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Seismic Behaviour of Reinforced Concrete Bridges Modelling
Seismic Behaviour
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RESUMO

Este trabalho foi desenvolvido no âmbito do programa de Investigação Prenormativa da Comissão Europeia de Suporte ao EuroCódigo N. 8 (EC8). Este código pretende ser o standard Europeu para o dimensionamento de estruturas de Engenharia Civil sujeitas a acções sísmicas. O programa incluiu uma campanha de ensaios Pseudo-dinâmicos realizados em pontes de betão armado projectadas de acordo com o referido código. O método Pseudo-dinâmico é um método híbrido que combina a integração numérica das equações de movimento com a medição experimental das forças de restituição da estrutura. Nos ensaios realizados o tabuleiro foi sub-estruturado e apenas os pilares foram testados no laboratório. Note-se que este foi o primeiro teste Pseudo-dinâmico realizado numa estrutura de grandes dimensões utilizando a técnica de sub-estruturação.

Testes preliminares realizados com o modelo de fibras permitiram antecipar o valor máximo das forças de restituição esperadas no laboratório. Estes resultados permitiram ainda verificar a capacidade do modelo para prever o comportamento de pilares de ponte sujeitos a acções cíclicas. A necessidade de desenvolver um modelo que representasse convenientemente o comportamento não linear às forças de corte resultou da comparação entre a resposta experimental e os resultados das previsões numéricas. Assim, acopulado ao modelo de fibras para as forças de flexão foi desenvolvido um modelo do tipo biela-tirante para as forças de corte. Esta formulação é baseada na analogia entre uma estrutura de betão armado com fissuração diagonal e uma treliça constituída por elementos diagonais de betão ligados por elementos longitudinais e transversais de aço.

A simulação dos ensaios no laboratório foi repetida utilizando o modelo desenvolvido e os resultados foram comparados com a resposta experimental. As diferenças ainda assim encontradas são discutidas e analisadas em detalhe. Para terminar, a resposta numérica de uma nova série de pontes com diferentes graus de irregularidade, foi projectada de acordo com o código EC8 e analisada para acções sísmicas de intensidade crescente.
RESUMÉ

Le travail ici présenté a été développé dans le cadre de la Recherche Prénormative, comme support du EuroCode8 (EC8), des Commissions Européennes. EC8 est le standard provisoire Européen pour le design de structures de génie civile, dans les zones de risque sismique. Ce programme inclut des tests expérimentaux pour un certain nombre de structures de ponts, testées sous conditions Pseudo-dynamiques. La méthode Pseudo-dynamique est une méthode hybride qui combine l'intégration numérique des équations du mouvement d'une structure, et les mesures expérimentales des forces de restitution correspondantes. De plus, le tablier a été substructuré et seules les piles ont été testées physiquement dans le laboratoire. En effet, ces expériences ont été la première série de tests Pseudo-Dynamiques effectués avec une technique de substructuration.

Des tests numériques réalisés avec le modèle de fibres ont été executés avant les expériences afin de prédire la réponse des structures. Ces résultats préliminaires permettent non seulement de vérifier la capacité du modèle pour la prediction du comportement de ponts en béton armé soumis à l'action de charges cycliques, mais aussi d'identifier les forces maximales admises par le système de contrôle du laboratoire. Le besoin d'améliorer un modèle pour l'étude du comportement au cisaillement non linéaire survient de la comparaison entre la réponse expérimentale et les résultats des prédictions numériques. Ainsi une formulation “strut-and-tie” accouplée au modèle de fibres classique a été développée. Cette formulation est basée sur l'analogie d'une structure en béton armé endommagée par des fissures diagonales, avec une poutre en treillis constituée de diagonales en béton et d'attaches en acier.

La série numérique a été répétée en employant le nouveau modèle et les résultats ont été comparés avec les réponses expérimentales. Les différences encore trouvées sont analysées dans ce travail. Finalement, une nouvelle série de ponts est projectée et leur réponse à des intensités sismiques croissantes est évaluée.
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8 CONCLUSIONS AND FUTURE RESEARCH
1 INTRODUCTION

1.1 GENERAL

Earthquakes damage civil engineering structures every year and bridges are no exception. Historically, bridges have proven to be vulnerable to earthquakes, sustaining damage to substructures and foundations. In some cases, due to unseating of the spans or local failure of the restrains, bridges are totally destroyed as superstructures collapse from their supporting elements [6]. The vertical component of the acceleration also contributes for this outcome. To prevent the falling out of bridge girders, restrainer tie plates are in general used as studied by Obata et al [55]. Shear failure or insufficient ductility in the piers due to poor confinement of the core concrete and poor detailing of the transverse and longitudinal steel, are also observed in several bridge piers emphasizing the importance of the shear strength in the design.

As a matter of fact, these seismic events represent an important ‘in situ’ source of information that should not be neglected and, furthermore, should always be present in the improvement of design codes and guidelines. In fact, significant advances in seismic design and strengthening of bridges have occurred after large earthquakes. The 1971 San Fernando earthquake caused substantial damage and exposed a large number of deficiencies in the design specifications of that time. During the Loma Prieta earthquake of October 1989, the dramatic collapse of the Cypress Street Viaduct in Okland and the damage of many elevated freeway bridge structures in the San Francisco bay area, highlighted weaknesses in bent joints, lack of ductility in beams and columns and poor resistance to longitudinal and transversal loads. Inspections (observed damage) after the earthquake revealed shear cracking and spalling of concrete especially in outrigger knee joints on
bends of several reinforced concrete viaducts [79]. In some cases, the shear force concentration in shorter piers caused by the irregular profile of some bridges was not properly taken into account in the design.

More recent events like the Northridge earthquake of January 1994 [51], [86], and the Kobe earthquake of January 1995 [27], [65], stressed once again those deficiencies. In particular, shear failure was still observed in several bridge piers emphasizing the importance of the design and the detailing of transverse steel to provide proper shear strength capacity. Changes in the design code and guidelines were and still are undertaken to compensate for these shortcomings.

Notice that, in agreement with the design codes, structures in seismic areas may experience inelastic behaviour during a large earthquake. The most recent approaches are based on the capacity of the structures to dissipate energy through the hysteretic behaviour of the piers; a certain degree of damage is thus acceptable. The amount of inelastic behaviour allowed in the design is "measured" through a behaviour factor $q$ that reduces the design forces computed through a linear elastic analysis. This factor depends on the importance of the bridge, i.e. on the requirements for post-earthquake functionality. Thus, different levels of acceptable ductility and damage are usually provided in the design codes.

A Pre-normative Research in support of EuroCode N. 8 (PREC8), the provisional European standards for the design of civil engineering structures in seismic prone areas, was launched by the European Commission (EC) to cover the topics of the EuroCode N. 8 (EC8) that needed to be clarified. Several working groups, each one dealing with different type structures, were established under this programme, namely: Reinforced concrete frames and walls, Infill frames, Bridges and Foundations and retaining walls. This activity was performed jointly with eighteen research organisations in the European Union grouped together in the PREC8 on Human Capital and Mobility (HCM) network. It included a large experimental campaign on different structural elements such as walls, frames and bridges. Moreover, a series of numerical analyses were also performed in order to extrapolate the experimental results to other input actions and structures.
1.2 MOTIVATION

The subject of the present work, the seismic behaviour of reinforced concrete (R/C) bridges, was the general topic under discussion by the Bridges Working Group (BWG) established within the PREC8 programme. Among other points, the programme for bridges included: the classification of structural regularity, the evaluation of behaviour factors, the improvement of methods of analysis and of capacity design procedures and the consideration of base isolation and asynchronous input motion. Notice that, although a general design code exists for civil engineering structures, bridges exhibit some characteristics that make them quite different from ordinary buildings, thus demanding special guidelines. Firstly, the mass of the bridge deck is an order of magnitude larger than the mass of a typical floor system. Secondly, bridge structural systems are not as redundant as typical building structures. Thus, in June 1994 a Pre-Standard for the seismic design of bridges, EC8 Part2: Earthquake Resistant Design of Bridges, was approved by the European Committee for Standardization (CEN).

The work follows closely this pre-normative research programme, responding to some of the needs of the bridges working group, namely: the analysis of the experimental response of a set of bridges, with and without isolating devices, tested under synchronous and asynchronous dynamic loading, the evaluation of behaviour factors and structural regularity and the development of numerical models for bending and shear forces to simulate bridge type structures, in particular the piers. The irregularity issue is discussed based on an expression developed by other authors that associate the combination of the deck and piers mode shapes with a single parameter.

1.3 THE ELSA LABORATORY

The European Laboratory for Structural Assessment (ELSA) in the Joint Research Centre (JRC) of the EC at Ispra in Italy, was the main supporter of the work. The excellent conditions of this laboratory, providing the hardware and software and all the necessary experimental equipment, allowed experimental tests on reinforced concrete structures to be carried out, and more suitable analytical models to represent the structures tested in the laboratory to be improved. Notice that to check and calibrate a numerical model it is not only necessary to have access to experimental data but also to have a good knowl-
edge of the characteristics of the specimens and testing conditions in the laboratory that permitted this data to be obtained. In the case of this work, the author had the opportunity to follow all the experimental tests performed on bridges in the ELSA laboratory, participating actively in the post-treatment of the results.

In this laboratory, the experiments are referred to as *Pseudo-dynamic tests*. The method associated to these tests is a hybrid method that combines the numerical integration of the equations of motion of a structure and the experimental measurement of the corresponding restoring forces. Since the inertial forces are simulated numerically, a reduced hydraulic power is required and the test is performed with a time scale enlarged with respect to real time. The equations of motion are solved on-line using a step-by-step numerical integration algorithm using the physical forces from the specimen and the inertial and the damping forces from the analytical model. Thus, to perform such a test it is necessary to possess a rigid structure to sustain the reaction forces applied to the structures. The European Laboratory for Structural Assessment has the most important reaction-wall facility in Europe.

Notice that a campaign of experimental tests on structures representing different configurations and design solutions is the ideal means to verify the adequacy of design strategies. However, the costs associated with an experimental campaign with such large number of test specimens make such programmes very expensive. Moreover, the progresses in non-linear modelling in the recent years confine the testing activity to special cases, verifying extreme design solutions or the behaviour of critical regions, and using the results as the basis for model calibration. It must be pointed out that numerical results are also used for predicting the response of the structures in the laboratory, thus helping to prepare the experimental campaign.

Therefore, the numerical analysis is complementary to the experimental analysis; the numerical models that are used for simulating structures not tested in the laboratory are calibrated with the experimental results. In the ELSA laboratory, the numerical analysis are performed with the computer code CASTEM 2000, an object oriented finite element code developed by a group of researchers of the *Commissariat à l'Energie Atomique* (CEA) at Saclay in France. Thanks to the generality of its data and the different levels of
modelling available, different analyses can be done within the same computational environment. Moreover, such computer code, being completely opened to the user, allows new improvements to be implemented.

1.4 DEVELOPED WORK

Thanks to the conditions of the laboratory described in the previous paragraphs, the work carried out in this thesis includes the experimental and the numerical analyses of bridges. The experimental campaign consisted of Pseudo-dynamic tests on a series of bridge structures using a substructuring technique: the piers were tested in the laboratory and the deck was simulated numerically on-line. The first part of the experimental campaign concerned the test of four bridges: one irregular bridge and one regular bridge designed according to the EC8, and two others representing two alternative design solutions to the irregular bridge.

The testing campaign in the ELSA laboratory was completed afterwards with three supplementary tests, namely: two bridges with isolating/dissipating devices and one bridge with asynchronous input motion. To prepare the experimental campaign, a quasi-static cyclic test was also performed on the short pier of the EC8 irregular bridge. The aim of this test was to evaluate the adequacy of the testing devices and instrumentation in the laboratory and to obtain the experimental response of a squat pier damaged by a controlled displacement history up to failure. The experimental results allowed not only the study of the behaviour of the set of bridges tested in the laboratory, but also the calibration of the analytical model used in the numerical analysis that preceded each experiment.

To simulate the bridge piers, a fibre model was proposed. This model is in between the local and the global formulations. Although the algorithm computes the global deformations at the level of the Gauss points of the structural elements, the response is given by the integral of the local forces calculated at different points representing the different materials in the transverse section. Thus, the fibre model can be regarded as a step further in the refinement of standard beam models. In fact, it uses the same algorithms to compute the deformation of the longitudinal axis of the finite elements: three rotations and three displacements at each node. The difference between this model and the stand-
ard beam model is in the procedure that it follows to calculate the resisting forces; instead of considering a global constitutive law at the level of the transverse sections, the fibre model goes deeper in the cross-section and computes the deformation and the stress of a set of points forming a mesh in the transverse section. The model was implemented in the object oriented computer code CASTEM 2000.

A series of numerical tests using the fibre model were performed before the experiments to predict the response of the structures. These preliminary results allowed us to verify the ability of the model to predict the behaviour of R/C bridges under dynamic loading. Furthermore, the maximum forces admitted by the control system in the laboratory were established taking into account these preliminary numerical results: an alarm system stops the Pseudo-dynamic procedure whenever the imposed forces go above that limit.

After the experimental campaign, the fibre model was improved, in particular to take into account the non-linear behaviour of R/C squat piers under high shear forces. The experimental tests performed in the ELSA laboratory on piers in this conditions showed that a quite different behaviour compared to piers under predominant bending moments would be expected. Therefore, to represent the response of such elements subjected to important transverse forces, a strut-and-tie formulation coupled with the classic fibre model for bending forces was developed. This formulation is based on the analogy of a R/C structure damaged with diagonal cracking with a truss made of concrete diagonals and steel ties. The experimental data were exhaustively used for checking and calibrating this model.

Subsequently, the numerical campaign was repeated using the fibre model with the inclusion of the non-linear behaviour in shear, and the results were compared with the experimental response. Topics like the flexibility of the foundation block and the importance of the shear displacements to the global response of the squat piers, are discussed. The differences still found between the numerical and the experimental results are analysed. Finally, the fibre model was again used to study the behaviour of a set of bridges representing different configurations in between the two bridge profiles tested at the ELSA laboratory: the so-called regular and irregular profiles, in order to extract conclusions about the regularity issue. The all set of bridges was designed for a behaviour fac-
tor of 2.5 and the responses were analysed for the accelerogram used in the experimental campaign multiplied by growing intensity parameters.

1.5 OUTLINE OF THE THESIS

The work is divided into seven more chapters. The Pseudo-dynamic technique, its potentials, advantages and disadvantages, in relation to other testing methods are described in chapter 2. The implementation procedures and the control system in the ELSA laboratory at Ispra, are also detailed in that chapter. The post-treatment of the experimental data and the analysis of the results is presented in chapter 3. Notice that this was the first Pseudo-dynamic testing campaign performed in the world using a substructuring technique on large-scale structures.

The description of the fibre model implemented at the ELSA laboratory is done in chapter 4. A series of numerical tests used for checking the model and highlighting its potentials are presented. The cracking of concrete and the simulation of bi-axial bending actions are also subjects under discussion in that chapter. The new model with the strut-and-tie formulation for shear forces coupled with the fibre model for bending forces, is described in chapter 5. At the end of this chapter, the numerical model is applied to a series of piers tested in the ELSA laboratory and the responses are compared to the experimental results.

In chapter 6, the analytical results of the bridges when the predictive model and the non-linear shear model are used, are analysed and compared with the experimental response. The chapter is divided into three parts: the pre-experimental tests, the post-experimental tests and, at the end, a supplementary set of numerical tests performed in order to analyse more in detail the differences still found between the post-experimental and the experimental results.

The numerical results of a new set of R/C bridges designed according to the EC8 and representing different configurations in between the so-called regular and irregular profiles tested in the ELSA laboratory, are analysed in chapter 7. The final conclusions and future research are drawn in chapter 8.
2 THE PSEUDO-DYNAMIC TEST METHOD

2.1 INTRODUCTION

The Pseudo-dynamic test method is a hybrid method which combines the numerical integration of the equations of motion of a structure and the experimental measurement of the corresponding restoring forces. Since the inertial forces are simulated numerically, a reduced hydraulic power is required [25] and the test is performed with a time scale enlarged with respect to the real time. This essentially means that the same equipment as conventional quasi-static tests is used; hydraulic actuators impose a history of prescribed displacements on the specimen and load-cells on the actuators measure the force necessary to apply the corresponding displacements. The equations of motion are solved online using a step-by-step numerical integration algorithm using the physical forces from the specimen and the inertial and damping forces from the analytical model.

In such a test, the structure is “condensed” on a reduced number of degrees-of-freedom (d.o.f.), those controlled by the actuators in the laboratory during the experiment. This static condensation performed indirectly in the test, only preserves exactly the static (elastic) behaviour of a structure, while it may alter considerably its dynamic behaviour due to the spurious mass and damping redistribution it induces in the structure. The use of reduced matrices not only imposes that the dynamic behaviour should be mainly controlled by the lowest modes of vibration but also that the lowest frequencies of the whole system should be as close as possible to the frequencies of the reduced system. Moreover, those modes of vibration should be well represented by the reduced d.o.f.

The concepts of the Pseudo-dynamic technique appear during the late 60s contempor-
neously to the installation of servo-controlled hydraulic actuators in structural testing laboratories. The Japanese were the first researchers to study and apply this technique to structural engineering. In fact, the largest facility in the world belongs to the Building Research Institute (BRI) of the Ministry of Construction of Japan.

The European Laboratory for Structural Assessment (ELSA) at Ispra has the most important reaction-wall facility in Europe and has been and will be used in the future for research activities on the design of civil engineering structures in seismic areas. The ELSA facility has been available since 1993. The experimental campaign that supplied most of the data used in this work was actually performed in this laboratory using the Pseudo-dynamic technique.

The technique, the implementation procedures and the control system as they are installed in the ELSA laboratory at Ispra, are detailed in this chapter. In section 2.2 the methodology is presented with its advantages and disadvantages in relation to other testing techniques. The more common numerical integration schemes are analysed in section 2.3. The implementation procedures and the control system are described in section 2.4. The characteristics of the ELSA laboratory are presented afterwards in section 2.5.

In section 2.6, the possibility of using the method coupled with substructuring techniques is discussed. A recent improvement implemented in the ELSA laboratory to take into account asynchronous input motion is analysed in section 2.7.

2.2 TESTING METHODOLOGY

The equation of motion of a structure with mass and damping matrices \([M]\) and \([C]\), respectively, and submitted to an external excitation \(\vec{f}\) is given by

\[
[M] \cdot \ddot{a} + [C] \cdot \dot{v} + r = \vec{f}
\]

where \(\ddot{a}\) represents the acceleration, \(\dot{v}\) the velocity and \(r\) the restoring force vectors.

In the case of a structure with a linear elastic behaviour, the restoring forces can be calculated numerically by multiplying the vector of nodal displacements, \(\ddot{a}\), by the stiffness matrix \([K]\). If the structures present a non-linear behaviour, the restoring forces can be
also computed provided the constitutive laws of the materials are known in detail. Finally, equation (2.1) is solved using a numerical step-by-step algorithm. Some of the algorithms available in the literature are described in section 2.3.

The main difficulty concerning the numerical analysis of structures under dynamic loading is not how to solve the equation of motion but how to predict the structural behaviour. The Pseudo-dynamic method actually emerges from this difficulty; instead of making use of numerical constitutive laws, the displacements $\vec{z}$ computed through the numerical algorithm are applied to the physical specimen and the corresponding restoring forces $\vec{f}$ are measured through load-cells placed in the hydraulic actuators. For the mass matrix, a finite element method simulating the distribution of mass in the structure can be used for computing the mass matrix condensed on the d.o.f. of the specimen in the laboratory. Because the hysteretic dissipation of energy is already included in the response of the structure, no numerical viscous damping is usually considered.

Although the Pseudo-dynamic method follows essentially the same steps as a full-numerical dynamic analysis, it uses the real stiffness and strength characteristics of the specimen. The two parallel systems: the integration step-by-step algorithm solved in the computer and the experimental set-up in the laboratory, intercommunicate. The first system computes the displacements and sends the information to the second system. The actuators apply the displacements to the structure and reply with the restoring forces from the load-cells. The first system receives this information and finds the displacements at the next time step.

This dual loop is illustrated in Figure 2.1 using a two floors building as an example. Only the horizontal displacements at the level of each floor are controlled in this hypothetic test. The broken lines represent the links that exist between the main components of the test apparatus and the arrows indicate the way the information flows.

The simplest system one can find to simulate a Pseudo-dynamic test is to have two operators: one to manipulate the computer that solves the integration algorithm and another to control the mechanical system that applies the displacements to the specimen. The first operator communicates the displacements to the second one which replies with the
restoring forces that come out from the application of these displacements.

To someone who is familiar with conventional quasi-static tests and step-by-step numerical integration algorithms in dynamic analysis, this method is straightforward. However, although, theoretically speaking, all structures that do not present high sensitivity to strain rate effects can be tested with such a technique, the test fits particularly well those which may be idealized as spring and mass discrete systems. Structures that present uniform distribution of mass and stiffness, such as arch dams, usually demand the control of a great number of d.o.f. that may overtake the actual capacity of the laboratory.

2.2.1 Advantages

The advantages of the Pseudo-dynamic technique concern the three following tests: quasi-static, full-numerical and shaking table. In relation to the first one, the advantage is clear: it allows carrying out tests taking into account the inertial and damping characteristics of the structure using, essentially, the same equipment as conventional quasi-static experiments.

In relation to the full-numerical analysis, the Pseudo-dynamic test does not previously require the knowledge of the mechanical properties of the materials or of any modelling or constitutive law; the structural damage and degradation of the strength and stiffness characteristics of the materials are implicitly taken into account in the restoring forces measured directly on the physical specimen.

Since no inertial or damping forces are applied to the specimen, because the structures are tested in the laboratory within a very large time scale, larger structures can be tested without the need of very high hydraulic power and fast acting actuators or control systems; the velocity and the acceleration are simulated numerically and only the static deformation of the structure is represented in the laboratory. The time period in a Pseudo-dynamic test is typically two or three orders of magnitude of the real time; a 10 seconds earthquake can last a couple of hours.

The costs of such an experimental hydraulic system are therefore much lower than those of an equivalent shaking table system. The specimens can be tested at a large or almost
full scale, avoiding problems such as those linked to the properties and bond characteristics of the materials that are difficult to represent in a small scale model. Moreover, since the time scale is greatly enlarged in relation to the real time, the damage of the specimens can be easily followed and the test can be stopped and restarted by the controllers as in any conventional quasi-static test.

Finally, another important advantage in relation to the shaking table refers to the possibility of considering the structure divided in several components and to test them separately. In this case, special care should be taken for the link between those components so that the structure responds as a whole. This subject is discussed later in section 2.6.

2.2.2 Disadvantages

The disadvantages of the Pseudo-dynamic test only refer to the shaking table test which is the most direct test one can perform to analyse the dynamic behaviour of a structure; the structure is placed on a platform which moves according to the real history of acceleration that is supposed to act at the basement of the specimen. Since only a small d.o.f. is usually controlled in a Pseudo-dynamic test, the method is not suitable for structures that require a refined distribution of d.o.f. to capture their dynamic behaviour in a realistic way. As an example, this technique is more appropriate for R/C or steel framed structures with heavy rigid floors supporting most of its mass than for masonry buildings.

An arch dam is a good example of a structure that presents a quite uniform distribution of mass and stiffness demanding the control of a big number of d.o.f. However, this problem can be avoided through the identification of the modes of vibration that have a higher participation in the response. In this way, the d.o.f. that need to be represented in the laboratory can be restricted to a smaller number. Nevertheless, one must not forget that the non-linear behaviour of the specimen changes the shape and the frequency of the vibration modes, and that simplification can introduce important errors in the results.

In a Pseudo-dynamic test of steel and reinforced concrete structures, the damping is mainly hysteretic; it is due to the dissipation of energy in the loading and unloading cycles and is implicitly taken into account in the experiment. However, if viscous damping exists, or the response of the structure is very sensitive to the viscous damping prop-
erties, this test is meaningless if these properties can not be identified and simulated accurately.

Moreover, if time is an important factor in the analysis of the specimen, such as in active control systems, the Pseudo-dynamic method cannot be applied. In the case of structures made of materials very sensitive to strain rate effects, the method is not suitable either. Actually, the method assumes that the errors due to strain rate effects in steel and reinforced concrete structures are rather small for the strain rate range of interest. In general, the strength of the materials is lower for lower loading rate and, therefore, the quasi-static loading rate of the Pseudo-dynamic test may lead to conservative results.

It should be pointed out that, in the case of a shaking table test, strain rate effects also exist when scale factors are used in the specimens; it is not possible to satisfy the dynamic similitude and maintain the same strain rate as the prototype at the same time. A small scale model is subjected to a higher strain rate than the prototype and in this case a non-conservative result may be expected.

2.3 NUMERICAL ALGORITHMS

The Pseudo-dynamic test requires a numerical step-by-step integration algorithm, as in any full-numerical analysis. Although different integration schemes can be found in the literature, they can be classified in two main groups: the explicit and the implicit algorithms.

The explicit methods are those which are naturally adapted to the Pseudo-dynamic test; the displacements on a certain time step only depend on the characteristics of the motion and restoring forces on the previous time steps. However, these methods are conditionally stable and the time steps have to be inferior to a value that depends on the highest frequency of vibration that participates in the response of the structure. Depending on the structures, this can decrease the incremental displacements to values that can go even below the precision of the control system and, consequently, increase the potential errors during the test.

Instead, implicit algorithms are unconditionally stable. However, the displacements and
the restoring forces at a time step depend not only on the motion at the previous steps but also at the present time step. In general, these algorithms require an iterative process to reach equilibrium, meaning that spurious hysteretic cycles may be applied to the structure. This non-realistic dissipation of energy can be avoided by imposing monotonically increasing or decreasing displacements within each time step; the incremental displacements evaluated using a stiffness higher that the tangent stiffness are multiplied by a reduction factor in order to ensure that the target displacements are not overtaken. Note that during an experiment the computation of a tangent stiffness is not feasible. However, non-iterative implicit algorithms exist and have been implemented in the Pseudo-dynamic algorithm.

As already mentioned, the big difference between the new methodology and a full-numerical analysis is in the way the restoring forces are determine. In the case of the Pseudo-dynamic method, they are not computed through a numerical model but are measured directly from the specimen in the laboratory. This difference requires the conventional step-by-step integration algorithms to be slightly modified in order to fit the Pseudo-dynamic testing procedures. In the following paragraphs the main integration schemes are detailed.

2.3.1 \( \alpha \)-Newmark algorithms

The integration algorithms that were implemented in the ELSA laboratory belong to the family of the "\( \alpha \)-Newmark" type schemes [37]. They are sustained by the discrete system of equations

\[
[M] \cdot a_{t+\Delta t} + (1 + \alpha) \cdot [C] \cdot v_{t+\Delta t} - \alpha \cdot [C] \cdot v_t + (1 + \alpha) \cdot r_{t+\Delta t} - \alpha \cdot r_t = (1 + \alpha) \cdot f_{t+\Delta t} - \alpha \cdot f_t
\]

2.2

where the displacement and velocity vectors at time \( t + \Delta t \) are given by

\[
v_{t+\Delta t} = v_t + [a_t + \gamma \cdot (a_{t+\Delta t} - a_t)] \cdot \Delta t
\]

\[
d_{t+\Delta t} = d_t + v_t \cdot \Delta t + [a_t + 2 \cdot \beta \cdot (a_{t+\Delta t} - a_t)] \cdot \frac{\Delta t^2}{2}
\]

2.3

Substituting equation (2.3) into equation (2.2), the acceleration at the time step \( t + \Delta t \) becomes

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\[ [\hat{M}] \cdot a_{t + \Delta t} = \hat{f}_{t + \Delta t} \]  

where

\[ [\hat{M}] = [M] + (\gamma \cdot (1 + \alpha)) \cdot [C] \]

\[ \hat{f}_{t + \Delta t} = (1 + \alpha) \cdot (f_{t + \Delta t} - r_{t + \Delta t}) - \alpha \cdot (f_t - r_t) - [C] \cdot \hat{v}_t \]

and

\[ \hat{v}_t = v_t + (1 - \gamma) \cdot (1 + \alpha) \cdot \Delta t \cdot a_t \]

To understand the meaning of the parameters \( \beta \) and \( \gamma \) in equation (2.3), the acceleration between step \( t \) and \( (t + \Delta t) \) is represented by a polynomial function of degree \( k \)

\[ a(x) = a_t + \frac{a_{t + \Delta t} - a_t}{\Delta t} \cdot (x - t)^k \quad t \leq x \leq t + \Delta t \]  

The integral of the acceleration gives the velocity and the integral of the velocity gives the displacement in the same time interval

\[ v(x) = v_t + a_t \cdot (x - t) + \frac{a_{t + \Delta t} - a_t}{(k + 1) \cdot \Delta t} \cdot (x - t)^{k+1} \]

\[ d(x) = d_t + v_t \cdot (x - t) + a_t \cdot \frac{(x - t)^2}{2} + \frac{a_{t + \Delta t} - a_t}{(k + 2) \cdot (k + 1) \cdot \Delta t} \cdot (x - t)^{k+2} \]

Substituting \( x = t + \Delta t \) into equation (2.8) and comparing the result with equation (2.3), gives

\[ \gamma = \frac{1}{(k + 1)} \quad \beta = \frac{\gamma}{(k + 2)} \]  

In the case that a constant acceleration \( a(x) = \bar{a}_{t + \Delta t} = (a_{t + \Delta t} + a_t)/2 \) is adopted in the time interval \( (t \leq x \leq t + \Delta t) \), all terms in \( k \) disappear and the acceleration \( a_t \) in equation (2.7) and equation (2.8) is substituted by the mean value \( \bar{a}_{t + \Delta t} \)

\[ v(x) = v_t + \bar{a}_{t + \Delta t} \cdot (x - t) \]

\[ d(x) = d_t + v_t \cdot (x - t) + \bar{a}_{t + \Delta t} \cdot \frac{(x - t)^2}{2} \]
which gives for \((x = t + \Delta t)\)

\[
\gamma = \frac{1}{2} \quad \beta = \frac{\gamma}{2}
\]

2.11

i.e. if a linear variation or a constant average acceleration is assumed within each time step, the parameter \((\gamma = 1/2)\) fully respects the integral given by equation (2.8) for the velocity. If a value \((\gamma > 1/2)\) is adopted, an artificial damping is included in the equation of motion. Although this suggests a decrease in the accuracy of the method, this is often the best means to correct the spurious excitation of higher modes of vibration that appears in the results due to experimental errors or the method itself. The parameter \(\alpha\) in equation (2.2) actually allows the user to include a supplementary dissipation of energy in the equilibrium to account for this spurious excitation.

**Central differences scheme (explicit)**

By choosing different values for \(\alpha, \beta\) and \(\gamma\), different integration schemes are found. The central differences scheme corresponds to \((\alpha = 0), (\gamma = 1/2)\) and \((\beta = 0)\). This formulation is explicit since the displacements and, consequently, the restoring forces at time step \((t + \Delta t)\) only depend on the characteristics of the motion at the previous time steps. As a counterpart, the method is conditionally stable. The numerical solution becomes unstable when \((\omega_o \cdot \Delta t > 2)\), \(\omega_o\) being the highest angular frequency of the multiple d.o.f. system in which the structure has been discretized.

**\(\alpha\)-method (implicit)**

By choosing \((\beta = (1-\alpha)^2/4), (\gamma = (1-2 \cdot \alpha)/2)\) and \((\alpha \in [-1/3, 0])\) the \(\alpha\)-method due to Hilber et al [37] is recovered. Since \((\beta \neq 0)\) the method is implicit; the displacements at time step \((t + \Delta t)\) not only depend on the characteristics of the motion at time step \(t\) but also on the acceleration at time step \((t + \Delta t)\). However, the algorithm is unconditionally stable.

Once more, the parameter \(\alpha\) allows the inclusion of some numerical damping in the equation of equilibrium to account for the spurious excitation observed at higher modes of vibration. According to Shing [76], this supplementary damping should only be included in the algorithm if no special measures are taken into account to avoid the errors.
in the Pseudo-dynamic test.

To implement the method in a Pseudo-dynamic formulation it is necessary to reformulate equation (2.3). Substituting equation (2.4) into equation (2.3), the expression for the displacements can be written as

\[ d_{t+\Delta t} = \bar{d}_{t+\Delta t} - [B] \cdot r_{t+\Delta t} \quad 2.12 \]

where

\[ [B] = (1 + \alpha) \cdot \beta \cdot \Delta t^2 \cdot [\hat{M}]^{-1} \quad 2.13 \]

and \( \bar{d}_{t+\Delta t} \) represents the explicit part of the displacement \( d_{t+\Delta t} \).

Three aspects make the algorithm iterative within each time step: the method is implicit, the structural behaviour is non-linear and the tangent stiffness matrix is not known nor can it easily be calculated during the test.

In each iteration there is a predictor phase and a corrector phase. In the first one, the explicit displacements, vector \( \bar{d} \) in the first iteration, are computed and applied to the structure. Then, the restoring forces are measured, the displacements are corrected though equation (2.12), corrector phase, and applied to the structure again. The solution is found when the displacements computed from two successive iterations are identical.

**Shing method (implicit)**

Shing [77] introduces some modifications in equation (2.4). The equation is rewritten in terms of incremental displacements in the time step \([t, t + \Delta t]\)

\[ [M_{\beta}] \cdot (\Delta d_{t+\Delta t})_j = [M_{\alpha}] \cdot (\bar{d}_{t+\Delta t} - (d_{t+\Delta t})_j) - \Delta t^2 \cdot \beta \cdot (1 + \alpha) \cdot (r_{t+\Delta t})_j \bar{f}_{t+\Delta t} \quad 2.14 \]

where \( j \) is the number of the iteration, \( \bar{d}_{t+\Delta t} \) is an explicit displacement in the time step,

\[ \bar{d}_{t+\Delta t} = d_t + v_t \cdot \Delta t + a_t \cdot \frac{\Delta t^2}{2} \quad 2.15 \]

and
\[ [M_\alpha] = [M] + (1 + \alpha) \cdot \gamma \cdot \Delta t \cdot [\dot{C}] \]
\[ [M_\beta] = [M_\alpha] + (1 + \alpha) \cdot \beta \cdot \Delta t^2 \cdot [K] \]

where \([K]\) is the stiffness matrix, and

\[
\tilde{f}_{t+\Delta t} = [\alpha \cdot r_t + (1 + \alpha) \cdot f_{t+\Delta t} - \alpha \cdot f_t - [M] \cdot a_t - [C] \cdot (v_t + (1 + \alpha) \cdot a_t \cdot \Delta t)] \cdot \beta \cdot \Delta t^2
\]

Just like in the previous method, this algorithm uses an iterative process to solve the displacements; the prediction value \(\tilde{d}\) is computed, applied to the structure and the restoring forces are measured. The new incremental displacement are calculated through equation (2.14) and the corrector phase, \((d_{t+\Delta t})_j = (d_{t+\Delta t})_{j-1} + (\Delta d_{t+\Delta t})_{j-1}\), goes on until the new incremental displacements are identical to zero.

In the implicit algorithms, the generalized mass matrix depends on the stiffness matrix. This means that if the structure undergoes non-linear deformations, the stiffness matrix should be updated and be as close as possible to the tangent stiffness. However, it is very difficult, if not impossible, to compute such a stiffness on-line during the experiment.

Moreover, the overshooting of the solution must be avoided. Overshooting means that during the iteration process the algorithm finds an intermediate solution that goes further than the target point; this induces the response of the structure include spurious hysteresis cycles that must be prevented. To avoid this phenomenon, the stiffness matrix must be always higher than or equal to the secant stiffness in the iteration step. For this reason, and because the higher stiffness of the structures is the elastic stiffness, this is the value generally used in the algorithms.

To prevent overshooting, Shing also suggests that the incremental value given by equation (2.14) be reduced by a factor \(\theta\) equal to 0.5

\[
(d_{t+\Delta t})_{j+1} = (d_{t+\Delta t})_j + \theta \cdot (\Delta d_{t+\Delta t})_j
\]

According to the same author, this approach, combined with the implementation procedure that considers approximate errors corrections, prevents spurious energy from being added in the solution. The corrections suggested by Shing consist in using two different

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transducers systems: one linked to the actuators to apply the displacements and another, exterior to the actuators, to measure the displacements actually applied to the structure. The incremental displacements are calculated using the real displacements and the equilibrium is found when the new increment is zero.

Modified Newmark method (explicit)

Shing and Mahin [76] improved an algorithm within the family of the “modified Newmark methods”. Instead of a viscous damping matrix, these two authors consider an additional term in the left hand side of equation (2.1) to model the damping of lower modes. At the same time, this additional term also prevents error propagation effects from amplifying the response of spurious higher modes. The new equation of motion is given by

\[
[M] \cdot a_{t+\Delta t} + r_{t+\Delta t} + \left( \gamma \cdot [K] + \frac{\varphi}{\Delta t^2} \cdot [M] \right) \cdot (d_{t+\Delta t} - d_t) = f_{t+\Delta t}
\]

where \(\gamma\) and \(\varphi\) are values that tune the amount of damping included in the numerical solution. Damping increases with \(\gamma\) and is zero for \((\sqrt{-\varphi/\gamma} = \omega \cdot \Delta t)\), where \(\omega\) represents the free-vibration angular frequencies of the structure. For values below this limit, the damping is negative and the solution becomes instable; in this case there is energy added in the numerical solution. The method was implemented with the explicit scheme of the Newmark method: \((\alpha = 0), (\beta = 0)\) and \((\gamma = 1/2)\).

The formulation is particularly interesting since damping increases with \((\omega \cdot \Delta t)\); it allows the inclusion of low damping in the lower modes and high damping in the higher modes of the structure.

Operator-Splitting scheme (implicit non-iterative)

Finally, an implicit non-iterative algorithm is presented. The equilibrium given by equation (2.2) is solved using the operator splitting scheme described by Nakashima [49]. The method maintains the stability of the \(\alpha\)-Newmark algorithm.

The operator-splitting formulation is based on an approximation of \(r_{t+\Delta t}\); the restoring force is split into a linear elastic part that remains implicit within the algorithm and an
explicit force that follows the physical behaviour of the structure,

\[ r_{t+\Delta t} = \ddot{r}_{t+\Delta t} + [K]_I \cdot (d_{t+\Delta t} - \ddot{d}_{t+\Delta t}) \]  \hspace{1cm} 2.20

where \( r_{t+\Delta t} \) is the restoring force that corresponds to the implicit fraction of the displacement in equation (2.3),

\[ \ddot{d}_{t+\Delta t} = d_t + v_t \cdot \Delta t + \left( \frac{1}{2} - \beta \right) \cdot a_t \cdot \Delta t^2 \]  \hspace{1cm} 2.21

and \([K]_I\) is the elastic stiffness matrix.

Just like in the Shing algorithm, to avoid overshooting of the solution and stabilize the convergence process, the elastic stiffness matrix is adopted. A very low amplitude displacement applied to the structure before the test allows the controller to know the value of this stiffness with great accuracy.

The approximation expressed in equation (2.20) makes the implicit algorithm non-iterative. The equations and a detailed description of the method can be found in annex C; there is a predictor phase, where the implicit displacements \( \ddot{d}_{t+\Delta t} \) are computed and applied to the structure, and a single corrector phase that modifies these displacements to take into account any possible non-equilibrated forces through the linear elastic model represented by \([K]_I\) in equation (2.20).

A series of numerical algorithms fitting the Pseudo-dynamic testing procedures have been presented: explicit, implicit, iterative and non-iterative methods. Although explicit algorithms are those that fit the Pseudo-dynamic tests best, implicit algorithms, in particular those non-iterative, have been largely used. The stability of the methods, that in the case of the explicit algorithms may impose incremental displacements which can go below the precision of the transducers, can be an important element in the decision of the most suitable method to use when testing a structure. On the other hand, when using implicit algorithms with iterative procedures, the equilibrium of forces may impose spurious hysteretic cycles which must be avoided.

Another important factor distinguishes the methods presented in this section: the proce-
dure they use for preventing spurious vibration modes from interfering with the results. This is usually done either by adding a parameter $\alpha$ in the equation of motion to increase the dissipation of energy, see equation (2.2), or by increasing the damping by changing the velocity that is computed from the numerical integral of the acceleration (see equation (2.8) to (2.11)). The algorithm that has been adopted in the latest experiments at the ELSA laboratory is the $\alpha$-Newmark algorithm with the Operator-Splitting scheme [40].

### 2.4 IMPLEMENTATION

Before presenting the characteristics of the control system implemented at the ELSA laboratory, the general steps that concern the implementation of a Pseudo-dynamic method are detailed. The description that follows assumes that the solution at time step $t$ has already been found. Furthermore, the explicit and the implicit methods are referred to separately.

In the case of explicit algorithms, the controller system generates a ramp function represented by a set of discrete displacements $d_k$ confined between the time step $t$ and the time step $t + \Delta t$. In general, the procedure distinguishes the variable controlling the servo-valve of the actuator, $d_k$, from the variable that measures the actual displacement applied to the structure $x_k$. In each step $k$, the driving signal sent to the servo-valve, $\Delta d_k$, is computed in relation to the previous displacements installed on the structure $x(k-1)$,

$$\Delta d_k = d_k - x(k-1) \tag{2.22}$$

The incremental displacements are applied to the structure until the displacement $x$ coincides with the target displacement $d_{t+\Delta t}$. Then, with the restoring forces measured at the end of the step, $r_{t+\Delta t}$, the algorithm computes the displacements for the next time step.

If the algorithm is implicit, the ramp function is defined between the displacement $d_t$ and the displacement $\tilde{d}_{t+\Delta t}$ from the predictor phase. The displacement $d_k$ is applied to the structure and both the restoring force $r_k$ and the displacement $x_k$ are measured. The solution is found when the displacement $x_k$ coincides with $\tilde{d}_{t+\Delta t}$. If the algorithm is non-iterative, the displacement is corrected (e.g. equation (C.18) for the $\alpha$-Newmark method with the Operator-Splitting scheme) before the algorithm computes the predictor displacement for the next time step.
Instead, if the algorithm is iterative, a new prediction is computed (e.g. equation (2.12) and equation (2.18)) and a ramp function from the old and the new target displacement, \((d_{t+\Delta t})_j \) and \((d_{t+\Delta t})_{(j+1)} \), respectively, is established for the new iteration. The algorithm stops when the target displacements from two successive iterations are identical. Note that identical means that \(|(d_{t+\Delta t})_j - (d_{t+\Delta t})_{(j+1)}|\) is not greater than a certain tolerance.

Special attention must be paid to avoid overshooting of the solution, since the intermediate target displacements can go further than the final solution. To reduce the effects of overshooting and improve the efficiency of the algorithm, Thewalt and Mahin [81] propose a hybrid solution for the \(\alpha\)-method: instead of seeking the predictor displacement, the algorithm aims at finding directly the final solution through a correction in the driving signal sent to the actuators.

The driving signal at the level of each actuator is modified using the feedback voltage from the restoring force,

\[
\Delta d_k^c = d_k - (x_{(k-1)} + B \cdot r_{(k-1)}) \tag{2.23}
\]

where \(B\) is a value from the \([B]\) matrix in equation (2.12). This equation shows that when the displacement \(d_k\) tends to the predictor displacement, the displacement \(x\) measured on the structure tends to the final solution. The equilibrium is found when \((x = \ddot{d}_{t+\Delta t} - B \cdot r)\) for all d.o.f. of the structure.

The target displacement is not known beforehand. The driving signal depends on the force generated by the new position of the actuator and this makes controlling the error much more difficult. Moreover, in the case that a viscous damping matrix is included in equation (2.2), the modified mass matrix in equation (2.5) is no longer diagonal. This leads to an inter-dependence of the actuator circuits and other components of the matrix \([B]\) have to be included in equation (2.23). This problem can be solved by centralizing the control of the actuators in a digital unit.

In the case of the Shing method, the driving signal is computed through equation (2.14) using the real displacement \(x\) in the place of \((d_{t+\Delta t})_j\), and the reduction factor \(\theta\) to avoid
overshooting of the solution

\[ \Delta d_j^c = \theta \cdot (\Delta d_{t + \Delta t})_j \]

2.24

The equilibrium is reached when the incremental displacement, i.e. the driving signal in all actuators is equal to zero.

The evaluation of the stiffness matrix required by the implicit algorithms is "built" experimentally at the beginning of the test. A standard procedure is used for accessing the components of the stiffness matrix: a small displacement is applied to the structure on the d.o.f. corresponding to each column of the stiffness matrix, and the force on this actuator and on all others preventing the other d.o.f from moving is measured.

2.4.1 Characteristics of the control system

The control systems can be divided into two main groups: the digital and the analog systems. If an analog control is used, the target displacements are converted to an analog signal before the ramp function is established. Otherwise, a digital to analog (D/A) converter is placed after computing the digital signal \( d_k \) from the ramp function.

In both cases the incremental displacement is calculated and the signal is sent to the servo-valve that controls the oil flow that goes in or out of the actuator piston. The displacement is applied to the structure and the restoring force is measured through a load-cell that responds with an analog signal. The displacement actually applied to the structure, \( x \), is measured through a displacement transducer exterior to the actuator.

The displacement \( x \) is then compared with the target displacement and the information concerning the new incremental displacement or correcting signal (e.g. equations (2.22), (2.23) and (2.24)) is generated by the control system, digital or analog, and sent to the servo-valve. Note that the driving signal already integrates any residual offset that may exist.

In the ELSA laboratory the control system is digital; the ramp defining the set of points \( d_k \) and the correcting signal are digital. The displacements are also measured through digital transducers linked directly to the digital control system without demanding any
A/D conversion. The only A/D conversion refers to the restoring forces. A digital to analog (D/A) converter is used for reconstructing the continuous control signal required by the servo-valve. A schematic picture of a digital control system identical to the one implemented in the ELSA laboratory is illustrated in Figure 2.2. The grey zone represents the digital control. The broken line corresponds to the case where iterative algorithms are used (e.g. Shing method).

A digital system like the one in Ispra has many advantages in relation to an analog system: the control algorithm is more flexible allowing complex calculations to be performed easily. For example, the implementation of substructuring techniques or the inclusion of numerical viscous damping in Pseudo-dynamic tests is a straightforward matter. Moreover, the control algorithms and the ramp functions can be changed quickly, and the errors involved are greatly reduced in relation to an analog system. In general, a digital system is a more user friendly system.

2.4.2 Errors

In step-by-step algorithms, the propagation of errors is a subject that has to be carefully analysed. Note that displacements applied to the structure inferior to the target displacements correspond to a supply of energy to the structure which may amplify the response of spurious higher modes. Instead, displacements applied to the structure higher than the target displacements correspond to an unreal dissipation of energy.

The most important errors are in the components of the control loop, namely: the hardware that controls the electro-hydraulic circuit and the servo-valves, the displacement transducers and the A/D and D/A converters. As, in general, the displacements that are used in controlling the errors are measured using a reference structure independent from the reaction-wall and from the actuators, any possible shift between the target displacement and the applied displacement is always very small.

Moreover, the use of a digital control system with digital transducers, avoids analog to digital and digital to analog conversion errors in the displacements; any error due to the digital to analog conversion of the driving signal for the actuators is corrected indirectly using the displacements measured by the parallel digital transducers. The only conver-
sion error that may exist comes from the analog to digital conversion of the restoring forces measured by the load-cells on the actuators.

The friction between the physical parts in contact, namely: the actuator and the structure and the actuator and the reaction wall, may increase fictitiously the energy dissipated by the specimen. Finally, the phenomenon of relaxation of the restoring forces can not be neglected; the waiting periods between two time steps should be small and the forces at the end of each time step should be measured immediately after reaching the equilibrium.

2.5 THE ELSA FACILITY

The European Laboratory of Structural Assessment is a large scale facility located in the Northern Italy, at Ispra, belonging to the Joint Research Centre of the European Commission. The facility has been used for Pre-normative Research in Support of Eurocode 8 [21] (PREC8), the provisional European standards for the design of civil engineering structures in seismic areas; a full-scale reinforced concrete building with 10m per 10m and 4 floors corresponding to a total height of 13.30m, six (1:2.5) scale bridges with a 80m long deck (simulated numerically) and three intermediate piers with heights of 2.8m, 5.6m and 8.4m were tested as part of this programme.

The facility has been available since 1993; a further extension of the reaction platform concluded during the first semester of 1996 gave to the laboratory its actual dimension as illustrated in Figure 2.3. The ELSA laboratory is actually the most important Pseudo-dynamic testing facility in Europe.

2.5.1 Characteristics of the laboratory

The main characteristics of the reaction-wall at the ELSA laboratory are presented in Figure 2.3. Basically, it consists of a 16m high and 21m wide hollow-core wall with a total thickness of 4m. Two horizontal platforms are also part of the reaction structure. The facility allows full-scale buildings up to five floors in height to be tested quasi-statically or using the Pseudo-dynamic technique. It is designed to resist to forces of several thousand kN, enough to damage seriously large-scale structures. Outside the laboratory there is also a large platform that has been used in the construction of large structures.
The characteristics of the electro-hydraulic actuators available in the ELSA laboratory are also presented in Figure 2.3. The actuators can be controlled either in displacement or force. To a certain extent, dynamic tests on small scale structures can be performed using the dynamic capabilities of the actuators.

2.5.2 Architecture of the control system at the ELSA laboratory

The control system is fully digital and the architecture of the system was designed by the group working in the laboratory. Each actuator is connected to a digital control unit which, in turn, is connected to the main computer through optical fibres. The measurement of the displacements is performed through optical digital transducers which transmit the information to the control units without the need of a digital to analog conversion.

The control operation can be done in local or remote mode. In the first case, the sequence of $d_k$ displacements is set as constant but the operator can increase or decrease the ramp signal, changing the velocity of application of the displacement. The control parameters can be changed by the operator. Instead, in the case of a remote operation, the ramp and the control parameters are defined at the beginning of the test and are not modified after.

Each control unit receives the digital information concerning the target displacements and computes the set of values $d_k$ from the ramp signal. The information passes through a D/A converter and is sent to the actuators which, in turn, transmit the restoring forces from the load-cells as feedback. In the case of digital control units, as in the ELSA laboratory, an A/D converter has to be used for the restoring forces.

At the end of each time step, when the equilibrium between the target displacements and the displacements actually applied to the structure is achieved, the control unit transmits the digital signal of the restoring forces to the main unit. The information concerning the real displacements applied to the structure comes from optical digital transducers linked to an exterior reference frame independent from the reaction-wall. The transducers currently used in the ELSA laboratory have a resolution of either 0.004$mm$ or 0.002$mm$ and a stroke of 1.0$m$ or 0.5$m$, respectively.
Each control unit is independent from the others. They are equipped with a hard disk that makes them autonomous and allows them to store long sequences of data which, in case of operation problems, permits to reconstruct the problem scenario.

This type of architecture makes the system very flexible: the addition of another actuator corresponds to the addition of another control unit and the capacity of the system can be easily increased. The computational tasks are shared by the different processors, one per each single actuator. This allows each control unit to have different ramps with different parameters. Furthermore, each control unit can be located near by the source. This avoids series of long cables connecting each actuator to the main computer and reduces the danger of noise corruption due to the distance. Moreover, the space occupied by an equivalent analog system is much larger.

2.6 SUBSTRUCTURING

The Pseudo-dynamic formulation can be applied in conjunction with substructuring techniques, i.e. the specimen can be split into several parts. In Figure 2.4 the same building of Figure 2.1 is illustrated but now with isolation devices at the basement. As represented in the round circle at the right bottom side of the picture, the building does not have to be placed on the isolators if, instead of the total displacements, the relative displacements $d_i$ are applied to the floors. This is a very simple case of substructuring.

Another interesting application of the substructuring technique concerns those structures where damage is predicted to be located in a few critical regions; the rest of the structure remains in the linear elastic range or presents a behaviour than can be easily simulated with numerical tools. In this case the critical regions are tested in the laboratory while the rest of the structure is simulated through a numerical model [23]. Since the computer is already a fundamental component of the testing apparatus, this technique does not introduce important modifications in the Pseudo-dynamic formulation: instead of measuring all restoring forces in the laboratory, part of these forces are calculated through a numerical model. In this case, the Pseudo-dynamic method requires not only a step-by-step integration algorithm but also a model for the substructure.

Assuming that the structure under analysis is divided into a numerical substructure and a
physical structure, equation (2.4) can be rewritten splitting the components linked to the
d.o.f of the substructure “S” from those of the physical structure “T”

\[
\begin{bmatrix}
  \hat{S}_{Mij} & \hat{S}_{Mi\beta}
  \\
  \hat{S}_{M\alpha j} & (\hat{M}_{\alpha \beta} + \hat{T}_{\alpha \beta}) & \hat{T}_{\alpha j}
  \\
  0 & \hat{T}_{M\beta j}
\end{bmatrix}
\begin{bmatrix}
  a_j
  \\
  a_\beta
  \\
  a_I
\end{bmatrix} =
\begin{bmatrix}
  \hat{s}_i
  \\
  \hat{s}_\alpha + \hat{T}_\alpha
  \\
  \hat{T}_I
\end{bmatrix}
\]

\[2.25\]

where \(i\) and \(j\) refer to the d.o.f of the numerical substructure and \(I\) and \(J\) to those of
the physical structure. The values \(\alpha\) and \(\beta\) represent the degrees of freedom common to the
two sub-systems. The modified forces \(\hat{T}_f\) are calculated with the restoring forces measured
in the physical structure while forces \(\hat{s}_f\) are calculated with the restoring forces
computed with the numerical model.

The displacements are applied simultaneously to the physical and to the numerical
structure. The modified forces are calculated (the right hand side of equation (2.25)), the
acceleration vector is solved and the displacements for the next time step are computed.

Although the method remains basically the same, the implementation of this new tech-
nique demands some changes in the Pseudo-dynamic algorithm without substructuring.
Firstly, the system has more d.o.f. than those in the laboratory and the additional number
of d.o.f can even exceed the capacity of the PC controlling the process. Secondly, for the
system to be able to simulate the substructure, new numerical tools have to be imple-
mented in the PC and the existing software have to be extended to include the new
degrees of freedom.

In order to profit from the existing hardware and software in the laboratory and the finite
element code already available, CASTEM 2000 [19] that is briefly described in annex B,
a new methodology has been adopted. In the ELSA Laboratory and in addition to the PC
which controls the classic Pseudo-dynamic test, the setup for the substructuring actually
includes an UNIX work-station to handle the numerical substructure. The necessary
exchange of information between the two processes running in parallel is secured by the
local network [7] through an inter-process communication. This clear splitting of the two
main tasks, enables the use of the Pseudo-dynamic standard algorithm with minor

THE PSEUDO-DYNAMIC TEST METHOD
changes.

A static condensation has been considered to assure the simplicity of the method. Making use of equation (2.25) and condensing the degrees of freedom of the substructure into the d.o.f. common to the two structures, a new system appears

\[
\begin{bmatrix}
S_{ij}^\wedge & S_{i\beta}^\wedge & 0 \\
0 & (S_{\alpha\beta}^\wedge + M_{\alpha\beta}) & T_{\alpha j}^\wedge \\
0 & T_{i\beta}^\wedge & T_{i j}^\wedge
\end{bmatrix}
\begin{bmatrix}
a_j \\
a_\beta \\
a_j(t + \Delta t)
\end{bmatrix}
= \begin{bmatrix}
S_{i}^\wedge \\
f_{\alpha} + T_{\alpha i}^\wedge \\
f_{i}
\end{bmatrix}(t + \Delta t)
\]  \hspace{1cm} 2.26

where

\[
S_{\alpha\beta}^\wedge = S_{\alpha\beta}^\wedge - M_{\alpha j} \cdot (S_{ji}^\wedge - S_{i\beta}^\wedge) \\
S_{f_{\alpha}} = f_{\alpha} - M_{\alpha j} \cdot (S_{ji}^\wedge - S_{i}^\wedge)
\]  \hspace{1cm} 2.27

and the d.o.f. of the physical structure become "uncoupled" from the numerical structure. The algorithm used in the standard Pseudo-dynamic procedure can be used for testing the physical structure as long as the information concerning \(S_{\alpha\beta}^\wedge\) and \(S_{f_{\alpha}}\) in equation (2.26) is transmitted step-by-step to the main unit that controls the integration algorithm.

Note that the exchange of information is in both directions. The work-station dealing with the substructure transmits the modified mass matrix \(S_{\alpha\beta}^\wedge\) and the force vector \(S_{f_{\alpha}}\) to the PC that controls the Pseudo-dynamic algorithm. In turn, the PC replies with the acceleration \(a_\beta\) that the algorithm in the work-station dealing with the substructure needs to compute the new modified force vector \(S_{f_{\alpha}}\). In the case the modified mass is constant, the matrix \(S_{\alpha\beta}^\wedge\) needs to be transmitted to the PC only once at the beginning of the test. The data flow between the substructuring process and the Pseudo-dynamic process implemented at the ELSA laboratory are illustrated in Figure 2.5.

This procedure allows all software available in the UNIX work-station useable, namely CASTEM 2000. In the ELSA laboratory, the connection interface between the PC and the UNIX work-station is available via a Client/Server protocol referred to as the Berkeley sockets (see e.g. Stevens [78]).
Figure 2.6 shows a hypothetic test of a bridge in the ELSA laboratory: the piers are tested physically and the deck is simulated in the computer using the CASTEM 2000 code. The two processes communicate via network using the Berkeley sockets.

