FACULDADE DE ECONOMIA DO PORTO

MACROECONOMICS OF INVESTMENT DYNAMICS AND FINANCING

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TESE DE DOUTORAMENTO EM ECONOMIA

Porto, 2011

To Aurelia and Renato,
my parents

To Silvia and Riccardo,
my sister and brother

Vita

Fabio Verona was born on the 25 of August of 1978, in Brescia, Italy.

He graduated in Engineering from the *Facoltá di Ingegneria dell'Universitá degli Studi di Brescia* (Italy) in 2005. He has been a Ph.D. student at the *Faculdade de Economia da Universidade do Porto* since 2005. He was a visiting scholar at the Department of Economics of Columbia University (New York) from August 2009 to July 2010.

Summary

This thesis consists of three essays that share a common goal of analyzing investment decisions and their macroeconomic implications. In particular, the thesis focuses on two distinct issues building on two alternative theoretical frameworks. Chapter 1 examines the role of distortions in investment financing in the transmission of monetary policy using a sticky-prices dynamic stochastic general equilibrium (DSGE) model. Chapters 2 and 3 analyze the role of information costs in shaping investment dynamics, respectively in a partial equilibrium framework and in a sticky-information general equilibrium model.

The model developed in chapter 1 adds a microfounded shadow banking sector to a standard sticky-prices DSGE model. In contrast with the standard retail banking sector, the shadow banking sector is characterized by optimism and perverse incentives that could lead financial intermediaries to underwrite bonds at a discounted rate, diverting a fraction of the bank's stockholders profits for their own benefits. The model predicts that the combination of a persistent expansionary monetary policy with such microeconomic distortions in the financial system causes a boom-bust cycle.

The model developed in chapter 2 provides an alternative explanation for the well-established fact that microeconomic investment is lumpy. The traditional explanation relies on the assumption that firms face non-convex (fixed) capital adjustment costs. The alternative explanation suggested relies on inattentiveness, whereby firms make infrequent investment decisions due to the costs of gathering and processing information and making optimal plans. Introducing such information/planning costs into a frictionless investment model induces lumpy capital adjustments. The model fits the quantitative facts on plant-level investment rates remarkably well.

In chapter 3, capital investment decisions with inattentiveness are embedded in a DSGE model, assuming information costs as the only source of rigidity. The resulting model features pervasive inattentiveness, as consumption, wages, prices and capital investment decisions are all based, to some degree, on outdated information sets. The model yields aggregate dynamics substantially different from those of an otherwise identical model with frictionless investment, and much closer to the empirical evidence. These results thus strengthen the case for the relevance of lumpy micro-level investment for the business cycle.

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Chapter 1

Monetary policy shocks in a DSGE model with a shadow banking system[1]

This chapter is motivated by the recent financial crisis and addresses whether a "too low for too long" interest rate policy may generate a boom-bust cycle. We suggest a model in which a microfounded shadow banking sector is included in an otherwise state-of-the-art DSGE model. When faced with perverse incentives, financial intermediaries within the shadow banking sector can divert a fraction of stockholders' profits for their own benefits and extend credit at a discounted rate. The model predicts that long periods of accommodative monetary policy do create the preconditions for, but do not cause *per se*, a boom-bust cycle. Rather, it is the combination of a persistent monetary ease with microeconomic distortions in the financial system that causes a boom-bust.

[1] This is a joint work with Manuel M. F. Martins and Inês Drumond. We thank José Peres Jorge, Ricardo Reis and Michael Woodford for useful comments, as well as seminar participants at the University of Porto and Columbia University. I am grateful to the Fundação para a Ciência e a Tecnologia for financial support (Ph.D. scholarship).

1.1 Introduction

The Fed funds rate: too low for too long? Some observers have recently criticized the Fed for helping fuel the credit/house-price boom and thereby the subprime crisis by keeping interest rates too low for too long. If correct, this criticism would have important implications for the future conduct of monetary policy. Unfortunately, conventional dynamic stochastic general equilibrium (DSGE) models are not well suited to address this issue because of their rather simple modelling structure of the financial system and of its relation with the real economy. In particular, although financial intermediaries have been at the center of the subprime crisis, they have played so far a relatively passive role in macroeconomic models.

This chapter aims to determine whether long periods of loose monetary policy may have a part to play in generating a boom-bust cycle. We do so by building a DSGE model with a two-sector financial system – a retail banking sector and a "shadow" banking sector in which there may exist optimism and perverse incentives. Such a model, in which we explicitly model the behavior of financial intermediaries within the shadow banking system, aims at providing further insights into the transmission of monetary policy.

The chapter is organized as follows. Section 1.2 briefly reviews the two strands of literature on which our work builds, one on the causes of the subprime crisis and one on the role of the financial sector in DSGE models. Section 1.3 provides a general descriptive overview of our model. Section 1.4 describes the financial system and, in particular, the modelling of the shadow banking system based on microeconomic foundations. Section 1.5 details the calibration of the model and presents the impulse responses in both a) a one-period expansionary monetary policy shock and b) a "persistently low interest rate" scenario. Section 1.6 concludes.[2]

The central result of the chapter is that a "too low for too long" interest rate policy does create the preconditions for, but does not cause *per se*, a boom-bust cycle. In fact, fluctuations in both real and financial variables are markedly amplified only when a persistently accommodative monetary policy environment is coupled with perverse incentives in the financial sector.

[2] Appendix A presents the complete model, while technical details are described in appendices B, C and D.

1.2 Motivation

1.2.1 The subprime crisis

Following the 2007 collapse of the U.S. subprime mortgage market and the resulting global financial and economic crisis, several authors have discussed the causes and consequences of the house price bubble and the boom-bust cycle. These analyses, either coming from the academia (*e.g.*, Borio, 2008 and Blanchard, 2009) or from policy-makers (*e.g.*, Trichet, 2009, Bean et al., 2010 and Bernanke, 2010), have overall concluded that the seeds of the crisis lay in a combination of both micro and macro factors.[3]

Microeconomic factors are mostly related to recent innovations in financial instruments, institutions and markets. A non-exhaustive list of these factors includes: the reduced incentives for lenders to properly screen and monitor borrowers due to pay packages encouraging the pursuit of short-term returns; the under-estimation of the true risk of complex (and often not transparent) structured financial products arising from the replacement of sound risk management practices with mathematical and statistical models of risk; the distorted incentives faced by ratings agencies; the moral hazard behavior of financial institutions considered too big or too important to fail; and, additionally, an inadequate regulation and supervision of individual financial institutions and markets and of the financial system as a whole.

Potential macroeconomic factors include a protracted period of very low (and in some cases negative) real interest rates and plentiful liquidity; large international payments imbalances resulting from a "savings glut" in surplus countries; and the benign macroeconomic environment at the beginning of the 21*st* century as side effect of the Great Moderation.

Evidence for assigning a central role, as cause of the subprime crisis, to excessively loose monetary policy is, nevertheless, mixed. To date, the 2010 Jackson Hole Symposium provides the most recent debate on this issue. On the one hand, Bean et al. (2010) argue that low policy rates played only a modest direct role. As they state, "although monetary policy may have played a role in the credit/house-price boom that preceded the crisis, it is rather more Rosencrantz than Hamlet." On the other hand, Taylor (2010) disagrees that the role of monetary policy was only a modest one without implications for future policy.

In this chapter we do not attempt to explicitly model the subprime crisis. First, it is unlikely that any of the aforementioned factors in isolation could explain the crisis. Second, it would

[3] See Borio (2008), Brunnermeier (2009) and FED (2009) for a chronology of the events relating to the subprime crises.

be too complex to comprise all of them in a DSGE model. Nevertheless, we do try to capture some micro factors (related in particular with the behavior of financial intermediaries) relevant to analyzing whether the Fed's policy to keep interest rates low for a prolonged period may have played a key role for the run-up of the crisis. To explore this hypothesis, we rely on a model in which the financial sector, rather than being passive, plays a central role in driving the boom-bust cycle.

1.2.2 Financial system in DSGE models

The DSGE model, currently the state-of-the-art macroeconomic model, results from a fusion of the Real Business Cycle models of the 1980s with the New Keynesian sticky-price models of the early 1990s. In its primordial version, this model incorporated no role for credit and financial factors at all. Works that followed have continued to assume frictionless financial markets so that financial intermediaries played a passive role, despite of the increasing awareness about their importance in affecting the performance of the economy, including though the transmission of monetary policy. For example, in the DSGE models currently used for monetary policy analysis at the main central banks – *e.g.*, the SIGMA model at the FED (Erceg et al., 2006), the Smets and Wouters model at the ECB (Smets and Wouters, 2003) and the Bank of England's Quarterly Model (Harrison et al., 2005) – the financial sector hardly plays a prominent role.

A first attempt to introduce a financial sector in a New Keynesian DSGE framework has been made by Bernanke et al. (1999). In their model, the financial sector is limited to a banking sector that amplifies the effects of the shocks *via* the financial accelerator effect. More recently, some authors have enhanced the structure and role of the financial sector in DSGE models. Iacoviello (2005) extends the Bernanke et al. (1999) model by introducing collateral constraints for firms, as in Kiyotaki and Moore (1997). Christiano et al. (2003, 2008, 2010) and Goodfriend and McCallum (2007) consider a perfectly competitive banking sector that offers agents a variety of financial assets with different returns, while Kobayashi (2008) and Gerali et al. (2010) consider imperfect competition in the banking sector so as to model the setting of interest rates by banks. Cúrdia and Woodford (2010) also allow for a time-varying spread between deposits and lending rates. Finally, a number of papers (see, for instance, Van den Heuvel, 2008, Gertler and Karadi, 2009, de Walque et al., 2010 and Meh and Moran, 2010) study the role of bank capital in the transmission of macroeconomic shocks.[4,5]

While most of the literature focuses on financial frictions that arise from the behavior of borrowers, the subprime crisis has highlighted the need to analyze the behavior of financial intermediaries themselves. In this chapter we take a step toward determining whether the financial sector plays an active role in the boom-bust cycle. We do so by augmenting the Christiano et al. (2010) model with a shadow banking system. Our microfounded financial system is thus composed of two different financial intermediaries (retail and investment banks) that intermediate funds from households (lenders) to two groups of entrepreneurs (borrowers).

Following the Bernanke et al. (1999) framework, in the retail banking sector there is an "agency / information" problem between borrowers and lenders. Information is asymmetric, in that the entrepreneur's realized return may be observed at no cost only by the entrepreneur, while it can be observed by the retail bank only after paying a monitoring cost. Thus, the model is of the costly state verification type.

In the shadow banking sector we introduce an "agency / money" problem, in that the investment bank manager may pursue his own private objectives, which need not coincide with those of the stockholders. This problem arises because the manager faces perverse incentives – in the form of side payments – to boost his private revenue at the expense of stockholders' profits, *i.e.* the bank manager can divert a fraction of stockholders' profits for his own benefit.

We then use the model to address the following questions:

1. How do perverse incentives in the financial sector affect the transmission of monetary policy shocks through the economy? How different are our findings from those of a workhorse DSGE model?

2. Does a "too low for too long" interest rate policy cause a boom-bust cycle?

3. What are the effects of perverse incentives in the financial sector when coupled with a persistently low interest rate environment?

[4] Drumond (2009) provides an exhaustive survey on the theoretical literature on the bank capital channel of propagation of exogenous shocks as well as on the regulatory framework of capital requirements under the Basel Accords.

[5] This brief review only focuses on DSGE models and, in view of the growth of this literature, does not aim to be exhaustive.

1.3 An overview of the model

The core of our framework is a simplified version of the *Financial Accelerator Model* described in Christiano et al. (2010), hereafter CMR.[6] It essentially corresponds to the models in Smets and Wouters (2003) and Christiano et al. (2005) enlarged with the financial accelerator mechanism developed by Bernanke et al. (1999). To this we add a shadow banking system that intermediates funds between households and an additional set of entrepreneurs. The model is thus composed of government, households, firms, capital producers, entrepreneurs, and banks. Figure 1.1 sketches the structure of the model. Agents drawn in black are those already present in the CMR model, while the "new agents" are drawn in blue.

Government expenditures represent a constant fraction of final output and are financed by lump-sum taxes imposed to the households. The central bank sets the nominal interest rate.

Households consume, save and supply labor services monopolistically. Two types of financial instruments, offered by banks, are available to households: time deposits and corporate bonds. To keep this part of the model as simple as possible, we assume that the rate of return is the same for both financial instruments, so households are indifferent between holding deposits or bonds.

On the production side, monopolistically competitive intermediate-good firms use labor (supplied by households) and capital (rented from entrepreneurs) to produce a continuum of differentiated intermediate goods. Perfectly competitive final-good firms buy intermediate goods and produce the final output, which is then converted into consumption, investment and government goods.

Capital producers combine investment goods with undepreciated capital purchased from entrepreneurs to produce new capital, which is then sold back to entrepreneurs.

Capital services are supplied by entrepreneurs, who own the stock of physical capital and choose how intensively to use it. Entrepreneurs purchase capital using their own resources – net worth, or equity, resulting from net proceeds of their activities from one period to the next – as well as external finance. In fact, entrepreneurs' net worth is not enough to finance the full amount of capital they acquire, so they finance a part of their capital expenditures either by issuing bonds or by means of bank loans.

[6] The simplified version excludes long-run growth, the fixed cost in the production function and distortionary taxes on capital and labor income and on household consumption. While not changing the model's dynamic responses to monetary policy shocks, these simplifications reduce its complexity.

The setting up of the shadow banking system is paralleled by the division of the entrepreneurial sector into two groups: the riskier entrepreneurs and the safer entrepreneurs, who have access to two different sources of external funding.[7] We assume that the riskier entrepreneurs obtain financing via retail bank loans, while the safer entrepreneurs issue bonds resorting to investment banks.[8] In particular, we consider the entrepreneurs of the CMR model as riskier because they may default (since they face an idiosyncratic productivity shock), while we consider the additional set of entrepreneurs as safer because we assume that they always have enough wealth to repay their debt and thus never default. Accordingly, we calibrate the model so as to guarantee that, in equilibrium, safer entrepreneurs finance themselves at a lower interest rate than riskier entrepreneurs.

Lending to riskier entrepreneurs involves an agency/information problem, because they costlessly observe their idiosyncratic shocks, whereas the retail bank must pay a monitoring cost to observe those shocks. The optimal lending contract is of the costly state verification type. In particular, a standard debt contract is set up specifying a loan amount and an interest rate to pay whenever the entrepreneur is solvent. If the entrepreneur cannot pay the required interest because of an unfavorable realization of his productivity shock, he goes into bankruptcy and turns over his remaining equity to the retail bank, after being monitored. The rate of return paid by solvent entrepreneurs must thus be high enough to cover the cost of funds to the bank, as well as the monitoring costs net of the resources that the bank can recover from bankrupt entrepreneurs (for further details see Bernanke et al., 1999).

The investment banking sector is the core part of our shadow banking system.[9] We assume that the bond market is populated by a continuum of monopolistic competitive investment banks, who set the coupon rate on bonds in order to maximize profits, which are then rebated

[7] A number of papers (see, among many others, Diamond, 1991, Chemmanur and Fulghieri, 1994, Holmstrom and Tirole, 1997, Berlin and Loeys, 1988, Bolton and Freixas, 2000, 2006, Repullo and Suarez, 2000, Chakraborty and Ray, 2007, Hale, 2007 and Gerber, 2008) characterize equilibria when bank lending and direct financing through securities issues are both present. Usually, in equilibrium, firms are segmented by risk classes in their choice of funding, with safer firms choosing bond financing and riskier firms preferring bank loans.

[8] Typically, a firm going public hires an investment bank to sell its securities. The investment bank (the underwriter) acts as an intermediary between the issuing firm and the ultimate investors. The most common type of underwriting arrangement is the "firm commitment" underwriting, according to which the underwriter buys the entire stock of bonds from the firm and resells it to investors at a higher price (*i.e.*, at a lower interest rate). This spread represents the investment bank's profits. See Ellis et al. (2000) for an in-depth analysis of the underwriting process.

[9] The expression "shadow banking system" has been suggested originally by Paul McCulley of PIMCO at the 2007 Jackson Hole conference, where he defined it as "the whole alphabet soup of levered up non-bank investment conduits, vehicles, and structures" (McCulley, 2007, pag. 2). See Pozsar et al. (2010) for a comprehensive and up-to-date description of the shadow banking system.

to the stockholders, *i.e.* to the households.[10] Within each investment bank, the agent that makes the decision is the investment bank manager, whom we call henceforth the underwriter.

Two distinct mechanisms – optimism and perverse incentives – are at work in the investment banking sector. First, we consider that an optimistic underwriter is willing to underwrite bonds at a lower – relatively to its "normal" value – coupon interest rate.[11] We assume that the underwriter turns out to be optimistic when the entrepreneur pledges more collateral and, accordingly, we model underwriter's optimism as a positive function of the entrepreneur's net worth. An unexpected increase of the entrepreneur's net worth – as a result of a monetary easing or a favorable productivity shock – triggers optimism and *may* result in a lower bond coupon interest rate. Second, we introduce perverse incentives by assuming that the safer entrepreneur offers side payments to the underwriter in order to borrow at a more favorable interest rate. In exchange of those side payments, an optimistic underwriter may *de facto* facilitate the extension of credit by setting a "discounted" – relatively to the "normal" – bond coupon rate. Defining how much the coupon rate deviates from the normal rate depends upon the underwriter's utility function, in which the trade-off between maximizing his private revenue and the investment bank's profits (hence, the stockholders' profits) is explicitly modeled. An agency conflict between investment bank managers and stockholders arises because side payments represent a compensation for the underwriter to the sacrifice of stockholders' profitability.[12]

Having briefly presented the main features of our model, in the next section we describe the financial system, with particular emphasis on the shadow banking system. The rest of the model is standard in the literature and is set out in appendix A.

[10] The empirical evidence on the U.S. market of bond underwriting suggests an oligopolistic market structure. For example, Fang (2005) shows that the largest five investment banks underwrite more than 60% of all deals, and the largest fifteen banks account for roughly 95% of all deals.

[11] There is considerable evidence that economic agents may be too optimistic. See De Bondt and Thaler (1994) for an exhaustive survey on behavioral finance and Puri and Robinson (2007) for an empirical analysis on how optimism affects economic decisions.

[12] The agency conflict between managers and stockholders is a central theme in the corporate-finance literature (see Stein, 2003, for a survey). The manager-stockholder agency conflict arises because the managers may pursue their own private objectives rather than those of outside stockholders. Studies on conflicts of interest in investment banking industry include, among others, Michaely and Womack (1999) and Mehran and Stulz (2007).

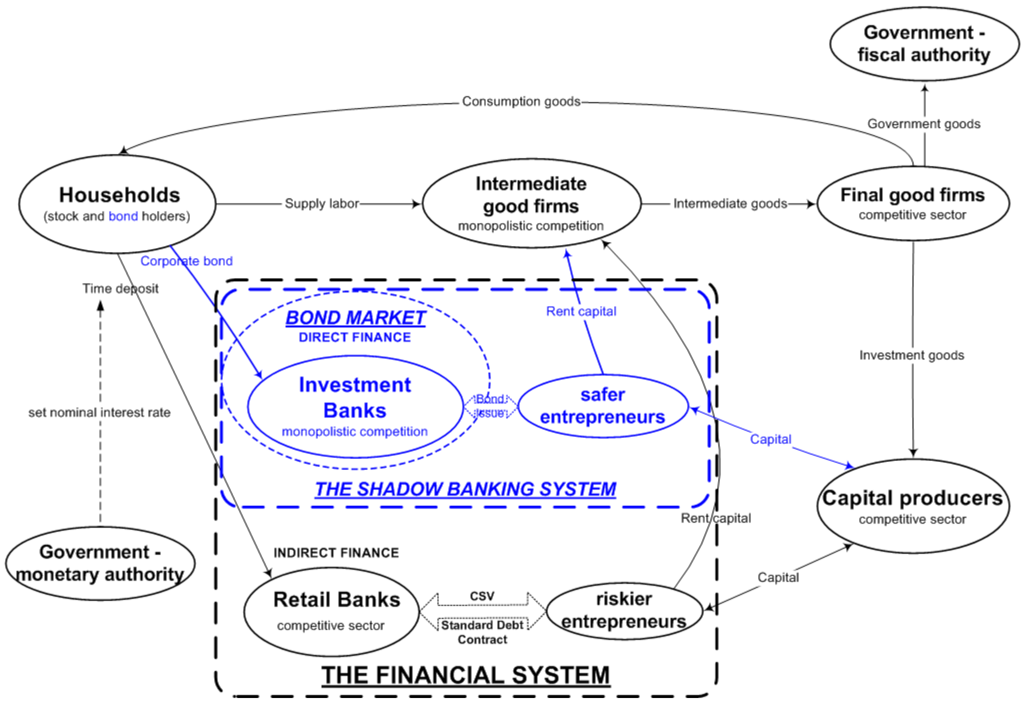

Figure 1.1: Structure of the model

1.4 The financial system

We assume that riskier entrepreneurs represent a fraction η of the total population of entrepreneurs, while safer entrepreneurs represent the remaining fraction, $1-\eta$. In what follows, the superscripts "H" and "H,r" ("L" and "L,l") refer to variables associated with the riskier (safer) entrepreneurs.

1.4.1 Riskier entrepreneurs and retail banks

Riskier entrepreneurs own a share of the economy's stock of physical capital. Entrepreneurs' net worth is enough to finance only a part of their holdings of physical capital, the rest being financed by loans from a representative retail bank. Entrepreneurial loans are risky because the returns on their investments are subject to idiosyncratic shocks. In particular, entrepreneurs who suffer a large unfavorable shock and who therefore cannot pay the required interest, go bankrupt. Financial frictions arise because the idiosyncratic shock is observed by the entrepreneurs at no cost, and by the bank only if it incurs in a fixed monitoring cost. To mitigate costs stemming from this source of asymmetric information, entrepreneurs and bank sign a standard debt contract, according to which the entrepreneur commits to pay back the loan principal and a non-default interest rate, unless he declares default. In case of default, the bank conducts a verification of the residual value of the entrepreneur's assets and takes in all of the entrepreneur's net worth, net of monitoring costs.

The debt contracts extended by the bank to entrepreneurs are financed by bank's issuance of time deposits to households. Although individual entrepreneurs are risky, the bank itself is not: by lending to a large number of entrepreneurs, the bank can diversify the idiosyncratic risk and thus can guarantee a safe return on households' deposits. Nevertheless, financial frictions – reflecting the costly state verification problem between entrepreneurs and the bank – imply that bank hedges against credit risk by charging a premium over the rate at which it can borrow from households.

In particular, as shown by Bernanke et al. (1999), the first order conditions of the contracting problem yield the following relationship linking the expected return on capital ($R^{k,H}_{t+1}$) relative to the risk-free interest rate (R^{e}_{t+1}) and the entrepreneur's leverage ratio ($\frac{Q_{\bar{k}',t}\bar{K}^{H,r}_{t+1}}{N^{H,r}_{t+1}}$):

$$\frac{E_t\left(1+R^{k,H}_{t+1}\right)}{1+R^{e}_{t+1}} = \Psi\left(\frac{Q_{\bar{k}',t}\bar{K}^{H,r}_{t+1}}{N^{H,r}_{t+1}}\right) \quad , \tag{1.1}$$

where $Q_{\bar{k}',t}$, $\bar{K}^{H,r}_{t+1}$ and $N^{H,r}_{t+1}$ denote, respectively, the price of capital, the entrepreneur's stock of capital and the entrepreneur's net worth and the function Ψ is such that $\Psi' > 0$ for $N^{H,r}_{t+1} < Q_{\bar{k}',t}\bar{K}^{H,r}_{t+1}$. The ratio $\frac{E_t\left(1+R^{k,H}_{t+1}\right)}{1+R^{e}_{t+1}}$, which Bernanke et al. (1999) interpreted as the external finance premium faced by the entrepreneur, depends positively on the entrepreneur's leverage ratio. Intuitively, all else equal, higher leverage means higher exposure, implying a higher probability of default, hence a higher credit risk, which the bank translates into a higher required return on lending.

The cost of borrowing fluctuates endogenously with the cycle due to two general equilibrium mechanisms.

The first one is the Bernanke et al. (1999) "financial accelerator" effect, whereby induced changes in the asset price alter the value of the collateral that the entrepreneur can pledge and, hence, the contractual loan rate. Specifically, a positive shock to the asset price – as a result of a monetary easing or a favorable shock to productivity – increases the entrepreneur's net worth and decreases the external finance premium, which in turn stimulates the demand for investment. The increase in net worth also reduces the expected default probability and allows the entrepreneur to take on more debt and to further expand investment. An accelerator effect arises, since the investment boom raises the asset price, further pushing up the entrepreneur's net worth and investment.

The second mechanism – which CMR refer to as the "Fisher deflation" effect – is absent in Bernanke et al. (1999) and works through a debt-deflation effect.[13] This effect arises because of the assumption that the return received by households on time deposits is nominally non-state contingent, while loans to entrepreneurs are state-contingent. Therefore, unexpected movements in the price level alter the *ex-post* real burden of entrepreneurial debt and, hence, the entrepreneur's net worth. Namely, following an unexpected increase in inflation, the total real resources transferred from the entrepreneur to households are reduced and, as a consequence, the entrepreneur's net worth increases.

As CMR point out, the "accelerator" and "Fisher" effect mechanisms reinforce each other in the case of shocks that move inflation and output in the same direction (*e.g.*, monetary policy shocks), whereas they dampen the macroeconomic transmission of shocks that move inflation and output in opposite directions (*e.g.*, technology shocks).

1.4.2 The shadow banking system

1.4.2.1 Safer entrepreneurs

Profit maximization

At the beginning of period t, the representative l-th entrepreneur provides capital services to intermediate-good firms. Capital services, $K_t^{L,l}$, are related to the entrepreneur's stock of physical capital, $\bar{K}_t^{L,l}$, by $K_t^{L,l} = u_t^{L,l}\bar{K}_t^{L,l}$, where $u_t^{L,l}$ denotes the level of capital utilization. In choosing the capital utilization rate, the entrepreneur takes into account the increasing and convex utilization cost function $a\left(u_t^{L,l}\right)$, that denotes the cost, in units of final goods, of setting the utilization rate to $u_t^{L,l}$.[14]

[13] Fisher (1933) emphasizes the "debt deflation" effect that arises when debt contracts are set in nominal terms. Other papers that analize the debt-deflation effect include Iacoviello (2005) and Gerali et al. (2010).

[14] The functional form that we use is $a\left(u_t^{L,l}\right) = \frac{r^{k,L}}{\sigma_a^L}\left[\exp^{\sigma_a^L\left(u_t^{L,l}-1\right)} - 1\right]$, where $r^{k,L}$ is the steady state value of the rental rate of capital, $a(1) = 0$, $a''(1) > 0$ and $\sigma_a^L = a''(1)/a'(1)$ is a parameter that controls the degree of convexity of costs.

Then, at the end of period t, the entrepreneur sells the undepreciated capital to capital producers at price $Q_{\bar{k}',t}$, pays the nominal coupon rate (R_t^{coupon}) on bonds issued and purchases new capital from capital producers at price $Q_{\bar{k}',t}$. The capital acquisition is financed partly by his net worth, $N_{t+1}^{L,l}$, and partly by issuing new bonds. The amount of bonds issued, $BI_{t+1}^{L,l}$, is given by:

$$BI_{t+1}^{L,l} = Q_{\bar{k}',t}\bar{K}_{t+1}^{L,l} - N_{t+1}^{L,l} \ . \tag{1.2}$$

The entrepreneur's time-t profits, $\Pi_t^{L,l}$, are given by:

$$\begin{aligned}\Pi_t^{L,l} = &\left[u_t^{L,l} r_t^{k,L} - a\left(u_t^{L,l}\right)\right] \bar{K}_t^{L,l} P_t + (1-\delta)\, Q_{\bar{k}',t}\bar{K}_t^{L,l} \\ &- Q_{\bar{k}',t}\bar{K}_{t+1}^{L,l} - R_t^{coupon}\left(Q_{\bar{k}',t-1}\bar{K}_t^{L,l} - N_t^{L,l}\right) \qquad ,\end{aligned}$$

where $r_t^{k,L}$ denotes the real rental rate, P_t the price of the final good and δ the depreciation rate.

In period t the entrepreneur chooses the capital utilization rate and the desired capital to use in period $t+1$ so as to maximize $\Pi_t^{L,l}$, taking as given the coupon rate to be paid on the bonds issued. The first order conditions with respect to $u_t^{L,l}$ and $\bar{K}_{t+1}^{L,l}$ are, respectively:

$$r_t^{k,L} = a'\left(u_t^{L,l}\right) \tag{1.3}$$

$$Q_{\bar{k}',t} = \beta E_t\left\{\left[u_{t+1}^{L,l} r_{t+1}^{k,L} - a\left(u_{t+1}^{L,l}\right)\right] P_{t+1} + (1-\delta)\, Q_{\bar{k}',t+1} - R_{t+1}^{coupon} Q_{\bar{k}',t}\right\} \ . \tag{1.4}$$

Equation (1.3) states that the rental rate on capital services equals the marginal cost of providing those services. As the rental rate increases it becomes more profitable to use capital more intensively up to the point where the extra profits match the extra utilization costs. The capital Euler equation (1.4) equates the value of a unit of installed capital at time t to the expected discounted return of that extra unit of capital in period $t+1$.

The entrepreneur's equity at the end of period t, $V_t^{L,l}$, is given by

$$V_t^{L,l} = \left\{\left[u_t^{L,l} r_t^{k,L} - a\left(u_t^{L,l}\right)\right] P_t + (1-\delta)\, Q_{\bar{k}',t}\right\}\bar{K}_t^{L,l} - \left(1 + R_t^{coupon}\right)\left(Q_{\bar{k}',t-1}\bar{K}_t^{L,l} - N_t^{L,l}\right) \ .$$

The first term represents the rental income of capital, net of utilization costs, and the proceeds from selling undepreciated capital to capital producers. The second term represents the payment (coupon and principal) of the bonds issued in period $t-1$.

To avoid a situation in which the entrepreneur accumulates enough net worth to become self-financed, we assume that there is a constant probability of death. Namely, in each period the entrepreneur exits the economy with probability $1-\gamma^L$. In that case, the entrepreneur rebates his equity to households in a lump-sum way:

$$transfer\ to\ households = \left(1-\gamma^L\right) V_t^{L,l} \ .$$

To keep the entrepreneurs' population constant, a new entrepreneur is born with probability $1-\gamma^L$.

The total entrepreneur's net worth $N_{t+1}^{L,l}$ combines total equity and a transfer, $W_t^{e,L,l}$, received from households, which corresponds to the initial net worth necessary for the entrepreneur's activity to start. The law of motion for the entrepreneur's net worth is:

$$N_{t+1}^{L,l} = \gamma^L V_t^{L,l} + W_t^{e,L,l} \ .$$

Financing cost minimization problem and funds demand curve

We assume that each investment bank z has some market power in conducting its intermediation services. An entrepreneur seeking an amount of borrowing for period $t+1$ equal to $BI_{t+1}^{L,l}$, defined by (1.2), would therefore allocate his borrowing among different investment banks, $BI_{t+1}^{L,l}(z)$, so as to minimize the total repayment due. At the end of period t, the entrepreneur chooses how much to borrow from bank z by solving the following problem:

$$\min_{BI_{t+1}^{L,l}(z)} \int_0^1 \left[1+R_{t+1}^{coupon}(z)\right] BI_{t+1}^{L,l}(z)\, dz$$

subject to the Dixit-Stiglitz aggregator

$$BI_{t+1}^{L,l} = \left\{ \int_0^1 \left[BI_{t+1}^{L,l}(z)\right]^{\frac{\varepsilon_{t+1}^{coupon}-1}{\varepsilon_{t+1}^{coupon}}} dz \right\}^{\frac{\varepsilon_{t+1}^{coupon}}{\varepsilon_{t+1}^{coupon}-1}} \ ,$$

where $R_{t+1}^{coupon}(z)$ is the interest rate charged by the z-th bank and $\varepsilon_{t+1}^{coupon} > 1$ is the time-varying interest rate elasticity of the demand for funds. The first order condition yields the following entrepreneur's demand for funds:

$$BI_{t+1}^{L,l}(z) = \left(\frac{1 + R_{t+1}^{coupon}(z)}{1 + R_{t+1}^{coupon}} \right)^{-\varepsilon_{t+1}^{coupon}} BI_{t+1}^{L,l} \ ,$$

where R_{t+1}^{coupon} is the nominal average coupon rate prevailing in the market at time $t+1$, defined as:

$$1 + R_{t+1}^{coupon} = \left\{ \int_0^1 \left[1 + R_{t+1}^{coupon}(z) \right]^{1-\varepsilon_{t+1}^{coupon}} dz \right\}^{\frac{1}{1-\varepsilon_{t+1}^{coupon}}} \ .$$

As expected, the funds demand curve has a negative slope: when the interest rate that the z-th bank sets increases relatively to the average rate, the entrepreneur decides to borrow less funds from that bank.

1.4.2.2 Investment banks

The investment banking sector comprises a continuum of monopolistic competitive investment banks, indexed by $z \in [0,1]$, owned by households. To keep the analysis as simple as possible, we follow the recent DSGE banking literature and assume perfect competition in the market for households' deposits in these banks.[15] We also rule out the entry and exit of investment banks. The investment bank therefore maximizes its profits, taking as given the return to pay to the households. The appendix A shows that the required return on bonds by households is equal to the risk-free rate, *i.e.* the central bank nominal interest rate R_t^e (see equations A.10 and A.11).

At the end of period t, the z-th investment bank thus solves the following profit maximization problem:

$$\max_{R_{t+1}^{coupon}(z)} \Pi_{t+1}^{IB}(z) = \left\{ \left[1 + R_{t+1}^{coupon}(z) \right] BI_{t+1}^{L,l}(z) - \left[1 + R_{t+1}^{e} \right] BI_{t+1}^{L,l}(z) \right\} \tag{1.5}$$

$$subject\ to \qquad BI_{t+1}^{L,l}(z) = \left(\frac{1 + R_{t+1}^{coupon}(z)}{1 + R_{t+1}^{coupon}} \right)^{-\varepsilon_{t+1}^{coupon}} BI_{t+1}^{L,l} \ .$$

[15] See, for instance, Aliaga-Diaz and Olivero (2007), Andrés and Arce (2009), Kobayashi (2008) and Teranishi (2008).

The first order condition is

$$\left(\frac{1+R_{t+1}^{coupon}(z)}{1+R_{t+1}^{coupon}}\right)^{-\varepsilon_{t+1}^{coupon}} - \varepsilon_{t+1}^{coupon}\frac{1+R_{t+1}^{coupon}(z)-\left(1+R_{t+1}^{e}\right)}{1+R_{t+1}^{coupon}}\left(\frac{1+R_{t+1}^{coupon}(z)}{1+R_{t+1}^{coupon}}\right)^{-\varepsilon_{t+1}^{coupon}-1} = 0 \ .$$

Imposing a symmetric equilibrium and rearranging yields

$$1+R_{t+1}^{coupon} = \frac{\varepsilon_{t+1}^{coupon}}{\varepsilon_{t+1}^{coupon}-1}\left(1+R_{t+1}^{e}\right) \ , \tag{1.6}$$

that is, the coupon rate is set as a markup, $\frac{\varepsilon_{t+1}^{coupon}}{\varepsilon_{t+1}^{coupon}-1}$, over the policy interest rate. The profits of the investment banking sector in period $t+1$ are given by

$$\Pi_{t+1}^{IB} = \left(R_{t+1}^{coupon}-R_{t+1}^{e}\right)(1-\eta)BI_{t+1}^{L,l} \ , \tag{1.7}$$

and are rebated to households.

Assuming that the interest rate elasticity of the demand for funds is constant, $\varepsilon_{t+1}^{coupon} = \varepsilon^{coupon,a}$, the coupon rate becomes a constant markup applied to the required return by households:

$$1+R_{t+1}^{coupon,a} = \frac{\varepsilon^{coupon,a}}{\varepsilon^{coupon,a}-1}\left(1+R_{t+1}^{e}\right) \ . \tag{1.8}$$

In what follows, we consider $R_{t+1}^{coupon,a}$ as the "normal" interest rate on bonds.

1.4.2.3 Optimism and perverse incentives in the shadow banking sector

In this subsection we extend the model presented in the previous subsection in two respects. First, we introduce optimism among underwriters in the investment banking sector by considering that an optimistic underwriter is willing to underwrite bonds at a lower – than the normal – coupon interest rate. Second, we introduce perverse incentives by assuming that the representative safer entrepreneur offers side payments to the underwriter in order to borrow at a more favorable interest rate. In exchange of those side payments, an optimistic underwriter may *de facto* facilitate the extension of credit by setting a "discounted" – relatively to its normal value – bond coupon rate.

We do so by endogenizing the choice of the interest rate elasticity underlying the demand for funds, $\varepsilon_{t+1}^{coupon}$. Note, from (1.6), that an increase in the interest rate elasticity leads to, *ceteris paribus*, a lower coupon rate. This relation between the elasticity and the coupon rate allows us to separate and solve the investment bank's profit maximization problem in two steps. First, the underwriter chooses the interest rate elasticity according to his preferences. Second, he solves the maximization problem (1.5), which leads to equation (1.6). In practice, after determining $\varepsilon_{t+1}^{coupon}$, the underwriter has implicitly determined the coupon rate that solves the investment bank's profit maximization problem (1.5).

Optimism

First, we assume that the underwriter becomes optimistic if the entrepreneur pledges a higher value as collateral. We thus model underwriter's optimism, χ_t, as a positive function of the entrepreneur's net worth. To take into account the fact that human beliefs are highly correlated and persistent (Carlson, 2007), we furthermore model optimism as an $AR(1)$ process with high persistence. Accordingly, the law of motion for optimism is given by

$$\chi_t = \rho_\chi \chi_{t-1} + \left(1-\rho_\chi\right)\left[\bar{\chi} + \alpha_3\left(N_{t+1}^{L,l} - N^{L,l}\right)\right] \ , \tag{1.9}$$

where $\bar{\chi}$, $\bar{\chi} = 0$, is the steady state level of optimism, ρ_χ captures the degree of persistence in optimism and $\alpha_3 > 0$ the sensitivity of optimism with respect to the deviation of the entrepreneur's net worth from its steady state value ($N^{L,l}$).

Second, we assume that the interest rate elasticity of the demand for funds is computed as follows:

$$\varepsilon_{t+1}^{coupon,biased} = \varepsilon^{coupon,a}\left(1+\chi_t\right) \ , \tag{1.10}$$

which means that positive deviations of optimism from its steady state level increase the interest rate elasticity of the demand for funds, relatively to its normal value of $\varepsilon^{coupon,a}$. The biased elasticity results in a lower coupon rate, which may be seen substituting (1.10) into (1.6), yielding the following expression

$$1 + R_{t+1}^{coupon,biased} = \frac{\varepsilon_{t+1}^{coupon,biased}}{\varepsilon_{t+1}^{coupon,biased} - 1}\left(1 + R_{t+1}^{e}\right) \ , \tag{1.11}$$

where $R_{t+1}^{coupon,biased}$ is the biased coupon rate that an optimistic underwriter would set on the bonds issued. Comparing (1.11) and (1.8), it is clear that the optimistic underwriter would underwrite bonds at a lower than the normal interest rate.

The coupon rate set by the underwriter (R_{t+1}^{coupon} in 1.6) thus varies from a maximum of $R_{t+1}^{coupon,a}$ (corresponding to $\varepsilon_{t+1}^{coupon,a}$) to a minimum of $R_{t+1}^{coupon,biased}$ (corresponding to $\varepsilon_{t+1}^{coupon,biased}$). In between these extremes, the value of the coupon rate chosen corresponds to a specific value of $\varepsilon_{t+1}^{coupon}$. In the next subsection we thus describe how such interest rate elasticity and, as a consequence, the bond coupon rate, are determined.

Perverse incentives and the optimal choice of the coupon rate

Suppose that the entrepreneur offers side payments to the underwriter in order to borrow at a more favorable coupon rate, *i.e.* at an interest rate lower than the normal rate $R_{t+1}^{coupon,a}$ defined by (1.8). Suppose also that households are not aware of this possibility. We assume that the amount of side payments paid to the underwriter at the end of period $t+1$ is given by

$$side\ payments_{t+1} = \Omega\left(R_{t+1}^{coupon,a} - R_{t+1}^{coupon}\right) V_{t+1}^{L,l}\ , \tag{1.12}$$

that is, side payments represent a fixed share, Ω, of the entrepreneurial equity and are proportional to the difference between $R_{t+1}^{coupon,a}$ and R_{t+1}^{coupon}. In principle, the underwriter should ignore these side payments and protect stockholders' interests, that is, the underwriter should maximize the bank's profits setting $R_{t+1}^{coupon} = R_{t+1}^{coupon,a}$. In that case, as equation (1.12) shows, he would not receive any side payments. However, the underwriter may alternatively choose to underwrite a bond at a lower rate ($R_{t+1}^{coupon} < R_{t+1}^{coupon,a}$) benefitting from those side payments. Clearly, side payments lead to an agency conflict within the investment bank, between its stockholders (*i.e.*, households) and its staff (*i.e.*, the underwriters). Note that the lower the R_{t+1}^{coupon} (compared to $R_{t+1}^{coupon,a}$), the higher will be the side payments that the underwriter receives and the lower will be the stockholders' return for a given level of $BI_{t+1}^{L,l}$ (as given by equation 1.7).[16]

[16] If there are side payments, before knowing whether exiting the economy, the entrepreneur transfers a share $\Omega\left(R_t^{coupon,a} - R_t^{coupon}\right)$ of his equity to the underwriter as side payments. After that, with probability $1-\gamma^L$ the entrepreneur exits the economy and rebates his equity to households in a lump-sum way:

$$transfer\ to\ households = \left(1-\gamma^L\right)\left[1-\Omega\left(R_t^{coupon,a} - R_t^{coupon}\right)\right] V_t^{L,l}\ .$$

Entrepreneurial net worth is thus given by

$$N_{t+1}^{L,l} = \gamma^L\left[1-\Omega\left(R_t^{coupon,a} - R_t^{coupon}\right)\right] V_t^{L,l} + W_t^{e,L,l}\ .$$

The top part of figure 1.2 sketches the trade-off faced by the underwriter – maximization of his own benefit versus maximization of stockholders' profits. To endogenize the choice of $\varepsilon_{t+1}^{coupon}$, we model this trade-off considering the following quadratic utility function for the underwriter:

$$u\left(\varepsilon_{t+1}^{coupon}\right) = -r_2\left(\varepsilon_{t+1}^{coupon,biased} - \varepsilon_{t+1}^{coupon}\right)^2 - (1-r_2)\left(\varepsilon_{t+1}^{coupon} - \varepsilon^{coupon,a}\right)^2, 0 \leq r_2 \leq 1 \ . \tag{1.13}$$

The first part mirrors the underwriter's private objective to maximize the amount of side payments received. Recall that, from equation (1.12), side payments are maximized when R_{t+1}^{coupon} is as low as possible, that is, when $R_{t+1}^{coupon} = R_{t+1}^{coupon,biased}$. This happens when $\varepsilon_{t+1}^{coupon} = \varepsilon_{t+1}^{coupon,biased}$. Parameter r_2 represents the importance the underwriter attaches to his private objective. The second part displays the underwriter's objective to maximize stockholders' profits by setting an $\varepsilon_{t+1}^{coupon}$ that is as close as possible to $\varepsilon^{coupon,a}$. This objective enters the underwriter's utility function with a weight of $(1-r_2)$.

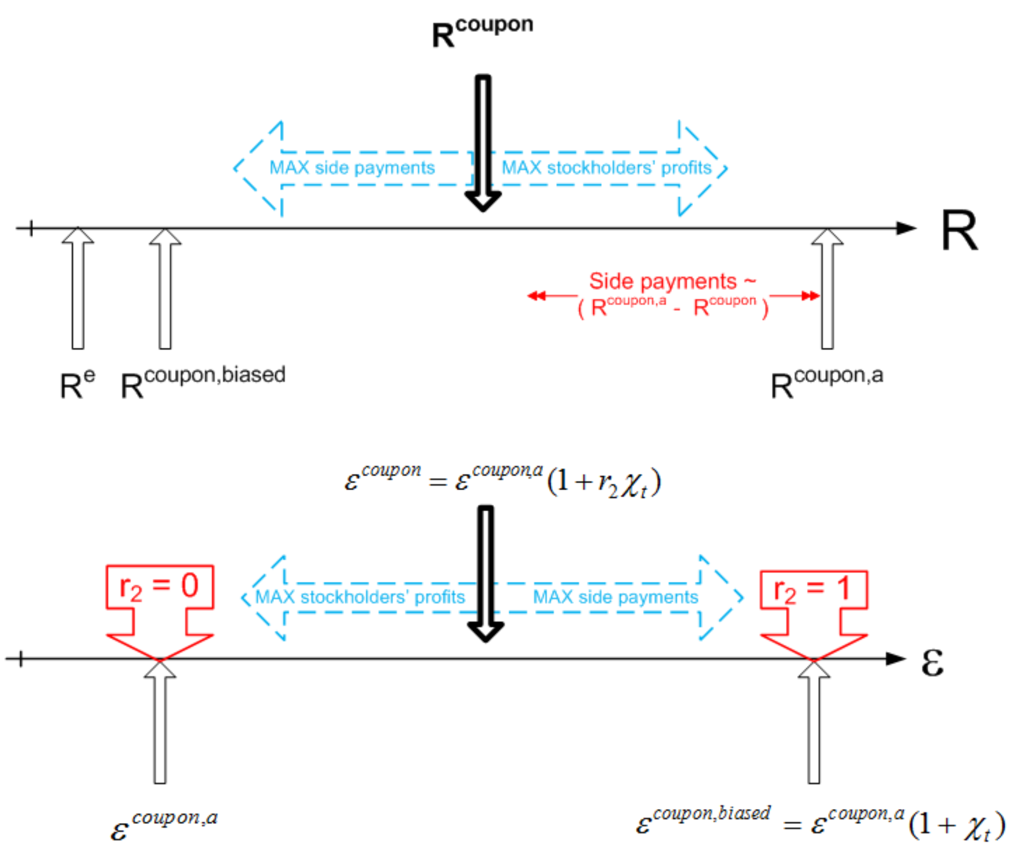

Figure 1.2: The underwriter's trade-off

The underwriter chooses $\varepsilon_{t+1}^{coupon}$ so as to maximize (1.13). The first order condition is:

$$2r_2\left(\varepsilon_{t+1}^{coupon,biased}-\varepsilon_{t+1}^{coupon}\right)-2\left(1-r_2\right)\left(\varepsilon_{t+1}^{coupon}-\varepsilon^{coupon,a}\right)=0\ .$$

Using (1.10) and rearranging, the first order condition then becomes

$$\varepsilon_{t+1}^{coupon}=\varepsilon^{coupon,a}\left(1+r_2\chi_t\right)\ . \tag{1.14}$$

Substituting (1.14) into (1.6) yields the following expression for the coupon interest rate

$$1+R_{t+1}^{coupon}=\frac{\varepsilon^{coupon,a}\left(1+r_2\chi_t\right)}{\varepsilon^{coupon,a}\left(1+r_2\chi_t\right)-1}\left(1+R_{t+1}^{e}\right)\ . \tag{1.15}$$

The coupon rate is therefore a time-varying markup, $\frac{\varepsilon^{coupon,a}(1+r_2\chi_t)}{\varepsilon^{coupon,a}(1+r_2\chi_t)-1}$, over the policy rate and is influenced both by the level of optimism and by the weight that the underwriter attaches to his private benefit. As a result, the optimistic underwriter may *de facto* set a coupon rate for the issued bonds that is lower than the rate that maximizes the bank's profits in the context of no optimism and no side payments. Note, in particular, that it is the combination of underwriter's optimism and his willingness to receive side payments ($\chi_t > 0$ and $r_2 > 0$) that leads to a discounted coupon rate.

