to the optimum mode of operation of the part studied';

(ii) 'although the states of the ignored parts are not optimal yet with respect to the optimum mode of operation of the part studied, the system as a whole will move towards an overall optimum by successive rounds of changes and adjustments of individual parts or subsystems' (this condition is usually referred to by stating that the overall system is 'well behaved').

An important aspect of Simpson's expression 'to forge effective links' becomes now clear. When optimizing or 'deriving good solutions' from any model representing a non-separable part of the overall system, inputs to the model which are outputs from other models, must be set at levels that are optimal (or near optimal) with respect to the optimum mode of operation of the part under study. An example that illustrates this procedure was given earlier, in section 4.3, when considering the blending problem: each raw material is costed in the objective function on the basis of the prices that are expected to be paid under the best possible purchasing policy. An
iterative procedure (hopefully convergent) would be required if changes in the quantity of each raw material that is to be purchased affect significantly the prices that are expected to be paid under the best possible purchasing policy and, conversely, if changes in the purchasing prices affect significantly the required quantities of each raw material.

So far only the broad objectives of the structuring process have been mentioned. But how can one economically go about the task of identifying the structure of the system (and then of representing it adequately)? In this respect there are two aspects of the approach that are worth mentioning.

The first one is what Beer \(^8\) called 'the cones of resolution'. The analysis started by considering a low resolution model representing the substitution between maize, sorghum and other raw materials of the animal feed industry. In this model, the purchasing, delivery and storage costs were aggregated under overall 'unit costs'. Approximate estimates of these costs were obtained — for example from published series of 'cif' prices of the different raw materials, as delivered in European ports — and the ranges of relative costs of the different imported raw materials were estimated. Just by varying those relative costs within the identified ranges it was possible, with an LP, to learn important properties of the substitution between maize and sorghum, on the one hand, and between these feed grains and other raw materials, on the other. As will be shown later, the recognition of those properties had a considerable influence on the way the overall system was partitioned — for analysis at higher levels of resolution — and on the choice of methods of linking the sub-systems so obtained.
Whatever efforts are made to learn the structure of the problem progressively (from low to high levels of resolution) there are situations where, due to the complexity of the system, progress can only be reached by making strong assumptions on its mode of operation. This leads to the second aspect of the approach to be discussed.

Braat, quoted in Simpson (p.10) states in this respect: 'in designing complex control systems we found it best to first decide on the structure of the control system and then optimize this system by means of setting the values of the control parameters'. Implicit in this statement is the idea that, due to the complexity of the system, there is no realistic way of covering all the available alternative courses of action. The analyst has then to pre-select a sub-set of them and limit the analysis to that sub-set. This pre-selection is based to a large extent on what can be described as 'intuition' (certainly helped by an association with closely related systems, whose behaviour is known). This procedure was adopted for example when deriving the feed grains delivery-inventory policy. It was assumed that

(i) the size and timing of shipments could be defined independently of the purchasing operations, as for the simplest configuration of the commodity purchasing problem presented in sub-section 4.1 (this assumption was primarily based on the 'recognition' of the small storage capacity at the grain terminals);

(ii) in spite of the ever changing structure of the delivery and inventory costs and the need to readapt the grains requirements, from time to time, the delivery-inventory policy could be adequately represented through a
'Static' model (this assumption was made considering the general robustness of inventory systems to changes in their parameters).

Obviously, for the sake of 'objectivity', assumptions based on 'intuition' have to be tested (i.e. one must ensure that they do not lead to significant suboptimization). However, the type of tests that can be performed have an obvious limitation that arises from the very fact that some courses of action were not objectively evaluated (it should be remembered that their non evaluation was the reason for making the assumptions). Means have then to be found of carrying out tests which, if not definitive, can give valid evidence supporting the analyst's intuition. For example, the first of the assumptions mentioned above was tested by establishing a comparison between

(i) the extra delivery-inventory costs that would arise as a result of leaving aside parts of the silo capacity in order to carry 'speculative stocks' (i.e. stocks built up when price rises are anticipated), and

(ii) the potential savings in the purchasing costs that would result from that practice (the magnitude of these savings was inferred from previous studies in commodity purchasing).

This and other tests of major assumptions involved in deriving the overall planning and control system will be described later in detail.

5.4 The Model Building Process

Most of what was said before is obviously related to the model building process. Not much more can be said without going into the details of each area of the overall planning and control system. Figure 6 gives an overview of the models developed in the course of the research, showing the basic input-output relations within each of the models and the relevant links between them. Their detailed analysis will be covered in subsequent chapters.
Figure 6 - Chart of the system models

EXPLORATORY MODELS:

FEED GRAIN IMPORTS PLANNING AND CONTROL SYSTEM:

Diagram showing the system models and planning process.
CHAPTER 4

THE SUBSTITUTION BETWEEN MAIZE, SORGHUM AND OTHER RAW MATERIALS OF THE ANIMAL FEED INDUSTRY

1. Introduction

The substitutability between the imported feed grains and other raw materials of the animal feed industry was identified earlier as central to the overall feed grain imports planning and control system. In this chapter an LP model (model 1, in Figure 6) is developed with the objective of identifying general characteristics of that substitution process.

The definition of the model is preceded in section 2, by an analysis of the relevant aspects of the animal feed production in Portugal, covering in detail

(i) the analysis of the structure of production (types of feeds, quantities produced, required characteristics);

(ii) the characterisation of the raw materials employed (whether produced internally or imported).

In section 3 a static LP formulation of the 'import mix problem' is presented. Special emphasis is given to the definition of the 'optimality criteria' and to selection of procedures aimed at reaching an acceptable compromise between model accuracy, on the one hand, and the effort involved in the collection of data and in the computation of results, on the other.

The next section presents an analysis of prices of the imported raw materials in the recent past. In the absence of published series reflecting the prices of the imported raw materials as
delivered at the Portuguese ports, other series - e.g. 'cif' prices at other European ports - are used (some of them after being rescaled) as approximations of the raw materials' import prices in Portugal.

Based on those price relations and using the model described in section 3, the characteristics of the process of substitution between maize, sorghum and the other raw materials are explored in section 5.

The chapter is rounded off with a discussion of the implications of the identified properties of that substitution process in the design of the overall planning and control system for the feed grains imports.

2. The Production of Animal Feeds in Portugal

2.1 General

The Portuguese animal feed industry has grown from a handful of grain and by-product mixers in the 1950's to one of the largest industries devoted exclusively to supplying goods and/or services to agriculture.

In 1976 the total number of manufacturers in the country was 78, most of them small and medium size enterprises. Out of them only 8 had an individual contribution of 3% or more to the total national production. The biggest one manufactured 10.7% of the total production (Source: IACA - Situação do Sector em 1976, Sept. 1977).
Figure 7 shows the geographical distribution of the manufacturing plants in the country. Their concentration around the Lisbon area is obvious. In 1976, the plants installed in this district alone manufactured 41.9% of the total production. In Lisbon and the three surrounding districts (Setubal, Leiria, Santarém) the production was 74.4% of the total.

This geographical distribution is strongly connected with the heavy dependence of the industry on raw materials directly imported from abroad. Most of these imports are channeled through the two main ports of the country. Figure 7 shows the areas supplied through
Lisbon and Oporto. The corresponding animal feed productions were, in 1976, 80.9% and 19.1% of the total, respectively (Source: IACA - Situação do Sector em 1976, Sept. 1977).

Another general point about the industry, which is relevant in the context of this chapter, concerns the economics of animal feed production. Some of its important features are apparent from Figure 8 - showing the breakdown of the industry's total turnover in 1976 - and can be summarized as follows:

(i) the costs of the raw materials make up a very large proportion of the total production value (they accounted for 83% of the total turnover; 'transport' was the next biggest intermediate cost item with only 3% of the total);

(ii) the gross value added by the industry in the process of manufacturing the animal feeds is comparatively modest (9% of the total production value in 1976);

Figure 8 - Breakdown of the animal feed industry's total turnover (1976)

Source: IACA - Situação do Sector em 1976 (Sep.1977)
(iii) the industry operates with an extremely thin gross profit margin (in 1976 it was about 0.8%; this figure should however be regarded as lower than usual; for example in 1975 the gross profit margin was 2.2%).

2.2 Breakdown of the Total Production by Type of Feed

An animal feed (or compound animal feed) is a mixture of different raw materials designed to be palatable to the animals, be capable of being processed, be acceptable to the market and meet the animals' nutritional requirements most economically. The desired levels of each nutrient factor (energy, protein, etc.) are established according to the animals to be fed (hogs, poultry, cattle, etc.), their sex and stage of development (e.g. starter, growing, finishing, etc.) and the production purpose for which the animals are fed (breeding, milk, eggs, etc.). Portuguese legislation (Portarias no. 22 921/67 and 417/76) classifies animal feeds in more than one hundred marketable standard categories.

In spite of the tremendous growth in the global production of animal feeds in the recent past (shown in Table 3, chapter 2), the distribution of the production among the different animal feed categories has remained nearly stationary, with a moderate annual seasonality. Table 6 shows how the global production was distributed by the main animal species - hogs, poultry, and cattle - between 1971 and 1976. The annual contribution of poultry feeds to the total production remained consistently around 53% (with the lowest quarterly share at around 31% in the first quarter). The annual proportions
of hogs and cattle feeds oscillated moderately in the past around 40% and 25%, respectively (with their highest quarterly shares in the fourth and first quarters, respectively). Other feeds (mainly for rabbits, sheep and horses) have usually contributed around 2% of the total production.

Table 6 - Distribution of the animal feed production by animal species (1971-1976)

| Year (Quarter) | Hogs Quart. Annual | Poultry Quart. Annual | Cattle Quart. Annual | Others Quart. Annual | Total Quart. Annual [1000 m.t.]
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>38.1</td>
<td>33.1</td>
<td>26.7</td>
<td>2.1</td>
<td>1195</td>
</tr>
<tr>
<td>1972</td>
<td>27.6</td>
<td>34.2</td>
<td>26.4</td>
<td>1.8</td>
<td>1380</td>
</tr>
<tr>
<td>1973</td>
<td>43.7</td>
<td>33.9</td>
<td>26.8</td>
<td>2.0</td>
<td>1527</td>
</tr>
<tr>
<td>1974</td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>42.3</td>
<td>30.7</td>
<td>24.9</td>
<td>2.1</td>
<td>441</td>
</tr>
<tr>
<td></td>
<td>34.7</td>
<td>21.6</td>
<td>2.8</td>
<td>1.8</td>
<td>424</td>
</tr>
<tr>
<td></td>
<td>33.6</td>
<td>23.5</td>
<td>2.0</td>
<td>1.6</td>
<td>457</td>
</tr>
<tr>
<td></td>
<td>32.4</td>
<td>25.1</td>
<td>1.9</td>
<td>1.6</td>
<td>471</td>
</tr>
<tr>
<td>1975</td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>36.2</td>
<td>30.7</td>
<td>30.9</td>
<td>2.2</td>
<td>457</td>
</tr>
<tr>
<td></td>
<td>36.7</td>
<td>27.1</td>
<td>2.0</td>
<td>1.7</td>
<td>441</td>
</tr>
<tr>
<td></td>
<td>37.5</td>
<td>26.3</td>
<td>2.1</td>
<td>1.7</td>
<td>458</td>
</tr>
<tr>
<td></td>
<td>33.8</td>
<td>23.3</td>
<td>2.0</td>
<td>1.7</td>
<td>475</td>
</tr>
<tr>
<td>1976</td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38.1</td>
<td>30.9</td>
<td>28.6</td>
<td>2.4</td>
<td>513</td>
</tr>
<tr>
<td></td>
<td>36.6</td>
<td>25.9</td>
<td>25.0</td>
<td>2.5</td>
<td>501</td>
</tr>
<tr>
<td></td>
<td>37.8</td>
<td>26.4</td>
<td>2.5</td>
<td>1.7</td>
<td>588</td>
</tr>
<tr>
<td></td>
<td>32.6</td>
<td>24.4</td>
<td>1.7</td>
<td>616</td>
<td></td>
</tr>
</tbody>
</table>

Source: IACA records

The total animal feed production is highly concentrated on a small number of feeds. This is illustrated in Figure 9 through

**Figure 9 - Lorentz curve of animal feed production (1976)**

Source: IACA - Situação do Sector em 1976 (Sep.1977)
the 'Lorentz curve' of the 1976 production.

In that year 15% of the total number of feeds contributed more than 90% of the global production. The share of each of those high production feeds is shown in Table 7. The 1975 figures were also included to show the degree of stability of the animal feed production structure.

Table 7 - Breakdown of the animal feed production (1975-1976)

<table>
<thead>
<tr>
<th>Animal feed categories</th>
<th>1975 Production</th>
<th>Cumulative production (% of total)</th>
<th>1976 Production</th>
<th>Cumulative production</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-815: Hoggs-grower(30 to 60/70 kg)</td>
<td>14.58</td>
<td>14.58</td>
<td>17.90</td>
<td>17.90</td>
</tr>
<tr>
<td>B-320: Milk cows (16 % protein)</td>
<td>6.99</td>
<td>37.04</td>
<td>7.58</td>
<td>38.70</td>
</tr>
<tr>
<td>S-820: Hoggs-finisher (more than 100 kg)</td>
<td>6.23</td>
<td>43.27</td>
<td>6.03</td>
<td>44.23</td>
</tr>
<tr>
<td>S-816: Hoggs-grower (60/70 to 100 kg)</td>
<td>7.43</td>
<td>50.70</td>
<td>5.85</td>
<td>50.58</td>
</tr>
<tr>
<td>B-332: Pullets-finisher</td>
<td>4.08</td>
<td>54.78</td>
<td>5.78</td>
<td>56.36</td>
</tr>
<tr>
<td>A-104: Broilers-starter (up to 6wks.)</td>
<td>6.25</td>
<td>61.03</td>
<td>5.69</td>
<td>62.05</td>
</tr>
<tr>
<td>A-120: Laying hens (ground)</td>
<td>5.85</td>
<td>66.88</td>
<td>5.05</td>
<td>67.10</td>
</tr>
<tr>
<td>B-350: Pullets-grower</td>
<td>3.98</td>
<td>70.86</td>
<td>4.44</td>
<td>71.54</td>
</tr>
<tr>
<td>B-334: Beef cattle</td>
<td>3.54</td>
<td>74.40</td>
<td>4.11</td>
<td>75.55</td>
</tr>
<tr>
<td>S-901: Hoggs-starter (up to 30 kg)</td>
<td>3.93</td>
<td>78.33</td>
<td>3.68</td>
<td>79.93</td>
</tr>
<tr>
<td>S-830: Sows in gestation</td>
<td>3.66</td>
<td>81.99</td>
<td>3.52</td>
<td>82.85</td>
</tr>
<tr>
<td>A-125: Laying hens (battery)</td>
<td>2.85</td>
<td>84.84</td>
<td>3.05</td>
<td>85.89</td>
</tr>
<tr>
<td>B-321: Milk cows (20 % protein)</td>
<td>1.77</td>
<td>86.61</td>
<td>2.26</td>
<td>88.87</td>
</tr>
<tr>
<td>A-150: Hens (large breeds)-breeder</td>
<td>1.85</td>
<td>88.46</td>
<td>1.56</td>
<td>89.72</td>
</tr>
<tr>
<td>A-111: Pullets (small breeds)-layer/breeder</td>
<td>1.23</td>
<td>89.69</td>
<td>1.15</td>
<td>90.87</td>
</tr>
<tr>
<td>Other feeds</td>
<td>10.51</td>
<td>100.00</td>
<td>9.15</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Source: IACA - Situação do Sector em 1976 (Sep.1977) IACA records

2.3 Feeds' Nutritional Requirements

Statements of the amounts of nutrients required by animals are currently described by the general term 'feeding standards'. The establishment of these standards is essentially based on input-output relationships between individual nutrients and some measure of performance, (e.g. liveweight gain in meat producing animals, volumes of milk in dairy cows, number of eggs in laying hens).
There are some theoretical objections to the ways in which the feeding standards are obtained from experimental data and are presented to the livestock grower. The main ones can be summarized as follows:

(i) to a large extent, they assume independence of the effects of different nutrients on the production output (which is not strictly correct);

(ii) the feeding standards are obtained from experiments conducted under rigorously controlled laboratory conditions; they do not take into account the characteristics which are peculiar to each particular real world situation (environment, local animal breeds involved, etc.);

(iii) the feeding standards are defined for specific conditions concerning the economics of the production process; to what extent does a farmer or a country require the livestock to perform at the level for which the standards were prepared is questionable.

These points illustrate why the application of feeding standards to any group of animals will inevitably involve inaccuracies and approximations. For this reason they must be regarded as basic guides to feeding practice rather than inflexible rules. They are nevertheless widely recognized as the only practical methodology for solving problems concerned with the planning of animal feeding (see Besse 9, Chappel 17, Dent et al 22, Griffiths et al 28, McDonald et al 45, Risse 51, A.E.C. 2).

The conversion of feeding standards - expressing animal's
nutritional requirements - into appropriate levels of each nutrient factor in manufactured feeds is a relatively straightforward process for the 'complete feeds' (nothing but these feeds, except water, needs to be fed to the animals). This is the case of all the hogs and poultry feeds produced in Portugal in significant quantities. For non-complete feeds (most of the cattle feeds) the balance of the nutrient factors is obtained by mixing the feeds with other products (for example, hay). The levels of the several nutrient factors in each feed will therefore depend on the practices adopted in the preparation of the final product fed to the animals; from the point of view of the manufacturers those levels are fixed essentially to satisfy either legal constraints or just label specifications acceptable to the market.

The choice of the nutritional constraints that are to be imposed on each feed involves a compromise between what is theoretically desirable and what is practical. To consider a high number of nutritional constraints can complicate excessively the formulation of a feed for an illusive gain that will be neutralised by the reality of the industrial operations.

Table 8 presents the nutritional constraints used throughout the study as a basis for the formulation of the animal feeds included in Table 7 ('high production feeds'). These constraints define maximum and minimum levels of different nutrient factors, expressed as a percentage of the total feed weight.

Part of the constraints are imposed by Portuguese legislation (Portarias no. 663/73 and 732/73) : maximum moisture, maximum ash,
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Moisture (%)</td>
<td>Max</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
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<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
</tr>
<tr>
<td>Crude protein (%)</td>
<td>min</td>
<td>14.00</td>
<td>12.00</td>
<td>13.00</td>
<td>14.00</td>
<td>16.00</td>
<td>18.00</td>
<td>20.00</td>
<td>15.50</td>
<td>16.00</td>
<td>16.00</td>
<td>16.00</td>
<td>14.00</td>
<td>16.00</td>
<td>15.00</td>
<td>12.00</td>
<td>16.00</td>
</tr>
<tr>
<td>Crude fibre (%)</td>
<td>Max</td>
<td>5.00</td>
<td>5.00</td>
<td>8.00</td>
<td>8.00</td>
<td>4.50</td>
<td>4.00</td>
<td>4.00</td>
<td>5.50</td>
<td>5.50</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
<td>8.50</td>
<td>7.00</td>
<td>7.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Crude fat (%)</td>
<td>min</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Calcium (%)</td>
<td>min-Max</td>
<td>0.60</td>
<td>0.50</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.80</td>
<td>0.90</td>
<td>2.60</td>
<td>2.60</td>
<td>2.40</td>
<td>2.40</td>
<td>0.90</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Total phosph. (%)</td>
<td>min-Max</td>
<td>0.45</td>
<td>0.40</td>
<td>0.40</td>
<td>0.45</td>
<td>0.45</td>
<td>0.60</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Ash (%)</td>
<td>Max</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
<td>10.00</td>
<td>10.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>13.00</td>
<td>10.00</td>
<td>8.00</td>
<td>9.00</td>
<td>8.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Poultry-Metab. energy (Kcal/Kg)</td>
<td>min</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hog-Met energy (Kcal/Kg)</td>
<td>min</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cattle-Met energy (Kcal/Kg)</td>
<td>min</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.95</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Methionine (%)</td>
<td>min</td>
<td>0.30</td>
<td>0.28</td>
<td>0.28</td>
<td>0.20</td>
<td>0.34</td>
<td>0.38</td>
<td>0.50</td>
<td>0.31</td>
<td>0.31</td>
<td>0.32</td>
<td>0.34</td>
<td>0.34</td>
<td>0.27</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Methionine + cystine (%)</td>
<td>min</td>
<td>0.61</td>
<td>0.56</td>
<td>0.55</td>
<td>0.40</td>
<td>0.60</td>
<td>0.79</td>
<td>0.87</td>
<td>0.56</td>
<td>0.58</td>
<td>0.59</td>
<td>0.59</td>
<td>0.50</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lysine (%)</td>
<td>min</td>
<td>0.84</td>
<td>0.72</td>
<td>0.71</td>
<td>0.50</td>
<td>0.97</td>
<td>1.01</td>
<td>1.13</td>
<td>0.61</td>
<td>0.64</td>
<td>0.61</td>
<td>0.60</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Source: Diário do Governo - Portarias no.663/73, 732/73
A.M.G. - Document technique no. 120
IAGA records
minimum crude protein, minimum-maximum calcium and minimum-maximum total phosphorus. The others were included in the attempt to reflect, as closely as possible, the industrial formulation practices commonly used in Portugal. They were based on a report by A.E.C. 2 (a basic source of reference among Portuguese nutritionists) and on IACA documents made available for consultation. These constraints set minimum energy (metabolisable energy, for poultry, and net energy, otherwise), minimum contents of methionine, methionine + cystine and lysine (only for hogs and poultry feeds), maximum crude fibre and minimum crude fat (for cattle feeds, only).

It has been shown in previous studies (see A.E.C. 1 and Griffiths et al. 28 ) that, from all these constraints, the ones with the highest effect on the costs of animal feeds are those specifying

(i) the energy content, and
(ii) the protein content (crude protein and amino acids - methionine, cystine and lysine).

Two significant groups of nutrient factors are not considered explicitly in the constraints included in Table 8, for the reasons explained below:

(i) aminoacids other than methionine, cystine and lysine: although their presence in the feeds is essential, their level is not explicitly imposed under the widely used assumption that they are not limiting factors in the formulation; in contrast to methionine, cystine and lysine, the feeds' crude protein content is normally sufficient to keep their levels at or above the desirable minima;
(ii) vitamins, oligo-elements, salts; in view of the small quantities involved, these ingredients are very cheap; it is standard practice to provide them in premixes and common salt which are added to the feeds in fixed proportions that will ensure adequate levels of each nutrient factor, irrespectively of the contributions made by the other raw materials included in the mix; the premixes include usually other ingredients that need to be added in the feeds in very small proportions like, for example, antibiotics.

Table 9 shows 'standard' proportions of premix (of a make widely used in Portugal) and salt. These figures were adopted in the formulation presented later in this chapter.

<table>
<thead>
<tr>
<th></th>
<th>Hogs</th>
<th>Poultry</th>
<th>Cattle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premixes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simontal</td>
<td>0.5%</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Salt</td>
<td>0.4%</td>
<td>0.2%</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Source: Simontal - Technical documents IACA records

2.4 Raw Materials

2.4.1 Imports and Home Production

The enormous growth in animal feed production has not been matched by a significant increase in the home production of raw materials employed by that industry. Consequently, animal feed production became heavily dependent on imports. As shown in Table 10, during 1976 over 65% of the raw materials consumed by the animal feed
industry were directly imported. In addition, from the home produced raw materials a considerable proportion consists of by-products of industries that are entirely dependent on imports (this is the case, for example, for the oilseed meals, which are imported to provide vegetable oils).

Table 10 - Raw materials used by the animal feed industry (1976)

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Consumption expressed as % of total</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Home produced</td>
<td>Imported</td>
<td>Total</td>
</tr>
<tr>
<td>Maize</td>
<td>-</td>
<td>46.22</td>
<td>46.22</td>
</tr>
<tr>
<td>Sorghum</td>
<td>-</td>
<td>20.04</td>
<td>20.04</td>
</tr>
<tr>
<td>Fish meal</td>
<td>0.32</td>
<td>1.32</td>
<td>1.64</td>
</tr>
<tr>
<td>Maize gluten meal</td>
<td>-</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>Peanut meal</td>
<td>1.15</td>
<td>1.15</td>
<td>2.29</td>
</tr>
<tr>
<td>Soybean meal</td>
<td>4.52</td>
<td>4.52</td>
<td>9.05</td>
</tr>
<tr>
<td>Sunflower meal</td>
<td>1.09</td>
<td>1.09</td>
<td>2.17</td>
</tr>
<tr>
<td>Safflower meal</td>
<td>1.05</td>
<td>-</td>
<td>1.05</td>
</tr>
<tr>
<td>Alfalfa meal</td>
<td>-</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>Wheat bran</td>
<td>5.74</td>
<td>-</td>
<td>5.74</td>
</tr>
<tr>
<td>Carob</td>
<td>1.40</td>
<td>-</td>
<td>1.40</td>
</tr>
<tr>
<td>Cane molasses</td>
<td>0.58</td>
<td>0.24</td>
<td>0.82</td>
</tr>
<tr>
<td>Calcium carbonate</td>
<td>0.91</td>
<td>-</td>
<td>0.91</td>
</tr>
<tr>
<td>Dicalcium phosphate(1)</td>
<td>-</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>Premixes(2)</td>
<td>-</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>Others</td>
<td>6.58</td>
<td>-</td>
<td>6.58</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>23.34</td>
<td>76.67</td>
<td>100.00</td>
</tr>
</tbody>
</table>

(1) This product is manufactured in Portugal from imported phosphate rock; the reason to classify it as 'imported' is discussed in the text.
(2) Although part of the value of the premixes is added in the country their main constituents are imported.

Source: IACA - Relatório e Contas, Exercício de 1976

For the purpose of the analysis that will be carried out later, it is relevant to identify some characteristics of the raw materials' home production.
Most of the home produced raw materials consumed by the animal feed industry - more than 80%, during 1976 (Source: IACA - Relatório e Contas. Exercício de 1976) - fall under the category of by-products, coming from industries as diverse as oil crushing, flour milling, fish canning, livestock slaughtering or rice husking.

Their production rates, fairly regular over time, are essentially determined by the demand for the main products (oil, flour, canned fish, etc.) - much more expensive than the by-products - rather than by the needs of the animal feed industry.

Produced in small quantities, without any relevant outlet apart from the animal feed industry (exports were never contemplated in view of their limited production), these by-products will be absorbed entirely by the animal feed industry.

The supply of the by-products can be assumed, at least in the short/medium term, to be perfectly inelastic with respect to price, above what can be called the 'floor' production price (below which it is not worthwhile to produce them). This price is unlikely to be reached in practice since the by-products' marginal production costs are small - the operations required to transform the 'crude by-product' into a product acceptable by the animal feed industry are either non-existent or of low cost (e.g. drying, mixing, milling). The demand for the home produced by-products will be a function of the degree of restriction imposed on imports. The more severe these restrictions are, the higher the demands will be and so the higher the prices will be (within the limits indirectly imposed by the feeds' maximum selling prices, fixed by Government). The general price
formation mechanism for these raw materials is presented in Figure 10.

The home produced by-products that are imported by Government agencies with roles similar to EPAC - IAPO for oilseed meals and AGAA for cane molasses (*) are the exceptions to the price mechanism described. Their prices are fixed by Government at the same levels as the internal prices of the imported raw materials. Clearly, by relating these prices to the animal feeds' maximum selling prices, the Government makes sure that these prices of home production and imports are attractive enough for them to be entirely consumed.

There is a much more restricted group of raw materials that, in contrast with the industrial by-products, are made available to the animal feed industry in quantities matching the industry’s needs. Examples of such raw materials are calcium carbonate and dicalcium phosphate. Changes in the 'import mix' can affect the demand for those raw materials and can therefore have effects on their consumption.

(*) IAPO: Instituto do Azeite e Produtos Oleaginosos

AGAA: Administração Geral do Açúcar e do Alcool
(and production). For example, to import raw materials less rich in phosphorus will imply the need to incorporate in the feeds more dicalcium phosphate which, in turn, will imply higher production levels for this raw material.

Among these raw materials it is relevant to make a distinction between two groups: those, such as calcium carbonate, whose production does not involve any significant direct foreign currency expenditure and those whose production depends directly on imports – dicalcium phosphate is an example. It is produced neutralising phosphoric acid with limestone; the phosphoric acid is, in turn, obtained from imported phosphate rock.

2.4.2 Analysis of the Raw Materials' Basic Ingredients

Table 11 gives the ingredient analysis for the main raw materials consumed by the animal feed industry. The ingredients considered in the table are those corresponding to the nutritional constraints considered earlier in Table 8. The raw material's grades were selected to reflect, as closely as possible, the ones produced or imported by Portugal and are based essentially on a report by A.E.C. 1

Table 11 shows quite clearly the close similarity between maize and sorghum. These grains are the main sources of energy of the animal feeds, for two reasons:

(i) the price per unit of energy obtainable through feed grains is always among the lowest obtainable through any raw material;

(ii) their energy content is among the highest ones (which, in view of the high energy requirements of the feeds, makes their presence in them indispensable).
<table>
<thead>
<tr>
<th>Raw materials</th>
<th>Maize</th>
<th>Sorghum</th>
<th>Wheat bran</th>
<th>Cane molasses</th>
<th>Calcium carb.</th>
<th>Dicalc. phosph.</th>
<th>Methionine #98</th>
<th>Lysine #98</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ingredients</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moisture (%)</td>
<td>13.00</td>
<td>13.00</td>
<td>8.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Crude Protein (%)</td>
<td>9.00</td>
<td>10.00</td>
<td>72.00</td>
<td>60.00</td>
<td>50.00</td>
<td>45.00</td>
<td>30.00</td>
<td>25.00</td>
</tr>
<tr>
<td>Crude fibre (%)</td>
<td>2.50</td>
<td>2.50</td>
<td>0.00</td>
<td>1.50</td>
<td>7.00</td>
<td>7.00</td>
<td>28.00</td>
<td>35.00</td>
</tr>
<tr>
<td>Crude fat (%)</td>
<td>4.00</td>
<td>3.00</td>
<td>8.50</td>
<td>2.50</td>
<td>1.00</td>
<td>1.00</td>
<td>1.50</td>
<td>0.60</td>
</tr>
<tr>
<td>Calcium (%)</td>
<td>0.02</td>
<td>0.04</td>
<td>4.00</td>
<td>0.01</td>
<td>0.15</td>
<td>0.25</td>
<td>0.25</td>
<td>0.34</td>
</tr>
<tr>
<td>Total phosph. (%)</td>
<td>0.30</td>
<td>0.50</td>
<td>2.50</td>
<td>0.40</td>
<td>0.60</td>
<td>0.60</td>
<td>1.20</td>
<td>0.84</td>
</tr>
<tr>
<td>Ash (%)</td>
<td>1.50</td>
<td>2.00</td>
<td>11.00</td>
<td>1.50</td>
<td>5.50</td>
<td>6.00</td>
<td>6.00</td>
<td>6.50</td>
</tr>
<tr>
<td>Poultry-M.E. (Kcal/Kg)</td>
<td>3.37</td>
<td>3.30</td>
<td>3.20</td>
<td>3.60</td>
<td>2.75</td>
<td>2.24</td>
<td>0.90</td>
<td>1.00</td>
</tr>
<tr>
<td>Hogs-M.E. (Kcal/Kg)</td>
<td>1.15</td>
<td>1.16</td>
<td>1.15</td>
<td>1.06</td>
<td>0.92</td>
<td>0.94</td>
<td>0.38</td>
<td>0.50</td>
</tr>
<tr>
<td>Cattle-M.E. (Kcal/Kg)</td>
<td>1.15</td>
<td>0.95</td>
<td>-</td>
<td>-</td>
<td>0.91</td>
<td>1.02</td>
<td>0.61</td>
<td>0.50</td>
</tr>
<tr>
<td>Methionine (%)</td>
<td>0.17</td>
<td>0.17</td>
<td>2.00</td>
<td>1.54</td>
<td>0.55</td>
<td>0.64</td>
<td>0.64</td>
<td>0.37</td>
</tr>
<tr>
<td>Meth.+Cyst. (%)</td>
<td>0.39</td>
<td>0.35</td>
<td>2.70</td>
<td>2.52</td>
<td>1.26</td>
<td>1.38</td>
<td>1.10</td>
<td>0.77</td>
</tr>
<tr>
<td>Lysine (%)</td>
<td>0.26</td>
<td>0.22</td>
<td>5.40</td>
<td>0.98</td>
<td>1.73</td>
<td>2.90</td>
<td>1.08</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Source: A.E.C. - Document technique no. 111
Feedstuffs, Vol.49 (1977), no. 30 (July)
Amongst the other raw materials, the several meals (fish, maize gluten, peanut, soybean, sunflower, safflower, alfalfa) can be broadly classified as 'sources of proteins'. It is noticeable from the table that these raw materials make up a group much less homogeneous than maize and sorghum.

Raw materials less rich in energy and proteins - for this reason cheaper than the previous ones - also play a significant role in the formulation of animal feeds. Wheat bran, cereals or cane molasses contribute to make volume at a low price and their presence in the feeds is required if energy and proteins are not to be unnecessarily wasted.

Synthetic methionine and lysine are used by the animal feed industry to meet the feeds' requirements of those aminoacids most economically. These aminoacids are usually in short supply in most of the non-synthetic raw materials. To meet their requirements, using exclusively non-synthetic raw materials, would generally imply, for a large number of hogs and poultry feeds, an increase in the crude protein level above that required only because of the need to meet the nutritional requirements of methionine and lysine (this point is raised for example by Griffiths et al.\textsuperscript{28}).

Finally, calcium carbonate and dicalcium phosphate are the basic sources used by the industry to meet the calcium and phosphorus feeds' requirements.

### 2.4.3 Limits on the Utilisation of Raw Materials

Although the nutrient factors considered in Table 11 are the basis for the formulation of the animal feeds, there are other factors that need to be taken into account, namely:

(i) the influence of the raw materials on the palatability of feeds to the animals;
(ii) their influence on the quality of the animal tissues or products (e.g. on the taste of the meat);

(iii) the unreliability of their composition; and

(iv) the difficulties posed by their physical characteristics for the processing, handling and storage of the feeds.

It is current industrial practice to consider all these factors in the formulation, by limiting the presence of the raw materials in each feed to levels considered 'safe', in the light of the nutritionists' experience. This limitation may be imposed individually to each raw material and/or to groups of similar raw materials.

Table 12 presents the limits on the utilisation of the major raw materials for each of the feeds considered earlier in Table 8. These figures were based essentially on the report by A.E.C. described earlier as a basic source of reference among Portuguese nutritionists.

3. Formulation of the Import Mix Problem

3.1 The LP 'Static' Model

Among the characteristics of the animal feed production process identified in the previous section was the 'near stationarity' of both

(i) the structure of production (i.e. its distribution by different feeds) and

(ii) the rate at which the home produced industrial by-products are made available to the industry.

As a direct consequence of this feature of the production process, it is acceptable to base the analysis of the properties of the substitution process between maize, sorghum and the other raw materials on a model that reflects that 'near stationarily'.
<table>
<thead>
<tr>
<th>Raw material Feed</th>
<th>(i) Individual limits</th>
<th>(ii) Group limits</th>
<th>Alfaíra feeds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maize</td>
<td>Sorghum</td>
<td>Fish meal 72</td>
</tr>
<tr>
<td>S-815</td>
<td>n 40</td>
<td>n 3</td>
<td>n 10</td>
</tr>
<tr>
<td>S-820</td>
<td>n 40</td>
<td>n 3</td>
<td>n 10</td>
</tr>
<tr>
<td>S-816</td>
<td>n 40</td>
<td>n 3</td>
<td>n 10</td>
</tr>
<tr>
<td>S-801</td>
<td>n 20</td>
<td>n 3</td>
<td>n 10</td>
</tr>
<tr>
<td>S-830</td>
<td>n 40</td>
<td>n 3</td>
<td>n 10</td>
</tr>
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<td>n 8</td>
<td>n 10</td>
</tr>
<tr>
<td>A-104</td>
<td>n 40</td>
<td>n 8</td>
<td>n 10</td>
</tr>
<tr>
<td>A-120</td>
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<td>n 5</td>
<td>n 10</td>
</tr>
<tr>
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<td>n 5</td>
<td>n 10</td>
</tr>
<tr>
<td>A-130</td>
<td>n 40</td>
<td>n 5</td>
<td>n 10</td>
</tr>
<tr>
<td>A-111</td>
<td>n 40</td>
<td>n 5</td>
<td>n 10</td>
</tr>
<tr>
<td>B-320</td>
<td>n 40</td>
<td>n 0</td>
<td>n 0</td>
</tr>
<tr>
<td>B-332</td>
<td>n 40</td>
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<td>n 0</td>
</tr>
<tr>
<td>B-334</td>
<td>n 40</td>
<td>n 0</td>
<td>n 0</td>
</tr>
<tr>
<td>B-322</td>
<td>n 40</td>
<td>n 0</td>
<td>n 0</td>
</tr>
</tbody>
</table>

n : no limit

Source: A.E.O. - Document technique no. 120

The model represents the problem of defining the import mix that, for a given set of raw materials' import prices, meets 'optimally' the country's requirements associated with given levels of

(i) the global production target and production structure, and

(ii) the internal availability of industrial by-products.

In view of the difficulties of the Portuguese economy, identified earlier in Chapter 2, the optimality criterion was defined as the minimization of the foreign currency expenditure (FCE) associated with the overall mix of raw materials.
Given the characteristics of the home production of industrial by-products it was found appropriate to plan the import mix assuming that those raw materials are fully utilized. It is assumed that the by-products of industries depending on imported raw materials do not contribute to the 'variable FCE'. Any indirect FCE so incurred is, in fact, independent of the import mix planning decision.

Clearly that assumption cannot be extended to the home produced raw materials that are made available to the industry in quantities that are determined by its needs. In times when unemployment and the deficit of the balance of payments (or trade deficit) are critical factors of the economy, it seems acceptable to attempt to include in the feeds as much value added in Portugal as possible. This attempt is consistent with the objective of minimizing the total FCE in the production of animal feeds and pricing those raw materials in the objective function with the marginal FCE associated with each extra unit of home production. In the remainder of the thesis, home produced raw materials involving a significant marginal FCE (e.g. dicalcium phosphate) will be referred to as 'imported' and will be priced according to the above criterion. When the marginal FCE is negligible (e.g. calcium carbonate) they will be referred to as home produced with 'unlimited supply' in contrast with the by-products, referred to as home produced with 'limited supply'.
Using the following notation

\begin{align*}
i & \quad \text{: index for imported raw materials} & (i=1, \ldots, I) \\
n & \quad \text{: index for home produced raw materials} & (n=1, \ldots, N) \\
j & \quad \text{: index for animal feeds} & (j=1, \ldots, J) \\
m & \quad \text{: index for nutrient factors (or ingredients)} & (m=1, \ldots, M_j) \\
p_i & \quad \text{: import price of raw material } i & \text{(US$/m.t.)} \\
q_j & \quad \text{: production target of feed } j & \text{(m.t.)} \\
A_n & \quad \text{: quantity available of raw material } n & \text{(m.t.)} \\
L_{mj} & \quad \text{: limit of nutrient factor } m \text{ in feed } j \text{ (min-max)} & \text{(\%)} \\
c_{mi}, c_{mn} & \quad \text{: content of ingredient } m \text{ in raw material } i \text{ or } n & \text{(\%)} \\
u_{ij}, u_{nj} & \quad \text{: upper bound on raw material } i \text{ or } n \text{ in feed } j & \text{(\%)} \\
(T_{i1}, \ldots, i_k, n_1, \ldots, n_k) & \quad \text{: group limit on raw materials} \\
i_1, \ldots, i_k, n_1, \ldots, n_k & \quad \text{in feed } j & \text{(\%)} \\
x_{ij}, x_{nj} & \quad \text{: quantity of raw material } i \text{ or } n \text{ in feed } j & \text{(m.t.)} \\
\end{align*}

The static import mix problem can be defined through the following LP:

**Minimize**: \( \sum_i p_i \cdot \sum_j x_{ij} \)

**Subject to**: 

(i) balance equalities:
\( \sum_i x_{ij} + \sum_n x_{nj} = q_j \)

(ii) constraints expressing limited supply of home produced raw materials:
\( \sum_j x_{nj} \leq A_n \)

(...)
(i) nutritional constraints:

\[
\sum_i q_{mi} \cdot x_{ij} + \sum_n q_{mn} \cdot x_{nj} \geq L_{mj} \cdot Q_j
\]

(iv) upper bounds on the utilisation of raw materials in feeds (if existent):

- if raw material \( i \) is identical to raw material \( n \):

\[
x_{ij} + x_{nj} \leq \left( U_{ij}/100 \right) \cdot Q_j
\]

- otherwise:

\[
x_{ij} \leq \left( U_{ij}/100 \right) \cdot Q_j
\]

\[
x_{nj} \leq \left( U_{nj}/100 \right) \cdot Q_j
\]

(v) limits on the utilisation of groups of raw materials (if existent):

\[
x_{i_1 j} + \ldots + x_{i_k j} + x_{n_1 j} + \ldots + x_{n_k j}
\]

\[
\leq \left[ \left( T_{i_1}, \ldots, i_k, n_1, \ldots, n_k \right)_j/100 \right] \cdot Q_j
\]

In this formulation, the constraints expressing the limited supply of home produced raw materials were written as inequalities. However, in view of the comparatively small amounts available of these raw materials, these constraints were found to be always active for the optimal solutions. Therefore, no changes would be introduced in the LP by writing those constraints as equalities.

For this reason, the number of variables of the LP can be reduced when there are imported raw materials and home produced raw materials with limited availability that can be assumed to have identical characteristics. If this is the case, for example,
with raw materials $i_o$ and $n_o$, the variables $x_{i_o j}$ and $x_{n_o j}$ can be condensed into a single variable $z_{i_o j}$ (amount of raw material $i_o$ - imported or home produced - in feed $j$) and the following changes have to be made in the LP:

(i) objective function: the term $(p_{i_o} \cdot \sum_j x_{i_o j})$ is replaced by $\left[ p_{i_o} \cdot (\sum_j z_{i_o j} - A_{n_o}) \right]$

(ii) constraint expressing limited availability of raw material $n_o$: the constraint $\sum_j x_{n_o j} \leq A_{n_o}$ is replaced by $\sum_j z_{i_o j} \geq A_{n_o}$

If some raw materials are added to the feeds on a fixed percentage basis, the LP structure remains unchanged. A 'correction' of the right hand sides of the balance equalities and nutritional constraints is required in this case. If $K$ different raw materials are added in fixed percentages $\delta_{kj}$ ($k = 1, \ldots, K$) to feed $j$ and if their content of ingredient $m$, expressed in percent terms, is $\gamma_{mk}$, then

(i) the balance equalities become

$$\sum_i x_{ij} + \sum_n x_{nj} = \left[ 1 - \sum_k (\delta_{kj}/100) \right] \cdot Q_j$$

and

(ii) the nutritional constraints become

$$\sum_i a_{mi} \cdot x_{ij} + \sum_n a_{mn} \cdot x_{nj} \leq \left[ L_{mj} - \sum_k (\gamma_{mk} \cdot \delta_{kj}/100) \right] \cdot Q_j$$

Clearly, since the raw materials are added onto the feeds in fixed proportions, any FCE incurred with them is invariant with respect to the mix. For this reason the objective function - reflecting the 'variable FCE' - does not need to be changed.
3.2 Discussion of Basic Assumptions Involved in the Model

The LP model just described was not designed to be used directly in the planning of the import mix. Extensions of the model necessary to include the uncertainty associated with the import prices and with the dynamics of the planning process will be considered later in the thesis. However there are two basic assumptions underlying both the static LP model and its extensions that are worth discussing.

The first one concerns the objective of minimizing the 'variable FCE' associated with the raw materials' costs. The FCE involved in other production costs is not considered in the computation of the total 'variable FCE'. What is implied in this procedure is that the contribution of those costs to the sector's total FCE is practically invariant with respect to the import mix decision. This assumption is identical to the one widely adopted in the blending problem facing animal feed manufacturers when defining the optimal mix as the one that minimizes the total raw materials' costs. The assumption was found acceptable in view of the nature of the operations involved (mostly transporting and mixing different raw materials totalling a specified amount of animal feeds) and in view of the structure of the total production costs (presented earlier in section 2.1).

The second point that deserves attention concerns the degree of control exercised by the Government and its agencies on the animal feed mix. In the solutions derived from the LP it is assumed that the raw materials are allocated 'optimally' to each of the feeds considered. However, the manufacturers, in the attempt of meeting their own individual
objectives — not necessarily in agreement with the defined government objective — are free to adopt animal feed compositions different from the ones assumed in the LP. In the notation of the previous section, the Government or its agencies control \( \sum x_{ij} \) (i.e. the total amount imported of each raw material). The type of control exercised by EPAC on the imports of feed grains was already described. The imports of oilseed meals and cane molasses are controlled in an identical way by IAPO and AGAA. Imports of other raw materials (either by CAIACA — a cooperative of the feed grain manufacturers that makes purchases in the international markets for its members — or by premixes manufacturers) are all subject to Government approval. However the Government or its agencies do not control the amount of each raw material that goes into each feed (in the LP, the values of \( x_{ij} \) or \( x_{nj} \)).

The difficulties of modelling the individual and collective behaviour of the manufacturers, with respect to the choice of the feed composition, can be understood easily in view of the following reasons:

(i) there is a large number of manufacturers, with different production capacities, different production outputs and, therefore, different production cost structures;

(ii) they are differently located in relation to the industries where home produced raw materials are made available;
(iii) also associated with their geographical location is the definition of their market (for example, a factory located far from the main ports is unlikely to be competitive in the Lisbon and Oporto areas, as a result of unfavourable transport costs);

(iv) the non-compulsory nature of many of the nutritional constraints is likely to lead to different formulation criteria by different manufacturers.

All these factors are likely to influence differently, for each manufacturer,

(i) the quantities and types of available home produced industrial by-products;

(ii) what feeds and how much of each type will be manufactured, for a given availability of raw materials (imported or home produced);

(iii) choosing between producing more (with less quality) or producing less (with better quality) by incorporating or not in the feeds low quality raw materials.

The Government or its agencies do not control these 'parameters' of the global formulation problem. They also do not control the prices of many of the home produced raw materials. Under this 'institutional' lack of control (it is part of the system under which the industry operates), it is appropriate for the Government and its agencies

(i) to adopt the import mix defined according to the LP model presented earlier (or according to the extensions of that model, discussed later) : the 'optimal' import mix so obtained can be interpreted as the one that, for the structure of feed production and the nutritional
standards assumed in the LP - both regarded as desirable - maximizes the total amount of feed than can be produced; this objective appears to be consistent both with the role assumed by Government and its agencies in the overall planning process and with the global interests of the industry;

(ii) to take complementary actions so as to induce the feed manufacturers to produce feeds with compositions as close as possible to the ones considered desirable from the country's point of view. These measures can be taken in different areas such as

(a) the allocation of quotas to the individual manufacturers: these should be defined as much as possible as a function of the production and availability of by-products envisaged for each producer;

(b) the prices controllable by the Government: the internal prices of imported raw materials should reflect the relations between their import prices;

(c) bonuses attributable to manufacturers complying with feeds' compositions close to the ones obtained through the import mix planning model; and

(iii) to monitor the actual production of animal feeds against the production targets.

3.3 Selection and Aggregation of Feeds and Raw Materials to be Considered in the LP Model

The LP model presented earlier can be extended to as many
animal feeds and raw materials as required, within the limits imposed by the availability of data and computational capacity.

It was mentioned earlier that the total number of animal feeds produced in the country is above one hundred. The number of different raw materials that can contribute to the optimal mix is of the same order of magnitude. To consider all these feeds and raw materials in the LP would bring it to an intolerable size and would involve an excessive data collection effort. Model simplifications are therefore required. The basic objective is to reduce the model size - either by eliminating unnecessary constraints or by reducing the number of variables (by excluding or aggregating feeds or raw materials) - ensuring that the significant features of the real world problem are adequately represented in the model.

Clearly the most appropriate level of detail for the model is dependent on the proposed use of the model. For the purpose of the analysis described in this chapter it seemed acceptable to consider in the LP the feeds included earlier in Table 7. The global production was assumed to be adequately represented by that sub-set of feeds, in relative proportions as for 1976, as shown in Figure 11. In that year, out of the 9.13% of the feeds left out of the LP, 6.83% were hogs, poultry and cattle feeds, in proportions almost identical to the ones within the group of selected feeds. For the remainder 2.3% of feeds (mostly for rabbits, sheep and horses) no data could be obtained.

The criterion adopted for the selection of feeds is simple. Others could be devised, eventually more accurate, but certainly at the expense of more data requirements. One alternative procedure for dealing with the small production feeds would be, for example,
(i) to pool them in homogeneous groups (that eventually could be added on to one or more of the feeds already considered in the LP), and

(ii) to consider each group in the LP as a compound feed, with nutritional requirements set equal to the average over the feeds within the group, weighted by their production shares.

This selection criterion has the advantage of ensuring an accurate representation of the average nutritional requirements
of the small production feeds in the LP. It seems advisable to consider this procedure in the detailed implementation of a model for the actual planning of the imports.

The selection of raw materials involves similar problems.

Figure 12 shows how, during 1976, the consumption of raw materials was distributed among the following categories:

(i) added to the feeds in fixed percentages (salt and premixes);
(ii) home produced 'with limited supply' (mostly industrial by-products);
(iii) imported and home produced 'with unlimited supply'.

Figure 12 - Structure of the consumption of raw materials (1976)

Source: IACA - Relatório e Contas, Ano do feito de 1976
Salt and premixes made up 0.26% and 0.47%, respectively, of the total raw materials' consumption. The figures that would be obtained by applying the 'standard' percentages presented previously in Table 9 to the production structure assumed in the LP are 0.39% and 0.50%. The 'standard' percentages of salt appear therefore to overestimate considerably the 1976 usage. This discrepancy, for which no explanation was obtained, is however meaningless in the context of the problem under analysis. The 'standard' percentages were used in the LP model.

The home produced raw materials 'with limited supply' contributed about 22% of the total consumption. From these, the most significant ones were wheat bran, soybean meal ($45, i.e., with 45% crude protein), carob, peanut meal ($40), sunflower meal ($30) and safflower meal ($25). All these raw materials, making up nearly 15% of the total production, were included separately in the LP. The remaining ones (all with individual contributions below 1% of the total consumption) were aggregated on a 'residual compound'. The nutritional properties of this compound were estimated by computing the weighted average of the nutritional characteristics of the raw materials whose ingredient analysis was available. The 'residual compound' was included in the LP, allocated in equal proportions to all the feeds considered (like a raw material added to the feeds in a fixed percentage). The reasoning behind this procedure is connected with the fact that each component of the compound is produced in very small quantities and is made available to the feed manufacturers in conditions that are quite different from the ones prevailing for those raw materials produced in larger quantities (e.g. they are likely to be distributed only
near the places of production or they will probably have a composition less reliable than the others). These circumstances are likely to ensure a spread of their usage among the whole range of produced feeds. For this reason it seemed more sensible to consider the 'residual compound' as allocated in equal proportions to all the feeds rather than allocating it in total to the LP which may lead to the compound being allocated to only a single feed, which is obviously unrealistic.

The remaining 77% of the total consumption was made up by raw materials either imported or home produced 'with unlimited supply'. The problem of selection of the raw materials that should be included in the LP presents one obvious difficulty: unless one raw material is explicitly included in the LP there is no way of knowing whether it can contribute to the optimal solution or not. Therefore, to exclude any raw material from the LP involves a risk of departure from optimality. This risk can be minimized by including in the formulation a group of raw materials as large as possible and covering a wide range of different nutritional characteristics (different contents of energy, proteins and other ingredients, combined in different ways). The heterogeneity of the selected group of raw materials is essential to guarantee an adequate flexibility in formulating the feeds' compositions. Another condition that must be satisfied by all the selected raw materials is their acceptability among the manufacturers.

Having in mind these criteria, the following raw materials were considered in the LP:

(i) 'sources of energy': maize and sorghum (barley, another
major feed grain, was not included in the LP for two main reasons:

(a) previous imports of this grain caused difficulties to the animal feed manufacturers, as mentioned in Chapter 2;

(b) for importing countries on the periphery of the EEC, partially as a consequence of the community's pricing policy, barley is considerably overpriced in relation to other feed grains - this was shown by Griffiths et al 28 in a study concerning animal feed formulation in the Republic of Ireland in the early 70's; only after EEC membership did barley become attractive in the animal feed formulation in that country);

(ii) 'sources of proteins': fish meal ($72), maize gluten meal ($60), peanut meal ($50), sorghum meal ($45), sunflower meal ($30), alfalfa meal ($17) (the grades are the ones that have been imported in the past by Portugal);

(iii) sources of calcium and phosphorus: calcium carbonate (the only home produced raw material 'with unlimited supply' considered in the LP) and dicalcium phosphate (although manufactured in Portugal, treated as 'imported', for reasons presented earlier);

(iv) synthetic amino-acids: methionine ($98) and lysine ($98) (the grades that have been imported in the past).

(v) other imported raw materials: cane molasses (it was considered that the presence of this raw material, together with other raw materials with relatively low contents of energy or proteins - e.g. wheat bran, carob, 'residual
compound', alfalfa meal, safflower meal - was sufficient to ensure an adequate flexibility in formulating the mix so as to avoid exceeding significantly the feeds' requirements on the most expensive nutrient factors).

3.4 Computer Implementation

All the computations were performed with a CDC 7600 using the MPOS (version 3) package 70. Two auxiliary programmes - a matrix generator and a report writer - were written in FORTRAN and added to the package in order to convert raw data into inputs acceptable to the package and to re-write (aggregate and simplify) the output report.

The LP model-considering explicitly 16 feeds and 19 raw materials (13 imported or home produced 'with unlimited supply' and 6 home produced 'with limited supply') and all the nutritional constraints and limits on the utilisation of raw materials in feeds presented earlier - comprises 275 variables, 230 constraints and 163 upper bounds. It was solved using the 'Prevised' algorithm - a version of the two-phase revised simplex method in which the inverse basis matrix is stored in sparse form. The amount of computer memory required to solve the LP was nearly 50k words and the computation time involved in phase I + phase II was on average about 11.5 CPU seconds.

4. Prices of Imported Raw Materials

The most general rule concerning the relations between raw materials' prices can be stated simply by saying that the more similar their properties and applications are, the closer their prices will move together.
Figure 13(i) shows, for the period April 1969 - January 1976, average monthly prices of US Yellow Maize no. 3 and US Yellow Sorghum no. 2, cif Rotterdam, for 30 to 60 days future delivery. This series was obtained averaging weekly quotes (Wednesday closing offer prices at the Hamburg Borse, as reported by Foreign Agriculture, USDA). The series was extended, for maize, until December 1977 with average monthly prices reported by Oil World Weekly.

These series, closely related to the feed grains import prices in Portugal, cover a period of marked price instability. Major
crop failures in the US (by far the major producer and exporter of feed grains) and an enormous increase in the world demand for animal feeds (mainly in the Soviet Union, Japan and EEC) transformed a situation of relative abundance that characterised the 1960's and early 1970's to one of relative shortage from the middle of 1973 onwards. This transformation brought about a substantial increase in the feed grain price level and variability, as shown in Figure 13(i).

The ratio sorghum price/maize price (monthly averages) is plotted in Figure 13(ii). Over the period of time covered by the diagram that ratio remained within the range (0.84, 1.08). The diagram suggests a strong seasonal variation of the ratio, with a low peak in June and reaching the highest values in November. Figure 14 presents the values of the ratio on each Wednesday of 1974. The important fact to observe in this figure is that the daily fluctuations around the 'current level' of the ratio (measured, for example, by a 5-term centred moving average) are small when compared with the changes in that level over time. During 1974, the standard error of the daily fluctuations around the 5-term moving average was 0.012 (i.e. 7% of the difference between the highest and the lowest value of the moving average).

The prices of the main 'sources of proteins' at the major European ports are plotted in Figure 15. After the summer of 1972 their prices started to climb to reach an all time record in the summer of 1973. Some of the reasons behind this price movement were
common to the feed grains: a considerable rise in demand and crop failures of oilseeds in major producing countries. This happened at a time when the Peruvian fish meal production collapsed, leading to a major shortage of 'sources of proteins' for animal feeds.

Source: Foreign Agriculture, USDA

Source: Oil World Weekly
It is noticeable in the figure how closely related the prices of oilseed meals were during the period represented. The fish meal price movements are obviously less correlated with the others, which is not surprising in view of the differences between the nutritional properties of fish meal and oilseed meals (shown previously in Tables 11 and 12).

The grades of fish meal (64-65% protein) and sunflower meal (38% protein) considered in Figure 15 do not coincide with the ones usually imported by Portugal (72% and 30% protein, respectively). Price series concerning these grades could not be obtained. Estimates of the average price ratios

fish meal $72/soybean meal $45 (2.50)
sunflower meal $30/soybean meal $45 (0.62)

were obtained from IACA and IAPO (they correspond to the internal price ratios, in turn kept in line with the long-term average import price relations). The way in which these ratios, and others presented below, were used in the analysis will be explained later.

No published series of the European prices of other 'sources of proteins' imported by Portugal could be obtained. Price series of maize gluten meal (60% protein) and alfalfa meal (dehydrated, 17% protein) at major US markets (Chicago and Kansas City, respectively) were made available by the USDA Livestock, Poultry, Grain and Seed Division. The price movements of these raw materials were found to follow closely those of fish meal and oilseed meals respectively. Estimates of the average import price ratios in Portugal
maize gluten meal $60 / fish meal $72 (0.65)
alalfa meal $17 / soybean meal $45 (0.50)

were obtained from IACA, as before.

In the absence of any published series concerning synthetic aminoacids, the general price movements were identified following the import prices (cif Portugal) paid by a major Portuguese importer (Figure 16). One can notice the similarity between the lysine price movements and those of fish meal prices, in Figure 15 (fish meal is a major natural source of lysine, as shown earlier in Table 11).

Figure 16 - Prices of synthetic aminoacids (as paid in Portugal by a major importer)

For cane molasses, only US market prices could be obtained (published by the USDA Agricultural Marketing Service, in Molasses Market News). The most valuable ingredient in cane molasses is its energy. For this reason its price movements are related to the feed
grain ones. Based on a rough energy equivalence of 6½ gallons of molasses to 1 bushel of corn, USDA publishes (in Molasses Market News) regular comparisons between the prices of energy obtainable from each of those raw materials. During the period between 1969 and 1977 – in which the average monthly maize prices cif European ports varied between 52 and 168 US$/m.t. – the ratio

\[
\frac{(\text{unit energy price})_{\text{molasses}} - (\text{unit energy price})_{\text{maize}}}{(\text{unit energy price})_{\text{maize}}}
\]

varied between -18% and +27%, at Chicago. An estimate of the average import ratio in Portugal cane molasses/US Yellow maize no. 3 (0.65) was again obtained from IACA.

Finally, for dicalcium phosphate, only the internal Portuguese market prices were available. The records of a major buyer of that raw material (Simontal) showed that the prices paid to the Portuguese manufacturers rose steadily from about Esc. 3.50/Kg in 1969 to Esc. 10.32/Kg in 1977, in ten successive price adjustments. In the absence of a reliable estimate for the marginal FCE/Kg of dicalcium phosphate, its value was assumed to lie within the range 10 to 70% of the manufacturers’ selling price.

5. The Substitution Between Maize, Sorghum and Other Raw Materials

5.1 The Substitution Between Maize and Sorghum and Between these and the 'Sources of Proteins'

Energy and protein (either crude protein or each specified aminoacid) are the nutrient factors with the highest effects on the costs of animal feeds. It seemed therefore natural to start the analysis
of the optimal mix properties by studying the effect on the optimal mix resulting from changes in the relative prices of 'sources of energy' (feedgrains) and 'sources of proteins'.

The analysis was carried out with the LP model described in section 3 and was based on the import price sets defined in Table 13.

Table 13 - Basic price sets used to analyse the substitution between maize, sorghum and the 'sources of proteins'

<table>
<thead>
<tr>
<th>Price set 1</th>
<th>100</th>
<th>80-110</th>
<th>200</th>
<th>260</th>
<th>160</th>
<th>160</th>
<th>100</th>
<th>80</th>
<th>65</th>
<th>100</th>
<th>3500</th>
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<td>Price set 2</td>
<td>100</td>
<td>80-110</td>
<td>275</td>
<td>179</td>
<td>110</td>
<td>110</td>
<td>69</td>
<td>55</td>
<td>65</td>
<td>100</td>
<td>2406</td>
<td>2888</td>
</tr>
<tr>
<td>Price set 3</td>
<td>100</td>
<td>80-110</td>
<td>825</td>
<td>536</td>
<td>330</td>
<td>330</td>
<td>206</td>
<td>165</td>
<td>65</td>
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<td>7219</td>
<td>8663</td>
</tr>
</tbody>
</table>

Those prices were defined according to the following criteria:

(i) the price of maize was set at a reference level (100) throughout the analysis;

(ii) the sorghum price was varied, in each price set, between 80 and 110 (in line with the sorghum/maize price ratios identified in section 4);

(iii) the price of canemolasses was fixed, in the three price sets, at 65 (0.65 is the average price ratio cane molasses maize);

(iv) the soybean meal price was fixed in relation to the one of maize considering the average price ratio in periods of stable prices (1.60, in set 1), and the lowest and the highest price ratios observed between 1969 and 1977 (1.10 and 3.30, in sets 2 and 3, respectively);
(v) the prices of other meals were fixed in all price sets at levels corresponding approximately to the average price ratios with soybean meal; these ratios, obtained either from published price series or from IACA and IAPO, as mentioned in section 3, were set as follows:

(a) fish meal : 2.50
(b) maize gluten meal : 1.63
(c) peanut : 1.00
(d) sunflower : 0.63
(e) alfalfa : 0.50

(vi) the price of lysine was defined setting the price ratio lysine/fish meal near its most frequent value, 10.5; the price of methionine was derived setting the price ratio lysine/methionine at 1.20 (near its average value before the fish meal crisis and after the end of 1976);

(vii) in the absence of any data on the marginal FCE associated with the production of dicalcium phosphate, its price was fixed at 100 in Portugal, over the period 1969-1973 the average internal price ratio dicalcium phosphate/maize was about 2.5; the marginal FCE was assumed to be around 40% of the Portuguese internal price).

Figure 17 (i) gives, for each price set, the total amount of feed grains (maize + sorghum) in the optimal mix, expressed as a proportion of the global feed production. The figure shows that, for constant prices of raw materials other than maize and sorghum, the
Figure 17 - Substitution between maize, sorghum and 'sources of proteins'

(i) (Maize + Sorghum) content as a proportion of the total animal feed production

(ii) Sorghum content as a proportion of the (Maize + Sorghum) content

(iii) Maize content as a proportion of the total animal feed production

Note: the diagrams are based on solutions derived for discrete sets of values of the price ratios:
- sorghum/maize: from 0.80 to 1.10, with increments of 0.01
- soy.meal/maize: from 1.10 to 3.30, with increments of 0.20

Any differences between solutions evaluated at two successive points of each grid are represented on the diagrams as step changes occurring at the mid points of the grid intervals.
The total amount of feed grains is not sensitive to changes in the relative prices of maize and sorghum. As sorghum price decreases, the amount of sorghum that enters into the optimal solution replaces an approximately equal amount of maize. This maize-sorghum substitution process is shown in Figure 17 (ii), for each price set. For all levels of the prices of the 'sources of proteins', the ratio sorghum content/(maize + sorghum) content varies from 0 to 45-55% as the sorghum price decreases in relation to the maize one. In this process, the maize is partially replaced by sorghum first in the hogs feeds, then in the poultry ones and finally in the cattle feeds. The extent of the replacement is set by the limits imposed on the usage of sorghum in each feed.

The second point raised by Figure 17 (i) is the low substitutability between the feed grains and the 'sources of proteins'. The rate of substitution between these two groups of raw materials is small and far from constant over the whole range of price relations. This is shown clearly in Figure 17(iii), which corresponds to a 'cut' on Figure 17(i) as illustrated. The ratio sorghum price/maize price was set at 1.05, ensuring that no sorghum is present in the mix (to separate the process of substitution between grains and sources of proteins from the substitution between maize and sorghum). The total amount of maize was plotted in Figure 17(iii) against different price levels of the 'sources of proteins' (the relative prices between these were kept constant, as for price sets 1, 2 and 3).

Figure 18 shows the impact on the total FCE resulting from adopting maize and sorghum imports that are non-optimal in relation
Figure 18 - Extra foreign currency expenditure incurred as a result of fixing the feed grain contents of the animal feeds at non-optimal levels

(i) Extra FCE incurred as a result of fixing the maize- sorghum contents as indicated (basis: price set 1)

(ii) Extra FCE incurred as a result of fixing the maize content as indicated (basis: price relations as for Figure 16 (iii))

Note:
(i) Maize and sorghum contents expressed as a proportion of (Maize + Sorghum) content;
(ii) Maize content expressed as a proportion of the total animal feed production
to their relative prices or to the price relations between them and the 'sources of proteins'. This will occur when

(i) the planning of feed grain imports is non-adaptive to those price changes (as currently happens), or when,

(ii) under an adaptive planning system, price forecasting errors are (inevitably) introduced into the planning model.

Figure 18(i) shows that, for a given maize-sorghum proportion, the relative difference

$$\left[ \frac{\text{FCE} - (\text{FCE})_{\text{op}}}{(\text{FCE})_{\text{op}}} \right] = \Delta \text{FCE}/(\text{FCE})_{\text{op}}$$

is very flat around its minimum. Extending the analysis to maize-sorghum proportions not represented in the figure it was possible to conclude that for any value of the ratio

$$R = \text{sorghum price/maize price}$$

one can always find a maize-sorghum proportion that, inside an interval $R \pm 0.05$, leads to an FCE that never exceeds the minimum by more than 0.25%. These figures provide an indication of the accuracy required in forecasting $R$ in order to stay close to the minimum FCE.

As shown in Figure 18(ii) and due to the low substitutability between feed grains and 'sources of proteins', for any given amount of feed grains in the mix the relative difference $\Delta \text{FCE}/(\text{FCE})_{\text{op}}$ is even flatter around its minimum, when expressed as a function of

$$R' = \text{Soybean meal price/maize price}.$$

For all values of $R'$, it is possible to find a feed grain proportion
such that, inside the interval $R + 25\% R$, the FCE never exceeds
its minimum by more than 0.25%.

5.2 Effects of Cane Molasses and Dicalcium Phosphate Price Changes
on the Content of Maize and Sorghum

In the price sets 1, 2 and 3 the prices of cane molasses
dicalcium phosphate were arbitrarily fixed in relation to the price
of maize. The effects of price changes in those raw materials in the
optimal content of maize and sorghum were found to be very limited.
This result is not surprising, in view of these raw materials'
ingredients:

(i) although energy is the most valuable ingredient in cane
molasses this raw material can by no means replace
directly the feed grains in the mix (first, because its
energy content is considerably lower than those of the
feed grains and, secondly, because of its extremely
high moisture content); taking price set 1 as a basis
(for which the content of imported cane molasses in the
mix is 2.96%), price changes in that raw material within
the range $\pm 30\%$ of its original price led to changes
in the total feed grains' content not exceeding 0.75%;
changes in the process of substitution between maize
and sorghum were found equally limited (this point will
be analysed later);

(ii) dicalcium phosphate only has value in the mix due to
its phosphorus content (calcium is freely available
through calcium carbonate); whatever the value of its
marginal FCE (expressed as a percentage of its internal
price in Portugal), the FCE per unit of phosphorus
obtainable from dicalcium phosphate is always much smaller than the one obtainable from any other raw material; this is the reason why changes in its marginal FCE were found to have negligible effects on the mix, in general, and on the feed grains' content, in particular.

5.3 The Robustness of the Substitution between Maize, Sorghum and the Other Raw Materials

It was shown before, in section 5.1, that the savings in FCE that can be made by adapting optimally the amounts of maize and sorghum as their prices change are rather small, when expressed as a percentage of the minimum FCE. In these circumstances, and however attractive those savings may sound in absolute terms, it is appropriate to ask whether the approximations and errors inevitably introduced into the model can eliminate or reduce significantly the advantage of adapting optimally the maize and sorghum imports as their prices change (between them or in relation to other raw materials).

Taking, for example, the substitution between maize and sorghum, that would be the case if the ratio

\[
\frac{[\text{Sorghum imports}/(\text{Maize + Sorghum imports})]}{\text{optimal}} = f(R)
\]

where

\[R = \frac{\text{Sorghum price}}{\text{Maize price}},\]

was highly sensitive to errors introduced in the LP model. Figure 19 (diagrams (i) and (ii)) illustrate what is meant by higher and lower
sensitivity. The shaded area represents possible shifts of the optimal f(R) line resulting from potential errors introduced in the model. From the diagram on the left, it is clear that even if R could

Figure 19 - Sensitivity of a process of substitution between two raw materials to changes in the model parameters

![Diagram showing sensitivity to changes in R](image)

\[ f(R) : \text{e.g. } \frac{\text{Sorghum content}}{\text{(Maize+Sorghum) content}} \text{ optimal} \]

\[ R : \text{ Sorghum price / Maize price} \]

be predicted accurately, the benefits resulting from making adjustments on f(R) following its assumed optimal values would be questionable over a wide range of values of R, such as the one defined by \( R_1 \) and \( R_2 \) (within this range it would make sense to adopt an average value \( \bar{f}(R_{1,2}) \), as shown in the diagram, irrespectively of the value of R). Clearly, for a lower sensitivity of f(R) to errors in the model (diagram (ii)), the benefits resulting from adjusting the proportion of sorghum from \( f(R_1) \) to \( f(R_2) \) when R changes from \( R_1 \) to \( R_2 \) become more significant.

The sensitivity of the optimal substitution between maize, sorghum and the other raw materials in relation to errors in the
parameters of the LP model was analysed considering the following major sources of error:

(i) relative prices of the imported raw materials;
(ii) structure of animal feed production; and
(iii) characteristics of the 'residual compound'.

Both processes of substitution - between maize and sorghum and between these feed grains and the other raw materials - were found to be rather insensitive to changes in the parameters of the model likely to occur during the planning process. This is illustrated for the substitution between maize and sorghum in Figure 20. The optimal substitution between the feed grains considered before in Figure 17(ii) for price set 1 is compared with the ones obtained assuming the following changes:

(i) changes in the raw materials prices
   (Price set 4, defined in Figure 20, corresponds to (a) a 20% increase in the feed grain prices, in relation to the other raw materials' prices, and (b) changes in the relative prices of the raw materials other than maize and sorghum, implying considerable substitutions among them);

(ii) changes in the animal feed production structure
   (Following the process of substitution between maize and sorghum for price set 1, it was observed that, as sorghum prices decrease, the feed in which maize is first replaced by sorghum is S-830, followed by all the hogs and poultry feed and, finally, the cattle feeds; the last feeds in which the substitution takes
Figure 20 - Sensitivity of the optimal substitution between maize and sorghum to changes in the model parameters

(i) Changes in raw materials' import prices (from price set 1 to price set 4)

| Price set 1 | 100 | 50 | 25 | 12.5 | 6.25 | 3.125 | 1.5625 | 0.78125 |
| Price set 2 | 100 | 50 | 25 | 12.5 | 6.25 | 3.125 | 1.5625 | 0.78125 |

(ii) Changes in the animal feed production structure

<table>
<thead>
<tr>
<th>Feeds (as supplied before)</th>
<th>Production (alternative considered)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-470</td>
<td>7.87 %</td>
</tr>
<tr>
<td>R-720</td>
<td>6.73 %</td>
</tr>
<tr>
<td>R-112</td>
<td>6.49 %</td>
</tr>
<tr>
<td>Others</td>
<td>6.72 %</td>
</tr>
</tbody>
</table>

(iii) Changes in the ingredients of the 'residual compound'

<table>
<thead>
<tr>
<th>Moisture</th>
<th>Crude protein</th>
<th>Crude fibre</th>
<th>Crude fat</th>
<th>Calcium</th>
<th>Total phosphorus</th>
<th>Ash</th>
<th>Ph.P. (mol/l)</th>
<th>Ret. ash (kg)</th>
<th>Ret. ash (calc)</th>
<th>Ret. p.'</th>
<th>Ret. s.'</th>
<th>Lysine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asabell</td>
<td>12.70</td>
<td>15.90</td>
<td>10.90</td>
<td>5.40</td>
<td>0.39</td>
<td>0.77</td>
<td>6.12</td>
<td>0.81</td>
<td>0.87</td>
<td>0.87</td>
<td>0.11</td>
<td>0.80</td>
</tr>
<tr>
<td>Altern., considered</td>
<td>*</td>
<td>17.71</td>
<td>*</td>
<td></td>
<td>*</td>
<td>*</td>
<td>1.67</td>
<td>0.84</td>
<td>0.61</td>
<td>0.20</td>
<td>0.17</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Note: the diagram is based on solutions derived for a discrete set of values of the sorghum/maize price ratio from 0.80 to 1.10, with increments of 0.01. Any differences between solutions evaluated at two successive points of the grid are represented on the diagram as step changes occurring at the mid points of the grid intervals.
place are B-320 and B-321. It was shown before that
the major non-seasonal changes in the animal feed
production structure correspond to switching part of
the hogs feeds' share to cattle feeds and vice-versa;
the figures presented earlier in Table 6 show that the
biggest shift in the past was about 4%. The biggest
change in the optimal substitution between maize and
sorghum will occur if that shift takes place between
S-830 and B-320/321, as assumed in Figure 20);

(iii) changes in the ingredients of the 'residual compound'
(The aggregate nutritional characteristics of the
'residual compound' are probably difficult to estimate
accurately. To test the influence of the accuracy
of that estimate on the process of substitution
between sorghum and maize, large changes in the ingredients
of the compound were considered, especially in those
corresponding to active constraints in the LP and
particularly those with highest marginal costs. The
changes considered in Figure 20 were among the ones
with largest effect on the substitution between maize
and sorghum - they correspond to a 30% reduction of
energy and proteins relative to the levels considered
originally).

5.4 Effect of Import Price Changes Over Time on the Mix and on the FCE

The analysis of the effects of price changes on the optimal
import mix presented so far was based on parametrizations conducted
with the purpose of illustrating general properties of the mix, particularly with respect to the feed grains. Clearly there is an important aspect not covered in the previous analysis: it concerns the way in which raw materials' price changes over time affect

(i) the optimal import mix, and

(ii) the value of the unit FCE associated with the production of animal feeds.

The analysis of this aspect of the problem was based upon the average monthly price series presented in section 4. The series concerning raw materials with specifications different from the ones imported by Portugal—fish meal and sunflower meal—were rescaled (multiplied by 1.13 and 0.77, respectively) in order to bring the average price ratios

fish meal $72/soybean meal $45 and
sunflower meal $30/soybean meal $45,

over the 1969-1977 period considered before, to the levels identified earlier as the average ones (2.50 and 0.62, respectively).

In the absence of price series of maize gluten meal, alfalfa meal and cane molasses, the content of these raw materials in the mix was fixed at the levels corresponding to average price relations, as defined in price set 1 (with sorghum price/maize price set at 0.95). To keep in perspective the limited role played by these raw materials in the mix, it should be noticed that

(i) even when their prices are at minimum levels, relative to the other raw materials, they make up less than 7% of the total weight of the mix;

(ii) the FCE originating from these raw materials never exceeded
5% of the total FCE for all price combinations considered in the analysis.

Estimates of the monthly prices of synthetic aminoacids were obtained by linear interpolation, from those shown earlier in Figure 16 (prices actually paid by a major Portuguese importer).

Figure 21 summarises the effects of price changes, from month to month, on the optimal imports of maize and sorghum, over the period 1970-1975. One can notice the effect of the rise in the prices of 'sources of proteins' that took place in 1972. The total amount of feed grains increases from around 60% to 64%. Later in 1973, due to the simultaneous increase in the feed grain prices and decrease in the prices of the sources of proteins, that figure comes down to about 59%. In the meantime, frequent shifts between maize and sorghum in the optimal mix take place. With a few exceptions, the optimal amount of sorghum, when expressed as a proportion of the total amount of feed grains, is either zero or above 30%. The extent of the contribution of the feed grains to the total FCE is also shown in the figure. Excluding maize gluten meal, alfalfa meal and cane molasses, that contribution varies, over the period considered, between 62.7 and 79.6%. Its average value is 73.5% (around 70% if those raw materials are included).

Figure 22 shows, month by month, the unit FCE incurred with the import mix (excluding maize gluten meal, alfalfa meal and cane molasses) at the prevailing raw materials' prices. The minimum FCE - the one corresponding to the current optimal solution - is
Figure 21 - Maize and sorghum in the optimal import mix (monthly, over the period 1970-1975)

compared with the FCE that would be incurred if the solution that was optimal six months earlier was adopted instead.

The figure illustrates clearly the high variability of the minimum FCE per unit of animal feed production over the period of time covered. It is noticeable that, in spite of that considerable variability, the two lines are, most of the time, very close. Over the whole period, the average relative differences between them is 1.42%. Although this figure is small in relative terms, it becomes impressive when converted into money: considering a unit FCE of 110 US$/m.t. and an annual animal feed production as for 1976 (2.218 million metric tonnes), 1.42% corresponds to about US$ 3.5 million. The small relative difference between the two lines has however an important implication: by acting on the mix, it is not
possible to reduce significantly the risk of facing a sudden large FCE increase. When, after the mix is optimized for a given set of prices, considerable price changes take place (such as the ones occurring over 6-month periods), the mix will generally become non-optimal. However, as shown in Figure 21 it will generally lead to a FCE that, in relative terms, is close to the minimum. Therefore, even if one attempted to guard against the possibility of price changes leading to a significant increase in the total FCE, the risk could
not be significantly reduced by altering the mix of raw materials.

This characteristic is hardly surprising in view of the analyses carried out previously concerning the raw materials' prices, on the one hand, and the substitutability between imported raw materials, on the other. In fact it was shown that for raw materials that are substitutable in large proportions (such as maize and sorghum) their prices move closely together. Conversely, for raw materials whose relative prices change considerably (e.g. feed grains and 'sources of proteins') their substitutability in the mix is very limited.

6. Discussion of Implications of the Identified Characteristics of the Mix on the Feed Grains' Import Planning Process

The LP model on which the analyses presented earlier were based involves a number of simplifications that make it unsuitable for the actual planning of the feed grain imports. Amongst them is the inclusion in the objective function of import prices that are assumed to be independent of the amounts actually imported. Clearly this is an oversimplification of the problem, at least as far as the feed grains are concerned. As will be shown later, there are economies of scale in the sea transport, unloading and storage operations that result in a dependence of the import prices on the actual amounts imported of each grain. The LP model did not include explicitly this dependence. However it provided information which played a central role in further developments of the model. Particularly relevant in this respect were the following characteristics of the process of substitution between maize, sorghum and the other raw materials, identified earlier:

(i) the total amount of feed grains in the optimal mix is practically independent of the ratio sorghum price/maize
price (this implies that in order to define the total amount of feed grains required, initially sorghum can be excluded from the import mix formulation);

(ii) the total amount of feed grains in the optimal mix is rather insensitive to adjustments in the relative prices between feed grains and the other raw materials (if sorghum is excluded from the mix, the optimal imports of maize are restricted to a limited range - between 58 and 64% of the total amount of feed to be produced; within this range, economies of scale in the maize delivery process are necessarily limited and, therefore, have a small impact on the total maize import price and on the optimal imports of maize);

(iii) the substitution between maize and sorghum implies, for small changes in the relative prices of these grains, substantial changes in their optimal imports (therefore, economies of scale in the delivery process of each grain are likely to influence the optimal substitution process).

In Chapter 7, following the analysis of the feed grains delivery-inventory process, the LP model presented earlier is extended to include the economies of scale in that process. Naturally, the properties of the optimal mix mentioned above are central in the development of the new model. Possible economies of scale in the imports of other raw materials were not analysed. However relevant they may be for the definition of those raw materials' imports, they will not have a significant effect on the feed grains' optimal
imports (in view of the low sensitivity of the optimal imports of maize and sorghum to changes in the relative prices of other raw materials).