2.7 ASYNCHRONOUS INPUT MOTION

In the case of an asynchronous input motion, the equation of motion can not be expressed in terms of the relative motion to the basement but in terms of the total displacement. This means that in a quasi-static test, all nodes of the structure, including those at the basement, have to move. Although in a Pseudo-dynamic test this is theoretically feasible, it is not an easy task to move the basement of a large structure. However, if the nodes at the basement are not coupled or if coupling exists but can be simulated through a numerical model, the relative motion can still be applied to each of the structures.

An example of a structure satisfying this condition is a bridge with the piers physically represented in the laboratory and the deck substructured and simulated through a numerical model; the structural elements touching the foundation (the piers) present no physical links with each other and the relative motion to the basement can still be applied to the piers. The actual links are established at the level of the numerical substructure.

The algorithm that has been used in the ELSA laboratory to test uncoupled structures under asynchronous type motion considers different boundary conditions for the physical and the numerical structures. In the case of the physical structure, as it is made of a set of independent elements actuated by different acceleration time histories at the basement, the equilibrium can be expressed in terms of relative motion. Instead, the numerical substructure is seen as a new structure with supporting nodes at the d.o.f shared with the physical structure and the equilibrium must be established in terms of absolute motion. The equilibrium expressed in equation (2.25) can be split into two systems,

\[
\begin{bmatrix}
\dot{S}_{ij} & \dot{S}_{i\beta} \\
\dot{S}_{\alpha j} & \dot{S}_{\alpha\beta}
\end{bmatrix}
\begin{bmatrix}
A_j \\
A_{\beta}
\end{bmatrix}_{(t+\Delta t)} =
\begin{bmatrix}
S_{f_i} \\
S_{f_{\alpha}}
\end{bmatrix}_{(t+\Delta t)}
\]

2.28
\[
\begin{bmatrix}
T^\alpha & T^\beta \\
M_{\alpha\beta} & M_{\alpha\beta}
\end{bmatrix}
\begin{bmatrix}
a^\alpha \\
a^\beta
\end{bmatrix}
= 
\begin{bmatrix}
T^\alpha \\
T^\beta \\
T^\gamma \\
T^\delta
\end{bmatrix}
\begin{bmatrix}
f^\alpha \\
f^\beta \\
f^\gamma \\
f^\delta
\end{bmatrix}
\]

where \( a \) and \( A \) are the relative and the absolute acceleration values, respectively. The relation between the relative acceleration and the absolute acceleration at each point of the physical structure, in particular at the d.o.f. shared with the substructure, is given by

\[
A^\beta = a^{base} + a^\beta
\]

where the vector \( a^{base} \) is the acceleration at the basement and presents as many degrees of freedom as \( \beta \). Note that, for the acceleration at the basement to be consistent with the \( \alpha \)-Newmark formulation, \( a^{base} \) have to follow the same procedure as the applied forces in equation (2.2),

\[
a^{base} = (a^{base})^\alpha = (1 + \alpha) \cdot a^{base}_{t+\Delta t} - \alpha \cdot a^t
\]

Substituting equation (2.30) into equation (2.28) and joining the two structures together, the new equilibrium is expressed by

\[
\begin{bmatrix}
S^\alpha_{Mij} & S^\alpha_{M\beta} & 0 \\
S^\beta_{Mij} & S^\beta_{M\alpha\beta} & T^\alpha_{M\alpha\beta} \\
0 & T^\alpha_{M\beta} & T^\alpha_{M\beta}
\end{bmatrix}
\begin{bmatrix}
A^\alpha \\
a^\beta \\
a^\gamma
\end{bmatrix}
= 
\begin{bmatrix}
S^\alpha_{f^\alpha} - S^\beta_{M\alpha\beta} \cdot (a^{base})^\alpha \\
S^\beta_{f^\alpha} - S^\beta_{M\alpha\beta} \cdot (a^{base})^\alpha + f^\alpha \\
f^\beta
\end{bmatrix}
\begin{bmatrix}
T^\alpha \\
T^\beta \\
T^\gamma \\
T^\delta
\end{bmatrix}
\]

The main difference between this system and the one described in equation (2.25) is in the right hand side vector which present new components from the substructure. Therefore, the procedure described in section 2.6 for the Pseudo-dynamic test coupled with the substructuring technique can be also applied to the new structure; the mass matrix transferred from the work-station to the PC is the same and only the vector with the modified forces from the substructure is different. Since the differences refer to the algorithm handling with the numerical structure, the Pseudo-dynamic algorithm in the PC does not have to be modified and the application of an asynchronous motion is straightforward.

Finally, it should be pointed out that the modified force vector that refers to the substructure, \( S^\alpha_{f} \), does not include the inertial forces "\(-[M] \cdot [1] \cdot (a^{base})^\alpha\)" , "[1]" being a matrix
with zeros and ones that account for the direction of the input motion at each d.o.f.

2.8 APPLICATIONS IN ENGINEERING

The Pseudo-dynamic method is a general testing method; it can be applied to structures that are not strongly affected by time-scale and that can be idealized as discrete systems with a limited number of d.o.f. to be controlled. Typically, reinforced concrete and steel frame structures with the mass mainly concentrated at a discrete number of levels: the floors in the case of buildings and the deck in the case of bridges, for example, are the most suitable structures to be tested with this methodology.

There are many possible applications to this method. Full scale structures can be tested quasi-statically but still simulating the dynamic characteristics of the materials. Since the test happens with a very large time-scale in relation to real time, it is possible to follow the different stages the structure undergoes, giving to the test a very interesting research applicability.

Furthermore, the substructuring technique coupled with the Pseudo-dynamic method allows the consideration of only a part of the structure in the laboratory and to test the whole structure establishing numerical links with the numerical components of the structure. This is particularly useful when only a few components of the specimen are forecast to be seriously damaged.

This technique is also particularly interesting when it is important to consider soil-structure interaction and isolating/dissipating devices at the bottom of the structure. In these cases, the soil and the isolating/dissipating devices can be simulated with a numerical model or can be “built” in the laboratory without the structure mounted on them (e.g. Figure 2.4).

At the ELSA laboratory, two full scale buildings were tested: a steel bare frame structure and a R/C frame structure. The first one was tested several times with different beam to column joints [42]. The second building is part of the Pre-Normative Research Programme in Support of Eurocode 8 (see in [1] and [52]). This structure was tested with and without infilled panels and isolating devices. Retrofitting techniques were applied to
the beam to column joints damaged at previous stages.

Other structures of great interest are the (1:2.5) scale R/C bridges with a 80\textit{m} long deck supported by three intermediate piers and abutments at the extremities [50]. The deck was substructured and simulated numerically in the computer. The bridges were the largest structures ever tested using such a technique. The results of these tests are presented and discussed in chapter 3. They were exhaustively used in the verification of the model described in chapters 4 and 5.
Figure 2.1 - Pseudo-dynamic testing

1 - Test specimen
2 - Actuators
3 - Displacement transducers
4 - Data acquisition
5 - Reference frame
6 - Load cells
Figure 2.2 - Digital control system
Figure 2.3 - Reaction-Wall facility at the ELSA Laboratory
Figure 2.4 - Pseudo-dynamic testing with substructuring. An isolated two-storey building (see Figure 2.1)

\[ s \cdot W = f + v \cdot C + d \cdot W \]
SUBSTRUCTURING PROCESS

Compute $S^M$ (supposed constant during the test)
Condense $S^M$ on the interface d.o.f. -> $SC^M$

Time loop, index i:
- Compute $S^{-i+1}_d$ and $S^{-i+1}_v$ (predicted values)
- Compute $S^{-i+1}_r$ (through the numerical model)
- Compute $S^{i+1}_j$
- Condense $S^{i+1}_j$ on the interface d.o.f. -> $SC^{i+1}_j$
- Solve $S^{i+1}_d$ using $C^{i+1}_d$
- Compute $S^{i+1}_d$ and $S^{i+1}_v$
- Make $i = i + 1$ and loop again

Figure 2.5 - Pseudo-dynamic test coupled with the substructuring technique

PSD CONTROLLER PROCESS

Initialize $T^M$
Add $SC^M$ to $T^M$

Time loop, index i:
- Compute $T^{-i+1}_d$ and $T^{-i+1}_v$ (predicted values)
- Impose $T^{-i+1}_d$ and measure $T^{-i+1}_r$
- Compute $T^{i+1}_j$
- Add $SC^{i+1}_j$ to $T^{i+1}_j$
- Solve $T^{i+1}_d$ (and obviously $C^{i+1}_d$ too)
- Compute $T^{i+1}_d$ and $T^{i+1}_v$
- Make $i = i + 1$ and loop again

S - Numerical substructure / T - Physical structure
C - Interface nodes
Figure 2.6 - Pseudo-dynamic testing with substructure - a four pier bridge case

1. Measure forces and impose displacements
2. Compute actuators
3. PSD controller
4. Process
5. Network communication via Berkeley Sockets
6. (numerical substructure)

DECK

Various deck cross-sections

PIERS (physical structure)

Pier cross-section

BRIDGE
3 AN EXPERIMENTAL CAMPAIGN ON R/C BRIDGES

3.1 INTRODUCTION

The ELSA reaction-wall facility of the Joint Research Centre in Ispra, Italy, is being used for Prenormative Research in support of EuroCode 8 (PREC8) [21], the provisional European standards for the design of civil engineering structures in seismic prone areas (see Pinto [64]). The research is performed jointly with 18 research organisations in the European Union grouped together in the PREC8 network under the programme of the European Commission on Human Capital and Mobility.

Among other priority topics such as: Reinforced concrete frames and walls, Infilled frames, Foundations and retaining walls, this project also covered the specific aspects of the code concerning the design of Bridges. The research programme included the Pseudo-dynamic testing of six bridges at the ELSA Laboratory at Ispra, Italy, using substructuring techniques. This was the first testing campaign performed in the world using such a technique on large scale structures [67], [68].

This programme was actually the precursor of all the tests, analysis and improvements described in this thesis. The work followed closely the PREC8 programme, trying to cover the demands and needs of the Bridges Working Group.

The first part of the experimental campaign concerned the test of four bridges: one irregular and one regular bridge designed according to the EC8, and two others representing two alternative design solutions to the irregular bridge. The test campaign in the ELSA laboratory was subsequently completed with three supplementary tests, namely: two
bridges with isolating/dissipating devices and one bridge with asynchronous input motion. Note that during an earthquake the foundation points move differently. The differences are not only due to the distance between the support elements, piers and abutments, but also to the geologic and topographic characteristics of the surrounding area [54].

To prepare the experimental campaign, a quasi-static cyclic test was performed on the short pier of the EC8 irregular bridge. The aim of the test was to evaluate the adequacy of the testing devices and instrumentation in the laboratory and, at the same time, to obtain the experimental response of a squat pier (aspect ratio = 1.75) damaged by a controlled displacement history up to failure. These results enabled the calibration of the analytical model used in the numerical analyses that preceded each experiment.

The PREC8 research programme for bridges, justifying the experimental campaign at the ELSA laboratory, is briefly presented in section 3.2. The design and the construction of the bridge piers and the choice of the scale factor are presented in section 3.3. The test set-up, the procedures and instrumentation used in the experiments, as well as the materials and the geometrical characteristics of the specimens, are described in section 3.4.

The subsequent sections follow the experimental campaign in chronological order. In section 3.5, the preliminary cyclic test on the short pier of the reference irregular bridge is presented and the results are analysed. The sequence of the Pseudo-dynamic tests is described in section 3.6, and the results of the tests on the four bridges appear afterwards in section 3.7. Cyclic tests imposing top displacements up to failure on some of the piers previously tested under Pseudo-dynamic conditions, were carried out. The results are discussed in section 3.8. Ultimate deformation capacity, plastic hinge length and ductility demands are then identified. The response of the three last irregular bridges, two with isolating/dissipating devices and one with asynchronous input motion, are briefly presented in section 3.9.

The author had the unique opportunity of following these experiments, participating actively in the post-treatment of the results. The summary and the main conclusions of the experimental campaign, are drawn in section 3.10. The figures are presented in sec-
3.2 THE PREC8 PROGRAMME - BRIDGES' WORKING GROUP

The prenormative research in support of EC8 is a programme that was launched by the European Commission to cover the topics of the European standard design code EC8 that needed to be clarified. Among other topics the programme included the classification of structural regularity, the evaluation of behaviour factors, the improvement of methods of analysis and of capacity design procedures. The working groups were formed according to the four different structural elements they represent: Reinforced concrete frames and walls, Infilled frames, Bridges and Foundations and retaining walls.

The place of ELSA in this programme was to perform the large-scale tests that have been carried out on a smaller scale (or component level), or by analytical methods, by other partners of the PREC8 network. The ELSA laboratory also participated actively in the analytical campaign within the programme.

Considering the main objectives for bridges, essentially related to regularity and behaviour factor procedures, the research programme of the bridges working group included the following tasks: evaluation of ductility demand, definition of parameters of regularity, development of design methods for irregular bridges and execution of experimental tests to verify the structural behaviour of a set of bridges. Secondary objectives related to capacity design procedures, second-order effects, asynchronous motion and isolating/dissipating devices, were also included in the PREC8 programme [63].

Apart from the Pseudo-dynamic tests of the six bridge models performed in the ELSA laboratory, shaking table tests performed in the Istituto Sperimentale Modelli E Strutture (ISMES) in Bergamo, Italy, were also part of the experimental programme. The other members of the working group were the Laboratório Nacional de Engenharia Civil (LNEC) in Lisbon, Portugal, the Universidad Politecnica de Madrid in Spain, Imperial College in London, United Kingdom and the Università di Pavia and the Università di Roma - La Sapienza in Italy.
3.3 THE TESTING CAMPAIGN AT THE ELSA LABORATORY

3.3.1 Objectives

A campaign of experimental tests on structures representing different configurations and design solutions is the ideal means to verify the adequacy of design strategies. However, the costs associated with an experimental campaign using large number of test specimens make such programmes very expensive. Moreover, the progresses in non-linear modelling occurred in the recent years has reduced the testing activity to special cases: to verify extreme design solutions and to constitute the basis for numerical modelling calibration.

In the case of the experimental campaign at the ELSA laboratory, a reduced number of bridges was tested. The objectives pursued with this campaign included the evaluation of the seismic performance of bridges (regular and highly irregular bridges) designed in accordance with the present version of the EC8, the comparison of two alternative design solutions to the EC8 conforming irregular bridge and the evaluation of safety margins for each solution.

An important additional objective was also foreseen with this campaign: the calibration of non-linear analytical models for bridge piers under cyclic loading. Furthermore, the effectiveness of isolating/dissipating devices in highly irregular bridges as well as the effect of the asynchronous input motion in the behaviour of bridges, was also evaluated in a second phase of the experimental programme.

Finally, the ELSA laboratory was interested in demonstrating its testing capabilities, especially the potential of the Pseudo-dynamic technique with substructuring applied to large scale structures such as bridges.

3.3.2 The bridges

Six bridges were selected for Pseudo-dynamic testing. The full scale bridges consist of three piers with different heights, ranging from 7 to 21 meters, and identical cross sections, supporting a 200m long continuous deck divided into four 50m long spans.

The models of these six bridges are illustrated in Figure 3.1 and described in Table 3.1.
The numbers given in parentheses correspond to the characteristics of the transverse section illustrated in Figure 3.4. The labels refer to the bridges profile and to the piers reinforcement layout; each number represents the number of times each pier is higher than the shortest pier of the six bridges, i.e. 7m; the letters A, B and C refer to the EC8 conforming solution and to the two alternative design solutions already referred, respectively.

Concerning the boundary conditions, the piers are linked to the deck through hinges that only transmit horizontal transverse forces. In the vertical direction, the deck is simply supported on devices placed at the piers top level. Thus, no interaction is considered in this direction between the piers and the deck. The deck is simply supported on the abutments.

The last two bridges in Table 3.1 correspond to two supplementary alternative design solutions to bridge B213A: one with isolating/dissipating devices on the top of all three piers and two abutments and the other with only the central pier isolated.

<table>
<thead>
<tr>
<th>Bridge</th>
<th>Pier 1</th>
<th>Pier 2</th>
<th>Pier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Height [m] (Section)</td>
<td>Long. Steel [%]</td>
<td>Height [m] (Section)</td>
</tr>
<tr>
<td>^2B232</td>
<td>14.0 (4)</td>
<td>1.15</td>
<td>21.0 (2)</td>
</tr>
<tr>
<td>^B213A</td>
<td>14.0 (1)</td>
<td>0.50</td>
<td>7.0 (3)</td>
</tr>
<tr>
<td>^2B213B</td>
<td>14.0 (1)</td>
<td>0.50</td>
<td>7.0 (5)</td>
</tr>
<tr>
<td>^B213C</td>
<td>14.0 (4)</td>
<td>1.15</td>
<td>7.0 (1)</td>
</tr>
<tr>
<td>^B213A-5Dev</td>
<td>14.0 (1)</td>
<td>0.50</td>
<td>7.0 (3)</td>
</tr>
<tr>
<td>^B213A-1Dev</td>
<td>14.0 (1)</td>
<td>0.50</td>
<td>7.0 (3)</td>
</tr>
</tbody>
</table>

- a. Regular bridge
- b. Irregular bridge designed according to EC8
- c. As in b. but with increased reinforcement in the central (short) pier
- d. As in b. but with increased reinforcement in the higher piers and decreased in the short pier
- e. As in b. but with isolating/dissipating devices on the 3 piers and abutments
- f. As in b. but with an isolating/dissipating device on the central pier

All these bridges were tested under synchronous input conditions. The numerical algorithm used in the Pseudo-dynamic tests was the $\alpha$-Newmark method with the Operator-Splitting scheme described in annex C.2: an implicit method with a non-iterative formu-
lation. The parameter $\alpha$ adopted in the algorithm is ($\alpha = -0.1$).

At the end of the experimental campaign, the bridge B213A was also tested under asynchronous input motion conditions and the two results were compared.

3.3.3 Design of the bridges

The bridges in the experimental campaign are representative of typical multi-span continuous-deck motorway bridges and describe two different situations, two different profiles corresponding to different levels of structural regularity in relation to the behaviour of the structure in the transverse direction. The deck is characterized by a hollow-core prestressed concrete beam, with a full-scale width of 14m. The piers are linked to the deck only in the horizontal transverse direction of the bridge. The weight of the deck is applied to the piers through vertical actuators. The deck is free to rotate at the extremities on the abutments. The piers have a constant rectangular hollow-core cross-section with 2.0m by 4.0m and 0.4m thick walls.

The regular configuration refers to a bridge for which the fundamental mode shape of the deck alone is similar to the mode shape of the complete structure. This corresponds to a symmetric structure having the tallest pier in the middle. This structure was labelled B232, meaning a profile with two extreme piers 14.0m high ($2 \times 7.0m$) and a central pier 21.0m high ($3 \times 7.0m$) (see Table 3.1).

The irregular bridge corresponds to a bridge for which the first mode shape of the deck with the piers is quite different from the first mode shape of the deck alone. This configuration was labelled B213, meaning the three piers are 14m, 7m and 21m high, respectively (1:1 scale). The bridge is intended to be highly irregular; besides the fact that the structure is not symmetrical in the transverse direction, the fundamental mode shapes of the deck induce much higher forces in the central, stiffer pier.

According to Calvi and Pinto [11] and to Calvi [9], the bridges were designed for a peak ground acceleration of 0.35g in medium soil conditions (soil type B) and applying the EC8 provisions. Notice that in the case of some of these bridges, the minimum reinforcement ratio was reached in one or more piers. Therefore, and because a higher reinforce-
ment ratio attracts higher forces to the piers, contributing to confuse the general trend of the results, a minimum longitudinal steel ratio half the value prescribed in the code was adopted: 0.5% instead of the 1.0%. All the bridges were designed assuming a linear elastic behaviour for the deck.

The bridge with the regular configuration was designed for a ductility demand of around two and the irregular bridge for a ductility demand of the central pier of around four. The two other irregular bridges with the same profile as the standard bridge B213A, were designed according to two alternative solutions to the EC8 [22]: the first one was designed to get a decrease of the ductility demand at the short pier to 2/3 of the value found for bridge B213A. This imposed an increase of the short pier reinforcement steel ratio from 0.92% to 1.69%. The analysis was done through an iterative non-linear dynamic calculation. This lower ductility demand solution was labelled B213B.

The second alternative solution for the irregular bridge was purely based on empirical considerations, incrementing the reinforcement of the lateral piers and decreasing the reinforcement of the central pier. The idea was to transfer to the lateral piers the forces attracted by the shortest pier. Several researchers involved in the working group shared the feeling that a beneficial effect could come from this solution, provided the global strength was not reduced. This bridge was labelled B213C and corresponds to the solution with the least reinforced short pier.

The reference irregular bridge B213A was also the object of tests involving seismic isolating devices and asynchronous input motion. The two alternative solutions with isolating devices correspond to: bridge B213A with devices at the top of the three piers and the two abutments, referred to as bridge B213A-5Dev, and bridge B213A with an isolating device at the top of the short pier only, B213A-1Dev. However, no special provisions were included in the design to account for the isolating devices.

3.3.4 Scaling of the structures

The choice of the scale factor at which the specimens should be built is determined not only by the maximum capacity of the laboratory but also by the costs of the specimen and testing apparatus. Moreover, since standard concrete and reinforcing bars are used in
the specimens, the model should be as close as possible to the prototype not to jeopardise the representativeness of the test. This is particularly important in the case of hollow-core piers due to the relatively small thickness of their walls.

In a Pseudo-dynamic tests, the inertial forces are represented numerically and, therefore, the mass of the specimen is not a limiting factor. Moreover, the assumption of a linear elastic behaviour for the deck allowed it to be simulated in the computer through a numerical model and included in the response through a substructuring technique. So, since the deck was not physically represented in the laboratory, the overall size of the bridges was not a limiting factor to the scale of the model either. The only practical limitation was given by the number of actuators that could be placed on top of the piers to apply the necessary horizontal and vertical forces.

A scale factor (1:2.5) was then selected (see Table 3.7 for the appropriate scaling factors). This factor was small enough to allow three actuators deform the strongest pier (2500kN total force) but not so small that it could risk the representativeness of the test; the thickness of the piers walls, 160mm, was large enough to place the steel bars and cast the concrete without major problems. The minimum diameter for the longitudinal bars and the stirrups was 8mm and 5mm, respectively. Even if some differences in the bond mechanisms were foreseen, they were not expected to interfere significantly with the results.

3.3.5 Construction of the piers

Due to the high accuracy required and to the reduced size of the elements, the construction of the piers was done in a pre-cast concrete workshop. The piers were cast in the horizontal position using a steel formwork. The casting of the piers was carried out in two stages: first, the casting of the three sides of the hollow core section and, afterwards, the casting of the top side. The reinforcing steel of a short pier is illustrated in Figure A.1 in annex A.

Then, the piers were transported to the working area of the ELSA laboratory and were positioned on the plinth with the crane. The casting of the plinths was done leaving the longitudinal steel bars coming from the pier properly embedded in the concrete. In order
to avoid the plinths from being too flexible, they were designed to resist the full strength of the piers without any important damage, such as shear cracking. The reinforcing steel of the plinths is illustrated in Figure A.2. Note that the plinths represent the piers foundation and an excessive flexibility would compromise the final results. To reduce the size of the plinths and the weight of the complete piers, some steel bars were placed inside the plinths to prestress them.

The different phases of the construction are documented by the photographs presented in Figures A.3, A.4, A.5 and A.5. The average mechanical properties of the concrete and steel are presented in section 3.4.1.

3.4 THE TEST SET-UP

The Pseudo-dynamic tests of the bridges were performed according to the substructuring technique implemented in the ELSA laboratory and presented in section 2.6; the deck was simulated numerically through 32 constant length linear elastic Timoshenko eccentric beam elements and the piers were tested physically. The interaction between the two parts of the structure is automatically taken into account in the algorithm.

In the case of the asynchronous input motion, other supplementary transformations were required at the level of the algorithm dealing with the substructure. The necessary changes are referred to in section 2.7.

The piers were aligned in front of the reaction wall with the shortest one in the middle as illustrated in Figure A.6. The plinths of the piers were rigidly attached to the strong floor by means of steel bars passing through the reaction floor. A stiff steel cap connected with bolts and epoxy resin to the tip of the piers, was used for applying the weight of the deck and the horizontal forces required by the Pseudo-dynamic algorithm.

The axial force \( P = 1700kN \) - corresponding to a normalized axial force, \( v = 0.1 \) was applied to the top of the piers through four actuators connected to the cap and to steel bars embedded in the pier plinth and placed inside the pier (see Figure 3.2 and Figure A.7). The vertical force was applied at the beginning of the test and it remained practically constant thanks to the hydraulic accumulators placed to compensate for the effects
of the horizontal displacement.

Two double-acting servo-hydraulic actuators (three in the case of the short pier of bridge B213B), capable of a load of 1.0MN each, were used for imposing the horizontal displacements on the top of each pier. The two extremities of the actuators were connected by spherical joints to the steel cap and to a steel plate attached to the reaction wall. The displacements were measured with respect to two independent steel frames using digital optical transducers. Only the top displacements were controlled by the algorithm in the laboratory.

The Pseudo-dynamic method can be regarded as a test on a structure condensed on a restricted number of degrees of freedom (d.o.f.), those controlled in the laboratory during the experiment. This static condensation only preserves exactly the static (elastic) behaviour of the structures, while it may alter considerably, by spurious mass and damping redistribution, its dynamic behaviour. The use of the Pseudo-dynamic method with reduced matrices obtained in this way, imposes that the dynamic behaviour be mainly controlled by the lowest modes of vibration and that these modes be well represented by the restricted degrees of freedom. Moreover, the lowest natural frequencies of the whole system should be as close as possible to the natural frequencies of the reduced system.

In order to evaluate the influence of this simplification on the results, a numerical simulation of a Pseudo-dynamic test was performed on a bridge considering that the piers stiffness was statically condensed on their top nodes. In order to reproduce the ELSA testing conditions, only horizontal transverse forces to the bridge were transferred from the piers to the deck (hinge pier-deck connection). The comparison of these results with those of the classic dynamic analysis is presented in [33]. A very good agreement between the two results was obtained.

3.4.1 Test specimens

Piers

The geometrical characteristics of the pier models are presented in Figure 3.3. Figure 3.4 shows the reinforcing steel layout for the five different transverse sections. The total height of the specimens is 2.8m, 5.6m and 8.4m for the short, medium and high piers,
respectively. However, due to the position of the steel loading device, the force is applied to the piers at a point 0.6m above the reinforced concrete top section. To take this into account, the height of the specimens was shortened to 2.2m, 5.0m and 7.8m for the short, medium and tall piers, respectively. However, in the case of the isolated bridges, since the isolators are placed between the reinforced concrete pier and the steel loading device, the isolated piers are 0.32m higher than the original piers.

The mechanical characteristics of the materials: a B500 Tempcore steel and a C25/30 concrete, were evaluated on the basis of tests performed on material samples. The results are presented in Tables 3.2, 3.3 and 3.4.

### Table 3.2: Steel mechanical properties (average values from 3 tests)

<table>
<thead>
<tr>
<th>Bar Diameter [mm]</th>
<th>Yielding Stress $\sigma_y$ [MPa]</th>
<th>Ultimate Stress $\sigma_u$ [MPa]</th>
<th>Yielding Strain $\varepsilon_y$ [%]</th>
<th>Ultimate Strain $\varepsilon_u$ [%]</th>
<th>Young Modulus $E$ [MPa]</th>
<th>Hardening Ratio $a=E_u/E$</th>
<th>Ultimate Ratio $\sigma_u/\sigma_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$8</td>
<td>529</td>
<td>585</td>
<td>0.26</td>
<td>10.09</td>
<td>203461</td>
<td>0.0028</td>
<td>1.10</td>
</tr>
<tr>
<td>$\phi$10</td>
<td>490</td>
<td>575</td>
<td>0.25</td>
<td>11.25</td>
<td>196000</td>
<td>0.0039</td>
<td>1.17</td>
</tr>
<tr>
<td>$\phi$14</td>
<td>468</td>
<td>571</td>
<td>0.23</td>
<td>11.23</td>
<td>203478</td>
<td>0.0046</td>
<td>1.22</td>
</tr>
</tbody>
</table>

### Table 3.3: Concrete mechanical properties (average values from samples of the pier A1)

<table>
<thead>
<tr>
<th>Concrete Cubic Compression Strength [MPa] (16 cubs)</th>
<th>Concrete Tensile Strength [MPa] Brazilian Test (5 cylinders)</th>
<th>Initial Tangent Modulus (E_i) [GPa]</th>
<th>Poisson Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.4</td>
<td>3.1</td>
<td>29.4</td>
<td>0.15 - 0.2</td>
</tr>
</tbody>
</table>

### Table 3.4: Concrete cubic compression strength (average values)

| Pier Type (see Figure 3.17) | A1  | A2  | A3  | B   | C   | D   | E1  | E2  | F   | G   | H   |
|-----------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Strength [MPa]              | 35.4| 37.0| 35.2| 40.2| 41.2| 38.9| 37.0| 44.4| 50.5| 44.1| 48.5|

Figure 3.5 shows the results of the cyclic test on one of the concrete samples (a concrete cylinder) and Figure 3.6 presents the axial tensile stress versus strain diagrams for some of the steel bars.

**Deck**

The deck was simulated with linear elastic Timoshenko eccentric beam elements [34].
The mechanical characteristics of the deck transverse section presented in Table 3.5 were computed using the computer code CASTEM 2000 (see annex B) and the material characteristics of the concrete: Young modulus \( (E = 25 \text{GPa}) \) and transverse modulus \( (G = 10 \text{GPa}) \). A specific weight \( (\rho = 2.5 \text{ton/m}^3) \) was adopted to evaluate the mass of the deck.

### Table 3.5: Deck cross-section geometrical and mechanical characteristics (1:2.5 scaled model)

| Barycentre \((G)\) | \(|Eds|\) [MN] | \(|Eyds|\) [MNm] | \(|Ex^2ds|\) [MNm²] | \(|Ey^2ds|\) [MNm²] | \(|Gds|\) [MN] | \(|Gyds|\) [MNm] | \(|G(x^2 + y^2)ds|\) [MNm²] |
|-----------------|-------------|-------------|---------------|----------------|-------------|-------------|----------------|
| \(x=0\) \n \(y=0.602\) | 27837 | 16772 | 56517 | 13544 | 11132 | 6709 | 22601 + 5416 |

### 3.4.2 Instrumentation

The distribution of the displacement transducers on the specimens is schematically illustrated in Figure 3.8. The transducers are represented by straight lines with two black points at the extremities. Each measuring point is signalled by three channel numbers that correspond to the three piers of each bridge: the first one refers to the medium pier, the second to the short pier and the third to the tall pier (e.g. channels 23, 47 and 71 measure the diagonal elongation of the medium, short and tall piers, respectively). The vertical transducers from 1 to 16, in the case of the medium pier, define the horizontal slices represented in Figure 3.3.

The instrumentation of the short piers of the irregular bridges included a higher number of displacement transducers in order to enable the splitting of bending and shear deformations. For each test, 93 passive channels were used for measuring the displacements and axial forces in the bridge piers. No strain gauges were placed on the transverse and longitudinal reinforcing bars.

Keeping in mind the purpose of the measurements, the displacement transducers were assembled into three main groups: rotations, horizontal displacements and transverse
section warping.

Rotations

A set of displacement transducers (LVDTs) placed along the two external opposite faces of the specimen allowed the computation of the pier slices curvature. The formulation that was used for accessing the rotations and the bending displacements is described in annex D. In addition, a supplementary set of transducers was placed for monitoring any possible rotation in the direction perpendicular to the imposed displacements (e.g. channels 17 to 20 for the medium piers) and to measure both the total rotation of the pier (R/C part) and its elongation (e.g. channels 21 and 22 for the medium pier).

Horizontal displacements

Another set of transducers (e.g. channels 73 to 77 for the short piers) measured the horizontal displacements at different levels. They were linked to a rigid bar of height $H$ hinged at both extremities: at the top and at the bottom of the pier as illustrated in Figure 3.8. The total horizontal displacement $d_i$ of a point $i$ located at a distance $h_i$ from the base, was computed using the top displacement $d$ and the displacement $a_i$ measured by the horizontal transducer at this point

$$d_i = \left(\frac{h_i}{H}\right) \cdot d + a_i$$  \hspace{1cm} 3.1

The instrumentation set-up also included the measurement of the top displacement and the corresponding horizontal reaction forces at each pier. These values are actually part of the control system of the Pseudo-dynamic test. The reaction forces were quantified through load cells placed at the piston-rod ends of each actuator and the horizontal top-displacements were measured in relation to a reference frame, external to the pier, using Heidenhein optical transducers. For the vertical forces, they were measured through load cells mounted in series with the vertical bars that transmitted the forces to the foundation. Each vertical piston was equipped with a hydraulic accumulator that should absorb any possible variation of the vertical force with the horizontal displacement. Thus, an almost constant force was expected during the experiments.
3.5 QUASI-STATIC TEST OF A SQUAT PIER

3.5.1 Test objectives and testing procedure

In order to prepare the Pseudo-dynamic testing campaign, a squat pier was previously tested [67]. The main objectives of this test were: to verify the adequacy of the testing devices and instrumentation (position, fixing systems, loading conditions, etc.) and to obtain experimental results from a well known cyclic displacement history enabling the calibration of the analytical models that were used afterwards in the preliminary numerical analyses. The short pier of bridge B213 was chosen.

From the point of view of the analytical models, this was an interesting pier to test; the shear behaviour was expected to be important and, thus, the calibration of more advanced models taking into account the non-linear behaviour due to the flexural and shear forces, would be possible.

The history of horizontal displacements imposed on the top of the pier was selected based on numerical analyses and on the behaviour of the pier during the loading process (see Figure 3.7). It consisted of:

- a set of cycles of increasing amplitude until the foreseen analytical yielding point (8mm): 1, 2, 4, 6 and 8mm. Due to other phenomena not included in the numerical analysis, such as: flexibility of the basement, shear deformation and pull-out effects, no yielding was detected from the observation of the global transverse force versus top displacement experimental curve. The same result was obtained for a supplementary 10mm amplitude cycle;

- a subsequent cycle of 18mm amplitude to catch the yielding point of the pier. The yielding of the steel bars was reached for a top displacement around 12mm. Two additional cycles with the same amplitude (ductility demand = 1.5) were applied to the pier afterwards.

- two sets of three cycles each with amplitudes of 36mm (μ = 3.0) and 72mm (μ = 6.0), where μ is the ductility demand.
A constant axial load \( (P = 1700kN) \) was applied to the pier. It corresponded to the normalized axial load that would be applied to the piers during the Pseudo-dynamic tests.

### 3.5.2 Test specimen

The geometrical characteristics of the specimen correspond to those presented in Figure 3.3 for the short pier (S): a 2.8m high pier with 2.2m of reinforced concrete plus 0.6m for the steel loading device, giving an aspect ratio (L/D = 1.75). The reinforcing steel layout corresponds to section type 3 in Figure 3.4. The mechanical characteristics of the materials: a B500 Tempcore steel and a C25/30 concrete, are presented in Tables 3.2 and 3.3.

### 3.5.3 Processing of the test results

**Force-displacement diagrams and damage**

The force displacement diagrams shown in Figure 3.9 illustrate the global behaviour of the pier within the uncracked, cracked, yielding, low and high ductility zones. The physical phenomena that occurred during the test, namely: cracking, yielding, spalling of the cover concrete and buckling of the rebars, are also indicated in that figure. They can be summarized as follows:

- initial flexural cracking during the 2\( mm \) amplitude cycle followed by diagonal cracking - Figure 3.9 b) c);

- yielding for 12\( mm \) top displacement - Figure 3.9 d). For ductility demand 1.5 and 3.0, there was a general increase of the diagonal and flexural cracking - Figure 3.9 d) e);

- spalling of the cover concrete in the transition from ductility demand 3.0 to 6.0 - Figure 3.9 a);

- buckling of the main longitudinal steel bars near the basement for the first reversal cycle at ductility demand 6.0 and of the lateral internal rebars along the main diagonal cracks during the last two cycles at ductility demand 6.0 (see photographic documentation in annex A).

Therefore, from the response diagrams, a good seismic performance of the pier in terms
of ductility and energy dissipation could be expected. In fact, neither spalling nor buckling occurred within the 1.0 to 3.0 ductility range. The failure of the pier was dictated by the rupture of the rebars, after buckling in compression, that occurred at ductility 6.0. After spalling of the cover concrete and failure of the unconfined concrete between stirrups, the rebars that were not properly tied to the stirrups, or not completely embraced by the transverse steel, buckled to the sides with a length two and three times the space between the stirrups. Spalling started for a ductility demand close to 5.0.

**Flexural and shear deformations**

Following the standard procedure in annex D, the horizontal displacements of the pier due to the bending moments were estimated from equation (D.4). The displacements due to the shear forces were calculated from the difference between the total and the bending displacements.

However, as the horizontal displacements were not measured at the top of all slices (see Figure 3.8), the calculation of the shear displacements was restrained to a smaller number of points along the pier height. In order to estimate the “missing” values, a parabolic distribution was assumed for the total displacements within the slices. In the interpolation procedure, three available displacements were used systematically: one just below and two just above the point being analysed.

Figure 3.10 a) shows the displacement profiles along the pier height while Figure 3.10 b) shows the shear and bending displacements separately. As expected, shear deformations in squat piers contribute significantly to the total displacement. In the case of this squat pier, the shear deformation was responsible for approximately 40% of the total displacement at the level of slice 8 (1.6m height). This contribution tended to decrease with the increase of the ductility level as illustrated in Figure 3.11.

The distribution of the average curvature and shear strain within the slices is also presented in Figure 3.10.

**Energy dissipation**

The flexural moment versus curvature and the shear force versus distortion diagrams are
presented in Figure 3.12; while the shear force versus distortion curves exhibit a smooth
decrease of the hysteretic energy dissipated along the pier height, the flexural moment
versus curvature diagrams show that most of the dissipation of energy is concentrated in
the first slice.

Figure 3.13 presents the ratio of the energy dissipated in each slice to the total energy, a),
and the dissipation of energy accumulated from the base of the pier up to the slices, b),
for the different displacement ductility levels. In fact, most of the hysteretic energy is
dissipated in the first 0.6m of the pier (more than 80% for 1.5 ductility demand and more
than 90% for 3.0 and 6.0 ductility demand). Furthermore, 50% of the total energy is dis-
sipated in the first slice. It is also clear from these diagrams that, in terms of dissipation
of energy, the bending mechanism dominates.

**Ductility**

The maximum values of curvature ductility demand in the slices for the different dis-
placement ductility levels and for both positive and negative displacements, are pre-
sent in Table 3.6. The yielding curvature of the slices was assumed equal to the
yielding curvature estimated for the first slice.

<table>
<thead>
<tr>
<th>Slice</th>
<th>Yield Curvature (C_yb)*1E-2</th>
<th>C_m/C_yb (μ_c)</th>
<th>C_m*1E-2 (μ_c)</th>
<th>( μ_d = 1.5 )</th>
<th>( μ_d = 3 )</th>
<th>( μ_d = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C_m^*10^2 ) [1/m]</td>
<td>(μ_c)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.65</td>
<td>0.61</td>
<td>1.3</td>
<td>1.5</td>
<td>2.2</td>
<td>2.6</td>
</tr>
<tr>
<td>2</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
</tr>
<tr>
<td>3</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
</tr>
<tr>
<td>4</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
</tr>
<tr>
<td>5</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
</tr>
</tbody>
</table>

a. Inferior to \( C_yb \)

**Other measurements**

As previously mentioned, one of the objectives of this test was to evaluate the adequacy
of the testing and measurement devices in order to simplify and improve the testing sys-
tem in the subsequent Pseudo-dynamic tests. Thus, a pair of transducers eccentric to the vertical line of transducers 25 to 40 in Figure 3.8, was used for following any possible out of plane rotations. The results in Figure 3.14 show that no important differences exist between the displacements measured by the left and the right side transducers, i.e. the out of plane deformations were negligible. Therefore, these transducers were not placed on the bridge piers during the Pseudo-dynamic tests.

Figure 3.14 also shows that, although hydraulic accumulators were placed to compensate the effects of the horizontal displacement, the vertical force imposed on the top of the piers presents differences between the maximum and the minimum values of around 15% of the initial value.

3.5.4 Summary

The results of this preliminary test on the squat pier of bridge B213A confirmed the ductile behaviour expected from the design. Furthermore, the test highlighted the following aspects:

- extensive shear cracking was observed from the beginning of the test just after the first flexural cracking. It led to a decrease of the pier stiffness and an increase of the yielding displacement (12mm) when compared to the analytical value (8mm);

- for almost all ductility levels, the shear displacement was approximately 30% of the total displacement. However, the corresponding dissipation of energy was less than 10% of the total dissipated energy. This is evident from the analysis of the shear force versus shear strain diagrams that show a pronounced pinching effect from diagonal crack closing. In this case, the presence of diagonal reinforcement would have improved the cyclic behaviour of the pier;

- the dissipation of energy occurred mainly in the first slice of the pier near the base (140mm) that corresponds to a height of 8.8% of the section depth. No clear spread of plasticity to the upper slices was observed;

- the spalling of the cover concrete occurred between ductility 4.0 and 5.0 and was confined to the first slice. However, the corresponding crack pattern was already visible
at the last 3.0 ductility cycle;

- buckling of the φ12 mm rebars occurred during the first reversal cycle at ductility 6.0. Moreover, during the last two cycles at ductility 6.0 the rebars φ8 mm located in the sides walls of the pier buckled between the two faces of the main diagonal cracks. The spalling of the cover concrete observed in this zone contributed to that unusual result;

- failure occurred during the third cycle at ductility 6.0. In this analysis, failure corresponds to the instant for which the load carrying capacity of the pier decreases of 30%.

Finally, this experiment showed that the loading and instrumentation devices adopted in the test were adequate for the purposes of the experiments and that they could be maintained in the subsequent Pseudo-dynamic tests.

3.6 THE PSEUDO-DYNAMIC TESTS

3.6.1 Input motion

Following the hypothesis of the bridges design described in section 3.3.3, a stationary artificial accelerogram fitting the EC8 response spectrum was generated for the Pseudo-dynamic tests with synchronous input motion. The accelerogram, with a duration of 10 seconds and a maximum amplitude of (0.35g), was referred to as Design Earthquake (DE). It was adjusted to the scale of the test specimens according to the relations presented in Table 3.7: a peak acceleration of (0.35g × 2.5 = 0.875g) and a duration of (10s / 2.5 = 4s). Figure 3.15 shows the accelerogram and the corresponding response spectrum for the full-scale structure (Prototype).

For the asynchronous motion, the five input accelerograms, one for each pier and the two abutments, were also generated to fit the same EC8 response spectrum but also to respect a certain coherence between them. The procedure is referred to later in section 3.9.2.

Each bridge was tested twice under Pseudo-dynamic conditions: first using the DE and then using the same input motion multiplied by a higher intensity factor (not initially previewed in the PREC8 programme) in order to impose a major damage in the piers (High
Level Earthquake - HLE). For the set of the irregular bridges, the intensity of the high level earthquake was tailored to the weakest design solution (bridge B213C) so that the response of all the bridges could be compared at the end.

Table 3.7: Cauchy similitude relationships between Prototype (P) and Model (M)

<table>
<thead>
<tr>
<th>Label</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>( L_P = (2.5) \cdot L_M )</td>
</tr>
<tr>
<td>Area</td>
<td>( A_P = (2.5)^2 \cdot A_M )</td>
</tr>
<tr>
<td>Volume</td>
<td>( V_P = (2.5)^3 \cdot V_M )</td>
</tr>
<tr>
<td>Mass</td>
<td>( M_P = (2.5)^4 \cdot M_M )</td>
</tr>
<tr>
<td>Velocity</td>
<td>( \nu_P = \nu_M )</td>
</tr>
<tr>
<td>Acceleration</td>
<td>( a_P = (2.5)^{-1} \cdot a_M )</td>
</tr>
<tr>
<td>Force</td>
<td>( F_P = (2.5)^2 \cdot F_M )</td>
</tr>
<tr>
<td>Time</td>
<td>( t_P = (2.5) \cdot t_M )</td>
</tr>
<tr>
<td>Frequency</td>
<td>( f_P = (2.5)^{-1} \cdot f_M )</td>
</tr>
<tr>
<td>Strain</td>
<td>( \varepsilon_P = \varepsilon_M )</td>
</tr>
<tr>
<td>Stress</td>
<td>( \sigma_P = \sigma_M )</td>
</tr>
</tbody>
</table>

The choice of the intensity factors was based on predictive numerical analyses: 1.2 for the irregular bridges and 2.0 for the regular bridge. The values 1.2 and 2.0 were estimated, through numerical analyses, to cause failure of the short pier of bridges B213C and B232. Table 3.8 summarizes the Pseudo-dynamic tests carried out in the laboratory.

Table 3.8: Scaling factors and maximum nominal input accelerations for the Pseudo-dynamic tests

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Design Earthquake (DE)</th>
<th>High Level Earthquake (HLE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intensity Factor</td>
<td>Max. Nominal Acceleration [m/s²]</td>
</tr>
<tr>
<td>B232</td>
<td>1.0</td>
<td>3.5</td>
</tr>
<tr>
<td>a B213A, B213B, B213C</td>
<td>1.0</td>
<td>3.5</td>
</tr>
</tbody>
</table>

a. with or without isolating/dissipating devices and for synchronous and asynchronous input motion

3.6.2 The tests sequence

Predictive numerical analysis showed that very low damage should be expected in the medium and tall piers of bridges B213A and B213B for the DE input motion. Based on these results, the re-utilization of the two extreme piers of the standard irregular bridge B213A on bridges B213B and B213A with isolating devices and asynchronous input
motion, was foreseen. Consequently, to prevent these piers from being seriously damaged, the high level earthquake test on bridges B213A and B213B was performed simulating the behaviour of the medium and tall piers analytically. Based on the envelop response curves from the design earthquake test and, again, on predictive numerical analysis indicating that a very low dissipating bi-linear behaviour curve should be expected for the two piers, the non-linear elastic force displacement curves represented by a thick broken line in Figure 3.16 were adopted for the HLE tests.

This underlines the large interaction that exists, or should exist, between the numerical and the experimental analyses. In fact, all the experimental campaign was preceded by numerical computations which turned out to be an important decision support, allowing the improvement of the experimental procedures and the reduction of the global costs of the programme.

The experimental campaign was divided in three parts: the first set of tests corresponded to the experiments with synchronous input motion without isolating/dissipating devices, the second one corresponded to the isolated bridges with synchronous input motion and the third and last one corresponded to the experiments on bridge B213A with asynchronous input motion. The sequence of the tests is schematically described in Figure 3.17. Figure 3.18 illustrates the seismic tests performed on the bridges:

- bridge B213A; design earthquake test with the deck substructured, followed by the high level earthquake test with not only the deck but also the medium and the high piers substructured;

- bridge B213B; design earthquake test with the deck substructured and using the medium and the high piers from bridge B213A. The high level earthquake test was performed afterwards with the deck and the medium and the high piers substructured;

- bridge B213C; design earthquake test followed by the high level earthquake test, both with the deck substructured;

- bridge B232 (regular); design earthquake test followed by the high level earthquake test, both with the deck substructured.
To evaluate the safety margins associated with each pier and, subsequently, with each bridge, at the end the piers that were not needed in subsequent experiments were tested cyclically until failure. This campaign included the medium and tall piers of the regular bridge and of the irregular bridge B213C.

The second part of the experimental campaign included four tests on the EC8 conforming bridge B213A isolated with two different configurations: with isolating/dissipating devices at the top of all piers and abutments or only at the top of the short pier; these two configurations were labelled B213A-5Dev and B213A-1Dev, respectively. The sequence of tests was:

- bridge B213A-5Dev; design earthquake test followed by the high level earthquake test, both with the deck substructured and the three piers physically represented in the laboratory. The medium and tall piers were the same of bridges B213A and B213B;

- bridge B213A-1Dev; design earthquake test followed by the high level earthquake test. The same input actions and testing conditions described for bridge B213A-5Dev were applied to bridge B213A-1Dev. Moreover, all three piers in the two tests came from the bridge B213A-5Dev.

Finally, the third part of the campaign consisted in testing bridge B213A, slightly damaged after the previous input actions and without the isolating devices, with an asynchronous input motion. New accelerograms were generated for this test but always for the same response spectrum in Figure 3.15. In order to compare the response with the results with synchronous motion, the same intensity factors of the input action considered in bridge B213A were adopted.

In the test, only the horizontal transverse forces were transmitted from the deck to the piers and vice-versa. The weight of the deck was imposed directly on the top of the piers and no interaction between the two structures was considered in the vertical direction.

3.6.3 Inertia and damping forces

In a Pseudo-dynamic test, the inertia and the viscous damping forces are simulated numerically and the corresponding matrices may be estimated from preliminary dynamic
identification tests performed on the structure (e.g. stiffness and free vibration tests). However, in the case of Pseudo-dynamic tests with substructuring, as only part of the structure exists in the laboratory and the connections between the substructure and the physical structure only exist in the numerical algorithm, this initial dynamic identification is impossible.

The solution adopted in the case of the bridges was to evaluate the mass matrix of the numerical substructure through the finite element model of the deck, and to include its contribution to the mass matrix of the Pseudo-dynamic algorithm as described in section 2.6. In the case of the piers, the mass matrix was computed as if it was condensed at the piers top nodes.

Concerning the numerical damping, no contribution from the piers was considered; the main source of energy dissipation was the material hysteresis that is automatically taken into account during the test. However, because a linear elastic model was assumed for the deck, some numerical damping was included in the algorithm to account for any possible dissipation of energy from the deck. The damping matrix of the substructure was computed by the Rayleigh method and involving only the mass and the stiffness of the deck. Thus, a damping ratio ($\zeta=1.6\%$) was chosen for the two lower natural frequencies of the bridge in the transverse direction. The same procedure was adopted for the piers simulated through a numerical model, i.e. the medium and tall piers of bridges B213A and B213B for the HLE action.

However, in the case of the asynchronous input motion, other more consistent damping matrixes had to be used [61]; although the contribution of the stiffness matrix to the Rayleigh damping is also valid for absolute motions, the mass matrix introduces damping forces even when only a rigid body type motion exists.

3.7 TESTING RESULTS FROM THE FIRST SET OF BRIDGES

3.7.1 Global response

The dynamic response of the bridges in terms of the piers top displacements is shown in Figures 3.19 to 3.22. Table 3.9 presents the maximum top displacements and the maximum bending moments of each pier.
Table 3.9: Global response for design earthquake and high level earthquake tests

<table>
<thead>
<tr>
<th>Bridge</th>
<th>Pier</th>
<th>H [m] (Section)</th>
<th>Design Earthquake (DE)</th>
<th>High-Level Earthquake (HLE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Maximum Displ. [m]</td>
<td>Flexural Moment [MNm]</td>
<td>Maximum Displ. [m]</td>
</tr>
<tr>
<td>B232</td>
<td>1</td>
<td>5.6 (4)</td>
<td>0.044</td>
<td>4.03</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4 (2)</td>
<td>0.062</td>
<td>2.92</td>
</tr>
<tr>
<td>B213A</td>
<td>1</td>
<td>5.6 (1)</td>
<td>0.022</td>
<td>2.36</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (3)</td>
<td>0.024</td>
<td>3.67</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>0.028</td>
<td>2.24</td>
</tr>
<tr>
<td>B213B</td>
<td>1</td>
<td>5.6 (1)</td>
<td>0.030</td>
<td>2.39</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (5)</td>
<td>0.023</td>
<td>5.09</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>0.028</td>
<td>2.14</td>
</tr>
<tr>
<td>B213C</td>
<td>1</td>
<td>5.6 (4)</td>
<td>0.017</td>
<td>2.73</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (1)</td>
<td>0.020</td>
<td>2.55</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (4)</td>
<td>0.026</td>
<td>2.49</td>
</tr>
</tbody>
</table>

a. () Values from the bi-linear elastic numerical model

The response of the regular bridge is mainly controlled by the first mode of vibration with components in the horizontal transverse direction of the bridge. For the irregular bridges, the response includes the two first modes of vibration with components in the same direction. This is very clear from the analysis of the mode shapes of the bridges represented in Figure 3.23 and the envelop displacement profiles of the deck illustrated in Figure 3.24. In fact, the two modes shapes of the irregular bridge that participate more actively in the response have similar natural frequencies. These values were computed numerically considering for the piers the material properties of the deck. As expected, the synchronous input motion imposes symmetrical, or close to symmetrical deformations on the structures.

Figure 3.25 shows the "power spectra" of the piers top displacements for the two earthquake intensity levels. The frequency content of the responses of the design earthquake and the high level earthquake is quite different. Due to the non-linear behaviour of the piers, the average predominant frequency of the response decreases.
3.7.2 Piers force-displacement diagrams and damage

The force versus displacement diagrams at the top of the piers for the four bridges are illustrated in Figures 3.26 to 3.29. Figure 3.30 groups the diagrams of the short piers of the irregular bridges. The physical phenomena that occurred during the tests, namely: cracking, yielding, spalling of the cover concrete and buckling of the rebars, for the different bridges and the two earthquake levels, are described in Table 3.10. Only cracking

<table>
<thead>
<tr>
<th>Bridge</th>
<th>Pier</th>
<th>H [m] (Section)</th>
<th>Design Earthquake (DE)</th>
<th>High-Level Earthquake (HLE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B232</td>
<td>1</td>
<td>5.6 (4)</td>
<td>Cracking at the bottom</td>
<td>Slight spalling at the section corners</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Slight yielding</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4 (2)</td>
<td>Cracking up to half height</td>
<td>Slight spalling and buckling of the ‘free’ rebars at the section corners</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Slight yielding</td>
<td></td>
</tr>
<tr>
<td>B213A</td>
<td>1</td>
<td>5.6 (1)</td>
<td>Slight cracking</td>
<td>Simulated analytically</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (3)</td>
<td>Cracking and slight yielding</td>
<td>Spalling and Buckling of the ‘free’ rebars</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Slight yielding</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>Slight cracking</td>
<td>Simulated analytically</td>
</tr>
<tr>
<td>B213B</td>
<td>1</td>
<td>5.6 (1)</td>
<td>No additional cracking(^a)</td>
<td>Simulated analytically</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (5)</td>
<td>Diagonal cracking</td>
<td>Diagonal cracking</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Very slight yielding</td>
<td>Yielding</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>No additional cracking(^a)</td>
<td>Simulated analytically</td>
</tr>
<tr>
<td>B213C</td>
<td>1</td>
<td>5.6 (4)</td>
<td>Slight cracking</td>
<td>Slight yielding</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (1)</td>
<td>Cracking and yielding</td>
<td>Buckling of rebars, spalling</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Crack opened at the base</td>
<td>Failure of rebars at the base</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (4)</td>
<td>Slight cracking</td>
<td>Cracking</td>
</tr>
</tbody>
</table>

\(^a\) Piers from Bridge B213A

and slight yielding was observed in the piers during the DE input action. Instead, during the HLE action and apart from a very clear yielding of the longitudinal bars, there was spalling of the cover concrete and, in the case of the irregular bridges, buckling of the rebars. Moreover, the short pier of bridge B213C reached failure for the second input motion.

The final state of the piers, with the different cracking patterns, can be seen in Figures A.8 and A.9. Although the three squat piers present a clear shear cracking pattern, while in the pier of bridge B213B there is a quite uniform distribution of cracks, in the pier of
bridge B213C (the least reinforced) the cracks are much more concentrated.

Figure 3.31 illustrates the displacement profiles along the height of the pier. The shear and the bending responses were split using the procedure described in section 3.5.3. As expected, the shear deformation of the short piers contributed significantly to the total displacement. This is particularly evident in the most reinforced pier, i.e. the short pier of bridge B213B. In fact, approximately 40% of the displacement at the level of slice 8, placed at 1.6m from the base, results from shear deformations.

3.7.3 Ductility

The values of the displacement ductility demand in the bridge piers for the two earthquake tests are presented in Table 3.11. The objectives of the HLE were partly achieved:

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>H [m] (Section Type)</th>
<th>Design Earthquake</th>
<th>aHigh-Level Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Maximum Displ. [m]</td>
<td>Ductility</td>
<td>Maximum Displ. [m]</td>
</tr>
<tr>
<td>B232</td>
<td>1</td>
<td>5.6 (4)</td>
<td>.044</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4 (2)</td>
<td>.062</td>
<td>1.9</td>
</tr>
<tr>
<td>B213A</td>
<td>1</td>
<td>5.6 (1)</td>
<td>.022</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (3)</td>
<td>.024</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>.028</td>
<td>&lt;1</td>
</tr>
<tr>
<td>B213B</td>
<td>1</td>
<td>5.6 (1)</td>
<td>.025</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (5)</td>
<td>.023</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>.028</td>
<td>&lt;1</td>
</tr>
<tr>
<td>B213C</td>
<td>1</td>
<td>5.6 (4)</td>
<td>.017</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (1)</td>
<td>.020</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (4)</td>
<td>.026</td>
<td>0.5</td>
</tr>
</tbody>
</table>

a. 1.2 and 2.0 times the DE for bridges B232 and B213, respectively
b. (.) Values obtained from the numerical model (pier substructured)

the irregular bridge B213C failed for the 1.2 intensity factor and the regular bridge increased its ductility demand to twice the value from the DE but without failure of the piers.

The irregular bridges show a high concentration of ductility demand at the short pier not
shared by the two other piers. The increase of 20% in the intensity of the input action corresponded to an increase of the displacement ductility demand of almost three times in the case of bridge B213C, and two times in the case of bridge B213A.

