Entrepreneurial Finance and Economic Growth in the Presence of Trade Unions

by

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Biography

Luís Carlos Monteiro Silva was born in Guimarães, in the 9th of December of 1993. In July of 2015, he concluded his barchelor degree on Economics in Minho University. Currently he is attending the 2^{nd}year of the Master in Management in the Faculty of Economics of Porto. in which he obtained an average of 16 out of 20 in the curricular part.

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Aknowledgments

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Abstract

This study aims to build a novel setup combining the process of entrepreneurial finance (i.e. the process of search and matching between entrepreneurs and capitalists), with the standard monopoly union framework, within a general endogenous growth model.

The idea motivating this research can be described as follows. One the one hand, entrepreneurs need to raise capital for their ventures; on the other hand, capitalists are the ones who provide that capital to the projects they think are the most valuable. Hence, through the process of matching entrepreneurs with capitalists, trade unions might influence this relationship by raising (lowering) wages and, thus, increasing (decreasing) costs and reducing (increasing) profitability.

This study follows the standard endogenous growth approach where the economy is composed by three sectors: (1) a final good sector in which the good is produced competitively employing unionized labour, to whom is paid a wage according to the monopoly union model; (2) an intermediate good sector, in which each intermediate good is produced and sold by a monopolistic firm; and (3) a R&D sector, which employs entrepreneurs and capitalists to introduce new varieties of intermediate goods. The innovations selected by capitalists in the R&D sector to be financed are then patented and sold to the intermediate good firms that monopolize the market for that particular product. Trade unions influence the wages of final good sector workers.

This type of framework allows to study the relationship between unions, entrepreneurs and capitalists through an endogenous growth perspective. In the end, it aims to provide some highlights on this relationship as well as on its impacts on growth, allocation of work between sectors and innovation.

Keywords: Financing of innovation; Trade unions; Endogenous growth

JEL-Codes: 031; J51; Q56

Resumo

Este estudo tem como objetivo construir um modelo que combine o processo de financiamento dos empreendedores (ou seja, o processo de correspondência entre empreendedores e capitalistas), com o modelo do sindicato monopolista, dentro de um modelo de crescimento endógeno geral.

A ideia que motiva este estudo pode ser descrita a seguir. Por um lado, os empreendedores precisam de obter capital para seus empreendimentos; por outro lado, os capitalistas fornecem esse capital aos projetos que analisam como sendo os mais valiosos. Assim, através deste processo de correspondência entre empreendedores e capitalistas, os sindicatos podem influenciar esta relação ao aumentar (diminuir) os salários e, assim, aumentar (diminuir) os custos e reduzir (aumentar) a lucro empresarial.

Este estudo segue a abordagem padrão do crescimento endógeno, no qual a economia é composta por três setores: (1) um setor final, no qual o bem é produzido de forma competitiva empregando mão-de-obra sindicalizada, a quem é pago um salário de acordo com o modelo do sindicato monopolista; (2) um setor intermediário, no qual cada bem intermediário é produzido e vendido por uma empresa monopolista; e (3) um setor de I&D, que emprega empreendedores e capitalistas para introduzir novas variedades de bens intermediários. As inovações selecionadas para serem finanicadas pelos capitalistas no setor de I&D são então patenteadas e vendidas para as empresas intermediárias que monopolizam o mercado para essa variedade em particular. Os sindicatos influenciam os salários dos trabalhadores do setor final.

Este modelo permite estudar a relação entre sindicatos, empreendedores e capitalistas através de uma perspectiva de crescimento endógeno. No final, é pretendido obter conclusões sobre esta relações, bem como sobre seus impactos no crescimento, alocação de trabalho entre setores e inovação.

Palavras-chave: Financiamento da inovação; Sindicatos; Crescimento endógeno

Códigos JEL: 031; J51; Q56

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1 Introduction

The process of the entrepreneurs "creating" innovation typically faces multiple obstacles that several times drive them to failure. One of the major problems is the difficulty that entrepreneurs find in gathering enough capital to finance their ideas, i.e., the problem of financing the innovation.

This dissertation focuses on how trade unions affect the interaction between entrepreneurs looking for capital and capitalists who provide it. Trade unions might influence this relationship between these two agents by affecting wages and, consequently, changing the willingness of entrepreneurs to invest time in coming up with new ideas and the decision of capitalists to finance them.

Indeed, evidence shows that unions can influence the level of wages and the general conditions for their members, even if we take into account that their membership has been declining over the past years.[1] For instance, a study found for UK that the union wage premium – the gap between the hourly earnings of union members and non-members – was 16% in the public sector and 7% in the private sector.[2] Moreover, their actions are also of a major importance in bringing media attention to new legislation that can potentially harm workers. For example, trade unions have recently taken actions in UK to increase "workers voice" in companies[3], whereas in France they are raising awareness to Macron's liberal agenda, a debate which would be difficult to have without them.[4]

Hence, we propose a novel novel setup combining the process of entrepreneurial finance, combining the contribution of Giordani (2015) with the standard monopoly union framework. Our main conclusions can be summarized as follows: (a) a higher trade union's markup corresponds to a higher (lower) level of employment in the final good (entrepreneur and capitalist) sector(s); (b) it is possible to infer that an a higher bargaining power of entrepreneurs effects the effectiveness of trade union's markup of reallocating

[1]The Guardian view on trade unions: needed as much as ever. (2016, December). Retrieved from: https://www.theguardian.com/commentisfree/2016/dec/19/the-guardian-view-on-trade-unions-needed-as-much-as-ever

[2]Holmes, R. (2017, February). With women at the forefront, the unions are relevant again. Retrieved from: https://www.theguardian.com/commentisfree/2017/feb/22/women-trade-unions-gender-pay-gap-discrimination

[3]O'Connor, S. (2017, July). Taylor report to call for more 'voice' for UK workers. Retrieved from: https://www.ft.com/content/422b3c9a-6248-11e7-91a7-502f7ee26895?mhq5j=e1

[4]Stothard, M. (2017, May). French unions move quickly to resist Macron's labour reforms. Retrieved from: https://www.ft.com/content/5f80f0ae-34a9-11e7-99bd-13beb0903fa3?mhq5j=e1

individuals from being an entrepreneur to a worker in the final good sector.

The rest of this dissertation proceeds as follows. Section 2 provides the first bibliometric exercise on the topic. Section 3 presents and explains the proposed theoretical model, following the contributions of Giordani (2015) and Neto et. al. (2017). Section 4 analyzes the model under the balanced growth path and provides some comparative statistics of the model. Finally, Section 5 concludes.

2 Literature Review

This study is broadly related with the literature regarding entrepreneurship, innovation, trade unions and economic growth.

In the first section, we develop a bibliometric analysis of the literature in which it is explained the criteria used to choose the articles that were firstly analyzed.

Next, it will be presented the complete literature review. The concept of entrepreneurship is clarified, followed by the literature regarding financing of entrepreneurship activities (financing of innovation) and the effects of entrepreneurship in employment, productivity and economic growth.

In the third section, trade unions are introduced and it is discussed their role on wages and labour as well as their effects on economic growth.

Finally, in the fourth and final subsection, some concluding remarks are presented regarding the reviewed literature, and it is explained the gap that exists in the literature that we are proposing to explore in this study.

2.1 Bibliometric Analysis

2.1.1 Introduction

This chapter discusses how the literature reviewed was selected for this study and the details on the search procedure are shown and discussed in detail. The section closes with some statistics of the selected sample of studies.

2.1.2 Initial Search - Sample Search Procedure

The initial search for articles was based on a review of the abstracts from articles published in all economic journals gathered from the Scopus database. The first terms from 1993 onwards introduced in the Scopus database were “trade unions” and “financing of innovation”, using the subject/keywords search procedure. However, no articles relating these topics were found.

Consequently, the strategy changed and the terms "entrepreneur" and "endogenous growth" were used as the search keywords. Once again the search was done for articles from 1993 and limited only to subject/keywords to minimize the number of non-related articles. This search resulted in a total number of 93 papers. These papers were analyzed individually and the articles that did not specifically treat the entrepreneurship-endogenous growth question were disqualified from the bibliometric exercise. In the end, 39 papers remained.

A similar exercise was done in Scopus using the terms "trade unions" and "economic growth" from 1993. This search resulted a total of 89 papers. After disqualification of non-relevant articles, 33 papers remained to be analyzed.

2.1.3 Main Findings

Following Neto and Silva (2013) all the publications that remained in this research were categorized in terms of methodology, namely within five possible alternatives:

. Formal (F): if the main purpose of the paper is to build a theoretical, abstract model/argument attempting to explain the relationship between entrepreneurship and economic growth;

. Empirical (E): if an econometric and/or statistical analysis is applied;

. Formal and Empirical (F+E): if there is a theoretical model/argument and an econometric/statistical application of the model/argument;

. Appreciative (AP): if the paper develops an argumentative analysis without any abstract model or econometric analysis;

. Survey (S): if a literature review or an appraisal is presented.

2.1.4 Main Findings regarding entrepreneurship-growth research

Regarding the link between entrepreneurship and growth, Figure 1 shows that the categories "empirical" and "formal" account for about 66% of the papers analyzed. Nevertheless, the categories "formal and empirical" and "appreciative" represent an important part with, respectively, 18% and 16%. Finally, the remaining category, "survey", has a residual role with only 5% of the research.

Considering the evolution pattern over time, represented in Figure 5, it seems that the categories "formal" and "empirical" have dominated the method of entrepreneurship-growth research for most of the period analyzed. These categories are not only represented among others in 2000 and 2007. "Formal and empirical" and "appreciative" are represented in the years 2004, 2007, 2008, 2009, 2013, 2015 and 2016. The category "survey" is represented in the years 2000 and 2008. We can conclude that, taking into account the number of papers of each category, there is a balenced representation through the considered time period.

Regarding the number of the relevant articles regarding the relationship between entrepreneurship and growth and the total gathered number of articles (represented in Figure 5 by a black line with the percentage values on the right vertical axis), it is

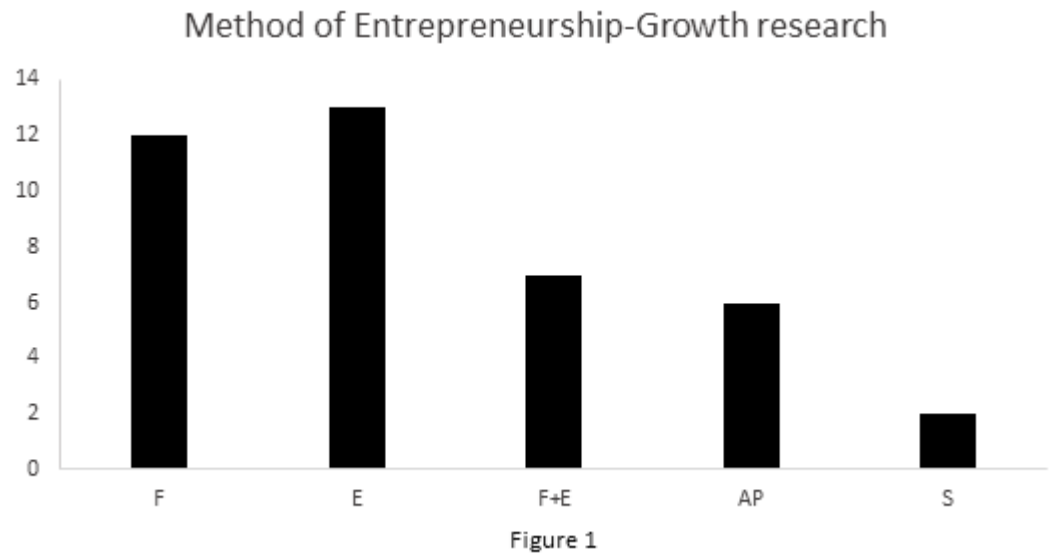

Figure 1: Method of entrepreneurship-growth research

possible to track the evolution of the published papers on entrepreneurship-growth research. On average, approximately 42% of the articles that present "entrepreneurship" and "endogenous growth" as keywords are directly related with the entrepreneurship-growth relationship. From Figure 2, it is possible to verify that the percentage of papers related with the research was very inconsistent in the first five years of analysis. Note that, in 2000 it was 100%, while in 2001 and 2002 it was 0%. After these two years, the percentage increased from 2003 to 2005, when it reached 100%. From 2005 onwards it followed a decreasing pattern that oscillated between 30% and 70% until 2014, when the percentage of entrepreneurship-growth related research was 0%. In the following years, the percentage was consistent and around 60%. [5]

2.1.5 Main Findings regarding trade unions-growth research

Regarding the link between trade unions and growth, Figure 4 shows that the categories "formal", "empirical" and "appreciative" account for 87% of the papers analyzed, representing "24%, 30% and 27%, respectively. The category "formal and empirical" is the fourth most important with 12% of the trade unions-economic growth related research.

[5]For presentation issues the results between the years 1993-1999 were excluded from Figure 5 In this period only two papers were found, one in 1993 and another in 1994.

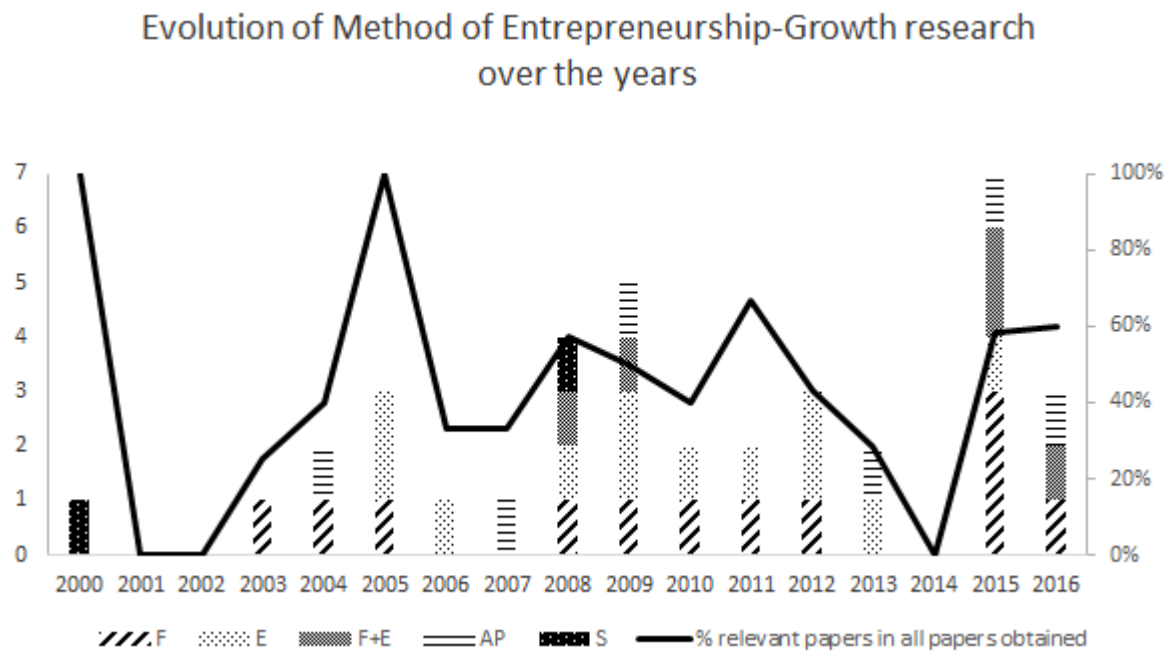

Figure 2: Evolution of method of entrepreneurship-growth research over the years

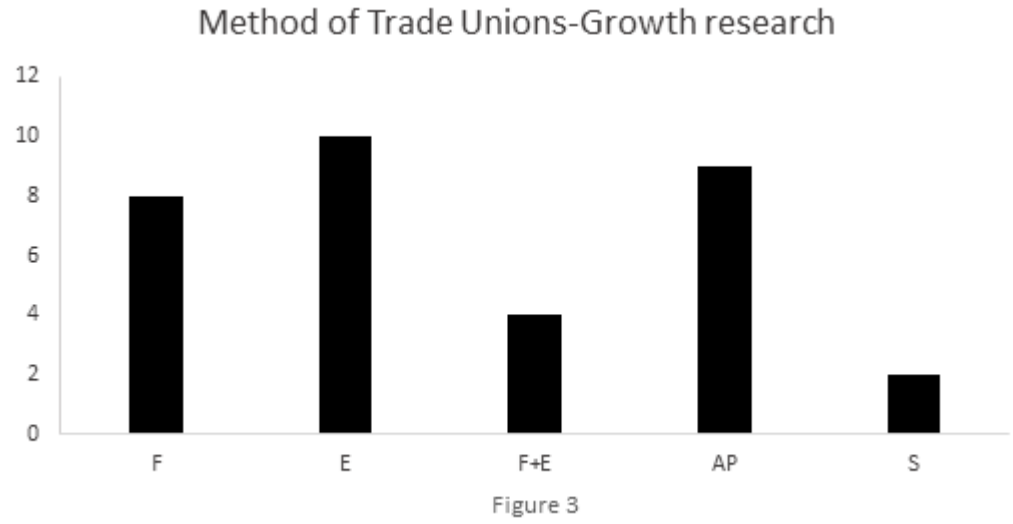

Figure 3: Method of unions-growth research

Considering the evolution pattern over time, represented in Figure 7, it seems that the categories "formal", "empirical" and "appreciative" are relatively evenly distributed, being at least one of them represented in all the years with representation of some category, with the exception of 2012. "Formal and Empirical" increased its weight in the literature after the first decade of the century, being represented in the years 2010, 2011 and 2012. Besides these years, its also represented in 2000. "Survey" is represented in distanced moments in time in 2008 and 2014.