Another property of the mix identified through the LP model - the inability of reducing significantly the risk of facing an unexpected large FCE increase, by altering the mix of raw materials - has far-reaching implications in the design of the feed grains import planning process. Any protection against that risk can only be achieved through the buying operations, by anticipating the raw materials purchases in relation to the country's needs. As will be shown in Chapter 9 that characteristic of the mix implies a considerable simplification of the formal treatment of risk in the import mix planning problem.
CHAPTER 5

THE FEED GRAINS SHIPPING OPERATION : UNCERTAINTY OF VESSELS' ARRIVALS AND SHIPPING COST STRUCTURE

1. Introduction

This chapter contains an analysis of two important aspects of the grain shipping operation: the uncertainty associated with the arrivals of vessels delivering grain at the Portuguese ports and the structure of the shipping costs. As will be shown in the next chapter, these aspects are central in the development of a policy to plan the delivery of grains and other raw materials handled at common port terminals.

Section 2 includes a study of the uncertainty of vessels' arrivals. A significant change in the shipping operation occurred during the period covered by the analysis (1976-1977). Until March 1977 the feed grains were purchased c&f Portugal (cost and freight to Portugal). It was the seller's responsibility to ship the grains to Portugal and to make the delivery at the Portuguese grain terminals within a period specified in each purchasing contract - the 'period of delivery'. In March 1977 a contract was signed between EPAC and the Portuguese nationalised shipowners and charterers - Companhia Nacional de Navegação (CNN) and Companhia de Transportes Marítimos (CTM) - for the shipment of maize, sorghum and wheat. Since then grains have been bought on a fob basis (free on board). Each purchasing contract specifies the 'period of loading' at a port of an exporting country. It is the responsibility of the Portuguese shipping companies to find a vessel arriving at the loading port within the specified period of loading.
In the process of planning the vessels' arrivals, the variable controlled by EPAC was, before March 1977, the period of delivery. Since then, it has been replaced by the period of loading. By comparing the actual arrivals with those controllable variables, the objective of the analysis presented in section 2 is to characterise the uncontrollable parameters of the process of arrivals. This is done first by defining variates relating the actual arrivals with the corresponding periods of delivery or loading and then by proposing hypotheses about the distribution of those variates and testing them with the available data on past grain shipments.

In section 3, the structure of the shipping costs is analysed with the aim of

(i) identifying the major shipping cost components;

(ii) building up a model relating each of these components to various operating parameters (e.g. size of vessel, ports of loading and unloading, location of the vessel when hired) and cost factors (e.g. state of the shipping charter market, price of fuel); and

(iii) deriving a generalised shipping cost function, independent of the features that are particular to each vessel's trip charter contract.
2. The Uncertainty of Vessels' Arrivals

2.1 Grain Deliveries before March 1977 (Purchases C&F)

2.1.1 General. Definition of a Variate Relating Actual Arrivals to the Corresponding Periods of Delivery (6)

Among other clauses, the contract between EPAC and the grain exporters included the definition of the period of delivery and of the penalties incurred for late delivery (i.e. a vessel arriving after the last day of the period of delivery). From the seller's point of view there were also penalty costs involved in the case of early deliveries (EPAC had the choice of keeping a vessel waiting in port, until the first day of the delivery period, at the seller's expense).

The length of the delivery period normally accepted by the grain exporters is one month. However, in view of the limited storage capacity at the grain terminals, EPAC always requested shorter periods of delivery (usually between 10 and 15 days). As will be shown later, this attempt of setting limits on the delivery dates stricter than the ones normally accepted in the grain trade did not succeed. During the period covered by the analysis more than 50% of the vessels delivering feed grains at the Portuguese ports arrived after the last day of the specified delivery periods. What this means is that either the penalty costs for late deliveries were too low to be effective or, more likely, as a result of pressure put on EPAC by the grain exporters, they were not strictly enforced.

There are many sources of uncertainty in the grain delivery process, which cause difficulties to the exporters in complying with tight delivery periods. Among those sources of uncertainty are:

(i) the difficulty of obtaining an appropriate vessel (in
location, size and cost) for each particular delivery, within too tight a schedule;

(ii) delay (or early release) in the trip in which the vessel due to carry the grain was previously engaged;

(iii) delays in the port of loading (e.g. due to congestion, dockers' industrial actions, bad weather); and

(iv) delays at sea, due to bad weather (the effect of bad weather on the time spent by vessels in the sea is very limited; crossing the Atlantic a vessel can be delayed, at most, 2 days — unless, of course, it sinks ... ).

The aggregate effect of all these factors on the date of arrival was studied by analysing the variate

\[ \delta = \text{date of arrival} - \text{last day of the delivery period}. \]

The adoption of such a variate was based on a preliminary analysis of the available sample of arrivals. This analysis showed that the distribution of the actual arrivals was centred near the last day of the delivery period. The influence of the length of the delivery period on the distribution of \( \delta \) was tested and was found not significant (this test will be presented below). Had the result of this test been different, another variate would have to be used (e.g. date of arrival - mid date of the delivery period, or this variate rescaled by a factor dependent on the length of the delivery period).

2.1.2 Statistical Analysis of \( \delta \)

The analysis was based on a sample of feed grain shipments which were scheduled to arrive at the Portuguese ports during 1976 (data obtained from EPAC records). 'Exceptional' deliveries
('aflotes' and shipments with delivery dates rescheduled after purchase) were eliminated. The sample finally used in the analysis is presented in Appendix 1.

The values of δ over time are shown in diagram (i) of Figure 23. Diagram (ii) gives the sample distribution of the length of the delivery periods.

The sample includes 37 shipments of maize and 15 of sorghum. The importing processes concerning these two grains are practically identical (involving the same importing agency, the same group of exporters and the same exporting country). One can therefore expect the delays in maize and sorghum shipments to follow identical distributions. This hypothesis was tested using the Kolmogorov-Smirnov test - two samples, small and different sizes (see Kim et al. 35). Figure 23(iii) summarizes the structure and the result of the test, which led to the acceptance of the hypothesis (*) . Following this result the shipments of maize and sorghum were pooled into a common sample for further statistical analyses.

(*) A 5% significance level was adopted in all statistical tests presented in the thesis.
Figure 23 - Delays (d) and lengths of delivery periods. Test for equality of the distribution of delays of maize and sorghum shipments (Sample: Feed grain shipments scheduled to arrive at Portuguese ports during 1976)

(i) Values of d over time

- Values of d over time

\[ \bar{d} = 1.01 \text{ days} \]
\[ \sigma = 17.26 \text{ days} \]

- Maize shipments
- Sorghum shipments

- Last day of period of delivery

(ii) Distribution of lengths of delivery periods

- Length of delivery period (days)

- Average = 15

(iii) Kolmogorov-Smirnov test for equality of the distributions of the delays of maize and sorghum shipments

Cumulative sample distributions

- \( D = 0.263 \)
- \( \bar{d}_1 = 0.35 \)
- \( \bar{d}_2 = 0.37 \)
- \( \bar{d}_3 = 0.57 \)
- \( \bar{d}_4 = 0.55 \)

- Test statistic: \( D = \text{Max.} | F_1(d) - F_2(d) | \)
- \( D = 0.263 < D_0 = 0.05 \)
- \( c/(25 \times 25) = 22.5/(25 \times 25) = 0.336 \)
- \( D_0 \) accepted

(e) Kim, P.J., and Jenrich, R.I., "Tables of the Exact Distribution of the Two Sample Kolmogorov-Smirnov Criterion, \( D_{n,m} = \max | F_1 - F_2 | \), in 'Selected Tables in Mathematical Statistics', Vol. I, Institute of Mathematical Statistics (1976)
As pointed out before, the sources of uncertainty in the process of arrivals are numerous. Since most of them are unlikely to be significantly correlated one can expect $\delta$ to follow a normal distribution. This hypothesis was tested, and accepted, using Lilliefors's test 43 (a Kolmogorov-Smirnov type test, applicable when the parameters of the normal distribution are estimated from the sample). Figure 24 summarizes the structure and the results of the test.

**Figure 24 - Lilliefors test for normality of the distribution of $\delta$**

Cumulative distributions:
- $F(\delta)$: Normal($\mu=1.61$, $\sigma=12.26$)
- $F_0(\delta)$: Sample distribution

$D = 0.084$

$H_0 : \delta \sim F(\delta)$
$H_1 : \delta \sim \text{other distribution}$

Test statistic: $D = \text{Max.} |F(\delta) - F_0(\delta)|$

$D = 0.084 < D_0 (\alpha = 0.05) = 0.886 / \sqrt{N} = 0.886 / \sqrt{52} = 0.123$

$H_0$ accepted

Having accepted the normality of $\delta$, the homogeneity of its variance was tested, using Bartlett's test. The observations were grouped in four quarterly sub-samples of a sufficient dimension to comply with the requisites of the test. As shown in Table 14, the null hypothesis was accepted.

Table 14 - Bartlett's test for homogeneity of the variance of $\delta$

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>$\sigma_1^2 = \ldots = \sigma_k^2$ (k=4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>$\sigma_1^2 \neq \ldots \neq \sigma_k^2$</td>
</tr>
<tr>
<td>Test statistic</td>
<td>$X = \left{ \bar{y}_j \ln s_j^2 - \frac{k}{j=1} \bar{y}_j \ln s_j^2 \right} / c$</td>
</tr>
<tr>
<td>where</td>
<td>$\bar{y}_j = n_j - 1$ (n_j: no. of observations in group j)</td>
</tr>
<tr>
<td></td>
<td>$\bar{y}_j = \frac{\sum y_j}{j=1} \quad j=1$</td>
</tr>
<tr>
<td></td>
<td>$s_j^2 = \frac{1}{\nu_j} \sum_{i=1}^{\nu_j} (y_{ij} - \bar{y}_j)^2$</td>
</tr>
<tr>
<td></td>
<td>$s^2 = \frac{1}{\nu} \sum_{j=1}^{k} \nu_j s_j^2$</td>
</tr>
<tr>
<td></td>
<td>$c = 1 + \left[ \frac{1}{(3k-1)} \right] \left[ \frac{k}{j=1} (1/\nu_j) - 1/\nu \right]$</td>
</tr>
<tr>
<td>$X = 1.89 &lt; \chi^2_2 (\alpha = 0.05) = 7.815$</td>
<td></td>
</tr>
<tr>
<td>$H_0$ accepted</td>
<td></td>
</tr>
</tbody>
</table>

A test for the first order stationarity of $\delta$ was also performed, using the analysis of variance technique. The observations were grouped monthly and the following one-way random effect model was assumed:

$$\delta_{ij} = \delta_j + \varepsilon_{ij}$$

$$= \mu + \alpha_j + \varepsilon_{ij}$$

with

$$\alpha_j \sim N(0, \sigma_a^2)$$

$$\varepsilon_{ij} \sim N(0, \sigma_i^2)$$

(\(\alpha_j\)'s and \(\varepsilon_{ij}\)'s all mutually independent)

where

$$j = 1, \ldots, 12$$

(month index)

$$i = 1, \ldots, n_j$$

(n_j: no. of observations in group j).
As shown in Table 15, the null hypothesis (first order stationarity) was accepted.

Table 15 - Test for 1st. order stationarity of \( \delta \)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between months (BM)</td>
<td>( \sum (\hat{\delta}_j - \bar{\delta})^2 ) = 2032</td>
<td>11</td>
<td>184.7</td>
</tr>
<tr>
<td>Within months (WM)</td>
<td>( \sum (\hat{\delta}<em>{ij} - \bar{\delta}</em>{ij})^2 ) = 5634</td>
<td>40</td>
<td>140.8</td>
</tr>
<tr>
<td>Total</td>
<td>( \sum (\hat{\delta}<em>{ij} - \bar{\delta}</em>{ij})^2 ) = 7666</td>
<td>51</td>
<td>150.3</td>
</tr>
</tbody>
</table>

\( H_0 : \sigma^2 = 0 \)
\( H_1 : \sigma^2 > 0 \)

Test statistic: \( F = \frac{BMSS}{WMSS} \)

\( F = 1.31 < F_{11,40} (\alpha = 0.05) = 2.04 \)

\( H_0 \) accepted

As stated above, the model assumes mutual independence of all \( a_j \)'s and \( \epsilon_{ij} \)'s. The assumption of independence between the \( \epsilon_{ij} \)'s seems, a priori, realistic, in view of the high number of different vessels and loading ports involved in the grain shipping operation. In fact, since the main sources of uncertainty are connected with the vessel and the loading port involved in each shipment and since they change frequently from shipment to shipment, consecutive \( \epsilon_{ij} \)'s can be expected to be uncorrelated. The mutual independence between \( a_j \)'s and \( \epsilon_{ij} \)'s seems perfectly acceptable. However, some positive correlation can exist between successive \( a_j \)'s. And it can be shown that, if this was true, it could lead to the acceptance of \( H_0 \) in situations where, at 5% significance level, it should actually be rejected.

Mainly for this reason, a further check was made by testing whether consecutive \( \delta_{ij} \)'s were significantly correlated. Any positive correlation between consecutive \( a_j \)'s (if different from zero) or \( \epsilon_{ij} \)'s would, in fact, imply a positive correlation between successive \( \delta_{ij} \)'s. In the significance test, the grain shipments were ordered according to the
last days of their delivery periods. When they coincided, the shipments were ordered randomly (as shown in Appendix 1). As shown in Table 16 the null hypothesis (uncorrelated $\delta_{ij}$’s) could not be rejected.

Table 16 - Test for correlation between consecutive $\delta_{ij}$’s

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>$\rho = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>$\rho &gt; 0$</td>
</tr>
</tbody>
</table>

Test statistic: $t = (r \sqrt{N-2}) / \sqrt{1-r^2}$

where $r$: estimate of correlation coefficient (0.129)
$N$: no. of observations (51)
$t = 0.911 < t_{49} (\alpha = 0.05, \text{ one sided }) = 1.67$

$H_0$ accepted

A final test was carried out to check whether the variate $\delta$ was significantly affected by the length of the delivery period. From the total number of shipments (52), two sub-samples were selected according to the following criterion:

(i) sub-sample 1: shipments with delivery periods of 10, 11 or 12 days (28 observations);
(ii) sub-sample 2: shipments with delivery periods of 15, 16 or 17 days (19 observations) (see Figure 23 (ii)).

The Kolmogorov-Smirnov test (two small samples of different size) was used to test the differences between the two distributions. As shown in Figure 25, the null hypothesis of equally distributed $\delta$’s could not be rejected.
2.2 Grain Deliveries After March 1977 (Purchases fob)

2.2.1 General. Definition of a Variate Relating Actual Arrivals to the Corresponding Periods of Loading (δ')

Since March 1977 the imported grains have been shipped to Portugal in vessels either owned or, on most occasions, chartered by CNN and CTM. The contract between EPAC and these shipping companies states, for each shipment, the period of loading at a port of an exporting country. The length of this period is usually between 15 and 17 days (near the minimum
normally accepted by the shipping companies). Extra costs (payable to the grain sellers) are incurred by CNN and CTM when the vessels fail to arrive at the loading port within the loading period.

When interviewed in January 1978, the manager of CNN in charge of the planning of grain shipments defined the target date for the arrival of a vessel at the port of loading as the middle of the first half of the loading period. This target corresponds to what normally is, from the shipper's point of view, the least cost compromise. Clearly, circumstances may bring about departures from this target (e.g. a favourable contract involving a vessel that can only reach the port later than the target date).

The date of arrival at the loading port (DALP) can be expressed as follows:

\[
\text{DALP} = \text{TDALP} + \text{deviation} (\delta'_1)
\]

where

\text{TDALP} : target date of arrival at the loading port.

Following the definition of this target date, one can write

\[
\text{DALP} = \left[ \text{EDLP} + \frac{1}{4} (\text{LDLP} - \text{EDLP}) \right] + \delta'_1
\]

(1)

where

\text{EDLP} : earliest day of the loading period

\text{LDLP} : latest day of the loading period.

The date of arrival at the Portuguese port (DAPP) can be written:

\[
\text{DAPP} = \text{DALP} + \text{TLP} + \text{VT}
\]

(2)

where

\text{TLP} : time spent at the loading port

\text{VT} : voyage time (from the loading port to the Portuguese port).
These times can, in turn, be expressed as follows:

\[ \text{TLP} = \text{NLT} + \delta_2' \]  
\[ \text{VT} = E(\text{VT}) + \delta_3' \]

where

NLT : nominal loading time, defined as the ratio of the size of shipment, Q (m.t.) to the nominal loading rate, LT (m.t./day, 'shinc' - Sundays, Saturdays and holidays included);

E(\text{VT}) : expected value of the voyage time (under 'average' weather conditions and under an average cruise speed of 13.5 knots).

Substituting expressions (1), (3) and (4) in (2), one obtains

\[ \text{DAPP} = \left\{ \left[ \text{EDLP} + \frac{1}{4} (\text{LDLP} - \text{EDLP}) \right] + \text{NLT} + E(\text{VT}) \right\} + (\delta_1' + \delta_2' + \delta_3') \]

or, rearranging,

\[ \delta' = \text{DAPP} - \left\{ \left[ \text{EDLP} + \frac{1}{4} (\text{LDLP} - \text{EDLP}) \right] + Q/LR + E(\text{VT}) \right\} \]  

(5)

The variate \( \delta' \) relates the actual date of arrival at the Portuguese port (DAPP) with

(i) the controllable variables (EDLP, LDLP, Q) and with

(ii) the known uncontrollable variables (the standard value of LR in the US ports is 5000 m.t./day, shinc ; E(\text{VT}) is approximately 11.5 days, from the Atlantic coast to Portugal, or 13 days, from the Gulf to Portugal).

2.2.2 \hspace{1cm} \textbf{Statistical Analysis of} \( \delta' \)

The analysis was based on a sample of feed grain shipments planned to be loaded - all in US ports - during the period April-November 1977 (data obtained from EPAC ). 'Exceptional' shipments (e.g. with unrecorded loading period) were eliminated. The sample finally used in the analysis is presented in Appendix 2.
In expression (5) DAPP, EDLP, LDLP and Q were obtained directly from EPAC records. The nominal loading rate was assumed to be 5000 m.t./day (shinc), since all the shipments originated at US ports.

In the absence of information specifying the loading ports of each grain shipment, the expected voyage time, E(VT), was defined by averaging the expected voyage times from the Atlantic Coast and from the Gulf to the Portuguese ports.

The values of δ' over time and the sample distribution of the length of the loading periods are shown in Figure 26. A formal test for the equality of the distributions of δ' for sorghum and maize shipments is clearly meaningless, in view of the small number of sorghum shipments involved (only 6). This hypothesis was accepted following the inspection of the values of δ' shown in diagram (i) of Figure 26.

Figure 26 - Delays (t') and lengths of loading periods
(Sample: Feed grain shipments planned to be loaded at US ports during the period April-November 1977)

![Diagram showing values of δ' over time and distribution of lengths of loading periods.](image-url)
Tests for normality, homogeneity of the variance and stationarity of the mean of $\delta'$ were made in the same way as for $\delta$. The results, identical to the ones obtained earlier, are summarised in Figure 27. The effect of the length of the loading period on the distribution of $\delta'$ could not be tested meaningfully because most of the loading periods were either 16 or 17 days, as shown in Figure 26 (ii).

Further tests were conducted to check whether or not the variate $\delta'$ is affected by the time made available to the shipping companies (by EPAC) to obtain a vessel suited for each particular shipment. In Figure 28(i) the variate $\delta'$ is plotted against the time interval between

(i) the date of each purchase (this is the moment when EPAC specifies to the shipping companies the port and period of loading) and

(ii) the target date for the arrival of the vessel at the loading port (defined as before).

In order to test whether the variance of $\delta'$ is dependent on $\Delta$, the data was divided into two sub-samples, as defined in Figure 28(i). An F-test was used and, as shown in Figure 28(ii), the null hypothesis of equal variance was accepted. From the scatter diagram in Figure 28(i) it is apparent that there is no relation between $\delta'$ and $\Delta$ (a linear regression model fitted to the data confirmed this; its coefficient of determination was close to zero).
Figure 27 - Summary of tests on \( z' \) (notation as in previous figures and tables)

(i) Lilliefors test for normality

\[ H_0 : z' \sim F(z') \]
\[ H_1 : z' \sim \text{other distribution} \]
\[ D = 0.110 < D_0(\alpha=0.05) = 0.159 \]
\[ H_0 \text{ accepted} \]

(ii) \( F \)-test for homogeneity of variance

( two sub-samples: Apr/May - 18 observations, Aug/Nov - 13 observations)

\[ H_0 : \sigma_1^2 = \sigma_2^2 \]
\[ H_1 : \sigma_1^2 \neq \sigma_2^2 \]

Test statistic: \( F = \frac{s_1^2}{s_2^2} \)
\[ F = 2.64 < F_{17,12}(\alpha=0.05) = 2.83 \]
\[ H_0 \text{ accepted} \]

(iii) \( t \)-test for equality of means

( sub-samples as in (ii) )

\[ H_0 : \mu_1 = \mu_2 \]
\[ H_1 : \mu_1 \neq \mu_2 \]

Test statistic:
\[ t = \frac{\bar{x}_1 - \bar{x}_2}{(s_1^2/s_2^2)\sqrt{1/N_1 + 1/N_2}} \]
\[ t = 1.46 < t_{30}(\alpha=0.05) = 2.04 \]
\[ H_0 \text{ accepted} \]

( Test for independence of consecutive values of \( z' \):

\[ H_0 : \rho = 0 \]
\[ H_1 : \rho > 0 \]
\[ r = 0.145 \]
\[ N = 30 \]
\[ t = (r\sqrt{N-2})/\sqrt{1-r^2} = 0.621 < t_{28}(\alpha=0.05, \text{one sided}) = 1.70 \]

\( r \) is not significantly different from zero.)
Figure 28 - Delay \( \hat{d} \) and the shipping companies' advance notice of contract, \( \Delta \)

(i) Scatter diagram of \( \hat{d} \) v. \( \Delta \)

- Sub-sample 1 (15 observations)
- Sub-sample 2 (16 observations)

\[ -\Delta = \text{target at loading port} - \text{date of purchase} \] [days]

(ii) F-test for homogeneity of variance

( two sub-samples, as defined in (i) )

\[ H_0 : \sigma_1^2 = \sigma_2^2 \]
\[ H_1 : \sigma_1^2 \neq \sigma_2^2 \]

Test statistic : 
\[ F = \frac{\hat{s}_2^2}{\hat{s}_1^2} \]
\[ F = 1.42 < F_{15,14}(\alpha = 0.05) = 2.46 \]

\( H_0 \) accepted

(iii) Regression \( \hat{d} = a + b \Delta \)

ANOVA table

<table>
<thead>
<tr>
<th>Source</th>
<th>S.S.</th>
<th>d.f.</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Due to regression (DR)</td>
<td>0.7</td>
<td>1</td>
<td>0.7</td>
</tr>
<tr>
<td>About regression (AR)</td>
<td>1776.9</td>
<td>29</td>
<td>61.3</td>
</tr>
<tr>
<td>Total</td>
<td>1777.6</td>
<td>30</td>
<td>59.3</td>
</tr>
</tbody>
</table>

Test for the significance of \( b \):

\[ F = \frac{\text{MSR}}{\text{MSE}} = 0.012 < F_{1,29}(\alpha = 0.05) = 4.18 \]

\( b \) not significantly different from zero
2.3 Interpretation of Results

The statistical analyses of variates $\delta$ and $\delta'$ showed the existence of important similarities between the two processes of delivery, before and after the contract between EPAC and the Portuguese shipping companies. Both variates were found to follow stationary normal distributions. Clearly the reason for such similarities lies on the fact that both processes involve the same sources of uncertainty, irrespectively of the company in charge of contracting the vessels (either the Portuguese shipping companies, on the one hand, or the grain exporters or their shipping agencies, on the other). This was, in the first place, the reason for extending the statistical analysis to the available sample of feed grain shipments taking place prior to March 1977.

In spite of these similarities in the nature of the grain delivery process it is clear that differences in the way the penalty costs were defined and/or enforced to the grain exporters (before March 1977) and to the Portuguese shipping Companies (after March 1977) could lead to a significant difference in the unreliability of the grain deliveries. For this reason the variances of the two variates $\delta$ and $\delta'$ were compared. The hypothesis of identical variances of $\delta$ and $\delta'$ was tested (using an F-test) and was rejected. Table 17 shows the computations and the result of the test carried out.
Table 17 - F-test for equality of the variances of $\delta$ and $\delta'$

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>$\sigma^2 = \sigma'^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>$\sigma^2 \neq \sigma'^2$</td>
</tr>
</tbody>
</table>

$$F = \frac{s^2}{s'^2} = \frac{150.31}{73.27} = 2.05 > F_{51,30} (\alpha = 0.05) = 1.94$$

$H_0$ rejected

It is obvious that the observed reduction in the variance could be attributed to a general change in the situation of the world grain shipping market, occurring between the end of 1976 and March 1977, that would affect the shipment of grains to Portugal, irrespective of the change in the shipping procedures that took place. However, this hypothesis can hardly be accepted. In fact, the dry bulk seaborne movement was similar in both years covered by the analysis (low in comparison with the available transport capacity) and no significant changes in the shipping market situation were reported in the specialised press.

The parameters of the distribution of $\delta'$ - as estimated from the sample of feed grain shipments between April and November of 1977 - will be used in the next chapter to derive a delivery-inventory policy for the imported feed grains. If EPAC's management wishes to plan a delivery of grain with an expected date of arrival $d_o$, then the target date of arrival at the loading port will be determined according to the expression:

$$TDALP = d_o - (E(VT) + NLT + \bar{\delta}')$$

Where

- $E(VT)$: expected voyage time (under 'normal' weather conditions)
- $NLT$: nominal loading time ($NLT = Q/LR$)
- $\bar{\delta}'$: mean value of $\delta'$ (according to Figure 26, $\bar{\delta}' = 5.54$ days; rounding to the nearest integer, $\bar{\delta}' \approx 6$ days)
From this target date and for a specified length of the loading period (LLP) the earliest and the latest day of the period of loading can be determined (applying the definition of the target date of arrival at the loading port):

\[
\text{EDLP} = \text{TDALP} - \frac{1}{4}\times \text{LLP} \\
\text{LDLP} = \text{TDALP} + \frac{3}{4}\times \text{LLP}
\]

For this loading period, the actual date of arrival is given by

\[
d = \text{TDALP} + \text{EVT} + \text{NLT} + \delta'
\]

\[
= \left[ d_0 - (\text{EVT} + \text{NLT} + \delta') \right] + \text{EVT} + \text{NLT} + \delta' =
\]

\[
d_0 + \delta' - \delta'
\]

and follows a normal distribution with mean \(d_0\) (the 'planned' date of arrival) and standard deviation \(\sigma_d = \sqrt{\sigma_{\delta'}^2 + \sigma_\delta^2}\).

Since \(\sigma_{\delta'}^2 = 8.56\) days (see Figure 26), the estimated standard deviation of \(d\) is

\[
\sigma_d = \sqrt{8.56^2 + \frac{8.56^2}{31}} = 8.70\text{ days}
\]

According to the management of CNN, the shipping company's advance notice of contract (\(\Delta\)) should not be less than one month, in order to give the companies adequate flexibility in the search for the most appropriate vessel. This implies that a grain shipment from the US should be specified by EPAC at least

\[
30\text{ days} + \left[ \text{EVT} + Q/LR + 6\text{ days} \right] \approx 1.5\text{ to } 2\text{ months}
\]
in advance of the planned date of arrival.

As mentioned before, given the high number of different vessels and loading ports involved in the shipment of grains, the distributions of the dates of delivery of different shipments can be assumed to be
mutually independent. Implied in this independence is the possibility of occurrence of 'crossed deliveries', i.e. deliveries that actually take place in an order which is different from their planned dates of delivery. Such crossed deliveries do actually occur in practice. This is shown in Figure 29 for the sample of feed grain shipments taking place under the contract between EPAC and the Portuguese shipping companies.

3. The Structure of Shipping Costs

3.1 General

After EPAC informs CNN or CTM of a shipment that is to be made (specifying the grain to be shipped, the amount involved, the ports of loading and unloading, the period of loading and the rates of loading and unloading), the company in charge of the shipment starts the search for the most economical dry bulk carrier. This can be:

(i) one of its own (if available and well located for the job);

(ii) a vessel currently chartered by the company under a 'time charter' contract (vessel hired for a specified period of time, during which the charterer is free to use it on any route he chooses);

(iii) a vessel currently under a renewable 'trip charter' contract (under a trip charter contract a vessel is hired for a specific trip); or
(iv) any other vessel chartered for the particular shipment in question.

The vast majority of the grain shipments to Portugal are made under trip charter contracts, with the Portuguese shipping companies acting as charterers (rather than as shipowners). For this reason the contract between EPAC and those companies is based on the freight rates quoted on the open trip charter market. The analysis of the structure of the shipping costs is, for the same reason, based on that type of contract.

The option given to both the shipowners and shipcharterers to switch from time charter contracts to trip charter contracts ensures that the shipping costs are generally similar whatever charter contract is adopted. When Portuguese flag vessels are used, costing the shipping operation on the basis of international market rates is still valid. In fact, since the shipping companies can, as shipowners, sell their services in the open market, the costs based on international rates represent, from the point of view of the country, a measure of the 'opportunity costs' of the services provided.

3.2 The Basic Cost Components

From the point of view of the Portuguese shipping companies when acting as trip charterers, the costs incurred in a shipment of grain can be classified as follows:

(i) Variable costs

(a) vessel hiring cost (this cost is essentially a hire charge for the use of the vessel, paid by the shipcharterer in the form of a trip charter rate per day, multiplied by the total number of days the vessel is under contract);
(b) fuel cost (under trip charter contracts all the fuel costs, either at sea or in port, are chargeable to the charterer);

(c) insurance cost (only against the damages chargeable to the charterer, as specified in standard charter contracts);

(d) port charges;

(ii) Fixed costs (administrative costs, communications and other overhead costs incurred by the shipping company which are allocated to the grain shipments according to some company criterion).

The fixed costs are very small when compared with the variable ones. As their name suggests, they will not change (at least significantly) if the ship sizes and the frequency of deliveries are changed. For these reasons they were excluded from the cost structure analysis.

In addition to the costs mentioned above, EPAC will have to pay the profit margin earned by the Portuguese shipping companies. However, it should be noted that a uniform profit margin (i.e. a constant percentage of the variable costs for all shipment sizes) has practically no effect on the total shipping costs paid by the country or by the nationalised companies - EPAC, CNN and CTM - taken as a whole. However, if the profit margin is not uniform, it may induce choices of shipments that are not 'optimal' from the country's point of view. This was the case when the study of the shipping costs was undertaken. As shown in Figure 30, the rate charged to EPAC by the Portuguese shipping companies for the transport of grains was a step function of the size of the shipment which clearly involved an uneven profit margin, encouraging artificially the adoption of shipment sizes close to the break points. For this reason it is inappropriate to use
the contract rates as a basis for the study of grain shipping policies. In addition, a contract price schedule such as the one presented in Figure 30 is only valid for a particular set of conditions prevailing at a particular time. It does not provide an adequate basis for the analysis of aspects such as

(i) the effect of changes in the unloading rate on the shipping cost structure (a point obviously relevant to the study of the grain delivery policy at the new port terminals);

(ii) the effect of changes in the trip charter rates, fuel costs or other parameters on the shipping cost structure (which is necessary for the
characterisation of grain delivery policies over time).

3.3 Derivation of the Variable Shipping Costs for Each Trip Charter Contract

3.3.1 General

Each trip charter specifies in detail all the conditions of the agreement between the shipowner and the shipcharterer concerning the hiring of a particular vessel for a particular trip. Especially relevant for deriving the variable shipping costs are the contract clauses defining:

(i) the location of the vessel when delivered to the charterer (i.e. the point at which the vessel becomes under contract);

(ii) the shipment size;

(iii) the ports and rates of loading and unloading;

(iv) the location of the vessel when returned to the shipowner (i.e. the point at which the contract ceases - usually at the unloading port after the unloading operation is completed);

(v) the average speed at sea;

(vi) the type(s) of fuel burnt by the engines of the vessel and the rate of fuel consumption at sea and in port; and

(vii) the trip charter rate (the daily rate paid by the charterer from the day the vessel is delivered to him until the vessel is released from the contract).

Once these conditions are specified, the variable shipping costs can be obtained from the expressions presented in the subsequent sections.
3.3.2 **Vessel Hiring Cost**

As stated before, the vessel hiring cost is obtained by multiplying the agreed trip charter rate by the total number of days the vessel is under contract. The payment is calculated on the basis of the estimated number of days required for the whole trip (assuming an average vessel speed at sea and nominal rates of loading and unloading as specified in the charter contract). Since CNN and CTM pay this amount at the beginning of the trip, the vessel hiring cost chargeable to EPAC, at the completion of the shipment, will include the cost of financing the operation. If, at the end of the trip, the total time involved in the whole operation is different from the one estimated at the beginning, the payment made initially to the shipowner is corrected, on the basis of daily rates of demurrage or early release of the vessel. These rates are specified in each contract and are usually equal to the trip charter rate.

Using the notation

- \( C_v \): vessel hiring cost (US$)
- \( R \): trip charter rate (US$/day)
- \( T \): total trip time (from the day the vessel is hired to the charterer to the day it is returned to the shipowner) (days)
- \( \alpha \): interest paid by the charterer on working capital (in January 1978, according to the management of CNN, the rate of interest was 15% per year) (%/year)
- \( T_s \): time the vessel spends at sea (days)
- \( T_p \): time the vessel spends in port (days)
- \( v \): vessel average speed at sea (knots)

(1 knot = 1 nautical mile/hour)
VL : total voyage length (nautical miles)

(VL can be derived from the contract conditions, using Reed's Marine Distance Tables \(14\), which give the distances between any two major ports in the world)

Q : shipment size (1,000 m.t.)

LR : nominal loading rate (shinc) (1,000 m.t./day)

UR : nominal unloading rate (shinc) (1,000 m.t./day)

(the standard values of these rates in January 1978 were defined earlier in Figure 30),

the vessel hiring cost is calculated as

\[ C_v = R.T \]

or, including the cost of financing the operation,

\[ C_v = R.T \left(1 + \frac{a}{100} \cdot \frac{T}{365}\right); \]

but

\[ T = T_s + T_p = \frac{VL}{24 \cdot v} + \left(\frac{Q}{LR} + \frac{Q}{UR}\right); \]

substituting this expression into the previous one, the vessel hiring cost becomes

\[ C_v = R_v \left[\frac{VL}{24 \cdot v} + \left(\frac{Q}{LR} + \frac{Q}{UR}\right)\right] \cdot \left\{ 1 + \frac{a}{100} \cdot \frac{1}{365} \left[\frac{VL}{24 \cdot v} + \left(\frac{Q}{LR} + \frac{Q}{UR}\right)\right]\right\}. \]

3.3.3 Fuel Cost

The fuel cost can be derived from the contract conditions specifying the type(s) of fuel burnt by the vessel and the respective daily rates of consumption at sea and in port. Most vessels burn either ordinary marine fuel oil or marine diesel oil or both. For vessels consuming diesel oil, a cost equivalent to 'all fuel oil' rates of consumption at sea (FCS) or in port (FCP) can be defined as follows:
\[ FCS = FCS_{fo} + FCS_{d'} \left( \frac{P_{fo}}{P_d} \right) \]

or

\[ FCP = FCP_{fo} + FCP_{d'} \left( \frac{P_{fo}}{P_d} \right) \]

where

\[ FCS_{fo} (FCS_{d'}) : \text{rate of consumption of fuel (diesel) oil at} \]
\[ \text{sea (m.t./day)} \]

\[ FCP_{fo} (FCP_{d'}) : \text{rate of consumption of fuel (diesel) oil in} \]
\[ \text{port (m.t./day)} \]

\[ P_{fo} (P_d) : \text{unit prices of fuel (diesel) oil (US$/m.t.)} \]

(according to the management of CNN, the unit prices paid by the Portuguese shipping companies in January 1978 were approximately US$ 90/m.t. (fuel oil) and US$ 145/m.t. (diesel oil), or the equivalent in Escudos).

Using previous notation, the total fuel cost \((C_T)\), expressed in US$, is given by

\[ C_T = P_{fo} \cdot (FCS \cdot T_S + FCP \cdot T_P) \]

\[ = P_{fo} \cdot \left[ FCS \cdot \frac{VL}{24} + FCP \cdot \left( \frac{Q}{LR} + \frac{Q}{UR} \right) \right] . \]

### 3.3.4 Insurance Cost

The structure of the insurance cost depends on the nature of the contracts between the shipping companies and their insurance brokers. The insurance cost \((C_i)\) covering CNN's risks, when the company acts as shipcharterer, is proportional to the vessel's deadweight tonnage and depends on the total trip time, following the expression
\[ C_1 = IR \times DWT \times \text{Max} \left( \frac{T}{365}, \frac{60}{365} \right) \] (US$)

where

IR : insurance rate \hspace{1cm} \text{(US$/(1000 m.t. \text{ Year})})

(in January 1978, the rate paid by CNN was Esc. 160 000 = US$4000 \text{(*)}, per thousand m.t. of deadweight and per year).

DWT : vessel's deadweight tonnage \hspace{1cm} (1000 m.t.)

3.3.5 Port Charges

The structure of the charges imposed by port authorities varies from port to port. For the purpose of this study of the global shipping cost structure it did not seem relevant to analyse in detail how the different cost components (pilotage, mooring and demoorings, towage, length of quay occupied, etc.) affect the structure of the port charges. Instead, a crude model provided by the management of CNN in January 1978 was adopted. A distinction was made between the US and Argentinian ports, on the one hand, and the Portuguese ports, on the other. The port charges \( C_p \) were defined, in each case, as follows:

(i) US and Argentinian ports:

\[ C_p = 15 000 + 250. \text{DWT} \] (US$)

with DWT following previous notation;

(ii) Portuguese ports:

\[ C_p = \text{Esc. 200 000 = US$ 5000} \] (US$)

\text{(*) Using the January 1978 rate of exchange, Esc. 40/US$.}
3.3.6 **Estimation of the Variable Shipping Costs Based on the Information Reported by the Shipping Press**

The expressions presented in the previous sections would enable the estimation of the total variable shipping costs from the trip charter contracts. However, since these could not be obtained from the shipping companies, the analysis of the costs was instead based on reports of the contracts in the shipping press. But, as illustrated in Table 18, these press reports do not include all the contract conditions required to estimate the total variable costs.

### Table 18 - A sample of trip charter reports (dry bulk carriers chartered by CNN and CTM)

<table>
<thead>
<tr>
<th>Date of report</th>
<th>Vessel</th>
<th>DWT (t.)</th>
<th>Speed/fuel consumption(*)</th>
<th>Voyage</th>
<th>Rate (US $/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 Nov 1977</td>
<td>John Lykes</td>
<td>34 750</td>
<td>14.0 / 40 fo + 2.5 d</td>
<td>Hamburg / Transatlantic / Portugal</td>
<td>4 000</td>
</tr>
<tr>
<td>25 Nov 1977</td>
<td>King James</td>
<td>57 550</td>
<td>13.0 / 42 fo + 3.0 d</td>
<td>Fasching Gibraltar / USSR / US Gulf / Portugal</td>
<td>3 500</td>
</tr>
<tr>
<td>7 Dec 1977</td>
<td>Cedrilla</td>
<td>30 422</td>
<td>14.5 / 39 fo + 1.5 d</td>
<td>US Gulf / - / Portugal</td>
<td>4 500</td>
</tr>
<tr>
<td>26 Dec 1977</td>
<td>Jalorjays</td>
<td>52 594</td>
<td>14.5 / 41 fo + 3.0 d</td>
<td>Portugal / Transatlantic / Portugal</td>
<td>3 000</td>
</tr>
<tr>
<td>29 Dec 1977</td>
<td>Georgina</td>
<td>41 035</td>
<td>12.0 / 36 fo + 1.0 d</td>
<td>Salvador / - / Portugal</td>
<td>3 500</td>
</tr>
<tr>
<td>27 Jan 1978</td>
<td>Good Luck</td>
<td>42 800</td>
<td>15.0 / 47 fo + 2.0 d</td>
<td>US Gulf / US Gulf / Portugal</td>
<td>4 500</td>
</tr>
<tr>
<td>31 Jan 1978</td>
<td>Meraklie</td>
<td>53 850</td>
<td>15.0 / 53 fo + 3.0 d</td>
<td>Fasching Gibraltar / USSR / US Gulf / Portugal</td>
<td>3 000</td>
</tr>
</tbody>
</table>

(*): fo = ordinary marine fuel oil  
d = marine diesel oil

Source: Shipping Statistics and Economics

Normally the sizes of the shipment (Q) are not reported. They were derived from the vessel’s deadweight tonnage (DWT), using the relation

\[ Q/DWT = 0.825 \]

(this ratio was obtained in a study by Hidrotécnica Portuguesa \(^{31}\), by averaging the values of Q/DWT observed in a sample of grain shipments made by CNN and CTM).

Estimates of the rate of fuel consumption in port, also not reported, were obtained from the management of CNN (and were later found to be in close agreement with figures published by Goss \(^{27}\)). According to the management of CNN, the rate of fuel consumption in port (and fuel oil cost equivalent) can be derived from each vessel's deadweight tonnage,
using the relation

\[
\text{FCP (m.t./day)} = \begin{cases} 
0.5 + 0.1 \times \text{DWT}, & \text{if DWT} \leq 25 \\
3.0 & \text{otherwise}
\end{cases}
\]

where DWT is, as before, the deadweight tonnage of the vessel, expressed in 1000 m.t..

When the various details of the voyage are adequately reported, the voyage length (VL) - necessary for the calculation of the total variable costs - can be estimated using Reed's Marine Distance Tables. Unfortunately, a substantial number of the trip charter reports do not give a clear definition of the voyage. Examples of such reports were given earlier in Table 18: for several of them it is unclear whether the vessel was loaded in the Gulf, Atlantic Coast or both. For this reason, out of the 40 trip charters reported between May 1977 and January 1978 concerning the shipment of grain between US ports and Portugal, the total variable costs could be estimated for only 15 of them.

3.4 Derivation of the Expected Variable Shipping Costs as a Function of the Size of the Shipments

In the previous section it was shown how the variable shipping costs could be estimated for each particular trip charter contract. It now remains to define a relationship that, at any given point in time (i.e. for the shipcharter market conditions prevailing at any given time), gives the expected variable shipping cost as a function of the size of a shipment.

In theory, with data on a high number of trip charter contracts, involving a wide range of shipment sizes, it should be possible to define
the expected cost function and to isolate random deviations due to circumstances that are peculiar to each contract. However, the lack of data makes this procedure infeasible. In fact, as mentioned earlier, the frequency of reported shipments in the US - Portugal routes is limited and in many cases the total variable costs cannot be estimated. Furthermore, the shipment sizes are predominantly in a limited range of values - the ones that have been most attractive to EPAC.

Under these circumstances an indirect approach was called for in trying to estimate the relation between the expected variable shipping costs and the size of shipments, over time. The approach adopted involved two basic steps:

(i) the definition, for each trip charter contract, of a 'standard trip charter rate';

(ii) the attempt to relate the obtained standard rates to regularly published rates (which provide, at any time, a measure of the state of the trip charter world market).

The first of these steps is required because the trip charter contracts take place under widely differing conditions, which are bound to affect the value of the trip charter rate. As shown previously in Table 18, the vessels can be hired at different locations and can have different speed/fuel consumption performances. It is only natural that a vessel with lower fuel consumption than another will normally be hired to the charterer at a higher rate, other factors being equal. Or a vessel already near the US coast when delivered to CNN or CTM will be charged at a higher daily rate than an identical vessel hired passing Gibraltar, for example. The latter makes a round trip with
one leg on ballast at the expense of the charterer; the former is hired for a single leg voyage, involving less time and less fuel for the charterer.

Since the trip charter rates can conceal different contract conditions it is meaningless to make direct comparisons between them, or between them and the published rates. Before these comparisons can be made it is necessary to reduce them to a 'common scale'. In line with this purpose the following 'standard contract conditions' were defined:

(i) standard voyage: a round voyage starting and finishing at the Portuguese port of unloading and calling at the loading port(s) specified in each charter contract;

(ii) standard rate of fuel consumption at sea: the expected rate of fuel consumption at sea of a bulk carrier with size and average speed as specified in the charter contract.

The standard rate of fuel consumption at sea (all fuel oil cost equivalent) was estimated from a sample of 87 trip charter contracts, involving dry bulk carriers consuming ordinary fuel and diesel oils, reported between May 1977 and January 1978 in Shipping Statistics and Economics (the sample is presented in Appendix 3). The rate of fuel consumption at sea was found to be dependent on the vessel size and speed, according to the model

\[ FCS = (-16.3 + 2.54 \cdot V + 3.62 \cdot 10^2 \cdot \text{DWT} \cdot V) + \varepsilon \text{ (m.t./day)} \]

where, as before,

\[ V : \text{vessel speed (knots)} \]

\[ \text{DWT} : \text{Vessel deadweight tonnage (1000 m.t.)} \]

\[ \varepsilon : \text{residual term (m.t./day)} \]
The statistical analysis showed that, although a significant part of the total variance of FCS is explained by the multiple regression model, the deviations from the fitted model were still considerable (the standard error of the residuals was 4.1 m.t./day). This implies the existence of considerable differences in efficiency among the many dry bulk carriers that are available for charter, as far as their fuel consumption is concerned.

The relationship between the standard rate of fuel consumption at sea, and the vessel's size and speed is graphed in Figure 31.

![Figure 31 - Standard fuel consumption at sea](image)

Having defined for each trip charter contract an equivalent 'standard contract' – specifying a standard voyage length and a standard fuel consumption – one can then derive from the trip charter rate actually paid (R) an equivalent 'standard rate' (R*). For each particular contract, with a given total cost C, the standard rate is defined
as the rate that under the equivalent standard contract would lead
to the same total variable cost C. The standard rate can thus be
calculated from the relation

\[
C = C_V + C_f + C_i + C_p =
\]

\[
= R.T. \left( 1 + \frac{a}{100} \cdot \frac{T}{365} \right) + (FCS \cdot T_s + FCP \cdot T_p) + C_i + C_p \quad \text{(actual contract)}
\]

\[
= R^* \cdot T^* \left( 1 + \frac{a}{100} \cdot \frac{T^*}{365} \right) + (FCS^* \cdot T_s + FCP \cdot T_p) + C_i + C_p \quad \text{(standard contract)}
\]
or

\[
R^* = \frac{R.T. \left( 1 + \frac{a}{100} \cdot \frac{T}{365} \right) + FCS \cdot T_s - FCS^* \cdot T_s^*}{T^* \left( 1 + \frac{a}{100} \cdot \frac{T^*}{365} \right)}
\]

where the variables and parameters corresponding to the actual contract
are written according to previous notation and the ones which are
'standard' are marked with an asterisk.

Table 19 shows the standard rates corresponding to the trip
charters for which the total variable costs could be estimated.

Table 19 - The standard rate (grain shipments chartered by CRN and CTW,
reported by Shipping Statistics and Economics)

<table>
<thead>
<tr>
<th>Month of report</th>
<th>Vessel</th>
<th>DWT m.t.</th>
<th>Speed knots [knots]</th>
<th>F,con,b at sea (fo eq. Eq.) Actual Standard [m.t./day]</th>
<th>Trip length Actual Standard [naut.miles]</th>
<th>Rate</th>
<th>Actual Standard [US $/m.t.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jun.1977</td>
<td>Aegean M.</td>
<td>40 074</td>
<td>14.5</td>
<td>45.2</td>
<td>10 000</td>
<td>8 800</td>
<td>3 100</td>
</tr>
<tr>
<td>Jul.1977</td>
<td>Laconicos G.</td>
<td>24 310</td>
<td>15.0</td>
<td>29.0</td>
<td>8 200</td>
<td>8 800</td>
<td>3 100</td>
</tr>
<tr>
<td>Aug.1977</td>
<td>Master P.</td>
<td>40 501</td>
<td>14.0</td>
<td>46.0</td>
<td>9 000</td>
<td>8 800</td>
<td>2 650</td>
</tr>
<tr>
<td></td>
<td>&quot;Michalakis&quot;</td>
<td>40 501</td>
<td>14.0</td>
<td>46.0</td>
<td>6 400</td>
<td>6 200</td>
<td>2 650</td>
</tr>
<tr>
<td></td>
<td>&quot;Narottam M.&quot;</td>
<td>53 546</td>
<td>14.5</td>
<td>46.2</td>
<td>9 900</td>
<td>8 200</td>
<td>2 500</td>
</tr>
<tr>
<td>Oct.1977</td>
<td>Laura M.</td>
<td>46 810</td>
<td>15.0</td>
<td>53.2</td>
<td>7 200</td>
<td>6 200</td>
<td>2 700</td>
</tr>
<tr>
<td></td>
<td>&quot;Ocean M.&quot;</td>
<td>55 840</td>
<td>15.0</td>
<td>41.2</td>
<td>7 200</td>
<td>6 200</td>
<td>3 100</td>
</tr>
<tr>
<td></td>
<td>&quot;Jalavijaya&quot;</td>
<td>52 599</td>
<td>14.2</td>
<td>46.8</td>
<td>10 400</td>
<td>8 900</td>
<td>2 900</td>
</tr>
<tr>
<td>Nov.1977</td>
<td>King James</td>
<td>52 558</td>
<td>13.0</td>
<td>46.8</td>
<td>9 100</td>
<td>8 900</td>
<td>3 500</td>
</tr>
<tr>
<td></td>
<td>&quot;Mereklis&quot;</td>
<td>53 836</td>
<td>15.0</td>
<td>56.8</td>
<td>8 900</td>
<td>8 900</td>
<td>3 350</td>
</tr>
<tr>
<td></td>
<td>&quot;S. Trader&quot;</td>
<td>28 500</td>
<td>15.0</td>
<td>58.2</td>
<td>9 000</td>
<td>8 800</td>
<td>4 050</td>
</tr>
<tr>
<td>Dec.1977</td>
<td>Konkar R.</td>
<td>44 492</td>
<td>13.5</td>
<td>37.2</td>
<td>9 000</td>
<td>8 800</td>
<td>3 950</td>
</tr>
<tr>
<td></td>
<td>&quot;G. Light&quot;</td>
<td>23 804</td>
<td>14.0</td>
<td>31.7</td>
<td>9 800</td>
<td>8 800</td>
<td>3 100</td>
</tr>
<tr>
<td>Jan.1978</td>
<td>Mereklis N.</td>
<td>53 830</td>
<td>15.0</td>
<td>57.8</td>
<td>9 100</td>
<td>8 900</td>
<td>3 000</td>
</tr>
<tr>
<td></td>
<td>&quot;Mereklis&quot;</td>
<td>53 830</td>
<td>15.0</td>
<td>57.8</td>
<td>9 100</td>
<td>8 900</td>
<td>3 000</td>
</tr>
</tbody>
</table>
The trip charter rates, once standardized, can then be compared with a published series reflecting the average trip charter rates prevailing in the world shipping market at the times the contracts are undertaken. The tramp trip charter series published by the General Council of British Shipping (GCBS) was used as term of reference.

For each of four different groups of bulk carriers,

(i) 'small' (12 - 19999 DWT),
(ii) 'handy size' (20 - 34999 DWT),
(iii) 'medium' (35 - 49999 DWT) and
(iv) 'Panamax' (50 - 84999 DWT),

the GCBS publishes monthly

(a) the average trip charter rate \( r_i \) (in US$/\text{deadweight m.t.} \times \text{month})

obtained from a sample of 'representative' contracts;

(b) the average deadweight tonnage \( \text{DWT}_i \) (in m.t.) of the vessels included in that sample.

For each trip charter contract shown earlier in Table 19, a 'GCBS rate' was derived from the published rates, as follows:

(i) gross rates \( R_i \) (in US$/\text{day}) were derived from the published values of \( r_i \) and \( \text{DWT}_i \):

\[
R_i \ (\text{US$/\text{day})} = \frac{r_i \ (\text{US$/\text{m.t.} \times \text{month})} \times \text{DWT}_i \ (\text{m.t.})}{30 \text{ days/month}}
\]

(ii) for a shipment involving a vessel with a given deadweight tonnage \( \text{DWT} \), the GCBS rate was interpolated or extrapolated linearly from the published average values \( \text{DWT}_i \), \( R_i \).

The standard and GCBS rates are compared in Figure 32. Using the 15 data points available, a simple linear regression model, in which the regression line was constrained to pass through the origin, was fitted, giving

\[
\text{standard rate} = (0.943) \times \text{GCBS rate} + c
\]
The 5.7% reduction from the GCBS rate to the expected 'standard rate' was found significantly different from zero (t ratio = -11.1). A number of reasons can justify the detected difference between the rates compared; among them are

(i) the individual characteristics of the routes US - Portugal (it is possible that as a result of the geographical location of the Portuguese ports, close to the mainstream of the bulk sea transport from the US to Europe, vessels hired in the US - Portugal routes are readily contracted for new trips, after the shipments to Portugal are completed; this would tend to lower the US - Portugal
trip charter rates in comparison with the average world based rates);
(ii) the performance of the Portuguese shipping companies when acting as charterers (they may be better than average).

The available data was insufficient to derive any positive conclusions about whether or not the ratio between the rates changes over the whole range of observed values of the GCBS rate. A linear regression model with two parameters was fitted to the data (see Figure 32) but it failed to improve significantly the explanation of the variance of the standard rate. Clearly the data was also insufficient to check whether the relation between the compared rates changes significantly over time.

3.5 Comparison Between the Estimated Total Variable Costs and the Prices Charged by CNN and CTM to EPAC in January 1978

Using the cost formulae presented in section 3.3 and assuming standard rates 5.7% below the GCBS rates, the expected variable costs per metric tonne of grain were estimated, as of January 1978, for the routes US (Atlantic Coast) - Portugal and US (Gulf) - Portugal. The total costs were calculated for the range of speeds considered previously in Figure 31 and were found to be rather insensitive to the value of the speed (within the range of speeds considered previously, it was found that a 1 knot increase in the speed had the effect of reducing the total costs only by 0.5 - 0.8%, depending on the route and the size of the vessel).

Estimates of total variable costs for both Atlantic Coast and Gulf routes, based on an average speed of 14 knots, were compared with the prices
charged by the Portuguese shipping companies to EPAC. The comparison suggests that, in January 1978, the shipping companies' gross profit margin increased, for both routes, with the size of the shipments. This is illustrated in Figure 33 (diagram (i)) for the Atlantic Coast route. This discrepancy between the prices charged to EPAC and the estimates of the total variable costs incurred by CNN and CTM has to be judged cautiously, in view of the limited data on which the costing model was based. When submitted to the management of CNN the model was accepted as valid and the intention by CNN to extend the analysis to more data was declared.

Figure 33 - The variable shipping cost for the US(Atlantic Coast)-Portugal route (one port at each side) (January 1978)

(ii) Breakdown of the variable cost

- vessel cost $c_v$
- fuel cost $c_f$
- insurance cost $c_i$
- port charges $c_p$

- shipment size = 15 000 m.t.
- shipment size = 50 000 m.t.
Figure 33 shows, in diagram (ii), the relative importance of the several variable cost components. In spite of the comparatively low trip charter rates charged in January 1978, the vessel hiring cost was, by far, the most important one, followed by the fuel cost.

3.6 Changes in the Shipping Costs over Time

The shipping costs vary considerably over time, mainly as a result of changes in their two major components - vessel hiring cost and fuel cost.

The level of the trip charter rate is determined by a dynamic interaction between the world tramp shipping supply and demand factors (for a discussion on the price factors and mechanisms of the tramp shipping world market, see Sturmeys and Taylor ). Changes in those factors affect differently the rate for vessels of different sizes, leading to considerable changes in the structure of the vessel hiring cost.

The quarterly averages of the GCBS trip charter rate are plotted in Figure 34 since the beginning of its publication (1975) until 1980. In this figure the time period was extended back to 1973 by plotting a similar series - the time charter rate for dry bulk carriers published by Shipping Statistics and Economics.
During the period represented in the figure, considerable changes in the rates took place. Following a rapid expansion of the major Western economies between 1972 and the end of 1973, and a rapid increase in the grain world trade, the demand for sea transport increased substantially, bringing the freight rates to a record high in the middle of 1974. With the recession caused by the oil crisis and with the largely subsidised world shipbuilding industry producing vessels well above world requirements, the rates fell considerably after 1974. Particularly affected were the large bulk carriers. Their rates remained at extremely low levels, when compared with the rates paid for small vessels, reflecting a marked imbalance of the shipping supply/demand factors amongst the different vessel sizes. In the third quarter of 1975 the gross rate per day paid for vessels as large as 60,000 DWT was practically the same as the one paid for vessels of 15,000 DWT (about US$3000/day). Only in 1978 did the rates start recovering, reaching new record levels in 1980.

The second major shipping cost component — fuel cost — has suffered successive increases since 1973. Between this year and 1979 the fuel oil price rose at an average rate of around 25% per year (from about US$30/m.t. in 1973, to about US 120$/m.t. in 1979).

3.7 Use of the Shipping Cost Model

The shipping cost model proposed in this chapter will be used in the next chapter in the study of delivery-inventory policies for grains and other raw materials handled and stored at the Portuguese port terminals.
The cost model was based on a limited amount of data made available by EPAC and CNN or reported in the shipping press between May 1977 (when the contract between EPAC and the Portuguese shipping companies was agreed) and January 1978 (when the analysis was carried out). Clearly, for the purpose of defining the contract prices, the shipping companies will need cost estimates which are more accurate than the ones obtained earlier in this chapter. In fact, even a small error of evaluation of these costs can have serious consequences for the shipping companies. It is obviously desirable that the cost analysis should be extended using more reliable and abundant data and should be carried out by EPAC and the Portuguese shipping companies as a matter of routine. In particular it seems desirable to refine the method of estimating the relation between the standard rates and the published ones, so as to incorporate new information as it becomes available, in an adaptive way. The adaptive regression model proposed by Gilchrist is, in principle, well suited for that purpose.

However important the accuracy of the estimation procedure may be for the shipping companies, it is not critical for the purpose of planning the shipment of grains and other raw materials imported through common port terminals. In fact, as will be shown later, the 'optimal' delivery-inventory policies are extremely robust with regard to changes in the shipping cost structure.

In the development of the delivery-inventory model presented in the next chapter, the standard rate and the GCBS rate were assumed to be related according to the expression derived earlier in section 3.4,

Standard rate = (0.943) \times \text{GCBS rate}.

Adjustments in this relationship can obviously be incorporated in the model without any difficulty.
CHAPTER 6

PLANNING THE SHIPMENTS OF GRAINS AND OIL SEEDS
TO THE PORTUGUESE GRAIN TERMINALS

1. Introduction. The Overall Modelling Process

The present chapter deals with the development of a model (model 2, in Figure 6) for planning the shipment of grains and oilseeds to Portugal. These raw materials - maize, sorghum, wheat, soybean, peanut and sunflower - are brought by sea and handled at the grain terminals of the country's main ports (Lisbon and Oporto), where they share common silos and unloading facilities.

A simulation - optimization model was developed with the objective of defining, for each terminal, delivery-inventory policies that minimize the expected variable shipping and inventory costs per unit of time, for acceptable 'shortage levels' (these are measured in terms of both the proportion of demand not satisfied immediately and how frequently shortages occur). For each raw material, a delivery-inventory policy states

(i) the size of shipments and

(ii) the timing of their planned arrivals.

Setting the timing of the planned arrivals is equivalent, in terms of the model, to specifying a buffer stock for each raw material. This buffer stock is to cover the delays between planned and actual dates of arrival of vessels carrying that raw material and is defined as the average net stock 'in port' immediately prior to the arrival of shipments of that raw material. 'In port' includes the raw material in the silo and in the vessels, whether unloading, berthing or waiting to be unloaded; the net stock in port is obtained
at any time, subtracting the demands not satisfied (assumed to be backordered) to the actual stock in port.

At the core of the overall model is a 'multi-product' simulation model which estimates the expected costs associated with different policies that satisfy the specified shortage levels. These costs are then used as inputs to a direct search routine. Due to the stochastic nature of the delivery-inventory problem and to the characteristics of method proposed to solve it, the derived policy leads to an expected cost that is not necessarily the true minimum. However, for all practical purposes, the difference between them is negligible. For this reason, the derived policy will be referred to simply as optimal and the corresponding expected cost as the minimum one.

The model was derived considering, in turn, the operational characteristics of

(i) the proposed new terminal in Lisbon, as defined from a study commissioned by EPAC and carried out by Hidrotécnica Portuguesa 31 (November 1977), and

(ii) the new terminal in Oporto (which started operating at the beginning of 1979).

For Lisbon, the analysis was extended to cover the concurrent operation of the new terminal with the one currently in use - the Beato terminal.

Figure 35 gives a general outline of the overall formulation and modelling process. Starting from the formulation of the general objective
Figure 35 - The modelling process

General objective
Definition of the delivery-inventory policy that maximizes the efficiency of the system, specifying:
(i) shipment sizes
(ii) timing of planned arrivals

Initial basic assumptions
(i) No speculative storage
(ii) Static model
(iii) No coordination of planned arrivals of vessels carrying different raw materials

Initial formulation
Min. \( VC = f(Q_1, \ldots, Q_6, \theta_1, \ldots, \theta_6) \)
S.t.: \( (A_1)_{ij} = f(Q_1, \ldots, Q_6, \theta_1, \ldots, \theta_6) \leq 2 \% \)
\( (A_2)_{ij} = f(Q_1, \ldots, Q_6, \theta_1, \ldots, \theta_6) \leq 5 \% \)
where
\( VC \) : expected variable costs per year
\( i \) : index for raw materials (i.e., \( i = 1, \ldots, 6 \))
\( Q_i \) : size of shipments of raw material \( i \)
\( \theta_i \) : buffer stock of raw material \( i \)
\( (A_1)_{ij} \) : expected fraction of the demand of raw material \( i \) not satisfied immediately from stock
\( (A_2)_{ij} \) : expected fraction of replenishment cycles in which a shortage occurs

Auxiliary models
(i) Shipping and inventory costs
(ii) Distribution of vessel arrivals: actual vs. planned

Difficulties
(1) Complexity of relations → simulation
(2) Too many variables and constraints → infeasibility of optimization

Problem reformulation (new)
Impose constraints on 'shortage levels' in port rather than in silo, for raw materials \( i \):
\( (A_1)_{ij} = h(Q_i, \theta_i) \leq 2 \% \)
\( (A_2)_{ij} = h(Q_i, \theta_i) \leq 5 \% \)
where
\( (A_1)_{ij} \) : expected fraction of time during which the net stock in port is negative
\( (A_2)_{ij} \) : expected fraction of replenishment cycles in which a shortage occurs in port

Single-product simulation model
Objective: for any raw material \( i \), find the minimum buffer stock \( \theta_i \), ensuring, for each shipment size \( Q_i \), shortage levels in port not exceeding the acceptable limits:
\( \theta_i = r(Q_i) \)

Multi-product simulation model
Primary objective: define the relationship
\( VC = f(Q_1, \ldots, Q_6, Q_1, \ldots, Q_6) \)
\( = f(Q_1, \ldots, Q_6) \)
(constraints on shortage levels in port satisfied implicitly)

Multi-product simulation model - direct search routine
Objective: minimize \( VC = f(Q_1, \ldots, Q_6) \)

Analysis of the solutions and model extensions
(i) Test assumptions
(ii) Extend simulation-optimization model:
(a) Consider constraints on \( Q_i \) (i.e., \( i = 1, \ldots, 6 \))
(b) Return to initial formulation (i.e., consider constraints on shortage levels in silo rather than in port)
(c) Consider the staggering of arrivals of vessels carrying different raw materials ('Equal order interval method')
(box 1), namely the definition of optimal delivery-inventory policies, some basic assumptions, later justified, were made initially (box 2):

(i) for all the raw materials – purchased under fluctuating prices – the decisions on the size of the stocks carried in port are not affected by expectations concerning their price movements (in other words, the policies are derived assuming no 'speculative storage'); as shown later, the delivery-inventory problem can, under this assumption, be dealt with separately from the purchasing problem;

(ii) the problem is treated as a 'static' one, in that the demands for the different raw materials, the shipping cost structure and other model parameters are considered stationary when deriving a delivery-inventory policy;

(iii) the delivery-inventory policies do not involve any attempt to co-ordinate the planned arrivals of vessels carrying different raw materials (i.e., no attempt is made to avoid the scheduling of arrivals of different vessels for the same dates).

Furthermore, due to the characteristics of the problem under analysis, the demands were considered deterministic and, when not satisfied immediately from the silo, they were assumed to be fully backordered, at a rate limited by the total capacity of delivery of raw materials from the silo to the consumers.

Under these assumptions, the size of successive shipments and of the buffer stock of each raw material could be assumed to be constant over time (for each raw material i, they will be denoted by \( Q_i \) and \( BS_i \), respectively). The problem could then be formulated (box 4), using 12 variables (\( Q_i, BS_i, i=1, \ldots, 6 \)), to minimize the expected variable
shipping and inventory costs, subject to constraints related to the need to keep the shortages in silo within acceptable levels. The models discussed in the previous chapter (box 3) are required for the definition of costs and shortage levels.

Because of the complexity of the problem, an analytical formulation of the objective function and constraints was not possible. Simulation was the only way of expressing the relations between control variables, on the one hand, and costs and shortage levels, on the other. However, the results provided by a simulation model could not possibly be used to select the policy that, satisfying specified levels of shortage in silo, would lead to the minimum total cost. The reasons for this are

(i) the high number of control variables \((Q_i, BS_i, i=1, \ldots, 6)\);

(ii) the existence of constraints on the shortage levels (two for each raw material);

(iii) the inability to express analytically the dependence of each shortage level in silo on all the control variables; and

(iv) the difficulty of estimating accurately each of these relations via simulation (the estimation is based on the occurrence of 'rare events' - the shortages; therefore the accurate estimation of the shortage levels, for each set of values of \(Q_i, BS_i, (i=1, \ldots, 6)\), would require prohibitively long simulation runs).

These difficulties (box 5) led to the re-formulation of the problem (box 6): in the constraints, the shortage levels in silo were replaced by those in port (shortage in silo means silo empty; shortage in port means silo empty and no raw materials inside any vessel in port). These levels would coincide only if there was no time lag between the arrival
of a vessel and the start of unloading. As this time interval increases so will the difference between shortage in silo and port.

It may be argued that this procedure is artificial since what really matters is not the occurrence of shortages in port, but in silo, since it is the latter that affects customers directly. However, the management of each terminal have several options open to them. They can, for example, change the priority of a vessel in the queue, start to unload using a floating crane or prepare for the late arrival of a ship by leaving a berth free. All these are ways of reducing the difference between shortage in silo and in port. A further procedure is analysed in this chapter: after the buffer stocks are defined to ensure that given shortage levels in port will not be exceeded, they are 'corrected' to provide for extra protection against the time spent by vessels in port before unloading. The shortage levels in silo can in this way be transformed to those initially assumed for shortages in port.

For each raw material \( i \), the shortage levels in port are a function of the shipment size and of the buffer stock of that raw material alone \( Q_i, BS_i \). Using a simple single-product simulation model it was possible to derive, for each \( Q_i \) the minimum value of \( BS_i \) that satisfies the constraints on the shortage levels in port. In other words, under the new formulation it was possible to express the minimum buffer stock as a function of the corresponding shipment size (box 7).

The next step in the solution of the problem (box 8) was the formulation of a multi-product simulation model relating the total costs to the size of the shipments of the various raw materials. The
main simplifications introduced were:

(i) the number of variables was halved; once the sizes of shipments are defined, the buffer stocks are directly determined through the relation obtained from the single-product simulation model;

(ii) by relating the buffer stocks and size of shipments in this way, the constraints imposed on the shortage levels are thus implicitly satisfied (optimally, as far as the buffer stocks are concerned).

These simplifications were crucial for the next step of the process. The multi-product simulation model was used in conjunction with a direct search technique to find the solution that minimizes the total costs, for the specified shortage levels in port (box 9).

The optimal solutions were then analysed, the assumptions initially made were tested and the model was extended to take into account:

(i) constraints on shortages in silo rather than in port, as originally formulated;

(ii) constraints on the sizes of the shipments;

(iii) the possibility of staggering the planned arrivals of vessels carrying different raw materials whenever advantageous (as shown later, the proposed model is an extension of Page and Paul's model based on the 'Equal Interval Method' (box 10).
The layout of this chapter is closely related with the structure of the modelling process briefly described above. Section 2 presents some background information concerning the characteristics of the new terminals and the raw material imports. Section 3 focuses on the general structure of the delivery-inventory systems and presents the initial assumptions and formulation of the problem. Section 4 discusses the difficulties associated with setting constraints on the shortage levels in silo, which lead to the re-formulation of the problem. The single-product simulation model is then discussed in section 5 and the expression relating the minimum buffer stock of any raw material to the corresponding size of shipments is defined (for the acceptable values of the shortage levels in port). The multi-product simulation model is then presented in section 6 and, in section 7, it is combined with a hill climbing technique to produce an optimization model. The solutions are then analysed, in section 8, and two of the basic model assumptions (stationarity of the model, no speculative storage) are tested and validated. Section 9 discusses extensions of the model and shows that no benefits can be expected from staggering the planned replenishment cycles of different raw materials - hence shows the validity of the third basic assumption. The chapter is concluded with an overview of the model, in section 10. Possible refinements and adaptations of the model are discussed in this section and the question of expanding the terminals' capacities is also briefly considered.
2. **Background Information (*)**

2.1 **The New Terminal at Lisbon**

Currently, grains and oil seeds are unloaded in an inefficient manner at the Lisbon port, situated in the estuary of the Tagus. Limitations on the port handling capacity and storage facilities combine with a high uncertainty in vessel arrivals to determine low rates of unloading and high costs of handling operations.

The existing installations include one terminal owned by EPAC - the Beato terminal - and several small warehouses and silos along the Tagus estuary, rented to either EPAC or IAPO by the port authority or by private firms. These warehouses and small silos have quays that, due to the limited water depth, give access only to barges. Even the quay at the Beato terminal has a very limited water depth (8 metres). This implies that bulk carriers of reasonable size have to be partially unloaded onto barges, with floating cranes, before they can be moored to the Beato quay.

In view of the limitations of the existing facilities, EPAC decided to build a new terminal. Hidrotécnica Portuguesa carried out a study which led to the definition of the specifications of the new terminal (used, in 1978, as the basis for submission of preliminary tenders and the selection of a short list of contractors).

(*) The information presented in this section was obtained from interviews with the management of EPAC and from Hidrotécnica Portuguesa.
According to those specifications the grain terminal consists basically of

(i) a quay prepared to receive simultaneously two bulk carriers with capacities up to 80,000 DWT;

(ii) two identical suction units for unloading vessels
    (these units can be either allocated simultaneously to the same vessel or used separately, unloading two vessels at the same time);

(iii) a silo.

The total unloading-elevating nominal capacity is 2 x 1000 m.t. of 'heavy grain'/hour ('heavy grain' is assumed to have a density of 0.78 m.t./m³). For a raw material with density \( \nu \) (in m.t./m³) the unloading capacity is given by 2000. (\( \nu/0.78 \)) m.t./hour. Behind this relation is the assumption that the rate of volume unloaded per unit of time does not depend on the particular raw material unloaded (which is an adequate approximation to reality). Estimates of the densities of grains and oil seeds are presented in Table 20.

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>0.75</td>
</tr>
<tr>
<td>Sorghum</td>
<td>0.63</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.78</td>
</tr>
<tr>
<td>Soybean</td>
<td>0.75</td>
</tr>
<tr>
<td>Peanut</td>
<td>0.33</td>
</tr>
<tr>
<td>Sunflower</td>
<td>0.55</td>
</tr>
</tbody>
</table>

The effective annual unloading capacity can be estimated (following Hidroética Portuguesa 31) as

\[
0.9 \times (2 \times 1000 \text{ m.t./hour} \times 12.5 \text{ hours/day} \times 232 \text{ work days/year})
\]

= 5 220 000 m.t./year (heavy grain).
(the coefficient 0.9 allows for breakdowns of equipment, set up
times, etc.; 12.5 hours is considered to be the effective amount
of work per day that can be obtained from 2 shifts; the number of
work days/year is calculated by excluding Saturdays, Sundays and
holidays as well as assuming an average of 16 days of interruptions
of the operations, due to bad weather.

The silo consists of nearly 100 cells with a total capacity of
175 000 m³. Out of this total, 700 m³ comprise cells specially
equipped to store oil seed meals, which show a tendency to compact
under pressure and, consequently, to block up the outflow devices.
All the other cells can store any of the raw materials handled at
the terminal. Because of this distinction between the storage facilities,
the problem was decomposed, with the analysis concentrating on the grains
and oil seeds, as discussed later.

After arrival, vessels wait in the port until one 'unloading
station' becomes free. Each vessel can be berthed only after the
vessel previously unloading at that station has finished the
deberthing operation. It is estimated that each of the berthing and
deberthing operations takes 0.25 working days.

Under normal circumstances vessels start their unloading
operation according to their order of arrival (only in exceptional
situations — shortages in silo — will the management be prepared to
change this rule).

Raw materials can be either unloaded to the silo elevators or
or, whenever appropriate, transferred directly to barges (for delivery
to firms located along the banks of the Tagus) or small coasters (serving the islands of Madeira and Azores). From the silo, the raw materials are loaded onto trucks mainly, but also onto wagons, barges, and coasters for onward dispatch to the consumers.

According to the conclusions of the study by Hidrotécnica Portuguesa, the Beato terminal should be kept only as a 'strategical capacity reserve'. It was argued that, due to the existing severe water depth restriction, the use of this terminal in the day-to-day unloading and storage operations would not be cost effective. In accordance with this conclusion, the analysis of the delivery-inventory policies concerning the raw materials imported through Lisbon started by considering that all the operations were concentrated in the new terminal alone. However, in view of the solutions obtained, this modus operandi was questioned and the possibility of operating both terminals in parallel was explored.

2.2 The New Terminal at Oporto

Prior to the middle of 1979, warehouses rented from the port authority were used to store the imported raw materials. Since then a new silo has started operating and the warehouses were returned to the port authority for other uses.

The new terminal - consisting of a quay for two vessels, two unloading units and a silo - has an operational structure which is identical to the one of the new terminal at Lisbon. The only difference between them concerns their size.
Figure 36 shows the basic operating characteristics of both new terminals.

2.3 Raw Material Imports

According to EPAC's and IAPO's plans available at the time the analysis was carried out (1978), the imports of the following raw materials will be, in future, handled at the grain terminals of Lisbon and Oporto:
(i) grains: maize, sorghum, wheat

(ii) oilseeds: soybean, peanut, sunflower

(iii) oilseed meals: soybean, peanut, sunflower.

Table 21 presents EPAC and IAPO forecasts for the annual imports of each of these categories of raw materials, through each of the two ports, in 1980. According to those Government Agencies (see Hidrotécnica Portuguesa 31), the volume of imports of these raw materials is expected to stabilize after 1980.

<table>
<thead>
<tr>
<th>Raw materials</th>
<th>Lisbon</th>
<th>Oporto</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed grains (maize + sorghum)</td>
<td>1930</td>
<td>480</td>
<td>2410</td>
</tr>
<tr>
<td>Wheat</td>
<td>350</td>
<td>150</td>
<td>500</td>
</tr>
<tr>
<td>Oil seeds</td>
<td>360</td>
<td>150</td>
<td>510</td>
</tr>
<tr>
<td>(Soybean)</td>
<td>(162)</td>
<td>(68)</td>
<td>(230)</td>
</tr>
<tr>
<td>(Peanut)</td>
<td>(90)</td>
<td>(37)</td>
<td>(127)</td>
</tr>
<tr>
<td>(Sunflower)</td>
<td>(108)</td>
<td>(45)</td>
<td>(153)</td>
</tr>
<tr>
<td>Oil seed meals (soy + pean + sunf)</td>
<td>240</td>
<td>100</td>
<td>340</td>
</tr>
<tr>
<td>Total</td>
<td>2880</td>
<td>880</td>
<td>3760</td>
</tr>
</tbody>
</table>

Note: the figures in brackets were estimated using the relative proportions of oilseed imports published in FAO Trade Yearbook 1977


The breakdown of imports of feed grains and oil seed meals by individual products will vary according to their relative import prices, as discussed earlier in chapter 4. On the other hand, the patterns of consumption of wheat and oil seeds are in practice unaffected by price fluctuations (wheat, used as human food, has no direct substitutes; the oil seeds are used by the oil crushing industry
in proportions which are quite rigidly set by the oil brands marketed in the country).

As pointed out before, the oilseed meals require special storage cells, due to their tendency to compact. Brought by sea in small shipments the oil seed meals are delivered to the customers almost as soon as they become available in the silos. The small storage capacity available for them in the new terminals is essentially there to provide the silo operators with some flexibility during the unloading/delivery operations. The only major interaction between the imports of oil seed meals and the imports of other raw materials arises from the use of common unloading facilities. As shown later, this interaction was taken into account in the study of the delivery-inventory policies for grains and oil seeds by considering that only part of the total unloading capacity is available for unloading these raw materials.

The grains and oilseeds are sold to industry (animal feeds, flour, oils) either under a regimen of monthly quotas imposed to each 'consumer' (grains) or through deliveries that are agreed in advance between IAPO and the industrialists (oil seeds). For given monthly quotas or 'agreed deliveries', the weekly or even daily demand is fairly regular. The main reasons for this regularity are:

(i) the stability of internal prices;

(ii) the fact that, since the raw materials are paid for by the consumers at the moment of delivery from the silos, they tend to use the port storage capacity (free of charge) as much as they can, not carrying significant stocks of their own;
(iii) the high number of consumers, most of which place large numbers of small orders (usually transported by truck); and
(iv) the limited transport capacity to deliver the raw materials to the consumers.

3. The Structure of the Delivery-Inventory Systems. Initial Problem Formulation

3.1 Basic Assumptions

The problem of planning the deliveries of grains and other raw materials handled at each of the new port terminals is very complex when stated in its most general form. As pointed out earlier in Chapter 3, section 4.3, the interactions between the delivery-inventory decisions and the purchasing decisions can be a major source of complexity. The interactions between the delivery-inventory decisions concerning the different raw materials handled and stored in the same port terminals aggravate the difficulty of the problem.

Clearly some simplification was required before the problem could be analysed and solved. As pointed out earlier in Chapter 3, section 5.3, progress towards finding a solution to the problem could only be achieved by

(i) making assumptions about the mode of operation of the system (with the objective of simplifying the analysis, without introducing a significant suboptimization);
(ii) carrying out the analysis of the problem and deriving solutions;
(iii) testing the validity of the assumptions by investigating the solutions obtained.
In the study of the delivery-inventory policies that follows, three basic assumptions, later justified or relaxed, were made. Firstly, it was assumed that, for all raw materials, decisions on the size and the timing of the shipments (and therefore on the stocks carried in port) are not affected by expectations concerning the raw materials' price movements.

Behind this assumption was the belief that, due to the limitations in the port storage capacity, the potential reduction in the purchasing costs resulting from speculative storage (i.e. from increasing stocks in anticipation of expected price rises, to be consumed later when prices are expected to stabilize or fall) would be negated by the increase in the total delivery-inventory costs in relation to their minimal value.

Following this assumption, the overall delivery-inventory-purchasing problem was partitioned into two sub-problems:

(i) the definition of a delivery-inventory policy: specifying the size and timing of the shipments of all raw materials, with the objective of minimizing the total delivery and inventory costs (this is the sub-problem dealt with in the current chapter);

(ii) the definition of purchasing (or buying) policies: once the size and the timing of shipments are specified, the purchasing policies state when should each of those shipments be bought (this sub-problem will be analysed later in Chapter 6).

Secondly, the problem was assumed to be static in that the demands for the different raw materials, the shipping cost structure and other model parameters were considered stationary when deriving the minimum cost
delivery-inventory policies. Since the system is analysed in a 'steady state', it can then be assumed that, for each raw material \( i \), the control variables \( Q_i \) and \( BS_i \) are invariant over time. In making such assumptions, the following points were taken into account:

(i) The rates of consumption of wheat and oilseeds are kept at nearly constant levels for long periods of time (6 months or more) and the adjustments made in those rates are usually moderate (therefore unlikely to cause considerable changes in the delivery-inventory policies).

(ii) As discussed earlier in chapter 4, it is desirable to change the rates of consumption of maize and sorghum according to their relative prices. However, in view of the difficulties associated with the process of controlling and, above all, redefining each consumer's individual import quotas of maize and sorghum, it is not feasible to change the rates of consumption more than a few times a year - quarterly adjustments are, according to the management of EPAC and IACA, about the tolerable limit. Clearly the static model does not provide a direct answer on how to adapt the system from one optimal solution to the next. An answer to this problem will be discussed after the solutions are analysed.