### 3.7.4 Earthquake performance of the bridges

Concerning the performance of the bridges under earthquake type loading, some aspects must be highlighted. Thus, the behaviour of the bridges is discussed in detail in the next paragraphs.

**Irregular bridges (B213)**

The behaviour of the three irregular bridges was mainly controlled by the short piers which, due to their higher stiffness, attracted most of the shear forces and the ductility demand of the bridges. The imposition of a minimum longitudinal steel ratio does not always comply with a global behaviour factor procedure: the strength of the piers may have to be increased beyond the value imposed by the behaviour factor procedure, thus changing the conditions of the design. It should be noticed that the three alternative solutions adopted in the design of the irregular bridges corresponded, in practice, to the application of local behaviour factors instead of a unique global factor. Actually, this may be a suitable approach to the irregularity issue.

Moreover:

- in the case of bridge B213A, the dissipation of energy was limited to the short pier. This corresponds to the situation for which a warning is issued in EC8, part 2: "*behaviour factors > 1.2 should not be used without special consideration, if the development of plastic hinges in the piers is not probable, because the piers do not reach yielding under the design seismic action*". The ductility demand on the short pier is, however, reasonably small for the design earthquake;

- in the test of bridge B213B, the short pier attracted much larger forces, but experienced considerably lower ductility demands. Again, energy dissipation only took place in the short pier. The distribution of cracks was much more uniform than in the case of bridge B213A (see Figures A.8 and A.9);
finally, for bridge B213C, the goal to obtain a more regular shear forces distribution was achieved. The ductility demands on the short pier were, however, much higher than in the case of bridge B213A. The advantages of the corresponding design approach are therefore debatable.

The test results confirmed that the strategy adopted in the design of bridge B213B, that corresponded to the application of a lower local behaviour factor, is the best of the three. However, an additional test with a higher intensity input motion is required to evaluate the ultimate capacity of the structure and, subsequently, to assess its safety against collapse.

Regular bridge (B232)

The results of the test on bridge B232 comply with the regularity issue, as mentioned in the EC8, part 2. The dissipation of energy took place in all three piers and the ductile behaviour was correctly assumed. An input signal with an intensity twice the design earthquake was applied without loss of capacity.

The maximum values of ductility demand for the DE were quite similar for all the bridges except for Bridge B213C. However, the irregular bridges, tested with an input motion slightly higher than the design earthquake (1.2 times) and much lower than the intensity used for the regular bridge (2.0 times), were heavily damaged (see the vulnerability functions in Figure 3.32). Thus, safety against collapse of the irregular bridges was quite low when compared to the regular bridge.

Finally, bridges B232 and B213A, designed in accordance to the EC8 [21], [22], did not comply with the general principle: "uniform safety margins against ultimate limit states should be provided to structures assumed to fulfil the same requirements".

3.8 ULTIMATE CAPACITY OF THE PIERS

3.8.1 Cyclic testing of the bridge piers

After the Pseudo-dynamic test of the bridges with synchronous input motion and without isolators, the piers not needed in subsequent tests were tested cyclically up to failure. The purpose of these tests was to quantify the ultimate deformation capacity of the piers and,
therefore, to evaluate the safety margins of the bridges.

Four piers were tested: the medium and the tall piers of bridges B213C and B232. The history of displacements consisted in:

- bridge B213C; medium pier: two cycles at 6, 9, 12 and 15mm. Failure was achieved during the third cycle at 15mm. Tall pier: two cycles at 6, 12, 18 and 24mm. Failure was achieved during the third supplementary cycle at 24mm.

- bridge B232; medium pier: two cycles at 9, 12 and 15mm. Failure was achieved during the first cycle at 18mm. Tall pier: two cycles at 15mm with immediate failure.

### 3.8.2 Flexural and shear deformation

Figure 3.33 shows the transverse force versus displacement diagrams at the top of the piers. Table 3.12 presents the ultimate curvature ductility for the tested piers. The short

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>H[m] (Section)</th>
<th>D_y+</th>
<th>D_y-</th>
<th>C_y+</th>
<th>C_y-</th>
<th>F_y+</th>
<th>F_y-</th>
<th>Max. Displ. Ductility</th>
<th>Ultimate Curvature Ductility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Slice #1</td>
</tr>
<tr>
<td>B232</td>
<td>1</td>
<td>5.6</td>
<td>0.32</td>
<td>0.032</td>
<td>0.0051</td>
<td>0.0054</td>
<td>0.68</td>
<td>0.66</td>
<td>5.6</td>
<td>19.8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4</td>
<td>0.35</td>
<td>0.032</td>
<td>0.0043</td>
<td>0.0039</td>
<td>0.30</td>
<td>0.30</td>
<td>4.3</td>
<td>21.6</td>
</tr>
<tr>
<td>B213C</td>
<td>1</td>
<td>5.6</td>
<td>0.31</td>
<td>0.032</td>
<td>0.0074</td>
<td>0.0070</td>
<td>0.67</td>
<td>0.67</td>
<td>4.8</td>
<td>19.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8</td>
<td>0.0051</td>
<td>0.0048</td>
<td>0.0040</td>
<td>0.0040</td>
<td>0.78</td>
<td>0.77</td>
<td>9.6</td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>0.57</td>
<td>0.065</td>
<td>0.0048</td>
<td>0.0048</td>
<td>0.45</td>
<td>0.44</td>
<td>4.2</td>
<td>22.1</td>
</tr>
</tbody>
</table>

pier of bridge B213C, since it reached failure during the Pseudo-dynamic test, was also included in this table. A large curvature ductility demand concentrated in the first slice occurred in the five piers. This is confirmed by the moment versus curvature diagrams in Figures 3.34 to 3.37 that reveal an important dissipation of energy at the level of the first slice when compared with the upper slices.
3.8.3 The piers performance

As mentioned before, the bridge piers were tested until collapse in order to evaluate their cyclic performance and their ultimate capacity. The final look of the medium and tall piers of the regular bridge B232 are illustrated in Figure A.10. From the test results the following aspects are underlined:

- the piers exhibited a ductile behaviour with stable hysteretic loops. A minimum displacement ductility of about 4.0 would be expected in the case of monotonic loading;

- the degradation of the load carrying capacity was quite low. An appropriate confinement of the core concrete contributed to this result;

- the dissipation of energy was mainly concentrated at the bottom slice. A sudden decrease of the curvature ductility demand was observed in the upper slices. Nevertheless, in the case of the short pier of bridge B213A, the diagonal cracking transferred, apparently, the dissipation of energy from slice two to slice three.

- finally, the piers collapse was generally initiated by the buckling of the rebars placed close to the section corners but not completely embraced by the transverse steel. After spalling of the cover concrete, these rebars buckled not only towards the outside but also towards the sides of the pier. The importance of a good detailing of the steel bars is once more highlighted.

3.9 TESTING RESULTS FROM THE SECOND AND THIRD SET OF BRIDGES

This section gives an overview of the last part of the PREC8 experimental campaign on bridges. It includes the test of the irregular bridge B213A with isolating/dissipating devices and, afterwards, with an asynchronous input motion; from strong motion arrays installed in seismic areas it is clear that the ground motion of points close to the surface is not synchronous [56]. Two alternative solutions for the irregularity issue are presented in the next section. Detailed description of the test can be found elsewhere [66].
3.9.1 Bridge B213A isolated

The scope of an isolating device is to shift the natural frequencies of the system it is supposed to protect to a frequency range less sensitive to the input action, to dissipate energy maintaining the relative displacements to acceptable values and, at the same time, to provide rigidity under service load levels. From the expected results and the effective response of the irregular bridges, two alternative design solutions were tested in the laboratory: bridge B213A with isolating devices on the top of the three piers and abutments and bridge B213A with an isolating device just on the top of the short pier. These two solutions were referred to as the five isolators system and the one isolator system, bridges B213A-5Dev and B213A-1Dev, respectively.

As illustrated in Figure 3.39, the devices are made of two elements: a rigid sliding cylinder that transmits the vertical force applied to the top of the device to the pier, and a flexible vertical steel element linking the upper-structure to the lower-structure in the horizontal transverse direction. When submitted to cyclic loading, this second element is highly dissipating. Figure 3.41 illustrates the non-linear hysteretic behaviour law of the isolators through the response curves registered at the abutments. Furthermore, it shifts the frequencies of the structure, modifying the stiffness of the pier in the transverse direction, and it cuts the maximum shear force transmitted to the piers through a bi-linear behaviour law with a very low hardening ratio. The number of dissipating elements positioned in parallel in the device is given by the design.

**Test set-up**

The devices were mounted in series with the piers. They were positioned at the top of the reinforced concrete structure and below the steel cap. The test set-up and the procedures used in the test of the bridges with isolating/dissipating devices were identical to those used in the previous tests. However, in the case of the bridge with five isolators, apart from the piers top section two other d.o.f. that correspond to the movement of the deck on the devices at the abutments had to be represented in the laboratory. These two devices were fixed on the ground floor and two actuators were placed on their top to impose the corresponding displacements. Therefore, two supplementary displacement transducers had also to be placed to measure the actual displacements applied on the top
of each device.

The height of the piers changed due to the inclusion of the devices; each isolated pier grew about 34 cm in relation to the non-isolated piers. In terms of the number of disputative elements placed inside the devices, bridge B213-5Dev had two elements on the abutments and four elements on the piers. In the case of bridge B213A-1Dev, the device had eight disputative elements.

The horizontal and the vertical mechanisms of the device are not coupled and no up-lifting effect was considered. Therefore, no actuators were placed on the devices at the abutments in the vertical direction.

The changes induced by the devices on the frequencies and on the mode shapes of the bridges are illustrated in Figure 3.38. The stiffness measured at the beginning of the first experiment on each of the three solutions: B213A, B213A-5Dev and B213A-1Dev, is compared in Figure 3.39.

**Test results**

The diagrams illustrated in Figure 3.40 show the displacement time histories for the solutions of bridge B213A with and without isolators. The maximum displacements are presented in Table 3.13. Since the same testing procedures and the same input motion were used for the three bridges, the comparison is straightforward. Notice that the displacements measured on the top of the isolated piers were split into the displacements due to the deformation of the devices and due to the deformation of the piers. The diagrams illustrated in Figure 3.41 show the global force versus displacement response curves on the top of the piers of the two isolated bridges, B213-5Dev and B213-1Dev, for the 1.2DE input motion. The examples presented in this section were only designed for unidirectional transverse actions. Any possible up-lifting effect on the devices was not taken into account.

From the analysis of the results it is possible to conclude that:

- almost all the dissipation of energy in the two bridges and for the two input motions occurred in the devices. In the case of bridge B213-5Dev, the high ductility demand at
the level of the devices corresponded mainly to a rigid body motion of the deck (see Figure 3.42) and no important damage was observed in the reinforced concrete elements. However, the abutments, as well as the piers, must be prepared to accommodate the high transverse displacements of the deck;

- the behaviour of the bridges was fully controlled by the devices. In the case of bridge B213-5Dev, the behaviour of the deck was quite independent from the piers. Furthermore, and admitting that 8cm is the upper limit for the horizontal displacement of the dissipating devices (value observed in the tests made by the constructor), input earthquake intensity much higher that 1.2DE may have caused failure of the devices destroying the link between the deck and the piers of the bridge B213A-5Dev. The ductility demand of the devices in bridge B213A-5Dev was higher than in bridge B213A-1Dev;

Table 3.13: Maximum displacements [m] for the DE and HLE (1:2.5 scale bridges)

<table>
<thead>
<tr>
<th>Test</th>
<th>Bridge</th>
<th>Abut. (left)</th>
<th>Medium Pier</th>
<th>Short Pier</th>
<th>Tall Pier</th>
<th>Abut. (right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td>B213A</td>
<td>Pier</td>
<td>2.16e-2</td>
<td>2.42e-2</td>
<td>2.79e-2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B213A-1Dev</td>
<td>Dev, Pier</td>
<td>2.50e-2</td>
<td>2.76e-3</td>
<td>3.68e-2</td>
<td></td>
</tr>
<tr>
<td>HLE</td>
<td>B213A</td>
<td>Pier</td>
<td>2.95e-2</td>
<td>5.15e-2</td>
<td>4.99e-2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B213A-1Dev</td>
<td>Dev, Pier</td>
<td>2.99e-2</td>
<td>2.92e-3</td>
<td>4.23e-2</td>
<td></td>
</tr>
</tbody>
</table>

- the increase of flexibility of the short pier of bridge B213A-1Dev transformed the irregular bridge in a bridge closer to the regular configuration. This is verified either by the mode shapes in Figure 3.38 or by the response of the deck in Figure 3.42. No increase in the damage occurred in the two piers not isolated in relation to bridge B213A; the decrease of stiffness and strength of the short pier did not cause any redistribution of transverse forces to the lateral piers. The dissipation of energy occurred
again in the short pier but at the level of the isolating device. No damage was found at
the base of the pier. Moreover, and for the high-level earthquake, lower displacements
were observed at the top of the medium and tall piers of the bridge B213A-1Dev in
relation to the bridge B213A.

The use of seismic isolators can thus be an effective way of protecting bridges against
the damaging effects of earthquakes. Nevertheless, as the devices change the characteristics
of the structures, non-linear numerical analysis must be performed taking into
account the new dynamic characteristics of the isolated bridges. In the particular case of
this irregular configuration, the solution with one isolating device seems to be the most
effective one; it gives to the irregular bridge a regular configuration and, furthermore,
does not leave the deck completely free to “leave” the piers or the abutments. Due to the
specific character of the tests, other more general conclusions can not be easily extracted
from these results.

3.9.2 Bridge with asynchronous input motion

The aim of the two tests performed on bridge B213A with an asynchronous input motion
and the two levels of intensity, was to show that the traditional design for synchronous
motion does not always lead to the most conservative results for all structural elements.
These tests were not meant to be conclusive about the effects of the variability of the
seismic action on bridges.

Artificial spatially correlated accelerograms generated from the EC8 response spectrum
for soil type B and 5% damping were used as input motion. However, since no coherence
loss matrix function was available within the research programme, a very particular pro-
cedure based on the studies referred in [36] was adopted [66]:

• generation of five independent accelerograms respecting the EC8 response spectrum;

• computation of the discrete Fourier Transform of the five signals through the Fast
Fourier Transform algorithm;

• imposition of a perfect coherence for the range of frequencies $[0, 0.4]$ Hz on the five
diagrams; the amplitude and the phase of one of the signals within this frequency
band was adopted in all the other four accelerograms. Above 0.4Hz, no spatial coherence was considered in the signals;

- computation of the new correlated input motions through an inverse Fast Fourier Transform algorithm. The final displacements are illustrated in Figure 3.43.

Test set-up

The test set-up in the laboratory was the same used in the Pseudo-dynamic tests with synchronous input motion. This was possible because no physical links existed between the piers. As described in section 2.7, the difference between the two algorithms, considering synchronous or asynchronous input motion, is in the equations that represent the numerical substructure. Although an intermediate step is required during the exchange of information between the numerical and the physical structures, the transformation is done at the level of the analytical structure. Thus, no changes had to be introduced in the algorithm that dealt with the physical structure when the asynchronous motion was considered.

Two tests for the 1.0 and the 1.2 times the five input accelerograms previously described, were preformed. The piers used in these two experiments were the same used in the tests with isolating/dissipating devices. The main results are presented in the next paragraphs. Details on these tests can be found elsewhere [66].

Test results

The diagrams in Figure 3.44 illustrate the transverse force versus displacement response curves at the top of the piers for the two earthquake intensities. The results are compared with the synchronous test. The maximum displacements are summarized in Table 3.14.

However, and before going further in the analysis of the results, two aspects must be commented upon: firstly, the stiffness of the two extreme piers for the first input action was not the same for the two tests. Secondly, in the case of the synchronous motion and the HLE, the two extreme piers were not represented physically but were simulated analytically through an elastic bilinear behaviour law (without plateau).
The global response shows that the asynchronous input motion can, in fact, imply a redistribution of damage in the piers. In the case of this bridge, it corresponded to a decrease of the maximum ductility demand and energy dissipation in the short pier and to an increase of the displacement of the medium pier that went clearly in the plateau zone for both input actions. Note that the comparison with the synchronous tests makes more sense for the DE than for the HLE, since the non-linear elastic behaviour law imposed on the extreme piers for the second accelerogram avoided any incursion in the plateau zone. Nevertheless, a higher damage was already visible in the medium pier at the end of the first input asynchronous motion. The results are illustrated in Figure 3.45.

Table 3.14: Maximum displacements [m] for the asynchronous DE and HLE (1:2.5 scale bridges)

<table>
<thead>
<tr>
<th>Test</th>
<th>Motion</th>
<th>Abut. (left)</th>
<th>Medium Pier</th>
<th>Short Pier</th>
<th>Tall Pier</th>
<th>Abut. (right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td>Synchronous Relative</td>
<td>2.16e-2</td>
<td>2.42e-2</td>
<td>2.79e-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Relative</td>
<td>4.92e-2</td>
<td>2.40e-2</td>
<td>4.50e-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HLE</td>
<td>Synchronous Relative</td>
<td>2.95e-2</td>
<td>5.15e-2</td>
<td>4.99e-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Asynchronous Absolute</td>
<td>7.27e-2</td>
<td>1.06e-1</td>
<td>1.13e-1</td>
<td>9.46e-2</td>
<td>7.96e-2</td>
</tr>
<tr>
<td></td>
<td>Relative</td>
<td>7.25e-2</td>
<td>3.95e-2</td>
<td>5.00e-2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Finally, according to the same figure, higher horizontal displacement drifts were found between the top section of each pair of contiguous piers when the bridge was submitted to the asynchronous input motion, i.e. higher stresses would have been expected in the deck in this case. Notice that, according to Kiureghian and Neuenhofer [43], although in most cases the spatial variability tends to reduce the response, this rule can not be generalized since, under certain conditions the response may actually amplify due to an increase in the pseudo-static component of the response.

3.10 SUMMARY AND CONCLUSIONS

Six reinforced concrete bridge models (1:2.5 scale) were tested in the ELSA laboratory using the Pseudo-dynamic test method with substructuring. Two bridge configurations were considered: one regular (bridge B232) and another irregular with five alternative design solutions (bridges B213A, B213B e B213C, B213A-5Dev, B213A-1Dev). The
testing campaign showed the versatility of the Pseudo-dynamic test method that was able to deal with different situations with minor changes in the algorithm controlling the physical structure in the laboratory.

Two Pseudo-dynamic tests were performed on each bridge using different magnitude factors for the same accelerogram. The first intensity factor corresponded to the design seismic action. The choice of the second, higher, intensity factor was based on numerical predictions that pointed out: the ultimate capacity of the weakest solution for the irregular bridges and the ultimate capacity of the regular bridge B232. Additionally, an asynchronous input motion was considered on bridge B213A.

Concerning the seismic behaviour of the structures, the following aspects are underlined:

- the behaviour of the irregular bridges was mainly commanded by the behaviour of the short pier; almost all the dissipation of energy took place in this pier which presented, at the end of the HLE input action, a high structural damage. However, this is against the EC8 rule that says that behaviour factors superior to 1.2 should not be adopted in such cases. Nevertheless, all piers presented quite ductile behaviour with quite stable hysteretic loops. In the case of the short piers, the diagonal cracking pattern was in agreement with the expectations and evidenced the importance of the shear mechanism to the global behaviour of the pier. The distribution of diagonal cracking was, however, much more spread in the piers with higher longitudinal and transverse steel ratio. Therefore, shear deformations should be taken into account in the modelling of squat piers;

- from the comparison of the two non-isolated alternative solutions to the EC8 conforming irregular bridge B213A, the solution B213B turned out to be the best of the three. The “strengthening” of the short pier decreased the high ductility demands observed at bridge B213A. For the weak solution B213C, although a more regular distribution of forces was achieved, the demand of ductility in the short pier was much higher than in bridge B213A, causing failure. In reality, these three alternative solutions corresponded to the adoption of different local behaviour factors for each one of the piers. Note that the imposition of a minimum longitudinal steel ratio does
not always comply with a global behaviour factor procedure: the strength of the piers may have to be increased beyond the value imposed by the behaviour factor procedure changing the conditions of the design;

- in the case of the irregular bridges, although the short piers presented quite ductile behaviour, the response demanded a very high ductility from the short piers that was not shared by the extreme piers. Therefore, almost all the dissipation of energy took place in the short pier which presented high structural damage at the end of the HLE input action;

- the results of the test on bridge B232 complied with the regularity issue mentioned in the EC8, part 2. The dissipation of energy took place in all piers and a ductile behaviour could be assumed. The mechanism turned out to be stable and efficient, with demands roughly proportional to the capacities. An input signal two times larger than the design earthquake was applied without loss of capacity.

An additional dynamic test should have been performed on the bridge B213B in order to estimate its ultimate capacity and, consequently, evaluate the safety margins associated to this alternative design solution that seems to be the most appropriate for highly irregular bridges.

In the case of the alternative solutions with isolating/dissipating devices, it is underlined that:

- almost all the dissipation of energy occurred in the dissipating devices. The solution with five isolating devices disconnected almost completely the deck from the piers. In this case the abutments and the piers must be prepared to accommodate the high transverse displacements of the deck. Although no ductility was demanded from the reinforced concrete elements, the high ductility demand in the isolating devices took the bridge to a point close to failure;

- the solution with one isolator presented a much better global behaviour with lower ductility demands in the isolating devices in relation to the previous bridge. Moreover, although a re-distribution of forces in the bridge would have been expected
imposing higher shear forces in the two piers without isolators, no increase of damage was found in these piers in relation to bridge B213A.

In relation to the asynchronous input motion, the tests performed on bridge B213A indicated that this type of motion can imply a re-distribution of ductility demand in the structures, increasing or decreasing the damage in some of their structural elements. In the case of bridge B213A, and for the selected input motion, it corresponded to a decrease of the maximum displacement of the short pier and to an increase of the displacement ductility demand of the medium pier.

Finally, one must underline the potential of the Pseudo-dynamic test method with sub-structuring used in this experimental campaign. The on-line analytical simulation of the deck and some of the piers of the bridge have evidenced the usefulness of the method implemented in the ELSA laboratory for the dynamic testing of large scale bridges.
Figure 3.1 - Full-scale scheme of the bridges
Figure 3.3 - Geometrical characteristics of the piers (M - Medium, S - Short and T - Tall). Representation of the pier slices.
Figure 3.5 - Stress versus strain diagrams of a concrete sample
Figure 3.6 - Axial tensile stress versus strain diagrams for the steel bars

Figure 3.7 - Preliminary test on a squat bridge pier - Imposed top displacement time history
Figure 3.8 - Instrumentation
Figure 3.9 - Cyclic test on the 2.8m high pier. Force versus displacement diagrams: a) total curve, b) linear regime and first cracking, c) within the cracking zone, d) up to ductility 1.5, e) up to ductility demand 3.0
Figure 3.10 - Displacement and deformation profiles: a) total displacement, b) shear and bending displacements, c) flexural curvature, b) shear strain
Figure 3.11 - Relative displacements due to: a) bending moments, b) shear forces
Figure 3.12 - Flexural moment versus curvature and shear force versus shear strain diagrams
Figure 3.13 - Relative energy dissipation: a) at the different slices, b) accumulated from the base up to the slice
Figure 3.14 - Diagrams measuring: a) out-of-plane deformations, b) the vertical force and the axial displacement at the pier axis.

Figure 3.15 - Design accelerogram. Comparison with the EC8 response spectrum (5% damping)
Figure 3.16 - Non-linear elastic curves (broken line) adopted for the behaviour of the medium and the tall piers in the HLE cases.
Cyclic Quasi-static Test on the bridge B213A squat pier

Figure 3.17 - Piers: Test specimens and testing sequence
Figure 3.18 - Schematic representation of the performed pseudo-dynamic tests (see Figure 3.17)

- Substructure Path
- Physical Structure
- Isolation/Dissipating Device

Bridge B213A-1DeV
DE / HIL

Bridge B213A-1DeV
DE / HIL

Bridge B213A-1DeV
DE / HIL

Bridge B232 (Regular)
DE / HIL

Bridge B213B
DE

Bridge B213C
DE / HIL

Bridge B213D
DE / HIL

Bridge B213A
DE
Figure 3.19 - Displacement time histories for bridge B213A
Figure 3.20 - Displacement time histories for bridge B213B
Figure 3.21 - Displacement time histories for bridge B213C
Figure 3.22 - Displacement time histories for bridge B232 (regular)
Table 3.15: Numerical frequencies of the ten first modes of vibration

<table>
<thead>
<tr>
<th>Modes of Vibration</th>
<th>Bridge Type (Scale 1:2.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B232</td>
</tr>
<tr>
<td>1</td>
<td>4.21 - H/H/S</td>
</tr>
<tr>
<td>2</td>
<td>5.57 - V/V/AS</td>
</tr>
<tr>
<td>7</td>
<td>10.36 - H/S</td>
</tr>
<tr>
<td>10</td>
<td>17.60 - V/V/AS</td>
</tr>
</tbody>
</table>

a. Main displacements of the deck: Horizontal in the transverse direction (H), Vertical (V), Symmetric (S) or Anti-Symmetric (AS).

Mode Shapes in the Horizontal Transverse direction of the Deck

B232 (Regular)  

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2 Hz</td>
<td>![Shape 1]</td>
</tr>
<tr>
<td>6.8 Hz</td>
<td>![Shape 2]</td>
</tr>
<tr>
<td>10.4 Hz</td>
<td>![Shape 3]</td>
</tr>
<tr>
<td>12.6 Hz</td>
<td>![Shape 4]</td>
</tr>
<tr>
<td>16.8 Hz</td>
<td>![Shape 5]</td>
</tr>
</tbody>
</table>

B213 (Irregular)  

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4 Hz</td>
<td>![Shape 6]</td>
</tr>
<tr>
<td>6.8 Hz</td>
<td>![Shape 7]</td>
</tr>
<tr>
<td>11.8 Hz</td>
<td>![Shape 8]</td>
</tr>
<tr>
<td>13.5 Hz</td>
<td>![Shape 9]</td>
</tr>
<tr>
<td>16.8 Hz</td>
<td>![Shape 10]</td>
</tr>
</tbody>
</table>

Figure 3.23 - Frequencies and mode shapes of the bridges
Figure 3.24 - Envelop displacement profiles of the deck for the regular and the irregular bridges
Figure 3.25 - Power spectrum of the piers top displacements
Figure 3.27 - Force-displacement diagrams for the bridge B213B
Bridge B232 (Regular)

Figure 3.29 - Force-displacement diagrams for the bridge B232 (regular)
Figure 3.30 - Force-displacement diagrams for the short pier of the integral bridge (B213a, B213b, and B213c)
Figure 3.31 - Flexural and shear deformations of the short piers for the HLE (10 peak values)
Figure 3.32 - Vulnerability functions - Demands for the two earthquake levels: a) pier drift ratio b) ductility c) energy dissipation
Figure 3.33 - Force displacement diagrams for the piers tested cyclically.
Figure 3.34 - Moment-curvature diagrams - Short pier of the bridges 213A and 213B
Figure 3.35 - Moment-curvature diagrams - bridge 213C
Figure 3.36 - Moment-curvature diagrams - bridge 213C
Figure 3.37 - Moment-curvature diagrams - bridge 232
Figure 3.38 - Numerical frequencies and mode shapes of the bridge models for the three design solutions: B213A, B213A-5Dev and B213A-1Dev
Figure 3.39 - Stiffness tests on the bridge piers (and abutments) before the first Pseudo-dynamic test on each of the three design solutions
Figure 3.40 - Displacement Time Histories for the Three Design Solutions and for the L2 DE
Figure 3.41 - Force versus displacement diagrams for the two isolated bridges and for the 1.2 DE
Figure 3.42 - Maximum displacements of the R/C piers top section and deck
DISPLACEMENT[cm] TIME [s] Histories: Asynchronous input motion

Figure 3.43 - Displacement time histories for the reference bridge B213A with asynchronous input motion
Figure 3.44 - Force versus displacement diagrams for the asynchronous input motion.
Figure 3.45 - Maximum displacement drifts and dissipated energy: synchronous versus asynchronous
4 THE FIBRE MODEL

4.1 INTRODUCTION

Over the last few years, the numerical modelling of reinforced concrete structures has been a topic of great research interest in civil engineering. Although the experimental tests are the best way to understand the behaviour of a structure, the price of such campaigns can be extremely high demanding numerical analysis to be performed contemporaneously. In fact, the numerical analyses are complementary to the experimental tests, in other words, the experimental results are used for checking and calibrating the numerical models that are exploited afterwards in more extensive analyses.

Many analytical models: from the more local to the more global models, have been proposed to simulate different structural elements. In the first group, a different model and mesh are associated to each material. The different meshes are linked through joint elements that characterize the interface between the materials. Instead, in the case of the more global models, the behaviour of the structure is represented by an overall law that integrates the structural behaviour of the composite material. In both cases the models are based on experimental results. Fillipou [30] and CEB [18] describe some of these models for reinforced concrete members like beams and columns.

Fibre type models are in between the local and the global formulations [4]. Although the algorithm computes the global deformations at the level of the Gauss points of the structural elements, the response is given by the integral of the local forces calculated at different points representing different materials and positions in the transverse section.
The structures are divided into three-dimensional (3D) geometrical linear elements that are sub-divided into longitudinal elements, named fibres. These longitudinal elements describe a mesh in the cross-section of the 3D elements and respond only to axial and shear deformations. No interface forces exist between the fibres. Each fibre occupies a certain position in the cross-section and responds to the constitutive law of the material it represents.

This chapter is dedicated to the fibre model implemented in the ELSA laboratory. Section 4.2 describes the algorithm and sections 4.3 and 4.4 present the constitutive laws adopted for the concrete and steel, for monotonic and cyclic loading. Numerical applications regarding these two types of loading are reported in section 4.5. Notice that the aim of section 4.5 is not only to validate the model but also to illustrate the potential of the fibres formulation with the adopted constitutive laws. The final conclusions are drawn in section 4.6.

4.2 FIBRE MODEL

The fibre model can be regarded as a step further in the refinement of standard beam models. In fact, it uses the same kinematic formulation to compute the deformation of the longitudinal axis of an element: three rotations and three displacements at each node. The difference to the standard beam models is in the procedure that it follows to calculate the resisting forces; instead of considering a global constitutive law at the level of the transverse sections, the fibre model computes the deformation and the stress of a set of points describing a mesh in the transverse section. The structure is thus divided not only in 3D beam type elements, but each of these elements is sub-divided into longitudinal fibres.

The model was implemented in a three-dimensional Timoshenko beam element [34], so that different interactions between axial and shear forces and bending moments could be considered. Each 3D element, with two nodes and six d.o.f. per node, is divided in other longitudinal elements, named fibres, that react to axial and shear displacements according to axial and shear stress versus strain constitutive laws. Each fibre defines a four or three nodes element in the transverse section with two per two Gauss points. The behaviour of the 3D elements is integrated using one Gauss point per element.
The model, implemented in the reference code of the ELSA laboratory: 'the object oriented computer code CASTEM 2000 [19] briefly described in annex B, was formulated for a longitudinal axis eccentric to the axis of gravity of the element. The basic assumptions of the Timoshenko beam theory, as well as the main implementation steps in CASTEM 2000, are presented in annex E.

4.2.1 Compatibility equations

Given a loading history, the finite element code uses the Timoshenko formulation to compute the displacements and rotations at the nodes of the elements. With the axial and shear deformations at the Gauss points, the program calculates the deformation at each Gauss point of each fibre through the equations of compatibility of displacements. This presumes that the deformation at any point of the transverse section only depends on the three displacements and three rotations computed at the Gauss point of the element. In the case of the Timoshenko formulation, the assumption is that plane sections remain plane after being deformed but do not, necessarily, maintain the initial angle with the beam axial axis (see Figure 4.1). Therefore, fibre strains are given by

\[
\begin{align*}
\varepsilon_x &= \varepsilon_x - y_i \cdot c_z + z_i \cdot c_y \\
\gamma_{xy} &= \gamma_{xy} - z_i \cdot c_x \\
\gamma_{xz} &= \gamma_{xz} + y_i \cdot c_x
\end{align*}
\]

4.1

where \( \varepsilon_x \) is the longitudinal strain and \( \gamma_{xy} \) and \( \gamma_{xz} \) are the transverse strains in the two main directions of the beam cross-section (see equation (E.3) and equation (E.14)). The curvature along the three axis is represented in Figure 4.1 by \( c_x \), \( c_y \) and \( c_z \). No warping of the cross-section is considered: a constant unitary function was adopted for the distribution of shear strain in the transverse section.

4.2.2 Equilibrium equations

Knowing the deformation at any point \( i \) in the cross-section (equation (4.1)), the axial stress \( (\sigma_x)_i \) and the shear stresses, \( (\tau_{xy})_i \) and \( (\tau_{xz})_i \), are computed through the constitutive laws of the materials, and the generalized forces expressed in equation (E.8) are calculated using a two per two points Gauss-Legendre rule. The equilibrium of forces is imposed within the beam algorithm described in annex C.
Figure 4.1 - Fibre model. Deformation of the transverse section

Therefore, the behaviour laws are defined not at the global level of the cross-section but at the level of the longitudinal fibres. The response curves represent the state of the transverse section at that local level; the reacting moment versus curvature diagram does not follow pre-defined rules (e.g. Takeda-like models) but integrates the behaviour of the fibres in the cross-section. Any variation of axial force or biaxial bending moments is explicitly taken into account in the response curves.

The constitutive laws implemented for the concrete and steel are described in sections 4.3 and 4.4. They are axial stress versus strain laws independent from shear behaviour which, in a standard fibre model, is assumed to be linear elastic.

4.2.3 Tangent stiffness matrix

As referred to in annex C, the tangent stiffness matrix is often required to solve non-linear problems. To compute this matrix, the elastic Young modulus $E$ in equation (E.20) have to be substituted by the tangent modulus given by
\[ E_i = \frac{d\sigma_x}{de_x} \] 4.2

4.3 CONCRETE CONSTITUTIVE LAWS

The complexity of the fibre model depends on the complexity of the constitutive laws that are selected to represent the behaviour of the materials. Typically, a fibre model considers axial stress versus strain constitutive laws uncoupled with linear elastic shear behaviour laws at each fibre. However, to take into account the non-linear behaviour in shear, a multi-dimension formulation for monotonic loading using the Mazars concrete model was already attempted by Combescure and Pegon [17] with promising results.

In the constitutive laws described in this chapter, the shear behaviour is linear elastic, i.e.

\[ \tau = G \cdot \gamma \] 4.3

where \( G \) is the distortional modulus given by

\[ G = \frac{E}{2 \cdot (1 + \nu)} \] 4.4

and \( E \) is the initial slope of the axial stress versus strain compression curve and \( \nu \) is the Poisson coefficient.

4.3.1 Monotonic loading

*Compression stresses*

When the concrete is submitted to a load of increasing intensity, it undergoes different levels of damage from micro-cracking up to ultimate failure. Figure 4.2 illustrates two schematic axial stress versus strain response curves of a concrete specimen under increasing axial deformation. After an almost linear region up to around half the compression strength, the concrete presents a pronounced non-linear behaviour with a strong degradation of the stiffness due to micro-cracking.

The peak stress zone in the stress-strain curve is relatively sharp for high-strength concrete and has a flat top for low-strength concrete. Due to the internal cracking parallel to the compression loading direction, the transverse strain increases considerably in the
peak zone. The strain at the maximum stress is, for standard concrete, around 0.2%. Although after the peak the concrete can still sustain compression stresses, it may present high decreasing strength ratios for increasing strain.

The evolution of the post-peak curve depends mainly on the degree of confinement of the concrete, e.g. on the transverse steel ratio and spacing between transverse steel bars. The confinement effect is explained by the three-axial behaviour of the concrete. When a concrete element is compressed, due to the Poisson ratio the transverse strain increases. Although at the beginning the volume of the specimen decreases, for stresses close to the compression strength the transverse strain becomes so large that the volume of the specimen with the cracks actually grows. If transverse steel bars exist, the core concrete is prevented from expanding and the lateral compression forces impose a three-axial compression stress-state in the concrete.

This phenomenon, referred to as the confinement effect, improves the ductile performances of the structures. In this case, the post-peak curve presents not only a lower decreasing strength ratio but also a residual strength for important strains. According to Park and Paulay [57], this reflects the coupling that exists between the confinement effect of the transverse steel and the interlocking effect of the aggregate. The results of a series of compression loading tests on reinforced concrete specimens described by Hoshikuma [39] illustrates quite well all these phenomena.

Different analytical approaches can be used for modelling concrete: ranging from classical *Plasticity* models to recent *Continuum Damage Mechanics* models [29]. In this work, a uniaxial stress versus strain behaviour law following empirical based rules was adopted for the concrete under monotonic compression loading: a two branches law of Hognestad type [38]. The first branch, a polynomial of degree two, defines the ascending part of the diagram and goes from zero to the peak stress point, \((e_{co}\sigma_{co})\):

\[
\frac{\sigma}{\sigma_{co}} = \frac{\varepsilon}{\varepsilon_{co}} \cdot \left(2.0 - \frac{\varepsilon}{\varepsilon_{co}}\right)
\]

for \(e_{co} < \varepsilon \leq 0\). The second branch, a descending straight line representing the concrete softening behaviour is given by
\[ \frac{\sigma}{\sigma_{co}} = 1.0 + Z \cdot (\varepsilon - \varepsilon_{co}) \]  

for \( \varepsilon \leq \varepsilon_{co} \) until failure. The equations in this chapter follow the standard convention: negative sign for compression stress and strain values and positive sign for tensile values.

As already mentioned, the slope of the post-peak branch depends on the degree of confinement of the concrete. This phenomenon also increases the peak value of the compression strength modifying the corresponding strain as illustrated in Figure 4.2.

![Figure 4.2 - Confinement effect in the concrete](image)

According to the Eurocode specifications [20] and Tassios [80], this effect can be included in the model through a confinement parameter \( \beta \) that depends on the characteristics of the transverse section and is given by

\[ \beta = \min(1 + 2.5 \cdot \alpha \cdot \omega_{w}, 1.125 + 1.25 \cdot \alpha \cdot \omega_{w}) \]  

in which \( \omega_{w} \) represents the mechanical volumetric ratio of the stirrups

\[ \omega_{w} = \frac{A_{w} \cdot \sigma_{y}}{b_{c} \cdot h_{c} \cdot \sigma_{co}} \sum (l_{w}/s) \]  

and \( A_{w} \) and \( l_{w} \) are the cross-sectional area and the total length of the stirrups, respectively, \( s \) is the stirrups spacing along the member axis, \( b_{c} \) and \( h_{c} \) are the dimensions of the confined concrete measured from the centre line of the stirrups and \( \sigma_{yw} \) is the yielding strength of the stirrups. The coefficient \( \alpha \) expresses the effect of the longitudinal bars
and of the density of the stirrups on the degree of confinement of the core concrete, i.e.

\[
\alpha = \left(1 - \frac{8}{3 \cdot n}\right) \cdot \left(1 - \frac{s}{2 \cdot b_c}\right) \cdot \left(1 - \frac{s}{2 \cdot h_c}\right)
\]

\[4.9\]

\(n\) being the number of longitudinal bars on the perimeter of the cross-section that are placed in an angle of a stirrup.

Following these specifications, the confinement effect is considered in the constitutive laws of the concrete through the relations

\[
\sigma_{co}^* = \beta \cdot \sigma_{co} \quad \varepsilon_{co}^* = \beta^2 \cdot \varepsilon_{co}
\]

\[4.10\]

\[
Z^* = \frac{\beta - 0.85}{\beta \cdot (0.1 \cdot \alpha \cdot \omega + 0.0035 + \varepsilon_{co}^*)}
\]

\[4.11\]

To improve the model for confined concrete, a third branch is considered after the softening compression branch and before failure: a zero slope straight line defining a compression plateau (see Figure 4.3). This additional branch accounts for the residual strength of the core concrete for important axial post-peak deformations. The strength of this new branch is a parameter of the model, \(\sigma_{pt}\). Park *et al.* [58] suggest (\(\sigma_{pt} = 0.2 \cdot \sigma_{co}^*\)).

**Tensile stresses**

Tensile tests on concrete specimens are not easy to perform; indirect methods are generally adopted (e.g. bending tests, Brazilian tests). Nevertheless, the tensile strength of the concrete is, in general, less than 20% of the compression strength.

When the tensile strength is reached, the concrete opens cracks and the strength reduces suddenly to almost zero presenting a clear non-ductile behaviour. Because of this behaviour and the low tensile strength of the concrete, the tensile strength is often ignored. However, to obtain a more realistic response, the tensile behaviour has to be taken into account in the concrete constitutive laws.

Moreover, in R/C structural elements, a new phenomenon occurs due to the ability of intact concrete between adjacent cracks to carry out tensile stresses: the tension-stiffen-
ing effect. When the concrete opens cracks, part of the tensile forces are transferred to the concrete between cracks through the steel bars and the bond mechanism, giving to the concrete a fictitious post-cracking strength (Park and Paulay [57]). Thus, in the pre-cracking phase the structural behaviour is strongly influenced by the tensile and, according to the importance of the tension stiffening effect and the bond stresses, this influence can go until yielding of the longitudinal steel bars. Gupta and Maestrini [35] developed an analytical formulation for modelling the tension stiffness phenomenon taking into account the bond mechanism.

Given the above considerations, a bilinear stress versus strain curve was adopted. From zero to the maximum tensile strength, point \((\varepsilon_t, \sigma_t)\), the model presents a linear elastic behaviour with a slope equal to the initial compression Young modulus, \((E_o = 2.0 \cdot (\sigma_{co}/\varepsilon_{co}))\). The second branch expresses the post-peak softening behaviour and is represented by a straight line that goes from point \((\varepsilon_t, \sigma_t)\) to a zero stress point at \(\varepsilon_m\), as illustrated in Figure 4.3.

The constitutive equations are given by

\[
\sigma = E_o \cdot \varepsilon
\]

for \(0 < \varepsilon < \varepsilon_t\), and by

\[
\sigma = \sigma_t \cdot \left( \frac{r - (\varepsilon / \varepsilon_t)}{r - 1} \right)
\]

for \(\varepsilon_t \leq \varepsilon < \varepsilon_{tm}\), with

\[
r = \varepsilon_{tm} / \varepsilon_t
\]

The adoption of the second branch of the tensile stress versus strain diagram, i.e. the post-cracking softening behaviour law, not only accounts for the tension-stiffening effect but also contributes to improve the performance of the non-linear algorithm. In fact, a very irregular response curve is obtained if a sudden cracking is assumed, \((\varepsilon_{tm} = \varepsilon_t)\) (see section 4.5.2). The strain \(\varepsilon_m\) is a model parameter and depends on the physical bonds between the concrete and the longitudinal steel, i.e. on the concrete and steel characteristics. Barzegar et al [3], for instance, suggest that when the reinforcing steel bars
intersect the cracks at right angles, an appropriate value for $\varepsilon_{tm}$ is the yielding strain of these bars.

![Concrete response curves for monotonic loading](image)

**Figure 4.3 - Concrete response curves for monotonic loading**

### 4.3.2 Cyclic loading

Under dynamic testing conditions, the concrete is submitted to cyclic loading. Thus, to allow dynamic tests to be performed, the non-linear model presented in the previous section had to be extended to other loading histories.

Experimental results show that the envelop of the concrete stress versus strain curve under cyclic loading may be considered unique and identical to the monotone loading curve [12]. Unloading from the envelop reflects the stiffness degradation due to concrete damage and the reloading curve from zero or tensile stresses tends to the envelop showing some strength degradation [18]. The intersection point between the unloading and the reloading curves defines the common points line: stresses above this line produce additional plastic strains while internal cycles below this line correspond to stable loops.

A small hysteresis effect and strength degradation is observed in the response of concrete specimens. The strength degradation reflects the loss of resisting capacity of the specimens due to cyclic loading. it is measured by the difference between the unloading and the reloading stress at the maximum reversal strain from the envelop (see $\Delta \sigma$ in Figure 4.4).
The envelop of the concrete tensile stress-strain curves under cyclic loading may also be considered unique and identical to the monotonic curve. Before reaching the tensile strength the concrete presents an almost linear elastic behaviour. After this point, experimental results on R/C elements show that the unloading and reloading branches follow a line with increasing stiffness degradation for increasing maximum reversal strain on the envelop.

No experimental tests have yet been performed subjecting the concrete simultaneously to cyclic compression and tensile stresses. This lack of information prevents a good knowledge of the crack closing phenomenon, i.e. how the two sides of an opened crack become effective in compression during a reversal strain loading.

![Figure 4.4 - Concrete response under compression cyclic loading](image)

The cyclic behaviour law adopted in the present work is based on experimental results. The compression monotonic curve described in section 4.3.1 represents the envelop of the stress-strain behaviour curve of the concrete under compression cyclic loading.

Two different set of rules were implemented: a very simple model that was used in the predictive analysis before the experimental campaign (see chapter 6) and a more sophisticated model improved afterwards. The two models were named the *preliminary model* and the *improved model*, respectively.

**Preliminary model**

To achieve a simple model, the unloading and reloading cycles from the compression
envelop follow a law similar to the one proposed by Mercer [47]: a straight line with a slope depending on the maximum strain ever reached during the loading history, $\varepsilon_{\text{max}}$.

$$E_d = E_o \left(1.0 - \frac{(\varepsilon_{\text{max}}/\varepsilon_{\text{co}})^2}{1.0 + (\varepsilon_{\text{max}}/\varepsilon_{\text{co}})^2 + (\varepsilon_{\text{max}}/\varepsilon_{\text{co}})^2} \right) \quad 4.15$$

Figures 4.5 and 4.6 illustrate an arbitrary cyclic strain history. The decreasing of $E_d$ with the increasing of the maximum strain, expressed in equation (4.15), tries to represent, in a simple way, the degradation of the material stiffness. The reloading compression curve, also represented by a straight line (see line (10-11) in Figure 4.5), goes from the zero stress point at the plastic strain $\varepsilon_{\text{pl}}$ to the maximum reversal strain from the envelop, $\varepsilon_{\text{max}}$. No strength degradation is considered and the common points line coincide with the envelop curve.

Regarding the tensile stresses, the model considers an envelop going from the zero stress point at $\varepsilon_{\text{co}}$ to the tensile strength point on the monotonic curve, $(\varepsilon_{\text{t}}, \sigma_{\text{t}})$, as illustrated by the broken line in Figure 4.6. The tensile stresses follow a straight line from the zero stress point at the plastic strain with a slope such that the envelop is reached for a strain $(\varepsilon_{\text{tr1}} = \varepsilon_{\text{pl}} + \varepsilon_{\text{t}})$. This first branch is elastic. The second branch, a straight line defining the softening post-cracking behaviour, has an inclination such that the intersection with the zero stress line is reached for a strain $(\varepsilon_{\text{tr2}} = \varepsilon_{\text{pl}} + \varepsilon_{\text{im}})$, as illustrated in Figure 4.6.

![Figure 4.5 - Numerical model for the concrete behaviour under cyclic loading](image)

Any unloading from the tensile curve follows a slope equal to the secant modulus of the
concrete at the plastic strain point, \((\varepsilon_{pl}, 0)\). With this model, the strength and stiffness degradation of the concrete under tensile stresses is explicitly taken into account. After reaching the maximum tensile strength, no more tensile stresses can be carried out by the concrete in subsequent loading cycles (see line (9-10) in Figure 4.5 and line (5-6) in Figure 4.6).

![Figure 4.6 - Numerical model for the concrete under tensile stresses (detail from Figure 4.5)](image)

If no tensile strength is adopted in the concrete constitutive law or its peak value has already been reached in previous cycles, the plastic strain is determined by the zero stress point from the compression unloading curve, \(\varepsilon_{pl}\). Otherwise, if during a loading cycle the concrete reaches the tensile strength and the strain goes beyond \(\varepsilon_{tr2}\), the plastic strain given by the zero stress point from the compression unloading curve must be corrected, \((\varepsilon_{pl} = \varepsilon_{pl} + \varepsilon_t)\).

**Improved model**

The need to improve the previous model came from the analysis of the results of the experiments described in chapter 3 and simulated analytically in chapter 6. In particular, the development of the non-linear shear model described in chapter 4 demanded smoother axial stress versus strain constitutive laws to “hide”, as much as possible, the singular points in the behaviour of the materials. Special care was put in the crack closing and tension stiffening phenomena.

Moreover, a clear overestimation of the pinching effect was found in the numerical tests.
simulated with the preliminary model described in chapter 6 in relation to the experiments. This overestimation is related to the crack closing effect that was not properly taken into account in the preliminary model. Finally, a more smooth constitutive law without abrupt variations of the stiffness, was also aimed in order to improve the performance of the numerical algorithm. Thus, new rules were implemented for the unloading and reloading curves under the envelop. Phenomena like: crack closing, stiffness degradation and tension stiffening, are considered and represented by exponential curves. Different parameters are prescribed depending on the loading direction.

An arbitrary axial stress versus strain history is illustrated in Figure 4.7. The circles sign either the extremities of the different branches of the hysteretic curves or the points where the loading changes direction. Before reaching the softening branch of the tensile stress-strain curve, the unloading curve from the compression envelop is represented by a straight line with growing stiffness degradation as described by equation (4.15). If after reaching the tensile strength the loading direction changes, the response follows an exponential curve that links the reversal point from the tensile curve to a pre-defined point in the unloading curve from the compression envelop. In the case of Figure 4.7, it corresponds to points (5=7=11) and 21.

Taking the curve that goes from point 7 to point 8 in Figure 4.7 as an example, the axial stress is given by

$$\sigma(\varepsilon) = (\sigma^* - \sigma^8 \cdot \varepsilon^*) \cdot \left(\frac{\varepsilon - \varepsilon_8}{\varepsilon^*} \cdot \frac{\varepsilon^*}{\sigma^* - \sigma^8 \cdot \varepsilon^*}\right) + \sigma^8 \cdot (\varepsilon - \varepsilon_8) + \sigma_8 \quad 4.16$$

where the subscripts refer to the points number of, $$(\sigma^* = \sigma_7 - \sigma_8), (\varepsilon^* = \varepsilon_7 - \varepsilon_8)$$ and $\sigma^T_7$ and $\sigma^T_8$ measure the tangent stiffness at points 7 and 8, respectively. Note that the curves (4-5) and (7-8) going in opposite directions are identical. The tangent stiffness is calculated through equation (4.2), i.e.

$$E_t(\varepsilon) = (\sigma^T_7 - \sigma^T_8 \cdot \varepsilon^*) \cdot \left(\frac{\varepsilon - \varepsilon_8}{\varepsilon^*} \cdot \frac{\varepsilon^*}{\sigma^* - \sigma^8 \cdot \varepsilon^*}\right) + \sigma^8 \quad 4.17$$
To define the exponential curve, four parameters, $F_1$, $F'_1$, $F_2$ and $F'_2$, two per each extreme point, were included in the model. The curves (7-8) and (10-11) in Figure 4.7 exemplify each of two possible situations: unloading from compression and from tensile stresses, respectively:

- curve (7-8):

$$(\sigma_7 = \sigma_{max}^c / F_1) \Rightarrow (\varepsilon_7 = \varepsilon_{max}^c - (\sigma_{max}^c - \sigma_7) / E_d)$$  \hspace{1cm} 4.18

$$(\sigma_8 = \sigma_{max}^t > 0) \Rightarrow (\varepsilon_8 = \varepsilon_{max}^t \land \varepsilon_8 \geq \varepsilon_{pla})$$

$$\sigma'_7 = \frac{\sigma_7}{(\varepsilon_7 - \varepsilon_8) / F'_1}$$  \hspace{1cm} 4.19

$$\sigma'_8 = \frac{\sigma_7}{(\varepsilon_7 - \varepsilon_8) \cdot F'_2}$$

where $\varepsilon_{max}^t$ is the maximum tensile strain ever reached since the last reversal loading from the compression envelop and $\sigma_{max}^t$ is the tensile strength still carried out by the concrete. The point $(\varepsilon_{max}^c, \sigma_{max}^c)$ signs the last position reached on the compression
envelop; in the case of curve (7-8) this point coincides with point 2. The plastic strain \( \varepsilon_{pla} \) is given by

\[
\varepsilon_{pla} = \varepsilon_{max} - \frac{\sigma_{max}}{E_d}
\]

In addition, the following conditions must hold true

\[
\begin{align*}
\left( \sigma'_{7} < \frac{\sigma_{7}}{(\varepsilon_{7} - \varepsilon_{pla}) - 1/10 \cdot (\varepsilon_{pla} - \varepsilon_{8})} \right) \land (\sigma'_{7} < E_d) \\
\left( \sigma'_{8} > \frac{\sigma_{8}}{9/10 \cdot (\varepsilon_{8} - \varepsilon_{pla})} \right)
\end{align*}
\]

If \( (\sigma'_{max} = 0) \), as in the case of curve (17-18), the strain at the zero stress point is given by

\[
\varepsilon_{18} = \varepsilon_{pla} + (F_{2}/2) \cdot (\varepsilon'_{max} - \varepsilon_{pla})
\]

\( \varepsilon'_{max} \) being equal to \( \varepsilon_{15} \), and the unloading and reloading curves are no longer the same (e.g. curve (17-18) and curve (20-21)).

- reloading curve (10-11):

\[
\begin{align*}
\sigma_{10} = \sigma'_{max} = 0 & \Rightarrow (\varepsilon_{10} = \varepsilon_{pla} + F_{2} \cdot (\varepsilon'_{max} - \varepsilon_{pla})) \\
\sigma_{11} = \sigma'_{max}/F_{1} & \Rightarrow (\varepsilon_{11} = \varepsilon_{max} - (\sigma'_{max} - \sigma_{11})/E_d)
\end{align*}
\]

\[
\begin{align*}
\sigma'_{10} = \frac{\sigma_{11}}{(\varepsilon_{11} - \varepsilon_{10}) \cdot F_{2}} \\
\sigma'_{11} = \frac{\sigma_{11}}{(\varepsilon_{11} - \varepsilon_{10})/F_{1}}
\end{align*}
\]

where \( \varepsilon'_{max} \) is equal to \( \varepsilon_{9} \) and point \((\varepsilon'_{max}, \sigma'_{max})\) coincides with point 2. In addition, the following conditions must hold true
\[ \sigma'_{10} > \frac{\sigma_{10}}{9/10 \cdot (\varepsilon_{10} - \varepsilon_{\text{pla}})} \]

\[ \left( \sigma'_{11} < \frac{\sigma_{11}}{(\varepsilon_{11} - \varepsilon_{\text{pla}}) - 1/10 \cdot (\varepsilon_{\text{pla}} - \varepsilon_{0})} \right) \wedge (\sigma'_{11} < E_d) \]

If during the loading history, and after reaching the peak tensile strength, the response curve returns to the compression envelop, point \((\varepsilon'_{\text{max}}, \sigma'_{\text{max}})\) is modified to

\[ (\sigma'_{\text{max}})_{\text{new}} = \mathcal{R} \cdot (\sigma'_{\text{max}})_{\text{old}} \]

\[ (\varepsilon'_{\text{max}} - \varepsilon_{\text{plas}})_{\text{new}} = \mathcal{R} \cdot (\varepsilon'_{\text{max}} - \varepsilon_{\text{plas}})_{\text{old}} \]

where the words "old" and "new" refer to the two last values on the envelop, \((\varepsilon_{\text{new}} < \varepsilon_{\text{old}})\), and

\[ \mathcal{R} = \begin{cases} 
\frac{(1 + \varepsilon_{\text{plas}})_{\text{new}}}{(1 + \varepsilon_{\text{plas}})_{\text{old}}} & \text{if } (\varepsilon_{\text{plas}})_{\text{new}} > -1 \\
0 & \text{if } (\varepsilon_{\text{plas}})_{\text{new}} \leq -1 
\end{cases} \]

The modification in equation (4.25) tries to reproduce the effect of the compression forces on the tensile strength, i.e. the decrease of the tensile strength with the increase of the compression strain.

Finally, any unloading or reloading between two exponential curves (e.g. curve (17-18) and curve (20-21)) follows a linear branch with a slope equal to \(E_d\) in equation (4.15) (e.g. lines (16-17) and (19-20)).

The choice of the parameters of the exponential curve must take into account that the axial stress-strain behaviour law of the concrete should represent what happens throughout the length of the fibres. In other words, it should integrate the bi-axial stress state of the concrete. Thus, the parameters may assume different values when representing members or materials with different characteristics. The crack closing and the tension-stiffness effect are implicitly taken into account through these parameters.
4.4 STEEL CONSTITUTIVE LAW

The constitutive laws implemented for the steel fibres are axial behaviour laws independent of shear behaviour. The shear forces are represented by the linear elastic law given by equation (4.3).

Two axial stress-strain behaviour laws: one for monotonic and another for cyclic loading, were implemented for the steel bars. The constitutive laws are described in the two next sections.

4.4.1 Monotonic loading

In general, the steel stress versus strain curves for monotone increasing strain come from tensile tests. After a linear elastic range, the steel exhibits a yielding plateau, a point beyond which the stress stays almost constant for increasing strains. In this stage, the material suffers important and irreversible changes in its internal matrix and the Poisson ratio, generally ranging from 0.2 to 0.3, becomes equal to 0.5, i.e. the steel bars do not change volume.

Two different stress values define the plateau: the upper and the lower yielding stress, the second one being the value usually adopted. The upper value is determined by the geometrical characteristics of the bar and the speed of the test. The length of the plateau is mainly determined by the strength and internal characteristics of the steel: higher strength and carbon steel presents a shorter plateau.