Taking into consideration the number of the relevant articles regarding the relationship between trade unions and growth and the total gathered number of articles, it is possible to track the evolution of the published papers on trade unions-growth research. On average, approximately 37% of the articles that present "trade unions" and "economic growth" as keywords are directly related with the trade unions-growth relationship. From Figure 4 it is possible to identify two different phases in the evolution pattern. The first one between 2000 and 2003 is very inconsistent. The years 2000 and 2002 have a percentage of 100% in the relation between total gathered number of articles and the relevant ones and 2001 and 2003 have 0%. The second phase is between 2004 and 2016 and its more regular than the previous one. The percentages of total/relevant papers range from approximately 20% to 75%, reaching peak in 2015. [6]

[6]For presentation issues the results between the years 1993-1999 were excluded from Figure 7. In this period only one paper was found, in 1993.

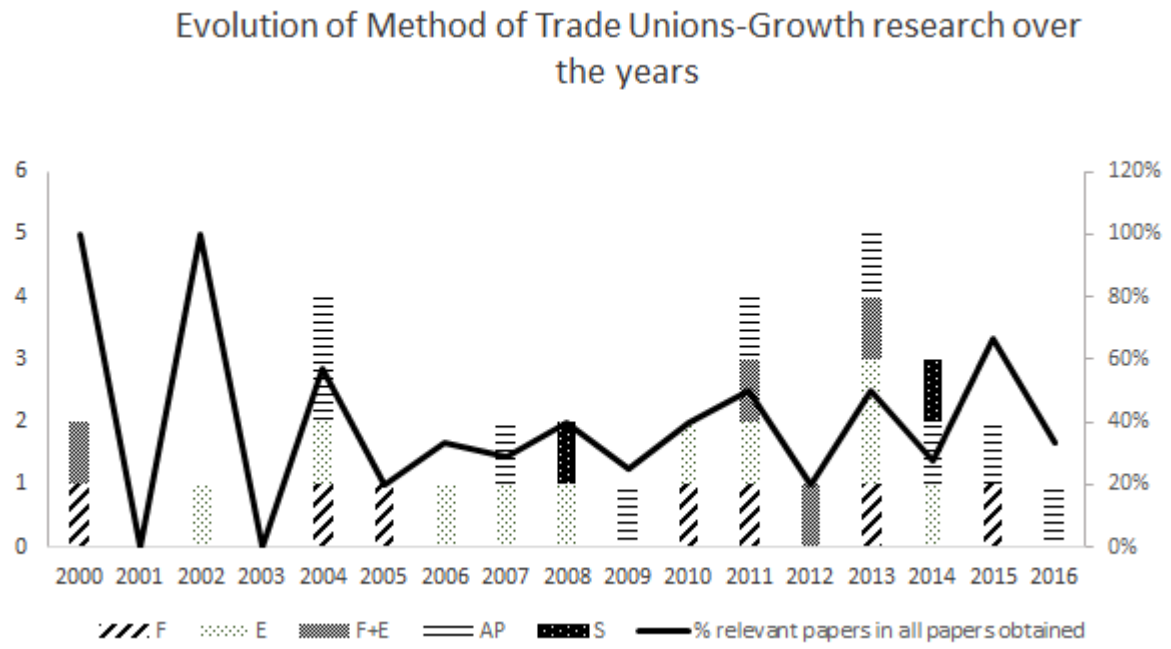

Figure 4: Evolution of method of unions-growth research over the years

2.1.6 Final search

It is worth-noting that the initial research presented on the previous search was complemented through the use of others databases (e.g., B-on), as well as with other research methods, namely cross-reference checking and existing surveys on the topic. Hence, the potential errors and lack of information in the sample of papers used in previous literature review reduced.

2.2 Literature relating Entrepreneurship, Finance and Economic Growth

2.2.1 Entrepreneurship

An entrepreneur can be defined as "individual who pursues opportunities for financial or social gain, often at great financial risk".[7]

Empirically, van Praag and Cramer (2001) states two definitions that need to be taken into account for a firm to be considered entrepreneurial: (a) size; and (b) age. Regarding the former, for personnel, the most often used measure, the common cutoff point is 100 employees. However, various size classes are often observed and analyzed separately, such as 10–20, 20–50, 50–100, 100+ employees. In such cases, again 100 employees is used as the boundary between small and large. In what relates with the latter, most literature refers to entrepreneurial firms as with less than 5 to 7 years.

Entrepreneurship ability is very important to a nation's economic development. Knowledge by itself is only a necessary condition for the exercise of successful enterprise. The ability to transform new knowledge into economic opportunities involves a set of skills, aptitudes, insights, and circumstances that is neither uniformly nor widely distributed in the population (Braunerhjelm et al, 2009). In this sense, the entrepreneurship ability is extremely valued in society.

Schumpeter (1947) stresses the distinction between the inventor and the entrepreneur. In Schumpeter's view the inventor produces ideas, while the entrepreneur transforms ideas into products. An idea or scientific principle is not, by itself, of any importance for economic practice. Hence, to grow, an economy requires both researchers who produce inventions and entrepreneurs who implement them. In other words, scientific knowledge has no economic impact unless some effort is made to spread and apply it (Michelacci, 2003).

If agents are heterogeneous in the ability to come up with ideas and extract their returns, might be the case that some should specialize themselves in innovation and others in implementation. The transfer of ideas from innovators to entrepreneurs can lead to a more efficient use of resources, making all parties better off, therefore increasing

[7]Definition of entrepreneur (20167, July). Retrieved from: http://lexicon.ft.com/Term?term=entrepreneur

incentives to investment in research and revelopment (Silveira et al, 2010).

2.2.2 Financing of Innovation

As stated in the introduction, this study has a particular focus on the process of financing entrepreneurs, who have new business ideas and need to gather capital to finance them.

Assymetries of information between entrepreneurs and capitalists represent a serious constraint in the market of innovation. According to Antony et al (2011), smaller firms are relatively more innovative but face more obstacles in accessing the necessary financial leverage. A similar result was obtained by Gertler (1988), Devereux and Schiantarelli (1990), Cabral and Mata (2003) and Beck et al. (2006): younger firms are more "credit constraint" than mature ones. Mata (1996) finds that the biggest part of young firms, which typically are of small size, are established below their desired size. This is mainly due to financial constraints. These results are in line with the results of Bernanke et al. (1994), who finds that smaller firms must provide more collateral than larger ones.

Wagenvoort (2003) and Angelini and Generale (2008) found that, for Europe and United States, respectively, smaller firms are more credit constrained. Indeed, several obstacles arise during this process and theoretical arguments proposed to explain financial market imperfections in the innovation sector range from transaction costs and tax advantages to agency costs due to informational asymmetries between the innovator and the financer (Giordani, 2014). These aspects are common to any financing relation, but for a number of factors, frictions are even higher in the financing of innovative ventures.

The literature regarding the subject highlights three main reasons that make financing innovation harder than other financing processes.

The first problem associated with capitalists providing capital to entrepreneurs is that a specific innovation is a unique event, and the process of producing them is uncertain and largely unpredictable (Cozzi and Giordani, 2011). Therefore, the risk is bigger in this type of loans, which makes the agreement between the financer and entrepreneur more expensive to the entrepreneur and riskier to the capitalist.

The second reason relates with the fact that most of the R&D expenditure is likely to be less tangible and might not serve as a good collateral for external finance (Almeida and Campello, 2007). Bankruptcy costs are likely to be relatively low for firms with a high proportion of tangible capital among their assets, particularly property and equipment

associated with generally applicable technologies, while these costs are likely to be higher for innovative firms with a higher proportion of intangible assets, such as knowledge and reputation, and with more specialized equipment. For a given level of debt, the risk of bankruptcy may also be higher. Both factors suggest that more innovative firms are likely to be less reliant on debt finance, to minimize expected bankruptcy costs (Aghion et al, 2004). Once again, a higher risk of credit default of innovators leads to a lower probability of an agreement between the parts.

Finally, a quality-signaling strategy, which could be used to decrease the problem of asymmetric information between the entrepreneur and the financier and attenuate his notion of risk, is hardly implementable in the market of innovation because entrepreneurs may be reluctant to signal the quality of their own project, as they may reasonably fear that competitors copy their new idea (Bhattacharya and Ritter 1983).

Because of this problem, the relationship between investors and entrepreneurs have several agency costs and informational asymmetries that other types of financing relations usually do not have.

Several authors investigated other types of problems that this asymmetric information issue can bring into this relationship. For instance, Myers and Majluf (1984) point to dilution costs of issuing outside equity when managers are better informed than outside investors about the firm's financial prospects. Specifically, by selling equity to outside investors, the firm's current owners may send a signal to the markets that its future prospects of revenue are not as good, otherwise they would have chosen instead to keep being full residual claimant on the firm's profits (e.g., by issuing debt, rather than equity). This problem of signaling leads to new share issues being underpriced, which imposes a dilution cost on the firm's initial owners. Hence, it is likely that for more innovative firms there will be a greater degree of asymmetric information between the capital markets and the firm's management, increasing these dilution costs that will tend to be even higher. In this case, new equity will be a particularly expensive source of finance for these firms. On the other hand, more innovative firms are also likely to generate more attractive investment opportunities than firms that are less innovative. If so, innovative firms are also likely to be more reliant on external finance from either debt or equity than less innovative firms. Less innovative firms are more likely to have sufficient internal funds to finance all the investment expenditures which they want to pursue. Myers and

Majluf (1984), with their "pecking order theory of capital structure", suggest that more innovative firms are likely to be more reliant on external sources of funds, but are likely to favor debt over new equity among external sources, to avoid these relatively high dilution costs.

The literature on this subject is very clear in stating that financing innovation is a very complex activity that depends on several factors, requiring a deep analysis by both capitalists and entrepreneurs.

Aghion et al. (2004) analyzed the financial structure of R&D conducting firms. Using a firm level data set for the U.K., the authors found that the share of traditional debt finance in the firms' balance sheets presents an inverted U-shaped form. They state that firms that report positive but low R&D investments use more debt finance than firms that report no R&D, but the use of debt finance falls with R&D intensity among those firms that report R&D activities. They also found that firms that report R&D are more likely to raise funds by issuing shares than firms that report no R&D, and this probability increases with R&D intensity. The authors argue that the reason for this is the higher intangibility of innovative activity which puts a limit on traditional bank lending (the same results mentioned above by Almeida and Campello, 2004).

Scellato (2007), through the analysis of the Italian manufacturing sector, find out that the total number of patents granted per year to an establishment is significantly negatively influenced by the degree of financial constraints the firm is faced with.

Besides all these factors, there are also external conditions that influence the process of financing by entrepreneurs. For instance, the current state of the financial markets is a major determinant in the access to credit. The venture capital industry is highly cyclical, with periodic changes in supply and demand conditions (Gompers and Lerner, 1999; Lerner, 2002).

Gompers and Lerner (2000) find a positive relation between the valuation of new ventures and capital inflows, implying that increases in the supply of venture capital may result in greater competition to finance companies and rising valuations.

This fact suggests that the rate of innovation and entrepreneurship ventures depends on the current conditions of the markets and economy. But the opposite is also true, economic growth is also deeply influenced by the level of innovation and new companies that arise in the economic landscape. The ability to go on entrepreneurship ventures is

important to determine the growth rate of an economy and the role that financers assume is widely discussed in the literature.

In traditional development economics, there were two schools of thought with sharply differing perspectives on the potential importance of finance in economic growth.

One one side, Robinson (1952) states that finance is essentially the handmaiden to industry, responding passively to other factors that produce cross-country differences in growth. According to this author, economies with growth prospects effectively promote supporting institutions such as banks and venture capitalists, to provide external funds to make these systems more efficient. In addition, where economic growth takes place, financial intermediaries follow. Robinson views the causality between financial development and economic growth as reversed, where the economic growth the prompter of financial development. Several economists agree with Robinson's view, including Robert Lucas (1988), who feels that the role of finance is over-emphasized in the literature concerning the determinants of the process of economic growth.

On the other side, economists such as McKinnon (1973) and Shaw (1973) saw financial markets as playing a key role in economic activity. In their way of thinking, differences in the quantity and quality of services provided by financial institutions could partly explain why countries grew at different rates. Goldsmith (1969, p. 400), states that financial development "accelerates economic growth and improves economic performance to the extent that it facilitates the migration of funds to the best user, i.e., to the place in the economic system where the funds will earn the highest social return". Also, Schumpeter (1969) states that financial intermediaries play an important role in promoting innovations. He argues that economies with more developed and efficient forms of financial systems tend to grow more quickly than those that do not. The financial sector provides services which include reallocating capital to the most efficient activities and highest returns without a high risk of loss through adverse selection, moral hazard or transaction costs and are seen as an essential catalyst to economic growth (Rajan and Zingales, 1998).

King et al (1993) states that better financial systems improve the probability of successful innovation and consequently accelerate the process of economic growth. Similarly, financial sector distortions reduce the rate of economic growth by their negative effect on the rate of innovation. The same author also states that a more developed

financial system enhances productivity improvement by (a) choosing higher quality entrepreneurs and business projects; (b) mobilizing more effectively external financing for these entrepreneurs; (c) providing superior vehicles for diversifying the risk of innovative activities; and (d) revealing more accurately the potentially large profits associated with the uncertain business of innovation. Hence, better financial systems stimulate economic growth by accelerating the rate of productivity enhancement. Laeven et al (2013) states that financial development leads to higher rates of growth in the capital stock, income, and productivity. This happens because if capital is redirected away from the less productive investment opportunities in the economy toward more productive ones, productivity will go up and the economy's output will rise further.

Chiu at al. (2017) states that the generation of new ideas and their implementation are the major factor underlying economic performance and growth, and financial development plays an important role in this process. Government policies toward financial systems may also have an important causal effect on long-run growth. Without financial innovations that enhance the screening of entrepreneurs, economic growth will slow (Laeven et al, 2013).

To sum up, recent studies have employed time-series, cross-country and panel analysis to take stands in the debate, but overall, while economists agree that financial development and economic growth are related, they disagree on the direction of causation (Masih et al, 2011).

2.2.3 How Entrepreneurship affects Innovation, Employment, Productivity and Economic Growth

Contributions to Innovation

To measure innovation of a firm or country, researchers normally use three different measures: R&D expenditure, number of patents produced, and number of new products/technologies introduced in the market.

Castany et al. (2005) compares the average R&D expenditure per employee of large and small Spanish firms (being the cut-off point established at 200 employees), between 1990 and 1994, and finds that large firms have allocated around 2.5 times more resources to research than small firms.

In contrast to this result, Arvanitis (1997) finds identical levels of R&D expenditure per employee for the largest part of the Swiss firms with less than 300 employees. For firms with more than 300 employees, the proportional R&D expenditure per employee is decreasing with firm size.

Yang and Huang (2005) finds evidence that R&D expenditures induces higher growth rates, especially for small firms. This implies that each dollar spent on research in a small firm is more valuable than a dollar spent in a large firm and justifies that, proporcionally, small firms engage in more R&D than large ones.

Regarding the number of patents, there are two streams of thought in the literature.

On one hand, Almeida and Kogut (1997) finds that entrepreneurs produce fewer patents than established firms in the semiconductor industry in the US. In the same line, Sørensen and Stuart (2000) provides evidence that the time between patent applications decreases with firm age and size in that sector and also biotechnology.

On the other hand, Plehn-Dujowich (2007) finds that smaller firms produce up to four times more patents per dollar spend on R&D than large companies. Additionally, patents of small firms are significantly more cited by research professionals than patents of large companies, indicating a higher innovative content of the first ones. Lotti and Schivardi (2005) studies the patenting behavior of countries, sectors and firms. Through an exhaustive analysis of data, the author finds that while the probability of patenting rises with firm size, the patenting intensity (granted patents relative to employment) falls

with employment.

The last measure to evaluate innovation is related to new products and technologies introduced in the economy.