(iii) The shipping costs structure changes over time, mainly due to changes in the trip charter rate (analysed in the previous chapter) and to fuel price increases. The inventory costs, which are a function of the raw materials' prices, will change with time as well. However, one can anticipate that the effect of those changes on the optimal solutions will be small. In fact, problems such as the one under analysis have
optimal solutions that, in general, are robust to changes in the cost parameters.

The third assumption made initially was that, due to the large uncertainty associated with the arrival of each shipment, no significant improvement in the usage of the storage and unloading facilities (and therefore on the efficiency of the system) could be achieved by staggering the planned arrivals of shipments of different raw materials.

3.2 Operational Characteristics of the Systems

3.2.1 Interruptions in the Operation of the Terminals

The operation of the grain terminals is frequently interrupted during week-ends and holidays or due to bad weather and dredging operations. However, in order to simplify the analysis of the delivery-inventory systems, these were treated as continuous.

While the operations at the terminals take place in working days only, costs such as those associated with the stay of vessels in port are incurred whether the terminals are operating or not. When analysing the system as continuous, the time related operational parameters of the terminals and the time related costs must be assessed in a common basis: either days (365 per year) or work days. In this research, the former basis was adopted. As a consequence, some of the terminal's operational parameters had to be adjusted, in order to take into
account the effects of representing the real systems (with their operations frequently interrupted) by equivalent continuous operation systems (operating 365 days/year)

(i) unloading capacity: the total daily unloading capacity at each terminal was obtained by dividing the annual capacity by 365 days/year. The unloading facilities are shared between oil seed meals, on the one hand, and grains and oil seeds, on the other. Therefore the unloading operations of the latter are interrupted frequently, while oil seed meals are being unloaded. These interruptions were taken into account by reducing the total unloading capacity by a factor reflecting the proportion of time spent unloading oil seed meals; the capacity available for grains and oil seeds at each terminal was estimated using the relation

\[ U = \frac{\text{flow of grains and oil seeds (in volume)}}{\text{total flow of imported raw materials (in volume)}} \times \text{unloading capacity} \]

Underlying this estimate is the assumption that the time that

(*) Clearly the interruptions in the operation of the terminals, in particular the ones due to bad weather, are not evenly distributed over the year; this fact should be taken into account by adjusting the parameters differently from quarter to quarter, say. However, in this study an annual average correction factor was used.
unloading facilities are allocated to each group of raw materials is proportional to the corresponding volume that is unloaded, which is not necessarily true. However, it should be noted that the accuracy of the estimate of the time spent unloading oil seed meals is not critical in the analysis of the delivery-inventory system of grains and oil seeds. In fact, since the flow of oil seed meals handled at the terminals is small relative to the total flow of imported raw materials, the value of U is not greatly affected by changes in the estimate of the time that the unloading berths are allocated to oil seed meals. Furthermore, as shown later, the solutions of the delivery-inventory model are rather insensitive to changes in the values of the unloading capacity U.

(ii) Berthing and deberthing times: these times, estimated to be 0.25 working days for all terminals, were converted, in the continuous operation systems, into

\[ 0.25 \times \left( \frac{365 \text{ days/year}}{\text{no. of working days/year}} \right) \text{ days.} \]

3.2.2 Size of Shipments

For each raw material \( i \) \( (i=1, \ldots, 6) \) the size of successive shipments was assumed to be constant \( (Q_i) \) - this is a natural consequence of representing the system through a static model.
The shipment sizes were assumed to be related to the sizes of the vessels (their deadweight tonnage) according to the following expression, obtained from the management of CNN:

\[ Q_i = 0.90 \cdot (v_i / 0.78) \cdot DWT_i \]

where, as before, \( v_i \) is the density of raw material \( i \) (in m.t./m\(^3\)) and \( Q_i \) and \( DWT_i \) are expressed in 1000 m.t..

### 3.2.3 Uncertainty of Arrivals of Vessels

Following the results of the analysis presented in the previous chapter, it was assumed that

(i) if a vessel is planned to arrive at a date \( t_o \), the actual date of arrival follows a normal distribution with mean \( t_o \) and known (estimated) standard deviation;

(ii) the actual dates of arrival of different shipments are independently distributed.

These assumptions were tested for maize and sorghum shipments only. However, since the shipping operation is essentially the same for all raw materials, any differences in the pattern of uncertainty associated with the shipment of the various raw materials can be expected to be small. For this reason, and for the purpose of analysing the general characteristics of the delivery-inventory system, the assumptions were extended to all raw materials and the standard deviations were all assumed to be equal to 9 days.
3.2.4 Queue Discipline

On arrival at the terminals, vessels that cannot start the berthing operation immediately join a queue assumed to have FIFS (first in first served) discipline. Although this discipline will not generally lead to the most economical operating policy, only exceptionally will the management of the grain terminals consider changing it, in view of the problems that might arise as a consequence, with the vessels' charterers or owners. A departure from the FIFS discipline will only be decided in extreme situations, for example, in the case of an immediately foreseeable shortage in silo of a raw material currently available in a vessel 'far behind' in the queue.

3.2.5 Storage Capacity

From the total volume available to store grains and oil seeds at each terminal 10% were assumed to be lost due to operational requirements (e.g. cells out of service for maintenance and repairs of equipment, impossibility of filling up all the available cells without mixing the different raw materials). The 'effective' silo capacities were thus estimated to be

(i) at the Oporto new terminal: \( 0.90 \times 102 000 = 91 800 \text{ m}^3 \);

(ii) at Lisbon new terminal: \( 0.90 \times 162 000 = 145 800 \text{ m}^3 \).

For each terminal it was assumed that the silo can store at any moment in time, any volumes of the several raw materials \( V_i \) that satisfy the constraint

\[
\sum_{i=1}^{\delta} V_i \leq V \quad \text{(effective capacity of the silo)}
\]
3.2.6 Unloading Operation

In each terminal, each of the two unloading units (of equal capacity) can be allocated freely to either of the two unloading quays. Two alternative allocation rules were considered:

(i) Rule 'O' - if there are two vessels berthed at the unloading quays, one unit is allocated to each of them; if there is only one vessel, both unloading units will be allocated to that vessel;

(ii) Rule 'l' - the two unloading units are always allocated simultaneously to the same vessel; when two vessels are berthed at the same time, all the unloading capacity is allocated to the vessel which arrived first.

The unloading rate at each unloading station (expressed in units of volume per unit of time) will depend on

(i) whether the silo is full or not (if the silo is full, the volume inflow equals the volume outflow; otherwise the total unloading rate equals the total unloading capacity when at least one vessel is berthed);

(ii) the state of the unloading stations (none, one or two vessels berthed).

Depending on the state of the silo and unloading stations and on the allocation rule adopted, the unloading rates (expressed in 1000 m.t./day) are, according to previous notation, given by:

(i) silo not full/rule 'O':

(a) one vessel berthed (with raw material i)
\[ U_1 = \left( \frac{v_i}{0.78} \right) \cdot U \quad (U \text{ is the total unloading capacity, heavy grain, 1000 m.t./day}) \]

\[ U_2 = 0 \]

(b) two vessels berthed (with raw materials i, j):

\[ U_1 = \left( \frac{v_i}{0.78} \right) \cdot \frac{U}{2} \]

\[ U_2 = \left( \frac{v_j}{0.78} \right) \cdot \frac{U}{2} \]

(ii) Silo not full/rule 'i':

(a) one vessel berthed (with raw material i):

\[ U_1 = \left( \frac{v_i}{0.78} \right) \cdot U \]

\[ U_2 = 0 \]

(b) two vessels berthed (the one containing raw material i having arrived first):

\[ U_1 = \left( \frac{v_i}{0.78} \right) \cdot U \]

\[ U_2 = 0 \]

(iii) silo full/rule 'O':

(a) one vessel berthed (with raw material i):

\[ U_1 = v_i \cdot VF \quad (\text{where VF is the volume flow leaving the silo, in 1000 m}^3/\text{day}) \]

\[ U_2 = 0 \]

(b) two vessels berthed (with raw materials i, j):

\[ U_1 = \frac{v_i}{2} \cdot VF \]

\[ U_2 = \frac{v_j}{2} \cdot VF \]
(iv) silo full/rule '1' :

(a) one vessel berthed (with raw material i) :

\[ U_1 = v_1 \cdot VF \]
\[ U_2 = 0 \]

(b) two vessels berthed (the one containing raw material i having arrived first) :

\[ U_1 = v_i \cdot VF \]
\[ U_2 = 0 \]

3.2.7 Demands

In view of the way in which the raw materials are distributed to the consumers (under monthly quotas or 'agreed deliveries' and regularly over time), all the demands were assumed to be deterministic and constant. The rates of demand, expressed in 1000 m.t./day, will be denoted by \( d_i \) (i=1, ...,6).

Furthermore, it was assumed that demands not satisfied immediately from the silos are fully backordered. The speed at which the back-ordering process - or process of 'recovery from shortage' - takes place, is limited by the available capacity for delivering the raw materials from the silo to the consumers.

Normally, when there are raw materials in the silo, the total volume flow from the silo to the consumers is

\[ NVF = \sum_i \left( d_i / v_i \right) \quad (1000 \text{ m}^3/\text{day}). \]
According to an estimate provided by the management of EPAC, with the available fleet of trucks, wagons and barges, the capacity of delivery from the silo to the customers can be stretched up to about 15% over the normal volume flow (NVF). The extra capacity
\[
e = 0.15 \cdot \sum \left( \frac{d_i}{\nu_i} \right) \quad (1000 \text{ m}^3/\text{day})
\]
can be used to recover from shortages, whenever necessary.

The process of recovery from shortage - as assumed in the delivery-inventory model - can be described considering first the simplest situation, in which only one raw material (say, raw material 1) undergoes a shortage, with normal demands \(d_i\) for all the other raw materials. As soon as raw material 1 is unloaded and becomes available in the silo, the shortage will be recovered at the rate
\[
r_{r1} = e \cdot v_1 \quad (1000 \text{ m.t./day})
\]
This delivery of backorders is achieved in addition to the normal demand \(d_1\), as illustrated in Figure 37.

This figure shows, over a period of time, the following stocks of raw material 1:

(i) stock in port (SP): amount stored in silo or in any vessel unloading, berthing or waiting to be unloaded;

(ii) net stock in port (NSP): (stock in port) - (backorders still not met);

(iii) stock in silo (SS): amount stored in silo;

(iv) net stock in silo (NSS): (stock in silo) - (backorders still not met).
In the case illustrated in the figure, the process of recovery from shortage is assumed to take place at a constant rate ($r_{r_1}$). However, this would not be the case had a shortage of another raw material occurred between $t_2$ and $t_4$. In fact, when there are raw materials undergoing a shortage (raw materials $k$, say) their (unused)
volume rate of delivery $\sum_{k} (d_{k}/v_{k})$ is free to be used
in the recovery of shortages and should therefore be added on to
$e = 0.15 \cdot \sum_{i} (d_{i}/v_{i})$.

When several raw materials undergo processes of recovery from
shortage at the same time (raw materials $i$, say) the individual
rates of recovery are assumed to be proportional to the raw materials' normal demands $d_{i}$. Therefore the rate of recovery of a raw material $i$ can be expressed as

$$rr_{i} = \left[ e + \sum_{k} (d_{k}/v_{k}) \right] \cdot \frac{v_{i}}{\sum_{i} v_{i}} \cdot \frac{d_{i}}{d_{i}}.$$  

This expression needs to be corrected when, with no stock of raw material $i$ inside the silo, it leads to a rate of delivery from the silo to the customers larger than the rate of unloading from a vessel to the silo, i.e., when

$$d_{i} + rr_{i} > u_{i}.$$  

In this case $rr_{i}$ is corrected to $rr'_{i}$ satisfying

$$d_{i} + rr'_{i} = u_{i}.$$  

The difference $rr_{i} - rr'_{i}$ is then reallocated to increase the rate of recovery of other raw materials (if there are any undergoing recovery from shortage).

3.3 The Variable Costs Associated with the Operation of the Systems

3.3.1 General

Considering still the situation in which the grains and oil seeds are unloaded and stored only at the new terminals, the delivery-inventory policy for each terminal is entirely specified, within the framework of the assumptions presented in previous sections, by
(i) the size of shipments $Q_i$ (i=1, ..., 6) and
(ii) the size of the buffer stocks $BS_i$ (i=1, ..., 6).

Once the 'uncontrollable' parameters of each delivery-inventory system are specified (e.g. demands(*) , unloading and storage capacities, etc.), the total operating costs per unit of time (per year, say) provide a measure of the efficiency of the system. Clearly, for the purpose of comparing alternative policies, only the components of the total annual cost that depend on the control variables $Q_i$, $BS_i$ (i=1, ..., 6) are worth considering.

For example, the costs of the unloading and dispatch operations (from the vessels to the silo and from the silo to the customers, respectively) were excluded from the analysis. In fact since they were assumed to be proportional to the volume of the raw materials handled at each terminal, their annual values are not affected by the policy that is adopted, depending only on the values of the demands (considered 'given').

The variable annual costs involved in the operation will be analysed considering separately the shipping, inventory and shortage costs.

3.3.2 **Shipping Costs**

The variable shipping costs were derived from the formulae presented in the previous chapter, with some adaptations, namely:

(i) the average time spent by vessels in the Portuguese ports previously was assumed to be given by $Q_i/UR$, where $UR$ was

(*) in this chapter, all demands are considered given or imposed externally to the delivery-inventory model; in the next chapter, the question of how to set the demands for maize and sorghum (the only substitutable raw materials) will be analysed.
the nominal unloading rate specified in the contract; clearly that time will depend on the delivery-inventory policy adopted and therefore, is treated in the analysis that follows as a function of \( Q_i \) and \( BS_i \) (\( i=1, \ldots, 6 \)) and of the system parameters;

(ii) the insurance costs were excluded from the shipping costs; the reason behind this lies in the assumption made (and later verified) that all vessels are hired for less than 60 days. In fact, under this assumption, the insurance cost per vessel transporting raw material \( i \) is, according to the expression presented in Chapter 5, section 3.3.4,

\[
(C_i)_i = \frac{60}{365} \cdot IR \cdot DWT_i
\]

or, since \( Q_i = 0.90 \cdot (\nu_i/0.78) \cdot DWT_i \),

\[
(C_i)_i = \frac{K}{365} \cdot \frac{Q_i}{\nu_i} \quad (\text{with } K = \frac{60 \cdot IR \cdot 0.78}{0.90} = \text{constant});
\]

the total annual insurance cost becomes

\[
\sum [\frac{(C_i)_i \cdot d_i \cdot 365}{Q_i}] = K \cdot \sum \left( \frac{d_i}{\nu_i} \right);
\]

i.e. this annual cost is invariant with respect to the delivery-inventory policy that is adopted.

If \( CSH_i \) (US$) is used to denote the expected variable cost (vessel hiring, fuel and port charges) incurred for each shipment \( Q_i \), the expected variable shipping costs per year are given by
\[ VSC = \sum_{i} \left[ CSH_i \cdot \left( \frac{d_i}{Q_i} \cdot 365 \right) \right] \] (US$)

where, as before, \( d_i \) and \( Q_i \) are expressed in 1000 m.t./day and 1000 m.t. respectively.

3.3.3 Inventory Costs

For the existing storage facilities at each grain terminal the only significant variable inventory costs incurred are the holding costs (cost of the capital tied up in raw materials).

The raw materials, purchased on a fob basis, are paid for when they are delivered (loaded) at the exporting ports. Clearly a holding cost is incurred during the trip between the exporting and the unloading port. However, it is not part of the variable costs since it remains unchanged whatever delivery-inventory policy is adopted. The variable holding cost can therefore be calculated on the basis of the stocks held in Portuguese ports (i.e. from the moment the raw materials arrive at each terminal until they are finally delivered to the customers).

Using the notation

\[ p_i : \text{average unit price of raw material } i \] (US$/1000 m.t.)

\[ (H_{SP})_i : \text{average stock in port of raw material } i \] (1000 m.t.)

\[ \alpha : \text{rate of interest on capital} \] (%/year)

the expected variable inventory cost per year is given by

\[ VIC = \frac{\alpha}{100} \cdot \sum_{i} \left[ P_i \cdot (H_{SP})_i \right] \] (US$)
3.3.4 Shortage Costs: Difficulties in their Evaluation

For each given delivery-inventory policy, the management of each terminal will face, from time to time, the prospect of a shortage in silo. In such a situation, the management can resort to a number of exceptional measures aimed at avoiding or at least reducing the extent of the shortages. These measures will include altering the discipline of the queue of vessels waiting to be unloaded, leaving a berth free to receive a vessel which is expected to arrive late, increasing the deliveries of one feed-grain to customers when the other is out of stock or attempting to buy 'afloats'.

Ideally, the costs associated either with the effects of the shortages or with the actions undertaken with the aim of avoiding them (or reducing their extent) should be included in the total variable costs, as part of the overall measure of the efficiency of the system under analysis. However, this procedure could not be pursued since reliable estimates of those costs could not be obtained. The following points illustrate some of the basic difficulties associated with their estimation:

(i) the effect of each shortage on the industrial production of the final users is extremely difficult to model (that effect will depend on the stocks held by the industrialists at the time the shortage occurs and on the way these stocks are distributed among them);

(ii) it is difficult to estimate both the cost and the effectiveness of some of the 'exceptional measures' that can be undertaken by management when facing the prospect of a shortage (a good example is the attempt of buying afloats: how much can actually be bought? To be delivered when (i.e. with what effectiveness)? At what price?).
(iii) there are intangible factors associated with the shortages (e.g. the image of the government agency in question) which cannot be ignored but which are extremely difficult to convert into monetary terms.

3.4 Initial Problem Formulation

In the absence of reliable estimates of the shortage costs the problem was initially formulated as a minimisation of all the other expected annual variable costs (these can be called simply the expected variable costs, VC) while keeping the shortages in silo at or below specified levels.

It should be observed that the minimization of the expected variable costs VC is, for all practical purposes, consistent with the overall objective of minimizing the country's foreign currency expenditure (FCE) involved in the import operations. In fact, with the exception of the port charges at the Portuguese ports, all costs included in VC are equivalent to an expenditure of foreign currency:

(i) Shipping costs:

(a) vessel hiring cost: this cost was evaluated on the basis of the open market freight rates; when a foreign vessel is hired, the hiring cost is paid directly in foreign currency; for reasons discussed in chapter 5, section 3.1, when a Portuguese flat vessel is hired, an identical 'opportunity FCE' is incurred - equivalent to the foreign currency that would be earned if the vessel was hired to a foreign company, in the open market;

(b) fuel cost: this cost is met directly in foreign currency, when vessels are refuelled abroad; regarding the Portuguese marine fuel as exchangeable with that produced abroad (their prices are nearly
identical), the 'opportunity FCE' argument used above is also applicable to the case of vessels refuelled at Portuguese ports;

(c) port charges: clearly only the charges paid at foreign ports involve a direct FCE;

(ii) inventory costs: the costs included in VC were interest charges on the capital tied up in acquired raw materials; since these are bought with foreign currency, the interest charges must be regarded as a FCE, as well.

By including the charges at the Portuguese ports in VC, the overall objective of minimizing the FCE involved in the import operations is, in principle, overridden. According to this objective, only the indirect FCE component of those charges, if it exists at all, should be considered. However, the value of the charges is so small when compared with the other costs that their potential effect on the delivery-inventory policies is clearly negligible.

The first step involved in setting constrains on the shortages in silo for each raw material is the selection of an appropriate measure (or combination of measures) for the shortages. Peterson and Silver 48, after considering a number of different plausible measures of service (which are equivalent to measures of shortage) recognize (p.212) that '(...) there are no hard and fast rules for selecting the appropriate approach and/or measure of service. Which to use depends upon the environment of the particular company under consideration as well as management's attitude (...)'. Clearly the difficulty in defining sound rules for the selection of those measures is an immediate consequence of the difficulty in specifying the structure of the shortage costs in the first place.
In order to take into account the effects of both the total duration and the frequency of the shortages, the following measures were considered, for each raw material $i$:

(i) $(\beta'_s)_i$: the expected fraction of the demand that is not satisfied immediately from silo (i.e. that is backordered);

(ii) $(\beta'_w)_i$: the expected fraction of replenishment cycles in which a shortage in silo (i.e. a period in which the demand is not satisfied immediately) occurs.

Constraints fixing the maximum levels of both these measures of shortage were considered in the initial problem formulation, for each raw material $i$:

$$(\beta'_s)_i \leq B'_i$$

$$(\beta'_w)_i \leq B''_i$$

One point should be noted about the meaning of these constraints. It was mentioned earlier that the management of the government agencies can take exceptional action when facing the prospect of a shortage in silo. This exceptional action was not explicitly considered in the modelling of the delivery-inventory systems. Therefore, the inclusion of the constraints in the model represents an attempt at limiting the extent to which situations of potential shortage can occur (i.e. situations in which, depending on the judgement of the management, either exceptional measures are taken or the shortages will actually occur, as predicted by the model).

The maximum shortage levels adopted throughout the analysis were, for all raw materials,

$B'_i = 1\%$ (i.e. 3.65 days of consumption/year)

$B''_i = 5\%$ (i.e. 5 shortages/100 shipments)
Both measures of shortage and their maximum levels attempt to reflect the views expressed by EPAC managers in the course of interviews.

4. Difficulties Arising from the Initial Formulation: Replacement of the Constraints on Shortages in Silo by Constraints on Shortages in Port

4.1 Infeasibility of an Analytical Approach to the Solution of the Problem

Following the formulation presented in the previous section, the problem can be stated, using previous notation, as follows:

Minimize: \( VC = VIC + VSC \)
\[
= \frac{a}{100} \cdot \sum_i \left( p_i \cdot (u_{SP})_i \right) + \sum_i \left[ CSH_i \cdot \left( \frac{d_i \cdot 365}{Q_i} \right) \right]
\]

Subject to:
\[
\begin{align*}
(\beta^s)_i & \leq B^s \\
(\beta^-)_i & \leq B^- \quad \text{(for } i=1, \ldots, 6) \end{align*}
\]

A prerequisite to derive a solution to the problem by mathematical analysis (numerical analysis included) is the definition of the objective function and constraints explicitly in terms of the controllable and uncontrollable variables.

Clearly the objective function will satisfy this condition only if both the average stock in port, \((u_{SP})_i\), and the expected cost associated with each shipment, \(CSH_i\), can, for all raw materials, be expressed as functions of the problem variables and parameters.

Most of the time the stock in port coincides with the net stock in port. As illustrated earlier in Figure 37, only during periods of shortage in silo or the subsequent periods of recovery from shortage
will those stocks be different. Given the very small fraction of
time each raw material will be in such situations, the average stock
in port is, for all practical purposes indistinguishable from the
average net stock in port. The latter, in turn, is obtained, for
each raw material, by adding the average working stock \( Q_i / 2 \) to
the buffer stock (defined, for each raw material, as the average
net stock in port immediately prior to the arrival of shipments of
that raw material). Therefore, the average stock in port can be
expressed by

\[
(\mu_{\text{SP}})_i = (\mu_{\text{NSP}})_i = BS_i + Q_i / 2.
\]

Unlike \((\mu_{\text{SP}})_i\), the expected cost of each shipment \(CSH_i\) cannot
be expressed explicitly in terms of the controllable and uncontrollable
variables. As pointed out before, \(CSH_i\) is a function of the average
time \(TUP_i\) each vessel carrying raw material i spends in the port
of unloading, queueing, berthing, berthed and waiting to start
unloading, unloading and deberthing. \(TUP_i\) can be regarded as the
expected queueing-plus-service time in a queueing system, which,
in view of its extreme complexity, is not amenable to analytical
treatment by queueing theory. A number of factors contribute to its
complexity, namely:

(i) the arrivals of the vessels are not Poisson (it is
obvious that the memoriless nature of the Poisson
phenomena is not compatible with the planned nature
of the arrivals);

(ii) vessels carrying different raw materials can be regarded
as 'clients' with different service characteristics;
(iii) the service times are not exponential; in fact the
time spent by each vessel of an unloading station
(and therefore the service time) depends on:
(a) the state of the other station (occupied/
non-occupied) and
(b) the state of the silo (not full/full),
as well as on the unloading rate.

The impossibility of expressing $\mathbf{TP}_i$ in terms of the problem
variables rules out an explicit representation of the objective
function. The same problem occurs in the attempt at representing
the constraints. In fact it is clear that, for each raw material, once
the buffer stock is specified, the shortage levels in silo will depend
on the time that vessels carrying that raw material spend in port
before starting the unloading operation. This time is a function of
all the control variables $BS_i$ and $Q_i$ ($i=1, \ldots, 6$) (since any of these
affects the state of the silo which, in turn, influences the size of
the queue). Again this function cannot be defined explicitly in
view of the complexity of the queuing system. Another factor that causes
problems in representing the constraints analytically is that of crossed
deliveries (this will be discussed later).

The inability to define the model relations for an analytical
approach leaves simulation as the only practical alternative to
explore the problem.
4.2 Simulation Approach: Difficulties in the Selection of the Best Policy, Under the Initial Formulation

Essentially, from a simulation model one can derive, for given values of the uncontrollable variables, a correspondence between sets of values of the control variables \((Q_i, BS_i, i=1, \ldots, 6)\) and the relevant measures of the system's performance (expected variable costs and shortage levels). Clearly, any attempt to select the best policy has to be made externally to the simulation model (this model only evaluates alternative policies).

Two basic sources of difficulty determine the impracticality of selecting the best policy, under the problem formulation considered so far, on the basis of the results provided by a simulation model. The first one is the high number of control variables involved: \(Q_i, BS_i (i=1, \ldots, 6)\). Obviously, as the number of control variables increases so will increase the number of alternative policies that will have to be evaluated in the search for the optimal solution, at the expense of an increase in the computational effort.

The second source of difficulty is associated with the nature of the constraints on the shortage levels in silo. In itself the presence of constraints in the problem formulation is not a major problem. This will become clear when ways of handling constraints in the search for the optimal solution will be discussed later in section 9. The real difficulty arises from the particular nature of the constraints. It was shown before that the relation between the shortage levels in silo and the control variables could only be obtained via simulation. However, the accurate estimation of the shortage levels, for each set of values of \(Q_i, BS_i (i=1, \ldots, 6)\), would require prohibitively long
simulation runs. This is a direct consequence of the rare occurrence of shortages over time. As it will be shown later, the length of the simulation runs that would be required to estimate accurately the shortage levels in silo is of such a magnitude that would rule out, for all practical purposes, the possibility of selecting the best policy.

4.3 Reformulation of the problem

Given the inability to solve the problem as stated earlier, either analytically or with the help of simulation, a new formulation was adopted with the objective of achieving a simplification of the model - in order to be able to solve it - whilst at the same time representing the system, or the original problem, in terms which are acceptably close to the real ones.

The problem was reformulated, calculating constraints on shortages in port rather than in silo, as follows:

Minimize : \( VC \)

Subject to : 
\[ (\beta_1^p)_i \leq B_i \]
\[ (\beta_2^p)_i \leq B_i \]

where

\( VC \) : expected variable costs (as before);

\( \beta_1^p \) : the expected fraction of time during which the net stock in port is negative;

\( \beta_2^p \) : the expected fraction of the replenishment cycles in which a shortage in port (i.e. a period of negative net stock in port) occurs.

These shortage levels were limited, as before, to 1% and 5%, respectively. Clearly this means that, by definition, the levels
of shortage in silo will be higher than the ones implied in the initial formulation. They would coincide only if there was no time lag between the arrival of a vessel and the moment the raw material carried in it becomes available in the silo, at a rate that is high enough to satisfy the demand.

Accepting the initial formulation as valid, the two central questions about the new one are:

(i) is it an acceptable proxy of the initial formulation?

(ii) does it lead to a 'solvable' problem?

Answers to these questions, both affirmative, will be provided in the remainder of the chapter.

5. The Relations Between the Control Variables and the Shortage Levels in Port

5.1 Single-Product Inventory Model. The Operating Doctrine

The major simplification resulting from considering shortages in port (rather than in silo) in the new formulation is that, for each raw material, each shortage level becomes dependent on the shipment size and the buffer stock of that raw material alone, being independent of the control variables concerning other raw materials. This is a direct consequence of the fact that there is no interference between shipments until the vessels are in port. In fact interactions only arise when the vessels compete for limited berthing/unloading and storage facilities.

Under these circumstances, the relation between $Q_i$ and $BS_i$, on
the one hand, and the shortage levels in port \((\beta_p^1)\) and \((\hat{\beta}_p)\), on the other, can be analysed considering a single-product inventory model.\(^(*)\)

The situation to be analysed, and hence the type of model needed, differ from those covered in classical inventory theory in two respects. The first one concerns the ordering process. It is current practice in the bibliography dealing with inventory models (for example Hadley and Whitin\(^29\) or Peterson and Silver\(^48\)) to present the lead time - separating the moment of placing an order from the moment of its delivery - as either constant or following a distribution that is invariant from order to order and cannot be modified by the purchaser. Clearly this does not apply to the ordering process that was described earlier. Two shipments can be ordered at the same time for delivery at different planned future dates of delivery. The distribution of the actual dates of delivery is directly connected with those planned delivery dates and not with the date on which the orders are placed.

As mentioned in the previous chapter (section 2.3) the orders must be placed at least one and half months before the planned dates of delivery\(^(**)\). However, for reasons concerning the planning and

\(^(*)\) In the remainder of section 5, the index specifying the raw material \((i)\), will be dropped from all symbols representing demands, shipment sizes, stocks, etc.; all the considerations and results are equally applicable to all raw materials.

\(^(**)\) This was shown to be the case only for feed grains; however for the other raw materials the minimum time lag between the placement of an order and its planned date of delivery is approximately the same.
the execution of the purchasing operations (explained in chapter 8), the size of the shipments and their planned delivery dates will, in many instances, have to be specified well in advance of that deadline. The inventory problem will therefore be analysed with the primary objective of specifying, at each point in time and over a given time horizon, the size of shipments and their planned dates of delivery. The question of when each of these shipments should actually be ordered will be analysed later in chapter 8 in the context of the purchasing problem.

The second aspect where the model differs from the classical ones is the explicit consideration of the occurrence of crossed deliveries. It will be shown later that the classical approximation — with successive dates of delivery following independent distributions but not crossing — can lead to a gross misrepresentation of the system under analysis.

In order to define the system’s operating doctrine it is useful to introduce first the concept of 'planned net stock in port' (at any time t) : this stock is defined here as the net stock that would exist in port at any time t if all the shipment arrivals took place on the planned dates (clearly the actual net stock will be generally different from the 'planned' one, since the actual arrivals will not usually coincide with the planned ones).

The system is assumed to use 'transactions reporting' (i.e. all transactions of interest are recorded as they occur and the information is immediately made known to the decision maker) and is assumed to operate under the following doctrine: at any moment in time, consecutive shipments of equal size Q will be planned to arrive at times when the planned net stock in port reaches a level \( S_0 \) — this level will be called
the 'planned replenishment point'.

The policy is illustrated in Figure 38 considering an inventory system at time \( t=0 \). The state of the system can be defined by the

![Figure 38 - Planned and actual net stock in port over time and, at time \( t=0 \), determination of the planned arrival of the 'next shipment' \( t_{4} \)](image)

**Figure 38 - Planned and actual net stock in port over time and, at time \( t=0 \), determination of the planned arrival of the 'next shipment' \( t_{4} \)**

following parameters:

\( s \): actual net stock in port

\( n_1 \): the number of shipments that, although planned to arrive before \( t=0 \), are still to be delivered (in the situation represented in Figure 38, \( n_1 = 1 \) - the shipment planned to arrive at \( t = t_0 \) has not arrived yet);

\( n_2 \): the number of shipments already ordered and planned to arrive after \( t = 0 \) (\( n_2 = 3 \));

\( n_3 \): the number of these \( n_2 \) shipments that have actually arrived before \( t = 0 \) (\( n_3 = 2 \));
$t_j$'s: the planned dates of arrival of shipments already ordered ($..., t_1, t_2, t_3$).

At time $t = 0$, the planned date of arrival ($t_4$) of the 'next shipment' (i.e. of the first shipment still not ordered) can be derived from the definition of the operating doctrine:

Planned net stock in port immediately prior to $t_4$ ($= s_o$)

$= \text{current planned net stock in port} + n_2.Q - (t_4 - 0).d$

or, since the current planned net stock in port is, by definition,

$s + n_1.Q - n_3.Q,$

$s_o = (s + n_1.Q - n_3.Q) + n_2.Q - t_4.d$ ;

hence,

$t_4 = \left[ s + (n_1 + n_2 - n_3).Q - s_o \right]/d = (s + 2Q - s_o)/d$ .

It is worth noting that, since $s + (n_1 + n_2 - n_3).Q = s + 2Q$ is the stock 'on hand plus ordered', $t_4$ could be defined as the time at which this stock reaches the 'planned replenishment point'. The only reason why this stock cannot be used to define $t_j$ ($j>4$) is that, when each of these dates is set, there is no guarantee that the previous ones were already ordered. This is the basic justification for using the concept of planned net stock in port in the definition of the operating doctrine.

According to this doctrine, the planned dates of arrival $t_j$ ($j>4$) are

$t_j = t_{j-1} + Q/d.$

Although, in theory, $t_4$ could as well be derived from $t_3$ according to

$t_4 = t_3 + Q/d,$

in practice this is not advisable. In fact, in doing so, if one
shipment size was different from \( Q \) (for example, if the size of the shipment arriving last was \( Q - \Delta Q \)), the replenishment point at \( t_4 \) would have changed from \( s_o \) to \( s_o - \Delta Q \). This would not be the case if the planned dates of arrival were instead derived from the expression considered above. In fact, after the arrival of the shipment of size \( Q - \Delta Q \), the difference \( \Delta Q \) would be reflected in the value of \( s \) and therefore would be taken into account in deriving \( t_4 \). Clearly, if deviations of the shipment sizes from their assumed value \( Q \) could be predicted before the shipment arrivals, they should be taken into account in the expression of \( t_4 \), by replacing \( (n_1 + n_2 - n_3)Q \) by the sum of the predicted sizes of shipments already ordered but yet to be delivered.

Leaving aside this practical aspect of the inventory policy (considered later in chapter 8) there is one theoretical implication of the operating doctrine that is worth discussing. It concerns the use (or, better, the lack of use) of the information available to the decision maker about the current state of the system, at the time the planned dates of delivery are set.

The situation considered previously in Figure 38 can be used to illustrate this point. At time \( t=0 \), the decision maker 'knows' that, from all the shipments already ordered, only those planned to arrive at \( t_o \) and \( t_3 \) are still to be delivered. What difference would it make to the definition of \( t_4 \) (and the following \( t_j \)'s) if, for example, the shipments planned to arrive at \( t_1 \) and \( t_2 \) were also yet to be delivered? According to the assumed operating doctrine, \( t_4 \) (and the other \( t_j \)'s) would still be defined in the same way (note that the stock on hand plus ordered, at \( t=0 \), would not change). In other
words, the information about whether or not shipments ordered previously have been actually delivered is treated as irrelevant as far as the planning of new dates of delivery is concerned.

The justification for this can be seen by reference to Figure 39. As shown there, the deadline to order a shipment with planned date of arrival $t_j$ is

$$t_j - 1.5 \text{ months} \approx t_j - 5. \sigma$$

where $\sigma$ ($\approx 9$ days) is the standard deviation of the distributions of the shipments' actual arrivals. Clearly, at that time $t = t_j - 5. \sigma$, the shipments planned to arrive within the internal $\Delta_2$ will (with virtual certainty) not have arrived yet. This means that the information about the number of shipments that actually have arrived at that stage will concern only those planned to arrive during $\Delta_1$ or earlier. But all these shipments are bound to arrive (with virtual
certainty) before time \( t = t_j \). Consequently, the state of the inventory system at time \( t = t_j \) is independent of the arrivals that took place before time \( t = t_j - 5.0 \).

It is therefore justifiable to plan the arrival of any shipment on the basis of a constant planned replenishment point \( s_0 \), whether the planning of that arrival takes place at the deadline for ordering the shipment or at an earlier stage.

Before concentrating on the analysis of the relation between the control variables and the shortage levels, it is important to make an observation about the value of the buffer stock under the defined operating doctrine.

The buffer stock was defined earlier - as is conventional - as the expected net stock (in port) immediately prior to the arrivals (of vessels). It is obvious that, if the deliveries did not cross, the buffer stock would be equal to the planned replenishment point (since the distributions of arrivals are symmetrical and the demand is constant). It can be shown, through trivial but tedious mathematics, that the above equality still holds in the case of crossed deliveries, provided that the interval between successive planned dates of delivery is kept constant (which is the case under the assumed operating doctrine). In view of this equality, the planned replenishment point can be regarded as a reinterpretation of the buffer stock, adequate for the purpose of defining the operating doctrine.
5.2 **Estimation of the Relationship Between the Control Variables and the Shortage Levels, via Simulation**

As a result of considering crossed deliveries in the inventory model the relationships between $Q$ and $BS$, on the one hand, and the shortage levels, on the other, become extremely complex. Although these relationships could be expressed using methods of numerical analysis, the computational procedure involved is too complicated to be practical. For this reason, the inventory system was simulated. A simulation model of extreme simplicity was written in FORTRAN IV and implemented on a CDC 7600. Apart from enabling the estimation of those relationships, the model provided other relevant information that will be discussed later in section 9.3.1.

The inventory system is entirely specified by the values of $BS$ and $Q$ (controllable variables) and $d$ and $\sigma$ (uncontrollable variables). Therefore the shortage levels can be described as (implicit) functions of those variables

$$
\beta'_p = h_1'(BS, Q, d, \sigma)
$$

$$
\beta''_p = h_1''(BS, Q, d, \sigma)
$$

or, equivalently, as

$$
\beta'_p = h_2'(BS, Q, T, \sigma)
$$

$$
\beta''_p = h_2''(BS, Q, T, \sigma)
$$

where $T = Q/d$ is the 'planned replenishment cycle' (it should be noted that, except for the irrelevant case of $d = 0$, any set of values of $BS$, $Q$, $d$, $\sigma$ corresponds to one - and only one - set of values of $BS$, $Q$, $T$, $\sigma$).
Since the shortage levels are dimensionless, it is clear that they will not be affected by changes in the units in which both stocks (BS, Q) or both times (T/σ) are expressed. In other words, they will not change if both stocks or both times are multiplied by a constant factor. As a result, the shortage levels can be expressed as functions of BS/Q and T/σ:

\[ \beta_p^' = h_3^' (BS/Q, T/\sigma) \]

\[ \beta_p^" = h_3^" (BS/Q, T/\sigma) \]

(this representation does not cover the situations in which Q = 0—clearly irrelevant—and σ = 0—the deterministic case, in which the solution is trivial).

Obviously the reduction in the number of the parameters upon which the shortage levels depend (from 4 to 2) led to a major simplification in the analysis and treatment of the results obtained from the simulation model. This will be illustrated considering the relation

\[ \beta_p^' = h_3^' (BS/Q, T/\sigma). \]

The values of \( \beta_p^' \) were estimated initially for a grid of points including all the combinations of the ratios

- BS/Q : 0.0 to 1.5 (steps of 0.25)
- T/σ : 1.0 to 5.0 (steps of 0.5).

Later the size of the grid was reduced in the areas identified as relevant.
For each set of values BS/Q and T/σ, each simulation run covered 2000 'planned replenishment cycles', taking on average, approximately 0.6 CPU seconds. The length of each simulation was defined so as to lead to practically error free estimates of \( \beta_p \).

In diagram (i) of Figure 40, \( \beta_p \) is plotted against BS/Q and T/σ; the plotted curves represent graphical interpolations of the grid of simulated points. Intersecting these curves by horizontal lines corresponding to given shortage levels (1%, 2.5% and 5%, in diagram (i)), a number of points defining, for different values of T/σ, the corresponding values of BS/Q that lead to those shortage levels is obtained. These points are represented by dots in diagram (ii) (it should be mentioned that they were actually obtained only after the curves of diagram (i) were represented in an appropriate scale, enabling accurate graphical interpolations).

Finally, for each shortage level, a curve was fitted to the points obtained from diagram (i). The fitted relation between BS/Q and T/σ is given in diagram (ii) for \( \beta_p = 1\% \) and its goodness of fit is shown in Figure 40 (iii).

The relation between BS/Q and T/σ, for \( \beta_p = 5\% \), was obtained using an identical procedure. The fitted relation is shown in Figure 41 (i) (bold line) and is compared with the one that would be obtained under the classical assumption of independent arrivals not crossing (interrupted line). This expression can be derived easily as follows:

(i) for a given buffer stock BS and a given demand d, a shortage will occur whenever the arrival of a vessel is delayed by more than BS/d;
Figure 40 - Relationship between $\beta_P'$, BS/Q and T/Q

(i) Shortage level $\beta_P'$ as a function of BS/Q and T/Q

(ii) Relationship between BS/Q and T/Q for $\beta_P' = 1.0, 2.5, 5.0\%$

(iii) Goodness of fit of the relation $\text{BS/Q} = -1.252 + 2.19/(T/Q)^{0.30}$

<table>
<thead>
<tr>
<th>T/Q</th>
<th>BS/Q (actual)</th>
<th>BS/Q (fitted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.140</td>
<td>1.138</td>
</tr>
<tr>
<td>1.5</td>
<td>0.875</td>
<td>0.864</td>
</tr>
<tr>
<td>2.0</td>
<td>0.685</td>
<td>0.689</td>
</tr>
<tr>
<td>2.5</td>
<td>0.550</td>
<td>0.564</td>
</tr>
<tr>
<td>3.0</td>
<td>0.460</td>
<td>0.467</td>
</tr>
<tr>
<td>3.5</td>
<td>0.385</td>
<td>0.398</td>
</tr>
<tr>
<td>4.0</td>
<td>0.330</td>
<td>0.325</td>
</tr>
<tr>
<td>4.5</td>
<td>0.275</td>
<td>0.270</td>
</tr>
<tr>
<td>5.0</td>
<td>0.230</td>
<td>0.223</td>
</tr>
</tbody>
</table>
(ii) the delay

\[ \epsilon = \text{actual date of arrival} - \text{planned date of arrival} \]

follows a normal distribution \( N(0, \sigma^2) \);

(iii) the constraint \( \beta_p = 5\% \) can be expressed as

\[ P(\epsilon > BS/d) = 5\% \]

or, equivalently, as

\[ BS/d = 1.65 \cdot \sigma \]

(iv) replacing \( d \) by \( Q/T \) in this expression and rearranging the terms, it becomes

\[ (BS/Q) \cdot (T/\sigma) = 1.65 \]

From the analysis of Figure 41 it is clear that for low values of the ratio \( T/\sigma \) (i.e., when the probability of crossed deliveries becomes significant) this relation grossly overestimates the ratio \( BS/Q \).
The expression $(BS/Q) \cdot (T/\sigma) = 1.65$ is practically exact for values of $T/\sigma > 6$ (the probability of crossed deliveries is virtually zero for these values of the ratio). This fact was used to check the accuracy of the relation derived via simulation and graphical interpolation. Within the range $T/\sigma > 6$, the buffer stocks derived via simulation are within 3% of their theoretical value. Deviations of such a magnitude can cause errors in $\beta_p$ of no more than 0.5% (which was considered acceptable in the context of the problem under analysis).

Figure 42 presents the relations obtained via simulation between $BS/Q$ and $T/\sigma$ for shortage levels $\beta_p^' = 1\%$ and $\beta_p^" = 5\%$.

![Figure 42 - Minimum value of BS/Q that, for each T/\sigma , leads to $\beta_p^' < 1\%$ and $\beta_p^" < 5\%$](image)

From these relations one can define, for each value of $Q$, what is the minimum buffer stock, $BS = r(Q)$, that satisfies simultaneously
both constraints

\[ \beta_p \leq 1\% \]

\[ \beta^*_p \leq 5\%. \]

The minimum value is given by

\[ BS = r(Q) = \begin{cases} Q \cdot \left[-1.252 + 2.390/[Q/(d,\sigma)]^{0.30}\right], & \text{if } Q \leq 1.4. \ (d,\sigma) \\ Q \cdot \left[-0.211 + 1.327/[Q/(d,\sigma)]^{0.55}\right], & \text{otherwise} \end{cases} \]

One aspect of this relationship BS = r(Q) that deserves attention is the sensitivity to changes in the value of the standard deviation. Figure 43 shows the relation between the ratio BS/Q and T, for standard deviations of 8, 9 and 10 days. A change in \( \sigma \) of \( \pm 1 \) day leads to changes in the value of BS/Q varying from about \( \pm 8\% \) on the left of the diagram (T=10 days) to about \( \pm 15\% \) on the right (T=70 days).

Figure 43 - Sensitivity of BS/Q to changes in the standard deviation \( \sigma \)
In view of the magnitude of the changes in BS/Q, it seemed worthwhile to investigate how errors of estimation of $\sigma$ can affect the shortage levels in port. The analysis was carried out considering the situation where

(i) the standard deviation $\sigma$ is estimated to be 9 days (this was the case, in chapter 5, for the feed grain shipments taking place after March 1977);

(ii) its actual value is 10 days (it should be noted that, in spite of the size of the sample of those shipments (31), the probability of $\sigma > 10$ days is still about 15%).

For values of $T/\sigma > 1.4$, i.e. when the constraint $\beta_p \leq 5\%$ is active, such a small error in the estimation of $\sigma$ would lead to an increase of $\beta_p$ of about 2.0 to 2.5% (i.e. from 5% to up to 7.5%). For values of $T/\sigma < 1.4$, the shortage level $\beta_p$ would suffer an increase from 1.0% to about 1.4%. These figures illustrate clearly the need for an accurate estimation of $\sigma$.

6. The Multi-product Simulation Model

6.1 General Model Inputs and Outputs

The objective of the simulation model is to provide, for each delivery-inventory policy and for specified values of the model parameters (treated as 'uncontrollable' variables), estimates of efficiency of the systems under study (with the characteristics described in section 3).
Particularly important in the context of the problem under analysis is the estimation of the expected variable shipping and inventory costs (VC) for policies \( \{ Q_i, BS_i = r_i(Q_i), i=1, \ldots, 6 \} \) (i.e. policies in which, for given shipment sizes, the buffer stocks are kept at minimum values, compatible with satisfying the constraints on shortage levels in port). However, the model can be used to evaluate other policies \( \{ Q_i, BS_i = x_i(Q_i) + EBS_i, i=1, \ldots, 6 \} \) (where \( EBS_i \) can be interpreted as 'corrections' to the buffer stocks in relation to their 'minimal values') and to estimate measures other than costs (e.g. shortage levels in port and in silo, distributions of stocks in port and in silo, distributions of the time spent by vessels queueing or unloading, etc.).

The model inputs and outputs are listed in Tables 22 and 23, respectively.

The remainder of section 6 focuses on the most relevant aspects of the model, namely:

(i) the generation of vessel arrivals;
(ii) the initialization of the system;
(iii) the organization of the simulation (model structure);
(iv) the method of estimation of costs;
(v) the computer implementation.

6.2 Generation of Vessel Arrivals

The generation of vessel arrivals involves

(i) setting the planned dates of arrival at equal time intervals, and
(ii) for each planned date of arrival, sampling the actual date of arrival from a normal distribution.
### Table 22 - Inputs of the multi-product simulation model

**1. Controllable variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_i )</td>
<td>shipment size (of raw material ( i ), ( i=1,...,N )) [1000 m.t.]</td>
</tr>
<tr>
<td>( EBS_i )</td>
<td>'correction' to the buffer stock [days of consumption at a rate ( d_t )]</td>
</tr>
<tr>
<td>( IU )</td>
<td>parameter specifying the rule for allocating the unloading capacity to vessels (0 or 1, as defined in section 5.2.6)</td>
</tr>
</tbody>
</table>

**2. Uncontrollable variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>number of raw materials</td>
</tr>
<tr>
<td>( d_t )</td>
<td>demand [1000 m.t./day]</td>
</tr>
<tr>
<td>( e_1 )</td>
<td>standard deviation of the distributions of arrivals [days]</td>
</tr>
<tr>
<td>( v_1 )</td>
<td>raw materials' specific gravity</td>
</tr>
<tr>
<td>( (B_1^1) )</td>
<td>maximal expected fraction of time during which the net stock in port is negative (1 or 2.5%)</td>
</tr>
<tr>
<td>( (B_2^1) )</td>
<td>maximal expected fraction of replenishment cycles in which a shortage in port occurs (5 or 10%)</td>
</tr>
<tr>
<td>( VL_1 )</td>
<td>standard voyage length [nautical miles]</td>
</tr>
<tr>
<td>( LR_1 )</td>
<td>loading rate at the port of origin [1000 m.t./day shinc]</td>
</tr>
<tr>
<td>( v_1 )</td>
<td>vessels' speed [knots]</td>
</tr>
<tr>
<td>( U )</td>
<td>unit purchasing price (fob) [US $ / m.t.]</td>
</tr>
<tr>
<td>( V )</td>
<td>effective storage capacity of the silo [1000 m³]</td>
</tr>
<tr>
<td>( U_1 )</td>
<td>effective unloading capacity (total) [1000 m.t./(h.gr.)/day]</td>
</tr>
<tr>
<td>( e_1 )</td>
<td>extra capacity of delivery from the silo to the customers (expressed as a proportion of the 'normal' rate of delivery)</td>
</tr>
<tr>
<td>( b )</td>
<td>berthing / deberthing times [days]</td>
</tr>
<tr>
<td>( RO_1, P_1, R_2 )</td>
<td>coefficients of the standard trip charter rate ( R_s^t = R_0 + R_1 DWT_1 + R_2 DWT_2 ), with ( R_1 ) in [1000 US $/day] and ( DWT_1 ) (vessels' deadweight tonnage) in [1000 m.t.]</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>rate of interest on capital [% / year]</td>
</tr>
<tr>
<td>( P_{fo} )</td>
<td>unit cost of fuel oil [US $ / m.t.]</td>
</tr>
</tbody>
</table>

**3. Simulation control parameters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( TST )</td>
<td>total simulation time [days]</td>
</tr>
<tr>
<td>( NOB )</td>
<td>number of 'snap shots' (observations of the state of the system) over ( TST )</td>
</tr>
<tr>
<td>( IHIST )</td>
<td>parameter controlling the printout</td>
</tr>
</tbody>
</table>
Table 23 - Outputs of the multi-product simulation model

(1) Point estimates

(a) For each raw material:
   - shortage levels in port $(\bar{y}_1)_{ij}$, $(\bar{y}_2)_{ij}$ [%]
   - shortage levels in silo $(\bar{y}_3)_{ij}$, $(\bar{y}_4)_{ij}$ [%]
   - average stock in port [1000 m.t.]
   - expected variable cost per unit of imported r. material [US $/m.t.]
   - expected variable shipping cost per unit of imported r.m. [US $/m.t.]

(b) Aggregate:
   - expected annual variable cost $VC$ [1000 US $]

(11) Histograms (including mean and standard error)

(a) For each raw material:
   - lengths of shortages in port [days]
   - lengths of shortages in silo [days]
   - vessels' queuing time [days]
   - vessels' time in berth (waiting) [days]
   - vessels' time in berth (unloading) [days]
   - vessels' time in berth (total) [days]
   - stock in port (sample of NGB 'snap shots') [1000 m.t.]
   - stock in silo (sample of NGB 'snap shots') [1000 m$^3$]

(b) Aggregate:
   - lengths of continuous unloading periods [days]
   - lengths of full silo periods [days]
   - stock in silo (sample of NGB 'snap shots') [1000 m$^3$]
   - length of the queue (sample of NGB 'snap shots')
The standard approach of generating, at each arrival, the 
time of the next one could not be adopted due to the possibility 
of deliveries crossing. In view of the large storage capacity available 
on the computer used to run the simulations, the whole set of 
arrivals could be generated and stored at the beginning of each 
simulation.

In generating the arrivals of vessels over a simulation period 
\([0, TST]\) consideration was given to the fact that

(i) vessels planned to arrive before time \(t=0\), can 
actually arrive after \(t=0\), and,

(ii) similarly, those planned to arrive after \(t=TST\), can 
arrive before that time.

In the multi-product simulation model, the planned dates of 
arrival of vessels carrying any raw material \(i\) (\(i=1, \ldots , N\)) are set 
at times

\[
\{-N_{1i}T_i, (-N_{1i}+1)T_i, \ldots , -T_i, 0, T_i, \ldots , (N_{2i}-1)T_i, N_{2i}T_i\}
\]

where

\(T_i\) : average replenishment cycle \((Q_i/d_i)\);

\(N_{1i}\) : largest integer in \((3\sigma_i/T_i)\);

\(N_{2i}\) : largest integer in \([TST + 3\sigma_i/T_i]\);

\(\sigma_i\) : standard deviation of the distributions of vessel 
arrivals.

Vessels planned to arrive before \(-N_{1i}T_i\) or after \(N_{2i}T_i\) 
would, with virtual certainty, arrive outside the interval
\[ 0, TST \] since
\[-(N_{i} + 1), T_{i} < -3\sigma_{i}, \text{ and} \]
\[(N_{2,i} + 1), T_{i} > TST + 3\sigma_{i}. \]

For this reason these arrivals can be ignored in the simulation.

The actual arrivals are sampled from independent normal distributions \( N(\mu_{ij}, \sigma_{i}^2) \) (\( j = N_{1,i}, \ldots, N_{2,i} \)) where \( \mu_{ij} \) are the planned dates of arrival defined above. The normal variates are generated using a standard random number generator coupled with a subroutine performing the Box-Muller transformation.

6.3 Initialization

The procedure adopted in the model was designed to generate an initial state that is

(i) compatible with the 'logic' of the system;

(ii) simple to derive, and

(iii) close to steady-state conditions.

In order to simplify the generation of the initial state it is assumed that:

(i) if there are any vessels inside the port, they have still not initiated their berthing operations;

(ii) no raw materials are undergoing a process of recovery from shortage; therefore the rate of supply from the silo to customers is either \( d_i \) (normal supply) or zero (shortage).
Once the vessel arrival times are generated and put into ascending order, the stocks in port (actual, or on hand, and net) of all raw materials \( i = 1, \ldots, N \), at time \( t = 0 \), are computed as follows:

(i) if \( K_i \) is the number of vessels carrying raw material \( i \) with actual arrival times preceding \( t = 0 \), then the net stock in port is
\[
NSP_i = (K_i - N1_i) \cdot Q_i + BS_i;
\]

(ii) the actual stock in port is
\[
SP_i = \begin{cases} 
0, & \text{if } NSP_i \leq 0 \text{ (i.e. shortage in port)} \\
NSP_i, & \text{otherwise.}
\end{cases}
\]

The stocks in silo (actual and net) are then initialized as follows:

(i) if \( NSP_i \leq 0 \), then the stocks in silo are set at
\[
SS_i = 0 \quad \text{(actual stock in silo)}
\]
\[
NSS_i = NSP_i \quad \text{(net stock in silo)};
\]

(ii) if the actual and net stocks in port are positive, then the actual stocks in silo are set equal to the net stocks in silo, and are calculated according to the following procedure:

(a) for each raw material \( i \), find the largest number of full shipments, \( NSH_i \), contained in \( SP_i \)
\[
NSH_i = \text{largest integer in } (SP_i/Q_i);
\]

(b) the actual stock in port \( SP_i \) can therefore be expressed as
\[
SP_i = NSH_i \cdot Q_i + DIF_i \quad (Q_i > DIF_i \geq 0);
\]

(c) it is assumed, for all raw materials \( i \), that the amounts \( DIF_i \) are stored inside the silo (if this
is not possible — i.e. if \( \sum_{i=1}^{N} \left( \frac{DIF_i}{V_i} \right) > V \) (effective silo storage capacity) — the arrivals are generated again and the initialization process restarted; at the end of this step

\[ SS_i = NSS_i = DIF_i, \]

this leaves only full shipments outside the silo

(d) if \( NSH_1 > 1 \), a shipment (of size \( Q_1 \)) is placed inside the silo whenever possible; if it is possible then

\[ NSH_1 = NSH_1 - 1, \]

\[ SS_1 = NSS_1 = \text{previous value of } SS_1 + Q_1; \]

the process is repeated for raw materials 2, 3, ..., \( N,1,2,\ldots,N \), ..., until either all \( NSH_i = 0 \) or it is not possible to place any additional \( Q_i \) inside the silo.

The final value of \( NSH_i \) (i=1, ..., N) represents the number of full vessels of raw material i, which are assumed to have joined the queue at time \( t = 0 \). The vessels are placed in the queue in the following order: choosing first one vessel of those containing raw material 1 (if it exists), then one vessel of each of raw materials 2, 3, ..., N, and repeating the process as many times as required (e.g. if there were 2 vessels of raw material 1 (2 of 1), 1 of 2 and 3 of 5, the raw materials in the ordered vessels would be 1, 2, 5, 1, 5, 5).

6.4 The Structure of the Simulation Model

The model was based on the discrete event simulation approach and was structured as a three-phase model (following Tocher 66):
(i) 'A' phase - comprising:
   (a) time scan: the extraction from a list of scheduled
       future events (i.e. times at which the system is to
       be inspected) of the time and nature of the event
       that is to take place next;
   (b) clock advance: the clock is advanced to the time of
       the next event;

(ii) 'B' phase - including the execution of the 'Bound' activities
     (i.e. carrying out the set of programme instructions that
     produce the state changes which are direct consequences
     of the occurrence of the current event).

(iii) 'C' phase - including the testing and, if appropriate, the
      execution of the 'Conditional' activities (these activities
      are materialized by sets of programme instructions
      associated with state changes that will occur at the current
      event if and only if certain conditions are satisfied).

Table 24 presents the different types of events 'E' (and the
   corresponding 'B' activities) and the 'C' activities considered in the
   simulation model.

Two observations must be made on these events and activities and
on the way in which the sequence A-B-C is organized. The first
concerns the events \( E_3, E_5, E_6 \) and \( E_7 \): these events, once scheduled
   to happen at a given future time \( t \) may need to be rescheduled later.
This can be illustrated with a simple example, considering the situation
where all raw materials except one are being supplied at the 'normal'
rate, the exception being a raw material (say, 1) undergoing a recovery
Table 24 - Types of events, 'B' activities and 'C' activities

(i) Events 'E' (and 'B' activities)

- \( E_1 (B_1) \): Vessel arrival
- \( E_2 (B_2) \): End berthing operation
- \( E_3 (B_3) \): Begin deberthing operation (* End unloading operation *)
- \( E_4 (B_4) \): End deberthing operation
- \( E_5 (B_5) \): Begin shortage in silo
- \( E_6 (B_6) \): End recovery from shortage
- \( E_7 (B_7) \): Begin full silo period
- \( E_8 (B_8) \): Snap shot

(ii) 'C' activities

- \( C_1 \): Begin berthing operation
- \( C_2 \): Begin unloading operation
- \( C_3 \): Update rates of demand and unloading
- \( C_4 \): End shortage in silo (* Begin recovery from shortage *)
- \( C_5 \): Update times of events \( E_3, E_4, E_5, E_7 \)
- \( C_6 \): End shortage in port
- \( C_7 \): End full silo period
- \( C_8 \): Begin shortage in port

from shortage. Under these circumstances the event \( E_6 \) (end recovery from shortage) is programmed (at a time \( t_0 \), say) to happen at a time

\[
t = \frac{(SS_1(t_0) - NSS_1(t_0))}{rr_1}
\]

as shown in Figure 44(i)

where

\[
rr_1 = e^{-v_1}
\]

Figure 44 - Rescheduling of events
is the rate of recovery from shortage (as defined earlier, in section 3.2.7).

However, if between \( t_0 \) and \( t \) (at \( t_1 \), say), the event \( E_5 \) (begin shortage in silo, for another raw material \( k \), say) occurs, then \( E_6 \) has to be rescheduled to a new time

\[
  t' = \left[ SS_1(t_1) - NSS_1(t_1) \right] / rr_1
\]

(see Figure 44(ii))

where

\[
  rr_1 = \left( e + d_k / v_k \right), \quad v_1
\]

Since, at time \( t_1 \), the event \( E_6 \) cannot be rescheduled for a time \( t < t_1 \) it is always safe to schedule event \( E_6 \) (or any others) taking into account only the current state of the system and to include in the model a 'C'-activity which will check, whenever appropriate, whether the rescheduling of events is necessary (activity \( C_5 \)-Update times of events \( E_3, E_5, E_6, E_7 \)).

The second observation concerns the organization of the transition between the 'B' and 'C' phases. In order to increase the efficiency of the simulation (critical, since the model was to be embedded into a direct search optimization routine) the transition between those phases was organized so as to by-pass, after each 'B' activity, those 'C' activities that cannot take place until the state of the system is changed. As with the 'cellular structure' proposed by Carvalho and Crookes, the objective to avoid going through those 'C' activity tests which are guaranteed to fail.

The change from an initial 'B \rightarrow all C' structure to the 'selective B \rightarrow C' structure presented in Figure 45 led to a reduction in the computation time of about 40%.
Figure 45 - Structure of the multi-product simulation model

\[ E_1 : \text{Vessel arrival (update } T_{E_1}) \]
\[ E_2 : \text{Berthing operation (update } T_{E_2}) \]
\[ E_3 : \text{Begin deberting operation (update } T_{E_4}) \]
\[ E_4 : \text{End deberting operation} \]
\[ E_5 : \text{Begin shortage in silo} \]
\[ E_6 : \text{End recovery from shortage} \]
\[ E_7 : \text{Begin full silo period} \]
\[ E_8 : \text{Snapshot (update } T_{E_8}) \]
\[ C_1 : \text{Begin berthing operation (update } T_{E_2}) \]
\[ C_2 : \text{Begin unloading operation} \]
\[ C_3 : \text{Update rates of demand and unloading} \]
\[ C_4 : \text{End shortage in silo} \]
\[ C_5 : \text{Update } T_{E_3}, T_{E_5}, T_{E_6}, T_{E_7} \]
\[ \text{Time scan: Find next event } (E_1, ..., E_8) \]
\[ \text{Advance time (update stocks and vessels' loads)} \]
\[ C_6 : \text{End shortage in port} \]
\[ C_7 : \text{End full silo period} \]
\[ C_8 : \text{Begin shortage in port} \]
6.5 **Estimation of the Variable Costs**

The variable costs, VC, were defined earlier in section 3.3 as the sum of two components: the inventory costs and the shipping costs.

The first component—inventory costs—requires the evaluation of the average stock of each raw material in port. Following the arguments presented by Conway, such evaluation is obtained by integration rather than by periodic sampling, in order to reduce the variance of the errors of estimation. The integration is carried out from the beginning of the simulation period, since, as far as the stocks in port of each raw material are concerned, the system is initialized in a 'steady state'.

The evaluation of the second component—shipping costs—is based on the costs incurred by all vessels that arrive after time \( t=0 \) and leave before \( t=TST \). The vessels arriving shortly after \( t=0 \) were included in the sample because, as concluded from the results of a few pilot runs, their lengths of stay in port are not significantly different from those of vessels arriving later on.

6.6 **Computer Implementation**

The model was written in FORTRAN IV on a CDC 7600. The programme compilation time (including all the auxiliary subroutines used) was 2.7 CPU seconds.

The execution time depends obviously on the length of the simulation (TST) and on the values attributed to the input variables. References to the execution times will be made in the next section.
7. **Combination of the Simulation Model with a Direct Search Routine**

7.1 **General**

Using the simulation model it is possible to estimate the expected variable costs \( VC \) for different delivery-inventory policies, in particular, \( \{ Q_i, BS_i = r_i(Q_i), i = 1, \ldots, 6 \} \). In order to select, among these policies, the one that leads to the minimum expected costs \( VC \), the simulation model was combined with a direct search routine. In this section the procedure adopted will be described and the difficulties involved, and the way in which they were overcome, will be discussed.

The presentation is illustrated with the example of the optimization of operations at the Oporto new terminal under the scenario presented in Table 25 (these conditions are similar to those prevailing when the new terminal began operations).

7.2 **The Direct Search Routine. Computer Implementation**

The search procedure is based on Powell's multivariate search method 49 combined with the univariate search method proposed by Davies, Swann and Campey 21.

The 'Powell-DSC' method was chosen in view of its high efficiency. In a comparative analysis of different optimization algorithms, conducted by Himmelblau 33, using a uniform set of standards and test problems, the 'Powell-DSC' algorithm ranked first, together with the 'Stewart-DSC' algorithm, among the search (i.e. derivative free) algorithms.

A short computer programme was written in FORTRAN IV combining:
Table 25 - Oporto new terminal: scenario for optimization

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Rule for allocation of unloading capacity: rule 'O'</td>
</tr>
<tr>
<td>(ii)</td>
<td>Raw materials (N=6): maize, sorghum, wheat, soybean, peanut, sunflower</td>
</tr>
</tbody>
</table>
| (iii) | Demands:  
(a) demands for feed grains (maize+sorghum), wheat and oil seeds as given in Table 21  
(b) demand for maize / demand for sorghum: 66.6/33.3 (approx., the 'optimal' ratio under average relative price ratio) |
| (iv) | Standard deviations of the distributions of vessel arrivals: 9 days, for all raw materials |
| (v) | Shortage levels in port: (Δ%) = 1% and (Δ%) = 5%, for all raw materials |
| (vi) | Grain terminal parameters:  
(a) effective storage capacity: V = 91,800 m³  
(b) effective unloading capacity: 
\[ U = (1,872,000/365)/(0.85) = 4,399 \text{ m.t.}/(h, \text{grain})/\text{day} \]  
(0.85 is an estimate of the ratio volume flow of grains and oil seeds / total volume flow, obtained assuming an average specific gravity of 0.50 for the oil seed meals)  
(c) extra capacity of delivery: e = 0.15  
(d) berthing/deberthing times: b = 0.25. (365/208) = 0.439 days |
| (vii) | Trip charter rate: as of July 1979 |
| (viii) | Ports of loading:  
(a) grains and soybean: US (av. Gulf/Atlantic Coast)  
(b) peanut: Gulf of Guinea  
(c) sunflower: Yugoslavia/Bulgaria/Romania  
(round trip lengths, rates of loading and port charges adjusted accordingly) |
| (ix) | Vessels' speeds: \( v = 14 \text{ knots} \) |
| (x) | Prices of raw materials (in US $/m.t.):  
maize (120), sorghum (112), wheat (150), soybean (240), peanut (550), sunflower (350)  
(av. export prices (fob) during 1975-1978, FAO Trade Yearbook, rounded figures) |
| (xi) | Rate of interest on capital: \( r = 15\% \)/year |
| (xii) | Unit cost of fuel oil: \( p_f = 90 \text{ US $/m.t.} \) |
the simulation model described in the previous section
written as a 'function' of the main programme and used
to evaluate the variable costs associated with alternative
delivery-inventory policies) with

(ii) a standard 'Powell-DSC' subroutine (used to conduct the
search).

The simulation-optimization model was implemented on a CDC 7600.

The search path may lead to points \( \{ Q_i, i=1,\ldots,6 \} \) corresponding
to policies which are

(i) infeasible: if one or more of the shipment sizes are
negative - clearly the system cannot be simulated for
these points, or

(ii) feasible but unattractive: if one or more of the shipment
sizes are positive but too small to be cost effective -
although the system can be simulated, the computational
effort per unit of simulated time is considerable in
view of the high number of (small) shipments involved.

A 'barrier' type method was incorporated in the programme with the
objective of avoiding the simulation of the system for such points.
This was achieved by defining the cost function, for any point
\( \{ Q_1, i=1,\ldots,6 \} \), as follows:

\[
VC(Q_1,\ldots,Q_6) = VC(Q_1^*,\ldots,Q_6^*) + \sum_i M_i (Q_i^*-Q_i)
\]

where

\[
VC(Q_1^*,\ldots,Q_6^*) : \text{estimate of the annual variable}
\]

cost derived, via simulation, for the point

\( \{ Q_i^*, i=1,\ldots,6 \} \);
\[ Q_i^* = \begin{cases} Q_{\text{min}}, & \text{if } Q_i < Q_{\text{min}} \\ Q_i, & \text{otherwise} \end{cases} \]

\[ M : \text{a large number (}10^{20}\text{ was the number adopted);} \]

\[ Q_{\text{min}} : \text{set, by trial and error, at a positive value} \]
\[ \text{below the range of interesting shipment sizes (the} \]
\[ \text{adopted value was 3000 m.t.).} \]

Clearly, the effect of such a function is to penalize solutions
which are either infeasible or unattractive and, consequently, to
redirect the search to the interesting range of shipment sizes.

7.3 Difficulties Due to the Stochastic Nature of the Function to be
Minimized and Proposed Way of Overcoming Them

For any delivery-inventory policy \( \{ Q_i, BS_i = r_i(Q_i), i = 1, \ldots, 6 \} \) - or,
equivalently, for any point of the search space \( \{ Q_i, i = 1, \ldots, 6 \} \) - one can
obtain, via simulation, estimates of the expected value of the variable
cost VC. The estimate obtained from each simulation run will follow
a distribution centred around the expected value of VC (assuming that
the model does not introduce any estimation bias) with a standard
deviation which will depend on the length of the simulation.

The efficiency of the search algorithm can be seriously affected
by random differences between the expected value of the function to be
minimized and the point estimates obtained from successive
independent simulation runs. The effect of these differences is
equivalent to persistently changing the shape of the function as the
search proceeds. As a result

(i) the search path can go through the same point several
times without converging (since the function can be
evaluated differently at the same point \( Q_i, i=1,...,6 \), at different stages of the search process), and

(ii) the algorithm can converge to a point which is not the true minimum (but is only a minimum of a 'transient configuration' of the estimated function).

This problem can be minimized - though not eliminated - by increasing the length of the simulation, thereby reducing the standard deviation of the estimation errors, as the algorithm gets nearer to the minimum. This procedure was tried but it was found rather impractical (difficult to 'tune') and it cannot guarantee a high efficiency of the algorithm in the final stages of the search (where small errors of evaluation can disrupt the convergence considerably).

These difficulties led to the adoption of an alternative procedure: all the way through the search the simulations were run with the same length and the same set of streams of random numbers. The purpose of this procedure is to ensure that, if the function is evaluated twice at the same point, along the search path, then the results are identical. In other words, the original function is replaced by a 'fixed image', which is then minimized. Clearly, the algorithm can perform this minimization as efficiently as for any (other) deterministic function. It remains only to be checked whether the image chosen is a good representation of the original function or whether it distorts this function to a significant extent.

There are two basic types of distortion, that can result in the convergence of the algorithm to a point significantly away from the true minimum. They are illustrated in Figure 46, considering the simple case of a univariate search. In diagram (i) the original
function, assumed to be located midway through the shaded confidence band, is replaced by a corrugated image that may cause the algorithm to stop in a local minimum. Diagram (ii) illustrates possible shifts of the image in relation to the original function, leading to poor estimates of the true minimum.

These distortions can be minimized, at the expense of an increase in computational effort, by increasing the length of the simulation runs.

By trial and error, an acceptable compromise between the reliability of the estimates of the minimum and the computing time required was found, for each of the systems that was to be optimized (Oporto new terminal, Lisbon new terminal alone and, later, Lisbon new terminal in parallel with the Beato terminal).
Tests on the robustness of the estimates of each minimum were performed along the following lines:

(i) effect of local minima (either on images of the expected VC function or on the original function itself): the optimization process was run using the same set of streams of random numbers, but starting the algorithm at different points and along different search directions;

(ii) effect of different sets of streams of random numbers: the search routine was run, for given initial conditions (point and search directions), using different sets of random numbers.

The results presented below, concerning the optimization of the Oporto new terminal, under the scenario presented earlier in section 7.1, give an indication of what was considered to be an acceptable compromise between reliability of the results (i.e. solutions sufficiently near the optimum) and computational effort.

Table 26 shows typical differences among the solutions obtained starting from different initial conditions, using always the same set of streams of random numbers and fixing the total simulation time at 1825 days (5 years). These differences were considered to be negligible, in the context of the problem under analysis.

<table>
<thead>
<tr>
<th>Table 26 - Influence of local minima on the search for the optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trial '1'</strong> (out of streams '1')</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td>Metric</td>
</tr>
<tr>
<td>-----------------------------------</td>
</tr>
<tr>
<td>Starting point: 0 (1000 m.s.l.)</td>
</tr>
<tr>
<td>Order of search (1st iteration)</td>
</tr>
<tr>
<td>Number of iterations</td>
</tr>
<tr>
<td>No. of evaluations (incl. runs)</td>
</tr>
<tr>
<td>Optimal solution: 0 (1000 m.s.l.)</td>
</tr>
<tr>
<td>Minimum variable cost [1000 m.s.l]</td>
</tr>
<tr>
<td>Total execution time [CPU sec.]</td>
</tr>
<tr>
<td>Av. execution time / (CPU sec.)</td>
</tr>
</tbody>
</table>

Note: VT7 kept always at 2505 days (5 years)
The effect, on the final solution, of adopting different sets of streams is illustrated in Table 27. Clearly it does not make sense in this case to compare directly the estimates of the variable cost obtained, for each solution, from each distinct set of random numbers.

Table 27 - Influence of the set of streams of random numbers on the search for the optimum

<table>
<thead>
<tr>
<th>Trial '1'</th>
<th>Trial '4'</th>
<th>Trial '5'</th>
</tr>
</thead>
<tbody>
<tr>
<td>(set of streams '1')</td>
<td>(set of streams '4')</td>
<td>(set of streams '5')</td>
</tr>
<tr>
<td>Mali</td>
<td>Sor</td>
<td>Vae</td>
</tr>
<tr>
<td>50.00</td>
<td>20.00</td>
<td>20.00</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>No. of evaluations (total, runs)</td>
<td>90</td>
<td>94</td>
</tr>
</tbody>
</table>

Optimal solution: Q (1000 m³) | 24.19 | 15.09 | 17.97 | 11.57 | 4.37 | 6.14 | 24.19 | 17.70 | 15.18 | 11.89 | 5.60 | 5.92 | 24.19 | 14.20 | 20.00 | 11.00 | 5.00 | 5.19 |

Note: TST kept always at 1825 days (≈ 5 years)

Instead, for each solution, more reliable estimates of VC were obtained from series of 15 independent simulation runs. The estimates obtained are compared in Table 28. Again, the differences between them were considered meaningless.

Table 28 - Comparison between the solutions obtained in trials '1', '4' and '5' (based on 15 independent runs for each solution)

<table>
<thead>
<tr>
<th>Solution</th>
<th>Average variable cost VC</th>
<th>St. error of variable cost VC S.E.(VC)</th>
<th>St. error of av. variable cost S.E.(VC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial '1'</td>
<td>17 857</td>
<td>127</td>
<td>33</td>
</tr>
<tr>
<td>Trial '4'</td>
<td>17 897</td>
<td>128</td>
<td>47</td>
</tr>
<tr>
<td>Trial '5'</td>
<td>17 908</td>
<td>207</td>
<td>53</td>
</tr>
</tbody>
</table>

Note: TST kept always at 1825 days (≈ 5 years)

8. Analysis of the Solutions: Test of Assumptions

8.1 General

The operations at the new terminals at Lisbon and Oporto were analysed for scenarios reflecting different operating conditions.
of the systems under analysis. In this section one of the optimal solutions obtained for the Oporto new terminal will be described in detail and will be used to illustrate

(i) how the buffer stocks can be adjusted in order to bring the shortages in silo to the levels assumed for those in port in the current formulation (or, in other words, how one can return to the initial problem formulation);

(ii) how two of the basic model assumptions - the exclusion of speculative storage and the model's static nature - were tested (the justification for the third basic assumption - no attempt to stagger the planned arrivals of vessels carrying different raw materials - will be given later in section 8).

After the detailed analysis of this particular solution, the general results obtained under different scenarios, for the Oporto and Lisbon terminals, will be discussed and the major conclusions will be presented.

8.2 The Operations at the Oporto New Terminal (Scenario Presented in Table 25, Section 7.1)

8.2.1 The Optimal Solution

Table 29 shows the shipment sizes, and other related parameters, for the optimal solution obtained in what was described earlier in Table 26, section 7.3, as trial '1'.
Table 29 - The optimal solution obtained in trial '1' (Oporto): shipment sizes and related parameters

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Shipment size (1000 m.t)</th>
<th>Vessel size (DWT)</th>
<th>No. of vessels per year</th>
<th>Av. replenishment cycle [days]</th>
<th>Ratio 2/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>21.85</td>
<td>25.25</td>
<td>14.64</td>
<td>24.93</td>
<td>2.77</td>
</tr>
<tr>
<td>Sorghum</td>
<td>15.00</td>
<td>20.63</td>
<td>10.66</td>
<td>34.22</td>
<td>3.80</td>
</tr>
<tr>
<td>Wheat</td>
<td>17.57</td>
<td>19.52</td>
<td>8.53</td>
<td>42.75</td>
<td>4.75</td>
</tr>
<tr>
<td>Soybean</td>
<td>11.57</td>
<td>13.57</td>
<td>5.87</td>
<td>62.10</td>
<td>6.90</td>
</tr>
<tr>
<td>Peanut</td>
<td>4.33</td>
<td>11.37</td>
<td>8.55</td>
<td>42.71</td>
<td>4.74</td>
</tr>
<tr>
<td>Sunflower</td>
<td>6.14</td>
<td>9.68</td>
<td>7.32</td>
<td>49.80</td>
<td>5.85</td>
</tr>
</tbody>
</table>

The estimates presented below - concerning costs, stocks in port and silo, length of stay of vessels in port, shortage levels in port and in silo - were based on a simulation run with TST = 7300 days.

The breakdown of the total annual variable costs into shipping costs (vessel hiring, fuel, port charges) and inventory costs is given in Table 30. Although the figures concern a particular scenario, they illustrate the relative magnitude of the several cost components: shipping costs are clearly dominant (90% of total, in the table) and within those, vessel hiring costs have, by far, the largest share.

Table 30 - Breakdown of shipping and inventory costs (1000 US $)

<table>
<thead>
<tr>
<th>Cost item</th>
<th>16 014 (90 %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shipping</td>
<td></td>
</tr>
<tr>
<td>vessel hiring</td>
<td>11 654 (73 %)</td>
</tr>
<tr>
<td>fuel</td>
<td>1 197 (20 %)</td>
</tr>
<tr>
<td>port charges</td>
<td>1 163 (7 %)</td>
</tr>
<tr>
<td>Inventory</td>
<td>1 843 (10 %)</td>
</tr>
<tr>
<td>Total</td>
<td>17 857</td>
</tr>
</tbody>
</table>

Table 31(i) presents the observed mean stocks in port for each raw material. These figures are compared with the average net stocks in port

\[ \frac{Q_i}{2} + BS_i = \frac{Q_i}{2} + r_i(Q_i). \]

As expected, in view of the low levels of shortages in port, the
differences between them are small.

Table 31 - Stocks in port and in silo

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Av. stock in port</th>
<th>Buffer stock</th>
<th>Av. net stock in port</th>
</tr>
</thead>
<tbody>
<tr>
<td>(estimated)</td>
<td>(Bm)</td>
<td>(BS),</td>
<td>(Qn/2 + BS)</td>
</tr>
<tr>
<td>Maize</td>
<td>23.28</td>
<td>11.95</td>
<td>22.87</td>
</tr>
<tr>
<td>Sorghum</td>
<td>14.97</td>
<td>6.19</td>
<td>13.88</td>
</tr>
<tr>
<td>Wheat</td>
<td>14.72</td>
<td>6.19</td>
<td>14.97</td>
</tr>
<tr>
<td>Soybean</td>
<td>8.67</td>
<td>2.84</td>
<td>8.63</td>
</tr>
<tr>
<td>Peanut</td>
<td>3.77</td>
<td>1.55</td>
<td>3.72</td>
</tr>
<tr>
<td>Sunflower</td>
<td>5.01</td>
<td>1.88</td>
<td>4.85</td>
</tr>
</tbody>
</table>

(ii) Distribution of the aggregate stock in silo

<table>
<thead>
<tr>
<th>Volume Range</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 40 000 m³</td>
<td>0.0%</td>
</tr>
<tr>
<td>40-50 000 m³</td>
<td>1.1%</td>
</tr>
<tr>
<td>50-60 000 m³</td>
<td>2.3%</td>
</tr>
<tr>
<td>60-70 000 m³</td>
<td>4.5%</td>
</tr>
<tr>
<td>70-80 000 m³</td>
<td>14.4%</td>
</tr>
<tr>
<td>80-91 800 m³</td>
<td>27.4%</td>
</tr>
<tr>
<td>&gt; 91 800 m³</td>
<td>50.3%</td>
</tr>
</tbody>
</table>

The distribution of the aggregate stock of all raw materials in the silo, expressed in volume units, is shown in Table 31(ii). The silo is full for about 50% of the time and its average occupation (given by the ratio average stock in silo/silo effective capacity) is 92.6%. These figures (similar to others obtained under different cost scenarios) clearly illustrate the tightness of the silo capacity constraint.