The bottom part of figure 1.2 shows that the exact value of the coupon rate that is chosen – corresponding to a unique value of ε^{coupon} – depends upon the value of r_2 and the degree of optimism.

1.5 The response to monetary policy shocks

Having presented the model, we now analyze its dynamics. We first describe the calibration of the model and then present the impulse responses to two types of monetary policy shocks.

The central bank sets the short-term nominal interest rate, R_t^e, following a Taylor-type interest rate rule. Specifically, the monetary policy rule allows for interest rate smoothing and interest rate responses to deviations of expected inflation ($E_t\pi_{t+1}$) and current output (Y_t) from their steady states:

$$R_t^e=\left(R_{t-1}^e\right)^{\tilde{\rho}}\left[R^e\left(\frac{E_t\pi_{t+1}}{\bar{\pi}}\right)^{\alpha_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\alpha_y}\right]^{(1-\tilde{\rho})}\varepsilon_t^{MP}\ , \tag{1.16}$$

where R^e, $\bar{\pi}$ and $\bar{Y}$ are the steady state values of R_t^e, π_t and Y_t, respectively, α_π and α_y are the weights assigned to expected inflation and output, $\tilde{\rho}$ captures interest rate smoothing and ε_t^{MP} is a white noise monetary policy shock.

We first solve numerically the model, for the steady state, using the computational procedure described in appendix C. We then compute the first-order Taylor series approximation to the equilibrium conditions in the neighborhood of the steady state.[17]

We compare the responses to monetary impulses under three different variants of the above-described model:

- variant 1: the simplified version of the *Financial Accelerator Model* of CMR, which corresponds to setting the share of riskier entrepreneurs η equal to 1;
- variant 2: our model including the shadow banking system but excluding optimism and side payments, which is obtained setting $r_2 = 0$;
- variant 3: our model including optimism and side payments in the shadow banking system, assuming $r_2 = 1$, *i.e.* the version in which the underwriter only cares about his own benefit and maximizes the amount of side payments received.

Recall from subsection 1.4.1 that the transmission mechanism in variant 1 is affected by two general equilibrium mechanisms. The first one is the Bernanke et al. (1999) “financial accelerator” effect, whereby induced changes in asset prices alter the value of the collateral that the entrepreneur can pledge and, hence, the contractual loan rate. The second mechanism is a CMR-type “Fisher deflation” effect, whereby unexpected movements in the price level alter the *ex-post* real burden of entrepreneurial debt and, hence, the entrepreneur’s net worth.

To these two channels, variant 2 adds a new set of monopolistic investment banks. The monopolistic power in setting bond interest rates affects the credit supply conditions of a set of entrepreneurs through the introduction of a constant interest rate spread. Hence, this variant allows us to analyze whether the interest-rate-setting by banks interacts with the aforementioned channels and to what extent it does modify the monetary transmission mechanism.[18]

[17] All simulations in this chapter have been conducted with Dynare.

[18] We should note that, as far as we know, at most two out of these three effects – “financial accelerator”, “Fisher deflation” and “monopolistic banking competition” – have been analyzed within a single model. As regards studies that address only one of the effects, when compared to standard models with frictionless financial markets, CMR show that the “financial accelerator” effect, as well as the “Fisher deflation” effect, amplify and propagate the transmission of monetary policy shocks, while Andrés and Arce (2009) find that monopolistic competition in the banking sector dampens the macroeconomic transmission of policy shocks. As regards studies that combine two of these effects, Iacoviello (2005) and CMR show that the “accelerator” and the

Finally, variant 3 adds optimism and perverse incentives to the "monopolistic banking competition" effect. In this variant, the underwriter diverts a fraction of stockholders' profits for his own benefit and extends credit at a lower interest rate. In this framework, the behavior of the underwriter influences the credit supply conditions of a set of borrowers, which in turn influences the real economy through a countercyclical time-varying interest rate spread. This variant allows us to study the importance of the bank manager's behavior in shaping the monetary transmission mechanism, which has been virtually ignored in the literature so far.

We conduct two policy experiments. The first experiment is a one-period expansionary monetary policy shock. It allows us to assess whether (and how) the transmission mechanism of monetary policy is affected by the presence of a shadow banking system using as a benchmark the impulse responses of a workhorse DSGE model (variant 1).

In the second experiment, we create a "persistently low interest rate" scenario by forcing the nominal interest rate to be 25 basis points lower than its steady state value during 8 quarters. This experiment allows for a) determining if an extended period of loose monetary policy generates *per se* a boom-bust cycle and b) analyzing whether the interaction between long periods of accommodative monetary policy and perverse incentives in the financial sector causes and/or amplifies fluctuations in real and financial activity.

The next subsection briefly describes the calibration, before turning to the analysis of the impulse response functions in subsections 1.5.2 and 1.5.3.

1.5.1 Calibration

The model is calibrated to the U.S. economy, assuming that a period is a quarter. The values are chosen so that the model's steady state reproduces some key features in the U.S. data. In this subsection we only describe the calibration of the parameters related with the shadow banking system. The values of the remaining parameters are calibrated within the range usually considered in the New Keynesian literature.[19] Table 1.1 reports the values of the

"Fisher" effect reinforce each other in what concerns the response of the economy to monetary policy shocks, whereas Mandelman (2010) finds that the assumption of an imperfectly competitive banking system in the Bernanke et al. (1999) framework magnifies the propagation and amplification of policy shocks to the economy.

[19] The values of the parameters related with the riskier entrepreneurial sector are primarily chosen to match the cost of external finance, *i.e.*, the contractual, no-default interest rate on entrepreneurial debt (Z_t resulting from A.7). Setting the fraction of realized payoffs lost in bankruptcy, μ, to 0.15 and the standard deviation of the entrepreneur idiosyncratic productivity shock, σ, to 0.55 yields $Z = 6.8\,\%/year$, which is close to observed data. This in turn also guarantees that, in equilibrium, bond financing is cheaper than bank financing (safer entrepreneurs finance themselves at a more favorable interest rate). To match the observed leverage ratio, we

calibrated parameters, and tables 1.2 and 1.3 report the steady state implications of the model and their empirical counterparts.

To match the return on time deposits (which is also equal to the steady state central bank nominal interest rate), we set the discount factor β to 0.9875. Equation (1.6) shows that the steady state spread between the coupon rate and the risk-free rate (the yield spread) depends on the interest rate elasticity ε^{coupon}. Chen et al. (2007) report an average annual yield spread of *AAA* bonds of 84 basis points. Accordingly, we set ε^{coupon} to 510 so that the annual yield spread is around 80 basis points. As a result, the coupon rate paid by safer entrepreneurs is $5.9\%/year$.

To match the observed average leverage ratio, we set the survival probability of safer entrepreneurs γ^L to 0.96. In the law of motion for optimism (1.9), we set the persistence parameter ρ_χ to 0.9 and the sensitivity to entrepreneur's net worth α_3 to 40.[20]

The parameter Ω (the fraction of equity that the entrepreneur is willing to pay as side payments) is chosen so as to guarantee that the entrepreneur is always better off when he pays side payments. In principle, the safer entrepreneur may choose between two options. He can either pay the coupon rate $R^{coupon,a}_{t+1}$ given by equation 1.8 or he can offer side payments and obtain a lower coupon rate (R^{coupon}_{t+1} given by equation 1.15). This choice depends on the value of Ω. Given our baseline calibration, in appendix D we show that, in the steady state, the entrepreneur is better off whenever Ω is smaller than a threshold level $\bar{\Omega} = 0.25$. Accordingly, we set Ω to 0.1, thus assuming that the entrepreneur gives away 10% of his equity as side payments to obtain a lower coupon rate.

Finally, we calibrate the parameter η (the share of riskier entrepreneurs in the economy) by replicating the ratio of bond finance to bank finance in the U.S. economy which, according to De Fiore and Uhlig (2005), is equal to 1.34. We closely match this ratio by setting η to 0.3.

As tables 1.2 and 1.3 show, the model is successful in reproducing most of the salient features of the U.S. economy: key macroeconomic and leverage ratios, interest rates and, importantly, its financial market structure.

set the survival rate γ^H to 0.97.

[20] This calibration guarantees that $\varepsilon^{coupon}_{t+1} = \varepsilon^{coupon,a}(1 + r_2\chi_t) > 1\ \forall t$.

1.5.2 The economy's response to an unanticipated one-period expansionary monetary policy shock

In this subsection we study the transmission of a monetary policy shock by analyzing the impulse responses to a one-period innovation in the short-term nominal interest rate (ε_t^{MP} in 1.16), corresponding to a 25 basis points reduction of the annualized nominal interest rate. Figures 1.3-1.5 illustrate the impulse responses of the key variables under the three variants of the model (variant 1: blue solid line; variant 2: red crossed line; and variant 3: black circled line).

In all figures presented, variables are expressed in percent deviation from their steady state values, except for inflation, that is expressed as annualized percent deviation from its steady state, and interest rates, that are expressed in percentage points at annual rate. The horizontal axis represents time on a quarterly scale.

The responses of aggregate variables in variant 1 are qualitatively standard. After the initial drop, the nominal interest rate gradually returns to its steady state value. Aggregate quantities – output, consumption and investment – as well as inflation display a hump-shaped response and peak after about three to six quarters. The price of capital shows maximum upward reactions at impact before returning to its steady state. The effects on aggregate variables are long-lived despite the fact that the effects on the nominal interest rate only last for roughly two years.

Overall, the response of lending activity is weak at the aggregate level: although entrepreneurs accumulate more capital ($stock\ of\ capital\ \uparrow$), the sharp increase in the aggregate net worth ($N\uparrow$) leads to a decrease of total credit ($q\bar{K}-N$) below its steady state level.

While in most cases the responses in variant 2 are pretty similar to those in variant 1, it is notable that the impact of the monetary policy shock is somewhat dampened under this variant. We find, in line with other studies, that the introduction of market power in banking results in smoother effects. A striking difference is evident, however, when we compare the responses in variant 3 with those in the other two variants of the model. First, the business cycle is amplified – in particular, the peak in investment is two times greater than under variant 2. Second, at its height, the response of investment is roughly twice as big, in percent terms, as the response of output (while it is nearly the same in the other variants). Finally, and in contrast with the other two variants of the model, total credit increases: the rise in aggregate entrepreneurial capital purchases more than compensates the more pronounced (compared to the other variants) increase in the aggregate net worth, so that the net effect is an increase of total credit above its steady state value. This can be explained by analyzing each type of entrepreneur separately.

Therefore, turning to the variables specific to the entrepreneurial sector, we conclude that under the three variants of the model the riskier entrepreneur's net worth increases in response to the shock because of both the "accelerator" and the "Fisher" effects. The rise in the price of capital leads to a boost in the value of the assets of the entrepreneur, which in turn reduces the probability of bankruptcy ($\bar{\omega} \downarrow$). Moreover, because of the drop in entrepreneur's leverage, retail bank charges a lower interest rate on loans ($Z \downarrow$). This reflects the fact that the cost of external financing depends on the borrower's leverage: as predicted by equation (1.1), all else equal, the lower the leverage, the lower the external finance premium, hence the lower the interest rate on loans. This "accelerator" effect is then reinforced by the "Fisher" effect: the *ex-post* value of existing entrepreneurial debt decreases as inflation rises. As a consequence, the entrepreneur's net worth increases further.

In both variants 2 and 3, the monetary policy shock leads to a lower coupon rate paid by the safer entrepreneur. Under variant 2 the coupon rate is set as a constant markup over the policy rate (recall equation 1.8), therefore R^{coupon} follows the nominal interest rate path. In variant 3, however, the coupon rate also depends on the underwriter's behavior (equation 1.15). In particular, the rise in the price of capital increases the value of the collateral held by the entrepreneur (*net worth* $\uparrow$), which in turn triggers optimism (equation 1.9). The underwriter's optimism, combined with his willingness to receive side payments ($r_2 = 1$), leads to a drop in the bond coupon rate larger than that in variant 2. As figure 1.5 shows, $R^{coupon}\mid_{variant\,3}$ is smaller than $R^{coupon}\mid_{variant\,2}$ for about 20 quarters, that is, the underwriter persistently extends credit at a lower interest rate in exchange of side payments.

Thus, in both variants 2 and 3, the monetary policy shock leads to a lower borrowing cost for both types of entrepreneurs – both Z and R^{coupon} decrease. However, note that the underwriter's behavior and preferences influence the spread between the cost of financing for the riskier entrepreneur and the coupon rate on bonds for the safer entrepreneur ($Z - R^{coupon}$, in figure 1.5). This spread in turn strongly influences the allocation of funds between safer and riskier entrepreneurs and, consequently, total capital and total credit dynamics.

In fact, in variant 2 the drop in Z is larger than the drop in R^{coupon} ($Z - R^{coupon} \downarrow$). That is, financing in the loan market becomes relatively cheaper than funding in the bond market. As a result, there is an increase in the amount of borrowing from retail banks (*loans* $\uparrow$) and a reduction in the flow of funds to the safer entrepreneur (*bond amount* $\downarrow$). Differently, in variant 3, $Z - R^{coupon}$ increases, *i.e.* given the marked reduction in the coupon rate due to optimistic behavior, bond financing becomes relatively cheaper than bank financing, leading safer entrepreneurs to invest more (*bond amount* $\uparrow$), while the amount of borrowing from retail banks drops (*loans* $\downarrow$). Riskier entrepreneur thus prefer to use capital more intensively (when compared with variant 2). Overall, the increase in capital stock is higher than under variant 2 since the increase in bonds issued more than offsets the decrease of loan amount.

These findings suggest that financial market frictions alone – in the form of monopolistic competition in banking system – do not change significantly the model's dynamics, whereas the behavior of the financial intermediary – driven by optimism and perverse incentives – does play a role in the transmission of the monetary policy shock: the effects of monetary policy on real and financial activity are in fact amplified in the variant of the model in which the financial intermediary plays a more active role.

1.5.3 The economy's response in a "persistently low interest rate" scenario

At the macroeconomic level, it has been recognized that accommodative monetary policies have historically been a key factor in driving boom-bust cycles of all types.[21] Although the low level of the federal funds rate in the early 2000s is generally considered to have helped fuel the housing bubble that burst in 2007, it is still an open debate whether lax monetary policies played a key role in generating the boom-bust cycle.[22]

In addition to and interacting with the low interest rate environment prevailing at the beginning of the 2000s, microeconomic factors related to recent innovations in the financial market structure and products may have also contributed to the subprime crisis. Even though the interaction between microeconomic distortions in the financial sector and a persistently loose monetary policy environment seems to have been relevant in generating and/or amplifying the boom-bust cycle, the relative importance of each of these factors is still open to debate.

Our model is well-suited to analyze the interaction between long periods of accommodative monetary policy and financial market distortions, as well as to disentangle their relative importance. To do so, in this subsection we create a "persistently low interest rate" scenario and analyze the model's dynamics. We reproduce such a scenario by combining the right

[21] See, for instance, Bordo (2008) and Calomiris (2008).

[22] The Fed funds rate was gradually reduced from around 6.50% in November 2000 to around 1.75% in December 2001 and was kept at that level until December 2002. Then, after two policy interventions (November 2002 and June 2003), it was reduced and kept to 1% until June 2004.

sequence of monetary policy innovations (ε_t^{MP} in equation 1.16) in order to hold the nominal interest rate 25 basis points lower than its steady state value during 8 quarters. There are thus eight consecutive monetary policy shocks, each coming as a surprise to the agents. The overall impulse responses are then obtained by summing up the responses to each of the successive monetary policy shocks.

Figures 1.6-1.8 display the impulse responses of several variables under the three variants of the model. By construction, the nominal interest rate deviates from its steady-state value by 25 basis points during 8 quarters. Then, from period 9 onwards, its dynamics is governed by the Taylor rule with response to deviations of expected inflation and current output from their respective steady states. In period 8, inflation and output are well above their steady state values. Hence, starting from period 9, the nominal interest rate rises and gradually reverts to its steady state value.

The dynamic responses of aggregate variables are qualitatively similar across the three variants of the model. Output, investment, consumption, inflation and the price of capital rise until period 8. The subsequent monetary tightening leads to a contraction of output, consumption and investment and a rapid decline in the price of capital.

Nevertheless, variant 3 exhibits responses that are quantitatively different. The effects of monetary policy shocks on the real economy are considerably amplified – the peak in output is about 35% higher and the response in investment is about twice as large as that in the other variants of the model. The effects on financial variables are magnified as well. The percentage increase in the price of capital, at its peak, is roughly the double of the increase that occurs in the other variants. Moreover, after the initial jump, the price of capital rises 60% during the boom phase, which is more than four (eleven) times the increase in variant 1 (2) during the same period. Note also that the pattern of the price of capital mimics the typical shape of an asset price bubble: the large and rapid asset price increase is followed by a burst and then a collapse.

The most striking difference which, of course, underlies the dynamics of the other aggregate variables, is that, whereas lending activity is weak at the aggregate level under variants 1 and 2, the persistent monetary easing leads to a lending boom in variant 3 that lasts well after the roughly 4 years it takes for the effects on the nominal interest rate to die away. Looking at the entrepreneurial variables, it is evident that the boom in total credit is driven by the safer entrepreneur's demand for funds (*bond amount* $\uparrow$).

Figure 1.8 allows us to trace the monetary transmission mechanism in variant 3. The rise in the price of capital leads to a boost in the safer entrepreneur's net worth, which in turn triggers an optimistic sentiment by the underwriter. This optimism, when combined with the underwriter's willingness to receive side payments – as well as with the increase in the amount of these payments induced by the increase in the entrepeneur's net worth (equation 1.12) – leads the underwriter to set a significantly lower coupon rate on the bonds issued. In particular, the discount relative to the "normal" rate occurs on impact and continues further during the period of persistently low interest rate, as R^{coupon} is well below the normal coupon rate $R^{coupon,a}$ and further declines as time goes on. The protracted opportunity for the safer entrepreneur to have access to an abnormally cheap source of funds – both in absolute terms ($R^{coupon} \downarrow$) and relatively to the cost of borrowing in the loan market ($Z - R^{coupon} \uparrow$) – leads him to accumulate capital aggressively. As a result, the safer entrepreneur's demand for capital rises, pushing up aggregate demand and causing a boom in the price of capital. The rise in safer entrepreneur's capital purchases more than compensates the increase in his net worth, so that the net effect is an increase of bond issued much above its steady state value. Finally, in general equilibrium, relatively higher borrowing cost for the riskier entrepreneur (when compared to bond coupon rate) induces him to cut capital expenditures and to use his capital more intensively.

Overall, these findings indicate that a persistently loose monetary policy does not cause *per se* a boom-bust cycle. In fact, neither in the CMR model, nor in its augmented version with monopolistic banking competition, a "too low for too long" interest rate policy generates a boom-bust cycle. However, monetary policy does create the preconditions for a boom-bust: optimism and perverse incentives in the financial sector, when coupled with a persistently low interest rate environment, result in greatly amplified fluctuations in both real and financial variables.[23]

1.6 Conclusion

This chapter has analyzed whether long periods of loose monetary policy play a key role in generating a boom-bust cycle, as well as the role of perverse incentives in the financial

[23] We have checked the sensitivity of our results to changes in the values of parameters η and Ω. We have considered three different values of η, namely 0.368, 0.45 and 0.81, which imply a bond to bank finance ratio of, respectively, 1, 0.7 and 0.13 (the latter value reproduces the financial market structure of the Euro Area). Although the qualitative responses are quite similar to those of the baseline calibration, quantitatively the effects on both real and financial variables become dampened as bank financing becomes more important. Finally, changing the value of parameter Ω does not affect the overall dynamics of the model, as long as $\Omega < \bar{\Omega}$.

sector in causing and/or amplifying fluctuations in real and financial activity during periods of accommodative monetary policy.

Starting from a model that nests most contemporary DSGE monetary models, we have introduced a microfounded bond market comprised of a monopolistically competitive investment banking sector. The underwriter within the investment bank, who sets the coupon rate on the bonds issued either as a constant markup over the nominal interest rate, or at a discounted rate due to the likelihood of receiving side payments, is the pivotal agent in our model.

We have first analyzed the responses to a one-period expansionary monetary policy shock. The results show that financial market frictions alone – in the form of monopolistic competition in the banking sector – do not change significantly the model's dynamics (when compared with a workhorse DSGE model). Yet, the effects of monetary policy on economic activity are amplified in the model in which the underwriter facilitates the extension of credit when optimism and perverse incentives are taken into account.

We have then simulated a "persistently low interest rate" scenario by keeping the central bank nominal interest rate 25 basis points below its steady state value for 8 quarters. Our main result is that a "too low for too long" interest rate policy does create the preconditions for, but does not cause *per se*, a boom-bust cycle. In fact, fluctuations in both real and financial variables are markedly amplified only when optimism and perverse incentives in the financial sector are coupled with such a persistently accommodative monetary policy environment. These findings suggest that, to reduce the odds of future booms, busts and asset price bubbles, policymakers should focus on tuning the financial architecture and reinforcing the financial supervision to restrain optimistic behaviors and perverse incentives. In doing so, policymakers will protect the financial system and the economy as a whole from the negative and often disruptive effects associated with economic booms and busts.

Appendix A - The complete model

Final-good firms

Perfectly competitive firms produce the final good that is converted into household consumption goods, investment goods, government goods, goods used up in capital utilization and in bank monitoring as well as for underwriter consumption goods.

The representative firm produces the final good Y_t, using the intermediate goods $Y_{i,t}$, and the production technology

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f} ,$$

where λ_f, $\infty > \lambda_f \geq 1$, is the markup for the intermediate-good firms. The representative firm chooses $Y_{i,t}$ to maximize its profits, taking the output price, P_t, and the input prices, $P_{i,t}$, as given. The maximization problem of the representative firm is thus given by:

$$\max_{Y_{i,t}} \quad P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

$$subject\ to \quad Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f} .$$

Solving the profit maximization problem yields the following demand function for the intermediate good i

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_f}{1-\lambda_f}} Y_t .$$

Perfect competition in the final goods market implies that he price of the final good can be written as:

$$P_t = \left[\int_0^1 P_{i,t}^{\frac{1}{1-\lambda_f}} di \right]^{1-\lambda_f} . \tag{A.1}$$

Intermediate-good firms

Monopolistic competitive firms, indexed by $i \in [0,1]$, produce differentiated intermediate goods using the following production function:

$$Y_{i,t} = (K_{i,t})^{\alpha} (L_{i,t})^{1-\alpha} \quad , \tag{A.2}$$

where $0 < \alpha < 1$ and $K_{i,t}$ and $L_{i,t}$ denote, respectively, the capital and labor input for the production of good i.

The capital input is assumed to be a composite of two entrepreneur-specific bundles of capital services, $K_{i,t}^{H}$ and $K_{i,t}^{L}$ which in turn combine the capital services of the individual members of the two entrepreneur sectors, $K_{i,t}^{H,r}$ and $K_{i,t}^{L,l}$. Formally,

$$K_{i,t} = \left[\eta^{1-\rho} \left(K_{i,t}^{H}\right)^{\rho} + (1-\eta)^{1-\rho} \left(K_{i,t}^{L}\right)^{\rho}\right]^{\frac{1}{\rho}} \quad , \tag{A.3}$$

where ρ denotes the degree of substitutability between the two entrepreneur-specific bundles of capital services and, since all entrepreneurs are identical within each group, $K_{i,t}^{H} = \eta K_{i,t}^{H,r}$ and $K_{i,t}^{L} = (1-\eta) K_{i,t}^{L,l}$.

The i-th firm hires labor and rents capital in competitive markets by minimizing its production costs, taking as given the nominal wage rate, W_t, and the real rental rates of capital, $r_t^{k,H}$ and $r_t^{k,L}$. The firm i's optimal demand for capital and labor services must thus solve the following cost minimization problem:

$$\min_{\left\{L_{i,t}, K_{i,t}^{H}, K_{i,t}^{L}\right\}} C(\cdot) = \frac{W_t L_{i,t}}{P_t} + K_{i,t}^{H} r_t^{k,H} + K_{i,t}^{L} r_t^{k,L} \tag{A.4}$$

subject to (A.2) and (A.3).

Since all firms i face the same input prices and since they all have access to the same production technology, real marginal costs s_t are identical across firms and are given by

$$s_t = \left[\frac{\tilde{w}_t}{1-\alpha}\right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left[\frac{\alpha}{r_t^{k,H}} \left(K_t^{H,r}\right)^{\rho-1}\right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} (Y_t)^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}} \frac{\rho}{\rho+\alpha(1-\rho)} \ ,$$

where $\tilde{w}_t$ denotes the real wage.

Price setting

Prices are determined through a variant of the Calvo's (1983) mechanism. In particular, every firm faces a constant probability, $1-\xi_p$, of reoptimizing its price in any given period, whereas the non-reoptimizing firms set their prices according to the indexation rule

$$P_{i,t} = P_{i,t-1}\,(\bar{\pi})^{\iota_1}\,(\pi_{t-1})^{1-\iota_1} \quad ,$$

where $\bar{\pi}$ represents the steady state inflation rate, $\pi_{t-1} = P_{t-1}/P_{t-2}$ is the inflation rate from $t-2$ to $t-1$ and the parameter ι_1, $0 \leq \iota_1 \leq 1$, represents the degree of price indexation to steady state inflation. The i-th firm that optimizes its price at time t chooses $P_{i,t} = \tilde{P}_{i,t}$ that maximizes the present value of future expected nominal profits:

$$\max_{P_{i,t}} \; \Pi_t^{IGF} = E_t \sum_{\tau=0}^{\infty} (\beta\xi_p)^{\tau}\,\lambda_{t+\tau}\left[\left(P_{i,t+\tau} - S_{t+\tau}\right)Y_{i,t+\tau}\right]$$

$$subject\ to \qquad Y_{i,t+\tau} = \left(\frac{P_{i,t+\tau}}{P_{t+\tau}}\right)^{\frac{\lambda_f}{1-\lambda_f}} Y_{t+\tau} \ ,$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\lambda_{t+\tau}$ the multiplier in the households' budget constraint, $S_{t+\tau}$ the firm's nominal marginal cost and $\beta \in (0\,,\,1)$ the discount factor. At the end of each time period, profits are rebated to households.

We consider only the symmetric equilibrium at which all firms choose the same $\tilde{P}_t = \tilde{P}_{i,t}$. Thus, from (A.1), the law of motion for the aggregate price index is

$$P_t = \left\{ (1-\xi_p)\,\tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p\left[P_{t-1}\,(\bar{\pi})^{\iota_1}\,(\pi_{t-1})^{1-\iota_1}\right]^{\frac{1}{1-\lambda_f}} \right\}^{1-\lambda_f} \quad .$$

Capital producers

A continuum of competitive capital producers produce the aggregate stock of capital $\bar{K}_t$. New capital produced in period t can be used in productive activities in period $t+1$. At the end of period t, capital producers purchase existing capital, $x_{K,t}$, from entrepreneurs and investment goods in the final good market, I_t , and combine them to produce new capital, $x^{'}_{K,t}$, using the following technology:

$$x^{'}_{K,t} = x_{K,t} + A\left(I_t, I_{t-1}\right) \ .$$

Old capital can be converted one-to-one into new capital, while the transformation of the investment good is subject to quadratic adjustment costs. The function $A(\cdot)$ summarizes the technology that transforms current and past investment into installed capital.

Investment goods are purchased in the final good market at price P_t. Let $Q_{\bar{k}',t}$ be the nominal price of new capital. Since the marginal rate of transformation between new and old capital is unity, the price of old capital is also $Q_{\bar{k}',t}$. The representative capital producer's period-t profit maximization problem is thus given by

$$\max_{\left\{I_{t+\tau}, x_{K,t+\tau}\right\}} \quad E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \lambda_{t+\tau} \Big\{ Q_{\bar{k}',t+\tau} \left[x_{K,t+\tau} + A\left(I_{t+\tau}, I_{t+\tau-1}\right)\right] \\ - Q_{\bar{k}',t+\tau} x_{K,t+\tau} - P_{t+\tau} I_{t+\tau} \Big\} \ . \tag{A.5}$$

Let δ denote the depreciation rate and note that, from (A.5), any value of $x_{K,t+\tau}$ is profit maximizing. Thus considering $x_{K,t+\tau} = (1-\delta)\bar{K}_{t+\tau}$ is consistent with both profit maximization and market clearing.

The first order condition with respect to I_t is:

$$E_t \left[\lambda_t \left(Q_{\bar{k}',t} A_{1,t} - P_t\right) + \beta \lambda_{t+1} Q_{\bar{k}',t+1} A_{2,t+1}\right] \quad = 0 \ ,$$

where

$$A_{1,t} = \frac{\partial A\left(I_t, I_{t-1}\right)}{\partial I_t} \ ; \ A_{2,t+1} = \frac{\partial A\left(I_{t+1}, I_t\right)}{\partial I_t} \ .$$

This is the standard Tobin's Q equation that relates the price of capital to the marginal costs of producing investment goods.[24]

[24] We adopt the following investment adjustment costs function:

$$A\left(I_t, I_{t-1}\right) = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \ , \ S\left(\frac{I_t}{I_{t-1}}\right) = \frac{S^{''}}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

so that $S(1) = S^{'}(1) = 0$ and $S^{''}(1) = S^{''} > 0$ in steady state. Therefore

$$A_{1,t} = \frac{\partial A\left(I_t, I_{t-1}\right)}{\partial I_t} = 1 - \frac{S^{''}}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - S^{''}\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)$$

The aggregate capital stock evolves according to

$$\bar{K}_{t+1} = \eta \bar{K}^{H,r}_{t+1} + (1-\eta)\bar{K}^{L,l}_{t+1} = (1-\delta)\left[\eta \bar{K}^{H,r}_{t} + (1-\eta)\bar{K}^{L,l}_{t}\right] + A(I_t, I_{t-1}) \ .$$

Riskier entrepreneurs and retail banks

The role of the representative retail bank in the model is to collect time deposits from households in order to finance riskier entrepreneur's investment project. The bank hedges against credit risk by charging a premium over the risk-free rate at which it can borrow from households. The risk-free rate that the bank views as its opportunity cost to lending is a contractual nominal interest rate that is determined at the time the bank liability to households is issued. Unlike in Bernanke et al. (1999), this rate is not contingent on the shocks that intervene before the entrepreneurial loan matures.

At each point in time there is a continuum of heterogeneous entrepreneurs of total measure η, indexed by (H,r). At the end of time t, each entrepreneur is characterized by his net worth, $N^{H,r}_{t+1}$, which is used, in combination with a bank loan, to purchase the time-$(t+1)$ stock of capital, $\bar{K}^{H,r}_{t+1}$. After the purchase, the entrepreneur experiences an idiosyncratic productivity shock, $\omega^{H,r}_{t+1}$, which transforms the purchased capital $\bar{K}^{H,r}_{t+1}$ into $\omega^{H,r}_{t+1}\bar{K}^{H,r}_{t+1}$. By assumption, $\omega^{H,r}$ is independently and identically distributed over time and across entrepreneurs and follows a log-normal distribution,

$$\ln\left(\omega^{H,r}\right) \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right) \ ,$$

where σ is the standard deviation of $\ln\left(\omega^{H,r}\right)$.

Capital utilization decision

At the beginning of period t, the representative entrepreneur provides capital services to intermediate-good firms. Capital services, $K^{H,r}_t$, are related to the entrepreneur's stock of physical capital, $\bar{K}^{H,r}_t$, by $K^{H,r}_t = u^{H,r}_t \bar{K}^{H,r}_t$, where $u^{H,r}_t$ denotes the level of capital utilization. In choosing the capital utilization rate, the entrepreneur takes into account the increasing and

and

$$A_{2,t+1} = \frac{\partial A(I_{t+1}, I_t)}{\partial I_t} = S''\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right) \ .$$

convex utilization cost function $a\left(u_t^{H,r}\right)$, that denotes the cost, in units of final goods, of setting the utilization rate to $u_t^{H,r}$.[25]

The entrepreneur chooses $u_t^{H,r}$ solving the following maximization problem:

$$\max_{u_t^{H,r}} \left[u_t^{H,r} r_t^{k,H} - a\left(u_t^{H,r}\right)\right] \omega^{H,r} \bar{K}_t^{H,r} P_t \ .$$

After determining the utilization rate of capital and earning rent on it, the entrepreneur sells the undepreciated part to capital producers at price $Q_{\bar{K}',t}$. The entrepreneur's nominal gross rate of return on capital purchased at time $t-1$, $1+R_t^{k,H,r}$, is given by

$$1+R_t^{k,H,r} = \frac{\left[u_t^{H,r} r_t^{k,H} - a\left(u_t^{H,r}\right)\right] P_t + (1-\delta)\, Q_{\bar{K}',t}}{Q_{\bar{K}',t-1}} \omega^{H,r} \ .$$

Because the mean of $\omega^{H,r}$ across entrepreneurs is unity, we may define the average nominal gross rate of return on capital across all entrepreneurs as follows

$$1+R_t^{k,H} = \frac{\left[u_t^{H,r} r_t^{k,H} - a\left(u_t^{H,r}\right)\right] P_t + (1-\delta)\, Q_{\bar{K}',t}}{Q_{\bar{K}',t-1}} \ . \tag{A.6}$$

Loan decision and the standard debt contract

At the end of period t, the entrepreneur has available net worth, $N_{t+1}^{H,r}$, which he uses to finance his capital expenditures, $Q_{\bar{K}',t}\bar{K}_{t+1}^{H,r}$. To finance the difference between expenditures and net worth, he borrows an amount $B_{t+1}^{H,r} = Q_{\bar{K}',t}\bar{K}_{t+1}^{H,r} - N_{t+1}^{H,r}$ from the retail bank.

After the purchase, the entrepreneur experiences an idiosyncratic productivity shock, $\omega_{t+1}^{H,r}$, which transforms the purchased capital $\bar{K}_{t+1}^{H,r}$ into $\omega_{t+1}^{H,r}\bar{K}_{t+1}^{H,r}$. Financial frictions arise from asymmetric information between entrepreneur and bank. In particular, the entrepreneur costlessly observes his idiosyncratic shock, whereas the bank must pay a monitoring cost – which represent a fraction μ, $0<\mu<1$, of the entrepreneur's gross return – to observe it. The optimal financing mechanism is a standard debt contract which gives the lender the right to all liquidation proceeds in the event of an entrepreneur's default.

[25] The functional form that we use is $a\left(u_t^{H,r}\right) = \frac{r^{k,H}}{\sigma_a^H}\left[\exp^{\sigma_a^H\left(u_t^{H,r}-1\right)} - 1\right]$, where $r^{k,H}$ is the steady state value of the rental rate of capital, $a(1)=0$, $a''(1)>0$ and $\sigma_a^H = a''(1)/a'(1)$ is a parameter that controls the degree of convexity of costs.

At the end of time t, the bank offers a debt contract to the entrepreneur, which specifies the loan amount, $B_{t+1}^{H,r}$, and the gross interest rate on the loan, $Z_{t+1}^{H,r}$. At time $t+1$, the entrepreneur declares bankruptcy if $\omega_{t+1}^{H,r}$ is smaller than the default threshold level, $\bar{\omega}_{t+1}^{H,r}$, defined by

$$\bar{\omega}_{t+1}^{H,r}\left(1+R_{t+1}^{k,H,r}\right)Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}=Z_{t+1}^{H,r}B_{t+1}^{H,r}\ . \tag{A.7}$$

Therefore, if $\omega_{t+1}^{H,r}>\bar{\omega}_{t+1}^{H,r}$, the entrepreneur pays the lender the amount $Z_{t+1}^{H,r}B_{t+1}^{H,r}$ and keeps the remaining $\left(\omega_{t+1}^{H,r}-\bar{\omega}_{t+1}^{H,r}\right)\left(1+R_{t+1}^{k,H,r}\right)Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}$. On the other hand, if $\omega_{t+1}^{H,r}<\bar{\omega}_{t+1}^{H,r}$, the entrepreneur defaults and receives nothing, while the bank monitors the entrepreneur at cost $\mu\left(1+R_{t+1}^{k,H,r}\right)\omega_{t+1}^{H,r}Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}$ and receives all of the residual net worth $(1-\mu)\left(1+R_{t+1}^{k,H,r}\right)\omega_{t+1}^{H,r}Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}$.

The bank raises the funds that are necessary to finance the entrepreneurs activities issuing time deposits to households, and pays them a nominal rate of return R_{t+1}^{e}. Perfect competition in the banking sector implies that the following bank's zero profit condition holds in each period:

$$\underbrace{\left[1-F_t\left(\bar{\omega}_{t+1}^{H,r}\right)\right]Z_{t+1}^{H,r}B_{t+1}^{H,r}}_{revenue\,from\,non-bankrupt\,entrepreneurs}+\underbrace{(1-\mu)\int_0^{\bar{\omega}_{t+1}^{H,r}}\omega^{H,r}dF\left(\omega^{H,r}\right)\left(1+R_{t+1}^{k,H,r}\right)Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}}_{revenue,\,after\,monitoring\,cost,\,from\,bankrupt\,entrepreneurs}=$$

$$\underbrace{\left(1+R_{t+1}^{e}\right)B_{t+1}^{H,r}}_{payment\ to\ households}\quad , \tag{A.8}$$

where $F_t\left(\omega^{H,r}\right)$ is the cumulative distribution function of $\omega^{H,r}$.

Let $k_{t+1}^{H,r}=\frac{Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}}$ denote the ratio of capital expenditures to net worth. Combining (A.7) with (A.8) and using the definition of k_{t+1} yields

$$\left[\Gamma_t\left(\bar{\omega}_{t+1}^{H,r}\right)-\mu G_t\left(\bar{\omega}_{t+1}^{H,r}\right)\right]k_{t+1}^{H,r}\frac{1+R_{t+1}^{k,H}}{1+R_{t+1}^{e}}=k_{t+1}^{H,r}-1\ ,$$

where $G_t\left(\bar{\omega}_{t+1}^{H,r}\right)=\int_0^{\bar{\omega}_{t+1}^{H,r}}\omega^{H,r}dF\left(\omega^{H,r}\right)$ and $\Gamma_t\left(\bar{\omega}_{t+1}^{H,r}\right)=\bar{\omega}_{t+1}^{H,r}\left[1-F_t\left(\bar{\omega}_{t+1}^{H,r}\right)\right]+G_t\left(\bar{\omega}_{t+1}^{H,r}\right)$. The term $\Gamma_t\left(\bar{\omega}_{t+1}^{H,r}\right)$ represents the share of entrepreneurial earnings received by the bank and $\mu G_t\left(\bar{\omega}_{t+1}^{H,r}\right)$ the expected monitoring costs. Therefore $1-\Gamma_t\left(\bar{\omega}_{t+1}^{H,r}\right)$ is the share of profits going to the entrepreneur.

The contract determines the division of the expected profits between borrower and lender. In particular, the optimal contract maximizes the entrepreneur's expected return at time $t+1$ subject to the zero profit condition on banks. The optimal contracting problem may be written in the following way:

$$\max_{\left\{k_{t+1}^{H,r},\bar{\omega}_{t+1}^{H,r}\right\}} \qquad E_t\left\{\left[1-\Gamma_t\left(\bar{\omega}_{t+1}^{H,r}\right)\right]\frac{1+R_{t+1}^{k,H}}{1+R_{t+1}^{e}}k_{t+1}^{H,r}\right\}$$

$$subject\ to \quad \left[\Gamma_t\left(\bar{\omega}_{t+1}^{H,r}\right)-\mu G_t\left(\bar{\omega}_{t+1}^{H,r}\right)\right]k_{t+1}^{H,r}\frac{1+R_{t+1}^{k,H}}{1+R_{t+1}^{e}}=k_{t+1}^{H,r}-1\ .$$

The first order conditions of the contracting problem yield the following relationship between the leverage ratio, $\frac{Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}}$, and the expected discounted return to capital (see Bernanke et al., 1999 for details):

$$\frac{E_t\left(1+R_{t+1}^{k,H}\right)}{1+R_{t+1}^{e}}=\Psi\left(\frac{Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}}\right)\ ,$$

where $\Psi'>0$ for $N_{t+1}^{H,r}<Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}$. The ratio $\frac{E_t\left(1+R_{t+1}^{k,H}\right)}{1+R_{t+1}^{e}}$, which Bernanke et al. (1999) interpreted as the external finance premium faced by entrepreneur, depends positively on the entrepreneur's leverage ratio. Intuitively, all else equal, lower leverage means lower exposure, implying a lower probability of default, hence a lower credit risk, which the bank translates into a lower required return on lending.

Entrepreneurial net worth

After the loan contract received in $t-1$ is settled, the entrepreneurial equity, $V_t^{H,r}$, is given by

$$V_t^{H,r}=\left(1+R_t^{k,H}\right)Q_{\bar{k}',t-1}\bar{K}_t^{H,r}-$$
$$\left[1+R_t^e+\frac{\mu\int_0^{\bar{\omega}_t^{H,r}}\omega^{H,r}dF_{t-1}\left(\omega^{H,r}\right)\left(1+R_t^{k,H}\right)Q_{\bar{k}',t-1}\bar{K}_t^{H,r}}{Q_{\bar{k}',t-1}\bar{K}_t^{H,r}-N_t^{H,r}}\right]\left(Q_{\bar{k}',t-1}\bar{K}_t^{H,r}-N_t^{H,r}\right)\ .$$

Equity depends on the profits accumulated from the entrepreneur's activities. The first term represents the proceeds from selling undepreciated capital to capital producers, plus the rental income of capital, net of the costs of utilization (see equation A.6). The term in squared brackets represents the gross rate of return paid by entrepreneur on time-$(t-1)$ loans.

At this point, to ensure that entrepreneur does not accumulate enough net worth to be fully self-financed, CMR assume that there is a constant probability of death. Namely, in each period entrepreneur exits the economy with probability $1-\gamma^H$. In this case, entrepreneur rebates his equity to households in a lump-sum way:

$$transfer\ to\ households = \left(1-\gamma^H\right) V_t^{H,r} \ .$$

To keep the population constant, $1-\gamma^H$ entrepreneurs are born each period.

Entrepreneurial net worth, $N_{t+1}^{H,r}$, combines the total equity and a transfer received from households, $W_t^{e,H,r}$, and is given by

$$N_{t+1}^{H,r} = \gamma^H V_t^H + W_t^{e,H,r} \ .$$

A feature of the debt contract is that entrepreneurs with no net worth receive no loans. Thus, if newborn entrepreneurs receive no transfers, they would have zero net worth and would therefore not be able to purchase any capital. The same happens with the fraction of entrepreneurs who are bankrupt due to a low realization of ω. To avoid this situation, the $1-\gamma^H$ entrepreneurs who are born and the γ^H who survive receive the subsidy $W_t^{e,H,r}$ from households.

Households

There is a continuum of infinitely lived risk averse households, indexed by $j \in [0,1]$. Each household consumes, supplies a differentiated labor input and allocates his savings between riskless time deposit and corporate bonds. As households differ in hours worked and in income, one would expect that they would also differ in consumption and asset allocations. However, each household j is assumed to hold state-contingent securities that provide insurance against household-specific wage-income risk. As a result, households are homogeneous with respect to consumption and asset holdings in equilibrium. Therefore, in what follows, consumption and saving decisions are not indexed by j.[26]

[26] See Erceg et al. (2000) for a discussion about the existence of state-contingent securities.

Consumption and saving decisions

The instantaneous utility function of a given household is separable in consumption and hours worked and given by:

$$u(\cdot) = \log\left(C_{t+\tau} - bC_{t+\tau-1}\right) - \psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L} \ , \tag{A.9}$$

where C_t denotes the household consumption at time t and $h_{j,t}$ denotes its hours worked in period t. The parameter $b > 0$ measures the degree of external habit formation in consumption, $\sigma_L > 0$ is the inverse of the Frisch elasticity of labor supply and $\psi_L > 0$ is a preference parameter that affects the disutility of supplying labor.

At the end of period t, household allocates his savings into time deposits, T_t, and corporate bonds, CB_t. At the end of period $t+1$, time deposits pay a riskless rate of return equal to R^e_{t+1}, while the rate of return on corporate bonds is R^F_{t+1}. We assume that both rates are known when household makes his saving decision and are not contingent on the realization of period-$(t+1)$ monetary policy shock.

The household budget constraint at time t, written in nominal terms, is given by

$$\begin{gathered}
\left(1+R^e_t\right)T_{t-1} + \left(1+R^F_t\right)CB_{t-1} + W_{j,t}h_{j,t} \\
+\left(1-\gamma^L\right)\left[1-\Omega\left(R_t^{coupon,a} - R_t^{coupon}\right)\right]\left(1-\eta\right)V_t^{L,l} + \left(1-\gamma^H\right)\eta V_t^{H,r} \\
+\Pi_t^{IGF} + \Pi_t^{IB} + NCS_t - CB_t - T_t - P_tC_t - W_t^e - Lump_t \geq 0
\end{gathered} \quad ,$$

where $W_{j,t}$ is the wage earned by the household j, $NCS_{j,t}$ represents the net payoff of the state contingent securities that the j^{th} household purchases to insulate itself from the uncertainty associated with the ability to re-optimize its wage, Π_t^{IGF} and Π_t^{IB} are the profits received from, respectively, intermediate-good firms and investment banks, $\left(1-\gamma^H\right)\eta V_t^{H,r}$ are the lump-sum transfers received from riskier entrepreneurs who exit the economy, $\left(1-\gamma^L\right)\left[1-\Omega\left(R_t^{coupon,a} - R_t^{coupon}\right)\right]\left(1-\eta\right)V_t^{L,l}$ are the lump-sum transfers (net of side payments) received from safer entrepreneurs who exit the economy, W_t^e is the total transfer payment to entrepreneurs and $Lump_t$ are lump-sum taxes paid to finance government expenditures.

The representative household takes its consumption and saving decisions so as to maximize the expected lifetime utility subject to its intertemporal budget constraint. The optimization problem is given by

$$\max_{\{C_{t+\tau},T_{t+\tau},CB_{t+\tau}\}} \quad E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\log\left(C_{t+\tau} - bC_{t+\tau-1}\right) - \psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L} \right]$$

$$\begin{aligned} subject\ to \quad & \left(1+R^e_{t+\tau}\right) T_{t-1+\tau} + \left(1+R^F_{t+\tau}\right) CB_{t-1+\tau} + W_{j,t+\tau} h_{j,t+\tau} \\ & + \left(1-\gamma^L\right) \left[1 - \Omega\left(R^{coupon,a}_{t+\tau} - R^{coupon}_{t+\tau}\right)\right] V^L_{t+\tau} + \left(1-\gamma^H\right) V^H_{t+\tau} \\ & + \Pi^{IGF}_{t+\tau} + \Pi^{IB}_{t+\tau} + NCS_{t+\tau} - CB_{t+\tau} - T_{t+\tau} \\ & - P_{t+\tau} C_{t+\tau} - W^e_{t+\tau} - Lump_{t+\tau} \geq 0 \ . \end{aligned}$$

The first order conditions with respect to T_t, CB_t and C_t are, respectively,

$$\lambda_t = \beta\left(1+R^e_{t+1}\right) E_t\left(\lambda_{t+1}\right) \tag{A.10}$$

$$\lambda_t = \beta\left(1+R^F_{t+1}\right) E_t\left(\lambda_{t+1}\right) \tag{A.11}$$

$$P_t \lambda_t = \frac{1}{\left(C_t - bC_{t-1}\right)} - \beta b \frac{1}{\left(C_{t+1} - bC_t\right)} \ , \tag{A.12}$$

where λ_t is the Lagrange multiplier associated to the households' budget constraint. Equation (A.10) represents the standard Euler equation. The right hand side of (A.12) is the marginal utility of consumption, taking into account habit persistence. Comparing (A.10) and (A.11), it must hold that $R^F_{t+1} = R^e_{t+1}\, \forall t$, *i.e.* the return on corporate bonds equals the return on time deposits, which in turn is equal to the central bank nominal interest rate. This result is due to the assumption that both interest rates are known when household makes his optimal decision and are not contingent on the realization of period-$(t+1)$ monetary policy shock.