After the yielding plateau, the steel exhibits a strain-hardening behaviour up to the maximum strength, followed by a strain-softening behaviour pointing to failure. In the case of cool worked or a high-strength high-carbon steel, the tensile monotonic curve may not present the yielding plateau, going immediately from the linear elastic range to the strain-hardening curve. In this case, the steel presents a very fragile behaviour, as illustrated by the thinner line in Figure 4.8, and the yielding stress is given by the point in the curve that presents a deflection to the linear elastic curve equal to a chosen value, usually 5%.

When the steel bars are subjected to compression forces, a supplementary aspect has to
be taken into account: the inelastic buckling. Experimental tests by Monti and Nuti [48] show that when the ratio between the length and the diameter of the bars is superior to five, this phenomenon occurs and the compression behaviour is quite different from the tensile behaviour; the yielding plateau and the strain-hardening branches may disappear and the stresses may decrease after a first linear elastic branch. This phenomenon occurs in R/C elements when, after spalling of the cover concrete, the space between stirrups is not small enough to prevent buckling of the longitudinal steel bars.

![Diagram](image)

**Figure 4.8 - Steel bars under monotone increasing deformation**

In terms of the numerical implementation in the fibre model, the monotonic tensile stress versus strain curve proposed for the reinforcing steel is a five parameters law defining three different range curves (see Figure 4.8): a linear elastic branch defined by the Young modulus $E$

$$\sigma = E \cdot \varepsilon$$  \hspace{1cm} 4.27

the yielding plateau from the yielding strain $\varepsilon_{sy}$ up to the hardening strain $\varepsilon_{sh}$, defined by the yielding stress ($\sigma_{sy} = E \cdot \varepsilon_{sy}$)

$$\sigma = \sigma_{sy}$$  \hspace{1cm} 4.28

and, finally, the hardening curve up to the peak stress-strain point ($\varepsilon_{su}, \sigma_{su}$) (usually named the ultimate point) represented by a fourth degree polynomial.

$$\sigma = \sigma_{su} - (\sigma_{su} - \sigma_{sy}) \cdot \left(\frac{\varepsilon_{su} - \varepsilon}{\varepsilon_{su} - \varepsilon_{sh}}\right)^4$$  \hspace{1cm} 4.29
In the case the buckling effect is not considered, the axial compression forces also follow the constitutive law expressed in equations (4.27), (4.28) and (4.29). Otherwise, another behaviour law must be adopted. A model developed by Monti and Nuti [48] is presented in the next section.

4.4.2 Cyclic loading

Basic model

According to experimental results, before yielding the steel bars present a linear elastic behaviour. After yielding, the unloading branch from the monotonic curve starts from a straight line with a slope equal to the initial Young modulus. However, it becomes non-linear for a stress much lower than the yielding stress in the loading direction. This phenomenon, referred to as the Bauschinger effect, depends on the history of displacements applied to the specimen.

Therefore, no hysteretic effect with dissipation of energy occurs before the steel bars reach the yielding stress. Instead, after yielding, the response curves observed from experimental tests show that a strong hysteretic effect with large dissipation of energy occurs during the cyclic behaviour.

Following the experimental results, the explicit formulation proposed by Giuffré and Pinto and implemented by Menegotto [46], was chosen to represent the behaviour of the reinforcing steel bars under cyclic loading. The cyclic model is activated and the monotonic model is no longer valid, when, after going further the yielding strain, the bar unloads to a strain $\varepsilon$ such that

$$|\varepsilon_{max} - \varepsilon| \geq \frac{|\varepsilon_{yf}|}{3}$$  \hspace{1cm} 4.30

where $\varepsilon_{max}$ is the maximum reversal strain from the monotonic curve. From this point the monotonic model is no longer valid. Otherwise, the unloading and the reloading branches follow a straight line with a slope equal to the elastic Young modulus and the monotonic curve represents the envelop of the steel behaviour law.

The Giuffré-Pinto model was chosen because of its simplicity, numerical efficiency and
good agreement with experimental results on reinforcing steel bars. Note that a single smooth function is enough to represent the stress versus strain response curves in both loading directions,

$$\sigma^* = b \cdot \varepsilon^* + \left( \frac{(1-b)}{(1+(\varepsilon^*)^{1/R})} \right) \cdot \varepsilon^*$$ \hspace{1cm} 4.31

where

$$\sigma^* = \frac{\sigma_s - \sigma_r}{\sigma_o - \sigma_r}$$ \hspace{1cm} 4.32

$$\varepsilon^* = \frac{\varepsilon_s - \varepsilon_r}{\varepsilon_o - \varepsilon_r}$$ \hspace{1cm} 4.33

and

$$R = R_o \cdot \frac{a_1 \cdot \xi}{a_2 + \xi}$$ \hspace{1cm} 4.34

As illustrated in Figure 4.9, equation (4.31) defines a family of transition curves between two asymptotes with slopes $E$ and $E_h$, $(\varepsilon_o, \sigma_o)$ being the intersection point. The pair of values $(\varepsilon_r, \sigma_r)$ are the coordinates of the last reversal loading point, $b$ represents the ratio between the hardening slope $E_h$ and the initial slope $E$ and $R$ is a parameter that defines the shape of the transition curve. This parameter actually tunes the influence of the Bauschinger effect in the response, increasing or decreasing the length of the pseudo linear elastic branch during the reversal loading. It depends on $\xi$ that measures the difference between the maximum strain ever reached in the loading direction and $\varepsilon_o$ divided by $(\varepsilon_o - \varepsilon_r)$ (see Figure 4.9).

The hardening slope can be estimate through the expression

$$E_h = \frac{\sigma_{su} - \sigma_{sy}}{\varepsilon_{su} - \varepsilon_{sy}}$$ \hspace{1cm} 4.35

The parameters $a_1$ and $a_2$ and $R_o$ should be obtained from experimental results. Nevertheless, Menegotto [46] suggests
When the cyclic model is activated, the reversal loading point in the opposite direction of the initial loading coincides with the yielding point. In the case of Figure 4.9 this point corresponds to \((\varepsilon_r, \sigma_r)_o\).

![Figure 4.9 - Numerical model for the steel under cyclic loading](image)

**Inelastic buckling**

Tests made by Monti and Nuti [48] on several steel bars showed that for ratios \((L/D)\) lesser or equal to five, where \(L\) is the length of the bar between the two fixed extremities and \(D\) is the diameter, the compression and the tensile response curves are similar and no buckling effect is observed. These tests were made on steel bars without the concrete, i.e. as if the cover concrete had already been destroyed.

If the ratio \((L/D)\) is greater than five, the cyclic stress versus strain curves must be modified to take into account the inelastic buckling effect. According to the same author, the post-yielding compression branch presents a softening behaviour and a new \(b\) factor is considered, as illustrated in Figure 4.10

\[
b_c = a \cdot (5 - L/D) \cdot e \cdot \left( b \cdot \xi^* \cdot \frac{E}{\sigma_{yy} - \sigma_y} \right)
\]
where \( (\sigma_{\infty} = 4 \cdot \frac{\sigma_{s y}}{L/D}) \) and \( \xi' \) represents the major difference between the reversal strains that occurred in both directions during the loading history.

\[ E_r = E \cdot (a_5 + (1 - a_5) \cdot e^{(-a_5 \cdot \xi'^2)}) \]  \hspace{1cm} 4.38

where \( (a_5 = 1.0 + (5 - L/D)/7.5) \) and \( (\xi'' = \xi' \cdot (\varepsilon_o - \varepsilon_r)) \). Consequently, a new \( b \) factor was considered in equation (4.31) for the tensile stresses in order to maintain the same hardening stiffness \[ b_t = (b \cdot E)/E_r \]  \hspace{1cm} 4.39

If one defines \( \varepsilon_s \) as the strain at which the compression curve diverges 5% from the tensile curve towards lower values, an additional stress given by \[ \sigma_s^* = \gamma_s \cdot E \cdot \frac{b - b_c}{1 - b_c} \]  \hspace{1cm} 4.40

where
\[ \gamma_s = \varepsilon_o - \varepsilon_5 = \frac{11 - L/D}{10 \cdot (\varepsilon^c_0 \cdot (L/D) - 1)} \]

must be added to the compression yielding stress so that the new asymptote given by the softening stiffness \((b_c E)\) could be correctly positioned in the diagram. Figure 4.10 shows that this shifting allows the steel to follow first the original curve without the buckling effect and only after the modified compression branch.

The model is valid for \((5.0 < L/D \leq 11.0)\). The parameters \(a\), \(c\) and \(a_6\) depend on the characteristics of the steel and should be determined from experimental results. Nevertheless, Monti [48] suggests for bars with yielding stresses around 450MPa the following values:

\[ a = 0.008 \quad c = 0.500 \quad a_6 = 620.0 \]

4.5 NUMERICAL APPLICATIONS

A series of monotonic and cyclic numerical tests were performed on R/C structural elements to validate the fibre model and the potential of the constitutive laws detailed in sections 4.3 and 4.4. According to the type of loading, the tests are divided into two groups, namely: monotonic and cyclic.

The first group includes two tests. The first one simulates the response curve of a rectangular section for seven different reinforcement ratios. The concrete and steel stress versus strain behaviour laws were chosen according to the values adopted by Park and Paulay [57] in an identical numerical experiment. Notice that the aim of this test is not to validate the constitutive laws but to check the fibre algorithm procedures implemented in CASTEM 2000.

The second test, also performed on a rectangular R/C structural element, aimed at underlining the potential of the fibre model showing that phenomena like cracking of R/C structures could be well represented. The importance of the concrete tensile strength to the global response of R/C elements subjected to bending moments, and the sensitivity of the results to the parameters of the tensile constitutive law, are discussed.

In the second group, two other numerical tests were computed. The first test was per-
formed on a R/C cantilever T beam with a B500 steel. It simulates an experiment carried out at the LNEC laboratory on specimen S1V3, as described by Pipa and Carvalho [69]. From the analysis of the results, some considerations concerning the crack closing phenomenon are presented.

Classical global behaviour models usually fail when biaxial bending is considered. In order to validate the fibre model for this type of loading, the numerical simulation of an experiment carried out at the ELSA laboratory and described by Bousias [5] is presented as second test. Further investigation is suggested so that strength degradation at high strain level could be properly taken into account in the Giuffré-Pinto model.

4.5.1 Numerical solution

Numerical tests performed on several structures showed that when the tangent stiffness is adopted in the non-linear numerical algorithm, the convergence process is, in general, very sensitive to the slope hardening ratio of the steel: a small b factor in equation (4.31), together with a softening behaviour of some of the concrete fibres, may decrease the stiffness of a cross-section to values close to zero. Most of the numerical problems that occurred during the calculations performed with the fibre model were actually due to this circumstance.

To overtake this situation, the tangent stiffness matrix used in the integration algorithm is combined with the elastic stiffness

\[
[K]_t = x \cdot [K] + (1 - x) \cdot [K]_t, \quad 0 \leq x \leq 1
\]

4.43

where \([K]\) and \([K]_t\) are the elastic and the tangent stiffness matrices, respectively. In general, values of \(x\) around 0.01% provided very good results.

4.5.2 Monotonic loading tests

**Reinforced concrete rectangular cross sections**

The first numerical test was performed on a reinforced concrete rectangular section. A set of different volumetric steel ratios is considered on both sides of the beam (see Table 4.1 and Figure 4.11). An increasing curvature path was imposed on the cross-sections
and the bending moments were computed through the fibre algorithm.

<table>
<thead>
<tr>
<th>Beam</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>3.75</td>
<td>3.75</td>
<td>3.75</td>
<td>2.50</td>
<td>2.50</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>ρ'</td>
<td>2.50</td>
<td>1.25</td>
<td>---</td>
<td>1.25</td>
<td>---</td>
<td>1.25</td>
<td>---</td>
</tr>
</tbody>
</table>

The response curves illustrated in Figure 4.11 are compared with the theoretical results obtained by Park and Paulay [57] using the same behaviour stress versus strain laws for the steel and concrete.

![Moment/(b*d^2) (MN/m^2)](image)

Curvature (1/cm)

Figure 4.11 - numerical results using the fibre model in CASTEM 2000 (Park and Paulay test)

According to equations (4.5) and (4.6), the concrete is characterized by a compression strength ($\sigma_{co} = -27.6 MPa$) and a peak strain ($\epsilon_{co} = -0.2\%$). The softening behaviour is described by ($Z = 100.0$). No compression plateau nor tensile stresses or confinement of the core concrete were considered. The failure of the cross-section is commanded by the behaviour of the concrete in compression which occurs when the deformation of the extreme fibre is ($\epsilon \leq -0.4 \%$).
An elastic perfectly plastic law was adopted for the steel. This two parameters law is characterized by a yielding stress \((\sigma_{sy} = 276.0MPa)\) and a Young modulus \((E = 207.5GPa)\). Figure 4.11 shows the number and distribution of fibres in the cross-section, as well as the moment versus curvature results for the different reinforcement ratios presented in Table 4.1.

A perfect agreement with the results presented by Park and Paulay in [57] was achieved.

Influence of the concrete tensile strength

In the second test, the influence of the concrete tensile behaviour law on the global response of R/C structural elements subjected to bending moments is analysed. A square doubly symmetrical section was chosen as illustrated in Figure 4.12.

The compression strength and the peak strain adopted for the concrete are, respectively, \((\sigma_{co} = (-30.0)MPa)\) and \((\varepsilon_{co} = -0.2\%)\), and the tensile strength is \((\sigma_t = 3.0MPa)\). For the steel, a Young modulus \((E = 203.0GPa)\) and a yielding stress \((\sigma_{sy} = 460.0MPa)\) were chosen.

The distribution of the fibres in the cross-section is illustrated in Figure 4.12. The cover concrete, with a thickness of 0.19m, was modelled with one fibre at each extremity of the cross-section. A confinement factor \((\beta = 1.331)\) was adopted for the core concrete. Since the aim of the test is to analyse the moment versus curvature response curve of the cross-section during the opening of the cracks, the other characteristics of the steel and of the concrete constitutive laws, being irrelevant, are not presented.

Four numerical tests were performed. In each one a different parameter \(r\) in equation (4.13) was adopted. In the first test a sudden rupture, \((\varepsilon_{tm} = \varepsilon_t)\), was chosen. In the second, third and fourth tests the ratio \(r\) is equal to 1.5, 2.0, and 22.7, respectively. The fourth value follows the suggestion of Barzegar [3]: the value \(\varepsilon_{tm}\) is equal to the yielding strain of the steel bars. The results are drawn with thick lines in Figure 4.12. The dashed line illustrates the response of the section if zero tensile strength had been considered.

When a transverse section is subjected to rotations of increasing amplitude, the moment versus curvature response curve before cracking is represented by an almost straight line.
During these first steps the R/C has a linear behaviour and the four results are identical. When the first set of fibre Gauss points reaches the tensile strength, the cracks start opening and the response curve inflects tending to the response curve of the zero tensile strength case. In the case of curve 1, when a crack opens the stiffness of the cross-section decreases to a value that corresponds to the stiffness of a new cross-section with a layer of concrete less. This is particularly well illustrated in Figure 4.12 by the thin lines pointing to the origin.

![Graph](image)

**Figure 4.12 - Influence of the tensile parameters in the section global response**

Note that the non-ductile behaviour given by \( r = 1 \) gives a discontinuous response; when a crack opens the tensile strength goes immediately to zero, changing instantaneously the internal equilibrium in the section. Since the applied axial force is constant (in the case of this test, is equal to zero), the tensile forces from the cracked fibres must be transferred to other fibres, concrete and steel. However, part of these tensile forces is "eliminated" by the movement of the neutral axis that decreases the compression area and, therefore, the total compression force. The temporary decrease of the bending moment observed in the response curve illustrated in Figure 4.12, is actually explained by these two effects.

These results underline the importance of the post peak softening behaviour of the concrete under tensile stresses. The rough behaviour observed in curve 1 affects the per-
performances of the non-linear algorithm. Moreover, it does not correspond to the reality; each fibre must represent the average axial behaviour along the length of the 3D element that contains it. Instead, the response curves illustrated in Figure 4.12 by numbers two, three and four when a post-cracking ductile behaviour is adopted, are closer to reality. In R/C elements, the capacity of the concrete between cracks to resist tensile forces, referred to as the tension-stiffening effect, can be simulated with such ductile behaviour laws.

The ideal $r$ parameter does not exist, it depends on the concrete and steel characteristics and on the physical bonds between the two materials. Nevertheless and to conclude this topic, Barzegar [3] suggestion gives, for this example where no important axial forces are applied to the cross-section, a ratio $r$ which seems to be particularly high.

4.5.3 Cyclic tests

Uniaxial loading of a cantilever T beam

A series of tests on R/C cantilever T beams were performed in the LNEC laboratory within Task D of the Cooperative Research Program on Seismic Response of R/C Structures (2\textsuperscript{nd} phase). The aim of this experimental campaign was to provide information on the non-linear behaviour of the beams of a four storey full scale building that was tested later at the ELSA laboratory using the Pseudo-dynamic technique.

In this section, the experimental test on specimen S1V3 is simulated with the fibre model. The characteristics adopted for the materials are the mean values found from tests performed on the concrete and steel specimens. The unconfined concrete is characterized by a compression strength ($\sigma_{co} = (-35.0)MPa$), a peak strain ($\varepsilon_{co} = -0.2\%$) and a descending branch of the compression envelop defined by ($Z = 100.0$). All the concrete has been considered to be confined with a factor ($\beta = 1.33$). A Poisson ratio ($\nu = 0.25$) has also been adopted. No compression plateau nor tensile strength have been adopted in the concrete behaviour law. The decision of considering that the beam was already fully cracked when the experiment took place was based on the experimental results: the first branch of the trilinear curve that corresponds to the concrete without cracks does not appear in the response curve.
The numerical tests have been performed using both the preliminary and the improved models for the concrete. The four parameters adopted in the cyclic laws of the improved model are: \((F_1 = 2.0)\), \((F_1' = 20.0)\), \((F_2 = 0.75)\) and \((F_2' = 20.0)\).

The characteristics of the steel bars are: the yielding stress \((\sigma_{sy} = 550.0\, MPa)\), the Young modulus \((E = 203.0\, GPa)\), the Poisson ratio \((\nu = 0.25)\), the hardening and the ultimate strain equal to 0.22\% and 9.0\%, respectively, the ultimate stress \((\sigma_{su} = 610.0\, MPa)\) and the hardening ratio \((b = 0.03)\).

![Diagram of T beam cross-section](image)

**Figure 4.13 - Transverse section of the T beam - S1V3 test**

Figure 4.13 illustrates the cross-section of the cantilever T beam and Figure 4.14 the distribution of fibres adopted in the numerical test. The structure has been divided into seven Timoshenko beam elements: four 0.15\(m\) high elements at the base followed by three 0.30\(m\) high elements with one Gauss point each. A transverse displacement history equal to the one imposed during the experiment is applied to the free top of the beam. No axial compression forces have been considered.

The numerical response curve illustrated in Figure 4.14-a) by the top force versus displacement diagram, presents a good global agreement with the experimental results. The anomalous behaviour observed in the experimental curve results from buckling of some of the bottom steel bars. However, a more careful analysis points out the following aspects:

- the pinching effect is much more pronounced in the numerical results using the pre-
liminary model than in the experiments. This is illustrated in Figure 4.14-a). Actually, this model does not take into account the crack closing effect. The importance of this phenomenon is confirmed by the numerical response using the improved model (where this effect is included) that is represented by the thick line in Figure 4.14-b). Furthermore, the good agreement between this numerical result and the experimental response confirms the ability of the model to represent this effect;

• the whole numerical response is shifted by a displacement proportional to the applied force. This indicates that during the test there was a rotation of the bottom section of the specimen due to the flexibility of the basement or to the bond slippage of the longitudinal bars. In fact, the numerical results become quite close to the experimental response when a linear elastic rotational spring calibrated to the initial stiffness in the experimental test is placed at the basement of the beam. The two results, with and without flexible foundation and using the improved model, are illustrated in Figure 4.14-b);

• in contrast with the numerical results, it seems that no hardening of the steel bars occurs in the laboratory during the first load in the negative direction. As the steel constitutive law is based on experimental tests carried out on the steel specimens, one is forced to conclude that there is yielding penetration inside the block foundation which artificially increases the plateau length.

Finally, two numerical tests were performed using different set of parameters for the Giuffré-Pinto model; the first, referred to as $R_{20}$ in Figure 4.14-b), adopts the parameters in equation (4.36) and the second, referred to as $R_{10}$ in the same figure, considers the new parameters, $(R_p = 10.0), (a_1 = 9.0)$ and $(a_2 = 0.15)$. The comparison of the two results shows that the second set of parameters smooths slightly the numerical curve and gives a response that is closer to the results in the laboratory. This second set of parameters has been used later in chapter 6.

**Biaxial loading of a R/C square section**

The structure tested by Bousias at the ELSA laboratory [5]: a 1.49$m$ high R/C element framing into an enlarged end block of reinforced concrete, intended to simulate part of a
column between the foundation and the point of inflection. From the twelve specimens tested in the laboratory, specimen S9 was chosen. The cross-section, a doubly symmetrical square section, is represented in Figure 4.15.

![Graphs showing force displacement curves](image)

**Figure 4.14 - Force displacement curve on the top of the cantilever T beam**

The axial force and the biaxial transverse displacements imposed on the free top of the column during the experimental test have been adopted in the numerical test. The values
presented in Figure 4.16 have been collected directly from a database. This displacement history is usually referred to as a "shrinking" path: after an important displacement, there is a progressive movement in both directions back to the initial zero displacement position. The loading follows a four squares displacement path centred at the origin with half-side lengths equal to 100mm, 80mm, 60mm and 40mm in each direction.

The characteristics adopted for the materials are the mean values found from laboratory tests made on the concrete and steel specimens.

![Diagram of column transverse section tested by Bousias. Distribution of fibres in the cross-section](image)

Figure 4.15 - Column transverse section tested by Bousias. Distribution of fibres in the cross-section

Around the section, a 19mm thick cover concrete with a compression strength ($\sigma_{co} = (-30.0)MPa$), a peak strain ($\varepsilon_{co} = -0.2\%$) and a slope defining the descending branch of the envelope ($Z = 100.0$), is considered. A confinement factor ($\beta = 1.33$) is adopted at the core concrete. The Poisson ratio is ($\nu = 0.25$). As for the tensile parameters, a strength ($\sigma_{t} = 3.0MPa$) and a ratio ($r = 3.0$), were adopted. No compression plateau was considered. The preliminary model was used for the concrete.

The steel is characterized by the following mean values: the yielding stress ($\sigma_{sy} = 440.0MPa$), the Young modulus ($E = 203.0GPa$), the Poisson ratio ($\nu = 0.25$), the hardening and the ultimate strain equal to 0.8% and 13.0%, respectively, and the ultimate stress ($\sigma_{su} = 760.0MPa$). The hardening ratio is given by ($b = 0.13$).

The structure has been divided into seven elements with increasing lengths from the
basement to the free top: 0.05m, 0.10m, 0.15m, 0.20m, 0.25m 0.35m and 0.39m. The distribution of fibres adopted in the transverse section is represented in Figure 4.15.

**Figure 4.16 - Loading path: axial force and horizontal displacements**

The numerical response curves, expressed in Figures 4.17 and 4.18, present a good agreement with the experimental results. The decrease of the strength observed in the experiments from one cycle to the other, is due to the degradation of the concrete and steel properties during the cyclic loading.

**Figure 4.17 - Free top force versus displacement response curves in OZ direction**

However, the high strength degradation which results from the high strain level imposed
to the column at the beginning of the test is not well expressed in the numerical results. After the first square path, most of the concrete is destroyed and the response curve is mainly due to the steel. Since almost no strength degradation is considered in the steel behaviour model, the fibre response curves for the second and third square loading path tend to the same asymptote. This characteristic of the Giuffré-Pinto model implemented in this work, justifies the most important differences found between the experimental and the numerical results illustrated in Figure 4.17 and Figure 4.18. However and although the model can be improved, the lack of information on steel strength degradation during high strain cycles does not give enough confidence to modify the steel model as discussed in section 4.4.

![Diagram of force Y vs displacement UY](image)

**Figure 4.18** - Free top force versus displacement response curves in OY direction

### 4.6 CONCLUSIONS

Fibre type models, such as the one exposed in this chapter, are associated with axial stress versus strain constitutive laws of the materials. The shear and the axial stresses are uncoupled, and the three-dimensional effect caused by the Poisson coefficient is neglected or taken into account through simplified models that consider the confinement effect of the stirrups in the core concrete.

Nevertheless, fibre models are a powerful tool in the analysis of the behaviour of struc-
tural elements. In particular, R/C beams and columns are composite elements particularly suited to being modelled through these algorithms; in these elements, both the steel and the concrete present an important axial behaviour and the global reaction forces result mostly from the integral of the tensile and the compression forces developed in the cross-sections. This underlines the importance of having a good knowledge of the characteristics of the materials; a poor estimation of the strength or stiffness of the concrete or steel may significantly affect the final results. This is particularly evident when performing dynamic analysis, as discussed in detail in chapter 6.

The constitutive laws gave, in general, very good results and the numerical applications show the great potential of such models. Although more complete and multi-dimensional constitutive laws can be implemented in the fibre model, the constitutive laws presented in this chapter are already powerful enough for the applications under this work: linear structures in general and bridge piers in particular.

The fibre model in CASTEM 2000 has been implemented in a Timoshenko beam element so that the distortional effect due to shear forces could be taken into account. However, classical fibre type models, like the one herein presented, do not consider non-linear behaviour laws for shear stresses and this is the most important restriction of the model. If shear stresses exist and assume an important role in the global behaviour of a structure, some attention must be paid in the analysis of the results. Therefore, the improvements described in chapter 5, where the fibre model has been coupled with a non-linear model for shear stresses, represent an important advance.
5 NON-LINEAR SHEAR MODEL FOR R/C PIERS

5.1 INTRODUCTION

Bridges are civil structures that proved to be vulnerable to earthquakes. Recent events like the Northridge earthquake of January 1994 [51], [86], and the Kobe earthquake of January 1995 [27], [65], "illustrated" the most common damage and failure patterns experienced by such structures, namely: damage of the superstructures that in some cases, due to unseating of the spans or local failure of the restrains, were totally destroyed as they collapse from their supporting elements, shear failure or insufficient ductility in the piers due to poor confinement of the core concrete and poor detailing of the transverse and longitudinal steel. In some cases, the shear force concentration in shorter piers caused by the irregular profile of some bridges was not properly taken into account in the design.

It should be pointed out that although a general design code exists for civil engineering structures, bridges exhibit some characteristics that make them quite different from ordinary buildings and, therefore, demand special guidelines. Firstly, the mass of the bridge deck is an order of magnitude larger than the mass of a typical floor system. Secondly, bridge structural systems are not as redundant as typical building structures. This means that, although in a beam sides-way mechanism the plastic hinges would normally be located at the extremities of the beams, in bridge structures it is neither feasible nor desirable to locate them in the superstructure. Instead, it is preferable to place them in the columns, which thus become the primary sources of energy dissipation.

The general design approach is to protect the superstructure and foundations concentrat-
ing the inelastic behaviour in the columns, i.e. to provide the piers with sufficient deformation capacity and shear strength to ensure a ductile behaviour and shear resisting capacity under seismic loads. It is now generally accepted that the ductile behaviour of a reinforced concrete column can be reached by supplying adequate transverse reinforcement in the plastic hinge regions to provide adequate shear strength, properly confine the core concrete in the section and prevent premature buckling of the longitudinal reinforcement.

The behaviour of bridge structures subjected to important transverse forces is now more than ever a matter of great experimental and numerical concern. The aim of this chapter is to describe the non-linear shear model developed by the author at the European Laboratory of Structural Assessment at Ispra, Italy, for piers with low shear span ratio.

After a general and brief description of non-linear shear models, the developed model: a strut-and-tie type formulation coupled with the fibre model described in chapter 4, is presented. Several applications are carried out in order to assess the performance of the developed tools. The numerical response of bridge piers is compared with experimental results, in particular with those from the experimental campaign carried out at the ELSA laboratory [68]: four R/C structures with two different profiles representing a regular and an irregular bridge under horizontal transverse cyclic loading. The main conclusions are drawn in the last section.

5.2 NON-LINEAR SHEAR MODELS

Experimental tests on low shear span ratio R/C columns under important shear forces have shown a quite different behaviour compared to columns under predominant flexural moments, especially when combined with high axial forces. A brittle behaviour with opening of inclined cracks and sudden crushing of diagonal concrete, caused by the combination of high axial and shear stresses, is often observed in laboratory tests. Seismic events like the Northridge and the Kobe earthquakes, confirmed these results and underlined the need to consider the influence of shear forces on the resistance and ductility capacity of reinforced concrete structures. This is particularly relevant in structures where members with different characteristics are set together: short columns are a good example of structural elements that, due to their higher initial stiffness compared to other
elements, take most of the horizontal load.

To develop a model capable of representing the response of such elements subjected to important transverse forces, it is essential to have a good understanding of the physical phenomena involved. Hence, before considering the modelling aspects, the shear mechanism and the damage process of R/C squat piers are reviewed.

5.2.1 Shear mechanisms in R/C structural elements

Experimental tests on reinforced concrete columns (e.g. Li [44]) show that, before cracking, the shear force is mainly carried by the concrete; the formation of cracks reduces the concrete active area, decreasing the shear resisting capacity of the concrete. However, the formation of diagonal cracks induces a truss mechanism in the R/C element that reacts to shear forces. This truss is made of diagonal concrete struts and longitudinal and transverse steel ties (the reinforcing bars) that establish the equilibrium in the element. Moreover, the diagonal cracking also activates, through shear deformation, the aggregate interlock action along the cracks and the dowel force from longitudinal bars, giving a supplementary shear resistance to the cracked structure. The effectiveness of these effects depends on the capacity of transverse steel to maintain both sides of the crack in contact, i.e. on the transverse steel ratio and spacing of the stirrups along the R/C element.

Although before cracking the amount of transverse reinforcement has little effect on the shear carrying capacity of the structures, after cracking, and especially in the post-yielding phase, it increases the shear strength of the column and improves its performance. Therefore, the role of the transverse reinforcement is not only to provide confinement of the core concrete and to prevent premature buckling of the longitudinal bars, but also to resist shear forces. Experimental tests carried out in the ELSA laboratory showed that to prevent buckling it is important to provide the proper amount and spacing of the stirrups and still to adequately embrace the longitudinal bars; after crushing of the cover concrete, some longitudinal bars not embraced by the stirrups buckled laterally, presenting an effective length two or three times the distance between stirrups.

According to Park and Paulay [57], when the two surfaces of a crack of moderate width
slide one against the other, a number of coarse aggregate particles projecting across the crack enable shear forces to be transmitted. When the shear load increases, the interface forces produce local crushing with sliding of the crack surfaces and an important decrease in the shear stiffness occurs. Upon reversal loading, the surfaces of the crack remain in contact until an opposite shear force pushes the surfaces in the other direction. Then, the two surfaces slide with little resistance until the aggregate particles come into contact again [18].

![Diagram of diagonal cracking and dowel and interlock effect](image)

**Figure 5.1 - Diagonal cracking: dowel and interlock effect**

The longitudinal reinforcement also has an active role in the shear resisting mechanism. Firstly, it participates in the truss mechanism activated by diagonal cracking. Secondly, it contributes to shear strength through its dowel force capacity. The effectiveness of the dowel mechanism depends on many variables, namely: on the crack width, the bar diameter, the level of stress in the bar, the confinement of the concrete around the bar and the space between stirrups. A detailed description of the involved phenomena can be found in [57] and [60].

The axial load also has an important effect on the shear strength capacity. While axial tensile forces favour the opening and delay the closure of diagonal cracks and, therefore, increase the shear strength degradation, axial compression forces, up to a certain level,
delay the opening of cracks and increase the shear strength of the structural elements. Experimental results indicate that a more severe degradation of shear strength is observed in the case of varying axial load with cyclic bending.

The experimental results obtained by Li [44] confirm the interaction between shear strength and displacement ductility capacity; under cyclic loading the shear carried by the concrete decreases with the increase of the flexural displacement ductility. The reduction of shear strength is caused by the degradation of the shear resisting mechanism during reversal cyclic loading. Shear mechanism is the term commonly used to cover typical phenomena such as the aggregate interlock effect, the dowel action of flexural reinforcement and the shear transfer by the concrete through the truss mechanism [57].

Concerning shear failure, although it can be expressed in different ways, in general it is associated to crushing or splitting of the diagonal concrete. It corresponds to the development of an unstable truss mechanism with cracks propagating through the compression zone. However, if the transverse steel bars yield, they impose an unrestricted widening of the diagonal cracks that cause the aggregate interlock action to become ineffective. In this case, the dowel effect and the truss mechanism are pushed to their limits and failure occurs with little further deformation. As for the inclination of the critical cracks to the column axis, the observation shows that the angle can be smaller than 45° and, in general, the inclination becomes steeper as cracks propagate further into the compressed concrete zone. The experimental campaign carried out at the ELSA laboratory on squat bridge piers subjected to cyclic static loading confirms these results [68].

Experimental tests on R/C structures are the best way to understand the shear mechanisms involved in the behaviour of short column type structures. Although some data already exists, there is still much to investigate. The numerical simulation of the physical phenomena described is not an easy task and it is a present-day research topic. The next paragraphs give an overview of different numerical models dealing with the problem of the non-linear behaviour in shear.

5.2.2 Numerical modelling

Some authors suggest very simple models to represent the response of reinforced con-
crete structures under important transverse forces. The behaviour is simulated through
global shear force-displacement curves where each of the phenomena involved is repre-
presented, e.g. the aggregate interlock and the dowel effect, as described by Fardis [28] and
Jimenez [41]. These curves are usually based on experimental formulas.

Other authors propose the superimposition of different models for flexural and shear
behaviour and it is not rare to find empirical laws for both or at least one of these two
models, in particular to predict shear strength. Chang [13] presents a model of this kind.

Chang model

According to this author, two formulations are used for computing the response of col-
umns due to bending and shear forces. At each loading step, the equilibrium of axial
forces and flexural moments is imposed on the column independently of the applied
shear force. Then, according to the level of damage the column presents, different zones
with different elastic equivalent shear stiffness are considered in the structure as illus-
trated in Figure 5.2: not cracked, cracked, inside the elastic hinge or the plastic hinge
region or outside. For a cantilever beam of length $L$, the elastic shear stiffness is given
by:

- prior to cracking,

$$K_{ve} = \frac{G \cdot A_q}{f} \quad 5.1$$

where $A_q$ is the area that contributes to the shear stiffness and $f$ is a section form fac-
tor. The shear modulus is ($G = 0.4 \cdot E_c$) and it arises from equation (4.4) in chapter
4, and from the assumption of a Poisson ratio ($\nu = 0.25$) for the concrete;

- after cracking,

$$K_v = \frac{b_w \cdot d \cdot \cot \theta}{\frac{1}{E_s \cdot \rho_v} + \frac{1}{E_c \cdot (\sin \theta)^4}} \quad 5.2$$

where $\theta$ is the inclination angle of the cracks with respect to the longitudinal axis,
($\rho_v = A_v / (s \cdot b_w)$) is the volumetric ratio of the shear reinforcement, $A_v$ is the total
area of the stirrups, and $b_w$ and $s$ are the width of the concrete web and the stirrups spacing, respectively. These two expressions are applied to regions where no yielding of the longitudinal bars occurred.

Figure 5.2 - Chang model

The shear displacement is computed through the integral

$$
\int_{L} (V/K) \, dx
$$

where $V$ is the transverse force, $K$ is the shear stiffness and $L$ is the total length of the column along the longitudinal axis $\bar{x}$, as illustrated in Figure 5.2. Thus, the total elastic shear displacement at the top of the column is given by

$$
\Delta_{se} = V \cdot L \cdot \left[ \frac{1}{K_{ve}} \cdot \frac{M_{cr}}{M_{max}} + \frac{1}{K_{vh}} \cdot \frac{L_h}{L} + \frac{1}{K_{vc}} \cdot \left(1 - \frac{M_{cr}}{M_{max}} \cdot \frac{L_h}{L}\right) \right]
$$

where $K_{vh}$ is the shear stiffness within the hinge zone with length $L_h$. $K_{vc}$ is the shear stiffness outside the hinge zone but inside the cracked region. Both values are calculated through equation (5.2) for the corresponding space between stirrups, inside or outside the hinge zone. $M_{cr}$ and $M_{max}$ are, respectively, the cracking and the maximum flexural moments in the structure. Thus, the flexural response determines the shear flexibility but without any feedback; the shear behaviour does not interfere in the flexural response of the structure.
When the longitudinal steel bars yield, the behaviour is no longer elastic and an iterative process, using a strut-and-tie type model, is used for calculating the inelastic shear deformation. This method is based on a truss analogy where shear forces are resisted by a mechanism made of concrete struts and tensile ties that carry compression and tensile forces. From the compatibility of displacements and the equilibrium of forces in the truss, a plastic shear deformation $\gamma$ is computed and the total displacements are then given by

$$\Delta_s = \Delta_{se} + \gamma \cdot L_p$$  5.5

$L_p$ being the length of plastic zone.

** Priestley model **

Some authors consider other semi-empirical models, most of them being based on truss analogies. In general, they are used for calculating the shear strength of R/C elements and not for establishing the hysteretic shear force-displacement response curves; given a structure with a pre-defined reinforced concrete cross-section, these models predict the resisting mechanism and compute the contribution of the concrete and steel to the shear strength within this mechanism. Priestly [70], for instance, defines shear strength through the superimposition of three independent components that account for the contribution of the concrete, $V_c$, of the transverse steel via a truss mechanism, $V_s$, and of the axial load through an arch action, $V_p$,

$$V = V_c + V_s + V_p$$  5.6

There are several expressions available in the literature and in the design codes [20] to account for the two first contributions to shear strength. In the case of the concrete, most of the available expressions can be grouped in the general equation

$$V_c = K \cdot \sqrt{f_c} \cdot b_w \cdot d$$  5.7

where $d$ is the effective depth of the section, $f_c$ is the compression strength of the concrete and $K$ is a parameter that includes the contribution of other factors to the concrete shear strength. For example, some codes consider the effect of the axial force through $K$. 

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The contribution of the transverse steel $V_s$ is evaluated within a truss mechanism defined by a cracking angle $\theta$,

$$V_s = A_{sw} \cdot f_{sw} \cdot \frac{d}{s} \cdot \cot \theta$$  \hspace{1cm} (5.8)

$A_{sw}$ being the cross-sectional area of the stirrups, $f_{sw}$ the stirrups yielding stress and $s$ the stirrups spacing. Priestly considers for the diagonals of the truss an inclination angle to the column axis ($\theta = 30^\circ$).

The contribution of the axial load, $V_p$, is calculated through the horizontal component of a diagonal developing from the top to the bottom of the column as illustrated in Figure 5.3. Priestly considers two different inclination angles for this diagonal: one for the first loading and another one for reverse loading.

This model only estimates the shear strength capacity of the structural elements. To capture the mechanics of the involved phenomena, more accurate models have to be used.

A *Strut-and-Tie model for 2D elements*

A strut-and-tie type model is often adopted for membrane elements. It assumes that the direction of the principal stresses coincides with the direction of the principal strains which, in turn, is computed through the equations of compatibility of deformations in the membrane. The equations of equilibrium are established by making use of the uniaxial constitutive laws of the materials applied to each principal direction.

Two different modelling assumptions are commonly adopted: a rotating or a fixed crack angle formulation. In the first case, the crack angle follows the direction of the principal compression stresses in the concrete. In a fixed angle formulation, the direction of the first crack defines the crack angle for the rest of the loading history. In this case, the compatibility of displacements and the equilibrium of forces introduce an additional variable in the model: a shear strain together with a shear stress-strain constitutive law.

Note that the uniaxial stress-strain laws assume that the material is homogeneous and damage is uniform. Thus, to represent properly the global behaviour of the concrete struts these laws should integrate the state of the concrete in a wide zone of the structure.
In the Compression Field Theory (CFT), Collins proposes the first modified stress-strain curve for the concrete in compression [83], [84], [85]. This uniaxial law tries to represent the average stress in the element.

![Diagram](Image)

Figure 5.3 - Contribution of axial load to shear strength (Priestly model [70])

The characteristics of the curve, strength and stiffness, depend on the principal tensile strain $\varepsilon_1$ (see Figure 5.4). The constitutive equations for compression stresses are expressed by

$$\sigma_c = \sigma'_c \cdot (0.8 - 0.34(\varepsilon_1/\varepsilon'_c))^{-1} \cdot \left[2\left(\frac{\varepsilon_2}{\varepsilon'_c}\right)^2 - \left(\frac{\varepsilon_2}{\varepsilon'_c}\right)^2\right]$$

and by

$$\sigma_c = \begin{cases} \sigma_{cr} & (\varepsilon_1 \geq \varepsilon_{cr}) \\ E_c \cdot \varepsilon_1 & (\varepsilon_1 < \varepsilon_{cr}) \end{cases}$$

for tensile forces. The values in these equations are illustrated in Figure 5.4.

Since strut-and-tie type models make use of uniaxial stress-stain laws to represent the behaviour of concrete struts, the use of modified constitutive laws, taking into account the global state of the concrete, represented a considerable improvement in these models.
Before that, the uniaxial compression strength of standard concrete cylinders was used, giving an overestimation of shear and torsional strength.

Figure 5.4 - Average stress-strain response of concrete. Modified model due to Vecchio and Collins

**Fibre based models**

The use of modified behaviour curves to represent the heterogeneous state of a material through a single homogenized law is now common practice in several models. Garstka [31] superimposes a strut-and-tie model based on the CFT for shear deformations with a classic fibre model for flexural deformations. The model defines a truss made of concrete struts that follow a uniaxial stress-strain law modified to take into account the cracks in the direction of the compression forces.

For structures with a typical one-dimensional geometry (e.g. beam, columns), Petrangeli and Pinto [62] propose a fibre model with three-dimensional constitutive laws for the concrete fibres. Such a model allows the consideration of the effect of shear forces and diagonal cracking in the structure. Apart from the global equilibrium of forces, to compute the strain at the stirrups the model establishes the equilibrium of internal forces in the transverse direction at the level of each fibre in the cross-section; an iterative process is used for this purpose. After having established the internal equilibrium, the two or three-dimensional state of the concrete is defined.
The main difficulty in such a model is to find a three-dimensional constitutive law for the concrete that covers the entire range of possible cyclic loads. Moreover, the computation phase can be far more time consuming and the number of variables defining the internal state of the cross-section can be much greater than in a standard model.

5.3 STRUT-AND-TIE MODEL IN FIBRE MODELLING

A series of experimental tests on reinforced concrete bridge piers under cyclic loading were performed in the ELSA laboratory at Ispra; simultaneously, a classic fibre model was used for predicting their response. The comparison of the experimental response with the numerical results pointed out the need to develop a model to represent the non-linear shear behaviour of squat piers. In this context, a model based on the procedure presented by Garstka was developed and implemented in CASTEM 2000. It superimposes the classic fibre model described in chapter 4 that gives very good results for bending moments and axial forces, with a strut-and-tie model to simulate the non-linear shear behaviour.

The strut-and-tie formulation and the implementation procedures of this new model are presented and discussed in the next sections.

5.3.1 Non-linear shear modelling

The global behaviour of reinforced concrete columns with low shear span ratio depends on the level of applied shear forces which must be considered in the response. Classic fibre models, like the one implemented in CASTEM 2000, consider a linear elastic behaviour law for shear stresses; the fibres belonging to the same section "see" the same shear strain and contribute to the final force proportionally to their area and distortional modulus. Only the confinement effect the stirrups have in the core concrete is taken into account in these models; the stirrups do not contribute to the shear strength and stiffness of the cross-section.

Figure 5.5 illustrates the main internal resisting forces in a short cantilever beam damaged by a transverse and longitudinal force applied at the free extremity. The resisting elements along the cracks are: the steel reinforcing bars that tie both sides of the crack together and the concrete through the aggregate interlock effect (see zoomed section 2 in
Figure 5.5). Furthermore, the concrete also contributes to shear strength with its own shear resisting capacity and, above all, through a truss mechanism effect that is described hereafter.

Figure 5.6 illustrates another cantilever beam under important shear forces. From the analysis of this figure, one can observe that due to diagonal cracking a new structure appears analogous to a truss made of transverse and longitudinal steel ties and diagonal concrete struts. The non-linear shear model presented in this chapter is based on this analogy. In the present formulation, the interlocking forces along the cracks and the dowel effect illustrated in Figure 5.5 are neglected. However, they can be implicitly taken into account in the constitutive laws of the concrete.

![Diagram](image)

**Figure 5.5 - Cracking pattern - internal acting forces**

The fibre model and the strut-and-tie model interact through the *average axial strain* computed at the level of each transverse section. The coupling between the two models is, in fact, ensured by this deformation. The average axial strain was chosen because, as the name itself suggests, it represents the average axial deformation of the cross-section due to the axial force and bending moments. Other values were tried with very little success.

The next paragraphs describe in detail the equations of compatibility of displacements...
and equilibrium of forces in the cross-section. Other subjects such as the shear cracking angle and the constitutive laws of the materials that are fundamental to the numerical implementation of the algorithm, are also discussed. In the text, the words diagonal and strut are used indistinctly to refer to the concrete elements of the truss.

![Real Structure and Truss Diagram](image)

**Figure 5.6 - Truss analogy**

**Equations of compatibility of displacements**

Referring to Figure 5.6, the compatibility of displacements in the transverse direction is established through the geometric analysis of the truss deformed by the axial force $P$, the transverse force $V$ and the bending moment $M$. Two concrete diagonals are considered in the formulation for each loading direction: one strut under compression forces and another under tensile forces. In the case of shear forces exist in both transverse directions of the element, it is the author’s believe that the procedure described for one direction is valid for the perpendicular direction and the two responses can be superimposed.

The struts are represented by their central axis. The same cracking angle is adopted for both diagonals, i.e. the two struts are symmetric in relation to the axis of the column. Notice that this angle is a parameter of the model, in other words, it does not change during the loading history. This subject is discussed later in the chapter.

The diagonals represent the direction of the pseudo principal stresses. The word pseudo
refers to the fact that the principal directions were estimated and not calculated through the Mohr's circle. This also means that the strut under tensile stresses is not always perpendicular to the compressed strut. The angle between the two pseudo principal directions is \((2 \cdot \theta)\) and, therefore, the two concrete diagonals are perpendicular one to the other only when \((\theta = 45^\circ)\) (see Figure 5.7).

![Figure 5.7 - Compatibility of displacements](image)

To establish the equations of compatibility of displacements, the shear strain is set apart from the average axial deformation due to bending moments and axial forces. Assuming a cracking angle \(\theta\), as illustrated in Figure 5.7, the components \(j\) of the displacement of the diagonal \(i, \Delta_{ij}\), are given by

\[
\begin{align*}
\Delta_{11} &= (h_v^* - h_v) \cdot (\sin \theta) \\
\Delta_{21} &= (h_v^* + h_v) \cdot (\sin \theta) \\
\Delta_{12} &= h_{oe} \cdot (\cos \theta) \\
\Delta_{22} &= h_{oe} \cdot (\cos \theta)
\end{align*}
\]

5.11

where \(h_v, h_v^*\) and \(h_{oe}\) are the displacements due to the stirrups strain \(\varepsilon_{wy}\), to the shear deformation \(\gamma\) and to the average axial strain \(\varepsilon_{oe}\), respectively,
\[ h_v = \Delta_{wy} = \varepsilon_{wy} \cdot h \]
\[ h_v^* = l \cdot \tan \gamma \]
\[ h_{oe} = \varepsilon_{oe} \cdot l \]

The superimposition of the two partial displacements, \((\Delta_i = \Delta_{i1} + \Delta_{i2})\), gives the total displacement of each diagonal

\[ \Delta_1 = (\varepsilon_{wy} \cdot h - l \cdot (\tan \gamma)) \cdot (\sin \theta) + \varepsilon_{oe} \cdot l \cdot (\cos \theta) \]
\[ \Delta_2 = (\varepsilon_{wy} \cdot h + l \cdot (\tan \gamma)) \cdot (\sin \theta) + \varepsilon_{oe} \cdot l \cdot (\cos \theta) \]

These equations represent the compatibility of displacements that has to be respected in the transverse sections. The uniform strain at each diagonal is obtained by dividing both displacements by the length of the diagonals, \((l/(\cos \theta))\)

\[ \varepsilon_1 = \frac{\Delta_1}{l/(\cos \theta)} = \varepsilon_{oe} \cdot (\cos \theta)^2 + \varepsilon_{wy} \cdot (\sin \theta)^2 - \frac{\tan \gamma}{2} \cdot (\sin(2\theta)) \]
\[ \varepsilon_2 = \frac{\Delta_2}{l/(\cos \theta)} = \varepsilon_{oe} \cdot (\cos \theta)^2 + \varepsilon_{wy} \cdot (\sin \theta)^2 + \frac{\tan \gamma}{2} \cdot (\sin(2\theta)) \]

The strains \(\gamma\) and \(\varepsilon_{oe}\) come directly from the 3D Timoshenko beam algorithm. Instead, the stirrups strain \(\varepsilon_{wy}\) is computed internally at the level of the transverse section. This value must respect the equilibrium of forces inside the 3D element. The procedure that was used to calculate \(\varepsilon_{wy}\) is presented in section 5.4.2.

**Equations of equilibrium of forces**

With the deformation of each diagonal calculated through equation (5.14), the resisting forces of the struts, \(F_{c1}\) and \(F_{c2}\), and of the stirrups, \(F_{wy}\), are computed through the constitutive laws of the materials

\[ F_{c1} = \sigma_c(\varepsilon_1) \cdot A_{sturt} = \sigma_c(\varepsilon_1) \cdot A_{shear} \cdot (\cos \theta) \]
\[ F_{c2} = \sigma_c(\varepsilon_2) \cdot A_{sturt} = \sigma_c(\varepsilon_2) \cdot A_{shear} \cdot (\cos \theta) \]
\[ F_{wy} = f_{sw}(\varepsilon_{wy}) \cdot \frac{h/(\tan \theta)}{s} \cdot (2 \cdot A_{sw}) = f_{sw}(\varepsilon_{wy}) \cdot A_{shear} \cdot (\tan \theta) \cdot \rho_{sw} \]

\((\rho_{sw} = (2 \cdot A_{sw})/(b_w \cdot s))\) being a measure of the transverse steel ratio and \(\sigma_c\) and \(f_{sw}\) the stresses in the concrete (struts) and in the stirrups (ties), respectively. The value \(A_{sw}\)
is the cross-sectional area of the transverse steel, \( s \) is the stirrups spacing and \( b_w \) is the transverse dimension of the column shear resisting area \( A_{shear} \), as illustrated in Figure 5.5.

![Figure 5.8 - Internal forces in the section](image)

The forces given by equation (5.15) and illustrated in Figure 5.8 have to respect the internal and the external equilibrium in the structure:

- equilibrium of internal forces at point A:

\[
F_{wy} + (F_{c1} + F_{c2}) \cdot (\sin \theta) = 0
\]

5.16

- equilibrium of total forces at the cross-section:

\[
V + (F_{c1} - F_{c2}) \cdot (\sin \theta) = 0
\]

5.17

This system of equations is solved in two stages. Firstly, the fibre formulation gives the values of the axial strain and the shear strain in the cross-section. Then, equation (5.16) is used for computing the deformation of the stirrups, i.e. the strain at the transverse steel satisfying the compatibility of displacements and, at the same time, respecting the internal equilibrium in the element. Finally, with the stirrups strain \( \varepsilon_{wy} \), the shear resisting force in the transverse section is computed through equation (5.17) and the equilibrium with the applied transverse force is checked within the Timoshenko beam formulation.
Substituting equation (5.15) into equations (5.16) and (5.17), we obtain

\[ f_{sw}(e_{wy}) \cdot \rho_{sw} + (\sigma_c(e_1) + \sigma_c(e_2)) \cdot (\sin\theta)^2 = 0 \]  \hspace{1cm} 5.18

\[ (\sigma_c(e_2) - \sigma_c(e_1)) \cdot b_w \cdot h \cdot (\sin\theta) \cdot (\cos\theta) = V \]  \hspace{1cm} 5.19

The transverse steel ratio is explicitly taken into account in equation (5.18) and it contributes to the shear strength in equation (5.19) through the struts strains \( e_1 \) and \( e_2 \).

**Damage of the struts**

The concrete damage due to flexural and axial forces can be included in the non-linear shear model not only through the average axial strain of the cross-section, but also through a damage parameter that would represent the state of the transverse section due to the axial forces applied to the fibres. With this procedure, the decrease of the concrete shear strength for increasing flexural displacement ductility could be described in a more suitable way.

Different cumulative damage parameters can be used; from a simple parameter proportional to the maximum compression strain computed at the core concrete of the cross-section, to more sophisticated parameters that consider the amount of energy dissipated in the fibres during the loading history. Such damage parameters would establish a supplementary link between the fibre model and the strut-and-tie formulation.

In this case, two internal variables \( D_1 \) and \( D_2 \), one per diagonal, would have to be included in the model, allowing damage due to loading in one direction not to be completely dependent from loading in the opposite direction. In this case, equation (5.15) representing the force at the concrete struts would be given by

\[ F_{ck} = \sigma_c(e_k) \cdot (1 - D_k) \cdot A_{shear} \cdot (\cos\theta) \]  \hspace{1cm} 5.20

which corresponds to substitute the stresses \( \sigma_c(e_k) \) in the diagonals in equations (5.18) and (5.19) by

\[ (\sigma_c^*)_k = \sigma_c(e_k) \cdot (1 - D_k) \]  \hspace{1cm} 5.21

where \( (k = 1, 2) \) refers to each strut.
However, a damage parameter is always very difficult to calibrate. Because no experimental data was available to evaluate the degradation of shear strength during a well known loading history, in the numerical tests illustrated in this work this link was never considered, i.e. $D_k$ was always equal to zero.

Constitutive laws

The axial stress-strain law used for the concrete in the strut-and-tie model was the same implemented for the fibres in CASTEM 2000 and described in sections 4.3 in chapter 4. In particular, the improved model was adopted. However, the parameters that control the crack-closing and the tension-stiffening effect may present different values. Notice that a non-linear shear model such as the one considered in this work, attempts to reproduce the response of a structure through the behaviour of just three elements per cross-section: two made of concrete and one made of steel. This means that it is not possible for the model to “disguise” any singularity present in the axial stress-strain response curve of these three elements. For example, cracking of a diagonal or yielding of the transverse steel occurs throughout the whole cross-sectional area of the strut or tie. Instead, for the fibres this only occurs at one Gauss point of one of the fibres in the cross-section.

Therefore, to properly represent the global shear stress-strain response curve of the structure and to improve the performances of the numerical algorithm, smooth axial stress-strain constitutive laws were used for “hiding”, as much as possible, the singular points present in the behaviour laws of the steel and concrete. In fact, this was one of the main reasons for the development of the improved model described in chapter 4.

The parameters of the exponential curve of this concrete model must be chosen so that the axial stress-strain behaviour law integrates the bi-axial state of the concrete in the whole diagonal element. Thus, the parameters may assume different values when representing the behaviour of the struts or of the concrete fibres. The interlocking and the dowel effect can be implicitly taken into account through these parameters.

For the transverse steel, the adopted constitutive law is the explicit formulation of Mene-gotto-Pinto described in sections 4.4 in chapter 4. The monotonic curve was not considered to avoid any of its singular points from interfering negatively in the shear stress-
strain response curve. Note that the Menegotto-Pinto model is a very smooth model.

**Tensile strut**

Concerning the contribution of the tensile stresses to the shear response, it is pointed out that:

- to represent properly the shear force-displacement response curve of a reinforced concrete structure before yielding of the longitudinal steel bars, the tensile stresses should be considered in the constitutive laws of the struts;

- the tensile constitutive law of the diagonals should be carefully chosen to prevent the anomalous behaviour illustrated in Figure 5.9. When after an important monotonic loading there is a reversal loading (from point 1 to 4 in Figure 5.9, i.e. 1-4), the diagonal under tensile stresses (already fully cracked and still virgin in compression) follows the almost zero stress line represented by 4-7 in Figure 5.9 and the shear stress is controlled by the compressed strut (see equation (5.17)). After decreasing to zero, the strut stress increases up to the tensile peak (still uncracked) and then decreases through the softening branch of the tensile concrete curve, as represented by line 5-6. Since the shear stresses follow the behaviour of this strut, the cross-section responds accordingly: a peak shear stress-strain point followed by a softening curve appears on the shear stress-strain response curve. This behaviour is, of course, anomalous and should be avoided;

- since the non-linear shear model considers the diagonals to be symmetric to the column axis, only when \( \theta = 45^\circ \) the diagonals are perpendicular one to the other. This angle defines the direction of the principal compression stress. Thus, when the cracking angle is different to 45\(^\circ\), the strain at the tensile strut does not correspond to the strain in the direction perpendicular to the compressed strut. Instead, it corresponds to a value closer to the compressed strut strain, i.e. the peak tensile stress is reached at the diagonal after being reached in the pseudo principal tensile direction; the diagonal under tensile strains is delayed in relation to the principal direction;

- although shear and flexural constitutive models follow similar concrete constitutive
laws, there is no link between them concerning tensile behaviour. The fibres and the diagonals may reach cracking at different steps of the loading history.