Table 32 shows the length of stay of vessels in port. It is interesting to notice the comparatively small queueing times (the queue is empty over 80% of the time). The unloading operations, necessarily affected by the fact that the unloading capacity cannot be fully used when the silo is full (i.e. 50% of the time), take most of the time that vessels spent in port. The optimal nominal
unloading rates vary substantially amongst the different raw materials, according to their specific gravity.

Table 32 - Length of stay of vessels in port. Optimal nominal rates of unloading

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Av. queuing time</th>
<th>Av. unloading time</th>
<th>Berthing+berth. time</th>
<th>Av. length of stay in port (days)</th>
<th>Nominal unloading rate [t./t./day]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>1.14</td>
<td>10.99</td>
<td>0.88</td>
<td>13.01</td>
<td>1679</td>
</tr>
<tr>
<td>Sorghum</td>
<td>1.29</td>
<td>9.27</td>
<td></td>
<td>11.44</td>
<td>1311</td>
</tr>
<tr>
<td>Wheat</td>
<td>1.06</td>
<td>9.04</td>
<td></td>
<td>10.98</td>
<td>1600</td>
</tr>
<tr>
<td>Soybean</td>
<td>2.12</td>
<td>6.26</td>
<td></td>
<td>9.26</td>
<td>1249</td>
</tr>
<tr>
<td>Peanut</td>
<td>1.56</td>
<td>5.37</td>
<td></td>
<td>7.81</td>
<td>554</td>
</tr>
<tr>
<td>Sunflower</td>
<td>2.04</td>
<td>4.82</td>
<td></td>
<td>7.74</td>
<td>793</td>
</tr>
</tbody>
</table>

In deriving the solution that is being described, the buffer stocks were set at

$$BS_i = T_i(Q_i).$$

Since, for all raw materials, $T_i / C > 1.40$ (see Table 29) only the constraints on the fraction of replenishment cycles involving a shortage

$$\left(\frac{\beta_i}{s_i}\right) \leq 5\% \quad (i=1,\ldots,6)$$

are active (for reasons presented in section 5.2).

Table 33 shows, for each raw material, the shortage levels in port and in silo. In order to keep the differences between the shortage levels ($\beta_i / s_i$) and ($\beta_i / s_i$) in the right perspective, the absolute number of shortages in port (expected and observed) and in silo were included in the table.

Table 33 - Shortages in port and in silo

<table>
<thead>
<tr>
<th>Raw material</th>
<th>No. of shortages (over a period of 750 days)</th>
<th>$\beta_i / s_i$ - fraction of replenishment cycles with shortages (%)</th>
<th>$\beta_i / s_i$ - fraction of time under shortage (%)</th>
<th>Average length of 'observed' shortages (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port Expected 'Observed'</td>
<td>Silo 'Observed'</td>
<td>Port Expected 'Observed'</td>
<td>Silo 'Observed'</td>
</tr>
<tr>
<td>Maize</td>
<td>14.7</td>
<td>16</td>
<td>17</td>
<td>3.0</td>
</tr>
<tr>
<td>Sorghum</td>
<td>10.6</td>
<td>7</td>
<td>11</td>
<td>3.0</td>
</tr>
<tr>
<td>Wheat</td>
<td>8.6</td>
<td>9</td>
<td>12</td>
<td>5.0</td>
</tr>
<tr>
<td>Soybean</td>
<td>5.8</td>
<td>8</td>
<td>8</td>
<td>5.0</td>
</tr>
<tr>
<td>Peanut</td>
<td>8.7</td>
<td>4</td>
<td>8</td>
<td>5.0</td>
</tr>
<tr>
<td>Sunflower</td>
<td>7.4</td>
<td>9</td>
<td>10</td>
<td>5.0</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>47</td>
<td>66</td>
<td>-</td>
</tr>
</tbody>
</table>
The comparison between the expected and the 'observed' number of shortages - quite different for some of the raw materials - illustrates the difficulty of obtaining reliable estimates of the shortage levels from the multi-product simulation model (a consequence of the fact that shortages are 'rare events').

Nevertheless, the results show that the difference between the total number of shortages in silo and in port is, in absolute terms, quite small: over the simulated 20-year period, the observed number of shortages per year in port and in silo were, respectively, \( \frac{47}{20} = 2.35 \) and \( \frac{66}{20} = 3.30 \). This result is consistent with the small time spent by the vessels in port before starting the unloading operation.

The table shows, in addition, the shortages levels \( \beta_p' \) (naturally smaller than 1%) and \( \beta_s' \), as well as the average length of the 'observed' shortages in port and in silo.

8.2.2 Adjustment of the Buffer Stocks

The small magnitude of the difference between the number of shortages per year in silo and in port - one shortage a year in the solution just described - reinforces the arguments presented in favour of the current problem formulation (in which the shortage levels in port, rather than in silo, are restricted to remain within specified levels). It seems acceptable to assume that, if required, management will take exceptional action to minimize the difference between the shortage levels in port and in silo.

An alternative procedure is to increase each buffer stock \( BS_i = r_i(Q_i) \) by an amount \( EBS_i \) sufficiently large to cover consumption during the
average time $\text{lag}(\bar{y})$ between a vessel's arrival and the beginning of its unloading operation. In theory, such a procedure would lead to shortage levels in silo identical to those initially assumed for the shortages in port only if all the vessels carrying each raw material spent the same time in port before the beginning of their unloading operations. In this situation, vessels arriving on dates $d \sim N(d_0, \sigma^2)$ would start unloading on dates $d' \sim N(d'_0 = d_0 + \bar{y} \ , \ \sigma^2)$. Clearly for each shortage in port occurring when the buffer stocks are set at $BS_i = I_i(Q_i)$, there would be an identical shortage in silo if the buffer stocks were increased by $EBS_i$ (as defined above).

When the time lag $\bar{y}$ between the vessels' arrivals and the starting of their unloading operations is not constant, the shape of the distribution of $d'$ will be different from that of $d$. Consequently, the shortage levels in silo obtained after increasing the buffer stocks by $EBS_i$ will be generally different from those in port when $BS_i = I_i(Q_i)$. However, in reality, this difference is negligible since the variance of the distribution of the delays in the arrivals of vessels ($\sigma^2 = 81$) is much larger than the variance of $\bar{y}$ (between 1 and 4, according to the simulation results).

Given the tightness of the silo volume constraint one can expect that increasing the buffer stocks without changing the vessel sizes (therefore increasing the average stock in port) will affect the vessel's queueing time. This is confirmed by the results shown in Figure 47(i). Solution '1' is the solution described earlier, with all $EBS_i$ set at zero. In solution '2' the $EBS_i$'s are set at levels just large enough to cover each raw material's consumption during the queueing and berthing time (estimated from solution '1'). As expected, as the stocks increase so does the queueing time and a new correction of the $EBS_i$'s is therefore required. The 'nearly right' values of the
Figure 47 - Adjustment of the optimal solution to satisfy given levels of shortage in silo

(1) Reduction of the shortage levels in silo resulting from the adjustment of the buffer stocks

<table>
<thead>
<tr>
<th>Raw material</th>
<th>SOLUTION 1</th>
<th></th>
<th>SOLUTION 2</th>
<th></th>
<th>SOLUTION 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EBS$_1$</td>
<td>No. of</td>
<td>No. of</td>
<td>EBS$_1$</td>
<td>No. of</td>
</tr>
<tr>
<td></td>
<td>(days of</td>
<td>shortages</td>
<td>shortages</td>
<td>(days of</td>
<td>shortages</td>
</tr>
<tr>
<td></td>
<td>consumpt.)</td>
<td>in port</td>
<td>in silo</td>
<td>consumpt.)</td>
<td>in silo</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.00</td>
<td>16</td>
<td>17</td>
<td>1.58</td>
<td>1.58</td>
</tr>
<tr>
<td>Sorghum</td>
<td>0.00</td>
<td>7</td>
<td>11</td>
<td>1.73</td>
<td>1.73</td>
</tr>
<tr>
<td>Soybean</td>
<td>0.00</td>
<td>9</td>
<td>12</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Peanut</td>
<td>0.00</td>
<td>4</td>
<td>8</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Sunflower</td>
<td>0.00</td>
<td>9</td>
<td>10</td>
<td>2.48</td>
<td>2.48</td>
</tr>
<tr>
<td>Total variable cost (*)</td>
<td>17 856</td>
<td></td>
<td>18 437</td>
<td></td>
<td>18 683</td>
</tr>
</tbody>
</table>

(*) Expressed in [1000 US $]

(1i) Extrapolation method to determine EBS$_1$
(from solutions 1 and 2 to solution 3)

- Extra buffer stock (EBS) [days of consumption]
EBS\(_i\)'s adopted in solution '3' were derived, for each raw material, with the aid of the trivial extrapolation method shown in Figure 47(ii).

The reduction of the number of shortages in silo from solution '1' to solution '3' - to levels similar to those in port in solution '1' - is achieved at the expense of a considerable cost increase (about US$ 800 000 per annum, i.e. about 4.5% of the total variable costs). In the procedure described above no attempt was made to redefine the optimal shipment sizes, after the buffer stocks were increased. It is therefore conceivable that some improvement in relation to the solution '3' could be achieved, by 'reoptimizing' the shipment sizes \(Q_i\) after each adjustment of the buffer stocks. However, by adopting this procedure no significant cost reduction was achieved in relation to solution '3' (the shipment sizes underwent only slight adjustments in relation to that solution and the total variable cost was reduced from US$ 827 000 to US$ 794 000).

The question that remains to be answered is what is more acceptable for the management of the grain terminal:

(i) to adopt solution '1' and to take exceptional measures aimed at reducing the shortage levels in silo, or

(ii) to pay around 800 000 US$/year to avoid the need for such measures.

The small number of occasions in which exceptional action would be required if policy (i) was adopted and the substantial extra costs involved in policy (ii) strongly suggest that the former policy is more favourable than the latter.
8.2.3 The Unattractiveness of Speculative Storage

The delivery-inventory policies derived with the aid of the simulation-optimization model assume no speculative storage. In other words, the stocks are never raised above the levels required to insure a regular supply of the imported raw materials at the minimum expected delivery and inventory cost. In order to check whether the assumption is acceptable the volume of the silo was parametrized and the increase in the minimum delivery-inventory cost resulting from the use of part of the silo capacity for speculative storage was evaluated.

The parametrization was carried out running the simulation-optimization model for different values of the effective volume capacity \( V \), setting \( TST = 1825 \) days and \( EBS_i = 0 \) (\( i = 1, \ldots, 6 \)). Figure 48 shows the minimum variable shipping and inventory costs \( (VC) \) obtained using a particular set of streams of random numbers. The parametrization was repeated with other sets and the curves obtained were nearly parallel to the one shown (this means that the errors of the cost estimates derived using the same random numbers are positively correlated; this correlation is obviously beneficial, since it increases the accuracy of the cost comparisons).

The magnitude of the cost associated with the use of volume capacity for speculative storage is illustrated in the diagram: if the volume equivalent to only two weeks of consumption of the imported raw materials was set aside for that purpose, the variable cost would increase by about US$ 5.7 million/year. This is equivalent to 4.4% of the total annual purchasing cost of all raw materials.
The first step in the analysis of whether or not speculative storage can be of any benefit is to compare, at any point in time, the costs associated with

(i) buying a raw material in the 'cash' market (i.e. for immediate delivery) before it is strictly needed to satisfy demand and to hold the delivered quantity (a speculative stock) until this moment comes;

(ii) buying the same quantity of the raw material in the 'futures' market setting the delivery date as late as possible (this is equivalent to buy the raw material and keep it stored abroad until it is actually required for consumption).

The cost of carrying a speculative stock equivalent to one month of consumption of maize, for example, can be estimated as follows:

(i) volume required to store the equivalent to one month of consumption:

\[
\frac{320,000 \text{ m.t./year}}{12 \text{ months/year}} \times \frac{1}{0.75 \text{ m.t./m}^3} = 35,556 \text{ m}^3
\]
(ii) extra variable cost incurred annually (from Figure 48):
US$ 4.0 million

(iii) cost of capital employed on the speculative stock
(15% per annum): (\frac{320\ 000\ \text{m.t./year}}{12\ \text{months/year}} \times 120\ \text{US$/m.t.$}).0.15 =\]
= 3\ 200\ 000.\ 0.15 = \text{US$}\ 480\ 000

(iv) total cost/year
\text{US$}\ (4\ 000\ 000 + 480\ 000) = \text{US$}\ 4.48\ million

(v) monthly cost expressed as a percentage of the purchasing
cost of the amount of maize held:
\(\frac{\text{US$}\ 4.48\ million}{12}\) 
\text{US$}\ 3.2\ million) \times 100 = 11.7\%\ per\ month

Table 34 gives estimates of the monthly cost of carrying speculative
stocks equivalent to one month of consumption of each of the imported
raw materials.

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Monthly holding rate (%age of the value of the stock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>11.7</td>
</tr>
<tr>
<td>Sorghum</td>
<td>12.7</td>
</tr>
<tr>
<td>Wheat</td>
<td>7.9</td>
</tr>
<tr>
<td>Soybean</td>
<td>5.0</td>
</tr>
<tr>
<td>Peanut</td>
<td>5.4</td>
</tr>
<tr>
<td>Sunflower</td>
<td>5.7</td>
</tr>
</tbody>
</table>

These monthly holding rates are extremely high (particularly for
the cheaper raw materials, i.e. the grains). This is an obvious
consequence of the tightness of the storage capacity constraint.

An approximate measure of the cost of storing the raw materials abroad
can be obtained, at any particular point in time, by comparing

(i) their 'cash' price (price quoted for immediate delivery)

with

(ii) their quoted 'futures' prices (prices quoted for delivery
at future dates).
A 'relative monthly margin' can be defined, for each 'future', as

$$\left\{ \left[ \frac{\text{'future' price} - \text{'cash' price}}{\text{'cash' price}} \right] \times 100 \right\} / m \%$$

where

$m$ is the time interval, in months, between the current date (i.e. the date on which the prices are quoted) and the middle of the month in which the 'future' contract reaches maturity.

Figure 49 shows the distributions of this margin for maize and soybean, at Chicago (the major US market for these commodities). The distributions were based on the average monthly prices (for maize) and mid-month prices (soybeans) and cover the periods 1967-1976 and 1970-1976, respectively. In both cases this margin never reached the monthly holding rates presented in Table 34 (11.7% for maize and 5% for soybean). This means that, over the period considered, it would never be beneficial to anticipate deliveries in relation to their deadlines and to carry speculative stocks in port.

No price data was available to derive the distributions of the relative monthly margin between 'futures' and 'cash' prices of the other grains and oilseeds. Although there are no reasons to suspect that they will differ substantially from those derived for maize and sorghum it is worthwhile considering the situation where that margin becomes larger than the cost of holding speculative stocks.

In this situation no advantage can result from buying the raw materials in the 'futures' market. However, this does not necessarily mean that a purchasing policy that involves holding speculative stocks when prices are expected to rise is better than the policy assumed throughout this study, in which the stocks are never raised above
Figure 49 - Distribution of the relative margin between 'futures' and 'cash' prices of maize and soybean.

(i) Maize, yellow no. 2, Chicago, monthly average prices, difference between all futures and cash, 1967-1976

(ii) Soybean, yellow no. 1, Chicago, mid month prices, difference between Jan, May and Sep futures and cash, 1970-1976

Source of prices: Chicago Board of Trade - Statistical Annuals
the levels required to insure a regular supply of the raw materials
at minimum delivery and inventory costs. In order to check whether
the former policy can be advantageous it is necessary to establish
a comparison between

(i) the likely savings in the purchasing costs resulting
from its adoption (under the latter policy the average
purchasing costs will tend, in the long term, to the
average market prices), and

(ii) the increase in the delivery - inventory costs resulting
from carrying speculative stocks in the silo.

Kingsman and Taylor developed models for the 'tactical'
buying of commodities in the 'cash' markets (i.e. purchases for
immediate delivery). These models, based on price forecasts derived
from 'statistical' models, were applied to a number of different
commodities. Their results indicate that the potential savings
that can be made in relation to the average market prices depend
on the maximum amount of the commodity that can be stored (for speculative
purposes) at any time. Typically, the largest savings are obtained
when the speculative stocks can be raised to about two months of consumption
and, when the stock holding cost is taken into account, those savings
do not exceed 2% of the average market prices. Larger values of the
maximum allowable stock do not lead to larger savings in view of the
inability to produce sufficiently accurate statistical price forecasts for
more than two months ahead.

When behavioural price models are used to derive medium term price
forecasts (3 months ahead or more) the potential savings could be further
increased, if the maximum allowable speculative stock could be increased
to more than two months' consumption. However this is not practicable
in the situation under analysis, since the silo storage capacity is very
limited.
Table 35 shows the extra delivery-inventory costs that would be incurred if the silo volume required to store half, one or two months of consumption of each raw material was set aside permanently (for maize only half and one month's consumption were considered; storage of two months' consumption would not be practicable with the current silo capacity).

Table 35 - Extra cost associated with the use of part of the silo for 'speculative storage'

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Volume set aside as half a month of consumption of each r. material</th>
<th>Volume set aside as one month of consumption of each r. material</th>
<th>Volume set aside as two months of consumption of each r. material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>4.2</td>
<td>10.5</td>
<td>-</td>
</tr>
<tr>
<td>Sorghum</td>
<td>5.0</td>
<td>11.5</td>
<td>26.9</td>
</tr>
<tr>
<td>Wheat</td>
<td>2.9</td>
<td>6.6</td>
<td>15.5</td>
</tr>
<tr>
<td>Soybean</td>
<td>1.8</td>
<td>3.8</td>
<td>8.3</td>
</tr>
<tr>
<td>Peanut</td>
<td>1.9</td>
<td>4.2</td>
<td>9.3</td>
</tr>
<tr>
<td>Sunflower</td>
<td>1.9</td>
<td>4.5</td>
<td>9.7</td>
</tr>
</tbody>
</table>

These extra costs were derived by:

(i) calculating the volume required to store half, one or two months' consumption of each raw material;

(ii) estimating, from the diagram given in Figure 48, the increase in the minimum annual delivery-inventory costs associated with the reduction of the effective storage capacity, in each case;

(iii) expressing each cost increase as a percentage of the annual average purchasing cost of the raw material stored.

If the maximum allowable speculative stock of one raw material (for example sorghum) is set at, say, two months' consumption, the
the speculative stock carried will vary over time between zero and two months' consumption and its average value will approach one month's consumption, in the long run.

Under these conditions, it would clearly be unreasonable to set aside permanently the volume required to store two months' consumption, not making full use of the silo capacity when the sorghum speculative stock held is below its maximum.

If that space, when it becomes available, is used to store any raw material, in the long run, the average reduction in the effective capacity of the silo would be equivalent to one month of sorghum consumption. The increase in the delivery-inventory cost (ΔVC) ensuing in this situation is difficult to estimate. However, a lower bound on its value is provided by the cost increase that would be incurred if the volume equivalent to one month of consumption of sorghum was set aside permanently (Δ'VC = 11.5% of the sorghum purchasing cost, according to Table 35).

This cost is necessarily smaller than ΔVC for two reasons:

(i) Δ'VC is the extra cost that would be incurred if a constant speculative stock of one month's consumption of sorghum was kept permanently in silo; in reality the stock will vary over time, around this (expected) value; it can easily be shown that the expected cost increase in this case is larger than ΔVC, since the curve represented in Figure 48 has a positive second derivative (\( \frac{d^2 \text{VC}}{\partial V^2} > 0 \));

(ii) Δ'VC was derived assuming that the sizes of shipments (and the buffer stocks) are 'optimal' when, in reality, this cannot be achieved; in fact, due to the nature of the
tactical purchasing problem (under uncertain prices)
it is not possible to predict when the speculative stocks
will be raised and to what levels; this implies that
deliveries set 'optimally' before a decision to raise
the speculative stock is taken will become generally
'non-optimal' after the stock is increased.

The extra costs incurred by setting aside the volume required
to store one month's consumption of each raw material - shown in
Table 35 - represent therefore underestimates of the extra delivery-
inventory costs incurred under purchasing policies with maximum
allowable speculative stocks of two months' consumption. The magnitude
of these costs (better, their underestimates) is such as to render
these policies clearly unprofitable. A comparison between the extra
costs and the potential savings for maximum allowable speculative
stocks of one month's consumption leads to the same conclusion (the
savings, in this case, typically do not exceed 1% of the average market
prices).

8.2.4 Robustness of the Optimal Solution. Validation of the Model's
Static Assumption

The optimal solutions obtained from the proposed model were derived
under stationary conditions. However, many of the parameters of the
model change considerably over time. In order to check the validity
of the model it is therefore necessary to verify whether the optimal
solutions are robust with regard to parameter changes which are likely
to occur, particularly those concerning:

(i) vessel hiring costs (through the trip charter rate)
(ii) fuel costs
(iii) prices of raw materials
(iv) shifts between the demands of maize and sorghum (the
aggregate demand of these two raw materials and the 
demands of the others are likely to suffer only small 
adjustments from time to time).

The effect on the optimal solutions resulting from changes in the 
vessel hiring cost is shown considering the most significant change 
in the GCBS trip charter rates that occurred in any 6-month period between 
the start of the published series (January 1975) and September 1980 
(the average quarterly values of the rates were shown in the previous 
chapter, in Figure 34). The largest change occurred between January 
1979 and July 1979. The GCBS trip charter rates for these months are 
shown in Table 36(i).

The optimal solutions obtained in each case—derived using the 
same set of streams of random numbers and setting TST = 1825 days 
and EBS_i = 0 (i=1, ..., 6)—are shown in Table 36(ii). Note that 
solution 'I' is the optimal solution considered in the previous sections.

As relevant as comparing the changes in the optimal shipment sizes 
is to compare, under one of the vessel hiring cost structures (July 
1979, say), the total annual variable cost corresponding to the 
adoption of each solution. This is shown in Table 36 (iii). The 
cost estimates where obtained using identical sets of streams of random 
numbers and setting TST = 7300 days.

Clearly, the departure from optimality has a very small effect on 
the total costs (within the range of errors introduced by the model 
itself).
Table 36 - Effect of the change in the trip charter rate from January to July 1979 on the optimal solution

<table>
<thead>
<tr>
<th>(i) GCBS trip charter rate</th>
<th>Average rate [US $/(m.t.DWT.month)]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>15,702</td>
<td>15,834</td>
</tr>
<tr>
<td>Handy size</td>
<td>26,267</td>
<td>27,496</td>
</tr>
<tr>
<td>Medium</td>
<td>36,779</td>
<td>39,742</td>
</tr>
<tr>
<td>Panamax</td>
<td>62,996</td>
<td>61,835</td>
</tr>
</tbody>
</table>

Source: General Council of British Shipping (1st Aug.1979)

(ii) Optimal solutions (shipment sizes in [1000 m.t.]) (*)

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Solution 'I' (Jul.1979)</th>
<th>Solution 'II' (Jan.1979)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>21.85</td>
<td>18.34</td>
</tr>
<tr>
<td>Sorghum</td>
<td>15.00</td>
<td>13.00</td>
</tr>
<tr>
<td>Wheat</td>
<td>17.57</td>
<td>15.45</td>
</tr>
<tr>
<td>Soybean</td>
<td>12.57</td>
<td>12.69</td>
</tr>
<tr>
<td>Peanut</td>
<td>4.33</td>
<td>5.70</td>
</tr>
<tr>
<td>Sunflower</td>
<td>6.14</td>
<td>5.09</td>
</tr>
</tbody>
</table>

(*) Solutions derived using the same random numbers and setting TST = 1825 days

(iii) Cost comparison under the July 1979 trip charter rate (*)

<table>
<thead>
<tr>
<th>Solution</th>
<th>Total Variable Cost [1000 US $]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (optimal)</td>
<td>17,857</td>
</tr>
<tr>
<td>II (non optimal)</td>
<td>17,940 (±0.47 %)</td>
</tr>
</tbody>
</table>

(*) Cost estimates based on simulation runs with the same random numbers and TST = 7300 days

The second major component of the total variable costs, the fuel cost, has suffered considerable increases since 1973. In Table 37 solution 'I' (optimal under the scenario described earlier in Table 25,
assuming a unit cost of fuel oil of US$ 90/m.t. is compared with solutions 'III' and 'IV' (which are optimal under fuel unit costs of 45 and 135 US$/m.t., respectively).

Table 37 - Effect of a ± 50% change in the fuel price on the 'optimal' solution

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Solution 'III'</th>
<th>Solution 'I'</th>
<th>Solution 'IV'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p_{fo} = 45 \text{ US$ /m.t.} )</td>
<td>( p_{fo} = 90 \text{ US$ /m.t.} )</td>
<td>( p_{fo} = 135 \text{ US$ /m.t.} )</td>
</tr>
<tr>
<td>Maize</td>
<td>21.03</td>
<td>21.85</td>
<td>22.84</td>
</tr>
<tr>
<td>Sorghum</td>
<td>15.05</td>
<td>15.00</td>
<td>16.67</td>
</tr>
<tr>
<td>Wheat</td>
<td>16.38</td>
<td>17.57</td>
<td>14.86</td>
</tr>
<tr>
<td>Soybean</td>
<td>11.03</td>
<td>11.57</td>
<td>13.00</td>
</tr>
<tr>
<td>Peanut</td>
<td>4.55</td>
<td>4.33</td>
<td>5.01</td>
</tr>
<tr>
<td>Sunflower</td>
<td>5.79</td>
<td>6.14</td>
<td>6.13</td>
</tr>
</tbody>
</table>

(*) Solutions derived using the same random numbers and setting TST = 1825 days

(iii) Cost comparisons (*)

(a) \( p_{fo} = 90 \text{ US$ /m.t.} \)

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Total variable cost [1000 US$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I ('optimal')</td>
<td>17 897</td>
</tr>
<tr>
<td>III (non 'optimal')</td>
<td>17 932 (+0.42 %)</td>
</tr>
</tbody>
</table>

(b) \( p_{fo} = 135 \text{ US$ /m.t.} \)

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Total Variable Cost [1000 US$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV ('optimal')</td>
<td>19 445</td>
</tr>
<tr>
<td>I (non 'optimal')</td>
<td>19 455 (+0.051 %)</td>
</tr>
</tbody>
</table>

(*) Cost estimates based on simulation runs with the same random numbers and TST = 7500 days

Again the differences between them are extremely small, with no significant impact on the total variable costs.
In Table 38, solution 'I' is compared with solutions 'V' and 'VI', derived after the prices of raw materials were respectively increased and decreased by 25%. The differences between them are very small, showing the robustness of the optimal solution to changes in the raw material's prices. Identical results were obtained changing the raw material's prices one by one within the ±25% range.

Table 38 - Effect of ±25% change in the price of the raw materials on the optimal solution

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Solution 'I' (p₁)</th>
<th>Solution 'V' (1.25 p₁)</th>
<th>Solution 'VI' (0.75 p₁)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>21.85</td>
<td>20.00</td>
<td>22.47</td>
</tr>
<tr>
<td>Sorghum</td>
<td>15.00</td>
<td>16.69</td>
<td>14.81</td>
</tr>
<tr>
<td>Wheat</td>
<td>17.57</td>
<td>15.51</td>
<td>17.95</td>
</tr>
<tr>
<td>Soybean</td>
<td>11.57</td>
<td>15.00</td>
<td>15.00</td>
</tr>
<tr>
<td>Peanut</td>
<td>4.33</td>
<td>4.77</td>
<td>5.00</td>
</tr>
<tr>
<td>Sunflower</td>
<td>6.14</td>
<td>5.59</td>
<td>5.72</td>
</tr>
</tbody>
</table>

(*) Solutions derived using the same random numbers and setting TST = 1825 days

(ii) Cost comparisons (*)

(a) Under 1.25 p₁

<table>
<thead>
<tr>
<th>Solution</th>
<th>Total variable cost [1000 US $]</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (optimal)</td>
<td>18 317</td>
</tr>
<tr>
<td>I (non optimal)</td>
<td>18 362 (+0.25 %)</td>
</tr>
</tbody>
</table>

(b) Under 0.75 p₁

<table>
<thead>
<tr>
<th>Solution</th>
<th>Total variable cost [1000 US $]</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI (optimal)</td>
<td>17 406</td>
</tr>
<tr>
<td>I (non optimal)</td>
<td>17 396 (-0.057 %) (***)</td>
</tr>
</tbody>
</table>

(*) Cost estimates based on simulation runs with the same random numbers and TST = 7500 days

(**) This negative value clearly indicates that the cost difference is below the margin of error introduced in the 'optimisation' and cost estimation processes.
The effect on the optimal solution resulting from shifts between the demands of maize and sorghum is shown in Table 39. In deriving the solutions for different values of the ratio sorghum demand/maize demand the fob prices of both these grains were kept constant, with the values defined earlier in Table 25. The justification for this procedure was given earlier: once the demands are set, the optimal solutions are not sensitive to changes in the prices of raw materials (the question of how to set the demands optimally for given 'fob' prices of maize and sorghum will be analysed in Chapter 7). The values of the ratio sorghum demand/maize demand were set at 1 : 1, 1 : 2 and 0. It will be shown in Chapter 7 that the 'optimal' values of that ratio are, in fact, either 0 or somewhere within the range between about 1 : 2 and 1 : 1 (depending on the relative fob prices of sorghum and maize).

The figures presented in Table 39(i) and (ii) again suggest a considerable robustness in the optimal solutions. The most noticeable difference between the solutions is the (inevitable) absence of sorghum shipments in solution 'VIII'. Even in this situation ($d_{sorghum} = 0$), the shipment sizes of the other raw materials are only moderately larger than those obtained in the other optimal solutions. The cost increase resulting from adopting, in this situation, shipments sizes as defined in solution 'VII' rather than the optimal ones, is only about 1.0% (see cost comparison (b) in Table 39 (ii)).

There is, however, one major difference between the parameter changes contemplated in Tables 36 to 38 and those considered in Table 39. In the former ones since the demands ($d_i$) were kept constant, small changes in the size of shipments ($Q_i$) imply small
Table 39 - Effect of changes in the mix maize-sorghum on the 'optimal' solution

(1) 'Optimal' solutions (shipment sizes in [1000 m.t.]) (*

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Solution 'VII'</th>
<th>Solution 'I'</th>
<th>Solution 'VIII'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($d_{scor}d_{mai}$ = 1 : 1)</td>
<td>(1 : 2)</td>
<td>(0)</td>
</tr>
<tr>
<td>Maize</td>
<td>21.41</td>
<td>21.85</td>
<td>25.00</td>
</tr>
<tr>
<td>Sorghum</td>
<td>15.05</td>
<td>15.00</td>
<td>-</td>
</tr>
<tr>
<td>Wheat</td>
<td>14.99</td>
<td>17.57</td>
<td>17.97</td>
</tr>
<tr>
<td>Soybean</td>
<td>12.02</td>
<td>11.57</td>
<td>13.04</td>
</tr>
<tr>
<td>Peanut</td>
<td>5.55</td>
<td>4.33</td>
<td>6.86</td>
</tr>
<tr>
<td>Sunflower</td>
<td>5.44</td>
<td>6.24</td>
<td>8.32</td>
</tr>
</tbody>
</table>

(*) Solutions derived using the same random numbers and setting TST = 1825 days

(ii) Cost comparisons (*

(a) $d_{scor} : d_{mai} = 1 : 2$

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Total variable cost [1000 US $]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I ('optimal')</td>
<td>17 897</td>
</tr>
<tr>
<td>VII (non 'optimal')</td>
<td>17 910 (+ 0.30 %)</td>
</tr>
</tbody>
</table>

(b) $d_{scor} = 0$

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Total variable cost [1000 US $]</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIII ('optimal')</td>
<td>15 954</td>
</tr>
<tr>
<td>VII with $Q_{cor} = 0$ (non 'optimal')</td>
<td>16 125 (+ 1.07 %)</td>
</tr>
</tbody>
</table>

(*) Cost estimates based on simulation runs with the same random numbers and TST = 7300 days

(iii) T/6 ratios and buffer stocks for maize and sorghum

<table>
<thead>
<tr>
<th>Raw materials</th>
<th>Solution 'VII'</th>
<th>Solution 'I'</th>
<th>Solution 'VIII'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T/6 BS [1000 m.t.]</td>
<td>T/6 BS [1000 m.t.]</td>
<td>T/6 BS [1000 m.t.]</td>
</tr>
<tr>
<td>Maize</td>
<td>3.62 9.48</td>
<td>2.77</td>
<td>11.95</td>
</tr>
<tr>
<td>Sorghum</td>
<td>2.54 6.98</td>
<td>3.80</td>
<td>6.38</td>
</tr>
</tbody>
</table>
adjustments of the ratios $T_i/q_i = Q_i/(d_i \cdot c_i)$. As a consequence, the buffer stocks suffer only minor adjustments.

In the situation where maize and sorghum demands change, the ratios $T/q$ and, consequently, the buffer stocks of these raw materials, can differ substantially from each optimal solution to the next. The buffer stocks corresponding to solutions 'VII', 'I' and 'VIII' are shown in Table 39(iii). As expected, the largest differences occur between solutions 'VII' and 'VIII'.

The transitions between such solutions can, in principle, cause problems. In fact, the relationships

(i) $s_o = BS$ (planned replenishment point = buffer stock; see section 5.1), and

(ii) $\beta_p = h_3$ (BS/Q, T/q) and $\beta_p = h_3$ (BS/Q, T/q)

(relationships between shortage levels in port and the relevant variables and parameters of the system; see section 5.2)

only hold in the 'steady state' situation. If the shipments are planned ignoring the perturbations arising during changes of state, the shortage levels in port can take values that are different from those specified initially. This is illustrated in Figure 50 considering the planning of sorghum shipments during the transition between solutions 'VII' $\rightarrow$ 'VIII' and 'VIII' $\rightarrow$'VII'.

Diagram (i) shows the 'planned stock' in port following the adoption of the operating doctrine described in section 5.1.
During the first period in which $d_{\text{sorghum}} = d_{\text{maize}} = d$, shipments of size $Q_{\text{sorghum}} = 15\ 050$ m.t. are planned to arrive at times $t_0$, $t_1$, $t_2$, $t_3$ (i.e. the times when the planned stock in port reaches the planned replenishment point $s_o = 8980$ m.t.).

After the interruption of sorghum supply (when $d_{\text{sorghum}} = 0$) the arrivals of sorghum shipments restart, planned following the same principle. The first shipment is planned to arrive at $t_4$, $\Delta = s_o/d$ days before the sorghum demand is due to restart.
(second period in which \( d_{sorghum} = d \)) and is followed by shipments planned to arrive at \( t_5, t_6, t_7, \) etc.

Clearly this planning procedure ignores the effects on the shortage levels resulting from the transition between the different states of the system. In order to check the magnitude of this effect, the probabilities of shortage for replenishment cycles were calculated. Since \( T = 22.9 \) days = \( 2.54 \cdot \sigma \), it can be assumed that a shipment planned to arrive at \( t_j \) will actually arrive (with near certainty) within the interval \( (t_{j-1}, t_{j+1}) \). This implies that

(i) within any planned replenishment cycle \( (t_j, t_{j+1}) \) only one shortage can occur;

(ii) this shortage will occur if and only if both shipments planned to arrive at \( t_j \) and \( t_{j+1} \), actually arrive after \( t_j + \Delta \);

(iii) the probability of shortage is therefore

\[
P = \left[ \int_{t_j + \Delta}^{\infty} f_j(t) \, dt \right] \cdot \left[ \int_{t_j + \Delta}^{\infty} f_{j+1}(t) \, dt \right]
\]

where

\( f_j(t) \) and \( f_{j+1}(t) \) are the probability density functions of the actual arrivals of shipments planned to arrive at \( t_j \) and \( t_{j+1} \) respectively.

Clearly for the planned replenishment cycle starting at \( t_3 \), the probability of shortage is simply given by

\[
P' = \int_{t_3 + \Delta}^{\infty} f_3(t) \, dt
\]

The probabilities of shortage for each planned replenishment
cycle are plotted in Figure 50(ii). Except for the planned replenishment cycle starting at $t_3$, they are all equal. The fact that their value differs from 5% (the calculated value was 5.49%) is attributable to errors in the definition of the relationship $BS_i = r_i(\theta_i)$. For the exceptional replenishment cycle, the calculated value of the probability of shortage was 6.49%. The difference between this figure and 5% (or 5.49%) can be eliminated, by adjusting the planned arrival $t_3$ to $t'_3 = t_3 - \delta_3$ (see Figure 50(i)). The value of $\delta_3$ required to reduce the probability of shortage to 5.49% - 0.77 days - is so small that it does not affect significantly the probability of shortage in the previous replenishment cycle.

The case just analysed illustrates how the transition between successive optimal solutions can be made smoothly without major problems. Clearly, for values of the ratio $T/\theta$ lower than the one considered, the calculation of the probabilities of shortage becomes more complicated (as a result of more interactions between successive arrivals). However the type of analysis required to plan the transition in such cases is basically the same.

In addition to the sensitivity analyses described - concerning major parameter changes likely to occur over time - the robustness of the optimal solution was submitted to further tests covering changes in

(i) the unloading capacity (varied between 90% and 110% of the assumed value), and in

(ii) the extra capacity of delivery from silo to the customers, used during periods of recovery from shortage (this capacity, previously assumed to be 15% of the normal delivery rate, was varied between 10% and 20% of this rate).
The effects of these changes on the optimal solutions were imperceptible (within the margin of error introduced either in the optimization or in the cost estimation). The interpretation of these results is simple. The insensitivity of the optimal solution to changes in the unloading capacity can be attributed to the near full utilization of the silo capacity. In fact, when the silo becomes full during the unloading of a vessel the rate of unloading becomes dependent on the outflow of raw materials from the silo, rather than on the installed unloading capacity.

The insensitivity of the optimal solution to changes in the value of the extra capacity of delivery from the silo can be attributed to the reduced number of occasions in which that extra capacity is necessary - i.e. the small number of shortages in silo.

The general implications of the results shown in this section can be summarised as follows:

(i) as some of the parameters of the system change over time, the transition between successive optimal delivery-inventory policies can be made smoothly, most times through small adjustments - this is the basic justification for the acceptance of the 'static' model as a useful tool to derive delivery-inventory policies;

(ii) given the considerable robustness of the optimal solutions, it is not critical (for the purpose of deriving delivery-inventory policies) to predict or to estimate accurately parameters such as raw materials' prices, shipping cost factors, unloading capacity or extra delivery capacity from the silo to the customers.
8.3 The Operations at the Oporto New Terminal (Other Scenarios)

Although section 8.2 was mainly based on the scenario presented earlier in Table 25, it also included results concerning other scenarios. Of particular importance was the recognition that the optimal solutions (i.e. optimal vessel sizes and buffer stocks) are extremely robust to changes in the highly variable cost parameters.

The results presented earlier indicate that even large changes in these parameters, such as those observed over recent years, would not lead to substantial changes in the optimal solutions. These were derived for widely different values of the main cost parameters:

(i) the GCBS trip charter rate was set at the levels prevailing at January 1979 - representing an average low value of the series since the beginning of its publication in 1975 - and July 1979 - the highest value of the rate available at the time of the study (see Figure 34, chapter 5); the rate reached an all-time record level in May 1980 but even the increase is not expected to alter the optimal solution significantly;

(ii) the fuel costs were varied within the range 45 to 135 US$/m.t.;

(iii) the raw materials' prices were scaled by ±25% in relation to their values in the basic scenario.

The fact the optimal solution for each of these situations was quite close to that obtained for the basic scenario implies that this solution - extensively discussed in the previous section - is fairly representative for the period analysed. Properties of this basic solution, such as the relation between shortage levels in silo
and in port, the tightness of the silo volume constraint or the
time that vessels spend in port, can therefore be generalised to
the other optimal solutions. However there is one aspect of the delivery-
inventory operations that changes considerably from scenario to
scenario, namely the magnitude of the costs involved.

From the several results presented earlier - concerning
the particular scenario defined in Table 25 - the only one
depending critically on the assumed values of the cost parameters
was that showing the unattractiveness of speculative storage.
The analysis carried out involved establishing comparisons between

(i) the cost of holding speculative stocks at the Oporto
terminal and the cost of keeping them abroad (i.e. 
to buy for forward delivery);

(ii) the potential savings in purchasing costs and the increase
in the delivery-inventory costs that would result from
carrying speculative stocks.

For the purpose of checking whether speculative storage is
generally unattractive, it is clearly relevant to extend these comparisons
to a scenario in which the ratio (raw materials' prices/shipping rates)
is close to its highest value, either for grains or oilseeds. The
reason for choosing the highest value of the ratio is that

(i) as the shipping rates decrease so will the extra
shipping cost incurred as a result of carrying speculative
stocks and

(ii) as the raw materials' prices increase so will the potential
savings associated with the practice of speculative storage.
Table 40 provides a comparison between the magnitude of the raw materials' prices (maize and soybean) and the biggest component of the shipping costs - the vessel hiring cost (shown for the most relevant range of vessel sizes).

Table 40 - Raw materials' prices v. shipping freight rates (1971-1980)

<table>
<thead>
<tr>
<th>Period (1)</th>
<th>US maize av. export price (fob - Gulf)(2) [US $ / m.t.]</th>
<th>US soyb. av. export price (fob - Gulf)(3) [US $ / m.t.]</th>
<th>Freight rates of small bulk carriers(4) [US $/(DWT, month)]</th>
<th>F,mat.prices/freight rates (Index = 100 for scenario defined in table 25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971-72</td>
<td>50.30</td>
<td>(130.0)</td>
<td>(3.32)</td>
<td>149 186</td>
</tr>
<tr>
<td>1972-73</td>
<td>95.59</td>
<td>(247.7)</td>
<td>(7.22)</td>
<td>117 162</td>
</tr>
<tr>
<td>1973-74</td>
<td>127.58</td>
<td>(241.7)</td>
<td>(6.90)</td>
<td>136 129</td>
</tr>
<tr>
<td>1974-75</td>
<td>130.68</td>
<td>252.9</td>
<td>(7.95)</td>
<td>161 151</td>
</tr>
<tr>
<td>1975-76</td>
<td>137.29</td>
<td>206.4</td>
<td>6.72</td>
<td>172 145</td>
</tr>
<tr>
<td>1976-77</td>
<td>100.61</td>
<td>284.2</td>
<td>6.78</td>
<td>147 199</td>
</tr>
<tr>
<td>1977-78</td>
<td>104.33</td>
<td>242.5</td>
<td>7.40</td>
<td>159 155</td>
</tr>
<tr>
<td>1979-79</td>
<td>110.63</td>
<td>275.4</td>
<td>9.52</td>
<td>115 136</td>
</tr>
<tr>
<td>1979-80</td>
<td>n.a.</td>
<td>256.2</td>
<td>13.75</td>
<td>n.a. 88</td>
</tr>
</tbody>
</table>

(2) Source : USDA Foreign Agriculture Service
(3) Source : USDA Foreign Agriculture Service
Figures in brackets : derived from the Chicago cash prices (in Chicago B.T. Statistical Annuals) ;
7.5% margin added on
(4) 1971-75 : Shipping State & Economics time charter rate (multi-deckers, 10-19999 DWT)
1975-80 : GCBS trip charter rate (bulk carriers, 12-19999 DWT)

During the 1970's, the highest value of the ratio soybean price/vessel hiring cost occurred in 1976-77. A scenario reflecting the conditions prevailing in this period - the one in which speculative storage of oilseeds is potentially most attractive - was chosen to test the assumption. This scenario differs from that defined in Table 25 in the values adopted for the trip charter rate, the raw materials prices and the fuel cost. These were set as follows:

(i) trip charter rate: average GCBS rate over the period covering the last quarter of 1976 and the three first quarters of 1977;
(ii) raw materials' prices:
   (a) maize and soybean: average export prices (fob) during the 1976-77 crops (as defined in Table 40);
   (b) other raw materials: price ratios sorghum/maize, wheat/maize, peanut/soybean and sunflower/soybean as in the scenario defined in Table 25.

(iii) unit cost of fuel oil: US$60/m.t.

Figure 51 presents, for this new scenario,

(i) the curve relating the variable delivery-inventory costs with the effective capacity of the silo;

(ii) the cost of holding speculative stocks equivalent to one month of consumption of each raw material;

(iii) the extra delivery-inventory cost associated with the use of part of the silo for speculative storage.

The costs shown in Figure 51(ii) and (iii) are substantially lower than those derived for the previous scenario (see Tables 34 and 35). Nevertheless, they are still of such a magnitude as to render speculative storage unattractive.

All the results presented so far were derived assuming that the vessels are unloaded following what was defined earlier, in section 3.2.6, as rule '0'. The alternative rule defined in that section as rule '1' was analysed, with all other parameters set as in the scenario defined in Table 25, but it was found to be slightly inferior to rule '0':

(i) it led to an increase in the number of shortages in silo, in relation to rule '0';
Figure 51 - The cost of 'speculative storage' (scenario: 1976-1977)

1. Variable shipping and inventory costs v. silo effective capacity

- Total annual variable cost [in US $]
- Extra annual cost = US $ 3.3 million
- Volume required to store the equivalent to 2 weeks of consumption of all raw materials
- Current full silo capacity
- Silo effective capacity

2. Cost of holding 'speculative stocks' equivalent to one month of consumption of each raw material

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Monthly holding rate</th>
<th>(Percentage of the value of the stock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>Sorghum</td>
<td>8.6</td>
<td></td>
</tr>
<tr>
<td>Wheat</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>Soybean</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td>Peanut</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>Sunflower</td>
<td>3.1</td>
<td></td>
</tr>
</tbody>
</table>

3. Extra shipping and inventory costs associated with the use of part of the silo for 'speculative storage'

Annual extra cost (expressed as percentage of each raw material's annual purchasing cost)

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Volume set aside as half a month of consumption of each raw material</th>
<th>Volume set aside as one month of consumption of each raw material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>2.8</td>
<td>6.6</td>
</tr>
<tr>
<td>Sorghum</td>
<td>3.3</td>
<td>7.4</td>
</tr>
<tr>
<td>Wheat</td>
<td>1.7</td>
<td>4.3</td>
</tr>
<tr>
<td>Soybean</td>
<td>0.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Peanut</td>
<td>0.8</td>
<td>1.7</td>
</tr>
<tr>
<td>Sunflower</td>
<td>0.9</td>
<td>1.9</td>
</tr>
</tbody>
</table>
(ii) when the shortage levels in silo were reduced to those obtained under rule '0' (by increasing the buffer stocks), the variable delivery-inventory costs were about 1% larger than those obtained using rule '0'.

8.4 The Operations at Lisbon

The analysis of the delivery-inventory operations at Lisbon was initially carried out assuming that all imported grains and oilseeds were to be handled at the new terminal alone (as proposed in the study commissioned by EPAC from Hidrotécnica Portuguesa 31 ).

Such an analysis revealed that, under this modus operandi, the efficiency of the delivery-inventory operations would be severely impaired by the limited storage capacity planned for the new silo. This can be illustrated by considering the optimal solution obtained under a scenario similar to the one presented earlier in Table 25 (changing only the demands and the grain terminal operating parameters, as appropriate for Lisbon). Table 41 shows some important characteristics of the Lisbon solution, namely:

(i) the shipment sizes and related parameters,
(ii) the distribution of the aggregate stocks in silo, and
(iii) the shortage levels in port and in silo.

The figures presented in the table demonstrate clearly that, if the terminal was used to unload and store all the imported raw materials, it would become highly congested. With a silo storage capacity equivalent to approximately two weeks of supply of the imported raw materials, these raw materials inevitably would have to be transported in small and frequent shipments - about 120 shipments per year. Even for these small shipment sizes, the silo
Table 41 - Optimal solution for Lisbon: new terminal alone and scenario identical to the one defined in Table 25 (with demands and operating parameters of the terminal adapted as appropriate) (*)

### (i) Shipment sizes and related parameters

<table>
<thead>
<tr>
<th>Raw material size (DWT)</th>
<th>Shipment size</th>
<th>No. of vessels replenish per year</th>
<th>Av. cycle days</th>
<th>Ratio T/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 m.t.</td>
<td>29.85</td>
<td>34.49</td>
<td>43.10</td>
<td>8.47</td>
</tr>
<tr>
<td>Maize</td>
<td>24.24</td>
<td>33.25</td>
<td>26.54</td>
<td>13.75</td>
</tr>
<tr>
<td>Sorghum</td>
<td>21.98</td>
<td>24.42</td>
<td>15.92</td>
<td>22.92</td>
</tr>
<tr>
<td>Wheat</td>
<td>16.23</td>
<td>18.75</td>
<td>9.28</td>
<td>36.57</td>
</tr>
<tr>
<td>Soybean</td>
<td>7.55</td>
<td>19.30</td>
<td>12.24</td>
<td>29.80</td>
</tr>
<tr>
<td>Peanut</td>
<td>8.70</td>
<td>13.71</td>
<td>12.41</td>
<td>29.40</td>
</tr>
</tbody>
</table>

### (ii) Distribution of the aggregate stock in silo

- < 100,000 m³: 0.0 %
- 100-110,000 m³: 4.0 %
- 110-120,000 m³: 0.5 %
- 120-130,000 m³: 4.4 %
- 130-140,000 m³: 5.1 %
- 140-151,000 m³: 10.3 %
- 151,000 m³: 74.8 % (full silo)

### (iii) Shortage levels in port and in silo [%]

<table>
<thead>
<tr>
<th>Raw material</th>
<th>T/s</th>
<th>$T/\bar{s}$ - fraction of time under shortage</th>
<th>$T/\bar{s}$ - fraction of rep. cycles with short.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Port</td>
<td>Silo</td>
</tr>
<tr>
<td>Maize</td>
<td>0.94</td>
<td>1.26</td>
<td>2.90</td>
</tr>
<tr>
<td>Sorghum</td>
<td>1.53</td>
<td>-</td>
<td>5.59</td>
</tr>
<tr>
<td>Wheat</td>
<td>2.55</td>
<td>-</td>
<td>5.62</td>
</tr>
<tr>
<td>Soybean</td>
<td>4.06</td>
<td>-</td>
<td>2.96</td>
</tr>
<tr>
<td>Peanut</td>
<td>3.31</td>
<td>-</td>
<td>3.28</td>
</tr>
<tr>
<td>Sunflower</td>
<td>3.27</td>
<td>-</td>
<td>7.20</td>
</tr>
</tbody>
</table>

### (iv) Annual variable cost

\[ VC = \text{US}\$ 45,548,000 \]

(*) Solution derived setting TST = 1095 days; estimates of stocks, shortage levels and costs based on a simulation run with TST = 3650 days.
would be full about 75% of the time and its average rate of utilization would be about 97%.

As a result of the tightness of the silo capacity constraint and the consequent congestion of the terminal, the gap between the shortage levels in port and in silo would widen considerably as shown in Table 41 (iii).

These results led to the questioning of the conclusions of the study by Hidrotécnica Portuguesa 31, which implied that the use of the Beato terminal in the day-to-day unloading and storage operations would not be cost effective, in view of the existing severe water depth restriction at its quay side (8 meters). The analysis was extended to cover the concurrent operation of the Beato terminal with the new one.

Except for the water depth restriction, the operational structure of the Beato terminal is similar to that of the new terminals of Oporto and Lisbon. The unloading of vessels in these terminals was assumed to take place only after they were berthed. For the Beato terminal this will only be possible when the draught of the vessels does not exceed the limit imposed by the water depth at the unloading quays (for the 8 meter water depth, the limit on draught was assumed to be 7 meters). When this limit is exceeded, the vessels have to be partially unloaded by floating cranes onto barges that carry the raw material to the quays, as shown in Figure 52. Only after the draught is brought down to 7 m can the vessels be directed to the quays to be unloaded directly to the silo.
Figure 52 - Basic operating characteristics of the Beato grain terminal

Note: \((Q_i)_{7m}\) are shipment sizes leading to 7 meter vessel draughts

### Basic parameters of the terminal

- Water depth at the quay side: 8 m (7 m maximum draught)
- Total unloading capacity: 280 000 m\(^3\), heavy grain / year
- Volume of the silo: 90 000 m\(^3\)
- Berthing and deberthing times: 0.25 working days

### Unloading operation

1. \(Q_i \leq (Q_i)_{7m}\) : direct unloading at the quay side
2. \(Q_i > (Q_i)_{7m}\) : two-stage unloading operation:
   - I - transhipment to barges (until \(Q_i = (Q_i)_{7m}\))
   - II - direct unloading at the quay side
For any vessel size (DWT), the load that leads to a draught of 7 m ($Q_{7m} = f(DWT)$) was derived considering the physical characteristics of dry bulk carriers according to the 1966 Standard Rule of Design, broadly described in Goss. Figure 53 gives the relevant characteristics of dry bulk carriers and the relationship obtained by regressing $Q_{7m}$ against DWT.

Figure 53 - Derivation of $Q_{7m}$ as a function of the size of dry bulk carriers

(1) Physical characteristics of dry bulk carriers (*). Shipment sizes leading to a draught of 7 m ($Q_{7m}$)

<table>
<thead>
<tr>
<th>Vessel size (DWT)</th>
<th>Weight / mm of immersion</th>
<th>Full load draught</th>
<th>Estimate of weight of spares, bunkers, etc., at arrival</th>
<th>Draught with spares, bunkers, etc.</th>
<th>$Q_{7m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A) [m.t.]</td>
<td>(B) [m.t.]</td>
<td>(C) [m]</td>
<td>(D) [m.t.]</td>
<td>(E) [m]</td>
</tr>
<tr>
<td>15 410</td>
<td>2.60</td>
<td>8.295</td>
<td>290</td>
<td>2.487</td>
<td>11 774</td>
</tr>
<tr>
<td>25 750</td>
<td>2.48</td>
<td>9.943</td>
<td>490</td>
<td>2.692</td>
<td>14 992</td>
</tr>
<tr>
<td>41 190</td>
<td>4.92</td>
<td>11.208</td>
<td>780</td>
<td>3.003</td>
<td>19 665</td>
</tr>
<tr>
<td>66 550</td>
<td>6.92</td>
<td>12.395</td>
<td>1260</td>
<td>2.970</td>
<td>27 893</td>
</tr>
</tbody>
</table>

(E) = (D) - [(A) - (D)] / 1000 . (B)
(F) = 1000 . (E) . [7 - (E)]


(11) Relationship between $Q_{7m}$ and DWT

\[ Q_{7m} = 6.81 + 0.315 \times \text{DWT} \]
Assuming, as before, that for each raw material $i$

\[ Q_i = 0.90 \cdot (\sqrt{v_i}/0.78) \cdot DWT_i \]  

(see section 3.2.2)

the maximum shipment size not requiring some preliminary unloading onto barges, can be obtained from

\[ Q_i = 0.90 \cdot (\sqrt{v_i}/0.78) \cdot DWT_i \]

\[ = Q_{7m} \]

\[ = 6.83 + 0.315 \cdot DWT_i \text{ (from Figure 53(ii))} \]

which leads to

\[ (Q_i)_{7m} = \frac{6.83}{1 - \frac{0.315 \cdot 0.78}{0.90 \cdot \sqrt{v_i}}} \quad (1000 \text{ m.t.}) \]

Clearly, this expression is strictly valid only for the 1966 Standard Rule of Design of dry bulk carriers. However, the safety margin introduced by limiting the vessels' draught at the quay side to 7 meters (1 meter less than the water depth) is thought to be adequate to cover other bulk carrier designs involving larger draughts.

The operation of the Beato terminal in parallel with the new terminal was analysed assuming the following policy:

(i) all the grain imports are handled at the new terminal (together with the imports of oil seed meals);

(ii) all the oil seed imports are handled at the Beato terminal.

In choosing this policy (as an alternative to the modus operandi proposed by Hidrotécnica Portuguesa\textsuperscript{31} ) consideration was given to the following points:

(i) each group of raw materials handled at each terminal - grains and oil seeds - is imported by a different Government agency (EPAC and IAPO, respectively);
(ii) the raw materials with highest consumption (grains) are handled at the more efficient terminal (the new one);

(iii) the Beato terminal is left with the raw materials whose delivery operations are less affected by the water depth constraint (the oil seeds are imported in smaller vessels than those used in the delivery of grains; for two of them – peanut and sunflower – the vessels carry lighter loads, in view of their lower densities);

(iv) the policy assumes independent operations at each of the terminals; for this reason it can be analysed applying the simulation-optimization model independently to each terminal (a new simulation model would have to be developed to analyse policies involving the joint operation of the terminals – e.g. involving the transhipment of raw materials from one terminal to the other by barge or the unloading of vessels first at the new terminal and then at the Beato one).

For the above policy, the operations at the new terminal were analysed using the simulation-optimization model without any modification apart from restricting the raw materials to grains only.

The analysis of the delivery-inventory operations at the Beato terminal – oil seeds only – was carried out under the following assumptions:

(i) in the first stage of the unloading operation the floating crane(s) and the barges provide an unloading capacity larger than that installed at the quays; in other words,
the availability of crane(s) and barges is not a
limiting factor in the unloading operation (this is
currently the case);

(ii) the time lost in the berthing operation is 0.25 days
regardless of whether the unloading operation involves
one or two stages (although it may seem that the two-
stage operation should involve more time lost, this
is not necessarily the case, since the transhipment
onto barges can be initiated before the berth becomes
free).

Under these assumptions, the effect of the water depth constraint can
be taken into account in the model just by including in the variable
cost expression a term for the extra cost incurred when the draught
of the vessels exceeds 7 meters. The objective function to be
minimized is, in this case,

$$VC'(Q_1, Q_2, Q_3) = VC(Q_1, Q_2, Q_3) + \sum_i \left[ EUC \cdot \frac{Q_i - Q_i'}{Q_i} \cdot \frac{365 \cdot d_i}{v_i} \right]$$

where

$$VC(Q_1, Q_2, Q_3) : \text{annual variable cost, as defined earlier}
(1000 \text{ US$})$$

$$Q_i' = \begin{cases} Q_i, & \text{if } Q_i \leq (Q_i)_m \\ (Q_i)_m, & \text{otherwise} \end{cases} 
(1000 \text{ m.t.})$$

$$EUC : \text{unit extra cost involved in the first}
\text{stage of the unloading operation when}
Q_i > (Q_i)_m \text{ (cost of hiring the floating}$$
crane(s) and the barges required to tranship raw material i) (US$/m^3)

\[ d_i : \text{rate of demand of raw material } i \quad \text{(1000 m.t./day)} \]

\[ v_i : \text{density of raw material } i \quad \text{(m.t./m}^3) \]

The unit extra unloading cost (EUC) was set initially at US$1/m^3 (≈ US$ 1.28/m.t. heavy grain) and a combined solution new terminal + Beato terminal was derived for a scenario identical to the one leading to the solution presented in Table 41 (new terminal alone). The main characteristics of both solutions are compared in Table 42.

The comparison obviously favours the parallel operation of both terminals. The annual delivery-inventory costs are reduced by about US$ 2.69 million (the standard deviation of this estimate is about US$ 0.08 million). In view of the unreliability of the estimate of EUC, the analysis was repeated for the Beato terminal considering a practically infinite value for that cost (this is equivalent to imposing the constraints \( Q_i \leq (Q_i)_m \) for all the raw materials). Under these circumstances the cost reduction associated with the use of both terminals rather than the new one alone would still be US$ 2.08 million. This reduction will clearly outweigh any extra costs (personnel, equipment repairs and maintenance, depreciation) incurred as a result of keeping the Beato terminal operational (note that these extra costs were not explicitly considered in the comparison between the variable costs associated with each solution).

The other major advantage of keeping the Beato terminal in use is the reduction of the shortages in silo to levels closer to those in port (the gap between them is even smaller than for the Oporto terminal: this is to a large extent the result of having four unloading
Table 42 - Delivery-inventory operations at Lisbon: combined operation of both terminals (solution 'N+B') v. new terminal alone (solution 'N') (scenario identical to that considered in Table 41; operating parameters of the Bento terminal as specified in Figure 52, with $EUC = US \$ 1 / m^3$) (*)

### (i) Shipment sizes and ratio T/O

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Shipment size [1000 m.t.]</th>
<th>'N+B'</th>
<th>'N'</th>
<th>'N+B'</th>
<th>'N'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>45.30</td>
<td>29.85</td>
<td>1.43</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>Sorghum</td>
<td>28.48</td>
<td>24.24</td>
<td>1.60</td>
<td>1.53</td>
<td></td>
</tr>
<tr>
<td>Wheat</td>
<td>28.33</td>
<td>21.98</td>
<td>1.29</td>
<td>2.55</td>
<td></td>
</tr>
<tr>
<td>Soybean</td>
<td>19.97</td>
<td>16.23</td>
<td>5.00</td>
<td>4.06</td>
<td></td>
</tr>
<tr>
<td>Peanut</td>
<td>10.56</td>
<td>7.34</td>
<td>4.75</td>
<td>3.31</td>
<td></td>
</tr>
<tr>
<td>Sunflower</td>
<td>11.00</td>
<td>8.70</td>
<td>4.33</td>
<td>3.27</td>
<td></td>
</tr>
</tbody>
</table>

### (ii) Annual variable costs [1000 US $]

| Solution 'N' : | VC = 45 548 |
| Solution 'N+B' : | VC = 32 564 (N) + 10 298 (B) = 42 862 |
| Δ = 2 586 |

### (iii) Shortages in port and in silo [m2]

<table>
<thead>
<tr>
<th>Raw material</th>
<th>β'/γ - fraction of time under shortage</th>
<th>β''γ - fraction of rep. cycles with short.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>'N+B'</td>
<td>'N'</td>
</tr>
<tr>
<td>Maize</td>
<td>1.26</td>
<td>2.90</td>
</tr>
<tr>
<td>Sorghum</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wheat</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Soybean</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Peanut</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sunflower</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(*) Solution 'N+B' derived setting TST = 3650 days; solution 'N' as defined in Table 41
stations - rather than two - for handling the six imported raw materials).

The effective silo capacity planned for the new terminal (151 200 m$^3$) is quite small when compared with the flow of imports even if the oil seeds are handled, as assumed above, through the Beato terminal. In fact, the entire volume of the silo can only store the equivalent to approximately

(i) one month of supply of maize (assuming a ratio $d_{\text{maize}}/d_{\text{sorghum}} = 2 : 1$), or

(ii) two months of sorghum, or

(iii) four months of wheat.

A comparison between the costs and the potential benefits associated with the practice of speculative storage was carried out as for the Oporto new terminal. The analysis covered only the speculative storage of sorghum (maximum allowable speculative stock of one month of consumption) and wheat (maximum allowable speculative stock of one of two months of consumption). The holding of higher stocks of each of these raw materials or of any significant stocks of maize would lead to considerable congestion of the terminal, with the inevitable increase in the number and the extent of the shortages in silo for the other raw materials. The results of the analysis led to a conclusion identical to the one reached for the Oporto new terminal. For the planned storage capacity, no potential benefits can result from holding speculative stocks of grain in the silo.
9. Model Extensions

9.1 General

In this section two extensions of the delivery-inventory model will be described:

(i) the incorporation of constraints on the size of shipments, and

(ii) the representation of policies involving the attempt to coordinate the planned arrivals of vessels carrying different raw materials.

For the Portuguese grain terminals the constraints on the size of shipments were found to be non-active. However their incorporation in the model might become necessary if, for example, the silos were to be expanded. The second extension of the model enabled the validation of one of the basic assumptions made at the beginning of the study: as will be shown in section 9.3, for the Portuguese delivery-inventory operations, no significant benefits in relation to the policies discussed so far can be obtained from the coordination of the planned arrivals of vessels carrying different raw materials.

9.2 Incorporation of Constraints on the Size of Shipments

The need for incorporating constraints into the model can arise for different reasons, namely:

(i) limitations in the docks or quays where the vessels are unloaded;

(ii) similar limitations in the ports in the exporting countries; and

(iii) difficulty of avoiding spoilage of the imported raw materials in the Portuguese ports.
The constraints arising from (i) and (ii) have the general form

\[ DWT_i \leq MVS_i \]

where

- \( DWT_i \): deadweight tonnage of vessel carrying raw material \( i \)
- \( MVS_i \): maximum vessel size either in Portugal or abroad

(as far as the Portuguese new terminals are concerned, the maximum vessel sizes are equal for all raw materials: 40–45 000 m.t. DWT and 80 000 m.t. DWT at Oporto and Lisbon, respectively; at the Beato terminal the constraint on the vessels' draught—considered in section 8.4—is more restrictive than that on the vessel sizes).

The shipment sizes \( Q_i \) are related to the vessel sizes through the expression

\[ Q_i = 0.9 \cdot (\nu_i/0.78) \cdot DWT_i \]

(see section 3.2.2.); hence, the constraints can be expressed in terms of the controllable variables \( Q_i \) as follows:

\[ Q_i \leq 0.9.(\nu_i/0.78) \cdot MVS_i \]

The need for other constraints can arise from the difficulty of avoiding spoilage of the imported grains and oil seeds in port. This difficulty is different for each of the imported raw materials and depends on their temperature and moisture content. These two factors can vary from shipment to shipment, depending on the condition of the imported raw materials at the country of origin and on the weather conditions during the loading and unloading operations.
The perishability of the imported raw materials can be taken into account in the problem formulation by limiting their average storage time in port. This can be expressed as

\[(\text{av. stock in port/demand})_i \leq \text{MAST}_i\]

where

\[\text{MAST}_i : \text{maximum average storage time of raw material } i\]

in port;

the above inequality can be expressed approximately as

\[\frac{Q_i}{2} + B S_i \leq d_i \cdot \text{MAST}_i\]

or

\[\frac{Q_i}{2} + r_i(Q_i) \leq d_i \cdot \text{MAST}_i\]

The maximum average storage times can be extended considerably — for some of the raw materials practically without limit — by a suitable treatment of the imported raw materials. Drying, cooling, chemical treatments or combinations of them can be used for that purpose.

Both types of constraints can be expressed as

\[f_i(Q_i) \leq 1\]

where \(f_i(Q_i)\) represents, for each raw material \(i\), a function of the control variable \(Q_i\) and of a number of fixed parameters of the system.

Such constraints can be incorporated into the model using the penalty function method. For each constraint \(f_i(Q_i) \leq 1\), an artificial penalty term

\[P_i(Q_i) = \begin{cases} 0, & \text{if } f_i(Q_i) \leq 1 \\ M \cdot [1-f_i(Q_i)]^2, & \text{otherwise (with } M \text{ very large, e.g. } 10^{20} \end{cases}\]

is added to the cost function being minimized. The effect of this term
is to insure that infeasible solutions are non-attractive and are therefore rejected during the search for the minimum cost point.

One important feature of the penalty function method is the fact that the presence of non-active constraints does not affect significantly the computational time required to reach the optimal solution, if the search procedure is started on a point belonging to the feasible region. This means that all the constraints, whether likely to be active or non-active, can always be included in the model, without the danger of increasing the computational burden significantly beyond what is strictly necessary.

In the study of the delivery-inventory operations at the Portuguese terminals the constraints on the size of vessels turned out to be non-active. In fact, as a result of the limited storage capacity at the silos or, in the case of the Beato terminal, as a result of the moderate inflows of oil seeds, the solutions derived involve vessel sizes that are well below the limits imposed by the Portuguese docks or quays. Furthermore, since those sizes are quite common in the export trade of each of the raw materials considered, they will not restrict significantly the choice of exporting ports to the Portuguese importing agencies.

The new terminals at Oporto and Lisbon include special cells where part of the imported raw materials - those in 'bad condition' - can be treated to avoid spoilage. Since the introduction of these facilities is a novelty in Portuguese grain terminals, no reliable estimates of the perishability of the raw materials could be obtained.
It is believed that the existence of such facilities will not completely eliminate the problem of spoilage, particularly for the raw materials for which this problem is specially acute (e.g. maize).

For the solutions derived earlier in this chapter, the average storage times do not differ significantly from those observed in the old terminals (in poorer conditions and without treatment) - under the policies adopted in the past by the Portuguese importing agencies. The perishability of the raw materials is therefore unlikely to be critical. However, if, for example, the storage capacities at the silos were to be increased in relation to the current ones, that aspect of the storage operations could become significant.

Earlier, even ignoring the extent to which raw materials can be preserved in port and the respective cost involved, it was possible to show the unattractiveness of speculative storage. Clearly, for storage capacities larger than the existing or planned ones, consideration should be given to the perishability of the raw materials and the problems that it poses, in addition to all the other factors considered in the analysis carried out.

9.3 Coordination of Planned Arrivals of Vessels Carrying Different Raw Materials

9.3.1 The Method of 'Equal Order Intervals' (EOI)

The simulation-optimization model, in the version presented so far, is unable to represent delivery-inventory policies involving the attempt to coordinate the planned arrivals of vessels carrying
different raw materials. The potential benefits of such type of policy can be explained by outlining the EOI method, proposed by Page and Paul 47, for the solution of the deterministic multi-product inventory problem with instantaneous replenishment and with one constraint (e.g. on storage space).

The solution of this problem by the classical method, using Lagrange multipliers, leads to an average rate of utilization of the available storage space of 50%. This is the consequence of reducing the order quantities to sizes such that the constraint will not be violated even if all the products are replenished at the same time.

One way of insuring that all products are not replenished at the same time is to group them in one or more categories, with all the products within the same category having a common 'order interval'. Once a common period of replenishment is established, the replenishments of the products can be staggered in such way as to minimize the maximum storage space required at any time, for those products (Page and Paul defined such a policy for the general case of any number of products with the same order interval).

By grouping different products in this way, a better utilization of the available storage space will generally be obtained. However this does not mean that the overall inventory costs will necessarily be reduced, since that better utilization of the restricted facility is achieved at the expense of the introduction of additional constraints in the problem (those required to equalize the order intervals of each group of raw materials).

In order to see what benefit (if any) can be derived from a better utilization of the storage space, the solution obtained using the
classical method (i.e. ungrouped raw materials) should, in principle, be compared with those solutions corresponding to all the possible combinations of raw materials into different EOQ groups.

When the number of raw materials is large, complete enumeration of groupings becomes impracticable. For such situations, Page and Paul suggest the evaluation of a sub-set of solutions that are more likely to lead to an overall optimal solution, selected according to a heuristic procedure described in their paper.

The delivery-inventory problem considered in this thesis differs from that tackled by Page and Paul in several respects:

(i) the storage space constraint is not 'strictly binding', i.e. the shipment sizes do not have to satisfy rigidly any overall constraint; on the contrary, as far as the storage space is concerned, they can take any value, but penalty costs are incurred whenever the silo becomes full (through the delay of vessels in port and, hence, the payment of extra shipping costs);

(ii) the replenishment in the silo is not instantaneous;

(iii) another facility shared by vessels carrying different raw materials is restricted: the unloading stations;

(iv) the cost structure is more complex than that assumed by Page and Paul (the latter is the classical cost structure in inventory problems);

(v) the shipment arrivals are not deterministic.

In situations where the replenishments take place at dates defined with certainty - such as in Page and Paul's problem - the occurrence of simultaneous replenishments of raw materials with equal order intervals can always be avoided by staggering their replenishment
cycles. However, this cannot be generally guaranteed when the
dates of arrival are defined probabilistically. In this situation,
the objective is more limited: one can only attempt to match
periods of high probability of occurrence of the arrivals of one
raw material with periods of low probability of occurrence of arrivals
of other raw materials.

The effectiveness of policies involving the staggering of
replenishment cycles will be impaired by the uncertainty of arrivals,
to an extent which depends on the value of the ratio $T/\sigma$. This is
illustrated in Figure 53 considering two different values of that
ratio: 2.5 and 5.0.

The effect of the uncertainty of arrivals on the net stock
in port of any raw material is shown in diagrams (a). In these
diagrams the function $\mu_s(t)$ represents the expected value of that
stock at any time $t$. The width of the shaded band $\mu_s(t) \pm \sigma_s(t)$
(where $\sigma_s(t)$ is the standard deviation of the net stock in port at
time $t$) provides a measure of the dispersion of the distribution
of the stock, at any point in time, around its expected value
$\mu_s(t)$. The values of both $\mu_s(t)$ and $\sigma_s(t)$ were derived using the
single-product simulation model considered earlier in section 5.2.

Diagrams (b) show the function $g(t) = \sum f_j(t)$, where $f_j(t)$
are the density functions of the actual arrivals of the shipments
with planned dates of delivery $t_j$. For any infinitesimal time interval
dt, the value $dQ = g(t). dt$ represents the expected number of arrivals
taking place during $dt$. This can be shown, considering the interval
represented in diagram (i)(b). For the sake of simplicity it will be
assumed that only the vessels with planned delivery dates $t_o$ and $t_1$
Figure 53 - The net stock in port and the combined density function of vessels' arrivals for different values of the ratio $T/s$.
can actually arrive during \( dt \). The probabilities of occurrence of 0, 1 and 2 arrivals during \( dt \) are:

\[
P_0 = [1-f_0(t).dt] \cdot [1-f_1(t).dt] = 1 \text{ (ignoring terms in } dt \text{ and } dt^2)
\]

\[
P_1 = [1-f_0(t).dt].f_1(t).dt + f_0(t).dt \cdot [1-f_1(t).dt]
\]

\[
= [f_1(t) + f_2(t)].dt.
\]

\[
P_2 = f_0(t).f_1(t).dt^2
\]

The expected number of arrivals is

\[
E_{dt} = 0.P_0 + 1.P_1 + 2.P_2 =
\]

\[
= [f_0(t) + f_1(t)].dt = g(t).dt
\]

It is clear that \( E_{dt} \) is still given by \( g(t).dt \) when the number of vessels that can arrive during \( dt \) is different from two.

The expected number of arrivals taking place during any finite interval \( \Delta t \) is

\[
E_{\Delta t} = \int(\Delta t) g(t).dt
\]

If the integral is extended over an entire planned replenishment cycle \([t_{j-1}, t_j]\), the expected number of arrivals is

\[
\int_{t_{j-1}}^{t_j} g(t).dt = \sum_m \left[ \int_{t_{j-1}}^{t_j} f_m(t).dt \right] = \int_{-\infty}^{+\infty} f_j(t).dt = 1 \text{ (see diagram (i)(b))}
\]

(i.e. the expected number of arrivals per planned replenishment cycle is one).

As shown in the diagrams, as \( T/\sigma \) becomes smaller, all the functions \( \mu_s(t), \sigma_s(t) \) and \( g(t) \) become less variable over time. For \( T/\sigma \leq 2.5 \) there
is not much distinction between the values taken by them, for example, at the beginning or in the middle of the planned replenishment cycles. It is obvious that in such situations no significant benefits can be expected from staggering the planned replenishment cycles of different raw materials with the same order interval.

As T/σ increases, those functions start showing some variation. Clearly, from the results shown in Figure 53 it is not possible to identify the range of values of T/σ for which the staggering of replenishment cycles becomes an attractive proposition. The answer to this question can only be obtained through the application of the EOI method to stochastic problems.

9.3.2 Extension of the Simulation-Optimization Model

In order to illustrate how the EOI method proposed by Page and Paul can be adapted to the solution of the more complex problem analysed in this chapter, the case of a three-product problem (products 1, 2, 3) will be considered. In this case the raw materials can be combined under five different 'policies':

(i) 1, 2, 3
(ii)(a) 1 - 2, 3
(b) 1 - 3, 2
(c) 1, 2 - 3
(iii) 1 - 2 - 3

where the raw materials belonging to the same 'order interval' group are connected by an hyphen.

The first policy corresponds to the one that has been considered
so far, with no attempt to group the raw materials. The control variables are, in this case, $Q_1$, $Q_2$, $Q_3$.

In the second set of policies, two of the three raw materials are placed in the same group. The essential difference in the way of specifying the control variables in the simple problem dealt with by Page and Paul and in the more complex problem considered in this chapter can be illustrated considering, for example, policy 1-2, 3. In Page and Paul's problem, the control variables would be $Q_1$ and $Q_3$ or $Q_2$ and $Q_3$. After $Q_1$ (or $Q_2$) is set, $Q_2$ (or $Q_1$) can be derived from $Q_1/d_1 = Q_2/d_2$ where, as before, the $Q$'s and $d$'s represent shipment sizes and demands, respectively. Once a particular group of raw materials is specified (1-2, in the case considered) the staggering of the replenishment cycles is automatically fixed according to an optimal rule designed to minimize the maximum space required to store the raw materials belonging to that group.

In the problem considered in this chapter, given the complexity of the relations involved, such a rule could not be derived. For the policy 1-2, 3 a new variable needs to be considered, in addition to $Q_1$ (or $Q_2$) and $Q_3$, in the optimization of policy 1-2,3. This variable, called here the phase difference between cycles 1 and 2 and denoted by $\theta_{12}$, is defined as the time lag between

(i) any one planned date of delivery of raw material 1 (say $t_1$) and

(ii) the first planned date of delivery of raw material 2 that succeeds $t_1$.

This time lag will be expressed as a percentage of the length of the planned replenishment cycles of raw materials 1 and 2.
In general - for any policy, involving any grouping - the total number of variables to be considered in the optimization is always equal to the number of raw materials. The set of variables chosen is to some extent arbitrary: for example, for policy (iii) the variables could be \( Q_3, \theta_{13}, \theta_{23} \) or \( Q_2, \theta_{12}, \theta_{13} \), among many other equivalent sets.

In order to derive the optimal solutions for the different policies, only minor changes had to be introduced in the simulation-optimization programme described earlier. These changes can be described considering again the policy 1-2, 3.

After the set of independent variables is defined - for example, \( Q_2, Q_3 \) and \( \theta_{12} \) - at each entry of the simulation segment of the programme (i.e. the evaluating function of the programme) the variables \( Q_2 \) and \( Q_3 \) are set at values determined by the search algorithm. \( Q_1 \), no longer an independent variable, is set at \( Q_1 = (d_1/d_2) \cdot Q_2 \).

The arrivals of vessels are generated following normal distributions \( N(t_{ji}, \sigma_i^2) \), where the planned dates of arrival \( t_{ji} \) (\( j \)th vessel, raw material \( i \)) are

\[
(n - \theta_i/100) \cdot T_i
\]

where

- \( n \) is an integer between \(-N_{1_i} \) and \( N_{2_i}^* \);
- \( \theta_i \) is in the range \([0, 100]\);
- \( T_i, N_{1_i} \) are as defined in section 6.2;
- \( N_{2_i}^* \) is equal to \( N_{2_i} + 1 \), where \( N_{2_i} \) is the integer defined earlier in section 6.2 (i.e., one more arrival is generated in the new version of the simulation programme).
For the raw materials whose shipment sizes are considered independent variables (raw materials 2 and 3, in the case being considered) the values of $\theta_1$ are set equal to zero ($\theta_2 = \theta_3 = 0$). The value of the remaining $\theta_1$ ($\theta_1 = \theta_{12} = \theta_1 - \theta_2$) represents the phase difference between the planned replenishment cycles of raw materials 1 and 2. At each entry of the simulation programme, $\theta_1$ is set at value determined by the search algorithm. If, in the course of the search, any $\theta_1$ takes a value outside the interval $[0, 100]$, then it is replaced, in the simulation segment of the programme, by $\theta_1 = \text{largest integer in } (\theta_1/100)]$.

Finally, and still keeping to the notation introduced in section 6.2, the expressions of the net stocks in port were altered, in the initialization of the simulation, to

$$\text{NSP}_1 = (K_1 - N_1) \cdot Q_1 + BS_1 - \left(\theta_1/100\right) \cdot T_1 \cdot d_1$$

9.3.3. Application of the EOI Method to the Delivery-Inventory Operations at the Portuguese Terminals

Table 43 shows, for some of the policies derived earlier in this chapter, the values of the ratio $T/\sigma$ corresponding to each of the imported raw materials.

<table>
<thead>
<tr>
<th>Table 43 - Values of T/σ for policies derived earlier for the different terminals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw material</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>(solution 'N+E' in Table 42)</td>
</tr>
<tr>
<td>Maize</td>
</tr>
<tr>
<td>Sorghum</td>
</tr>
<tr>
<td>Wheat</td>
</tr>
<tr>
<td>Soybeans</td>
</tr>
<tr>
<td>Peanut</td>
</tr>
<tr>
<td>Sunflower</td>
</tr>
</tbody>
</table>
In the solution obtained for the Beato terminal, the ratio \( T/C \) is above 4 for all raw materials. For the other solutions the ratio is below that figure, either for all raw materials (at the Lisbon new terminal) or for those which account for most of the flow of imports and most of the costs (maize and sorghum, at Oporto). Given the fact that the effectiveness of EOI policies increases with the value of \( T/C \), the method was first applied to the operations at Beato (considering oil seeds only). The results are summarized in Table 44.