In this subject, Love and Ashcroft (1999) finds that the number of innovations increases with plant size in a sample of more than 300 Scottish plants. Huergo and Jaumandreu (2004) examines the probability that a Spanish firm introducing a product or process innovation and found that large firms have a significantly higher probability of introducing innovations than small firms. The difference is 37 percentage points for process innovations and 27 percentage points for product innovations.

The result that larger firms are more likely to introduce more innovations is not surprising due to the fact that they have more product lines and human resources to improve upon. Taking this in account, Love and Ashcroft (1999) use a second measure of innovativeness, which they call innovations per employee, and found that this measure actually decreases with firm size. Hence, according to these authors, smaller plants are indeed more innovative than large ones.

Contributions to Employment

Most of the literature on the generation of employment by entrepreneurs states that these firms have a disproportionately high contribution to the creation of jobs.

In one of the first articles analyzing this relationship, Fritsch (1997) shows a positive correlation between start-up rates and employment growth in all German industries based on regional data in the period 1986–1989. His conclusion was that a region characterized by a higher start-up rate is associated with a higher employment increase. Folster (2000) demonstrates the same positive effect for Sweden, as he states that "an increased share of self-employment by one percent of the population increases total employment by about 1.3 percent of the population" (p. 143). Baldwin (1998) examines the changes in employment shares of certain size classes of Canadian manufacturing plants. He concludes that the smallest size class, i.e., plants employing fewer than 100 employees, has increased its employment share by 0.57 percent annually over the period 1973–1992, whereas the larger size classes experienced decreasing employment shares. These articles are among the many that correlates entrepreneurship with an increase in job creation rates.

A recent stream of research, initiated by Fritsch (2008), studies aggregated data, mostly at the regional level to analyze the direct and indirect effects of entrepreneurial activity on employment growth (and other outcomes). Start-ups, or market entries lead to new business development, whereas incumbent firms might be forced to be dissolved because of the increased competition of these new entrants. Hence, indirectly, combining the entry of new companies and the disappearing of older and less efficient ones, might boost competitiveness and economic growth.

Mueller et al. (2008) relates start-up rates in Britain (measured by the number of new firms per 1000 employees created annually) to regional employment changes (over two years). Higher start-up rates are associated with higher immediate levels of employment (in these new firms themselves). However, the same authors state that after some years the relationship of start-up rates with employment growth is negative due to the competitive pressure that leads inefficient incumbents to fire workers or exit the market. In the longer run though, the net effect is positive due to increased competitiveness.

Acs and Mueller (2008) finds a similar pattern for the United States. The higher the regional start-up rate, the higher is the regional employment growth rate. The positive

effect is stronger in the first year, then it comes down to zero after some years and increases thereafter. Fritsch and Mueller (2008), Baptista et al. (2008) and Van Stel and Suddle (2008) corroborate this result based on German, Portuguese and Dutch data, respectively.

Carree and Thurik (2008) finds evidence for the same pattern based on country level data. They find that increased business ownership rates are shown to go together with an instantaneous small effect on employment generation, a mid-term negative effect and a long term positive effect.

Relevant studies using disaggregated data to examine the relationship between firm size or age and the proportional number of jobs a firm has created have come to the same conclusion: smaller firms have the highest percentage-rate growth. Thus, proportional to their size, small firms created more jobs than large firms. Some examples of these studies include Calvo (2006), Hart and Oulton (1996), Konings (1995) and Oliveira and Fortunato (2006).

All these studies are clear in indicating a positive relationship between entrepreneurship and employment. However, it is also important to discuss the role of entrepreneurship on wages and job satisfaction.

Wunnava and Ewing (2000) finds that in 1989, U.S. firms with between 100 and 500 employees, paid male employees 18 percent more than identical employees of firms with less than 100 employees. The average wage differential between firms employing more than 500 individuals and entrepreneurial firms amounts to 27 percent for male employees.

An explanation for this gap might be the higher skill levels of employees of large firms, due to their higher experience and education. Brown and Medoff (2003), which studies firm wage differentials based on age, shows that the positive correlation between firm age and employee wages turns into a negative relationship when controlling for the workers' heterogeneity (differences in experience, education, etc.). He states that the higher level of wage paid by established firms is completely explained by the characteristics of their workers. Troske (1999), argues that another explanatory factor for the wage gap between workers in small and large firms is the presence of a complementarity between capital and skill. The complementarity implies that workers in more capital-intensive firms are paid higher wages. Hence, if larger firms are more capital intensive than smaller firms, this could explain part of the gap.

Regarding job satisfaction, there are ambiguous results in the literature. On one hand, Winter-Ebmer and Zweimuller (1999), in a study done in Switzerland, finds that on-the-job search and actual job changes are undertaken less frequently by employees of large firms. Thus, small firms have lower satisfaction levels than large firms.

On the other hand, Benz (2003) examines actual scores on a job satisfaction questionnaire, finding employees of smaller German, British, and Swiss firms to have higher average job satisfaction scores than employees of larger firms in the same countries. In a different study, Clark and Oswald (1996) obtains the same result for U.K. employees.

Contributions to Productivity and Economic Growth

Starting by making a literature analysis of the dynamics between entrepreneurship and labour productivity, Brouwer et al. (2005) relates the firm's value added and the firm's gross output to the cost of labour based on a cross-section of about 4500 Dutch manufacturing firms for the year 1999. The authors find that both value added and gross output per euro of labour increase with firm size, so they conclude that entrepreneurs appear to have a lower level of labour productivity compared to larger firms in the Dutch manufacturing industry.

Jensen et al. (2001) recognizes several difficulties in comparing productivity levels across plants of different ages.[8] These authors state that three different effects occur in manufacturing plants as they grow older. The first one is the positive age effect, in the sense that older plants are more productive due to the management accumulating experience, gains from learning by doing, or the achievement of economies of scale. The second effect is that older plants are likely to be more productive due to a process of natural selection: while younger firms have to prove their worth, older firms have already survive it. Third, there is a possibly offsetting negative effect related with older plants: since the best-practice technologies are embodied in new capital, it is more likely to find them in start-up plants. Hence, younger plants in a given year tend to have more productive technologies. Taking into account these three effects together, the authors find a relatively low contribution of younger firms to labour productivity.

However, Disney et al. (2003) provides evidence in contrast to the conclusions above. These authors find that, in the U.K. manufacturing sector between 1980 and 1992, establishments younger than one year have an average annual labour productivity (output per person hour) 2.4 per cent higher than incumbent establishments, and five percent higher than for exiting establishments.

A similar result was obtained by Pagano and Schivardi (2003). They analyze the impact of the firm size distribution on the growth of labour productivity in several European countries and find a positive relationship between the concentration of a sector on fewer and larger firms and labour productivity growth.

Foster et al. (2006) comes to the same conclusions fot the U.S. retail sector in the

[8]Note that age is positively correlated with size.

90's. Their results show that exiting establishments are far less productive than entering establishments, and entering and incumbent establishments have similar productivity levels.

As one can see from the previously mentioned literature, there is no consensus wheter entrepreneurship has a positive or a negative effect on labour productivity, though the evidence suggests that large firms seem to be more productive.

It is interesting to note that innovation process and economic growth can both suffer from the lack of either research effort or entrepreneurial skills. Economic history provides examples of both cases. Mokyr (1990) argues that the Islamic civilization, at the end the twelfth century, severely declined its economic growth because it was not capable of adding much new to the existing stock of ideas it retrieved and applied so brilliantly. Mokyr also discusses notorious examples of societies, such as the Greeks and the Romans, that suffered because of their lack of entrepreneurial skills. Such a deficit may also well explain the economic decline of the late Victorian Britain usually attributed to the inability of British entrepreneurship in promoting the diffusion of new and advanced production methods.

Baldwin (1998) examines the changes in shipment shares of Canadian plant size classes in manufacturing during the period 1973-1992. The smallest size class has increased its shipment share by 0.18 percent on average per year, while the shares of larger size classes have decreased or remained the same. Actually, the author shows that economic activity has been shifted from large towards small firms. Whether the effect of such a shift is positive in terms of economic value added depends on the relative performance of small versus large firms and the performance improvements of large firms due to the improved competitiveness as a consequence of more small firms.

Carree (2002) states that an increase in large firm employment shares leads to lower value added index. Consequently, on average, a shift towards from large to small units has led to an increase in economic growth. Brouwer et al. (2005) provides additional support to Carree's results as they find that the percentage growth rates in productivity, in terms of output as well as value added relative to the costs of the factors of production decrease with firm size.

Carree and Thurik (1998) investigates the consequences of the transformation process in Europe's manufacturing industries, particularly in the late 1980s and early 1990s, from

large firms toward small firms. Based on an industry panel covering 14 manufacturing industries in 13 European countries, they find that, on average, the employment share of relative large firms has a negative impact on output growth. In a more recent study, the same authors (Carree and Thurik, 2008) relate the growth of the number of business owners as a percentage of the labour force to GDP growth. They establish that the initial effect on GDP growth of a higher business ownership rate is positive and there is no significant evidence of business ownership having an indirect effect later on. This means that an increase in entrepreneurship activity leads to a positive effect in economic growth.

Audretsch and Thurik (2001) comes to a similar result. In their panel study for European countries they found that a concentration towards smaller firms positively affects growth.

Audretsch and Keilbach (2004) using regional data for 8 OECD economies, points out that firm entry and exit affects positively regional output growth. Depending on the region, they conclude that up to 40% of growth is associated with firm turnover. With firm entry and exit they associate entrepreneurship and young innovative firms since entry and exit rates are usually positively correlated.

In the same line of the previous authors, Baumol et al. (2007) argues that the development and growth of innovative entrepreneurial companies was the driving force behind the advanced countries technological revolution, contributing to productivity growth in the past 15 years. Van Praag and Versloot (2008) provides a meta-study of 57 high-quality studies on the contribution of entrepreneurs (young firms with less than 100 employees) in economic growth measured through their macroeconomic performance. They conclude that entrepreneurial firms engender relatively more productivity growth and produce and commercialize high quality innovations.

Thurik (1999) provides empirical evidence from a 1984-1994 cross-sectional study of 23 OECD countries. He shows that increased entrepreneurship, as measured by business ownership rates, is associated with higher rates of employment growth at the country level.

Braunerhjelm et al (2004) implements different regression techniques and found surprisingly robust support for entrepreneurship as one important source of growth, while no such relationship could be established for R&D.

Contrariwise to the effect on labor productivity, entrepreneurship seems to have a positive effect on growth and foster economic development.

2.3 Literature relating Trade Unions, labour and Economic Growth

2.3.1 Trade Unions

According to the Economic Times, Trade Unions are "organizations formed by workers from related fields that work for the common interest of its members. They help workers in issues like fairness of pay, good working environment, hours of work and benefits. They represent a cluster of workers and provide a link between the management and workers".[9] Pemberton (1988, p.9) provides a microeconomic foundation for the union objective function as "the outcome of an internal bargain between the leadership and membership".

As stated in the introduction of this study, unions assume a leading role as a crucial economic institution, representing a sizable portion of the workforce in most developed countries. Union participation rates in 2001 range from 12.8% in the United States to 85.1% in Iceland (Lawson, 2011).

The economic analysis of wage and employment determination in labor markets with the intervinience of trade unions was first explored with the work of Dunlop (1944). His formulation assumes that the union maximizes the welfare of its members by choosing a point on the firm's labor demand curve. This theory formed the basis of what is now commonly referred to as the Monopoly Union model, which will be used in this study (Section 3.2.)

According to Dunlop and its monopoly union model, the firm chooses the level of employment depending on the level of wages that the union previously have chosen for their members. Hence, the union can choose their optimal point on the firm's labour demand curve. This choice depends on several factors such as the scarcity and uniqueness of workers and varies between industries.

[9]Definition of Trade Union (July, 2017). Retrieved from: http://economictimes.indiatimes.com/definition/trade-union

2.3.2 How Trade Unions affect Labour, Wages and Economic Growth

The conventional point of view is that trade unions exercise their monopoly power to improve welfare of workers, by raising their wages above the competitive level. On the one hand, this higher demanded wage alters the distribution of income between firms and workers. On the other hand, it might increase unemployment level which, in turn, might slow down economic growth. This viewpoint is accepted among most of theoretical scholars, but the empirical evidence does not support it unequivocally.

In general, growth and unemployment are intimately related for two reasons. First, unemployment affects the scale of operation of the economy and thereby the growth rate. Second, growth affects inter-temporal decisions of workers about where to allocate on the labour market once they are laid off, and thereby it affects equilibrium unemployment (De Groot, 2001).

Although traditional evidence supports an unfavorable effect of trade unions on employment (Nickell, 1997), its impact on firm's decisions to innovate and their investment and hence the long-term economic growth is not so consensual (Neto and Silva, 2013).

Shister (1954), Booth (1995), Menezes-Filho and Van Reenen (2003), state that the collective bargaining promoted by trade unions has only a residual impact on firm's investment decisions. From the survey of the evidence from firm level data, Freeman and Medoff (1984) and Hirsch (2004) conclude that the average union productivity effects are statistically insignificant and, although collective bargaining governs the distribution of incomes between workers and firms, it leaves economic growth and productivity unchanged (Clark 1984).

A similar result was obtained by Ji et al (2016), who found that labour union has both a negative growth effect via unemployment and a positive growth effect via endogenous market structure, and these two effects exactly offset each other leaving an overall neutral effect on growth. This growth neutrality of wage-oriented unions provides a novel implication, given that increasing wage-oriented behavior of unions may lead to what is referred as the "Cheshire cat" phenomenon (Burda 1990), in which the union members support a wage policy that may be inimical to the long-run survival of the union.

Recently, using aggregate national data for OECD countries, the evidence suggests a few signs of a relationship between unionization and economic growth, either positive or

negative.

The contradictory results motivate several studies, such as Palokangas (1996), Ramos-Parreno and Sanchez-Losada (2002), Irmen and Wigger (2003), and Chang et al (2007). In contrast to the traditional notion, these studies seem to highlight the possibility of a positive effect of unionization on growth and employment.

Palokangas (1996) incorporates union wage bargain into a Romer (1990)-type model and analyses a union bargaining over the wage of low-skilled and high-skilled labour in the intermediate sector and he shows that the existence of the union might foster economic growth.

In the OECD Jobs Study analyzed by Lundvall (1994), the relationship between high-tech and high-wage sectors is explored. It is argued that production of manufacturing goods increasingly takes place in conditions of imperfect competition. This imperfect competition results in rents that are often shared with workers, and in wages that differ considerably between sectors, even after controlling for age, education, occupation and gender. These wage premiums are stable over time and their structure is roughly similar across countries (Krueger and Summers 1988). An important reason for firms to engage in rent sharing may be the presence of trade unions (De Groot, 2001). The potential consequences of this are that the unemployed may prolong their job search in the hope of getting into the high-wage firms and sectors, and displaced workers from the high-wage firms and sectors may have very high replacement rates and hence reservation wages when compensations are based on previous earning.

For instance, Van Reenen (1996) looks at the relationship between labour market institutions and growth from a totally different angle. Based on a panel of British firms, he finds strong evidence of the importance of rent-sharing. In particular, he finds that innovating firms generally pay higher wages, while rival innovations tend to depress their own wages. This evidence can be seen as an indication that firms engage in rent-sharing to enhance productivity growth.

However, there are some authors who disagree with these conclusions.

Bean and Crafts (1995) shows how the operation of trade unions may result in lower investments and growth in the economy. Firms invest to increase their profits and an essential characteristic of many types of investments is that they have sunk-costs associated with them. This fact puts trade unions, once investments have been made, in a

strong bargaining position. Due to the sunk-cost character of the investment, the trade union can extract part of the rent that is associated with the investment. Aware of this behavior, firms will invest less under the assumption that firms and unions cannot sign a complete contract in which the division of rents resulting from investments to be made in the future is agreed upon. The problem described here is known as the "hold-up problem" and was formally modeled by Grout (1984). Bean and Crafts integrate this insight into a model of endogenous growth in which R&D is aimed at developing new products. The reward for investments resulting in the development of new products is a monopoly profit to be earned by the inventor of the new product. In the presence of trade unions, some of the profits are extracted by them, which reduces the incentive to develop new products and consequently reduces growth. The main conclusions pointed out by these authors are that the presence of unions significantly depresses total factor productivity and that workplaces with multiple unions experience about one percentage point lower growth of total factor productivity than single union workplaces.

The central idea in Daveri and Tabellini (2000) is that the presence of unions results in high real wage costs. They developed an overlapping generations model in which both labour and capital are used as inputs in the production process. Endogenous growth results from an external effect related to the accumulation of physical capital and wages are set by monopolistic trade unions. Higher wage costs will push firms to a more capital intensive production process, and will result in lower employment and a reduction in the marginal product of capital. This reduction induces a decrease in savings and growth. Growth and unemployment are thus negatively correlated in both cross-sections and time series, where the correlation stems from differences in wage costs between countries or time-periods.