Labor supply and wage setting

Each household is a monopolistic supplier of a differentiated labor service, $h_{j,t}$, to the production sector. Labor services are bundled together using the aggregator function

$$L_{i,t} = \left[\int_0^1 \left(h_{j,t}\right)^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w} \ , \tag{A.13}$$

where λ_w, $\infty > \lambda_w \geq 1$, represents the wage markup. The demand curve for the j^{th} household specialized labor services is

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t} \ ,$$

and the aggregate nominal wage, W_t, is given by

$$W_t = \left[\int_o^1 \left(W_{j,t}\right)^{\frac{1}{1-\lambda_w}} dj\right]^{1-\lambda_w} \ . \tag{A.14}$$

In each period, a fraction ξ_w of households cannot reoptimize their wages and, therefore, set their wages according to the indexation rule

$$W_{j,t} = W_{j,t-1} \left(\bar{\pi}\right)^{\iota_{w1}} \left(\pi_{t-1}\right)^{1-\iota_{w1}} \ ,$$

where i_{w1}, $0 \leq \iota_{w1} \leq 1$, represents the degree of wage indexation to steady state inflation rate. The fraction $1-\xi_w$ of reoptimizing households set their wages by maximizing

$$\begin{aligned} \max_{W_{j,t}} \qquad & E_t \sum_{\tau=0}^{\infty} \left(\beta \xi_w\right)^{\tau} \left[-\psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L} + \lambda_{t+\tau} W_{j,t+\tau} h_{j,t+\tau}\right] \\ subject\ to \qquad & h_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t} \ . \end{aligned}$$

We only consider the symmetric equilibrium in which all households choose the same $\tilde{W}_t = W_{j,t}$. Thus, given (A.14), the law of motion of the aggregate wage index is given by

$$W_t = \left\{\left(1-\xi_w\right) \tilde{W}_t^{\frac{1}{1-\lambda_w}} + \xi_w \left[W_{t-1} \left(\bar{\pi}\right)^{\iota_{w1}} \left(\pi_{t-1}\right)^{1-\iota_{w1}}\right]^{\frac{1}{1-\lambda_w}}\right\}^{1-\lambda_w} \ .$$

Resource constraint

The aggregate resource constraint is

$$\begin{aligned} & C_t + I_t + GC_t + \eta \mu \int_0^{\bar{\omega}_t} \omega dF\left(\omega\right) \left(1 + R_t^{k,H}\right) \frac{Q_{\bar{k}',t-1} \bar{K}_t^{H,r}}{P_t} \\ & + UC_t + \eta a \left(u_t^{H,r}\right) \bar{K}_t^{H,r} + \left(1-\eta\right) a \left(u_t^{L,l}\right) \bar{K}_t^{L,l} = Y_t \ . \end{aligned}$$

Government expenditures, GC_t, are determined exogenously as a constant fraction, η_g, of final output: $GC_t = \eta_g Y_t$ and are financed by lump-sum taxes to the households. The fourth term represents final output used by banks in monitoring riskier entrepreneurs, and $UC_t = \Omega\left(R_t^{coupon,a} - R_t^{coupon}\right)(1-\eta)V_t^{L,l}$ represents the underwriter's consumption in period t. Finally, the last two terms on the left hand side capture capital utilization costs.

Aggregate variables and market clearing conditions

Aggregate net worth (N_{t+1}^{TOT}) and aggregate leverage (lev_{t+1}^{TOT}) are defined, respectively, as

$$N_{t+1}^{TOT} = \eta N_{t+1}^{H,r} + (1-\eta) N_{t+1}^{L,l}$$

and

$$lev_{t+1}^{TOT} = \eta\, lev_{t+1}^{H,r} + (1-\eta)\, lev_{t+1}^{L,l} = \eta \frac{Q_{\bar{k}',t}\bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}} + (1-\eta)\frac{Q_{\bar{k}',t}\bar{K}_{t+1}^{L,l}}{N_{t+1}^{L,l}} \ .$$

Total credit (B_{t+1}^{TOT}) is defined as the sum of bank loans and bonds issued and is given by:

$$B_{t+1}^{TOT} = \eta B_{t+1}^{H,r} + (1-\eta) BI_{t+1}^{L,l} \ .$$

The capital rental market clearing conditions are:

$$\int_0^1 K_{i,t}^{H} di = K_t^H = \eta K_t^{H,r} = \eta u_t^{H,r} \bar{K}_t^{H,r}$$

and

$$\int_0^1 K_{i,t}^{L} di = K_t^L = (1-\eta) K_t^{L,l} = (1-\eta) u_t^{L,l} \bar{K}_t^{L,l} \ .$$

Loan and bond market clearing conditions are, respectively, $T_t = \eta B_{t+1}^{H,r}$ and $CB_t = (1-\eta) BI_{t+1}^{L,l}$.

The market clearing condition in the labor market is: $L_t = \int_0^1 \left\{ \left[\int_0^1 \left(h_{j,t}\right)^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w} \right\} di$.

Finally, the total transfer from households (W_t^e) to entrepreneurs must satisfy

$$W_t^e = \eta W_t^{e,H,r} + (1-\eta) W_t^{e,L,l} \ .$$

Appendix B - Technical details

Final-good firms

The maximization problem solved by the representative final-good firm is the following:

$$\max_{Y_{i,t}} \qquad \Pi_t^{FGF} = P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

$$subject\ to \qquad Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f} \ .$$

The first order condition is:

$$\frac{\partial \Pi_t^{FGF}}{\partial Y_{i,t}} = 0 \Leftrightarrow P_t \lambda_f \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f - 1} \frac{1}{\lambda_f} Y_{i,t}^{\frac{1}{\lambda_f} - 1} - P_{i,t} = 0 \ .$$

A simple algebra shows that the demand function for the intermediate good i is given by

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_f}{1-\lambda_f}} Y_t \ . \tag{B.1}$$

Substituting (B.1) into the expression for Π_t^{FGF} yields

$$\Pi_t^{FGF} = P_t Y_t - \int_0^1 P_{i,t} \left(\frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_f}{1-\lambda_f}} Y_t di \ . \tag{B.2}$$

Perfect competition in the final-good market implies that $\Pi_t^{FGF} = 0$. Imposing this condition in (B.2) gives the following expression for the price of the final good:

$$P_t = \left[\int_0^1 P_{i,t}^{\frac{1}{1-\lambda_f}} di \right]^{1-\lambda_f} \ . \tag{B.3}$$

Intermediate-good firms

Cost minimization problem

The i-th firm's cost minimization problem, in real terms, is given by

$$\min_{\left\{L_{i,t},K_{i,t}^{H},K_{i,t}^{L}\right\}} C(\cdot)=\frac{W_t L_{i,t}}{P_t}+K_{i,t}^{H}r_t^{k,H}+K_{i,t}^{L}r_t^{k,L} \tag{B.4}$$

$$subject\ to\ \ Y_{i,t}=\left(K_{i,t}\right)^{\alpha}\left(L_{i,t}\right)^{1-\alpha} \tag{B.5}$$

$$K_{i,t}=\left[\eta^{1-\rho}\left(K_{i,t}^{H}\right)^{\rho}+(1-\eta)^{1-\rho}\left(K_{i,t}^{L}\right)^{\rho}\right]^{\frac{1}{\rho}}\ . \tag{B.6}$$

Solving (B.5) for L_t and using (B.6), the minimization problem may be rewritten as

$$\begin{aligned}\min_{\left\{K_{i,t}^{H},K_{i,t}^{L}\right\}} C(\cdot)=\frac{W_t}{P_t}\left(Y_{i,t}\right)^{\frac{1}{1-\alpha}}&\left[\eta^{1-\rho}\left(K_{i,t}^{H}\right)^{\rho}+(1-\eta)^{1-\rho}\left(K_{i,t}^{L}\right)^{\rho}\right]^{-\frac{\alpha}{\rho(1-\alpha)}}\\ &+K_{i,t}^{H}r_t^{k,H}+K_{i,t}^{L}r_t^{k,L}\end{aligned}\ .$$

The first order conditions with respect to $K_{i,t}^{H}$ and $K_{i,t}^{L}$ are, respectively,

$$\begin{aligned}r_t^{k,H}&=\frac{W_t}{P_t}\left(Y_{i,t}\right)^{\frac{1}{1-\alpha}}\\ &\frac{\alpha}{1-\alpha}\eta^{1-\rho}\left[\eta^{1-\rho}\left(K_{i,t}^{H}\right)^{\rho}+(1-\eta)^{1-\rho}\left(K_{i,t}^{L}\right)^{\rho}\right]^{-\frac{\alpha}{\rho(1-\alpha)}-1}\left(K_{i,t}^{H}\right)^{\rho-1}\end{aligned} \tag{B.7}$$

$$\begin{aligned}r_t^{k,L}&=\frac{W_t}{P_t}\left(Y_{i,t}\right)^{\frac{1}{1-\alpha}}\\ &\frac{\alpha}{1-\alpha}(1-\eta)^{1-\rho}\left[\eta^{1-\rho}\left(K_{i,t}^{H}\right)^{\rho}+(1-\eta)^{1-\rho}\left(K_{i,t}^{L}\right)^{\rho}\right]^{-\frac{\alpha}{\rho(1-\alpha)}-1}\left(K_{i,t}^{L}\right)^{\rho-1}\ .\end{aligned} \tag{B.8}$$

Taking the ratio of (B.7) and (B.8), the following arbitrage condition for the choice of capital services may be derived:

$$\frac{r_t^{k,H}}{r_t^{k,L}}=\left(\frac{\eta}{1-\eta}\right)^{1-\rho}\left(\frac{K_{i,t}^{H}}{K_{i,t}^{L}}\right)^{\rho-1}\ . \tag{B.9}$$

Since

$$K_{i,t}^{H}=\eta K_{i,t}^{H,r}=\eta\left(u_t^{H,r}\bar{K}_{i,t}^{H,r}\right)\ , \tag{B.10}$$

then the arbitrage condition may be rewritten in terms of entrepreneur-specific capital services as

$$\frac{r_t^{k,H}}{r_t^{k,L}} = \left(\frac{u_t^{H,r}\bar{K}_{i,t}^{H,r}}{u_t^{L,l}\bar{K}_{i,t}^{L,l}}\right)^{\rho-1} \quad .$$

From (B.7) we can derive the following expression for $K_{i,t}$:

$$K_{i,t} = \left[\frac{W_t}{P_t}\frac{1}{r_t^{k,H}}\frac{\alpha}{1-\alpha}\eta^{1-\rho}\left(K_{i,t}^H\right)^{\rho-1}\right]^{\frac{1-\alpha}{\rho+\alpha(1-\rho)}}\left(Y_{i,t}\right)^{\frac{1}{\rho+\alpha(1-\rho)}} \quad . \tag{B.11}$$

Now compute $r_t^{k,L}$ from (B.9) and $K_{i,t}^L$ from (B.6). Using these results and equation (B.11) to substitute in (B.4), it takes a few steps to obtain the following expression for the cost function $C(\cdot)$:

$$C(\cdot) = \left[\frac{W_t}{P_t}\frac{1}{1-\alpha}\right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}}\left[\frac{\alpha\eta^{1-\rho}}{r_t^{k,H}}\left(K_{i,t}^H\right)^{\rho-1}\right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}}\left(Y_{i,t}\right)^{\frac{\rho}{\rho+\alpha(1-\rho)}} \quad .$$

Real marginal costs are thus given by

$$s_{i,t} = \frac{\partial C(\cdot)}{\partial Y_{i,t}} = \left[\frac{W_t}{P_t}\frac{1}{1-\alpha}\right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}}\left[\frac{\alpha\eta^{1-\rho}}{r_t^{k,H}}\left(K_{i,t}^H\right)^{\rho-1}\right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}}\left(Y_{i,t}\right)^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}}\frac{\rho}{\rho+\alpha(1-\rho)} \quad .$$

Efficient input choice by firm i also implies that real marginal costs must be equal to the cost of renting one unit of capital divided by the marginal product of capital ($\partial Y/\partial K$). Since

$$\frac{\partial Y_{i,t}}{\partial K_{i,t}^H} = \alpha\left(\frac{L_{i,t}}{K_{i,t}}\right)^{1-\alpha}\eta^{1-\rho}\left(K_{i,t}^H\right)^{\rho-1}\left[\eta^{1-\rho}\left(K_{i,t}^H\right)^{\rho}+(1-\eta)^{1-\rho}\left(K_{i,t}^L\right)^{\rho}\right]^{\frac{1}{\rho}-1}$$

and

$$\frac{\partial Y_{i,t}}{\partial K_{i,t}^L} = \alpha\left(\frac{L_{i,t}}{K_{i,t}}\right)^{1-\alpha}(1-\eta)^{1-\rho}\left(K_{i,t}^L\right)^{\rho-1}\left[\eta^{1-\rho}\left(K_{i,t}^H\right)^{\rho}+(1-\eta)^{1-\rho}\left(K_{i,t}^L\right)^{\rho}\right]^{\frac{1}{\rho}-1} \quad ,$$

then

$$s_{i,t} = \frac{r_t^{k,H}}{\frac{\partial Y_{i,t}}{\partial K_{i,t}^H}} = \frac{r_t^{k,L}}{\frac{\partial Y_{i,t}}{\partial K_{i,t}^L}} \quad .$$

Since all firms i face the same input prices and since they all have access to the same production technology, real marginal costs $s_{i,t}$ are identical across firms, *i.e.*, $s_{i,t} = s_t$ with

$$s_t = \left[\frac{W_t}{P_t}\frac{1}{1-\alpha}\right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left[\frac{\alpha\eta^{1-\rho}}{r_t^{k,H}}\left(K_t^H\right)^{\rho-1}\right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left(Y_t\right)^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}} \frac{\rho}{\rho+\alpha(1-\rho)} ,$$

or, using equation (B.10),

$$s_t = \left[\frac{W_t}{P_t}\frac{1}{1-\alpha}\right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left[\frac{\alpha}{r_t^{k,H}}\left(K_t^{H,r}\right)^{\rho-1}\right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left(Y_t\right)^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}} \frac{\rho}{\rho+\alpha(1-\rho)} .$$

Price setting

Every firm faces a constant probability, $1-\xi_p$, of reoptimizing its price in any given period, whereas the non-reoptimizing firms set their prices according to the indexation rule $P_{i,t} = P_{i,t-1}\left(\bar{\pi}\right)^{\iota_1}\left(\pi_{t-1}\right)^{1-\iota_1}$. The i-th firm that optimizes its price at time t chooses $P_{i,t} = \tilde{P}_{i,t}$ that maximizes the present value of future expected nominal profits. The maximization problem is given by:

$$\max_{P_{i,t}} \Pi_t^{IGF} = E_t \sum_{\tau=0}^{\infty} \left(\beta\xi_p\right)^{\tau} \lambda_{t+\tau}\left[\left(P_{i,t+\tau} - S_{t+\tau}\right)Y_{i,t+\tau}\right]$$

$$subject\ to \qquad Y_{i,t+\tau} = \left(\frac{P_{i,t+\tau}}{P_{t+\tau}}\right)^{\frac{\lambda_f}{1-\lambda_f}} Y_{t+\tau} .$$

Substituting the demand function and rearranging yields

$$\max_{P_{i,t}} \Pi_t^{IGF} = E_t \sum_{\tau=0}^{\infty} \left(\beta\xi_p\right)^{\tau} \lambda_{t+\tau} Y_{t+\tau} P_{t+\tau} \left[\left(\frac{P_{i,t+\tau}}{P_{t+\tau}}\right)^{1+\frac{\lambda_f}{1-\lambda_f}} - s_{t+\tau}\left(\frac{P_{i,t+\tau}}{P_{t+\tau}}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right] . \quad \text{(B.12)}$$

We make use of the following definitions:

$$\tilde{p}_{t+\tau} = \frac{\tilde{P}_{t+\tau}}{P_{t+\tau}},\ p_{i,t+\tau} = \frac{P_{i,t+\tau}}{P_{t+\tau}},\ \lambda_{n,t+\tau} = \lambda_{t+\tau} P_{t+\tau} .$$

Then

$$\frac{P_{i,t+\tau}}{P_{t+\tau}} = \frac{\tilde{\pi}_{t+\tau}\dots\tilde{\pi}_{t+1}\tilde{P}_t}{\pi_{t+\tau}\dots\pi_{t+1}P_t} = X_{t,\tau}\tilde{p}_t \ , \tag{B.13}$$

where

$$X_{t,\tau} = \begin{cases} \frac{\tilde{\pi}_{t+\tau}\dots\tilde{\pi}_{t+1}}{\pi_{t+\tau}\dots\pi_{t+1}} & \tau > 0 \\ 1 & \tau = 0 \end{cases}$$

and $\tilde{\pi}_{t+\tau} = (\bar{\pi})^{\iota_1}(\pi_{t+\tau-1})^{1-\iota_1}$. Using (B.13) to substitute out $\frac{P_{i,t+\tau}}{P_{t+\tau}}$ in (B.12), then the profit maximization problem may be rewritten as

$$\max_{\tilde{p}_t} \ \Pi_t^{IGF} = E_t \sum_{\tau=0}^{\infty} (\beta\xi_p)^\tau J_{t+\tau}\left[X_{t,\tau}(\tilde{p}_t)^{1+\frac{\lambda_f}{1-\lambda_f}} - s_{t+\tau}(\tilde{p}_t)^{\frac{\lambda_f}{1-\lambda_f}}\right] \ ,$$

where $J_{t+\tau} = \lambda_{n,t+\tau}Y_{t+\tau}(X_{t,\tau})^{\frac{\lambda_f}{1-\lambda_f}}$ is exogenous from the point of view of the firm. The first order condition is

$$\frac{\partial \Pi_t^{IGF}}{\partial \tilde{p}_t} = 0 \Leftrightarrow E_t \sum_{\tau=0}^{\infty} (\beta\xi_p)^\tau \frac{J_{t+\tau}}{1-\lambda_f}(\tilde{p}_t)^{\frac{\lambda_f}{1-\lambda_f}-1}\left[X_{t,\tau}\tilde{p}_t - \lambda_f s_{t+\tau}\right] = 0 \ .$$

After rearranging, the first order condition becomes

$$\tilde{p}_t = \frac{E_t \sum_{\tau=0}^{\infty}(\beta\xi_p)^\tau J_{t+\tau}\lambda_f s_{t+\tau}}{E_t \sum_{\tau=0}^{\infty}(\beta\xi_p)^\tau J_{t+\tau}X_{t,\tau}} = \frac{K_{p,t}}{F_{p,t}} \ . \tag{B.14}$$

For computational tractability, it is crucial to write the infinite sums, $K_{p,t}$ and $F_{p,t}$, in a recursive representations. After some manipulations, one can show that

$$K_{p,t} = \lambda_{n,t}Y_t\lambda_f s_t + \beta\xi_p\left(\frac{\pi_t^{1-\iota_1}}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} K_{p,t+1}$$

and

$$F_{p,t} = \lambda_{n,t}Y_t + \beta\xi_p\left(\frac{\pi_t^{1-\iota_1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} \ .$$

Note that, when prices are fully flexible ($\xi_p = 0$), then $K_{p,t} = F_{p,t}$ and $s_t = 1/\lambda_f$, that is, the real marginal cost is the reciprocal of the markup.

We have derived the optimum price from the firm's first order condition. We now identify a consistency condition that must hold across all firm prices, which allows us to express $\tilde{p}_t$ in terms of aggregate variables only. Expanding (B.3) yields

$$P_t = \left[\int_0^1 P_{i,t}^{\frac{1}{1-\lambda_f}} di \right]^{1-\lambda_f} = \left[\int_{1-\xi_p} (P_{i,t})^{\frac{1}{1-\lambda_f}} + \int_{\xi_p} (P_{i,t})^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f} .$$

Regarding the limits of integration, $1-\xi_p$ refers to the firms that reoptimize prices in period t, while ξ_p refers to the firms that do not. Making use of the fact that whether firms are selected to reoptimize or not is determined randomly, we can rewrite the previous expression as follows:

$$P_t = \left\{ (1-\xi_p)\tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left[P_{t-1}\tilde{\pi}_t\right]^{\frac{1}{1-\lambda_f}} \right\}^{1-\lambda_f} .$$

Dividing both sides by P_t, it takes a few step to obtain

$$\tilde{p}_t = \left[\frac{1-\xi_p \left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right]^{1-\lambda_f} . \tag{B.15}$$

Finally, combining (B.15) with (B.14) we obtain

$$\frac{K_{p,t}}{F_{p,t}} = \left[\frac{1-\xi_p \left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right]^{1-\lambda_f} .$$

This expression relates the inflation rate to aggregate variables only.

Households

The wage decision

Each household j supplies a differentiated labor input to the production sector. Following Erceg et al. (2000), we assume that there is a representative employment agency that combines households' specialized labor, $h_{j,t}$, into homogeneous labor employed by firm i, $L_{i,t}$, using the following constant returns to scale technology:

$$L_{i,t} = \left[\int_0^1 \left(h_{j,t}\right)^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w} ,$$

where $\infty > \lambda_w \geq 1$ represents the wage markup. The representative employment agency hires $h_{j,t}$ in order to maximize its time-t profits:

$$\max_{h_{j,t}} \qquad W_t L_{i,t} - \int_0^1 W_{j,t} h_{j,t} dj$$

$$subject\ to \qquad L_{i,t} = \left[\int_0^1 \left(h_{j,t} \right)^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w} .$$

The first order condition leads to the following demand curve for the j^{th} household specialized labor services:

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t} \ .$$

Zero profit condition for the perfectly competitive employment agencies gives the following relation between the aggregate nominal wage and the wage earned by the household j:

$$W_t = \left[\int_o^1 \left(W_{j,t} \right)^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w} . \tag{B.16}$$

In each period, a fraction ξ_w of households cannot reoptimize their wages and, by assumption, set their wages according to the indexation rule $W_{j,t} = W_{j,t-1} \left(\bar{\pi} \right)^{\iota_{w1}} \left(\pi_{t-1} \right)^{1-\iota_{w1}}$. The fraction $1 - \xi_w$ of reoptimizing households set their wages solving the following problem

$$\max_{W_{j,t}} \qquad E_t \sum_{\tau=0}^{\infty} \left(\beta \xi_w \right)^{\tau} \left[-\psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L} + \lambda_{t+\tau} W_{j,t+\tau} h_{j,t+\tau} \right]$$

$$subject\ to \qquad h_{j,t+\tau} = \left(\frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \ .$$

Substituting out for $h_{j,t}$ using the labor demand curve yields:

$$\max_{W_{j,t}} E_t \sum_{\tau=0}^{\infty} \left(\beta \xi_w \right)^{\tau} \left\{ -\frac{\psi_L}{1+\sigma_L} \left[\left(\frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{1+\sigma_L} + \lambda_{t+\tau} W_{j,t+\tau} \left(\frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right\} .$$

This equation can be rewritten as:

$$\max_{W_{j,t}} \quad E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^{\tau} \left\{ -\frac{\psi_L}{1+\sigma_L} \left[\left(\frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{1+\sigma_L} \right.$$
$$\left. +\lambda_{t+\tau} \frac{P_{t+\tau}}{P_{t+\tau}} W_{j,t+\tau} \frac{W_{t+\tau}}{W_{j,t+\tau}} \left(\frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}+1} L_{i,t+\tau} \right\} . \tag{B.17}$$

We adopt the following definitions:

$$W_{j,t+\tau} = \tilde{W}_{t+\tau} \ , \ \tilde{w}_{t+\tau} = \frac{W_{t+\tau}}{P_{t+\tau}} \ , \ w_{t+\tau} = \frac{\tilde{W}_{t+\tau}}{W_{t+\tau}} \ ,$$

$$\lambda_{n,t+\tau} = \lambda_{t+\tau} P_{t+\tau} \ , \ \tilde{w}_{t+\tau} w_{t+\tau} = \frac{\tilde{W}_{t+\tau}}{P_{t+\tau}} \ .$$

Then

$$\frac{W_{j,t+\tau}}{W_{t+\tau}} = X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \ , \tag{B.18}$$

where

$$X_{t,\tau} = \begin{cases} \frac{\tilde{\pi}_{w,t+\tau} \dots \tilde{\pi}_{w,t+1}}{\pi_{t+\tau} \dots \pi_{t+1}} & \tau > 0 \\ 1 & \tau = 0 \end{cases}$$

and $\tilde{\pi}_{w,t+\tau} = (\bar{\pi})^{\iota_{w1}} (\pi_{t+\tau-1})^{1-\iota_{w1}}$. Substituting (B.18) in (B.17), we obtain

$$\max_{W_{j,t}} \quad E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^{\tau} \left\{ -\frac{\psi_L}{1+\sigma_L} \left[\left(X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{1+\sigma_L} \right.$$
$$\left. +\lambda_{n,t+\tau} \tilde{w}_{t+\tau} L_{i,t+\tau} \left(X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}+1} \right\} . \tag{B.19}$$

Maximizing (B.19) with respect to w_t yields[27]

$$E_t \sum_{\tau=0}^{\infty} (\beta\xi_w)^{\tau} \left\{ -\psi_L \left[\left(X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{\sigma_L} \frac{\lambda_w}{1-\lambda_w} L_{i,t+\tau} \left(X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}-1} X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right.$$
$$\left. + \lambda_{n,t+\tau} \tilde{w}_{t+\tau} L_{i,t+\tau} \frac{1}{1-\lambda_w} \left(X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right\} = 0 \ .$$

or, after rearranging,

$$E_t \sum_{\tau=0}^{\infty} (\beta\xi_w)^{\tau} L_{i,t+\tau} \left(X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\lambda_w}{1-\lambda_w} w_t^{\frac{\lambda_w}{1-\lambda_w}-1} \left\{ -\psi_L \left[\left(X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{\sigma_L} \right.$$
$$\left. + \lambda_{n,t+\tau} \frac{\tilde{w}_t w_t}{\lambda_w} X_{t,\tau} \right\} = 0 \ .$$

Multiplying this expression by $w_t^{-\frac{\lambda_w \sigma_L}{1-\lambda_w}}$ we obtain, after some manipulations,

$$\begin{aligned} E_t \sum_{\tau=0}^{\infty} (\beta\xi_w)^{\tau} L_{i,t+\tau} \left(X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \lambda_{n,t+\tau} \frac{\tilde{w}_t}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} X_{t,\tau} &= \\ E_t \sum_{\tau=0}^{\infty} (\beta\xi_w)^{\tau} L_{i,t+\tau}^{1+\sigma_L} \left(X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \psi_L \ . & \end{aligned} \tag{B.20}$$

Equation (B.20) can be rewritten as

$$F_{w,t} \tilde{w}_t w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} = \psi_L K_{w,t} \ ,$$

where

$$K_{w,t} = E_t \sum_{\tau=0}^{\infty} (\beta\xi_w)^{\tau} L_{i,t+\tau}^{1+\sigma_L} \left(X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}}$$

and

$$F_{w,t} = E_t \sum_{\tau=0}^{\infty} (\beta\xi_w)^{\tau} L_{i,t+\tau} \left(X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \lambda_{n,t+\tau} \frac{X_{t,\tau}}{\lambda_w} \ .$$

[27] Whether the household chooses w_t or $\tilde{W}_t = W_{j,t}$ makes no difference, since w_t is $\tilde{W}_t$ scaled by a variable over which the household has no control.

Therefore the optimal wage rate results

$$w_t = \left[\frac{\psi_L}{\tilde{w}_t} \frac{K_{w,t}}{F_{w,t}} \right]^{\frac{\lambda_w - 1}{\lambda_w(1+\sigma_L)-1}} . \tag{B.21}$$

We have derived the wage rate from the household's first order condition. We now derive an expression for the aggregate real wage, $\tilde{w}_t$, just in terms of aggregate variables.

Expanding equation (B.16) yields

$$W_t = \left[\int_o^1 \left(W_{j,t} \right)^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w} = \left[\int_{1-\xi_w} \left(W_{j,t} \right)^{\frac{1}{1-\lambda_w}} + \int_{\xi_w} \left(W_{j,t} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} .$$

Regarding the limits of integration, $1-\xi_w$ refers to the households that reoptimize in period t, while ξ_w refers to the households that do not. Making use of the fact that whether households are selected to optimize or not is determined randomly, we can rewrite the previous expression as follows:

$$W_t = \left[(1-\xi_w) \left(\tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(W_{t-1} \tilde{\pi}_{w,t} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} .$$

After dividing both sides by W_t, it takes few steps to obtain

$$w_t = \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w} , \tag{B.22}$$

where $\pi_{w,t} = W_t / W_{t-1} = \pi_t \tilde{w}_t / \tilde{w}_{t-1}$. Equating expressions (B.21) and (B.22) yields

$$K_{w,t} = \frac{F_{w,t} \tilde{w}_t}{\psi_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{\lambda_w(1+\sigma_L)-1} .$$

This expression relates the real wage to aggregate variables only. Note that, when wages are fully flexible ($\xi_w = 0$), the last expression becomes

$$\tilde{w}_t = \lambda_w \frac{\psi_L L_t^{\sigma_L}}{\lambda_{n,t}} ,$$

that is, the real wage in units of the consumption good, $\tilde{w}_t$, is a markup, λ_w, over the household's marginal cost of leisure, $\psi_L L_t^{\sigma_L}/\lambda_{n,t}$, also expressed in terms of the consumption good.

For computational tractability, it is crucial to write the infinite sums, $K_{w,t}$ and $F_{w,t}$, in a recursive representations. After some manipulations, one can show that

$$K_{w,t} = h_t^{1+\sigma_L} + \beta \xi_w \left[\frac{\pi_t^{1-\iota_{w1}}}{\pi_{t+1}\frac{\tilde{w}_{t+1}}{\tilde{w}_t}} \right]^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} K_{w,t+1}$$

and

$$F_{w,t} = h_t \frac{\lambda_{n,t}}{\lambda_w} + \beta \xi_w \left(\frac{1}{\pi_{t+1}\frac{\tilde{w}_{t+1}}{\tilde{w}_t}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\left(\pi_t^{1-\iota_{w1}}\right)^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} \ .$$

Appendix C - Model solution

This appendix reports the details on how we solved the model. The solution strategy involves linearization around the model's nonstochastic steady state. We first solve numerically the model, for the steady state, using the computational procedure described later in this appendix. We then employ the Dynare software package to compute the first-order Taylor series approximation of the equilibrium conditions in the neighborhood of the steady state.

In what follows, we adopt the following scaling notation:

$$q_t = \frac{Q_{\bar{k}'}}{P_t} \ , \ \lambda_{n,t} = \lambda_t P_t \ , \ w_t^{e,L,l} = \frac{W_t^{e,L,l}}{P_t} \ , \ w_t^{e,H,r} = \frac{W_t^{e,H,r}}{P_t} \ ,$$

$$n_{t+1}^{H,r} = \frac{N_{t+1}^{H,r}}{P_t} \ , \ n_{t+1}^{L,l} = \frac{N_{t+1}^{L,l}}{P_t} \ .$$

The equations that characterize the model's equilibrium, expressed in scaled form, are listed below.

- Investment bank
  - coupon rate (constant markup over the nominal interest rate)

$$1 + R_{t+1}^{coupon,a} = \frac{\varepsilon^{coupon,a}}{\varepsilon^{coupon,a} - 1}\left(1 + R_{t+1}^{e}\right) \tag{C.1}$$

  - law of motion for optimism

$$\chi_t = \rho_\chi \chi_{t-1} + \left(1 - \rho_\chi\right)\alpha_3\left(n_{t+1}^{L,l} - n^{L,l}\right) \tag{C.2}$$

  - coupon interest rate elasticity (with optimism)

$$\varepsilon_{t+1}^{coupon,biased} = \varepsilon^{coupon,a}\left(1 + \chi_t\right) \tag{C.3}$$

  - coupon rate (with optimism)

$$1 + R_{t+1}^{coupon,biased} = \frac{\varepsilon_{t+1}^{coupon,biased}}{\varepsilon_{t+1}^{coupon,biased} - 1}\left(1 + R_{t+1}^{e}\right) \tag{C.4}$$

– coupon interest rate elasticity (with optimism and side payments)

$$\varepsilon_{t+1}^{coupon} = \varepsilon^{coupon,a} \left(1 + r_2 \chi_t\right) \tag{C.5}$$

– coupon rate (with optimism and side payments)

$$1 + R_{t+1}^{coupon} = \frac{\varepsilon_{t+1}^{coupon}}{\varepsilon_{t+1}^{coupon} - 1} \left(1 + R_{t+1}^{e}\right) \tag{C.6}$$

- Intermediate-good firms

– arbitrage condition for the choice of capital services

$$\frac{r_t^{k,H}}{r_t^{k,L}} = \left(\frac{u_t^{H,r} \bar{K}_t^{H,r}}{u_t^{L,l} \bar{K}_t^{L,l}}\right)^{\rho - 1} \tag{C.7}$$

– two measure of marginal costs

$$s_t = \frac{\rho}{\rho + \alpha\left(1 - \rho\right)} \left[\frac{\tilde{w}_t}{1 - \alpha}\right]^{1 - \frac{\alpha}{\rho + \alpha(1 - \rho)}} \left[\frac{\alpha}{r_t^{k,H}} \left(u_t^{H,r} \bar{K}_t^{H,r}\right)^{\rho - 1}\right]^{-\frac{\alpha}{\rho + \alpha(1 - \rho)}} \left(Y_t\right)^{\frac{\alpha(\rho - 1)}{\rho + \alpha(1 - \rho)}} \tag{C.8}$$

$$s_t = \frac{r_t^{k,H}}{\alpha \left(\frac{h_t}{K_t}\right)^{1 - \alpha} \left(u_t^{H,r} \bar{K}_t^{H,r}\right)^{\rho - 1} \left[\eta \left(u_t^{H,r} \bar{K}_t^{H,r}\right)^{\rho} + \left(1 - \eta\right) \left(u_t^{L,l} \bar{K}_t^{L,l}\right)^{\rho}\right]^{\frac{1}{\rho} - 1}} \tag{C.9}$$

where

$$K_t = \left[\eta \left(u_t^{H,r} \bar{K}_t^{H,r}\right)^{\rho} + \left(1 - \eta\right) \left(u_t^{L,l} \bar{K}_t^{L,l}\right)^{\rho}\right]^{\frac{1}{\rho}} \tag{C.10}$$

- Capital producers

- first order condition with respect to investment

$$\lambda_{n,t} q_t \left[1 - \frac{S^{''}}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2 - S^{''} \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1\right)\right] - \lambda_{n,t}$$
$$+ \beta \lambda_{n,t+1} q_{t+1} S^{''} \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right) = 0 \tag{C.11}$$

- law of motion for aggregate stock of physical capital

$$\eta \bar{K}_{t+1}^{H,r} + (1-\eta) \bar{K}_{t+1}^{L,l} = (1-\delta) \left[\eta \bar{K}_t^{H,r} + (1-\eta) \bar{K}_t^{L,l}\right]$$
$$+ \left[1 - \frac{S^{''}}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t \tag{C.12}$$

- Riskier entrepreneur and retail bank

  - first order condition with respect to capital utilization

$$r_t^{k,H} = a^{'} \left(u_t^{H,r}\right) \tag{C.13}$$

  - definition of rate of return on capital

$$1 + R_t^{k,H} = \frac{\pi_t}{q_{t-1}} \left\{\left[u_t^{H,r} r_t^{k,H} - a\left(u_t^{H,r}\right)\right] + (1-\delta) q_t\right\} \tag{C.14}$$

  - standard debt contract

$$E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1})\right] \frac{1 + R_{t+1}^{k,H}}{1 + R_{t+1}^e} + \frac{\Gamma_t^{'}(\bar{\omega}_{t+1})}{\Gamma_t^{'}(\bar{\omega}_{t+1}) - \mu G_t^{'}(\bar{\omega}_{t+1})} \right.$$
$$\left. \left[\left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right] \frac{1 + R_{t+1}^{k,H}}{1 + R_{t+1}^e} - 1\right] \right\} = 0 \tag{C.15}$$

  - zero profit condition for bank

$$\left[\Gamma_t(\bar{\omega}_t) - \mu G_t(\bar{\omega}_t)\right] \frac{q_{t-1} \bar{K}_t^{H,r}}{n_t^{H,r}} \frac{1 + R_t^{k,H}}{1 + R_t^e} = \frac{q_{t-1} \bar{K}_t^{H,r}}{n_t^{H,r}} - 1 \tag{C.16}$$

- law of motion for net worth

$$n_{t+1}^{H,r} = \gamma^H \frac{q_{t-1}}{\pi_t} \bar{K}_t^{H,r} \left[R_t^{k,H} - R_t^e - \mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \left(1 + R_t^{k,H}\right) \right]$$
$$+\gamma^H \frac{n_t^{H,r}}{\pi_t} (1 + R_t^e) + w_t^{e,H,r} \tag{C.17}$$

- Safer entrepreneur
  - first order condition with respect to capital utilization

$$r_t^{k,L} = a' \left(u_t^{L,l} \right) \tag{C.18}$$

  - definition of rate of return on capital

$$1 + R_t^{k,L} = \frac{\pi_t}{q_{t-1}} \left\{ \left[u_t^{L,l} r_t^{k,L} - a\left(u_t^{L,l} \right) \right] + (1-\delta) q_t \right\} \tag{C.19}$$

  - first order condition with respect to capital (using the definition of rate of return on capital)

$$R_{t+1}^{coupon} - R_{t+1}^{k,L} - 1 + \frac{1}{\beta} = 0 \tag{C.20}$$

  - law of motion for net worth

$$n_{t+1}^{L,l} = \gamma^L \left[1 - \Omega\left(R_t^{coupon,a} - R_t^{coupon} \right) \right] \frac{q_{t-1}}{\pi_t} \bar{K}_t^{L,l} \left(R_t^{k,L} - R_t^{coupon} \right)$$
$$+ \left[1 - \Omega\left(R_t^{coupon,a} - R_t^{coupon} \right) \right] \frac{\gamma^L}{\pi_t} \left(1 + R_t^{coupon} \right) n_t^{L,l} + w_t^{e,L,l} \tag{C.21}$$

- Households
  - first order condition with respect to time deposits

$$\lambda_{n,t} = \frac{\beta}{\pi_{t+1}} \left(1 + R_{t+1}^e \right) \lambda_{n,t+1} \tag{C.22}$$

  - first order condition with respect to consumption

$$\lambda_{n,t} = \frac{1}{(C_t - bC_{t-1})} - \beta b \frac{1}{(C_{t+1} - bC_t)} \tag{C.23}$$

- Aggregate resource constraint and production function

$$C_t + I_t + \eta \left[\mu \int_0^{\bar{\omega}_t} \omega dF(\omega) \left(1 + R_t^{k,H}\right) \frac{q_{t-1}\bar{K}_t^{H,r}}{\pi_t} \right] + UC_t$$
$$+\eta a \left(u_t^{H,r}\right) \bar{K}_t^{H,r} + (1-\eta) a \left(u_t^{L,l}\right) \bar{K}_t^{L,l} = (1-\eta_g) Y_t \tag{C.24}$$

$$Y_t = K_t^{\alpha} h_t^{1-\alpha} \tag{C.25}$$

- Conditions associated with Calvo sticky prices and wages

$$\lambda_{n,t} Y_t + \beta \xi_p \left(\frac{\pi_t^{1-\iota_1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t} = 0 \tag{C.26}$$

$$\lambda_{n,t} Y_t \lambda_f s_t + \beta \xi_p \left(\frac{\pi_t^{1-\iota_1}}{\pi_{t+1}} \right)^{-\frac{\lambda_f}{\lambda_f - 1}} K_{p,t+1} - K_{p,t} = 0 \tag{C.27}$$

$$h_t \frac{\lambda_{n,t}}{\lambda_w} + \beta \xi_w \left(\frac{1}{\pi_{t+1} \frac{w_{t+1}}{w_t}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\left(\pi_t^{1-\iota_{w1}}\right)^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t} = 0 \tag{C.28}$$

$$h_t^{1+\sigma_L} + \beta \xi_w \left[\frac{\pi_t^{1-\iota_{w1}}}{\pi_{t+1} \frac{w_{t+1}}{w_t}} \right]^{\frac{\lambda_w (1+\sigma_L)}{1-\lambda_w}} K_{w,t+1} - K_{w,t} = 0 \tag{C.29}$$

$$K_{p,t} = F_{p,t} \left[\frac{1 - \xi_p \left(\frac{\pi_{t-1}^{1-\iota_1}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} \tag{C.30}$$

$$K_{w,t} = F_{w,t} \frac{\tilde{w}_t}{\psi_L} \left\{ \frac{1 - \xi_w \left[\frac{\pi_{t-1}^{1-\iota_{w1}}}{\pi_t \frac{w_t}{w_{t-1}}} \right]^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right\}^{1-\lambda_w (1+\sigma_L)} \tag{C.31}$$

- Other variables
  - External finance premium

$$P_t^e = \bar{\omega}_{t+1} \left(1 + R_{t+1}^{k,H}\right) \frac{q_t \bar{K}_{t+1}^{H,r}}{q_t \bar{K}_{t+1}^{H,r} - n_{t+1}^{H,r}} - \left(1 + R_{t+1}^e\right) \tag{C.32}$$

– Contractual, no-default interest rate on entrepreneurial debt

$$Z_t = \bar{\omega}_{t+1}\left(1 + R^{k,H}_{t+1}\right)\frac{q_t \bar{K}^{H,r}_{t+1}}{q_t \bar{K}^{H,r}_{t+1} - n^{H,r}_{t+1}} \tag{C.33}$$

– Aggregate net worth

$$n^{TOT}_{t+1} = \eta n^{H,r}_{t+1} + (1-\eta)\, n^{L,l}_{t+1} \tag{C.34}$$

– Bond amount

$$BI^{L,l}_{t+1} = q_t \bar{K}^{L,l}_{t+1} - n^{L,l}_{t+1} \tag{C.35}$$

– Bank loans

$$B^{H,r}_{t+1} = q_t \bar{K}^{H,r}_{t+1} - n^{H,r}_{t+1} \tag{C.36}$$

– Safer entrepreneur's leverage

$$lev^{L,l}_{t+1} = \frac{q_t \bar{K}^{L,l}_{t+1}}{n^{L,l}_{t+1}} \tag{C.37}$$

– Riskier entrepreneur's leverage

$$lev^{H,r}_{t+1} = \frac{q_t \bar{K}^{H,r}_{t+1}}{n^{H,r}_{t+1}} \tag{C.38}$$

– Aggregate leverage

$$lev^{TOT}_{t+1} = \eta\, lev^{H,r}_{t+1} + (1-\eta)\, lev^{L,l}_{t+1} \tag{C.39}$$

– Total credit (bank loans + bonds)

$$B^{TOT}_{t+1} = \eta B^{H,r}_{t+1} + (1-\eta)\, BI^{L,l}_{t+1} \tag{C.40}$$

• Monetary policy rule

$$R^e_t = \left(R^e_{t-1}\right)^{\tilde{\rho}}\left[R^e\left(\frac{E_t \pi_{t+1}}{\bar{\pi}}\right)^{\alpha_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\alpha_y}\right]^{(1-\tilde{\rho})}\varepsilon^{MP}_t \tag{C.41}$$

Steady state

The strategy used for computing the steady state in this model follows the approach used by Christiano et al. (2003). We set one of the endogenous variables of the model to a value that seems reasonable based on empirical evidence, making this variable exogenous in the steady state calculation. We then move a model's exogenous variable into the list of variables that are endogenous in the steady state calculation. This approach allows us to simplify the problem of computing the steady state.

We set the steady state rental rate of capital of the riskier entrepreneur, $r^{k,H}$, to 0.0504, in line with the value used by CMR, and we choose the parameter ψ_L in (A.9) as endogenous variable. The set of endogenous variables is:

$$\begin{gathered}\pi_t\,,\, s_t\,,\, I_t\,,\; \bar{\omega}_t\,,\, R_t^{k,H}\,,\, R_t^{k,L}\,,\, \bar{K}_t^{H,r}\,,\, \bar{K}_t^{L,l}\,,\, K_t\,,\, n_t^{H,r}\,,\, n_t^{L,l}\,, q_t\,,\, \lambda_{n,t}\,,\, C_t\,,\, \tilde{w}_t\,,\, h_t\,,\\ r_t^{k,L}\,,\, R_t^e\,,\, F_{p,t}\,,\, F_{w,t}\,, K_{p,t}\,,\, K_{w,t}\,,\, Y_t\,,\, \psi_L\,, u_t^{H,r}\,,\, u_t^{L,l}\,,\\ \varepsilon_t^{coupon}\,,\, \varepsilon_t^{coupon,biased}\,,\, R_t^{coupon}\,,\, R_t^{coupon,a}\,,\, R_t^{coupon,biased}\,,\, \chi_t\,,\\ P_t^{ext}\,,\, Z_t\,,\, B_t^{H,r}\,,\, BI_t^{L,l}\,,\, B_t^{TOT}\,,\, lev_t^{H,r}\,,\, lev_t^{L,l}\,,\, lev_t^{TOT}\,,\, n_t^{TOT}\;\;,\end{gathered}$$

and the equations available for computing the steady state value for these variables are (C.1)-(C.41).

As in Woodford (2003), steady state inflation is set to zero, that is, $\bar{\pi} = 1$. By assumption, $u^{H,r} = u^{L,l} = 1$ and $\chi = 0$. Solve for R^e and q using (C.22) and (C.11). Use (C.5) and (C.3) to compute ε^{coupon} and $\varepsilon^{coupon,biased}$. Solve for the steady state interest rates R^{coupon}, $R^{coupon,a}$ and $R^{coupon,biased}$ using, respectively, (C.6), (C.1) and (C.4). Take the ratio of (C.26) and (C.27) to obtain the value for s. Equations (C.20) and (C.19) can be used to obtain $R^{k,L}$ and $r^{k,L}$. Now we set $r^{k,H} = 0.0504$ and solve for $R^{k,H}$ using (C.14). Then solve the non-linear system composed by equations (C.15)-(C.17) to obtain the values for $n^{H,r}$, $\bar{\omega}$ and $\bar{K}^{H,r}$. From (C.7) we get the value for $\bar{K}^{L,l}$. Solve for $n^{L,l}$, K and I using (C.21), (C.10) and (C.12), respectively. Solve (C.9) for h and then (C.25) for Y. Then use (C.8) and (C.24) to solve for $\tilde{w}$ and C. Get λ_n using (C.23). Equations (C.26), (C.28) and (C.29) can be used to obtain F_p, F_w and K_w. It then follows from (C.30) that $K_p = F_p$. Finally, solve for ψ_L using (C.31). The remaining variables are trivial functions of the structural parameters and other steady state values and are computed using equations (C.32)-(C.40).

In these calculations, all variables must be positive, and $\bar{K}^{H,r} > n^{H,r} > 0$, $\bar{K}^{L,l} > n^{L,l} > 0$ and $Z > R^{coupon}$.

Appendix D - Calibration: threshold level for side payments

In this appendix we define the threshold for Ω below which the entrepreneur would always be better off when offering side payments.

The entrepreneur has two options. He can:

1. issue bonds at the "normal" coupon rate $R_{t+1}^{coupon,a}$ (equation 1.8);
2. offer side payments and obtain a lower coupon rate ($R_{t+1}^{coupon,biased}$, in equation 1.11).

In the first case, $R_t^{coupon} = R_t^{coupon,a}$, so entrepreneur's equity and net worth are given by, respectively,

$$V_t^{L,l,a} = revenues - \left(1 + R_t^{coupon,a}\right) BI_t^{L,l}$$

$$N_{t+1}^{L,l,a} = \gamma^L V_t^{L,l,a} + W_t^{e,L,l} \quad ,$$

where $revenues = \left\{ \left[u_t^{L,l} r_t^{k,L} - a\left(u_t^{L,l}\right)\right] P_t + (1-\delta)\, Q_{\bar{k}',t} \right\} \bar{K}_t^{L,l}$ and $BI_t^{L,l} = Q_{\bar{k}',t-1} \bar{K}_t^{L,l} - N_t^{L,l}$.