Table 44 - Application of the EOI method to the operations at the Beato terminal, with oil seeds only (*).

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Policy</th>
<th>Shipment size ( Q_i ) [1000 m.t.]</th>
<th>Ratio ( T/C )</th>
<th>Phase ( \theta_i ) [0-100]</th>
<th>Annual variable cost [1000 US $]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Soybean</td>
<td>19.97</td>
<td>5.00</td>
<td>-</td>
<td>10 298</td>
<td></td>
</tr>
<tr>
<td>2-Peanut</td>
<td>10.56</td>
<td>4.75</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-Sunflower</td>
<td>11.00</td>
<td>4.13</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Soybean</td>
<td>(21.03)</td>
<td>5.26</td>
<td>35.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-Peanut</td>
<td>11.70</td>
<td>5.26</td>
<td>(0.00)</td>
<td>10 273</td>
<td></td>
</tr>
<tr>
<td>3-Sunflower</td>
<td>10.00</td>
<td>3.75</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Soybean</td>
<td>(15.53)</td>
<td>4.14</td>
<td>25.35</td>
<td>10 333</td>
<td></td>
</tr>
<tr>
<td>2-Peanut</td>
<td>10.00</td>
<td>4.50</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-Sunflower</td>
<td>11.02</td>
<td>4.14</td>
<td>(0.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Soybean</td>
<td>23.88</td>
<td>5.98</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-Peanut</td>
<td>(9.54)</td>
<td>4.29</td>
<td>26.95</td>
<td>10 369</td>
<td></td>
</tr>
<tr>
<td>3-Sunflower</td>
<td>11.43</td>
<td>4.29</td>
<td>(0.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Soybean</td>
<td>(19.41)</td>
<td>4.86</td>
<td>15.12</td>
<td>10 281</td>
<td></td>
</tr>
<tr>
<td>2-Peanut</td>
<td>(10.80)</td>
<td>4.86</td>
<td>77.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-Sunflower</td>
<td>12.94</td>
<td>4.86</td>
<td>(0.00)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(* All solutions derived with the same set of streams of random numbers and TST = 3650 days; EUC = US $ 1 / m³.

Although 'on paper' policy 1-2, 3 appears to lead to the lowest costs, the difference between that policy and the one derived earlier (1, 2, 3) is negligible (although a formal test of significance of that difference could have been carried out, it did not seem a worthwhile exercise).
Two points which emerged from the analysis, both concerning the staggering of the replenishment cycles, are worth noting:

(i) For values of the ratio $T/\sigma$ such as the one corresponding to policy 1-2, 3 ($T/\sigma = 5.26$) the effect of staggering the replenishment cycles is quite noticeable. This result, shown in Figure 54, contrasts with those obtained for the other terminals, which will be discussed later.

(ii) During the optimization of policy 1-2-3, two local optima were found, corresponding to different sequences of arrival of raw materials: the policy presented in Table 44 (sequence: $\cdots -1-2-3, \cdots$) led to the overall minimum: for the other sequence ($\cdots -1-3-2 \cdots$) the annual cost exceeded the minimum by only US$ 24 000. Although the difference is not of much real significance, the existence of more than one local minimum shows that difficulties can arise as the number of raw materials within a common
EOI group increases. The method cannot therefore be regarded as leading automatically to an overall minimum. Rather, it requires careful manipulation of the search in order to induce different local optima.

For the other terminals, as expected, the staggering of replenishment cycles did not lead to any reduction in the total costs. For the Oporto terminal - involving six raw materials - the optimization of all the possible EOI groupings could not be carried out because of the enormous computational effort that would have been required. Instead, the study concentrated on the grouping of those raw materials with similar values of T/σ in the original solution (following, in an ad hoc way, the principle that is behind the heuristic rule proposed by Page and Paul 47 for the selection of the EOI groupings). Even for those with the highest values of T/σ, the staggering of replenishment cycles had no significant effect on costs. This is illustrated in Figure 55, where the annual variable cost is plotted against the phase difference between the soybean and sunflower cycles. In spite of the fact that the ratio T/σ for these cycles (6.83) is larger than that for cycles 1 and 2 in Figure 54 (5.62), the effect of the phase differences on costs is now negligible. This clearly demonstrates that the value of the ratio T/σ for the grouped raw materials is not the only factor affecting the effectiveness of EOI policies. The difference between the shapes of the curves presented in Figures 54 and 55 can be attributed to the difference in the number of shipments left out of the EOI groups in each case. In the Oporto situation, where that number is much higher, any attempt to decrease the probability of direct interference between shipments of soybean and sunflower becomes less worthwhile, since the probability of interference between these shipments and those left out of the EOI group is quite considerable.
Figure 55 - Effect of staggering the replenishment cycles of soybean and sunflower on the annual costs (Oporto new terminal)

The analysis carried out served adequately the purpose of showing that, as far as the operations at the Portuguese terminals are concerned, no real benefits can be expected from BOI policies. Although focused on a restricted set of problems - involving particular values of critical parameters such as costs, demands, storage and unloading capacity - the analysis provided results which indicate the general effect of uncertainty on the effectiveness of BOI policies. A more comprehensive analysis of this problem is outside the scope of this research. The model presented earlier could be used for such an analysis.

10. Summary. Concluding Remarks

10.1 Summary

In this chapter the delivery-inventory problem facing the Portuguese importing agencies was examined and a model was proposed to help their management in the planning and decision making involved.

The complexity of the problem required the development of a specially tailored simulation-optimization model. The use of simulation permitted a realistic representation of aspects of the problem such as
(i) the uncertainty of vessels' arrivals (with the occurrence of 'crossed deliveries'),
(ii) the interaction between the shipping, unloading and inventory operations of different raw materials, and
(iii) the (complex) structure of delivery-inventory costs.

The problem of selecting the best delivery-inventory policy was solved by combining the simulation model with a direct search routine. A method was proposed to overcome the difficulties associated with the uncertainty of the cost function to be minimized.

The success of the simulation-optimization approach was critically dependent on:

(i) setting constraints on shortages in port rather than in silo;
(ii) the ability to estimate the relationship between control variables and shortage levels in port outside the multi-product simulation-optimization model.

For the solutions derived both for Oporto (new terminal) and Lisbon (combined operation of both the new terminal and the Beato terminal) it was shown that the shortage levels in silo were acceptably close to those in port. The difference between them can be further reduced either by resorting to what was described as 'exceptional measures' or, if so required, by raising the buffer stocks in relation to the values assumed initially. A method was proposed to 'correct' the buffer stocks so as to obtain shortage levels in silo as specified initially.

The relationship between shortage levels in port on the controllable variables was derived with the help of a single-product
simulation model. From the results of this model it was possible to derive the relationship between shipment sizes and the minimum buffer stock compatible with the specified shortage levels in port. The minimum buffer stock was found to be sensitive to changes in the value of the standard deviation of the distribution of vessel arrivals. The accurate estimation of this parameter is therefore important.

A considerable simplification of the problem was achieved by restricting the analysis to policies

(i) involving no speculative storage and

(ii) derived under 'static' conditions.

Without evaluating explicitly other alternative policies, it was possible to show (indirectly) that they would not lead to an improvement in the efficiency of the system. A further set of policies - those involving the attempt to coordinate the planned arrivals of vessels carrying different raw materials - was found to be equally unattractive. This conclusion was reached after an explicit evaluation of these policies, using an extension of the 'Equal Order Interval' method.

One important implication of the unattractiveness of speculative storage is the possibility of decomposing the overall purchasing-delivery-inventory problem into two sub-problems, without introducing suboptimization. The delivery-inventory sub-problem was dealt with in this chapter. Under the proposed operating doctrine, based on the concept of 'planned replenishment point', the aim is to define (at any point in time and over a given time horizon)

(i) the size of the shipments and

(ii) their planned dates of delivery,
with the objective of minimizing the expected delivery-inventory costs.

The question of when each shipment should be ordered - at or before a deadline set implicitly when planning the date of delivery - is the main concern of the purchasing sub-problem, and will be discussed later in Chapter 8.

The analysis of the model solutions revealed their low sensitivity to changes in parameters that are likely to vary considerably over time (e.g. cost parameters) or those for which reliable estimates are difficult to obtain (e.g. the limit up to which the capacity of delivery from the silos to the customers can be stretched). The robustness of the model solutions has two major implications, namely

(i) the validity of deriving the solutions through a static model (the transition between successive optimal policies can be made smoothly, most times through small adjustments of the control variables);

(ii) the effectiveness of the model does not depend critically on the accuracy of the estimates or forecasts of those parameters which vary considerably over time (shipping costs, raw materials' prices).

10.2 Possible Refinements and Adaptations of the Model

The model achieved its primary objective of defining 'optimal' delivery-inventory policies, for the current operational characteristics of both terminals. However, it is possible to identify a few areas where some improvements could be made.

The simulation-optimization approach clearly involves a substantial computational effort. This effort will become more critical with computers
slower than the CDC 7600 (which covers most of the computers currently available). Some aspects of the design of the simulation model clearly reflect the attempt to increase its computational efficiency: treating the terminal's operation as continuous - saving a considerable number of simulation events - and the way of organizing the transition between the 'B' and 'C' phases are examples of that. One area where there is scope for improvement is the generation of vessel arrivals. These were obtained with a standard random number generator coupled with a Box–Muller transformation subroutine. No attempt was made to reduce the variance of the expected cost estimate by 'controlling' the discrepancy between the sample distribution of arrivals and the theoretical one. In situations where the sample size can be specified in advance of the simulation - which is true in the case considered in this chapter - that can be best achieved by resorting to the 'descriptive sampling' method proposed recently by Salibi54.

Another area which is worthy of further analysis concerns the specification of the constraints on shortage levels in port. The way in which these constraints were specified, although attempting to reflect the views of management, involved a somewhat arbitrary choice of the measures of shortage ($\beta_p'$ and $\beta_p^*$) and of the particular numerical values assigned to them (1% and 5%, respectively).

As far as the selection of measures of shortage, the model is particularly flexible:

(i) in includes two complementary measures, in an attempt to take into account both the frequency and the total duration of shortages (whichever is more restrictive);
(ii) other measures can be incorporated in the model either instead of or in addition to those considered, if management so require after the analysis of the results; although it is easier to handle dimensionless measures of shortage, the analysis can be extended to dimensional ones (e.g. number of shortages per year), at the expense of a larger computational effort.

For any suitable measures of shortage, it is clearly desirable to present management with the minimum operating costs achievable for different levels of shortage. In this way, the choice of the final level(s) can be made less arbitrary. Such a task involves a straightforward, but time consuming, parametrization exercise not covered in the analysis described in this chapter.

A substantial effort was put on checking all the major assumptions upon which the model is based. The model inevitably entails simplifications of reality which require further testing (e.g. the way in which parameters such as the 'effective storage capacities' were defined). However, this testing can, and should, be carried out only within the context of a wider model validation exercise, during the implementation phase, with close involvement of management.

10.3 The Need for Expanding the Capacity of the Terminals

The emphasis of the research was on the development of an operational model in which the storage and handling capacities of each terminal were treated as given inputs. The research did not address directly the problem of determining whether
(i) those capacities are appropriate or, alternatively, whether
(ii) financial resources should be committed to expand them
(and, if so, up to what levels).

In broad terms, this problem involves the comparison, over the investment's life time period between

(i) the costs associated with increasing each of those capacities to different levels (capital costs and extra running costs), and
(ii) the corresponding benefits (measured in terms of reductions in the delivery, inventory and purchasing costs).

Once all costs are reduced to a common basis - usually their 'present values' - the 'optimal' capacities are those leading to the minimum overall cost. Clearly, the definition of the 'optimal' value of any given parameter of one terminal - e.g. its storage capacity - cannot be dissociated from the determination of the 'optimal' values of the others (e.g. the unloading capacity or the size of the quays).

However, for the purpose of illustrating

(i) the complexity of the problem in question and
(ii) how the model proposed in this chapter could be used in the analysis of that problem,

it is easier to consider the determination of the 'optimal' level of the storage capacity, assuming that all the other parameters are fixed (conceptually there is no difference between this and the overall problem).

The capital investment decision in question poses a highly complex problem deserving, on its own, a substantial research effort.

The major difficulties of the problem concern the evaluation of the long
term benefits associated with the expansion of the storage capacity.
One source of difficulty is the uncertainty of many of the parameters affecting the delivery-inventory-purchasing policies and the corresponding costs. Raw materials' prices, fuel costs and vessel hiring costs are examples of parameters varying widely over time. The evaluation of the long term benefits achievable in the future, by increasing the storage capacity from one level to another, involves the difficult but inevitable choice of one or more 'cost scenarios'. These scenarios should reflect the conditions that might occur during the relevant time horizon (either expected conditions or pessimistic or otherwise, depending on the criterion chosen).

Clearly the difficulty is not exclusive to cost parameters. Similar choices concerning other parameters - e.g. volume of imports of each raw material - may prove to be equally difficult.

Another source of difficulty concerns the evaluation of the long term benefits that can be achieved through the practice of speculative storage, when the storage capacity is increased from its current value. A point may be reached when speculative storage may become profitable and the separation of the overall delivery-inventory-purchasing problem into two sub-problems is no longer valid. Further research will be required with the objective of devising a policy (JD) aiming at the optimal joint determination of deliveries (timing and size of shipments) and purchases (for immediate or forward delivery).

By definition, the optimal policy so derived will lead to delivery-inventory-purchasing costs not larger than those obtained from the type of policy (SD) considered in this chapter (involving sequential decisions first on deliveries and then on purchases). The difference between the costs associated with the 'SD' and 'JD' policies - attributable to the
practice of speculative storage, whenever beneficial - will increase (or, at least, will not decrease) as the storage capacity increases. This can be expressed symbolically as

$$\left[ \frac{\partial C}{\partial V} \right]_{JD} < \left[ \frac{\partial C}{\partial V} \right]_{SD}, \text{ for any } V$$

where

- \( C \) : present value of the delivery + inventory + purchasing costs over the life of the investment, for any given scenario
- \( V \) : storage capacity.

Figure 56 illustrates conceptually, for any given scenario and over any assumed life of the investment, the effect of increasing the storage capacity on the present value of the total costs under policies 'SD' and 'JD'.

Figure 56 - Expansion of the storage capacity: theoretical effect on costs, under 'SD' and 'JD' policies
The values of \( V \) which minimize the total costs (\( V_o \), under policy 'SD', and \( V_1 \), under 'JD') are either

(i) the current storage capacity (obviously not the case in the diagram), or

(ii) those for which \( \left| \frac{\partial C}{\partial V} \right|_SD \) and \( \left| \frac{\partial C}{\partial V} \right|_JD \) equal the marginal cost of storage capacity.

Since

\[
\begin{bmatrix}
\frac{\partial C}{\partial V}_JD \\
\frac{\partial C}{\partial V}_SD
\end{bmatrix} \leq 
\begin{bmatrix}
\frac{\partial C}{\partial V}_SD \\
\frac{\partial C}{\partial V}_JD
\end{bmatrix}, \text{ for any } V,
\]

then it follows that

\( V_o \leq V_1 \)

The value \( V_o \) can be derived with the help of the model proposed in this chapter. In fact, under policy 'SD' the purchasing costs are not part of the variable costs; therefore

\[
\begin{bmatrix}
\frac{\partial C}{\partial V}_SD \\
\frac{\partial VC}{\partial V}_SD
\end{bmatrix} = 
\begin{bmatrix}
\frac{\partial VC}{\partial V}_SD
\end{bmatrix}
\]

where VC denotes, as before, the total (variable) delivery-inventory costs. These costs can therefore be used to obtain \( V_o \). This means that the model proposed in this chapter can be used to derive, for any scenario, a lower bound on the 'optimal' storage capacity (the justification for any further increase depends on the ability to evaluate the benefits of speculative storage).

The relevance of the capital investment problem discussed above
can be illustrated by presenting some preliminary results derived for the Oporto and Lisbon new terminals.

The values of $V_0$ were calculated for both terminals (at Lisbon considering the imports of grains only), under the following assumptions:

(i) The scenarios were those reflecting the cost conditions prevailing in the middle of 1979 (scenario defined in Table 25 for Oporto; for Lisbon the scenario was identical to that considered in deriving the solution presented in Table 41).

(ii) Estimates of the capital involved in building and equipping silo storage capacity were obtained from the director of the civil engineering project at the Oporto new terminal. As of 1980 the initial capital per $m^3$ was estimated as follows:

- building : Esc. 3500
- equipment : Esc. 2333

**total** : **Esc. 5833**

These estimates, relatively crude and deliberately overestimating rather than underestimating the actual costs, were converted into US$. Assuming a rate of exchange of Esc.40/US$, the total initial capital per $m^3$ becomes US$ 146.

(iii) The running costs associated with the extra storage capacity were neglected.

(iv) The life of the investment was assumed to be 20 years (quite a conservative figure).

(v) The rate of interest on capital was taken to be 15% per year.

(*) Given the difficult balance of payments situation in Portugal, caution is required in assessing the value of the Escudo in relation to other currencies. The exchange rate in itself may not be an accurate indicator; however, it is adequate for the present purpose.
Under these assumptions, the values of \( V_o \) can be determined by equating the present values of the marginal benefits and costs of expanding the storage capacity:

\[
\left[ \frac{\partial V_C}{\partial V} \right] \cdot \left[ \frac{1 - (1 + \alpha)^{-n}}{\alpha} \right] \bigg|_{V_o} = \text{US$ 146}
\]

Present value of the marginal reduction in delivery-inventory costs over the life of the investment

where

- \( V_C \) : annual delivery-inventory cost (under policy 'SD') (US$/year)
- \( \alpha \) : rate of interest on capital (0.15) (/year)
- \( n \) : life of investment (20) (years)

The term \( \left[ \frac{1 - (1 + \alpha)^{-n}}{\alpha} \right] \bigg|_{V_o} = 6.26 \) is the present value of US$ 1 received per annum, over the whole life of the investment (see Bierman et al.\(^{10}\)). Replacing this term by its numerical value in the above equation and rearranging, gives

\[
\left[ \frac{\partial V_C}{\partial V} \right] \bigg|_{V_o} = \frac{146}{6.26} = 23.3
\]

For the Oporto terminal, the value of \( V_o \) was determined using the diagram presented earlier in Figure 48. The slope of the cost curve equals 23.3 when \( V = V_o \approx 140,000 \text{ m}^3 \) (currently the effective storage capacity is 91,800 m\(^3\)).

For the Lisbon new terminal \( V_o \) was derived similarly:

\( V_o \approx 190,000 \text{ m}^3 \) (the capacity that is to be installed is 151,200 m\(^3\))
In order to keep these results in perspective it is important to note that:

(i) the unloading capacity was kept constant; one can expect that the 'optimization' of this parameter would have the effect of increasing the benefits from raising the storage capacity and, therefore, would lead to a larger $V_o$;

(ii) for $V = V_o$, the 'optimal' vessel sizes did not exceed the allowable limits in both terminals;

(iii) since the middle of 1979, both the fuel costs and the vessel hiring costs have increased significantly; for the current values of these costs the values of $V_o$ would probably be higher still.

The results clearly point to the need for an expansion of the storage capacity at both terminals. Further research, on the lines suggested above, is clearly needed on the capital investment problem discussed.

10.4 Problems Associated with the Specification of Maize and Sorghum Demands

As with the terminals' operating parameters, the demands were treated as given inputs. They were considered deterministic in view of the regime currently adopted of monthly import quotas or 'agreed deliveries'. This system of regulating the demands appears to be the most satisfactory answer to the difficulties posed by the high uncertainty of vessel arrivals, the low storage capacity in the port terminals and the large time lag required between placing an order for a shipment and its arrival.
The demands were also assumed to remain unchanged (over given periods of time). Their values were assumed to be defined externally to the model presented in this chapter. This is appropriate for wheat and the oil seed meals because

(i) their demands are rather inelastic with respect to prices and

(ii) the component of the price affected by the delivery-inventory decisions is small, compared with the total import price.

However maize and sorghum pose a special problem. As shown earlier in Chapter 4 small changes in their relative import prices can produce considerable shifts in their optimal demands. The demand levels themselves have a direct bearing on their import prices through economies of scale in the delivery-inventory operations. For these reasons the interdependence between the delivery-inventory costs and the demands becomes significant in determining the optimal values of the latter. The modelling of this interdependence is the subject of the next chapter.
CHAPTER 7

DETERMINATION OF THE OPTIMAL IMPORTS OF MAIZE AND SORGHUM TAKING INTO ACCOUNT ECONOMIES OF SCALE IN THEIR DELIVERY-INVENTORY OPERATIONS

1. Introduction

The substitution between maize, sorghum and the other raw materials of the animal feed industry was analysed earlier in Chapter 4. A 'static' LP model was developed for the purpose of identifying the general characteristics of that substitution process, under the objective of minimizing the foreign currency expenditure (FCE) associated with the overall mix of raw materials.

If, as before, \(x_{ij}\) is used to denote the quantity of imported raw material \(i\) used in feed \(j\) (in 1000 m.t./year), then the total amount of raw material \(i\) that is imported is \(X_i = \sum_{j} x_{ij}\). If maize and sorghum are denoted by \(i=1\) and \(i=2\), respectively, the objective function of the LP analysed in chapter 4 can be expressed as

\[
f = p_1X_1 + p_2X_2 + \sum_{i=3}^{I} p_iX_i \quad (1000 \text{ US$/year})
\]

where \(p_i\) (\(i=1, \ldots, I\)) are the 'import prices', defined earlier as the variable FCE incurred per unit of each imported raw material (in US$/m.t.). Following the convention introduced in chapter 4, the home produced raw materials involving a significant marginal FCE are referred to as 'imported' and are priced according to the above criterion.

(*) Throughout this chapter the objective function will be represented in this concise form; however it should be borne in mind that the decision variables used in the mathematical models discussed are, as before, the variables \(x_{ij}\).
A major simplification of the model was to assume that the import prices included in the objective function were independent of the amounts of each raw material actually imported. In this chapter, the LP model discussed in Chapter 4 is extended—becoming model 3, in Figure 6 (Chapter 3)—to represent the dependence of the feed grain import prices on the imports of maize and sorghum. The model does not take into account possible economies of scale in the import operations of other raw materials. As pointed out before (Chapter 4, section 6), however relevant they may be for the purpose of defining these raw materials' imports, they will not have a significant effect on the optimal imports of maize and sorghum.

The foreign currency spent on the feed grain imports comprises two basic components. The first one is the cost of acquiring the grains, free on board (fob) at the exporting ports. The feed grains' fob prices do not depend (at least perceptibly) on the quantities actually imported for two main reasons:

(i) in the US, all major grain transactions are 'hedged' with numerous small 'futures' contracts (the 'hedging' operations, discussed later in chapter 8, practically eliminate the possibility of quantity price discounts);

(ii) the inland transport in the US is made in barges, railwagons and trucks, which are much smaller than the ship loads that are exported (economies of scale in the inland transport to the exporting ports are therefore negligible).

The first component of the feed grains' FCE can therefore be expressed as $p_1^* \cdot X_1 + p_2^* \cdot X_2$ where $p_1^*$ and $p_2^*$ denote the fob prices of maize and sorghum (in US$/m.t.).

The second component is the FCE incurred in the insurance, transport and storage of both grains, up to the moment of delivery to the industry. If this component is denoted by ITS ($X_1^*, X_2^*$) (1000 US$/year), the total foreign currency expenditure spent on the feed grain imports
is given by

\[ p_1^* \cdot x_1 + p_2^* \cdot x_2 + \text{ITS}'(X_1, X_2) \]

Replacing \( p_1^* \cdot x_1 + p_2^* \cdot x_2 \) by this expression in the objective function, this becomes

\[ f = p_1^* \cdot x_1 + p_2^* \cdot x_2 + \text{ITS}'(X_1, X_2) + \sum_{i=3}^{I} p_i^* \cdot x_i \]

For any given values of \( X_1 \) and \( X_2 \), the term \( \text{ITS}'(X_1, X_2) \) will depend on the delivery-inventory policy adopted for maize and sorghum. According to the objective of minimizing the overall FCE spent on the import mix, the delivery-inventory policy should be chosen so as to minimize \( \text{ITS}'(X_1, X_2) \). However, as shown in the previous chapter, the delivery-inventory operations of maize and sorghum interact, at each grain terminal, with those of wheat and oilseeds. A delivery-inventory policy devised with the objective of minimizing the foreign currency spent on the feed grains would generally achieve that objective at the expense of an increase in the FCE incurred with the other raw materials. Clearly, in order to minimize the country's overall FCE, the foreign currency spent on the insurance, transport and storage of all raw materials handled at each terminal must be included in the objective function. For each terminal, the objective function becomes

\[ f = p_1^* \cdot x_1 + p_2^* \cdot x_2 + \text{ITS}(X_1, X_2) + \sum_{i=3}^{I} p_i^* \cdot x_i \]

where \( \text{ITS}(X_1, X_2) \) denotes the minimum FCE involved in the insurance, transport and storage of all raw materials handled at the terminal (with maize and sorghum imports equal to \( X_1 \) and \( X_2 \), respectively; the imports of wheat and oilseeds are treated as given inputs).
The function ITS \((X_1', X_2')\) cannot be derived analytically, due to the complexity of the delivery-inventory operations. By exploiting properties of the substitution between maize, sorghum and the other raw materials of the animal feed industry, it is possible to identify a small region on the plane \((X_1', X_2')\) that includes the optimal values of the maize and sorghum imports. The function ITS \((X_1', X_2')\) can then be defined approximately over that region, using the simulation-optimization model described in the previous chapter. As will be shown later, due to economies of scale in the delivery-inventory operations of maize and sorghum, ITS \((X_1', X_2')\) is a non-linear function in \(X_1', X_2'\). However, the form of this function is such that the problem can be handled as a set of easily 'solvable' LP's.

The model will be presented considering the scenario defined in Table 45. This scenario was chosen with the objective of simplifying the explanation, by drawing on results derived in previous chapters. Clearly the model can be applied, in the same way, to any other relevant scenario.

Table 45 - Scenario chosen for illustration of the model

(i) Imports for the region supplied through the Oporto terminal
   (a) Maize (and sorghum) imported to supply the animal feed industry only;
   (b) Total amount of feeds produced: 790 000 m.t./year
   (c) Structure of animal feed production (i.e., quantities to be produced of each feed) as assumed for the whole country in chapter 4

(ii) Prices of the raw materials imported for the animal feed industry:
   (a) Maize (fob price): US $120 / m.t.
   (b) Sorghum (fob price): US $112 / m.t.
   (c) Other raw materials (import prices, i.e., marginal PCE per m.t. imported): prices in US $ / m.t. obtained by multiplying those in price set 1, Table 13, chapter 4, by a factor of 1.56 (this factor was chosen so that results presented earlier in chapter 4 could be used)

(iii) Parameters of the delivery-inventory model as specified in Table 25, chapter 4 (except the demands of maize and sorghum, which are treated here as decision variables)
The procedure adopted in the formulation of the problem relies on an important property of the substitution between maize and sorghum. As shown earlier in chapter 4, the total amount of feed grains in the optimal mix is virtually independent of the ratio between the import prices of sorghum and maize (i.e. whether or not sorghum is present in the optimal mix). This implies that in order to estimate the total amount of feed grains required, sorghum can be excluded initially from the import mix formulation. This is the procedure followed in section 2.

Once the total amount of feed grains in the mix is estimated, the problem of defining the optimal maize/sorghum mix is considered in section 3. Firstly, the effect of replacing maize by sorghum on the overall minimum delivery-inventory costs is analysed. The simulation-optimization model developed in the previous chapter is used to derive a penalty function reflecting the difference between

(i) the minimum delivery-inventory costs achievable for different levels of sorghum imports, and

(ii) the minimum delivery-inventory costs obtained when only maize is imported.

A piecewise linear approximation of this function is then used in the formulation of a set of LP models that enable the determination of the optimal imports of maize and sorghum.

Section 4 contains an analysis of how economies of scale in the delivery-inventory operations of maize and sorghum affect the optimal substitution between these grains. In this section some of the
approximations involved in deriving the LP models are validated.

2. Formulation of the Problem Excluding Sorghum from the Import Mix

2.1 Preliminary Estimation of a Range of Maize Imports \( X_1 \) that includes the Optimal Value \( X_{1\text{op}} \)

When sorghum is excluded from the import mix, the objective function becomes

\[
f = p_1^* X_1 + \text{ITS} (X_1, 0) + \sum_{i=3}^I p_i X_i
\]

In this expression, \( p_1^* \) (maize fob price) and \( p_i \) (import prices of the other raw materials \( i=3, \ldots, I \)) are assumed to take the values defined in Table 45. At the outset, the form of the function \( \text{ITS} (X_1, 0) \) is unknown.

Clearly, any attempt to estimate \( \text{ITS} (X_1, 0) \) should be primarily concerned with the range of possible values of \( X_{1\text{op}} \). This range was estimated earlier in chapter 4. As shown in Figure 17(iii), when the import price of maize varies in relation to the other raw materials' prices between its (near) maximum and minimum values, \( (X_{1\text{op}}) \) varies between 57.7% and 64.5% of the total feed production. For the scenario assumed this production is 790,000 m.t./year and, therefore, \( X_{1\text{op}} \) will lie in the range defined by

\[
(i) \quad (0.577) \cdot 790\ 000 \approx 455\ 000\ \text{m.t./year}
\]
\[
(ii) \quad (0.645) \cdot 790\ 000 \approx 510\ 000\ \text{m.t./year}
\]

This range is quite narrow - the maximum value exceeds the minimum by about 12%. It is therefore unlikely that significant economies of scale in the delivery-inventory operations of maize would be achieved by increasing the level of imports from 455 000 to 510 000 m.t./year. It is therefore reasonable to approximate \( \text{ITS} (X_1, 0) \) by a linear function:
ITS\( (X_1, 0) = C + \Delta_1 \cdot X_1 \)

where

\[ C \quad : \quad \text{a constant term (1000 US$)} \]

\[ \Delta_1 \quad : \quad \text{marginal FCE involved in the insurance, transport and storage of maize (approximately constant over the relevant range of values of } X_1) \quad \text{(US$/m.t.)} \]

Later it will be shown that this approximation of \( \text{ITS}(X_1, 0) \) is valid.

Substituting \( \text{ITS}(X_1, 0) \) in the objective function, this becomes linear:

\[ f = (p_1^* + \Delta_1) \cdot X_1 + \sum_{i=3}^{1} p_i \cdot X_i + C \]

The constant \( C \) is clearly irrelevant in the formulation of the import mix and can be dropped; \( p_1^* + \Delta_1 \) is the variable FCE incurred per unit of imported maize, i.e., the maize import price (\( p_1 \)).

Another result derived earlier can now be used, in the attempt to further reduce the range of values of \( X_1 \) including \( (X_1)_{op} \). Figure 17 (iii) (chapter 4) shows that \( (X_1)_{op} \) is rather insensitive to changes in \( p_1 \) or, for a given \( p_1^* \), is insensitive to changes in \( \Delta_1 \). This can be illustrated deriving \( (X_1)_{op} \) for \( \Delta_1 \) equal to 0 and 30 US$/m.t. (with \( p_1^* = \text{US$ 120/m.t.} \)).

For the scenario defined in Table 45, \( (X_1)_{op} \) can be derived, for each value of \( \Delta_1 \), directly from diagram (iii) in Figure 17. For example, for \( \Delta_1 = 0 \), \( (X_1)_{op} \) can be determined as follows:
(i) \( p_1 = p_1^* = \text{US}\$ 120/\text{m.t.}; \)

(ii) the import price of soybean meal is, for the scenario defined in Table 45,
\( (1.36)(160) = \text{US}\$ 217.6/\text{m.t.}; \)

(iii) the ratio (soybean meal import price/maize import price) becomes \( 217.6/120 = 1.81; \)

(iv) from Figure 17 (iii), for the above price ratio \( (x_1)_{op} \) is 61.5% of the total feed production, i.e. 
\( (0.615)(790\ 000) \approx 485\ 600\ \text{m.t./year}. \)

For \( \Delta_1 = \text{US}\$30/\text{m.t.}, \) the 'optimal' imports of maize are 
\( (x_1)_{op} = (0.595)(790\ 000) \approx 470\ 100\ \text{m.t./year.} \)

In view of the low sensitivity of \( (x_1)_{op} \) to changes in \( \Delta_1 \), any rough interval estimate of the actual value of \( \Delta_1 \) (say, between 0 and 30 US$/m.t.) is adequate for the purpose of defining a narrow range within which \( (x_1)_{op} \) is likely to lie (in the situation above, between 470 000 and 486 000 m.t./year).

2.2 Estimation of \( \Delta_1 \). Determination of the Optimal Maize Imports

The function ITS \((x_1, 0)\) gives, for each value of \( x_1 \), the total FCE incurred in the insurance, delivery and inventory of all raw materials handled at the Oporto grain terminal (this is the terminal considered in the scenario defined in Table 45).

In order to estimate ITS\((x_1, 0)\) it is necessary to investigate where, in those operations, foreign currency is actually spent. This will be done considering, one by one, the insurance, shipping and inventory costs associated with the imports of all raw materials concerned. These costs and the corresponding FCE are as follows:
(i) Insurance of the imported raw materials (at sea).

The importing agencies insure the raw materials with Portuguese insurance brokers. For the policy currently adopted by the importing agencies - 'total loss' policy - the insurance fee is about 0.10% of the fob price of the imported commodity. Although this fee is paid in Escudos, part of it is to cover the risk of losing (and, hence, the risk of having to replace) imported goods. Therefore, part of the fee should be converted into FCE.

(ii) Shipping

The relevant costs incurred in this operation are:

(a) vessel hiring cost: this cost is met, in full, in foreign currency (see chapter 6, section 3.4);

(b) fuel cost: again, this cost is met, in full, in foreign currency (see chapter 6, section 3.4);

(c) vessel insurance cost: the shipping companies insure vessels with Portuguese insurance brokers; the insurance fee is paid in Escudos; however, as for the raw materials' insurance, part of the fee should be converted into FCE;

(d) port charges: clearly only the charges paid at the foreign ports involve a direct FCE; any indirect FCE incurred at the Portuguese ports is likely to be negligible.
(iii) Storage:

The relevant FCE incurred in the storage operations is the cost of capital tied up from the moment the raw materials are bought (fob, at an exporting port) until they are delivered to the industry.

The variable delivery-inventory costs considered in the previous chapter (denoted there by VC) correspond very closely to those identified as relevant for the calculation of ITS(X,0). There are, however, some minor differences between them, arising from two sources:

(i) The delivery-inventory model presented in the previous chapter took the quantity imported of each raw material as a fixed input. Therefore, costs that were strictly proportional to the quantities imported were fixed and could be ignored for the purpose of defining an optimal policy. The costs in this category were the insurance costs (of both raw materials and vessels) and the stock holding costs incurred during the trip between the ports of loading and the Portuguese ports.

(ii) In the delivery-inventory model, charges incurred at the Portuguese ports were included in VC. By including these charges the overall objective of minimizing the country's FCE was, in principle, overridden. However, as noted in chapter 6, section 3.4, these charges are so small that their effect on the delivery-inventory policies derived is negligible.

In view of the nature of these differences, the policy that minimizes the variable delivery-inventory costs (VC) in practice also minimizes the FCE incurred in the insurance, transport and storage of all raw materials handled at the port terminal.
In order to derive IST(X₁,0) it would therefore be sufficient to calculate, for each value of X₁, the minimum cost VC and simply add the extra insurance and stock holding costs and deduct the charges at the Portuguese ports. However, as shown later, this adjustment is so small that VC can be used directly instead of ITS(X₁,0) to estimate Δ₁.

Figure 57 shows – for the scenario defined in Table 45 and assuming no sorghum imports – the relationship between the imports of maize and the minimum variable delivery-inventory costs VC. The diagram covers the range of values of X₁ between 464 000 and 512 000 m.t./year (note that for values of Δ₁ between 0 and 30 US$/m.t. the relevant range of values of X₁, identified in the previous section, was 470 000 and 486 000 m.t./year).

Figure 57 – Relationship between variable delivery-inventory costs and maize imports (sorghum imports excluded)
The minimum variable delivery-inventory costs were estimated for a number of values of $X_1$ using the simulation-optimization model described in the previous chapter. A linear regression model was fitted to the points obtained. No significant improvement in the goodness of fit of the regression could be achieved by fitting higher order polynomials in $X_1$.

The value of $\Delta_{1}$ - the marginal increase in the FCE incurred in the insurance, transport and storage of all raw materials handled at the Oporto terminal, per extra unit of maize - is slightly larger than the slope of the straight line ($s_1 \approx \text{US}\$16/m.t.$). The difference between can be expressed as

$$\epsilon_1 = \Delta_1 - s_1$$

$$= \epsilon_{11} + \epsilon_{12} + \epsilon_{13} - \epsilon_{14}$$

where

$\epsilon_{11}$: unit FCE incurred in the insurance of maize

(if the entire insurance fee were convertible to FCE, $\epsilon_{11} = 0.10\% \cdot \text{US}\$ 120/m.t. = \text{US}\$ 0.12/m.t.);

$\epsilon_{12}$: unit FCE incurred in the insurance of maize vessels

(if the entire insurance fee were convertible to FCE, $\epsilon_{12} = \text{US}\$ 0.75/m.t., according to the expression of the vessel insurance cost presented earlier in chapter 5);

$\epsilon_{13}$: interest charges on the value of each unit of imported maize, incurred during the trip between the exporting ports and the Portugues ports (assuming a rate of interest of 15% per year and US exporting ports (Gulf-A. Coast), $\epsilon_{13} = \text{US}\$ 0.59/m.t.$);
\( \varepsilon_{14} \): port charges in Portugal per extra unit of imported maize (the optimal vessel sizes are not sensitive to changes in \( X_1 \), over the relevant range of maize imports; following the expression of Portuguese port charges given in chapter 5 and assuming a size of maize vessels of 20,000 m.t., \( \varepsilon_{14} = \text{US$ 0.25/m.t.} \)).

In total,

\[ \varepsilon_1 = \Delta_1 - s_1 = \text{US$ 1.21/m.t.} \approx 1\% \text{ of the fob maize price.} \]

In view of the small magnitude of the difference between \( \Delta_1 \) and \( s_1 \), the latter can be used to estimate \( \Delta_1 \). The effect of the error introduced in the determination of the total quantity of maize in the 'optimal' mix is negligible. In fact, the magnitude of \( \varepsilon_1 \) is below the margin of error achievable when forecasting \( p_1^* \), the fob maize price (this will become clear in the next chapter). Furthermore, as shown in the previous section, \( (X_1)_{op} \) is rather insensitive to changes in \( p_1 \) or, for a given \( p_1^* \), to changes (or errors) in \( \Delta_1 \).

Taking \( s_1 \) as an approximation of \( \Delta_1 \) and ignoring the constant \( C \), the objective function becomes

\[
f = (p_1^* + s_1) \cdot X_1 + \sum_{i=3}^{I} P_i \cdot X_i =
\]

\[
= 136 \cdot X_1 + \sum_{i=3}^{I} P_i \cdot X_i
\]

For the scenario defined in Table 45, the optimal value of \( X_1 \) is

\( (X_1)_{op} = (60.8\%) \cdot (730,000) \approx 480,000 \text{ m.t./year} \)

(the value 60.8\% was that obtained for price set 1, defined in Table 13, chapter 4, when the import price of sorghum was sufficiently high to make this grain unattractive).
3. **Formulation of the Problem Including Sorghum in the Import Mix**

As shown earlier in chapter 4, the total imports of feed grains in the optimal mix - $(X_1 + X_2)_{op}$ - are virtually independent of the ratio between the import prices of sorghum and maize (i.e. whether or not sorghum is present in the optimal mix). The value of $(X_1)_{op}$ derived in the previous section can therefore be adopted as an initial estimate of $(X_1 + X_2)_{op}$, when sorghum is included in the mix formulation.

The 'optimal' imports of each grain - $(X_1)_{op}$ and $(X_2)_{op}$ will naturally depend on their relative import prices. As shown earlier in chapter 4, Figure 17(ii) the optimal value of the ratio

$$\gamma = \frac{X_2}{X_1 + X_2}$$

can vary between 0 and about 60%. But the imports of each grain have themselves a direct bearing on their import prices through economies of scale in their delivery-inventory operations.

Assuming that the total amount of the imported grains is constant, Figure 58(i) illustrates the general effect of changing $\gamma$ on the total delivery-inventory costs. If the delivery-inventory operations were identical for both raw materials, the cost curve would be necessarily symmetrical, with a maximum at $\gamma = 50\%$ (broken line). In reality, because sorghum is less dense than maize, for the same imports, sorghum requires more (or larger) ships and more storage space in the silos. Consequently, the unit delivery-inventory costs are greater for sorghum and the cost curve becomes assymetrical (solid line, in diagram (i)).

For the scenario defined in Table 45 and assuming total feed grain imports of 480 000 m.t./year, the minimum variable delivery-inventory costs
Figure 58 - Effect of changes in the feed grain mix on the variable delivery-inventory costs (given fixed total feed grain imports)

(i) General form of the cost curve

- $\gamma$ = sorghum imports / (maize-sorghum) imports [%]

(ii) Piecewise linear approximation of the cost curve for the scenario defined in Table 45 with total feed grain imports of 480,000 m.t./year

- $AB$ (0.0 $\leq X_2 < 25.2$) : $VC = 16,035 + 30.83 \cdot X_2$
- $BC$ (25.2 $\leq X_2 < 96.7$) : $VC = 16,035 + (440 + 13.37 \cdot X_2)$
- $CD$ (96.7 $\leq X_2 < 288.0$) : $VC = 16,035 + (1,413 + 3.31 \cdot X_2)$

- Sorghum imports ($X_2$) [1000 m.t./year]

- Delivery-inventory costs ($VC$) [1000 US $/year]
were estimated for different values of $X_2$ (or, equivalently, $\gamma$), using the simulation-optimization model presented in the previous chapter. As shown in Figure 58(ii) the relationship between the minimum variable delivery-inventory costs (VC) and $X_2$ was approximated by three linear segments (each of them derived by linear regression).

The smallest value of VC is obtained when no sorghum is imported ($X_2 = 0$). The increase in VC incurred by replacing maize by an equal amount $X_2$ of sorghum is given by the penalty function

$$\text{Pen} \ (X_2) = \text{VC (maize/sorghum mix : } X_1, X_2) - \text{VC (no sorghum imported)}$$

Replacing the minimum variable delivery-inventory costs by their values given in Figure 58(ii), $\text{Pen} \ (X_2)$ becomes

$$\text{Pen}(X_2) = \begin{cases} 30.83 \cdot X_2, & 0 \leq X_2 \leq 25.2 \ (1000 \text{ m.t./year}) \\ 440 + 13.37 \cdot X_2, & 25.2 \leq X_2 \leq 96.7 \\ 1413 + 3.31 \cdot X_2, & 96.7 \leq X_2 \leq 288.0 \\ \end{cases} \ (1000 \ \text{US}\$/\text{year})$$

This penalty function is strictly valid only when $X_1 + X_2 = 480 \ 000$ m.t./year. However, as shown earlier in Figure 17(i), chapter 4, as the import price ratio between sorghum and maize changes, $(X_1 + X_2)_{op}$ suffers small adjustments. For example for price set 1, $(X_1 + X_2)_{op}$ varies between 60.8 and 63.8% of the total animal feed production. In order to study the effect of such adjustments in the feed grain imports on $\text{Pen}(X_2)$, this function was derived for $X_1 + X_2 = 504 \ 000$ m.t./year = 63.8% of 790 000 m.t./year. This function is compared, in Figure 59, with that obtained previously for $X_1 + X_2 = 480 \ 000$ m.t./year = 60.8% of 790 000 m.t./year. The relative difference between them does not exceed 2.9% for any value of $X_2$. As will be discussed later, this
difference is negligible for the determination of the optimal imports of maize and sorghum.

Figure 59 - Penalty function \( \text{Pen}(X_2) \) for different levels of the total feed grain imports (scenario defined in Table 45)

When sorghum is included in the mix formulation, the objective function to be minimized is

\[
f = p_1 * X_1 + p_2 * X_2 + \text{ITS}(X_1, X_2) + \sum_{i=3}^{I} p_i X_i
\]

Let \( T \) denote the total feed grains imports (i.e. \( T = X_1 + X_2 \)). If the term

\[
\text{ITS}(X_1 = T, X_2 = 0) = \text{ITS}(T, 0) = C + \Delta_1 T
\]

is added to and subtracted from the objective function, this becomes

\[
f = p_1 * X_1 + p_2 * X_2 + \text{ITS}(T, 0) + \left[ \text{ITS}(X_1, X_2) - \text{ITS}(T, 0) \right] + \sum_{i=3}^{I} p_i X_i
\]

The difference \( \left[ \text{ITS}(X_1, X_2) - \text{ITS}(T, 0) \right] \) is the extra FCE incurred in the insurance, transport and storage of all raw materials handled at the Oporto terminal when, rather than importing \( T \) units of maize alone, the mix maize/sorghum \( (X_1, X_2) \) units is imported.
If, as before, \( \Delta_1 \) is approximated by \( s_1 \) and if, similarly, 
\[ \text{ITS}(X_1, X_2) - \text{ITS} (T, 0) \] 
is approximated by \( \text{Pen}(X_2) \), the objective function becomes
\[
f = p_1^* X_1 + p_2^* X_2 + (C + s_1 \cdot T) + \text{Pen}(X_2) + \sum_{i=3}^{I} p_i^* X_i
\]
(it will be shown later that the effect of these approximations in the determination of \( (X_1)_{\text{op}} \) and \( (X_2)_{\text{op}} \) is negligible).

Substituting \( T \) by its value \( X_1 + X_2 \), dropping the constant \( C \) and rearranging terms, the objective function becomes
\[
f = (p_1^* + s_1) X_1 + (p_2^* + s_1) X_2 + \text{Pen}(X_2) + \sum_{i=3}^{I} p_i^* X_i
\]
or, replacing \( p_1^*, p_2^* \) and \( s_1 \) by their values,
\[
f = (120 + 16) X_1 + (112 + 16) X_2 + \text{Pen}(X_2) + \sum_{i=3}^{I} p_i^* X_i
\]
\[
= 136 X_1 + 128 X_2 + \text{Pen}(X_2) + \sum_{i=3}^{I} p_i^* X_i
\]

For each linear section (I, II, III) of the function \( \text{Pen}(X_2) \), the best mix can be derived solving an LP:

(i) \( \text{LP}_1 \) (for \( 0 \leq X_2 \leq 25.2 \) (1000 m.t./year)):

\[
\text{minimize : } f = 136 X_1 + (128 + 30.83) X_2 + \sum_{i=3}^{I} p_i^* X_i
\]
\[
= 136 X_1 + 158.83 X_2 + \sum_{i=3}^{I} p_i^* X_i \text{ (1000 US $/year)}
\]

subject to: 
\[
[A] \cdot \{x\} \leq \{b\} \text{ (constraints considered in Chapter 4)}
\]
\[
X_2 \leq 25.2
\]
(ii) \( \text{LP}_{II} \) (for 25.2 \( \leq X_2 \leq 96.7 \)):

\[
\begin{align*}
\text{minimize} & : f = 440 + 136 \cdot X_1 + 141.37 \cdot X_2 + \sum_{i=3}^{I} p_i \cdot X_i \\
\text{subject to} & : [A] \cdot \{x\} \geq \{b\} \\
25.2 \leq X_2 & \leq 96.7
\end{align*}
\]

(iii) \( \text{LP}_{III} \) (for 96.7 \( \leq X_2 \leq 288.0 \)):

\[
\begin{align*}
\text{minimize} & : f = 1413 + 136 \cdot X_1 + 131.31 \cdot X_2 + \sum_{i=3}^{I} p_i \cdot X_i \\
\text{subject to} & : [A] \cdot \{x\} \geq \{b\} \\
96.7 \leq X_2 & \leq 288.0.
\end{align*}
\]

Clearly, the optimal solution is the one that leads to the overall minimum value of \( f \); for the scenario assumed, the optimal imports of maize and sorghum are

\[
\begin{align*}
(\bar{X}_1)_{\text{op}} & = 60.8\% \cdot (790 \, 000) = 480 \, 000 \, \text{m.t./year} \\
(\bar{X}_2)_{\text{op}} & = 0
\end{align*}
\]

and the overall minimum value of \( f \) is 93.2 million US$/year (\( f \) is a proxy for the variable FCE; in the remainder of this chapter it will be referred to simply as 'FCE').

4. Effect of the Economies of Scale in the Delivery-Inventory Operations on the Optimal Substitution Between Maize and Sorghum

4.1 Comparison of Substitution Curves for FOB and Import Prices

The previous sections presented a method of determining the optimal import mix, taking into account economies of scale in the delivery-inventory operations of maize and sorghum. This section discusses

(i) the effect of these economies of scale on the optimal substitution between maize and sorghum, as their relative FOB prices vary;
(ii) the validity of the approximations involved in deriving the proposed LP models.

The analysis will be illustrated considering the scenario defined in Table 45 (with maize fob price = US$ 120/m.t.) but allowing the fob sorghum price to vary between US$ 112/m.t. (shown earlier to be unattractive) and US$ 90/m.t..

The penalty function \( \text{Pen}(X_2) \) was estimated, in the previous section, considering the fob sorghum price of US$ 112/m.t.. Clearly, as this price changes, the sorghum inventory costs will vary, affecting the penalty function. In order to test its sensitivity with respect to changes in the fob sorghum price, the penalty function was re-estimated setting this price at US$90/m.t.. The new estimate of \( \text{Pen}(X_2) \) is compared in Figure 60 with that derived in the previous section with the fob sorghum price set at US$112/m.t..

![Figure 60 - Penalty function Pen(X₂) for different values of the sorghum fob price (scenario defined in Table 45, with \( X_1 + X_2 = 480,000 \) m.t./year)]

The difference between the estimated functions varies between

(i) US$ 27 200/year (= 3.7% of \( \text{Pen}(X_2) \)), for \( X_2 = 25200 \) m.t./year and
(ii) US$ 44 300/year (=1.9% of \( \text{Pen}(X_2) \)), for \( X_2 = 288000 \) m.t./year.
In view of the small magnitude of these differences, Pen ($X_2$) can be replaced, for the whole range of sorghum prices, by the penalty function derived in the previous section (with $p_2^* = \text{US}\$ 112/m.t.). As will be discussed later, the errors introduced by this approximate procedure in the determination of the optimal imports of maize and sorghum are negligible.

Using the method described in the previous section, the optimal imports of maize and sorghum were calculated for the whole range of sorghum f.o.b. prices. Figure 61 shows for each value of the sorghum/maize f.o.b. price ratio, the optimal values of

(i) the total feed grain imports, expressed as a proportion of the global feed production (diagram (i), solid line);

(ii) the sorghum imports, expressed as a proportion of the total feed grain imports (diagram (ii), solid line).

The broken lines presented in the diagrams show the relationship between the optimal feed grain imports (expressed as above) and the import price ratio between sorghum and maize.

The total feed grain imports are rather insensitive to changes in the relative prices between sorghum and maize — whether import prices or f.o.b. prices are used. As shown in diagram (i), the solid and broken lines do not differ significantly.

However, as shown in diagram (ii), the relationship between the optimal value of $\gamma = X_2/(X_1 + X_2)$ and the f.o.b. price ratio (solid line) differs markedly from that between $\gamma$ and the import price ratio (broken line).
Figure 61 - Optimal feed grain imports for different values of the sorghum/maize price ratio

(1) (Maize-sorghum) imports expressed as a proportion of the total animal feed production

Sorghum/maize price ratio:
- FOB prices (scenario defined in Table 45, with sorghum FOB price allowed to vary)
- Import prices (as for price set 1, in Figure 17, chapter 4)

(2) Sorghum imports expressed as a proportion of the total feed grain imports ($Y$)

Sorghum/maize price ratio:
- FOB prices (scenario defined in Table 45, with sorghum FOB price allowed to vary)
- Import prices (as for price set 1, in Figure 17, chapter 4)

Note: the diagrams are based on solutions derived for discrete sets of values of the price ratios:
- FOB prices: from 0.75 to 1.10, with increments of 0.01.
- Import prices: from 0.80 to 1.10, with increments of 0.01.

Any differences between solutions evaluated at two successive points of each grid are represented on the diagrams as step changes occurring at the midpoints of the grid intervals.
For values of the fob price ratio up to about 0.885 the solid line has nearly the same shape as the broken line, but it is displaced to the left. The same happens with the lines plotted in diagram (i), although in this case the effect of the displacement is less noticeable.

The displacement between the solid and the broken lines can be derived by calculating, for each fob price ratio, the import price ratio between maize and sorghum that leads to the same optimal solution. For the fob price ratios smaller than 0.885, the optimal sorghum imports lie between 33.5 and 58% of the total feed grain imports (see diagram (ii) in Figure 61). This means that the optimal solution is derived from LP_{III}, i.e. the LP including in its objective function the following section of the penalty function:

\[ \text{Pen} \left( X_2 \right) = 1413 + 3.31 \cdot X_2, \quad 96.7 \leq X_2 \leq 288.0 \]

(or \( 20.1\% \leq Y \leq 60.0 \)).

Using the notation introduced previously, the objective function of that LP is

\[ f = \left( p_1^* + s_1 \right) \cdot X_1 + \left( p_2^* + s_2 \right) \cdot X_2 + \text{Pen}(X_2) + \sum_{i=3}^{I} p_i \cdot X_i \]

\[ = 136 \cdot X_1 + (p_2^* + 16) \cdot X_2 + (1413 + 3.31 \cdot X_2) \sum_{i=3}^{I} p_i \cdot X_i \]

hence

\[ f - 1413 = 136 \cdot X_1 + (p_2^* + 16 + 3.31) \cdot X_2 + \sum_{i=3}^{I} p_i \cdot X_i \]

From this expression it is clear that the optimal solution for a fob price ratio

\[ (R)^*_{fob} = \frac{p_2^*}{p_1^*} = \frac{p_2^*}{120} \]
is identical to that obtained for an import price ratio

\[(R)_{\text{import}} = (p_2^* + 19.31)/136 = [120. (R)_{\text{fob}} + 19.31]/136.\]

The displacement between the solid and broken lines, for
for \((R)_{\text{fob}} \leq 0.885\), is given by

\[\Delta R = (R)_{\text{import}} - (R)_{\text{fob}} = 0.142 - 0.112. (R)_{\text{fob}}^*\]

For the relevant range of values of \((R)_{\text{fob}}\) the displacement between
the solid and broken lines varies between

(i) \(\Delta R = 0.053\) (for \((R)_{\text{fob}} = 0.75\)) and

(ii) \(\Delta R = 0.038\) (for \((R)_{\text{fob}} = 0.885\)).

The displacement \(\Delta R\) decreases continuously as \((R)_{\text{fob}}\) increases up
to 0.885. However, since the lines shown in diagrams (i) and (ii) of
Figure 61 were derived from the solutions obtained for a discrete set
of values of the price ratios (grid size = 0.01), they cannot
show that continuous change in \(\Delta R\). Instead, the diagrams show only two
distinct values of \(\Delta R\): either 0.04 or 0.05 (see Figure 61(ii)).

When the sorghum/maize fob price ratio reaches 0.885, sorghum
becomes unattractive. In diagram (ii) of Figure 61, the solid line
changes abruptly from 33.5% to 0%. This is in clear contrast with the
relationship shown by the broken line which gives a number of optimal
values of \(\gamma\) between 0 and 33.5%.

The explanation for this difference is given in Figure 62. For
each linear section of the penalty function (shown in diagram (i)),
Figure 62 - Minimum \( FCE \) on total imports vs. fob price ratio between maize and sorghum (scenario defined in Table 45, with sorghum fob price allowed to vary)

(1) Penalty cost function \( \text{Pen}(X_2) \)

(total feed grain imports = 480 000 m.t./year)

\[
\begin{align*}
\text{Section I:} & \quad \{ 0.0 < X_2 < 25.2 \} : \quad \text{Pen}(X_2) = 30.63 \cdot X_2 \\
\text{Section II:} & \quad \{ 25.2 < X_2 < 96.7 \} : \quad \text{Pen}(X_2) = 440 + 13.37 \cdot X_2 \\
\text{Section III:} & \quad \{ 96.7 < X_2 \leq 288.0 \} : \quad \text{Pen}(X_2) = 1413 + 3.31 \cdot X_2
\end{align*}
\]

- \( e' = 30.6 \) US $/m.t. \\
- \( e'' = 12.1 \) \\
- \( e''' = 8.2 \)

- Sorghum imports \( X_2 \) [1000 m.t./year]

- Sorghum imports / (maize+sorghum) imports \( \gamma \) [%]

(11) Minimum \( FCE \) within each section of the penalty function \( \text{Pen}(X_2) \) and global minimum \( FCE \)

\[
\begin{align*}
\gamma = 2.5 \% & \quad \gamma = 0 \% & \quad \gamma = 20 \% & \quad \gamma = 53.5 \% & \quad (I) \\
\gamma = 20 \% & \quad \gamma = 58 \% & \quad \gamma = 57.5 \% & \quad \gamma = 47 \% & \quad (II) \\
\gamma = 58 \% & \quad \gamma = 58 \% & \quad \gamma = 58 \% & \quad \gamma = 58 \% & \quad (III)
\end{align*}
\]

- Sorghum fob price / Maize fob price

(Maize fob price = 120 US $ / m.t.)
the corresponding LP was solved for the whole range of values of the fob price ratio considered earlier. The minimum 'FCE' obtained from each LP is plotted against the fob price ratio in diagram (ii).

For \((R)_{\text{fob}} \leq 0.885\), the solutions with the smallest 'FCE' are those derived from LP\( _{III}\). For \((R)_{\text{fob}} = 0.75\) the optimal value of \(\gamma\) is 58% (as shown earlier in Figure 61(ii)). As the sorghum fob price increases, \(\gamma\) decreases, reaching 33.5% for \((R)_{\text{fob}} = 0.885\) (as \(\gamma\) decreases, the slope of the minimum 'FCE' line (III) decreases as well). For values of the price ratio larger than 0.885 the smallest 'FCE' is obtained for the solutions derived from LP\( _{I}\) (with \(\gamma = 0\%\)). Solutions involving values of \(\gamma\) between 0 and 33.5 are not optimal in view of the high penalty costs incurred when small amounts of sorghum are imported. As shown in diagram (i), for values of \(\gamma\) in the range \([0,33.5\%]\), the unit penalty cost incurred by replacing maize by sorghum in the import mix varies between \(s' = \text{US}\ 30.8/\text{m.t.}\) and \(s'' = \text{US}\$ 12.1/\text{m.t.}\).

4.2 Validation of the Approximations Used in the Derivation of the Optimal Imports of Maize and Sorghum

In the process of determining the optimal imports of maize and sorghum several approximations were made. Firstly, the FCE incurred in the insurance, transport and storage of all the raw materials handled at the Oporto terminal was replaced by the delivery-inventory costs. In section 2, the difference between them was identified and was shown to be negligible for the purpose of defining the total feed grain imports. This difference will now be shown to be equally negligible for the determination of the optimal imports of each grain.
In the objective function
\[
\begin{align*}
f &= \mathbf{p}_1^* X_1 + \mathbf{p}_2^* X_2 + [C + A_1(X_1 + X_2)] + [\mathrm{ITS}(X_1, X_2) - \mathrm{ITS}(T, 0)] + \sum_{i=3}^{I} \mathbf{p}_i^* X_i
\end{align*}
\]

\(\Delta_1\) was approximated by \(s_1\) and \([\mathrm{ITS}(X_1, X_2) - \mathrm{ITS}(T, 0)]\) was approximated by \(\mathrm{Pen}(X_2)\). As shown earlier in section 2, the marginal FCE per m.t. of maize \(\Delta_1\) exceeds the marginal delivery-inventory cost per m.t. of maize \(s_1\) by

\[
\epsilon_1 = \Delta_1 - s_1 = \text{US$1.21/m.t.}$
\]

The marginal delivery-inventory cost per m.t. of sorghum \(s_2\) underestimates the marginal FCE \(\Delta_2\) by a similar difference. This difference \((\epsilon_2 = \Delta_2 - s_2)\) was calculated, as for maize, for the relevant range of sorghum fob prices \([0.75, \mathbf{p}_1^*, 0.885 \cdot \mathbf{p}_1^*]\) and was found to vary between

\[
\begin{align*}
(i) & \quad \epsilon_2 = \text{US$1.10/m.t.}$, \quad \text{for} \quad \mathbf{p}_2^* = 0.75 \cdot \mathbf{p}_1^*, \quad \text{and} \\
(ii) & \quad \epsilon_2 = \text{US$1.21/m.t.}$, \quad \text{for} \quad \mathbf{p}_2^* = 0.885 \cdot \mathbf{p}_1^*.
\end{align*}
\]

Given that \([\mathrm{ITS}(X_1, X_2) - \mathrm{ITS}(T, 0)]\) and \(\mathrm{Pen}(X_2)\) both concern the replacement of maize by an equal amount \((X_2)\) of sorghum in the import mix, the difference between them becomes

\[
\begin{align*}
[\mathrm{ITS}(X_1, X_2) - \mathrm{ITS}(T, 0)] - \mathrm{Pen}(X_2) &= \\
&= (\Delta_2 - s_2) \cdot X_2 \quad \text{(sorghum entering the mix)} \\
&\quad - (\Delta_1 - s_1) \cdot X_2 \quad \text{(maize leaving the mix)} \\
&= (\epsilon_2 - \epsilon_1) \cdot X_2
\end{align*}
\]

Substituting

\[
\Delta_1 = s_1 + \epsilon_1
\]

and

\[
[\mathrm{ITS}(X_1, X_2) - \mathrm{ITS}(T, 0)] = \mathrm{Pen}(X_2) + (\epsilon_2 - \epsilon_1) \cdot X_2
\]

in the objective function and ignoring the constant \(C\),
\[ f = (p_1^* + s_1 + \varepsilon_1)X_1 + (p_2^* + s_2 + \varepsilon_2)X_2 + \text{Pen}(X_2) + \sum_{i=3}^{I} p_i X_i. \]

For section III of Pen \((X_2)\) the objective function is

\[ f = 1413 + (120 + 16 + \varepsilon_1)X_1 + (p_2^* + 16 + 3.31 + \varepsilon_2)X_2 + \sum_{i=3}^{I} p_i X_i \]

\[ = 1413 + (136 + \varepsilon_1)X_1 + (p_2^* + 19.31 + \varepsilon_2)X_2 + \sum_{i=3}^{I} p_i X_i. \]

For any given sorghum fob price \(p_2^*\), the optimal feed grain import mix derived from the LP with this objective function is the one corresponding to the import price ratio.

\[ (R)_{\text{import}} = \frac{p_2^* + 19.31 + \varepsilon_2}{136 + \varepsilon_1} = \frac{120 \cdot (R)_{\text{fob}} + 19.31 + \varepsilon_2}{136 + \varepsilon_1} \]

For the same fob price of sorghum, when, as before, \(\varepsilon_1\) and \(\varepsilon_2\) are neglected, the feed grain import mix derived from LP\(_{III}\) is that corresponding to

\[ (R)_{\text{import}} = \frac{(120 \cdot (R)_{\text{fob}} + 19.31)}{136} \]

For the range of sorghum prices \([0.75.p_1^*, 0.885. p_1^*]\) and for \(\varepsilon_1 = \text{US}$. 1.21/m.t. and \(\varepsilon_2\) between 1.10 and 1.21 US$/m.t., the absolute value of the difference between \((R)_{\text{import}}\) ('true' ratio) and \((R)_{\text{import}}\) (approximate ratio) is always less than 0.0018. The effect of this difference on the solutions derived through LP\(_{III}\) is clearly negligible.

It now remains to be shown that the sorghum fob price at which the solutions derived from LP\(_{III}\) become non-optimal is practically the same whether \(\varepsilon_1\) and \(\varepsilon_2\) are included in the objective function or are neglected. Referring back to diagram (ii) of Figure 62, when \(\varepsilon_1\) and \(\varepsilon_2\) were neglected, that price was derived from the
intersection of lines III and I. If \( \epsilon_1 \) and \( \epsilon_2 \) were included in the objective function, since the solutions derived from LP_{III} would not have been affected, line III would be shifted upwards by an amount \( \Delta f = \epsilon_1 X_1 + \epsilon_2 X_2 \). The displacement of line III in the neighbourhood of \( p_1^* = 0.885.p_2^* \) is represented in Figure 63 together with those of lines I and II. As shown in the diagram, the vertical displacement is almost identical for all three lines. This is a consequence of the fact that \( \epsilon_1 = \epsilon_2 \) and \( X_1 + X_2 \) is nearly constant for the solutions derived from LP_{I}, LP_{II} and LP_{III}. As a result, the lines I and III intersect at a sorghum fob price which does not differ significantly from that corresponding to the intersection of I' and III'. The estimated difference between those prices (0.004. US$/120/m.t. = US$ 0.48/m.t.) is clearly negligible for all practical purposes.

*Figure 63 - Inclusion of \( \epsilon_1 \) and \( \epsilon_2 \) in the objective function: effect on 'FCE'*

Other approximations involved in the determination of the optimal imports of maize and sorghum concern the shape of the penalty function \( \text{Pen}(X_2) \).
Firstly this function was assumed to be piecewise linear. The choice of the linear sections of Pen($X_2$) was to some extent arbitrary. This was particularly so for small values of $X_2$ (sections I and II), where the theoretical penalty function has the largest curvature. However, as shown earlier, this is precisely the part of the penalty function where its accuracy is less critical: when sorghum is imported in small quantities, its unit delivery-inventory cost becomes prohibitively high. The critical region of the penalty function is section III where, as shown earlier in Figure 58, the linear approximation is most appropriate.

Secondly, changes in the penalty function resulting from

(i) changes in the total feed grain imports in relation to the value assumed when deriving Pen($X_2$), and

(ii) changes in the fob sorghum price in relation to this value assumed when deriving Pen($X_2$)

were neglected. The effect of these changes was studied in a way similar to that used to ascertain the validity of ignoring $\varepsilon_1$ and $\varepsilon_2$ in the objective function. The errors resulting from neglecting the changes (i) and (ii) above were found to be of the same order of magnitude as those from neglecting $\varepsilon_1$ and $\varepsilon_2$, and, therefore, are equally negligible.

5. Conclusion

The import mix model was extended to take into account the dependence of the import prices of maize and sorghum on the imported quantities of both grains.

The results derived in this chapter demonstrated that the delivery-inventory costs - higher for sorghum than for maize and varying considerably with the quantity of each grain that is actually imported -
have a significant impact on the optimal feed grain import mix. In particular it was shown that

(i) importing sorghum in small quantities (for the scenario considered, quantities less than 33.5% of the total feed grain imports) is always disadvantageous, in view of the high delivery-inventory costs that would be incurred in such a situation;

(ii) sorghum only becomes competitive with maize when its fob price is substantially lower than that of maize (for the scenario considered, when $p_2^* \leq 0.885. p_1^*$).

In recent years shipping costs have been substantially higher than those considered in the scenario defined in Table 45. This has increased the economies of scale in the delivery-inventory operations, further inhibiting the substitution of maize by sorghum. In other words, sorghum imports only become advantageous in larger quantities and for even lower relative prices sorghum/maize.

Two further important aspects of the import mix planning problem remain to be covered. These are

(i) the uncertainty of the raw materials' prices, and

(ii) the dynamic nature of the problem.

Further developments of the model concerning these aspects will be discussed in Chapter 9, after the feed grain purchasing problem is analysed in detail in the next chapter.
CHAPTER 8
FEED GRAIN PURCHASING

1. Introduction

This chapter focuses on the study of the feed grain purchasing operations and on the development of a model - model 4, in Figure 6 (Chapter 4) - covering the most significant aspects of the problem associated with the buying decisions.

According to the diagram of Figure 6, in order to derive a purchasing policy for the feed grains it is necessary to specify in advance the size and timing of the shipments of maize and sorghum. The diagram also suggests that, because of the existing feedback loop involving models 2, 4 and 5, the specification of the feed grain shipments depends, in principle, on the purchasing policy that is adopted. However, as shown later in Chapter 9, this latter dependence is rather weak and will be ignored in the present chapter (the model considered here takes therefore the size and the timing of shipments - determined 'optimally' using the delivery-inventory model - as given inputs). The other feed back loop shown in the diagram (upper loop, involving models 4 and 5) will be equally ignored in this chapter. Its meaning and its implications for the planning and control system will be discussed later in chapter 9.

Following this brief introduction, the general purchasing environment in which an importer such as EPAC operates is discussed in section 2. Having examined the basic options open to the importer for conducting the buying operations, a purchasing 'operating doctrine' is chosen. The purchasing of maize 'futures' is shown to be the central issue in the whole process.
Section 3 concentrates on the short-term price forecasting of maize 'futures' prices. Two different models are discussed: Trigg and Leach's adaptive exponential smoothing and Taylor's price trend model. Both these models are used later as a basis to derive what, in Kingsman's terminology (see Chapter 3, section 4.3), are called the 'tactical' buying decisions.

The medium-term forecasting of maize prices is studied in section 4. Firstly, a behavioural model developed by Kingsman for the forecasting of maize 'cash' prices at Chicago and at the Gulf ports is presented. This model is then adapted to the forecasting of maize 'futures' prices at Chicago, to be used later on as a basis for deriving what Kingsman calls the 'active buying periods' of the feed grain shipments.

The 'tactical' buying problem is analysed in detail in section 5. After examining a purchasing rule proposed earlier by Kingsman, a new rule is developed and evaluated. It is shown to be an improvement on Kingsman's rule and to lead to statistically significant savings in relation to a policy yielding average purchasing costs.

In section 6 a decision rule is proposed to assist the purchaser in deriving the shipments' active buying periods, according to his attitudes towards risk. The results of a simulation study of the purchasing operations indicate that, under the proposed rule, substantial savings in the purchasing costs can be achieved.

The chapter is concluded in section 7 with a brief summary and with the discussion of further possible developments of the model.
2. The General Purchasing Environment. Selection of a Purchasing Operating Doctrine

2.1 Specification of Size and Timing of Feed Grain Shipments: an Illustration

The general purchasing environment in which an importer such as EPAC operates and the different ways in which the buyer can conduct the purchasing operations will be illustrated by considering the purchase of a hypothetical shipment. It will be assumed that on a given date \( t_0 \) — say 15th October 1981 — amongst the shipments specified using the delivery-inventory model, there is one with the following characteristics:

(i) raw material: maize (Yellow, US # 3);
(ii) size of shipment (Q): 30 000 m.t.;
(iii) planned date of delivery (\( d_0 \)): 28th May 1982;
(iv) port of loading: US, Gulf;
(v) length of loading period (LLP): 16 days;
(vi) rate of loading (LR): 5 000 m.t./day (shinc).

Using expressions presented earlier in Chapter 5 (sections 2.2 and 2.3), it is possible to derive from conditions (ii) to (vi) the following dates:

(vii) 'target' date at the loading port:

\[
TDALP = \frac{d_0 - [E(VT) + Q/LR + 6]}{4} = \frac{d_0 - (13 + 30/5 + 6)}{4} = \frac{d_0 - 25}{4} = 3rd\ May\ 1982.
\]

(viii) earliest day of loading period:

\[
EDLP = TDALP - \frac{1}{4} LLP = TDALP - 4\ days = 30th\ April\ 1982.
\]
These dates are represented in the diagram in Figure 64. From these, the purchaser can derive the dates that are most critical to the process of importing the maize shipment, namely

(ix) the deadline for the shipping contract (or simply 'shipping deadline') — one month before the 'target' date at the loading port: \( t_1 = 3\text{rd April 1982} \);

(x) the deadline for the purchasing contract with an exporter (or simply 'purchasing deadline') — 15 days before the first day of the loading period; this is about the minimum time required by an exporter to buy the grain in the US market and to move it to a port terminal: \( t_2 = 15\text{th April 1982} \).

Figure 64 - Definition of deadlines for shipping and purchasing contracts

Once all the conditions defining a shipment are specified, the purchaser must decide on how to conduct the purchasing operation. The options available to him and the basic mechanisms of the purchasing operations will be analysed below, following a brief description of the way in which commodity exchanges operate.
2.2 Commodity Exchanges: 'Cash' and 'Futures' Markets

The bulk of the international grain trade is conducted within or under the direct influence of commodity exchanges. Amongst these, the Exchanges operating in Chicago are the largest and the most influential ones in the world grain trade.

A commodity exchange consists of two organised markets - 'cash' and 'futures' - that interact and are mutually dependent. The 'cash' market concerns the transaction of physical material between a seller and a buyer. If a purchase is made on a particular day at the 'cash' (or 'spot') price, the buyer assumes ownership of the actual physical material and receives delivery immediately. He then stores it at his own expense until he uses or resells the commodity.

Dealing in the 'futures' market means making a contract to delivery (if the seller elects) or to receive (if the buyer elects) a specified quantity of a particular commodity at some stated time in the future. In most exchanges 'futures' are traded only for specific months. For example, at Chicago maize 'futures' are traded for delivery in March, May, July, September and December. Unlike contracts in the 'cash' market, 'futures' contracts are standardized: quantity, grade, delivery specifications are set by each exchange and are applicable to all 'futures' transactions. In the Chicago maize exchange each contract corresponds to 5000 bushels (approximately 127 m.t.) of yellow maize grade no. 2, deliverable at a number of warehouses approved by the Chicago Board of Trade. When a 'futures' contract is settled by delivery, this must be made between the first and the last business day of the delivery month. Within this period, the time of delivery is at the discretion of the seller.
The 'futures' contracts are temporary substitutes for 'cash' commodities. If they are held until maturity, delivery of the physical commodity is guaranteed by the exchange. This implies that when maturity is reached, the 'cash' and 'futures' prices will be approximately equal. In practice, the cost of making or taking a delivery generally causes a small difference between the 'cash' and 'futures' prices, even on the last trading day of an expiring contract.

Although the right of delivery is always guaranteed to both buyers and sellers of 'futures', the volume of 'futures' contracts settled by delivery is actually very small. Instead, most buyers and sellers liquidate their original positions before the 'futures' reach maturity. An original purchase or sale of a number of contracts can be liquidated, at any time before maturity, by an offsetting sale or purchase of the same number of contracts. This is possible because the clearing house of the exchange assumes full responsibility for the fulfilment of all contracts made in the exchange. Making use of the 'principle of offset', producers, traders, users or mere speculators trade in the 'futures' markets either to protect themselves against adverse future price changes or to attempt to profit from price changes, when these can be successfully predicted.

Ways in which 'futures' markets can be used in the import-export operations will be discussed below, considering the two basic options open to an importer such as EPAC, when buying grain shipments:

(i) purchasing directly the 'physical' commodity from an exporter, and

(ii) purchasing 'futures' and later converting them to 'physical'.
2.3 Purchasing directly in 'physical'

This option, adopted in the past by EPAC, is suitable when the purchasing operations are conducted without the support of effective quantitative methods. In the absence of reliable price forecasts and without a sound method to convert them to purchasing decisions, the (cautious) purchaser will generally buy each shipment near the corresponding deadline (for the maize shipment considered in section 2.1, near \( t_2 \) - See Figure 64). This policy, usually called 'hand-to-mouth', leads to purchasing costs that, in the long run, will tend to be the average market price (fob, prompt delivery).

Clearly, if the purchaser has access to reliable price forecasts and is confident that prices will rise progressively from their current levels (at \( t_0 \)) until the purchasing deadline is reached (at \( t_2 \)), then he will naturally try to anticipate the purchase. This can still be achieved in the 'physical' market by making a purchase (at time \( t_0 \), say) for 'forward delivery' (with the loading period specified as before).

Between the date of purchase (\( t_0 \)) and the date (\( t_2 \)) when the exporter will have to start the delivery process (i.e. acquiring maize in the US market and moving it to a terminal in the loading port), the price of maize will generally change (in the situation considered here, the buyer expects the price to rise). The exporter, who neither produces maize himself nor stores it in significant quantities and who does not regard speculation in the maize market as part of his business, will protect himself against such a price rise by 'hedging' the sale with 'futures'. Figure 65 illustrates the principle of the operation, considering the hypothetical situation where:
Figure 65 - Illustration of the 'perfect' buying hedge (by the exporter)

(1) Hypothetical maize price movement

![Graph showing hypothetical maize price movement with lines for Chicago May 'futures' and Chicago 'cash'.]

(2) Exporter's account

<table>
<thead>
<tr>
<th></th>
<th><strong>CASH</strong></th>
<th></th>
<th><strong>FUTURES</strong> (May delivery)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>t₀</strong> : 15th October, 1981</td>
<td><strong>t₀</strong> : 15th October, 1981</td>
<td></td>
</tr>
<tr>
<td>sell</td>
<td></td>
<td>buy 30,000 m.t. of 'futures'</td>
<td></td>
</tr>
<tr>
<td>30,000 m.t. of maize</td>
<td>Price / m.t.</td>
<td>Price / m.t.</td>
<td></td>
</tr>
<tr>
<td>Chicago 'cash'</td>
<td>US $ 110</td>
<td>Chicago May 'futures'</td>
<td>US $ 115</td>
</tr>
<tr>
<td>Extra fob costs</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit margin</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t₂ : 15th April, 1982</td>
<td></td>
<td>t₂ : 15th April, 1982</td>
<td></td>
</tr>
<tr>
<td>buy 30,000 m.t. of maize</td>
<td>Price / m.t.</td>
<td>sell 30,000 m.t. of 'futures'</td>
<td>Price / m.t.</td>
</tr>
<tr>
<td>and deliver fob</td>
<td></td>
<td>Chicago May 'futures'</td>
<td>US $ 110</td>
</tr>
<tr>
<td>Chicago 'cash'</td>
<td>US $ 125</td>
<td>Extra fob costs</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Profit margin</td>
<td>132</td>
</tr>
<tr>
<td>'Cash' result :</td>
<td>30,000 m.t. of maize @ US $ -14 / m.t.</td>
<td>'Futures' result :</td>
<td>30,000 m.t. of 'futures' @ US $ -15 / m.t.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Profit margin : US $ 1 / m.t.</td>
<td></td>
</tr>
</tbody>
</table>
(i) the exporter buys 'physical' maize at Chicago
    (the Chicago 'cash' market is used as a term of
    reference; the operation would be identical if the
    purchases in 'physical' were made in some other market);
(ii) the 'cash' price and the May 'futures' price at Chicago
    move exactly in parallel between times \( t_0 \) and \( t_2 \);
(iii) the costs incurred by the exporter in moving maize
    from Chicago to the exporting port and delivering it
    fob remain unchanged between \( t_0 \) and \( t_2 \) (these costs
    will be referred to as 'extra fob costs'.

In this situation, the hedge is commonly described as perfect.

The exporter would sell, at time \( t_0 \), 30 000 m.t. of maize at the
current fob 'cash' price (a price which would be profitable, from his
point of view, if the maize were to be delivered promptly). At the
same time he would buy an equivalent quantity of 'futures'. These
would be offset later near time \( t_2 \), when he would have to buy
30 000 m.t. of maize in the 'cash' market.

Clearly, if the 'futures' are to be resold at time \( t_2 \), they
cannot reach maturity before that date. It is common practice to
hedge with the nearest 'futures' after \( t_2 \) (May 'futures' in the
situation considered).

The hedging operation was illustrated considering the case of a forward
sale in which the time lag between the date of purchase \( (t_0) \) and
the 'purchasing deadline' \( (t_2) \) is considerable. It should be noted
that the general practice is for exporters to hedge any sales they
make, even if they are for prompt delivery. In fact, even in this
situation there will be a time lag between the sale to the importer and
the purchase of the 'physical' maize in the internal 'cash' market of the exporting country. However small this time lag may be, the exporter will still protect himself against prices changes that can occur, during that period (prices can in fact change considerably from one day to the next or even during the same day).

In the perfect hedge situation - where both the difference between the Chicago 'cash' and 'futures' prices and the 'extra fob costs' are assumed to remain unchanged and where the cost of 'carrying the hedge' between \( t_0 \) and \( t_2 \) is ignored - the price quoted to the importer (EPAC) at time \( t_0 \) would be the same whether for forward or for prompt delivery.

In reality neither the 'extra fob costs' nor the difference between 'cash' and 'futures' prices at Chicago remain constant (the latter tends to vanish as the maturity of the futures approaches). However, since they are much smaller and less volatile than the 'cash' and the 'futures' prices themselves, the risks of the overall operation to the exporter are considerably reduced through hedging. Clearly, predictable changes in these margins will be taken into account by the exporter, when establishing the selling price to EPAC.