In the same line as these previous authors, Addison and Hirsch (1989) state that rent-seeking behavior of unions lowers firms' investments in physical capital, as well as in R&D and other risky activities. This results in productivity growth being relatively low in strongly unionized firms and industries.

Chu et al (2016) finds that an increase in the bargaining power of a wage-oriented union leads to a decrease in employment in the domestic economy. This result has two important implications on innovation. The first one is that unions reduce the rates of innovation and economic growth; the second is that they cause innovation to be directed to

the foreign economy, which in turn causes a negative effect on domestic wages relative to foreign wages in the long run. Their model is able to explain about half of the decrease in relative wage between the US and the UK from 1980 to 2007. Furthermore, the decrease in unions' bargaining power leads to quantitatively significant welfare gains in the two countries. This result has two important implications on innovation. Firstly, by decreasing employment, an increase in the bargaining power of a wage-oriented union reduces the rates of innovation and economic growth. This theoretical implication is consistent with empirical studies that find negative effects of unions on innovation and growth. Secondly, by decreasing employment and the market size of the domestic economy, an increase in the bargaining power of a wage-oriented union causes innovation to be directed to the foreign economy, which in turn causes a negative long run effect on domestic wages relative to foreign wages. In the long run, the negative effect on relative wage income across countries would dominate the positive effect of labour unions on wages only in the case that the elasticity of substitution between domestic and foreign goods is large enough. .

In a more recent paper, Boone (2000) demonstrates that the existence of a union dampens the growth rate of the economy because firms will invest too many resources in the reduction of overhead costs so that the growth rate of the quality of a product is too low.

Empirical studies often find that increasing the degree of unionization has a negative effect on employment. Examples are Montgomery (1989), Blanchflower et al.(1991), Nickell and Layard (1999) and Kroland Svorny (2007).

Other authors emphasize that the relationship between the effect of bargaining by trade unions and economic growth is not linear and depends on several factors.

According to Lingers (2003), the union changes the allocation of high-skilled labour between the research and the intermediate goods sector, hence, the union affects economic growth. Two opposite effects can be observed. Firstly, there is a negative effect because the union lowers the profits which can be earned in the intermediate sector. Therefore, the incentive to perform R&D declines, which dampers economic growth. Secondly, the wage of high-skilled labour declines in the intermediate sector because low-skilled labour employment in the intermediate sector is smaller than in the competitive case. This triggers a migration of former intermediate high-skilled worker into the research sector thereby increasing the growth rate of the economy. Which one of these two effects

dominates depends on the elasticity of substitution in the intermediate good production function.

Chu et al (2016) finds that the effects of labour unions on employment, innovation and economic growth are theoretically ambiguous and depend on their wage orientation. As I stated above, these authors believe that in the case of wage-oriented unions, decreasing the bargaining power of unions stimulates employment and economic growth. However, when it comes to employment-oriented unions, increasing their bargaining power would raise employment and growth.

These theoretical findings suggest that there is no one-size-fits-all policy when it comes to reforming existing labour-market institutions, such as labour unions. Therefore, policy makers should make an effort to understand the country-specific or even industry-specific effects of labour unions.

2.4 Literature Review conclusion and goals of the study

This study casts a bridge between these streams of literature. It develops an endogenous growth model in which analyzes the role that trade unions exert in the R&D sector and the process of financing innovation and their effects on employment and economic growth. The economy is characterized by three sectors: (a) final good sector, in which a good is produced employing labour and intermediate goods and the wages of workers are defined by a trade union; (b) intermediate good sector, in which each intermediate good is produced by a firm that monopolizes the market; and (c) R&D sector which employs entrepreneurs and capitalists that introduce new varieties of intermediate goods in the market. This new varieties are introduced as a result of a successful process of search and matching in the R&D sector between entrepreneurs and capitalists. Once a successful matching occurs, they bargain over the distribution of the monopoly rents associated with the discovery of the new intermediate good variety.

The model studies the occupational choice of economic agents between these three setors. They can work in the final good sector in which they earn a wage defined by the union. Alternatively, they can work in the R&D sector, in which they can become entrepreneurs and devote their time to came up with a new business idea or capitalists and select the ideas they found the most valuable and finance them. In equilibrium the expected returns of these three activities are identical.

This model is apt to descrive the funcioning of the venture capital industry, in which entreprenerus propose their business to venture capital firms that provide funds and technical/managerial expertise to help them launch these ideas.

This literature review presents a broad understanding and knowledge of the topics addressed in this study. In order to have a good understanding of the concepts and different ideas, it is important to make an exaustive analysis of the literature and, to that effect, we gathered a vast number of papers covering different lines of tought in innovation, entrepreneurship and their effects on employment, economic growth and productivity, financing of innovation and trade unions.

In all the literature analyzed and to the best of our knowledge, no articles were found addressing the specific topic we choose to study, which is the influence of trade unions in entrepreneurial activities. Indeed, in the previous section, were presented sereval

papers connecting and descriving the impacts of entrepreneurship on economic growth, innovation, produtivity between others, as well as other relations, but no study was found dealing specifically with trade unions, entrepreneurs and capitalists. Due to the importance of this topic and the existent gap in the literature, we propose to analyze this relationship using a standard endogenous growth model with trade unions, capitalists and entrepreneurs.

The study is organized as follows: (a) Section 3 presents the model; (b) Section 4 presents the balanced growth path; and (c) Section 5 presents the conclusions.

3 The Model

Following Giordani (2015), in our model the economy is composed by three different sectors that interact with each other. These sectors are: (a) the final good sector, in which the final product is produced competitively employing labour and intermediate goods, present in Section 3.1; (b) the intermediate good sector, in which a specific intermediate good is produced and sold by a firm that has the monopoly for that particular variety, presented in Section 3.3; (c) the R&D sector, which employs entrepreneurs and capitalists that together introduce new varieties of intermediate goods, presented in Section 3.4.

Trade unions represent workers on the final good sector and bargain their wages, changing the reallocation of labour between these three sectors. These labour institutions are introduced in Section 3.2.

The innovations created by the interaction between entrepreneurs and capitalists in the R&D sector are patented and sold to intermediate good firms, which monopolize the market for that particular variety of product. Finally, the product goes to the final good sector where is produced to the general consumer.

In this economy the total population is denoted by L and is assumed to be growing exponencially at a constant rate η. The agents in the economy have to choose whether they want to work in the R&D sector, where they can act as entrepreneurs or as capitalists, or in the final good sector. Denoting by L_E, L_F, L_Y respectively the number of entrepreneurs, capitalists and final good workers, the labor market clearing condition can be written as:

$$L_E + L_F + L_Y = L. \tag{1}$$

We will start by caracterizing the final good sector, then the intermediate good sector and we wiil finish with the caracterization of the R&D sector.

3.1 Final good sector

The final good for consumption Y is homogenoeous and produced by competitive firms that combine labor and intermediate goods according to the following Cobb-Douglas technology:

$$Y = L_Y^{1-\gamma} \int_0^A x_j^{\gamma} dj,$$

in which L_Y represents the total labour in the final good sector, x_j represents the jth intermediate input and A is the measure of varieties of these inputs.

For profit maximization, firms solve the following problem:

$$max_{L_Y, xj} = \left[L_Y^{1-\gamma} \int_0^A x_j^{\gamma} dj - \omega L_Y - \int_0^A \rho_j x_j dj \right].$$

In this equation, x denotes the jth intermediate input, ωL_Y represents the labour costs in the final good sector and p_j is the price of the jth intermediate good.

The wage of the final good sector workers, ω, comes from the wage set by trade unions, which is defined in the next section.

3.2 Labour market framework: Monopoly trade union

In most industrielized countries, employees have the right to organize and to select exclusive bargaining agents to negotiate collective agreements to define their wages, hours of work, and other general conditions. They engage in concerted activity in order to achieve mutual aid and protection. The institutions that provide this protection are called trade unions and their main goal is to maintain or improve workers conditions through collective bargaining with employers. These labour institutions seek to achieve a mark-up on wages for their members compared to the situation without union presence. According to Freeman and Medoff (1981), trade unions lead to a higher union wage premium . In order to have power in the negociation process a union must have some control over the total labour supply because unions' bargaining strength is enhanced by the percentage of workers that they represent in a determinate sector.

In this section we introduce trade unions into the model. We used the monopoly trade union, firstly proposed by Dunlop (1944) and Ross (1948). Within this framework, the union decides uniterally the level of wages, leaving firms with the decision of the level of employment afterwords. Following Neto et al (2017), the utility function of the monopoly trade union demanded by unions is assumed to have the following form:

$$U^{MU} = (\omega_t - \bar{\omega_t})^{1-v}(L^D_{y,t})^v.$$

In the previous equation ωrepresents the final good workers real wage, while$\bar{\omega}$represents the perfect competition wage. The value of $v(0 \leq \nu \leq 1)$ states whether the union is more employment or wage-oriented. A higher ν indicates a more employment-oriented union. Within the Monopoly trade union framework wages are defined by the unions before the firm's decision on the level of employment, in a way that trade union institutions can anticipate the impact that their demanded wages levels will have on the employment level.

The final good wage with the monopoly trade union is equal to:

$$\omega = \Phi\bar{\omega},$$

in which$\Phi = \frac{1}{1-\frac{1-v}{v}\frac{1}{\varepsilon_{L,\omega}}} \geq 1$,can be interpreted as a mark-up over this perfect competition wage. This markup can be a proxy of trade union density and countries/industries

with higher trade union density would present a higher markup due to the higher bargaining power that their unions have. As we can infer from the previous equation, a markup higher than one ($\Phi > 1$) will make the wage in the final good sector higher than the perfect competition salary, while a markup equal to one ($\Phi = 1$) will make the final good sector wage equal to the perfect competition wage.

From the final good sector, we obtain that, without the action of unions, $p_j = \gamma(x_j/L_Y)^{\gamma-1}$and $\bar{\omega} = (1-\gamma)\frac{Y}{L_Y}$.

Introducing the mark-up in the final good wage equation, we end up with:

$$\omega = \Phi \left[(1-\gamma)\frac{Y}{L_Y} \right]. \tag{2}$$

3.3 Intermediate good sector

There are an infinite number of firms producing j-different intermediate goods distributed along the measure $[0, A]$ .The intermediate good firms dominate the market for a particular variety of a good, in which they become monopolists. This happens because they purchased the unique and infinite-lived patent for a variety of an intermediate good from a particular entrepreneur in the R&D sector. One unit of raw capital, rented at rate r, produces one unit of any intermediate input. For simplification purposes, we assume that there is no depreciation.

The jth intermediate good producer maximizes the following standard monopoly problem:

$$max_{xj} \left[\rho_j(x_j)x_j - rx_j\right].$$

Substituing for p_j given above and maximizing it is obtained:

$$x_j = \left(\frac{r}{\gamma^2}\right)^{\frac{1}{\gamma-1}} L_Y \equiv x \text{ and } \rho_j = \frac{r}{\gamma} \equiv \rho \forall j.$$

The monopoly profits can be defined as:

$$\pi_j = \gamma(1-\gamma)\frac{Y}{A} \equiv \pi \forall j. \tag{3}$$

Quantities, prices and profits are independent of the specific variety produced. Hence,

$$K = \int_0^A x_j dj = \int_0^A x dj = Ax.$$

As a result, the aggregate production function can be written as:

$$Y = (K)^{\gamma}(AL_Y)^{1-\gamma}. \tag{4}$$

Taking into account that the functional distribution of income implies

$$\omega L_Y + rK + \pi A = Y,$$

by using (2) and (3), i.e., the values for $\omega$and π respectively, we get:

$$r = \frac{Y - Y\Phi + Y\Phi\delta - Y\delta + Y\delta^2}{K}.$$

3.4 R&D sector

Entrepreneurs come up with ideas for new innovations, however they need the financial support of capitalists to provide the necessary capital to introduce this new varieties of products in the market. A patent is granted to each new design, and a successful innovator gets exclusive rights over the use of his unique product. Thus, a successful match between an entrepreneur and a capitalist generates profits from the introduction and sale of a new innovation.

Following Giordani (2015), in our model the innovation is the result of a process of succesfull matching between an entrepreneur and a capitalis.

The aggregate innovation function, that represents the production function of knowledge, is a constant return to scale Cobb-Douglas technology, represented by:

$$\dot{A} = \beta \left(L_E\right)^a \left(L_F\right)^{1-a}. \tag{5}$$

In this aggregate innovation function, the A represents the (increasing) measure of ideas that are lately incorporated into intermediate input varieties, $\beta > 0$, is a productivity parameter that captures the efficiency of the matching process.

When $a = 1$, the model colapses into a semi-endogenous growth model and the measure of ideas will be equal to the number of entrepreneurs in the market times the efficiency of the matching process.

Given the matching function in (5), the instantaneous probability of matching of entrepreneurs and capitalists is:

$$\alpha_E = \frac{\dot{A}}{L_E} = \beta.(\frac{L_F}{L_E})^{1-a},$$

while the one for capitalists is

$$\alpha_F = \frac{\dot{A}}{L_F} = \beta.(\frac{L_F}{L_E})^{a}.$$

Now, we are going to analyze the occupational choice of the agents in this economy. An agent working in the final good sector gains a wage, denoted by w, while agents that choose to become entrepreneurs or capitalists gain, respectively, v_E^0 and v_F^0. v_E^0 represents

the chance of a successfull matching with a capitalist for an entrepreneur, whereas v_F^0 represents the chance of a successfull matching with a entrepreneur for a capitalist. Hence, in equilibrium,$w = v_E^0 = v_F^0$ must hold.

The expected payoff associated with becoming a capitalist is defined by:

$$rv_F^0 = \alpha_F(v_F^1 - v_F^0) + \dot{v}_F^0,$$

in which v_F^1 represents the value of a successfull matching for a capitalist. This value is given by the following expression:

$$rv_F^1 = (1 - \bar{\theta})\pi + v_F^0 - v_F^1 + \dot{v}_F^1.$$

In this equation, $(1 - \bar{\theta})\pi$ represents the fraction of monopoly rents going to the capitalist prevailing in that particular market.

Solving the system in v_F^0 and v_F^1, it is obtained

$$v_F^0 = \frac{(1 - \bar{\theta})\pi\alpha_F}{(r - \frac{\dot{v}_F^0}{v_F^0}) + \alpha_E(r - \frac{\dot{v}_F^1}{v_F^1}) + (r - \frac{\dot{v}_F^1}{v_F^1})(r - \frac{\dot{v}_F^0}{v_F^0})}$$

.

The value v_F^1 can be found by sustituting for the expression above into:

$$v_F^1 = \frac{(1 - \bar{\theta})\pi + v_F^0}{1 + r - \frac{\dot{v}_F^1}{v_F^1}}$$

The expected payoff associated with becoming an entrepreneur is defined by the following equation:

$$rv_E^0 = \alpha_E(v_E^1 - v_E^0) - \dot{v}_E^0. \tag{6}$$

The variable r represents the rental rate of capital, while v_E^1 represents the value of a sucessfull matching with a capitalist for an entrepreneur. Notice that v_E^0 is different from v_E^1.The first one represents an expected value while the second one represents the value of the match (the same logic can be applied to v_F^0, v_F^1). Equation (6) states that the value of choosing to become an entrepreneur must be equal to the probability of a

sucessfull matching with a capitalist times the payoff associated with this chance (plus the capital gains/losses that may occur over time).

The value of v_E^1 is defined by:

$$rv_E^1 = \bar{\theta}\pi + v_E^0 - v_E^1 + \dot{v}_E^1. \tag{7}$$

In this equation, $\bar{\theta}\pi$ is the fraction of the monopoly profits going to the entrepreneur prevailing in that particular market. An analogous interpretation of equation (6) can be made for the equation (7).

Solving the system in v_E^0 and v_E^1, we obtain

$$v_E^0 = \frac{\bar{\theta}\pi\alpha_E}{(r-\frac{\dot{v}_E^0}{v_E^0})+\alpha_E(r-\frac{\dot{v}_E^1}{v_E^1})+(r-\frac{\dot{v}_E^1}{v_E^1})(r-\frac{\dot{v}_E^0}{v_E^0})}$$

The value of v_E^1can be obtained substituting the expression of v_E^0given above into the following:

$$v_E^1 = \frac{\bar{\theta}\pi+v_E^0}{1+r-\frac{\dot{v}_E^1}{v_E^1}}$$

3.5 Consumption decisions

The economy is populated by an infinitely-lived consumes and collects income from supplying labor. The representative household maximizes:

$$U = \int_0^\infty exp\left[-\rho t\right] \frac{c(t)^{1-\phi} - 1}{1-\phi} dt,$$

in which c represents C/L (consumption per capita), under the usual budget constraint: $\dot{K} = Y - C$. $p > 0$ is the subjective discount rate, and $\phi > 0$ is the inverse of the inter-temporal elasticity of substitution. The time path of consumption per capita must obey the following Euler equation:

$$\frac{\dot{c}}{c} = \frac{1}{\phi}(r(t) - \rho - \eta) \tag{8}$$

4 Balanced growth path

From this section on we will focus on the steady state. According to the labour market clearing condition must had along the balanced growth path, $gL_F = gL_E = gL_Y = \eta$.