Figure 5.9 - Influence of the tensile behaviour law in the shear stress-strain response curve

To take these aspects into consideration, the behaviour law of the concrete for the diagonal was modified; the tensile strength of the struts is no longer a parameter of the model and depends on the deformation of the fibres; the strut reaches the peak tensile strength when the average axial deformation of the cross-section, \( \varepsilon_{oe} \), reaches the strain at the peak tensile stress of the fibres concrete. This procedure introduces an additional link between the flexural and the shear model. Finally, as the constitutive law should represent the global behaviour of the struts, the post-peak tensile softening branch is usually much smoother than for the fibres.

*Shear cracking angle*

In this non-linear shear formulation, the cracking angle \( \theta \) is a parameter of the model. It
is computed through the geometric characteristics of the structure and of the longitudinal and transverse steel reinforcement ratio; knowing the structure and the type of loading, with or without important axial forces, semi-empirical equations are used to find a suitable value for $\theta$. A very simple approach described [13] is presented in the next paragraphs.

**Cracking limit analysis**

In this formulation, the crack inclination angle is calculated through the analysis of three possible shear failure modes in membrane type elements, namely: a) yielding of the steel in both directions without crushing of the concrete, b) yielding of the steel only in the weak direction with crushing of the concrete and, finally, c) crushing of the concrete without yielding of the steel. In the case of columns, the in-plane steel grid corresponds to the longitudinal and transverse steel bars. The strong and weak directions are the longitudinal and transverse directions of the beam, respectively (see Figure 5.10).

![Figure 5.10 - Analogy with a membrane](image)

For each of the failure modes a different expression is found for the inclination angle $\theta$ of the principal compression stresses at failure [45]:

- yielding of the steel in both directions without crushing of the concrete gives

$$\tan \theta = \frac{\rho_{sw} f_{sw}}{\rho_s f_{sy}}$$  \hspace{1cm} 5.22

where
\[
\rho_s = \frac{A_{sl}}{hb_w} \quad \rho_{sw} = \frac{A_{sw}}{sb_w}
\]

are the longitudinal, \(\rho_s\), and transverse steel ratios, \(\rho_{sw}\). \(A_{sl}\) and \(A_{sw}\) are the reinforcing steel areas in the two directions, \(f_{sw}\) and \(f_{sy}\) are the corresponding yielding stresses, \(b_w\) and \(h\) are the width and the length of the core of the cross-section and \(s\) is stirrups spacing. The shear stress corresponding to this failure mode is

\[
\tau_u = \sqrt{\rho_{sw} \cdot \rho_s \cdot f_{sy} \cdot f_{sw}}
\]

- yielding of the transverse reinforcement and crushing of the concrete without yielding of the longitudinal steel bars gives

\[
\sin \theta = \frac{\rho_{sw} \cdot f_{yw}}{f_{cd}}
\]

\(f_{cd}\) being the concrete strength. The shear stress that corresponds to failure is given by

\[
\tau_u = \sqrt{(f_{cd} - \rho_{sw} \cdot f_{sw}) \rho_{sw} \cdot f_{yw}}
\]

- crushing of the concrete without yielding of both transverse and longitudinal reinforcements. In this case, the element is subjected to pure shear, i.e.

\[
(\theta = 45^o) \land (\tau_u = \frac{1}{2} \cdot f_{cd})
\]

From the three possible failure modes, the governing mode is the one that corresponds to the lowest \(\tau_u\) value. Nevertheless, and according to the formulation, the crack inclination angle is not to be taken less than a minimum value given by

\[
\tan \theta_{min} = \frac{h}{2L}
\]

Experimental results obtained at the ELSA laboratory on squat piers showed cracking angles that were not in agreement with the values calculated with this formulation and presented in section 5.5.2. Thus, another algorithm making use of the compression forces in the struts and of the tensile forces in the longitudinal and transverse steel bars was developed for columns. The cracking angle is computed through the equilibrium of the
horizontal and vertical forces in a cracked member. The formulation was named \textit{cracking equilibrium model}.

\textbf{Cracking equilibrium model}

In this approach, the critical cracking angle is established through the equilibrium of forces in two resisting mechanisms: one made of a concrete strut going from the top to the bottom of the column and transmitting the applied axial forces directly to the basement (see Priestly in [70]), and a truss made of concrete struts and transverse and longitudinal steel ties that react to shear forces. Since in the analysis the column is assumed to be fully cracked, no tensile stresses are considered in the concrete.

The equilibrium is established in the transverse and longitudinal direction of the column. As illustrated in Figure 5.11 the equilibrium in the transverse direction is given by

\[ V = -F_{cs} \cdot \sin \theta - P \cdot \tan \alpha \]  

5.29

\(F_{cs}\) being the force in the strut within the truss mechanism and \(P\) the axial force on the top of the column.

![Diagram of Cracking equilibrium model](image)

\textbf{Figure 5.11 - Cracking equilibrium model}

The equilibrium of forces in the longitudinal direction of the column gives

\[ F_{cs} \cdot \cos \theta + P = F_c \]  

5.30

Substituting equation (5.29) into equation (5.30), the angle \(\theta\) is calculated through
\[ \tan \theta = \frac{V + P \cdot \tan \alpha}{P - F_c} \quad 5.31 \]

Knowing the distribution of flexural moments \( M(x) \) along the longitudinal axis of the structure \( \overline{ax} \), equation (5.31) becomes

\[ \tan[\theta(x)] = \left( \frac{dM(x)}{dx} + P \cdot \tan \alpha \right)/(P - F_c) \quad 5.32 \]

To simplify the next steps, a cantilever with a transverse force \( V \) at the top, \( (M(x) = V \cdot x) \), was adopted as illustrated in Figure 5.11. Therefore, given a structure and the applied axial force \( P \), the flexural moment \( M(\phi) \) and the compression force at the concrete \( F_c(\phi) \), for a curvature loading history, can be computed at any transverse section, in particular at the critical section (in this case, at the bottom near the basement where \( x = L - L_o/2 \)). Substituting these two values in equation (5.32), the expression of the cracking angle becomes

\[ \tan[\theta(\phi)] = \left( \frac{M(\phi)}{(L - L_o/2)} + P \cdot \tan \alpha \right)/(F(\phi)) \quad 5.33 \]

where \( (F(\phi) = P - F_c(\phi)) \).

As illustrated in Figure 5.11, the length \( L_o \) also depends on the cracking angle

\[ L_o = \frac{D - c(\phi)}{\tan \theta} \quad 5.34 \]

\( c(\phi) \) being the width of the compression zone. Substituting this value and \( (\tan \alpha = (D - c(\phi))/(2L)) \) in equation (5.33), a polynomial of the second degree is obtained for \( \tan \theta \)

\[ F(\phi) \cdot (\tan \theta)^2 - \left( \tan \alpha \cdot (F(\phi) + P) + \frac{M(\phi)}{L} \right) \cdot \tan \theta + P \cdot (\tan \alpha)^2 = 0 \quad 5.35 \]

The history of flexural moments, compression forces and compression zone width, are calculated through the fibre model.

The resulting transverse force, \( (F_c \cdot \sin \theta) \), must also be in equilibrium with the tensile force in the stirrups that cross the diagonal crack, \( F_{sw} \). Using equation (5.29) and assum-
ing an elastic perfectly plastic material for the transverse steel, the equilibrium gives

\[
\frac{M}{(L - L_0/2)} + P \cdot \tan \alpha \leq f_{yw} \cdot \rho_{sw} \cdot b_w \cdot L_o
\]

where \(f_{yw}\) is the yielding stress of the transverse steel, \(\rho_{sw}\) is the volumetric transverse steel ratio and \(b_w\) is the width of the transverse section. Substituting equation (5.34) into equation (5.36), we obtain

\[
\left( P \cdot \tan \alpha + \frac{M(\Phi)}{L} \right) \cdot (\tan \theta)^2 - (\tan \alpha \cdot (P \cdot \tan \alpha + B)) \cdot \tan \theta + B \cdot (\tan \alpha)^2 < 0
\]

where \((\tan \theta > 0)\) and \((B = 2 \cdot L \cdot f_{yw} \cdot \rho_{sw} \cdot b_w)\).

The cracking angle is the angle satisfying equation (5.35) and equation (5.37), for a curvature corresponding to yielding of the longitudinal or the transverse steel bars (see the example in Figure 5.22).

**Cross-sectional area of the struts**

The angle \(\theta\) defines the direction of the cracks in relation to the column axis. Therefore, the orientation of the diagonal that contributes the most to shear strength, i.e. the compressed strut, is correct. However, as both diagonals are symmetric to the member axis, the strut under tensile stresses does not necessarily respect the perpendicularity to the direction of cracking. This means that, for angles not far from 45° the model is coherent but some adjustments must be made whenever angles closer to 0° or 90° are forecasted. In particular, the area of the transverse section \(A_{strut}\) must be carefully analysed.

As shown in Figure 5.12, the cross-sectional area of the diagonals is given by

\[
A_{strut} = A_{shear} \cdot (\cos \theta)
\]

If the two extreme values \((\theta = 0^\circ)\) and \((\theta = 90^\circ)\) are considered in equation (5.38), the area of the cross-section of the struts is \(A_{shear}\) and zero, respectively, i.e. for low cracking angles the total active area of the two diagonals is almost twice the core concrete area and it tends to zero when \(\theta\) tends to 90°. Therefore, equation (5.38) was modified to take all possible range of angles from 0° to 90° into account.
Figure 5.12 - Cross-sectional area of the struts

As the two diagonals should represent the principal strain directions, they should be, at each loading step, perpendicular to each other

\[
(A_{\text{Strut}})_1 = b_w \cdot h \cdot (\cos \theta) \\
(A_{\text{Strut}})_2 = b_w \cdot h \cdot (\cos (90^\circ - \theta))
\]

Moreover, as the model accounts for cyclic loading, i.e. a compressed strut becomes a tensile strut and vice versa, the two diagonals should have the same cross-sectional area. Hence, the average value of the two areas in equation (5.39) was adopted

\[
A_{\text{Strut}}^* = \frac{(A_{\text{Strut}})_1 + (A_{\text{Strut}})_2}{2} = b_w \cdot h \cdot \frac{\cos \theta + \sin \theta}{2}
\]

and the transverse area given by equation (5.38) was multiplied by

\[
k_{A_x} = \frac{\cos \theta + \sin \theta}{2 \cdot \cos \theta} = \frac{1 + \tan \theta}{2}
\]

5.3.2 Summary

A strut-and-tie type model coupled with the classic fibre model was presented. To compile the information given in section 5.3.1, the main steps within the formulation are described step by step:
1) calculate the average axial strain $\varepsilon_{oe}$ in the cross-section using the axial strain of the fibres computed through equation (4.1)

$$
\varepsilon_{oe} = \frac{\int_A \varepsilon dA}{A}
$$

where $A$ is the area of the cross-section;

2) admit an initial value for the transverse steel strain $\varepsilon_{wy}$;

3) calculate the strain of the struts and ties through equation (5.14) using the shear strain computed within the hypothesis of the Timoshenko beam element;

4) compute the forces in the struts and ties using equation (5.15) and the constitutive laws of the materials;

5) establish the equilibrium of internal forces in the transverse section through equation (5.18). If the equilibrium is respected, go to 6). Otherwise, estimate another value for $\varepsilon_{wy}$ and go to 2);

6) compute the shear reaction force in the transverse section through equation (5.19).

The shear reaction forces $V$ computed at 6) substitute the transverse forces $F_y$ and $F_z$ in equation (E.8) and the algorithm detailed in annex E for the classic fibre model is used in the equilibrium of external forces of the 3D elements.

### 5.4 NUMERICAL IMPLEMENTATION

To implement the strut-and-tie model, a new fibre with special characteristics was added to the transverse section. It is made of concrete and steel and reacts to shear forces according to equation (5.17). Four Gauss points were considered in this fibre. The reason for using more than one Gauss point per fibre is discussed later in section 5.4.4. To distinguish this new element from the classic longitudinal fibres, it will be referred to as shear fibre. In the case that shear forces exist in both principal directions of the transverse section, one shear fibre needs to be considered in each direction. Moreover, if torsional moments also exist, the position and geometric characteristics of these fibres must
be carefully chosen. This topic is also discussed in section 5.4.4.

In a classic fibre model, the algorithm is straightforward; the program calculates at each Gauss point of each three-dimensional element, the axial deformation, the rotation along the three main directions of the element and the shear strain in the two transverse directions of the cross-section. Then, the program computes at each Gauss point of each fibre, the axial strain through the equations of compatibility of displacements and the internal stresses via the constitutive laws of the materials that are explicit in terms of strain. However, since the struts strain depends on the stirrups strain that is computed through the equilibrium of internal forces in the cross-section, an intermediate step, requiring an iterative algorithm, has to be considered.

This and other aspects linked to the implementation procedures, such as: the parameters of the model in CASTEM 2000 and the shear tangent stiffness, are presented in the next sections. The possibility to represent torsional moments is also discussed.

5.4.1 Parameters of the model in CASTEM 2000

The parameters of the shear model in CASTEM 2000 are divided into three groups, namely: one containing the parameters of the concrete constitutive law for the struts, another group containing the parameters of the steel law for the ties and, finally, a third group having the specific parameters of the non-linear shear model. The parameters of the first and the second group were already described in sections 4.3 and 4.4. The third group includes three parameters: the transverse steel ratio \( \rho \), the shear cracking angle \( \theta \) and a parameter defining the ultimate state of the concrete (deformation, energy, etc.).

5.4.2 Strain at the transverse steel

Knowing the shear and the average axial deformation arising from the 3D Timoshenko element, the equilibrium of internal forces on the cross-section is established and the stirrups strain is obtained through an iterative algorithm as referred to in section 5.3.2 and illustrated in Figure 5.13. This procedure is internal to the equilibrium of external forces.

The algorithm that was used for solving this intermediate step requires the equations of compatibility of displacements to be formulated not in terms of total displacements but
in terms of incremental displacements. The next sub-section describes the algorithm, the modifications performed on equation (5.14).

![Shear fibre model diagram]

Figure 5.13 - Shear fibre model

*Equations of compatibility for incremental displacements*

If $i$ is the loading step under analysis, one can write
\[
\varepsilon_{oe(i)} = \varepsilon_{oe(i-1)} + \Delta\varepsilon_{oe(i)} \\
\varepsilon_{wy(i)} = \varepsilon_{wy(i-1)} + \Delta\varepsilon_{zz(i)} \\
\tan \gamma(i) = \tan \gamma(i-1) + \Delta\gamma(i) = \frac{\tan \gamma(i-1) + \tan \Delta\gamma(i)}{1 + \tan \gamma(i-1) \cdot \tan \Delta\gamma(i)}
\]

Since the cracking angle \( \theta \) does not change during the loading history and \( (1 + \tan \gamma(i-1) \cdot \tan \Delta\gamma(i) \equiv 1) \), the deformation at loading step \( i \) can be written as a function of the deformation at loading step \( (i-1) \) plus an incremental value \( \Delta \)

\[
\varepsilon_k(i) = \varepsilon_k(i-1) + \Delta\varepsilon_k(i) \quad k = 1, 2
\]

where

\[
\Delta\varepsilon_k(i) = \left[ \Delta\varepsilon_{oe} \cdot (\cos \theta)^2 + \Delta\varepsilon_{wy} \cdot (\sin \theta)^2 + (-1)^k \cdot \frac{\tan \Delta\gamma}{2} \cdot (\sin(2\theta)) \right]_{(i)}
\]

and the two formulations, considering incremental displacements (equation (5.44)) and total displacements (equation (5.14)), are similar.

**Equilibrium of internal forces - Iterative algorithm**

The behaviour of the shear fibres depends on the average axial strain and shear strain in the transverse section and on the stirrups strain. The first two values come from the flexural fibre model. To calculate the third one, the equilibrium of internal forces is established in the cross-section through equation (5.18). Since functions \( \sigma_c \) and \( f_{sw} \) are nonlinear, an iterative process has to be used.

The system to be solved is constituted by one unknown equation and, therefore, a very simple algorithm was chosen. It starts from the equilibrium at loading step \((i-1)\) and establishes the equilibrium at the next step looking for the new strain at the stirrups. If \( j \) corresponds to the iteration, the compatibility of displacements for the two diagonals becomes

\[
[\varepsilon_{k(i)}]_{(j)} = [\varepsilon_{k(i)}]_{(j-1)} + [\Delta\varepsilon_{wy(i)}]_{(j)} \cdot (\sin \theta)^2 \quad k = 1, 2
\]

where
\[
[e_{wy(i)}]_{(j)} = [e_{wy(i)}]_{(j-1)} + [\Delta e_{wy(i)}]_{(j)}
\]

and

\[
[e_k(i)]_{(0)} = e_{k(i-1)} + \left[ \Delta e_{oe} \cdot (\cos \theta)^2 + (-1)^k \cdot \frac{\tan \Delta \gamma}{2} \cdot (\sin(2\theta)) \right]_{(i)}
\]

\[
[e_{wy(i)}]_{(0)} = e_{wy(i-1)}
\]

Considering now equation (5.18), the internal equilibrium gives

\[
R([e_{wy(i)}]_{(j)}) = \left[ \sigma([e_1(i)]_{(j)} + \sigma([e_2(i)]_{(j)}) \right] \cdot (\sin \theta)^2 + f_{sw}([e_{wy(i)}]_{(j)}) \cdot \rho_{sw}
\]

\[|R([e_{wy(i)}]_{(j)})| \leq Tol\]

where \(R(e_{wy})\) is a residual function that should not be greater than an established tolerance “Tol”. Ignoring pointer \(i\) and knowing that for small incremental values \(\Delta e\) one can write

\[
F(\varepsilon + \Delta \varepsilon) \equiv F(\varepsilon) + \frac{d}{d\varepsilon} F(\varepsilon) \cdot \Delta \varepsilon
\]

equation (5.18) becomes linear in \(\Delta e_{wy(j)}\)

\[
\left[ \sigma(e_1(j-1)) + \sigma(e_2(j-1)) \right] + \left[ E_{cl}(e_1(j-1)) + E_{cl}(e_2(j-1)) \right] \cdot \Delta e_{wy(j)} \cdot (\sin \theta)^2
\]

\[
= -f_{sw}(e_{wy(j-1)}) - E_{st}(e_{wy(j-1)}) \cdot \Delta e_{wy(j)} \cdot \rho_{sw}
\]

\[
(\sin \theta)^2
\]

i.e

\[
\Delta e_{wy(j)} = \frac{-f_{sw}(e_{wy(j-1)}) \cdot \rho_{sw} - [\sigma(e_1(j-1)) + \sigma(e_2(j-1))] \cdot (\sin \theta)^2}{E_{st}(e_{wy(j-1)}) \cdot \rho_{sw} + [E_{cl}(e_1(j-1)) + E_{cl}(e_2(j-1))] \cdot (\sin \theta)^4}
\]

or

\[
\Delta e_{wy(j)} = \frac{-R(e_{wy(j-1)})}{E_{R(j-1)}}
\]

where \((E_{R(j-1)} = \partial R(e_{wy(j-1)})/\partial(e_{wy(j-1)})\) and \(E_{cl}(\varepsilon)\) and \(E_{st}(\varepsilon)\) are the axial tangent stiffness of the struts and ties, respectively.
In the numerical tests performed with this algorithm, the convergence of the algorithm was, occasionally, very slow; the residual function had a very smooth tangent that increased abruptly in a very narrow band around the solution of the problem. In these cases and whenever the solution was not found after a pre-defined number of iterations, the bisection method was used in equation (5.49). The convergence criterion is given by

\[ |\varepsilon_{wy(j)} - \varepsilon_{wy(j-1)}| < 10^{-4} \cdot \varepsilon_{w,yield} \]  

where \( \varepsilon_{w,yield} \) is the yielding strain of the stirrups.

### 5.4.3 Shear tangent stiffness

The resolution of non-linear structural problems requires, very often, the tangent stiffness at the Gauss points of the 3D elements. In the case of the shear fibres, the tangent modulus is given by

\[
\frac{d}{d\gamma} \tau(\gamma) = \frac{d}{d\gamma} (V/A_{shear})
\]

Substituting equation (5.19) in equation (5.55), one can write

\[
\frac{d}{d\gamma} \tau(\gamma) = \left( \frac{\partial}{\partial \varepsilon} \sigma_c(\varepsilon_2) \cdot \frac{\partial \varepsilon}{\partial \gamma} - \frac{\partial}{\partial \varepsilon} \sigma_c(\varepsilon_1) \cdot \frac{\partial \varepsilon}{\partial \gamma} \right) \cdot (\sin \theta) \cdot (\cos \theta)
\]

Taking into account the dependence of \( \varepsilon_{oe} \) and \( \varepsilon_{wy} \) on \( \gamma \)

\[
\frac{\partial \varepsilon_k}{\partial \gamma} = \frac{\partial \varepsilon_k}{\partial \varepsilon_{oe}} \cdot \frac{\partial \varepsilon_{oe}}{\partial \gamma} + \frac{\partial \varepsilon_k}{\partial \varepsilon_{wy}} \cdot \frac{\partial \varepsilon_{wy}}{\partial \gamma} + \frac{\partial \varepsilon_k}{\partial \gamma} \quad k = 1, 2
\]

and using equation (5.14) that establishes the compatibility of displacements, the partial derivatives become

\[
\frac{\partial \varepsilon_k}{\partial \gamma} = (\cos \theta)^2 \cdot \frac{\partial \varepsilon_{oe}}{\partial \gamma} + (\sin \theta)^2 \cdot \frac{\partial \varepsilon_{wy}}{\partial \gamma} + \frac{(-1)^k}{2} \cdot \frac{\partial \varepsilon_{wy}}{\partial \gamma} \cdot (\sin(2\theta))
\]

Substituting now equation (5.58) into equation (5.56), the shear tangent stiffness is given by
\[
\frac{d}{d\gamma} \tau(\gamma) = \frac{(\sin(2\theta))^2}{4 \cdot (\cos\gamma)^2} \cdot (E_{ct}(e_1) + E_{ct}(e_2))
\]
\[
-\left( (\cos\theta)^2 \cdot \frac{\partial e_{oe}}{\partial \gamma} + (\sin\theta)^2 \cdot \frac{\partial e_{wy}}{\partial \gamma} \right) \cdot (E_{ct}(e_1) - E_{ct}(e_2)) \cdot \frac{\sin(2\theta)}{2}
\]

The partial derivatives of the axial strain \( e_{oe} \) and of the transverse steel strain \( e_{wy} \) with respect to the shear strain \( \gamma \), are calculated through a sensitive analysis on the equation of equilibrium of internal forces. To calculate \( \frac{\partial e_{wy}}{\partial \gamma} \), the algorithm checks what happens to \( e_{wy} \) when in equation (5.18) the shear strain is incremented by \( \Delta \gamma \)

\[
f_{sw}(e_{wy} + \Delta e_{wy}) \cdot \rho_{sw} + (\sigma_c(e_1 + \Delta e_1) + \sigma_c(e_2 + \Delta e_2)) \cdot (\sin\theta)^2 = 0 \quad 5.60
\]

with

\[
\Delta e_k = \Delta e_{wy} \cdot (\sin\theta)^2 + (-1)^k \cdot \frac{\tan \Delta \gamma}{2} \cdot (\sin(2\theta)) \quad k = 1, 2 \quad 5.61
\]

Using equation (5.18) and the hypothesis expressed in equation (5.50), the equation becomes linear in \( \Delta e_{wy} \)

\[
\Delta e_{wy} \cdot \left[ E_{st}(e_{wy}) \cdot \rho_{sw} + (E_{ct}(e_1) + E_{ct}(e_2)) \cdot (\sin\theta)^4 \right]
\]
\[
= (E_{ct}(e_1) - E_{ct}(e_2)) \cdot (\sin\theta)^2 \cdot \frac{\sin(2\theta)}{2} \cdot \tan \Delta \gamma \quad 5.62
\]

Since for small incremental values it is \( \frac{\partial e_{wy}}{\partial \gamma} \equiv \Delta e_{wy} / \Delta \gamma \) and \( (\tan \Delta \gamma \equiv \Delta \gamma) \), the derivative of the transverse steel strain with respect to the shear strain becomes

\[
\frac{\partial e_{wy}}{\partial \gamma} \equiv \frac{(E_{ct}(e_1) - E_{ct}(e_2)) \cdot (\sin\theta)^3 \cdot \cos\theta}{E_{st} \cdot \rho_{sw} + (E_{ct}(e_1) + E_{ct}(e_2)) \cdot (\sin\theta)^4} \quad 5.63
\]

Using the same procedure for the axial strain (i.e. check what happens to \( e_{oe} \) when in equation (5.18) the shear strain is incremented by \( \Delta \gamma \)), the partial derivative of \( e_{oe} \) with respect to the shear strain is given by

\[
\frac{\partial e_{oe}}{\partial \gamma} \equiv \frac{(E_{ct}(e_1) - E_{ct}(e_2))}{(E_{ct}(e_1) + E_{ct}(e_2))} \cdot \tan \theta \quad 5.64
\]

In the linear elastic case, the elastic modulus of the two struts is equal to the concrete
elastic modulus \( E_{e_1} = E_{e_2} = E_c \). Thus,

\[
\frac{\partial \varepsilon_{wy}}{\partial \gamma} = \frac{\partial \varepsilon_{oe}}{\partial \gamma} = 0
\]

Moreover, as \( \cos \gamma \equiv 1 \) equation (5.56) becomes

\[
\frac{d}{d\gamma} \tau(\gamma) = \frac{E_c}{2} \cdot (\sin(2\theta))^2
\]

Comparing with the classic expression of the elastic shear modulus \( G = \frac{E_c}{2(1 + v)} \) an equivalent Poisson ratio is derived

\[
v = \frac{1}{(\sin(2\theta))^2} - 1
\]

5.4.4 Modelling and implementation remarks

Some comments on the behaviour of the strut-and-tie model arise from the analysis of the equations of equilibrium of forces and compatibility of displacements. They concern the yielding of the transverse steel, the possibility of representing torsional moments and, finally, the snap-back phenomenon that may occur in the shear stress-strain response curve as illustrated in Figure 5.17.

Yielding of transverse steel

If the stirrups do not yield, a reduction on the transverse steel ratio \( \rho_{sw} \) does not substantially modify the shear force-displacement response curve; it increases the stirrups strain proportionally to the decrease of \( \rho_{sw} \) in order to maintain the same level of reaction force at the transverse steel, and it only introduces minor changes in the diagonals strain.

Instead, if the transverse steel yields the response depends on the state of the diagonal under tensile strain. Substituting the compression force \( F_{c_1} \) from equation (5.16) into equation (5.17), one can write

\[
V = F_{wy} + 2 \cdot F_{c_2} \cdot \sin \theta
\]

If the tensile strut is completely cracked, \( (F_{c_2} = 0) \), the structure slides and the incremental top displacement of the pier is mainly due to the shear strain. The shear resisting
force at the cross-section follows the constitutive law of the stirrups.

If the diagonal under tensile strain still resists to tensile forces and goes through the softening branch after peaking, the increase of the tensile strain at the diagonal implies, through equation (5.16) and equation (5.68), the decrease of the compression force in the other diagonal. Consequently, the global shear resisting force decreases until no residual tensile strength exists in the strut (line 3-4 in Figure 5.14). From this point, the structure is fully cracked and responds accordingly (line 4-5 in the same figure). In most of the cases that were studied, the link between the tensile constitutive law of the shear and the longitudinal fibres described in section 5.3.1 was enough to avoid this problem.

**Torsional moments**

As illustrated in Figure 5.15, the torsional moment in the transverse section $S$ of an element with longitudinal axis $\bar{\alpha}$, is given by the integral of the moment of transverse forces $(\tau_{xy} \cdot dS)$ and $(\tau_{xz} \cdot dS)$ in relation to $\bar{\alpha}$. Since the shear model simulates the response of structural elements submitted to these forces, it should also be able to simulate the response due to torsional moments. However, in this case the truss must take into account the existence a non-uniform distribution of shear deformations in the cross-section. Two solutions are proposed: to increase the number of shear fibres to properly simulate the distribution of shear forces in the transverse section or to expand or shrink the cross-sectional area of the shear fibres to make the Gauss points coincide with representative points of the structure.

In order to explain the two solutions, a 3D beam element with the rectangular hollowed core section illustrated in Figure 5.16 is considered. The first solution consists in representing each of the four side walls of the beam by a different shear fibre with a transverse sectional area proportional to the zone it represents on the cross-section. Each Gauss point of each shear fibre "sees" different shear strains, according to the torsional rotation and the position they occupy in the cross-section, and contributes differently to resist torsional moments (see Figure 5.16-a)). This explains the adoption of four Gauss points per shear fibre and not only one as would seem more appropriate.
Figure 5.14 - Influence of yielding of the stirrups in the shear stress-strain response curve

The second solution is to change the cross-sectional area of the shear fibres so that the position of each one of the four Gauss points would represent the average shear stress at each quarter of the total cross-section (see Figure 5.16-b)). In this case, a corrective factor would have to be used to convert the expanded area to the real transverse area. In the model implemented in CASTEM 2000, the shape factor $\alpha$ mentioned in annex E that accounts for the warping of the cross-section, being equal to the quotient between the real area and the expanded area, could represent this role.

This procedure has to be applied to both transverse directions of the 3D element and thus at least two shear fibres, one per each main transverse direction, have to be considered. In Figure 5.16-a) the stresses in the shear fibres are represented by one vector per fibre and not one per each Gauss point, as illustrated in Figure 5.16-b), so as not to overcharge the picture.
The snap-back phenomenon

For certain cyclic loadings, a snap-back phenomenon was observed at the shear stress-strain response curve after cracking. The reason for this phenomenon can be found in the equations of compatibility of displacements and equilibrium of forces of the algorithm; the diagonals strain is the combination of three different deformations: the average axial strain $\varepsilon_{oe}$, the stirrups strain $\varepsilon_{wy}$, and the shear strain $\gamma$.

\[
\bar{M}_x = \int \left( -\bar{V}_z \otimes \bar{\tau}_{xy} + \bar{V}_y \otimes \bar{\tau}_{xz} \right) dS
\]

Figure 5.15 - Torsional moments

During cyclic loading and after cracking, the average axial strain grows contemporarily with the applied load. When the load decreases, $\varepsilon_{oe}$ also decreases until a minimum value that corresponds, more or less, to the pinching zone of the flexural force-displacement curve. This is particularly evident after yielding of the longitudinal steel bars. However, while some cracks close, others open and $\varepsilon_{oe}$ increases again until the peak displacement in the other direction is reached. Instead, since growing shear forces correspond to, or should correspond to, growing shear deformations, the shear strain should increase and decrease following the loading history. Thus, when the structure unloads after cracking, the absolute values of $\varepsilon_{oe}$ and $\gamma$ may progress in opposite directions.

In fact, in the example in Figure 5.17 the shear strain changes sign before the minimum average axial strain is reached (see circle 1). After this point, while the absolute value of the shear strain increases, the average axial strain goes on decreasing and demanding higher shear strain in order to respect the compatibility of displacements and the equilibrium of forces in the structure. This is true until the minimum average axial strain occurs.
Then, it increases giving, together with the shear strain, a "positive" contribution to the deformation of the compressed strut.

Figure 5.16 - Numerical simulation of torsional moment resisting mechanisms

To respect the equilibrium of forces and the compatibility of displacements in the structure, the change in the growing direction of the average axial strain may force the shear strain to decrease in order to compensate the increase of the mean axial strain that, some-
times, is quite abrupt. This gives rise to the snap-back phenomenon in the shear stress-strain response curve, i.e. the decrease of the shear strain for an increase of the shear force illustrated in Figure 5.17 by the thin line in circle 3. The curves correspond to the analytical behaviour of the pier illustrated in this figure submitted to a cyclic horizontal displacement applied to the top. The cross-section and other characteristics of the pier are presented in section 5.5 and correspond to those of section type 3.

![Diagram showing the 'snap-back' phenomenon](image)

**Figure 5.17 - The 'snap-back' phenomenon**

To confirm this analysis a checking procedure was implemented: the value of the average axial strain due to the axial forces and flexural moments was "frozen" each time the shear strain changed sign, i.e. each time the shear stress-strain response curve crossed the shear stress axis. This value represented the minimum axial strain allowed in the strut in
the following steps and was corrected each time a higher axial strain occurred in the same circumstances. The result is illustrated in the upper right side diagram of Figure 5.17 and by the thick line in the shear stress-strain diagram. No snap-back effect is observed in this case.

Actually, for most all the examples solved with this procedure the snap-back phenomenon did not occur. This procedure seems to confirm the previous analysis and, at the same time, stresses the weight the average axial strain has in the shear response. Numerical tests performed with the same pier showed that the shear cracking angle also plays an important role in this process: a higher cracking angle reduces the weight of the average axial strain in the struts strain “disguising” the snap-back phenomenon.

5.5 MODEL VALIDATION

To check the proposed non-linear shear model, a series of squat bridge piers tested at the ELSA laboratory in Ispra, Italy (see chapter 3), were simulated. Since the aim of this section is not to simulate the bridges response but to check the behaviour of the non-linear shear model, instead of applying the seismic action to the global structure, the history of displacements observed during the tests is applied directly to the top of the piers. Furthermore, only the short pier of the irregular bridges, the one with the lowest shear span ratio, is analysed (see Figure 5.18).

![Figure 5.18 - Profile of the irregular bridge tested at the ELSA laboratory (scale 1:1)](image)

The numerical results of the short piers of the irregular bridges B213A, B213B and B213C of the PREC8 programme, considering the non-linear and the linear shear model, are presented and compared with the experimental results.
5.5.1 The experimental tests - some remarks

Although the experimental campaign is described in detail in chapter 3, some aspects of the tests that are particularly relevant for the numerical simulation of the bridge piers are once more highlighted.

Four experimental results corresponding to a static cyclic test and to Pseudo-dynamic tests on the three irregular bridges are presented. A vertical force equal to $1.72 MPa$ was applied to the top of all the piers to simulate the dead load of the deck. The mechanical properties of the materials interested in the tests: the steel (B500 Tempcore) and the concrete C25/30, are presented in Tables 3.2 to Tables 3.4, respectively. These values were evaluated on the basis of tests performed on material samples. The piers are named P3S, P1D, P3D and P5D, where the numbers refer to the section type in agreement with the nomenclature presented in Figure 3.4, and the last letter refers to the static test (S) or the Pseudo-dynamic test (D).

The piers in the laboratory are build in plinths rigidly attached to the strong floor. The local deformation of the plinths introduces a supplementary rotation at the bottom of the pier. This effect is taken into account in the numerical tests through a flexible foundation represented by a linear elastic rotational spring calibrated for the experimental tests. This is analysed more in detail in chapter 6.

Table 5.2 presents the volumetric ratios for the longitudinal and the transverse steel bars of the three different cross-section types. During the experiments displacement transducers were distributed along the sides of the piers to enable the splitting of bending and shear displacements. The displacements due to bending forces were computed through the method described in annex D. The difference to the total displacements, also measured in the laboratory, gave the shear displacements. With this approach, the displacements from any other source, such as the flexibility of the foundation, appear in the shear displacements.
Table 5.2: Longitudinal and transverse reinforcement in the piers cross-section

<table>
<thead>
<tr>
<th>Volumetric ratio of the longitudinal steel bars, $\rho_l$ (%)</th>
<th>Section Type</th>
<th>S1</th>
<th>S3</th>
<th>S5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical volumetric ratio of the confining hoops, $\rho_c$ (%)</td>
<td></td>
<td>24.7</td>
<td>20.5</td>
<td>35.5</td>
</tr>
</tbody>
</table>

5.5.2 Numerical applications

The cross-sections of the three piers are divided in bending and shear fibres. Since the applied load only introduces transverse forces in one direction of the pier and, furthermore, no torsional moments exist, a $I$ type cross-section was adopted. The parameters of the model defining both the characteristics of the steel and concrete are presented in Figures 5.19 and 5.20. The piers were divided into five plus two Timoshenko elements from the basement to the top with lengths of $0.34m$ and $0.55m$, respectively.

![Diagram](image)

<table>
<thead>
<tr>
<th>Piers</th>
<th>$\sigma_{c0}$ (MPa)</th>
<th>$\sigma_{c0}^*$ (MPa)</th>
<th>$\varepsilon_{c0}$ (%)</th>
<th>$\varepsilon_{c0}^*$ (%)</th>
<th>$Z$</th>
<th>$Z^*$</th>
<th>$\sigma_t$ (MPa)</th>
<th>$\varepsilon_{tm}/\varepsilon_t^a$</th>
<th>$\varepsilon_{tm}/\varepsilon_t^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>36.2</td>
<td>45.6</td>
<td>0.25</td>
<td>0.40</td>
<td>100</td>
<td></td>
<td>27.7</td>
<td>1.6</td>
<td>10</td>
</tr>
<tr>
<td>S3</td>
<td>32.0</td>
<td>40.2</td>
<td>0.25</td>
<td>0.39</td>
<td>100</td>
<td></td>
<td>28.3</td>
<td>1.2</td>
<td>7</td>
</tr>
<tr>
<td>S5</td>
<td>35.2</td>
<td>43.3</td>
<td>0.25</td>
<td>0.38</td>
<td>100</td>
<td></td>
<td>30.3</td>
<td>0.1</td>
<td>5</td>
</tr>
</tbody>
</table>

a. Longitudinal fibres

b. Shear fibres

Figure 5.19 - Parameters of the confined and unconfined concrete
Model parameters

The four parameters that define the loading and unloading rules of the improved model for the concrete described in section 4.3.2, are \((F_1 = 2.0), (F_2 = 0.75), (F_1' = 20.0)\) and \((F_2' = 20.0)\) for the longitudinal fibres and \((F_1 = 2.0), (F_2 = 0.75), (F_1' = 20.0)\) and \((F_2' = 3.0)\) for the shear fibres.

The parameters adopted in the Menegotto-Pinto model are \((R_o = 10.0), (a_1 = 9.0)\) and \((a_2 = 0.15)\).

![Graph showing stress-strain relationship for concrete](image)

<table>
<thead>
<tr>
<th>Piers</th>
<th>(\sigma_{sy} [\text{MPa}])</th>
<th>(\sigma_{ul} [\text{MPa}])</th>
<th>(\varepsilon_{sh}^{*} [%])</th>
<th>(\varepsilon_{ud} [%])</th>
<th>(E_o [\text{GPa}])</th>
<th>(E_h/E_o [%])</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 Long. Fibres</td>
<td>490.0</td>
<td>570.0</td>
<td>2.0</td>
<td>14.5</td>
<td>206.0</td>
<td>0.55</td>
</tr>
<tr>
<td>S1 Shear Fibres</td>
<td>700.0</td>
<td>730.0</td>
<td>0.34</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3 Long. Fibres</td>
<td>540.0</td>
<td>630.0</td>
<td>2.0</td>
<td>13.2</td>
<td>206.0</td>
<td>0.55</td>
</tr>
<tr>
<td>S3 Shear Fibres</td>
<td>700.0</td>
<td>730.0</td>
<td>0.34</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S5 Long. Fibres</td>
<td>480.0</td>
<td>580.0</td>
<td>2.0</td>
<td>13.0</td>
<td>206.0</td>
<td>0.55</td>
</tr>
<tr>
<td>S5 Shear Fibres</td>
<td>365.0</td>
<td>430.0</td>
<td>2.0</td>
<td>15.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.20 - Parameters of the longitudinal and transverse steel

When the cracks are diagonal to the column axis, the principal strain directions are also diagonal and the physical structure opens cracks before the longitudinal fibres reach the tensile strength. Thus, the tensile strength in the model should not be the value from the axial test, or any equivalent test like the Brazilian test, but a lower value. In order to adequately represent the pre-yielding branches of the global force-displacement curve, the tensile strength adopted in the concrete constitutive law is the value that fits the envelope of the top force-displacement experimental curve (see Figure 5.19).

The transverse steel ratio, it is given by
\[ \rho_{sw} = \frac{A_{sw} \cdot r}{b_w \cdot s} \]

where \( b_w = 0.32m \), \( s \) is the stirrups spacing, \( r \) measures the ratio between the total length of the stirrups in the direction of the shear stresses and the length of the cross-section, \( r = (4 \times 6 \times 0.492m)/(1.60m) \) and \( A_{sw} \) is the cross-sectional area of the stirrups. The values in equation (5.69) for the three different sections are presented in Table 5.3.

<table>
<thead>
<tr>
<th>Pier</th>
<th>( A_{sw} \text{ [m}^2\text{]} )</th>
<th>( s \text{ [m]} )</th>
<th>( \rho_{sw} \text{ [%]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1*5</td>
<td>( 1.963 \times 10^{-5} )</td>
<td>0.050</td>
<td>0.905</td>
</tr>
<tr>
<td>S3 5</td>
<td>( 1.963 \times 10^{-5} )</td>
<td>0.060</td>
<td>0.754</td>
</tr>
<tr>
<td>SS 6</td>
<td>( 2.827 \times 10^{-5} )</td>
<td>0.050</td>
<td>1.304</td>
</tr>
</tbody>
</table>

*a. Stirrups diameter in [mm]*

**Fibre discretization**

To represent the flexibility of the plinth at the basement, a linear elastic rotational spring was placed at the bottom of the pier. It was simulated through a fictitious element with a length of 0.01m (1:2.5 scale), with the same cross-section of the pier and a Young modulus of 0.2GPa. This issue is detailed in chapter 6. The distribution of the longitudinal and shear fibres in the piers cross-section is illustrated in Figure 5.21.

![FIBRES](image)

Figure 5.21 - Distribution of the fibres in the cross-section (bending + shear)
Note that, in the case of the unconfined concrete, the fibres at the middle correspond to the superposition of the fibres in the exterior and in the interior of the cross-section.

The transverse section of the shear fibre only includes the side-walls of the pier in the direction of the shear forces. The unconfined concrete was also included. Thus, it measures 0.32m per 1.44m.

**Struts inclination angle**

The cracking angle was calculated using the two models described in section 5.3.1: the *cracking limit analysis* and the *cracking equilibrium model*. In both models, only the side walls of the pier were considered to contribute to the shear strength. The values required by the cracking limit analysis to compute the angle \( \theta \) for the three different cross-sections are presented in Tables 5.3 and 5.4.

<table>
<thead>
<tr>
<th>Pier</th>
<th>( f_{sw} ) [MPa]</th>
<th>( \rho_s ) [%]</th>
<th>( b_{fy} ) [MPa]</th>
<th>( f_c ) [MPa]</th>
<th>( c_{\tau u} ) [MPa]</th>
<th>( d_{\tau u} ) [MPa]</th>
<th>( e_{\tau u} ) [MPa]</th>
<th>( f_\theta ) [degrees]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>700</td>
<td>0.276</td>
<td>364</td>
<td>41.2</td>
<td>2.52</td>
<td>14.86</td>
<td>20.6</td>
<td>68.3</td>
</tr>
<tr>
<td>S3</td>
<td>700</td>
<td>0.491</td>
<td>503</td>
<td>37.0</td>
<td>3.61</td>
<td>12.94</td>
<td>18.5</td>
<td>55.6</td>
</tr>
<tr>
<td>S5</td>
<td>364</td>
<td>0.491</td>
<td>503</td>
<td>42.2</td>
<td>3.42</td>
<td>13.33</td>
<td>21.1</td>
<td>54.2</td>
</tr>
</tbody>
</table>

a. \( \rho_s = (10 \cdot \pi \cdot D^2)/(1.28m \cdot 0.32m) \), with \( D = 6mm \) on S1 and \( D = 8mm \) on S3 and S5.

b. It only refers to the steel in the side walls of the structure

c. \( \tau_u = \sqrt{\rho_{sw} \cdot f_{sw} \cdot f_{fy}} \)

d. \( \tau_u = \sqrt{f_{cd} - \rho_{sw} \cdot f_{sw} \cdot f_{fy}} \)

e. \( \tau_u = f_{cd}/2 \)

f. \( \theta = \arctan[\sqrt{(\rho_{sw} \cdot f_{sw})/(\rho_s \cdot f_{fy})}] \)

The results from applying the cracking equilibrium model to the three cross-sections are presented in Figure 5.22. The curves represented by broken lines establish the limits given by equation (5.37). The angle adopted in the non-linear shear model is the value given by equation (5.35) that corresponds to yielding of the longitudinal steel bars but still within the limits established by equation (5.37). This is what happens to sections type 1 and 3. In the case of section type 5, it is the yielding of the transverse steel that determines the value of the cracking angle. The yielding point is indicated in the figure by a vertical broken line. The corresponding cracking angles are summarized in Table 5.5.
Table 5.5: Cracking angles

<table>
<thead>
<tr>
<th>Section type 1</th>
<th>Section type 3</th>
<th>Section type 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>46.0°</td>
<td>44.5°</td>
<td>46.8°</td>
</tr>
</tbody>
</table>

The angles calculated with the two methods are quite different; the results from the cracking equilibrium model agree quite well with the experimental values whereas the angles from the cracking limit analysis are far from the experimental angles observed in the piers at the laboratory (around 45° for the three piers: P1, P3 and P5).

Figure 5.22 - Cracking angle computed through the cracking equilibrium model

The numerical tests were performed considering the cracking angles computed with the
two different procedures. However, in the case of the cracking equilibrium model, the angle of 45° was adopted instead of the values in Table 5.5. The results are compared with the experimental responses.

5.6 RESULTS

The behaviour of the piers was simulated using the fibre model and the two different approaches for the shear forces: the linear elastic law and the non-linear shear model described in this chapter. The histories of displacement registered at the top of the four piers during the cyclic static test and the Pseudo-dynamic tests are applied to the top of the numerical structures. These diagrams are illustrated in Figure 5.23.

For each of the tests the following response curves are presented: transverse force-total displacement at the top of the pier and at a height of 1.70m, transverse force-shear and flexural displacements at a height of 1.70m, and, finally, the peak displacement profiles for both loading directions. For pier P5S, and apart from the two analytical cracking angles, two other angles were adopted: (θ = 48°) and (θ = 51°). For pier P5D and for the cracking angle (θ = 45°), two additional transverse steel ratios were considered: (ρ_sw = 1.15%) and (ρ_sw = 1.0%).

5.6.1 General results

Although the piers have the same concrete cross-section and the same shear span ratio, (r_sp = 1.75), the shear forces are quite different from one pier to the other. Since the flexural moments increase with the longitudinal steel ratio and higher flexural strength means higher transverse forces at the top of the pier, a stronger participation of the shear mechanisms in the response is expected when the longitudinal steel ratio increases. This is confirmed by Figure 5.24: the ratio between the shear and the flexural displacements increase from section type 1 to section type 5.

Furthermore, the presence of transverse forces changes the orientation of the principal directions which becomes diagonal to the longitudinal axis of the structure, increasing the weight of the transverse steel in the shear resisting mechanism; the longitudinal and the transverse steel form a steel grid that, together with the concrete, define a truss mechanism that reacts to the deformation of the structure. Therefore, there is a strong correla-
tion between shear strength and the amount of transverse steel in the pier. An example using pier P5D for different transverse steel ratios is illustrated in Figure 5.25; an increase of 0.15% on the transverse steel ratio increased significantly the displacements computed through the shear mechanism. This issue is discussed more in detail when the results of the high strength squat pier P5 are analysed.

The non-linear shear model uses the average axial deformation that comes from the flexural model which, in turn, depends on the length of the 3D elements that define the structure. Thus, in order to verify the dependence of the response of the model to the mesh discretization, four tests were performed on pier P5 considering elements with different lengths near the basement: two elements of 0.85\(m\) on the first test, three elements of 0.56\(m\) on the second test, five elements of 0.34\(m\) on the third test and four elements of 0.17\(m\) plus three elements of 0.34\(m\) on the last test. A history of displacements was applied to the top of the pier: a 2\(cm\) amplitude cycle followed by a 4\(cm\) cycle. The results presented in Figure 5.26 show a very good agreement between the four responses. Even the case with two 0.85\(m\) length elements gave a force-displacement diagram close to the more refined solution.

**Low strength pier - P1**

Pier P1 is the least reinforced of the three piers. The first comment is that, in spite of the general good agreement between the numerical and the experimental results, the ultimate state of the pier was not forecasted by the numerical model. The decrease of strength registered in the experimental force-displacement curve was due to the rupture of the longitudinal steel bars that did not occur in the numerical test. After this loading step, the comparison between the experimental and the analytical results makes no sense.

The simulation of the ultimate state of a structure caused by an abrupt event like the rupture of a steel bar is a very difficult subject. Small variations in the geometry or in the materials can explain the differences to the experimental results. In the numerical simulation of a structure submitted to uniaxial bending, such as the one under analysis, one bar can not break alone leaving all the others at the same level still resisting, unless different fibres are considered for each one of the bars. Moreover, the position and the characteristics of those bars would have to be known with extreme accuracy and the
degradation of the characteristics of the materials with cyclic loading would have to be properly taken into account.

From the comparison of the results considering the linear elastic behaviour law and the non-linear shear behaviour model illustrated in Figures 5.27 to 5.30 and in Figures 5.44 and 5.45, it seems that no evident benefit came from the non-linear shear model. The shear displacements ratio is very low and the stiffness of the structure was not strongly affected by the degradation of the shear stiffness. However, and in spite of the large over-estimation of the shear displacements, the total force-displacement curves at the height of 1.70m for the non-linear shear model are closer to the experimental results. The over-estimation of the numerical shear displacements was partly due to a more flexible response after cracking and before yielding of the longitudinal bars. Although this is true for both cracking angles, (θ = 68°) and (θ = 45°), the first angle gave a behaviour in shear closer to the experimental response. Nevertheless, the global force-displacement curves for the two angles, experimental and analytical, were similar and very close to the experimental response.

Two comments must be made. Firstly, the split of total displacements in flexural and shear displacements is fully legitimate only in the numerical analysis; in a real structure there is a perfect coupling between the two displacements. Secondly, since in the numerical model the flexural behaviour assumes that cracking is perpendicular to the longitudinal axis of the structure, the diagonal cracking on the physical structure changes the conditions that make the splitting fully comparable with the numerical results. Therefore, the first comparison between the analytical and the experimental results should be done at the level of the global force-displacement curves and only after should the flexural and the shear behaviour be analysed separately. Nevertheless, the splitting of the total experimental displacements gives a good idea of how much the behaviour of the structure is influenced by shear forces and, furthermore, how far the numerical model is from predicting the non-linear shear behaviour.

The profiles of maximum displacements are closer to the experimental results when the angle from the cracking limit analysis was adopted (see Figures 5.24 and 5.52). In spite of the more flexible behaviour of the model near the basement in relation to the experi-
ment, the two results are quite close.

**Medium strength pier - P3**

Pier P3 was tested for two different top displacement histories. The first one corresponds to the displacement measured during the Pseudo-dynamic test of bridge B213A. The second one is the growing peak displacement history described in section 3.5.1 and illustrated in Figure 5.23.

The global transverse force-displacement curves for both cracking angles followed quite well the experimental results; although the analytical angle \( \theta = 56^\circ \) gives a less dissipative shear curve, the test with the angle \( \theta = 45^\circ \) shows a transverse force-shear displacement curve that is closer to the results found in the laboratory (see Figures 5.31 to 5.38 and Figures 5.46 to 5.49). The snap-back phenomenon is quite clear when the second displacement history was applied to the top of the pier. This is especially evident when \( \theta = 45^\circ \), since in this case the participation of the average axial deformation in the non-linear shear model is more important (see equation (5.14)). Regarding the flexural behaviour, the fibre model represented the response due to the flexural moments with great accuracy.

As a higher longitudinal steel ratio enables greater shear forces on the structure, pier P3 shows a behaviour that was clearly more influenced by shear forces than pier P1. This is apparent in the analysis of the global response curve. For the same displacement imposed at the top of the pier, the results obtained with the linear shear model are more dissipative showing a typical flexural behaviour. This model did not reproduce the narrowing and lengthening of the hysteretic force-displacement curve typical of structures subjected to important transverse forces.

The profiles of maximum displacements in Figures 5.24 and 5.52 show that the pier with the angle computed through the cracking limit analysis is less flexible near the basement, giving better results. Nevertheless, the non-linear shear model for the two angles represents well the experimental results.
High strength pier - P5

From the three piers analysed in the present work, P5 is the pier with the higher longitudinal and transverse reinforcement ratio. The first comment on Figures 5.39 and 5.40 is that the analytical angle \( \theta = 54^\circ \) gives a very flexible response mainly controlled by the truss mechanism. When the maximum shear strength of the truss is reached, i.e. the transverse steel bars yield, the structure cannot respond to the new demand of shear force and slides near the basement. Thus, the maximum flexural moment was controlled by the truss mechanism and the contribution of the flexural mechanism to the global behaviour of the structure was significantly reduced.

The second comment on the results comes from the analysis of Figure 5.43: the analytical results for this particular pier are quite sensitive to the cracking angle adopted in the strut-and-tie model. A decrease of 3° on the cracking angle corresponded to a decrease on the shear displacement of more than 30%.

Two circumstances contributed to the high sensitivity of the analytical results to small variations of the cracking angle at pier P5: the low yielding stress of the transverse steel bars, 365.0MPa, and the proximity between the maximum transverse force from the truss and the flexural mechanism. Thus, a small decrease of the cracking angle corresponded to an increase of the amount of transverse steel involved in the internal equilibrium of the structure (see equation (5.15)) enough to allow the flexural mechanism develop much further in the inelastic range. This effect is well illustrated in Figures 5.39, 5.40, 5.50 and 5.51.

The comparison of the numerical response with the experimental results clearly shows that the angle computed through the cracking limit analysis did not properly represent what happened in the physical pier. Instead, when the angle from the cracking equilibrium model was adopted the numerical response was quite close to the experimental response. The profiles of maximum displacements illustrated in Figures 5.24 and 5.52 confirm this result. The analysis also shows that the equations of the cracking limit analysis are unsuitable for representing the cracking pattern observed in the experiments and for computing the cracking angle for the non-linear shear model.
Finally, the model represented well the non-symmetric behaviour in shear due to the yielding of the transverse steel bars; although two struts are considered to represent the state of the concrete in the two loading directions, the stirrups are the same regardless of the loading direction. Thus, if after imposing shear forces in one direction the stirrups yield, the plastic deformation of the steel bars is transmitted to the other direction. This phenomenon, that also occurs in the physical structures, is not so evident in the flexural behaviour since the longitudinal bars under tensile forces are not the same when the structure is loaded in opposite directions.

The results using the linear elastic law in shear (see Figures 5.41 and 5.42) show a much more dissipative global force-displacement curve in opposition to the experimental results.

5.7 CONCLUSIONS

In a strut-and-tie based model like the one described in this chapter, the determination of the cracking angle is very important. Special care must be taken when dealing with structures where the applied transverse force is near its maximum shear capacity, such as pier P5. In this case, small variations of the shear cracking angle can greatly modify the numerical response. The same thing happens with the transverse steel reinforcement ratio. Numerical tests on pier P5D, illustrated in Figures 5.25 and 5.43, show the different transverse force-displacement diagrams obtained for small variations of these two parameters.

Furthermore, the numerical tests presented in this chapter show that the analytical model referred to as the cracking limit analysis is not suitable to estimate the critical cracking angle for the non-linear shear model; angles much higher than the experimental values were found with this formulation. Worse, the corresponding numerical results for the critical pier P5 were quite far from the experimental results. Instead, the cracking equilibrium model gave, for the three piers, cracking angles in very good agreement with the experimental values.

The results expressed in Figures 5.44 to 5.51, show, in general, a very good agreement between the numerical and the experimental results. The comparison is made at two lev-
els: at the top and at 1.70m from the bottom of the pier. The numerical transverse force versus displacement curves at both levels reproduce quite well the experimental results. The behaviour of structures where the effect of shear forces is not negligible is well reproduced by the non-linear shear model; the narrowing and lengthening of the force versus displacement curves observed in the experimental results is well represented. However, as the decrease of the concrete and steel strength due to the dissipation of energy is not taken into account in the cyclic behaviour laws, the damage of the structure within the cycles is not well represented in the response curves. This is particularly evident in the results of pier P3S.