In the second case, $R_t^{coupon} = R_t^{coupon,biased}$, so entrepreneur's equity and net worth are now given by, respectively,

$$V_t^{L,l,b} = revenues - \left(1 + R_t^{coupon,biased}\right) BI_t^{L,l}$$

$$N_{t+1}^{L,l,b} = \gamma^L \left[1 - \Omega\left(R_t^{coupon,a} - R_t^{coupon,biased}\right)\right] V_t^{L,l,b} + W_t^{e,L,l} \quad .$$

The entrepreneur is therefore better off offering side payments whenever

$$
\begin{aligned}
& N_{t+1}^{L,l,b} \geq N_{t+1}^{L,l,a} \\
\Leftrightarrow \quad & \left[1-\Omega\left(R_t^{coupon,a}-R_t^{coupon,biased}\right)\right] V_t^{L,l,b} \geq V_t^{L,l,a} \\
\Leftrightarrow \quad & V_t^{L,l,b}-V_t^{L,l,a} \geq \Omega\left(R_t^{coupon,a}-R_t^{coupon,biased}\right) V_t^{L,l,b} \\
\Leftrightarrow \quad & \left(R_t^{coupon,a}-R_t^{coupon,biased}\right) BI_t^{L,l} \geq \Omega\left(R_t^{coupon,a}-R_t^{coupon,biased}\right) V_t^{L,l,b} \\
\Leftrightarrow \quad & BI_t^{L,l} \geq \Omega V_t^{L,l,b} \\
\Leftrightarrow \quad & \Omega \leq \frac{BI_t^{L,l}}{V_t^{L,l,b}} \ .
\end{aligned}
$$

Given the calibration in table 1.1, in the steady state it results that

$$
\Omega \leq \frac{BI^{L,l}}{V^{L,l,b}} = \frac{\bar{K}^{L,l}-n^{L,l}}{\left(r^{k,L}-\delta-R^{coupon,biased}\right)\bar{K}^{L,l}+\left(1+R^{coupon,biased}\right)n^{L,l}} = 0.25 = \bar{\Omega} \ .
$$

Table 1.1: Model parameters (time unit of model: quarterly)

Households	Value	Source	Description
β	0.9875	our calibration	discount factor
ψ_L	(36)	(endogenous)	weight on disutility of labor
σ_L	1	CMR	curvature of disutility of labor
b	0.63	CMR	habit persistence in consumption
ξ_w	0.75	Erceg et al. (2000)	fraction of households that cannot reoptimize wage
λ_w	1.05	CMR	markup, workers
ι_{w1}	0.29	CMR	weight of wage indexation to steady state inflation
Firms			
α	0.36	Levin et al. (2005)	capital share in the production function
ξ_p	0.75	Erceg et al. (2000)	fraction of firms that cannot reoptimize price
ι_1	0.16	CMR	weight of price indexation to steady state inflation
λ_f	1.2	CMR	markup, intermediate good firms
S''	29.3	CMR	curvature of investment adjustment cost function
δ	0.03	CMR	depreciation rate on capital
ρ	0.6	our calibration	degree of substitutability between capital services
Entrepreneurs			
σ_a^H, σ_a^L	18.9	CMR	curvature of capital utilization cost functions
μ	0.15	our calibration	fraction of realized profits lost in bankruptcy
σ^H	$\sqrt{0.3}$	our calibration	standard deviation of productivity shock
$\omega^{e,H,r}$, $\omega^{e,L,l}$	0.02	CMR	transfer from households
γ^L	0.96	our calibration	survival probability of safer entrepreneurs
γ^H	0.97	our calibration	survival probability of riskier entrepreneurs
η	0.3	our calibration	share of riskier entrepreneurs
Ω	0.1	our calibration	percentage of equity paid as side payments
Bond Market			
$\varepsilon^{coupon,a}$	510	Chen et al. (2007)	interest rate elasticity of the demand for funds
ρ_x	0.9	our calibration	degree of persistence in optimism
α_3	40	our calibration	sensitivity of optimism to entrepreneur's net worth
$\bar{\chi}$	0	our calibration	steady state level of optimism
Policy			
$\tilde{\rho}$	0.88	CMR	interest rate smoothing
α_π	1.82	CMR	weight of expected inflation in Taylor rule
α_y	0.11	CMR	weight of output gap in Taylor rule
η_g	0.2	CMR	share of government consumption

Table 1.2: Steady State Properties, Model vs U.S. Data

Variable	***Model***	***U.S. data***
K/Y	5.48	10.7
C/Y	0.63	0.56
I/Y	0.17	0.25
G/Y	0.2	0.2
leverage ratio = $Q\bar{K}/N$ [1]	$safer$ 1.26 $riskier$ 1.35	[1.21 ; 1.77]
bond to bank finance ratio [2]	**1.36**	**1.34**

When not specified, the source for U.S. data is CMR and the sample period is 1998Q4-2003Q4. [1] CMR compute the leverage as $N/(Q\bar{K}-N)$. We compute the leverage as in Bernanke et al. (1999). [2] Source: De Fiore and Uhlig (2005).

Table 1.3: Interest Rates, Model vs U.S. Data

Variable	***Model***	***U.S. data***
Rate of return on capital, R^k	$safer$ 11.38% $riskier$ 8.40%	10.32%
Cost of external finance, Z	6.81%	[7.1 ; 8.1]%
Time deposit, R^e	5.16%	5.12%
Cost of bond finance, R^{coupon}	5.99%	5.96% [1]

When not specified, the source for U.S. data is CMR and the sample period is 1987Q1-2003Q4. [1] Chen et al. (2007) find an average yield spread of *AAA* bonds over the period 1995-2003 of 84 basis points. Adding this spread to the risk-free rate (R^e) gives the value displayed in the table.

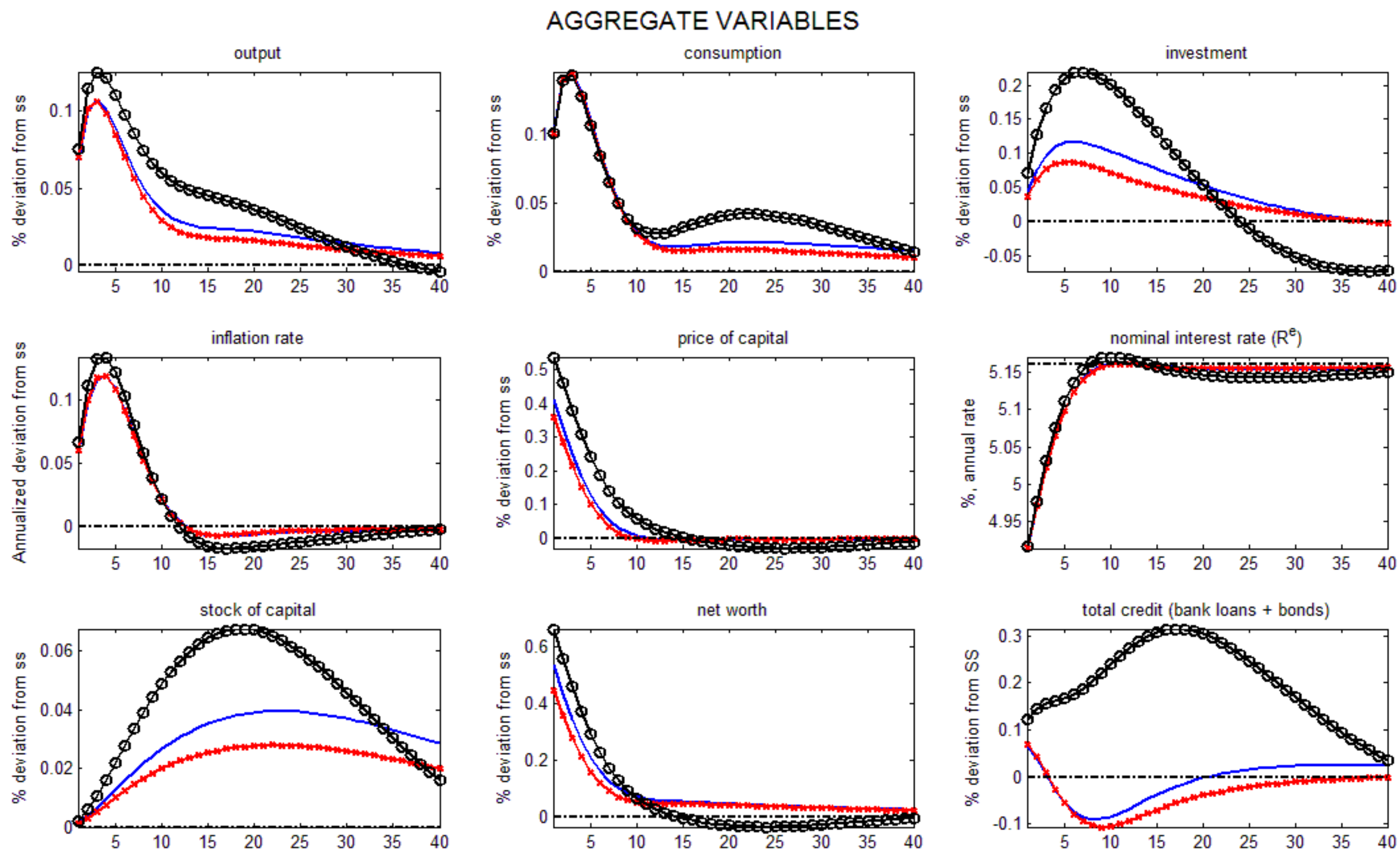

Figure 1.3: Impulse responses of aggregate variables to a 25 basis point decrease to the nominal interest rate

Note. Values expressed as percentage deviation from steady state values. Inflation is expressed as annualized percent deviation from its steady state and the interest rates are expressed as annual percentage points. Variant 1 (CMR, *Financial Accelerator Model*): blue solid line. Variant 2 (model with $r_2 = 0$): red crossed line. Variant 3 (model with $r_2 = 1$): black circled line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.1.

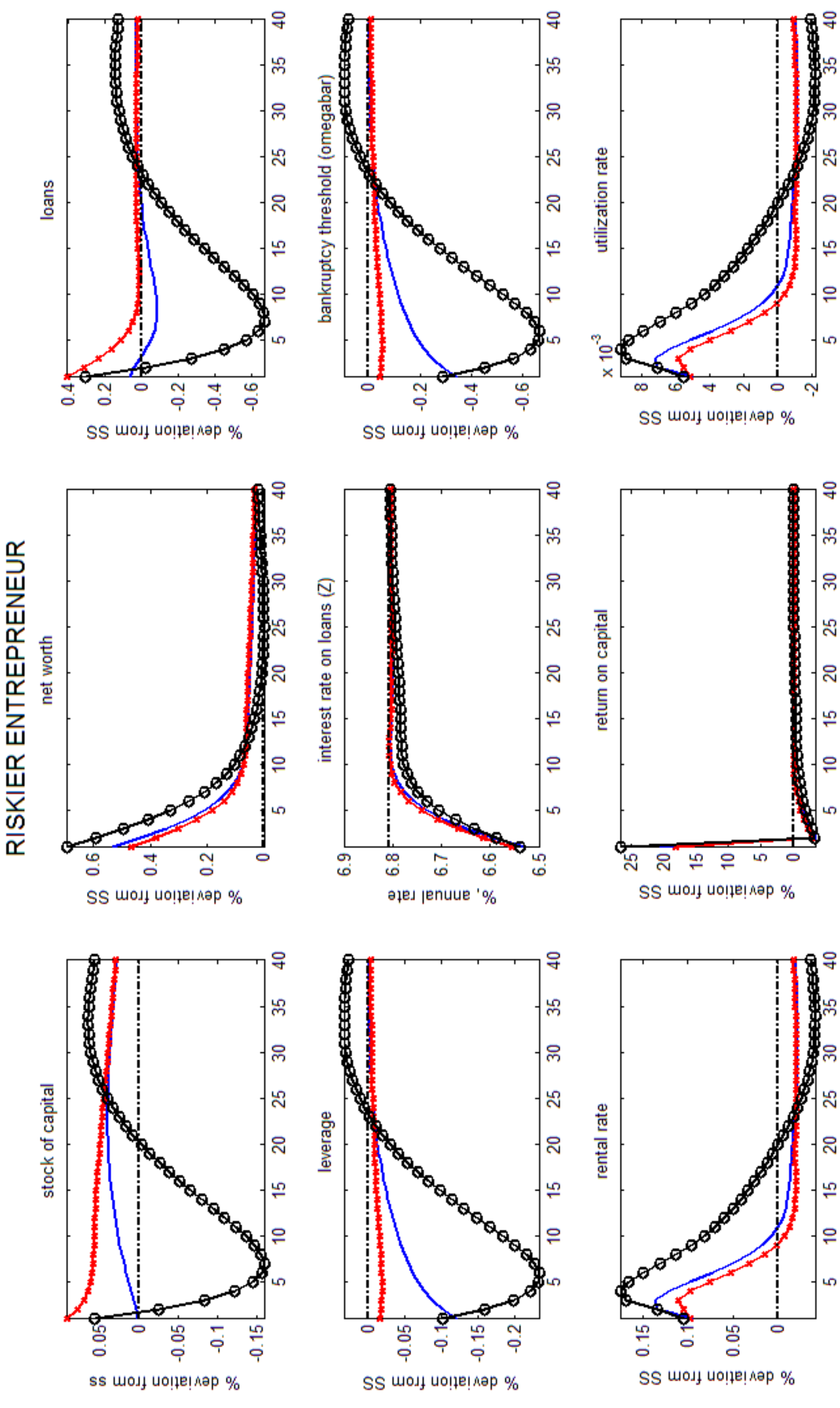

Figure 1.4: Impulse responses of riskier entrepreneur variables to a 25 basis point decrease to the nominal interest rate

Note. Values expressed as percentage deviation from steady state values, except for interest rates which are expressed as annual percentage points. Variant 1 (CMR, *Financial Accelerator Model*): blue solid line. Variant 2 (model with $r_2 = 0$): red crossed line. Variant 3 (model with $r_2 = 1$): black circled line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.1.

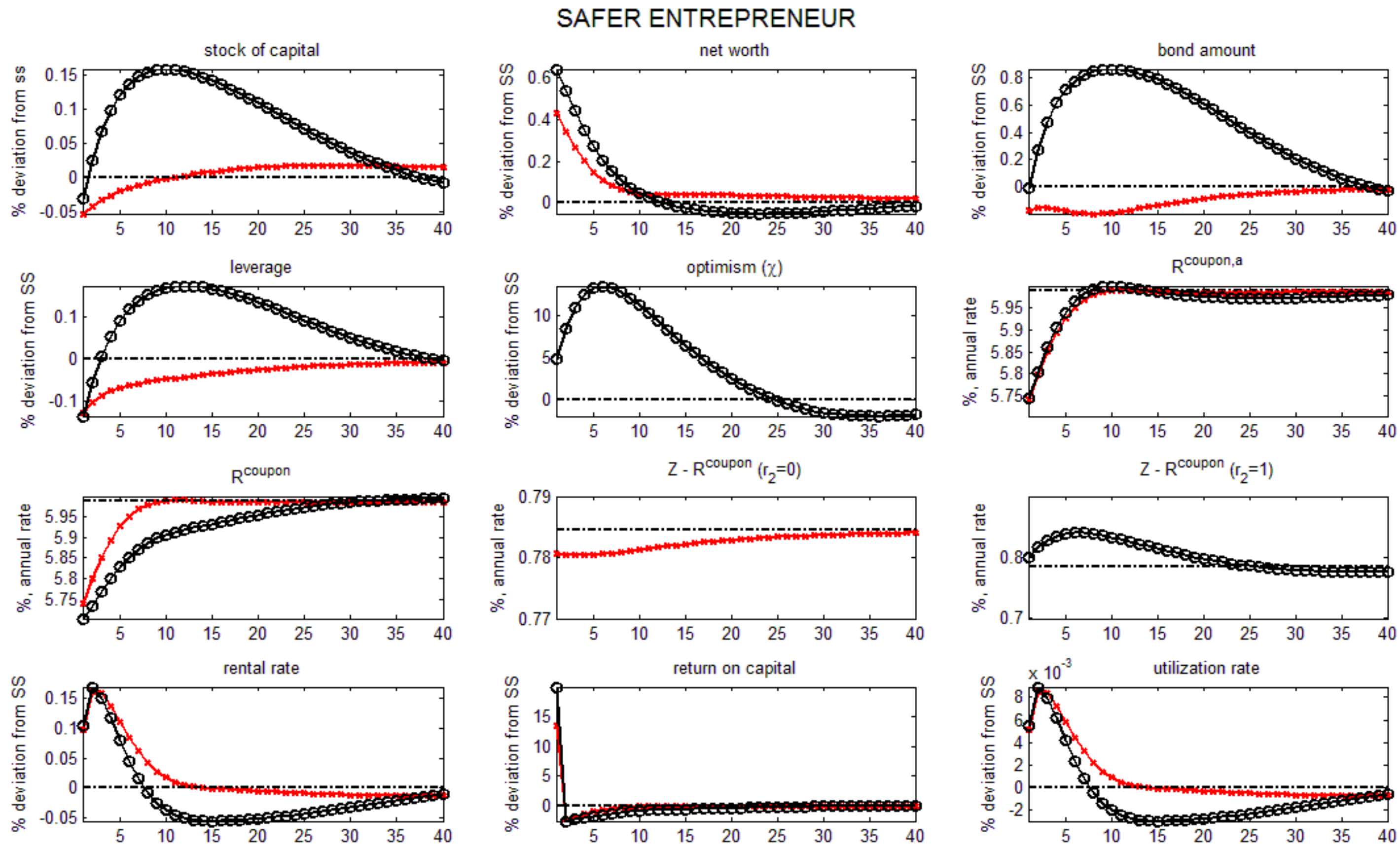

Figure 1.5: Impulse responses of safer entrepreneur variables to a 25 basis point decrease to the nominal interest rate

Note. Values expressed as percentage deviation from steady state values, except for interest rates which are expressed as annual percentage points. Variant 2 (model with $r_2 = 0$): red crossed line. Variant 3 (model with $r_2 = 1$): black circled line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.1.

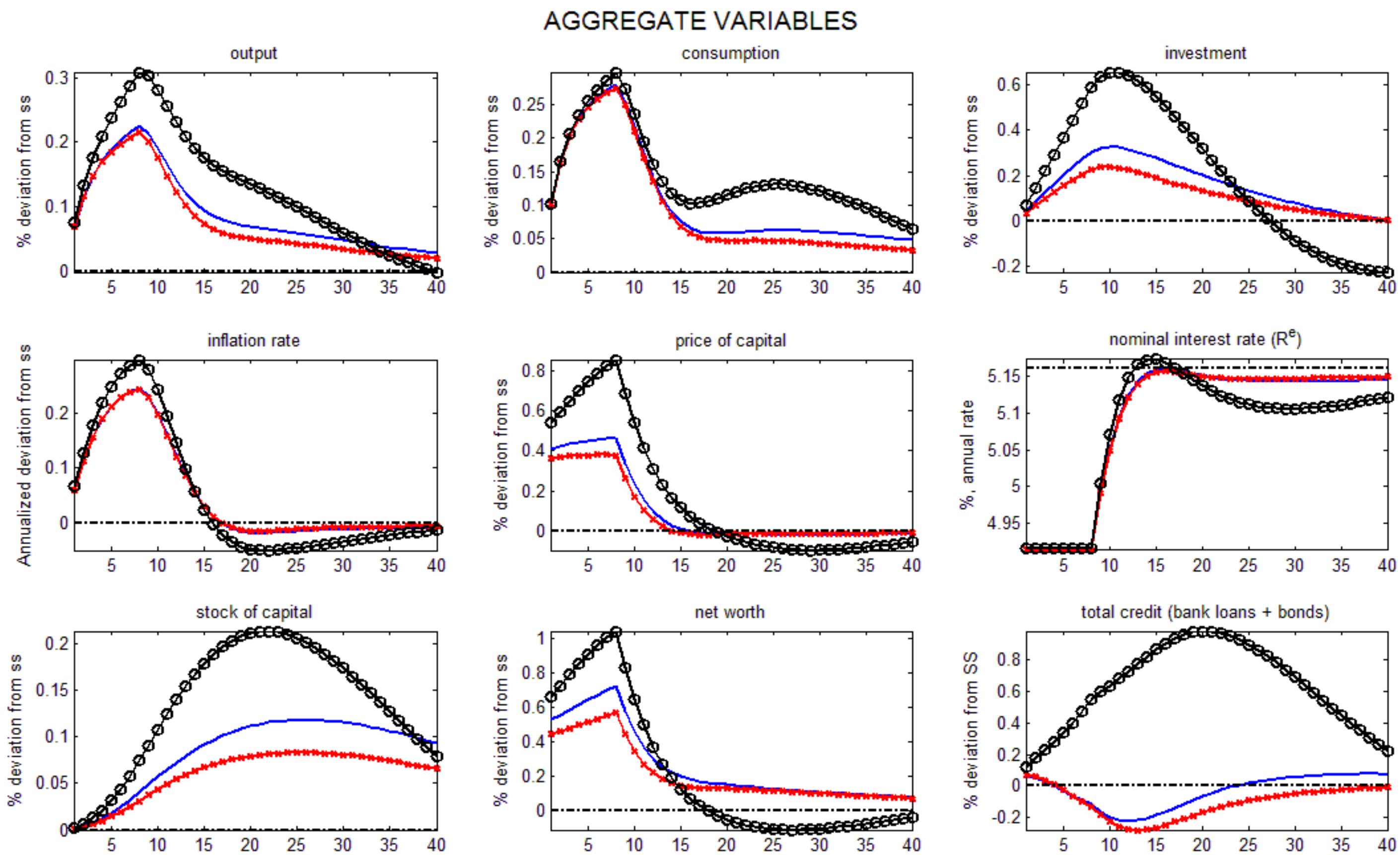

Figure 1.6: Impulse responses of aggregate variables to a persistent decrease in the nominal interest rate

Note. Values expressed as percentage deviation from steady state values. Inflation is expressed as annualized percent deviation from its steady state and the interest rates are expressed as annual percentage points. Variant 1 (CMR, *Financial Accelerator Model*): blue solid line. Variant 2 (model with $r_2 = 0$): red crossed line. Variant 3 (model with $r_2 = 1$): black circled line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.1.

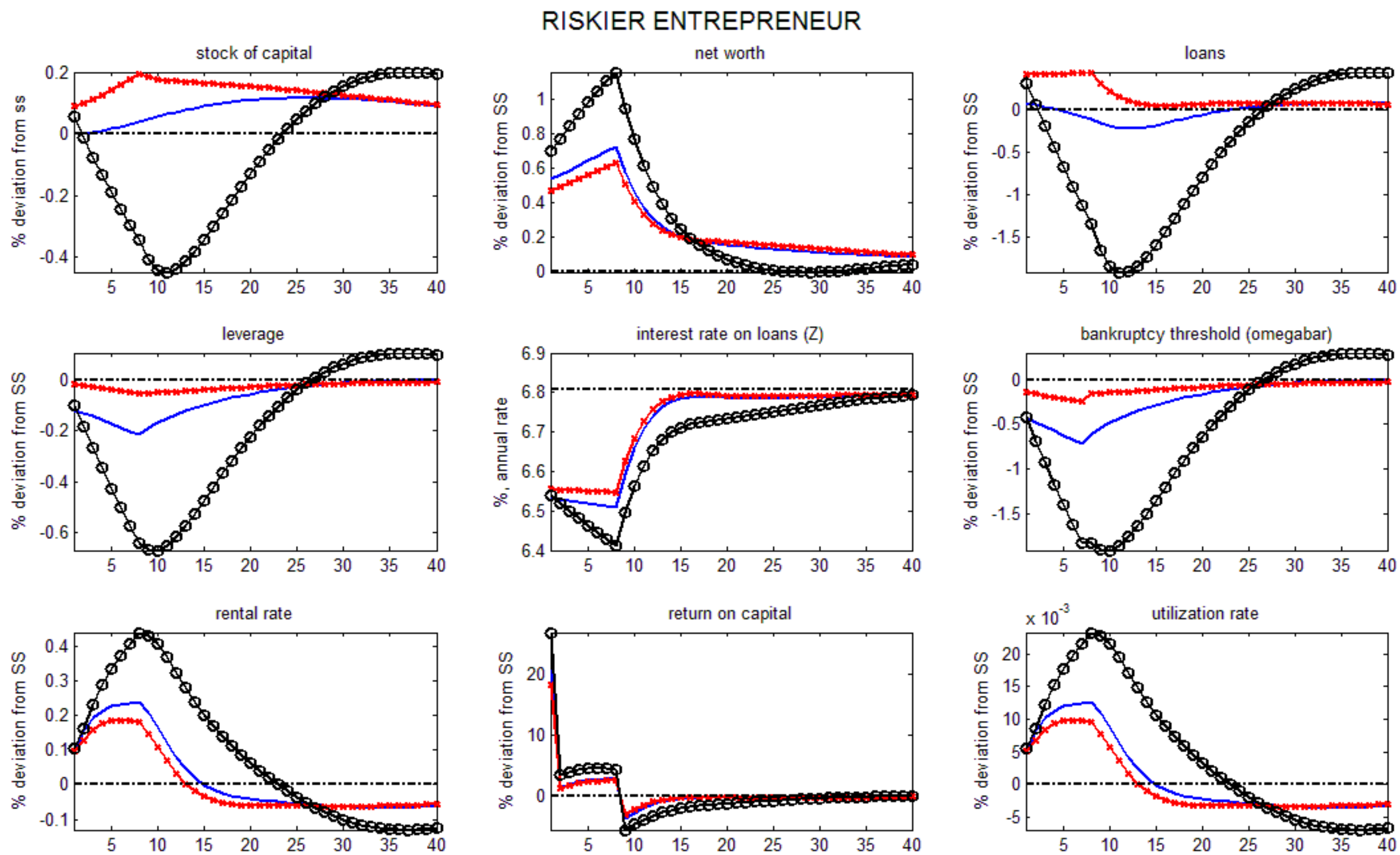

Figure 1.7: Impulse responses of riskier entrepreneur variables to a persistent decrease in the nominal interest rate

Note. Values expressed as percentage deviation from steady state values, except for interest rates which are expressed as annual percentage points. Variant 1 (CMR, *Financial Accelerator Model*): blue solid line. Variant 2 (model with $r_2 = 0$): red crossed line. Variant 3 (model with $r_2 = 1$): black circled line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.1.

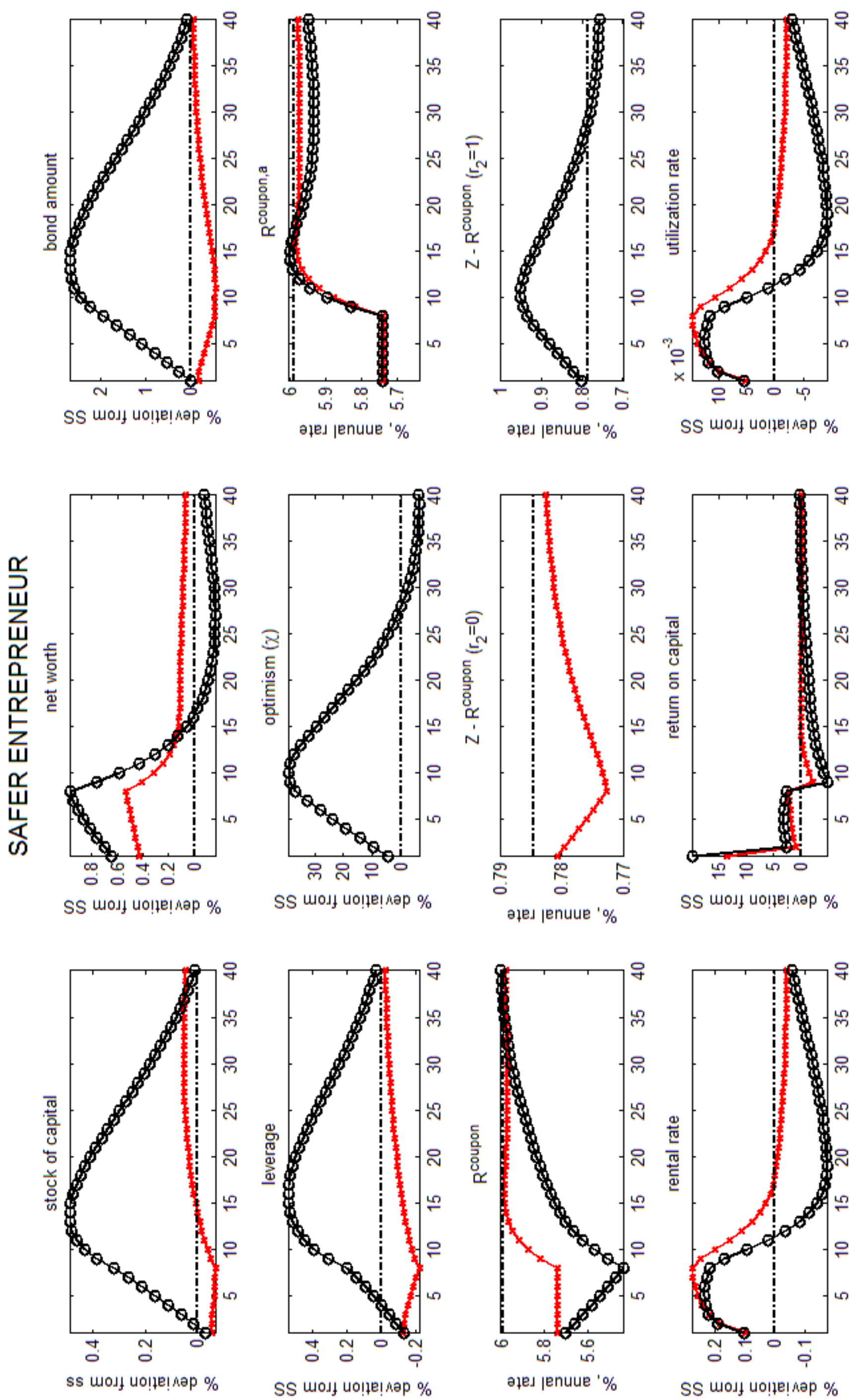

Figure 1.8: Impulse responses of safer entrepreneur variables to a persistent decrease in the nominal interest rate

Note. Values expressed as percentage deviation from steady state values, except for interest rates which are expressed as annual percentage points. Variant 2 (model with $r_2 = 0$): red crossed line. Variant 3 (model with $r_2 = 1$): black circled line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.1.

Chapter 2

Inattentiveness: an alternative explanation of investment gulps[1]

This chapter provides an alternative explanation for the microeconomic lumpiness of capital adjustment. Previous research has argued that capital adjustment is lumpy because firms face fixed capital adjustment costs. The new explanation proposed here is inattentiveness, whereby firms make infrequent investment decisions due to a cost of gathering and processing information. Introducing such information costs into an otherwise frictionless investment model induces infrequent and lumpy capital adjustments. The model fits the quantitative facts on plant-level investment rates remarkably well, and it also matches some higher order moments of aggregate investment rates. Moreover, inattentiveness enhances the cash flow sensitivity of investment.

[1] I am extremely grateful to Ricardo Reis for his invaluable guidance and to Emi Nakamura, Jón Steinsson, Luminita Stevens and Michael Woodford for helpful comments. This research was conducted while I was visiting Columbia University, whose hospitality I gratefully acknowledge. I thank the Fundação para a Ciência e a Tecnologia for financial support (Ph.D. scholarship). Any errors are my own.

2.1 Introduction

Two key facts about aggregate and micro-level investment adjustment have been emphasized in the literature (Caballero, 1999). On the one hand, at the aggregate level, investment series exhibit gradual and smooth adjustment. On the other hand, adjustment at the plant-level is occasional and large, or lumpy. That is, investment at the micro level is characterized by periods of relative low activity interrupted by sporadic episodes of large adjustments, which have usually been denoted as investment gulps or spikes.

To reproduce these behaviors, two different specifications of physical capital adjustment costs have traditionally been considered. While a simple neoclassical model with convex adjustment costs provides a good description for the smooth behavior observed at the macro level, the lumpy micro-level capital adjustment has been interpreted as evidence supporting (S,s) adjustment policy rules generated by non-convex adjustment costs. Over the past 20 years, substantial improvements in analytical techniques which could handle these types of non-linear adjustments have led to an extensive use of models with lumpy investment. Notwithstanding, it has been difficult to get a model that was able to reproduce both investment gulps and periods of inactivity of the frequency found in the data. In fact, after numerous attempts and refinements, only the most recent generation of lumpy investment models – Cooper and Haltiwanger (2006) and Khan and Thomas (2008) – has been successful in fitting the micro data well.

An alternative (and to some extent complementary) explanation for such micro lumpy behavior that has been recently pointed out – although not yet analyzed – is that investment planning and information processing can be costly activities in terms of required time, effort and expenses (Basu and Kimball, 2005 and Iacoviello and Pavan, 2007). That is, both costs of planning and costs of acquiring information might make investment lumpy at the micro level, even in absence of non-convex adjustment costs. Motivated by this argument, this chapter develops a new model of capital adjustment and tests whether it is able to fit the aforementioned facts about investment. We do so by drawing on recent behavioral models based on the assumption that agents update their information and plans infrequently.

This chapter is related to two strands of literature, which are reviewed in section 2.2. The first is the literature on investment in physical capital, and the second one is on informational frictions in macroeconomics. After briefly describing two standard capital adjustment costs models, section 2.3 presents a specific model based on the inattentiveness model of limited information *à la* Reis (2006b). Section 2.4 tests the implications of these models with both plant-level and aggregate data. The inattentiveness model is also contrasted with two leading lumpy investment models. Beyond the empirical findings on capital adjustment behavior, this chapter has a contribution to the empirical Q literature. In particular, section 2.5 analyzes whether and to what extent inattentiveness affects the sensitivity of investment to Tobin's Q and cash flow. Section 2.6 concludes.

2.2 Motivation

This research combines the works from two literatures – one on adjustment costs and investment dynamics and one on the inattentiveness approach of limited information. We therefore start the analysis with an overview of these two broad literatures.

Adjustment costs and investment dynamics[2]

Ever since the pioneer analysis of Eisner and Strotz (1963), the workhorse model of the investment literature has been, partly for analytical tractability, a neoclassical model with strictly convex – most often quadratic – costs of adjustment. This model provided a theoretical microfoundation to justify the inclusion of lagged dependent variables into empirical models of (otherwise static) factor demand (the flexible accelerator model of Clark, 1944, or the flexible user-cost model of Hall and Jorgenson, 1967). Convex costs of adjusting capital induce firms to spread their investment out over time, since a series of small adjustments is cheaper than a single large adjustment. Despite the relative success in reproducing the smooth adjustment of investment in physical capital observed at the aggregate level, empirical models based on convex adjustment costs have not performed so well along other dimensions (Abel and Blanchard, 1986 and Caballero, 1999). For example, estimations of such models have generally yielded unreasonably large adjustment costs coefficients, suggesting implausibly slow adjustment speeds.[3] Moreover, there is mounting empirical evidence confirming that capital adjustment at the micro level is sporadic and large. As several studies in many different countries have documented, investment at the plant-level is characterized by long

[2] See Chirinko (1993) and Bond and Van Reenen (2007) for excellent surveys of traditional investment models, and Hamermesh and Pfann (1996) for a more general survey on factor adjustment costs.

[3] Summers (1981) reports that 20 years after an unexpected economic shock, the capital stock would have only reached about 75% of its long-run steady-state level. Moreover, as Chirinko (1993) points out, most studies using panel data indicate much slower convergence rate.

periods of relative low activity broken by infrequent and possibly large adjustments in capital stocks.[4]

This very different picture of investment adjustment has led economists to question even more strongly the convex adjustment costs assumption. The micro evidence suggests that the predominant adjustment frictions at the micro level may be non-convex, rather than convex, in nature. Scarf (1960) shows that this type of nonlinear microeconomic adjustment can arise when firms face non-convex adjustment technologies. In Scarf's model, the adjustment cost is a fixed cost incurred at any time a firm wants to adjust her stock of inventories. To avoid the payment of such lump-sum cost, the firm only invests when capital's deviation from a target level exceeds a certain threshold. The optimal adjustment policy in the presence of fixed adjustment costs thus implies periods of inaction interrupted by infrequent episodes of large capital adjustments.

Beginning with Scarf, researchers have studied richer forms of adjustment costs structures to achieve greater consistency with the lumpy micro evidence.[5] The main difficulty among quantitative models of lumpy investment has been in reproducing the empirical observations of investment spikes versus inaction. In fact, after numerous attempts and refinements, only two (to the best of our knowledge) recent lumpy investment models – Cooper and Haltiwanger (2006) and Khan and Thomas (2008) – have been successful in matching the moments from the cross-sectional distribution of plant-level investment rates. Cooper and Haltiwanger (2006) estimate a model which nests alternative specifications of adjustment costs. They find that a model which combines both convex and non-convex components of adjustment costs (as well as irreversible investment) fits the data reasonably well. Khan and Thomas (2008) instead solve the problem in reproducing observations of both spikes and inaction by assuming that plants may undertake low levels of investment without incurring any adjustment costs.

[4] Evidence for U.S. comes from Caballero et al. (1995), Doms and Dunne (1998), Cooper et al. (1999), Cooper and Haltiwanger (2006) and Gourio and Kashyap (2007) (LRD database), Bayraktar (2002) and Bayraktar and Sakellaris (2006) (S&P's Compustat database) and Becker et al. (2006) (Census Bureau's Annual Survey of Manufactures data). For the U.K. see Attanasio et al. (2003) and Bayer (2006b); for Norway see Nilsen and Schiantarelli (2003); for Spain see Alonso-Borrego and Sánchez-Mangas (2008); for Germany see Bayraktar et al. (2005) and Bayer (2008); for Italy see Del Boca et al. (2008); for Sweden see Carlsson and Laséen (2005); for the Netherlands see Letterie and Pfann (2007); for Hungary see Reiff (2010); for Chile see Fuentes et al. (2006) and Gourio and Kashyap (2007); for Mexico see Gelos and Isgut (2001); for Colombia see Gelos and Isgut (2001) and Contreras (2008); for Cameroon, Ghana, Kenya, Zambia and Zimbabwe see Bigsten et al. (2005); finally, for Ethiopia see Gebreeyesus (2009).

[5] Some recent lumpy investment models with a single component of adjustment costs include Caballero and Engel (1999), Thomas (2002) and Veracierto (2002). Bertola and Caballero (1990), Abel and Eberly (1994), Cooper et al. (1999), Le and Jones (2005), Cooper and Haltiwanger (2006) and Bloom et al. (2007), among many others, consider combinations of convex and non-convex adjustment costs and/or irreversible investment.

Inattentiveness as a microfoundation for lumpy investment adjustment

Within the literature on informational frictions in macroeconomics, a strand called the inattentiveness approach has emerged recently. The term inattentiveness, coined by Reis (2006b), describes a particular type of learning mechanism. It analyzes the infrequent adjustment of choice variables that arises because gathering and processing information and making decisions and plans are costly activities.[6] The inattentiveness literature interprets the concept of "menu cost" introduced by Mankiw (1985) as a fixed cost of acquiring, absorbing and processing information and making decisions based on that information. Inattentiveness is the optimal response to such information/planning costs: agents rationally choose to update their information sets and plans only sporadically at optimally chosen dates, and to be inattentive to new information in between adjustment dates.

The inattentiveness literature is expanding rapidly and this approach has been successfully applied in several contexts, including the behavior of price-setting firms (Mankiw and Reis, 2002, Reis, 2006b and Alvarez et al., 2010b), workers (Mankiw and Reis, 2003), consumers (Reis, 2006a) and investors in financial markets (Gabaix and Laibson, 2002, Abel et al., 2007, 2009 and Alvarez et al., 2010a).[7] However, so far, the behavior of inattentive firms accumulating physical capital has not been studied.

Rather than relying on fixed physical costs of adjustment, the key assumption in this chapter (which also represents the only departure from an otherwise frictionless model) is that it is costly for the firm to acquire, absorb and process information and make plans for capital adjustment. Such costs lead the firm to make infrequent investment decisions. On the one hand, in between adjustment dates, the firm is "inactive" and only undertakes planned maintenance investment. On the other hand, when the firm does update her information and plan, the stock of capital immediately jumps to its optimal level. That is, at those planning dates it is likely to observe an investment gulp.

[6] A related strand of the literature, introduced by Sims (2003) and known as "rational inattention", analyzes infrequent adjustments that arise because of the limited ability of agents to absorb information. That is, agents cannot observe and process all available information when making economic decisions. Although rational inattention and inattentiveness sound almost the same, they are completely different learning technologies. Under rational inattention, agents receive a flow of noisy information every period. Consequently, agents update their plans frequently but incompletely. With inattentiveness instead, agents get no information almost all of the time and then occasionally observe the entire history of event perfectly. Plans are thus updated infrequently but completely.

[7] The aforementioned papers analyze the consequences of inattentiveness in partial equilibrium frameworks. Examples of general equilibrium models with inattentiveness are Ball et al. (2005), Mankiw and Reis (2006, 2007) and Reis (2009a,b).

Inattentiveness – because of either costs of acquiring information or costs of recomputing optimal plans – provides an alternative microfoundation for the lumpiness of capital adjustments at the plant-level.

2.3 A tale of three dynamic models of capital adjustment

We begin this section by describing a neoclassical investment model without capital adjustment costs of any form. This model serves as a reference for building more realistic investment models. Next, we describe a model with convex (*i.e.*, quadratic) and one with non-convex (*i.e.*, fixed) adjustment costs.[8] Finally, we introduce the model with information/planning costs and derive some theoretical results describing the firm's capital adjustment behavior. In the last subsection we compare the capital adjustment dynamics implied by these models.

2.3.1 The frictionless neoclassical investment model

Here we describe a simplified version of the model presented in Abel and Eberly (2008). Let $\widetilde{\Pi}_t = Z_t^{1-\alpha} K_t^{\alpha}$ be a reduced form revenue function obtained from the firm's optimization over freely adjustable factors of production. K_t denotes the current capital stock, Z_t the current period profitability shock and $0 < \alpha < 1$ the degree of monopoly power.[9]

The firm can purchase or sell capital instantaneously and frictionlessly, without any adjustment costs, at a constant price normalized to one. Hence the Jorgensonian user cost of capital equals the discount rate of the firm, r, and the depreciation rate, δ.[10] Operating profits, which

[8] We do not model combinations of convex with fixed costs of adjustment as well as irreversible investment. For a unified approach see references in footnote 5.

[9] Let $P_t = h_t Y_t^{-\frac{1}{\varepsilon}}$ be the demand curve where $h_t > 0$ is a demand shock and $\varepsilon > 1$ is the price elasticity of demand. Let $Y_t = A_t \left(K_t^{\gamma} N_t^{1-\gamma}\right)^s$ be the production function where γ is the capital share in a Cobb-Douglas production function and s the degree of returns to scale. Maximization of profit over the flexible labor factor, N_t, leads to a reduced form revenue function, $\widetilde{\Pi}_t = Z_t^{1-\alpha} K_t^{\alpha}$, where Z_t reflects productivity, the demand for the firm's output and/or the wage rate as well as structural parameters. The exponent on capital is $\alpha \equiv \gamma s \left(1 - \frac{1}{\varepsilon}\right) / \left[1 - (1-\gamma) s \left(1 - \frac{1}{\varepsilon}\right)\right]$. See Abel and Eberly (2008) for further details.

[10] Jorgenson (1963) shows that, if the level of capital inputs can be freely adjusted, the firm should invest until the marginal profit from an extra unit of capital is equal to the user cost of capital. The user cost represents the opportunity cost of holding one unit of capital for a period. It is usually defined as the sum of three terms: the firm's required rate of return, the depreciation rate and the expected rate of change in the price of capital goods. Here the latter term is zero since, by assumption, the price of capital is constant.

are revenue minus the user cost of physical capital, are given by

$$\Pi_t = Z_t^{1-\alpha} K_t^{\alpha} - (r+\delta) K_t \ . \tag{2.1}$$

Since there are no capital adjustment costs and investment is completely reversible, the optimal capital stock at each point in time is determined by maximizing static operating profits in equation (2.1) with respect to K_t. The first order condition yields the firm's optimal stock of capital

$$K_t^* = M^{\frac{1}{\alpha}} Z_t \ , \tag{2.2}$$

where $M \equiv \left(\frac{r+\delta}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}$. Throughout this chapter, we will refer to K_t^* as the "desired" or "frictionless" stock of capital interchangeably. According to the model, the frictionless stock of capital will tend to be as volatile as the profitability shock, and the response of capital stock mimics that of the shock. These two predictions are at odds with the conventional wisdom that capital stock adjusts either continuously and slowly (macro level), or infrequently and in a lumpy fashion (micro level).

2.3.2 The models with capital adjustment costs

Given the mismatch between the frictionless model's predictions and the data, researchers have introduced different forms of capital adjustment costs. Beyond other costs associated with the purchase of capital (*i.e.*, the user cost), the very act of adjusting the capital stock entails real costs. Ideally, the optimizing firm would choose a stochastic process for K_t, such that $K_t = K_t^* \ \forall t$. However, if altering the level of capital is costly, the firm has to trade off the benefits of tracking K_t^* more closely and the costs of doing so.

To make the analysis consistent with that of the inattentiveness model, an assumption that we maintain in this subsection is that, at the beginning of each period, the firm chooses the stock of capital to use in the current period. That is, after paying the adjustment cost, firm's investment instantly yields usable capital (there is no time-to-build aspect of investment).[11]

[11] The models we describe in this subsection are slightly different versions of commonly used investment models. For instance, we could assume instead that investment becomes productive with a lag of one period (that is, a one period time-to-build lag). This would require some straightforward modifications to the analysis but it would not affect the adjustment dynamics or the empirical results.

2.3.2.1 Convex adjustment costs and partial adjustment

The traditional investment models have assumed that adjustment costs are convex, and quadratic cost functions have been by far the most extensively used specification. For purposes of exposition, let us assume that the firm faces two types of costs – a cost of departing from the target stock level and a transaction cost incurred when changing the level of capital stock. Because of these costs, the firm will tend to adjust her capital stock slowly over time rather than instantaneously. Putting these two costs together in a loss function and using a discrete time setup, the dynamic programming problem is given by

$$\hat{V}\left(K_{t-1}, K_t^*\right) = \min_{K_t} \frac{\left(K_t - K_t^*\right)^2}{2} + \frac{\varphi}{2}\left(K_t - K_{t-1}\right)^2 + \beta E_t \hat{V}\left(K_t, K_{t+1}^*\right) ,$$

where $\varphi > 0$ is the capital adjustment costs parameter and $\beta \equiv \frac{1}{1+r}$. The firm's optimal policy consists in reducing the gap between the desired and the previous period level of capital stock by a fraction ν, $\nu \leq 1$, each period.[12] In particular, capital accumulates according to the following equation:

$$K_t - K_{t-1} = \nu\left(K_t^* - K_{t-1}\right) .$$

The change in capital, $K_t - K_{t-1}$, is thus proportional to the difference between the previous level of capital and the target, where ν parameterizes how quickly the gap is closed. The firm faces a trade-off between the speed at which she adjusts and the cost of making such adjustment: everything else equal, the larger is the adjustment cost (φ), the smaller is the size of adjustment (ν), and the slower is the speed of adjustment. Finally, investment in period t is given by

$$I_t = K_t - (1-\delta) K_{t-1} = \delta K_{t-1} + \nu\left(K_t^* - K_{t-1}\right) .$$

2.3.2.2 Non-convex adjustment costs and state-dependent (lumpy) adjustment

As first shown by Scarf (1960), lumpy behavior arises naturally when adjusting the stock of capital entails a fixed cost. Here we outline a simple model of this nature.