Dividing the aggregate innovation function (5) by A, is is obtained:

$$g_A = \frac{\beta(L_E)^a(L_F)^{1-a}}{A},$$

which, taking logs and derivatives, gives us

$$g_A = \eta.$$

This expression states that the rate of technological progress in the economy is equal to the growth rate of the population. Taking into account the expression of the aggregate production function represented in (4), the growth rate of income per capita must be equal to the growth rate of the population ($\frac{Y}{L} = \gamma = \eta$). From (8), and knowing that $\dot{c}/c = \dot{y}/y$, we can determine the value of r along the steady state, which is equal to:

$$r = \eta(\phi + 1) + \rho. \tag{9}$$

Using equation (6), in the steady state we have $\dot{v}_E^0/v_E^0 = \dot{v}_E^1/v_E^1$. Dividing (7) by v_E^1 and rearranging we get:

$$1 + r - \frac{v_E^0}{v_E^1} - \frac{\dot{v_E}^1}{v_E^1} = \frac{\bar{\theta}\pi}{v_E^1}.$$

Since all terms on the left side of the equation ($1 + r - \frac{v_E^0}{v_E^1} - \frac{\dot{v_E}^1}{v_E^1}$) are constant along the steady state, then $\dot{v}_E^1/v_E^1 = \dot{\pi}/\pi$. From (3) we know that $\dot{\pi}/\pi = \eta$, which implies that $\dot{}^0_E/\dot{v_E^0} = \boldsymbol{v}_E^1/v_E^1 = \eta$. Along the steady state, and using equation (9), the values of v_E^0 and v_E^1 can be expressed as:

$$v_E^0 = \frac{\bar{\theta}\pi\alpha_E}{(\eta\phi + \rho)(1 + \alpha_E) + (\eta\phi + \rho)^2}; v = \frac{\bar{\theta}\pi + v_E^0}{1 + \eta\phi + \rho}. \tag{10}$$

Repeting the same steps for v_F^0 and v_F^1, it is obtained:

$$v_F^0 = \frac{(1-\bar{\theta})\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2} and v_F^1 = \frac{(1-\bar{\theta})\pi + v_F^0}{1+\eta\phi+\rho}. \tag{11}$$

Defining employment across the three activities in share values, we can characterize the steady state equilibrium as the triple (l_Y, l_E, l_F) that solves the following system:

$$\begin{cases} v_E^0 = w \\ v_F^0 = w \\ l_E + l_F + l_Y = 1, \end{cases} \tag{12}$$

where the expressions for v_E^0, v_F^0 and w are respectively given in (10), (11) and (2).

The equations $v_E^0 = w$ and $v_F^0 = w$ state that, in equilibrium, the returns in the final good sector must be equal to the expected returns from developing entrepreneurial or capitalist activities, respectively. Notice that w is a certain value while v_E^0 and v_F^0 are expected values. These three variables must be equal in equilibrium, even for risk-averse agents. As in Giordani (2015), the two first equations can be interpreted as the best response functions of entrepreneurs and capitalists while the third equation of (13) represents the labour market-clearing condition. Analizying this system we can conclude that there are a strategic complementarity between entrepreneurs and capitalists ($\partial l_E/\partial l_F > 0$; $\partial l_F/\partial l_E > 0$), meaning that an increase in l_F raises the probability of matching α_E for an entrepreneur, raising the expected returns of becoming an entrepreneur which induces a rise in the number of agents that want to become entrepreneurs. The exactly same logic can be applied to capitalists.

Using (11), (12), (2) and (3) and $\dot{A}/A = n$, $\alpha_E = \dot{A}/L_E$, $\alpha_F = \dot{A}/L_F$, we can rewrite the system (12) as

$$\begin{cases} \frac{\frac{1}{\Phi}\eta\bar{\theta}\gamma\frac{l_Y}{l_E}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} = 1 \\ \frac{\frac{1}{\Phi}\eta(1-\bar{\theta})\gamma\frac{l_Y}{l_F}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2} = 1 \\ l_E + l_F + l_Y = 1, \end{cases} \tag{13}$$

where $\alpha_E = \beta.(l_F/l_E)^{1-a}$and $\alpha_F = \beta.(l_E/l_F)^{a}$.

It is important to note that, as in Giordani (2015), this equation system cannot be solved explicitly in the three unknown variables, l_E, l_F, l_Y. Hence, in line with the economic literature on search and matching (Pissarides, 1984), we assume that entrepreneurs and capitalists enrol in a bargaining process over the distribution of the monopoly rents. This is analysed in the next section.

4.1 Steady-state under Nash bargaining of monopoly rents

When a successful matching occurs between an entrepreneur and a capitalist, both start a bargaining process over the profits generated by the match (π). Hence, they negociate over their respective stare of monopoly profits $(\theta, 1-\theta)$, taking as given the the shares prevailing in the market $(\bar{\theta}, 1-\bar{\theta})$. Using (10) and (11), we can define the surpluses for the capitalists and entrepreneurs as:

$$S_F = v_F^1 - v_F^0 = \frac{(1-\theta)\pi + v_F^0}{1+\eta\phi+\rho} - v_F^0 = \frac{(1-\theta)\pi - v_F^0(\eta\phi+\rho)}{1+\eta\phi+\rho}. \tag{14}$$

$$S_E = v_E^1 - v_E^0 = \frac{\theta\pi + v_E^0}{1+\eta\phi+\rho} - v_E^0 = \frac{\theta\pi - v_E^0(\eta\phi+\rho)}{1+\eta\phi+\rho}, \tag{15}$$

The Nash bargaining problem is the one of maximizing the following expression with respect to θ:

$$(S_E)^{\sigma}.(S_F)^{1-\sigma},$$

where σ represents the bargaining power of the entrepreneur in the market. Taking the first order condition and simplifying the equation, we obtain:

$$\sigma S_F - (1-\sigma)S_E = 0.$$

Introducing the values for S_E, S_F represented in (14,15) into the expression above and simplifying we obtain:

$$\theta\pi = \sigma\pi + (1-\sigma)v_E^0(\eta\phi+\rho) - \sigma v_F^0(\eta\phi+\rho). \tag{16}$$

Taking into account that

$$v_E^0 = \frac{\bar{\theta}\pi\alpha_E}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} \ , v_F^0 = \frac{(1-\bar{\theta})\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2},$$

in which $\bar{\theta}$ is the profit share of the entrepreneur prevalent in the market, and plugging these values into (16), we obtain

$$\theta\pi = \sigma\pi + \frac{(1-\sigma)\bar{\theta}\pi\alpha_E}{1+\alpha_E+\eta\phi+\rho} - \frac{\sigma(1-\bar{\theta})\pi\alpha_F}{1+\alpha_F+\eta\phi+\rho}$$

Finally, knowing that in equilibrium $\theta = \bar{\theta}$, we can solve for $\bar{\theta}$ to obtain:

$$\bar{\theta}^b = \frac{\sigma(\alpha_E + 1 + \eta\phi + \rho)}{1 + \eta\phi + \rho + \sigma\alpha_E + (1-\sigma)\alpha_F}. \tag{17}$$

In this expression b stands for bargaining.

Given that $l_E^b/l_F^b = \sigma/(1-\sigma)$, the probabilities of matching along the steady state can be written as:

$$\alpha_E = \beta(\tfrac{l_F^b}{l_E^b})^{1-a} = \beta(\tfrac{1-\sigma}{\sigma})^{1-a} \text{ and } \alpha_F = \beta(\tfrac{l_E^b}{l_F^b})^{a} = \beta(\tfrac{1-\sigma}{\sigma})^{a}.$$

Plugging (17) into the system (13) and introducing the values of α_E and α_F, we can obtain:

$$\begin{cases} \frac{\frac{1}{\Phi}\eta\gamma l_Y}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = \frac{l_E}{\sigma} \\ \frac{\frac{1}{\Phi}\eta\gamma l_Y}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = \frac{l_F}{1-\sigma} \\ l_E + l_F + l_Y = 1. \end{cases} \tag{18}$$

From this system of equations one can show that $l_E^b/l_F^b = \sigma/(1-\sigma)$. As in Giordani (2015), this implies that the relative importance of entrepreneurs over capitalists reflects their respective bargaining power. In particular it is $l_E^b > l_F^b$ if and only if $\sigma > 1/2$, that is, if and only if the bargaining power of entrepreneurs is higher that the one of capitalists.

Solving (18) for l_E^b, l_F^b and l_Y^b we get:

$$\begin{cases} l_E^b = \frac{\eta\gamma\sigma}{\eta\gamma+\Phi\left[(\eta\phi+\rho)(1+\eta\phi+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})\right]} \\ l_F^b = \frac{\eta\gamma(1-\sigma)}{\eta\gamma+\Phi\left[(\eta\phi+\rho)(1+\eta\phi+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})\right]} \\ l_Y^b = \frac{\left[(\eta\phi+\rho)(1+\eta\phi+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})\right]}{\Phi^{-1}\eta\gamma+\left[(\eta\phi+\rho)(1+\eta\phi+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})\right]}. \end{cases} \tag{19}$$

4.1.1 Comparative Statistics

As stated in the introduction, one of the main novelties of this thesis is that it allows the possibility to study the impact of trade unions on the relationship between entrepreneurs and capitalists. In particular, this can be done by analyzing the distribution of labor across the three sectors.

Firstly, regarding the final good sector $\partial l_Y^b/\partial\Phi > 0$, which means that a higher trade union markup corresponds to a higher level of employment in this sector. Although this result might seem straightforward, it is interesting to note that, by demanding a higher wage than the perfect equilibrium scenario, which is associated with a higher markup, unions are also able to increase employment. This is due to a reallocation effect: a a higher wage in the final good sector deters individuals to become entrepreneurs and capitalists - note that $\partial l_E^b/\partial\Phi < 0$ and $\partial l_F^b/\partial\Phi < 0$. This seems to be in line with Figure 5 and Figure 6 which present a positive relationship between trade union's markup and the employment level in the service sector, a proxy of the employment level in the final goods sector.

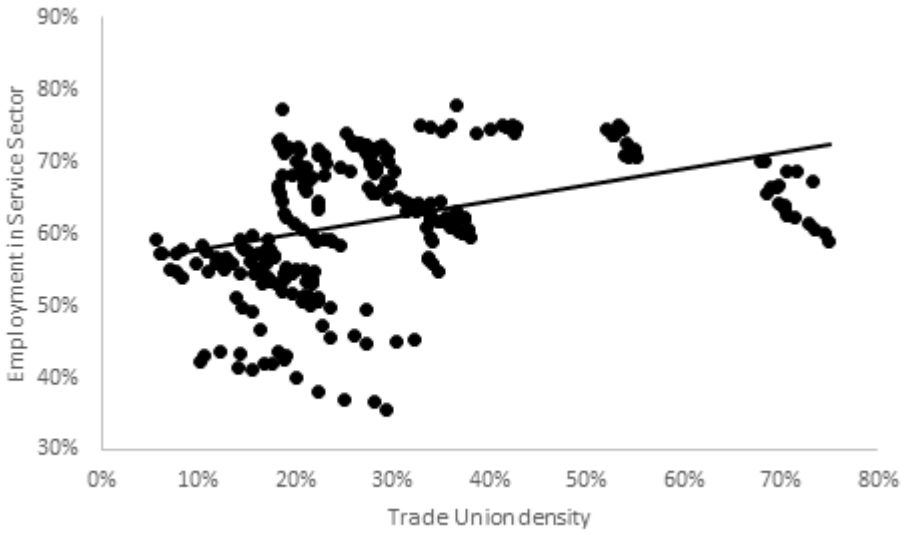

Figure 5: Relation of people working in the services sector with trade union density in OECD between 2000 and 2014, available at http://stats.oecd.org (accessed on July 2017)

Secondly, in line with Giordani (2015) , one can show that an increase in the entrepreneurs' bargaining power leads to a higher (lower) number o entrepreneurs (cap-

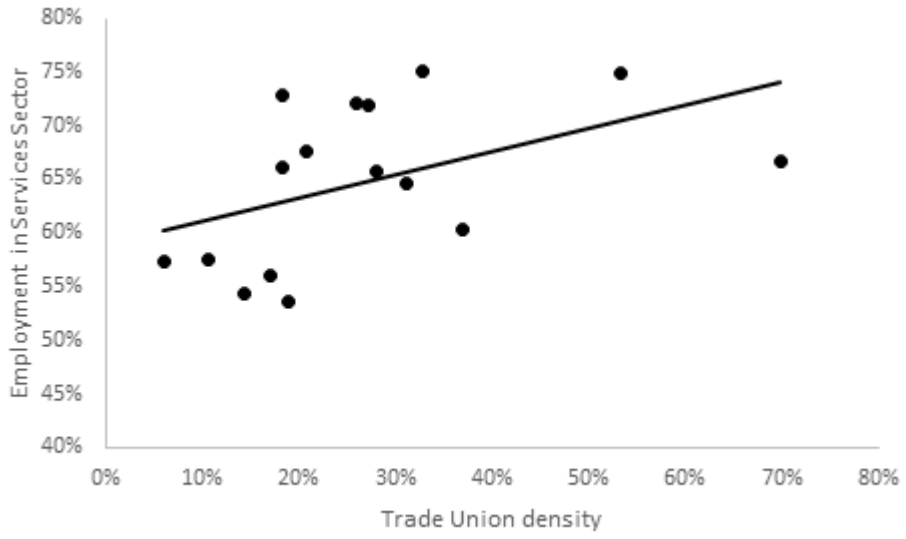

Figure 6: Relation of people working in the services sector with trade union density in OECD in 2014, available at http://stats.oecd.org (accessed on July 2017)

italists) - $\partial l_E^b/\partial\sigma > 0$ ($\partial l_F^b/\partial\sigma < 0$). Interestingly, since our model include both the bargaining power of entrepreneurs (or, inversely, of capitalists) and the trade union's markup, it is possible to infer that a higher bargaining power of entrepreneurs affects the effectiveness of the trade union's markup of reallocating individuals from being an entrepreneur to a worker in the final good sector. The economic reasoning behind is the following: a higher bargaining power of entrepreneurs implies a higher surplus and a lower willingness from individuals to move to the final good sector.

Finally, following Giordani (2015), it is possible to analyze the relationship between employment in the R&D sector (l_E, l_F) and the remaining parameters of the model.

Analyzing the system (19), we know that $\partial l_E^b/\partial\rho < 0$ and $\partial l_F^b/\partial\rho < 0$, which means that the higher the discount rate, the lower the amount of resources devoted to innovative activities. It is also possible to conclude that $\partial l_E^b/\partial\phi < 0$ and $\partial l_F^b/\partial\phi < 0$, implying that the lower the willingness to substitute consumption over time (higher ϕ), the lower the employment in the R&D sector.

Furthermore, $\partial l_E^b/\partial\beta < 0$ and $\partial l_F^b/\partial\beta < 0$, which means that a higher productivity of the innovation function shifts resources from the R&D sector to the final good sector. Moreover, it is possible to infer that $\partial l_E^b/\partial\eta > 0$ and $\partial l_F^b/\partial\eta > 0$, which means that the effect of the growth rate on the R&D sector is positive. Finally, it is interesting to note that$\partial l_E^b/\partial a > 0$ and $\partial l_F^b/\partial a > 0$ if and only if $\sigma > 1/2$, implying that a higher

productivity of entrepreneurs has a positive effect on the overall employment in the R&D sector if and only if their bargaining power is higher than the one of capitalists.

5 Conclusions

In this thesis we have studied the impact of trade unions on the relationship between entrepreneurs and capitalists, within an economic growth model. Section 2 provided the first bibliometric exercise on the topic. From this section the main findings are the following: (a) 66% of the papers regarding the link between entrepreneurship-growth are categorized, in terms of methodology, as "formal" and "empirical"; (b) almost 90% of the papers relating the link trade unions-growth are categorized as "formal", "empirical" and "formal and empirical". From the additional literature review, to the best of our knowledge, we concluded that was no theoretical study dealing specifically with trade unions, entrepreneurs, and capitalists. Section 3 presented a theoretical model, following the contributions of Giordani (2015). By introducing a trade union on the final good sector (as in Neto et. al., 2017), it was possible to study the reallocation of labor among the three sectors in the model. Hence, from the theoretical model, the main conclusions are the following: (a) a higher trade union's markup corresponds to a higher (lower) level of employment in the final good (entrepreneur and capitalist) sector(s); (b) it is possible to infer that an a higher bargaining power of entrepreneurs effects the effectiveness of trade union's markup of reallocating individuals from being an entrepreneur to a worker in the final good sector. Finally, as future work, it would be interesting to extend this analysis to a Schumpeterian growth model, where the success of innovations could depends on the matching between entrepreneurs, capitalists, and trade unions.