The other factor that will be considered by the exporter when fixing this price is the cost of 'carrying the hedge' between \( t_0 \) and \( t_2 \). Essentially this cost is the interest on the deposit required by the broker who buys and sells 'futures' at the exchange on his behalf. When buying 'futures', the exporter does not have to pay the full value of the contract. Instead he will be required to make a deposit that usually does not exceed 10% of the value of the 'futures' transacted (the value of the deposit depends, to some extent, on the particular arrangements between the broker and his client). The deposit is refunded
when the purchase is offset. The unit cost of carrying the hedge between \( t_o \) and \( t_2 \) is given by:

\[
cch(t_o, t_2) = \left( \frac{\alpha}{100} \right) \times D \times \left( \frac{t_2 - t_o}{365} \right)
\]

where

\( \alpha \) : rate of interest on capital (\%/year)

\( D \) : deposit

Having established the difference between the 'real hedge' and the 'perfect hedge', it is now relevant to analyse what would be the price that, in a real situation, the exporter would charge (at time \( t_o \)) for the maize shipment with forward delivery.

If the shipment was bought at time \( t_2 \), for prompt delivery, the exporter would quote the corresponding fob 'cash' price \( P_{\text{fob-prompt}}(t_2) \) (i.e. a price that would cover his expenses and would reward him with an adequate profit margin). If the exporter were to sell the shipment with the same profit, at time \( t_o \), then he would have to quote the following price:

\[
P_{\text{fob-forward}}(t_o) = P_{\text{fob-prompt}}(t_2) - (\text{May 'futures' price } (t_2) - \text{May 'futures' price } (t_o)) + cch(t_o, t_2)
\]
But at time $t_0$ the exporter will only know the May 'futures' price ($t_0$) and the unit cost of carrying the hedge between $t_0$ and $t_2$. The best he can do about the difference

$$(P_{\text{fob-prompt}}(t_2) - \text{May 'futures' price}(t_2))$$

is to obtain an estimate of its value. He cannot avoid the risk of misjudging this margin on each particular shipment. However small this risk may be, the exporter (generally regarded as risk averse) will transfer it to the importer by raising the price fob-forward ($t_0$) by an extra 'risk charge'. In other words, the exporter will increase his expected profit margin to cover the risks involved in sales in 'physical' for forward delivery. In the long term the importer (EPAC) is expected to pay the following price for the shipment with forward delivery:

$$P_{\text{fob-forward}}(t_0) = \text{Expected} \left[ P_{\text{fob-prompt}}(t_2) - \text{May 'futures' price}(t_2) \right] + \text{May 'futures' price}(t_0) + \left[ cch(t_0, t_2) + rc(t_0, t_2) \right]$$

where

$cch(t_0, t_2)$ : Unit cost of carrying the hedge between $t_0$ and $t_2$

$rc(t_0, t_2)$ : unit risk charge for a forward delivery sale (at $t_0$, with purchasing deadline $t_2$)

For any two dates $t_0$, $t_2$, the term $[cch(t_0, t_2) + rc(t_0, t_2)]$ will be small when compared with the potential changes in the 'futures' prices. If the 'futures' price movements of maize could be predicted accurately, potential reductions in the purchasing costs in relation to the 'hand-to-mouth' policy could be achieved by the importer (EPAC). In
simple terms, the purchase would be made in advance of the deadline (say at $t_0$, for forward delivery) if the May 'futures' prices were expected to increase by more than $[cch(t_0,t_2) + rc(t_0,t_2)]$. Otherwise the purchaser would wait.

However there is a practical drawback in purchasing directly in 'physical'. Purchasing in advance of the deadline commits the buyer to a contract with the exporter. This makes it difficult to revise the timing and the size of shipments after they were purchased (which can happen long before the deadline for the shipping contract). Such revisions may be desirable if it is realized that, due to unpredictable changes in the feed grain prices, their consumptions should be changed from the values originally assumed or that some extra maize or sorghum needs to be imported to make up for some grain spoilage.

Ideally a purchasing operating doctrine should allow the possibility of altering the size and the timing of the shipments before their purchasing deadline is reached, with no significant extra costs, while permitting maximum potential savings over the average purchasing cost policy. This can be achieved if the importer purchases 'futures' first and then converts them to 'physical' (on the deadline of the purchasing contract), rather than buying directly in 'physical'.

2.4 Purchasing 'Futures' and Converting them Later to 'Physical'

This method of conducting the purchasing operations can be illustrated ignoring initially the problems caused by possible revision of the size and timing of shipments. For the maize shipment considered earlier in section 2.1 (see Figure 64) the whole importing operation would involve the following steps:
(i) at any time before the purchasing deadline \( t_2 \) the importer (EPAC) buys May 'futures' equivalent to a total of 30 000 m.t.;

(ii) as before, EPAC contracts a ship with one of the Portuguese shipping companies at (or near) the shipping deadline \( t_1 \);

(iii) at (or near) the purchasing deadline \( t_2 \) EPAC exchanges 'futures' for 'cash' maize, with an exporter; in this type of transaction, the exporter quotes the price in terms of a 'basis' (in US$/m.t. or equivalent) over or under the 'future' in question (May); the 'basis' is the difference between the current cash price (fob, prompt delivery) and the current May 'futures' price.

Clearly the transactions of 'futures' and 'cash' in this operation would coincide with those of a purchase in 'physical' (say at date \( t_0 \), for forward delivery) if EPAC was to buy all 'futures' at that date. The final unit price of maize paid by the importer would be

\[
p = \text{May 'futures' price} \ (t_0) \\
+ \text{basis} \ (t_2) \ \text{(over/under May 'futures' price} \ (t_2)) \\
+ \text{unit cost of holding the May 'futures' position between} \\
t_0 \ \text{and} \ t_2;
\]

but, since

\[
\text{basis} \ (t_2) = P_{fob-prompt} \ (t_2) - \text{May 'futures' price} \ (t_2)
\]

and

\[
\text{unit cost of holding the May futures between} \ t_0 \ \text{and} \ t_2 = cch \ (t_0, t_2),
\]

then the unit price of maize becomes, after rearranging terms,
\[ P = P_{\text{fob-prompt}}(t_2) - [\text{May 'futures' price}(t_2) - \text{May 'futures' price}(t_0)] + \text{cch}(t_0, t_2) \]

Comparing this expression with the one of the price fob-forward \((t_0)\) derived in the previous section, it becomes clear that, in the long term, the price paid by the importer when he buys 'futures' will be lower than that paid if the purchase was conducted in 'physical'. The difference - equal to the risk charge - will be small. It results from eliminating the exporter's risk of a change in the basis 

\[ P_{\text{fob-prompt}} - \text{May 'futures' price} \]

between \(t_0\) and \(t_2\).

A more significant advantage of purchasing 'futures' first and then converting them to 'physical' results from an extra degree of freedom gained by the buyer when deciding how much to purchase at each buying opportunity. In the previous price expressions it was assumed that all 'futures' were bought on the same date \(t_0\). But from the importer's point of view there is no need - and indeed no advantage - in doing so. The purchaser can choose to buy as many 'futures' contracts as he finds convenient at any buying opportunity preceding the purchasing deadline \(t_2\) (subject to the constraint of buying the equivalent to the shipment size up to that deadline). As shown later, this degree of freedom - not available when buying directly in 'physical' - can be used by the importer to increase the efficiency of the purchasing operations.

Another significant advantage of purchasing 'futures' and converting them to 'physical' near the purchasing deadline is the possibility
of adjusting easily the size and the timing of each shipment before
the shipping deadline is reached. Adjustments in the size of shipments
can be handled just by buying or selling 'futures' contracts, whenever
necessary:

(i) if, after a revision of the delivery-inventory policy,
the size of a shipment is increased in relation to the
previous value, the buyer will simply have to cover that increase
by purchasing more futures;

(ii) if the size of shipment is reduced, he can either use the
'futures' in excess (if any) to cover the purchase of
other shipments or, when this is not possible, he
will resell those extra futures near the purchasing
deadline (generally at a profit, if the forecasting of
'futures' price movements is adequate).

Changes in the timing of a shipment can only cause problems when they
imply the need to change the 'futures' contracts used to cover the purchase.
For example, for the maize shipment considered in Figure 64, the purchase
could be covered with May 'futures'. However, if the loading period was
delayed by one month, the May 'futures' would reach maturity one
month before the purchasing deadline. As mentioned earlier,
in this situation, the exporter will generally prefer to quote the
price in relation to the next 'futures' contract (July, in the
Chicago maize exchange). The importer would be left with two basic
courses of action. The first would consist of switching May 'futures'
for July 'futures' (i.e. offsetting his 'May' position and buying
the same volume of July 'futures') and then proceeding as before.
Such an operation would involve a small cost: the commission paid to the
broker acting on his behalf at the exchange (in the Chicago maize
exchange the total commission is only US$ 20.00 per contract bought and sold in the same day).

The second course of action available to the importer would be

(i) to offset his May 'futures' position before the 1st of May (generally at a profit, if the forecasting of 'futures' prices is adequate), and

(ii) to buy 'physical' near the new purchasing deadline on a 'flat price' basis (i.e. with the price quoted in US$/m.t. or equivalent, instead of being defined in relation to a particular 'futures' price).

Finally, the purchasing of 'futures' first and then 'physical' offers another significant advantage over the direct purchasing in 'physical'. It results from the possibility of converting maize 'futures' either to 'physical' maize or to 'physical' sorghum. This possibility can be of a major importance for the importer in situations such as the following one. If the importer expects both maize and sorghum prices to rise (earlier in chapter 4 it was shown that these move closely together), he may wish to commit himself to the purchase of maize and sorghum shipments well in advance of their corresponding purchasing deadlines (six months in advance, say). Then he would start to buy maize 'futures' equivalent to the several maize and sorghum shipments in question. By doing so he would not commit himself necessarily to the purchase of a specific amount of 'physical' maize and a specific amount of 'physical' sorghum. Instead he would commit himself to the total amount of feed grains. If later he realizes that it would be beneficial to switch some maize to sorghum or vice-versa he could do so without restrictions (as far as the purchasing operations are concerned, until the purchasing deadline of each shipment). When
this deadline is reached the importer will convert the 'futures' to the
'physical' grain he elects. The exporter will quote the price of the
particular grain chosen in terms of a 'basis' over or under the maize 'futures' price. Later, in chapter 9, it will be shown
that the possibility of switching to one grain or the other is of
considerable importance in the whole import planning process.

2.5 The Purchasing Operating Doctrine Chosen

For all the reasons pointed out in section 2.4, it is clear that
the importer (EPAC) benefits from conducting the purchasing operations
first through the maize 'futures' market and converting 'futures'
to 'physical' near the purchasing deadline of each maize or sorghum
shipment. The purchasing operating doctrine considered in this chapter
assumes that:

(i) once the size and the timing of the feed grain shipments
are specified or revised (using the delivery-inventory
model defined in chapter 6), the 'purchasing deadline'
\( t_2 \) of each shipment is derived following the procedure
defined in section 2.1;

(ii) the importer buys 'futures' in the Chicago maize exchange;
for each shipment, the month of delivery of the corresponding
'futures' will depend on the purchasing deadline of the shipment,
as defined in Figure 66;

(iii) for each shipment, the corresponding 'futures' can be
purchased at any buying opportunity between a date
\( t_0 < t_2 \) and \( t_2 \); the interval \( t_2 - t_0 \) - called by
Kingsman 41 the 'potential buying period' - will be
set by the senior management of the Importing Agency, according to their attitude towards risk and uncertainty. In this chapter the potential buying period of any shipment will be assumed not to exceed six months and will be regarded as a given input (a discussion of criteria for the selection of the length of the buying period will be presented later in chapter 9);

Figure 66 - 'Futures' traded v. shipments' purchasing deadlines

(iv) a number of 'futures' contracts equivalent to the shipment size must be bought until the deadline $t_2$ is reached;

(v) except for the adjustments imposed by revisions of the delivery-inventory policy, once the 'futures' are bought, they are not resold by the importer;

(vi) for each shipment, the corresponding 'futures' will be converted to 'physical' maize or sorghum on the purchasing deadline.

This operating doctrine was selected according to the code of conduct of EPAC: a Government Agency participating in the market with the objective of acquiring grain efficiently, to meet the country's specific demands. It is clear that if savings
in purchasing costs can be achieved in the context of the selected operating doctrine, then profits would naturally ensue from a purely speculative activity (i.e. buying and selling 'futures' per se, without restrictions, in a way unrelated to the actual demands that have to be met). This merely speculative stance – rather risky and difficult to fit within EPAC's corporate objectives – is not contemplated in the study of the maize 'futures' purchasing policies discussed in this thesis.

According to the selected operating doctrine, the final unit fob price paid by the importer for a feed grain shipment will be

\[ p = \text{average 'futures' price paid by the importer between } t_0 \text{ and } t_2 + \text{basis at time } t_2 \text{ (over/under the 'futures' price quoted at } t_2) + \text{the average unit cost of holding acquired 'futures' until } t_2. \]

But, since the basis at time \( t_2 \) is

\[ \text{basis } (t_2) = \text{'cash' price at time } t_2 \text{ (fob-prompt delivery)} - \text{'futures' price at time } t_2, \]

it follows that

\[ p = \text{'cash' price at time } t_2 \text{ (fob-prompt-delivery)} - [\Delta f - \text{chf}] \]

where

\[ \Delta f : \text{ difference between the 'futures' price quoted at time } t_2 \text{ and the average 'futures' price paid by the importer between } t_0 \text{ and } t_2 \]

\[ \text{chf : average unit cost of holding acquired 'futures' until } t_2. \]
If, for each shipment, all the corresponding 'futures' contracts were bought at the purchasing deadline \( t_2 \) (and were immediately converted to 'physical'), such a policy would be equivalent to the 'hand-to-mouth' policy. Both \( \Delta f \) and \( \text{chf} \) would become zero and the price paid by the importer would be

\[
p = \text{'cash' price at time } t_2 \text{ (fob-prompt delivery)}
\]

A positive difference \( [\Delta f - \text{chf}] \) represents, for each feed grain shipment, a saving made on the price that would be paid under a 'hand-to-mouth' policy. Success in making savings consistently, for consecutive shipments, can only be achieved with good forecasts of maize 'futures' prices and an appropriate purchasing policy.

Sections 3 and 4 will focus on the short and medium-term forecasting of maize 'futures' prices. The objective of the analysis that follows thereafter, in sections 5 and 6, is to derive maize 'futures' purchasing policies that, within the framework of the chosen operating doctrine (with potential buying periods specified by senior management) and within constraints reflecting the purchaser's own conception of risk, seek to maximize the expected savings on the price paid under the 'hand-to-mouth' policy.

3. **Short-term Forecasting of Maize 'Futures' Prices at Chicago**

3.1 **General**

Commodity 'futures' prices fluctuate considerably over very short periods of time. It is not uncommon for a 'futures' price to change 5% or more from one day to the next. In the past, it has been widely accepted that daily 'futures' price series follow a trendless random walk. For a sequence of regular price observations \( (z_t) \), the trendless random
walk model stipulates that
\[ z_t = z_{t-1} + \epsilon_t \]
where the price changes \( \epsilon_t \) are assumed to have zero mean and form a sequence of independent random variables. The major premise of the random walk model is based on the concept of an 'efficient market'. This is defined as a market in which there are large numbers of equally informed, actively competing people, able to set the prices at levels that, at any moment, reflect all the available information about past events or events that are expected to happen in the foreseeable future.

In such a market - or, equivalently, if prices actually followed a trendless random walk - information based on past or current prices would be useless to predict future prices. Purchasing rules based on statistical models could not be successful - i.e. in the long run could not lead to purchasing costs lower than the average market price.

This is in clear contradiction with results obtained by Kingsman for a number of commodities. Using Trigg and Leach's adaptive exponential smoothing to derive price forecasts, and adopting a dynamic programming purchasing model, Kingsman showed that savings in relation to the average market prices can in fact be obtained. These results provided strong evidence against the trendless random walk model.

Later, Taylor and Kingsman presented conclusive evidence for the existence of price trends in a commodity price series (previously regarded as a trendless random walk) and proposed the first formal price-trend model.

This model, further developed by Taylor, will be studied after a brief review of Trigg and Leach's adaptive exponential smoothing.
3.2 Adaptive Exponential Smoothing Model (*)

3.2.1 Underlying Principles

The two basic principles underlying the adaptive exponential model are as follows:

(i) Level and trend

The model assumes that, after each observation of the price series \( z_t \), the future development of the series can be summarised by two parameters:

\[
\begin{align*}
    a_t & : \text{current level of the series} \\
    b_t & : \text{current trend of the series.}
\end{align*}
\]

The model assumes that future values of the series will be scattered randomly about the straight line 
\[ a_t + N b_t, \text{ where } N = 1, 2, \ldots \]

The model departs from the trendless random walk in two respects.

(a) the current level \( a_t \) does not coincide necessarily with the current observation of the series \( z_t \);

(b) a non-zero trend \( b_t \) is assumed.

(ii) Adaptive exponential smoothing

In the original non-adaptive exponential smoothing the current level \( a_t \) is computed from a weighted average of the current and past observations. The weights decay exponentially (at a constant rate) from the most

(*) This section follows closely Taylor 60 (pp.157-160)
recent to the more dated observation. In Trigg and Leach's adaptive model, the smoothing rate for the level \( \alpha_t \) is related to the reliability of preceding forecasts via a 'tracking signal'. The smoothing rate for the trend \( \beta_t \) is related to \( \alpha_t \), so as to minimise the sum of the exponentially weighted squares of deviations (see Brown\(^{13} \)).

3.2.2 Updating Formulae

The following notation will be used:

\[ z_t : \text{ price quotation at time } t; \]
\[ a_t, b_t : \text{ estimates of the price level and trend at time } t \text{ (i.e. after } z_t); \]
\[ \hat{z}_{t,N} : \text{ forecast of price } z_{t+N} \text{ made at time } t \text{ (N = 1,2,...);} \]
\[ e_t : \text{ forecasting error } z_t - \hat{z}_{t-1,1}; \]
\[ \alpha_t, \beta_t : \text{ level and trend smoothing constants at time } t; \]
\[ \text{MAD}_t : \text{ smoothed mean absolute deviation of the one-ahead forecasting errors } e_1, \ldots, e_t; \]
\[ \text{SMER}_t : \text{ smoothed mean of one-ahead forecasting errors } e_1, \ldots, e_t; \]
\[ \gamma : \text{ smoothing constant used to calculate MAD and SMER;} \]
\[ \text{TRACK}_t : \text{ tracking signal } \text{SMER}_t/\text{MAD}_t. \]

With the exception of \( z_t \) (the variable to be forecast) and \( \gamma \) (a chosen constant), all the above parameters are revised when a new price becomes available. The updating cycle consists of the following steps:
(i) calculate the forecasting error:
\[ e_t = z_t - \hat{z}_{t-1,1}; \]

(ii) revise the smoothed mean error and the mean absolute deviation and calculate the tracking signal:
\[ \text{SMER}_t = (1 - \gamma) \cdot \text{SMER}_{t-1} + \gamma \cdot e_t \]
\[ \text{MAD}_t = (1 - \gamma) \cdot \text{MAD}_{t-1} + \gamma \cdot |e_t| \]
and
\[ \text{TRACK}_t = \frac{\text{SMER}_t}{\text{MAD}_t}; \]

(iii) adjust the trend and level smoothing constants:
\[ \alpha_t = \left| \text{TRACK}_t \right| \]
\[ \beta_t = 2 - \alpha_t - 2 \cdot \sqrt{1 - \alpha_t}; \]

(iv) revise the level and the trend
\[ a_t = (a_{t-1} + b_{t-1}) + \alpha_t \cdot e_t \]
\[ b_t = b_{t-1} + \beta_t \cdot e_t \]

(v) forecast the future prices; the distributions of the future prices are
\[ z_{t+N} \sim \mathcal{N} \left( \mu_{t+N}, \sigma_{t+N}^2 \right) \quad (N = 1, 2, \ldots) \]
where the mean and the variance are estimated by
\[ \hat{E}_N = \hat{z}_t N = a_t + N.b_t \]

\[ \hat{\psi}_N^2 = (\lambda \text{MAD}_t)^2 [1 + (N-1)\sigma_t^2] \]

The parameter \( \lambda \) controls the variances of the distributions; for normally distributed errors \( \lambda = 1.25 \).

3.2.3 Specification of the Model Parameters

The only numbers to be specified are the smoothing constant and the initial values of the parameters that are updated at each price quote. The value of \( \gamma \) (generally less than 0.2) is chosen by experimentation, using the forecasting model in conjunction with a purchasing rule (as described later in section 5). The initial values of the adaptive parameters can be obtained simply by using a number of price quotes to 'run-in' the forecasting model (the first 20 quotes of a price series are sufficient for that purpose).

3.3 Taylor's Price-Trend Model

3.3.1 Definition

Earlier the trendless random walk was presented as a model widely accepted in the past, as the one best describing the behaviour of daily commodity price series. There was a good justification for such a belief: as shown later, such a model is, in fact, a good approximation of reality. For a sequence of prices \( \{z_t\} \), the trendless random walk, in its most basic form, stipulates that

\[ z_t = z_{t-1} + \varepsilon_t \]

where, as stated before, the price changes \( \varepsilon_t \) are assumed to have zero
mean and form a sequence of independent random variables.

It is a well documented fact (see, for example, Taylor and Kingsman\textsuperscript{65}) that the variance of commodity price changes $\epsilon_t$ is far from homogeneous (later, this will be shown to be true also for Chicago maize 'futures' prices). However, one of the most important assumptions underlying the time series methods based on autocovariance analysis is the homogeneity of variance. A popular method of trying to obtain variance homogeneity is to study not the prices themselves but their logarithms.

Taylor and Kingsman\textsuperscript{65}, tested the family of power transformations proposed by Box and Cox on a commodity 'futures' price series and found that the logarithmic transformation was the most suitable for variance homogenization. The difference of logarithms of successive prices $\{x_t\}$, commonly called 'daily returns', are approximately equal to the fractional price changes

$$x_t = \log (z_t) - \log (z_{t-1}) = \frac{z_t - z_{t-1}}{z_{t-1}}$$

Taylor and Kingsman\textsuperscript{65} noted that although the logarithmic transformation does remove much of the instability of the variance, it does not totally solve the problem of non-homogeneity.

The trendless random walk, after the transformation of prices, states that

$$\log (z_t) = \log (z_{t-1}) + x_t$$

where the successive daily returns $\{x_t\}$ have independent distributions with zero mean and time dependent variance.
Taylor and Kingsman proposed an alternative model - called here Taylor's price-trend model - incorporating a stochastic price trend in the day-to-day price change. It is supposed that on each trading day there is a small probability \( (1-p) \) that new information is circulated in the market causing a change in the price trend. It is further supposed that each new trend is independent of all the past values and is drawn from a probability distribution with mean \( \mu_0 \) (in practice indistinguishable from zero) and time dependent variance \( \nu_t^2 \). The model can be expressed as

\[
\log(z_t) = \log(z_{t-1}) + \mu_t + e_t
\]

or

\[
x_t = \mu_t + e_t
\]

where

\[
\mu_t = \begin{cases} 
\mu_{t-1} & \text{, with probability } p \\
\text{some new value} & \text{, with probability } 1-p
\end{cases}
\]

According to this trend changing process, the mean duration of the trend is

\[
m = \sum_{i=1}^{\infty} i(1-p)^{i-1} = (1-p)^{-1} \text{ trading days.}
\]

The model further assumes that:

(i) the residuals \( \{e_t\} \) are mutually independent and follow distributions with zero mean and time dependent variance \( \nu_t^2 \);

(ii) the residuals are independent of the price trends, i.e. for all \( t, s \), \( \text{cov}(e_t, \mu_s) = 0 \);

(iii) for all \( t \), the ratio

\[
A = \frac{\text{variance of price trend (} \mu_t \text{)}}{\text{variance of daily return (} x_t = \mu_t + e_t \text{)}} = \frac{\nu_t^2}{\nu_t^2 + \sigma_t^2}
\]

is constant.
3.3.2  **Test for the Significance of Trends**

For Taylor's price trend model (with parameters $p,A$) the theoretical autocorrelations of the daily returns $(x_t)$ are given by

$$\rho_i = A.p^i \quad \text{for all lags } i > 0$$

Providing that the ratio $A$ is small (which is always the case in commodity price series — see, for example, Taylor$^{62}$), the theoretical autocorrelation function is negligible and will not be identified by inspecting the sample coefficients individually. The estimation of these coefficients directly from the daily returns $(x_t)$ would be inefficient because of the fluctuation of the variances $\nu_t^2$ and $\sigma_t^2$. As suggested by Taylor$^{60}$, this problem can be overcome by transforming the series $(x_t)$ into a new series

$$y_t = x_t/s_t$$

where $s_t$ is an estimate of the mean absolute deviation of the random variable generating $x_t$. This estimate is derived using the inductive procedure

$$s_t = \left(0.9\right) \cdot s_{t-1} + \left(0.1\right) \cdot |x_{t-1}|$$

The sample autocorrelation coefficients $(r_i)$ of the series $y_t(t=t_1,\ldots,t_n)$ are calculated following the expression

$$r_i = \frac{\sum_{t=1}^{t_n-i} \left(y_t - \bar{y}\right) \left(y_{t+i} - \bar{y}\right)}{\sum_{t=1}^{t_n-i} \left(y_t - \bar{y}\right)^2}, \quad \text{where } \bar{y} = \frac{1}{t_n} \sum_{t=1}^{t_n} y_t$$
The sample autocorrelation coefficients can be used to test the significance of the price trends:

\[ H_0 : \rho_i = 0, \text{ for all } i > 0 \] (trendless random walk)

\[ H_1 : \rho_i = A \cdot p^i, \text{ for some } A > 0, 0 < p < 1 \] (price trend model).

Taylor\(^6\) proposed the following test statistic:

\[ U^* = 0.4649 \sqrt{n} \sum_{i=2}^{30} (0.92)^i \cdot r_i \]

where

\[ r_i : \text{ sample autocorrelation coefficients} \]

\[ n : \text{ size of the sample used in the estimation of } r_i \text{'s.} \]

If \( H_0 \) is true, the test statistic \( U^* \) is asymptotically distributed as \( N(0,1) \).

### 3.3.3 Estimation of Model Parameters

The parameters \( A \) and \( p \) can be estimated by minimizing

\[ S(p,A) = n \sum_{i=2}^{30} (r_i - A \cdot p^i)^2 \]

To minimize \( S \), a set of values of \( p \) (or, equivalently, \( m = 1/(1-p) \)) is chosen and, for each of these, the minimization over \( A \) is solved by

\[ \frac{\partial S}{\partial A} = 0 \]

or

\[ A = f(p) = \left( \sum_{i=2}^{30} p^i \cdot r_i \right) / \left( \sum_{i=2}^{30} p^{2i} \right) \]

The estimate of \( p \) — denoted by \( p_o \) — can be obtained by plotting \( S[p,f(p)] \) and finding its minimum. The estimate of \( A(A_o) \) is then determined by \( A_o = f(p_o) \).
The joint 95% confidence region for the two parameters can be shown to include all pairs \((p,A)\) for which

\[ S(p,A) < S(p_0, A_0) + 5.99 \]

(5.99 is the 5% point of the \(X^2\) distribution with 2 degrees of freedom).

3.3.4 Updating Formulae

For the price-trend model, the updating cycle (repeated when a new price \(z_t\) becomes available) consists of the following steps:

(I) calculate the current return \(x_t\):

\[ x_t = \log(z_t) - \log(z_{t-1}) \]

(II) calculate the estimate of the mean absolute deviation of the random variable generating \(z_{t+1}\):

\[ s_{t+1} = (0.9) \cdot s_t + (0.1) \cdot |x_t| \]

(III) revise the estimate of the price trend:

\[ \mu_{t+1} = (s_{t+1}/s_t) \cdot [q_o \cdot \hat{\mu}_t + (p_o - q_o) \cdot x_t] \]

where

\[ q_o : \text{root of the quadratic equation} \]

\[ q^2 - q \cdot \left\{ \frac{1 + (1-2A_o) \cdot (p_o)^2}{(1 - A_o) \cdot p_o} \right\} + 1 = 0 \]

which satisfies \(0 < q_o < 1\);

(IV) estimate the standard deviation of \(\hat{\mu}_{t+1}\):

\[ \hat{\sigma}_{t+1} = \sqrt{\frac{p_o \cdot (p_o - q_o)}{1 - p_o \cdot q_o}} \cdot \sqrt{\frac{A_o}{1 - A_o}} \cdot (\lambda \cdot s_{t+1}) \]

where

\[ \lambda : \text{ratio between the standard deviation and the mean absolute deviation of } e_{t+1} \ (\lambda = 1.25 \text{ for the normal distribution}; \text{Taylor}^{60} \text{ found that the distribution of the residuals is leptokurtic and recommends the use of } \lambda = 4/3 - \text{the ratio for a } t \text{ distribution with six degrees of freedom}); \]
forecast the future prices; the distributions of the future prices are

\[ z_{t+N} \sim \phi_{t+N} \left( EV_{t+N}^\prime, \varphi_N^2 \right) \quad (N = 1, 2, \ldots) \]

where the mean and the variance are estimated by

\[ EV_{t,N}^\prime = z_{t,N} = z_t^\prime \exp \left[ \frac{1 - (p_o)^N}{1 - p_o} \cdot \hat{\mu}_{t+1} \right] \]

and

\[ \varphi_N^2 = z_t^2 \cdot \left[ V_N - W_N \right] \]

with

\[ V_N = V_1^\prime \cdot \left( \frac{p_o}{1 - p_o} \right) \cdot A_0 \cdot \left[ N - \frac{1 - (p_o)^N}{1 - p_o} \right] \]

\[ W_N = V_1^\prime \left[ \frac{1 - (p_o)^N}{1 - p_o} \right]^2 \cdot \frac{p_o(p_o - q_o)}{1 - p_o \cdot q_o} \cdot A_0 \]

\[ V_1^\prime = (\lambda \cdot s_{t+1})^2 = \left( \frac{4}{3} \cdot s_{t+1} \right)^2 \]

The price forecasts N periods ahead \( \hat{z}_{t,N} \) (defined above) differ from those derived with adaptive exponential smoothing, in two significant respects:

(i) the current level of the price (denoted earlier by \( \hat{a}_t \)) coincides necessarily with the current price \( z_t \);

(ii) the prices are not projected linearly into the future.

3.3.5 Initialization of the Model

Once the parameters \( p^*, A^* \) are specified, the only numbers to be specified are the initial values of the parameters that are updated. As with the adaptive exponential smoothing, these values can be obtained by using a number of price quotes to 'run-in' the forecasting model. The first twenty quotes of a series \( \{ x_t \} \) are normally used to derive

\[ s_{21} = (0.05) \cdot \Sigma_{t=1}^{20} |x_t| \]
The value of $\hat{\mu}_{21}$ can be derived using the equation

$$\hat{\mu}_{t+1} = \left( s_{t+1}/s_t \right) \cdot \left[ q_o \cdot \hat{\mu}_t + (p_o - q_o) \cdot x_t \right]$$

setting

$$\begin{cases} s_t \quad (t = 1,20) = s_{21} \\ \hat{\mu}_t = 0. \end{cases}$$

Clearly the first twenty terms of $\{ x_t \}$, $\{ y_t \}$ and $\{ \mu_t \}$ are ignored in all calculations.

3.3.6 Application of the Model to Maize 'Futures' Prices

Taylor's model was applied to daily closing 'futures' prices at the Chicago Maize Exchange, between the first trading day of January 1963 and the last one of December, 1976.

A total of more than 18 000 price quotes were transcribed from the Chicago Board of Trade Annual statistical Reports (1963-1976) to a computer file. A programme was written in FORTRAN in order to

(i) read in, for each 'futures' contract quoted, a series $\{ z_t \};$

(ii) transform each series $\{ z_t \}$ to $\{ y_t \}$

(iii) from each set of $\{ y_t \}$ series concerning contracts with the same delivery month (e.g. all March contracts quoted between January 1963 and December 1976), calculate the sample autocorrelation coefficients;

(iv) calculate, for each of these samples (March, May, July, September and December), the statistic $U^*$;

(v) estimate the model parameters ($p_o$ and $A_o$), for each 'futures' sample and overall.
The programme was run on a CDC 7600. The amount of computer memory required to run the programme was nearly 50K words and the total computation time involved was about 15 CPU seconds.

The selection of the series \( \{ y_t \} \) to be considered in the analysis followed the criterion suggested by Taylor. Essentially the objective is to obtain, for each set of contracts with the same delivery month (e.g. March), a set of series \( \{ y_t \} \) satisfying the following conditions:

(i) for each contract (say, March 1970) the series \( \{ y_t \} \) finishes on the last trading day before the delivery month (March 1970) - the objective is to avoid spurious price effects that can occur due to the special trading conditions prevailing during the month of delivery;

(ii) the series \( \{ y_t \} \) of two successive contracts with the same delivery month (say, March 1970 and March 1971) do not overlap. For example, if the last term of \( \{ y_t \} \) Mar.70 refers to 27. Feb. 1970 (the last trading day before the delivery month) the series \( \{ y_t \} \) Mar.71 will start not before the next trading day (2. March, 1970\(^*\)). The objective of this procedure is to avoid the inclusion of highly correlated price changes (of successive contracts) in the overall sample used to estimate the autocorrelation coefficients for the March 'futures' contracts.

All the \( \{ y_t \} \) series finally considered in the analysis are defined in Appendix 4.

\(^*\) it will start on this day if prices \( z_t \) \( \) Mar.71 were quoted for the previous 21 days (these are required to derive the previous 20 daily returns \( x_t \) \( \) Mar.71, in turn used to estimate the current value of the mean absolute deviation); otherwise \( \{ y_t \} \) \( \) Mar.71 will start 21 days after the date of the first price quote.
For each set of contracts with the same delivery month (D) the sample autocorrelation coefficients \( r_{i}^{D}, i=1,\ldots,30 \) were calculated using the expression

\[
\begin{align*}
    r_{i}^{D} &= \frac{\sum_{j=1}^{M^{D}} \sum_{t=j+1}^{N_{j}^{D}} [(y_{j,t}^{D} - \bar{y}^{D}) \cdot (y_{j,t-i}^{D} - \bar{y}^{D})]}{\sum_{j=1}^{M^{D}} \sum_{t=1}^{N_{j}^{D}} (y_{j,t}^{D} - \bar{y}^{D})^{2}}
\end{align*}
\]

with

\[
\begin{align*}
    \bar{y}^{D} &= \frac{\sum_{j=1}^{M^{D}} \sum_{t=1}^{N_{j}^{D}} y_{j,t}^{D}}{\sum_{j=1}^{M^{D}} N_{j}^{D}}
\end{align*}
\]

and where

- \( M^{D} \): total number of contracts with the same delivery month D
- \( j \): index denoting each specific contract \((j=1,\ldots,M^{D})\)
- \( \{ y_{j,t}^{D} \} \): series of scaled daily returns for each contract \( j \), delivery month \( D \) \((t=1,\ldots,N_{j}^{D})\)

Table 46 presents, for each set of March, May, July, September and December contracts,

(i) the sample autocorrelation coefficients \((lags \ i=1,\ldots,30)\)

(ii) the statistic \( U^{*} \)
Table 46 - Sample autocorrelation coefficients ($r_i^D$) and $U^*$ statistics

<table>
<thead>
<tr>
<th>LAG</th>
<th>MAR</th>
<th>MAY</th>
<th>JUL</th>
<th>SEP</th>
<th>DEC</th>
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<td>0.0377</td>
<td>0.0377</td>
<td>0.0097</td>
<td>0.0047</td>
</tr>
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<td>0.0229</td>
<td>0.0404</td>
</tr>
<tr>
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<td>0.0443</td>
<td>0.0371</td>
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<td>0.0365</td>
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<td>0.0042</td>
<td>0.0079</td>
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<td>0.0278</td>
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<td>0.0180</td>
<td>0.0022</td>
<td>0.0045</td>
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<td>0.0333</td>
<td>0.0392</td>
<td>0.0162</td>
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<tr>
<td>30</td>
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<td>0.0376</td>
<td>0.0024</td>
<td>0.0118</td>
<td>0.0047</td>
</tr>
</tbody>
</table>

(ii) $(U^*)^D : 3.56562 \quad 1.97811 \quad 1.81355 \quad 2.67840 \quad 2.34119$

For all contracts, $U^*$ exceeds the 5% point of the N(0,1) distribution (1.65). Therefore, the trendless random walk hypothesis,

$H_0 : \rho_i = 0$, for all $i > 0$,

can be rejected in favour of Taylor's price trend hypothesis,

$H_1 : \rho_i = A \cdot p^i$, for some $A > 0$, $0 < p < 1$.

The existence of slight, yet significant, autocorrelation between maize 'futures' price changes has been proved. Since the prices do not follow a trendless random walk, it may be possible to define successful policies for the 'tactical' buying of maize 'futures' (i.e. policies leading to average purchasing prices lower than the average market prices).
The question of whether the autocorrelation that exists is sufficient to permit financially rewarding decisions will be analysed in section 5.

Having established the significance of price trends, the analysis was completed with the estimation of the parameters \( A, p \). The results are summarized in Figure 67. Firstly, the analysis was conducted separately for each set of contracts with the same delivery month (D):

\[
p^D_0 \text{ was found by minimizing } \quad S^D[p, f(p)] = \left[ \sum_{j=1}^{m^D} N_j^D \right] \cdot \left[ \sum_{i=2}^{30} (r_i^D - f(p) \cdot p_i)^2 \right]
\]

with

\[
f(p) = \left[ \sum_{i=2}^{30} p_i \cdot r_i^D \right] / \left[ \sum_{i=2}^{30} p_i^2 \right]
\]

and where all the other symbols are set according to the notation introduced earlier. \( A_D^O \) was then derived from

\[
A_D^O = f(p_0^D).
\]

Figure 67(i) gives the estimates of \( p_0^D \) (or \( m_0^D \)) and \( A_D^O \) for all contracts March to December. The differences between them may seem significant.

However, as illustrated in Figure 67(ii) for the March contract, the functions \( S^D[p, f(p)] \) are very flat about the minima. For mean trend durations of 17 to 50 days (respectively the smallest and the largest estimates of \( m_0^D \)) the March function \( S^D[p, f(p)] \) exceeds the minimum only by 1.41% and 0.46% respectively. The extreme flatness of \( S^D[p, f(p)] \) is inevitable, due to the small theoretical autocorrelation coefficients and the fact that the estimates \( p^D_0 \) are close to 1.

Figure 67(iii) shows the joint 95% confidence region of \( p \) (or \( m \)) and \( A \) for the March contracts. The 95% confidence regions were derived for all the other contracts and their intersection is represented by the shaded area. As shown in the diagram this intersection includes all
Figure 67 - Estimation of model parameters

(i) $y_0^1$ and $y_0^2$ and $u_0$ (D : March to December)

<table>
<thead>
<tr>
<th></th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
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<td>$y_0^1$</td>
<td>0.967</td>
<td>0.942</td>
<td>0.900</td>
<td>0.875</td>
<td>0.855</td>
<td></td>
<td></td>
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<tr>
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<td>1.77</td>
<td>1.86</td>
<td>2.05</td>
<td>2.24</td>
<td>2.43</td>
<td>2.62</td>
<td>2.81</td>
</tr>
<tr>
<td>$u_0$</td>
<td>1.99</td>
<td>1.99</td>
<td>1.99</td>
<td>1.99</td>
<td>1.99</td>
<td>1.99</td>
<td>1.99</td>
<td>1.99</td>
</tr>
</tbody>
</table>

(ii) Function $F[p, z]$ for March 'futures'

(iii) 95% Joint confidence regions for $p$ and $A$

(iv) Fitted autocorrelation functions ($f_1 - A_1, f_2$)
the points \( [p^D_0 \text{ (or } m^D_0), A^D_0] \) \((D = \text{March to December})\).

On this evidence it is not possible to reject the hypothesis of a common price-trend model to all 'futures' contracts. Such a model would be appropriate if information made available to the market caused similar price trends, both in duration and size, for all 'futures' contracts. This is in accordance with the observed behaviour of 'futures' price series, which appear to move closely together (the margins between successive 'futures' contracts are markedly less volatile than the prices themselves).

Under the hypothesis of a common price-trend model for all 'futures' contracts - adopted in the remainder of this research - the parameters of the overall model were estimated by minimizing the overall sum

\[
S(p, A) = \left[ \sum_{D} \sum_{j=1}^{N_j^D} \right] \cdot \left[ \sum_{D} \sum_{i=2}^{30} (m^D_i - A \cdot p^i)^2 \right]
\]

The overall parameter estimates obtained were

\( p_0 = 0.962 \) \((m_0 = 26 \text{ days})\)
\( A_0 = 1.49% \).

The fitted theoretical autocorrelation function, \( (\rho)_0 = A_0 \cdot p^i \), is compared in Figure 67(iv) with those obtained for each set of contracts with the same delivery month \( (\rho^D_i) = A^D_0 \cdot (p^D_0)^i \).

3.4 Distinguishing Features of the Two Forecasting Models

In contrast with adaptive exponential smoothing - which can be regarded as a 'general purpose' forecasting model, adopted by Kingsman as a basis to derive day-to-day buying decisions - Taylor's price-trend model was developed with the specific objective of describing the particular behaviour of commodity prices. Its structure, inferred directly from the
Figure 68 - Comparison of forecasts obtained from adaptive exponential smoothing and from Taylor's price-trend model


- Daily closing price [US $ / bushel]

<table>
<thead>
<tr>
<th>t</th>
<th>t+5</th>
<th>t+10</th>
<th>t+15</th>
<th>t+20</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.15</td>
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<tr>
<td>2.16</td>
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<td>2.17</td>
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<td>2.18</td>
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<tr>
<td>2.19</td>
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</tr>
</tbody>
</table>

- Current level in Ad.exp.smooth. (s_t)
- Current price (y_t)
- Price-trend model: 95% confidence envelope
- Ad.exp.smooth: 95% confidence envelope

(ii) March-1974 maize 'futures' (Chicago): price forecasting on the 16th Nov. 1973

- Daily closing price [US $ / bushel]

<table>
<thead>
<tr>
<th>t</th>
<th>t+5</th>
<th>t+10</th>
<th>t+15</th>
<th>t+20</th>
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<tr>
<td>3.70</td>
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<td>3.50</td>
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<td>3.30</td>
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<tr>
<td>3.10</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.90</td>
<td></td>
<td></td>
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</tbody>
</table>

- Note: in this diagram, prices are represented on a scale 20 times larger than that adopted in diagram (i)
- Ad.exp.smooth. 95% confidence envelope
- Price-trend model: 95% confidence envelope

(e) In the US grain trade, prices are usually quoted in US $/bushel; to convert into US $/mt, multiply by 79.77
analysis of commodity price series, differs markedly from that of adaptive exponential smoothing. The main distinguishing features of the two forecasting models will now be reviewed, considering the forecasts shown in Figure 68. The understanding of the differences between the two models is necessary to explain some of the results presented later in section 5, in the discussion of rules for the 'tactical' purchasing of maize 'futures'.

The diagrams of Figure 68 show forecasts of March 'futures' prices on two different occasions: 20 October 1965 and 16 November 1973. The price volatility, i.e. the magnitude of the day-to-day price changes, was much higher during the period covered in diagram (ii) (it should be noted that the price scale in this diagram is twenty times larger than that adopted in diagram (i)).

The price forecasts and the 95% confidence price forecast envelopes were derived using expressions presented earlier in sections 3.2.2 and 3.2.4 and setting the models' parameters as follows:

(i) adaptive exponential smoothing

\[ \gamma = 0.15 \]  
(the justification for the choice of this value will be given later in section 4);

\[ \lambda = 1.25 \]  
(standard choice for normally distributed forecasting errors);

(ii) price-trend model:

\[ p_o = 0.962 \]

\[ A_o = 1.49\% \] (estimates presented in the previous section)

\[ \lambda = 4/3 \]  
(as recommended by Taylor$^{60}$).
Two differences between the forecasts produced using the price-trend model and adaptive exponential smoothing were already mentioned earlier. The first one concerns the current price level \( a_t \) which, in Taylor's model, is assumed to coincide always with the current price \( z_t \). In the adaptive exponential smoothing \( a_t \) and \( z_t \) will generally not coincide (as illustrated in diagram (i) of Figure 68).

The second distinguishing feature of the two models is the way in which the price forecasts are projected into the future, linearly, in the exponential smoothing model, and non-linearly in Taylor's model (although with some difficulty, this can be observed in diagram (ii)).

Another important difference between the two models concerns the process of estimation of trends. As illustrated in both diagrams of Figure 68, the adaptive exponential smoothing model reacts faster than Taylor's model, after a sequence of price changes either predominantly positive or predominantly negative. In Taylor's model the rate at which the trend estimate is updated following any price change is extremely small. In fact, in the trend updating formula

\[
\hat{\beta}_{t+1} = (s_{t+1}/s_t) \cdot [q_0 \cdot \hat{\beta}_t + (p_0 - q_0) \cdot x_t]
\]

(see section 3.3.4) the ratio \((p_0 - q_0)/q_0\) (which 'controls' the speed of the updating process) is only 0.0125.

In the adaptive exponential smoothing, the trend updating formula is

\[
b_t = b_{t-1} + \beta_t \cdot e_t
\]

(see section 3.2.2). The value of \( \beta_t \) is, in turn, updated according to the formulae presented earlier in section 3.2.2. When the model 'identifies' a change in the trend, the value of \( \beta_t \) increases to values much larger than that of the ratio \((p_0 - q_0)/q_0\). For example, in the situation considered in diagram (ii)
of Figure 68, $\delta_t = 0.259$ (i.e. about 20.7 times larger than the ratio $(p_o - q_o)/q_o$). As a result, the trends estimated with adaptive exponential smoothing tend to fluctuate between extremes that, in absolute value, are much larger than those obtained using the price-trend model.

Another major difference between the two models is related to the specification of the forecasting envelopes. As illustrated clearly in Figure 68, the configuration of the forecasting envelopes derived using adaptive exponential smoothing changes considerably over time. These changes can be explained considering the expression of the variance of the forecasting errors $N$ periods ahead:

$$\hat{\sigma}^2_N = (\lambda \cdot \text{MAD}_t)^2 \cdot [1 + (N-1) \cdot \alpha_t^2]$$

(see section 3.2.2)

The value of the parameter

$$\alpha_t = |\text{TRACK}_t| = \left| \frac{\text{SMER}_t}{\text{MAD}_t} \right|$$

determines how much the forecasting envelope widens from the one-ahead forecast to the $N$-ahead forecast. In situations where, in the recent past, prices have been scattered around a nearly straight line, $\text{SMER}_t$ and, hence, $\alpha_t$ tend to be small. The situation is illustrated in diagram (i) (with $\alpha_t = 0.110$): the forecasting envelope is then defined by two lines which are nearly straight and parallel.

In the situation depicted in diagram (ii) (with $\alpha_t = 0.696$), the width of the forecasting envelope increases with $N$, as determined by the expression of $\hat{\sigma}^2_N$. 
In contrast with adaptive exponential smoothing, Taylor's model specifies envelopes in which the relative width for different values of N does not change over time:

\[ \varphi_N^2 = z_t^2 \cdot [v_N - w_N] = \]

\[ = z_t^2 \cdot (\lambda \cdot s_{t+1}^2) \cdot f(p_o, A_o, q_o, N) \] (see section 3.3.4).

The term \( s_{t+1}^2 \) sets the width of the one-ahead forecasting confidence interval (a role identical to that played by MAD \( t \) in the exponential smoothing model). The function \( f(p_o, A_o, q_o, N) \) - invariant with respect to time - determines the relative width of the forecasting envelope for different values of N.

4. Medium-term Forecasting of Maize 'Futures' Prices at Chicago

4.1 General

Statistical forecasting models, such as those described in the previous section, extrapolate recent price movements into the future, without attempting to 'explain' the reasons that determine price changes. Clearly with these models it is not possible to predict, in advance, reversals of price-trends. For this reason, the meaningful use of statistical models is restricted to short-term forecasts, up to about two months into the future (this will be shown later in section 5).

The limitations of statistical forecasting models led to the development of behavioural models. The principles underlying such models were reviewed earlier in Chapter 3 (section 4.2). They were first proposed by Kingsman, who developed a behavioral model for maize
'cash' prices at Chicago and at the US-Gulf ports \(^{40}\) (*). This model will be reviewed after presenting briefly some background information on the world maize market.

According to the purchasing operating doctrine defined earlier in section 2.5, all feed grain shipments are purchased initially in 'futures' (later converted to 'physical', on the shipments' purchasing deadlines). In the context of this operating doctrine and for the purpose of defining the active buying periods of the different shipments, the relevant forecasts are those of average monthly 'futures' prices at Chicago. These will be derived by adjusting the Chicago 'cash' price forecasts obtained from Kingsman's model, after the relationship between average monthly 'futures' and 'cash' prices is ascertained.

4.2 **Background on the Maize World Market**

The world production and trade of maize has been largely dominated by the United States of America. During the sixties and seventies this country has provided over 50% of the world production. Its exports rose sharply over these two decades, reaching over 75% of the total world trade in the seventies. Other producing or exporting countries (Argentina, France, South Africa) play incomparably smaller roles than the US in the world maize market. In one sentence, it can be said that, essentially, the world maize situation is the US maize situation.

Government programmes have played a dominant role in the US grain

(*) Kingsman introduced some minor modifications to the model since this research was carried out. The latest version will be described in a forthcoming book (Kingsman, B.G., 'Raw Materials Purchasing : An Operational Research Approach').
economy in the past, either through direct price support measures or through production adjustment programmes. During the fifties production was consistently above the requirements. During this period attempts by the US Government to limit the feed grain production proved ineffective. The feed grain carryover - mostly owned by the Government - reached an all time record in 1961. The first Feed Grain Programme was then introduced. According to its provisions, farmers had to divert acreage in order to qualify for the price support operations. In addition, diversion payments were paid to farmers complying with the programme. Under the new programme - successfully controlling production - the carryover stocks of maize (and other feed grains) were progressively reduced. After a crop failure in 1967 and comparatively high exports in the previous year, maize prices rose for the first time above the floor price guaranteed by Government. This was the beginning of the transition from a situation of large abundance - with prices staying at the 'floor' levels determined by the direct intervention of Government - to a 'free market' situation - in which prices responded to changes in the demand/supply situation.

In the spring of 1973 a new period in the USA maize market was to begin: a sharp departure from the pattern of the previous years was to take place. Although the US maize production (and, in general, the US feed grain production) was only slightly below the record crop of 1971, strong demand pushed total disappearance (*) well above

(*) This term is commonly used in the US maize market, meaning consumption or usage.
production. This was mainly due to a spectacular increase in exports, both to the industrialized countries of the West (Japan, EEC) and to the Soviet Union.

These large exports - resulting from a combined effect of an expansion of the livestock by those countries and poor crops abroad - were sustained and increased during the seventies becoming a significant proportion of total US maize disappearance. From a level of 7% of the total disappearance in the late fifties, exports rose to 12% in the sixties and reached over 30% by the late seventies.

The shift from relative abundance to relative scarcity, in 1973, had a dramatic effect on maize prices. As illustrated in Figure 69, there was a notable departure from low and stable prices (which characterized previous years), to high and extremely volatile prices (typical of the years that followed).

*Figure 69 - Maize 'cash' prices (daily closing prices, Chicago, yellow maize no. 2) (1972-1974)*

Note: 21 trading days = 1 month
4.3 **Kingsman's Maize 'Cash' Price Model**

The model was developed in 1976 and was based on the analysis of data covering the crop years 1967/68 to 1974/5 (the US maize crop year starts on the 1st of October).

The development of the model comprised four basic steps:

(i) the choice of a 'market barometer' (a variable - or combination of variables - attempting to measure the state of the supply-demand situation and to 'explain' past price changes);

(ii) the analysis of the flow of information to the market and the derivation of the market expectations of the supply-demand situation (expectations measured in terms of the market barometer, revised dynamically as new information becomes available);

(iii) the analysis of patterns for the seasonal movements of prices over the crop year and the identification of critical months in which turning points in any of the identified patterns tend to occur;

(iv) inference of relationships between

(a) market expectations of the value of the chosen 'barometer' (at particular times of the year, when relevant information becomes available), and

(b) maize 'cash' prices occurring later, in those months identified earlier as potential turning points in the seasonal price patterns.
A preliminary analysis involving an attempt to relate average annual prices to different combined measures of supply and disappearance of the US maize market led Kingsman to the choice of a market 'barometer' defined, for each crop year, as

\[ \text{ER (export ratio)} = 100 \times \frac{E}{TS - DD} \]  

(\%) 

where

- **E**: exports
- **TS**: total supply (since maize imports are negligible, the total supply for one crop year is \(TS = CO + P\), where \(CO\) is the carryover stock from the previous year and \(P\) is the current crop production
- **DD**: US domestic disappearance.

The next step in the model building process was a detailed analysis of the flow of information to the market. Essentially there is one major source of information to the market: the United States Department of Agriculture (USDA). This information is conveyed to the market in the form of reports which are circulated throughout the world within minutes of being released. For the maize market, the most important reports are:

(i) **Crop Production Reports (CP reports)**

They provide estimates of the US maize production as of the 1st of July, August, September, November and December and are released about the 10th of each month;

(ii) **Quarterly Stock of Grains in All Positions Reports (QSGAP reports)**

These reports provide estimates of US stocks, domestic disappearance and exports as of the 1st of January, April, June (July, before 1976) and October and are issued about the 24th of each month;
(iii) Feed Situation Reports (FS reports)
Published five times a year (February, April, May, August and November), they are a source of both current and historical information. Before 1973/74 the November issue ("Outlook") was particularly important: it contained the first forecasts of both domestic disappearance and exports for the current crop year.

(iv) Agricultural Supply and Demand Estimates (ASDE reports)
Published since September 1973, they constitute the single most important source of current information. They report practically without delay any changes in the USDA estimates or forecasts of the carryover stocks, production, exports and domestic disappearance.

Table 47 presents USDA estimates, published in successive issues of the ASDE Reports, of the 1975/76 carryover, crop production, total supply, exports and domestic disappearance. The corresponding values of the export ratio are shown in the last column.

The table illustrates clearly the nature of the flow of information to the market, involving frequent and significant changes in the estimates of the supply-demand factors. In the particular crop year considered in the Table it is noticeable that there was a systematic underestimation of the exports.

In this process there are some critical points in which the estimates of the export ratio assume particular importance:

(i) 10th August (or shortly afterwards): after the release of the first reliable crop production estimate;

(ii) 10th November: after the release of both
(a) the November CP report, and
(b) the October QSGAP report (providing estimates of the usages in the previous crop year and of the carryover);

(iii) 24th January: after the release of the January QSGAP report (providing estimates of the domestic disappearance and exports of the first quarter of the crop year); and

(iv) 24th April: after the release of the April QSGAP report (providing estimates of the first half year usages).

The USDA is generally regarded by the maize market as the most impartial and reliable source of information. Therefore, its estimates can generally be used to derive (retrospectively) the market expectations of the value of the export ratio. There are however two types of situations in which the market expectations must be derived in a different way. The first one occurs in cases where USDA estimates

Table 47 - USDA maize supply and demand estimates for the 1975/76 crop year.

<table>
<thead>
<tr>
<th>Date of release of the ASDE reports</th>
<th>Carryover stock</th>
<th>Crop production</th>
<th>Total supply</th>
<th>Domestic disappearance</th>
<th>Exports</th>
<th>Export ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CO</td>
<td>CP</td>
<td>TS</td>
<td>DD</td>
<td>E</td>
<td>ER</td>
</tr>
<tr>
<td></td>
<td>[ million bushels]</td>
<td>[%]</td>
<td>[ million bushels]</td>
<td>[ million bushels]</td>
<td>[ million bushels]</td>
<td>[ million bushels]</td>
</tr>
<tr>
<td>(…)</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
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<td>n.a.</td>
</tr>
<tr>
<td>18.Mar.75</td>
<td>360</td>
<td>6073</td>
<td>6434</td>
<td>4475</td>
<td>1200</td>
<td>61.26</td>
</tr>
<tr>
<td>11.Jul.75</td>
<td>360</td>
<td>6046</td>
<td>6407</td>
<td>4365</td>
<td>1200</td>
<td>56.77</td>
</tr>
<tr>
<td>12.Aug.75</td>
<td>375</td>
<td>5850</td>
<td>6186</td>
<td>4265</td>
<td>1300</td>
<td>67.67</td>
</tr>
<tr>
<td>12.Sep.75</td>
<td>375</td>
<td>5687</td>
<td>5983</td>
<td>4025</td>
<td>1400</td>
<td>71.14</td>
</tr>
<tr>
<td>14.Oct.75</td>
<td>295</td>
<td>5737</td>
<td>6033</td>
<td>4025</td>
<td>1400</td>
<td>69.38</td>
</tr>
<tr>
<td>28.Oct.75</td>
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<td>5737</td>
<td>6097</td>
<td>4015</td>
<td>1450</td>
<td>69.64</td>
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<td>5004</td>
<td>6164</td>
<td>4040</td>
<td>1450</td>
<td>68.27</td>
</tr>
<tr>
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<td>&quot;</td>
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<td>6164</td>
<td>4040</td>
<td>1450</td>
<td>68.27</td>
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<td>5767</td>
<td>6127</td>
<td>4215</td>
<td>1450</td>
<td>72.07</td>
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<tr>
<td>9.Mar.76</td>
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<td>&quot;</td>
<td>4065</td>
<td>1550</td>
<td>75.17</td>
<td></td>
</tr>
<tr>
<td>23.Apr.76</td>
<td>&quot;</td>
<td>&quot;</td>
<td>4115</td>
<td>1550</td>
<td>77.04</td>
<td></td>
</tr>
<tr>
<td>11.May.76</td>
<td>&quot;</td>
<td>&quot;</td>
<td>4115</td>
<td>1600</td>
<td>79.52</td>
<td></td>
</tr>
<tr>
<td>10.Jun.76</td>
<td>&quot;</td>
<td>&quot;</td>
<td>4115</td>
<td>1600</td>
<td>82.01</td>
<td></td>
</tr>
<tr>
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<td>(…)</td>
<td>(…)</td>
<td>(…)</td>
<td>(…)</td>
</tr>
</tbody>
</table>

Note:
(1) When USDA provided range estimates, their mid-points were taken
(2) TS includes 1 million bushels of imported maize
(3) n.a. : not available
(4) To convert bushels into m.t., divide by 39.37
were not published. Before 1973, the first USDA estimates of
(a) usages (domestic, exports) during the whole of the previous
year,
(b) carryover stocks from the previous year, and
(c) usages in the current crop year
were published in late October (a, b) and mid-November (c). In order
to derive the August export ratio, Kingsman conjectured that the market
used old crop year usages to evaluate the new crop situation. The old
crop year usages (domestic disappearance and exports) were derived using
a simple extrapolation method. For example, in August 1968, the exports
for the whole 1967/68 crop year were derived from the exports during the
first three quarters \( E_{1+II+III}^{1967/68} \) - released in the previous July
QSGAP report - as follows

\[
E_{1967/68} = \frac{E_{1+II+III}^{1967/68}}{E_{1+II+III}^{1966/67}} \cdot E_{1966/67}
\]

Comments published in the specialized press have suggested that,
ocasionally, the January and April USDA forecasts of the whole crop
year usages are regarded by the market as conservative. For this
reason Kingsman advocated the adoption of the above extrapolation method,
using the first quarter and first six months usages, released by the
USDA in January and April, respectively.

The next step of Kingsman's analysis was the study of seasonal
price movements. Having rejected the existence of a uniform seasonal
pattern for maize prices, Kingsman identified the critical periods
where turning points in any of the past observed patterns tended to occur:
October-November, January, March, April June. The prices at these periods specify almost completely the type of seasonal price movement that will occur. Prices at other times can be obtained by simple interpolation.

Not surprisingly, the turning points of the seasonal price movements occur at times just preceding those periods that were identified earlier as the most critical ones as regards the flow of information to the market.

The final stage of the modelling process consisted of regressing maize prices (at each of the critical periods identified above) against the August, November, January and April market expectations of the export ratio. The prices considered by Kingsman in the regression analyses were the mid-month 'cash' prices. After April 1971 these prices were adjusted for the subsequent 'floating' of the dollar value against other major currencies. Thus all prices were expressed in terms of the pre-1971 dollar value in the following way:

\[
\text{Adjusted price (at time } t) = DV \cdot \text{ Actual price (at time } t)\]

where

\[
DV = \frac{\text{Dollar value (at time } t)}{\text{Pre-1971 dollar value}}
\]

The value of the dollar (at any time) was defined as the weighted average of the rates of exchange against the currencies of the major importing countries, excluding the Soviet Union, with weights assigned proportionally to each country's imports in the previous crop year. Virtually identical values of the ratio \(\frac{\text{Dollar value (at time } t)}{\text{Dollar value (pre-1971)}}\) were obtained by considering the values of the dollar against the SDR - the paper gold unit of the International Monetary Fund.
Figure 70(i) summarizes the variables considered in each of the regressions carried out by Kingsman. For each of them a relationship between the price and the export ratio in question was fitted using data concerning the crop years 1967/68 to 1974/75. The regression model shown in Figure 70(ii) (model 1) is

\[
(ACP_{\text{Oct/Nov}})_{\text{low}} = \frac{a}{b - \text{ER}}
\]

where

- \((ACP_{\text{Oct/Nov}})_{\text{low}}\) : the lower of the October/November month adjusted 'cash' prices (US$/bushel)
- \(a, b\) : model parameters \((a = 82.6, b = 108.5)\)
- \(\text{ER}\) : export ratio (\%)

This model was derived by regressing linearly the reciprocal of \((ACP_{\text{Oct/Nov}})_{\text{low}}\) against \(\text{ER}\):

\[
y = \frac{1}{(ACP_{\text{Oct/Nov}})_{\text{low}}} = a + b(\text{ER} - \bar{\text{ER}}) + \text{error}
\]

Estimates of \(a\) and \(b\) were obtained using straightforward linear regression theory, assuming constant standard deviation of the errors. The other regression models (models 2 to 11) were derived similarly. In some cases, the transformation preceding the linear regression was logarithmic or exponential. In others, the untransformed adjusted prices were regressed directly against the export ratio.

4.4 Forecasts of the Average Monthly 'Cash' Prices

Ideally, in order to derive forecasts of the average monthly prices, the regression models should include, as dependent variables, the adjusted average monthly prices (rather than the mid-month prices, as considered by Kingsman). However, the forecasts obtained are virtually equivalent regardless of whether average monthly or mid-month prices are used.
Figure 70 - Price determination regression models

<table>
<thead>
<tr>
<th>(1) Models' variables</th>
<th>Explanatory variable (export ratio)</th>
<th>Independent variable (mid-month adj. price(s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>August (around the 10th)</td>
<td>The lower of Oct./November</td>
</tr>
<tr>
<td>(2)</td>
<td>&quot;</td>
<td>The higher of Oct./November</td>
</tr>
<tr>
<td>(3)</td>
<td>&quot;</td>
<td>January</td>
</tr>
<tr>
<td>(4)</td>
<td>&quot;</td>
<td>March/April (average)</td>
</tr>
<tr>
<td>(5)</td>
<td>&quot;</td>
<td>June</td>
</tr>
<tr>
<td>(6)</td>
<td>November (around the 10th)</td>
<td>January</td>
</tr>
<tr>
<td>(7)</td>
<td>&quot;</td>
<td>March/April (average)</td>
</tr>
<tr>
<td>(8)</td>
<td>&quot;</td>
<td>June</td>
</tr>
<tr>
<td>(9)</td>
<td>January (around the 24th)</td>
<td>March/April (average)</td>
</tr>
<tr>
<td>(10)</td>
<td>&quot;</td>
<td>June</td>
</tr>
<tr>
<td>(11)</td>
<td>April (around the 24th)</td>
<td>June</td>
</tr>
</tbody>
</table>

Regression model (1):
(October/November)_low price v. August export ratio

Fitted regression line:
Adj. price = 82.6 / (108.5 - export ratio)
The forecasts of the average monthly prices were derived using Kingsman’s model, with some minor modifications:

(i) the forecasts of the October/November average monthly prices were obtained by averaging the \((\text{Oct/Nov})_{\text{low}}\) and the \((\text{Oct/Nov})_{\text{high}}\) mid-month price forecasts; Table 48 shows that such forecasts are virtually identical to those that would be obtained from the regression of average monthly adjusted 'cash' prices versus the export ratio.

\[
\begin{align*}
(1) & \quad \hat{\text{ACP}}_{\text{Oct/Nov}}^{\text{low}} = 82.6 / (108.5 - \text{ER}) \quad \text{[US $/bushel]} \\
(2) & \quad \hat{\text{ACP}}_{\text{Oct/Nov}}^{\text{high}} = 88.8 / (109.6 - \text{ER}) \quad \text{[US $/bushel]} \\
(3) & \quad \text{ACP}_{\text{Oct/Nov}} = \text{forecast (1) + forecast (2)} / 2
\end{align*}
\]

Forecasts based on regressions of av. monthly adj. 'cash' prices against ER:

\[
\begin{align*}
(4) & \quad \hat{\text{ACP}}_{\text{Oct}} = 87.8 / (111.3 - \text{ER}) \quad \text{[US $/bushel]} \\
(5) & \quad \hat{\text{ACP}}_{\text{Nov}} = 83.3 / (106.0 - \text{ER}) \quad \text{[US $/bushel]} \\
\end{align*}
\]

Comparison of fitted prices over the period 1967-68 to 1974/75 (US $/bushel)

<table>
<thead>
<tr>
<th>Crop year</th>
<th>ER</th>
<th>Oct/Nov</th>
<th></th>
<th>Oct</th>
<th>Nov</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>low</td>
<td>high</td>
<td>av</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>1967/68</td>
<td>30.5</td>
<td>1.06</td>
<td>1.12</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td>1968/69</td>
<td>31.2</td>
<td>1.07</td>
<td>1.13</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>1969/70</td>
<td>36.3</td>
<td>1.24</td>
<td>1.21</td>
<td>1.26</td>
<td>1.27</td>
</tr>
<tr>
<td>1970/71</td>
<td>48.0</td>
<td>1.37</td>
<td>1.44</td>
<td>1.41</td>
<td>1.39</td>
</tr>
<tr>
<td>1971/72</td>
<td>30.5</td>
<td>1.06</td>
<td>1.12</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td>1972/73</td>
<td>42.0</td>
<td>1.24</td>
<td>1.31</td>
<td>1.28</td>
<td>1.27</td>
</tr>
<tr>
<td>1973/74</td>
<td>66.3</td>
<td>1.96</td>
<td>2.05</td>
<td>2.02</td>
<td>1.95</td>
</tr>
<tr>
<td>1974/75</td>
<td>81.0</td>
<td>3.01</td>
<td>3.11</td>
<td>3.06</td>
<td>2.90</td>
</tr>
</tbody>
</table>

(ii) the forecasts of June prices were extended to July as well;

(iii) the forecasts of average monthly prices in September, December, February and May were obtained by interpolation.

The average monthly 'cash' prices at Chicago are compared, in
Figure 71, with the prices fitted (up to 1974/75) or forecast (in 1975/76 and 1976/77) using the behavioural model.

All the predicted prices were derived using the adjustment factor $D_V$ at the time the forecasts were made (denoted henceforth by $D_{V_c}$ or, simply, $D_{V_c}$), not in the future months whose prices were being predicted (denoted by $D_{V_{\text{forward}}}$ or $D_{V_{f}}$); therefore, the forecasting errors shown in the diagram include a component due to unpredictable changes in the value of the dollar occurring after the forecasts were made.

Since the research described in this thesis was carried out, Kingsman used the model to derive forecasts in the subsequent crop years. These forecasts (to be published in Kingsman's forthcoming book) were extremely accurate (even better than these shown in Figure 71).

In conclusion, Kingsman's model is remarkably good at predicting in advance the general level of 'cash' prices and the varying price movements over the crop year.

4.5 Relationships between Average Monthly 'Cash' and 'Futures' Prices

As mentioned earlier in section 4.1, for the purpose of defining the active buying periods for the different shipments, the relevant forecasts are those of Chicago average monthly 'futures' prices. They can be derived from the average monthly 'cash' price forecasts, once the relationships between 'futures' and 'cash' prices have been established.

The way in which these relationships were analysed will be illustrated considering the March 'futures' prices. A summary of the results for the remaining 'futures' contracts will be presented thereafter.

The feed grain shipments purchased using March 'futures' are those having purchasing deadlines between the 15th of November and the 15th of February (see Figure 66, section 2.5). According to the operating doctrine chosen in section 2.5, 'futures' must not be bought more
Figure 71 - Maize 'cash' prices (Chicago): medium term forecasts derived using Kingsman's price behavioural model

- — — — — — o-—-—-—- forecast made around the 10th of August
- — — — — — 1-—-—-— forecast made around the 10th of November
- — — — — — — — — — forecast made around the 24th of January
+ — — — — — — — — — forecast made around the 24th of April

$ / bushel

than six months in advance of the purchasing deadline of each shipment. This implies that March 'futures' must not be bought before the 15th of May (i.e. six months before the 15th of November). However, the first reliable price forecast for the new crop year can only be obtained around the 10th of August. It would be extremely unwise to make, before then, purchase commitments for the new crop year. Consequently, in the purchasing strategy discussed later in section 6, a further constraint on the purchasing operations was imposed: 'futures' corresponding to shipments with purchasing deadlines after August are only bought after the August price forecast becomes available. This implies that all March 'futures' will be bought between

(i) August (after the price forecast becomes available) and

(ii) 15th of February.

The study of the relationship between March 'futures' prices and 'cash' prices was therefore restricted to the months from August to February.

Figure 72 (i) shows, for the August to February months of the crop years 1967/68 to 1974/75, the margin

\[ \Delta = \frac{\text{Av.monthly March'futures'price} - \text{av.monthly'cash'price}}{\text{Av.monthly'cash'price}} \times 100 \, (\%) \]

The diagram shows a distinct seasonal pattern for the relative margin: increasing up to a maximum in October/November, returning to a near zero average in January and February.

The positive value of the margins in October/November is not surprising. Since the bulk of the maize crop is harvested during these months, 'cash' prices tend to fall to a trough in October/November
Figure 72 - Analysis of the monthly margin of March 'futures' over 'cash'
(Chicago, 1967/68 to 1974/75)

(i) Monthly margin: \(( \text{Av.fut.price - Av.cash price}) / \text{Av.cash price} \)

(ii) Average monthly margins in Oct/Nov, Jan and Feb:

<table>
<thead>
<tr>
<th>March 'future' contract</th>
<th>Margins [%] (A1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oct/Nov</td>
</tr>
<tr>
<td>1968</td>
<td>4.2</td>
</tr>
<tr>
<td>1969</td>
<td>2.6</td>
</tr>
<tr>
<td>1970</td>
<td>3.3</td>
</tr>
<tr>
<td>1971</td>
<td>8.5</td>
</tr>
<tr>
<td>1972</td>
<td>10.6</td>
</tr>
<tr>
<td>1973</td>
<td>6.5</td>
</tr>
<tr>
<td>1974</td>
<td>4.1</td>
</tr>
<tr>
<td>1975</td>
<td>8.3</td>
</tr>
</tbody>
</table>

\[ \bar{\Delta} = 6.0 \]  
\[ \bar{\delta} = 2.9 \]  
\[ t = \frac{\bar{\Delta}}{\bar{\delta}} = 2.9 \]  
\[ (\alpha = 0.05) \text{ (sig)} \]  
[not sig]  
[not sig]
and to rise between then and March. The positive margins in October/November can be regarded as a partial compensation made by the market for the price rise that is expected to take place between October/November and March. As the month of delivery (March) approaches, the margins Δ become smaller, as expected.

The relative margins (Δₐ) were averaged, over the period 1967/68 to 1974/75, for

(i) October/November and January (i.e. the months for which 'cash' price forecasts were obtained using Kingsman's model);

(ii) February (the last relevant month for the March contract).

The significance of the average margins (Δ) in those months was tested, using the t-test (under the assumption that for each of the months considered, successive values of the margin, Δₐ, follow identical independent distributions). As shown in Figure 72 (ii), only the average margin $\bar{\Delta}_{\text{Oct/Nov}}$ was found to be significantly different from zero.

Further tests carried out showed that:

(i) there was no significant difference between the October and November margins (their aggregation was therefore acceptable);

(ii) there was no significant correlation between each year's margins in August, October/November, January and February (hence, the value of the margin on each of these periods could not be used to improve the forecasts of the margins in the subsequent periods of the same crop year);
(iii) for each period October/November, January and February, the value of the margin in each crop year ($\Delta_1$) was not related to the corresponding values of the August and November export ratio.

The analysis was extended to 'futures' contracts other than March, with identical results. Figure 73 shows, for all contracts, the estimates of the seasonal margins in relevant months,

$$\hat{\Delta} = \begin{cases} 
\Delta, & \text{if } \Delta \text{ is significantly different from zero} \\
0, & \text{otherwise.}
\end{cases}$$

Figure 73 - Seasonal margins of 'futures' prices over 'cash' prices (estimated over the period 1967/68 to 1974/75)
4.6 Forecasts of the Average Monthly 'Futures' Prices

The average monthly 'futures' prices (FP) can be expressed as

\[ \text{FP} = (1 + \Delta/100). \text{CP} = (1 + \Delta/100). \frac{\text{ACP}}{\text{DV}_{f}} \] (US$/bushel)

where

- CP : (unadjusted) average monthly 'cash' price (US$/bushel)
- \( \Delta \) : 100.(FP-CP)/CP
- ACP : adjusted average monthly 'cash' price (US$/bushel)
- \( \text{DV}_{f} \) : value of the dollar in the month whose price is being forecast, expressed as a proportion of the pre-1971 value.

At any given time when a forecast of FP is made, the expected value of FP is approximately given by

\[ \mu_{FP} = (1 + \mu_{\Delta}/100). \mu_{CP} = (1 + \mu_{\Delta}/100). \frac{\mu_{ACP}}{\mu_{DV}} \]

where \( \mu_{\Delta}, \mu_{CP}, \mu_{ACP} \) and \( \mu_{DV} \) denote the expected values of the variates \( \Delta, \text{CP}, \text{ACP} \) and \( \text{DV}_{f} \), respectively.

This relation, based on a linear approximation of the function \( \text{FP}(\Delta, \text{ACP}, \text{DV}_{f}) \) in the neighbourhood of the point \( (\mu_{\Delta}, \mu_{ACP}, \mu_{DV}) \), is valid for small variances of \( \Delta, \text{ACP} \) and \( \text{DV}_{f} \), and, hence, for a small variance of FP (in the next section this will be shown to be the case for all the medium term forecasts of FP considered).

According to the expression above, the forecasts of the 'futures' prices can be derived approximately simply by adjusting the 'cash' price forecasts with the corresponding estimates of the seasonal margin, i.e.,

\[ \hat{\text{FP}} = (1 + \hat{\Delta}/100). \hat{\text{CP}} = (1 + \hat{\Delta}/100). \frac{\hat{\text{ACP}}}{\text{DV}_{c}} \] (US$/bushel)
where
\[
\hat{\Delta} \quad \text{: seasonal margin estimate (as given in Figure 73) (\%)}
\]
\[
\hat{\text{CP}} \quad \text{: forecast of the (unadjusted) average monthly 'cash' price (\(\hat{\text{CP}} = \frac{\hat{\text{ACP}}}{\text{DV}}\)) (US$/bushel)}
\]
\[
\hat{\text{ACP}} \quad \text{: forecast of the adjusted average monthly 'cash' price (obtained either directly from Kingsman's regression models or by interpolation) (US $/bushel)}
\]
\[
\text{DV}_C \quad \text{: value of the dollar at the time the forecast is made, expressed as a proportion of the pre-1971 value (this value is assumed to be the expected value of \(\text{DV}_f\), i.e. \(\text{DV}_C = E(\text{DV}_f) = \mu_{\text{DV}}\).}
\]

In the case of the March 'futures', the average monthly 'futures' prices in October/November, January and February were defined as
\[
\hat{\text{FP}} = (1 + \hat{\Delta}/100) \cdot \hat{\text{CP}}
\]
with
\[
\hat{\Delta} = \begin{cases} 
6.0\%, & \text{for October/November} \\
0\%, & \text{for January and February} 
\end{cases} \quad \text{(see Figure 72(ii))}
\]

and where \(\hat{\text{CP}}\) are the relevant average monthly 'cash' price forecasts, obtained as described in section 4.4.

For the other months (September, December) the March 'futures' price forecasts were interpolated linearly, as shown in Figure 74 (the forecasts of the September prices were interpolated between the price at the time the forecasts were made - around the 10th of August - and the October/November price forecasts).

An identical procedure was adopted to derive the average monthly 'futures' price forecasts for the other contracts.
4.7 Analysis of the Forecast Errors of the Average Monthly 'Futures' Prices

4.7.1 General. Sources of the Forecast Errors

For the purpose of making rational purchasing decisions on the basis of price forecasts it is essential to have a clear indication of the reliability of the forecasts. In this section, the reliability of the average monthly 'futures' price forecasts will be characterized through an analysis of the distributions of the forecast errors.

The different sources of error in the forecasting of average monthly 'futures' prices can be illustrated considering, once again, the March 'futures' prices. For those months whose 'futures' prices are forecast directly by adjusting the 'cash' price forecasts \( \hat{P} \) with the seasonal margin \( \hat{\Delta} \) (i.e. October/November, January and February), each
forecast error arises from three different sources:

(i) the error in the forecast of the adjusted average monthly 'cash' price;

(ii) the deviation of the margin $\Delta$ in relation to the estimate of its seasonal value (shown earlier in Figure 73);

(iii) the error due to changes in the value of the dollar between the time when the forecast is made and the month whose price is being forecast.

For the months whose 'futures' price forecasts are obtained by linear interpolation (September and December, in the case of March 'futures') the forecast errors result from:

(i) deviations of the average 'futures' prices in these 'intermediate months' from the straight lines connecting the actual 'futures' prices in the interpolation extremes;

(ii) errors in the definition of the interpolation lines (or, equivalently, errors in the forecasts of those 'futures' prices used as a basis for interpolation).

The estimation of the distributions of the forecast errors poses considerable theoretical difficulties and demands a number of assumptions and approximations. The nature of the problems involved can be best illustrated considering firstly the errors of the adjusted average monthly 'cash' price forecasts.

4.7.2 Forecast Errors of the Adjusted Average Monthly 'Cash' Prices

For the months of the crop year identified earlier as critical - October/November, January, March/April and June - the forecasts of the adjusted average monthly 'cash' prices were derived using Kingsman's regression models. In these models, mid-month adjusted prices were used as dependent variables.
Earlier in section 4.4 it was shown that identical forecasts would have been obtained if the average monthly prices were regressed directly against the export ratios. It would appear that, by taking the average monthly prices (rather than mid-month prices) as dependent variables, it would be possible to estimate directly the standard deviations of the forecast errors. However, there are serious difficulties in this estimation process if the models are derived via linear regression. These difficulties can be illustrated by reference to Table 49.

In the model considered in the table, the dependent variable (i.e. the adjusted average monthly 'cash' price, ACP) was first transformed, as shown in (ii), and then regressed linearly against the export ratio. Following the common assumption of independent and normally distributed errors $\varepsilon_1'$, with constant standard deviation, the best unbiased estimates of $\alpha$ and $\beta$ were derived using a linear regression package.

As shown in (iii), the standard deviation of the forecast error of the transformed variable, $e_1' = y - \hat{y}$, is

$$S.D.(e_1') = S.D.(\varepsilon_1'). g(ER)$$

where $g(ER)$ varies between 1.06 and 1.16 within the relevant range of values of ER (30 to 80%).