Let the adjustment cost incurred when changing the capital stock at any given time T be proportional to the current capital stock (before adjustment): $\bar{\Phi} K_{T^-}$. At any normalized time

[12] See appendix F for details of the solution of the dynamic programming problem as well as for the analytical relationship between ν and the structural parameters (φ and β).

0, the firm's problem is

$$V(K_0, \mathbf{Z_0}) = \max_{\{K_t\}_{t=0}^{T}, T} E_0 \left\{ \int_0^T e^{-rt} \left[Z_t^{1-\alpha} K_t^{\alpha} - (r+\delta) K_t \right] dt + e^{-rT} \left[\bar{\Phi} K_{T^-} + V(K_T, \mathbf{Z_T}) \right] \right\} . \tag{2.3}$$

We describe the adjustment policy in terms of two trigger levels, L and U, where $L < U$.[13] Let κ_{t^-} be the deviation between the desired and the current capital before adjustment takes place: $\kappa_{t^-} = K_t^* - K_{t^-}$. If κ_{t^-} is smaller (larger) than or equal to the trigger level L (U), the firm will increase (decrease) her capital stock such that $\kappa_{t^+} = 0$ after adjustment.[14] There is therefore an investment gulp every time κ_{t^-} reaches these trigger levels. On the other hand, if the value of κ_{t^-} is between these trigger levels, the firm keeps the level of the capital stock constant by undertaking maintenance investment.[15] That is, the firm may undertake some small maintenance investment without incurring any adjustment costs. The optimal policy may thus be described by the following rule

$$K_t^{Ss} = \begin{cases} K_t^* & if\ \kappa_{t^-} \geq U\ or\ \kappa_{t^-} \leq L \\ K_{t-1} & if\ L < \kappa_{t^-} < U \end{cases} .$$

Consequently investment is given by

$$I_t = \begin{cases} K_t^* - (1-\delta) K_{t-1} & if\ \kappa_t \geq U\ or\ \kappa_t \leq L \\ \delta K_{t-1} & if\ L < \kappa_t < U \end{cases} .$$

Note that, in (2.3), the optimal stopping time for adjustment, T, depends on the evolution of the variable Z_t. Adjustment is therefore state-dependent: the firm observes the state of the economy every instant and, according to it, decides whether it is optimal to adjust or to stay inactive. Once the decision to act has been taken, adjustment is instantaneous and capital reaches the optimal frictionless level. The optimal adjustment policy in the presence of fixed

[13] The problem in (2.3) has an explicit analytical solution. See appendix F for the corresponding differential equation, value matching, and smooth pasting conditions.

[14] Here we assume $\kappa_{t^+} = 0$ for the sake of simplicity. However, as Caballero (1999) points out, the optimal dynamic target is generally different from the frictionless one. That is, in general the value after adjustment κ_{t^+} is different from 0.

[15] Khan and Thomas (2008) extend the Khan and Thomas (2003) model to allow, among other things, for low-level capital adjustments that are exempt from adjustment costs. They find that the extended model fits the plant-level data much better than the previous one. Bachmann et al. (2010) also make a similar assumption. An alternative approach (taken, for instance, in Bertola and Caballero, 1990, Caballero, 1993, Thomas, 2002 and Gourio and Kashyap, 2007) is to let capital be eroded by depreciation in the case the firm remains inactive.

adjustment costs thus implies periods of inaction followed by infrequent episodes of large capital adjustments.

2.3.3 The model with information/planning costs

In this subsection we present and solve an alternative model of capital adjustment based on the inattentiveness model of limited information *à la* Reis (2006b). The key assumption in this model is that information processing and investment planning are costly activities – rather than adjusting capital stock as it is common in the investment literature. Relatively to the model described in section 2.3.1, the firm faces one additional constraint: she must pay a cost in order to update her information sets and to make new plans. This cost can be interpreted as the cost in money and time of obtaining and assimilating information, or it could stand for the opportunity cost of taking time to compute optimal plans. Thus, in what follows we will refer to this cost as information or planning cost interchangeably.

Information costs give rise to rational inattentiveness to information in the sense that the firm rationally chooses to update her information about the state of the economy and to make new plans only sporadically at optimally chosen dates. That is, the firm forms expectations rationally, though she does not do so often. As a consequence, expectations conditional on old information continue to influence current choices. When the firm decides to incur such a cost, conditional on the information obtained she decides when next to plan, and the amount of capital to use during the period of inattentiveness.

The first decision at a planning date is on when to update information and plan again. The optimal length of inattentiveness trades off the costs of being inattentive to news and the costs incurred by planning. The firm is aware of the fact that while on the one hand, being inattentive saves on the costs of planning, on the other hand, it implies that decisions sufficiently far in the future are made with severely outdated information. At some point the costs of following an outdated plan become higher than the costs of updating information, so it becomes optimal for the firm to make a new plan.

The second decision is the plan for capital adjustments, which represents the path for the amount of physical capital to use until the next planning date. In between adjustment dates, the dynamics of capital follows a pre-determined plan, regardless of the news in the economy. At observation dates instead, information is revealed and optimal choices incorporate it, therefore it is likely to observe a jump in the stock of capital at those dates.

Having informally introduced the model of inattentiveness, we now described the formal problem and derive the optimality conditions describing capital adjustment behavior.

2.3.3.1 The inattentive firm's problem

Time is continuous and infinite. Let $\mathbf{x}_t$ be the state vector, which is generated by a continuous time stochastic process defined on a standard filtered probability space with filtration $F = \{F_t, t \geq 0\}$. We assume that $\mathbf{x}_t$ is a first-order Markov process. The state at a given date $t+\tau$ is then a function of $\mathbf{x}_t$ and a set of innovations $u^\tau = (u_t, u_{t+\tau}]$, so that $\mathbf{x}_{t+\tau} = \Psi(\mathbf{x}_t, u^\tau)$ denotes the transition between the state at date t and the state at date $t+\tau$, which is assumed to be differentiable.

The planning dates are denoted by $D(i)$, where $i \in \mathbb{N}_{\mathbb{O}}$ and $D(i+1) \geq D(i)$ for all i with $D(0) \equiv 0$. The periods of inattentiveness are defined recursively as $d(i) = D(i+1) - D(i)$. The firm's optimal choice of planning dates defines a new filtration $\Im = \{\Im_t, t \geq 0\}$ such that $\Im_t = F_{D(i)}$ for $t \in [D(i), D(i+1))$.

Whenever the firm decides to update her information and plans, she pays a non-negative finite planning cost given by $\theta_t \equiv \theta(\mathbf{x}_t)$. She then chooses when to plan again as well as a plan for capital until the next adjustment, $K(t) = K[D(i), D(i+1))$. Two remarks are worthwhile. First, note that the firm can choose the next planning date either at the current planning date or at any future date. However, since she does not observe any new information while inattentive, her choice will be the same irrespective of when it is made. Second, the firm's choices at any time t must be measurable with respect to $\Im_t$. That is, the firm's capital choices for time t is conditional on the information she has at time t, which by assumption coincides with the information available at the last planning date.

The firm maximizes the expected value of operating profits, net of planning costs. The firm's problem can be written as:

$$\max_{\{\mathbf{K(i)},\mathbf{D(i)}\}_{\mathbf{i=0}}^{\infty}} \quad E\left\{\sum_{i=0}^{\infty}\left[\int_{D(i)}^{D(i+1)} e^{-rt}\Pi_t dt - e^{-rD(i+1)}\theta\left(\mathbf{x}_{D(i+1)}\right)\right]\right\} \tag{2.4}$$

$$s.t. \quad \{D(i), K(i)\} \quad are\ \Im - adapted \tag{2.5}$$

$$\mathbf{x}_{t+\tau} = \Psi(\mathbf{x_t}, u^\tau) \tag{2.6}$$

$$\Pi_t = Z_t^{1-\alpha} K_t^\alpha - (r+\delta) K_t \ . \tag{2.7}$$

Note that if the costs of planning are always 0, the firm optimally chooses to be always attentive. The problem (2.4)-(2.7) has a recursive structure between adjustment dates. Let $\mathbf{x}$ denote the state at the current planning date and $\mathbf{x}^{'}$ the state at the next planning date. We can then rewrite the problem as

$$V(\mathbf{x}) = \max_{\{K_t\}_{t=0}^{d}, d} \left\{ \int_0^d e^{-rt} \left[Z_t^{1-\alpha} K_t^{\alpha} - (r+\delta) K_t \right] dt + e^{-rd} E \left[-\theta\left(\mathbf{x}^{'}\right) + V\left(\mathbf{x}^{'}\right) \right] \right\}$$

$$subject\ to \qquad \mathbf{x}^{'} = \Psi\left(\mathbf{x}, u^d\right) \tag{2.8}$$

where the measurability constraint (2.5) is imposed by having passed the expectation operator through $\{K, d\}$, so that these choices are made conditional on the information available at the current planning date. The solution to the problem in (2.8) is a pair of functions, $K(\mathbf{x}, t)$ and $d(\mathbf{x})$, determining the optimal plan for capital from time 0 to time d and when the next planning will take place.

At a first sight, the problem in (2.8) seems similar to the optimal stopping problem in (2.3). However, recall from section 2.3.2.2 that adjustment in (2.3) – *i.e.*, the choice of T – is state-contingent. In the inattentiveness model instead, in between adjustments, the firm rationally chooses not to collect new information. The inattentive firm adjusts at optimally chosen dates regardless of the state of the economy at those dates. Capital adjustment with inattentiveness – *i.e.*, the choice of d^* – is therefore recursively time-contingent, independent of the current state, but a function of the state at the past planning date.

2.3.3.2 The optimality conditions

The first order condition of (2.8) with respect to d is

$$\Pi(\mathbf{x}, d) = E\left\{ r\left[V\left(\mathbf{x}^{'}\right) - \theta\left(\mathbf{x}^{'}\right) \right] + \left[\theta_{\mathbf{x}}\left(\mathbf{x}^{'}\right) - V_{\mathbf{x}}\left(\mathbf{x}^{'}\right) \right] \frac{\partial \Psi\left(\mathbf{x}, u^d\right)}{\partial d} \right\}. \tag{2.9}$$

By weighting the benefit of adjusting against the costs incurred by planning, equation (2.9) implicitly defines the optimal length of inattentiveness. On the left-hand side is the value from extending the inattentiveness interval, which equals the profits from keeping to the outdated plan for capital. The right hand side captures the value from planning at time d, which is the sum of two terms. The first term is the present value of having obtained new information and re-planned, which is the difference between the value of having a new plan and the cost of writing it. The second term is the cost from updating the information at date d rather than in another instant in which the cost and benefit of a fresh plan may change.

Differentiating equation (2.8) with respect to K_t and setting the derivative equal to zero yields the optimal plan for capital:

$$K^{IN}(\mathbf{x},t) = M^{\frac{1}{\alpha}}\left[E\left(Z_t^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}} \quad . \tag{2.10}$$

The envelope conditions with respect to each component j of the state vector $\mathbf{x}$ are

$$V_j(\mathbf{x}) = \int_0^d e^{-rt}\Pi_j(\mathbf{x},t)\,dt + e^{-rd}E\left[\left(-\theta_{\mathbf{x}}\left(\mathbf{x}'\right) + V_{\mathbf{x}}\left(\mathbf{x}'\right)\right)\Psi_j\left(\mathbf{x},u^d\right)\right] \quad . \tag{2.11}$$

Equations (2.8)-(2.11) characterize the value function $V(\mathbf{x})$, the plan for capital $K^{IN}(\mathbf{x},t)$ and the optimal inattentiveness interval $d(\mathbf{x})$.

2.3.3.3 Model's predictions

We now derive some theoretical implications of the model by making assumptions that lead to a closed-form solution.[16] We assume that the profitability shock, Z_t, follows a trendless geometric Brownian motion with constant volatility $\sigma > 0$, so $dZ_t = \sigma Z_t dz$ where dz is the increment of a standard Wiener process. We further assume that the costs of planning are a fixed share Θ of profits at the time of planning.

Prediction #1: the length of inattentiveness

Following Reis (2006b), we compute a perturbation approximation of the optimal length of inattentiveness around the point where planning is costless.[17] In this case, the following result holds:

Proposition 1. *An approximation of the optimal inattentiveness interval is given by*

$$d^* = \sqrt{\frac{4\Theta}{\alpha\sigma^2}} \; .$$

This results shows the determinants of inattentiveness. First, $\frac{\partial d^*}{\partial \Theta} > 0$: the larger the costs of

[16] Appendix E contains the proofs of all the propositions.

[17] Results are sensitive to the point around which the approximation is taken. For instance, Jinnai (2007) solves Reis (2006b) model by approximating around the point where firms have asymmetric information. Overall Jinnai's approximation predicts shorter inattentiveness intervals than Reis' approximation.

planning are, the longer is inattentiveness. Second, $\frac{\partial d^*}{\partial \sigma} < 0$: more volatile profitability shock leads the firm to update her information more frequently since it is costly to not pay attention to news in a world that is quickly changing. Third, $\frac{\partial d^*}{\partial \alpha} < 0$: the higher the sensitivity of the profit function, the larger the costs of reacting with a delay to news so the firm avoids being inattentive for too long.

Prediction #2: capital adjustment dynamics with inattentiveness

Let $K^*_{D(i)}$ denote the capital chosen by an attentive firm at date $D(i)$, which is given by equation (2.2). The optimal capital chosen by an inattentive firm at planning date $D(i)$, $K^{IN}_{D(i)}$, has to be equal to the capital chosen by an attentive firm, since these decisions are made conditional on the same information set. Therefore, at adjustment dates, it must hold that

$$K^{IN}_{D(i)} = K^*_{D(i)} = M^{\frac{1}{\alpha}} Z_{D(i)} \ .$$

In between adjustment dates, the firm is inattentive and

Proposition 2. *The optimal plan for capital between adjustment dates, for $D(i) < t < D(i+1)$, obeys the equation*

$$K^{IN}_t = K^*_{D(i)} e^{\left\{-\alpha \frac{\sigma^2}{2}[t-D(i)]\right\}} \ .$$

The optimal adjustment policy with inattentiveness is thus fully described by the following equations

$$K^{IN}_t = \begin{cases} K^*_{D(i)} = M^{\frac{1}{\alpha}} Z_{D(i)} & \forall i \in \mathbb{N}_{\mathbb{O}} \\ \\ K^*_{D(i)} e^{\left\{-\alpha \frac{\sigma^2}{2}[t-D(i)]\right\}} & \forall\ D(i) < t < D(i+1) \end{cases} .$$

The dynamics of capital adjustment with inattentiveness thus implies periods of relatively low activity followed by infrequent episodes of possibly large capital adjustments.

Prediction #3: the dynamics of investment

If the firm enters period $D(i)$ with capital stock $K^-_{D(i)}$, her capital stock jumps instantly to $K^+_{D(i)} = K^-_{D(i)} + I_{D(i)}$, where the superscripts "+" and "−" on $K_{D(i)}$ denote, respectively, the stock of capital immediately after and immediately before $D(i)$, and $I_{D(i)}$ is the investment gulp at date $D(i)$. Between planning dates, the firm is inattentive and, in order to keep her capital stock in line with the optimal plan, she undertakes planned maintenance investment,

I_t^M, to fully compensate for depreciation. Therefore optimal investment by inattentive firm is:

$$\begin{cases} I_{D(i+1)} = K^+_{D(i+1)} - K^-_{D(i+1)} = K^*_{D(i+1)} - K^*_{D(i)} e^{-\alpha \frac{\sigma^2}{2} d^*} & \forall i \in \mathbb{N}_{\mathbb{O}} \\ \\ I^M_{t+1} = \frac{dK^{IN}_t}{dt} + \delta K^{IN}_t & \forall \; D(i) < t < D(i+1) \end{cases} .$$

2.3.4 Capital adjustment dynamics

Having presented the models, we now compare the capital adjustment dynamics implied by different adjustment policies. The top graph in figure 2.1 presents an example of a sample path of the frictionless capital stock and three capital adjustment behavior (partial, state- and time-dependent adjustment).

At first sight, two distinct capital adjustment patterns emerge. While quadratic adjustment costs lead to a smooth adjustment, the models with fixed – both adjustment and information – costs predict discrete and lumpy adjustments: periods of relative low investment activity (in which capital is constant or slightly decreases) are interrupted by large (both positive and negative) capital adjustments. In particular, the sizes of such adjustments are much larger than those implied by the convex adjustment costs model.

Looking more carefully, although adjustments in the models with fixed costs may look similar, they are actually different in one important respect – the timing of adjustment, *i.e.* when the firm decides to adjust. The bottom graph in figure 2.1 details the difference between state- and time-dependent adjustments. It presents an example of a sample path of the deviation between frictionless and actual capital stock in both models with fixed costs. In the inattentiveness model, full adjustments occur at fixed time intervals d^*, regardless of the state at those dates. In the (S,s) model, full adjustments instead only occur – at dates T_1 and T_2 – when the departure from the desired stock of capital becomes too large in absolute value, reaching the upper level U or the lower level L. That is, whenever there is a cost of gathering information and planning, and no direct costs of adjusting capital, the optimal adjustment is time as opposed to state dependent.

This simple example shows that a slightly different interpretation of menu cost – cost of adjusting versus cost of planning – leads to very different implications for the dynamics of capital adjustment. The distinction between time- and state-dependent adjustments can have crucial implications for important economic questions. For instance, monetary policy has long-lasting effects on the real economy if the firm makes her investment decisions according

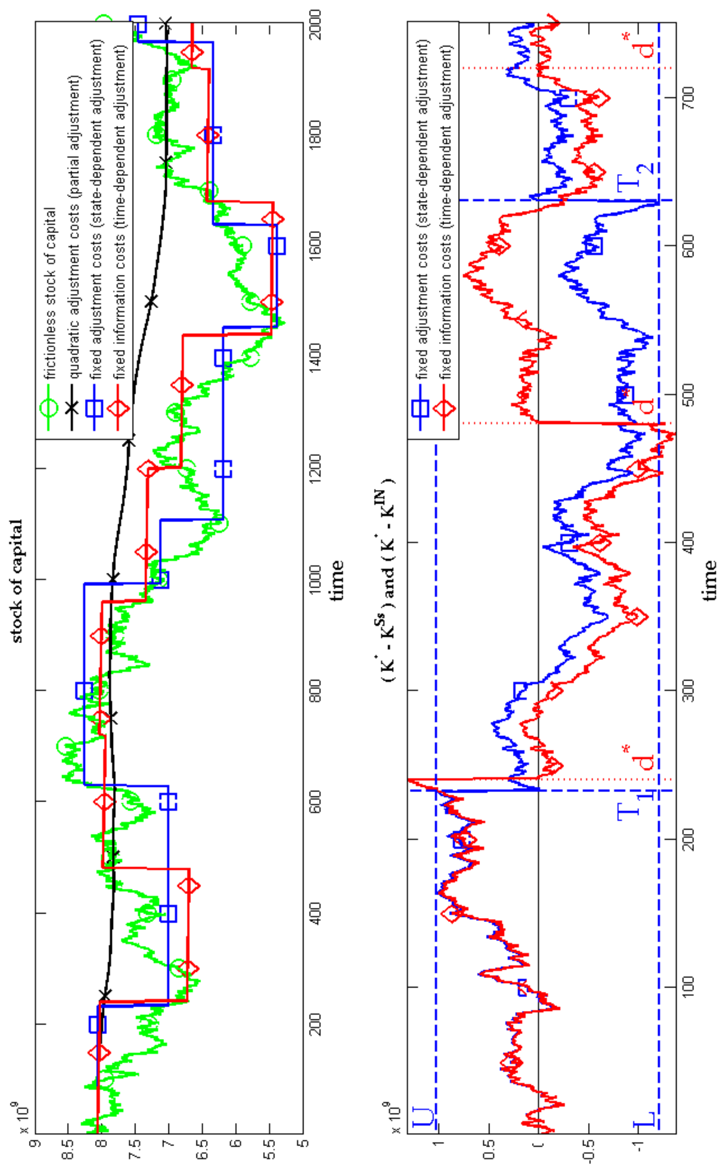

Figure 2.1: Examples of capital adjustment behavior under various adjustment policies (baseline parameters, table 2.1)

to a time-dependent rule, whereas if the firm adjusts her capital stock according to a state-dependent rule, then monetary policy may have little real effect.

2.3.5 Aggregation

This subsection aggregates individual investment decisions to obtain the predictions of the model for the time series of aggregate investment. To do so, we treat Z_t as an aggregate profitability shock and we make some assumptions about the distribution of the firms' decision dates.

Each inattentive firm in the economy gathers information and recomputes optimal plans slowly over time. That is, firms respond infrequently – and possibly asynchronously – to the aggregate shock. For each firm, the sequence of optimally chosen planning dates $D = \{D(i)\}_{i=0}^{\infty}$ forms a sequence of stochastic increasing events. The arrival of decision dates is therefore a stochastic process, whose properties may be described by a set of probability density functions for the length of the inattentiveness period, conditional on when the firm last planned. We denote these by $f_i(t)$ and assume that

Assumption 1. *The densities $f_i(t)$ describe random variables that are*

i) *mutually independent;*

ii) *independent across firms;*

iii) *the same for all firms;*

iv) *uniformly distributed.*

Independence of decision dates allows us to only keep track of when the last decision date for each firm occurred. Parts (ii) and (iii) of Assumption 1 in turn allow us to interpret $f_i(t)$ as the actual fraction of firms that are planning at a given point in time. Let ρ denote the mean number of planning dates in a unit of time. As a result, (iv) implies that, in each period, the share of firms planning is constant and equal to $\rho = \frac{1}{E(d^*)}$. Moreover, at any point in time t, the fraction of firms not having planned for n periods, $t - d^* < \forall n < t$, also equals ρ.[18]

At a given instant in time, the aggregate capital equals the sum of the capitals chosen by different firms. If the index of the firms, j, stands for how long it has been since the firm last

[18] Caballero and Engel (1991) present a generalized (S,s) model and derive conditions under which the aggregate distribution is uniform. Reis (2006b) shows that, under some (very strict) conditions, the arrival of decision dates in the aggregate economy tends to the exponential distribution with parameter $\rho = 1/E(d^*)$. Here we consider the uniform distribution to keep the computational burden manageable.

planned, then the aggregate stock of capital is defined by

$$K_t^{IN,A} = \int_{t-d^*}^{t} \frac{1}{d^*} M^{\frac{1}{\alpha}} \left[E_j \left(Z_t^{1-\alpha} \right) \right]^{\frac{1}{1-\alpha}} dj \ .$$

Aggregate investment at time t is given by

$$I_t^{IN,A} = \int_{t-d^*}^{t} \frac{1}{d^*} I_j^{IN} dj = \frac{1}{d^*} I_t + \int_{t-d^*}^{t} \frac{1}{d^*} I_{j-1}^{M} dj \ ,$$

where the first term represents the investment gulp of the $\frac{1}{d^*}$ firms that are planning at time t, while the second term represents the maintenance investment of the remaining inattentive firms. Finally, the aggregate investment rate is defined as the ratio of aggregate investment to aggregate capital stock.

2.4 How do the models fit the data on investment?

2.4.1 Plant-level data

This subsection evaluates the qualitative ability of the models to match some key moments characterizing the plant-level investment rates.

The models are calibrated to annual data, since the plant-level evidence is based on annual surveys. Our preferred choices are $r = 0.04$, which is the value typically used in the literature for the discount factor; the depreciation rate $\delta = 0.14$, to match the average investment rate at the plant-level; and the standard deviation of the profitability shock $\sigma = 0.12$, so as to have (in combination with the planning costs parameter) an average inattentiveness of at least one year. We also set the degree of monopoly power α to 0.7, consistent with the estimates in the literature (see *e.g.* Cooper and Haltiwanger, 2006). With regard to the convex adjustment costs model, we set the adjustment costs parameter φ to 11.76 so that the size of adjustment ν is 0.24. The value for ν lies between the one reported by Summers (1981) and the one estimated by Carlsson and Laséen (2005).[19] In the model with fixed adjustment costs, we pick the threshold values $U = L = 0.15$, that is, there is no adjustment as long as the current capital

[19] The evidence provided by Summers (1981, p. 101) implicitly gives $\nu = 0.07$, while Carlsson and Laséen (2005, p. 975) report $\nu = 0.45$.

stock (before adjustment) is within a $\pm 15\%$ band around the frictionless level.[20] Finally, we consider two values for the costs of planning in the inattentiveness model, $\Theta = 0.010$ and $\Theta = 0.017$, so that the average inattentiveness intervals are, respectively, 4 and 5 quarters. These planning costs values are in line with the evidence provided in Reis (2006b).[21]

Using this parametrization, we generate simulated capital and investment data for a panel of 1000 plants and 16 years.[22] As a technical matter, instead of generating one normally distributed value per year for the profitability shock Z, we divide each year into 240 intervals (days) and generate a normally distributed shock for each interval. Using a finer time grid allows us to choose more precise values for planning dates (recall that planning dates in the inattentiveness model are chosen from a continuous set). Investment rates are computed as total annual investment – the sum of gulps and maintenance investment in any given year – divided by the capital stock in the final day of the year.

Table 2.1 reproduces different moments of plants' investment rates in the U.S. data (second column) and the models' predictions (third to sixth column). For the sake of comparison, we also report the moments from two leading papers in the lumpy investment literature, Cooper and Haltiwanger (2006) (seventh column) and Khan and Thomas (2008) (last column).

Here inaction is defined as a plant-level investment rate less than 1% in absolute value. Positive investment rates are those at or exceeding 1%, while negative investment rates are those equal to or below -1%. Finally, as is common in this literature, positive spikes are observations in which the investment rate exceeds 20%, and negative spikes are episodes in which the investment rate is less than -20%. The second column of table 2.1, taken from Cooper and Haltiwanger (2006), documents the nature of capital adjustment behavior using data from the Longitudinal Research Database, a plant-level U.S. manufacturing data set. Clearly, the data exhibits both periods of inactivity and large positive bursts of investment activity, while little evidence of negative investment, especially large investment episodes. Also, there is a sharp asymmetry in positive versus negative investment rates as well as in positive versus negative investment spikes. For instance, positive spikes are observed 10 times as often as negative spikes. Finally, autocorrelation in plant-level investment rate is positive but very low.

The model with quadratic adjustment costs cannot produce the bursts of investment and in-

[20] Note that the threshold values map one-to-one to the adjustment costs parameter $\bar{\Phi}$. These values can be obtained solving numerically the non-homogenous system given by (F.7).

[21] Extensive sensitivity analysis confirmed that the properties of the inattentiveness model are not sensitive to variation in the parametrization, as long as the average inattentiveness interval is at least 4 quarters.

[22] This is roughly the length of the dataset analyzed by Cooper and Haltiwanger (2006). Adding more plants does not affect the results.

Table 2.1: Summary statistics, U.S. plant-level data and models

	data[a]	quadratic adjustment costs model[b]	fixed adjustment costs model[c]	fixed information costs model[d]	fixed information costs model[e]	CH06[f]	KT08[g]
inaction rate (%)	8.1	0	18.3	3.6	2.8	n.a.	7.3
negative investment (%)	10.4	0	8.8	13.8	12.7	n.a.	17.5
positive investment (%)	81.5	100	72.8	82.6	84.5	n.a.	75.2
positive spike (%)	18.6	4.1	19.9	28.3	24.1	13.2	18.5
negative spike (%)	1.8	0	0.18	0.97	1.3	2.3	1
serial correlation	0.058	0.647	-0.058	-0.064	-0.069	0.148	n.a.

Notes. Inaction rate: $|i/k| < 1\%$. Negative investment: $i/k \leq -1\%$. Positive investment: $i/k \geq 1\%$. Positive spike: $i/k > 20\%$. Negative spike: $i/k < -20\%$. Serial correlation: corr $\left[\left(\frac{i}{k}\right)_t, \left(\frac{i}{k}\right)_{t-1}\right]$. For each variable, we compute the time series average for each plant, and report the average across plants. The baseline parameters are $\alpha = 0.7$, $r = 0.04$, $\delta = 0.14$ and $\sigma = 0.12$. [a] Data are from Cooper and Haltiwanger (2006), table 1, sample period 1972-1988. [b] $\varphi = 11.76$. [c] $U = L = 0.15$. [d] $\Theta = 0.010$ ($d^* = 4$ quarters). [e] $\Theta = 0.017$ ($d^* = 5$ quarters). [f] Cooper and Haltiwanger (2006), table 5. [g] Khan and Thomas (2008), table II. n.a.: not available.

action observed in the data. Moreover, through the smoothing of investment, it creates excessively positive autocorrelation of investment rates. Not surprisingly, it fits quite well the persistence of the aggregate investment rate.[23] In fact, this model is widely used to describe the gradual adjustment observed at the aggregate level. Nevertheless, to match the capital adjustment observed in plant-level data different models of adjustment need to be considered. As column 4 shows, even the simple model with fixed adjustment costs considered here is able to create investment inactivity at the plant-level as well as to produce both positive and negative investment spikes.

With only one exception, the inattentiveness model does a reasonably good job. It matches both positive and negative investment of the frequency found in the data. It also produces some inactivity, although it underestimates the frequency of such episodes. Moreover, it generates positive and negative spikes. Also, column 6 reports the predictions of the model when the firm is inattentive for five quarters. Overall, the model's performance is similar to the one with four-quarter plans. The exception is that the model creates low negative serial correlation in investment. This moment of the data however warrants a digression, since the

[23] See table 2.2.

findings in the empirical investment literature are mixed.

On the one hand, one of the key findings in Doms and Dunne (1998) is that large investment episodes are often spread across few (usually two or three) years, that is, there is a large probability of having a spike in the period immediately following a spike. In technical language, adjustment hazard functions – the probability of adjusting – are downward-sloping with respect to the time since the prior spike. The evidence provided by Doms and Dunne (1998) therefore suggests positive serial correlation of investment at the plant-level and appears supportive of a convex adjustment costs model.

On the other hand, Cooper et al. (1999) argue that unobservable heterogeneity at the plant-level may yield downward-sloping hazard functions even if the hazard for any individual plant is upward sloping. In fact, after controlling for unobservable heterogeneity, Cooper et al. (1999) find evidence of upward-sloping hazard functions: the likelihood of a plant experiencing a large investment episode is increasing in the time since the previous spike passes by. Put differently, bursts of investment are followed, on average, by periods of low investment.[24]

The fact that the hazard slopes upward provides support for lumpy adjustment behavior. Therefore the negative serial correlation of the inattentiveness model (and of the non-convex adjustment costs model as well) is analogous to the upward sloping hazards. Perhaps not surprisingly, only a hybrid model presented in Cooper and Haltiwanger (2006), which mixes both convex and non-convex components of adjustment costs, is able to reproduce this moment of the data.

Finally, the last column reports the results in Khan and Thomas (2008). Note that the inattentiveness model performs at least as well as the Khan and Thomas model, which is "the first to succeed in matching the available moments from the cross-sectional distribution of plant investment rates", (Khan and Thomas, 2008, p. 408).

2.4.2 Aggregate data

Having established the consistency of the inattentiveness model with essential features of the microeconomic data, we now test the implications of the model with aggregate data. We use the same parametrization as before and generate simulated aggregate capital and investment data for a panel of 500 economies and 52 years.

[24] Gelos and Isgut (2001) provide support for the evidence in Doms and Dunne (1998), while Nilsen and Schiantarelli (2003) and Fennema et al. (2006) obtain results consistent with those of Cooper et al. (1999).

Table 2.2: Summary statistics, U.S. aggregate data and models

	data 1954-2005[a]	data 1984-2005[a]	fixed information costs model [b,c]	fixed information costs model [b,d]	KT08[e]
serial correlation	0.797	0.846	0.210	0.172	0.210
standard deviation	0.010	0.011	0.104	0.102	0.085
skewness	0.465	0.730	0.222	0.198	1.121
excess kurtosis	-0.094	-0.446	-0.078	-0.268	2.313

Notes. For each moment, we compute the time series average for each economy, and report the average across economies. [a] Data are annual private fixed nonresidential investment-to-capital ratio, computed using Bureau of Economic Analysis tables and following the procedure described in Bachmann et al. (2010), appendix B1. [b] The baseline parameters are $\alpha = 0.7$, $r = 0.04$, $\delta = 0.14$, $\sigma = 0.12$ and $\Theta = 0.010$ ($d^* = 4$ quarters). [c] Statistics are based on a 52-year simulated sample. [d] Statistics are computed using the last 22 years of the 52-year sample. [e] Khan and Thomas (2008), table III.

Table 2.2 shows the second and higher order moments of annual aggregate investment rate in the post-war U.S. data (1954-2005, second column) and in the Great Moderation period (1984-2005, third column). It also displays the inattentiveness model's predictions considering both the entire 52-year sample and the last 22 years of the full sample (fourth and fifth column, respectively). For the sake of comparison, the sixth column reports the moments of the Khan and Thomas (2008) partial equilibrium lumpy investment model.

The serial correlation of aggregate investment is about 0.8 in the data, much higher than that observed at the plant-level. Aggregate investment also exhibits near zero standard deviation, positive skewness and negative excess kurtosis. Note also that, while the second order moments are nearly the same in both periods, the higher moments are larger (in absolute value) during the Great Moderation period.[25]

The performance of the inattentiveness model at the aggregate level is not quite as successful as that at the plant-level. Although aggregation smooths out investment spikes and helps to generate positive serial correlation of investment rate, the model still predicts far too little persistence relative to the aggregate data. The model also overestimates (roughly 10 times) the variability of the investment rate. Nevertheless, over the other dimensions, the model

[25] The aggregate investment rate moments reported in table 2.2 are different from those reported by Khan and Thomas (2008). Examining annual private investment-to-capital ratio over the period 1954:2005, Khan and Thomas find persistence, standard deviation, skewness and excess kurtosis of 0.695, 0.008, 0.008 and -0.715, respectively.

does a better job. In fact, it matches well the excess kurtosis and it predicts about the right amount of skewness. Note also that the model fits the post-war data slightly better than the data for the Great Moderation period.

Despite the mixed performance of the inattentiveness model, one should note that the Khan and Thomas (2008) partial equilibrium lumpy investment model is also not successful in matching the moments of aggregate investment rate. In fact, the Khan and Thomas model predicts roughly the same serial correlation and standard deviation as the inattentiveness model, and it also sharply overstates the skewness and kurtosis. It is worthwhile to point out that Khan and Thomas improve the fit of their model when they include the effects of general equilibrium in the lumpy investment environment. In particular, their general equilibrium model yields an aggregate investment rate with persistence and volatility close to that observed in the data. In chapter 3 we embed capital investment decisions with inattentiveness in a general equilibrium framework and analyze whether (and to what extent) such a model improves the fit at the aggregate level.

2.5 Applying the inattentiveness model to Tobin's Q and cash flow

Regressions of investment on Tobin's Q and cash flow typically yield small positive coefficients on Q and larger coefficients on cash flow (see, for example, Moyen, 2004). The small coefficient on Q has traditionally been interpreted as evidence of very high adjustment costs, and the positive cash flow effect on investment has first been interpreted by Fazzari et al. (1988) as evidence that firms face financial constraints. Abel and Eberly (2008) cast doubts about these traditional interpretations of empirical investment equations. In fact, they show that investment may remain sensitive to both Tobin's Q and cash flow even when adjustment costs and financing constraints are absent from the model.[26]

In the following subsection we briefly describe the main assumptions and results of the theoretical model of Abel and Eberly (2008). Then we evaluate how the assumptions made by Abel and Eberly affect the predictions of the inattentiveness model. Finally we analyze the effects of Tobin's Q and cash flow on the investment rate in both models by means of

[26] The interpretation of cash flow effects as evidence of financing constraints has also been called into question by, among others, Kaplan and Zingales (1997), Gomes (2001), Alti (2003), Cooper and Ejarque (2003) and Eberly et al. (2008).

regression analysis.

2.5.1 How Q and cash flow may affect investment without frictions: the Abel and Eberly (2008) model

2.5.1.1 Functional form assumptions

The theoretical model is the same as the one presented in section 2.3.1. In order to derive an analytical relationship among investment, Tobin's Q and cash flow, Abel and Eberly make the following assumptions about the stochastic processes.

The profitability shock, Z_t, follows a geometric Brownian motion with a time-varying drift, μ_t, and a constant variance, σ^2:

$$dZ_t = \mu_t Z_t dt + \sigma Z_t dz \ .$$

The drift μ_t in turn follows a regime-switching process and remains constant for a random length of time. A new value of μ_t is drawn from an unchanging distribution $F(\tilde{\mu})$ with finite support $[\mu_L, \mu_H]$ with constant probability λ. The draws of new values of μ_t are i.i.d. and are independent from the realizations of the other stochastic processes in the model.

Next, variable M_t, defined as

$$M_t \equiv \left(\frac{r+\delta_t}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}} \ ,$$

follows a trendless geometric Brownian motion with a constant variance σ_M^2:

$$dM_t = \sigma_M M_t dz_M \ ,$$

where dz_M is the increment of a standard Wiener process.

2.5.1.2 Cash flow and Tobin's Q

From section 2.3.1, recall that $\widetilde{\Pi}_t = Z_t^{1-\alpha} K_t^{\alpha}$ represents revenue net of labor costs, hence $\widetilde{\Pi}_t$ also represents cash flow before investment expenditure. Let $c_t = \widetilde{\Pi}_t / K_t$ denote the cash flow before investment, normalized by the capital stock. Then the optimal cash flow per unit of capital, c_t^*, is given by

$$c_t^* = \frac{\widetilde{\Pi}_t^*}{K_t^*} = \frac{r+\delta_t}{\alpha} = M_t^{-\frac{1-\alpha}{\alpha}} \ . \tag{2.12}$$

That is, cash flow exhibits time-series variation because of the assumption that the depreciation rate (through the variable M_t) varies stochastically over time.

Tobin's Q is the ratio of the value of the firm – the present value of current and expected future operating profits – to the replacement cost of the firm's capital stock. In this model Tobin's Q is given by

$$Q_t^* = 1 + \frac{(1-\alpha)\,\omega^*}{r-\mu_t+\lambda} c_t^* \ , \tag{2.13}$$

where $\omega^* \equiv \left\{E\left[\frac{r-\mu_t}{r+\lambda-\mu_t}\right]\right\}^{-1} > 1$. Three remarks are in order. First, Q_t^* is a measure of average Q, rather than marginal Q, which equals one in this model. This distinction is noteworthy because average Q is readily observable, whereas marginal Q is not directly observable. Second, Tobin's Q exceeds one – even without any adjustment costs – for a firm that earns rents from monopoly power ($\alpha < 1$). Third, Tobin's Q exhibits time-series variation (that is not perfectly correlated with contemporaneous cash flow) because of the assumption that the growth rate of Z_t varies stochastically.

2.5.1.3 The effects of Tobin's Q and cash flow on investment

Gross investment I_t^* is the sum of net investment, dK_t^*, and depreciation, $\delta_t K_t^* dt$. Therefore gross investment rate is

$$\frac{I_t^*}{K_t^*}dt = \frac{dK_t^*}{K_t^*} + \delta_t dt = \left[\mu_t + \delta_t + \frac{1-\alpha}{2}\left(\frac{\sigma_M}{\alpha}\right)^2\right]dt + \sigma dz + \frac{1}{\alpha}\sigma_M dz_M \ .$$

The expected value of the investment-capital ratio at time t, ι_t^*, is

$$\iota_t^* = \mu_t + \delta_t + \frac{1}{2}\frac{1-\alpha}{\alpha^2}\sigma_M^2 \ . \tag{2.14}$$

Using equation (2.13) to express the growth rate μ_t in terms of the Q_t^* and c_t^*, and equation (2.12) to express δ_t in terms of c_t^*, then equation (2.14) may be rewritten as

$$\iota_t^*\left(Q_t^*, c_t^*\right) = \left[\alpha - \frac{(1-\alpha)\,\omega^*}{Q_t^*-1}\right]c_t^* + \lambda + \frac{1-\alpha}{2}\left(\frac{\sigma_M}{\alpha}\right)^2 \ . \tag{2.15}$$

Equation (2.15) shows the main result of the Abel and Eberly model: that Tobin's Q and cash flow can help account for movements in investment, even in a model in which there are no frictions whatsoever. These effects arise because of the fact that Tobin's Q reflects expectations about future revenue growth, while cash flow reflects the effects of the user cost of capital. Since both of these shocks (revenue growth and the user cost of capital) drive investment, investment turns out to be correlated with both Q and cash flow.

To analyze the effects of these variables on investment, let $\beta_{Q^*} = \frac{\partial \iota_t^*(Q_t^*, c_t^*)}{\partial Q_t^*}$ and $\beta_{c^*} = \frac{\partial \iota_t^*(Q_t^*, c_t^*)}{\partial c_t^*}$ denote, respectively, the response of the investment-capital ratio to a variation in Q_t^* and c_t^*. Then

$$\beta_{Q^*} = \frac{(1-\alpha)\,\omega^* c_t^*}{(Q_t^* - 1)^2} > 0 \ ,$$

that is, investment is an increasing function of Q_t^* even though there are no convex costs of adjustment. Similarly,

$$\beta_{c^*} = \alpha - \frac{(1-\alpha)\,\omega^*}{Q_t^* - 1} > 0$$

as long as $\mu_t + \delta_t > \lambda \ \forall t$, which is assumed to be the case. Thus cash flow has a positive effect on investment even though capital markets are perfect and there are no financing constraints.

2.5.2 Extending the inattentiveness model

Having modified the stochastic structure of the model, we first describe the new implications of the inattentiveness model. Then we examine the relationship among investment, Tobin's Q and cash flow in this framework.

2.5.2.1 Model's predictions

Proposition 3. *With stochastic depreciation, the optimal inattentiveness interval approximately equals*

$$d^* = \sqrt{\frac{4\alpha\Theta}{(\alpha\sigma)^2 + \sigma_M^2}} \ .$$

This results shows that optimal inattentiveness falls with the volatility of the depreciation rate shocks ($\frac{\partial d^*}{\partial \sigma_M} < 0$). That is, the firm adjusts her plans more frequently when the depreciation rate is more volatile since the cost of being inattentive is higher in a world that is rapidly changing.[27]

Proposition 4. *The optimal plan for capital between adjustment dates, for $D(i) < t < D(i+1)$, obeys the equation*

$$K_t^{IN} = K_{D(i)}^* e^{\left\{\left[\mu_{D(i)} - \alpha\frac{\sigma^2}{2} - \frac{1}{2}\left(\frac{\sigma_M}{\alpha}\right)^2\right][t-D(i)]\right\}}.$$

In this case, the value of the drift of the profitability shock at planning date, as well as the volatility of the depreciation rate, affects the plan for capital during the intervals of inattentiveness. The optimal adjustment policy with inattentiveness is thus described by the following equations

$$K_t^{IN} = \begin{cases} K_{D(i)}^* = M_{D(i)}^{\frac{1}{\alpha}} Z_{D(i)} & \forall i \in \mathbb{N}_{\mathbb{O}} \\ \\ K_{D(i)}^* e^{\left\{\left[\mu_{D(i)} - \alpha\frac{\sigma^2}{2} - \frac{1}{2}\left(\frac{\sigma_M}{\alpha}\right)^2\right][t-D(i)]\right\}} & \forall\ D(i) < t < D(i+1) \end{cases}.$$

2.5.2.2 Cash flow, Tobin's Q and investment

Proposition 5. *Cash flow per unit of capital and Tobin's Q are given by, respectively,*

$$c_t^{IN} = \begin{cases} c_{D(i)}^* = \left[M_{D(i)}\right]^{-\frac{1-\alpha}{\alpha}} & \forall i \in \mathbb{N}_{\mathbb{O}} \\ \\ c_{D(i)}^* e^{\left\{\left[\frac{1-\alpha}{2}\left(\frac{\sigma_M}{\alpha}\right)^2\right][t-D(i)]\right\}} & \forall\ D(i) < t < D(i+1) \end{cases}$$

$$Q_t^{IN} = \begin{cases} 1 + \frac{(1-\alpha)\omega^*}{r-\mu_{D(i)}+\lambda} c_{D(i)}^* & \forall i \in \mathbb{N}_{\mathbb{O}} \\ \\ 1 + \frac{(1-\alpha)\omega^{IN}}{r-\mu_{D(i)}-c_1+\lambda} c_t^{IN} & \forall\ D(i) < t < D(i+1) \end{cases},$$

where $c_1 \equiv -\alpha\frac{\sigma^2}{2} - \frac{1}{2}\frac{\sigma_M^2}{\alpha}$ and $\omega^{IN} \equiv \left\{E\left[\frac{r-\mu_t-c_1}{r-\mu_t-c_1+\lambda}\right]\right\}^{-1}$. Note that, during periods of inattentiveness, c_t^{IN} and Q_t^{IN} do not exhibit time-series variation. In particular, neither Q_t^{IN} is a function of the current growth rate, nor is cash flow related with the depreciation rate.

Finally, investment with inattentiveness combines investment gulps and maintenance invest-

[27] Similarly to the case analyzed in section 2.3.3, it also holds that $\frac{\partial d^*}{\partial \Theta} > 0$ and $\frac{\partial d^*}{\partial \sigma} < 0$, while the prediction $\frac{\partial d^*}{\partial \alpha} < 0$ now holds if and only if $\alpha > \frac{\sigma_M}{\sigma}$.

ment and is described by

$$\begin{cases} I_{D(i+1)} = K^*_{D(i+1)} - K^*_{D(i)} e^{\left\{\left[\mu_{D(i)} - \alpha\frac{\sigma^2}{2} - \frac{1}{2}\left(\frac{\sigma_M}{\alpha}\right)^2\right] d^*\right\}} & \forall i \in \mathbb{N}_{\mathbb{O}} \\ I^M_{t+1} = \frac{dK^{IN}_t}{dt} + \delta_t K^{IN}_t = \left[\mu_{D(i)} - \alpha\frac{\sigma^2}{2} - \frac{1}{2}\left(\frac{\sigma_M}{\alpha}\right)^2 + \delta_t\right] K_t & \forall\, D(i) < t < D(i+1) \end{cases}.$$

During intervals of time between consecutive planning dates, maintenance investment I^M_t varies continuously over time to fully compensate for depreciation and does not respond to possible variations of the growth rate. From this and the results above, it follows that investment during period of inattentiveness is not correlated neither with Tobin's Q , nor with cash flow.

Thus, since it is not possible to derive an analytical relationship between these variables, we analyze the effects of Tobin's Q and cash flow on the investment rate by means of regression analysis.

2.5.3 Simulation results

We simulate the models by first choosing a set of baseline parameters. As in section 2.4, we pick $r = 0.04$, $\delta = 0.14$, $\sigma = 0.12$ and $\alpha = 0.7$. The drift μ_t of the profitability shock follows a regime-switching process with finite support $[\mu_L, \mu_H]$. Condition 1 in Abel and Eberly (2008) states that the value of the firm is finite if $r > \mu_H$. To satisfy this condition, we set $\mu_H = 0.03$ and $\mu_L = 0$, that is, the growth rate varies between 0 and 3 percent per year.[28] We use the estimates of Eberly et al. (2008) to calibrate the parameter λ, which represents the probability of changing regime. Eberly et al. (2008) estimate alternative investment models in which the demand (or productivity) shock follows a regime-switching process and find that the probability of a regime switch is approximately 7% per year. This implies an average regime duration of 14.2 years. We thus set $\lambda = \frac{1}{14.2}$. Next, in the inattentiveness model, the condition $\frac{\partial d^*}{\partial \alpha} < 0$ holds if and only if $\sigma_M < \alpha\sigma$ (see footnote 27). Given our baseline parameters for α and σ, we pick $\sigma_M = 0.057$ so as to satisfy this condition. Finally we set $\Theta = 0.016$ implying an average inattentiveness of 1 year.