6 APPENDIX

6.1 Appendix A: Final good sector

$Y = L_Y^{1-\gamma} \int_0^A x_j^{\gamma} d_j$

$max_{L_Y,xj} = \left[L_Y^{1-\gamma} \int_0^A x_j^{\gamma} dj - \omega L_Y - \int_0^A \rho_j x_j dj \right]$

6.2 Appendix B: Labour market framework: Monopoly trade union

$\omega = (1-\gamma)\frac{Y}{L_Y}$

$U = (\omega_t\text{-}\bar{\omega})^{1-v}(L_y)^v$

$(=)(1-v)(\omega_t - \bar{\omega})^{-v} * 1 * (L_y)^D + (\omega_t - \bar{\omega})^{1-v} v (L_y^D)^{v-1} * \frac{\partial L_y}{2w} = 0\ (=)$

$(=)(1-v)(\omega_t - \bar{\omega}) * (L_y)^D + (\omega_t - \bar{\omega}) v (\frac{(Ly^D)^v}{Ly} * \frac{\partial L_y^D}{\partial w} * \frac{\omega_t}{\omega_t} = 0(=)$

$(=)(1-v)(\omega_t - \bar{\omega})^{-v} * (L_y)^D + (w_t - \bar{\omega})^{1-v} v + (L_y^D)^v * (-\varepsilon L_y, \omega) * \frac{1}{\omega_t} = 0(=)$

$(=)(1-v)(\omega_t - \bar{\omega})^{-v} = (\omega_t - \omega)^{1-v} v (\frac{(L_y^D)^v}{W}) \varepsilon L_y, \omega(=)$

$(=)(1-v)(L_y^D)^v = (\omega_t - \bar{\omega}) * v (\frac{(L_y^D)^v}{W_t}) \varepsilon L_y, \omega(=)$

$(=)(\omega_t - \bar{\omega}) = \frac{1-v}{v} * \frac{\omega_t}{\varepsilon L_{y,w}}(=)$

$(=)\omega_t - \left[\frac{1-v}{v} \frac{W_t}{\varepsilon L_{y,W_t}} \right] = \bar{\omega}(=)$

$(=)\omega_t (1 - \frac{1-v}{v} * \frac{1}{\varepsilon L_{y,W_t}})(=)$

$(=)\omega_t = \frac{1}{x}\bar{\omega}$

$\frac{1}{x} = \Phi$

$\omega_t = \Phi\bar{\omega}$

$\omega = \Phi(1-\gamma)\frac{Y}{L_Y}$

6.3 Appendix C: Intermediate good sector

The intermediate good producer solves the following problem

$max_{xj} \left[\rho_j(x_j) x_j - r x_j \right].$

$x_j = \left(\frac{r}{\gamma^2} \right)^{\frac{1}{\gamma - 1}} L_Y \equiv x$

$\rho_j = \frac{r}{\gamma} \equiv \rho \forall j.$

$\pi_j = \gamma(1-\gamma)\frac{Y}{A} \equiv \pi \forall j.$

$K = \int_0^A x_j dj = \int_0^A x dj = Ax.$

$$
\begin{aligned}
&Y = wL_Y + rK + \pi A \\
&(=)Y = \Phi w L_Y + rK + \pi A \\
&(=)Y = \Phi\left[\left(1-\delta\right)\frac{Y}{L_Y}\right]L_Y + rK + \left[\delta\left(1-\delta\right)\frac{Y}{A}\right]A \\
&(=)Y = \Phi\left(1-\delta\right)Y + rK + \delta(1-\delta)Y \\
&(=)Y - \Phi\left(1-\delta\right)Y - \delta(1-\delta)Y = rK \\
&(=)r = \frac{Y - Y\Phi + Y\Phi\delta - Y\delta + Y\delta^2}{K}
\end{aligned}
$$

6.4 Appendix D: R&D sector

$\dot{A} = \beta\left(L_E\right)^a\left(L_F\right)^{1-a}$

$\alpha_E = \frac{\dot{A}}{L_E} = \beta.(\frac{L_F}{L_E})^{1-a}$

$\alpha_F = \frac{\dot{A}}{L_F} = \beta.(\frac{L_F}{L_E})^{a}$

$rv_E^0 = \alpha_E(v_E^1 - v_E^0) - \dot{v_E^0}$

$rv_E^1 = \bar{\theta}\pi + v_E^0 - v_E^1 + \dot{v}_E^1$

$$
\begin{aligned}
&\begin{cases} rv_E^0 = \alpha_E(v_E^1 - v_E^0) + \dot{v_E}^0 \\ rv_E^1 = \theta\pi + v_E^0 - v_E^1 + \dot{v_E}^0 \end{cases} \\
&(=)\begin{cases} r(\frac{rv_E^0}{\alpha_E} + v_E^0 - \frac{\dot{v_E}^0}{\alpha_E}) = \theta\pi + v_E^0 - (\frac{rv_E^0}{\alpha_E} + v_E^0 - \frac{\dot{v_E}^0}{\alpha_E}) + \dot{v_E}^1 \\ - - - - - \end{cases} \\
&(=)\begin{cases} - - - - - \\ \frac{r^2 v_E^0}{\alpha_E} + rv_E^0 - \frac{r\dot{v_E}^0}{\alpha_E} = \theta\pi - \frac{rv_E}{\alpha_E}0 + \frac{\dot{v_E}^0}{\alpha_E} + \dot{v_E}^1 \end{cases} \\
&(=)\begin{cases} - - - - - \\ v_E^0 = \frac{\theta\pi - \frac{rv_E^0}{\alpha_E} + \frac{\dot{v_E}^0}{\alpha_E} + \dot{v_E}^1 - \frac{r^2 v_E^0}{\alpha_E} + \frac{r\dot{v_E}^0}{\alpha_E}}{r} \end{cases} \\
&(=)\begin{cases} - - - - - \\ v_E^0 = \frac{\theta\pi}{r} - \frac{rv_E^0}{\alpha_E r} + \frac{\dot{v_E}^0}{\alpha_E r} + \frac{\dot{v_E}^1}{r} - \frac{r^2 v_E^0}{\alpha_E r} + \frac{r\dot{v_E}^0}{\alpha_E r} \end{cases}
\end{aligned}
$$

$$(=)\begin{cases} - - - - - \\ v_E^0 = \frac{\theta\pi\alpha_E}{r - \frac{\dot{v_E}^0}{v_E^0} + \alpha_E r - \alpha_E \frac{\dot{v_E}^1}{v_E^1} + r^2 - r\frac{\dot{v_E}^0}{v_E^0} - r\frac{\dot{v_E}^1}{v_E^1} + (\frac{\dot{v_E}^1}{v_E^1})(\frac{\dot{v}^0}{v_E^0})} \end{cases}$$

$$(=)\begin{cases} \frac{r^2 v_E^0}{\alpha_E} + r v_E^0 + \frac{r v_E^0}{\alpha_E} = \theta\pi + \frac{r\dot{v_E}^0}{\alpha_E} + \frac{\dot{v_E}^0}{\alpha_E} + \dot{v_E}^1 \\ - - - - - \end{cases}$$

$$(=)\begin{cases} - - - - - \\ r^2 v_E^0 + r v_E^0 \alpha_E + r v_E^0 = \theta\pi\alpha_E + r\dot{v_E}^0 + \dot{v_E}^0 + \alpha_E \dot{v_E}^1 \end{cases}$$

$$(=)\begin{cases} - - - - - \\ v_E^0(r^2 + r\alpha_E + r) = \theta\pi\alpha_E + r\dot{v_E}^0 + \dot{v_E}^0 + \alpha_E \dot{v_E}^1 \end{cases}$$

$$(=)\begin{cases} - - - - - \\ v_E^0 = \frac{\theta\pi\alpha_E + r\dot{v_E}^0 + \dot{v_E}^0 + \alpha_E \dot{v_E}^1}{r^2 + r\alpha_E + r} \end{cases}$$

Different derivation:

$$\begin{cases} r v_E^0 = \alpha_E(v_E^1 - v_E^0) + v_E^0 \\ r v_E^1 = \bar{\theta}\pi + v_E^1 - v_E^0 + \dot{v_E^1} \end{cases}$$

$$(=)\begin{cases} - - - - - \\ v_E^1(r+1) = \bar{\theta}\pi + v_E^0 + \dot{v_E^1} \end{cases}$$

$$(=)\begin{cases} - - - - - \\ v_E^1 = \frac{\bar{\theta}\pi + v_E^0 + \dot{v_E^1}}{(r+1)} \end{cases}$$

$$(=)\begin{cases} r v_E^0 = \alpha_E \left[\left(\frac{\bar{\theta}\pi + v_E^0 + \dot{v_E^1}}{r-1} \right) - v_E^0 \right] + v_E^0 \\ - - - - - \end{cases}$$

$$(=)\begin{cases} r v_E^0 = \alpha_E * \frac{\theta\pi}{r+1} + \alpha_E \frac{v_E^0}{r+1} + \alpha_E \frac{\dot{v_E}^0}{r+1} - \alpha_E * v_E^0 + v_E^0 \\ - - - - - \end{cases}$$

$$(=)\begin{cases} r v_E - \alpha_E * \frac{v_E^0}{r+1} + \alpha_E * v_E^0 - v_E^0 = \alpha_E * \frac{\theta\pi}{r-1} + \alpha_E * \frac{\dot{v_E}^1}{r-1} \\ - - - - - \end{cases}$$

$$(=)\begin{cases} v_E^0(r - \frac{\alpha_E}{r+1} + \alpha_E - 1) \quad = \alpha_E * \frac{\theta\pi}{r+1} + \alpha_E * \frac{\dot{v_E}^1}{r+1} \\ - - - - - \end{cases}$$

$$(=)\begin{cases} v_E^0 = \frac{\frac{\alpha_E * \theta\pi + \alpha_E \dot{v_E}^1}{r+1}}{r - \frac{\alpha_E}{r+1} + \alpha_E - 1} \\ - - - - - \end{cases}$$

$$(=)\begin{cases} v_E^0 = \frac{\frac{\alpha_E * \theta\pi 0 + \alpha_E * \dot{v_E}^1}{r+1}}{\frac{r^2 + r - \alpha_E + \alpha_E r + \alpha_E - r + 1}{r+1}} \\ - - - - - \end{cases}$$

$$(=)\begin{cases} v_E^0 = \frac{\alpha_E * \theta\pi + \alpha_E \dot{v_E}^1}{r^2 - \alpha_E + \alpha_E r + \alpha_E + 1} \\ - - - - - \end{cases}$$

6.5 Appendix E: Consumption decisions

$U = \int_0^\infty exp - [-pt] \frac{c(t)^{1-\theta}}{1-\theta} dt$

$\frac{\dot{c}}{c} = \frac{1}{\phi}(r(t) - \rho - \eta)$

$(=) n = \frac{1}{\phi} r - \frac{1}{\phi}\rho - \frac{1}{\phi}\eta$

$(=) r = \eta + \frac{1}{\phi}\rho + \frac{1}{\phi}\eta$

$(=) r = \eta(1 + \phi) + \rho$

6.6 Appendix F: The balanced growth path

$gL_F = gL_E = gL_Y = n$

Dividing the production function of ideas by A

$g_A = \frac{\beta.(L_E)^a (L_F)^{1-a}}{A}$

$g_A = n$

Moreover, dividing the equation of rv_E^1 by v_E^1 and rearranging, we get

$\frac{rv_E^1 = \theta\pi + v_E^0 - v_E^1 + \dot{v_E}^1}{v_E^1}$

$(=) r = \frac{\theta\pi}{v_E^1} + \frac{v_E^0}{v_E^1} - 1 + \frac{\dot{v_E}^1}{v_E^1}$

$(=) 1 + r - \frac{v_E^0}{v_E^1} - \frac{\dot{v_E}^1}{v_E^1} = \frac{\theta\pi}{v_E^1}$

$v_E^0 = \frac{\bar{\theta}\pi\alpha_E}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}$

$v_E^1 = \frac{\bar{\theta}\pi + v_E^0}{1+\eta\phi+\rho}$

$v_F^0 = \frac{(1-\bar{\theta})\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}$

$v_F^1 = \frac{(1-\bar{\theta})\pi + v_F^0}{1+\eta\phi+\rho}$

We can characterize the steady state equilibrium as the triple (l_Y, l_E, l_F) that solves the following system:

$$
\begin{cases}
v_E^0 = \Phi w \\
v_F^0 = \Phi w \\
l_E + l_F + l_r = 1
\end{cases}
$$

$$
(=)\begin{cases}
\frac{\bar{\theta}\pi\alpha_E}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} = \Phi((1-\gamma)\frac{Y}{L_Y}) \\
\frac{(1-\bar{\theta})\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2} = \Phi((1-\gamma)\frac{Y}{L_Y}) \\
l_E + l_F + l_r = 1
\end{cases}
$$

$$
(=)\begin{cases}
\frac{\bar{\theta}\pi\alpha_E}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} = \frac{\Phi((1-\gamma)\frac{Y}{L_Y}).(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} \\
\frac{(1-\bar{\theta})\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2} = \frac{\Phi((1-\gamma)\frac{Y}{L_Y}).(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} \\
l_E + l_F + l_r = 1
\end{cases}
$$

$$
(=)\begin{cases}
\frac{\bar{\theta}\pi\alpha_E}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} = \frac{\Phi((1-\gamma)\frac{Y}{L_Y}).(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} \\
\frac{(1-\bar{\theta})\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2} = \frac{\Phi((1-\gamma)\frac{Y}{L_Y}).(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} \\
l_E + l_F + l_r = 1
\end{cases}
$$

$$
(=)\begin{cases}
\frac{\frac{\bar{\theta}\pi\alpha_E}{\Phi(1-\gamma)\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} = 1 \\
\frac{\frac{(1-\bar{\theta})\pi\alpha_F}{\Phi(1-\gamma)\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2} = 1 \\
l_E + l_F + l_r = 1
\end{cases}
$$

$$
(=)\begin{cases}
\frac{\frac{\bar{\theta}(\gamma(1-\gamma)\frac{Y}{A})(\frac{\dot{A}}{L_F})}{\Phi(1-\gamma)\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} = 1 \\
\frac{\frac{(1-\bar{\theta})(\gamma(1-\gamma)\frac{Y}{A})(\frac{\dot{A}}{L_E})}{\Phi(1-\gamma)\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2} = 1 \\
l_E + l_F + l_r = 1
\end{cases}
$$

$$
(=)\begin{cases}
\frac{\frac{\bar{\theta}\gamma(1-\gamma)Y\frac{\dot{A}}{A}\cdot\frac{1}{L_E}}{\Phi(1-\gamma)\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2} = 1 \\
\frac{\frac{(1-\bar{\theta})\gamma(1-\gamma)Y\frac{\dot{A}}{A}\cdot\frac{1}{L_F}}{\Phi(1-\gamma)\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2} = 1 \\
l_E + l_F + l_r = 1
\end{cases}
$$

$$
(=)\begin{cases}
\dfrac{\frac{\bar{\theta}\gamma(1-\gamma)Y.n.\frac{1}{L_E}}{\Phi(1-\gamma)\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\
\dfrac{\frac{(1-\bar{\theta})\gamma(1-\gamma)Y.n.\frac{1}{L_F}}{\Phi(1-\gamma)\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\
l_E+l_F+l_r=1
\end{cases}
$$

$$
(=)\begin{cases}
\dfrac{\frac{\bar{\theta}\gamma.Y.n.\frac{1}{L_E}}{\Phi.\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\
\dfrac{\frac{(1-\bar{\theta})\gamma.Y.n.\frac{1}{L_F}}{\Phi.\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\
l_E+l_F+l_r=1
\end{cases}
$$

$$
(=)\begin{cases}
\dfrac{\frac{1}{\Phi}\eta\bar{\theta}\gamma Y\frac{\frac{1}{L_E}}{\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\
\dfrac{\frac{1}{\Phi}\eta(1-\bar{\theta})\gamma Y\frac{\frac{1}{L_F}}{\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\
l_E+l_F+l_r=1
\end{cases}
$$

$$
(=)\begin{cases}
\dfrac{\frac{1}{\Phi}\eta\bar{\theta}\gamma Y\frac{\frac{1}{L_E}}{\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\
\dfrac{\frac{1}{\Phi}\eta(1-\bar{\theta})\gamma Y\frac{\frac{1}{L_F}}{\frac{Y}{L_Y}}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\
l_E+l_F+l_r=1
\end{cases}
$$

$$
(=)\begin{cases}
\dfrac{\frac{1}{\Phi}\eta\bar{\theta}\gamma Y\frac{1.L_Y}{Y.L_E}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\
\dfrac{\frac{1}{\Phi}\eta(1-\bar{\theta})\gamma Y\frac{1.L_Y}{Y.L_F}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\
l_E+l_F+l_r=1
\end{cases}
$$

$$
(=)\begin{cases}
\dfrac{\frac{1}{\Phi}\eta\bar{\theta}\gamma\frac{L_Y}{L_E}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\
\dfrac{\frac{1}{\Phi}\eta(1-\bar{\theta})\gamma\frac{L_Y}{L_F}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\
l_E+l_F+l_r=1
\end{cases}
$$

$$
(=)\begin{cases}
\dfrac{\frac{1}{\Phi}\eta\bar{\theta}\gamma\frac{l_Y}{l_E}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\
\dfrac{\frac{1}{\Phi}\eta(1-\bar{\theta})\gamma\frac{l_Y}{l_F}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\
l_E+l_F+l_r=1
\end{cases}
$$