Finally, tests performed on pier P5 show that the non-linear shear model can be considered mesh independent: four different meshes, more or less refined near the basement, led to similar top transverse force-displacement response curves.
5.8 FIGURES

Figure 5.23 - History of displacements imposed at the top of the piers
Figure 5.24 - Numerical versus experimental peak displacement profiles for the four piers and the non-linear shear model
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Figure 5.28 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier P1D, using the non-linear shear model.
Figure 5.29 - Numerical versus experimental force versus total displacement response curves for pier P1D at two different height levels and using the linear elastic shear model.
Figure 5.30 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier P1D, using the non-linear shear model
Figure 5.31 - Numerical versus experimental force versus total displacement response curves for pier P3D at two different height levels and using the linear elastic shear model.
Figure 5.32 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier P1D, using the non-linear shear model.
Figure 5.33 - Numerical versus experimental force versus total displacement response curves for pier P3D at two different height levels and using the linear elastic shear model.
Figure 5.34 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier P3D, using the linear elastic shear model.
Figure 5.35 - Numerical versus experimental force versus total displacement response curves for pier P3S at two different height levels and using the non-linear shear model
Figure 5.36 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier P3S, using the non-linear shear model
Figure 5.37 - Numerical versus experimental force versus total displacement response curves for pier P3S at two different height levels and using the linear elastic shear model
Figure 5.38 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier P3S, using the linear elastic shear model
Figure 5.39 - Numerical versus experimental force versus total displacement response curves for pier P5D at two different height levels and using the non-linear shear model
Figure 5.40 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier PSD, using the non-linear shear model
Figure 5.41 - Numerical versus experimental force versus total displacement response curves for pier P5D at two different height levels and using the linear elastic shear model.
Figure 5.42 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier P5D, using the linear elastic shear model
Figure 5.43 - Numerical versus experimental force versus total, flexural and shear displacement response curves for pier P5D with different transverse critical cracking angles
Figure 5.44 - Numerical versus experimental force versus total displacement response curves for pier P1D at two different height levels and using the non-linear shear model
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Figure 5.49 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier P3S, using the non-linear shear model
Figure 5.50 - Numerical versus experimental force versus total displacement response curves for pier P5D at two different height levels and using the non-linear shear model
Figure 5.51 - Numerical versus experimental force-flexural and shear displacement response curves at 1.7m from the bottom, for pier PSD, using the non-linear shear model.
Figure 5.52 - Numerical versus experimental peak displacement profiles for the four piers and the non-linear shear model
6 NUMERICAL SIMULATION OF THE EXPERIMENTAL CAMPAIGN

6.1 INTRODUCTION

The ELSA reaction-wall facility, together with other laboratories in Europe, has been used for Pre-normative Research in Support of Eurocode 8 [21]. This programme of the European Commission included a large experimental campaign on bridges. The results are presented and analysed in chapter 3. This campaign was preceded by a set of numerical tests performed with the fibre model described in chapter 4. The preliminary results allowed the verification, a posteriori, of the ability of the model to predict the non-linear behaviour of reinforced concrete bridges under dynamic loading. Furthermore, the maximum forces assumed in the control system in the laboratory were predicted taking into account these preliminary numerical results: an alarm system stops the Pseudo-dynamic procedure whenever the imposed forces go above this limit.

The present chapter describes the numerical tests performed within this programme and follows the sequence of the tests carried out in the laboratory. Thus, the chapter is divided into three parts: the pre-experimental tests, the post-experimental tests with the new model born from the analysis of the experimental results and a supplementary set of numerical tests where the main differences still found between the post-experimental results and the experimental response are analysed in more detail.

Therefore, the first part of the chapter reports the predictive numerical calculations. The specific aspects of the Pseudo-dynamic technique that should be properly taken into account in the numerical simulation of the experimental campaign are described in sec-
tion 6.2. The test set-up is referred to in the same context. The geometry of the bridges tested in the laboratory and the mechanical characteristics of the materials adopted in the numerical analysis are presented in section 6.3. The input accelerogram and the damping matrices are referred to in section 6.4.

Topics such as the flexibility of the foundation block and the importance of the shear displacements to the global response of the squat piers, are discussed. This discussion arose from the comparison of the numerical results with the experimental response of a short column submitted to a cyclic static test, as described in section 6.5. This experiment was particularly helpful to verify the model that was used afterwards in the predictive computations. The effect of the soil-structure interaction is an important subject that has been studied by several authors (e.g. Ciampoli [14]). Elnashai [26] shows that the inclusion of the soil-structure interaction in the static analysis could result in reductions of 15-30% in displacement ductility demand to the piers due to the large variations attained in the yield limit state. Nevertheless, this is not taken into account in the analysis hereafter presented. At the end of this first part, the numerical results of the bridges are analysed and compared with the experimental response in section 6.6.

The need to improve the classic fibre model to include the non-linear behaviour due to shear forces actually emerged for the comparison of the analytical and the numerical results. The details of this model were already presented in chapter 5. The analytical campaign was then repeated, applying the non-linear shear model to the squat piers of the bridges with the irregular configuration B213. The new results and the comparison with both the pre-experimental calculations and the experimental response are presented in the second part of the chapter in section 6.7.

The third part reports a supplementary set of numerical calculations carried out after the analysis of the results in the second part. This third campaign aimed at verifying the differences still found between the numerical and the experimental results when the non-linear shear model was used. This analysis is presented in section 6.8.

Finally, the main conclusions are summarized in section 6.9 and the pictures are shown in section 6.10.
6.2 TESTING PROCEDURES

Two objectives were forecast with the pre-experimental numerical tests: to give to the laboratory a prior information on the behaviour of the piers and, at the same time, to evaluate the capacity of the model to predict the non-linear response of reinforced concrete bridges. Thus, the analytical calculations had to reproduce, as close as possible, the testing conditions in the laboratory. The following sub-sections give an overview of the aspects of the Pseudo-dynamic method and testing procedures that were to or should provided in the numerical calculations.

6.2.1 The Pseudo-dynamic test

A Pseudo-dynamic test can be regarded as a test on a structure condensed at a restrict number of degrees of freedom (d.o.f.), those controlled by the actuators during the experiment. In the case of the bridge piers tested in the laboratory, only the top section of the piers in the horizontal transverse direction of the bridge was controlled (imposing displacements and measuring restoring forces). Thus, no mode shapes besides the first one could have been reproduced. To represent adequately this effect in the numerical simulations, the mass of the piers was statically condensed at the top nodes of the piers, i.e., at the same points, and in the same direction, of the actuators in the laboratory.

Regarding the substructuring technique used for testing the bridges: a numerical deck in interaction with physical piers in the laboratory (see Figure 2.6), no important changes had to be introduced in the Pseudo-dynamic algorithm in relation to the algorithm without substructuring; instead of measuring all restoring forces in the laboratory, part of these forces were calculated with a numerical model. Since in the full numerical simulations both forces are calculated through an analytical model, no special precautions had to be taken due to substructuring. However, in this case, a special attention must be paid to the connection between the numerical and the physical structure. The conditions of the tests in the laboratory are discussed in section 6.2.2.

The integration algorithm that was used for solving the equation of dynamic equilibrium in the experimental tests and in the numerical computations was not the same. While the \( \alpha \)-Newmark formulation coupled with the Operator-Splitting scheme was used in the
tests in the laboratory, the classic Newmark formulation was adopted in the numerical calculations, the first being the algorithm implemented at the ELSA laboratory for Pseudo-dynamic tests and the second the formulation available in the computer code CASTEM 2000. Both algorithms are described in annex C.

6.2.2 The test setup

According to the test setup in the laboratory, only shear forces along the transverse direction of the bridge were transferred through the pier-deck connection points; the nodes of the deck, in virtual contact with the piers, are prevented from moving either in the vertical or in the longitudinal direction. No interaction was considered in the vertical direction between the structure in the laboratory and the numerical substructure. The weight of the deck was imposed directly at the top of the piers and, apart from small variations due to the deformation of the piers caused by cyclic loading, it remained constant during the test. Furthermore, as the axial force was always oriented from the top to the basement of the piers, the second order effects in the laboratory were quite reduced. The piers moved in the vertical direction apart from the deck. No translation nor torsional movements were allowed at the extremities of the bridge. However, the deck was free to rotate around the vertical and horizontal transverse axis of the bridge.

Identical conditions were provided in the numerical structure: the deck was described by the same linear elastic model and the same boundary conditions. The top section of the piers was free to move but always respecting the links provided in the Pseudo-dynamic test between the piers and the super-structure.

6.3 GEOMETRIC CHARACTERISTICS

Although the information concerning the geometry of the bridges is already presented elsewhere, in particular in chapter 3, some are now repeated in the context of the numerical analysis. During the description of the two bridge profiles considered in the experimental campaign, the regularity issue is discussed; a parameter proposed by Calvi et al [8] is presented and applied to the structures. The modelling assumptions and the mechanical properties adopted for the concrete and steel are also presented. The mode shapes of the two profiles are computed at the end.
6.3.1 Geometric characteristics of the bridges - Regularity parameter

The bridges have a 200m long deck with four equal spans supported by three piers with
the same rectangular hollow-core cross-section and abutments at the extremities. Two
different configurations, corresponding to different levels of structural regularity, were
adopted in the design. Since the bridges within the PREC8 programme were only sub-
mitted to a seismic action in the horizontal transverse direction of the deck, the regularity
concept only regards the behaviour in this direction.

The definition of regularity is based on a parameter $R$ proposed by Calvi et al [8] which
essentially compares the mode shapes of the deck alone with the mode shapes of the
entire structure, deck and piers:

$$
R = \sqrt{\frac{\sum_{i=1}^{n} \left( \frac{\Phi_j}{\sqrt{\Phi_j^T[M]\Phi_j}} \right)^2}{n}}
$$

6.1

where $\Psi_j$ and $\Phi_j$ represent, respectively, the eigenvectors of the deck with and without
the stiffness of the piers included, $[M]$ is the mass matrix and $n$ is the number of eigen-
values taken into account in the analysis. From equation (6.1) it results that $R$ ranges
from zero to one as the bridge goes from a configuration giving mode shapes to the deck
far from those of the deck without the piers, to a configuration where the first $n$ mode
shapes of both structures are identical. According to Calvi, a bridge is close to the regular
configuration when $R$ is close to one.

The so called regular bridge, labelled B232, corresponds to a symmetric structure having
the tallest pier in the middle. Each number of the label name refers to the number of
times each pier is higher than the height-modulus (7.0m), i.e., $2x7m$ for the first pier,
$3x7m$ for the second and $2x7m$ for the third one.

Instead, the irregular bridge presents a configuration labelled B213, i.e. having piers with
2, 1 and 3 height-modulus, meaning $14m$, $7m$ and $21m$ high piers, respectively. This
bridge was intended to be highly irregular. In fact, not only the structure is not symmetric
but the fundamental mode shape of the deck also induces much higher forces at the cen-
tral and stiffer pier when compared to the regular profile. The two different configurations and the geometric characteristics of the deck and of one the pier cross-sections, are shown in Figure 3.1 in chapter 3.

The parameter $R$ in equation (6.1) for the two bridges, B232 and B213, is given in Table 6.1. It was computed supposing that the mass of the bridges is uniformly distributed on the deck. Furthermore, only the modes contributing to important horizontal transverse displacements of the bridge were considered; a static condensation of the piers at the connection points with the deck has been performed. The agreement between the mode shape vectors of bridge B232 and those of the deck without the stiffness of the piers included is quite good, ($R = 0.996$). According to Calvi et al, this bridge profile can be assumed to be regular.

Other two bridges with the same irregular profile were considered in the experimental campaign as described in chapter 3: B213B and B213C. To distinguish the reference bridge, designed according to the EC8, from these two, the bridge was labelled B213A. The difference between the three bridges is on the amount of longitudinal and, consequently, transverse steel in the piers. The geometric characteristics of the bridges were given in Tables 3.1 and 3.5 in chapter 3.

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Deck</th>
<th>B232</th>
<th>B213</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Phi_1$</td>
<td>$\Phi_2$</td>
<td>$\Phi_3$</td>
</tr>
<tr>
<td>P1</td>
<td>.537</td>
<td>.776</td>
<td>-.612</td>
</tr>
<tr>
<td>P2</td>
<td>.759</td>
<td>.000</td>
<td>.866</td>
</tr>
<tr>
<td>P3</td>
<td>.537</td>
<td>-.776</td>
<td>-.612</td>
</tr>
</tbody>
</table>

$\Phi_j[M]\Psi_j \Rightarrow .994 \quad 1.000 \quad .994 \quad .524 \quad .564 \quad .943$

$R = .996 \quad R = .703$

a. Mode shapes normalized to the mass matrix, i.e. $(\Phi_j[M]\Phi_j) = (\Psi_j[M]\Psi_j) = 1$
b. Nodes in agreement with the piers illustrated in Figure 3.1

6.3.2 Modelling

The numerical analyses were performed with the computer code CASTEM 2000 [19] described in annex B and using the fibre model detailed in chapters 4 and 5 the structures
are divided into one-dimensional geometric elements with six d.o.f. per node, each element is sub-divided into longitudinal fibres and each fibre behaves according to the material law it represents. The model is implemented in Timoshenko beam elements with one Gauss integration point. Linear elastic eccentric beam elements were adopted to represent the deck while the piers were modelled by fibre non-linear elements (see Figure 6.1). The predictive numerical calculations were performed with linear elastic constitutive relations for shear forces.

The concrete constitutive laws and the parameters used in the pre- and post-experimental numerical analyses are slightly different. They correspond to the simplified and the improved models referred to in section 4.3. The differences between these two laws are described in chapter 4. The standard Newmark integration algorithm is used in the dynamic calculations (see in annex C).

The deck is divided into thirty-two elements of equal length and is simulated with a linear elastic model with a longitudinal and shear deformation modulus equal to 25GPa and 10GPa, respectively. The mass of the structure is equal to the mass of the concrete, 2.5ton/m³, meaning 17.4ton/m and 10.4ton/m uniformly distributed along the deck and piers, respectively. As referred to in section 6.2.1, to assure that the piers only vibrate along the first horizontal transverse mode as in the Pseudo-dynamic test in the laboratory, the mass of the piers was statically condensed at their top nodes in this direction.

The ten first frequencies of bridges B232 and B213 are given in Table 3.15. Figure 3.23 shows the five first mode shapes involving the deformation of the deck in the horizontal transverse direction. For similar mode shapes, the three irregular bridges present differences in the frequencies not grater than 2%.

6.4 INPUT DATA AND NUMERICAL DAMPING

A stationary seismic input motion, characterized by a peak ground acceleration of 0.35g with a frequency content corresponding to the EC8 response spectrum for intermediate soils, was considered in the experimental campaign (see Figure 3.15) applied to the bridges in the horizontal transverse direction of the deck. This accelerogram corresponds to the design action and, therefore, to a damage control limit state. It will be systemati-
cally referred to in the text as the design earthquake (DE). A constant vertical force equal to 10.6MN, equivalent to a normalized axial load equal to 0.1, was applied at the top of each pier.

A second non-linear analysis using the same accelerogram multiplied by a factor of 1.2 in the case of the irregular bridge, and 2.0 in the case of the regular bridge, was applied to the four bridges formerly damaged by the DE. As already mentioned in chapter 3, the intensity factors 1.2 and 2.0 should correspond to the ultimate capacity of the irregular bridge B213C and the regular bridge B232. They were selected based on numerical calculations. This subject is referred later in section 6.6.1.

The damping matrix adopted in the experiments was also considered in the numerical calculations: a Rayleigh matrix evaluated for a damping ratio ($\xi = 1.6\%$) for the frequencies of 5.6Hz and 7.2Hz (at the 1:2.5 scale bridge),

$$[C] = 0.633 \cdot [M] + (0.398 \cdot 10^{-3}) \cdot [K]$$  \hspace{1cm} (6.2)

where $[C]$, $[M]$ and $[K]$ are, respectively, the damping, the mass and the stiffness matrices of the bridges.

6.5 PRELIMINARY STUDIES

In order to prepare the experimental campaign, a quasi-static cyclic test was performed on the short pier of bridge B213A [67]. The aim of this test was to evaluate the adequacy of the testing and instrumentation devices and to obtain experimental results from a squat pier (aspect ratio = 1.75) submitted to a well known increasing top displacement history up to failure. These preliminary results enabled to check the analytical models that were used in the subsequent predictive numerical analyses.

6.5.1 Material properties and piers finite element mesh

Based on the mechanical properties of the materials adopted in the design, the parameters shown in Figures 6.2 and 6.3 (phase 1) were chosen for the axial stress versus strain behaviour laws of the concrete and steel.

The piers cross-sections are composed of unconfined and confined concrete and steel
fibres. Along their height, the piers are divided into 3.5m length elements. To simulate properly the plastic hinges, the element closest to the foundation is sub-divided into five elements and the one above into two elements of equal length. Regarding the fibres for each of the materials, namely: the 2.0cm thick cover concrete, the confined concrete and the steel, they were distributed in the cross section as illustrated in Figure 6.4. The location and number of fibres should be such that they represent properly the transverse section without demanding excessive computation time.

The parameters \((R_o = 20.0), (a_1 = 18.5) \text{ and } (a_2 = 0.15)\), were considered for the Giuffré-Pinto model.

6.5.2 Results

A monotonic loading was applied to the top of the numerical pier and the response was superimposed to the envelop of the top force versus displacement curve obtained from the test in the laboratory. The comparison between the experimental and the analytical results, illustrated in the upper left side diagram in Figure 6.8, showed that:

- the numerical elastic stiffness was greater than the experimental stiffness. Although the contact between the foundation block and the floor was considered to be perfect, the block itself may deform increasing the flexibility of the pier. Note that post-tensioning steel bars connecting the foundation block to the reaction floor are used for avoiding any sliding or distortional movement of the basement;

- the numerical post-cracking curve, i.e. the second branch of the so-called three-linear curve, was much stiffer than in the experimental results. The fact that the shear span ratio of the pier is quite low, 1.75, and a linear elastic law was assumed for the shear behaviour may have contributed to this result.

These aspects are analysed in detail in the two following sections.

6.5.3 Flexibility of the foundation

The flexibility of the foundation was checked through a 3D linear elastic model. Both structures, pier with and without foundation block, were divided into cubic 20-node ele-
ments (see Figure 6.5) and a horizontal force was applied to the top of the structure, as represented in Figure 6.6. The response of the pier for both boundary conditions, built-in and supported by the foundation block, revealed differences in the stiffness of about 20%.

To take this flexibility into account, a linear elastic rotational spring simulated by a fictitious 0.025m length element (1:1 scale), with the transverse section of the pier without the steel, was placed at the bottom of the pier. One extremity of this new element is built-in and the other is fixed to the pier and is only free to rotate in the transverse direction of the deck. The stiffness of the element was calibrated to fit the linear elastic branch of the experimental response.

The flexibility of the foundation for the 14.0m and the 21.0m high piers was computed using the experimental results from cyclic tests performed at a very low displacement amplitude (inside the linear elastic range) on the three piers of the first bridge tested in the laboratory, bridge B213A. The geometric and mechanical characteristics of the fictitious elements are given in Figure 6.7.

The reason for the different stiffness values found at the foundation block of the three piers may be due to several factors, namely: to the characteristics of the concrete and steel, to the reinforcing steel layout, to the casting conditions, to the bond mechanism established between the concrete and the steel bars.

6.5.4 Shear contribution

The second aspect that was not properly represented in the numerical results was the post-cracking stiffness; the second branch of the numerical three-linear curve was much stiffer than the one from the experimental response. This can be explained by the fact that the fibre model in the predictive calculations did not take into account the non-linear behaviour due to shear forces.

To check this hypothesis, the displacements due to the flexural moments in the experiment were separated from the shear displacements, following the procedure described in chapter 3, and compared with the analytical results. The assumption that the total hori-
zontal displacements can be split is entirely true only for the numerical model. In fact, while in the model the two mechanisms are disconnected (Timoshenko element with a linear elastic shear behaviour law), in a physical structure they are linked. Nevertheless, this splitting gives a good idea of how in a physical structure the total displacement is shared by the shear and flexural mechanisms.

The experimental and the numerical results are illustrated in left side diagrams in Figure 6.8. Both curves present similar post-cracking flexural force versus displacement stiffness. The shifting of the post-cracking branch was due to an over estimation of the tensile strength in the numerical model and to a possible slippage of the longitudinal reinforcing bars in the physical pier. The first is responsible for the lengthening of the linear elastic branch of the numerical response curve and the second for the increase of the yielding displacement during the experiment.

However, the so-called experimental shear displacements show that the behaviour of the pier was clearly non-linear in shear. The comparison between the numerical and the experimental curves seems to indicate that, although the bond slippage effect also produces a more “flexible” behaviour after cracking, the main differences found at the post-cracking branch were mainly due to the non-linear behaviour in shear.

Since a non-linear shear model was not available in the code for the predictive computations, the fibre model was still used but with modified mechanical properties for the concrete and for the steel in order to fit the experimental envelop curve. A parametric study on the strength and stiffness of both materials showed that the only way to change significantly the slope of the post-elastic branch of the monotonic curve was to modify the steel Young modulus (see Figure 6.3 - phase 2). On the one hand, since the cracking of concrete pushes the compression area to a narrow band above the central axis of the cross section, if one changes the parameters of the concrete model this modifies the stiffness of pier mainly before cracking, when a large portion of the concrete in the cross-section is still active, and it does not significantly change the post-cracking branch of the response curve. On the other hand, under cracking conditions the behaviour of the transverse section is mainly controlled by the stiffness of the steel bars; a more “flexible steel” does not change significantly the pre-cracking branch of the response curve (the

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steel bars represent 15% of the total elastic stiffness) but, as it increases considerably the curvature of the cracked transverse sections, it reduces significantly the global post-cracking stiffness. Thus, a longitudinal steel with half of the standard stiffness was adopted. The reduction of the Young modulus, if properly calibrated, is an indirect way one can use to consider the bond slippage effect. Finally, in order to get a numerical response curve closer to the experimental results (see Figure 6.8), a tensile strength higher than the value adopted in the first calculations, 3.0\text{MPa} instead of 2.5\text{MPa} (see Figure 6.2 - phase 2), was also considered.

These results concerned only the squat piers. The shear span ratio of the 14.0m and 21.0m high piers are two and three times higher than the ratio of the squat piers. Therefore, no changes were considered in their mechanical characteristics.

6.6 NUMERICAL PREDICTIONS OF THE PSEUDO-DYNAMIC TESTS

Each of the four bridges was tested twice Pseudo-dynamically and each experiment was preceded by a predictive numerical calculation. The interaction between the experimental and the numerical tests is described in detail in the next paragraphs. The comparison of the results and the main conclusions are presented afterwards.

6.6.1 Numerical simulations and interaction with the testing programme

Each bridge is analysed separately. The description follows the experimental campaign as it occurred in the laboratory: first the irregular configuration, with bridges B213A, B213B and B213C, and afterwards the regular bridge B232.

The maximum forces allowed in the control system at the laboratory were selected from this predictive analysis: an alarm system stopped the Pseudo-dynamic procedure whenever a force over this limit was requested. During the test, the experimental and the numerical results were compared on-line: a computer graphic window showed the piers top numerical displacements simultaneously with the experimental displacements imposed on the specimens during the test.

Bridge B213A

The first structure tested in the ELSA laboratory was bridge B213A. The DE was applied
to the numerical structure and the results were analysed.

Since for the DE the numerical and the experimental results compared quite well (see Figures 6.9 and 6.13), the decision for the new peak ground acceleration was based on the numerical calculations; the state of the bridge after the DE was kept in memory and the structure already damaged was subjected to amplified seismic inputs. A peak ground acceleration 20% higher than the one of the DE was chosen; this amplified accelerogram was expected to cause important damage in the squat pier of bridge B213C, the less reinforced squat pier of the three irregular bridges. As the aim of the experimental campaign was to compare the three alternative solutions, the same earthquake intensity value was adopted for the other two irregular bridges, B213A and B213B.

The Pseudo-dynamic test for the second input motion was performed with only the central pier physically represented in the laboratory; since the numerical response for the new seismic input showed that a non-linear but almost elastic behaviour should be expected at the medium and tall piers, these two piers were simulated numerically through a non-linear elastic force versus displacement curve adjusted to fit the experimental response curve for the DE input action. This procedure was adopted to preserve the two external piers from suffering important damage, since they were to be used after in bridges B213B and B213A with isolating devices and asynchronous input motion. However, in the full numerical analyses, the fibre model was used for all three piers.

**Bridge B213B**

The second bridge tested in the laboratory was the bridge B213B. As mentioned in the previous section, the medium and tall piers are the same of bridge B213A. The “numerical test” tried to follow, as close as possible, this procedure, i.e. to impose the input motion on a structure with the external piers damaged and the central pier intact. Therefore, the medium and tall piers in the numerical simulation of bridge B213B started from their final state after the DE test on bridge B213A. The set of internal variables corresponding to the end of this test was kept in memory and was imposed on the two piers at the beginning of the numerical simulation of bridge B213B.

For the second test, corresponding to a peak ground acceleration 20% higher than the
design earthquake, since the medium and tall piers were to be reused again, the procedure described for the second experiment on bridge B213A was also followed for bridge B213B. New non-linear elastic force versus displacement curves adjusted to the experimental results of the DE test on bridge B213B, were adopted at the medium and tall piers in the Pseudo-dynamic test. However, like in the case of bridge B213A, in the full numerical analyses the fibre model was used for all three piers.

**Bridges B213C and B232**

A straightforward scheme was followed for bridges B213C and B232. In this case, since the piers were not to be reused in other experiments, the DE was applied to the bridges, the results were analysed and, afterwards, the second intensity input motion: 20% higher than the DE for bridge B213C and twice the DE for bridge B232, was imposed on the two structures already damaged by the first seismic action. The same scheme was followed in the numerical computations.

The amplification factor of 2.0 was chosen for bridge B232 in order to reach its ultimate capacity. This choice was, once more, based on a set of numerical tests performed on bridge B232 after being damaged by the DE action.

**6.6.2 Remarks**

Before discussing and comparing the results, there are some aspects that were not equally taken into account in the numerical and in the Pseudo-dynamic tests that should be referred, namely:

- the starting conditions for the second test on each of the bridges were not the same in the numerical and in the Pseudo-dynamic test. The state of the piers after the first test was different: while in the laboratory the zero position corresponds to zero transverse force at the top of the piers, in the numerical simulation it corresponds to a global zero force on the whole structure. Thus, although the external forces were zero, the internal forces, in this case the horizontal forces at the top of the piers due to the interaction with the deck, were not compulsory zero;

- the second Pseudo-dynamic test started from zero displacement at all the control
points of the bridge, i.e. from a new equilibrium position different from the first experiment. This means that the plastic displacements remaining after the first test were not included in the displacements recorded during the experiment. In the numerical simulation, this corresponds to shift the experimental displacements from a quantity equal to the plastic displacements remaining after the first seismic load. This explains the slight discrepancies found between the experimental and the analytical results at the first time steps of the second test, as illustrated in Figure 6.10 for the short pier of bridge B213B;

- again, for the second input motion and for bridges B213A and B213B, while in the experiment the two external piers were substructured and a non-linear elastic top force versus displacement curve was used for simulating the piers, in the full numerical analysis the fibre model is used for simulating all three piers;

- finally, while in the numerical calculations the vertical force remained constant, being internally controlled by the algorithm, in the Pseudo-dynamic test it changed slightly with the deformation of the piers. This is illustrated in Figure 6.21.

6.6.3 Numerical results and comparison with the experimental response

The numerical simulations were carried out considering the real dimensions of the bridges and the results were converted to the scale of the piers in the laboratory, (1:2.5), according to the relations presented in Table 3.7.

A resume of the maximum transverse displacements, shear forces and ductility demands computed with the numerical model, is presented in Table 6.2. The mechanical properties of the materials were chosen to fit those of the physical model in the laboratory. The time histories of the piers top displacements and forces are presented in Figures 6.9 to 6.16. The numerical curves for the DE action are quite close to the experimental ones and show similar frequency contents. The agreement is specially notorious for the most important peak values. Generally speaking, the hysteretic curves, horizontal top force versus horizontal top displacement shown in Figures 6.17 to 6.20, confirm the good performance of the numerical model for the four bridges.
For the higher peak ground acceleration, the first time steps follow quite well the experimental results: the same frequencies and the same peak forces and displacements. This agreement proceed fairly well for the whole time history of bridge B232 (see Figures 6.12 and 6.16). However, this was not the case for the irregular bridges. The response curves (see Figures 6.9 to 6.11 and Figures 6.13 to 6.15) show that:

- bridge B213A: the peak displacements and transverse forces are quite similar, even if for the short and tall piers the results between the 1st and the 3rd seconds are hardly comparable. However, this is the time interval for which the peak values were not so important. Furthermore, at the end, the numerical and the experimental responses of the tall pier were clearly out of phase;

Table 6.2: Earthquake demands (preliminary values - numerical)

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>$H$ [m] (Section Type)</th>
<th>Design Earthquake</th>
<th>Shear Force [MN]</th>
<th>Max. Disp. [m]</th>
<th>Ductility</th>
<th>High-Level Earthquake</th>
<th>Shear Force [MN]</th>
<th>Max. Disp. [m]</th>
<th>Ductility</th>
</tr>
</thead>
<tbody>
<tr>
<td>B213A</td>
<td>1</td>
<td>5.6 (1)</td>
<td>.024</td>
<td>1.6</td>
<td>.44</td>
<td>.027</td>
<td>1.8</td>
<td>.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (3)</td>
<td>.026</td>
<td>1.8</td>
<td>1.38</td>
<td>.045</td>
<td>3.1</td>
<td>1.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>.027</td>
<td>&lt;1</td>
<td>2.59</td>
<td>.047</td>
<td>&lt;1</td>
<td>2.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B213B</td>
<td>1</td>
<td>5.6 (1)</td>
<td>.031</td>
<td>2.1</td>
<td>.44</td>
<td>.028</td>
<td>1.9</td>
<td>.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (5)</td>
<td>.026</td>
<td>1.5</td>
<td>2.13</td>
<td>.028</td>
<td>1.6</td>
<td>2.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>.030</td>
<td>&lt;1</td>
<td>.24</td>
<td>.042</td>
<td>&lt;1</td>
<td>.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B213C</td>
<td>1</td>
<td>5.6 (4)</td>
<td>.020</td>
<td>&lt;1</td>
<td>.58</td>
<td>.027</td>
<td>&lt;1</td>
<td>.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (1)</td>
<td>.027</td>
<td>4.2</td>
<td>.98</td>
<td>.056</td>
<td>11.9</td>
<td>1.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (4)</td>
<td>.032</td>
<td>&lt;1</td>
<td>.32</td>
<td>.051</td>
<td>&lt;1</td>
<td>.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B232</td>
<td>1</td>
<td>5.6 (4)</td>
<td>.047</td>
<td>1.9</td>
<td>.75</td>
<td>.078</td>
<td>3.1</td>
<td>.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4 (2)</td>
<td>.065</td>
<td>1.5</td>
<td>.33</td>
<td>.119</td>
<td>2.8</td>
<td>.35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. 2.0 and 1.2 times the design earthquake respectively for bridge B232 and for the irregular bridges (B213A, B213B and B213C).

- bridge B213B: the remarks made to bridge B213A are also addressed here. However, the differences between the numerical and the experimental results are much more evident and also include the medium pier too. At the end, the two responses are also out of phase. The numerical short pier presents a strength approximately 18% higher than the physical pier (see Figure 6.18). This factor changes the equilibrium conditions for the most important negative peak displacement which occurred around 0.9
seconds after the beginning of the test and quite after the yielding point of the physical short pier. Thus, this peak is reached for a displacement level much higher than in the numerical computations and, therefore, the damage of the short pier is considerably underestimated. After this time instant, the differences between both curves are quite important;

- bridge B213C: up to the first 3 seconds, the numerical response of bridge B231C is quite close to the experimental results. Then, the physical short pier collapsed and from the 3rd second up to the end of the test the numerical structure reacted with higher forces for lower peak displacements. The physical structure became much more flexible than the numerical structure and presented lower frequencies of vibration.

In spite of the good agreement between the experimental and the numerical results for bridge B232, the unloading and reloading branches of the global force versus displacement curve of the medium pier illustrated in Figure 6.20 are stiffer than in the experiments; the numerical curve seems to be rotated in relation to the curve found in the Pseudo-dynamic test in the laboratory. This behaviour can possibly be explained by the bond slippage of the longitudinal steel bars that introduces a supplementary rotation at the basement of the pier.

The time histories of the axial forces measured at the top of the three piers of bridge B213C are illustrated in Figure 6.21. These forces present maximum variations of 7% and follow the cyclic horizontal and vertical deformation of the piers.

6.6.4 Conclusions from the predictive analysis

The numerical simulation of the Pseudo-dynamic tests on bridges performed in the ELSA laboratory was presented and the numerical results were compared with the experimental response. A classic fibre formulation was used for modelling the piers. In spite of having considered a linear elastic behaviour for shear, the changes introduced in the steel properties led to quite good results for the design earthquake input motion. The numerical results of the irregular bridges B213 and the regular bridge B232 compared well with the experimental responses. The prediction of the magnitude of the maximum
displacements and forces was quite good.

However, the changes in the steel properties were oriented towards the experimental results available at the ELSA laboratory for the squat pier of bridge B213A and no information exists that would enable the extrapolation of these results to other structures. Nevertheless, this may be a simple way to consider effects such as the bond slippage of steel bars in reinforced concrete structures. The importance of the non-linear shear behaviour is highlighted by the diagonal crack pattern observed during the experiments.

In relation to the higher intensity input action, the results for the short piers of the irregular bridges show ductility demands much lower than in the experiments. This highlights the need to develop other tools to take into account other phenomena, such as the non-linear behaviour in shear, in the numerical analyses. Note that the major differences between the numerical and the experimental results occurred at the short piers with higher reinforcement ratios, i.e. the piers of bridges B213A and B213B.

Concerning the ultimate state, the results illustrated in Figure 6.19 for the short pier of bridge B213C show that failure was not properly taken into account in the numerical model; the damage accumulated during cyclic loading, for instance, was not considered in the behaviour laws of the steel bars. However, it is not easy to predict failure with extreme precision; the materials present variations in the mechanical characteristics and in the position they occupy in the physical structure which can be hardly taken into account in the numerical calculations. To consider these two factors in the model, the real characteristics and position of each steel bar should be known with precision and, therefore, each bar should be represented by one fibre element. In the case of the transverse section illustrated in Figure 6.4, the steel fails when the fibre representing all steel bars placed at the top or at the bottom of the section fail.

A careful calibration of the materials strength and stiffness is also indispensable. This is specially true when trying to catch yielding or failure of a structure. Moreover, for members under dynamic type loading, an incorrect evaluation of the stiffness of the materials may change significantly the results. This seems to be the case of the squat pier of bridge B213B for the second input motion. Notice that R/C members are complex composite
structures which response may be affected by many different causes. Among them there is pre-stressing, bond deterioration and temperature of the concrete, slipping and buckling of the steel bars.

6.7 THE POST-EXPERIMENT NUMERICAL ANALYSES

The need to develop a cyclic model for the non-linear shear behaviour of reinforced concrete elements emerged from the comparison of the previous results with the experimental results. The fibre model used in the predictive analyses was then improved to cover this insufficiency. This new formulation was described in chapter 5.

When the experimental campaign ended, the numerical simulation of the irregular bridges B213A, B213B and B213C, was repeated using the non-linear shear model for the squat piers. The results were compared with either the experimental response or the numerical response with the linear elastic behaviour in shear (and standard steel Young modulus). New mechanical properties based on updated tests on concrete and steel specimens were considered for the materials. Moreover, the concrete constitutive law was slightly modified in relation to the predictive analyses; instead of the simplified model, the improved model described in section 4.3, chapter 4, was used. Therefore, the response of bridge B232 was also re-computed for the new constitutive laws and mechanical properties of the materials.

Before entering into the analysis of the results, the properties of the materials used in this post-experiment numerical campaign are presented.

6.7.1 Material properties and piers finite element mesh

The mechanical properties of the materials were evaluated from tests on concrete and steel samples (see Tables 3.2 and 3.4 in chapter 3). The parameters of the model adopted in the numerical calculations are summarized in Figures 6.22 and 6.23. The parameters adopted in the Giuffré-Pinto model were: \( R_o = 10.0 \), \( a_1 = 9.0 \) and \( a_2 = 0.15 \). A Rayleigh damping matrix calibrated for the same critical damping and the same frequencies in the pre-experimental campaign was adopted.

Along their height, the short piers were divided into five elements of 0.85\( m \) near the
basement and into two elements of 1.375m on the top. The 3.5m high region on the bottom of the medium and tall piers is divided into 3 elements of equal length, being followed by two elements of 1.75m. The remaining part is divided into elements of 3.5m. A more refined mesh was considered near the foundation to simulate the plastic hinges.

Regarding the transverse section, the pier cross sections were divided into unconfined and confined concrete and steel fibres. Each one of the materials was distributed in the cross section as illustrated in Figure 5.21, chapter 5; shear fibres exist only in the short piers of the irregular bridges. The location and number of fibres should be such that it represents properly the transverse section without requiring excessive computation time. Therefore, because the seismic action only deformed the piers in the transverse direction of the bridge, instead of the hollow-core section adopted in the preliminary analyses, an “I” layered transverse section was chosen.

The mechanical properties of the concrete of the shear and longitudinal fibres are the same. The transverse steel ratio for each of the transverse sections is presented in Table 5.3, chapter 5. A cracking angle of 45° was considered at the squat piers of the three irregular bridges. This value is in agreement with the physical state of the piers observed after the experiments and the numerical analyses presented in section 5.5.2, chapter 5. A linear elastic shear model was adopted for the medium and tall piers.

6.7.2 The non-linear shear results - comparison with the experimental response

The response for the three irregular bridges and the regular bridge are presented in Figures 6.24 to 6.31. The numerical results are superimposed on the experimental results. The top force versus displacement response curve of the short piers of the B213 type bridges, using the linear elastic (with the steel standard stiffness) and the non-linear plastic model in shear, are compared in Figure 6.32 and the maximum displacements and shear forces are presented in Tables 6.3 and 6.4.

From the analysis of the results the following aspects are underlined for each of the bridges:

- bridge B213A: the ductility demand at the short pier is higher when the non-linear
shear model is used (see Figure 6.32), meaning, in this case, a response closer to the experimental results. However, no evident benefit rise from the non-linear shear model when compared with the model used in the preliminary analysis with the steel modified Young modulus for the steel (see Figures 6.17 and 6.28);

Table 6.3: DE demands (post-experimental values - linear and non-linear shear model)

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>H[m] (Section Type)</th>
<th>Design Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td>B213A</td>
<td>1</td>
<td>5.6 (1)</td>
<td>.020</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.6 (3)</td>
<td>.021</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.6 (1)</td>
<td>.027</td>
</tr>
<tr>
<td>B213B</td>
<td>1</td>
<td>5.6 (1)</td>
<td>.022</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.6 (1)</td>
<td>.018</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.6 (1)</td>
<td>.027</td>
</tr>
<tr>
<td>B213C</td>
<td>1</td>
<td>5.6 (4)</td>
<td>.016</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.6 (1)</td>
<td>.024</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.6 (4)</td>
<td>.028</td>
</tr>
<tr>
<td>B232</td>
<td>1</td>
<td>5.6 (4)</td>
<td>.039</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.6 (2)</td>
<td>.058</td>
</tr>
</tbody>
</table>

- bridge B213B: again, a higher ductility demand is computed at the short pier when the non-linear shear model was used (see Figure 6.32). Although the response is closer to the experimental results when compared to the linear elastic shear model with the standard stiffness for the steel, the ductility demand is still lower when compared to the results from the preliminary analyses (see Figures 6.18 and 6.29). However, the narrowing of the global force versus displacement curve, denoting a strong non-linear shear effect, is well represented;

- bridge B213C: the differences between the response of the bridge using the linear and the non-linear behaviour in shear illustrated in Figure 6.32 are not significant. Actually, this bridge has the least reinforced short pier, i.e. the pier where the effect of the non-linear behaviour in shear is less important. Although the ductility demand in the piers from the preliminary analyses is closer to the experimental results, the differences are not important and, furthermore, the quality of the results using the new
model is considerably better (see Figures 6.19 and 6.30). As for the global behaviour of the bridge, the remarks made in the preliminary analysis in section 6.6.2 are also addressed in this paragraph;

- bridge B232: the new properties of the materials did not improve the results. Actually, the ductility demand is lower than in the preliminary analysis. Moreover, the unloading stiffness of the medium pier is still much higher than in the experimental results. Nevertheless, the fit between the experimental and the numerical illustrated in Figure 6.31 results is still very good.

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>H [m] (Section Type)</th>
<th>Linear Shear Behaviour</th>
<th>Non-Linear Shear Behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>B213A</td>
<td>1</td>
<td>5.6 (1)</td>
<td>.021</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (3)</td>
<td>.035</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>.037</td>
<td>&lt;1</td>
</tr>
<tr>
<td>B213B</td>
<td>1</td>
<td>5.6 (1)</td>
<td>.024</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (5)</td>
<td>.021</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (1)</td>
<td>.030</td>
<td>&lt;1</td>
</tr>
<tr>
<td>B213C</td>
<td>1</td>
<td>5.6 (4)</td>
<td>.024</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8 (1)</td>
<td>.046</td>
<td>9.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4 (4)</td>
<td>.040</td>
<td>&lt;1</td>
</tr>
<tr>
<td>B232</td>
<td>1</td>
<td>5.6 (4)</td>
<td>.073</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4 (2)</td>
<td>.110</td>
<td>3.0</td>
</tr>
</tbody>
</table>

a. 2.0 and 1.2 times the design earthquake respectively for bridge B232 and for the irregular bridges (B213A, B213B and B213C).

Finally, although the quality of the results is considered to be quite good, the maximum values of the transverse top displacement for the higher intensity input action and the two short piers with higher longitudinal steel ratio, are clearly lower than the experimental ones.

### 6.8 A NEW NUMERICAL CAMPAIGN

As shown above, there are still differences between the experimental and the numerical
response using the new algorithm for the higher intensity seismic action that demanded further investigation. The consideration of the non-linear shear deformation did not explain the main differences found in the peak values of the squat piers. Since there are still aspects which are not equally taken into account in the two analyses, a new set of numerical tests was performed in order to find suitable justification for these differences.

6.8.1 Comparison between the two integration algorithms

A different integration algorithm was used in the Pseudo-dynamic tests in the laboratory and in the full numerical analyses: the $\alpha$-Newmark method with the Operator Splitting scheme and the standard Newmark method, respectively. Although no differences were expected from the use of different integration algorithms, to check the influence this could have in the response, bridge B213B, where the main differences between the numerical and the experimental results were found, was submitted to the higher intensity seismic action using both algorithms in a full numerical basis.

The numerical test with the Pseudo-dynamic algorithm followed the procedures in the laboratory. The deck was simulated apart from the piers and the two processes ran side by side, sending the information one to the other as in a real Pseudo-dynamic test with substructuring. The mass of the piers was condensed at the connection points with the deck. The elastic stiffness that the algorithm needs for solving the problem (see the details of the method in annex C) is the same used in the experiments: 46.5MN/m, 180.0MN/m and 15.1MN/m for the medium, short and tall piers, respectively. These values correspond to the piers transverse stiffness condensed at their top nodes and are similar to the elastic stiffness of the numerical piers after the first seismic action. To simulate the conditions in the experiment, the three piers were previously damaged by a two cyclic static action that imposed the maximum displacements measured in the experiment during the first seismic loading. This last procedure was also followed when the standard Newmark algorithm was used.

At the beginning of each time step, the $\alpha$-Newmark method with the Operator Splitting scheme predicts the displacements on the structure and impose them on the deck and on the piers top nodes. The resulting forces are computed through the fibre model; a linear elastic law was used for the deck and the non-linear plastic model of the previous analy-
sis was chosen for the piers. Furthermore, the squat pier was simulated through the non-linear shear behaviour model. Since in the Pseudo-dynamic test only the piers top nodes are controlled, the transverse elastic stiffness and the transverse forces are added to the deck and the acceleration vector is solved. The displacements and velocities are corrected and a new prediction is made for the displacements at the next time step. This is illustrated in the scheme drawn in Figure 6.33 and is in agreement with the substructuring technique described in section 2.6.

This simulation was compared with the numerical results performed with the Newmark method. As expected, the results using both algorithms are identical, being represented by the continuous line in the three first diagrams in Figure 6.34. Thus, the use of a different integration algorithm did not explain the differences found in the previous sections between the experimental and the numerical results.

6.8.2 The influence of the behaviour model

The numerical test with the Operator Splitting algorithm was then repeated for the higher intensity seismic action and still adopting the non-linear elastic laws used in the experiments for the two external piers. This second test checked the sensitivity of the results to the constitutive laws adopted in the laboratory for the tall and medium piers.

Again, the procedure described in the previous section was followed. However, instead of computing the force at the top of the three piers through the fibre model, now the forces at the tall and medium piers are given by the non-linear elastic laws adopted in the experiment. The comparison with the results in section 6.8.2, illustrated in the upper diagrams in Figure 6.34, shows that although the response is not the same, the differences are not important. Thus, and although a more flexible bridge is found when the non-linear elastic laws are chosen, this did not explain the main differences found between the experimental and the numerical results.

6.8.3 New material properties

Until now two possible explanations for the discrepancies observed between the numerical and the experimental results were excluded: the integration algorithm and the numerical simulation of the external piers by non-linear elastic behaviour laws. This left only
one possibility: the constitutive laws, or the non-linear shear model, did not represent properly the behaviour of the squat pier. However, the numerical results in section 5.6, chapter 5, showed that there was a good agreement between the experimental and the numerical results when the non-linear shear model was adopted. Note that the numerical results in that chapter were not computed from dynamic tests but correspond to the static application of the displacements measured during the Pseudo-dynamic tests on the bridge piers.

The results illustrated in Figure 6.29 show that the strength of the squat pier in the numerical analyses is higher than in the experiment. Furthermore, in spite of the non-linear shear model, the post cracking branch of the numerical pier is stiffer than in the experiment. To check the influence of these two parameters, two last numerical tests were performed using the Pseudo-dynamic integration algorithm and the non-linear elastic behaviour laws for the two external piers.

In the first test the strength of the concrete and of the steel was decreased to 91% of its initial value. This is in agreement with the experimental results. After this, a new calculation was performed considering not only the reduced strength but also half of the initial elastic stiffness of the longitudinal steel. The two results and the comparison with the case solved in section 6.8.2, are illustrated in the lower diagrams in Figure 6.34 and in the upper diagrams in Figure 6.35, respectively. Figure 6.35 superimposes the results from this second test on the experimental response. The results show that:

- firstly, using the strength measured at the squat pier in the experiment did not change substantially the numerical response. Although the displacements at this pier are higher than before, the differences between the numerical and the experimental results were still important;

- secondly, the decrease of the longitudinal steel stiffness modified significantly the numerical response. The displacements became quite close to the experimental ones.

As referred to in section 6.5.4, a more flexible steel produces a more flexible response after cracking. This increase of the curvature, that occurs mainly in cracked cross-sections, has a global effect similar to bond slippage, i.e. it makes the post-cracking stiffness
to decrease. From this analysis, and based on the previous results, it may be that it is not the non-linear shear model that does not simulate properly the behaviour of the squat pier but that there are other phenomena that should be also included in the formulation.

### 6.8.4 Analysis of the results

A resume of the maximum displacements and shear forces is presented in Table 6.5. Several important results arose from this supplementary analysis. Firstly, although a different numerical integration algorithm was used for simulating the experimental campaign, this did not change the results. Secondly, no important differences are found in the results when the two external piers of bridge B213B are substituted by the non-linear elastic laws adopted in the experiment.

However, it seems that a supplementary displacement exists at the top of the squat pier caused by the bond slippage of the steel bars at the bottom that decreases the post-cracking stiffness. The results also show that the response of the bridge at the level of the squat pier is very sensitive to this stiffness; differences of 37% were found in the displacements of the squat pier when the steel stiffness was modified.

<table>
<thead>
<tr>
<th>Piers Simulated with the fibre model</th>
<th>c-Newmark Method coupled with the OS scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum Displacements [m]</td>
</tr>
<tr>
<td></td>
<td>P1</td>
</tr>
<tr>
<td>P1, P2, P3</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>-2.44</td>
</tr>
<tr>
<td>P2</td>
<td>2.73</td>
</tr>
<tr>
<td></td>
<td>-2.43</td>
</tr>
<tr>
<td>P2 (Reduced Strength)</td>
<td>2.75</td>
</tr>
<tr>
<td></td>
<td>-2.48</td>
</tr>
<tr>
<td>P2 (Reduced, Strength and Stiff.)</td>
<td>2.65</td>
</tr>
<tr>
<td></td>
<td>-2.46</td>
</tr>
<tr>
<td>Experimental Results</td>
<td>2.79</td>
</tr>
<tr>
<td></td>
<td>-2.61</td>
</tr>
</tbody>
</table>

### 6.9 CONCLUSIONS

The numerical simulation of a set of Pseudo-dynamic tests performed in the ELSA labo-
ratory was presented. They were divided into three parts: the prediction, the post-experimental and the checking analyses. A fibre model was used to model the piers. In the predictive analysis, and in spite of having considered a linear elastic behaviour in shear, the modifications introduced in the steel properties led to quite good results for the design earthquake input motion. Note that this can be an easy way to consider the bond slippage of the longitudinal steel bars in reinforced concrete structures.

The numerical responses of the irregular bridges B213 and the regular bridge B232 compare well with the experimental results. The prediction of the magnitude of the maximum displacements and forces is in general quite good.

In relation to the higher intensity input action, the results for the short piers of the irregular bridges show ductility demands much lower than in the experiments. The main differences between the numerical and the experimental results are found in the short piers with higher reinforcement ratio, i.e. the piers within bridges B213A and B213B.

After the tests, the numerical campaign was repeated using the fibre model with the non-linear behaviour in shear. The improvements in the response were not enough to explain the differences between the experimental and the numerical response when the linear elastic behaviour in shear was considered.

A third campaign was launched on bridge B213B to try to explain these differences. Several results came out from this analysis. Firstly, although a different numerical integration algorithm was used for simulating the experimental campaign, this did not change the results. Secondly, no important differences were found in the results when the two external piers of bridge B213B under the HLE were substituted by the non-linear elastic laws adopted in the experiment. Finally, it seems that a supplementary rotation of the pier, possibly near the foundation, modifies the stiffness of the pier changing deeply the results. This flexibility could be explained by a bond slippage of the steel bars. The results show that the response of the bridge was very sensitive to the post-cracking stiffness of the short pier.

A careful calibration of the materials strength and stiffness is, therefore, indispensable. The numerical campaign illustrates the importance of simulating properly phenomena.
that may modify the stiffness of the structure. That is particularly relevant when dealing with the dynamic behaviour of bridges, since the global response is mainly controlled by a restrict number of structural elements: the piers. In this case, the response of the bridge may depend deeply on the stiffness of these elements. For bridge B213B and the high level intensity input action, this was true even when there was a good fitting between the analytical and the experimental strength.

Finally, concerning the ultimate state, the results for the shortest pier of bridge B213C show that failure is not properly considered in the numerical model; the damage accumulated during cyclic loading, for instance, was not considered in the behaviour laws of the steel bars. Nevertheless, it is quite difficult to predict failure with extreme precision; several reasons can be pointed out, namely: the materials present variations in the mechanical characteristics and in the position they occupy in the physical structure which can be hardly taken into account in the numerical calculations. To consider these factors in the model, the real characteristics and position of each steel bar should be known with extreme precision and one fibre should be used for representing each of these bars.
6.10 FIGURES

Figure 6.1 - Numerical analysis assumptions

<table>
<thead>
<tr>
<th>Piers</th>
<th>$f_{co}$ [MPa]</th>
<th>$f_{co}^*$ [MPa]</th>
<th>$\varepsilon_{co}$ [%]</th>
<th>$\varepsilon_{co}^*$ [%]</th>
<th>$Z_*$</th>
<th>$Z^*$</th>
<th>$f_t$ [MPa]</th>
<th>$\varepsilon_t$ [%]</th>
<th>$\varepsilon_{tm}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>25.0</td>
<td>26.8</td>
<td>0.20</td>
<td>0.23</td>
<td>100.0</td>
<td>47.1</td>
<td>2.5</td>
<td>0.012</td>
<td>0.060</td>
</tr>
<tr>
<td>Phase 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.2 - Confined and unconfined concrete constitutive laws and model parameters for the piers

<table>
<thead>
<tr>
<th>Piers</th>
<th>$E_0$ [GPa]</th>
<th>$f_{sy}$ [MPa]</th>
<th>$f_{u1}$ [MPa]</th>
<th>$\varepsilon_{sh}^*$ [%]</th>
<th>$\varepsilon_{u1}$ [%]</th>
<th>$E_h/E_o$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium and Tall (Short - phase 1)</td>
<td>206.0</td>
<td>500.0</td>
<td>625.0</td>
<td>2.30</td>
<td>9.80</td>
<td>1.16</td>
</tr>
<tr>
<td>Short - phase 2</td>
<td>103.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.3 - Numerical model for the steel under cyclic loading
Figure 6.4 - Distribution of the fibres in the piers and deck cross-section

Figure 6.5 - 3D representation of the pier and block foundation with cubic finite elements
Figure 6.6 - Top force-displacement curve with and without flexible foundation

Figure 6.7 - Numerical simulation of the flexible foundation
Figure 6.8 - Experimental and numerical results for the squat pier (displacements at 1.70m high)
Figure 6.9 - Analytical and experimental time histories of the top piers displacement for bridge B213A (preliminary results)
Figure 6.10 - Analytical and experimental time histories of the top piers displacement for bridge B213B (preliminary results)
Figure 6.11 - Analytical and experimental time histories of the top piers displacement for bridge B213C (preliminary results)
Figure 6.12 - Analytical and experimental time histories of the top piers displacement for bridge B232 (preliminary results)
Figure 6.13 - Time histories of the force at the top of the piers for bridge B213A
Figure 6.14 - Time histories of the force at the top of the piers for bridge B213B
Figure 6.15 - Time histories of the force at the top of the piers for bridge B213C
Figure 6.16 - Time histories of the force at the top of the piers for bridge B232 (regular)
Figure 6.17 - Force-displacement diagrams for bridge B213A (preliminary results)
Figure 6.18 - Force-displacement diagrams for Bridge B213B (preliminary results)
Figure 6.19 - Force-displacement diagrams for bridge B213C (preliminary results)
Figure 6.20 - Force-displacement diagrams for bridge B232 (preliminary results)

TALL PIER

<table>
<thead>
<tr>
<th>Force [MN]</th>
<th>Displ. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

MEDIAN PIER

<table>
<thead>
<tr>
<th>Force [MN]</th>
<th>Displ. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

2.0 x DE

1.0 x DE

Bridge B232 (Regular)
Figure 6.21 - Vertical forces in the 1.2xDE Pseudo-dynamic test for the three piers of bridge B213C
<table>
<thead>
<tr>
<th>Piers</th>
<th>$f_{co}$ [MPa]</th>
<th>$f_{co*}$ [MPa]</th>
<th>$\varepsilon_{co}$ [%]</th>
<th>$\varepsilon_{co*}$ [%]</th>
<th>$Z$</th>
<th>$Z^*$</th>
<th>$f_I$ [MPa]</th>
<th>$\varepsilon_{im}/\varepsilon_t^A$</th>
<th>$\varepsilon_{im}/\varepsilon_t^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>36.2</td>
<td>45.6</td>
<td>0.25</td>
<td>0.40</td>
<td>100</td>
<td>27.7</td>
<td>1.6</td>
<td>2.0</td>
<td>10</td>
</tr>
<tr>
<td>S2</td>
<td>43.5</td>
<td>55.5</td>
<td>0.25</td>
<td>0.41</td>
<td>100</td>
<td>29.1</td>
<td>1.6</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>S3</td>
<td>32.0</td>
<td>40.2</td>
<td>0.25</td>
<td>0.39</td>
<td>100</td>
<td>28.3</td>
<td>1.6</td>
<td>7</td>
<td>100</td>
</tr>
<tr>
<td>S4</td>
<td>39.0</td>
<td>47.8</td>
<td>0.25</td>
<td>0.38</td>
<td>100</td>
<td>35.0</td>
<td>1.6</td>
<td>7</td>
<td>100</td>
</tr>
<tr>
<td>S5</td>
<td>35.2</td>
<td>43.3</td>
<td>0.25</td>
<td>0.38</td>
<td>100</td>
<td>30.3</td>
<td>0.1</td>
<td>5</td>
<td>100</td>
</tr>
</tbody>
</table>

a. Longitudinal fibres
b. Shear fibres
c. Short pier

Figure 6.22 - Confined and unconfined concrete characteristics

<table>
<thead>
<tr>
<th>Piers</th>
<th>$f_{sy}$ [MPa]</th>
<th>$f_{ul}$ [MPa]</th>
<th>$\varepsilon_{sh}$ [%]</th>
<th>$\varepsilon_{ul}$ [%]</th>
<th>$E_o$ [GPa]</th>
<th>$E_f/E_{ol}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Long. Fibres</td>
<td>490.0</td>
<td>570.0</td>
<td>2.00</td>
<td>14.5</td>
<td>206.0</td>
</tr>
<tr>
<td></td>
<td>Shear Fibres</td>
<td>700.0</td>
<td>730.0</td>
<td>0.34</td>
<td>1.6</td>
<td>206.0</td>
</tr>
<tr>
<td>S2</td>
<td>Long. Fibres</td>
<td>490.0</td>
<td>570.0</td>
<td>2.00</td>
<td>14.5</td>
<td>206.0</td>
</tr>
<tr>
<td></td>
<td>Shear Fibres</td>
<td>700.0</td>
<td>730.0</td>
<td>0.34</td>
<td>1.6</td>
<td>206.0</td>
</tr>
<tr>
<td>S3</td>
<td>Long. Fibres</td>
<td>540.0</td>
<td>630.0</td>
<td>2.00</td>
<td>13.2</td>
<td>206.0</td>
</tr>
<tr>
<td></td>
<td>Shear Fibres</td>
<td>700.0</td>
<td>730.0</td>
<td>0.34</td>
<td>1.6</td>
<td>206.0</td>
</tr>
<tr>
<td>S4</td>
<td>Long. Fibres</td>
<td>495.0</td>
<td>595.0</td>
<td>2.00</td>
<td>13.0</td>
<td>206.0</td>
</tr>
<tr>
<td></td>
<td>Shear Fibres</td>
<td>365.0</td>
<td>430.0</td>
<td>2.00</td>
<td>15.0</td>
<td>206.0</td>
</tr>
<tr>
<td>S5</td>
<td>Long. Fibres</td>
<td>480.0</td>
<td>580.0</td>
<td>2.00</td>
<td>13.0</td>
<td>206.0</td>
</tr>
<tr>
<td></td>
<td>Shear Fibres</td>
<td>365.0</td>
<td>430.0</td>
<td>2.00</td>
<td>15.0</td>
<td>206.0</td>
</tr>
</tbody>
</table>

Figure 6.23 - Longitudinal and transverse steel characteristics
Figure 6.24 - Analytical and experimental time histories of the top piers displacement for bridge B213A using the non-linear shear model.
Figure 6.25 - Analytical and experimental time histories of the top piers displacement for bridge B213B using the non-linear shear model
Figure 6.26 - Analytical and experimental time histories of the top piers displacement for bridge B213C using the non-linear shear model
Figure 6.27 - Analytical and experimental time histories of the top piers displacement for bridge B232 (regular). New materials and the improved model for the concrete.
Figure 6.29 - Force-displacement diagrams for bridge B213B
Bridge B213C  (Shear Non-Linear)

Figure 6.30 - Force-displacement diagrams for bridge B213C

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- MEDIUM PIER
- SHORT PIER
- TALL PIER

Displ. [m] vs Force [MN] for different pier types.
Figure 6.31 - Force-displacement diagrams for bridge B232 (regular)

- Experimental
- Analytical

Bridge B232 (New Model)
Figure 6.32 - Force-displacement diagrams for bridge B213A

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non-linear shear model

linear shear model
Figure 6.3 - Numerical simulation of the Q-Newton method with the Operator Splitting (OS) scheme - data exchange

S - Deck / I - Piers / C - Interface nodes

Make $i + 1$ and loop again

Compute $p_{S}$ and $p_{S}$

Solve $p_{S}$

Compute $p_{S}$

Add $TC - l + 1 + i$

Compute $p_{S}$ (through the fibre model)

Add $TC - l + 1 + i$

Compute $p_{S}$ (predicted values)

Time loop, index $i$

Add $TC - l + 1 + i$

Compute $p_{S}$ (supposed constraint during the test)

Precondition process (Piers)

Paratel process (Piers)

Initialize $W$

Condense $W$ on the common dofs.