Using this parametrization, we generate simulated capital, investment, Tobin's Q and cash flow data for a panel of 1000 firms and 15 years. We then use these simulated data to run OLS

[28] Appendix E shows that the condition for the inattentiveness model is less restrictive.

regressions of the investment-capital ratio on Tobin's Q and normalized cash flow. Following the literature (*e.g.* Alti, 2003), we estimate the regression specification

$$\frac{I_t}{K_t} = \beta_0 + \beta_1 Q_t + \beta_2 \frac{c_t}{K_t} + \varepsilon_t \ , \tag{2.16}$$

where gross investment I_t is the sum of investment gulps and maintenance investment in year t, K_t is the capital stock at the beginning of year t, Q_t is the beginning-of-the-year t value of Tobin's Q, c_t is the sum of cash flows during year t, and ε_t is the error term.

Table 2.3 reports the sample means of the estimated regression coefficients, the standard errors, and the adjusted R^2. We report the results of investment regressions without and with cash flow (columns labelled, respectively, "univariate" and "multivariate").

Columns 2 and 3 show the results from the U.S. data. We report the results for two classes of firms, those identified by Fazzari et al. (1988) as financially constrained (left side) and those that are less financially constrained (right side). In the univariate regression, the coefficient on Q is quantitatively small – suggesting high adjustment costs – with modest explanatory power. Adding cash flow to the investment regression improves the fit – the adjusted R^2 increases – and reduces the effect of Q on investment. In the multivariate regression, the estimated coefficient on cash flow is much larger than the coefficient on Q, that is, investment is very sensitive to cash flow. Furthermore, investment is more sensitive to cash flow for firms that are identified to be more financially constrained. This is the main finding of Fazzari et al. (1988).

Columns 4 and 5 report the results for the Abel and Eberly (2008) model. The overall picture is consistent with the data and confirm the main theoretical prediction of the model: that Tobin's Q and cash flow do explain investment, even when adjustment costs and financing constraints are removed from the model.

The results for the inattentiveness model are shown in columns 6 and 7. There are two important results. First, the estimated coefficients are larger than in the data. This result should be not surprising since the optimal investment decision in the inattentiveness model involves infrequent and lumpy changes in investment rates. Therefore, due to the highly nonlinear nature of investment decisions, the estimated coefficients on the linear equation (2.16) turn out to be rather uninformative. Given these shortcomings, Gomes (2001) chooses the adjusted R^2 as indicator of additional informative content of the cash flow regressor in such nonlinear models. Thus, following Gomes (2001), the second result of this exercise is that the cash flow regressor significantly improves the predictive power of the regression as

Table 2.3: Investment regressions, U.S. data and models

	data[a]		Abel and Eberly (2008)[b]		Inattentiveness model[b]	
coefficient	univariate	multivariate	univariate	multivariate	univariate	multivariate
β_1	0.0045 / 0.0044	0.0008 / 0.0020	0.055	0.014	0.344	-0.329
	(0.0004) / (0.0002)	(0.0004) / (0.0003)	(0.023)	(0.003)	(0.406)	(0.138)
$\overline{R}^2$	0.23 / 0.11		0.297		0.035	
β_2		0.464 / 0.230		0.589		6.141
		(0.027) / (0.010)		(0.021)		(0.552)
$\overline{R}^2$		0.46 / 0.19		0.979		0.900

Notes. The fixed term effect β_0 is not reported. Standard errors are in parentheses. [a] Data are from Fazzari et al. (1988), table 5, class 1 (low dividend) / class 3 (high dividend), sample period 1970-1984. [b] The baseline parameters are $\alpha = 0.7$, $r = 0.04$, $\delta = 0.14$, $\sigma = 0.12$, $\mu_L = 0$, $\mu_H = 0.03$, $\lambda = \frac{1}{14.2}$, $\sigma_M = 0.057$ and $\Theta = 0.016$ ($d^* = 4$ quarters).

the adjusted R^2 also improves substantially when cash flow is included. The inattentiveness model therefore still finds a role for cash flow in explaining investment.

To better understand the large cash flow sensitivity of investment implied by the inattentiveness model, we run the following experiment. We consider a sudden and permanent rise in the plant-level productivity. We then derive the path for normalized cash flow and investment rate generated by the inattentiveness model and compare it with the path generated by the Abel and Eberly model.

The productivity shock Z_t is assumed to be constant at its steady state level and then, at year 4, unexpectedly rises by 10 percent and remains at this new level thereafter. The top graph in figure 2.2 shows the path of normalized cash flow predicted by the two models. In both models, a positive shock to productivity leads to increased cash flow in the period following the shock, and cash flow returns to its new steady state two periods after the shock. The models also exhibit similar dynamic paths.[29]

Differences among the models become apparent examining the response of investment rate in the bottom graph of figure 2.2. The inattentiveness model predicts an investment rate response that is much larger than that in the frictionless model. Thus, since the cash flow responses are almost the same in both models, the message from figure 2.2 is clear: investment

[29] This result comes from the fact that the depreciation rate is held constant throughout this experiment (*i.e.* variable M_t is constant).

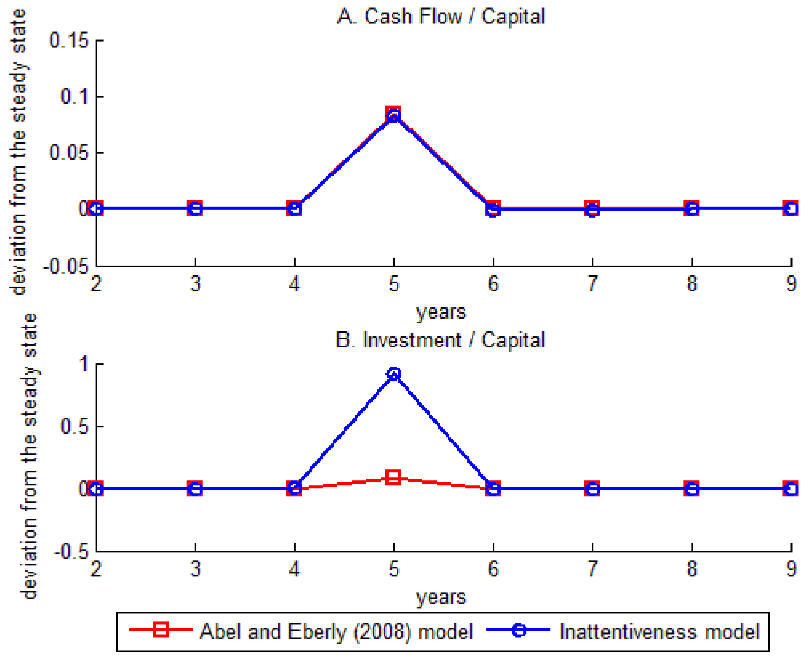

Figure 2.2: Dynamic paths after a 10 percent rise in the plant-level productivity at year 4

rate is much more sensitive to cash flow variations in the inattentiveness model. The intuition behind this is that the inattentive firm responds only infrequently, and by large amounts, to shocks. Inattentiveness thus enhances the cash flow sensitivity of investment.

2.6 Conclusion

This chapter has presented a novel microfoundation for the lumpiness of micro-level capital adjustment. Traditional explanations are based on fixed costs of adjusting the stock of capital. In this chapter capital adjustment is frictionless. Instead, the new explanation proposed here is inattentiveness, whereby firms make infrequent investment decisions due to a cost of gathering and processing information. Introducing such information/planning costs into an otherwise frictionless model is enough to induce infrequent and possibly lumpy capital adjustments. On the one hand, in between adjustment dates, the firm is inactive and only undertakes planned maintenance investment. On the other hand, when the firm does update her information and plan, the stock of capital immediately jumps to its optimal level. It is therefore likely to observe an investment gulp at those planning dates.

We have found that the inattentiveness model does a good job in matching key features of micro-investment, and it also fits some (but not all) features of investment at the aggregate level. We have also shown that the inattentiveness model still finds a role for cash flow in explaining investment and that inattentiveness exacerbates the cash flow sensitivity of investment.

Overall, we believe these results are encouraging for at least two reasons. First, the model developed here represents the first attempt to apply inattentiveness to the behavior of firms accumulating physical capital, while the literature that analyzes lumpy capital adjustments due to non-convex adjustment costs is much more mature. Second, the calibration is not fully optimized. It is likely that by changing some parameters, the inattentiveness model might match the moments more closely. In any case, the model does not seem to perform noticeably worse than one of the leading models in the literature.

The model so far is to some extent stylized and leaves much room for improvements. For example, in this chapter we have treated information/planning costs as a pure alternative to adjustment costs. As it is well known, whenever there is a cost of gathering information and planning, and no direct costs of adjusting capital, the optimal adjustment rule is time as opposed to state dependent. More recent work on information frictions in monetary economics integrates both state and time-dependent adjustment rules in one framework. Abel et al. (2009) and Alvarez et al. (2010a) study financial investment decisions with information and transaction costs, Alvarez et al. (2010b) analyze the price setting problem of a firm facing both observation and adjustment costs, and Woodford (2009) builds a hybrid model of rational inattention (state-dependent adjustment) and inattentiveness (time-dependent adjustment) to study the optimal price-setting decision of a firm. Therefore, it would be a worthy pursuit to follow this literature and extend this model to include adjustment costs. Furthermore, the model abstracts from general equilibrium considerations. In the next chapter we embed capital accumulation with inattentiveness into the Mankiw and Reis (2006, 2007) sticky information general equilibrium model and analyze whether such a model can account for key features of the business cycle.

Appendix E - Inattentiveness model

E1. Quick review of the solution of continuous-time stochastic processes (Brownian motions)

Let S_t be a geometric Brownian motion defined by the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \ , \tag{E.1}$$

where dW_t is a Wiener process, and μ and σ are, respectively, the drift and the variance parameter. The behavior of S_t can be derived by applying Ito's lemma to $d\ln S_t$:

$$\begin{aligned} d\ln S_t &= \frac{1}{S_t}dS_t - \frac{1}{2}\frac{1}{S_t^2}(dS_t)^2 = \\ &\quad \frac{1}{S_t}\left[\mu S_t dt + \sigma S_t dW_t\right] - \frac{1}{2}\frac{1}{S_t^2}S_t^2\sigma^2 dt \ , \end{aligned}$$

where the last equality follows from (E.1) and from the fact that $(dW_t)^2 = dt$ and $(dt)^2 = dt\,dW_t = 0$. Thus

$$d\ln S_t = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW_t \ . \tag{E.2}$$

Integrating (E.2) and applying the fundamental theorem of calculus yields

$$S_t = S_0 e^{\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\}} \ .$$

One can also easily derive the following relationships, which hold $\forall k$:

$$S_t^k = S_0^k e^{\left\{k\left(\mu - \frac{\sigma^2}{2}\right)t + k\sigma W_t\right\}} \tag{E.3}$$

$$E_0\left(S_t^k\right) = S_0^k e^{\left\{k\left[\mu + (k-1)\frac{\sigma^2}{2}\right]t\right\}} \tag{E.4}$$

$$\left[E_0\left(S_t^k\right)\right]^{\frac{1}{k}} = S_0 e^{\left\{\left[\mu + (k-1)\frac{\sigma^2}{2}\right]t\right\}} \tag{E.5}$$

E2. The inattentive firm's problem - Section 2.3.3

Applying the above properties to the stochastic processes for Z_t ($dZ_t = \sigma Z_t dz$) yields:

$$Z_t = Z_{D(i)} e^{\left\{-\frac{\sigma^2}{2}[t-D(i)]+\sigma z_t\right\}}$$

$$E_{D(i)}\left(Z_t\right) = Z_{D(i)} e^{[t-D(i)]} \tag{E.6}$$

$$\left[E_{D(i)}\left(Z_t^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}} = Z_{D(i)} e^{\left\{-\alpha\frac{\sigma^2}{2}[t-D(i)]\right\}} \tag{E.7}$$

The first order condition with respect to K_t is:

$$E\left[\alpha Z_t^{1-\alpha} K_t^{\alpha-1} - (r+\delta)\right] = 0 \Leftrightarrow K_t^{\alpha-1} E\left(Z_t^{1-\alpha}\right) = \frac{r+\delta}{\alpha} \Leftrightarrow$$

$$K_t = \left[\frac{r+\delta}{\alpha}\right]^{-\frac{1}{1-\alpha}} \left[E\left(Z_t^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}} = M^{\frac{1}{\alpha}} \left[E\left(Z_t^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}} \ . \tag{E.8}$$

Proof of proposition 2

Substituting (E.7) into (E.8) gives the result in the proposition. ||

Proof of proposition 1

Using (E.8) to evaluate the profit function shows that expected optimal operating profits are

$$\Pi(\mathbf{x},t) = \underbrace{(1-\alpha)\left(\frac{r+\delta}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}}_{\equiv\Xi} \left[E\left(Z_t^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}} = \Xi Z_0 \exp\left(-\alpha\frac{\sigma^2}{2}t\right) \ ,$$

where the second equality follows from (E.7). In this case the Bellman equation (2.8) may be rewritten as

$$V\left(\mathbf{x_0}\right) = \max_d \left\{\int_o^d e^{-rt}\Xi Z_0 e^{-bt} dt + e^{-rd} E\left[-\Theta\Pi\left(\mathbf{x},d\right) + V\left(\mathbf{x},d\right)\right]\right\} \ ,$$

where $b = \alpha\frac{\sigma^2}{2}$. Given that $\Pi(\mathbf{x},0) = \Xi Z_0$, we make the (educated) guess that the value function is linear: $V(\mathbf{x}) = AZ$, where A is a coefficient to be determined. The Bellman equation then becomes

$$AZ_0 = \max_d \left\{ \frac{\Xi Z_0 \left(1 - e^{-(r+b)d}\right)}{r+b} + e^{-rd} \left[-\Theta \Xi Z_0 e^{-bd} + AE\left(Z_d\right) \right] \right\} .$$

From (E.6), $E\left(Z_d\right) = Z_0$, then cancelling terms yields

$$A = \max_d \left\{ \frac{\Xi \left(1 - e^{-(r+b)d}\right)}{r+b} + e^{-rd} \left[A - \Theta \Xi e^{-bd} \right] \right\} . \tag{E.9}$$

The first-order condition from the maximization problem is

$$\frac{\partial A}{\partial d} = e^{-rd} \left\{ \Xi e^{-bd} \left[1 + \Theta\left(r+b\right)\right] - rA \right\} = 0 . \tag{E.10}$$

At the optimum d^*, (E.9) gives the solution for A:

$$\begin{aligned} A &= \frac{\Xi \left(1 - e^{-(r+b)d^*}\right)}{r+b} + e^{-rd^*} \left[A - \Theta \Xi e^{-bd^*} \right] \\ &\Leftrightarrow \\ A &= \frac{\Xi \left(1 - e^{-(r+b)d^*}\right) - \Theta \Xi \left(r+b\right) e^{-(r+b)d^*}}{\left(r+b\right)\left(1 - e^{-rd^*}\right)} . \end{aligned}$$

Using (E.10) and rearranging then yields the condition

$$\begin{aligned} \Gamma\left(b, \Theta, d^*\right) &= re^{bd^*} - \left[1 + \Theta\left(r+b\right)\right] \left(r + b - be^{-rd^*}\right) \\ &= re^{\alpha \frac{\sigma^2}{2} d^*} - \left[1 + \Theta \left(r + \alpha \frac{\sigma^2}{2}\right)\right] \left(r + \alpha \frac{\sigma^2}{2} - \alpha \frac{\sigma^2}{2} e^{-rd^*}\right) . \end{aligned}$$

Next, we check the second-order conditions for the maximization problem in (E.9). Note that

$$\begin{aligned} \frac{\partial^2 A}{\partial d^2} &= -re^{-rd} \left\{ \Xi e^{-bd} \left[1 + \Theta\left(r+b\right)\right] - rA \right\} \\ &\quad -b \Xi e^{-(r+b)d} \left[1 + \Theta\left(r+b\right)\right] \quad . \end{aligned}$$

At the optimal d^*, equation (E.10) implies that the first term in the sum is 0, while the second term is always negative. Therefore, $\frac{\partial^2 A}{\partial d^2} < 0$, which guarantees that the zero of the function

$\Gamma(b,\Theta,d^*)$ corresponds to a maximum.

The optimal choice of inattentiveness d^* is the zero of $\Gamma(\cdot)$. For $\Theta > 0$, $\Gamma(b,\Theta,0) = -r\theta(r+b) < 0$,

$$\Gamma_\Theta = -(r+b)\left[r+b-be^{-rd^*}\right] < 0\,\forall d$$

$$\Gamma_d(\cdot) = br\left\{e^{bd^*} - [1+\Theta(r+b)]e^{-rd^*}\right\}$$

$$\Gamma_d(b,\Theta,0) = -\Theta br(r+b) < 0; \Gamma_d(b,\Theta,+\infty) = +\infty; \Gamma_d(b,\Theta,d^*) > 0$$

$$\Gamma_b(\cdot) = -1 - 2\Theta(r+b) + rde^{bd^*} + e^{-rd^*}[1+\Theta r + 2\theta b]$$

$$\Gamma_b(b,\Theta,0) = -\Theta r < 0; \Gamma_b(b,\Theta,+\infty) = +\infty; \Gamma_b(b,\Theta,d^*) > 0\ .$$

For $d^* > 0$, the implicit function theorem implies that

$$\Gamma_\Theta(b,\Theta,d^*) + \Gamma_d(b,\Theta,d^*)\frac{\partial d^*}{\partial\Theta} = 0 \Leftrightarrow \frac{\partial d^*}{\partial\Theta} = -\frac{\Gamma_\theta(b,\Theta,d^*)}{\Gamma_d(b,\Theta,d^*)} > 0$$

$$\Gamma_b(b,\Theta,d^*) + \Gamma_d(b,\Theta,d^*)\frac{\partial d^*}{\partial b} = 0 \Leftrightarrow \frac{\partial d^*}{\partial b} = -\frac{\Gamma_b(b,\Theta,d^*)}{\Gamma_d(b,\Theta,d^*)} < 0\ .$$

Finally, to obtain the approximation, let define $\tilde{\Theta} = \sqrt{\Theta}$ so that

$$\Gamma\left(b,\tilde{\Theta},d^*\right) = re^{bd^*} - \left[1+\tilde{\Theta}^2(r+b)\right]\left(r+b-be^{-rd^*}\right)$$

and note that $\Gamma(b,0,0) = 0$, $\Gamma_d(b,0,0) = 0$ and $\Gamma_{\tilde{\Theta}}(b,0,0) = 0$. The implicit function theorem, $\Gamma_{\tilde{\Theta}} + \Gamma_d\frac{\partial d^*}{\partial\tilde{\Theta}} = 0$ therefore does not apply since $\Gamma_{\tilde{\Theta}} = \Gamma_d = 0$, so the point $\tilde{\Theta} = d = 0$ is a bifurcation point. One further round of differentiation plus the fact that $\Gamma_{d\tilde{\Theta}}(b,0,0) = 0$ lead to the conclusion that

$$\Gamma_{\tilde{\Theta}\tilde{\Theta}} + \Gamma_{dd}\left(\frac{\partial d}{\partial\tilde{\Theta}}\right)^2 = 0 \Rightarrow \frac{\partial d}{\partial\tilde{\Theta}} = \sqrt{-\frac{\Gamma_{\tilde{\Theta}\tilde{\Theta}}}{\Gamma_{dd}}}\ .$$

Since $\Gamma_{\tilde{\Theta}\tilde{\Theta}}(\cdot) = -2(r+b)\left(r+b-be^{-rd^*}\right)$ and $\Gamma_{dd}(\cdot) = br\left\{be^{bd} + r\left[1+\tilde{\Theta}^2(r+b)\right]e^{-rd}\right\}$, then $\Gamma_{\tilde{\Theta}\tilde{\Theta}}(b,0,0) = -2r(r+b)$ and $\Gamma_{dd}(b,0,0) = br(r+b)$, therefore

$$\frac{\partial d}{\partial\tilde{\Theta}} = \sqrt{-\frac{\Gamma_{\tilde{\Theta}\tilde{\Theta}}}{\Gamma_{dd}}} = \sqrt{\frac{2}{b}}\ .$$

Since a first-order Taylor approximation of d^* around $\tilde{\Theta} = 0$ is given by $d^* = \frac{\partial d^*}{\partial \tilde{\Theta}} \sqrt{\Theta}$, then

$$d^* = \sqrt{\frac{4\Theta}{\alpha\sigma^2}} \ . \quad ||$$

E3. The inattentive firm's problem - Section 2.5.2

Proof of proposition 4

The first order condition with respect to K_t is:

$$E\left[\alpha Z_t^{1-\alpha} K_t^{\alpha-1} - (r+\delta_t)\right] = 0 \Leftrightarrow K_t = \left[E\left(Z_t^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}} \left[\frac{E\left(r+\delta_t\right)}{\alpha}\right]^{-\frac{1}{1-\alpha}} \ .$$

Since $M_t \equiv \left(\frac{r+\delta_t}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}$, then $E\left(r+\delta_t\right)/\alpha = E\left(M_t^{-\frac{1-\alpha}{\alpha}}\right)$ and the first order condition becomes

$$K_t = \left[E\left(Z_t^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}} \left[E\left(M_t^{-\frac{1-\alpha}{\alpha}}\right)\right]^{-\frac{1}{1-\alpha}} \ .$$

Given the stochastic processes for Z_t ($dZ_t = \mu_t Z_t dt + \sigma Z_t dz$) and M_t ($dM_t = \sigma_M M_t dz_M$), applying the properties (E.3)-(E.5) gives the result in the proposition. ||

Proof of proposition 3

Abel and Eberly (2008) show that the optimal capital stock (K_t^*) and the optimal operating profit (Π_t^*) are given by, respectively, $K_t^* = Z_t M_t^{1/\alpha}$ and $\Pi_t^* = (1-\alpha) Z_t M_t$, while optimal cash flow is $c_t^* = M_t^{-\frac{1-\alpha}{\alpha}}$. Inheriting the stochastic properties of Z_t and M_t, optimal capital stock and operating profits follow a geometric Brownian motion with time-varying drift:

$$\frac{dK_t^*}{K_t^*} = \left[\mu_t + \frac{1-\alpha}{2}\left(\frac{\sigma_M}{\alpha}\right)^2\right] dt + \sigma dz + \frac{1}{\alpha}\sigma_M dz_M \tag{E.11}$$

and

$$\frac{d\Pi_t^*}{\Pi_t^*} = \mu_t dt + \sigma dz + \sigma_M dz_M \ , \tag{E.12}$$

while cash flow inherits the stochastic property of M_t and follows a geometric Brownian motion with constant drift:

$$\frac{dc_t^*}{c_t^*} = \frac{1-\alpha}{2}\left(\frac{\sigma_M}{\alpha}\right)^2 dt + \frac{\alpha-1}{\alpha}\sigma_M dz_M \ .$$

For the processes in (E.11)-(E.12), it is easy to show that the expected value of capital and profits are, respectively,

$$E_{D(i)}\left(K_t^*\right) = K_{D(i)} e^{\left[\mu_{D(i)} + \frac{1-\alpha}{2}\left(\frac{\sigma_M}{\alpha}\right)^2\right][t-D(i)]} \tag{E.13}$$

and

$$E_{D(i)}\left(\Pi_t^*\right) = \Pi_{D(i)} e^{\mu_{D(i)}[t-D(i)]} \ . \tag{E.14}$$

Let function $G(\mathbf{x},t)$ denote the expected difference between profits earned with full information and profits earned while following a pre-determined plan (Π_t^{IN}). To obtain an approximation of the optimal length of inattentiveness, we use the result in Proposition 4 in Reis (2006b), that states that a perturbation approximation of the optimal inattentiveness around the situation when planning is costless is

$$d^* = \sqrt{\frac{2\Theta}{G_t(\mathbf{x},0)}} \ . \tag{E.15}$$

For an inattentive firm, optimal profits are

$$\Pi_t^{IN} = (1-\alpha)\left[E\left(Z_t^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}}\left[E\left(M_t^{-\frac{1-\alpha}{\alpha}}\right)\right]^{-\frac{\alpha}{1-\alpha}} \ .$$

Given the stochastic processes for Z_t and M_t, applying the properties (E.3)-(E.5) gives

$$\Pi_t^{IN} = \Pi_{D(i)} e^{\left\{\left[\mu_{D(i)} - \alpha\frac{\sigma^2}{2} - \frac{1}{2}\frac{\sigma_M^2}{\alpha}\right][t-D(i)]\right\}} \ ,$$

where $\Pi_{D(i)} = (1-\alpha) Z_{D(i)} M_{D(i)}$. Using (E.13) and (E.14), $G(\mathbf{x},t)$ results

$$G(\mathbf{x},t) = E_{D(i)}\left(\Pi_t^*\right) - \Pi_t^{IN} = e^{\mu_{D(i)}[t-D(i)]}\left\{1 - e^{\left\{-\left(\alpha\frac{\sigma^2}{2} + \frac{1}{2}\frac{\sigma_M^2}{\alpha}\right)[t-D(i)]\right\}}\right\} \ . \tag{E.16}$$

Taking the derivative of (E.16) with respect to t and evaluating at $(\mathbf{x},0)$ yields $G_t(\mathbf{x},0) = \left(\alpha\frac{\sigma^2}{2} + \frac{1}{2}\frac{\sigma_M^2}{\alpha}\right) = \left[\frac{(\alpha\sigma)^2 + \sigma_M^2}{2\alpha}\right]$. Substituting this result in (E.15) gives the result in the proposition. $\Box$

Proof of proposition 5

For an inattentive firm, optimal cash flow is given by $c_t^{IN} = E\left(M_t^{-\frac{1-\alpha}{\alpha}}\right)$. Applying the properties (E.3)-(E.5) gives the expression for c_t^{IN} in the proposition.

Tobin's Q is defined as the ratio of the value of the firm to the firm's capital stock: $Q_t = V_t/K_t$. To compute the value of the inattentive firm, we follow the steps in Abel and Eberly (2008). The value of the firm can be derived by viewing the firm as composed of two divisions – a capital-owning division that owns K_t at time t and a capital-operating division that rents capital to produce and sell output at time t. On the one hand, capital can be instantaneously and costlessly bought or sold at a price of one at time t, so that the value of the capital-owning division at time t is K_t. On the other hand, the value of the capital-operating division is the present value of current and expected future operating profits.

The value of the firm at time t is thus given by

$$V_t^{IN} = K_t^{IN} + \int_t^{\infty} \Pi_{t+\tau} e^{-r\tau} d\tau \ ,$$

where

$$\Pi_{t+\tau} = \Pi_t \exp\left[\left(\underbrace{\mu_t - \alpha\frac{\sigma^2}{2} - \frac{1}{2}\frac{\sigma_M^2}{\alpha}}_{\mu_t + c_1}\right)(t+\tau)\right] \ ; \ c_1 \equiv -\alpha\frac{\sigma^2}{2} - \frac{1}{2}\frac{\sigma_M^2}{\alpha} < 0 \ .$$

Let $P_t = P(\mu_t, \Pi_t) \equiv \int_t^{\infty} E_t(\Pi_{t+\tau}) e^{-r\tau} d\tau$ be the price of a claim on the infinite stream of profits $\Pi_{t+\tau}$ for $\tau \geq 0$. Because the path of future growth rates of Π_t is independent of the current value of Π_t, $P(\mu_t, \Pi_t)$ may be rewritten as $p(\mu_t)\Pi_t$, where

$$p(\mu_t) \equiv E_t\left[\int_t^{\infty} \frac{\Pi_{t+\tau}}{\Pi_t} e^{-r\tau} d\tau\right] \ .$$

Let $p(\mu_t, T)$ be the value of $p(\mu_t)$ conditional on the assumption that the growth rate of Π_t remains equal to μ_t until time $t+T$, and after that a new value of the growth rate is drawn from the unconditional distribution. Therefore

$$\begin{aligned} p(\mu_t, T) &= \int_0^T e^{-(r-\mu_t-c_1)\tau} d\tau + e^{-(r-\mu_t-c_1)T} E_t \left[\int_T^\infty \frac{\Pi_{t+\tau}}{\Pi_{t+T}} e^{-r(\tau-T)} d\tau \right] \\ &= \frac{1-e^{-(r-\mu_t-c_1)T}}{r-\mu_t-c_1} + e^{-(r-\mu_t-c_1)T} E_t \left[\int_0^\infty \frac{\Pi_{t+T+\tau}}{\Pi_{t+T}} e^{-r\tau} d\tau \right] . \end{aligned}$$

Condition 1 The value of the firm is finite if $r > \mu^H + c_1$.

Since $c_1 < 0$, then Condition 1 is less restrictive than that in Abel and Eberly (2008).

Let p^* be the expectation of $p(\mu_t)$ when μ_t is drawn from its unconditional distribution, so that the last equation can be rewritten as

$$p(\mu_t, T) = \frac{1-e^{-(r-\mu_t-c_1)T}}{r-\mu_t-c_1} + e^{-(r-\mu_t-c_1)T} p^* . \tag{E.17}$$

Since a new regime arrives with constant probability λ, the density of T is

$$f(T) = \lambda e^{-\lambda T} \tag{E.18}$$

and

$$p(\mu_t) = \int_0^\infty p(\mu_t, T) f(T) dT . \tag{E.19}$$

Substituting (E.17) and (E.18) into (E.19) yields

$$\begin{aligned} p(\mu_t) &= \int_0^\infty \left[\frac{1-e^{-(r-\mu_t-c_1)T}}{r-\mu_t-c_1} + e^{-(r-\mu_t-c_1)T} p^* \right] \lambda e^{-\lambda T} dT \\ &= \frac{\lambda}{r-\mu_t-c_1} \left\{ \int_0^\infty \left[e^{-\lambda T} + (rp^* - \mu_t p^* - c_1 p^* - 1) e^{-(r-\mu_t+\lambda)T} \right] dT \right\} \\ &= \frac{1+\lambda p^*}{r-\mu_t-c_1+\lambda} . \end{aligned} \tag{E.20}$$

Since $p^* = E[p(\mu_t)]$, taking the unconditional expectation of (E.20) gives

$$p^* = (1+\lambda p^*) E\left[\frac{1}{r-\mu_t-c_1+\lambda} \right] ,$$

which can be rearranged to obtain

$$p^* = \omega^{IN} E\left[\frac{1}{r-\mu_t-c_1+\lambda}\right]$$

since

$$\omega^{IN} \equiv \left\{E\left[\frac{r-\mu_t-c_1}{r-\mu_t-c_1+\lambda}\right]\right\}^{-1} .$$

Therefore $\omega^{IN} = (1+\lambda p^*)$, so

$$p(\mu_t) = \frac{\omega^{IN}}{r-\mu_t-c_1+\lambda} .$$

Consequently

$$P(\mu_t,\Pi_t) \equiv \int_t^{\infty} E_t(\Pi_{t+\tau})\, e^{-r\tau} d\tau = \frac{\omega^{IN}}{r-\mu_t-c_1+\lambda}\Pi_t$$

and the value of the firm is given by

$$V_t^{IN} = K_t^{IN} + \frac{\omega^{IN}}{r-\mu_t-c_1+\lambda}\Pi_t^{IN} .$$

Tobin's Q for an inattentive firm is therefore given by

$$Q_t^{IN} = \frac{V_t^{IN}}{Q_t^{IN}} = 1 + \frac{(1-\alpha)\,\omega^{IN}}{r-\mu_{D(i)}-c_1+\lambda} c_t^{IN} . \quad \text{II}$$

Appendix F - Partial and state-dependent (lumpy) adjustment models

Convex adjustment costs and partial adjustment

Let us suppose that the firm incurs in two types of costs – a cost whenever the current capital stock differs from the frictionless level and one to adjust the current level of capital. Putting these two costs together in a loss function and using a discrete time setup, the dynamic programming problem is given by

$$\hat{V}\left(K_{t-1},K_t^*\right)=\min_{K_t}\frac{\left(K_t-K_t^*\right)^2}{2}+\frac{\varphi}{2}\left(K_t-K_{t-1}\right)^2+\beta E_t\hat{V}\left(K_t,K_{t+1}^*\right)\ , \tag{F.1}$$

where $\varphi>0$ is the adjustment costs parameter. The first-order condition of the optimization problem is

$$K_t-K_t^*+\varphi\left(K_t-K_{t-1}\right)-\beta\varphi E_t\left(K_{t+1}-K_t\right)=0\ , \tag{F.2}$$

where the last term comes from using (F.1) to solve $\partial\hat{V}/\partial K_t$. Given that the problem is quadratic, we make the (educated) guess that the the control variable (K_t) is linearly related to the two elements of the state vector (K_{t-1},K_t^*):

$$K_t=\nu_1K_t^*+\nu_2K_{t-1}\ ,$$

where ν_1 and ν_2 are coefficients to be determined. Using this conjecture in (F.2) and taking expectations of the future value of K_t yields

$$K_t-K_t^*+\varphi\left(K_t-K_{t-1}\right)-\beta\varphi\left[\nu_1K_t^*+\left(\nu_2-1\right)K_t\right]=0\ , \tag{F.3}$$

where we use the fact that $E_tK_{t+1}^*=K_t^*$ for a geometric Brownian motion. Solving (F.3) for K_t gives the solution for ν_1 and ν_2:

$$\nu_1=\frac{1+\beta\varphi\nu_1}{1+\varphi-\beta\varphi\left(\nu_2-1\right)}$$

and

$$\nu_2=\frac{\varphi}{1+\varphi-\beta\varphi\left(\nu_2-1\right)}\ .$$

It is easy to check that $\nu_2 = 1 - \nu_1$. The optimal adjustment is thus given by

$$K_t = \nu K_t^* + (1-\nu) K_{t-1} \ ,$$

where $\nu \equiv \nu_1$ is the (positive) solution of the following quadratic equation

$$\beta\varphi\nu^2 + (1+\varphi-\beta\varphi)\,\nu - 1 = 0 \ . \tag{F.4}$$

Non-convex adjustment costs and state-dependent (lumpy) adjustment

The firm has a stock of capital K_t and a strictly positive desired (or target) stock K_t^*. Let the adjustment cost incurred when changing the capital stock at any given time T be proportional to the current capital stock (before adjustment): $\bar{\Phi}K_{T^-}$. At any normalized time 0, the firm's problem is to minimize the present value cost:

$$\tilde{V}(K_0, K_0^*) = \inf_{K_T, T} E_0 \left\{ e^{-rT}\bar{\Phi}K_{T^-} + e^{-rT}\tilde{V}(K_T, K_T^*) \right\} \ , \tag{F.5}$$

where T is the first stopping time. Let $w_t \equiv K_t - K_t^*$ denote the departure from the desired optimum. Then it is possible to write $\tilde{V}(K_t, K_t^*)$ as $K_t J(w_t)$. If one assumes that the firm keeps the level of the capital stock constant by undertaking maintenance investment to fully compensate for depreciation, and defines $N \equiv \inf_w e^w J(w)$, then the problem can be written in terms of a single state variable:

$$J(w_0) = \inf_T E_0 \left\{ e^{-rT}\bar{\Phi} + e^{-rT} e^{-w_{T^-}} N \right\} \ .$$

Since $K_t^* = M^{\frac{1}{\alpha}} Z_t$, then the target stock level inherits the stochastic properties of Z_t and thus follows a geometric Brownian motion: $dK_t^* = \sigma K_t^* dz$. Then $dw_t = dK_t - dK_t^* = -\sigma dz$. After repeated application of Ito's lemma, one can show that $J(w)$ satisfies the following linear homogenous ordinary differential equation:

$$\frac{\sigma^2}{2} J'' - rJ = 0 \ . \tag{F.6}$$

The optimal policy consists of lower and upper trigger levels, denoted by L and U, respectively, and a common target point denoted by c, with $L \le c \le U$. The general solution of (F.6) is

$$J(w) = A_1 e^{\alpha_1 w} + A_2 e^{\alpha_2 w} \ ,$$

where $\alpha_1 = \sqrt{2r}/\sigma$, $\alpha_2 = -\alpha_1$ and A_1 and A_2 are constants of integration to be determined simultaneously with L, U and c from the boundary conditions of the problem. The latter are given by the value matching and smooth pasting conditions that, after some rearranging, yield the following non-homogenous full-rank system:

$$\begin{cases} J'(L) + J(L) = \bar{\Phi} \\ J'(U) + J(U) = \bar{\Phi} \\ J'(c) + J(c) = 0 \\ J'(L) + e^{(c-L)} J(c) = 0 \\ J'(U) + e^{(c-U)} J(c) = 0 \end{cases} \quad , \tag{F.7}$$

which can be solved numerically. It is well known that the inaction range is generally increasing with respect to σ and $\bar{\Phi}$.

Chapter 3

Lumpy investment in sticky information general equilibrium[1]

This chapter introduces lumpy micro-level investment into a sticky information general equilibrium model. Lumpy investment arises because of sticky information in capital decisions instead of the more popular assumption of non-convex adjustment costs. The only source of rigidity in the model is inattentiveness, which is pervasive to all markets and decisions. The model yields aggregate dynamics substantially different from those of an otherwise identical model with frictionless investment, and much closer to the empirical evidence. In delivering these results this model strengthens the case for the relevance of lumpy micro-level investment for the business cycle.

[1] I am extremely grateful to Ricardo Reis for his invaluable guidance, to Alper Çenesiz for helpful comments, and to Manuel M. F. Martins for extensive and critical comments on an early draft of this chapter which have considerably improved its content and readability. I would also like to thank professor Bachmann for providing the data on quarterly investment rates, Assia Ezzeroug and Maik Wolters for discussions on the implementation of sticky information models in Dynare, and the Fundação para a Ciência e a Tecnologia for financial support (Ph.D. scholarship). Any errors are my own.

3.1 Introduction

Figure 3.1 plots output and investment over the U.S. business cycle. The figure shows that aggregate investment is strongly procyclical, very persistent and much more volatile than output. Underlying such smooth aggregate investment dynamics are nevertheless infrequent and large, or lumpy, capital adjustments at the microeconomic level. Doms and Dunne (1998) show that about 50 % of an average plant's cumulative investment over 15 years is concentrated in a period of two or three (contiguous) years.

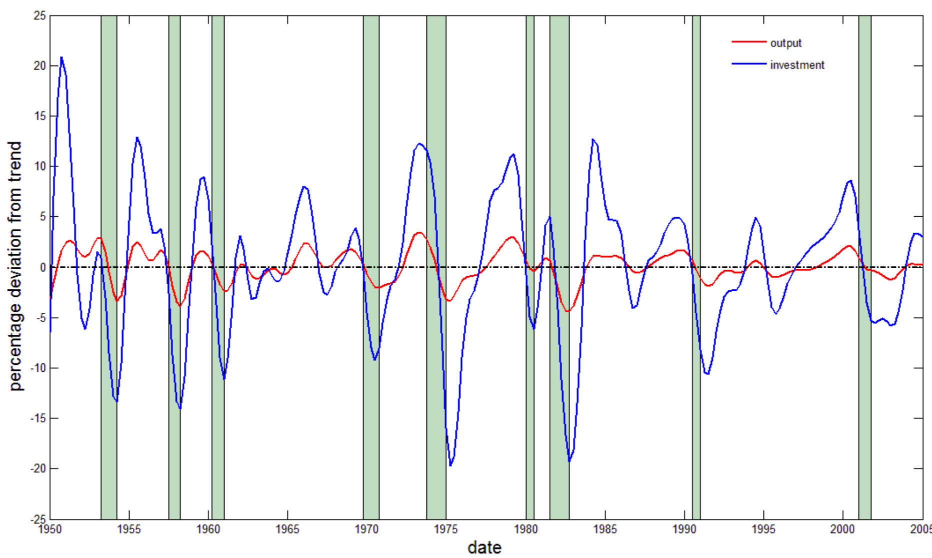

Figure 3.1: Output and investment over the U.S. business cycle

Note. The figure displays detrended quarterly real GDP and real private domestic investment in the U.S. over the period 1950-2005. The trends have been computed using the band-pass Baxter-King filter. Red line: output. Blue line: investment. Grey bars denote NBER recessions.

The volatility of investment is a prime contributor to aggregate fluctuations. According to Barro (1997, table 9.1), private investment accounts for about 93 % of the fluctuations in GDP, and thus "as a first approximation, explaining recessions amounts to explaining the sharp contractions in the private investment components".[2] Notwithstanding the importance of investment in explaining the business cycle (as well as, obviously, in determining economic long-term growth), capital accumulation has somewhat been ignored in canonical versions of the New Keynesian model (*e.g.* Galí, 2008). By now, however, standard DSGE

[2] Barro's findings come from analyzing the role of investment during five U.S. recessions (namely, those ending in 1961Q1, 1970Q4, 1975Q1, 1982Q4 and 1991Q4).

models do feature endogenous capital accumulation (*e.g.* Levin et al., 2005 and Smets and Wouters, 2007). Developing a microfounded model able to explain aggregate investment dynamics has nevertheless kept economists busy for years. To reproduce smooth aggregate investment dynamics, these DSGE models introduce convex investment adjustment costs. In doing so, however, the lumpy nature of plant-level investment is simply brushed away, and so these models' microfoundations for investment behavior seem rather weak. More recent theoretical research (*e.g.* Caballero, 1999) has proposed an alternative source for the smooth aggregate investment dynamics, suggesting that it may result from aggregation of infrequent, asynchronous and lumpy micro-level capital adjustments, which can be generated by fixed costs of capital adjustment.

An important debate running through the recent general equilibrium literature is whether micro-level lumpy capital adjustments have important implications for aggregate investment and, more generally, for the business cycle. The origin of the debate over the (ir)relevance of lumpy investment for aggregate dynamics dates back to Thomas (2002). Previously, partial equilibrium state-dependent lumpy adjustment models (Caballero et al., 1995, Doms and Dunne, 1998, Caballero and Engel, 1999, Cooper et al., 1999 and Doyle and Whited, 2001) had stressed important amplification and propagatory effects arising from infrequent plant-level investment activities. Thomas (2002) reassessed the impact of lumpy micro-level investment in a general equilibrium framework and concluded that firm-level investment lumpiness plays no important role for the aggregate dynamics of the model economy. In fact, her lumpy investment model generates business cycle dynamics and moments that are alike to those generated by an otherwise identical model characterized by frictionless investment. According to Thomas (2002, pag. 508), the irrelevance result arises because of the influence of general equilibrium forces: "in general equilibrium, households' preference for relatively smooth consumption profiles offsets changes in aggregate investment demand implied by the introduction of lumpy plant-level investment". Subsequently Gourio and Kashyap (2007) and Bachmann et al. (2010), among others, contrasted the Thomas result and found that lumpy investment matters for aggregate dynamics. Gourio and Kashyap (2007) re-calibrated Thomas' (2002) model and found that the recalibrated model has properties that differ from those of the standard RBC model. This result led them to conclude that the irrelevance result does not come only from general equilibrium effects, but also depends on how the model is calibrated. Bachmann et al. (2010) show that synchronization of plant-level investment activity in response to shocks can produce large effects on aggregate investment demand, so they argue that microeconomic investment lumpiness is relevant for macroeconomic analysis.[3]

Against this background, this chapter evaluates the relevance of lumpy investment in a sticky information general equilibrium environment. In chapter 2 we have shown that lumpy capital adjustment behavior arises naturally when firms face costs of gathering, absorbing, and processing information. We also have found that such partial equilibrium model is successful in fitting quantitative facts on plant-level investment rates. In this chapter we embed this theoretical framework into the Mankiw and Reis (2006, 2007) sticky information general equilibrium (SIGE) model. Specifically, we augment the SIGE model with a set of firms that make capital investment decisions with inattentiveness. In the capital-augmented version of the SIGE model, as in the original SIGE model, the only source of rigidity is inattentiveness, which is a pervasive feature of all markets and decisions – consumption, wages, prices and capital investment decisions are all made, to some degree, based on outdated information sets.

This chapter provides two main contributions.

First, enhancing the SIGE model with capital and investment overcomes one of its weaknesses pointed out in Reis (2009b). Such improvement narrows the gap between the sticky-information DSGE approach and the workhorse sticky-prices DSGE framework (*e.g.* Smets and Wouters, 2003 and Christiano et al., 2005), which have included capital and investment from the beginning. We provide a fully fledged microfounded DSGE model that relies only on pervasive inattentiveness to mimic the inertia present in the data, rather than on a large set of nominal and real rigidities as put forth by the sticky-prices approach – *e.g.* staggered price and wage setting with partial indexation, habit persistence in consumption, investment (or capital) adjustment costs and variable capital utilization.

Second, embedding into the SIGE model lumpy investment that is consistently microfounded on inattentiveness in capital adjustment decisions, reconciles general equilibrium modelling with the recent developments in the microeconomic theories of investment. As a consequence, this model also allows us to provide further contributions to the debate over the (ir)relevance of lumpy investment for the business cycle.

The chapter is organized as follows. Section 3.2 presents the capital-augmented sticky information general equilibrium (SIGEK) model, section 3.3 presents the key log-linearized

[3] Other papers supporting the relevance result include Bayer (2006a), Sveen and Weinke (2007), Iacoviello and Pavan (2007) and Fiori (2010). Khan and Thomas (2003, 2008) and House (2008) in turn provide additional robustness analysis in favor of the irrelevance result. A similar irrelevance result has been obtained in Veracierto (2002), who analyzes the role of plant-level irreversibilities in investment for aggregate fluctuations.

equations, and section 3.4 analyzes the business cycle implications of the model. Finally, section 3.5 concludes. Technical details are relegated to appendix G.

3.2 The capital-augmented sticky information general equilibrium model

There are three sets of agents: firms, households and government.

Within the firms sector, there are two types of firms, intermediate- and final-good firms, and there is a continuum of each indexed by i and f, respectively, in the unit interval. Monopolistic competitive intermediate-good firms have two departments: a hiring department that is always attentive and chooses how much of each variety of labor to hire, and a pricing/sales department that is only sporadically attentive and sets the price of the firm's output. Perfectly competitive final-good firms also have two departments: a purchasing department that is always attentive and chooses how much of each variety of intermediate goods to buy, and an inattentive producing department that produces the final good by combining its firm-specific capital with a Dixit-Stiglitz aggregator of varieties of intermediate goods.

Each household is made up of a consumer and a worker, and there is a continuum of each type of individual indexed by j and k, respectively, in the unit interval. Consumers consume, save and borrow by trading bonds between themselves. Each worker provides differentiated labor services to intermediate-good firms. Both consumers and workers are inattentive and make optimal decisions only sporadically.

Finally, monetary and fiscal policy follow exogenous rules and close the model.

Figure 3.2 sketches the structure of the model. Compared to the original SIGE model, the SIGEK model features a new set of agents, the final-good firms. To lay down the model formally, we start by describing the market clearing conditions and policy processes and then set out the agents' problems.