6.7 Appendix G: The steady state under Nash bargaining of monopoly rents

We can define the surpluses as:

$S_{E=}v_E^1 - v_E^0 = \frac{\theta\pi+v_E^0}{1+n\phi+\rho} - v_E^0 = \frac{\theta\pi - v_E^0(\eta\phi+\rho)}{1+\eta\phi+\rho}$

$S_F = v_F^1 - v_F^0 = \frac{(1-\theta)\pi+v_F^0}{1+\eta\phi+\rho} - v_F^0 = \frac{(1-\theta)\pi - v_F^0(\eta\phi+\rho)}{1+\eta\phi+\rho}$

The Nash bargaining problem is the one of maximizing the following expression in respect to θ

$(S_E)^\sigma.(S_F)^{1-\sigma} = 0$

$(=)\sigma(S_E)^{\sigma-1}(S_F)^{1-\sigma} + (S_E)^\sigma(1-\sigma)(S_F)^{(1-\sigma)-1}(-1)$

$(=)\sigma(S_E)^{\sigma-1}(S_F)^{1-\sigma} + (S_E)^\sigma(1-\sigma)(S_F)^{-\sigma}(-1)$

$(=)\sigma(S_E)^{\sigma-1}(S_F)^{1-\sigma} - (S_E)^\sigma(1-\sigma)(S_F)^{-\sigma}$

$(=)\sigma S_F - (1-\sigma)S_E = 0$

$\sigma S_F - (1-\sigma)S_E = 0$

Introduing the values for S_E, S_F

$\sigma(\frac{(1-\theta)\pi - v_F^0(\eta\phi+\rho)}{1+\eta\phi+\rho}) - (1-\sigma)(\frac{\theta\pi - v_E^0(\eta\phi+\rho)}{1+\eta\phi+\rho}) = 0$

$(=)\frac{\sigma(1-\theta)\pi - \sigma v_F^0(\eta\phi+\rho) - (1-\sigma)(\theta\pi) + (1-\sigma)(v_E^0(n\phi+\rho))}{1+n\phi+\rho} = 0$

$(=) -\sigma v_F^0(\eta\theta+\rho) + (1-\sigma)v_E^0(\eta\phi+\rho) + \sigma\pi = \theta\pi$

Knowing that

$v_E^0 = \frac{\bar{\theta}\pi\alpha_E}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}$

$v_F^0 = \frac{(1-\theta)\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}$

Plugging these values

$\theta\pi = \sigma\pi + (1-\sigma)(\frac{\theta\pi\alpha_E}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2})(\eta\phi+\rho)$-$\sigma(\frac{(1-\bar{\theta})\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}(\eta\phi+\rho)$

$(=)\theta\pi = \sigma\pi + (\frac{(1-\sigma)\bar{\theta}\pi\alpha_E}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2})(\eta\phi+\rho) - (\frac{\sigma(1-\bar{\theta})\pi\alpha_F}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}(\eta\phi+\rho)$

$(=)\theta\pi = \sigma\pi + (\frac{(1-\sigma)\bar{\theta}\pi\alpha_E}{\eta\phi+\rho+1+\alpha_E}) - (\frac{\sigma(1-\bar{\theta})\pi\alpha_F}{\eta\phi+\rho+1+\alpha_F})$

Finally, knowing that in equilibrium $\theta = \bar{\theta}$,we can solve to $\bar{\theta}$

$\theta\pi = \sigma\pi + (\frac{(1-\sigma)\bar{\theta}\pi\alpha_E}{\eta\phi+\rho+1+\alpha_E}) - (\frac{\sigma(1-\bar{\theta})\pi\alpha_F}{\eta\phi+\rho+1+\alpha_F})$

$(=)\frac{\bar{\theta}\pi(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)} = \frac{\sigma\pi(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$

$+\frac{((1-\sigma)\bar{\theta}\pi\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)} - \frac{(\sigma(1-\bar{\theta})\pi\alpha_F)(\eta\phi+\rho+1+\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$

$(=)\frac{\bar{\theta}(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)\bar{\theta}}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)} = \frac{\sigma\pi(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$

$+\frac{((1-\sigma)\bar{\theta}\pi\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)} - \frac{(\sigma(1-\bar{\theta})\pi\alpha_F)(\eta\phi+\rho+1+\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)} - \frac{\pi(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$

$(=)\frac{\bar{\theta}(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)\bar{\theta}}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)} = \frac{\sigma\pi(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)} + \frac{(1-\sigma)(\pi\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$

$$-\frac{(\sigma)(\pi\alpha_F)(\eta\phi+\rho+1+\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}-\frac{\pi(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$(=)\frac{\theta(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}=\frac{\sigma}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}+\frac{(1-\sigma)(\pi\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$-\frac{(\sigma)(\pi\alpha_F)(\eta\phi+\rho+1+\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$(=)\frac{\theta(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}=\frac{(1-\sigma)(\pi\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}-\frac{(\pi\alpha_F)(\eta\phi+\rho+1+\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$(=)\frac{\theta(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}=\frac{(1-\sigma)(\pi\alpha_E)(\eta\phi)+(1-\sigma)(\pi\alpha_E)\rho+(1-\sigma)(\pi\alpha_E)+(1-\sigma)(\pi\alpha_E)(\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$-\frac{(\pi\alpha_F)(\eta\phi)+(\pi\alpha_F)(\rho)+(\pi\alpha_F)+(\pi\alpha_F)(\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$(=)\frac{\theta(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}=$$

$$\frac{(-\sigma)(\pi\alpha_E)(\eta\phi)+(\pi\alpha_E)(\eta\phi)+(-\sigma)(\pi\alpha_E)(\rho)+(\pi\alpha_E)(\rho)+(-\sigma)(\pi\alpha_E)+(\pi\alpha_E)+(-\sigma)(\pi\alpha_E)(\alpha_F)+(\pi\alpha_E)(\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$-\frac{(\pi\alpha_F)(\eta\phi)+(\pi\alpha_F)(\rho)+(\pi\alpha_F)+(\pi\alpha_F)(\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$(=)\frac{\theta(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}=$$

$$\frac{(-\sigma)(\pi\alpha_E)(\eta\phi)+(\pi\alpha_E)(\eta\phi)+(-\sigma)(\pi\alpha_E)(\rho)+(\pi\alpha_E)(\rho)+(-\sigma)(\pi\alpha_E)+(\pi\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

$$(=)+\frac{(-\sigma)(\pi\alpha_E)(\alpha_F)+(\pi\alpha_E)(\alpha_F)-(\pi\alpha_F)(\eta\phi)-(\pi\alpha_F)(\rho)-(\pi\alpha_F)-(\pi\alpha_F)(\alpha_E)}{(\eta\phi+\rho+1+\alpha_E)(\eta\phi+\rho+1+\alpha_F)}$$

...

$$(=)\bar{\theta}^b=\frac{\sigma(\alpha_E+1+\eta\phi+\rho)}{1+n\phi+\rho+\sigma\alpha_E+(1-\sigma)\alpha_F}$$

Plugging the expression for $\bar{\theta}^b$given the above system and simplifying where possible we finally obtain

$$\begin{cases}\frac{\frac{1}{\Phi}\eta\bar{\theta}\gamma\frac{l_Y}{l_E}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\ \frac{\frac{1}{\Phi}\eta(1-\bar{\theta})\gamma\frac{l_Y}{l_F}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\ l_E+l_F+l_r=1\end{cases}$$

$$(=)\begin{cases}\frac{\frac{1}{\Phi}\eta\frac{\sigma(\alpha_E+1+\eta\phi+\rho)}{1+n\phi+\rho+\sigma\alpha_E+(1-\sigma)\alpha_F}\gamma\frac{l_Y}{l_E}}{(\eta\phi+\rho)(1+\alpha_E)+(\eta\phi+\rho)^2}=1\\ \frac{\frac{1}{\Phi}\eta(1-(\frac{\sigma(\alpha_E+1+\eta\phi+\rho)}{1+n\phi+\rho+\sigma\alpha_E+(1-\sigma)\alpha_F}))\gamma\frac{l_Y}{l_F}}{(\eta\phi+\rho)(1+\alpha_F)+(\eta\phi+\rho)^2}=1\\ l_E+l_F+l_r=1\end{cases}$$

$$\begin{cases}\frac{\eta\gamma l_Y}{(\eta\theta+\rho)(1+\eta\phi+\rho+\sigma\alpha_E+(1-\sigma)\alpha_F)}=\frac{l_E}{\sigma}\\ \frac{\eta\gamma l_Y}{(\eta\theta+\rho)(1+\eta\phi+\rho+\sigma\alpha_E+(1-\sigma)\alpha_F)}=\frac{l_F}{1-\sigma}\\ l_E+l_F+l_Y=1\end{cases}$$

Given that $l_E^b/l_F^b=\sigma/(1-\sigma)$, the probabilities of matching along the steady state can be written as

$$\alpha_E^b=\beta(\frac{l_F^b}{l_E^b})^{1-\alpha}=\beta(\frac{1-\sigma}{\sigma})^{1-a}$$

$$\alpha_F^b=\beta(\frac{l_E^b}{l_F^b})=\beta(\frac{\sigma}{1-\sigma})^a$$

$$
\begin{cases}
\frac{\frac{1}{\Phi}\eta\gamma l_Y}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = \frac{l_E}{\sigma} \\
\frac{\frac{1}{\Phi}\eta\gamma l_Y}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = \frac{l_F}{1-\sigma} \\
l_E + l_F + l_Y = 1
\end{cases}
$$

$$
(=)\begin{cases}
\frac{\frac{1}{\Phi}\eta\gamma l_Y(\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = l_E \\
\frac{\frac{1}{\Phi}\eta\gamma l_Y(1-\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = l_F \\
l_E + l_F + l_Y = 1
\end{cases}
$$

$$
(=)\begin{cases}
\frac{\frac{1}{\Phi}\eta\gamma l_Y(\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = l_E \\
\frac{\frac{1}{\Phi}\eta\gamma l_Y(1-\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = l_F \\
\frac{\frac{1}{\Phi}\eta\gamma l_Y(\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} + \frac{\frac{1}{\Phi}\eta\gamma l_Y(1-\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} + l_Y = 1
\end{cases}
$$

$$
(=)\begin{cases}
--- \\
--- \\
\frac{\frac{1}{\Phi}\eta\gamma l_Y(\sigma)+\frac{1}{\Phi}\eta\gamma l_Y(1-\sigma)+l_Y((\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))} = 1
\end{cases}
$$

$$
(=)\begin{cases}
--- \\
--- \\
\frac{\frac{1}{\Phi}l_Y(\eta\gamma\sigma+\eta\gamma-\eta\gamma\sigma)+l_Y((\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = 1
\end{cases}
$$

$$
(=)\begin{cases}
--- \\
--- \\
\frac{\frac{1}{\Phi}l_Y(\eta\gamma)+l_Y((\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = 1
\end{cases}
$$

$$
(=)\begin{cases}
--- \\
--- \\
\frac{1}{\Phi}l_Y = \frac{((\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})).((\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))}{((\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})).(\eta\gamma+(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))}
\end{cases}
$$

$$
(=)\begin{cases}
--- \\
--- \\
\frac{1}{\Phi}l_Y = \frac{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})}{\eta\gamma+(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})}
\end{cases}
$$

$$(=)\begin{cases} --- \\ --- \\ l_Y = \frac{\Phi(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})}{\eta\gamma+(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})} \end{cases}$$

$$(=)\begin{cases} \frac{\eta\gamma l_Y(\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} = l_E \\ \frac{\eta\gamma l_Y(1-\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} = l_F \\ --- \end{cases}$$

$$(=)\begin{cases} l_E = \frac{(\eta\gamma\sigma)(\frac{\Phi((\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a}))}{\eta\gamma+(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})})}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} \\ l_F = \frac{(\eta\gamma-\eta\sigma\gamma)(\frac{\Phi((\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})}{\eta\gamma+(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})})}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} \\ --- \end{cases}$$

$$(=)\begin{cases} l_E = \frac{(\eta\gamma\sigma)(\frac{\Phi((\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a}))}{\eta\gamma+(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})})}{(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})} \\ l_F = \frac{(\eta\gamma-\eta\sigma\gamma)(\frac{\Phi((\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a}))}{\eta\gamma+(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})})}{(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})} \\ --- \end{cases}$$

$$(=)\begin{cases} l_E = \frac{\Phi(\eta\gamma\sigma)}{\eta\gamma+(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})} \\ l_F = \frac{\Phi\eta\gamma(1-\sigma)}{(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})} \\ l_Y = \frac{\Phi((\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a}))}{\eta\gamma+(\phi\eta+\rho)^2+(\phi\eta+\rho)(1+2\beta\sigma^a(1-\sigma)^{1-a})} \end{cases}$$

Correção última derivação (resultdos desta foram os incluídos no modelo)

$$\begin{cases} \frac{\frac{1}{\Phi}\eta\gamma l_Y}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} = \frac{l_E}{\sigma} \\ \frac{\frac{1}{\Phi}\eta\gamma l_Y}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} = \frac{l_F}{1-\sigma} \\ l_E + l_F + l_Y = 1 \end{cases}$$

$$(=)\begin{cases} \frac{\frac{1}{\Phi}\eta\gamma l_Y(\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} = l_E \\ \frac{\frac{1}{\Phi}\eta\gamma l_Y(1-\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} = l_F \\ l_E + l_F + l_Y = 1 \end{cases}$$

$$(=)\begin{cases} \frac{\frac{1}{\Phi}\eta\gamma l_Y(\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} = l_E \\ \frac{\frac{1}{\Phi}\eta\gamma l_Y(1-\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} = l_F \\ \frac{\frac{1}{\Phi}\eta\gamma l_Y(\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} + \frac{\frac{1}{\Phi}\eta\gamma l_Y(1-\sigma)}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^a)} + l_Y = 1 \end{cases}$$

$$
\begin{aligned}
(=)&\begin{cases} - - - \\ - - - \\ \frac{\frac{1}{\Phi}\eta\gamma l_Y(\sigma)+\frac{1}{\Phi}\eta\gamma l_Y(1-\sigma)+l_Y((\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))} = 1 \end{cases} \\
(=)&\begin{cases} - - - \\ - - - \\ \frac{\frac{1}{\Phi}l_Y(\eta\gamma\sigma+\eta\gamma-\eta\gamma\sigma)+l_Y((\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a}))}{(\eta\theta+\rho)(1+\eta\theta+\rho+\sigma(\beta(\frac{1-\sigma}{\sigma})^{1-a})+(1-\sigma)(\beta(\frac{\sigma}{1-\sigma})^{a})} = 1 \end{cases} \\
(=)&\begin{cases} - - - \\ - - - \\ \frac{\frac{1}{\Phi}l_Y(\eta\gamma)+l_Y Z}{Z} = 1 \end{cases} \\
(=)&\begin{cases} - - - \\ - - - \\ \frac{l_Y(\frac{\eta\gamma}{\Phi}+Z)}{Z} = 1 \end{cases} \\
(=)&\begin{cases} - - - \\ - - - \\ l_Y(\frac{\eta\gamma}{\Phi}+Z) = Z \end{cases} \\
(=)&\begin{cases} - - - \\ - - - \\ l_Y = \frac{Z}{(\frac{\eta\gamma}{\Phi}+Z)} \end{cases} \\
(=)&\begin{cases} - - - \\ - - - \\ l_Y = \frac{Z}{(\theta^{-1}\eta\gamma+Z)} \end{cases} \\
(=)&\begin{cases} l_E = \frac{\theta^{-1}\eta\gamma l_Y(\sigma)}{Z} \\ l_F = \frac{\theta^{-1}\eta\gamma l_Y(1-\sigma)}{Z} \\ - - - \end{cases}
\end{aligned}
$$

$$
(=)\begin{cases} l_E = \frac{\theta^{-1}\eta\gamma\sigma\frac{Z}{(\theta^{-1}\eta\gamma+Z)}}{Z} \\ l_F = \frac{\theta^{-1}\eta\gamma(1-\sigma)\frac{Z}{(\theta^{-1}\eta\gamma+Z)}}{Z} \\ --- \end{cases}
$$

$$
(=)\begin{cases} l_E = \frac{\frac{\eta\gamma\sigma}{\Phi}}{\frac{\eta\gamma}{\Phi}+Z} \\ l_F = \frac{\frac{\eta\gamma(1-\sigma)}{\Phi}}{\frac{\eta\gamma}{\Phi}+Z} \\ --- \end{cases}
$$

$$
(=)\begin{cases} l_E = \frac{\eta\gamma\sigma}{\eta\gamma+Z\Phi} \\ l_E = \frac{\eta\gamma(1-\sigma)}{\eta\gamma+Z\Phi} \\ l_Y = \frac{Z}{(\theta^{-1}\eta\gamma+Z)} \end{cases}
$$