Following the assumptions made about the errors $\varepsilon_1'$, the standard deviation of the forecast error

$$e_1 = \hat{ACP} - ACP$$

is

$$S.D.(e_1) = S.D.(e_1'). \mu_{ACP}^2 = S.D.(\varepsilon_1'). g(ER). \mu_{ACP}^2.$$
Table 49 - Regression of adjusted average October 'cash' price w. August export ratio (1967/68 to 1974/75)

(1) Notation

\[ \text{ACP} : \text{adjusted average 'cash' price in October [US $/bushel]} \]
\[ \hat{\text{ACP}} : \text{forecast of ACP (ACP} = \hat{\text{ACP}} = \hat{\mu}_{\text{ACP}} \) \]
\[ \text{ER} : \text{August export ratio \%} \]
\[ n : \text{no. of regression points (n=6)} \]

(11) Transformation of the dependent variable

\[ y = f(\text{ACP}) = 1/\text{ACP} \]

(111) Linear regression model

\[ y = \alpha + \beta \cdot (\text{ER-ER}) + \varepsilon_y \quad (\text{S.D.}(\varepsilon_y) = \text{constant}) \]
\[ \hat{\alpha} = 0.746 \]
\[ \hat{\beta} = -0.0114 \]
\[ \text{ER} = 45.7 \]
\[ \text{var}(\varepsilon_y) = 1.79 \cdot 10^{-3} \]
\[ \delta (\text{forecast of } y) = \hat{\delta}(y) = \hat{\alpha} + \hat{\beta} \cdot (\text{ER-ER}) \]
\[ \varepsilon_y (\text{forecast error of } y) = y - \delta \]
\[ \text{var}(\varepsilon_y) = \text{var}(y) = \text{var}(\delta) \]
\[ \text{var}(\varepsilon_y) = \text{var}(\varepsilon_y) \cdot (\text{ER-ER})^2 \cdot \text{var}(\delta) \]
\[ = \text{var}(\varepsilon_y) \cdot \left[ 1 + \frac{1}{n} + \frac{\text{ER}}{n} \cdot (\text{ER-ER})^2 \right] \]
\[ = \text{var}(\varepsilon_y) \cdot \text{g(ER)}^2 \]
\[ \text{S.D.}(\varepsilon_y) = \text{S.D.}(\varepsilon_y) \cdot \text{g(ER)} \quad (*) \]

(iv) Forecasting model

\[ \hat{\text{ACP}} = \hat{\delta}(\text{ACP}) = \hat{\delta}(1/y) = 1/\hat{\delta}(y) \quad (**) \]
\[ \hat{\text{ACP}} = 1 / \left[ \hat{\alpha} + \hat{\beta} \cdot (\text{ER-ER}) \right] = 87.8 / (111.3 - \text{ER}) \]
\[ \varepsilon_y (\text{forecast error of ACP}) = \text{ACP} - \hat{\text{ACP}} \]
\[ \text{var}(\varepsilon_y) = \text{var}(\text{ACP}) = \text{var}(\hat{\text{ACP}}) \]
\[ = \text{var}(1/y) = \text{var}(1/y) \]
\[ \text{var}(1/y) = \left[ \frac{d(1/y)}{dy} \right]^2 \cdot \text{var}(y) \quad (**) \]
\[ = (1/y)^4 \cdot \text{var}(y) \]
\[ = \frac{\hat{\text{ACP}}^4}{\text{var}(\varepsilon_y)} \]
\[ \text{var}(1/y) = \hat{\text{ACP}}^4 \cdot \text{var}(\varepsilon_y) \quad (**) \]
\[ \hat{\text{ACP}}^4 \cdot \text{var}(\varepsilon_y) \cdot [\text{var}(\delta) + (\text{ER-ER})^2 \cdot \text{var}(\delta)] \]
\[ \text{var}(\varepsilon_y) = \text{var}(\varepsilon_y) \cdot \text{g(ER)}^2 \cdot \hat{\mu}^2_{\text{ACP}} \]
\[ \text{S.D.}(\varepsilon_y) = \text{S.D.}(\varepsilon_y) \cdot \text{g(ER)} \cdot \hat{\mu}^2_{\text{ACP}} \]

(*) For values of ER between 30 and 80 (see Table 49, section 4.4), g(ER) varies between 1.06 and 1.15

(**) Approximation valid for small variance of the variate y
As ER changes, the changes in $g(ER)$ are insignificant when compared to the changes in $\mu_{ACP}$. The standard deviation of $e_1$ can therefore be regarded as being approximately proportional to the square of $\mu_{ACP}$ (or its estimate, $\hat{ACP}$). However this result is critically dependent on the particular transformation used. For example, the logarithmic transformation (adopted in other regression models) would have led to a standard deviation $S.D.(e_1)$ approximately proportional to $\mu_{ACP}$ (or, $\hat{\mu}_{ACP} = \hat{ACP}$). In other models, when the dependent variable $ACP$ is not transformed,

$$S.D.(e_1) \approx \text{constant}.$$ 

These relationships between $S.D.(e_1)$ and $\mu_{ACP}$ differ for rather artificial reasons. They are, in fact, the direct consequence of

(i) the assumption of constant standard deviation of $e_1$

(an assumption which is difficult to justify on rational grounds and which is difficult to substantiate or to reject on the evidence provided by eight regression points only);

(ii) the use of linear regression models involving different transformations of the dependent variable.

It seems preferable to make a common assumption regarding the final relationship between $S.D.(e_1)$ and $\mu_{ACP}$ for all regression models, rather than postulating assumption (i) above and blindly accepting its consequences (i.e. the various functional relationship between $SD(e_1)$ and $\mu_{ACP}$ associated with each transformation).
Following the examination of the observed errors of Kingsman’s regression models, it was decided to assume, for all regression models and independently of the transformation used, that the standard deviation of the dependent variable

\[ \text{ACP} = \mu_{\text{ACP}} + \epsilon_1 \]  
(with \( \mu_{\text{ACP}} = \mu_{\text{ACP}}(\text{ER}) \) and independent errors \( \epsilon_1 \))

was proportional to the mean \( \mu_{\text{ACP}} \):

\[ \text{S.D.}(\text{ACP}) = \text{S.D.}(\epsilon_1) = (k_1/100) \cdot \mu_{\text{ACP}}. \]

For each regression, the constant \( k_1 \) is the standard deviation of the percent errors 100 (ACP - \( \mu_{\text{ACP}} \))/\( \mu_{\text{ACP}} \).

In theory, under the assumption made, the forecasts obtained using linear regressions involving transformations other than logarithmic are less efficient than those that would be obtained via non-linear regression. However, in practice, the differences between them are unlikely to be of any significance, in view of the excellent fit of the linear regressions.

The linear regression models, with the exception of those involving the logarithmic transformation, have one limitation. Under the assumption made about the standard deviation of ACP, they do not enable the estimation of the standard deviation of \( \hat{\text{ACP}} = \hat{\mu}_{\text{ACP}} \) and, hence, of \( \epsilon_1 = \text{ACP} - \hat{\text{ACP}} \). As will be shown in the next section, without an estimate of S.D.(\( \hat{\text{ACP}} \)), the standard deviations of the 'futures' prices forecast errors, can only be estimated approximately. Although, in principle S.D.(\( \hat{\text{ACP}} \)) could be estimated via non-linear regression, the computer programmes available were not suitable for that purpose. For this reason it was decided

(i) to adopt the forecasts of ACP obtained using Kingsman’s regression models (for October/November, January, March/April and June);

(ii) to assume, for each of these months, that the standard deviation of ACP is proportional to \( \mu_{\text{ACP}} \).
This assumption was extended to those months in which the forecasts of the average 'cash' prices were derived via interpolation.

4.7.3 Estimation of the Standard Deviations of the Forecast Errors of Average Monthly 'Futures' Prices

For those 'futures' price forecasts derived by adjusting the 'cash' price forecasts with the seasonal margins, the distributions of

\[ \text{APP} : \text{adjusted average monthly 'futures' price (US$/bushel)} \]

\[ \text{FP} : \text{actual average monthly 'futures' prices (US$/bushel)} \]

can be derived from the distributions of

\[ \text{ACP} : \text{adjusted average monthly 'cash' price (US$/bushel)} \]

\[ \Delta : \frac{\text{APP} - \text{ACP}}{\text{ACP}} \quad (\%) \]

\[ \text{DV}_f : \text{value of the dollar in the month whose 'futures' price is being forecast, expressed as proportion of the pre-1971 dollar value.} \]

In Table 50, the mean and the standard deviation of APP and FP are derived from those of ACP, \( \Delta \) and \( \text{DV}_f \). The mean and standard deviations of these variates are defined in Table 50(ii). In addition to the assumption made earlier about the standard deviation of ACP, two further assumptions were made. Firstly, the value of the dollar in the month whose price is being forecast (\( \text{DV}_f \)) was assumed to have, after April 1971, mean

\[ \mu_{\text{DV}} = \text{DV}_c \quad (\text{value of the dollar at the time the forecast is made}) \]

and standard deviation proportional to the mean:

\[ \text{S.D.}(\text{DV}_f) = (k_3/100) \cdot \mu_{\text{DV}} = (k_3/100) \cdot \text{DV}_c \]

The constant \( k_3 \) - the standard deviation of the percent change in the value of the dollar 100.(\( \text{DV}_f - \text{DV}_c \))/\( \text{DV}_c \) - clearly will depend on the time lag between the moment in which the forecast is made and the
### Table 50 - Mean and standard deviation of the adjusted and actual average monthly 'futures' prices

(1) **Notation**
- **ACP**: adjusted average monthly 'cash' price [US $ / bushel]
- **Δ**: relative margin between average monthly 'futures' and 'cash' prices (adjusted or unadjusted) [%]
- **DVₜ**: value of the dollar in the month whose price is being forecast, expressed as a proportion of the pre-1971 value
- **DVₑ**: value of the dollar at the time the forecast is made
- **AFF**: adjusted average monthly 'futures' price [US $/bushel]
- **FP**: (unadjusted) average monthly 'futures' price [US $/bushel]

(2) **Assumptions about the distributions of ACP, Δ and DV**

- (a) \( ACF = \mu_{ACP} + \varepsilon_1 \) \( S.D.(ACF) = S.D.(\varepsilon_1) = (k_1/100).\mu_{ACP} \)
- (b) \( \Delta = \mu_\Delta + \varepsilon_2 \) \( S.D.(\Delta) = S.D.(\varepsilon_2) = k_2 \)
- (c) \( DVₜ = \mu_{DV} + \varepsilon_3 (\mu_{DV} - DVₑ) \) \( S.D.(DVₜ) = S.D.(\varepsilon_3) = (k_3/100).\mu_{DV} \)
- (d) \( \text{cov}(\varepsilon_1, \varepsilon_2) = 0 \)
- \( \text{cov}(\varepsilon_1, \varepsilon_3) = 0 \)
- \( \text{cov}(\varepsilon_2, \varepsilon_3) = 0 \)

(3) **Mean and standard deviation of AFF**

- (a) \( AFF = (1 + \Delta/100).ACF \)
  \[ \mu_{AFF} = (1 + \frac{\mu_\Delta}{100}).\mu_{ACP} \]  \hfill (1)
- (b) \( \text{var}(AFF) = \left[ \frac{\Delta}{1(1 + \Delta/100)} \right]^2 \text{var}(1 + \Delta/100) + \left[ \frac{\Delta}{\Delta \mu_{ACP}} \right]^2 \text{var}(ACF) \)
  \[ = (1/100)^2(k_1^2 + k_2^2) \cdot \mu_{AFF}^2 \]
  \( \text{(where: } k_1 = k_2/(1 + \mu_\Delta/100) \text{) } \)
  \[ S.D.(AFF) = (k_{12}/100).\mu_{AFF} \]
  \( \text{(where: } k_{12} = \sqrt{k_1^2 + k_2^2} \text{) } \)

(4) **Mean and standard deviation of FP**

- (a) \( FP = AFF / DVₜ \)
  \[ \mu_{FP} = \frac{\mu_{AFF}}{\mu_{DV}} \]
  \[ = \frac{(1 + \mu_\Delta/100).\mu_{ACP}}{\mu_{DV}} \]  \hfill (2)
- (b) \( \text{var}(FP) = \left[ \frac{\mu_{FP}}{\mu_{AFP}} \right]^2 \text{var}(AFF) + \left[ \frac{\mu_{FP}}{\mu_{DV}} \right]^2 \text{var}(DVₜ) \)
  \[ = (1/100)^2(k_{12}^2 + k_2^2) \cdot \mu_{FP}^2 \]
  \[ S.D.(FP) = (k/100).\mu_{FP} \]
  \( \text{(where: } k = \sqrt{k_{12}^2 + k_2^2} \text{) } \)

(1) Formulae based on a linear approximation of \( AFF = AFF(\Delta, ACP) \) in the neighbourhood of \( (\mu_\Delta, \mu_{ACP}) \): approximation valid for small variances of \( \Delta \) and \( ACP \)

(2) Formulae based on a linear approximation of \( FP = FP(AFF, DVₜ) \) in the neighbourhood of \( (\mu_{AFF}, \mu_{DV}) \): approximation valid for small variances of \( AFF \) and \( DVₜ \)
month whose price is forecast. (*)

Secondly, it was assumed that the variates ACP, \( \Delta \) and \( DV_f \) are independent (no relevant connection between them could be found on rational grounds and the assumption could not be statistically rejected).

Expressions of the mean and standard deviation of AFP and FP were derived in (iii) and (iv) (the expression of \( \mu_{FP} \) is that shown earlier in section 4.6). According to the assumptions made, both standard deviations are approximately proportional to the respective means:

\[
\text{S.D. (AFP)} \approx \left( \frac{k}{12/100} \right) \mu_{\text{AFP}}
\]

\[
\text{S.D. (FP)} \approx \left( \frac{k}{100} \right) \mu_{FP}
\]

where \( k_{12} \) and \( k \) are the standard deviations of the percent errors \( \frac{100(\text{AFP}-\mu_AFP)}{\mu_AFP} \) and \( \frac{\mu_{FP}}{\mu_{FP}} \) respectively.

If, as before, \( \hat{FP} \) is used to denote the forecast of an average monthly 'future' price FP, the forecast error is

\[ e = FP - \hat{FP} \]

In earlier sections it was assumed that:

(i) for any given regression relating ACP to ER, the errors \( \epsilon_1 = ACP - \mu_{ACP} \) in the different crop years are independent;

(ii) for any given month, the errors \( \epsilon_2 = \Delta - \mu_{\Delta} \) in different crop years are independent.

Under these assumptions FP and \( \hat{FP} \) are independent and the standard

(*) Prior to April 1971, i.e. before the dollar was floated, clearly

\[ DV_f = DV_c \text{ and } \text{S.D.}(DV_f) = k_j = 0. \]
deviation of the forecast error is given by

\[
S.D.(e) = \sqrt{\text{var}(FP) + \text{var}(\hat{FP})}
\]

Recalling that \( FP \) is given by

\[
FP = \frac{(1+ \Delta/100) \cdot ACP}{DV_c}
\]

(with \( DV_c = \mu_{DV} \)),

and that \( \Delta \) and \( ACP \) (hence, \( \hat{\Delta} \) and \( \hat{ACP} \)) are independent, the variance of \( \hat{FP} \) is

\[
\text{var}(\hat{FP}) \approx (\frac{ACP}{100 \cdot DV_c})^2 \cdot \text{var}(\hat{\Delta}) + (\frac{1 + \Delta/100}{DV_c})^2 \cdot \text{var}(ACP).
\]

For reasons discussed in the previous section, \( \text{var}(\hat{ACP}) \) and, hence, \( \text{var}(\hat{FP}) \) could only be estimated for those months in which \( \hat{ACP} \) was derived from linear regression models involving the logarithmic transformation. The estimation of \( \text{var}(\hat{FP}) \) for such months revealed that this term could be neglected in the expression of \( S.D.(e) \):

\[
S.D.(e) = \sqrt{\text{var}(FP) + \text{var}(\hat{FP})}
\]

\[
= \sqrt{\text{var}(FP)} = S.D.(FP)
\]

The error incurred as a result of this approximation — between 4 and 11% — was found acceptable in view of the unreliability of the estimates of \( S.D.(FP) \) (derived from very small samples).

The expression

\[
S.D.(FP) \approx (k/100) \cdot \mu_{FP}
\]

\[
\approx \left[ \sqrt{k_{12}^2 + k_{3/100}^2} \right] \cdot \mu_{FP}
\]

was derived, in Table 50, for those 'futures' price forecasts obtained by adjusting the 'cash' price forecasts with the seasonal margins.

In the procedure adopted in the estimation of \( S.D.(FP) \) that expression was assumed to hold as well for those 'futures' price forecasts derived via interpolation.
For each particular forecast - i.e. made at a given date and concerning the average price of a particular 'futures' contract over a specific month - the value of

\[ \hat{k} = \text{S.D.} \left[ 100. \frac{FP - \mu_{FP}}{\mu_{FP}} \right] \]

was estimated from

\[ \hat{k} = \sqrt{\hat{k}_{12}^2 + \hat{k}_3^2} \]

where

\[ \hat{k}_{12} : \text{ estimate of S.D.} \left( 100. \frac{AFP - \mu_{AFP}}{\mu_{AFP}} \right), \]
derived over the period 1967/68 to 1974/75 (before the dollar was 'floated', i.e. before April 1971, \( D_{VF} = 1 \), hence \( AFP = FP \) and \( \hat{k} = \hat{k}_{12} \));

\[ \hat{k}_3 : \text{ estimate of S.D.} \left( 100. \frac{DV_{VF} - DV_C}{\mu_{AFP}} \right), (\hat{k}_3 = 0 \]
before April 1971; thereafter, \( \hat{k}_3 > 0 \), with a magnitude depending on the time lag between the time the forecast is made and the month whose price is being forecast).

Figure 75 gives, for all futures contracts, estimates of \( k_{12} \) corresponding to the different forecasts of the adjusted average monthly prices. The estimates were derived over the period 1967/68 to 1974/75.

Estimates of \( k_3 \), for different time lags (1 to 10 months) are presented in Figure 76. They were obtained from a series of 54 average monthly values of the dollar between April 1971 (when the floating of the dollar started) and September 1975 (the end of the 1974/75 crop year).

Estimates of \( k \) (with the dollar floating) were derived, for each particular average monthly 'future' price forecast, from the corresponding values of \( \hat{k}_{12} \) and \( \hat{k}_3 \) and are presented in Figure 77.
Figure 75 - Estimates of $k_{12}$ (derived over the period 1967/68 to 1974/75)

$$k_{12} = 100 \cdot \sqrt{\frac{1}{6} \cdot \sum_{i=1}^{8} \left( \frac{A_{i} - \hat{A}_{i}}{A_{i}} \right)^2}$$

- ○ forecasts made around the 10th of August
- △ forecasts made around the 10th of November
- □ forecasts made around the 24th of January
- + forecasts made around the 24th of April

(1) MARCH 'futures'

(2) MAY 'futures'

(3) JULY 'futures'

(4) SEPTEMBER 'futures'

(5) DECEMBER 'futures'
Several points may be made about these estimates. The first one concerns the relative accuracy of forecasts of the same 'futures' price, but made at different times (e.g. the forecasts made in August and November of the March 'futures' price in December). The diagrams show, as expected, that the more recent forecasts are generally more reliable. Occasionally, the estimates of $\hat{k}$ for the more recent forecasts were found to be larger than for previous ones. Estimation errors - quite significant, since the estimates $\hat{k}$ are based on small samples - are the most likely cause of such results. In reality, no decision maker would choose, in such circumstances, the older forecast in preference to the more up-to-date price prediction. Consequently, the most recent forecasts have always been used in this research, in the study of medium-term buying decisions (presented later in section 6).

In view of the unreliability of the estimates $\hat{k}$, some caution must be exercised when interpreting the diagrams presented in Figure 77. However these diagrams suggest that the accuracy of the forecasts depends not only on how far ahead the forecasts are made, but also on the particular months whose prices are being forecast. The less accurate forecasts are, according to the diagrams, those concerning the months January to March and June/July. In June/July the comparatively low accuracy of the forecasts can be attributed to the effect on prices
Figure 77 - Estimates of the standard deviations of the forecast errors of the average monthly 'futures' prices (with the dollar 'floating')

\[ e \text{ (forecast error)} = \bar{F}P - \bar{F} \]

\[ \text{S.E.}(e) = \left( \frac{\bar{e}}{100} \right) \cdot \bar{F} \]

\[ \hat{\sigma} = \sqrt{\frac{\sum e^2}{12} - \frac{\bar{e}^2}{5}} \]

- O- - - - - - O forecasts made around the 10th of August
- A- - - - - - A forecasts made around the 10th of November
- D- - - - - - D forecasts made around the 24th of January
- J- - - - - - J forecasts made around the 24th of April

(i) MARCH 'futures'

(ii) MAY 'futures'

(iii) JULY 'futures'

(iv) SEPTEMBER 'futures'

(v) DECEMBER 'futures'
of the new crop prospects. These can change markedly specially in July, a critical month of the maize growing season (when lack of rain can seriously affect the maize crop yield). In the period January to March the causes of the unreliability of the forecasts are less obvious. Part of it is due to the low accuracy of the 'cash' price forecasts over that period (possibly reflecting frequent over and under-reactions of the market in anticipation of or following the release of the 1st quarter consumption figures). For the 'futures' contracts other than March the low accuracy of the forecasts in the period January to March is partially due to a comparatively large variability of the relative margin between 'futures' and 'cash' prices in that period (this does not occur for the March 'futures' in view of the proximity of the maturity month).

A further point should be made about the shape of the distributions of the forecast errors. So far, only the expected values (assumed to be zero) and the standard deviations of those distributions have been discussed. Clearly not much can be inferred about their shape from the available samples of errors, given their small size. Therefore, the specification of the distributions must depend upon largely untested assumptions - either about the distribution of the final forecast errors \( e = FP - \hat{FP} \) or about their different components. The first course - simpler than the second - was chosen, for the analysis of medium-term buying decisions: the forecast errors, \( e \), were assumed to be normally distributed.

The points made above have centred on the accuracy of the 'futures' price forecasts, as measured by the magnitude of the forecast errors \( e = FP - \hat{FP} \) and, in particular, by the estimates of \( k \). This
measure of accuracy conceals one limitation of the forecasts derived, namely their inability to 'explain' adequately price changes taking place shortly after the forecasts are made (one or two months ahead). This inability can be illustrated considering, for example, the March 'futures' forecasts made in August, for September and October. On the face of the estimates presented in diagram (i) of Figure 77, the accuracy of those forecasts is good (nearly as good as the forecast for December, for example). However, the relative price changes taking place between the 10th August and September/October are themselves quite small (considerably smaller than those between 10th of August and December, for example). For September and October, the proportion of the variance of the relative price changes that is 'explained' by the forecasts is quite modest,

(i) August (10th) to September (average) : 52.1%
(ii) August (10th) to October (average) : 61.4%,

whereas, for December, that proportion is about 92.4% (the estimates were derived over the period 1967/68 to 1974/75).

Other forecasts, made at different times and concerning other 'futures' contracts, show a similar inability to 'explain' accurately the price changes taking place shortly after the forecasts are made. Therefore, their usefulness is restricted, in practice, to the prediction of medium-term price changes. In contrast, the forecasts derived from the time series models are of use only for predicting daily price changes occurring in the short-term. It is the complementary nature of the two types of forecasts that, in general, determines the need to
partition the buying decision process into two stages. In fact, once
the size and the date of delivery of each shipment is defined and
for a given potential (or strategic) buying period, the buying decisions
will generally involve:

(i) determining the active buying period (using medium-term
forecasts), and

(ii) developing a tactical buying policy for the day-to-day operations,
to arrive at the most appropriate decisions within the active
buying period (using short-term forecasts).

Clearly, if the purchaser is restricted by the senior management
to buy within a short period of time (i.e. if the potential buying period
is itself short, say one month), the first stage becomes meaningless
and the purchaser is left only with the short-term 'tactical' buying
problem. The next section will focus on this problem. The discussion
of the general two-stage buying policy will be considered later, in
section 6.

5. 'Tactical' Buying of Maize 'Futures'

5.1 The Problem Considered

The analysis of the buying decisions is initiated in this section,
by considering the following 'tactical' buying situation:

(i) for any given feed grain shipment of size \( Q \), the purchaser
will buy an equivalent number \( n_Q \) of maize 'futures' contracts
at the Chicago exchange (the month of delivery of the
'futures' contracts to be bought for any one shipment is
related to the shipment's purchasing deadline, as defined
earlier in section 2.5, Figure 66);
(ii) for each shipment, the 'futures' contracts will be bought over an 'active buying period' with $N$ trading days and finishing on the purchasing deadline (here, in section 5, the length of the period will be specified arbitrarily; it will be parametrized between half a month and 3 months or, equivalently, between 11 and 63 trading days);

(iii) on each trading day the purchaser can buy any number of 'futures' contracts at the price quoted at the exchange, subject to the constraint that the number of contracts acquired by the end of the buying period must be $n_Q$.

Essentially, this problem is identical to one of the 'tactical' purchasing problems considered by Kingsman: the purchasing of a specified amount of a commodity before a given deadline, with a number of opportunities in which purchases can be made.

The development and testing of 'tactical' buying policies poses some methodological problems. These will be discussed below in section 5.2, preceding the analysis of a policy proposed by Kingsman and of a new policy developed in the course of this research.

5.2 Methodological Considerations

5.2.1 Selection of a Measure of Purchasing Performance

The first problem arising in the development and testing of purchasing policies concerns the definition of a measure of purchasing performance. The measures proposed in previous work by Kingsman and Taylor present a common feature: for any given policy, they
rely on a comparison between

(i) the purchasing costs incurred under the policy in
question, and

(ii) those incurred under what can be described as a 'benchmark policy': involving the regular buying of the
commodity and leading to unit purchasing costs that will
tend, in the long run, to the average market prices.

As implied earlier in section 2.5, the policy used in this research as the
term of comparison is the 'hand-to-mouth' policy. In the context of
the 'tactical' buying problem under analysis (buying \( n_Q \) 'futures'
contracts within a period with \( N \) opportunities) such a policy was
shown to be equivalent to buying all \( n_Q \) 'futures' contracts on the
last opportunity. For any individual shipment, the savings in the unit
purchasing costs resulting from applying a buying policy distinct from the
'hand-to-mouth' one, are given by

\[
\text{unit savings} = \Delta f - \text{chf} \quad \text{(see section 2.5)}
\]

where

\( \Delta f \) : difference between the 'futures' price quoted in the
last buying opportunity and the average 'futures'
price paid over the active buying period;

\( \text{chf} \) : average unit cost of holding 'futures' until \( t_2 \).

Naturally the term \( \text{chf} \) will depend on the particular buying policy
adopted: the earlier the 'futures' contracts are bought, the higher this
cost will be. Generally, its value will be very small. For example,
for an active buying period of one month, if it is assumed that

(i) on average, the futures are held for half of the buying
period,
(ii) the broker requires a deposit of 10% of the price of the contracts bought by the importer, and

(iii) the rate of interest on working capital is 15%,

the term chf would be

\[
\text{chf} = \frac{(10\% \cdot \text{'futures' price paid})}{2} \cdot \frac{15\%}{12}
\]

\[
= 0.06\%. \text{'futures' price paid}
\]

Usually the deposit required by the broker is less than 10% and it can vary significantly from case to case, depending on existing agreements between broker and client.

In view of the small magnitude of chf and of its dependence on each individual contract between broker and client, the term chf was omitted from the expression of the unit savings, in the derivation of 'futures' buying policies. Later in section 5.5 it will become clear that the cost of holding 'futures' is, in fact, negligible.

Ignoring the term chf, the unit savings for each shipment become

\[
\text{unit savings} = \Delta f
\]

\[
= \text{'futures' price quoted on the last buying opportunity - average 'futures' purchasing cost.}
\]

At this point it is useful to introduce the following notation:

k : index denoting a shipment and the corresponding buying period (k = 1, \ldots )

n : index denoting the \( n^{th} \) day of any buying period (n = 1, \ldots , N)

\((z_n)_k\) : 'futures' price quoted on the \( n^{th} \) day of buying period \( k \)

\(\bar{z}_k\) : \(\frac{1}{N} \sum_{n=1}^{N} (z_n)_k\), i.e., average 'futures' price quoted over the buying period \( k \).
For each shipment, the purchasing deadline is defined using the delivery-inventory model. This date is chosen irrespectively of the way in which 'futures' prices move or are expected to move. Similarly, the length of the buying period is defined without regard for price movements (as stated earlier, in this section the number of buying opportunities is specified arbitrarily). Under such conditions, since the 'futures' prices do not drift systematically either upwards or downwards\(^(*)\), in the long run (i.e. for a large number of shipments \(K\)) the following identity holds asymptotically:

\[
\frac{1}{K} \sum_{k=1}^{K} (z_{n_k}) = \frac{1}{K} \sum_{k=1}^{K} (z_{n_1})_k, \quad \text{for any } n_1 \leq N
\]

\[
= \frac{1}{K} \sum_{k=1}^{K} z_k^*
\]

This implies that, over the long run, the unit savings defined by the expression presented above will be identical to those defined by

\[\text{unit savings} = \text{average 'futures' price over the buying period} - \text{average 'futures' purchasing cost}\]

(where the first term replaces the 'futures' price quoted on the last buying opportunity).

If the first measure of unit savings were to be adopted, this would be equivalent to comparing directly the particular buying policy

\(^(*)\) As stated earlier in section 3.3.1, when presenting Taylor's price-trend model, the mean of the daily price-trends is, in practice, indistinguishable from zero; inflationary factors could cause prices to drift slightly upwards; however, within short buying periods, the effect of such drift is negligible.
under consideration with the bench-mark policy (equivalent to buying all 'futures' on the last buying opportunity). The use of this measure would, however, give rise to a difficulty. When the average 'futures' purchasing cost of any one policy is compared with the 'futures' price on the last day, the erratic fluctuation of this price - which may, on occasion, depart significantly from the pattern of previous prices - will cause an undesirable variability (from period to period) in the unit savings measure. Clearly, the higher this variability, the more difficult it will be to establish the significance of the unit savings associated with the policy in question (i.e. to establish, over a number of buying periods, whether the observed savings reflect an improvement in the buying decisions or whether they can be the result of pure chance).

The second measure of unit savings - equivalent to the first in terms of expected value - has the advantage over it of being less variable; hence, it will allow the significance of unit savings to be established on the basis of fewer data. For this reason this measure was adopted and henceforth the term 'unit savings' will be reserved for it. Basically, the adoption of this measure means that the policy that is chosen as a direct term of comparison is one of spreading the purchases uniformly over the buying period. The comparison with the bench-mark policy defined above (buying all 'futures' in the last day of the active buying period) is made indirectly since, in the long-run, both these policies lead to the same average purchasing costs.

As noted by Kingsman\textsuperscript{38}, a measure such as the 'unit savings' alone is inadequate to define the performance of a buying policy, because greater savings can be made in buying periods with larger price fluctuations.
Small benefits may be the result of small price fluctuations rather than poor purchasing policies. Kingsman\textsuperscript{38} proposed the 'purchasing efficiency' as a measure of performance and defined it as follows:

\[
purchasing \text{ efficiency} = \frac{AP - AC}{AP - HC} \times 100\% = \frac{S}{PS} \times 100\%
\]

where,

- \( AP \) : average 'futures' price over the buying period
- \( AC \) : average 'futures' purchasing cost
- \( HC \) : unit 'futures' hindsight cost (defined as the price at which the buyer could have bought the 'futures' if he had known how prices would move; clearly, in the problems under analysis, \( HC \) is the minimum 'futures' price within the buying period)
- \( S \) : unit savings
- \( PS \) : potential savings

A 0\% value of the purchasing efficiency implies that the buyer has bought the 'futures' at their average price over the buying period (i.e. zero unit savings) and a 100\% value represents the goal of buying at the lowest possible price (i.e. unit savings equal to the potential savings). A negative efficiency would mean buying the 'futures' at an average cost greater than the average price over the buying period.

For the purpose of testing the performance of 'tactical' buying policies, the purchasing efficiency measure presents two basic difficulties. The first one, recognized by Taylor\textsuperscript{60}, is concerned with its asymmetry. In periods when prices are falling, there are invariably some large negative efficiencies, often less than -400\%, whilst the maximum efficiency is by definition +100\%. In the long run, the
occurrence of large negative efficiencies (less than \(-100\%\)) will introduce a negative bias in the average efficiency: a policy that, in the long run, leads to average purchasing costs identical to the average market prices could have a negative expected purchasing efficiency.

The second difficulty – arising in the context of the particular problem under analysis – concerns the definition of the potential savings. The hindsight cost is the minimum 'futures' price within each buying period. The fact that this is an extreme price value, quoted on a single day of the buying period, will cause an undesirable variability of the purchasing efficiency from period to period.

Having recognized the first of these limitations of the purchasing efficiency, Taylor\(^{60}\) proposed a new measure of performance, which will be called here, the 'rescaled absolute savings' (RAS). For each particular buying period RAS is defined as:

\[
RAS = \frac{S}{APV}
\]

where

- \(S\) : unit savings
- \(APV\) : 'absolute price volatility', defined as the average value of the absolute price changes during the buying period

If a given buying policy were applied consistently over consecutive buying periods, RAS would be a suitable statistic to test formally the long-term performance of the policy if its distribution were invariant with respect to time. This would be the case if the following assumptions held:

1. prices follow a stochastic process in which all parameters except the price volatility are invariant with time;
(ii) moments of order n of the distribution of S are proportional to \((APV)^n\) (in particular, the mean and the standard deviation are both proportional to APV).

The first of the assumptions, central in Taylor's price-trend model, was tested by that author for different commodities. The second assumption is no more than a (largely untested) conjecture. Even if it were untrue, this conjecture could be accepted as an approximation of reality. However, there is a danger involved in doing so: since the absolute price volatility changes markedly over time, any departure from the assumed proportionality could cause wide variations in the distribution of RAS (which, in turn, would imply difficulties in the formal testing of the long-term performance of buying policies). With the objective of reducing this risk, a new measure of performance was adopted - the 'rescaled relative savings' (RRS):

\[
RRS = \frac{RS}{RPV}
\]

where

- **RS**: relative savings \((RS = \text{S/av.'fut'. price over the buy.period})\)
- **RPV**: 'relative price volatility', defined as the average value of \(s_t\) during the buying period \((s_t\) was defined earlier in section 3.3.2 as an estimate, at time \(t\), of the mean absolute deviation of the random variable generating the daily return \(x_t\)).

If the logarithmic transformation of daily prices were sufficient to completely homogenize the variance of the daily returns \(\{x_t\}\), then the distribution of RPV would be invariant over time and the relative savings could be appropriately used as a measure of purchasing performance.
Although this is not the case, the variability of RPV over time is nevertheless smaller than that of APV. For this reason, RRS is likely to be more stable than ARS over time and, therefore, potentially more appropriate to test formally the long-term performance of buying policies.

5.2.2 Procedure Adopted in the Calibration and Testing of 'Tactical' Buying Policies

The definition of 'tactical' buying policies invariably depends on the specification of certain parameters whose values are chosen empirically. These are either parameters of the model used to derive price forecasts or parameters of the purchasing rule adopted. The selection of their values (or, equivalently, the calibration of the policies) involves

(i) simulating the policy using real price data, setting the parameters at different values, and

(ii) choosing those that lead to the best performance.

The dataset used in this research in the calibration and testing of 'tactical' buying policies consisted of the daily closing prices of all maize 'futures' contracts quoted between the first trading day of 1963 and the last trading day of 1976 (i.e. the same dataset used earlier to test and estimate the parameters of Taylor's price-trend model).

The dataset was divided into two subsets: the first one (1963-1972) was used to derive the best values of the parameters of the forecasting-buying models and the second one (1973-1976) was used to test the significance of the savings associated with the buying policies analysed. The basic criteria underlying this partition of the dataset were as follows:
(i) The dimension of the first subset (10 years of daily prices) is about the minimum required to derive meaningful estimates of Taylor's price-trend model parameters.

(ii) The periods covered by each subset are markedly different as far as the volatility of prices is concerned. As shown in Table 51 for the December 'futures' contracts, maize prices were much more stable during 1963-72 than during the following years (the transition between one subperiod and the other was pictured earlier in this chapter, in Figure 69, section 4.2). By calibrating the buying policies in the first subperiod and testing them on the second one, the robustness of the policies with respect to changes in the price volatility is subject to an extremely hard test.

Table 51 - Variance of price changes for the December 'futures' contracts (1963-76)

| Year | No. of price changes | Variance $\left(\sigma^2\right)$ | [(cents/bushel)$^2$] |
|------|----------------------|--------------------------------|--|---|
| 1963 | 210                  | 0.93                           |               |
| 1964 | 210                  | 0.29                           |               |
| 1965 | 210                  | 0.25                           |               |
| 1966 | 210                  | 1.64                           |               |
| 1967 | 210                  | 1.12                           |               |
| 1968 | 209                  | 0.57                           |               |
| 1969 | 208                  | 0.81                           |               |
| 1970 | 208                  | 2.60                           |               |
| 1971 | 252                  | 4.06                           |               |
| 1972 | 257                  | 1.20                           |               |
| 1973 | 248                  | 27.52                          |               |
| 1974 | 246                  | 37.08                          |               |
| 1975 | 249                  | 22.46                          |               |
| 1976 | 252                  | 12.68                          |               |

Max($\sigma^2$) / min($\sigma^2$) = 148.32
Within each of the two subperiods, each price series \( \{ z_t \} \) concerning any one 'futures' contract (e.g. December 1970) was divided into consecutive 'buying periods' of \( N \) trading days (the last period was made to coincide with the last \( N \) days of the series; \( N \) was set at different values between 11 and 63 trading days). These 'buying periods' formed the samples used in the calibration and testing of the buying policies analysed in this research. Before discussing the procedure adopted it is useful to introduce the following notation:

\[
\begin{align*}
    i & : \text{index denoting the delivery month of the 'futures' contracts (} i = 1, \ldots, 5 \text{ for March, \ldots, December, respectively);} \\
    j & : \text{index denoting the year in which each particular contract reaches maturity (considering for example the calibration subperiod 1963-72, } i = 5 \text{ and } j = 1, \ldots, 10 \text{ will denote the contracts December 1963, \ldots, December 1972, respectively);} \\
    k & : \text{index denoting the } k^{th} \text{ 'buying period' within any price series } \{ z_{t} \}_{ij} \\
\end{align*}
\]

Within the calibration subperiod (characterized by stable prices), the logarithmic transformation proved to be sufficient to solve the problem of variance homogeneity. The structure and the result of Bartlett's test \(^7\) for homogeneity of the variance of the daily returns \( \{ x_t \} = \{ \log (z_t) - \log (z_{t-1}) \} \) within that period is shown in Table 52, for the December 'futures'.

As noted in the previous section, under price returns with homogeneous variance, there is no point in rescaling the relative savings (RS), since the relative price volatility (RPV) is invariant over time. For this reason all the buying policies considered in this
research were calibrated initially by setting their parameters at the values that maximized the sample mean of the relative savings

$$\bar{RS} = \frac{\sum_i \sum_j \sum_k (RS)_{ijk}}{\text{total number of buying periods in the calibration sample}}$$

(the exercise was repeated later using the rescaled relative savings, with no significant differences in the results).

Once the policies were calibrated they were formally tested over the second sub-period. For each set of contracts with the same month of delivery (e.g. December 1973, ..., December 1976) a t-statistic based on the sample distribution of the rescaled relative savings was defined as:
\[ t_i = \frac{\overline{RRS}_i}{\hat{\sigma}(RRS_i)/\sqrt{n_i}} \]

where \( \overline{RRS}_i \) and \( \hat{\sigma}(RRS_i) \) are the sample mean and the standard error of the rescaled relative savings for buying periods concerning the contracts \( i \), and \( n_i \) is the sample size.

The price series concerning contracts with different delivery months are highly correlated. Therefore the rescaled relative savings concerning such contracts can only be expected to be correlated as well. For this reason they cannot be pooled together to derive a single \( t \) statistic, to test the significance of the rescaled relative savings over all contracts. A conservative test can however be produced, using the statistic

\[ \bar{t} = \frac{1}{5} \sum_{i} t_i \]

In fact, under the null hypothesis

\[ H_0 : \text{expected } RRS_i = 0 \text{ (all } i) \]

the variates \( t_i \) are essentially standardized normal and \( \sum t_i \) follows a normal distribution with zero mean, and variance not exceeding 25 (this would be the case if all the \( t_i \)'s were perfectly correlated). Hence, under the null hypothesis, the variate \( \bar{t} \) follows a normal distribution with zero mean and standard deviation less than or equal to one. If \( H_0 \) can be rejected assuming that \( \bar{t} \) is a standardized normal variate, then it can be rejected unconditionally.

5.3 Computations

A computer programme was written in FORTRAN IV to calibrate and test each of the different 'tactical' buying policies considered in
this research. The programme was run on a CDC 7600. For any of the policies considered, in either the calibration or testing phases, the computer memory requirement and computation time did not exceed 75K words and 8 CPU seconds, respectively.

5.4 Kingsman's Buying Policy

Kingsman\(^{37}\) proposed a dynamic programming (DP) model to derive policies for a number of different 'tactical' buying problems, including the one considered in this research (where 'futures' contracts can be purchased in a number \(N\) of buying opportunities, with a final deadline by which a specified number of contracts must be bought). Kingsman's buying policy will be described for this problem, using the following notation:

\[ M \quad : \quad \text{number of days remaining up to the end of the buying period, at the current buying opportunity;} \]
\[ m \quad : \quad \text{index denoting successive buying opportunities between the current one (} m = M \text{) and the end of the buying period (} m = 0 \text{);} \]
\[ z_M \quad : \quad \text{current price offer;} \]
\[ \{\phi_m(z)\} \quad : \quad \text{probability density functions of prices on the remaining buying opportunities (} m = M-1, M-2, \ldots, 0 \text{).} \]

Under the assumption of independent price distributions \(\{\phi_m(z)\}\), the best buying policy is defined as

(i) wait, if \(z_M \geq P_M\), or

(ii) buy the total number of 'futures contracts, if \(z_M < P_M\), where \(P_M\) is the lowest expected price at which the purchase can be made at the remaining \(M\) opportunities. \(P_M\) is defined by the recurrence relations
\[
\begin{align*}
P_1 &= \int_0^\infty z \phi_o(z) \, dz \\
P_m &= \int_0^{P_{m-1}} \phi_{m-1}(z) \, dz + P_{m-1} \int_{P_{m-1}}^{\infty} \phi_{m-1}(z) \, dz, \quad (m = 2, \ldots, M)
\end{align*}
\]

The major theoretical objection to this policy concerns the underlying assumption of independence of the price distributions \(\{\phi_m(z)\}\). As pointed out by Taylor, in reality, these distributions are not independent. Nevertheless, when applied in previous studies (Kingsman), the policy led to profitable decisions. In view of these results and in the absence of any other alternative applicable to the 'tactical' buying problem under analysis, Kingsman's policy was the first to be investigated.

Initially the policy was applied using Trigg and Leach's adaptive exponential smoothing to derive the price distributions \(\{\phi_m(z)\}\). These were treated as normal and, accordingly, the parameter controlling the variance of the distributions, \(\lambda\), was set equal to 1.25.

The only parameter of the forecasting-purchasing model requiring calibration was the smoothing constant, \(\gamma\), of the adaptive exponential smoothing (see section 3.2.3). Table 53 summarizes the results of the calibration, for buying periods with 21 trading days. Two points must be observed about these results. Firstly, they confirm that, due to the homogeneity of variance of the daily returns during the calibration period, it is practically equivalent to maximize \(\bar{RS}\) or \(\bar{RRS}\) (point raised earlier in section 5.2.2). Secondly, the results suggest that the standard deviation of the distributions of \(RS\) and \(RRS\) is not significantly affected by the value chosen for \(\gamma\). Therefore the increase in 'yield' (measured either by \(\bar{RS}\) or by \(\bar{RRS}\)) is not achieved at the expense of a decrease in the 'reliability' of the purchasing performance (measured
Table 53 - Kingsman's buying policy: calibration of the parameter $\gamma$ for one-month buying periods (21 trading days)

(1) Effect of $\gamma$ on the relative savings (RRS)

<table>
<thead>
<tr>
<th>'Futures' contracts</th>
<th>Sample size (no. of buying per$^2$)</th>
<th>Av. relative savings</th>
<th>S.E. of relative savings</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\bar{\gamma}_1$ (%)</td>
<td>$\hat{\gamma}_1$ (%)</td>
<td>$t_1 = \frac{\bar{\gamma}_1}{\hat{\gamma}_1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\gamma=0.05$</td>
<td>$\gamma=0.10$</td>
<td>$\gamma=0.15$</td>
</tr>
<tr>
<td>1 (Mar)</td>
<td>91</td>
<td>0.29</td>
<td>0.41</td>
<td>0.54</td>
</tr>
<tr>
<td>2 (May)</td>
<td>88</td>
<td>0.21</td>
<td>0.42</td>
<td>0.45</td>
</tr>
<tr>
<td>3 (Jul)</td>
<td>87</td>
<td>0.21</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>4 (Sep)</td>
<td>88</td>
<td>0.16</td>
<td>0.15</td>
<td>0.22</td>
</tr>
<tr>
<td>5 (Dec)</td>
<td>92</td>
<td>0.24</td>
<td>0.43</td>
<td>0.51</td>
</tr>
</tbody>
</table>

$\bar{\gamma}_5 = 0.21$ 0.31 0.36 0.20  $\bar{t} = 1.28 1.78 1.90 1.24$

(II) Effect of $\gamma$ on the rescaled relative savings (RRS)

<table>
<thead>
<tr>
<th>'Futures' contracts</th>
<th>Sample size</th>
<th>Av. rescaled relative savings</th>
<th>S.E. of rescaled relative savings</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_1$</td>
<td>$\bar{\gamma}_5$ (%)</td>
<td>$\hat{\gamma}_5$ (%)</td>
<td>$t_1 = \frac{\bar{\gamma}_5}{\hat{\gamma}_5}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\gamma=0.05$</td>
<td>$\gamma=0.10$</td>
<td>$\gamma=0.15$</td>
</tr>
<tr>
<td>1</td>
<td>91</td>
<td>0.48</td>
<td>0.60</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>88</td>
<td>0.31</td>
<td>0.58</td>
<td>0.68</td>
</tr>
<tr>
<td>3</td>
<td>87</td>
<td>0.39</td>
<td>0.28</td>
<td>0.33</td>
</tr>
<tr>
<td>4</td>
<td>88</td>
<td>0.29</td>
<td>0.13</td>
<td>0.36</td>
</tr>
<tr>
<td>5</td>
<td>92</td>
<td>0.29</td>
<td>0.64</td>
<td>0.72</td>
</tr>
</tbody>
</table>

$\bar{\gamma}_5 = 0.35$ 0.45 0.60 0.35  $\bar{t} = 1.14 1.70 2.13 1.25$
either by \( \hat{\sigma} (RS) \) or by \( \hat{\sigma} (RRS) \). The value of \( \gamma \) that maximizes \( RS \) and \( RRS \) also maximizes the corresponding \( t' \) and \( t \) statistics.

The calibration exercise was repeated for buying periods of different lengths. As shown in Table 54, the largest relative savings are obtained for lengths of 21 and 42 days, with \( \gamma = 0.15 \) in both cases.

<table>
<thead>
<tr>
<th>Length of</th>
<th>Best value</th>
<th>((RS)_{\text{Max}})</th>
<th>(t)-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>buying</td>
<td>of the</td>
<td></td>
<td></td>
</tr>
<tr>
<td>period</td>
<td>parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N)</td>
<td>(\gamma)</td>
<td>(%)</td>
<td>(%)</td>
</tr>
<tr>
<td>[trad. days]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.10</td>
<td>0.08</td>
<td>0.78</td>
</tr>
<tr>
<td>21</td>
<td>0.15</td>
<td>0.36</td>
<td>1.98</td>
</tr>
<tr>
<td>42</td>
<td>0.15</td>
<td>0.49</td>
<td>1.38</td>
</tr>
<tr>
<td>63</td>
<td>0.15</td>
<td>0.11</td>
<td>0.29</td>
</tr>
</tbody>
</table>

The small magnitude of the relative savings was to be expected, given the small volatility of prices during the testing period. However, as shown in Table 55, the performance of Kingsman's policy during this period (with \( \gamma = 0.15 \)) was disappointing.

| Length of | Mean relative | Mean rescaled | \(t\)-statistic |
| buying   | savings       | relative      |                 |
| period   | \(\%\)        | savings       |                 |
| \(N\)    | \(RS\)        | \(RRS\)       | \(t(=0.05) = 1.65\) |
| [trad. days] |          |            |                 |
| 21       | -0.066       | -0.125      | -0.33 (not sig.) |
| 42       | 0.112        | 0.188       | 0.35 (not sig.) |

The failure of Kingsman's policy under highly volatile prices, could be attributed to two basic reasons:

(i) the failure of adaptive exponential smoothing, as a price forecasting model, and/or

(ii) the inappropriateness of the assumption of independence of the price distributions \(\phi_m(z)\)
An attempt was made to use Kingsman's DP model with price forecasts derived from Taylor's price-trend model. This attempt was unsuccessful, showing an obvious incompatibility between the two models. According to Taylor's forecasting model,

(i) prices are projected into the 'future' from the current price (in other words, the current price level \( a_t \) coincides necessarily with the current price \( z_t \));

(ii) the price trends are extremely small (when compared with the standard deviations of the price forecast distributions \( \{\phi_m(z)\} \)).

As a result, the policy derived from the DP model under the assumption of independent \( \{\phi_m(z)\} \) leads invariably to one of two decisions: either to make the purchase on the last but one buying opportunity (if the trend is positive by then) or to make the purchase on the last opportunity (otherwise).

5.5 New Buying Policy

5.5.1 Underlying Principles and Definition

The buying policy developed in this research makes use of an idea first introduced by Taylor\(^{60}\), when deriving a policy for the 'tactical' buying of commodities in 'cash' markets. Essentially, the idea consists of basing purchasing decisions upon the degree of confidence in the positiveness of the current price-trend.

In the scenario considered by Taylor it is assumed that a purchaser buys a commodity for some production process which requires the same amount of the commodity every day (the exact amount used is unimportant and will be assumed to be one unit). On each day the
purchaser can buy the commodity at the 'cash' price quoted on an exchange, for delivery before the next day's production. The buyer is required to make the purchases subject to the constraint that there is always at least one unit in stock at the beginning of each day's production but not more than \( M \) units in stock.

The policy proposed by Taylor for this situation uses the concept of a 'target stock' \( (TS) \). The policy specifies, on each day \( t \), the value of the target stock at the beginning of day \( t+1 \) \( (TS_{t+1}^*) \). If the inventory at the beginning of day \( t+1 \) will equal or exceed \( TS_{t+1} \) without a further purchase, then no purchase is made. Otherwise the stock is raised up to \( TS_{t+1}^* \).

Using the following notation
\[
\hat{\mu}_t : \quad \text{estimate of the price-trend on day } t \text{ (derived using Taylor's model, as defined earlier in section 3.3.4)}
\]
\[
\hat{\sigma}_t : \quad \text{estimate of the standard deviation of } \hat{\mu}_t \text{ (see section 3.3.4)}
\]

the target stock \( TS \) is defined to be proportional to the estimated size of the current trend measured in standard deviation units. Denoting the constant of proportionality by \( k \) and considering that purchases can only be made in multiples of one demand unit, the target stock for the beginning of day \( t+1 \), \( TS_{t+1}^* \) is defined by

\[
\begin{align*}
TS_{t+1}^* &= k \cdot \frac{\hat{\mu}_t}{\hat{\sigma}_t} \\
TS_{t+1} &= \begin{cases} 
1 & , \text{if } TS_{t+1}^* < 1 \\
\text{the integer closest to } TS_{t+1}^* & , \text{if } 1 \leq TS_{t+1}^* < M \\
M & , \text{if } M < TS_{t+1}^* 
\end{cases}
\end{align*}
\]
To apply the rule, the value of $k$ must be specified and estimates of the price-trend model must be given. The value of $k$ is derived by experimentation and will generally depend on the value of the 'maximum stock cover', $M$.

The problem dealt with in this research differs fundamentally from that considered by Taylor. The type of policy required differs equally from that just presented. Nevertheless, as implied earlier, it draws on Taylor's idea of basing purchasing decisions upon the magnitude of $\frac{\hat{v}_t}{\hat{u}_t}$ (the more positive this ratio is, the higher will be the degree of confidence that the trend is actually positive).

In the 'futures' markets, the purchaser can only buy integer numbers of 'futures' contracts. However, in the proposed policy, the amount purchased at any buying opportunity was treated as a continuous variable. In the context of the problem under analysis, such an approximation is valid, since the size of a contract - about 127 m.t. - is much smaller than the size of the shipments that are to be bought - larger than 15,000 m.t..

The principles underlying the new policy are of extreme simplicity and can be illustrated considering initially the situation facing the decision maker on the first day of the buying period (i.e. with $N-1$ days left up to the end of the buying period). If $Q$ is used to denote the shipment size, the rule states that the amount that should be purchased on the opening day ($B_{N-1}$) is given by
\[
\begin{align*}
B_{N-1}^* &= k \left( \tilde{\nu}_{N-1}/\tilde{\nu}_{N-1} \right) \cdot Q \\
B_{N-1} &= \begin{cases} 
0 & \text{, if } B_{N-1}^* < 0 \\
B_{N-1}^* & \text{, if } 0 \leq B_{N-1}^* \leq Q \\
Q & \text{, if } Q < B_{N-1}^*
\end{cases}
\end{align*}
\]

Within the range \((0, Q)\), the amount to be bought when there are \(N-1\) days left is defined to be proportional to \(\tilde{\nu}_{N-1}/\tilde{\nu}_{N-1}\) and \(Q\). The inclusion of the term \(Q\) in the expression of \(B_{N-1}^*\) is necessary, if the rule is to lead to unit savings which are independent of the shipment size. The constant \(k\) (the proportion of \(Q\) that will be bought when \(\tilde{\nu}_{N-1}/\tilde{\nu}_{N-1} = 1\)) is derived by experimentation.

Having introduced the decision rule for the first day of the buying period, the next step is to question whether the expression presented above for \(B_{N-1}^*\) can be generalized to the other days of the buying period. The implications of setting

\[
B_{N-1}^* = B_{N-2}^* = \ldots = B_n^* = \ldots = k \cdot (\tilde{\nu}_n/\tilde{\nu}_n) \cdot Q
\]

can be assessed by considering firstly the following hypothetical situations arising in the purchasing of two shipments of identical size \(Q\):

(i) shipment 1 : \(\tilde{\nu}_n/\tilde{\nu}_n\) takes the first positive value (say, \(a\)) when there are \(n_1\) days left up to the end of the buying period;

(ii) shipment 2 : as for shipment 1 the first positive value of \(\tilde{\nu}_n/\tilde{\nu}_n\) is \(a\), but occurs when there are \(n_2 < n_1\) days left to buy the amount \(Q\).
If \( B^*_n \) = \( B^*_n \), then the buying decisions would be identical in both situations. The purchasing rule would ignore one important difference between them: in the second case there are fewer days left to buy the same amount \( Q \). In order to take this difference into account, the expression of \( B^*_n \) must be corrected by replacing the constant \( k \) by a function of the number of days left up to the end of the buying period:

\[
B^*_n = f(n) \cdot \left( \frac{\hat{u}_n}{\hat{v}_n} \right) \cdot Q
\]

The decision rule implied by this expression is satisfactory when referred to the first purchase made within any buying period. However, difficulties would ensue from applying it to decisions following purchases made in previous buying opportunities. These difficulties can be illustrated considering again two hypothetical situations:

(i) shipment 1: size \( Q_1 = Q + \Delta Q \), \( n \) days left, \( \Delta Q \) already bought in previous buying opportunities, \( \frac{\hat{u}_n}{\hat{v}_n} = a \);  
(ii) shipment 2: size \( Q_2 = Q \), \( n \) days left, no purchases made in previous buying opportunities, \( \frac{\hat{u}_n}{\hat{v}_n} = a \).

If the above expression of \( B^*_n \) were used to derive a buying decision in each case, the decisions reached would be different,

\[
(B^*_n)_1 = f(n) \cdot a \cdot (Q + \Delta Q) > (B^*_n)_2 = f(n) \cdot 2Q,
\]
when, in fact, there is no difference between the two decisions problems. This inconsistency of the decision rule can be eliminated by replacing the shipment size by the amount still left to be bought, in the expression of \( B^*_n \). In its final version, the proposed policy specifies that, with \( n \) days left up to the end of the decision period, the amount that should be bought \( (B_n) \) is
for \( n \geq 1 \):

\[
\begin{align*}
B_n^* &= f(n) \cdot (\hat{\Sigma}_n / \tilde{\Sigma}_n) \cdot q_n \\
B_n &= \begin{cases} 
0, & \text{if } B_n^* < 0 \\
B_n^*, & \text{if } 0 \leq B_n^* \leq q_n \\
q_n, & \text{if } q_n < B_n^*
\end{cases}
\end{align*}
\]

for \( n = 0 \):

\( B_0 = q_0 \)

where \( q_n \) is the amount left to be bought when there are \( n \) days to go up to the end of the buying period.

The function \( f(n) \), to be derived by experimentation, can only be expected to be monotonic decreasing. In fact, the further away from the deadline a decision is taken, the higher is the probability of a change in the sign of the trend (recognized as positive, when the decision is taken). For this reason, other factors being equal, one can expect the size of the purchases to be comparatively small when \( n \) is large and vice-versa. In theory, \( f(1) \) must be infinite: when \( \hat{\Sigma}_1 / \tilde{\Sigma}_1 \) is positive, however small, any amount left to be bought \( (q_1) \) should be purchased.

5.5.2 **Calibration and Testing**

As with Kingsman's buying policy, the period 1963-72 was used to estimate or calibrate the parameters upon which the new policy is based. The results are summarized in Figure 78.

As shown in Figure 78(i), the significance of the overall \( \hat{U}^* \) statistic and, hence, of price trends could be established from the calibration dataset alone. The estimates of Taylor's model parameters
(i) Estimation of the parameters of Taylor's price-trend model

(a) Significance of the $U^*$ statistics:

<table>
<thead>
<tr>
<th>Contract</th>
<th>$U^<em>$ statistic ($U^</em> (\alpha = 0.05) = 1.65$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mar</td>
<td>2.94 (sig.)</td>
</tr>
<tr>
<td>May</td>
<td>2.02 (sig.)</td>
</tr>
<tr>
<td>Jul</td>
<td>1.47 (not sig.)</td>
</tr>
<tr>
<td>Sep</td>
<td>2.33 (sig.)</td>
</tr>
<tr>
<td>Dec</td>
<td>2.06 (sig.)</td>
</tr>
<tr>
<td>Overall ($\bar{U}^*$)</td>
<td>2.16 (sig.)</td>
</tr>
</tbody>
</table>

(b) Parameter estimates:

$p_0' = 0.947$  \quad (m_0' = 19 \text{ days})

$A_0' = 1.77\%$

(ii) Calibration of the function $f(n)$ (maximization of RS during the calibration period)

(iii) Summary of statistics concerning the buying performance over the calibration period

<table>
<thead>
<tr>
<th>Length of buying period</th>
<th>Relative savings</th>
<th>Rescaled relative savings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.E.</td>
</tr>
<tr>
<td>N [trad., days] [%]</td>
<td>$\overline{RS}$</td>
<td>$\hat{G}(RS)$</td>
</tr>
<tr>
<td>21</td>
<td>0.42</td>
<td>1.48</td>
</tr>
<tr>
<td>42</td>
<td>0.48</td>
<td>1.81</td>
</tr>
</tbody>
</table>
are also shown. The differences between them and the estimates derived from the entire 1963-76 date set \( p_o = 0.962, \) or \( m_o = 26 \) days, and \( A_o = 1.49\% \) are clearly insufficient to suggest any significant changes in the values of the parameters after 1972. More likely, the differences are due to estimation errors.

The only value of the function \( f(n) \) which can be derived from theoretical considerations is \( f(1) \): as mentioned in the previous section, its value must be infinite. All the other values of \( f(n) \) can only be derived by experimentation. Clearly, given the sequential nature of the 'tactical' buying problem, any 'optimal' value \( [f(n_o)]_{op} \) can be derived only after all the 'optimal' values of \( f(n_o -1), f(n_o -2), \ldots, f(1) \) have been specified. The calibration must therefore be conducted in the following sequence:

(i) start with buying periods of 3 days (in the first day \( n = 2 \), i.e. there are 2 days left up to the end of the buying period); find the value \( [f(2)]_{op} \) that maximizes the relative savings when \( f(1) \) is set equal to its 'optimal' value \( [f(1)]_{op} = \infty \);

(ii) increase the length of the buying period one day at a time; at each stage find the value \( [f(n)]_{op} \) that maximizes the relative savings, when \( f(n-1), \ldots, f(n) \) are set at their 'optimal' values (derived in previous stages).

This procedure, although theoretically appropriate, is rather cumbersome and implies a considerable computational effort. In practice, it can be considerably simplified by replacing the 'optimal' function \( [f(n)]_{op} \) by a piecewise linear approximation. This was the procedure adopted in the calibration of \( f(n) \). As shown in Figure 58(ii), in the
range \([n = 2, n = 42]\), the function was approximated by three linear segments. Rather than 'optimizing' all the values of \(f(n)\), only four values were 'optimized' \((n = 2, 10, 20, 41)\) and all the others were interpolated linearly. The approximation was found to be acceptably, given the low sensitivity of the relative savings to changes in \(f(n)\) around its 'optimal' values. These were found by calculating the mean relative savings \(\overline{RS}\) for a grid of points

\[
\{ f(2), f(10) \leq f(2), f(20) \leq f(10), f(41) \leq f(20) \}.
\]

The size of the grid, originally set at 0.1, was progressively reduced, as the 'location' of the 'optimal' region became apparent.

Figure 78 (iii) summarizes the buying performance of the proposed rule over the calibration period. The results are only shown for periods with 21 and 42 days (the mean relative savings for 11-day and 63-day periods were quite modest). It is interesting to observe that the mean relative savings were nearly identical to those obtained during the calibration of Kingsman's policy (shown earlier in Table 54).

The difference between the two policies only became apparent over the testing period 1973-1976. The most important statistics concerning the performance of the new policy are shown in Table 56.

<table>
<thead>
<tr>
<th>Length of buying period</th>
<th>Mean relative savings</th>
<th>Rescaled relative savings</th>
<th>Mean</th>
<th>S.E.</th>
<th>(t)-statistic \((t(\alpha=0.05) = 1.65))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N) ([\text{days}])</td>
<td>(\overline{RS})</td>
<td>(\overline{R\overline{S}})</td>
<td>(\hat{\sigma}(R\overline{S}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>1.08</td>
<td>0.70</td>
<td>2.44</td>
<td>1.73 (sig.)</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>2.05</td>
<td>1.07</td>
<td>3.25</td>
<td>1.53 (not sig.)</td>
<td></td>
</tr>
</tbody>
</table>
The results presented in the table are of extreme importance, in several respects. Firstly, the distributions of the rescaled relative savings during 1973-76 are very similar to those obtained during 1963-72 (see Figure 78(iii)) : for $N = 21$ trading days the differences are negligible; $N = 42$ trading days they are not statistically significant. This confirms the validity of the arguments which led to the choice of the relative rescaled savings as a measure of performance.

Secondly, the average rescaled relative savings ($\overline{RRS}$) were shown to be significant, for buying periods of 21 trading days (i.e. one month). The conditions under which the t-test is valid – invariance of the distribution of RRS over time, independence of RRS in successive buying periods and normality of $\overline{RRS}_t$ (average RRS for each set of buying periods referring to contracts with a common delivery month $n$), were verified and were found to be realistic. For two-month buying periods, although $\overline{RRS}$ was larger than that obtained for the one-month periods, its significance ($\alpha=0.05$) could not be established. The value of $\overline{RRS}$ shown in Table 56 is highly unlikely to be the result of pure chance but more data are needed before its significance can be established.

Finally, the magnitude of the mean relative savings – approximately proportional to the mean relative price volatility – can, in times of unstable prices, become extremely attractive. As shown in Table 56, during 1973-76 the mean relative savings would have been between about 1 and 2% depending on the length of the buying period (the savings presented in the table should be corrected by deducting the costs incurred by holding 'futures' contracts during the buying periods; however, as shown earlier in section 5.2.1, these costs are very small; therefore the magnitude of the savings would not be significantly affected). Assuming an animal feed grain consumption of 2.4 million m.t. (see Table 21, in Chapter 6) and an average price of maize 'futures'
of US$ 130/m.t., relative savings of 1% would be equivalent to an annual reduction in the purchasing costs of US$ 3.12 million.

Having established the significance of the results, one point should be made about the distributions of RS and RRS. As shown in Figure 79, both distributions have a near zero median and are considerably skewed. The examination of these distributions revealed that the number of times in which losses are incurred is only slightly smaller than the number of occasions in which savings are made (47.6 - 52.4%). The implication of this fact is that the long-term savings are made essentially as a result of 'savings-larger-than-losses' rather than 'more-successes-than-failures'. This aspect of the policy should be well understood by the purchaser, if the temptation of abandoning the policy altogether (as a result of a high proportion of 'failures') is to be resisted.

5.5.3 Recalibration of the Policy Using the Entire Dataset

The policy was finally recalibrated over the entire period 1963-76. This involved

(i) resetting the parameters of Taylor's price -trend model at the values estimated over the entire period (p_o = 0.962, or m_o = 26 days, and A_o = 1.49%);

(ii) recalibrating the function f(n), using the maximization of the mean rescaled relative savings as the 'optimality criterion' (clearly, the maximization of the mean relative savings would now be inadequate, in view of the changes in the relative price volatility, taking place after 1972).
Figure 79 - Sample distributions of the relative savings (RS) and rescaled relative savings (RRS) under the new buying policy (one-month buying periods, all contracts, 1973-76)

(1) Relative savings

- Relative Frequency [%]

- Relative Savings (RS) [%]

<table>
<thead>
<tr>
<th>Losses</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.6%</td>
<td>52.4%</td>
</tr>
</tbody>
</table>

Mean: 2.08%, S.E.: 3.62%, Coeff. of skewness (*): 0.66

(2) Rescaled relative savings

- Relative Frequency [%]

- Relative Frequency (RRS)

<table>
<thead>
<tr>
<th>Losses</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.6%</td>
<td>52.4%</td>
</tr>
</tbody>
</table>

Mean: 0.70, S.E.: 2.44, Coeff. of skewness (*): 0.83

(*) estimated assuming that the distributions are Pearson (see Kendall et al.):

\[
sk = \frac{\text{mean} - \text{mode}}{\sigma} = \frac{\beta_1 \cdot (\mu_2 + 3)}{2(\mu_2^2 - \mu_1^2 - 9)}
\]

with

\[
\beta_1 = \frac{\mu_3}{\mu_2^3} / \mu_2^3
\]

\[
\beta_2 = \frac{\mu_4}{\mu_2^4} / \mu_2^4
\]

(\(\mu_n\) is the \(n^{th}\) moment about the mean)
The result of this recalibration is shown in Figure 80. The new function is noticeably different from that derived from the 1963-72 data subset. This difference, rather unexpected at first sight, can be explained easily. According to the proposed policy, the purchasing decisions are based upon the value of

$$B_n^* = f(n) \cdot \left( \frac{\hat{\mu}_n}{\hat{\nu}_n} \right) \cdot q_n^*$$

Figure 80 - New buying policy: parameters estimated/calibrated over the entire period 1963-1976

![Price-trend model parameters:]

- \( p_o = 0.962 \) (\( m_o = 26 \) days)
- \( A_o = 1.49 \% \)

Clearly, the changes introduced in the forecasting parameters (previously \( p_o, A_o \) and now \( p_o', A_o' \)) will cause changes in the day-to-day values of \( \left( \frac{\hat{\mu}_n}{\hat{\nu}_n} \right) \). Hence, it is only natural that \( f(n) \) should require an adjustment to take into account the differences in \( \left( \frac{\hat{\mu}_n}{\hat{\nu}_n} \right) \).

The magnitude of the adjustment (from Figure 78 (ii) to Figure 80) suggests that \( f(n) \) is sensitive to changes in the parameters of the forecasting model. A result with interesting practical implications emerged from the comparison between the values of RRS over 1963-76 (for one-month buying periods), under the following conditions:

(i) \( p_o, A_o, f(n) \) as defined in Figure 80: \( \overline{RRS} = 0.74 \)

(ii) \( p_o', A_o', f(n) \) as defined in Figure 78(ii): \( \overline{RRS} = 0.67 \)

(iii) \( p_o, A_o', f(n) \) as defined in Figure 78(iii): \( \overline{RRS} = 0.62 \)

The fact that the lowest savings were those obtained in (iii) indicates
that it would be better to keep all the parameters as estimated at the end of 1972 (i.e. to be consistent) than to adopt the best estimates of the forecasting parameters without recalibrating the function \( f(n) \). In practice this means that updating the estimates of the forecasting model is, by itself, insufficient to improve the performance of the policy. Such an improvement can only be expected if the function \( f(n) \) is recalibrated, according to the updated parameter estimates.
6. **Choice of Active Buying Periods.** Overall policy for Buying Maize 'Futures'

6.1 **General**

When the purchaser is restricted by senior management to buy within short potential buying periods (up to one or two months), the problem he faces is reduced to what has been described as the 'tactical' buying problem. However, if good medium-term price forecasts can be produced, further reductions in the purchasing costs can be achieved by extending the length of the potential buying periods to well over two months. In this situation, the 'tactical' buying problem is preceded by the selection, within each potential buying period, of an active buying period.

In previous research no attempt has been made to treat formally the decision problem involved in the choice of an active buying period. In this section, a decision rule will be proposed for that purpose. It seeks the minimization of the long-term purchasing costs, within constraints set according to the purchaser's conception of permissible risks in the buying operation.

The definition of an active buying period for any given shipment involves specifying

(i) when it starts (i.e. the first buying opportunity in which the purchasing of 'futures' for that shipment will be actively considered), and

(ii) its length.

The next section will focus on the selection of the first day of the active buying period. Initially, the length of this period will be set arbitrarily at one month (21 trading days). The question of how to select
the most appropriate length for that period will be discussed later in section 6.3.

6.2 Rule Proposed for Triggering the Purchase of a Shipment

6.2.1. Underlying Principles and Definition

If no medium-term price forecasts were available to the purchaser, naturally the active buying period would immediately precede the purchasing deadline. This period will be referred to as the latest possible active buying period. When the purchaser has access to medium-term price forecasts, and the potential buying period is extended to more than two months, he will have to decide whether or not it is worthwhile to make the purchase before the latest possible active buying period. In taking such a decision, the purchaser will have to establish a balance between

(i) the potential savings resulting from purchasing earlier that the latest possible active buying period, and

(ii) the risk that such a decision will, later on, prove to have been the wrong one (i.e. that prices that were expected to rise, do actually fall).

The rule proposed for triggering the purchases of shipments recognizes this essential aspect of the buying decisions. Its basic principles are illustrated in Figure 81, considering a hypothetical scenario: the purchase of a shipment with its purchasing deadline on the 31st of January, after the August forecast of maize 'futures' prices becomes available.

Diagram (i) represents a situation in which the profile of the average monthly price forecasts is monotonically increasing, between August and January. In this situation, starting the active buying period
earlier rather than later can be expected to lead to savings in the purchasing costs. However, from the point of view of the purchaser,

there is an unavoidable risk associated with the decision of making the purchase in advance of the latest possible active buying period: after the decision is taken, prices may actually fall rather than rise, contrary to the purchaser's expectation. In this event, he would later on regret the decision.

The proposed rule recognizes the purchaser's need to keep the probability of occurrence of such situations below a limit which is
acceptable from his point of view. This is achieved by initiating each shipment's purchasing only when the current price is less than or equal to a price $p^*$ satisfying the condition

$$\text{Probability}(\bar{p} < p^*) = \psi$$

where

$\bar{p}$ : average price during the latest possible active buying period (January, in the case considered in Figure 81)

$\psi$ : 'acceptable probability of regret' (a parameter to be specified by the purchaser).

Should the current price remain above $p^*$ throughout the potential buying period, the purchase of the shipment will take place during its latest possible buying period.

If all 'futures' contracts were to be bought on the same day, at the trigger price $p^*$, the value of $\psi$ would give the probability of making a decision which, from the point of view of the purchaser, would be regrettable - buying earlier at price $p^*$ when he should have waited until the latest active buying period (with an average price $\bar{p}$ lower than $p^*$). In a situation such as the one represented in diagram (i) of Figure 81, it might seem reasonable, on the evidence of the medium-term forecasts alone, to purchase the entire shipment on the day in which the trigger price is reached (marked with an arrow on diagram (i)). However, this would ignore possible short-term negative price trends at that point in time. In many cases, delaying a purchase by a few days might be beneficial. For this reason, the trigger price ($p^*$) is used to define the beginning of the active buying period, within which the tactical purchasing of maize 'futures' will take place, according to the policy proposed earlier in section 5. For the time being, the length of the buying
period will be assumed to be one month (the question of how best to specify this length will be discussed later in section 6.3).

The rule has so far been presented for the case in which the medium-term price forecasts are monotonically increasing, as shown in Figure 81(i). However, situations do occur in which prices are expected to fall at first and then to rise, as illustrated in Figure 81(ii). In the situation considered there, the average monthly prices are forecast to decrease to their lowest level $p_{low}$ in October-November and then to increase through December and January.

According to the proposed rule, when $p_{low} < p^*$ the tactical buying process is initiated whenever the current price is less than or equal to a trigger price defined as follows:

$$
\text{Trigger price} = \begin{cases} 
p_{low} & \text{up to the middle of the period in which the price is expected to reach its lowest value (in Figure 81(ii), end of October)} 
\vspace{1em} 
p^* & \text{thereafter}
\end{cases}
$$

(when $p_{low} > p^*$ the trigger price is, as before, $p^*$).

According to this rule, when $p_{low} < p^*$, the purchase can be triggered off in three different ways, depending on how prices move after the price forecasts become available (in Figure 81(ii) these different ways of triggering the purchase are represented by arrows (a), (b) and (c)):

(a) before the end of October the purchase will be triggered off only if the price $p_{low}$ is reached (for prices $p_{low} < p < p^*$, although the 'probability of regret' is below its acceptable value, it seems unreasonable to initiate the tactical buying, since the price is still expected to fall);

(b) if the purchase is not triggered off before the end of October and if, by then the current price is below $p^*$, the tactical buying is initiated (it seems unreasonable to wait
indefinitely until the price reaches the lowest predicted level \( p_{\text{low}} \), since this may never occur; the choice of the middle of October-November period as the date in which the trigger price is changed from \( p_{\text{low}} \) to \( p^* \) is clearly arbitrary);

(c) if prices remained above \( p^* \) up to the end of October, then the tactical buying will be initiated whenever the price \( p^* \) is reached.

One essential characteristic of the decision rule proposed to trigger off the 'tactical' buying process is its inbuilt flexibility. This is achieved by using the acceptable probability of regret (\( \psi \)) as an input, reflecting the purchaser's personal conception of permissible risk. Such a conception will certainly vary from purchaser to purchaser. Furthermore, for any given purchase, it is likely to change over time, depending on specific circumstances prevailing at the time when the buying decisions are taken. For example, a series of 'bad' purchases could determine a reduction in the acceptable probability of regret in the subsequent purchases. Besides, the purchaser may find that the value of \( \psi \) depends on how many future shipments have already been bought at the time of deciding whether or not to trigger off the purchase of a new shipment.

6.2.2. **Simulation of the Buying Operations Using the Proposed Rule**

In order to check the proposed rule, the buying operations were simulated for a sample of 215 weekly shipments with purchasing deadlines between 6 October 1972 and 12 November 1976. The exercise was conducted initially under the following conditions:
(i) Short-term price forecasts:
Derived using Taylor's price-trend model, with parameters estimated over the 1963-72 'calibration' period.

(ii) Medium-term 'futures' price forecasts and standard deviations of the corresponding forecast errors:
Derived as defined in section 4. The forecasts were assumed to be available for use in the buying decisions 5 days after the release of the information upon which they are based.

(iii) Potential buying periods
Defined as:
(a) the six-month period preceding each shipment's purchasing deadline (for deadlines between the 15th February and the 30th August);
(b) the period starting on the 15th August (when the August price forecasts are assumed to become available) and each shipment's purchasing deadline (for deadlines between the 15th September and the 14th February);
(c) the one-month period preceding each shipment's purchasing deadline (for deadlines between the 1st September and the 14th September).

(iv) Length of the active buying periods:
Set equal to one month, for all shipments.

(v) Acceptable probability of regret ($\psi$):
Under the assumption of normally distributed forecast errors, $\psi$ was set equal to $\frac{1}{4}$ (this value, assumed to be constant over time, was thought to be acceptable to most purchasers, in most circumstances).
(vi) Decision rule for triggering the active buying of each shipment:
As defined in section 6.2.1 (the mean and the standard deviation of the average prices of latest active buying periods not coinciding with calendar months, were derived by linear interpolation);

(vii) 'Tactical' buying:
According to the new policy proposed in section 5, with the function f(n) calibrated over the 1963-72 period.

The buying policy corresponding to this set of conditions will be referred to simply as
{ PBP = 6 months, ABP = 1 month, ψ = \frac{1}{4} }

where
PBP : maximum length of the potential buying period
ABP : length of the active buying period
ψ : acceptable probability of regret.

A computer programme was written in FORTRAN IV to simulate the buying operations. The programme was run on a CDC 7600, requiring 45 K words of computer memory and taking about 7 CPU seconds of compilation and execution time.

The results derived under the buying policy defined above are summarized in Figure 82 and Table 57.

The correspondence between 'futures' price movements and the timing of the purchases is illustrated in Figure 82. The way in which the average monthly 'futures' prices are presented in diagram (i) can be illustrated by considering, for example, the July 'futures' contracts. According to the purchasing operating doctrine adopted, the shipments purchased using July 'futures' are those with purchasing deadlines between the middle of
Figure 82 - 'Futures' price movements and purchases made under the policy
\{ PBF = 6 months, ABP = 1 month, V = 1/4 \}
(sample of 215 weekly shipments, with purchasing deadlines between 6 Oct 1972 and 12 Nov 1976)

(i) Average monthly 'futures' prices

(ii) Cumulative number of shipments either purchased or being purchased at the end of each month

- Purchasing in the earliest possible active buying period
- Purchasing in the latest possible active buying period
- Purchasing according to the policy considered
Table 57 - Purchasing performance under the policy
[PEF = 6 months, AEP = 1 month, \(\psi = 1/4\)]
(sample of 215 weekly shipments, with purchasing deadlines between 6 Oct.1972 and 12 Nov.1976)

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>Unit purchasing cost</th>
<th>Percent savings in relation to the 'hand-to-mouth' policy</th>
<th>Purchasing efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[US #/bushel]</td>
<td>[US #/bushel] [%]</td>
<td>[%]</td>
</tr>
<tr>
<td>1972</td>
<td>IV</td>
<td>144.18</td>
<td>132.61</td>
<td>8.02</td>
</tr>
<tr>
<td>1973</td>
<td>I</td>
<td>158.25</td>
<td>139.54</td>
<td>11.82</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>189.27</td>
<td>139.43</td>
<td>25.94</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>250.76</td>
<td>187.11</td>
<td>27.97</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>256.42</td>
<td>253.60</td>
<td>1.10</td>
</tr>
<tr>
<td>1974</td>
<td>I</td>
<td>301.60</td>
<td>276.62</td>
<td>8.28</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>269.69</td>
<td>269.26</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>348.10</td>
<td>258.28</td>
<td>17.57</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>366.44</td>
<td>358.06</td>
<td>2.61</td>
</tr>
<tr>
<td>1975</td>
<td>I</td>
<td>304.04</td>
<td>305.57</td>
<td>-0.50</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>276.36</td>
<td>254.71</td>
<td>-8.64</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>292.27</td>
<td>290.94</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>277.50</td>
<td>280.76</td>
<td>-1.17</td>
</tr>
<tr>
<td>1976</td>
<td>I</td>
<td>270.21</td>
<td>266.64</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>282.06</td>
<td>273.57</td>
<td>2.91</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>285.75</td>
<td>287.36</td>
<td>-0.66</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>257.39</td>
<td>265.68</td>
<td>-3.22</td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td>267.40</td>
<td>253.44</td>
<td>5.22</td>
</tr>
</tbody>
</table>

(*) 13 shipments per quarter, except for 1976-IV (7 shipments)

April and the middle of June. For this period (April to June), the July 'futures' prices are shown in the diagram by a bold line (these are the prices that would be paid under the hand-to-mouth policy). In the six months preceding April (i.e. in the potential buying period of the shipment with earliest purchasing deadline, amongst those covered with
July 'futures'), the July 'futures' prices are shown by a dotted line. The same criterion was used to present prices of other 'futures' contracts.

The approximate timing of the purchases over the simulation period is shown in diagram (ii) by the bold line. The diagram also indicates the freedom given to the purchaser under the policy considered: this freedom is represented by the gap between the broken line and the line limiting the shaded area.