Extract $p_{S}$ from the piers top node

Impose $p_{S}$ + $p_{C}$ - $p_{C}$ + $p_{C}$ via the fibre model

Compute the forces $TC - l + 1$

Equivalent to the displacements $TC - l + 1$. 
Figure 6.34 - Analytical and experimental time histories of the top piers displacement for bridge B213A using the non-linear shear model

NUMERICAL CAMPAIGN
Figure 6.35 - Analytical and experimental time histories of the top piers displacement for bridge B213B using the non-linear shear model.
7 NUMERICAL ANALYSIS OF A NEW SET OF BRIDGES

7.1 INTRODUCTION

The irregularity of bridges is a topic of great interest to civil engineering, particularly when seismic behaviour is involved. In general, design codes assume that elements with similar functions under the same loading conditions, shall present similar demands. If this assumption is not verified, other rules have to be applied to the structural elements; the design codes usually provide guide-lines to deal with these situations. However, this presumes that the designer is able to select the structures that can or cannot be designed according to certain rules, i.e. that the design code also provides the necessary elements for distinguishing the structures. In the case of bridges, this means that the design code should allow the designer distinguish a regular bridge from an irregular bridge.

Calvi et al. [8] proposed an empirical expression to quantify the irregularity of a bridge through a single parameter which accounts for the correlation between the mode shapes of the deck alone and of the deck with the piers. Although this parameter still needs to be improved, it can be a first attempt to quantify structural irregularity.

Therefore, a new set of bridges was selected to be tested using the fibre model described in chapter 4. The bridges present profiles that are in between the so-called irregular configuration of bridge B213 and the regular configuration of bridge B232. The purpose of this numerical campaign is to help understand how the irregular profile B213 develops into the regular profile B232. These bridges correspond to different levels of structural regularity and are submitted to the accelerogram used in the experiments in the ELSA laboratory at Ispra (see Figure 3.15). In this context, the regularity issue only concerns
the behaviour of the bridges in the horizontal transverse direction of the deck. To control the results and be able to extract conclusions, all the bridges were designed following the eurocode n. 8 [22] criteria.

The chapter is divided into five sections. In section 7.2 the design of bridges B213 and B232 is judged under the EC8 ([21], [22]) rules and the results are set against the structures that were tested in the laboratory. This allowed us verify the design conditions and, at the same time, to describe the EC8 design criteria. Then, a behaviour factor equal to 2.5 was chosen and the new set of bridges was designed, or re-designed, in section 7.3. A growing intensity factor for the accelerogram used in the experimental campaign was applied to the bridges. The transverse force versus displacement response curves at the top of the piers are analysed in section 7.4. Two different minimum steel ratios were adopted in the design. Furthermore, both the average and the characteristic properties of the materials were considered for each bridge. The results are analysed in section 7.5 and the figures with the main results are illustrated in section 7.6.

7.2 DESIGN OF THE PREC8 BRIDGES

The regular bridge B232 and the reference irregular bridge B213A referred to in the previous chapters were designed according to the EC8 rules. The design action corresponds to the EC8 response spectrum for soil type B and 5% critical damping. Since the same criteria were used in the design of the new set of bridges, and the behaviour factors chosen for the two bridges tested in the laboratory still needed to be verified, the EC8 design rules for bridges are described using the two bridges. All the analyses and results presented in this chapter concern the (1:2.5) scale models.

The response spectrum analysis is considered in the bridges design. According to the EC8, this corresponds to a linear elastic analysis of the peak dynamic responses of all significant modes of the structure using the ordinates of the site-dependent design spectrum. The overall response is obtained by combination of the maximum modal contribution of all the significant modes of the structure. This condition is satisfied if the sum of the effective modal masses for the modes amounts at least to 90% of the total mass of the structure. This ratio is referred to by $\rho_m$. 

304 NUMERICAL ANALYSIS OF A NEW SET OF BRIDGES
To determine the probable maximum value $S$ of an action effect, the complete quadratic combination method is used,

$$ S = \sqrt[n]{\sum_{i=1}^{n} \sum_{j=1}^{n} S_i \cdot r_{ij} \cdot S_j} $$  

7.1

where $S_i$ is the modal response of mode $i$, and $r_{ij}$ is the correlation factor between the $n$ considered modes,

$$ r_{ij} = \frac{8 \cdot \xi^2 \cdot (1 + \rho) \cdot \rho^{3/2}}{(1 - \rho^2)^2 + 4 \cdot \xi^2 \cdot \rho \cdot (1 + \rho)^2} $$  

7.2

$\xi$ being the viscous damping ratio and ($\rho = T_j / T_i$) the fraction of the natural periods of modes $i$ and $j$, with ($T_j \leq T_i$).

The bridges were designed to present a ductile behaviour, i.e. a behaviour factor $q$ between 1.5 and 3.0. In this case, and according to the eurocode n. 8, the effective stiffness for the ductile components, the piers, must be estimated on the basis of the secant stiffness at the theoretical yield point of the plastic joint. Following the first method of annex C of the EC8, the effective moment of inertia $J_{eff}$ of a pier of constant cross section can be estimated through the combination of the moment of inertia of the uncracked cross-section $J_{un}$ and the estimated moment of inertia of the cracked section $J_{cr}$,

$$ J_{eff} = 0.08 \cdot J_{un} + J_{cr} $$  

7.3

This expression derives from parametric analysis of simplified non-linear models of cantilever piers with hollow rectangular and hollow and solid circular cross-sections [22]. The value $J_{cr}$ is estimated through the expression

$$ J_{cr} = M_y / (E_c \cdot c_y) $$  

7.4

where $M_y$ and $c_y$ correspond to the yielding moment and curvature of the transverse section, and $E_c$ to the elastic modulus of the concrete. However, this expression requires prior estimation of the longitudinal steel ratio to access the yielding point.

For the (1:2.5) scaled piers, the effective moment of inertia was estimated to be about
25% of the uncracked moment of inertia, \((J_{\text{un}} = 0.1892m^4)\), i.e. \((J_{\text{eff}} = 0.0473m^4)\). According to equation (7.3), it corresponds to a moment of inertia of the cracked section \((J_{\text{cr}} = 0.0322m^4)\) which is close to the values found for the cross-sections built in the laboratory. Nevertheless, and after designing the piers, this value should be verified and, if necessary, modified and the analysis should be repeated.

The characteristic compression strength of the concrete is \((f_{ck} = 25MPa)\). This is the strength below which 5% of all possible strength test results for the specified concrete may fall; it corresponds to a Young modulus \((E_c = 30.5GPa)\).

The mass of the structure is mainly due to the mass of the deck. However, the mass of the piers was also included in the analysis; half of the total mass of the piers was added to the mass of the deck concentrated at the connection points. The cross-sectional area of the deck and piers is \(1.1135m^2\) and \(0.6656m^2\), respectively. The density of the concrete is \((\rho_c = 2.5ton/m^3)\), giving a total mass of 222.7ton for the deck. The piers have a mass per unit length equal to 1.664ton/m. This corresponds to condensed masses at the top of the piers equal to 2.2ton, 1.1ton and 3.3ton, for the medium, short and tall piers, respectively.

Knowing the geometry, the stiffness and the mass of the structure, the maximum shear forces, flexural moments and piers top displacements can be computed through the response spectrum analysis. In this case the seismic action corresponds to the EC8 response spectrum for soil type B and 5% critical damping applied in the horizontal transverse direction of the bridge. The participation factor of each mode \(i\) is given by

\[
P_i = \frac{\bar{\Phi}_i^l \cdot [M] \cdot \bar{r}}{\bar{\Phi}_i \cdot [M] \cdot \bar{\Phi}_i} \tag{7.5}
\]

where \(\bar{\Phi}_i\) is the mode shape vector \(i\), \([M]\) is the mass matrix and \(\bar{r}\) is a transformation vector that expresses the displacement of each degree of freedom of the structure due to the static application of a unit support displacement \([15]\). For the chosen action, it is a vector with the value 1.0 in the degrees of freedom that correspond to the direction of the seismic action and zero in all the others. The results for each of the mode shapes of the (1:2.5) scale structure that participate actively in the response are presented in Table 7.1.
The selected modes are those which have an effective modal mass ratio superior to 0.1%. Moreover, the number of mode shapes must be such that the sum of the effective modal masses represents at least 90% of the total mass

\[ \sum (P_j)^2 \cdot (\ddot{\phi}_j \cdot [M] \cdot \ddot{\phi}_j) \geq 0.90 \cdot (\ddot{r} \cdot [M] \cdot \ddot{r}) \]  

Three mode shapes were considered for the two bridges (see Table 7.1). The displacements in Table 7.1 correspond to the values of the EC8 response spectrum converted to the scale of the model, i.e. with frequencies and accelerations multiplied by the scale factor. The accelerogram and the corresponding EC8 acceleration response spectrum are illustrated in Figure 3.15.

To compute the mode shapes the bridges were simulated with beam elements with the geometry of the concrete. The moment of inertia of the piers in the transverse direction of the bridge is the effective moment \( J_{eff} \) already referred.

Note that the transverse mode shape with frequency 12.98Hz that deforms the deck of bridge B213 with two inflection points, since it has a modal mass inferior to 0.1% of the total mass it was not considered. Likewise, the mode shape with frequency 5.77Hz that deforms the deck of bridge B232 with one inflection point, being anti-symmetric, presents a participation factor equal to zero and it was not considered either.

The bridges linear elastic peak dynamic response is calculated through equation (7.1) and using the values in Table 7.1

<table>
<thead>
<tr>
<th>Mode</th>
<th>B213</th>
<th>B232</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.24</td>
<td>.793</td>
</tr>
<tr>
<td>2</td>
<td>6.01</td>
<td>1.460</td>
</tr>
<tr>
<td>3</td>
<td>29.72</td>
<td>.287</td>
</tr>
</tbody>
</table>

\( \rho_m = 92.9\% \)  
\( \rho_m = 92.8\% \)
\[ S_t = P_i \cdot \bar{\phi}_i \cdot X_i \] 7.7

where \( X_i \) is the value of the displacement, or the corresponding force, in the response spectrum in agreement with the frequency of mode shape \( i \). The flexural moments and shear forces at the basement of the piers and the horizontal displacements at the top are presented in Table 7.2.

A normalized axial force \((v = 0, 1)\) was considered on the top of the piers. It corresponds to the vertical force \((P = 1.72MN)\) from the design seismic combination

\[ S_d = G_k + E + \psi_{21} \cdot Q_{2k} \] 7.8

where \( G_k \) is the characteristic value of the permanent loads, \( E \) is the most unfavorable combination of the components of the earthquake and \( \psi_{21} \) is the combination factor for the characteristic value of the traffic load, \( Q_{2k} \).

**Table 7.2: Design base shear forces, base flexural moments and top displacements of the piers**

<table>
<thead>
<tr>
<th>Pier</th>
<th>B213A</th>
<th>B232</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>2.91</td>
<td>.52</td>
</tr>
<tr>
<td>b2</td>
<td>6.50</td>
<td>2.32</td>
</tr>
<tr>
<td>c3</td>
<td>1.16</td>
<td>.14</td>
</tr>
</tbody>
</table>

a. Medium pier for both bridges  
b. Short pier for bridge B213A and tall pier for bridge B232  
c. Tall pier for bridge B213A

The moment versus curvature diagrams illustrated in Figure 7.1 correspond to the design strength of the piers cross-section tested in the ELSA laboratory at Ispra within the PREC8 programme. The characteristic strength of the concrete and the steel bars is \((f_{ck} = 25MPa)\) and \((f_{sk} = 500MPa)\), with material safety factors of 1.5 and 1.15, respectively. This corresponds to design strengths, i.e. characteristic values divided by safety factors, \((f_{cd} = 16.7MPa)\) and \((f_{sd} = 434.8MPa)\). These curves were obtained adopting for the numerical behaviour of the concrete a parabola with a peak strain \((\varepsilon_{co} = 0.002)\) followed by a plateau. The coefficient of Poisson is \((v = 0.2)\). The steel is elastic perfectly plastic with a Young modulus \((E_s = 200GPa)\).
Figure 7.1 - Design flexural moment versus curvature diagrams for the piers transverse sections

The ratio of the design action in Table 7.2 to the design strength of the transverse sections tested in the laboratory illustrated in Figure 7.1, gives the behaviour factors that were probably assumed in the design of the piers. Hence, the piers of the regular bridge exhibit a behaviour factor \( q \geq 1.9 \). Instead, the short, medium and tall piers of bridge B213 exhibit behaviour factors \( (q = 2.5) \), \( (q = 1.7) \) and \( (q < 1) \), respectively. However, the longitudinal steel ratio of these two piers is below the minimum value fixed by the code; although some countries situated away from seismic areas allow minimum steel ratios that can go down to 0.3%, the longitudinal steel ratio of the two extreme piers of the irregular bridge, 0.5%, is already below the minimum fixed by the EC8: 0.8% of the total area of the concrete transverse section.

This analysis shows that, while in the case of the regular bridge the behaviour factors at the level of each pier correspond to an effective behaviour factor of the whole structure, when dealing with bridge B213A, since each pier is described by a different behaviour factor, this extrapolation is not so clear. However, if a unique behaviour factor has to be
attributed to the bridge, the highest value found at the different components of the structure should be chosen.

In this analysis, a linear elastic behaviour law was adopted for the deck, i.e. the seismic action was considered not to induce plastic deformations in the superstructure. The Young modulus of the deck is equal to the Young modulus of the concrete, \( E_c = 30.5 \text{GPa} \).

7.3 DESIGN OF THE NEW SET OF BRIDGES

Following the procedures described in section 7.2, four other bridges were designed for a behaviour factor \( q = 2.5 \) using the profile of bridge B213 as starting point and choosing growing heights for the central pier: 4.2\( m \), 5.6\( m \), 7.0\( m \) and 8.4\( m \). Then, with the medium pier at one extremity and the tall pier at the centre, the tall pier at the other extremity was shortened to 7.0\( m \) as illustrated in Figure 7.2. These 5 new bridges are labelled to by code names identical to those used for bridges B213 and B232: a "B" followed by a number equal to the number of times each pier is higher than the 2.8\( m \) high pier: B2(1.5)3, B223, B2(2.5)3, B233, B23(2.5).

The purpose of these 5 new bridges in between the so-called irregular and regular profiles, is to help understand how the irregular profile B213 develops into the regular profile B232. These bridges were designed for the EC8 response spectrum for soil type B and 5% critical damping. After they were submitted to the accelerogram used in the experiments in the ELSA laboratory at Ispra (see Figure 3.15) applied in the horizontal transverse direction of the deck.

7.3.1 The regularity parameter

The regularity is quantified by the parameter \( R \) proposed by Calvi et al. [8]. From equation (6.1) detailed in section 6.3.1, it is apparent that \( R \) ranges from zero to 1.0 as the bridge goes from a configuration with mode shapes far from those of the deck without the piers, to a configuration where the first \( n \) mode shapes are identical to those of the deck alone. According to Calvi, a bridge is close to the regular configuration if \( R \) is close to 1.0. In the next paragraphs the words deck and bridge refer to the deck without the piers and with the piers, respectively.
Figure 7.2 - Bridges profiles
Notice that equation (6.1) only makes sense if the mode shapes of the two structures are, somehow, comparable or correspond to similar situations; to combine a symmetric mode with an anti-symmetric mode seems to be senseless. To simplify this analysis and, at the same time, to include only the modes that contribute to important horizontal transverse displacements at the level of the piers, the stiffness and the mass of the deck and bridge are condensed at the top nodes of the piers in the transverse direction of the deck. A three d.o.f. system is obtained.

It is however important to note that in equation (6.1) the first mode of the deck should be combined with the first mode of the bridge and not the second one. In fact, the result can be quite different, as shown in Table 7.3. Two computations made with bridge B213 are presented in the table. In the first one (case 1) the modes are combined as they appear, i.e. the first mode of the deck with the first mode of the bridge, the second with the second and the third with the third. In the second computation (case 2), the first mode of the deck changes with the second mode. The two values ($R = 0.62$) and ($R = 0.85$), quite different from each other, show that although the parameter $R$ is used in the analyses, a more consistent expression should be found to define regularity of a bridge.

The values of the regularity parameters for the seven bridges are presented in Table 7.4.

### Table 7.3: Mode shapes and regularity parameters (see equation (6.1))

<table>
<thead>
<tr>
<th>Node</th>
<th>Deck</th>
<th>Deck + Piers (bridge B213)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Phi_1$</td>
<td>$\Phi_2$</td>
</tr>
<tr>
<td>P1</td>
<td>2.122</td>
<td>-3.067</td>
</tr>
<tr>
<td>P2</td>
<td>3.002</td>
<td>0.000</td>
</tr>
<tr>
<td>P3</td>
<td>2.122</td>
<td>3.067</td>
</tr>
</tbody>
</table>

$\Phi_j[M]\Psi_j\rightarrow$

<table>
<thead>
<tr>
<th>$\Phi_j[M]\Psi_j\rightarrow$</th>
<th>.401</th>
<th>.465</th>
<th>.877</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 $\rightarrow R = .618$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2 $\rightarrow R = .849$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Mode shapes normalized to the mass matrix, i.e. ($\Phi_j[M]\Phi_j = \Psi_j[M]\Psi_j = 1$)

b. In agreement with Figure 7.2

### Table 7.4: Regularity parameters

<table>
<thead>
<tr>
<th></th>
<th>B213</th>
<th>B2(1.5)3</th>
<th>B223</th>
<th>B2(2.5)3</th>
<th>B233</th>
<th>B23(2.5)</th>
<th>B232</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>.618</td>
<td>.771</td>
<td>.913</td>
<td>.955</td>
<td>.966</td>
<td>.983</td>
<td>.995</td>
</tr>
</tbody>
</table>
The parameter increases as the bridges profiles change from B213 to B232, being almost 1.0 for bridge B232. According to Calvi, this result indicates that the bridges become more and more regular as their profiles get closer to the profile of bridge B232.

7.3.2 The response spectrum analysis

The linear elastic response of the bridges to the design seismic action in Figure 3.15 was computed through the response spectrum method. The values in correspondence with those in Tables 7.1 and 7.2 are presented in Table 7.5. The same effective moment of inertia and material characteristics used in section 7.2 were considered in the analysis of these bridges. The results for bridges B213 and B232 are repeated in Table 7.5 to make the comparison of the results easier.

The transverse section of the piers is the hollow core rectangular section already considered in the previous bridges and illustrated in Figure 7.3. The piers longitudinal steel was calculated for the axial force (\(P = 1.72MN\)) and the flexural moments in Table 7.5 divided by the behaviour factor (\(q = 2.5\)). The design strengths and the concrete and steel constitutive laws are those referred to in section 7.2: a parabola followed by a rectangle for the concrete and an elastic perfectly plastic model for the steel. The minimum longitudinal steel ratio allowed in the piers was the value adopted in the piers tested in the laboratory, i.e. 0.50%. However, to verify the influence of the minimum steel ratio in the response, the piers were also designed for a minimum steel ratio of 0.29%.

The longitudinal steel consists of four layers of steel bars placed at the shorter sides of the cross-section, \(A_{sl1}\), plus additional steel bars distributed along both transverse sides of the cross-section, \(A_{sl2}\), as illustrated in Figure 7.3. The steel ratio of 0.29% corresponds to a uniform distribution of the longitudinal steel along the perimeter of the cross-section.

The amount of steel in the critical zones (where the plastic hinges are likely to occur) is calculated so that the design flexural strength \(M_{Rd}\) of the section respects the relation

\[ M_{Sd} \leq M_{Rd} \]

where \(M_{Sd}\) is the design moment that corresponds to the seismic loading combination.
given by equation (7.8) and presented in Table 7.5, divided by the behaviour factor. The same longitudinal steel ratio is adopted in sections outside the critical zones.

Table 7.5: Response spectrum analysis for the (1:2.5) scale bridges

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>B213</td>
<td>1</td>
<td>5.24</td>
<td>.793</td>
<td>43.95</td>
<td>1</td>
<td>2.91</td>
<td>.52</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.01</td>
<td>1.460</td>
<td>159.98</td>
<td>2</td>
<td>6.50</td>
<td>2.32</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>29.72</td>
<td>.287</td>
<td>9.18</td>
<td>3</td>
<td>1.16</td>
<td>.14</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>$\rho_m = 92.9%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B2(1.5)3</td>
<td>1</td>
<td>4.00</td>
<td>1.314</td>
<td>186.12</td>
<td>1</td>
<td>3.29</td>
<td>.59</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.47</td>
<td>.309</td>
<td>8.91</td>
<td>2</td>
<td>9.16</td>
<td>2.18</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11.39</td>
<td>-.326</td>
<td>11.15</td>
<td>3</td>
<td>2.52</td>
<td>.30</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>$\rho_m = 89.8%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B223</td>
<td>1</td>
<td>3.13</td>
<td>1.278</td>
<td>186.12</td>
<td>1</td>
<td>4.57</td>
<td>.82</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.44</td>
<td>.177</td>
<td>3.23</td>
<td>2</td>
<td>7.62</td>
<td>1.36</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11.02</td>
<td>.392</td>
<td>17.66</td>
<td>3</td>
<td>3.00</td>
<td>.36</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>$\rho_m = 89.8%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B2(2.5)3</td>
<td>1</td>
<td>2.69</td>
<td>1.282</td>
<td>185.00</td>
<td>1</td>
<td>5.16</td>
<td>.92</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.44</td>
<td>.145</td>
<td>2.24</td>
<td>2</td>
<td>5.66</td>
<td>.81</td>
<td>6.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10.88</td>
<td>.425</td>
<td>20.42</td>
<td>3</td>
<td>3.23</td>
<td>.39</td>
<td>5.3</td>
</tr>
<tr>
<td></td>
<td>$\rho_m = 89.9%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>B233</td>
<td>1</td>
<td>2.46</td>
<td>1.287</td>
<td>184.62</td>
<td>1</td>
<td>5.43</td>
<td>.97</td>
<td>3.9</td>
</tr>
<tr>
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<td>5.43</td>
<td>.132</td>
<td>1.89</td>
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<td>4.19</td>
<td>.50</td>
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<tr>
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<td>10.80</td>
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<td>3.34</td>
<td>.40</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>$\rho_m = 89.9%$</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>B23(2.5)</td>
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<td>2.61</td>
<td>1.287</td>
<td>184.54</td>
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<td>5.53</td>
<td>.99</td>
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<td></td>
</tr>
<tr>
<td>B232</td>
<td>1</td>
<td>2.86</td>
<td>1.294</td>
<td>183.48</td>
<td>1</td>
<td>5.77</td>
<td>1.03</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10.91</td>
<td>.454</td>
<td>23.94</td>
<td>2</td>
<td>3.91</td>
<td>.47</td>
<td>6.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>28.89</td>
<td>.233</td>
<td>6.50</td>
<td>3</td>
<td>5.77</td>
<td>1.03</td>
<td>4.2</td>
</tr>
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<td></td>
<td>$\rho_m = 90.0%$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>B232</td>
<td>1</td>
<td>2.86</td>
<td>1.294</td>
<td>183.48</td>
<td>1</td>
<td>5.77</td>
<td>1.03</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10.91</td>
<td>.454</td>
<td>23.94</td>
<td>2</td>
<td>3.91</td>
<td>.47</td>
<td>6.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>28.89</td>
<td>.233</td>
<td>6.50</td>
<td>3</td>
<td>5.77</td>
<td>1.03</td>
<td>4.2</td>
</tr>
<tr>
<td>$\rho_m = 92.8%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Left side pier (5.6m high)  
b. Central pier (variable height)  
c. Right side pier (variable height)

The longitudinal steel ratio estimated for each pier of the seven bridges is presented in Table 7.6. The values in parenthesis correspond to piers where the longitudinal steel ratio
required by equation (7.9) is below the minimum ratio of 0.29%. Concerning the distribution of longitudinal steel bars in the transverse section, the steel referred to as \( A_{st2} \) in Figure 7.3 is the same for all the piers and only the main steel \( A_{st1} \) changes from one pier to the other. The steel \( A_{st2} \) corresponds, in the model, to 40 rebars of 6mm diameter equally distributed at the longer sides of the cross-section. The main steel \( A_{st1} \) is equally distributed by the four layers disposed at the shorter sides.

![Figure 7.3 - Piers transverse section layout](image)

The moments of inertia computed after the design through equation (7.4) gave, via the response spectrum analysis, flexural moments different to the values in Table 7.5 in less than 5%. Therefore, the initial estimation of \( J_{eff} = 0.25 \cdot J_{un} \) was considered to be satisfactory and the longitudinal steel ratios in Table 7.6 were adopted. However, as some of the piers in Table 7.6 are over dimensioned, the hypothesis of full cracked section may not be verified for these piers.

<table>
<thead>
<tr>
<th>Pier</th>
<th>B213</th>
<th>B2(1.5)3</th>
<th>B2(2.5)3</th>
<th>B223</th>
<th>B2(2.5)3</th>
<th>B233</th>
<th>B23(2.5)</th>
<th>B232</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(.29)</td>
<td>(.29)</td>
<td>.38</td>
<td>.48</td>
<td>.52</td>
<td>.55</td>
<td>.62</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>.75</td>
<td>1.28</td>
<td>.98</td>
<td>.58</td>
<td>.29</td>
<td>.29</td>
<td>.29</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(.29)</td>
<td>(.29)</td>
<td>(.29)</td>
<td>(.29)</td>
<td>(.29)</td>
<td>.35</td>
<td>.62</td>
<td></td>
</tr>
</tbody>
</table>

A supplementary analysis was performed on bridge B213. The transverse sections of the two extreme piers were reduced in order to respect, at the same time, the yielding moments imposed by the behaviour factor of 2.5, i.e. 1.16\( MNm \) for the medium pier and 0.46\( MNm \) for the tall pier, and the longitudinal steel ratio of 0.29%. Thus, the cross-sec-
tions of the medium and tall piers were reduced to 57% and 85% of their initial area, respectively.

The response spectrum analysis was applied to this new bridge. A strong reduction of the flexural moments at the two extreme piers, with a small increase at the central pier, was found. Thus, new iterations with new reduced cross-sections for the two extreme piers were carried out. During this process the two transverse sections tended to zero, i.e. the bridge tended to the configuration with only the central pier linked to the deck in the direction of the seismic action.

Therefore, a supplementary bridge with the two extreme piers disconnected from the deck in the horizontal transverse direction of the bridge, was also designed for the behaviour factor of 2.5 and the response spectrum illustrated in Figure 3.15. The results from the response spectrum analysis are presented in Table 7.7.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.46</td>
<td>1.201</td>
<td>198.39</td>
<td>1.86</td>
</tr>
<tr>
<td>2</td>
<td>30.01</td>
<td>.294</td>
<td>9.24</td>
<td>.05</td>
</tr>
</tbody>
</table>

$P_m = 92.8\%$

<table>
<thead>
<tr>
<th>Pier</th>
<th>Flexural Moment [MNm]</th>
<th>Shear Force [MN]</th>
<th>Top Displ. [cm]</th>
<th>Long. Steel [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2</td>
<td>8.29</td>
<td>2.96</td>
<td>1.5</td>
<td>1.21</td>
</tr>
</tbody>
</table>

7.4 THE RESPONSE OF THE NEW BRIDGES

The non-linear dynamic response of the bridges for the accelerogram illustrated in Figure 3.15 was then computed. The fibre model described in chapter 4 was used in the simulation of the bridges. The shear stress versus shear deformation law is linear elastic (see equations (4.3) and (4.4) in the same chapter). The mean strength of the materials was adopted. In the case of the steel, the mean and the characteristic yielding stresses were assumed to be equal. A coefficient of variation ($c = 0.12$) was considered for the concrete giving, for a normal distribution, a mean strength

$$f_{cm} = \frac{f_{ck}}{1 - 1.64 \cdot c}$$  \hspace{1cm} 7.10
i.e. \( f_{cm} = 31.1\, MPa \). The other parameters of the concrete and steel behaviour laws are illustrated in Figures 7.4 and 7.5. These values are in agreement with the constitutive laws described in sections 4.3 and 4.4 in chapter 4.

<table>
<thead>
<tr>
<th>( f_{co} ) [MPa]</th>
<th>( \sigma_{co} ) [MPa]</th>
<th>( \varepsilon_{co} ) [%]</th>
<th>( \varepsilon_{co}^* ) [%]</th>
<th>( Z )</th>
<th>( Z^* )</th>
<th>( \sigma_i ) [MPa]</th>
<th>( \varepsilon_i ) [%]</th>
<th>( \varepsilon_{tm}/\varepsilon_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.1</td>
<td>39.2</td>
<td>0.20</td>
<td>0.32</td>
<td>100.0</td>
<td>29.1</td>
<td>3.1</td>
<td>0.010</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Figure 7.4 - Properties of the confined and unconfined concrete at the piers

<table>
<thead>
<tr>
<th>( E_0 ) [GPa]</th>
<th>( \sigma_{yy} ) [MPa]</th>
<th>( \sigma_{yy}/\sigma_{sy} )</th>
<th>( \varepsilon_{h}^* ) [%]</th>
<th>( \varepsilon_{ul} ) [%]</th>
<th>( E_h/E_0 ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200.0</td>
<td>500.0</td>
<td>1.14</td>
<td>2.0</td>
<td>13.0</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Figure 7.5 - Numerical model for the steel

The seismic input is the accelerogram illustrated in Figure 3.15. The two hypothesis of minimum longitudinal steel ratio, 0.29% and 0.50%, were considered. A Rayleigh damping for 0.5% critical damping on the two first frequencies in the transverse direction of the bridge, was adopted. In the case of bridge B232, the second frequency, 5.77Hz, do not correspond to the second mode shape in Table 7.5 but to the first anti-symmetric mode shape. A supplementary computation was performed considering 5.0% critical damping instead.
The transverse force versus horizontal displacement hysteretic curves at the top of the piers for the minimum longitudinal steel ratio of 0.50%, are illustrated in Figures 7.6 to 7.13. Four other transverse force versus displacement curves at the top of the piers are also presented in the figures. These last curves represent different steps of the design and calculation procedures and are displayed in the same graphic when referring to the same pier. The first curve, represented by a broken line, corresponds to the behaviour curve used in the response spectrum analysis. The first branch of the curve shows the stiffness used in this analysis. The plateau corresponds to the transverse force presented in Table 7.5 divided by the behaviour factor. The extremity of the plateau indicates the maximum displacement computed through the response spectrum analysis.

The second curve, with star marks, illustrates the monotonic response of the piers when the longitudinal steel ratio in Table 7.6 and the design strength of the materials are considered. The third curve, with square marks, represents the monotonic response of the piers for the same longitudinal steel ratio and the average strength of the concrete and steel. The difference between these two curves gives a measure of the safety factor that is considered in the design due to the variability of the materials properties. The diagram is drawn with a bold line. The fourth and last curve, with triangle marks, illustrates the behaviour of the bridges when a minimum steel ratio equal 0.50% is adopted. The average strength of the materials is considered. This curve corresponds to the response diagrams represented at the top of the figures.

At the end, the same accelerogram multiplied by a parameter of increasing value was applied to the set of bridges designed for a minimum steel ratio equal to 0.5%. The average strength of the concrete and steel was adopted. The results are discussed in the next section.

7.5 ANALYSIS OF THE RESULTS

The analytical results illustrated in Figures 7.6 to 7.13 and summarized in Tables 7.8 and 7.9, show that the displacement ductility demand for the more heavily damaged piers goes from 1.8 to 4.5. The difference between the values in the two tables is in the strength of the materials: average or design values and in the longitudinal steel ratio: 0.50% or 0.29%. These results correspond to 0.5% critical damping. The higher ductility
demand ($\mu = 4, 5$) was registered at the short pier of bridge B213 when the design strength was adopted. Notice that the central pier of bridge B010 presents a ductility demand of 3.4, lower than the ductility demand of bridge B213. Moreover, a great increase of ductility demand is observed when the bridges profile changes from B2(1.5)3 to B213. In fact, while bridges from B223 to B232 respond with similar maximum ductility demands, around 2.0 and not far from the design behaviour factor, bridges B010, B213 and B2(1.5)3 present ductilities of 3.4, 4.5 and 3.6, respectively. These results show that the design performed with the response spectrum analysis is on the safe side only for the so-called regular structures.

Table 7.8: Earthquake demands (numerical values for the average strength of the materials)

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>H [m]</th>
<th>Design Earthquake (Average Strength) - 0.5% Critical damping</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Minimum Steel ratio - 0.29%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Max. Disp. [m]</td>
</tr>
<tr>
<td>B010</td>
<td>2</td>
<td>2.8</td>
<td>.021</td>
</tr>
<tr>
<td>B213</td>
<td>1</td>
<td>5.6</td>
<td>.017</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8</td>
<td>.024</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>.025</td>
</tr>
<tr>
<td>B2(1.5)3</td>
<td>1</td>
<td>5.6</td>
<td>.028</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.2</td>
<td>.039</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>.031</td>
</tr>
<tr>
<td>B223</td>
<td>1</td>
<td>5.6</td>
<td>.038</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.6</td>
<td>.054</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>.043</td>
</tr>
<tr>
<td>B2(2.5)3</td>
<td>1</td>
<td>5.6</td>
<td>.038</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7.0</td>
<td>.057</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>.049</td>
</tr>
<tr>
<td>B233</td>
<td>1</td>
<td>5.6</td>
<td>.042</td>
</tr>
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<td></td>
<td>2</td>
<td>8.4</td>
<td>.061</td>
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<td></td>
<td>3</td>
<td>8.4</td>
<td>.045</td>
</tr>
<tr>
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<td>5.6</td>
<td>.036</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4</td>
<td>.054</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7.0</td>
<td>.044</td>
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<tr>
<td>B232</td>
<td>1</td>
<td>5.6</td>
<td>.042</td>
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<td></td>
<td>2</td>
<td>8.4</td>
<td>.059</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.6</td>
<td>.042</td>
</tr>
</tbody>
</table>

NUMERICAL ANALYSIS OF A NEW SET OF BRIDGES
Although in this analysis the material safety factors, i.e. the ratio between the design strength and the average strength of the concrete and steel are 1.15 and 1.9, respectively, the ratio between the design strength and the mean strength of the piers is not greater than 10%. The small increase of the piers strength is due to:

- firstly, after cracking, and in particular after yielding of the steel bars, the concrete compression area decreases considerably and the steel bars provide most of the piers strength. Thus, a ratio of the design strength to the mean strength much lower than 1.9 would be expected:

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>H [m]</th>
<th>Design Earthquake (Design Strength) 0.5% Critical damping</th>
<th>Design Earthquake (Design Strength) 5.0% Critical damping</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Max. Disp. [m] Ductility Demand Shear Force [MN]</td>
<td>Max. Disp. [m] Ductility Demand Shear Force [MN]</td>
</tr>
<tr>
<td>B010</td>
<td>2</td>
<td>2.8</td>
<td>.024 3.4 1.43</td>
<td>.012 1.7 1.39</td>
</tr>
<tr>
<td>B213</td>
<td>1</td>
<td>5.6</td>
<td>.020 1.0  .30</td>
<td>.013 &lt;1 .27</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8</td>
<td>.026 4.5 1.05</td>
<td>.013 2.2 1.05</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>.027 &lt;1  .18</td>
<td>.015 &lt;1 .14</td>
</tr>
<tr>
<td>B2(1.5)3</td>
<td>1</td>
<td>5.6</td>
<td>.033 1.6  .33</td>
<td>.019 &lt;1 .31</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.2</td>
<td>.050 3.6  .96</td>
<td>.030 2.1 .96</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>.039 &lt;1  .20</td>
<td>.027 &lt;1 .18</td>
</tr>
<tr>
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<td>1</td>
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<td>.044 2.2  .37</td>
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<td>.038 1.7 .59</td>
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<td>8.4</td>
<td>.045 &lt;1  .20</td>
<td>.032 &lt;1 .19</td>
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<td>5.6</td>
<td>.036 1.8  .40</td>
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<td>7.0</td>
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<td>.043 1.3 .35</td>
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<tr>
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<td>8.4</td>
<td>.041 &lt;1  .20</td>
<td>.035 &lt;1 .19</td>
</tr>
<tr>
<td>B233</td>
<td>1</td>
<td>5.6</td>
<td>.049 2.2  .42</td>
<td>.031 1.4 .41</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4</td>
<td>.073 1.6  .22</td>
<td>.052 1.2 .21</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>.058 1.3  .21</td>
<td>.039 &lt;1 .20</td>
</tr>
<tr>
<td>B23(2.5)</td>
<td>1</td>
<td>5.6</td>
<td>.040 1.9  .43</td>
<td>.027 1.3 .42</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.4</td>
<td>.055 1.2  .21</td>
<td>.043 &lt;1 .20</td>
</tr>
<tr>
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<td>3</td>
<td>7.0</td>
<td>.041 1.4  .28</td>
<td>.033 1.1 .27</td>
</tr>
<tr>
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<td>5.6</td>
<td>.043 2.0  .46</td>
<td>.031 1.5 .45</td>
</tr>
<tr>
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<td>2</td>
<td>8.4</td>
<td>.057 1.2  .21</td>
<td>.044 &lt;1 .20</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.6</td>
<td>.043 2.0  .46</td>
<td>.031 1.5 .45</td>
</tr>
</tbody>
</table>

Table 7.9: Earthquake demands (numerical values for the design strength of the materials)
secondly, most of the rebars along the sides of the pier do not reach the yielding strain and most of them do not even reach the design strength. Therefore, it is not strange that a strength ratio inferior to 1.15 was found either.

Another remark is that the displacement ductility demands at the piers for the two different minimum steel ratios and the design earthquake action, are similar; no important differences were found between the two results.

Table 7.10: Earthquake demands (numerical values for the 2.0DE and the 3.0DE accelerograms)

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>H[m]</th>
<th>2.0xDesign Earthquake 0.5% Critical damping</th>
<th>3.0xDesign Earthquake 0.5% Critical damping</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Max. Disp. [m]</td>
<td>Ductility Demand</td>
</tr>
<tr>
<td>B010</td>
<td>2</td>
<td>2.8</td>
<td>.081</td>
<td>11.8</td>
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<tr>
<td>B213</td>
<td>1</td>
<td>5.6</td>
<td>.067</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8</td>
<td>.087</td>
<td>15.0</td>
</tr>
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<td>.085</td>
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It is also noteworthy that the piers that are over-reinforced, i.e. the steel ratio given by the design is lower than the minimum value of 0.29%, respond to the design earthquake with little yielding or even no yielding of the steel bars. In this case, the conditions of the
design are no longer respected and the irregular dissipation of energy not equally distributed among the three piers was, in fact, expected. This is particularly evident in bridges B213, B2(1.5)3 and B223. The results are illustrated in Figure 7.14. Nevertheless, for the design earthquake action and the average strength of the materials, all the bridges except bridge B213 present acceptable ductility demands.

Table 7.11: Earthquake demands (numerical values for the 4.0DE accelerograms)

<table>
<thead>
<tr>
<th>BRIDGE</th>
<th>Pier</th>
<th>H (m)</th>
<th>4.0xDesign Earthquake 0.5% Critical damping</th>
<th>Max. Disp. [m]</th>
<th>Ductility Demand</th>
<th>Shear Force [MN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B010</td>
<td>2</td>
<td>2.8</td>
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<tr>
<td>B213</td>
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<td>5.6</td>
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<td>B2(1.5)3</td>
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<td>B2(2.5)3</td>
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<td>5.6</td>
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<td>B233</td>
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<td>.22</td>
<td>10.2 (F)</td>
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For seismic actions of higher intensity, the ductility demand of the short piers of the so-called three more irregular bridges: B010, B213 and B2(1.5)3, increases considerably. The two first bridges reach failure for an intensity of the seismic action around two times the design earthquake (see Table 7.10). In particular, these bridges present a clear non-uniform distribution of damage; only the short piers are severally damaged. Note that, in
the case of bridge B010, the short pier is the only one that contributes to the behaviour of the bridge in horizontal transverse direction of the deck. These results are summarized in Figures 7.15 and 7.14. The involvement of the three piers in the other five bridges is much clearer. Nevertheless, the collapse occurred always at the shortest pier. On the contrary, the highest pier is always the least damaged of all.

In these numerical analyses, the collapse of a pier corresponds to the collapse of an extreme steel bar. However, another more realistic damage functional taking into account the ductility demand and the dissipation of energy, should have been used to define the structural collapse, namely: the Banon and Veneziano [2] and the Park and Ang [59] damage functions. An analysis of failure based on the collapse of the steel under monotonic loading, as it is done in this chapter, most probably overestimates the piers failure.

When the parameters defining the physical state of the components of a bridge present different values and no other expressions are available, the most unfavorable value should be chosen to represent the state of the whole structure. Figure 7.16 illustrates the maximum displacement ductility demand computed at the piers of each bridge. From the analysis of the picture, it is apparent that all the bridges, apart from B010 and B213, present similar maximum displacement ductility demands. For these bridges, failure occurred for an intensity of the seismic action around three times the design action.

These results seem to be in agreement with the tendency of the regularity parameters presented in Table 7.4; the bridges become more regular as they get closer to the bridge B232 profile. However, it seems that only for values of $R$ greater than 0.95 there is a good distribution of ductility demand and dissipation of energy between the three piers. Note that the total energy dissipated in all the bridges is similar, as shown in Figure 7.17.

The response of the bridges was also computed for 5.0% critical damping. The displacement ductility demands of the eight bridges are illustrated in Figure 7.18. The results are quite different from those in Figure 7.15 when 0.5% critical damping is adopted; lower ductility demands were found and much higher seismic actions were allowed in the structures. In general, intensities of the seismic action around four and five did not cause
collapse of the bridges. The differences found in the results call the attention for the numerical simulation of the damping forces; higher damping forces prevent the structures from deforming further in the inelastic field. The critical damping should therefore be carefully chosen.

From this analysis, it is evident that the regularity issue is not an easy matter. Moreover, the results also show that:

- the change in the height of the central pier of bridge B213 to twice the initial value, modified considerably the behaviour of the bridge which became much more regular. In this context, being more regular means to present a more uniform distribution of ductility demand and dissipation of energy in the piers;

- since the design code may impose over-strength on some structural elements, it interferes with structural regularity. Piers demanding reinforcement ratios lower than the minimum values imposed by the code, being more resistant, cause damage to be deviated to the other piers of the bridge where most of the dissipation of energy will take place;

- the relation between the regularity parameter $R$ and the ductility demand is not very clear. Figure 7.19 illustrates the variation of the ductility demand of the bridges (the highest value found for the three piers) with the regularity parameter. Although the ductility demand tends to decrease with the increase of $R$, i.e. with the increase of the structural regularity, for the profiles of bridges B223, B2(2.5)3, B233, B23(2.5) and B232 under the same seismic action, the ductility demands fluctuate considerably;

Finally, a more complete analysis considering other configurations and bridges with different number of piers should be carried out so that the results could be extrapolated to other cases. Moreover, this study only concerns the behaviour of bridges with seismic actions in their horizontal transverse direction. Consequently, identical analysis should be performed for seismic actions in other directions in order to get more conclusive results on this issue.
Bridge B010

Force [MN]

Figure 7.6 - Force-displacement diagrams for bridge B010
Figure 7.7 - Force-displacement diagrams for bridge B213

- MEDIUM PIER
- CENTRAL PIER
- RIGHT SIDE PIER

- Average Strength - Min. Long. Steel Ratio = 0.50%
- Average Strength - Min. Long. Steel Ratio = 0.29%
- Design Strength - Min. Long. Steel Ratio = 0.29%
Figure 7.8 - Force-displacement diagrams for bridge B2(1.5)3

- Medium Pier
- Central Pier
- Right Side Pier

Key:
- ▼ Average Strength - Min. Long. Steel Ratio = 0.50%
- □ Average Strength - Min. Long. Steel Ratio = 0.29%
- ● Design Strength - Min. Long. Steel Ratio = 0.29%
Figure 7.9 - Force-displacement diagrams for bridge B223

- MEDIUM PIER
- CENTRAL PIER
- RIGHT SIDE PIER

△ Average Strength - Min. Long. Steel Ratio = 0.50%
□ Average Strength - Min. Long. Steel Ratio = 0.29%
★ Design Strength - Min. Long. Steel Ratio = 0.29%
Figure 7.10 - Force-displacement diagrams for bridge B2(2.5)3

- Medium Pier
- Central Pier
- Right Side Pier

Symbols:
- ▽ Average Strength - Min. Long. Steel Ratio = 0.50%
- □ Average Strength - Min. Long. Steel Ratio = 0.29%
- • Design Strength - Min. Long. Steel Ratio = 0.29%
Figure 7.11 - Force-displacement diagrams for bridge B233
Figure 7.12: Force-displacement diagrams for bridge B23(2.5)
Bridge B232

Figure 7.13 - Force-displacement diagrams for bridge B232

- MEDIUM PIER
- CENTRAL PIER
- RIGHT SIDE PIER

Symbols:
- ▼ Average Strength - Min. Long. Steel Ratio = 0.50%
- □ Average Strength - Min. Long. Steel Ratio = 0.29%
- ⭕ Design Strength - Min. Long. Steel Ratio = 0.29%
Figure 7.14 - Ratio between the energy dissipated by each pier and total dissipated energy
Figure 7.15 - Ductility demands of the piers for growing seismic actions (0.5% critical damping)
Figure 7.16 - Maximum ductility demands for the eight different bridge profiles and different earthquake intensity factors

Figure 7.17 - Total dissipated energy for the 1.0 and 2.0 earthquake intensities
Figure 7.18 - Ductility demands of the piers for growing seismic actions (5.0% critical damping)
Figure 7.19 - Maximum ductility demands of each bridge versus the regularity parameter
8 CONCLUSIONS AND FUTURE RESEARCH

8.1 INTRODUCTION

This work aimed the analysis of the seismic behaviour of reinforced concrete bridges, in particular the development of a numerical model for bending and shear forces to simulate such structures. Moreover, it aimed the analysis of the behaviour of a set of bridges tested under Pseudo-dynamic conditions and the evaluation of behaviour factors and structural regularity for these structures. The excellent conditions of the European Laboratory for Structural Assessment, the reaction-wall facility and the hardware and software available, allowed experimental tests to be carried out and more suitable analytical models to represent the structures tested in the laboratory to be improved.

Notice that the numerical and the experimental analysis are complementary; the numerical models that are used for simulating structures not tested in the laboratory are calibrated with the experimental results. In fact, the data obtained from the experimental campaign was exhaustively used for checking and calibrating the numerical model improved in this work: a fibre model for bending forces coupled with a strut-and-tie formulation for shear forces. The numerical analyses were performed with the computer code CASTEM 2000, an object oriented finite element code developed by a group of researchers of the Commissariat à l'Energie Atomique at Saclay in France.

8.2 THE EXPERIMENTAL CAMPAIGN

The experimental campaign consisted of six reinforced concrete bridge models (1:2.5 scale) tested at the ELSA laboratory using the Pseudo-dynamic test method coupled with a substructuring technique. This campaign is part of the PREC8 programme of the Euro-
pean Commission. Two bridge profiles with the deck supported by three intermediate piers were considered: a regular profile (bridge B232) and an irregular profile, with five alternative design solutions, with or without isolating/dissipating devices. These bridges are referred to as B213A, B213B, B213C, B213A-5Dev and B213A-1Dev. The numbers in between the letters refer to the relative length of piers and the position they occupy on the bridge.

The results of this testing campaign underlined the versatility and the potential of the Pseudo-dynamic test method. This testing method was able to deal with different situations with minor changes in the algorithm that controls the physical structure in the laboratory. The on-line analytical simulation of the deck and some of the bridge piers evidenced the usefulness of the method implemented in the ELSA laboratory for dynamic testing of bridges. Moreover, tests with isolating/dissipating devices and asynchronous input motion were still performed with the same apparatus. The author had the unique opportunity to participate in this experimental programme, which was the first Pseudo-dynamic testing campaign performed in the world using the substructuring technique on large scale structures.

Two Pseudo-dynamic tests were performed on each bridge using different intensity factors for the same accelerogram. The first intensity corresponds to the design seismic action. The choice of the second and higher intensity factors was based on preliminary numerical calculations that pointed out the ultimate capacity of the weakest solution for the irregular bridge and a ductility demand twice the value of the design seismic action for the regular bridge. The irregular bridge B213A was also tested with isolating devices on the top of the three piers and the abutments, or with only the central short pier isolated. Additionally, an asynchronous input motion was considered on the irregular bridge B213A.

Concerning the seismic behaviour of the non-isolated bridges tested in the laboratory with synchronous motion, the results pointed out that:

- the behaviour of the irregular bridges is mainly commanded by the behaviour of the short and central pier. Almost all the dissipation of energy comes from this pier which
presented, at the end of the high level earthquake input action, a high structural damage. However, this is against the EC8 rule that says that behaviour factors superior to 1.2 should not be adopted in such cases. Nevertheless, all the piers presented a quite ductile behaviour with stable hysteretic loops and ductility demands not inferior to 4.0. The diagonal cracking pattern observed is also in agreement with the expectations and evidences the importance of the shear mechanism to the global behaviour of the pier. The distribution of diagonal cracking is, however, much more spread in the piers with higher longitudinal and transverse steel ratio;

- from the comparison of the two non-isolated alternative solutions to the EC8 conforming irregular bridge B213A, the solution B213B turned out to be the best of the three. The strengthening of the short pier attracted higher forces, decreasing the high ductility demand observed at bridge B213A. For the weak solution B213C, although a more regular distribution of forces is achieved, the demand of ductility in the short pier was much higher than in bridge B213A causing failure. In reality, these three alternative solutions correspond to the adoption of different local behaviour factors for each pier. Notice that the imposition of a minimum longitudinal steel ratio do not always comply with a global behaviour factor procedure: the strength of the piers may have to be increased beyond the value imposed by the behaviour factor procedure changing the conditions of the design.

- the results of the experiment on bridge B232 comply with the regularity issue as mentioned in EC8, part 2. The dissipation of energy takes place in all piers and a ductile behaviour could be assumed. The mechanism turned out to be stable and efficient, with demands roughly proportional to the capacities. An input signal two times larger than the design earthquake was applied without loss of capacity of the piers.

In the case of the alternative solutions with isolating/dissipating devices, it is pointed out that:

- almost all the dissipation of energy occurred in the dissipation devices. The solution with the three piers and the abutments with isolating devices disconnect almost completely the deck from the piers. Although a very low ductility was demanded in the

CONCLUSIONS AND FUTURE RESEARCH
reinforced concrete elements, the high ductility demanded in the isolating devices may take the bridge to a point close to failure. Furthermore, in this case the abutments and the piers must be prepared to accommodate the high transverse displacements of the deck;

- the solution with only the central short pier isolated presented a much better global behaviour with lower ductility demands in the isolating device in relation to the previous bridge. Moreover, although a re-distribution of forces in the bridge would be expected, imposing higher shear forces in the two piers without isolators, no increase of damage was found in these piers in relation to bridge B213A.

Since in general the piers are separated from the deck and devices are used for controlling the forces transmitted from one structure to the other, the use of these devices as isolators and dissipators of energy is a normal procedure on bridge structures. As referred to in the previous paragraph, this may be an easy and economical way to solve the problem of the non-uniform distribution of ductility demand on the piers of an irregular bridge. However, these devices exhibit a clear non-linear behaviour that have to be considered in the design. In the case of dissipation devices, the energy dissipated arises from their hysteretic behaviour. Thus, because no guidelines exist in the code to account for these elements, the design should be done by experts in the non-linear analysis of structures.

In relation to the asynchronous input motion, the tests done on bridge B213A indicated that:

- for this bridge and for the selected input motion, it corresponded to a decrease of the maximum displacement of the short pier and to an increase of the maximum displacement of the medium pier.

Although this result can not be generalized, since it depends on the structure and on the input action that are considered, it shows that the asynchronous input motion may imply a re-distribution of ductility demand in the structures, increasing the damage in some structural elements and decreasing in others.

Concerning the analytical models, a fibre type model was implemented to simulate the
structures tested in the laboratory. These models are associated to axial stress versus strain constitutive laws of the materials. The shear and the axial stresses are uncoupled and the three-dimensional effect caused by the Poisson coefficient is neglected or taken into account through simplified models that consider the confinement effect of the stirrups in the core concrete.

8.3 THE NUMERICAL TESTS

The numerical analysis performed in this work showed that fibre type models are a powerful tool in the analysis of the behaviour of structural elements with axial forces and bending forces acting in both transverse directions of their cross-sections. In particular, R/C beams and columns are composite elements made to be modelled through these algorithms: both the steel and the concrete present an important axial behaviour and the global reaction forces result mostly from the integral of the tensile and the compression forces developed in the cross-sections. Thus it is important to have a good knowledge of the characteristics of the materials; a poor estimation of the strength or stiffness of the concrete or steel may change significantly the final results. This is particularly evident when performing dynamic analysis, as discussed in detail in chapter 6.

The fibre model was implemented in a Timoshenko beam element so that the distortional effect due to shear forces could be taken into account. However, classical fibre type models, like the one presented in this work, do not consider non-linear behaviour laws for shear stresses and this is the most important restriction of the model. If shear stresses exist and assume an important role in the global behaviour of a structure, some attention must be paid in the analysis of the results. Therefore, the improvements described in chapter 5, where the fibre model has been coupled with a non-linear model for shear stresses, represent an important advance in this area. Although more complete and multi-dimensional constitutive laws can be implemented in the fibre model, the constitutive laws presented in this chapter are already powerful enough for the applications under this work: linear structures in general and bridge piers in particular.

The results expressed in chapter 5, when the non-linear shear model is used, show, in general, a very good agreement between the numerical and the experimental results. The numerical transverse force versus displacement curves fit quite well the experimental
results. The behaviour of structures where the effect of shear forces is not negligible is well reproduced by the non-linear shear model; the narrowing and lengthening of the force versus displacement curves observed in the experimental responses is well represented. However, as the decrease of the concrete and steel strength due to the dissipation of energy is not taken into account in the cyclic behaviour laws, the structural damage within the cycles is not well represented in the response curves. Tests done on one of the piers tested in the laboratory show also that the non-linear shear model can be considered mesh independent: four different meshes, more or less refined near the basement, gave similar top transverse force versus displacement response curves.

In a strut-and-tie formulation like the one described in this chapter, the computation of the cracking angle is very important. To estimate the cracking angle, an algorithm that establishes the equilibrium of the internal and external forces within a pier damaged with diagonal cracking was improved with results in very good agreement with the values observed in the experiments.

The model was used for simulating the set of bridges tested under Pseudo-dynamic conditions in the ELSA laboratory. The analysis of the results was divided into three parts: the predictive, the post-experimental and the checking analyses. In the predictive analysis, and in spite of having considered a linear elastic behaviour for shear forces, the modification introduced in the steel properties described in chapter 6, led to results quite close to the experimental response. This modification consisted in reducing the steel Young modulus in order to increase the post-cracking flexibility of the pier that was observed at the transverse force versus horizontal top displacement diagram. Other phenomena like the pull-out effect can also be simulated in this way.

The numerical responses of the irregular bridges B213 and the regular bridge B232 compare well with the experimental results. The prediction of the magnitude of the maximum displacements and forces is, in general, good. In relation to the higher intensity input action, the results for the short piers of the irregular bridges show ductility demands much lower than in the experiments. The main differences between the numerical and the experimental results are found in the short piers with higher reinforcement ratio, i.e. the piers of bridges B213A and B213B.
After the tests, the numerical campaign was repeated using the fibre model with the non-linear behaviour in shear included. The modifications in the results were not enough to explain the discrepancies found between the experimental and the numerical response when the linear elastic behaviour in shear was considered.