3.2.1 Market clearing conditions and economic policy

The total output produced by final-good firms, Y_t^{FIN}, is divided into consumption, investment and government goods. Market clearing in the final goods market thus requires that:

$$Y_t^{FIN} = G_t \left(C_t + INV_t \right) \ ,$$

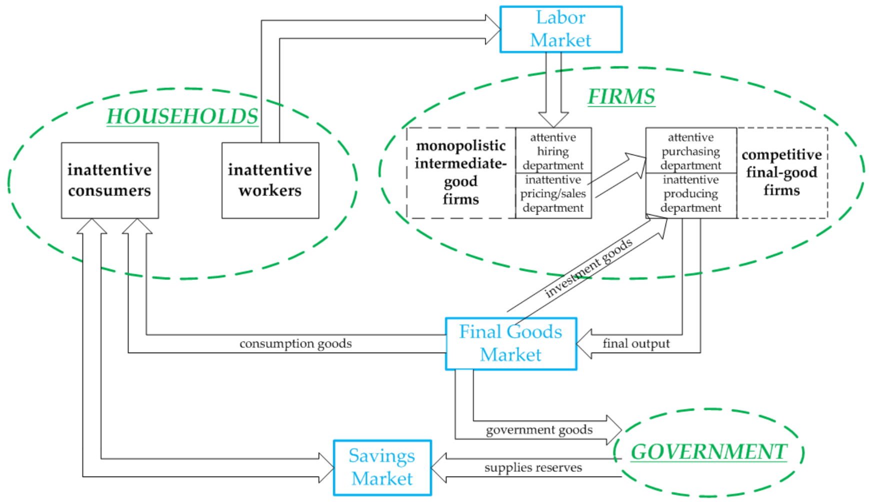

Figure 3.2: Structure of the model

where $1-1/G_t$ is the fraction of output consumed by the government, and $C_t = \int_0^1 C_{t,j}dj$ and $I_t = \int_0^1 INV_{t,f}df$ represent, respectively, total consumers consumption and total final-good firms investment. Government consumption G_t is financed by lump-sum taxes to households that keep the budget balanced at all dates. The fraction G_t is stochastic and shocks to it can be interpreted as aggregate demand shocks. The government also sets the nominal interest rate i_t according to:

$$i_t \equiv \log\left[E_t\left(\Pi_{t+1}P_{t+1}/P_t\right)\right] = \phi_\pi \log\left(\frac{P_t}{P_{t-1}}\right) - \varepsilon_t \ ,$$

where P_t denotes the price level, Π_{t+1} the real interest rate between t and $t+1$, and ε_t a discretionary monetary policy shock. The definition of the nominal interest rate follows the Fisher relation, whereas policy is set according to a Taylor rule that only reacts to inflation.

There is an intermediate goods market for each variety i, in which all final-good firms f are buyers and the seller is the intermediate-good firm that has the monopoly over its variety i . In equilibrium:

$$Y_{t,i}^{INT} = \int_0^1 Y_{t,f}^{INT}(i)\,df \ , \tag{3.1}$$

where $Y_{t,i}^{INT}$ is the total production of intermediate good i at time t, and $Y_{t,f}^{INT}(i)$ is the demand by final-good firm f of variety i at time t.

There is a labor market for each variety of labor k. The intermediate-good firms i demand labor, which is supplied by the household k that has the monopoly over its labor services.

Market clearing requires:

$$L_{t,k} = \int_0^1 N_{t,i}(k)\, di \ , \tag{3.2}$$

where $L_{t,k}$ is the total labor supply of variety k at time t, and $N_{t,i}(k)$ is the labor demand by intermediate-good firm i of variety k at time t. Total output and labor are defined by aggregating across all varieties: $Y_t^{FIN} = \int_0^1 Y_{t,f}^{FIN} df$ and $L_t = \int_0^1 L_{t,k} dk$.

Finally, nominal bonds are in zero net supply so the condition for the bond market to clear is $\int_0^1 B_{t,j} dj = 0$.

3.2.2 Final-good firms

3.2.2.1 Attentive purchasing departments

The purchasing department of the $f-$the firm buys a continuum of varieties i of intermediate goods in the amount $Y_{t,f}^{INT}(i)$ at price $P_{t,i}$, and combines them into a final input $Y_{t,f}$ according to a Dixit-Stiglitz aggregator with a random and time-varying elasticity of substitution $\hat{v}_t$. Each department solves the following problem, given current prices and a total desired amount of inputs $Y_{t,f}$:

$$\min_{\left\{Y_{t,f}^{INT}(i)\right\}_{i\in[0,1]}} \quad \int_0^1 P_{t,i} Y_{t,f}^{INT}(i)\, di$$

$$subject\ to \quad Y_{t,f} = \left[\int_0^1 Y_{t,f}^{INT}(i)^{\frac{\hat{v}_t-1}{\hat{v}_t}}\, di\right]^{\frac{\hat{v}_t}{\hat{v}_t-1}} .$$

Optimal behavior implies that the demand for each variety i by firm f is:

$$Y_{t,f}^{INT}(i) = Y_{t,f}\left[\frac{P_{t,i}}{P_t}\right]^{-\hat{v}_t} ,$$

where $P_t = \left[\int_0^1 P_{t,i}{}^{1-\hat{v}_t} di\right]^{\frac{1}{1-\hat{v}_t}}$ is the static price index. Integrating over the continuum of departments f and using the market clearing condition (3.1) gives the total demand for the intermediate-good of variety i:

$$Y_{t,i}^{INT} = \left(\frac{P_{t,i}}{P_t}\right)^{-\hat{v}_t} Y_t \ , \tag{3.3}$$

where $Y_t \equiv \int_0^1 Y_{t,f} df$.

3.2.2.2 Inattentive producing departments

The final good is the composite of two inputs – a homogeneous input Y_t, resulting from a Dixit-Stiglitz aggregator of varieties of intermediate goods, and the installed firm-specific capital, $K_{t-1,f}$. The producing department of the $f-$th firm produces the final good $Y_{t,f}^{FIN}$ according to the following technology:

$$Y_{t,f}^{FIN} = Z_t Y_t^{1-\alpha} K_{t-1,f}^{\alpha} \ , \tag{3.4}$$

where $\alpha < 1$ represents the share of capital in the firm's production function and Z_t an aggregate shock to the final goods production. The timing in (3.4) implies that capital becomes productive with a one-period delay.

The firm can purchase or sell capital instantaneously and frictionlessly, without any adjustment costs, at a constant price normalized to one. When the price of capital is constant, the Jorgensonian user cost of capital – *i.e.* the opportunity cost of holding one unit of capital for a period – is simply the sum of the discount rate of the firm and the depreciation rate.

Let us consider the problem faced by the producing department that last updated its information τ periods ago. Following the SIGE tradition, we assume that, in each period, a fraction η of firms, randomly drawn from the population, updates their information, so there are $\eta (1-\eta)^{\tau}$ firms in this situation. Each of these firms chooses the stock of capital $K_{t,\tau}$ to maximize expected real profits:

$$\max_{K_{t,\tau}} \qquad E_{t-\tau}\left[Y_{t,\tau}^{FIN} - (\Pi_t + \rho) K_{t-1,\tau}\right]$$

$$subject\ to \qquad Y_{t,\tau}^{FIN} = Z_t Y_t^{1-\alpha} K_{t-1,\tau}^{\alpha} \qquad ,$$

where ρ is the real depreciation rate and $(\Pi_t + \rho)$ represents the user cost of capital. The first-order condition is

$$E_{t-\tau}\left[\alpha Z_{t+1} Y_{t+1}^{1-\alpha} K_{t,\tau}^{\alpha-1}\right] = E_{t-\tau}\left(\Pi_{t+1} + \rho\right) \ .$$

If the firm observed all variables, this condition would state that the firm accumulates capital up to the point where the marginal product of capital equals the user cost of capital. After some rearrangements, the desired stock of capital is

$$K_{t,\tau} = \left[E_{t-\tau} \left(\frac{\Pi_{t+1} + \rho}{\alpha} \right) \right]^{-\frac{1}{1-\alpha}} \left[E_{t-\tau} \left(Z_{t+1} Y_{t+1}^{1-\alpha} \right) \right]^{\frac{1}{1-\alpha}} .$$

To attain the stock $K_{t,\tau}$ in period $t+1$, the firm demands the quantity $INV_{t,\tau}$ of final good in period t given by

$$INV_{t,\tau} = K_{t,\tau} - (1-\rho) K_{t-1,\tau} .$$

3.2.3 Intermediate-good firms

3.2.3.1 Attentive hiring departments

Each of the intermediate-good firms has a department that hires a continuum of labor varieties indexed by k in the amount $N_{t,i}(k)$ at price $W_{t,k}$. Labor services are combined into the labor input $N_{t,i}$ according to a Dixit-Stiglitz function with a random and time-varying elasticity of substitution $\hat{\gamma}_t$. The hiring department of the $i-$th firm solves the following problem, taken as given current wages and a total desired amount of inputs $N_{t,i}$:

$$\min_{\{N_{t,i}(k)\}_{k\in[0,1]}} \quad \int_0^1 W_{t,k} N_{t,i}(k)\, dk$$

$$subject\ to \quad N_{t,i} = \left[\int_0^1 N_{t,i}(k)^{\frac{\hat{\gamma}_t - 1}{\hat{\gamma}_t}} dk \right]^{\frac{\hat{\gamma}_t}{\hat{\gamma}_t - 1}} .$$

The solution to this problem is:

$$N_{t,i}(k) = N_{t,i} \left(\frac{W_{t,k}}{W_t} \right)^{-\hat{\gamma}_t} ,$$

where $W_t = \left[\int_0^1 W_{t,k}{}^{1-\hat{\gamma}_t} dk \right]^{\frac{1}{1-\hat{\gamma}_t}}$ is the static wage index. Summing over all firms i and using the market clearing condition (3.2) gives the total demand for labor of variety k:

$$L_{t,k} = \left(\frac{W_{t,k}}{W_t} \right)^{-\hat{\gamma}_t} N_t ,$$

where $N_t \equiv \int_0^1 N_{t,i} di$.

3.2.3.2 Inattentive pricing/sales departments

Let us consider now the problem faced by the pricing department of an intermediate-good firm that last updated its information τ periods ago. Each period, a randomly drawn fraction of firms λ updates their information, so there are $\lambda(1-\lambda)^{\tau}$ firms in this situation. They choose a nominal price $P_{t,\tau}$ to maximize expected real profits:

$$\begin{aligned} \max_{P_{t,\tau}} \quad & E_{t-\tau}\left[\frac{P_{t,\tau}Y_{t,\tau}^{INT}}{P_t} - \frac{W_t N_{t,\tau}}{P_t}\right] \\ subject\ to \quad & Y_{t,\tau}^{INT} = A_t N_{t,\tau}^{\beta} \\ & Y_{t,\tau}^{INT} = \left(\frac{P_{t,\tau}}{P_t}\right)^{-\hat{v}_t} Y_t \end{aligned}$$

The first constraint is the production function, where β measures the degree of returns to scale and aggregate productivity A_t is stochastic. The second constraint is the total demand for the firm's product in (3.3). The first order condition is:

$$P_{t,\tau} = \frac{E_{t-\tau}\left[\hat{v}_t W_t N_{t,\tau}/P_t\right]}{E_{t-\tau}\left[\beta\left(\hat{v}_t - 1\right)Y_{t,\tau}^{INT}/P_t\right]} \ .$$

If the firm observed all the variables on the right-hand side, this condition would state that the nominal price charged, $P_{t,\tau}$, is equal to a markup, $\hat{v}_t/(\hat{v}_t-1)$, over nominal marginal costs, which corresponds to the cost of an extra unit of labor, W_t, divided by its marginal product, $\beta Y_{t,\tau}^{INT}/N_{t,\tau}$.

3.2.4 Households

Households live forever and discount future utility by a factor $\xi \in (0,1)$. They obtain utility each period from consumption and leisure according to:

$$U\left(C_{t,j}, L_{t,k}\right) = \ln C_{t,j} - \frac{\chi L_{t,k}^{1+1/\psi}}{1+1/\psi} \ ,$$

where $C_{t,j}$ is consumption by consumers j at date t, $L_{t,k}$ is the labor supplied by worker k at date t, ψ is the Frisch elasticity of labor supply and χ captures relative preferences for consumption versus leisure.

At each date t, the household faces a budget constraint given by:

$$A_{t+1} = \Pi_{t+1}\left(A_t - C_{t,j} + \frac{W_{t,k}L_{t,k} + T_t}{P_t}\right) ,$$

where A_{t+1} denotes the real resources of households at the beginning of period $t+1$ and T_t are lump-sum transfers. These transfers comprise the profits received from intermediate-good firms, lump-sum taxes paid to the government, and payments for an insurance contract that households sign at the beginning of each time period so that they begin each period with the same wealth.

In the savings market, consumers face a probability δ of revising their plans every period, so at each period there are $\delta(1-\delta)^{\tau}$ of consumers in this situation. They choose a plan for current and future consumption, $\{C_{t+\tau,\tau}\}_{\tau=0}^{\infty}$, which is a sequence $\left\{C_{t,0}\,;C_{t+1,1}\,;C_{t+2,2}\,;\cdots\right\}$ where $C_{t,\tau}$ is the time-t expenditure of a consumer who last updated her information τ periods ago. The optimality conditions for consumers are:[4]

$$\frac{1}{C_{t,0}} = \xi E_t\left[\Pi_{t+1}\frac{1}{C_{t+1,0}}\right]$$

and

$$\frac{1}{C_{t,\tau}} = E_{t-\tau}\left[\frac{1}{C_{t,0}}\right] .$$

The first equation is the Euler equation for an attentive agent. It states that the marginal utility of consuming today equals the expected discounted marginal utility of consuming tomorrow times the return on savings. The second equation states that marginal utility of consumption for inattentive consumers equals the one they would expect in case there was full information.

In the labor market, a randomly drawn fraction of workers ω updates their plans each period, so at each period there are $\omega(1-\omega)^{\tau}$ of workers in this situation. They choose a plan for current and future wages, $\{W_{t+\tau,\tau}\}_{\tau=0}^{\infty}$, which is a sequence $\left\{W_{t,0}\,;W_{t+1,1}\,;W_{t+2,2}\,;\cdots\right\}$ where $W_{t,\tau}$ is the time-t wage set by a worker who last updated the information τ periods ago. The optimality conditions for workers are:

$$\frac{\hat{\gamma}_t}{\hat{\gamma}_t - 1}\frac{P_t L_{t,0}^{1/\psi}}{W_{t,0}} = \xi E_t\left[\Pi_{t+1}\frac{\hat{\gamma}_{t+1}}{\hat{\gamma}_{t+1}-1}\frac{P_{t+1}L_{t+1,0}^{1/\psi}}{W_{t+1,0}}\right]$$

[4] The dynamic problem solved by consumers and workers is more complicated than those solved by firms. We refer the reader to appendix G for details.

and

$$W_{t,\tau} = \frac{E_{t-\tau}\left[\hat{\gamma}_t L_{t,\tau}^{1+1/\psi}\right]}{E_{t-\tau}\left[\hat{\gamma}_t L_{t,\tau} L_{t,0}^{1/\psi}/W_{t,0}\right]} \ .$$

The first equation is the intertemporal labor supply Euler equation for an attentive worker. If $\hat{\gamma}_t$ was constant, the equation states that the marginal disutility of supplying labor today ($L_{t,0}^{1/\psi}$) divided by the real wage ($W_{t,0}/P_t$) is equal to the discounted marginal disutility tomorrow ($L_{t+1,0}^{1/\psi}$) divided by the corresponding real wage ($W_{t+1,0}/P_{t+1}$) times the real interest rate. With a time-varying $\hat{\gamma}_t$, the Euler equation takes into account the change in the markup charged by the monopolistic worker. The second condition notes that workers who are not perfectly informed set wages so that their expected disutility from working mirrors the disutility from working expected by the attentive workers.

3.3 The sticky information equilibrium

The detailed presentation of the model log-linearization is presented in appendix G. In this section we discuss the key reduced-form relations.

We log-linearize the equilibrium conditions around the non-stochastic steady state. Small caps denote the log-deviations of the respective large-cap variable from this steady state, with the exceptions of the following variables: v_t and γ_t, which are the log-deviations of $\hat{v}_t$ and $\hat{\gamma}_t$, respectively; r_t, which is the log-deviation of the short real interest rate $E_t\left[\Pi_{t+1}\right]$; and R_t, which is the log-deviation of the long real interest rate defined as $\lim_{T\to\infty} E_t\left[\bar{\Pi}_{t,t+1+T}\right]$, where $\bar{\Pi}_{t+l,t+1+k} = \prod_{z=t+l}^{t+k} \Pi_{z+1}$ is the compound return between two dates. Small letters with no subscript denote parameters and steady-state values.

The aggregate capital stock is:

$$k_t = \eta \sum_{\tau=0}^{\infty} (1-\eta)^{\tau} E_{t-\tau}\left[\frac{y_{t+1}^{FIN} - \alpha k_t}{1-\alpha} - \frac{r}{(r+\rho)(1-\alpha)} r_t\right] \ . \tag{3.5}$$

The level of capital stock (k_t) is positively related to the expected value of the firm's production (y_{t+1}^{FIN}) and negatively related to the current level of capital stock because of decreasing return to scale in production ($\alpha < 1$). A lower real interest rate (r_t) implies a lower opportunity cost of holding capital and therefore an incentive to increase the stock of capital. If many firms are informed (η is high), capital is instantly responsive to changes in these determinants, whereas otherwise capital adjustment takes place gradually over time.

Aggregate investment (inv_t) is:

$$inv_t = \frac{1}{\rho} k_t - \frac{1-\rho}{\rho} k_{t-1} \ . \tag{3.6}$$

The Phillips curve (or aggregate supply) is:

$$p_t = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} E_{t-\tau} \left[p_t + \frac{\beta (w_t - p_t) + (1-\beta) y_t - a_t}{\beta + \nu (1-\beta)} - \frac{\beta}{(\nu - 1)[\beta + \nu (1-\beta)]} \nu_t \right] \ . \tag{3.7}$$

The price level (p_t) depends on past expectations of its current value, real marginal costs and the desired markup. Marginal costs are larger the higher are the real wages paid to workers ($w_t - p_t$), the more is produced (y_t)(because of decreasing returns to scale, $\beta < 1$), and the lower is aggregate productivity (a_t). The desired markup falls with the elasticity of substitution across the varieties of goods (ν_t). Unexpected shocks to any of these variables only raise prices by λ because only this share of price-setting firms is attentive and, thus, aware of the news.

The IS curve is:

$$c_t = \delta \sum_{\tau=0}^{\infty} (1-\delta)^{\tau} E_{t-\tau} (c_t^n - R_t) \ , \tag{3.8}$$

where $c_t^n = \lim_{\tau \to \infty} E_t c_{t+\tau}$ is a measure of consumers' wealth and $R_t = \sum_{\tau=0}^{\infty} (i_{t+\tau} - \Delta p_{t+1+\tau})$ is the long real interest rate. Higher expected future wealth increases current spending, whereas higher expected interest rates encourage savings and lower spending. The higher is δ, the larger is the share of informed consumers who respond to shocks as they occur, therefore the more responsive consumption is to changes in these variables.

The wage curve is:

$$w_t = \omega \sum_{\tau=0}^{\infty} (1-\omega)^{\tau} E_{t-\tau} \left[p_t + \frac{\gamma}{\gamma + \psi} (w_t - p_t) + \frac{l_t}{\gamma + \psi} + \frac{\psi}{\gamma + \psi} (c_t^n - R_t) - \frac{\psi}{(\gamma + \psi)(\gamma - 1)} \gamma_t \right] \ . \tag{3.9}$$

Wages (w_t) increase one-to-one with the price level, as workers care about real income; they increase with real wages in the economy, since higher real wages push up the demand for a particular labor variety through substitution; they increase with labor supplied (l_t), because of the increasing marginal disutility of working; they increase with wealth, since leisure is a normal good; they decrease with interest rates, since lower interest rates decrease the return on savings and the incentive to work; and they fall as the elasticity of substitution across labor varieties increases (γ_t) since workers' desired markup falls. As ω increases, a larger fraction of workers is informed so wages become more responsive to changes in these determinants, whereas otherwise wages only respond gradually over time.

The aggregate resource constraint is

$$y_t^{FIN} = \alpha_c c_t + \alpha_i inv_t + g_t \ , \tag{3.10}$$

where $\alpha_c = c/(c+inv)$ and $\alpha_i = inv/(c+inv)$.

The policy rules are

$$r_t = i_t - E_t(\Delta p_{t+1}) \tag{3.11}$$

and

$$i_t = \phi_\pi \Delta p_t - \varepsilon_t \ . \tag{3.12}$$

Intermediate output is given by

$$y_t = \frac{y_t^{FIN} - z_t - \alpha k_{t-1}}{1-\alpha} \tag{3.13}$$

and labor is given by

$$l_t = \frac{y_t - a_t}{\beta} \ . \tag{3.14}$$

Equations (3.5)-(3.14) characterize the equilibrium for y_t^{FIN} (final output), c_t (consumption), w_t (wage), p_t (price), inv_t (investment), k_t (stock of capital), r_t (real interest rate), i_t (nominal interest rate), y_t (intermediate output) and l_t (labor) given exogenous shocks to ε_t (monetary policy), Δa_t (aggregate intermediate-good productivity growth), g_t (aggregate demand), v_t (intermediate-good markup), γ_t (labor markup) and z_t (aggregate final-good productivity). Each of these shocks follows an independent $AR(1)$ process: $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + e_t^\varepsilon$, $\Delta a_t = \rho_{\Delta a} \Delta a_{t-1} + e_t^{\Delta a}$, $g_t = \rho_g g_{t-1} + e_t^g$, $v_t = \rho_v v_{t-1} + e_t^v$, $\gamma_t = \rho_\gamma \gamma_{t-1} + e_t^\gamma$ and $z_t = \rho_z z_{t-1} + e_t^z$, where the shocks $e_t^s \sim N(0, \sigma_s^2)$ are i.i.d. with $E\left[e_t^s e_{t+k}^s\right] = 0$ for $k \neq 0$ and $E\left[e_t^s e_t^{s'}\right] = 0$ for $s \neq s'$.

3.4 Is lumpy investment relevant for the business cycle?

Having presented the SIGEK model's key relations, we now study the impact of lumpy micro-level investment on the aggregate business cycle. Through this section, we analyze and contrast the behavior of four models:

1. the SIGEK model with pervasive inattentiveness;
2. the SIGEK model with frictionless investment, which is obtained setting $\eta = 1$;
3. a classical model, *i.e.* the model when $\eta = \delta = \lambda = \omega = 1$ so that all agents are attentive;
4. the SIGE model.

Recall that the Thomas' (2002) irrelevance conclusion arises from the fact that her lumpy investment model generates business cycle dynamics (both impulse response functions and second moments) that are alike to those generated by an otherwise identical model characterized by frictionless investment. Accordingly, comparing the results from model 1 with those from model 2 allows for gauging whether lumpy investment consistently founded on inattentiveness is relevant for the business cycle.

Model 3 is used here as the simplest benchmark, with which all models with some source of informational inertia could be compared. Finally, comparing the results obtained with model 1 to those obtained with model 4 allows for assessing whether the inclusion of capital and investment in the SIGE model, developing the SIGEK model, modifies the performance of the sticky-information general equilibrium approach.[5]

We calibrate the model assuming that a period is a quarter. The share of consumption in total output α_c is assumed to be 0.85, so that the share of investment is $\alpha_i = 1 - \alpha_c = 0.15$. The steady state real depreciation rate and real interest rate, ρ and r, are set to 0.035 and 0.01, respectively, which implies a user cost of capital of $18\,\%/year$. The share of capital in the final-good firm's production function α is set to 0.33. The inattentiveness parameter η is

[5] All simulations have been conducted with Dynare. The results for the SIGE model have been obtained by simulating the Reis (2009a) model using the calibration in table 3.5. To make results comparable with those of other models, the simulation of the SIGE model has been conducted setting $\alpha_y = 0$ in the monetary policy rule, that is, dropping the interest rate response to the output gap, so that the nominal interest rate only responds to inflation.

assumed to be 0.1, which implies that final-good firms are inattentive on average for 10 quarters.[6] The serial correlation and the standard deviation of the final-good productivity shock, ρ_z and σ_z, are set to 0.75 and 0.5, respectively. The values for the remaining parameters are taken from table 2 in Reis (2009b). Those values have been obtained from the estimation of the SIGE model using Bayesian methods on post-86 U.S. data. Table 3.5 shows the baseline parameter values for the SIGEK model.

In what follows we first analyze the impulse responses to the various structural shocks as well as the contribution of those shocks to the forecast error variance of the endogenous variables, and then we investigate the ability of the models to match some second order moments of U.S. aggregate data.

3.4.1 Impulse response functions and variance decomposition

Figures 3.8 to 3.13 plot the impulse response functions to one-standard-deviation impulses to the six shocks. In all figures presented, variables are reported as percentage deviation from their steady state values, and the horizontal axis represents time on a quarterly scale. Blue-circle and blue-diamond lines represent the responses of the SIGEK model with pervasive inattentiveness and with frictionless investment, respectively, while red-cross lines represent the responses of the SIGE model. For the sake of clarity, we do not report the impulse responses of the classical model, which are way too large and essentially have no persistence. We first describe the dynamics of the SIGEK model with pervasive inattentiveness, and then we compare it with the dynamics of the SIGEK model with frictionless investment and of the SIGE model.

Figure 3.8 plots the effects of a positive (expansionary) monetary policy shock. The model with pervasive inattentiveness predicts that output, consumption, investment, capital, hours worked, real wage and inflation all increase in the short run in response to a monetary expansion. The responses however do not show any hump-shaped pattern and, with few exceptions, they also converge rapidly to their steady state levels. The fast reaction of macroeconomic variables to monetary policy is due to the fact that the policy shock is short-lived ($\rho_\varepsilon = 0.29$).

Figure 3.9 displays the responses to a positive intermediate-good productivity shock. By construction, the impact of this technology shock on output, consumption, investment and

[6] The value for η lies within the empirically plausible range for the lumpiness parameter indicated by Sveen and Weinke (2007). After analyzing the micro evidence reported by Doms and Dunne (1998), Sveen and Weinke suggest that the lumpiness parameter should take values between 0.06 and 0.12.

the real wage can be permanent. A positive productivity shock in fact permanently raises these variables but lowers hours worked and the output gap on impact, consistently with the findings in Galí (1999). Figure 3.10 displays the responses to a positive final-good productivity shock. Although the effect of this shock is transitory, the dynamics is qualitatively similar to that of the intermediate-good productivity shock.

Turning to the aggregate demand (government spending) shock, figure 3.11 shows that a positive innovation to aggregate demand raises inflation, output, and hours worked. While increasing investment significantly, this shock has a negative wealth effect that induces consumption to fall.

Figure 3.12 displays the effects of a positive shock to the price markup. The shock makes the economy more competitive (the desired price markup decreases) and so inflation falls while output, consumption and investment increase on impact. However, all variables respond quickly because the price markup shock is also quite short-lived ($\rho_v = 0.28$).

Figure 3.13 shows the effects of a positive wage markup shock (which corresponds to a fall in the desired markup). The real wage falls and there is an expansion in output, hours worked, consumption and investment. The fall in wages induces a fall in prices, so inflation falls and the central bank reduces the nominal interest rate over time to gradually push inflation back to its steady state value. Noticeably, the responses of most variables are both hump-shaped and delayed.

Even though the shape of the impulse response functions of the SIGEK model with frictionless investment is qualitatively similar to that of the model with pervasive inattentiveness, there are visible differences between them. As one would expect, the main quantitative difference is that the responses of some variables, especially those of capital and investment, are much larger because attentive final-good firms make their capital investment decisions every period, so they react instantly to shocks. Interestingly, a positive intermediate-good productivity shock in this economy raises hours worked instead of lowering them. The intuition is that, with capital fully flexible, final-good firms invest much more because they expect to produce more. However, in order to expand their production, they have to both accumulate more capital and purchase more intermediate goods (recall that these goods are one input in the final-good production function). This pushes up the production of intermediate goods (see equation 3.13), which more than compensates the increase in productivity and leads intermediate-good firms to hire more labor (see equation 3.14).

Figures 3.8-3.9 and 3.11-3.13 also report the impulse response functions implied by the SIGE model. Overall, the dynamics does not change significantly when the SIGE model is augmented by a microfounded lumpy investment model. The impulse responses of the SIGE model are in fact qualitatively, and in most cases also quantitatively, similar to those of the SIGEK model with pervasive inattentiveness.

Finally, table 3.1 presents the variance decompositions at different horizons for the SIGEK model with pervasive inattentiveness.[7] The monetary policy shock plays a small role in the variance of the error in forecasting most macroeconomic variables, with the exception of the nominal interest rate.[8] The aggregate intermediate-good productivity shock is important to explain consumption and investment, while the final-good productivity shock does not play a significant role. The demand shock is by far the most important shock and accounts for most of the variance of the error in forecasting output, investment and hours at all horizons. This persistent long-run effect arises because the demand shock is highly persistent ($\rho_g = 0.99$). Finally, while the labor markup shock is quite important to explain inflation, real wage and nominal interest rate especially at long horizons, the goods markup shock explains little of the variance of any of the variables, with the exception of inflation at very short horizon.

Overall, in contrast with Thomas' findings, this analysis seems to indicate that lumpy investment is relevant for the business cycle, since there are substantial quantitative differences between the models' responses with lumpy and with frictionless investment.

3.4.2 Second moments: models versus U.S. aggregate data

We now examine whether the SIGEK model yields empirically reasonable aggregate dynamics by comparing the model's predictions with some key second order moments characterizing the post-86 U.S. economy. In particular, we focus on the volatility and autocorrelations of output, investment, consumption, hours, real wage and inflation, as well as on the cross-correlation of output with the other variables.

3.4.2.1 Output and investment

Table 3.2 and figure 3.3 display output and investment moments in the U.S. data as well as the corresponding models' predictions. The main features of the data are well-known.

[7] The variance decomposition for the SIGE model is similar to the one in table 3.1.

[8] This result is consistent with the evidence in Bernanke et al. (2005) and Christiano et al. (2005).

Table 3.1: Variance decomposition (SIGEK model with pervasive inattentiveness)

variable	**shock**					
	monetary policy	intermediate-good productivity	aggregate demand	goods markup	labor markup	final-good productivity
A. Contribution to unconditional variance						
output	0	8	87	0	5	0
investment	0	17	78	0	5	0
consumption	0	45	47	0	8	0
hours	0	1	86	0	4	8
real wage	4	4	50	1	41	1
inflation	1	2	25	6	61	5
nominal interest rate	18	2	21	5	50	4
B. Contribution to one-quarter-ahead, one-year-ahead, and four-year-ahead variance						
output	1 , 0 , 0	12 , 14 , 15	86 , 80 , 76	0 , 0 , 0	2 , 5 , 8	0 , 0 , 0
investment	1 , 0 , 0	3 , 9 , 17	94 , 85 , 77	0 , 0 , 0	3 , 6 , 5	0 , 0 , 0
consumption	1 , 0 , 0	85 , 86 , 79	14 , 10 , 7	0 , 0 , 0	0 , 4 , 13	0 , 0 , 0
hours	0 , 0 , 0	2 , 2 , 2	84 , 81 , 81	0 , 0 , 0	1 , 4 , 6	13 , 13 , 11
real wage	18 , 8 , 6	18 , 7 , 6	0 , 6 , 24	8 , 3 , 2	55 , 74 , 62	0 , 0 , 1
inflation	5 , 2 , 1	6 , 3 , 2	36 , 29 , 25	41 , 13 , 6	6 , 46 , 61	5 , 7 , 5
nominal interest rate	60 , 34 , 19	3 , 2 , 2	15 , 20 , 21	17 , 9 , 5	3 , 31 , 50	2 , 5 ,4

Note. All numbers are in percentage units. In panel A and for each forecasting-horizon in panel B, rows may not add up to one due to rounding.

Table 3.2: Aggregate output and investment, models vs data in the post-86 U.S.

Series		**Standard deviation** absolute	relative to output	**coefficients of autocorrelation (order)** 1	2	3	4
output	data	0.90	1.00	0.93	0.76	0.52	0.29
	classical	3.16	1.00	0.60	0.51	0.48	0.46
	SIGEK ($\eta = 1$)	4.34	1.00	0.35	0.30	0.29	0.29
	SIGEK	2.67	1.00	0.90	0.83	0.79	0.75
investment	data	4.33	4.84	0.93	0.75	0.51	0.27
	classical	24.61	7.78	0.14	0.01	-0.01	-0.01
	SIGEK ($\eta = 1$)	23.39	5.39	0.05	0.00	0.00	0.01
	SIGEK	6.41	2.40	0.71	0.56	0.46	0.40

Note. The moments for U.S. data have been obtained applying the Baxter-King bandpass filter to the logarithm of the original series, with a band of 6 to 32 quarters.

Both output and investment are very persistent, with a first order serial correlation above 0.9. Investment is procyclical, with no phase shift, and is about 5 times as volatile as output.[9]

The classical model overestimates the volatility of output and investment and underestimates their persistence. It also does not perform well when it comes to fitting the lead-lag relation with output. The SIGEK model with frictionless investment ($\eta = 1$) does not perform much better than the classical model. In particular, the absence of sticky information in capital decisions makes investment too volatile (in both absolute and relative terms) with no persistence whatsoever. Even though the contemporaneous correlation with output is close to that observed in the data, all cross-correlations at lags other than zero are almost null. Pervasive inattentiveness, in turn, improves the ability of the model to fit the facts on output and investment. Output is less volatile and more persistent than in the classical model as well as in the frictionless investment model.[10] Although the model predicts that investment is only about two and a half times as volatile as output, it improves promisingly as regards fitting investment autocorrelations (even at high lags) and the shape of the cross-correlation curve (see figure 3.3).

In table 3.3 we report two moments of the aggregate investment rate. Columns 2 and 3 are

[9] All data have been taken from the FRED database available through the Federal Reserve Bank of St. Louis. The cyclical components of each series have been obtained applying the Baxter-King bandpass filter and they are similar to those obtained with the Hodrick-Prescott filter.

[10] The SIGE model yields similar moments for output.

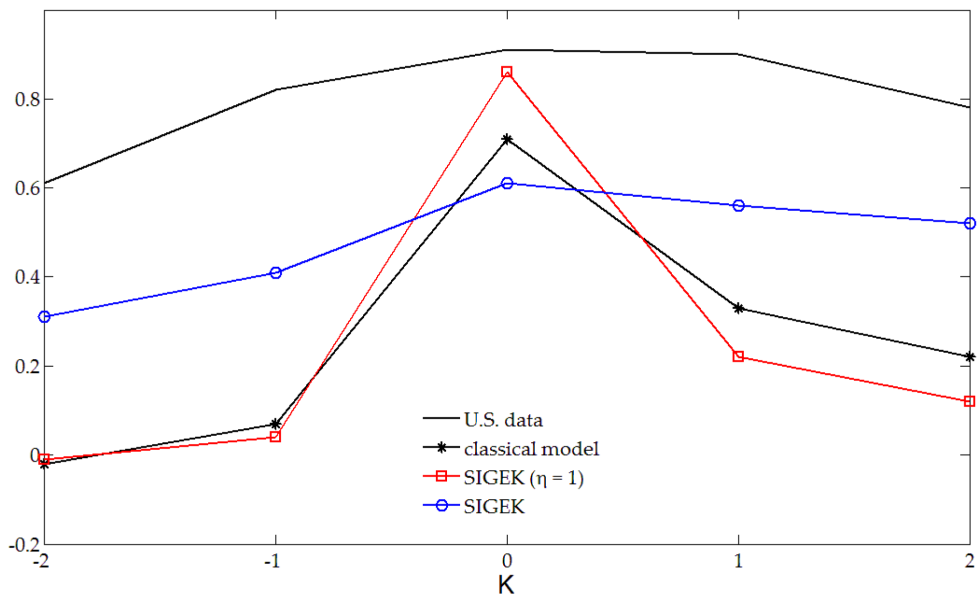

Figure 3.3: Cross-correlation of investment with output at lag K, $K = \{-2, -1, 0, 1, 2\}$

Note. The figure reports the cross-correlation of the cyclical component of investment with the $K-$quarter lag of the cyclical component of output. The moments for U.S. data have been obtained applying the Baxter-King bandpass filter to the logarithm of the original series, with a band of 6 to 32 quarters. U.S. data: black line. SIGEK model: blue-circle line. SIGEK model ($\eta = 1$): red-square line. Classical model: black-asterisk line.

Table 3.3: Aggregate investment rate, U.S. data and models

	annual data 1984-2005[a,b]	partial equilibrium model [b]	quarterly data post-86[c]	SIGEK model [d]
serial correlation (corr $\left[\left(\frac{i}{k}\right)_t, \left(\frac{i}{k}\right)_{t-1}\right]$)	0.846	0.210	0.970	0.724
standard deviation	0.011	0.104	0.137	0.07

[a] Data are annual private fixed nonresidential investment-to-capital ratio. [b] See table 2.2 in chapter 2. [c] Data are quarterly private fixed investment-to-capital ratio. [d] Baseline parameters: see table 3.5.

taken from table 2.2 in chapter 2 and show the moments in the data (annual values) and the respective moments implied by the partial equilibrium model with lumpy investment, while columns 4 and 5 report the moments in the data (quarterly values) and the respective moments implied by the SIGEK model with pervasive inattentiveness. Table 3.3 shows that there is an unambiguous improvement in the model fit moving from the partial equilibrium lumpy investment model to its general equilibrium counterpart with pervasive inattentiveness. In fact, while in a partial equilibrium framework the aggregate investment rate is less persistent and much more volatile than in the data, its persistence increases sharply (although still remaining lower than in the data) and its excessive volatility is virtually eliminated when lumpy investment is included in general equilibrium.[11]

By now, the results indicate that the SIGEK model is capable of delivering a plausible aggregate role for lumpy investment – the model's implied second moments of output and investment are significantly different from and closer to the data than those implied by an otherwise identical model with frictionless investment. We now analyze whether the quantitative differences in the models' output and investment moments extend to other key macroeconomic variables.

3.4.2.2 Consumption, hours, real wage and inflation

Table 3.4 reports the variabilities and autocorrelation coefficients of consumption, hours, real wage and inflation, and figure 3.4 plots the cross-correlations of these variables with output at different lags and leads.[12]

[11] Khan and Thomas (2008) obtain a similar result using a state-dependent lumpy investment model. They find that their general equilibrium model matches the data on aggregate investment rates much better than its partial equilibrium counterpart.

[12] The SIGE model yields similar predictions.

Table 3.4: Aggregate variables, models vs data in the post-86 U.S.

Series		**Standard deviation**		**coefficients of autocorrelation (order)**			
		absolute	relative to output	1	2	3	4
consumption	data	0.79	0.88	0.95	0.81	0.61	0.41
	classical	3.09	0.98	0.56	0.45	0.41	0.38
	SIGEK ($\eta = 1$)	1.96	0.45	0.94	0.89	0.83	0.79
	SIGEK	2.00	0.75	0.95	0.89	0.85	0.80
hours	data	1.39	1.55	0.96	0.86	0.70	0.52
	classical	6.42	2.03	0.43	0.32	0.29	0.27
	SIGEK ($\eta = 1$)	9.48	2.18	0.22	0.16	0.16	0.15
	SIGEK	5.33	2.00	0.83	0.73	0.65	0.60
real wage	data	1.00	1.11	0.91	0.69	0.44	0.25
	classical	2.22	0.70	0.44	0.31	0.27	0.26
	SIGEK ($\eta = 1$)	1.45	0.33	0.86	0.72	0.63	0.57
	SIGEK	1.56	0.59	0.88	0.77	0.69	0.63
inflation	data	0.23	0.26	0.65	0.58	0.58	0.57
	classical	2.03	0.64	0.23	0.08	0.04	0.03
	SIGEK ($\eta = 1$)	0.94	0.22	0.37	0.39	0.39	0.36
	SIGEK	0.70	0.26	0.88	0.81	0.74	0.67

Note. The moments for U.S. data have been obtained applying the Baxter-King bandpass filter to the logarithm of the original series, with a band of 6 to 32 quarters.

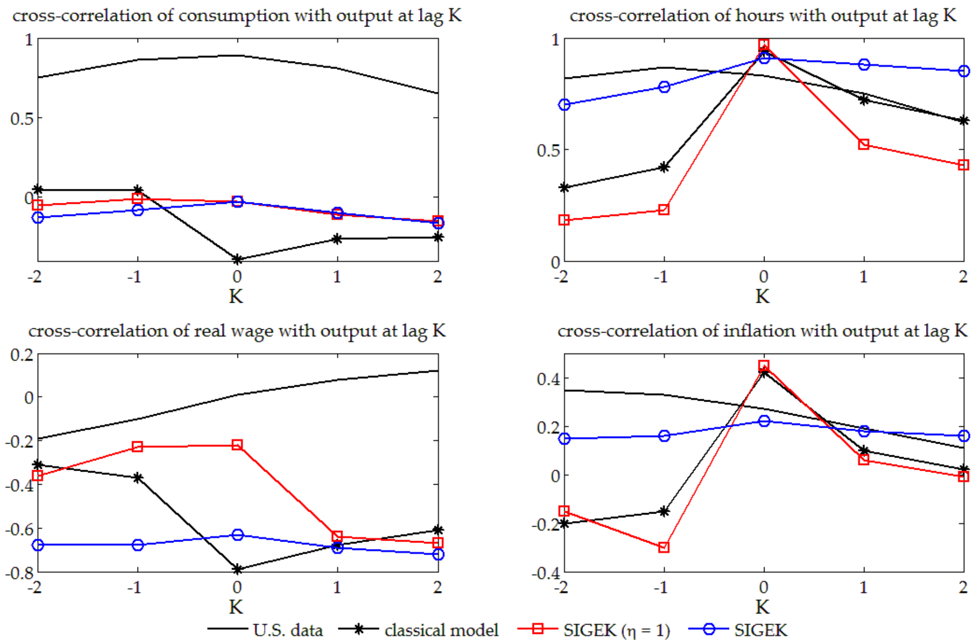

Figure 3.4: Cross-correlation of other variables with output at lag K, $K = \{-2,-1,0,1,2\}$

Note. The figure reports the cross-correlation of the cyclical component of the respective variable with the $K-$quarter lag of the cyclical component of output. The moments for U.S. data have been obtained applying the Baxter-King bandpass filter to the logarithm of the original series, with a band of 6 to 32 quarters. U.S. data: black line. SIGEK model: blue-circle line. SIGEK model ($\eta = 1$): red-square line. Classical model: black-asterisk line.

The results still exhibit interesting differences, both qualitatively and quantitatively, between the lumpy and the frictionless investment model. In particular, the inclusion of lumpy investment adjustment does improve and affect the model's performance along these dimensions. The crucial conclusion is that the model that overall best captures the moments of consumption, hours, real wage and inflation is the SIGEK model with pervasive inattentiveness. Hence, pervasive sticky information improves the ability of the SIGEK model to overall mimic the dynamics of key macroeconomic data (although some moments, especially the cross-correlations of consumption and real wage with output, seem hard to mimic).

We can thus conclude that, overall, the business cycle is clearly affected by lumpy investment at the micro-level.

3.4.2.3 How sensitive are the second order moments of investment to changes in the degree of inattentiveness η?

The previous results have been obtained by setting the degree of information stickiness η to 0.10 for final-good firms, in line with the suggestion of Sveen and Weinke (2007). To check for robustness, figures 3.5-3.7 contrast the SIGEK model's investment moments (our crucial variable) with their empirical counterparts, for different values of the parameter η.

Recall that the stickier information is, the smaller is the fraction of updating firms (smaller η) and the smaller is the impact of shocks on capital and investment. Thus, as the degree of information stickiness increases (η decreases), investment should become less volatile and more persistent. Figures 3.5 and 3.6 confirm this conjecture. The standard deviation decreases and the autocorrelation function shifts upward as η decreases.

The figures further show that the SIGEK model has difficulties in simultaneously mimicking the volatility and the persistence of investment. On the one hand, the model is able to match the high volatility of investment observed in the data only when firms are often attentive, updating their information on average once every eight months ($\eta \simeq 0.4$). One the other hand, a high degree of information stickiness ($\eta < 0.1$) is required to match the high persistence of investment.

While it remains true that the SIGEK model with pervasive inattentiveness is superior to the alternatives here studied in fitting the dynamic behavior of investment, it suffers from a trade-off between fitting the volatility and fitting the persistence of investment. It seems difficult to solve this trade-off by only fine-tuning one parameter – the degree of inattentiveness η – in the economy.

Finally, figure 3.7 plots the cross-correlation of investment with output at different lags and leads. As η increases, the model becomes better at matching the contemporaneous correlation with output but performs worse when it comes to matching cross-correlations at lags other than zero. Lower values of η (higher degrees of inattentiveness in investment) seem to improve the ability of the model to fit the overall lead-lag relation of investment with output.

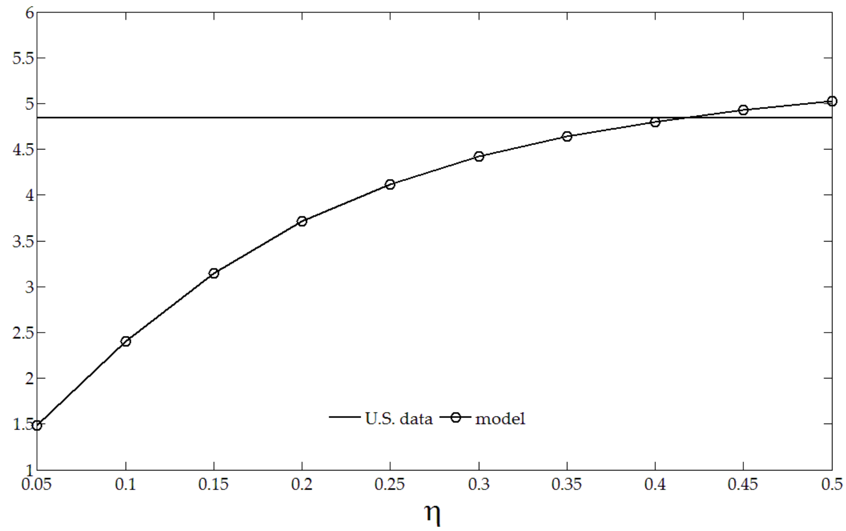

Figure 3.5: Standard deviation of investment (sensitivity analysis for different values of η)
Note. U.S. data: black line. SIGEK model: black-circle line. Other parameters than η: see table 3.5.

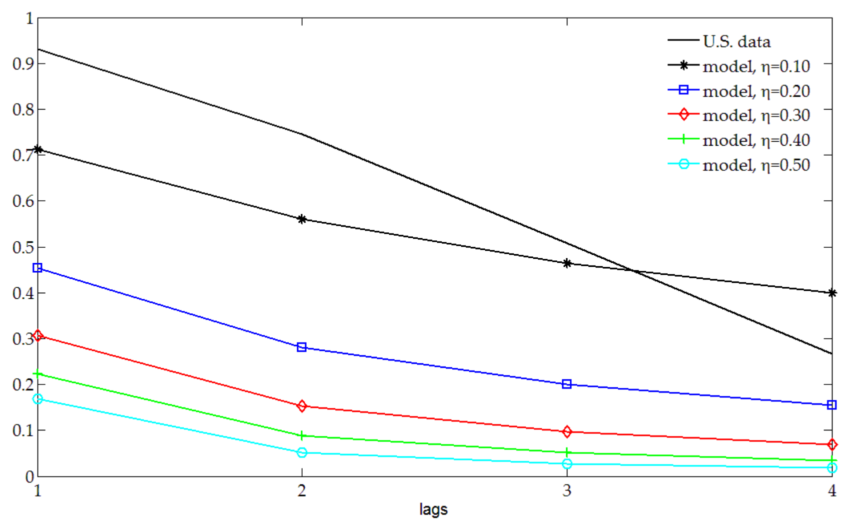

Figure 3.6: Autocorrelation of investment at lags 1 to 4 (sensitivity analysis for different values of η)
Note. U.S. data: black line. SIGEK model ($\eta = 0.1$): black-asterisk line. SIGEK model ($\eta = 0.2$): blue-square line. SIGEK model ($\eta = 0.3$): red-diamond line. SIGEK model ($\eta = 0.4$): green-plus line. SIGEK model ($\eta = 0.5$): cyan-circle line. Other parameters than η: see table 3.5.