It is interesting to notice the differences between the purchasing patterns in different crop years, under the policy considered. In 1972/73 the active buying periods were anticipated nearly as much as was compatible with the policy adopted, over the entire crop year. The completely opposite situation occurred during 1975/76. The crop year 1973/74 and 1974/75 was characterized by purchasing patterns that oscillated between those extremes.

The purchasing performance over the simulated period is summarized in Table 57. Two measures of performance are considered. The first one is the percent savings in relation to the 'hand-to-mouth' policy (for each shipment, all futures purchased on the last buying opportunity). As shown in the table the overall percent savings were 5.22%. Assuming annual feed grain imports of 2.4 million m.t. (see Table 21, Chapter 6) those savings would be equivalent, over the simulation period considered (4.125 years), to US$ 54.4 million. This impressive figure should be regarded with due caution, for two main reasons:

(i) the medium-term forecasts used in the simulation of the purchasing operations were derived from models fitted over the period 1967/68 to 1974/75 (which includes most of the simulation period);
(ii) In spite of covering slightly more than four years, the simulation period is far too short to allow any definite conclusions to be drawn, concerning the long-term performance of the policy considered (the simulation exercise could, however, lead to the straightforward rejection of the policy if its performance turned out to be worse or even no better than the 'hand-to-mouth' policy).

Nevertheless, in spite of these limitations, the results of the simulation are very encouraging. Table 57 shows that, out of the seventeen quarters covered in the simulation, losses in relation to the 'hand-to-mouth' policy were incurred in only five, and were quite modest in comparison with the savings achieved in other quarters.

In the last five quarters, the aggregate percent savings were practically negligible (0.06%). The fact that this was precisely the period left out when fitting the medium-term forecasting models could, at first sight, cast some doubts on the significance of the overall results. However, the reason for the small magnitude of the savings lies in the fact that, during most of this period, prices were falling: for a large number of shipments, the 'hand-to-mouth' policy was the best possible policy. The savings made when this was not the case (specially during the second quarter of 1976), were just sufficient to offset the losses incurred during the remaining of the last five quarters of the simulation period.

In addition to presenting the savings in relation to the 'hand-to-mouth' policy, Table 57 also includes Kingsman's purchasing efficiency measure, defined, for each shipment, as

\[
Purchasing \ Efficiency = \frac{\text{Average } AP - \text{ Average } AC}{\text{Average } AP - \text{ High } } \times 100\%\]

where

\[
AP : \text{ average 'futures' price over the potential buying period}
\]
AC : average 'futures' purchasing cost
HC : unit 'futures' hindsight cost (minimum 'futures' price within the potential buying period).

In the context of a medium-term policy such as the one considered, Kingsman's measure is particularly helpful in the evaluation of how good or how bad is the choice of the active buying period within each potential buying period. The quarterly and overall efficiencies - obtained by averaging the efficiencies of the corresponding individual shipment - show that such choices were consistently good. The efficiencies were negative for two quarters only. It is interesting to observe that in these quarters, the 'hand-to-mouth' policy would have led to results either significantly worse (1974, I) or practically equivalent (1976, III) to those obtained under the policy considered. Kingsman's measure also reveals that, over the last five quarters, the decisions of delaying the active buying periods as much as possible, were mostly right (the exceptions were the decisions concerning 1976-III shipments, where it would have been better to have earlier active buying periods, had the price rise taking place after April been forecast with confidence).

The simulation exercise was extended to other policies derived from that considered above by changing the values of the acceptable probability of regret, \( \psi \), and the maximum length of the potential buying period, PB. As shown in Figure 83(i), the purchasing performance over the simulation period considered was practically unaffected by changes in the parameter \( \psi \) (varied between \( \frac{1}{10} \) and \( \frac{1}{2} \)). Clearly, this result does not imply that the choice of \( \psi \) will not affect the long-term performance of a medium-term policy (indeed, it must affect it). The result only shows that different policies are applied over a specific period of time, due to the particular price movements taking place during that period, they can lead to identical purchasing decisions. This is what happened in the simulation
period considered for the vast majority of shipments (for this reason, the quarterly performances were almost as unaffected by changes in $\psi$ as the overall performance).

As for the parameter $\psi$, the overall purchasing performance remained practically unchanged when the maximum length of the potential buying period was changed from 6 to 4 months (see Figure 83(ii)). However, in this case, the quarterly performances of the two policies were moderately affected by changes in PBP. For PBP = 4 months, the savings were smaller specially during 1972-73 (when prices rose constantly). This was, however, compensated during 1975, when some of the bad purchases which would have been made with PBP = 6 month were avoided.

Figure 83 - Parametrization of $\psi$ and PBP: effect on the percent savings in relation to the 'hand-to-mouth' policy (sample of 215 weekly shipments, with purchasing deadlines between 6 Oct 1972 and 12 Nov 1976)

1. Parametrization of $\psi$

   - $\psi$ acceptable probability of regret
   - Percent savings
   - PBP = 6 months
   - PAB = 1 month

2. 'Parametrization' of PBP

<table>
<thead>
<tr>
<th>PBP</th>
<th>Percent savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 months</td>
<td>5.25 %</td>
</tr>
<tr>
<td>6 months</td>
<td>5.22 %</td>
</tr>
</tbody>
</table>

   (ABP = 1 month, $\psi = 1/4$)
6.3 The Length of the Active Buying Period (ABP)

In principle, the active buying period should be long enough to allow the purchaser to make the best use of short-term price forecasts by means of a 'tactical buying policy.' On the other hand, an excessively long active buying period may be disadvantageous. In fact if, say, prices are expected to rise in the medium-term, an excessively long active buying period is likely to lead to the postponement of purchases which should have been made earlier.

The best value of ABP will depend on the degree to which medium-term price forecasts 'explain' price changes occurring in the short-term. Clearly the better these changes are 'explained', the shorter the active buying period should be.

Throughout section 6.2 ABP was set equal to one month. In the simulation described there, the savings achieved with policy \( \{ \text{PBP = 6 months, ABP = 1 month, } \psi = \frac{1}{4} \} \) in relation to the 'hand-to-mouth' policy were 5.22%. In an attempt to evaluate the sensitivity of the purchasing performance to changes in ABP, the simulation was repeated setting \( \text{ABP = 2 months} \): the savings increased marginally, to 5.45%. This would appear to confirm that medium-term price forecasts do not, in general, 'explain' well price changes taking place in the short-term (since otherwise one would expect the performance of the policy to deteriorate as ABP increases).

Caution should be exercised, however, when interpreting this small increase in the purchasing performance. As illustrated earlier in Figure 82, the medium-term buying policies lead occasionally to the simultaneous triggering of purchases of large numbers of shipments (for
instance, the purchases of a total of 20 shipments were triggered on the 15th August 1972 alone). This means that the comparison of the savings achieved, over the entire period, with \( APB = 1 \) month and \( ABP = 2 \) months, will be strongly affected by what happened in those few periods where large numbers of shipments were purchased.

The simulation was repeated setting \( ABP = 1 \) day (i.e. the shortest possible active buying period), and in this case the savings obtained, 6.18%, were slightly larger than in the two other cases. Even if the increase in savings is interpreted as being largely the result of chance, the simple fact that this chance can occur over a period of more than four years indicates that, if \( ABP \) is kept constant for each particular policy, no real advantage will ensue from applying the tactical buying policy over active buying periods longer than one day. One possible explanation for this result is that, by keeping \( ABP \) constant over time, the policies considered fail to distinguish situations where it is highly unlikely that prices will fall after the purchase is triggered, from those where this is more likely to happen.

Advantages may arise from making \( ABP \) adaptable to the particular circumstances prevailing at the time the decision to initiate a purchase is made, rather than being kept fixed over time. In fact, although in general short-term price movements are not well 'explained' by medium term price forecasts, it is nevertheless reasonable to expect that when a purchase is triggered at a time when the medium-term forecast indicates a marked rise in prices, the likelihood of the price falling in the short-term is smaller than in cases where small medium-term price rises are forecast.
In this research, no attempt has been made to formulate a rule for defining the best length of the active buying period for each particular purchase. The development of such a rule would be an interesting research topic. To derive this rule empirically through simulation does not seem to be adequate since it is very difficult to establish the significance of the results even for long simulation periods. Rather, its development will require a clear understanding of how short and medium-term price forecasts explain price movements over active buying periods of various lengths.

Until further work on this topic yields results, the most appropriate medium-term buying policy seems to be one which indicates how the beginning of the active buying period should be determined, and leaves to the judgement of the purchaser the choice of its length, within the range [1 day, 2 months].

7. Summary and Conclusions

In this chapter, the most significant aspects of the feed grain purchasing operations have been analysed. Initially, a purchasing operating doctrine was chosen following an examination of the different ways in which the importer (EPAC) can conduct its buying operations. The doctrine chosen, which consists of conducting the purchasing operations initially through the 'futures' market and then converting 'futures' to 'physical' close to the purchasing deadline of each maize or sorghum shipment, was shown to be the most appropriate in terms of EPAC's objectives. In fact, it leads to no increase in purchasing costs over the policy which consists of buying 'physical' only, while it allows the purchaser to retain considerable flexibility for changing the sizes of shipments and the type of grain (maize or sorghum) until the shipping deadline of each shipment is reached. The question of how and when to decide on both the appropriate grain and the size of each shipment will be discussed in the next chapter, in the context of the import mix planning problem.
This was followed by an analysis of short and medium-term forecasting models for the Chicago maize 'futures' prices. For short-term forecasting, after a brief review of Trigg and Leach's adaptive exponential smoothing, Taylor's price-trend model was studied in greater detail. The statistical significance of short-term price trends in the maize 'futures' market was then established; although small, these trends later proved to be sufficient for successful buying decisions to be derived (i.e. decisions leading to purchasing costs lower than the average market prices).

For the medium-term forecasting, following a study of the relationships between Chicago maize 'cash' and 'futures' prices, Kingsman's behavioural model of 'cash' prices was adapted to the forecasting of maize 'futures' prices. This was followed by an analysis leading to the characterization of the distributions of the forecast errors. In the course of this analysis it became apparent that, if the necessary computer programmes were available, more efficient price forecasts and, above all, better estimates of the standard deviations of the forecast errors would be obtained if the behavioural price model were to be based on non-linear regression methods rather than linear regression with transformations of the dependent variables.

The 'tactical' buying problem of maize 'futures' contracts was then addressed. A new buying policy, based on a heuristic rule, was developed and was shown to lead to statistically significant savings in relation to the average market prices. This was done by means of a statistical test applied to a measure of efficiency (rescaled relative savings, RRS) which was introduced to avoid the shortcomings of existing measures.

The policy was shown to perform nearly equally to Kingsman's classical
dynamic programming policy in periods of price stability and better than it in times of high price volatility. The results of the simulation of the policy, using real prices, substantiate the conjecture that relative savings are approximately proportional to the relative volatility of prices. In periods of high price volatility, the savings in the purchasing costs turned out to have a magnitude of between 1 and 2% of the average maize 'futures' market prices, for active buying periods between 1 and 2 months.

Indirectly, the analysis and testing of the proposed 'tactical' buying policy supported the validity of the basic assumption of Taylor's price-trend model, namely that price changes follow a stochastic process which, except for their volatility, is invariant over time. It also showed that the measure of performance RRS is adequate for the purpose of carrying out statistical tests on the significance of savings.

The 'tactical' buying policy proposed is particularly relevant when the purchaser is restricted to buying over short potential buying periods or when no reliable medium-term price forecasts are available.

In the course of the analysis of the 'tactical' buying problem a number of areas deserving further investigation were identified, in particular:

(i) To explore other heuristics, namely by changing the relationship between the size of the purchase, $B_n^*$, and the estimated size of the current trend measured in standard deviations, $\hat{\mu}_t/\hat{\nu}_t$. For any given number of days left up to the end of the buying period, and for any given amount of 'futures' left to be bought, $B_n^*$.
was specified to be proportional to positive values of \( \hat{v}_t / \hat{v}_t \). There is no guarantee that the assumed relationship yields the best possible results; other relationships should be therefore explored.

(ii) To apply the proposed 'tactical' buying policy (or improved versions thereof) to commodities other than maize, with the objective of

(a) further testing the policy, and

(b) establishing how its performance is likely to be affected by the characteristics of each price series (e.g. Taylor's price-trend\:model parameters \( p_o \) and \( A_o \)).

The analysis was finally extended to the more general situation involving large potential buying periods. A policy was developed to derive the start of the active buying period from the medium-term price forecasts available to the purchaser. The policy takes into account the purchaser's conception of permissible risk - measured by the acceptable probability of regret \( \psi \). This parameter confers flexibility on the policy, making it adaptable to different purchasers and to different attitudes towards risk by the same purchaser in different circumstances.

A simulation of the buying policy was carried out over a period of about four years. Although the exercise is insufficient, on its own, to validate conclusively the policy and to evaluate its long-term performance, the results were very encouraging, showing the potential for substantial savings to be made.
The choice of the length of the active buying period and indeed the potential benefits resulting from the tactical buying policy when this is performed in the context of a medium term policy remain open questions. The success of the tactical buying policy in these conditions is likely to depend on the ability to vary the length of the active buying period according to the short and medium-term price forecasts available at any given point in time. The study of the way in which these forecasts should be combined in the process of defining rationally that length is an important research topic. Before this research is carried out, the choice of the length of the active buying period should remain a matter for the judgement of the purchaser.
CHAPTER 9

PLANNING OF THE IMPORT MIX

1. Introduction

In this chapter, the static model developed in Chapter 7 (model 3 in Figure 6, Chapter 3) is extended to form the import mix planning model (model 5 in the same figure).

In section 2 the model is made adaptive to the dynamics of the planning situation. A multi-period planning model is initially considered and the possibility of simplifying it into a set of single period planning models is discussed. In these models the import prices are still assumed to be deterministic.

In section 3 this assumption is relaxed when the model is extended to analyse the decision problem under risk. The problem is approached considering first the situation in which the import mix planner (or planning organization) does not exercise any influence over the choice of the purchasing strategies that are to be adopted by the various organizations which import raw materials for the animal feed industry. The role of the planner is restricted, in this situation, to the choice of the import mix that, for given forecasts of the raw materials' import prices (provided by the importing organizations), will best satisfy his objective. This situation, which, in terms of Figure 6 (chapter 3) would be equivalent to neglecting the feedback loop between models 5 and 4, will be referred to simply as 'planning without feedback'. Later in section 3 the analysis is extended to the situation where the import mix planner can influence the purchasing strategies adopted by the purchasing departments of the various importing organizations (a situation described herein as 'planning with feedback').
Section 4 concludes the chapter with an overview of the feed grain imports planning and control system and a discussion of the interrelationships between its three basic models: the import mix planning model, the delivery-inventory model and the purchasing model.

2. Dynamic Formulation of the Import Mix Planning Problem

2.1 General

As mentioned earlier in chapter 4, section 3.2, the raw materials imported for the animal feed industry are purchased in the international markets by different organizations.

(i) Government agencies: EPAC (feed grains), IAPO (oil seed meals), AGAA (cane molasses);

(ii) CAIACA, a cooperative of the animal feed producers (fish meal, maize gluten meal, alfalfa meal);

(iii) Premix manufacturers (synthetic aminoacids).

The Central government plays a major role in the importing process of each raw material by fixing the amounts that can be imported by each of those organizations. The purpose of the import mix planning model is to assist the Government in establishing, in a coherent way, the import quotas for the different raw materials.

The fluctuating nature of commodity prices will give rise to continuous changes of the 'optimal' import mix. Ideally the import quotas should therefore be adapted continuously over time. However, there are difficulties associated with the process of altering the import quotas, since these imply a process of

(i) reallocation of the individual quotas to each animal feed manufacturer (this is a rather delicate process
involving tough bargaining by the individual manufacturers); (ii) resetting the controls required to enforce the individual quotas; (iii) altering raw materials' internal prices, maximum selling prices of animal feeds and other Government policy parameters.

This process, time consuming and costly, implies that (i) in practice, changes in the import quotas will be made periodically rather than continuously; (ii) each change has to be planned in advance of its enforcement, to allow the resetting of the controls on the individual quotas and on the internal prices for the new situation.

In the import mix planning model discussed in this chapter, it is assumed that (i) the import quotas will be reviewed quarterly (as mentioned earlier in chapter 6, section 3.1, quarterly adjustments are about the tolerable limit); (ii) over each quarter, the imported and home produced raw materials are consumed at a constant rate (as discussed in chapter 6, section 3.2.7, this is a realistic assumption).

The process of planning the import mix for any given quarter will start long before its beginning, when the planner receives from the importing organizations the first estimates of the raw materials import prices (for delivery during the quarter in question). As new information becomes available - either on import prices or on newly committed purchases - the import mix will be readjusted. This process will finish, i.e. the raw material's import quotas will have to be finally defined, on what can be described as the 'planning deadline'. This deadline must precede the
beginning of the quarter by, at least, the minimum time required to readjust the controls on individual quotas and internal prices. Furthermore, it must be set before the shipping deadlines of those shipments that are to be delivered at the beginning of the quarter. The latter of these conditions (which is equivalent to setting the planning deadline about 1.5 months before the beginning of the quarter) is likely to be more restrictive than the former.

2.2 Multi-period Formulation

The multi-period formulation will be illustrated considering the planning of the import mix at time $t_0$ in the situation depicted in Figure 84.

![Figure 84 - Import mix planning: multi-period situation](image)

At time $t_0$ the import quotas for the current quarter (0) will already have been set. This quarter can therefore be ignored in the import mix.
planning process. Quarter 3 (and later periods) can also be ignored if, as was assumed for maize and sorghum, purchases are not to be made more than six months in advance of the purchasing deadlines.

Except for the synthetic aminoacids, which are of minor importance, all imported raw materials are transported by sea. Given the restrictions on the storage capacity at the Portuguese ports, it is proper to assume that the shipments of the imported raw materials will be scheduled to arrive regularly over time and that no significant stocks above those strictly required to maintain (economically) a regular supply will be carried at any moment. Purchases made in advance of the shipping deadlines will be made either through the 'futures' market (as proposed for maize and sorghum) or directly in 'physical' for forward delivery (if some of the buying organizations continue to operate in this way).

In the situation considered in Figure 84, the dynamic formulation of the import mix must take into account the possibility of reallocating 'optimally' committed purchases (i.e. purchases already made at time \( t_0 \)) amongst the several feeds that are to be produced in each of quarters 1 and 2. This includes the possibility of transferring to quarter 2 purchases which were originally scheduled to be delivered in quarter 1 and vice-versa. For those purchases made in 'futures' this may imply the need to replace the already acquired contracts by others reaching maturity later. When purchases are made in 'physical', their reallocation within and/or between periods will necessarily involve a renegotiation of the terms of contract with the exporter.

The home produced raw materials made available during quarter 1 could in theory be stored, and used in period 2. However, in practice, this does not occur (it should be remembered that these raw materials are produced and sold to the feed manufacturers practically without any
The import mix planning problem will be modelled using the following notation:

\( i \) : index for import raw materials \( (i = 1, \ldots, I) \)

\( n \) : index for home produced raw materials \( (n = 1, \ldots, N) \)

\( j \) : index for animal feeds \( (j = 1, \ldots, J) \)

\( p_i^1, p_i^2 \) : import prices of raw material \( i \) for new purchases of shipments that are to be delivered in quarters 1 and 2, respectively \( \text{(US$\text{/m.t.})} \)

\( B_i^1, B_i^2 \) : committed purchases of raw material \( i \) (quarters 1 and 2) \( \text{(m.t.)} \)

\( x_{ij}^1, x_{ij}^2 \) : quantity of imported raw material \( i \) used in feed \( j \) (quarters 1 and 2) \( \text{(m.t.)} \)

\( x_{nj}^1, x_{nj}^2 \) : quantity of home produced raw material \( n \) used in feed \( j \) (quarters 1 and 2) \( \text{(m.t.)} \)

\( t_i^{12}, t_i^{21} \) : committed purchases of imported raw material \( i \) transferred from quarter 1 to quarter 2 or vice-versa \( \text{(m.t.)} \)

\( c_{i' i}^{12}, c_{i' i}^{21} \) : unit cost (foreign currency expenditure) of transferring committed purchases of raw material \( i \) from one quarter to the other \( \text{(US$\text{/m.t.})} \)

Under this notation, and assuming complete freedom in the reallocation of committed purchases within and/or between the two quarters, the problem can be formulated as a mathematical programme as follows:
Minimize: \[ \sum_{i} \left( \sum_{j} p_i \cdot \left( x_{ij}^1 - (B_i - t_{i12}) - t_{i21} \right) \right) \]
+ \[ \sum_{i} \left( \sum_{j} p_i \cdot \left( x_{ij}^2 - (B_i - t_{i21}) - t_{i12} \right) \right) \]
+ \[ \sum_{i} c_{i}^{12} \cdot t_{i12} + \sum_{i} c_{i}^{21} \cdot t_{i21} \]

Subject to

(i) Non-negativity constraints on the amounts of each imported raw material \( i \) that are to be purchased for each quarter:
\[ \sum_{j} x_{ij}^1 - (B_i - t_{i12}) - t_{i21} \geq 0 \]
\[ -\sum_{j} x_{ij}^2 - (B_i - t_{i21}) - t_{i12} \geq 0 \]

(ii) Constraints limiting the amounts of each imported raw material \( i \) that can be transferred from one quarter to the other.
\[ t_{i12} \leq B_i \]
\[ t_{i21} \leq B_i \]

(iii) Balance equalities.

(iv) Constraints expressing the limited supply of home produced raw materials.

(v) Nutritional constraints.

(vi) Upper bounds on the utilization of raw materials in feeds (if existent).

(vii) Limits on the utilization of groups of raw materials if existent.)
where (iii) to (vii) are as before in the LP presented in Chapter 4, section 3.1, for each of the quarters 1 and 2.

Constraints (i) are defined assuming that purchases already made at time $t_0$ cannot be resold (i.e. they really represent a commitment). Although, in theory, it is always possible to resell purchases made in 'futures' (and not yet converted to 'physical'), it does not seem advisable, or indeed acceptable for the importing organizations, to indulge in such a practice as a matter of course, in view of the difficulties that would ensue in the control of purchasing operations.

Constraints (i) and (ii) are appropriate when the deliveries of committed purchases can be reallocated (both within each quarter and between quarters) with complete freedom. This applies to the situation where purchases are made in 'futures'. When these are conducted directly in 'physical', it may not be possible to reallocate them freely. For example it may not be possible to transfer all committed purchases $B_i^t$ to quarter 2 (as assumed in the model above). In this situation, the constraint

$$t_{12}^i \leq B_i^t$$

should be replaced by

$$t_{12}^i \leq (B_i^t)^*$$

where $0 \leq (B_i^t)^* \leq B_i^t$ represents that part of the committed purchases of raw material $i$ which is transferable from quarter 1 to quarter 2.

The other constraints should be adjusted similarly.

The formulation presented above will now be extended in two specific areas, to take into account

(i) the dependence of the import prices of maize and sorghum on the quantities that are imported, and
(ii) the fact that committed purchases of maize and sorghum are, in fact, maize 'futures' contracts already bought which are convertible to 'physical' maize, 'physical' sorghum or both.

As was seen in Chapter 7, the import prices of maize and sorghum are dependent on the quantities that are imported, because of economies of scale in the delivery-inventory operations. In the multi-period situation, this dependence can be modelled, for each of the Portuguese ports, in the same way as in chapter 7. For this purpose, the above notation will be augmented with the notation introduced earlier in chapter 7, namely:

\[ i = 1 \quad : \text{denotes maize} \]
\[ i = 2 \quad : \text{denotes sorghum} \]

\[ X^1_i, X^2_i \quad : \text{total quantity of imported raw material } i \]
\[ \text{used in quarters 1 and 2, respectively} \]
\[ (e.g. X^1_i = \sum_j x^1_{ij}) \quad \text{(m.t.)} \]

\[ (p_1^1)^*, (p_1^2)^* \quad : \text{fob prices for new purchases of maize} \]
\[ \text{shipments that are to be delivered in quarters 1 and 2, respectively} \quad \text{(US$/m.t.)} \]

\[ (p_2^1)^*, (p_2^2)^* \quad : \text{fob prices for new purchases of sorghum} \]
\[ \text{shipments that are to be delivered in quarters 1 and 2, respectively} \]

\[ s^1, s^2 \quad : \text{marginal delivery-inventory cost per unit of imported maize, when sorghum is not imported (quarters 1 and 2)} \quad \text{(US$/m.t.)} \]

\[ \text{Pen}(X^1_2), \text{Pen}(X^2_2) \quad : \text{penalty function giving the increase in the delivery-inventory costs} \]
incurred by replacing maize by an
equal amount \( X^1_2 \) or \( X^2_2 \) of sorghum

(quarters 1 and 2)

\[ f = \sum_{i=1}^{2} \left\{ \left( p^1_i \right)^{*} \left[ X^1_i - (B^1_i - t^1_{12}) - t^2_{11} \right] + s^1_i \cdot X^1_i \right\} + \frac{1}{2} \cdot \text{Pen} (X^1_2) \]

\[ + \sum_{i=3}^{1} \left\{ p^1_i \cdot \left[ X^1_i - (B^1_i - t^1_{12}) - t^2_{11} \right] \right\} \]

\[ + \sum_{i=1}^{2} \left\{ (p^2_1)^{*} \left[ X^2_i - (B^2_i - t^2_{21}) - t^1_{12} \right] + s^2_i \cdot X^2_i \right\} + \frac{2}{2} \cdot \text{Pen} (X^2_2) \]

\[ + \sum_{i=3}^{1} \left\{ p^2_i \cdot \left[ X^2_i - (B^2_i - t^2_{21}) - t^1_{12} \right] \right\} \]

\[ + \sum_{i=1}^{1} \left\{ c^1_{12} \cdot t^1_{12} + c^2_{21} \cdot t^2_{21} \right\} \]

Ignoring constant terms, the objective function can then be
written as follows

In chapter 7 the penalty functions were defined by three linear
segments (denoted by I, II, III). It was shown there that the optimal
solutions for the single-period situation were obtained either within
section I (with no sorghum imports) or within section III (significant
sorghum imports). Extending this result to the multi-period situation
(which is essentially the same, as far as the delivery-inventory operations
of maize and sorghum are concerned), the optimal solution can then be derived
solving the four LP's with

(i) the objective function defined as above,

(ii) all the constraints (i) to (vii) specified above for the
multi-period situation, and

(iii) one of the following sets of constraints :
(a) \( x_2^1 \in I_1 \) (i.e. \( x_2^1 \) is within section I of \( \text{Pen}^1(x_2^1) \)) and 
\( x_2^2 \in I_2 \);

(b) \( x_2^1 \in I_1 \) and \( x_2^2 \in III_2 \);

(c) \( x_2^1 \in III_1 \) and \( x_2^2 \in I_2 \);

(d) \( x_2^1 \in III \) and \( x_2^2 \in III_2 \);

Clearly, the optimal solution is the one that leads to the overall minimum value of \( f \).

In the model presented above (comprising a set of four LP's) one important aspect of the import mix planning problem was still ignored: the fact that committed purchases of maize or sorghum - i.e. maize 'futures' contracts already acquired - are convertible to 'physical' maize or 'physical' sorghum or both. For any given shipment with a purchasing deadline \( t \), the unit 'cost of conversion' of 'futures' into 'physical' will be the difference, at time \( t \), between

(i) the fob market 'cash' price of maize or sorghum, and

(ii) the maize 'futures' price (of the appropriate contract).

If the notation used above is augmented with the following one:

\[ B^1_{1,2}, B^2_{1,2} \]: total purchasing commitments of maize and sorghum (quarters 1 and 2);

\[ cc^1_1, cc^2_1 \]: average cost of conversion of maize 'futures' to 'physical' maize (periods 1 and 2); and

\[ cc^1_2, cc^2_2 \]: average cost of conversion of maize 'futures' to 'physical' sorghum (periods 1 and 2),
the multi-period import mix model can be extended to represent the possibility of converting maize 'futures' to 'physical' maize or 'physical' sorghum or both, by altering each of the four LP's defined above as follows:

(i) adding to the objective function the following terms, representing the costs of converting 'futures' to 'physical' (periods 1 and 2):

\[ \sum_{i=1}^{2} \left[ cc_i \left( (B_i^1 - t_i^{12}) + t_i^{21} \right) + cc_i \left( (B_i^2 - t_i^{21}) + t_i^{12} \right) \right] \]

(ii) setting constraints on the new problem variables \( B_i^1 \), \( B_i^2 \) (\( i = 1, 2 \)):

\[
\begin{align*}
B_i^1 + B_i^2 &= B_i^{1,2} \\
B_i^2 + B_i^2 &= B_i^{1,2}
\end{align*}
\]

2.3 Possible simplification of the multi-period model

The need for a multi-period model arises out of the possibility of transferring committed purchases of different raw materials from one period to another. With the objective of minimizing foreign currency expenditure it is clearly advantageous to explore that possibility to the full. However, from a practical point of view such transfers can cause difficulties for the purchasing departments even in the most flexible situation (i.e. when purchases are made through the 'futures' market). Such difficulties arise from the need for increased coordination and control of the purchasing operations, mainly due to the larger number of options open for consideration at each stage, and the disruptions caused when the allocation of 'futures' contracts to shipments is significantly changed. In the case where shipments are purchased directly in 'physical', the difficulties are obviously aggravated.
Under these circumstances, and considering that any planning process will imply a considerable effort of adaptation for the purchasing organizations, it seems wise to initially simplify the planning process as much as possible. One way of achieving this is to ignore the possibility of transfers of committed purchases from one period to another, in which case the two-period planning problem degenerates into two separate one-period planning problems (one for each quarter). Progressively, as the planning process is learnt by the organizations concerned, more flexibility should be introduced, relaxing the constraints on the transfers, as it becomes operationally viable to do so.
3. The Import Mix Planning Problem Under Risk

3.1 General

One of the assumptions underlying the import mix planning problem is that the purchasing departments of the importing organizations will produce forecasts of the import prices that will be paid for new purchases of shipments that are to be delivered within the relevant planning periods (for the case of maize and sorghum the relevant forecasts are those of the fob prices, from which import price forecasts can be derived). These forecasts will be derived from market price forecasts and will be dependent on the particular purchasing policies adopted by each department. If a hand-to-mouth policy were to be adopted for a given raw material, then the import price forecast for any given shipment would coincide with the forecast of the market price prevailing on its purchasing deadline. However, if a more efficient purchasing policy, based on good market price forecasts, is adopted, the forecasts of the import prices for some shipments, will be lower than those of the market prices prevailing on their purchasing deadlines. This will occur whenever the conditions under which the policy will lead to purchases made in advance the shipments' purchasing deadline are satisfied. The import price forecasts are necessarily subject to error. This error will generally vary according to

(i) the raw material (the prices of some raw materials will be more difficult to forecast than others);

(ii) the time of the year (for example, for maize, the price forecasts made before August for the following crop year are less reliable than those obtained after the August crop production estimate becomes available);

(iii) the importing organizations (the reliability of the forecasts will depend obviously on the forecasting methods that are adopted).
In this section the formulation of the import mix planning problem will be extended to take into account the stochastic nature of the import price forecasts, assuming that the import mix planner obtains estimates of their distributions from the purchasing departments of the importing organizations. This means that the import mix planning problem (previously treated as deterministic) will now be studied as a problem of decision making under risk.

The analysis will be carried out within the general framework of Benouillian Decision Theory. Admittedly, objections have been raised (e.g. Rivett, Tocher) against maximization of expected utility as a criterion for decision making under risk (Bernoulli's principle). The arguments used fall into two main categories:

(i) instances are shown in which 'reasonable' action appears to contradict the expected utility criterion; this is then interpreted as suggesting that while the axioms upon which the theory is based have immediate appeal, they conceal objectionable assumptions; and

(ii) the 'naive' assumption that people act, or should act, consistently and logically is criticized; since people do not in fact act in that way the assumption, and the theory based on it, are dismissed as useless or, even worse, misleading.

In spite of these criticisms, the fact remains that the maximum expected utility criterion, in the words of Dillon (p.7.) 'has the normative justification of being a logical deduction from a small number of postulates or axioms which many people agree are absolutely reasonable and should be met by a person who wishes to be consistent and rational in his workday decisions'. In addition to this argument of a normative nature, it is relevant also to quote a defence of the criterion by
Markowitz\textsuperscript{44} (p.210), on completely distinct grounds: 'This writer (the author) believes that the arguments in favour of the expected utility maxim are quite convincing (...). The maxim has to be stretched, perhaps intolerably, to apply to the making of decisions in which surprise and the fun of gambling are important motivations. These, however, are not important objectives (...) in the allocation of large amounts of other people's money'.

As was indicated in the introduction to this chapter, there are two different ways of dealing with the output of the import mix planning model. The next section focuses on what was called 'planning without feedback', where it is assumed that once the optimal import mix is obtained and the amount that should be imported of each raw material is specified to each purchasing department, the planner's role ends. This is equivalent to saying that the only way in which the import mix planner can act is by altering the amounts of raw materials that are to be imported, for a given set of purchasing policies (one for each raw material) over which he exercises no influence.

3.2 Planning Without Feedback

3.2.1 From Bernoulli's Principle to Realistic Criteria for the Choice of the Import Mix

Because of its generality, the principle of maximization of expected utility will not, by itself, tell much about what decisions should be made concerning the planning of the import mix. To specify which import mix should actually be adopted (rather than state what principles should be followed when planning the import mix), it is necessary to make assumptions about the planner's utility function, about the raw materials' import price distributions, or about both. Not surprisingly, the more restrictive those
assumptions can be made, the easier becomes the analysis of what import mix should be adopted.

The implications of Bernoulli's principle will be considered first for the case where the planner's utility is assumed to be a linear function of the foreign currency expenditure (FCE):

\[ U = a \cdot FCE + b \quad (a < 0, \text{implying that as FCE increases the utility will decrease}) \]

This type of function occurs when the planner adopts a 'risk neutral' attitude. This is the most likely attitude if the funds from which the FCE in the animal feed sector is drawn are considerably larger than any possible value of that FCE.

In this situation the maximization of expected utility is equivalent to the minimization of the expected FCE. For a given set of purchasing policies, the optimal import mix can therefore be obtained from the same model considered in the deterministic case, where the (determinist) import prices are replaced by the expected import prices.

Considering, for the sake of simplicity and without loss of generality, the single-period rather than the multi-period case, the import mix planning problem can be expressed symbolically as follows:

Maximize: \( E(U) = E[a \cdot FCE + b] = a \cdot E(FCE) + b \)

or, since \( a < 0 \),

Minimize: \( E(FCE) = \{E(p)\}^T \cdot \{x\} \)

Subject to: \([A] \cdot \{x\} \geq \{b\} \)

where

\( \{E(p)\} \) : vector of expected values of import prices for the assumed set of purchasing policies \( E(p) = 0 \) for all home produced raw materials; for maize and sorghum the \( E(p)'s \) are derived from the expected fob prices and
delivery-inventory costs;

\{ x \} : vector of the amounts of each raw material going into each feed

\{ A \} : matrix of coefficients of the constraints (as for the deterministic case).

In this formulation only the first moment of the distribution of FCE, i.e. \( E(FCE) \), plays a role in the choice of the optimal import mix. This will not generally be the case when the utility function is a polynomial of order higher than one. This becomes clear in the case when the utility is a quadratic function of FCE:

\[
U = a.(FCE)^2 + b. FCE + c
\]

with

\[
\frac{dU}{dFCE} = 2.a. FCE + b < 0
\]

The value \( \frac{dU}{dFCE} \) represents the marginal utility, which varies at the constant rate \( \frac{d^2U}{dFCE^2} = 2a \) as FCE increases (when a is negative, the planner is a 'risk avoider'). The application of Bernoulli's principle leads to:

Maximize: \( E(U) = E[a. FCE^2 + b. FCE + c] \)

\[
= a. E(FCE^2) + b. E(FCE) + c
\]

\[
= a. \{ V(FCE) - [E(FCE)]^2 \} + b. E(FCE) + c
\]

\[
= a. V(FCE) + \{ a. E(FCE)^2 + b. E(FCE) \} + c
\]

where \( V(FCE) \) denotes the variance of FCE. When \( a < 0 \), it becomes obvious from the expression above that the solution that maximizes \( E(U) \) belongs to the 'efficient set of solutions', defined as comprising all the solutions that for each given \( E(FCE) \) minimize \( V(FCE) \).
This fact is the basis for the 'E-V analysis' approach, first suggested by Markowitz in the context of portfolio analysis. In practical terms, its application to the import mix planning problem (without feedback) involves the following two steps:

(i) solve an LP to derive the solution that minimizes $E(FCE)$ (as for the linear utility function);

(ii) designating this minimum by $E_0^*$, for each $E(FCE) = E_0^* + \lambda \ (\lambda > 0)$, solve a quadratic programme (QP) to derive the solution that minimizes the variance of FCE; symbolically, the QP can be formulated as follows:

Minimize : $V(FCE) = \{x\}^T \cdot \begin{bmatrix} V_p \end{bmatrix} \cdot \{x\}$

Subject to : $[A]. \{x\} \leq b$

and

$\{E(p)\}^T \cdot \{x\} = E_0 + \lambda \ (\lambda \geq 0)$

where

$\{x\}, [A], \{b\}, \{E(p)\}, \lambda \ : \text{as defined above}$

$[V_p] : \text{variance-covariance matrix of import price forecasts.}$

The E-V efficiency curve (see Figure 85) is obtained by parametrizing $\lambda$. Under the assumption of a quadratic utility function (with $a < 0$), the decision maker will be able to find the optimal solution (i.e. the one that maximizes $E(U)$ by choosing on the E-V efficiency curve the point that, in his view, is the best trade-off between yield (measured by $E(FCE)$ and risk (measured by $V(FCE)$). This point will vary from decision maker to decision maker according to the parameters $a$, $b$ of their personal utility functions.
Quadradic and other simple polynomial utility functions have been criticized on theoretical grounds, following work by Arrow\textsuperscript{6} and Pratt\textsuperscript{50}. However, as argued by Dillon\textsuperscript{23} (pp. 33-34), the objections raised by those authors can be overcome following the hypothesis used by Anderson\textsuperscript{4} and Mossin\textsuperscript{46}. The approach suggested by these authors deserves from Dillon\textsuperscript{23}, the following comment (p. 34): 'From the practical point of view, the approach fits in well with the pragmatic approach of regarding a polynomial as an approximation to the unknown true utility function, recognizing that a new utility of net returns function [in the present case, a new utility function of the PCE] should be assessed whenever the decision maker's situation changes significantly'.
Markowitz \(^4^4\) (p.285) found that 'while it cannot be claimed that all (...) utility functions can be accurately approximated by a quadratic, the quadratic nevertheless shows a surprising flexibility in approximating smooth (...) curves' (in the present case, when the degree of risk aversion increases reasonably steadily with FCE).

For any non-quadratic utility functions, the 'E-V analysis' is still strictly valid when the import prices of all raw materials (or, in general, the unit 'returns' of risky assets) are assumed to follow normal distributions (see Dillon \(^2^3\)). In the context of the import mix planning problem, the normality assumption appears to be reasonable in most instances, given the high number of independent potential sources of error when forecasting the import prices. However, for some of the raw materials, there may be instances where the normality assumption is questionable. For example, when price mechanisms aimed at guaranteeing minimum prices to the producers operate, they will introduce considerable skewness in the distribution of the import price forecast errors.

The assumptions made either about the shape of the utility function or about the distributions of the import prices have an obvious intention: to reduce as much as possible the number of parameters of the distribution of FCE that are used for comparing alternative solutions (in the two cases considered above the relevant parameters are the first or the first and the second moments of the distribution of FCE). From the arguments presented before it is clear that the assumptions may be too restrictive. The inclusion of higher moments of the FCE distribution (namely its skewness) in the criterion for the choice of the import mix might therefore be desirable. However, this would mean a much enlarged computational effort, unacceptable
in the present case (as will be discussed below, in the import mix planning problem the computational effort required for the E-V analysis is already critical). The recognition of the value of the 'E-V analysis' as a last resort which, in many cases, has no real alternative, deserved from Samuelson 55, the following comments: 'These writers (Markowitz, Tobin and other major contributors to the 'E-V analysis') have realized that the results can only be approximate, but have also realized that approximate and computable results are better than none' (p.38).

3.2.2 The 'E-V analysis'

In the previous section, the 'E-V analysis' was presented as potentially more satisfactory than an analysis involving only the expected value of the distribution of FCE. However this will actually be the case only if the slope of the E-V efficiency curve is significantly negative, at least along part of the curve. In fact, if the slope is small - i.e. when V(FCE) is practically invariant - this measure of risk will play no useful role in the choice of the optimal import mix.

The slope of the (E,V) curve is a function of:

(i) the expected values, variances and covariances of the various import prices;

(ii) the set of constraints imposed when formulating the import mix planning problem (which limit the substitutability of the different raw materials in the mix); and

(iii) the variables defining the state of the system at the time the planning is done (namely those defining the amounts of raw materials already purchased at that time).

These factors, especially (i) and (iii), will vary with time and will have values which depend on the conditions prevailing at the particular moment when the import planning exercise is carried out.
In order to test whether there are conditions under which the slope of the E-V efficiency curve can have significantly negative values, and in the absence of information on the forecasts made in the past by the purchasing departments of the importing organizations, the 'E-V analysis' was carried out for a planning scenario designed to lead to a more negative slope for the E-V efficiency curve than would occur in most real circumstances.

The design of this planning scenario was carried out by selecting conditions likely to lead to a large negative slope of the E-V efficiency curve. The planning scenario chosen—a one-period planning scenario—is defined in Appendix 5.

As described earlier in section 3.2.1, the 'E-V analysis' involves two basic steps. Firstly the solution that minimizes $E(FCE)$ is derived using an LP model and then, once $E_o = \min E(FCE)$ is obtained, a QP model is solved to minimize $V(FCE)$ for different values of $E(FCE) \geq E_o$ (the constraints of this QP are those considered in the LP plus the one which sets the value of $E(FCE)$ at the desired levels).

For the same number of variables and constraints, a QP requires considerably more computer storage capacity than an LP (a comparison between the storage capacity requirements is given in the manual of the MPOS (version 3) package 70). In addition, the computational effort required by a QP is significantly larger than the one required by the equivalent LP. Within the limitations imposed by the computing facilities available, the QP model could not be solved with the number of variables and constraints adopted in the static LP formulation described in chapter 4. The size of the model was reduced by aggregating those animal feeds with highest degree of similarity into common 'compounds' (as shown in Appendix 5). In this way, the size of the QP (variables, constraints, upper bounds) was reduced
from (275, 231, 163) to (104, 79, 42). The effect of aggregating the animal feeds on the solution of the AP could not be tested directly (since the largest version could not be solved). However, an indirect test was made comparing the results of equivalent LP's (with feeds aggregated or not): this comparison showed that the solutions were practically unaffected.

An attempt was made initially to solve the reduced size QP using any of the four algorithms included in the MPOS (version 2) package. All four algorithms (extensions of the simplex method, based on Kuhn-Tucker theory) failed to find a solution. This failure resulted from a combination of three main factors, namely:

(i) the considerable size of the problem,
(ii) the numerical instability of the algorithms, and
(iii) the fact that, in the available version of the algorithms, these do not accept an initial solution fed in by the user.

The problem was finally solved using the algorithm proposed by Gill and Murray 26, based on a Newton-type method applicable to the solution of linearly constrained problems, in a version made available by the National Physical Laboratory.

After deriving the value of $E_O = \min E(FCE)$ and the corresponding value of $V(FCE)$, the minimization of $V(FCE)$ was carried out for four different values of $E$: those exceeding $E_O$ by 1, 2.5, 5 and 10%.

All the calculations were executed in an ICL 2600 computer. The amount of computer memory required to solve the QP model was about 107 K words and the total computation time required to carry out the 'E-V analysis' (i.e. solving one LP and four QP's) was about 1.5 CPU hours.
Figure 86 summarizes the results of the 'E-V analysis' for the planning scenario chosen. For ease of interpretation, \( \sqrt{V} \) is given (in addition to \( V \)) in the vertical scale.

Although the E-V curve may not follow precisely the one shown in Figure 86 (which was interpolated through points 0 to 4), the general shape of this curve clearly shows that, even in the extreme conditions of the scenario considered, a modest decrease in risk (as measured by \( V(FCE) \) or, equivalently, \( \sqrt{V(FCE)} \)) can only be achieved at the expense of a large increase in \( E(FCE) \). This result means that, given the conditions under which the E-V analysis is strictly valid (quadratic utility function or normally distributed import prices), the solution that minimizes \( E(FCE) \) is near-optimal if not optimal (in the sense of maximizing expected utility).
The result of the E-V analysis formally confirms earlier results (see Chapter 5, section 5) which suggested that the ability to decrease risk by changing the composition of the import mix is rather limited. This is a consequence of the fact that highly substitutable raw materials have strongly correlated prices and raw materials with uncorrelated prices are not substitutable to a significant extent. In view of this fact it is likely that the impossibility of reducing $V(FCE)$ is also true for other moments of the distribution of FCE. This, in turn, implies that the solution derived from the LP that minimizes $E(FCE)$ is likely to be near-optimal irrespectively of the assumptions made about the shape of the utility function or the distributions of the import prices.

These results, which have important practical implications, namely the simplification of the treatment of the import mix planning problem under risk, should be the object of further research aimed at testing their generality. In other words, it would be useful to test

(i) whether the results obtained arise from the special features of the problem analysed (where the home produced raw materials have cost coefficients equal to zero in the objective function which makes them attractive for inclusion in the mix and is likely to reduce the flexibility of the substitution between the imported raw materials), or

(ii) whether they would be maintained in a more general case where all raw materials are costed in the objective function at their real prices — as will be the case for most animal feed manufacturers.
3.3 Planning with Feedback

In section 3.2 the import mix planning problem was studied assuming that the planner's role is restricted to controlling the amounts of raw materials that are to be imported, for a given set of purchasing policies which he cannot influence. In the present section the case of 'planning with feedback' will be discussed, i.e. the import mix planning problem will be considered assuming that the planner does have the capacity to influence the purchasing policies.

The need for this feedback arises from potentially conflicting attitudes towards risk adopted by purchasers and planner. Purchasers usually make decisions on the basis of an attitude towards risk which can be represented by what was described as acceptable probability of regret. They are unaware of the fact that their performance will be judged almost continuously in function of the savings achieved in relation to an 'average policy' such as the 'hand-to-mouth' policy. Acting within the limits imposed on them by top management, purchasers will not necessarily see the country's scarcity of foreign currency as a major decision factor.

The planner (or the planning organization) aims primarily at minimizing the country's FCE and must be aware of the difficulties that unpredictable changes in the foreign currency spent with the animal feed industry can cause.

On occasions, the decision criteria adopted by the purchaser, on the one hand, and the planner, on the other, may clash. This may happen, for example, when at a given point in time the price of a raw material is expected to remain at the current level during the relevant planning horizon. Under these circumstances, the purchaser will usually decide to postpone the purchasing of shipments that are to be delivered sometime
in the future. By doing so, he avoids the risk of regret (i.e. the risk of coming later to regret a wrong decision) without any loss in expected yield (measured by the savings made in relation to the 'hand-to-mouth-policy). From the planner's point of view, however, the situation may be seen in a completely different way. If, for instance, the country is expected to face a shortage of foreign currency in the near future, it may be advantageous to buy the shipments at the present time rather than later. In this way, the expected FCE would not increase and the risk of an unexpected increase in FCE would be avoided.

In a situation such as the one described, the planner's point of view, which takes the national interest into account, should prevail. This can only be achieved under what has been described as 'planning with feedback'. This planning process starts at the importing organizations which, in the light of purchasing policies regarded by them as appropriate, provide forecasts of the import prices of the several raw materials. The planner will then derive the import mix that minimizes E(FCE) and will estimate the relevant measures of risk associated with that mix. Whenever appropriate, he can then suggest changes in the purchasing policies for any number of raw materials. These changes can only be implemented with the agreement of the top management of the importing organizations who, ultimately, are responsible for establishing the purchasing strategy for each raw material (i.e. for setting the potential buying periods within which the purchasers will choose buying policies according to their personal objectives and criteria).

This cyclical process (importing organizations - planner - importing organizations) is bound to give rise to organizational problems and will require special attention during implementation, given the delicate balance that is required in the relations between purchaser, top management and planner.
4. **Overview of the Feedgrains Import Planning and Control System**

In the section above, the analysis of the basic models of the overall planning and control system has been completed. Whilst the interactions between individual models have been taken into account when developing each of them, it is worthwhile to take a final view of the models from a global perspective.

The main interactions between the models will be discussed with reference to Figure 6 (see Chapter 4), taking as the starting point the optimal import requirements of maize and sorghum which are part of the output of the import mix planning model (model 5). These import requirements are inputs to the delivery-inventory model (model 2). On the basis of the maize and sorghum demands and of the demands for other raw materials handled at the grain terminals (exogenously determined) and other parameters which characterize the delivery-inventory system, model 2 yields a near-optimal delivery-inventory policy giving, in particular, the size and timing of the feedgrain shipments.

The size and timing of feed grain shipments is then used, together with short and medium-term forecasts of maize 'futures' prices, as an input to the maize 'futures' purchasing model (model 4). The forecast unit cost of purchasing maize 'futures' derived from this model is then converted to an estimate of the unit costs of maize and sorghum (fob, exporting port). These costs, together with the optimal delivery-inventory costs for various proportions of maize and sorghum (derived from model 2) will then be used as inputs to model 5. This model yields new values for the import mix that minimizes the expected FCE, as well as an estimate of the risk associated with the import mix selected.

In the case of planning with feedback, this estimate can then be used to revise the purchasing strategy adopted by the importing organizations.
For maize and sorghum, this revision will lead to a change in the output of model 4 and hence also of model 5, thus initiating a new iteration of the solution procedure.

In practice, the solutions derived from the integrated set of models are likely to converge rapidly, with minor adjustments taking place after the first iteration. This speedy convergence is partially due to the fact that the total import requirements of maize and sorghum have a low sensitivity to changes in the maize and sorghum import prices relative to the import prices of other raw materials. This low sensitivity introduces a dampening effect in the feedback loops which can be identified in the overall integrated model, since both these loops comprise the determination of the optimal maize and sorghum import requirements. In other words, after one iteration of the overall integrated model has been completed, changes in the import prices of raw materials whether caused by adjustments in the delivery-inventory policy or by changes in the purchasing policy or both) will not cause significant changes in the total import requirements of the two feedgrains.

It now remains to be shown that the relative proportions of maize and sorghum in the optimal mix will also converge rapidly. This can be done considering separately the two feedback loops represented in Figure 6.

(i) Feedback loop involving models 4 and 5: Maize and sorghum prices move closely together and the two feedgrains are purchased under a common maize futures buying policy. Hence, from the point of view of risk, the relative proportions in which they are present in the mix are not relevant. The only relevant aspect is, in fact, the total amount of maize plus sorghum in the mix. This implies that there is no reason for the feedback loop involving models 4 and 5 to give rise
to significant changes in the relative maize/sorghum proportions, after the first iteration is completed, i.e. after the strategy for both maize and sorghum is first defined in the light of risk considerations.

(ii) Feedback loop involving models 2, 4 and 5:
As far as this loop is concerned, the speedy convergence of the relative proportion of maize and sorghum in the optimal mix is guaranteed, since
(a) the unit purchasing cost f.o.b. of each grain is not significantly affected by the amounts that are bought;
(b) in the import price of each grain, the only component that could cause serious difficulties of convergence is the unit delivery-inventory cost, which depends on the amounts of each grain that are imported; however, the effect of economies of scale in the delivery-inventory operations has already been incorporated into model 5 for all possible proportions of maize and sorghum in the import mix and therefore cannot disturb the convergence.

The rapid convergence of the solutions derived from models 2, 4 and 5 is a property of great importance, in view of the simplification that it implies in the solution procedure. This is especially important in the context of the planning and control system under analysis, where:

(i) frequent revisions of the solutions will be required, mainly as a result of the need to adapt the system to frequent changes in the price forecasts of the imported raw materials;
(ii) each iteration of the solution procedure will imply a
t flow of information amongst different organizations.

An important aspect of the proposed planning and control system
is its in-built flexibility to adapt dynamically to changes in the environment.
This flexibility can be illustrated considering what is involved in the
planning and control process that, for any given quarter, leads to the
definition of

(i) import quotas for maize and sorghum (finally set at a date
called earlier the 'planning deadline');
(ii) a delivery-inventory policy (specifying the size and timing
 of shipments to be delivered during the quarter in question),
 and
(iii) a purchasing policy (specifying when the maize 'futures'
 contracts that are to be converted into 'physical' near to
 the purchasing deadline of each shipment should be bought).

For any given quarter, the process of planning the import mix starts
long before the planning deadline, when the planner receives from the
importing organizations the first forecasts of the raw materials'
import prices. As new information becomes available -either on import
prices or on newly committed purchases - the import mix will be readjusted;
committed purchases of maize and sorghum (in fact, maize 'futures' contracts
already acquired and yet to be converted to 'physical' maize or sorghum)
 can be reallocated 'optimally' within the quarter in question or can
be transferred to or from another quarter with maximum flexibility.
When an adjustment of the import mix takes place, the delivery-inventory policy is revised: at this stage of the planning and control process, the size and timing of each shipment of maize and sorghum are derived only for the purpose of providing the purchaser with a provisional schedule of shipments to be delivered during the quarter in question. Possible revisions of this schedule, taking place at a later stage of the planning process, will cause no difficulties to the purchaser if, as proposed, maize and sorghum are bought through the maize 'futures' market.

This process goes on until the planning deadline is reached: by then, the import quotas will have to be finally defined. From that moment onwards the only relevant decisions are those concerned with the delivery-inventory and the purchasing operations. Changes in the parameters of the delivery-inventory system may determine the need for successive revisions of the policy. In order to keep maximum control over the operations, shipping contracts should only be settled near to their deadlines and the 'futures' contracts should be converted to 'physical' near the shipments' purchasing deadlines.

An important aspect of the planning process is the derivation of forecasts of the prices that will be paid for each maize and sorghum shipment, fob at the exporting port, over the future specified quarters. Because of its importance, it is worthwhile discussing this aspect of the planning process in some detail.

According to the proposed purchasing operating doctrine for maize and sorghum, all shipments are first bought through the maize 'futures' market and later on, near to their purchasing deadlines, are converted into 'physical'. The final price paid by the importer for a shipment
can be regarded as a sum of two components:

(i) the average maize 'futures' purchasing cost and
(ii) the cost of converting the maize 'futures' into
    'physical' maize or 'physical' sorghum (fob, exporting
    port, prompt delivery)

In order to derive a forecast of the average maize 'futures'
purchasing cost the purchaser will need initially to forecast the
price movements of maize 'futures', from the current date up to the
shipment's deadline (this can be done using the medium-term forecasting
model discussed in Chapter 8, section 4). From these forecasts, and
according to the buying policy adopted, the purchaser will be able to
predict the active buying period over which the 'futures' will be bought.
The average market 'futures' price during the predicted active buying
period is a reasonable estimate of the average cost that he will pay
for the maize 'futures'.

The cost of converting maize 'futures' into 'physical' maize or
physical sorghum is the difference between the 'cash' fob price of
the feed grain in question and the maize 'futures' price, at the time
when the conversion takes place (i.e. near the shipment's purchasing
deadline). For maize, this cost difference can be forecast using
Kingsman's US-Gulf fob price forecasting model 40 and the medium-term
maize 'futures' price forecasting model analysed in Chapter 8, section 4.

The cost difference for maize and sorghum will generally be different
and will vary over time apparently according to a reasonably constant
seasonal pattern, as it can be inferred indirectly from Figure 13 (see
Chapter 4). It should be observed that the prices presented in that
figure are landed prices at Rotterdam and not as at the US exporting
ports. No historical FOB price series for sorghum (at the US exporting ports) was available during the study. Obviously such information should be recorded by EPAC in the future on a systematic basis so that a proper analysis can be carried out. Essentially such an analysis should be directed towards the development of a medium-term forecasting model for the relative prices of maize and sorghum. In fact, for commodities with prices moving as closely together as maize and sorghum, it is questionable whether medium-term forecasting models developed independently for each of them could provide accurate forecasts of their relative prices. In this situation it is probably better to explore the possibility of 'explaining' directly their relative prices (or prices differences).

In the meantime the best approach is to make an estimate of the relative price, extrapolating the prevailing values at the time of planning the maize and sorghum import requirements, taking into account the crude seasonal variations identified in Chapter 4. Since revisions of the maize and sorghum requirements for each quarter can be made until the planning deadline is reached, the final establishment of the maize and sorghum import quotas will be essentially dependent on the ability to produce forecasts of the sorghum/maize relative FOB prices only up to three months ahead. A relatively crude forecast based on extrapolation (with seasonal adjustment) will be acceptable for that purpose at least until further research is undertaken.
CHAPTER 10

IN CONCLUSION

1. The General Approach

Rather than concentrating on a narrowly defined problem area this thesis covers the whole range of operations involved in the importing of Portugal's requirements of maize and sorghum by a Government agency. The diversity of the operations that were considered required the adoption of a general approach which can be broadly characterized as follows:

(i) Definition of an appropriate boundary for the system to be analysed:
The boundary was set so as to isolate a nearly 'separable' system. In this process, strategic planning decisions have been excluded from the analysis. The research was mainly concerned with the planning and control of the short to medium-term decisions involved in the implementation of exogenously defined strategic objectives. At the same time, in order to avoid situations where an increase in the efficiency of the system would be achieved at the expense of a reduction in the efficiency of other related systems, it was necessary to include in the analysis other organizations and other commodity import operations which interact significantly with the system which is the main object of this study.

(ii) Identification of the structure of relations between the attributes of the set of entities composing the system:
Whenever necessary and possible, this was done through the development of exploratory models, at successively higher degrees of resolution. The objective was to take advantage of 'special features' of the problem that allowed the system to be modelled without giving rise to unmanageable complexity.
(iii) Development of an integrated set of models that takes into account the relevant interactions between the different components of the system:
The objective was to make it possible to derive from the models a set of the solutions for the different aspects of the general problem, which can be combined into a consistent overall decision policy.
In spite of being based on a particular problem, the approach provides a general framework for planning and controlling commodity import operations at a national level. Although the types of models may vary from commodity to commodity and from one organizational context to another, the same basic elements will be found in each case, namely:

(i) the definition of import requirements;
(ii) the planning of delivery-inventory operations
(iii) the formulation of purchasing policies.

For such problems, the approach adopted and the framework proposed in this thesis can play the essential role of allowing the integration of partial solutions into a consistent overall decision making policy.

2. The Models Developed

As a consequence of the operations orientation of the thesis, the models which have been developed in the course of the work are of different natures. The proposed planning and control system is based upon three basic models:

(i) the import mix planning model,
(ii) the delivery inventory model, and
(iii) the maize 'futures' purchasing model.
Important aspects of these models as well as relevant results derived from them will now be reviewed.

2.1 Import Mix Planning Model

Maize and sorghum, imported mainly to supply the animal feed industry, are substitutable with other imported and home produced raw materials.

Following an analysis of the structure of animal feed production and of the availability of home produced raw materials, a static LP model was developed to study the effect of changes in the prices of imported raw materials on the optimal import mix. The import prices were assumed to be independent of the amounts imported, and the optimal import mix was defined, for a given animal feed production target, as the mix that minimizes the country's foreign currency expenditure.

The analysis of the solutions of this static LP revealed important properties of the optimal mix which were exploited later in further developments of the model. It was found that the total quantity of feed grains (i.e. maize plus sorghum) in the optimal mix is practically independent of their relative import prices. The optimal imports of each feed grain were found to have low sensitivity to changes in the prices of other imported raw materials. It was also found that small variations in the relative import prices of the feed grains give rise to considerable changes in the maize/sorghum optimal import mix. For this reason, economies of scale in their delivery-inventory operations have, through their incidence on the import prices, a significant effect on the optimal maize/sorghum mix.
Making use of some of the identified properties of the optimal import mix, the economies of scale in the delivery-inventory operations of maize and sorghum (derived using the delivery-inventory model) were then incorporated in the static import mix model. The model became non-linear, being solved by a piecewise linear approximation (equivalent to a set of LP's).

This model was extended in two areas. Firstly it was made adaptive to the dynamics of the planning situation. A multi-period planning model was initially considered being later simplified into a set of single period planning models. These were formulated so that, at each revision of the import mix plan, committed purchases of each raw material could be reallocated 'optimally' among the different animal feeds that are to be produced. The formulation of the import mix planning problem was also extended so as to give explicit consideration to the risks arising from the fluctuating nature of the raw materials' import prices. A quadratic programming formulation was considered but was found unnecessary in the context of the problem analysed. In fact it was found that no significant reduction in the risk of facing an unexpected large increase in the foreign currency expenditure could be achieved by altering the import mix. Any protection against that risk can only be achieved by adopting appropriate strategies in the purchasing of imported raw materials.

2.2 Delivery-Inventory Model

The delivery-inventory operations of maize and sorghum were modelled together with those of the other raw materials which use the same unloading and storage facilities at the Portuguese port terminals. Prior to the development of the delivery-inventory model, two important aspects of the shipping operations were analysed in detail:
(i) The distribution of actual v. planning arrivals of vessels in port: the hypotheses that the actual dates of arrival of successive vessels are a) normally distributed around the respective planned dates of arrival, and b) independent from each other, were tested and accepted.

(ii) The shipping cost structure: the major shipping cost components were identified and a model was built relating each of these components to various operating parameters (e.g. size of vessel, ports of loading and unloading, location of vessel when hired) and cost factors (e.g. price of fuel, state of shipping charter market). On the basis of this model, a generalized shipping cost function, independent of the features that are particular to each trip charter contract, was derived.

The delivery-inventory problem was formulated as the minimization of variable shipping and inventory costs per unit of time, for acceptable shortage levels in port. A simulation model, providing a realistic representation of the complex delivery-inventory operations at each of the grain terminals, was built; this model was combined with a direct search routine to yield near-optimal delivery-inventory policies.

The success of the simulation-optimization procedure was critically dependent on the ability to estimate, with the help of a single-product simulation model, how each raw material's controllable variables (shipment size size and buffer stock) must be related so as to guarantee that the acceptable shortage levels are satisfied most economically.

The solutions derived from the simulation-optimization model were tested for sensitivity to changes in the main model parameters and were
found to be highly robust. This ensures the validity of deriving solutions from the model (which is static) and indicates that the effectiveness of the model does not depend critically on the accuracy of the estimates or forecasts of those parameters which vary considerably over time (shipping costs, raw materials' prices).

The analysis of the solutions derived from the model also revealed a number of results of practical importance; in this way, it was found that no benefits can be expected from carrying speculative stocks at the Portuguese terminals (indeed, a preliminary analysis revealed that, even in the absence of speculative storage, the current or planned sizes of the grain silos at both ports are already too small). It was also found that due to the high degree of uncertainty associated with the arrivals of vessels in port, no benefits can be expected to ensue from the attempt to coordinate the planned arrivals of vessels carrying different raw materials.

2.3 Maize 'Futures' Purchasing Model

This model, which is in fact a collection of different sub-models, covers the most significant aspects of the problem associated with the buying of feed grain shipments. For given sizes and timing of these shipments, the model allows the definition of a buying policy that seeks the minimization of the long-term purchasing costs, within constraints which are either imposed onto the purchaser by the top management of the importing agency or which arise from the purchaser's own attitude and conception of permissible risks in the purchasing operation.

A purchasing operating doctrine was selected by which the purchasing operations are conducted initially through the maize 'futures' market. The 'futures' contracts acquired are converted to 'physical' maize or sorghum close to each shipment's purchasing deadline. In this way, no extra costs
are incurred in relation to an operating doctrine in which purchases are made directly in 'physical' and the purchaser retains considerable flexibility for changing the sizes and timing of shipments and the type of grain, until the shipping deadline of each delivery is reached.

Preceding the analysis of the buying decisions a study of price forecasting models for the Chicago maize 'futures' market was carried out. Taylor's price-trend model was analysed in detail and the statistical significance of short-term price trends in the maize 'futures' market was established. Kingsman's behavioural price model of Chicago 'cash' prices was adapted to the medium-term forecasting of maize 'futures' prices.

The 'tactical' buying problem was then addressed and a new policy derived heuristically, was proposed. The policy was shown to lead to statistically significant savings in relation to the average market prices. In times of high price volatility, the policy proposed was shown to perform better than Kingsman's classical dynamic programming policy and to lead to sizeable savings.

The buying policy proposed is relevant when the purchaser has no access to reliable medium-term price forecasts or when he is restricted to buy over short potential buying periods.

When good medium-term price forecasts are available and the potential buying period is extended, the problem facing the purchaser is one of selecting an appropriate active buying period within which the day-to-day purchasing decisions are to be taken.

A policy was proposed to select the start of the active buying period, taking into account the purchaser's conception of permissible risk in the purchasing operations. No conclusive results were obtained concerning the choice of the length of the active buying period. Before further research is carried out it seems advisable to leave this choice to the purchaser's judgement.
The simulation of the proposed policy (with the length of the buying period set at an arbitrary constant value over time) although yielding very encouraging results is not sufficient, on its own, to validate conclusively the policy proposed for initiating the active buying periods or to evaluate the policy's long-term performance. However, they strongly suggest that

(i) substantial savings in relation to the hand-to-mouth policy can be derived from its adoption;

(ii) the magnitude of the savings is not significantly affected by the choice of the length of the active buying period, between its minimum possible value (1 day) and 2 months (i.e. the purchasing performance depends mainly on the choice of the start of the active buying period).

An improvement in the purchasing performance might ensue if a convenient way of adapting the length of the active buying period according to the current medium and short-term price forecasts could be devised.

3. **Areas for Further Research**

In previous chapters, several areas for further research have already been discussed. The most interesting of these, either in terms of their practical importance or in terms of the theoretical questions they raise, will now be summarized:

(i) Generalization of properties of the 'optimal' import mix:

In the import mix planning problem considered in this research, the optimal mix was found to have properties with important practical implications (e.g. the inability to reduce significantly the risk of facing unexpected increases in the foreign currency expenditure by acting on the mix). It would be useful to test
whether these properties arise from the special features of the problem analysed - with a sizeable proportion of raw materials with zero 'cost' - or, alternatively, whether they are generalizable to situations where all raw materials are costed in the objective function at real prices. In addition, it would be of practical interest to try to identify groups of raw materials which, like maize and sorghum, may have special substitution properties arising from the similarity of their nutritional characteristics.

(ii) Definition of the most appropriate operational characteristics for the grain terminals:

A preliminary analysis presented in Chapter 6 indicated that the storage capacity at the grain terminals are far short of the current needs. The definition of the most appropriate operational characteristics for those terminals poses a capital investment problem of high complexity. This complexity arises mainly as a result of

(a) the difficulty of making meaningful predictions of the return on investment, and

(b) the fact that, as the storage capacity is increased, the interaction between the planning of the raw materials' deliveries (size and timing of shipments) and the planning of purchases may become significant.

(iii) Medium-term forecasting of sorghum/maize relative US fob prices:

For commodities with prices moving as closely together as those of maize and sorghum it is questionable whether medium-term forecasting models developed independently for each of them can provide accurate forecasts of their relative prices.
In this situation it is probably better to develop models that 'explain' directly their relative prices. Factors such as differences in the timing of the harvests or in the storage costs, for example, are likely to 'explain' seasonal relative price movements. Year to year changes in the relative prices will be influenced by factors such as the relative abundancy or scarcity of each grain or even the quality of the grain.

(iv) Further testing and refinement of the 'tactical' buying policy:

The 'tactical' buying policy proposed in this thesis was derived heuristically. Attempts to further test and refine this policy should be pursued. It would be of practical interest to establish a relationship between the performance of the proposed policy (or improved versions thereof) and the parameters of the stochastic process generating the price series (parameters $p_o$ and $A_o$, in Taylor's price-trend model). Such a relationship, which could be derived through simulation, would allow the estimation of the potential savings, resulting from the application of the policy, directly from the characteristics of the commodity price series.

(v) Specification of the length of the active buying period, in the context of a medium-term buying policy:

In the context of a medium-term buying policy, the success of the tactical buying policy proposed could not be ascertained. Further research is required in order to explore the possibility of combining both short and medium-term price forecasts in the process of defining (rationally) the length of the active buying period. The results of such a study are unlikely to lead to a substantial increase in the purchasing performance of medium-
term policies. Nevertheless the research effort would be beneficial if progress could be made towards the combination of the short and medium-term forecasts (currently used separately) into a unified price forecast.

4. **Closing Remarks**

It is by now abundantly clear that both the inspiration and the orientation of this work have been of an eminently practical nature. It is, therefore, suitable to end these conclusions on an equally practical note.

Regardless of how interesting and potentially useful the models developed in the course of this research may be, they do not, in themselves, contribute towards the solution of the problems that inspired their formulation and development. If such a contribution is ever to take place, if the research work that has been presented in this thesis is ever to lead to actual, rather than potential, benefits, both the models and the approach will have to be subjected to the acid test of implementation. It is a sensible - and rather obvious - norm that the O.R. analyst should remain, at all times, aware of the difficulties and pitfalls of implementation. This is especially important in a case like the present, where interaction with the organizations which may come to benefit from the research done has been at a minimum. In fact, this means that the learning process which is so important for successful implementation has hardly started and that the organizations will suddenly be confronted with results and recommendations which are the outcome of a research project in which they did not participate actively (with the exception of a few individuals).
The sheer size of the system which was subject to analysis and the complexity of the interactions among its parts constitute an additional obstacle to a successful implementation. It is well known, in fact, that resistance to innovation is positively correlated with the magnitude, complexity and novelty of the innovations proposed.

On the positive side, the magnitude of the potential savings and the practical orientation of the research are assets for implementation, since the first provides a motivation for, and the second helps to ease the process of implementation. Also on the positive side, there are non-quantifiable benefits to be derived from the efforts involved in analysing the system, formulating its structure and modelling its parts. This effort cannot but have a positive impact upon the quality of management and decision-making, to the extent that it helps' ... to convert muddled thinking and amorphous deliberations into an orderly analysis, allowing crucial issues to be highlighted and purposely debated' (Eilon \textsuperscript{24}, p.38).

In practice, what these considerations amount to is that it will be necessary, and it will not be easy, to develop a strategy for implementation in which not only technical, but also personal and other considerations must play a crucial role, especially in overcoming the natural initial resistance to the innovations proposed.

FINIS LAUS DEO
REFERENCES


APPENDIX 1

SAMPLE OF FEED GRAIN SHIPMENTS SCHEDULED TO ARRIVE AT PORTUGUESE PORTS DURING 1976

(Source : EPAC records)

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(*) Sorghum shipments (maize shipments otherwise).
## APPENDIX 2

**SAMPLE OF FEED GRAIN SHIPMENTS SCHEDULED TO BE LOADED AT US PORTS DURING THE PERIOD APRIL–NOVEMBER 1977**

(Source: EPAC records)

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(A) - first port of destination in the Azores (otherwise: mainland)

(*) sorghum shipments (otherwise: maize)
### Appendix 3

**Sample of Dry Bulk Carriers with Trip Charters Reported Between May 1977 and January 1978: Deadweight Tonnage, Speed and Fuel Consumption at Sea**

(Source: Shipping Statistics and Economics)

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(*) Dates specified as six digits: Year __, Month __, Day __
APPENDIX 5 - SCENARIO FOR THE E-V ANALYSIS

1. General Conditions Leading to a Large Negative Slope of the E-V Efficiency Curve

1.1 The more negative the slope of the E-V efficiency curve is, the larger will be the possible trade-off between yield (measured by E) and risk (measured by V). This trade-off will be larger when the mix which minimizes E includes mainly those raw materials with large import price variances and when the mix that minimizes V relies mostly on those raw materials that in terms of their expected values are less attractive.

1.2 The possibility of a large trade-off between E and V is likely to increase as the differences between the raw materials' import price variances increase. Over a given planning horizon the largest differences between the import price variances will occur for the last period of the horizon when the best possible price forecast is produced for some raw materials (e.g. for purchases to be made in the very near future) and the worst possible forecast is produced for other raw materials (e.g. purchases left to be bought at the last buying opportunity and current price used as the forecast).

1.3 For two substitutable raw materials 1, 2 with given import price variances, \( \sigma_1^2 > \sigma_2^2 \), the possibility of achieving a maximum reduction in V with minimum increase in E, by replacing one raw material for the other, will be highest when:

(i) the ratio between the expected import prices \( \mu_1/\mu_2 \) is such that in the mix that minimizes E, raw material 1 is
near to its maximum possible amount in the mix and there 
is none of raw material 2 and when min E is least sensitive 
to changes in the amounts of raw materials 1 and 2;

(ii) For a given dynamic formulation of the import mix problem, the 
possibility of a large trade-off between E and V is highest 
when no purchases have been committed, since committed purchases 
reduce the flexibility of the import mix left to be bought.

2. Definition of the Scenario

Having in mind the conditions above, a scenario was defined with the aim 
of representing a hypothetical situation leading to a slope of the E-V 
efficiency curve more negative than in most real situations.

A single period formulation was adopted isolating from the whole 
time horizon, assumed to be six months, the last one-month period - the 
one for which the trade-off between E and V is potentially largest. 
Committed purchases were assumed to be zero.

The price of maize was arbitrarily fixed at 100; all the other prices 
were fixed in relation to this price.

The raw materials were classified into six categories:

(i) maize and sorghum;

(ii) oilseed meals;

(iii) other sources of proteins (fish meal, maize gluten meal, alfalfa 
meal, synthetic aminoacids);

(iv) can molasse;

(v) dicalcium phosphate;

(vi) home produced raw materials.
The two main categories of raw materials that can affect significantly the slope of the E-V efficiency curve are the feed grains (imported by EPAC) and the oilseed meals (imported by IAP0). These raw materials make up, in any price circumstances, nearly 90% of the total FCE.

The raw materials within each of these categories have closely related prices and are bought by the same organization. Their price forecasts cannot have very different reliabilities (e.g. it would be inconceivable to assume a 5% standard deviation for the import price of maize(*) and a standard deviation of 25% for the import price of sorghum).

Having the choice of considering the standard deviations of the import price forecasts to be

(a) low for feed grains and high for oilseed meals, or
(b) vice-versa

the first alternative was chosen for two reasons:

(i) If the sorghum and maize import prices were assumed to have high standard deviations, realistically they would have to be almost identical and the correlation between the import prices would have to be nearly one. This does not necessarily hold for more accurate forecasts. When the standard deviations are small, the relative difference between one and the other can be larger than in the first situation, and the substitution between maize and sorghum can contribute more significantly to a negative slope of the E-V efficiency curve.

(*) in this appendix all the standard deviations of the import prices are expressed as a percentage of the expected import prices.
(ii) The evidence of past market prices suggests that the variability of the prices of oilseed meals can be considerably larger than that of feed grain prices (see Figures 13 and 15, Chapter 4); for the price series shown in these figures, from September 1972 to December 1975 - the period of highest price volatility both for grains and oilseed meals - the standard errors of the 6-month ahead relative price changes (*) were about 25% for maize and sorghum and around 50% for the oilseed meals; the largest possible difference between import price variances of these raw materials would have occurred if reliable forecasts of maize and sorghum prices (e.g. with a 5-10% standard error) and the worst possible forecasts of oilseed meals (**) (i.e. with a 50% standard error) were produced at the same time.

Once the expected value and the standard deviation of the maize price were fixed (see Table A5.1), in order to achieve the largest reduction on V for the smallest increase in E (see condition 1.3 above), the parameters of the sorghum import price distribution were set as follows: the expected value was set at 105 (enough to ensure that sorghum

(*) The six-month ahead relative price changes are defined as $\frac{P_{t+6} - P_t}{P_t}$ were $P_{t+6}$ are average monthly prices.

(**) The worst possible 6-month ahead price forecast would be obtained using the current price as a predictor of the 6-month ahead price.
does not enter in the min E mix) and the standard deviation was set at 5% (considered a lower limit, given that for maize the assumed standard deviation was 10%); the correlation between maize and sorghum import prices was assumed to be zero (inducing a more negative slope of the E-V efficiency curve than if it was assumed positive - as in reality would happen).

Table A5.1 - Assumed expected values and standard deviations of the import prices

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Expected value</th>
<th>Standard deviation (percentage of expected value)</th>
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</thead>
<tbody>
<tr>
<td>Maize</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>Sorghum</td>
<td>105</td>
<td>5</td>
</tr>
<tr>
<td>Cane</td>
<td>45</td>
<td>30</td>
</tr>
<tr>
<td>Sunfl. meal</td>
<td>82</td>
<td>40</td>
</tr>
<tr>
<td>Soyb. meal</td>
<td>110</td>
<td>54</td>
</tr>
<tr>
<td>Pean. meal</td>
<td>118</td>
<td>45</td>
</tr>
<tr>
<td>Alfal. meal</td>
<td>80</td>
<td>25</td>
</tr>
<tr>
<td>M.glu. meal</td>
<td>190</td>
<td>25</td>
</tr>
<tr>
<td>Fish meal</td>
<td>250</td>
<td>50</td>
</tr>
<tr>
<td>Lysine</td>
<td>2 600</td>
<td>47</td>
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<tr>
<td>Methionine</td>
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<tr>
<td>Dic.phosph.</td>
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<td>H.produced</td>
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The standard deviations and the correlation coefficients of the import prices of the oilseed meals (see Tables A5.1 and A5.2) were set equal to the standard errors and to the correlation coefficients of the 6-month ahead relative price changes for the series shown in Figure 15, chapter 4, during the period September 1972 - December 1975 (this set of estimators reproduces what would happen in a period of high price variability if the current import prices were used to 'forecast' the 6-month ahead import prices). The expected values of the import prices of oilseed meals were defined as follows:
Table A5.3 - Ranges of prices of all raw materials relative to maize (maize price = 100)

<table>
<thead>
<tr>
<th>Raw material</th>
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<th>Max.</th>
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<tr>
<td>Sorghum</td>
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<td>95</td>
<td>110</td>
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<tr>
<td>Cane molasses</td>
<td>45</td>
<td>65</td>
<td>75</td>
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<tr>
<td>Alfalfa meal</td>
<td>55</td>
<td>80</td>
<td>165</td>
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<tr>
<td>Sunflower meal</td>
<td>70</td>
<td>100</td>
<td>205</td>
</tr>
<tr>
<td>Soybean meal</td>
<td>110</td>
<td>160</td>
<td>330</td>
</tr>
<tr>
<td>Peanut meal</td>
<td>110</td>
<td>160</td>
<td>330</td>
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<tr>
<td>Maize glu.mea</td>
<td>151</td>
<td>220</td>
<td>452</td>
</tr>
<tr>
<td>Fish meal</td>
<td>231</td>
<td>336</td>
<td>693</td>
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<tr>
<td>Lysine</td>
<td>2400</td>
<td>3500</td>
<td>6700</td>
</tr>
<tr>
<td>Methionine</td>
<td>1500</td>
<td>2100</td>
<td>4200</td>
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</table>

Note: for the sources of proteins the range was obtained by multiplying the soybean meal range by the relative price source of protein / soybean meal.

The parameters of the cane molasses import price distribution were set having in mind that, as far as this raw material is concerned, the most important substitution effect that can take place is with the feed grains. The expected value and the standard error of the import price were set following the criterion adopted for other raw materials in similar circumstances. The correlation coefficients were set at zero.

For dicalcium phosphate the parameters of the distribution were fixed arbitrarily at 100 (expected value) and 0 (standard deviation). Since this raw material is not substitutable to a significant extent with any of the other raw materials, the values attributed to those parameters cannot affect the analysis.

Zero prices were assumed for the home produced raw materials.

3. Aggregation of Feeds into 'Equivalent Compounds'

In order to reduce the size of the quadratic programme, animal feeds with similar specifications were grouped into 'equivalent compounds',
Table A5.4 - Aggregation of animal feeds into 'equivalent compounds'

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<td>CS-2</td>
<td>S-820, S-816, S-830</td>
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<td>CA-1</td>
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</tr>
<tr>
<td>CA-2</td>
<td>A-120, A-125, A-130</td>
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<tr>
<td>CB-1</td>
<td>B-320, B-321</td>
</tr>
<tr>
<td>CB-2</td>
<td>B-332, B-330, B-334</td>
</tr>
</tbody>
</table>

as shown in Table A5.4.

The characteristics of each compound (nutritional requirements, limits on the contents of the different raw materials) were set equal to the weighted average of each feed's characteristics (with weights proportional to each feed's production).