Thus, because differences existed between the numerical simulation and the tests performed in the laboratory, namely: in the behaviour curves of the piers simulated in the computer during the experiments, in the algorithm used for solving the non-linear dynamic equations and in the strength of the materials, a third analysis was carried out on bridge B213B. After several numerical tests where one by one all the conditions in the laboratory were reproduced, the conclusion seems to be that the central and stiffer pier exhibits a supplementary rotation close to the foundation block. The increase of flexibility can be caused by bond slippage of the steel bars which changes the stiffness of the pier, modifying the dynamic behaviour of the global structure. This analysis illustrates the importance of simulating properly all the phenomena that may modify the stiffness of the structure.

Concerning the ultimate state of the piers, the numerical results for the shortest pier of bridge B213C showed that failure is not properly considered in the numerical model. However, it is quite difficult to predict failure with extreme accuracy. Several reasons can be pointed out: the materials present variations in the mechanical characteristics and in the position they occupy in the physical structure which can hardly be taken into account in the numerical calculations. To consider these factors in the model, the real characteristics and position of each steel bar have to be known with extreme precision and each steel bar have to be represented by separated fibres.

At the end of this work, a new set of bridges with profiles between the so-called irregular and regular configurations: bridges B213 and B232, respectively, was designed for the EC8 design earthquake acting in the horizontal transverse direction of the deck. The results show that for these structures the regularity parameter $R$ by Calvi et al [8] is not far from the reality. For values of $R$ around or greater than 0.9, there is a better distribution of the ductility demand at the top of the three piers. The dissipation of energy in the piers is also in agreement with these results. Although the total energy is identical for all
the bridges, a much more uniform distribution of dissipated energy is achieved at the
three piers of the so-called more regular bridges. Note that the design code itself inter-
feres with the regularity issue, since it may impose over-strengthening on some of the
elements of the structure.

8.4 FUTURE RESEARCH

In spite of these results, the regularity issue is a matter that still needs to be clarified; a
more complete analysis considering other configurations with different number of piers
should be performed. Moreover, this study should also concern the behaviour of the
bridges in other directions apart from the horizontal transverse direction of the deck.

In terms of the numerical model, an internal damage parameter should be included in the
constitutive laws of the materials to take into account the degradation of the strength due
to cyclic loading. This is particularly relevant when dealing with the steel, since for high
ductility demand the steel participates more actively in the response than the concrete. In
the case of the Menegotto-Pinto model, the hardening branch should be modified to take
into account the higher displacement and the dissipation of energy occurred until that
moment. Notice that this seems to be one of the main shortcomings of the model. Para-
metric studies should be also performed in order to check the sensitivity of the results to
the constitutive laws of the materials.

Moreover, although the non-linear model for shear forces performed well, other phe-
nomena that modify the stiffness of the structural elements, like bond slippage of the
steel bars and the soil interaction, should be also considered. The modification of the
stiffness implies the modification of the natural frequencies of the structure and, there-
fore, its dynamic response.

The use of the numerical tool improved in this work, the fibre model coupled with the
strut-and-tie formulation, can be extrapolated to the analysis of other reinforced concrete
structures such as buildings. The study of new design procedures and the check of safety
margins for reinforced concrete structures designed according to the existent codes, can
be performed with this model.
9 REFERENCES


A PHOTOGRAPHIC DOCUMENTATION

The next pages show some photographs that were taken during the construction of the specimens and casting of the concrete plinths in the laboratory. A general view of the test setup with the three piers placed side by side without the deck is also shown. The eight last photographs reproduce the final aspect of the piers after being tested under Pseudo-dynamic conditions.

Figure A.1 - Reinforcing steel of the short pier (pier A1)
Figure A.2 - Reinforcing steel of the piers plinths

Figure A.3 - Positioning of the pier on the plinth
Figure A.4 - Casting of the plinth with the pier suspended by the crane

Figure A.5 - Casting of the concrete plinth
Figure A.6 - General view of the test setup
Figure A.7 - Top view of the test set-up
Figure A.8: Cracking patterns at the central piers of the irregular bridges - Lateral view

B-213A
As = 0.92%

B-213B
As = 1.69%

B-213C
As = 0.30%
Figure A.9 - Cracking patterns at the central piers of the irregular bridges - Front view
B CASTEM 2000

B.1 INTRODUCTION

CASTEM 2000 is an object oriented finite element code developed at Saclay, France, by a group of researchers of the Commissariat à l'Energie Atomique (CEA). It is a general code that manipulates meaningful data-structures through a high level language named Gibe. Thanks to the generality of its data different analyses can be performed within the same computational environment; different levels of modelling are available in the code.

The computer code CASTEM 2000 uses subroutines developed in the new language, Espe, which is an evolution of Fortran with some aspects of the C language. This new language introduces the concept of SEGMENT in Fortran; it associates groups of variables, called objects, to an unique variable POINTEUR. To dispose of an object oriented language it is necessary that variables representing a same entity be represented by a sing- le variable and, in this domain, the Fortran is still very restrictive.

In CASTEM 2000, the change of information is done through the POINTEUR variables; it is through them that the code has access to the set of variables they represent. The declaration of the SEGMENT instruction and the general use of the POINTEUR variable are detailed in [71].

B.2 ADVANTAGES OF CASTEM 2000

The main advantages are summarised in the following items: simplicity and clearness in using the software and in creating new operators, independence and compatibility of the
operators with no exceptions. The existence of documentation allowing a full understanding and use of the code is a supplementary advantage that must not be neglected. Finally, the code is completely opened to the user.

Concerning independence and compatibility, the operators in CASTEM 2000 are autonomous: one can modify an operator without conflicting with the others. This allows several users create new operators and procedures simultaneously without interfering with each other. Moreover, as the input objects are standard within CASTEM 2000, the operators can be combined together and be used sequentially with the same argument objects. Concerning clearness, the program gives the users the power to decide the sequence of steps and the selection of arguments that better suits the purpose of their work; this gives them the possibility to control and participate as much as possible in the results. However, this freedom also suggests a major responsibility for the user.

B.3 OPERATORS

OPERATOR is the general name given to the processes in CASTEM 2000. It contains a set of instructions in Esope which are compiled with the main program. The interface with the user is done through the Gibiane language. The general syntax is

\[ \text{OUTPUT} = \text{OPERATOR} \ \text{INPUT} ; \]

where the word INPUT refers to objects that already exist and that the OPERATOR needs to run. These objects can be created by the user or by another operator. The word OUTPUT refers to the objects the OPERATOR creates after running. Note that one of the most important characteristics of the operators is their independence and compatibility. From the point of view of the user that does not want to become a programmer, an OPERATOR is just a process that creates objects from other objects. The main steps the user has to follow to build a new operator are described in [72].

B.4 OBJECTS

An OBJECT is a collection of information with common characteristics. The information is grouped differently according to what it represents or the role it has in the code. These groups are referred to by their TYPE which can go from simple entities, like an
integer or a real number, to more sophisticated variables like fields of points, fields of
elements, models, most of them being tailored to the finite element modelling. Thus,
each TYPE is associated with a certain data that is described, created, accessed and sup-
pressed according to specific rules. In fact, the compatibility and independence of the
operators is guaranteed by the rigidity of these rules that must be respected with no
exceptions.

Therefore, each OBJECT is associated to a certain TYPE that determines the information
it contains and the rules to access it. In the Esope language the OBJECT variables are
supported by the instruction SEGMENT. As mentioned in section B.1, this instruction
has the important role of associating a POINTEUR variable to each OBJECT it creates.

Although OBJECTS belonging to other TYPE groups can be created and accepted by the
code, those already available in CASTEM 2000, and described in detail in [73], are con-
sidered to be enough for the purposes of the code.

B.5 THE GIBIANE LANGUAGE

CASTEM 2000 is a computer code that creates a new environment for the user. GIBIA-
NE is the specific language that allows the user communicate with the code within this
environment: data is grouped within objects and is manipulated through operators and
procedures. The elements that contribute the most for the success of this language are the
independence and compatibility of the commands. Once more, this means that input and
output objects of the same type must be created according to the same rules.

The guidelines of the GIBIANE language that are necessary to dialogue with the pro-
gram are described in detail in [74]. Nevertheless, an important aspect of the GIBIANE
language is pointed out: the reading and interpretation of the instructions is done line
after line, from the left side to the right side, until a semicolon mark is found; the first
operator on the line is the first one to be executed. In other words, the priority rules of
mathematics are not respected. An exception is made to the parentheses which have pri-
ority over all the other instructions: operators and procedures.

B.6 PROCEDURES
The word PROCEDURE has been mentioned in the text together with the word operator. In fact, the two words have identical meaning: both are processes that create objects from existing objects. However, an important difference exists: while an operator is written in Esope, a PROCEDURE is written in Gibiane, i.e. it does not have to be compiled or linked with the main program. Actually, it consists of a sequence of instructions, where other PROCEDURES may be also included, that are described according to the rules of the Gibiane language.

B.7 AN EXAMPLE

A three-dimensional structure simulated with CASTEM 2000 is presented as an example. The structure is made of three columns with (0.2 X 0.2)\(m^2\) cross-sections connected at the top by beams with (0.2 x 0.4)\(m^2\) cross-sections that support a (4.6 x 5.6 x 0.2)\(m^3\) slab. The piers are 2.8\(m\) high. The piers and the beams are simulated through Bernoulli linear elastic beam type elements. As for the slab, linear elastic thin shell elements were considered. Apart from the mass of the structure, there is a supplementary mass of 2100\(kg\) distributed on the slab. The geometric and material characteristics of the piers, beams and slab are presented in the Gibiane input file that solves the structure.

In this file, the lines that start with "*" are ignored by the program. Instead, the lines that are preceded by a black point correspond to the extra information that has been added in the text. The words that are written in bold represent operators or procedures. The words within ' ' correspond either to keywords the operators need to run or to character type objects. The mass, the force and the length units are "1e6kg", "1e6N" and "m", respectively. The pictures that appear on the screen when the file is executed are presented in section B.8.

CASTEM 2000

When the program is opened through the command "K2000", the specific environment of CASTEM 2000 is activated and the following messages appear on the screen:

****** CASTEM2000 ****** VERSION DU mer 20 sep 1995

*** GEMAT 10.0 NOV 91 *** ESOPE=18000000 (MOTS) BUF= 0 (MOTS)

************* INFORMATIONS GIBI *************

368  CASTEM 2000
Pour obtenir la notice de GIBI ==> faire : "NOTICE;"
Pour obtenir la notice de CASTEM2000 faire "NOTICE CASTEM2000;"

Débutants ==> faire : "INFO DEBU;"

ATTENTION! La procédure THERMIC pour les calculs transitoires est débranchée. Veuillez utiliser PASAPAS.

Dans la boucle "REPETER LAB1 100;" la variable "$LAB1" contient automatiquement la valeur de l'indice de boucle, ici de 1 a 100.

--- LOCAL NEWS ---
- New installation
- NOMLAN, INCRAME accessible
- Non symmetric solver available!
- The operator TEVO is now included in EXTR
- Exploring your 3D mesh
- Generating joints elements
- Generating blocks
- Computing mechanisms and limit loads

for the heavy cut-and-pasters:
opti donn 90;

The program is ready to receive the command lines which can be introduced on-line by the operator or through an ASCII file that the program reads. A very small example is presented; the first line "tells" the program that it should always give the prompt sign "$" and the "echo" of the instructions. Then, the values 2.0 and 3.0 are attributed to the variables "a" and "b" and the result of the operation "a + b" is written on the screen.

```
opti echo 1;
$a = 2;
*a = 2;
$b = 3;
*b = 3;
$c = a + b;
*c = a + b;
$list c;
*list c;
Entier valant: 6
```

The instructions for solving the displacements of the three columns structure due to the weight of the structure and the additional masses, were typed on a file which is presented hereafter.
Example of a three columns structure

opti 'ECHO' 1
opti 'DIME' 3 'ELEM' seg2;

- The structure is three-dimensional and the elements that are going to be
  used for the beams and piers have two points.
- Each command "line" is separated by a semicolon mark.

vibra = vrai;
*
pvz =  0.  0.  10e10 ;
pvl = 10e10 10e10 10e10;

- The points are defined by three co-ordinates

3D Mesh - Piers

dimz - height of the piers
dimx - distance between piers along OX
dimy - distance between piers along OY
*
rotp - height of the plastic hinge
cons - dimension of the "cantilever" of the slab

dimz = 2.60;
dimx = 4.00;
dimy = 2.50;
*
rotp = 0.50;
cons = 0.60;

nurp - number of elements at the plastic hinge (piers)
nupi - number of elements at the intermediate pier
nula - number of elements at the beam

nurp = 1;
nupi = 2;
nula = 4;

sec?? - cross-sectional area
iny?? - moment of inertia OY
inz?? - moment of inertia OZ
tor?? - torsional moment

?? = pi (the piers)
?? = be (the beams)

secpi = 0.04; inypi = 1.3e-4; inzpi = 1.3e-4; torpi = 2.25e-4;
secbe = 0.08; inybe = 2.7e-4; inzbe = 1.07e-3; torbe = 6.06e-3;
epsla = 0.20;

*----------------------------------------------
*    po1? - points of pier1
*    po2? - points of pier2
*    po3? - points of pier3
*----------------------------------------------

po10 = 0. 0. 0.;
polo = dimx 0. 0.;
pol30 = dimx dimy 0.;
pol40 = 0. dimy 0.;

* po11 = pol0 plus (0. 0. rotp);
po12 = pol0 plus (0. 0. (dimz-rotp));
pol3 = pol0 plus (0. 0. dimz);

* "plus" adds the co-ordinates of two points

po23 = pol0 plus (0. 0. dimz);
pol33 = pol0 plus (0. 0. dimz);
pol43 = pol0 plus (0. 0. dimz);

*----------------------------------------------
*    rot?1 - plastic hinge at the basement (piers)
*    rot?2 - plastic hinge at the top (piers)
*----------------------------------------------

rot11 = pol0 d nurp pol11; pil1 = pol1 d nupi pol12;
rot12 = pol1 d nurp pol13;

- The operator "d" creates a line between two points and divides
  it into an integer number of elements

rot21 = rot11 plus pol20; pil2 = pil1 plus pol20;
rot21 = rot11 plus pol20;
rot31 = rot11 plus pol30; pil3 = pil1 plus pol30;
rot32 = rot12 plus pol30;
rot41 = rot11 plus pol40; pil4 = pil1 plus pol40;
rot42 = rot12 plus pol40;

* 
* The operator "et" adds two variables

pilr = pil1 et pil2 et pil3;

* pilr = roto et pilr;

*----------------------------------------------
*    3D Mesh - Beams
*----------------------------------------------

beam = (pol0 d nula pol02 d nula pol03 d nula pol04 d nula pol10)
plus (0. 0. dimz);

* 3D Mesh - Slab

opti 'ELEM' qua4;

* The elements that are going to be defined are quadrangular with four
* points (shell elements for the slab)

lig0 = (po10 d nula po20 d nula po30 d nula po40 d nula po10)  
      plus (0. 0. dimz);

sup1 = surf lig0 'PLAN';

* A surface is created within the line (or the lines)

lig1 = (po10 d nula po40 d nula po30 d nula po20 d nula po10)  
      plus (0. 0. dimz);

lig2 = lig1 homo ((dimx+cons)/dimx) ((dimx/2) (dimy/2) (dimz));

sup2 = surf (lig0 et lig2) 'PLAN';

*

supt = sup1 et sup2;

*

elim (supt et pil1 et beam et lig0 et po10 et po20 et po30 et po40 
      et po13 et po23 et po33 et po43 
      et po14 et po24 et po34 et po44 
      et pmei) .001;

* Points with identical co-ordinates are recognised has being the same point

l1t = supt et pil1 et beam;

*

trac pvl (supt et pil1 et beam);

* The operator "trac" plots mesh type objects. In 3D analysis a point of
* view must be given, "pvl". See the picture in Figure B.1

*

Models

*

hom = modl pil1 'MECANIQUE' 'ELASTIQUE' 'POUT';

meba = modl beam 'MECANIQUE' 'ELASTIQUE' 'POUT';

* Bernoulli elements

mosla = modl supt 'MECANIQUE' 'ELASTIQUE' 'COQ3' 'COQ4';

* Thin shell elements

*

Characteristics

*

matr = matr hom 'YOUN' 30.e3 'NU' 0.25 'RHO' 2.5e-3;

cara = cara hom 'SECT' secp 'INRY' inypi 'INRZ' inzpi  
       'TORS' torpi 'VECT' (0. 1. 0.);
* 
mabea = matr mobea 'YOUN' 30.e3 'NU' 0.25 'RHO' 2.5e-3;
cabea = cara mobea 'SECT' secbe 'INRY' inybe 'INRZ' inzbe
    'TORS' torbe 'VECT' (0.0.1.);
*
masla = matr mosla 'YOUN' 30.e3 'NU' 0.25 'RHO' 2.5e-3;
casla = cara mosla 'EPAI' epsla;
* 
modto = mopil et mobea et mosla;
matto = mapil et mabea et masla et
capil et cabea et casla;
*---------------------------------------------------------------
* Stiffness and mass matrices
*---------------------------------------------------------------
riges = rigi modto matto;

• Stiffness matrix without boundary conditions

mases = mass modto matto;

• Mass matrix (due to the weight of the structure)

masad = mass 'DEPL' (21.0e-3 / (nbno supl)) supl;

• Mass matrix (concentrated on the slab)

masto = mases et masad;
masto = mases;
*---------------------------------------------------------------
* Boundary conditions
*---------------------------------------------------------------
rigba = bloq 'DEPL' 'ROTA' (po10 et po20 et po30);

• Stiffness due to the boundary conditions

rigto = riges et rigba;

• Total stiffness matrix

*---------------------------------------------------------------
* Vibration modes
*---------------------------------------------------------------

• An "IF" type cycle

si (vibra);
    nfreq = entier 2;
* 
soll = VIBRATION 'INTERVALLE' 1.200. 'BASSE' nfreq rigto masto;

• Natural frequencies and mode shapes

dzero = defo (pilt et beam et supt) (tire soll 'DEPL' 'NUME' 1) 0;
• A "DO" type cycle

\[ i = 0; \text{repe} \ BL1 \ \text{nfreq}; \ i = i + 1; \]
\[ \text{frel} = \text{tire} \ \text{sol} \ 'FREQ' \ 'NUME' \ i; \]
\[ \text{modl} = \text{tire} \ \text{sol} \ 'DEPL' \ 'NUME' \ i; \]
\[ \text{disl} = \text{defo} \ (\text{pilt \ et \ beam \ et \ supt}) \ \text{modl} \ 1 \ 'ROUG'; \]

• "Construction" of the deformed mesh to be plot

\[ \text{trac} \ \text{pvz} \ (\text{disl \ et \ dzero}); \]
\[ \text{trac} \ \text{pvl} \ (\text{disl \ et \ dzero}); \]

• See the pictures in Figures B.2 to B.5

\[ \text{fin} \ \text{BL1}; \]
\[ \text{finsi}; \]

*---------------------------------------------------------------
* PERMANENT FORCES
*---------------------------------------------------------------
\[ \text{chpop} = \text{manu} \ \text{chpo} \ \text{l1t} \ 1 \ 'UZ' \ (-9.81); \]
\[ \text{stafo} = \text{masto} * \ \text{chpop}; \]
\[ \text{forct} = \text{exco} \ 'FZ' \ \text{stafo} \ 'FZ'; \]

• Computation of the static force due to the masses

*---------------------------------------------------------------
* STATIC deformations
*---------------------------------------------------------------
\[ \text{stdef} = \text{reso} \ \text{rigto} \ \text{forct}; \]

• Computation of the displacements for the static force "forct"

\[ \text{sdvis} = \text{defo} \ \text{l1t} \ \text{stdef} \ 300; \]
\[ \text{trac} \ \text{pvl} \ \text{sdvis}; \]

• Deformed structure. See the picture in Figure B.6

\[ \text{list} \ (\text{extr} \ \text{stdef} \ 'UZ' \ \text{po44}); \]

• Lists the displacement at point "po14" (-2.336mm)

*
Figure B.1 - The three-dimensional structure

Figure B.2 - The first vibration mode (top view). Superimposition on the structure
Figure B.3 - The first vibration mode (another view). Superimposition on the structure

Figure B.4 - The second vibration mode (top view). Superimposition on the structure
Figure B.5 - The second vibration mode (another view). Superimposition on the structure

Figure B.6 - The deformed structure
C NON LINEAR DYNAMIC ALGORITHMS

C.1 NEWMARK FORMULATION

The formulation used in the non-linear dynamic numerical analyses presented in this work is the Newmark method [53]. It is an implicit step-by-step time integration algorithm that for non-linear computations becomes iterative. The velocity and displacement at time step \((t+\Delta t)\) depend on the characteristics of the motion at time step \(t\) and on the acceleration at the time step being analysed

\[
\begin{align*}
v_{t+\Delta t} &= v_t + [\gamma \cdot a_{t+\Delta t} + (1 - \gamma) \cdot a_t] \cdot \Delta t \\
d_{t+\Delta t} &= d_t + v_t \cdot \Delta t + \left[\beta \cdot a_{t+\Delta t} + \left(\frac{1}{2} - \beta\right) \cdot a_t\right] \cdot \Delta t^2
\end{align*}
\]  \hspace{1cm} \text{(C.1)}

where, \(a, v\) and \(d\) are, respectively, the acceleration, the velocity and the displacement vectors and \(\gamma\) and \(\beta\) are two parameters that tune the weight of the acceleration in the velocity and displacement expressions.

Knowing the dynamic equilibrium at time step \(t\)

\[
[M] \cdot a_t + [C] \cdot v_t + r_t = f_t
\]  \hspace{1cm} \text{(C.2)}

one can write the incremental equations in the time interval \([t, t+\Delta t]\)

\[
[M] \cdot \Delta a_{t+\Delta t} + [C] \cdot \Delta v_{t+\Delta t} + [K]_t \cdot \Delta d_{t+\Delta t} = \Delta f_{t+\Delta t}
\]  \hspace{1cm} \text{(C.3)}

where \([M]\) and \([C]\) are the mass and the damping matrices, \([K]_t\) is the tangent stiffness matrix at time step \(t\), \(r_t\) are the restoring forces in the structure at the same time step and \(\Delta f_{t+\Delta t}\) is the incremental loading vector in the time period under analysis.
To solve this equation, the acceleration and the velocity are expressed in terms of the incremental displacements in the time period \([t, t+\Delta t]\)

\[
\Delta a_{t+\Delta t} = a_{t+\Delta t} - a_t = -\frac{1}{\beta \cdot \Delta t} \cdot v_t - \frac{1}{2 \cdot \beta \cdot \Delta t^2} \cdot \Delta d_{t+\Delta t}
\]

\[
\Delta v_{t+\Delta t} = v_{t+\Delta t} - v_t = -\frac{\gamma}{\beta} \cdot v_t - \left( \frac{1}{2 \cdot \beta} - 1 \right) \cdot a_t \cdot \Delta t + \frac{\gamma}{\beta \cdot \Delta t} \cdot \Delta d_{t+\Delta t}
\]

where \((\Delta d_{t+\Delta t} = d_{t+\Delta t} - d_t)\). In the computations performed in this work, the two parameters assume values \((\gamma = 1/2)\) and \((\beta = \gamma/2)\). This fits the hypothesis of a constant acceleration equal to the mean value in the time period under analysis, \(((a_{t+\Delta t} + a_t)/2)\), i.e. a linear variation of the velocity and a quadratic variation of the displacement in the same time period.

Knowing the dynamic equilibrium at time step \(t\)

\[
[M] \cdot a_t + [C] \cdot v_t + r_t = f_t
\]

one can write the incremental equations in the time interval \([t, t+\Delta t]\)

\[
[M] \cdot \Delta a_{t+\Delta t} + [C] \cdot \Delta v_{t+\Delta t} + [K]_t \cdot \Delta d_{t+\Delta t} = \Delta f_{t+\Delta t}
\]

Substituting the \(\Delta\) quantities by the values into equation (C.4), equation (C.6) becomes

\[
[K]_t \cdot \Delta d_{t+\Delta t} = \Delta \tilde{f}_{t+\Delta t}
\]

where \([K]_t\) is the effective stiffness matrix and \(\Delta \tilde{f}_{t+\Delta t}\) the effective loading vector:

\[
[K]_t = [K]_t + \frac{2}{\Delta t} [C] + \frac{4}{\Delta t^2} [M]
\]

\[
\Delta \tilde{f}_{t+\Delta t} = \Delta f_{t+\Delta t} + 2 \cdot [C] \cdot v_t + [M] \cdot \left( \frac{4}{\Delta t} \cdot v_t + 2 \cdot a_t \right)
\]

Knowing from equation (C.5) that \((2 \cdot [M] \cdot a_t + 2 \cdot [C] \cdot v_t = 2 \cdot (f_t - r_t))\), the effective loading vector in equation (C.8) becomes

\[
\Delta \tilde{f}_{t+\Delta t} = f_{t+\Delta t} + \tilde{f}_t - 2 \cdot r_t + \frac{4}{\Delta t} \cdot [M] \cdot v_t
\]
After solving equation (C.7), the restoring forces at time step \((t+\Delta t)\) are computed using the displacements \((d_{t+\Delta t} = d_t + \Delta d_{t+\Delta t})\) and the subroutine with the non-linear behaviour model of the structure, and the equilibrium is checked.

Substituting the tangent stiffness matrix in equation (C.7) by the incremental restoring forces

\[
[K]_t \cdot \Delta d_{t+\Delta t} = r_{t+\Delta t} - r_t \quad \text{C.10}
\]

the residue is given by

\[
\Gamma = \Delta \hat{f}_{t+\Delta t} - (r_{t+\Delta t} - r_t) - \left(\frac{2}{\Delta t} \cdot [C] + \frac{4}{\Delta t^2} \cdot [M]\right) \cdot \Delta d_{t+\Delta t} \quad \text{C.11}
\]

For the equilibrium to be respected this value has to be inferior to a maximum tolerance, "Toler". Otherwise, an iterative process is required; the residue substitutes the loading vector in equation (C.7) and a new incremental displacement is computed. If the quantities at the \(i^{th}\) iteration are referred to by the index \(i\), the equation of equilibrium becomes

\[
[K]_t \cdot (\Delta d_{t+\Delta t})_{i+1} = \Gamma_i \quad i = 1, 2, \ldots \quad \text{C.12}
\]

where

\[
(\Delta d_{t+\Delta t})_1 = 0 \quad \text{C.13}
\]

\[
(r_{t+\Delta t})_1 = r_t \quad \text{C.13}
\]

the displacements at the end of iteration \((i+1)\) being given by

\[
(d_{t+\Delta t})_{i+1} = (d_{t+\Delta t})_i + (\Delta d_{t+\Delta t})_{i+1} \quad \text{C.14}
\]

The restoring forces are then re-calculated for the new displacements and a new residue \(\Gamma_{i+1}\) is computed,

\[
\Gamma_{i+1} = \Gamma_i - ((r_{t+\Delta t})_{i+1} - (r_{t+\Delta t})_i) - \left(\frac{2}{\Delta t} \cdot [C] + \frac{4}{\Delta t^2} \cdot [M]\right) \cdot (\Delta d_{t+\Delta t})_{i+1} \quad \text{C.15}
\]

The process stops when

\[
\Gamma_i < \text{Toler} \quad \text{C.16}
\]
C.2 $\alpha$-NEWMARK FORMULATION USING THE OPERATOR-SPLITTING METHOD

The algorithm that was used in the Pseudo-dynamic tests performed in the ELSA laboratory at Ispra is the $\alpha$-Newmark formulation with the Operator-Splitting scheme [16]. Knowing the characteristics of the motion at time step $t$, the value of the displacement and velocity at time step ($t+\Delta t$) is computed in two steps:

- the predictor phase,

\[
\tilde{d}_{t+\Delta t} = d_t + \Delta t \cdot v_t + \Delta t^2 \cdot \left(\frac{1}{2} - \beta\right) \cdot a_t
\]

\[
\tilde{v}_{t+\Delta t} = v_t + \Delta t \cdot (1 - \gamma) \cdot a_t
\]

- and the corrector phase,

\[
d_{t+\Delta t} = \tilde{d}_{t+\Delta t} + \Delta t^2 \cdot \beta \cdot a_{t+\Delta t}
\]

\[
v_{t+\Delta t} = \tilde{v}_{t+\Delta t} + \Delta t \cdot \gamma \cdot a_{t+\Delta t}
\]

In the $\alpha$-Newmark method the equilibrium at time step ($t+\Delta t$) is given by

\[
[M] \cdot a_{t+\Delta t} + (1 + \alpha) \cdot [C] \cdot v_{t+\Delta t} - \alpha \cdot [C] \cdot v_t + (1 + \alpha) \cdot r_{t+\Delta t} - \alpha \cdot r_t
\]

\[
= (1 + \alpha) \cdot f_{t+\Delta t} - \alpha \cdot f_t
\]

where the meaning of each symbol, except for the Greek letters, can be found in section C.1 and the parameters $\beta$ and $\gamma$ are given by

\[
\beta = (1 - \alpha)^2 / 4
\]

\[
\gamma = (1 - 2 \cdot \alpha) / 2
\]

and ($\alpha \in [-1/3, 0]$). This parameter tunes the numerical damping of the $\alpha$-Newmark method. As in the Newmark formulation described in section C.1, if the behaviour of the structure is non-linear the method becomes iterative. However, if the method is implemented with the Operator-Splitting scheme, although it remains implicit for the restoring forces, it does not require any iteration. On the other hand, the stability of the method is maintained.

The Operator-Splitting scheme establishes an approximation for the restoring forces at time step ($t+\Delta t$),
\[ r_{t+\Delta t} = \tilde{r}_{t+\Delta t} + [K]_f \cdot (d_{t+\Delta t} - \tilde{d}_{t+\Delta t}) \]

where the over-symbol "\(\tilde{\cdot}\)" refers to the predictor values. The matrix \([K]_f\) has, however, to be always equal or greater than the actual tangent stiffness of the structure \([K]_t\). Therefore, the value used in the calculations is, in general, the initial elastic stiffness.

This scheme starts with the predictor restoring forces calculated through the predictor displacements and corrects their value applying the stiffness matrix \([K]_t\) to the displacement error, i.e. to the vector that measures the difference between the predictor and corrector displacements.

Substituting the approximation given by equation (C.21) and the displacements and velocities in equation (C.17) and equation (C.18) into equation (C.19), the equation of equilibrium becomes

\[ [M] \cdot a_{t+\Delta t} = \dot{f}_{t+\Delta t} \]

where

\[ [\overline{M}] = [M] + \gamma \cdot \Delta t \cdot (1 + \alpha) \cdot [C] + \beta \cdot \Delta t^2 \cdot (1 + \alpha) \cdot [K]_f \]

and

\[ \dot{f}_{t+\Delta t} = (1 + \alpha) \cdot \dot{f}_{t+\Delta t} - \alpha \cdot f_t + \alpha \cdot \tilde{r}_t - (1 - \alpha) \cdot \tilde{r}_{t+\Delta t} + \alpha \cdot [C] \cdot \tilde{v}_t \]

\[ - (1 + \alpha) \cdot [C] \cdot \tilde{v}_{t+\Delta t} + \alpha \cdot (\gamma \cdot \Delta t \cdot [C] + \beta \cdot \Delta t^2 \cdot [K]_f) \cdot a_t \]
D EXPERIMENTAL BENDING DISPLACEMENTS

In the experiments, the two sides of the piers perpendicular to the loading direction had displacement transducers placed in the vertical direction at different heights along the pier. Displacement transducers were also placed in the horizontal direction to measure the transverse displacements at the level of the vertical transducers. Figure D.1 illustrates schematically the test of a pier with a horizontal force V on the top.

![Diagram showing displacement transducers and symbols for shear and bending displacements](image)

**Figure D.1 - Splitting of shear and bending displacements**

Although the shear and the bending behaviour are coupled, such instrumentation scheme allows the horizontal displacements that are mainly due to the bending rotations be computed separately. Furthermore, since during the experiment the total displacements were also measured, the difference to the bending displacements gives the shear deformation.
According to Figure D.1, the rotation of slice \( i \) is calculated through the vertical displacements measured by the transducers mounted on the two opposite sides of the pier

\[
\theta_i = \frac{(\Delta_l - \Delta_r)_i}{L}
\]

where \( \Delta \) are the displacements measured by the transducers at the slice opposite faces (\( l \)-left, \( r \)-right) and \( L \) is the distance between the two transducers centre line, as illustrated in Figure D.1. The total rotation on the top of the slice \( i \) is given by

\[
\Theta_i = \Theta_{(i-1)} + \theta_i
\]

where the number of the slice grows from the base to the top of the pier. The average curvature of the slice is computed dividing the slice rotation by the slice length \( z_i \)

\[
\phi_i = \frac{(\Delta_l - \Delta_r)_i}{L_i \cdot z_i}
\]

Hence, admitting a constant rotation in the slice equal to its mean value, the flexural displacement at the top of slice \( i \) is given by

\[
x_i = x_{(i-1)} + \frac{\Theta_i + \Theta_{(i-1)}}{2} \cdot h_i
\]

If one is interested in computing the displacements due to the bending rotations in other points than the extremities, a law of rotations must be adopted within the slices. Making \((\Delta \Theta_i = \Theta_i - \Theta_{(i-1)})\) and assuming an \( \alpha \)th degree polynomial for the rotations, it is

\[
\Theta(z) = \Theta_{(i-1)} + \frac{(z - z_{(i-1)})^\alpha}{h_i^\alpha} \cdot \Delta \Theta_i
\]

where \((z_{(i-1)} \leq z \leq z_i)\) is the coordinate along the pier axis (see Figure D.1). The curvatures are expressed by the derivative \( \frac{d}{dz} \Theta(z) \),

\[
\phi(z) = \alpha \cdot \frac{(z - z_{(i-1)})^{(\alpha - 1)}}{h_i^\alpha} \cdot \Delta \Theta_i
\]

The distribution of corresponding displacements is computed through the integral of
equation (D.6) with respect to $z$,

$$x(z) = x_{(i-1)} + (z - z_{(i-1)}) \cdot \Theta_{(i-1)} + \frac{(z - z_{(i-1)})^{(\alpha + 1)}}{\alpha \cdot (\alpha + 1) \cdot h_i^\alpha} \cdot \Delta \Theta_i$$ \hspace{1cm} D.7

If a linear variation is assumed for the rotations, i.e. ($\alpha = 1$), this expression gives at the top of the slice the displacement in equation (D.4).
E TIMOSHENKO BEAM FORMULATION

There are two main approaches to beam and column elements. They are sustained by two alternative theories: the Euler-Bernoulli beam theory and the Timoshenko beam theory. In the Euler-Bernoulli theory, the perpendicularity between the transverse section and the beam axis is always maintained no matter the type of structure and the input loading. Therefore, it is not adequate to structures presenting a high shear versus bending deformation ratio.

Instead, in the Timoshenko theory the transverse strain exists and is independent from the bending rotations. However, both approaches assume that plane sections remain plane after being deformed; the warping of the transverse section due to a non-uniform distribution of shear forces in the cross-section is not taken into account in the formulation. Nevertheless, shape factors can be introduced to account for this phenomenon.

In the next paragraphs the formulation of the Timoshenko beam theory as well as the procedures followed for its implementation in a finite element code are described in detail.

E.1 COMPATIBILITY OF DISPLACEMENTS

In the Timoshenko beam theory although plane sections remain plane after being deformed they do not maintain, necessarily, the initial angle with the beam axis. Therefore, two additional variables, $\beta_y$ and $\beta_z$ to the three rotations and the axial deformation of the element, are required. They correspond to the transverse deformation of the cross-section in the two in-plane directions, $\overline{\sigma_y}$ and $\overline{\sigma_z}$ of the cross-section, according to Figure
If \( U = (U_x, U_y, U_z) \) and \( \Theta = (\Theta_x, \Theta_y, \Theta_z) \) are the displacements and the rotations of the longitudinal axis of the beam \( \overrightarrow{\alpha x} \), the field of displacements of the beam \( (u = (u_x, u_y, u_z)) \) is given by

\[
\begin{align*}
    u_x(x, y, z) &= U_x(x) - y \cdot \Theta_z(x) + z \cdot \Theta_y(x) \\
    u_y(x, y, z) &= U_y(x) - z \cdot \Theta_x(x) \\
    u_z(x, y, z) &= U_z(x) + y \cdot \Theta_x(x)
\end{align*}
\]

E.1

In the case of the Timoshenko beam theory, the rotation of the transverse section is constituted by two components: a bending rotation, given by the derivative of the beam axis deformation line, and a fictitious rotation equivalent to the transverse shear deformation

\[
\Theta_z = \frac{dU_y}{dx} - \beta_y \quad \Theta_y = \beta_z - \frac{dU_z}{dx}
\]

E.2

The strains in the beam are then given by

\[
\begin{align*}
    \varepsilon_x &= \frac{\partial u_x}{\partial x} = \frac{dU_x}{dx} - y \cdot \frac{d\Theta_z}{dx} + z \cdot \frac{d\Theta_y}{dx} \\
    \gamma_{xy} &= \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = - \Theta_z + \frac{dU_y}{dx} - z \cdot \frac{d\Theta_x}{dx} \\
    \gamma_{xz} &= \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = \Theta_y + \frac{dU_z}{dx} + y \cdot \frac{d\Theta_x}{dx}
\end{align*}
\]

E.3

Note that if \( ((d\Theta_z)/(dx) = 0) \), it is \( (\gamma_{xy} = \beta_y) \) and \( (\gamma_{xz} = \beta_z) \).

The formulation assumes that the lateral stresses \( \sigma_y, \sigma_z \) and \( \tau_{yz} \) are null; in fact, no provisions were taken in equation (E.1) to account for the corresponding deformations. The missing strains are eliminated from the constitutive equations after imposing the stress boundary conditions.

**E.2 CONSTITUTIVE EQUATIONS - AN ELASTIC ISOTROPIC MATERIAL.**

In the case of an isotropic linear elastic material with and elastic modulus \( E \) and a Poisson ratio \( \nu \), the constitutive laws are given by
\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z
\end{bmatrix} = \frac{E}{(1 + \nu) \cdot (1 - \nu)} \cdot \begin{bmatrix}
(1 - \nu) & \nu & \nu \\
\nu & (1 - \nu) & \nu \\
\nu & \nu & (1 - \nu)
\end{bmatrix} \cdot \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z
\end{bmatrix}
\]

E.4

and

\[
\begin{bmatrix}
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix} = \frac{E}{2 \cdot (1 + \nu)} \cdot \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix}
\]

E.5

Imposing the boundary condition (\(\sigma_y = \sigma_z = \tau_{xy} = 0\)), the following relations are found for the strains

\[
\begin{align*}
\varepsilon_y &= \varepsilon_z = -\nu \cdot \varepsilon_x \\
\gamma_{xy} &= 0
\end{align*}
\]

E.6

Figure E.1 - Beam element

Substituting this equation into equation (E.4) and equation (E.5), the constitutive laws become much simpler and a direct relation between each strain and the corresponding stress is established

\[
\begin{bmatrix}
\sigma_x \\
\tau_{xy} \\
\tau_{xz}
\end{bmatrix} = \begin{bmatrix}
E & 0 & 0 \\
0 & G & 0 \\
0 & 0 & G
\end{bmatrix} \cdot \begin{bmatrix}
\varepsilon_x \\
\gamma_{xy} \\
\gamma_{xz}
\end{bmatrix}
\]

E.7
where \((G = E/(2 \cdot (1 + v)))\) is the distortional modulus.

In general, plasticity and damage models associate a generalized strain field \(\varepsilon\) with a generalized stress field \(\Sigma\) through a function \(\mathcal{S}\), \((\Sigma = \mathcal{S}(\varepsilon))\). In the formulation described in this annex, the components of the generalized stress field are

\[
F_x(x) = \int_S \sigma_x(x) dS \\
F_y(x) = \int_S \tau_{xy}(x) dS \\
F_z(x) = \int_S \tau_{xz}(x) dS \\
M_x(x) = \int_S (-z \cdot \tau_{xy}(x) + y \cdot \tau_{xz}(x)) dS \\
M_y(x) = \int_S (z \cdot \sigma_x(x)) dS \\
M_z(x) = \int_S (-y \cdot \sigma_x(x)) dS
\]

where \(F_x\) represents the longitudinal axial force, \(F_y\) and \(F_z\) the transverse forces, \(M_x\) the torsional moment and \(M_y\) and \(M_z\) the bending moments. The strain field is given by

\[
\tilde{\varepsilon} = (e_x, e_y, e_z, c_x, c_y, c_z) = \left(\frac{dU_x}{dx}, \beta_y, \beta_z, \frac{d\theta_x}{dx}, \frac{d\theta_y}{dx}, \frac{d\theta_z}{dx}\right)
\]

where \(e_x\) is the longitudinal strain, \(e_y\) and \(e_z\) the shear strain in both transverse directions, \(c_x\) the torsional curvature and \(c_y\) and \(c_z\) the bending curvatures.

**E.3 EQUILIBRIUM EQUATIONS - VIRTUAL WORK**

Let \(\overline{\sigma}\) be the internal stress vector in equilibrium with the external force vector \(\overline{q}(x)\) applied to the beam and \(\overline{\varepsilon}\) the corresponding strain vector. If the structure is submitted to a set of virtual displacements \(\delta U\), the formulation of the virtual work imposes that

\[
\int_L \int_S (\delta \overline{\varepsilon}' \cdot \overline{\sigma}) dS dL = \int_L (\delta \overline{U}' \cdot \overline{q}) dL
\]

where \(L\) and \(S(x)\) are the length and the cross-sectional area of the beam, respectively. The vector \(\delta \varepsilon\) is the virtual strain in agreement with \(\delta U\). This equation expresses the equilibrium that must exist between the external input energy and the internal energy.
Substituting \( \sigma = [D] \cdot \varepsilon \), established in equation (E.7), into equation (E.10), the equation of equilibrium becomes

\[
\int_S \int_S (\delta \varepsilon^t \cdot [D] \cdot \varepsilon) dS dL = \int_L (\delta U^t \cdot \bar{q}) dL \tag{E.11}
\]

This equation must be respected for all possible virtual displacements compatible with the deformation of the structure.

**E.4 FINITE ELEMENT FORMULATION**

In a finite element method the structure is divided into a finite number of elements that are represented by a particular number of points referred to as nodes. To implement the Timoshenko theory in this formulation it is thus necessary to establish an approximation for the displacements and rotations inside the elements. In the finite element method this is done through shape functions \( N \) that link the values at any point in the beam axis with the values at the nodes \( i \) of the element that contains it. Following the scheme in Figure E.1, it is

\[
U(x) = \sum_i N_i(x) \cdot U_i \tag{E.12}
\]

\[
\theta(x) = \sum_i N_i(x) \cdot \theta_i
\]

In the formulation described in this annex, two nodes are considered for each element, \( (i = 1, 2) \). Moreover, the shape functions are linear, being given by

\[
N_1(x) = \frac{x_2 - x}{L} \quad N_2(x) = \frac{x - x_1}{L} \tag{E.13}
\]

where \( L = x_2 - x_1 \) is the length of the element.

Substituting equation (E.12) and equation (E.13) into equation (E.3), the strain vector is expressed in terms of the node displacements and the node rotations of the element,
\[
\begin{bmatrix}
\varepsilon_x \\
\gamma_{xy} \\
\gamma_{xz}
\end{bmatrix} = \frac{1}{L} \cdot 
\begin{bmatrix}
-1 & 0 & 0 & 0 & -z & y & 1 & 0 & 0 & z & -y \\
0 & -1 & 0 & z & 0 & -A & 0 & 1 & 0 & -z & 0 \\
0 & 0 & -1 & -y & A & 0 & 0 & 0 & 1 & y & -B & 0
\end{bmatrix} \cdot \Phi = [B] \cdot \Phi
\]

where \((A = x_2 - x), (B = x - x_1)\) and \(\Phi\) is the vector with the displacements at the nodes.

\[
\Phi^\prime = \begin{bmatrix}
U_{x_1} & U_{y_1} & U_{z_1} & \theta_{x_1} & \theta_{y_1} & \theta_{z_1} & U_{x_2} & U_{y_2} & U_{z_2} & \theta_{x_2} & \theta_{y_2} & \theta_{z_2}
\end{bmatrix}
\]

Substituting now the relation \((\varepsilon = [B] \cdot \Phi)\) in equation (E.11), and knowing that

\[
\bar{U} = \frac{1}{L} \cdot 
\begin{bmatrix}
A & 0 & 0 & 0 & 0 & B & 0 & 0 & 0 & 0 \\
0 & A & 0 & 0 & 0 & 0 & B & 0 & 0 & 0 \\
0 & 0 & A & 0 & 0 & 0 & 0 & B & 0 & 0 \\
0 & 0 & 0 & A & 0 & 0 & 0 & 0 & B & 0 \\
0 & 0 & 0 & 0 & A & 0 & 0 & 0 & 0 & B
\end{bmatrix} \cdot \Phi = [N] \cdot \Phi
\]

it is

\[
\int_L \int_S (\delta \Phi^\prime \cdot [B]^\prime \cdot [D] \cdot [B] \cdot \Phi) dS dL = \int_L (\delta \Phi^\prime \cdot [N]^\prime \cdot \bar{q}) dL
\]

As equation (E.17) is independent of the set of virtual displacements \(\delta \Phi\) adopted in the structure, the equilibrium is expressed by

\[
\int_L \int_S ([B]^\prime \cdot [D] \cdot [B] \cdot \Phi) dS dL = \int_L ([N]^\prime \cdot \bar{q}) dL
\]

i.e.

\[
[K] \cdot \Phi = \bar{F}
\]

where \([K]\) represents the stiffness matrix and \(\bar{F}\) the vector of the equivalent applied forces on the element nodes.

The exact integral of the first member of equation (E.18) can be evaluated using a two-point Gauss-Legendre rule along the length of the element. However, and in particular
when the thickness of the beam becomes very small in one direction, such integration scheme may lead to an over-stiff solution. Hughes et al [40] demonstrated that the problem could be solved by using a 1-point Gauss-Legendre rule. This also corresponds to a selective integration on the components of the stiffness matrix involving the shear deformation.

Note that the only condition imposed on the 3 perpendicular axis defining the structure, \( \bar{o}\alpha, \bar{o}\gamma, \bar{o}\zeta \) in Figure E.1, is that the \( \bar{o}\alpha \) axis is parallel to the longitudinal axis of the beam. However, this axis can be eccentric to the axis of gravity of the element.

The stiffness matrix is symmetric and is given by

\[
[K] = \frac{1}{L} \begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} \\
K_{12} & K_{22} & K_{23} & K_{24} \\
K_{13} & K_{23} & K_{33} & K_{34} \\
K_{14} & K_{24} & K_{34} & K_{44}
\end{bmatrix}
\]

E.20

where

\[
[K_{11}] = \begin{bmatrix}
\int_S E dS & 0 & 0 \\
0 & \int_S G_\gamma dS & 0 \\
0 & 0 & \int_S G_\zeta dS
\end{bmatrix}
\]

E.21

\[
[K_{12}] = \begin{bmatrix}
0 & \int_S (E \cdot z) dS - \int_S (E \cdot y) dS \\
-\int_S (G_\gamma \cdot z) dS & 0 & \frac{l}{2} \cdot \int_S G_\gamma dS \\
\int_S (G_\zeta \cdot y) dS - \frac{l}{2} \cdot \int_S G_\zeta dS & -\frac{l}{2} \cdot \int_S G_\zeta dS & 0
\end{bmatrix}
\]

E.22
\[
\begin{bmatrix}
\int S(G_z \cdot y^2 + G_y \cdot z^2) dS & -\frac{l}{2} \cdot \int S(G_z \cdot y) dS & -\frac{l}{2} \cdot \int S(G_y \cdot z) dS \\
-\frac{l}{2} \cdot \int S(G_z \cdot y) dS & \int S \left( \frac{l^2}{4} \cdot G_z + E \cdot z^2 \right) dS & -\int S(E \cdot z \cdot y) dS \\
-\frac{l}{2} \cdot \int S(G_y \cdot z) dS & -\int S(E \cdot z \cdot y) dS & \int S \left( \frac{l^2}{4} \cdot G_y + E \cdot y^2 \right) dS
\end{bmatrix}
\]  
E.23

\[
\begin{bmatrix}
\int S(G_y \cdot z) dS & 0 & \frac{l}{2} \cdot \int S G_y dS \\
-\int S(G_z \cdot y) dS & -\frac{l}{2} \cdot \int S G_z dS & 0 \\
-\int S(E \cdot z) dS & \frac{l}{2} \cdot \int S G_z dS & 0 \\
\end{bmatrix}
\]  
E.24

\[
\begin{bmatrix}
\int S(G_y \cdot z) dS & -\int S(G_z \cdot y) dS \\
-\int S(E \cdot z) dS & 0 & \frac{l}{2} \cdot \int S G_z dS \\
\int S(E \cdot y) dS & -\frac{l}{2} \cdot \int S G_y dS & 0 \\
\end{bmatrix}
\]  
E.25

\[
\begin{bmatrix}
-\int S(G_z \cdot y^2 + G_y \cdot z^2) dS & -\frac{l}{2} \cdot \int S(G_z \cdot y) dS & -\frac{l}{2} \cdot \int S(G_y \cdot z) dS \\
\frac{l}{2} \cdot \int S(G_z \cdot y) dS & \int S \left( \frac{l^2}{4} \cdot G_z - E \cdot z^2 \right) dS & \int S(E \cdot z \cdot y) dS \\
-\frac{l}{2} \cdot \int S(G_y \cdot z) dS & \int S(E \cdot z \cdot y) dS & \int S \left( \frac{l^2}{4} \cdot G_y - E \cdot y^2 \right) dS
\end{bmatrix}
\]  
E.26

\[
\begin{bmatrix}
0 & \frac{l}{2} \cdot \int S(G_z \cdot y) dS & -\frac{l}{2} \cdot \int S(G_y \cdot z) dS \\
-\int S(G_z \cdot y) dS & 0 & \frac{l}{2} \cdot \int S G_y dS \\
\int S(G_z \cdot y) dS & \frac{l}{2} \cdot \int S G_z dS & 0 \\
\end{bmatrix}
\]  
E.27
\[
\begin{bmatrix}
\int_S (G_z \cdot y^2 + G_y \cdot z^2) \, dS
& \frac{1}{2} \cdot \int_S (G_z \cdot y) \, dS
& -\frac{1}{2} \int_S (G_y \cdot z) \, dS

\frac{1}{2} \cdot \int_S (G_z \cdot y) \, dS
& \int_S (\frac{I^2}{4} \cdot G_z + E \cdot z^2) \, dS
& -\int_S (E \cdot z \cdot y) \, dS

-\frac{1}{2} \int_S (G_y \cdot z) \, dS
& -\int_S (E \cdot z \cdot y) \, dS
& \int_S (\frac{I^2}{4} \cdot G_y + E \cdot y^2) \, dS
\end{bmatrix}
\]

E.28

and

\[
\begin{bmatrix}
K_{13}
\end{bmatrix} = -\begin{bmatrix}
K_{11}
\end{bmatrix} \quad \begin{bmatrix}
K_{33}
\end{bmatrix} = \begin{bmatrix}
K_{11}
\end{bmatrix}
\]

E.29

The values \(G_y\) and \(G_z\) represent the distortional modulus \(G\) multiplied by a shape factors \((\alpha_y, \alpha_z \leq 1)\), respectively, to account for the warping of the cross-section.

In the case of Takeda-like models, the constitutive laws represent the relation moment versus curvature decoupled in the two main directions of the transverse section. Thus, a new constitutive equation is established

\[
\begin{bmatrix}
M_y
M_z
\end{bmatrix} = \begin{bmatrix}
(E \cdot I)_y & 0 \\
0 & (E \cdot I)_z
\end{bmatrix} \cdot \begin{bmatrix}
c_y \\
c_z
\end{bmatrix} = \begin{bmatrix}
D_b
\end{bmatrix} \cdot \begin{bmatrix}
c_y \\
c_z
\end{bmatrix}
\]

E.30

where \(I\) is the moment of inertia of the transverse section, \(E \cdot I\) represents the bending stiffness and

\[
\begin{bmatrix}
c_y \\
c_z
\end{bmatrix} = \begin{bmatrix}
d\theta_y \\
d\theta_z
\end{bmatrix} = \begin{bmatrix}
dN_1 \\
dN_2
\end{bmatrix} \cdot \begin{bmatrix}
\theta_{y_1} \\
\theta_{y_2} \\
\theta_{z_1} \\
\theta_{z_2}
\end{bmatrix}
\]

E.31

i.e.

\[
\begin{bmatrix}
c_y \\
c_z
\end{bmatrix} = \frac{1}{L} \cdot \begin{bmatrix}
-1 & 0 & 1 & 0 \\
0 & -1 & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
\theta_{y_1} \\
\theta_{z_1} \\
\theta_{y_2} \\
\theta_{z_2}
\end{bmatrix} = \begin{bmatrix}
B_b
\end{bmatrix} \cdot \Phi_b
\]

E.32
Following a scheme analogous to the one used to deduce equation (E.18), a new stiffness matrix is found

\[
[K_b] = \int_L \left( [B_b]^t \cdot [D_b] \cdot [B_b] \right) dL
\]

that corresponds to

\[
[K_b] = \frac{1}{L} \cdot \begin{bmatrix}
(E \cdot I)_y & 0 & -(E \cdot I)_y & 0 \\
0 & (E \cdot I)_z & 0 & -(E \cdot I)_z \\
-(E \cdot I)_y & 0 & (E \cdot I)_y & 0 \\
0 & -(E \cdot I)_z & 0 & (E \cdot I)_z
\end{bmatrix}
\]

and where the components

\[
(E \cdot I)_y = \int_S (E \cdot z^2) dS
\]

\[
(E \cdot I)_z = \int_S (E \cdot y^2) dS
\]

are those expressed in equations (E.23), (E.26) and (E.28).

**E.5 TANGENT STIFFNESS MATRIX**

In general, the algorithms used to solve non-linear behaviour problems require the knowledge of the tangent stiffness. Referring to equation (E.20), the tangent matrix can be calculated by substituting the Young modulus $E$ and the distortional modulus $G$ by their tangent values. If the tangent modulus are not available, equivalent tangent parameters should be computed.

In the case of Takeda like models, the tangent stiffness matrix is build substituting the values $(E \cdot I)$ in equation (E.34) by the corresponding tangent values from the moment versus curvature law.

**E.6 MASS MATRIX**

To perform dynamic computations, it is also necessary to formulate the mass matrix in the same set of d.o.f. of the stiffness matrix. Returning to equation (E.10), the inertial
forces have to be added to the first member

\[ W = \int_L \int_S \left( \delta \ddot{U}^t \cdot \rho \cdot \frac{d}{dt^2} (\ddot{U}^2) \right) dS dL \]  \hspace{1cm} \text{E.36}

where \( \rho \) is the density of the material.

Substituting equation (E.16) into equation (E.36), the expression becomes

\[ W = \int_L \int_S \left( \rho \cdot \delta \ddot{\Phi} \cdot [N]^t \cdot [N] \cdot \frac{d\Phi}{dt^2} \right) dS dL \]  \hspace{1cm} \text{E.37}

Since equation (E.17), now with the inertial forces included, is independent of the set of virtual displacements \( \delta \Phi \), the equilibrium within the finite element gives

\[ \int_V ([B]^t \cdot [D] \cdot [B]) dV \cdot \ddot{\Phi} + \int_V ([N]^t \cdot \rho \cdot [N]) dV \cdot \frac{d\Phi}{dt^2} = \int_L ([N]^t \cdot \ddot{q}) dL \]  \hspace{1cm} \text{E.38}

where \( V \) is the volume of the element. Thus, the mass matrix is given by

\[ M = \int_V ([N]^t \cdot \rho \cdot [N]) dV \]  \hspace{1cm} \text{E.39}

i.e.

\[
M = \begin{bmatrix}
[M_{11}] & [M_{12}] & [M_{13}] & [M_{14}]
\end{bmatrix}
\begin{bmatrix}
[M_{12}]
[M_{22}]
[M_{23}]
[M_{24}]
\end{bmatrix}
\begin{bmatrix}
[M_{13}]
[M_{23}]
[M_{33}]
[M_{34}]
\end{bmatrix}
\begin{bmatrix}
[M_{14}]
[M_{24}]
[M_{34}]
[M_{44}]
\end{bmatrix}
\]  \hspace{1cm} \text{E.40}

where

\[
[M_{11}] = [M_{22}] = \begin{bmatrix}
Q_{11} & 0 & 0 \\
0 & Q_{11} & 0 \\
0 & 0 & Q_{11}
\end{bmatrix}
\]  \hspace{1cm} \text{E.41}
\[
[M_{13}] = [M_{24}] = \begin{bmatrix}
Q_{12} & 0 & 0 \\
0 & Q_{12} & 0 \\
0 & 0 & Q_{12}
\end{bmatrix} \quad \text{E.42}
\]

\[
[M_{33}] = [M_{44}] = \begin{bmatrix}
Q_{22} & 0 & 0 \\
0 & Q_{22} & 0 \\
0 & 0 & Q_{22}
\end{bmatrix} \quad \text{E.43}
\]

\[
[M_{12}] = [M_{14}] = [M_{23}] = [M_{34}] = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \quad \text{E.44}
\]

and \( \int_V (N_j \cdot \rho \cdot N_k) dV \). For the shape factors considered in this analysis it is \( Q_{11} = Q_{22} = (\rho \cdot V) / 3 \) and \( Q_{12} = Q_{11} / 2 \).