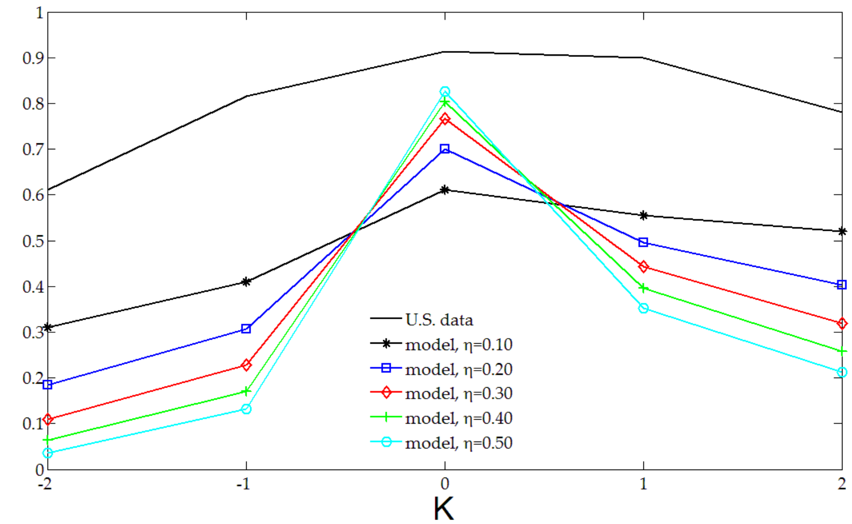

Figure 3.7: Cross-correlation of investment with output at lag K, $K = \{-2,-1,0,1,2\}$ (sensitivity analysis for different values of η)

Note. U.S. data: black line. SIGEK model ($\eta = 0.1$): black-asterisk line. SIGEK model ($\eta = 0.2$): blue-square line. SIGEK model ($\eta = 0.3$): red-diamond line. SIGEK model ($\eta = 0.4$): green-plus line. SIGEK model ($\eta = 0.5$): cyan-circle line. Other parameters than η: see table 3.5.

3.5 Conclusion

This chapter has analyzed the (ir)relevance of micro-level lumpy investment for the business cycle.

In chapter 2 we had shown that lumpy capital adjustment behavior arises naturally when firms face costs of gathering, absorbing, and processing information. In this chapter we have embedded that theoretical framework into the Mankiw and Reis (2006, 2007) SIGE model. Specifically, we have augmented the SIGE model with a set of firms that make capital investment decisions with inattentiveness. In the SIGEK model, as in the original SIGE model, inattentiveness is a pervasive feature of all markets and decisions, and it is the only feature that leads to a deviation from a classical model.

We have found that the model with lumpy investment yields business cycle dynamics (both impulse response functions and second moments) that are significantly different from those of its frictionless investment counterpart. We have also found that the model with pervasive inattentiveness is better at matching business cycle moments than is either a classical model or an otherwise identical SIGEK model with frictionless investment.

The SIGEK model has allowed for addressing how far inattentiveness alone affects macroeconomic dynamics. The findings are promising and confirm the Mankiw and Reis (2006) claim that pervasive inattentiveness is necessary to explain business cycle dynamics in sticky-information models. Moreover, in delivering these results this model has strengthened the case in favor of the relevance of lumpy micro-level investment for the business cycle. Introducing lumpy investment, with a microeconomic foundation based on inattentiveness, in a sticky information general equilibrium model seems to be a fruitful approach for further business cycle and monetary policy analysis.

Appendix G - Technical appendix

Inattentive consumers

Consumers, who revise their plans every period with a probability δ, have a value function $V(A_t)$ conditional on date t being a planning date. Consumers choose a plan for current $(\tau = 0)$ and future $(\tau \geq 1)$ consumption all the way into infinity $\{C_{t+\tau,\tau}\}_{\tau=0}^{\infty}$ considering that with a vanishingly small probability they may never update again:

$$V(A_t) = \max_{\{C_{t+\tau,\tau}\}_{\tau=0}^{\infty}} \left\{ \sum_{\tau=0}^{\infty} \xi^{\tau} (1-\delta)^{\tau} \ln C_{t+\tau,\tau} + \xi\delta \sum_{\tau=0}^{\infty} \xi^{\tau} (1-\delta)^{\tau} E_t \left[V(A_{t+1+\tau}) \right] \right\}$$

$$subject\ to \qquad A_{t+1+\tau} = \Pi_{t+1+\tau} \left(A_{t+\tau} - C_{t+\tau,\tau} + \frac{W_{t+\tau,\tau} L_{t+\tau,\tau} + T_{t+\tau}}{P_{t+\tau}} \right) \ for\ \tau = 0,1,2,...$$

The first term in the Bellman equation equals the expected discounted utility if the consumer never updates her information again. The second term includes the sum of the continuation values over all of the possible future dates at which the agent may plan again, each occurring with a probability $\delta(1-\delta)^{\tau}$. Since preferences are additively separable in consumption and leisure, and since consumers do not control labor supply, then the term in leisure drops out of the problem.

The optimality conditions are:

$$\xi^{\tau}(1-\delta)^{\tau} \tfrac{1}{C_{t+\tau,\tau}} = \xi\delta \textstyle\sum_{k=\tau}^{\infty} \xi^{k} (1-\delta)^{k} E_t \left[V'(A_{t+1+k}) \bar{\Pi}_{t+\tau,t+1+k} \right] \quad for\ \tau = 0,1,2,... \tag{G.1}$$

where

$$\bar{\Pi}_{t+\tau,t+1+k} = \prod_{z=t+\tau}^{t+k} \Pi_{z+1}$$

is the compound return between $t+\tau$ and $t+1+k$.

The envelope theorem condition is:

$$V'(A_t) = \xi\delta \sum_{k=0}^{\infty} \xi^{k} (1-\delta)^{k} E_t \left[V'(A_{t+1+k}) \bar{\Pi}_{t,t+1+k} \right] . \tag{G.2}$$

Combining (G.1) for $\tau = 0$ with (G.2) yields $1/C_{t,0} = V'(A_t)$, that is, the marginal utility of an extra unit of consumption equals the marginal value of an extra unit of wealth. Writing (G.2) recursively and using the last result gives

$$\frac{1}{C_{t,0}} = V'(A_t) = \xi E_t\left[\Pi_{t+1} V'(A_{t+1})\right] = \xi E_t\left[\Pi_{t+1}\frac{1}{C_{t+1,0}}\right] .$$

Then considering (G.1) for $\tau \geq 1$ and (G.2) for date $t+\tau$ yields

$$\frac{1}{C_{t+\tau,\tau}} = V'(A_{t+\tau}) = E_t\left[\frac{1}{C_{t+\tau,0}}\right] \Leftrightarrow \frac{1}{C_{t,\tau}} = E_{t-\tau}\left[\frac{1}{C_{t,0}}\right] .$$

Inattentive workers

Workers, who revise their plans every period with a probability ω, have a value function $\hat{V}(A_t)$ conditional on date t being a planning date. Workers choose a plan for current ($\tau = 0$) and future ($\tau \geq 1$) wage all the way into infinity $\{W_{t+\tau,\tau}\}_{\tau=0}^{\infty}$, considering that with a vanishingly small probability they may never update again:

$$\begin{aligned}\hat{V}(A_t) \quad = \quad & \max_{\{W_{t+\tau,\tau}\}_{\tau=0}^{\infty}} \left\{ -\sum_{\tau=0}^{\infty} \xi^{\tau}(1-\omega)^{\tau} E_t\left[\frac{L_{t+\tau,\tau}^{1+1/\psi}+1}{1+1/\psi}\right] \right. \\ & \left. +\xi\omega\sum_{\tau=0}^{\infty}\xi^{\tau}(1-\omega)^{\tau}E_t\left[\hat{V}(A_{t+1+\tau})\right] \right\}\end{aligned}$$

$$subject\ to \qquad A_{t+1+\tau} = \Pi_{t+1+\tau}\left(A_{t+\tau} - C_{t+\tau,\tau} + \frac{W_{t+\tau,\tau}L_{t+\tau,\tau} + T_{t+\tau}}{P_{t+\tau}}\right)\ for\ \tau = 0,1,2,...$$

$$L_{t+\tau,\tau} = \left(\frac{W_{t+\tau,\tau}}{W_{t+\tau}}\right)^{-\hat{\gamma}_{t+\tau}} N_{t+\tau}\ for\ \tau = 0,1,2,...$$

The first term in the Bellman equation equals the expected discounted utility if the worker never updates her information again. The second term includes the sum of the continuation values over all of the possible future dates at which the agent may plan again, each occurring with a probability $\omega(1-\omega)^{\tau}$.

The optimality conditions are:

$$\begin{gathered}\xi^{\tau}(1-\omega)^{\tau}E_t\left(\hat{\gamma}_{t+\tau}L_{t+\tau,\tau}^{1+1/\psi}\right)/W_{t+\tau,\tau}\\ = \\ \xi\omega\sum_{k=\tau}^{\infty}\xi^{k}(1-\omega)^{k}E_t\left[\hat{V}'(A_{t+1+k})\bar{\Pi}_{t+\tau,t+1+k}(\hat{\gamma}_{t+\tau}-1)L_{t+\tau,\tau}/P_{t+\tau}\right]\\ for\ \tau=0,1,2,...\end{gathered} \tag{G.3}$$

The envelope theorem condition is:

$$\hat{V}'(A_t)=\xi\omega\sum_{k=0}^{\infty}\xi^{k}(1-\omega)^{k}E_t\left[\hat{V}'(A_{t+1+k})\bar{\Pi}_{t,t+1+k}\right] \ . \tag{G.4}$$

Combining (G.3) for $\tau=0$ with (G.4) gives

$$W_{t,0}=\frac{\hat{\gamma}_t}{\hat{\gamma}_t-1}\frac{P_tL_{t,0}^{1/\psi}}{\hat{V}'(A_t)} \ \Rightarrow W_{t+\tau,0}=\frac{\hat{\gamma}_{t+\tau}}{\hat{\gamma}_{t+\tau}-1}\frac{P_{t+\tau}L_{t+\tau,0}^{1/\psi}}{\hat{V}'(A_{t+\tau})}$$

Writing (G.4) recursively yields

$$\hat{V}'(A_t)=\xi E_t\left[\hat{V}'(A_{t+1})\Pi_{t+1}\right] \ .$$

Combining these results implies

$$\frac{\hat{\gamma}_t}{\hat{\gamma}_t-1}\frac{P_tL_{t,0}^{1/\psi}}{W_{t,0}}=\xi E_t\left[\Pi_{t+1}\frac{\hat{\gamma}_{t+1}}{\hat{\gamma}_{t+1}-1}\frac{P_{t+1}L_{t+1,0}^{1/\psi}}{W_{t+1,0}}\right] \ .$$

Condition (G.3) for $\tau\geq 1$ and (G.4) for date $t+\tau$ then imply

$$W_{t,\tau}=\frac{E_{t-\tau}\left[\hat{\gamma}_tL_{t,\tau}^{1+1/\psi}\right]}{E_{t-\tau}\left[\hat{\gamma}_tL_{t,\tau}L_{t,0}^{1/\psi}/W_{t,0}\right]} \ .$$

The log-linear sticky-information equilibrium

Log-linearizing the aggregate resource constraint and policy rules yields:

$$y_t^{FIN}=\alpha_c c_t+\alpha_i inv_t+g_t \ , \tag{G.5}$$

$$r_t = i_t - E_t\left(\Delta p_{t+1}\right) \tag{G.6}$$

and

$$i_t = \phi_\pi \Delta p_t - \varepsilon_t \ . \tag{G.7}$$

From the attentive agents' problems:

$$y_{t,\tau}^{INT} = y_t - \nu\left(p_{t,\tau} - p_t\right) \tag{G.8}$$

and

$$l_{t,\tau} = l_t - \gamma(w_{t,\tau} - w_t) \ . \tag{G.9}$$

From the inattentive consumer's problem:

$$c_{t,\tau} = E_{t-\tau}\left[c_{t+1,0} - r_t\right] \ . \tag{G.10}$$

From the inattentive worker's problem:

$$w_{t,\tau} = E_{t-\tau}\left[p_t + \frac{1}{\psi} l_{t,\tau} - \frac{1}{\gamma - 1}\gamma_t - r_t + w_{t+1,0} - p_{t+1} - \frac{1}{\psi} l_{t+1,0} + \frac{1}{\gamma - 1}\gamma_{t+1}\right] \ . \tag{G.11}$$

From the inattentive pricing department's problem:

$$y_{t,\tau}^{INT} = a_t + \beta l_{t,\tau} \tag{G.12}$$

and

$$p_{t,\tau} = E_{t-\tau}\left[\omega_t - \left(y_{t,\tau}^{INT} - l_{t,\tau}\right) - \frac{\nu_t}{\nu - 1}\right] \ . \tag{G.13}$$

From the inattentive producing department's problem:

$$y_{t,\tau}^{FIN} = z_t + (1-\alpha)\, y_t + \alpha k_{t-1,\tau} \ , \tag{G.14}$$

$$k_{t,\tau} = E_{t-\tau}\left[y_{t+1} - \frac{r}{(r+\rho)(1-\alpha)} r_t + \frac{1}{1-\alpha} z_{t+1}\right] \tag{G.15}$$

and

$$inv_{t,\tau} = \frac{1}{\rho} k_{t,\tau} - \frac{1-\rho}{\rho} k_{t-1,\tau} \ . \tag{G.16}$$

Aggregating (G.12) gives the aggregate intermediate-good production function

$$y_t^{INT} = a_t + \beta l_t \ , \tag{G.17}$$

and the market clearing condition implies that $y_t^{INT} = y_t$.

Aggregating (G.16) and (G.14) over τ gives aggregate investment and the aggregate final-good production function:

$$inv_t = \frac{1}{\rho}k_t - \frac{1-\rho}{\rho}k_{t-1} \tag{G.18}$$

$$y_t^{FIN} = z_t + (1-\alpha)\, y_t + \alpha k_{t-1}\ . \tag{G.19}$$

Finally, the log-linearized definitions of price, wage, consumption and capital indices are:

$$p_t = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau}\, p_{t,\tau} \tag{G.20}$$

$$w_t = \omega \sum_{\tau=0}^{\infty} (1-\omega)^{\tau}\, w_{t,\tau} \tag{G.21}$$

$$c_t = \delta \sum_{\tau=0}^{\infty} (1-\delta)^{\tau}\, c_{t,\tau} \tag{G.22}$$

$$k_t = \eta \sum_{\tau=0}^{\infty} (1-\eta)^{\tau}\, k_{t,\tau} \tag{G.23}$$

This set of 19 equations (G.5 - G.23) provides the competitive equilibrium solution for the set of 19 variables (y_t^{FIN}, $y_{t,\tau}^{FIN}$, y_t, $y_{t,\tau}^{INT}$, y_t^{INT}, c_t, $c_{t,\tau}$, l_t, $l_{t,\tau}$, w_t, $w_{t,\tau}$, p_t, $p_{t,\tau}$, inv_t, $inv_{t,\tau}$, k_t, $k_{t,\tau}$, r_t, i_t) as a function of six exogenous shocks (Δa_t, g_t, ε_t, z_t, v_t and γ_t).

The reduced-form sticky-information equilibrium

Leading (G.19) one period and replacing for y_{t+1} in equation (G.15) gives

$$k_{t,\tau} = E_{t-\tau}\left[\frac{y_{t+1}^{FIN} - \alpha k_t}{1-\alpha} - \frac{r}{(r+\rho)(1-\alpha)}r_t\right]\ .$$

Replacing for $k_{t,\tau}$ in (G.23) gives the expression for aggregate capital stock:

$$k_t = \eta \sum_{\tau=0}^{\infty} (1-\eta)^{\tau} E_{t-\tau}\left[\frac{y_{t+1}^{FIN} - \alpha k_t}{1-\alpha} - \frac{r}{(r+\rho)(1-\alpha)}r_t\right]\ . \tag{G.24}$$

Combining equations (G.8), (G.12) and (G.13) to replace for $p_{t,\tau}$, $l_{t,\tau}$ and $y_{t,\tau}^{INT}$ in equation

(G.20) gives:

$$p_t = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} E_{t-\tau} \left[p_t + \frac{\beta (w_t - p_t) + (1-\beta) y_t - a_t}{\beta + v(1-\beta)} - \frac{\beta}{(v-1)[\beta + v(1-\beta)]} v_t \right] \; .$$

Using (G.19) to replace for y_t in the previous equation gives the aggregate supply relation:

$$\begin{aligned} p_t \;\; = \;\; & \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} E_{t-\tau} \left[p_t + \frac{\beta (w_t - p_t) + \frac{1-\beta}{1-\alpha} \left(y_t^{FIN} - z_t - \alpha k_{t-1} \right) - a_t}{\beta + v(1-\beta)} \right. \\ & \left. - \frac{\beta}{(v-1)[\beta + v(1-\beta)]} v_t \right] \; . \end{aligned} \tag{G.25}$$

Iterating forward on equation (G.10) yields

$$c_{t,\tau} = -\sum_{i=0}^{T} E_{t-\tau}(r_{t+1}) + E_{t-\tau}(c_{t+T+1,0}) \; .$$

Next, take the limit as $T \to \infty$. As time elapses to infinity all agents become aware of past news so $\lim_{i\to\infty} E_t(r_{t+i}) = \lim_{i\to\infty} E_t\left(r_{t+i}^n\right) = 0$. Moreover, since the probability of remaining inattentive falls exponentially with the length of the horizon, one can approach this limit fast enough to ensure that the sum in the first term converges. As for the second term, $\lim_{i\to\infty} E_t(c_{t+i,0}) = \lim_{i\to\infty} E_t\left(c_{t+i}^n\right) = c_t^n$. The first equality holds because consumers are fully insured every period and in the limit all are informed. The second equality holds because c_t^n follows a random walk. Using the definition of the long rate R_t, the expression above becomes:

$$c_{t,\tau} = E_{t-\tau}(c_t^n - R_t) \; .$$

Replacing for $c_{t,\tau}$ in (G.22) gives the IS curve:

$$c_t = \delta \sum_{\tau=0}^{\infty} (1-\delta)^{\tau} E_{t-\tau}(c_t^n - R_t) \; . \tag{G.26}$$

Performing analogous step, iterating forward on (G.11) and using the fact that $\left(w_t^n - p_t^n - \frac{l_t^n}{\psi}\right) = c_t^n$ results in

$$\psi w_{t,\tau} = E_{t-\tau}\left[\psi p_t + l_{t,\tau} - \frac{\psi \gamma_t}{\gamma - 1} - \psi R_t\right] + \psi c_t^n \; .$$

Using (G.9) to replace for $l_{t,\tau}$ and replacing $w_{t,\tau}$ in (G.21) gives the wage curve:

$$\begin{aligned} w_t &= \omega \sum_{\tau=0}^{\infty} (1-\omega)^{\tau} E_{t-\tau} \left[p_t + \frac{\gamma}{\gamma+\psi} (w_t - p_t) + \right. \\ &\left. + \frac{\frac{\left(y_t^{FIN} - z_t - \alpha k_{t-1}\right)}{1-\alpha} - a_t}{\beta(\gamma+\psi)} + \frac{\psi}{(\gamma+\psi)} \left(c_t^n - R_t\right) - \frac{\psi}{(\gamma+\psi)(\gamma-1)} \gamma_t \right] . \end{aligned} \tag{G.27}$$

Equations (G.24)-(G.27), together with aggregate investment (G.18), the aggregate budget constraint (G.5), the Fisher equation (G.6) and the Taylor rule (G.7) characterize the equilibrium for y_t^{FIN}, c_t, w_t, p_t, inv_t, k_t, r_t and i_t given exogenous shocks to Δa_t, ε_t, γ_t, v_t, z_t and g_t.

The classical equilibrium

In the classical economy, $\lambda = \omega = \delta = \eta = 1$ so all agents are attentive. The model collapses into the following system of 9 equations in 9 variables (y_t^{FIN}, y_t , c_t, inv_t, w_t, p_t, k_t, r_t and i_t):

$$c_t = E_t \left[c_{t+1} - r_t \right]$$

$$\beta (w_t - p_t) + (1-\beta) y_t - a_t - \frac{\beta}{(v-1)} v_t = 0$$

$$\psi (w_t - p_t) = \frac{y_t - a_t}{\beta} + \frac{\psi}{\theta} c_t - \frac{\psi}{(\gamma-1)} \gamma_t$$

$$y_t^{FIN} = \alpha_c c_t + \alpha_i inv_t + g_t$$

$$k_t = E_t \left[y_{t+1} - \frac{r}{(r+\rho)(1-\alpha)} r_t + \frac{1}{1-\alpha} z_{t+1} \right]$$

$$inv_t = \frac{1}{\rho} k_t - \frac{1-\rho}{\rho} k_{t-1}$$

$$y_t^{FIN} = z_t + (1-\alpha) y_t + \alpha k_{t-1}$$

$$r_t = i_t - E_t (\Delta p_{t+1})$$

$$i_t = \phi_\pi \Delta p_t - \varepsilon_t \ .$$

Defining:

$$\Omega_1 = 1 - \beta + \frac{1}{\psi} \; ; \; \Omega_2 = \frac{r}{(r+\rho)(1-\alpha)} \; ; \; \Omega_3 = \alpha_c + \frac{(1-\alpha)\beta}{\Omega_1}$$

$$\Omega_4 = -\frac{\alpha_i}{\rho}\left(\Omega_2 + \frac{\beta}{\Omega_1}\right) \; ; \; \Omega_5 = \Omega_3 + \frac{\alpha_i \Omega_2}{\rho} - \left(\Omega_2 + \frac{\beta}{\Omega_1}\right)\left[\alpha - \frac{\alpha_i(1-\rho)}{\rho}\right]$$

$$\Omega_6 = \Omega_2\left[\alpha - \frac{\alpha_i(1-\rho)}{\rho}\right] \; ,$$

then c_t is the solution of the following expectational difference equation:

$$\Omega_4 E_t(c_{t+1}) + \Omega_5 c_t + \Omega_6 c_{t-1} = f\,(a_{t-1}, a_t, \gamma_{t-1}, \gamma_t, v_{t-1}, v_t, g_t, z_t, z_{t-1}) \; .$$

Using the solution for consumption, one can show that the solutions for all the other real variables (y_t^{FIN}, y_t , inv_t, $w_t - p_t$, k_t and r_t) are determined as a function only of the exogenous real shocks (Δa_t, γ_t, v_t, z_t and g_t), independently of the monetary policy shock ε_t. The classical dichotomy holds in this economy. The monetary policy shock determines the nominal interest rate and inflation through the Taylor rule and the Fisher equation.

Table 3.5: Structural parameters

	value	source[a]	description
Preference and production			
β	0.67	RR	return to scale (intermediate-good firms)
ψ	5.15	RR	Frisch elasticity of labor supply
ν	10.09	RR	elasticity of substitution across goods varieties
γ	9.09	RR	elasticity of substitution across labor varieties
α	0.33	FV	share of capital in final-good firm's production function
Nonpolicy shocks			
$\rho_{\Delta a}$	0.03	RR	serial correlation of the intermediate-good productivity shock
$\sigma_{\Delta a}$	0.66	RR	standard deviation of the intermediate-good productivity shock
ρ_g	0.99	RR	serial correlation of the aggregate demand shock
σ_g	0.83	RR	standard deviation of the aggregate demand shock
ρ_ν	0.28	RR	serial correlation of the goods markup shock
σ_ν	1.06	RR	standard deviation of the goods markup shock
ρ_γ	0.86	RR	serial correlation of the labor markup shock
σ_γ	1.23	RR	standard deviation of the labor markup shock
ρ_z	0.75	FV	serial correlation of the final-good productivity shock
σ_z	0.5	FV	standard deviation of the final-good productivity shock

Table 3.5: Structural parameters (continue)

	value	source[a]	description
Monetary Policy			
ρ_ε	0.29	RR	serial correlation of the monetary policy shock
σ_ε	0.44	RR	standard deviation of the monetary policy shock
ϕ_π	1.17	RR	interest rate response to inflation
Inattentiveness			
δ	0.08	RR	fraction of consumers updating information every quarter
ω	0.74	RR	fraction of workers updating information every quarter
λ	0.52	RR	fraction of intermediate-good firms updating information every quarter
η	0.1	FV	fraction of final-good firms updating information every quarter
Others			
α_c	0.85	FV	steady-state share of consumption in GDP
α_i	0.15	FV	steady-state share of investment in GDP
ρ	0.035	FV	steady-state real depreciation rate (quarterly)
r	0.01	FV	steady-state real interest rate (quarterly)

Note. [a] RR: Reis (2009b, table 2). FV: our calibration.

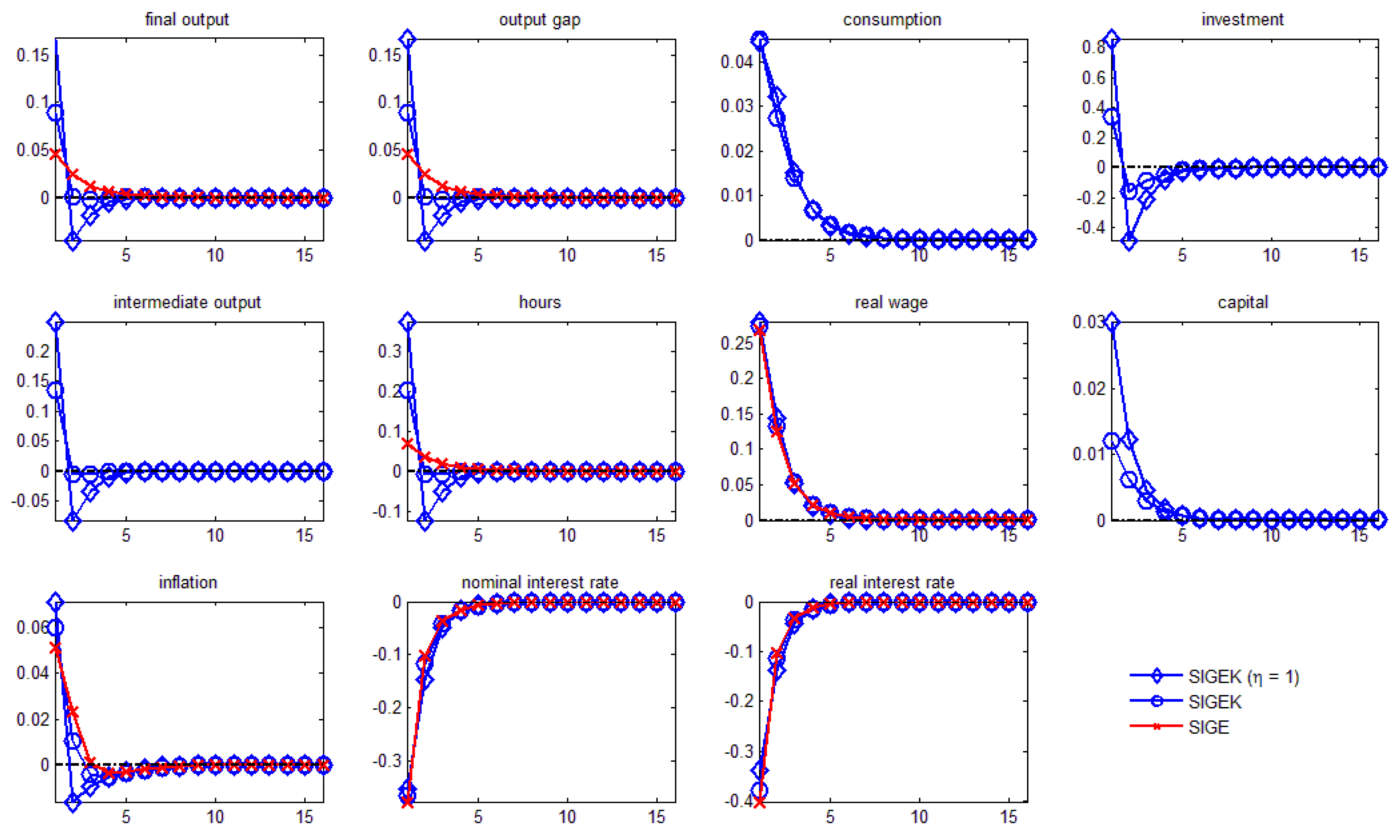

Figure 3.8: Impulse response functions to a monetary policy shock

Note. Values expressed as percentage deviation from steady state. SIGEK model with pervasive inattentiveness: blue-circle line. SIGEK model with frictionless investment ($\eta = 1$): blue-diamond line. SIGE model: red-cross line. Steady state: black dashed-dotted line. Baseline parameters: see table 3.5.

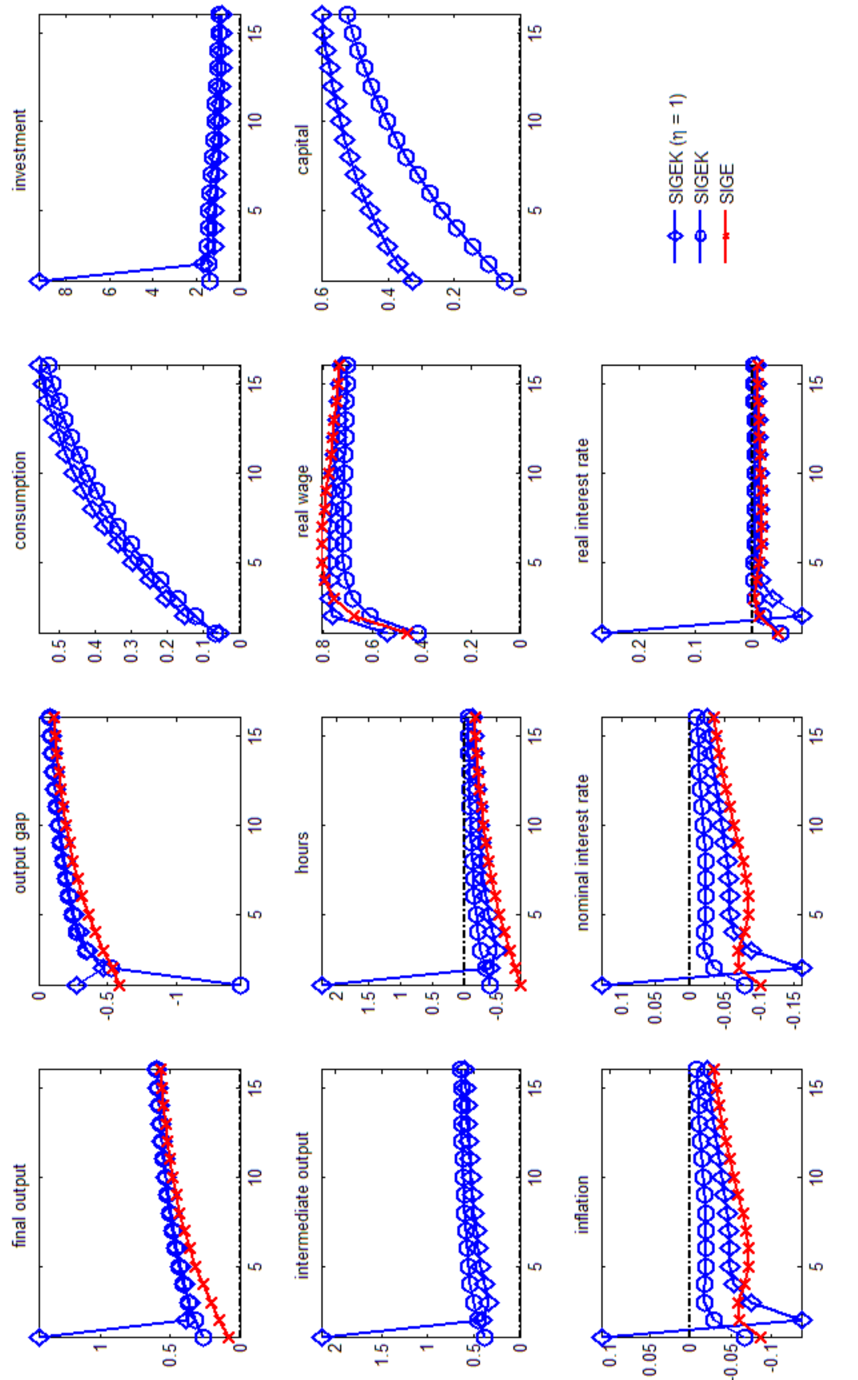

Figure 3.9: Impulse response functions to an intermediate-good productivity shock

Note. Values expressed as percentage deviation from steady state. SIGEK model with pervasive inattentiveness: blue-circle line. SIGEK model with frictionless investment ($\eta = 1$): blue-diamond line. SIGE model: red-cross line. Steady state: black dashed-dotted line. Baseline parameters: see table 3.5.

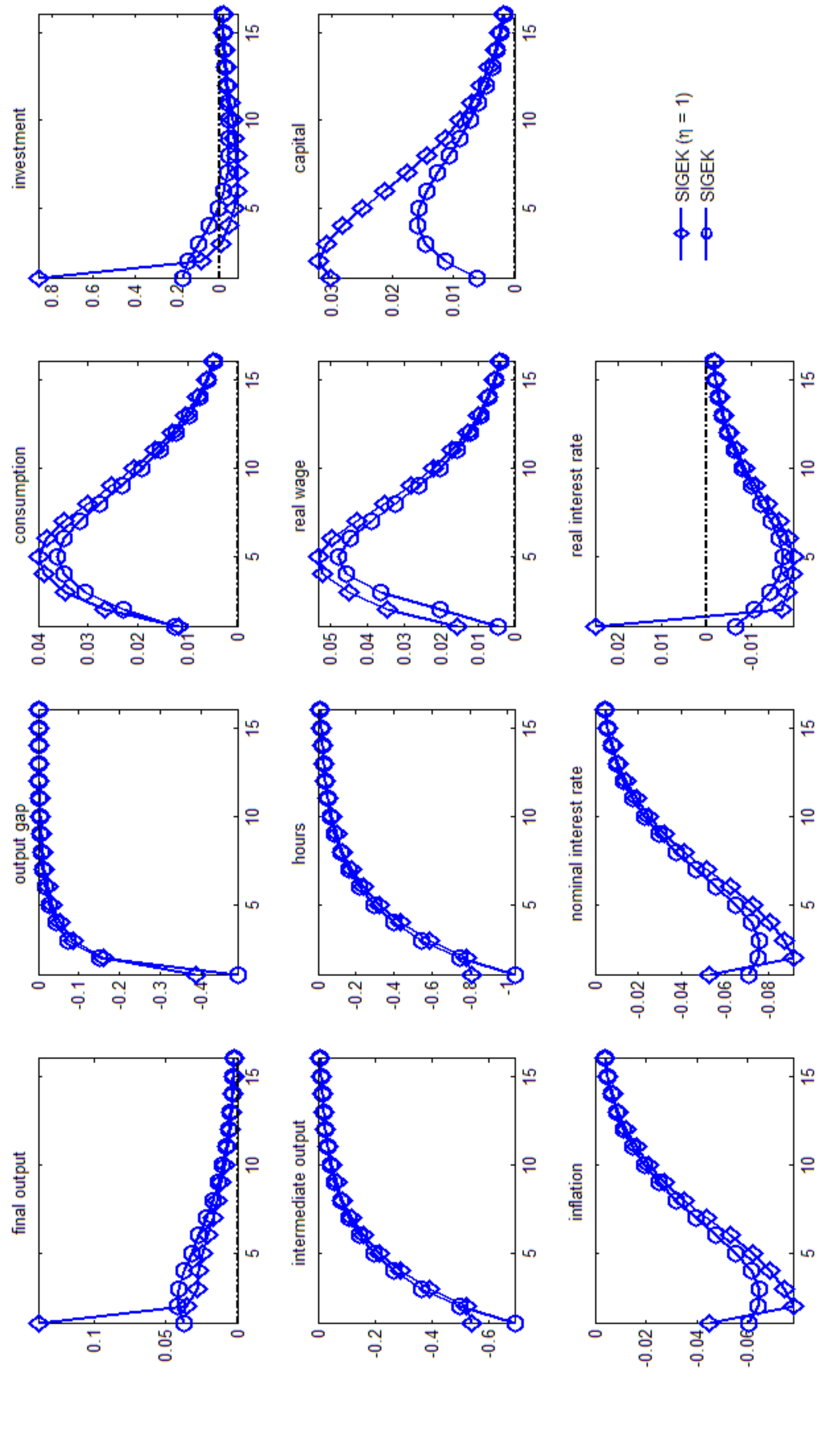

Figure 3.10: Impulse response functions to a final-good productivity shock

Note. Values expressed as percentage deviation from steady state. SIGEK model with pervasive inattentiveness: blue-circle line. SIGEK model with frictionless investment ($\eta = 1$): blue-diamond line. Steady state: black dashed-dotted line. Baseline parameters: see table 3.5.

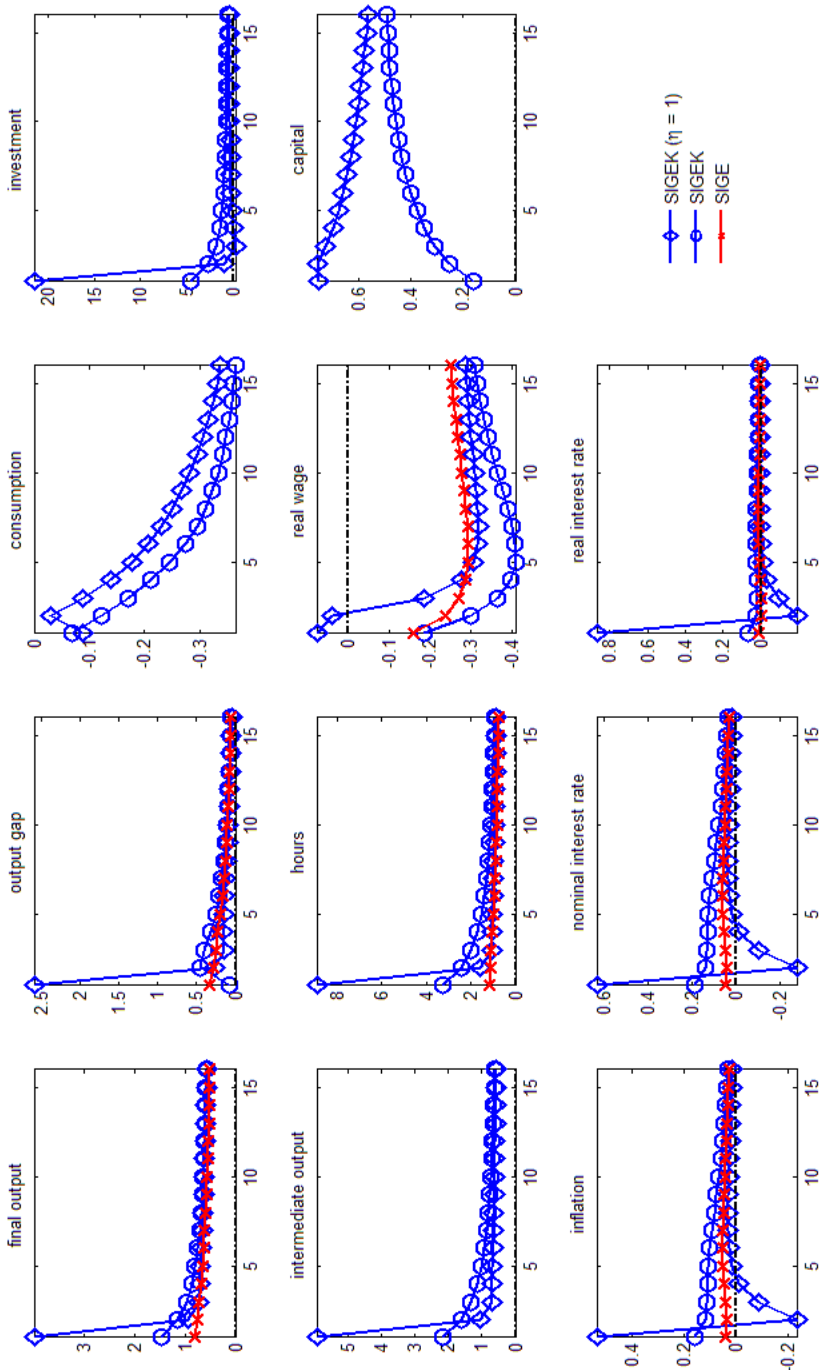

Figure 3.11: Impulse response functions to a demand shock

Note. Values expressed as percentage deviation from steady state. SIGEK model with pervasive inattentiveness: blue-circle line. SIGEK model with frictionless investment ($\eta = 1$): blue-diamond line. SIGE model: red-cross line. Steady state: black dashed-dotted line. Baseline parameters: see table 3.5.

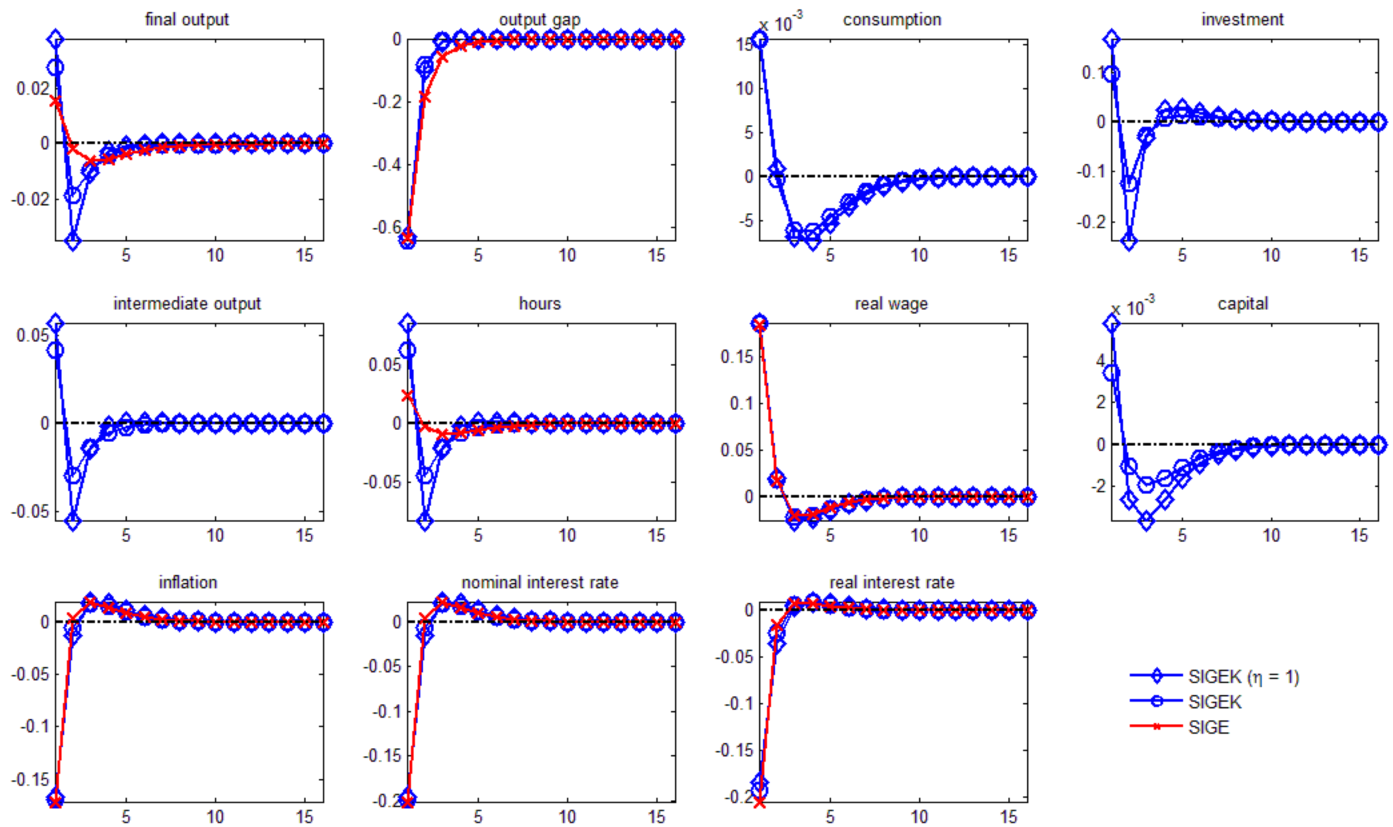

Figure 3.12: Impulse response functions to an intermediate-good markup shock

Note. Values expressed as percentage deviation from steady state. SIGEK model with pervasive inattentiveness: blue-circle line. SIGEK model with frictionless investment ($\eta = 1$): blue-diamond line. SIGE model: red-cross line. Steady state: black dashed-dotted line. Baseline parameters: see table 3.5.

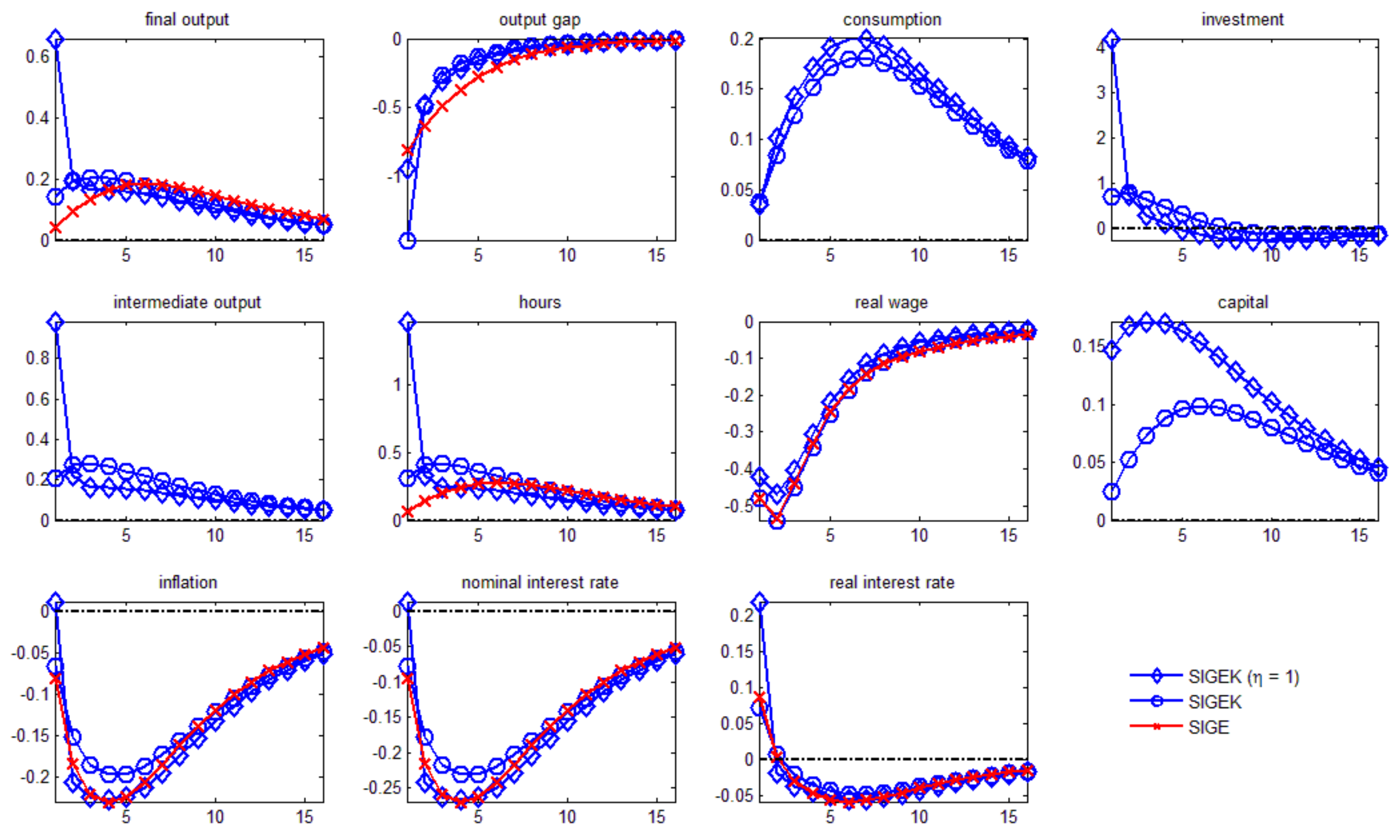

Figure 3.13: Impulse response functions to a wage markup shock

Note. Values expressed as percentage deviation from steady state. SIGEK model with pervasive inattentiveness: blue-circle line. SIGEK model with frictionless investment ($\eta = 1$): blue-diamond line. SIGE model: red-cross line. Steady state: black dashed-dotted line. Baseline parameters: see table 3.5.

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