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8 Attachments

8.1 Entrepreneurship-growth research papers

#	Authors	Source title	Title	Year	Rel.	F	E	F+E	AP	S
1	Cui W.	European Economic Review	Monetary-fiscal interactions with endogenous liquidity frictions	2016	x	x				
2	Wellalage N.H., Locke S.	Applied Economics	Informality and credit constraints: evidence from Sub-Saharan African MSEs	2016			x			
3	Gries T., Jungblut S., Naudé W.	International Journal of Economic Theory	The entrepreneurship Beveridge curve	2016			x			
4	Guerrero M., Urbano D., Fayolle A.	Journal of Technology Transfer	Entrepreneurial activity and regional competitiveness: evidence from European entrepreneurial universities	2016	x				x	
5	Urbano D., Aparicio S.	Technological Forecasting and Social Change	Entrepreneurship capital types and economic growth: International evidence	2016	x			x		
6	Peretto P.F.	European Economic Review	From Smith to Schumpeter: A theory of take-off and convergence to sustained growth	2015		x				
7	Huggins R., Thompson P.	Small Business Economics	Entrepreneurship, innovation and regional growth: a network theory	2015	x	x				
8	Aghion P., Akcigit U., Cagé J., Kerr W.R.	European Economic Review	Taxation, corruption, and growth	2015						
9	Mino K.	Economics Letters	A simple model of endogenous growth with financial frictions and firm heterogeneity	2015	x	x				
10	Bunten D., Weiler S., Thompson E., Zahran S.	Journal of Regional Science	Entrepreneurship, information, and growth	2015	x			x		
11	Kim D.K.	Seoul Journal of Economics	A schumpeterian analysis of the Saemaul Undong Movement in 1970s South Korea within the CDD framework	2015						
12	Laeven L., Levine R., Michalopoulos S.	Journal of Financial Intermediation	Financial innovation and endogenous growth	2015	x	x				
13	Gupta M.R., E	Hitotsubashi Journal of Economics	Environmental pollution, informal sector, public expenditure and economic growth	2015						
14	d'Agostino G., Scarlato M.	Regional Studies	Innovation, Socio-institutional Conditions and Economic Growth in the Italian Regions	2015	x			x		

Table 1: Entrepreneurship-growth research 1-14

#	Authors	Source title	Title	Year	Rel.	F	E	F+E	AP	S
15	Andrew J.	The Handbook of Service Innovation	Exposing an economic development policy clash: Predictability and control versus creativity and innovation	2015					x	
16	Montmartin B., Massard N.	Journal of Economic Surveys	Is financial support for private R&D always justified? A discussion based on the literature on growth	2015	x				x	
17	Giordani P.E.	Journal of Economics/ Zeitschrift fur Nationalokonomie	Entrepreneurial finance and economic growth	2015	x		x			
18	Klasing M.J.	Journal of Development Economics	Cultural change, risk-taking behavior and implications for economic development	2014						
19	Deligianni I., Voudouris I., Lioukas S.	IEEE Transactions on Engineering Management	The relationship between innovation and diversification in the case of new ventures: Unidirectional or bidirectional?	2014						
20	Godley A.C.	International Journal of the Economics of Business	Creativity and Growth in: Edmund Phelps (2013), Mass Flourishing. How Grassroots Innovation Created Jobs, Challenge and Change	2014						x
21	Nilsen R.	Norsk Geografisk Tidsskrift	Rural modernisation as national development: The Norwegian case 1900-1950	2014						
22	Fischer H., Burton R.J.F.	Sociologia Ruralis	Understanding Farm Succession as Socially Constructed Endogenous Cycles	2014						
23	Huggins R., Izushi H., Prokop D., Thompson P.	Zbornik Radova Ekonomskog Fakultet au Rijeci	Regional competitiveness, Economic growth and stages of development [Regionalna konkurentnost, gospodarski rast i faze razvoja]	2014						
24	Swagemakers P., Copena Rodríguez D., Domínguez García M.D., Simón Fernández X.	Geografisk Tidsskrift	Fighting for a future: an actor-oriented planning approach to landscape preservation in Galicia	2014						

Table 2: Entrepreneurship-growth research 15-24

#	Authors	Source title	Title	Year	Rel.	F	E	F+E	AP	S
25	Acs Z.J., Sanders M.W.J.L.	Small Business Economics	Knowledge spillover entrepreneurship in an endogenous growth model	2013	x		x			
26	Acs Z.J., Audretsch D.B., Lehmann E.E.	Small Business Economics	The knowledge spillover theory of entrepreneurship	2013	x				x	
27	Toprak M., Acar M.	Understanding the Process of Economic Change in Turkey: An Institutional Approach	Authoritarian nationalism and discrimination ending with immiserising modernization: Economic and social consequences of the republican power elite's fight	2013					x	
28	Runiewicz-Wardyn M.	Contributions to Economics	Knowledge flows, technological change and regional growth in the European Union	2013				x		
29	Lebson M.	International Migi	Why Refugees Rebel: Towards a Comprehensive Theory of Refugee Militarization	2013						
30	Carlin A.	Innovations	The effects of luxury consumption on economic growth: An approach based on non-homothetic preferences [Les Effets de la Consommation de Luxe sur la Croissance Économique : Approches par les Préférences Non-Homothétiques]	2013						
31	Urbano D., Guerrero M.	Gender and Society	Entrepreneurial Universities: Socioeconomic Impacts of Academic Entrepreneurship in a European Region	2013						
32	Muñoz F.-F.,	Proceedings - 26th European Conference on Modelling and Simulation, ECMS 2012	Linear modelling and simulation of an endogenous growth model with heterogeneous entrepreneurs	2012		x				
33	Acs Z.J., Sanders M.	Small Business Economics	Patents, knowledge spillovers, and entrepreneurship	2012	x		x			
34	Crescenzi R., Rodríguez-Pose A.	Innovation, Entrepreneurship, Geography and Growth	An "Integrated" Framework for the Comparative Analysis of the Territorial Innovation Dynamics of Developed and Emerging Countries	2012					x	
35	Acs Z.J., Audretsch D.B., Braunerhjelm P., Carlsson B.	Small Business Economics	Growth and entrepreneurship	2012	x		x			

Table 3: Entrepreneurship-growth research 24-35

#	Authors	Source title	Title	Year	Rel.	F	E	F+E	AP	S
36	Crescenzi R., Rodríguez-Pose A.	Journal of Econom	An 'integrated' framework for the comparative	2012						
37	Antony J., Klarl T., Maußner A.	Journal of Economics/ Zeitschrift fur Nationalokonomie	Firm heterogeneity, credit constraints, and en	2012	x	x				
38	Austvik O.G.	Governance	Landlord and Entrepreneur: The Shifting Roles of the State in Norwegian Oil and Gas Policy	2012						
39	Delmar F., Wennberg K., Hellerstedt K.	Strategic Entrepreneurship Journal	Endogenous growth through knowledge spillovers in entrepreneurship: An empirical test	2011	x	x				
40	Masih R., Khan S.F.	Applied Financial Economics	Is the finance led growth hypothesis robust to alternative measures of financial development?	2011	x		x			
41	Karlsen A.	European Planning Studies	"Cluster" creation by reconfiguring communities of practice	2011						
42	Henrekson M., Sanandaji T.	Ekonomiska Samfundets Tidskrift	Bridging the gap between entrepreneurship research and mainstream economics [William J. Baumols senaste försök att slå broar mellan entreprenörskapsforskningen och nationalekonomins huvudfåra]	2010						
43	Dosi G., Nelson R.R.	Handbook of the Economics of Innovation	Technical change and industrial dynamics as evolutionary processes	2010						
44	Strehle F., Katzy B.R., Davila T.	International Journal of Technology Management	Learning capabilities and the growth of technology-based new ventures	2010						
45	Jiang N., Wang P., Wu H.	Journal of Economic Dynamics and Control	Ability-heterogeneity, entrepreneurship, and economic growth	2010	x		x			
46	Braunerhjelm P., Acs Z.J., Audretsch D.B., Carlsson B.	Small Business Economics	The missing link: Knowledge diffusion and entrepreneurship in endogenous growth	2010	x	x				

Table 4: Entrepreneurship-growth research 36-46

#	Authors	Source title	Title	Year	Rel.	F	E	F+E	AP	S
47	Stimson R., Stough R.R., Salazar M.	Leadership and Institutions in Regional Endogenous Development	Leadership and institutions in regional endogenous development	2009						
48	Grossmann V.	Journal of Macroeconomics	Entrepreneurial innovation and economic growth	2009	x	x				
49	Haruyama T.	Scottish Journal of Political Economy	Competitive innovation with codified and tacit knowledge	2009						
50	Valliere D., Peterson R.	Entrepreneurship	Entrepreneurship and economic growth: Evidence from emerging and developed countries	2009	x			x		
51	Plehn -Dujowich J.M.	Journal of Economic Dynamics and Control	Endogenous growth and adverse selection in entrepreneurship	2009	x		x			
52	Bajona C., Locay L.	Review of Economic Dynamics	Entrepreneurship and productivity: The slow growth of the planned economies	2009						
53	Dekkers R.	Asia Pacific Business Review	Endogenous innovation in China: The case of the printer industry	2009						
54	Acs Z.J., Braunerhjelm P., Audretsch D.B., Carlsson B.	Small Business Economics	The knowledge spillover theory of entrepreneurship	2009	x				x	
55	Gries T., Naudé W.	Small Business Economics	Entrepreneurship and structural economic transformation	2009	x		x			
56	Audretsch D.B., Aldridge T., Oettl A.	Entrepreneurship, Growth, and Public Policy	Scientist commercialization and knowledge transfer?	2009						
57	Bahmani -Oskooee M., Galindo M.-A., Méndez M.T.	Entrepreneurship and Business: A Regional Perspective	Social capital and entrepreneurship in a regional context: The case of Spanish regions	2008						

Table 5: Entrepreneurship-growth research 47-57

#	Authors	Source title	Title	Year	Rel.	F	E	F+E	AP	S
58	Piotr D., Rekowski M.	Entrepreneurship and Business: A Regional Perspective	The relationship between entrepreneurship and economic growth: A review of recent research achievements	2008	x					x
59	Micek G.	Przeglad Geograficzny	Industrial clusters as the subject of analysis	2008						
60	Sequeira T.N.	Economic Modelling	On the effects of human capital and R&D policies in an endogenous growth model	2008						
61	Clemens C.	German Economic Review	Imperfect competition and growth with entrepreneurial risk	2008	x	x				
62	Naudé W., Gries T., Wood E., Meintjies A.	Entrepreneurship and Regional Development	Regional determinants of entrepreneurial start-ups in a developing country	2008	x		x			
63	Baldwin R.E., Robert-Nicoud F.	Journal of International Economics	Trade and growth with heterogeneous firms	2008	x			x		
64	Asselain J.-C.	Revue Economique	Business history and global approches: What convergences? [Histoire des entreprises et approches globales. Quelles convergences?]	2007						
65	Austvik O.G.	Tidsskrift for Samfunnsforskning	The government as a petroleum entrepreneur [Staten som petroleumsentreprenør]	2007						
66	Ha J., Howitt P.	Journal of Money, Credit and Banking	Accounting for trends in productivity and R&D: A Schumpeterian critique of semi-endogenous growth theory	2007	x				x	
67	Boschini A.D.	European Journal of Political Economy	The political economy of industrialisation	2006						
68	Voigtländer N., Voth H.-J.	Journal of Economic Growth	Why England? Demographic factors, structural change and physical capital accumulation during the Industrial Revolution	2006						
69	Clemens C., Heinemann M.	Journal of Economics/ Zeitschrift fur Nationalokonomie	On the effects of redistribution on growth and entrepreneurial risk-taking	2006	x		x			
70	Aghion P., Howitt P.	Handbook of Economic Growth	Chapter 2 Growth with Quality-Improving Innovations: An Integrated Framework	2005	x		x			

Table 6: Entrepreneurship-growth research 58-70

#	Authors	Source title	Title	Year	Rel.	F	E	F+E	AP	S
71	Henrekson M.	Industrial and Corporate Change	Entrepreneurship: A weak link in the welfare state?	2005	x		x			
72	Reiß J.P., Weinert L.	Journal of Macroeconomics	Entrepreneurs, moral hazard, and endogenous growth	2005	x	x				
73	Josten S.D.	Journal of Institutional and Theoretical Economics	Social capital, inequality, and economic growth	2004						
74	Audretsch D.B., Feldman M.P.	Handbook of Regional and Urban Economics	Chapter 61 Knowledge spillovers and the geography of innovation	2004	x				x	
75	O'Gorman C., Kautonen M.	Entrepreneurship and Regional Development	Policies to promote new knowledge-intensive industrial agglomerations	2004						
76	Alcouffe A., Kuhn T.	Journal of Evolutionary Economics	Schumpeterian endogenous growth theory and evolutionary economics	2004	x	x				
77	Buckler S., Dolowitz D.P.	New Political Economy	Can fair be efficient? New labour, social liberalism and British economic policy	2004						
78	Fagiolo G., Dosi G.	Structural Change and Economic Dynamics	Exploitation, exploration and innovation in a model of endogenous growth with locally interacting agents	2003						
79	Waelbroeck P.	Economics Letters	Innovations, production complexity and the optimality of R&D	2003						
80	Ozawa T.	Journal of Asian Economics	Pax American-led macro-clustering and flying-geese-style catch-up in East Asia: Mechanisms of regionalized endogenous growth	2003						
81	Michelacci C.	Economic Journal	Low returns in R&D due to the lack of entrepreneurial skills	2003	x	x				

Table 7: Entrepreneurship-growth research 71-81

#	Authors	Source title	Title	Year	Rel.	F	E	F+E	AP	S
82	Bishop K., Filatotchev I., Mickiewicz T.	Acta Oeconomica	Endogenous ownership structure: Factors affecting the post-privatisation equity in the largest Hungarian firms	2002						
83	Ferreira R.D.S., Lloyd-Braga T.	Economic Theory	Can market power sustain endogenous growth in overlapping-generations economies?	2002						
84	Hellwig M., Irmen A.	Journal of Economic Theory	Endogenous technical change in a competitive economy	2001						
85	Li C.-W.	Manchester School	Science, diminishing returns and long waves	2001						
86	Zweimüller J.	Empirica	Inequality, redistribution, and economic growth	2000	x					x
87	Belussi F.	Human Systems Management	Path-dependency vs. industrial dynamics: An analysis of two heterogeneous districts	1999						
88	Ritsilä J.J.	Entrepreneurship and Regional Development	Regional differences in environments for enterprises	1999						
89	Mouritsen J.	Management Acco	Driving growth: Economic Value Added versus Intellectual Capital	1998						
90	Corriveau L.	Economica	Innovation races, strategic externalities and endogenous growth	1998						
91	Musyck B.	Regional Studies	Autonomous Industrialization in South West Flanders (Belgium): Continuity and Transformation	1995						
92	Englmann F.C.	Journal of Evolutic	A Schumpeterian model of endogenous innovation and growth	1994	x			x		
93	King R.G., Levine R.	Journal of Moneta	Finance, entrepreneurship and growth	1993	x		x			

Table 8: Entrepreneurship-growth research 82-93

Papers relating trade unions - endogenous growth								
Ano	Número artigos	Relevantes	F	E	F+E	AP	S	
1993	1	1	x					
1994	1	0						
1995	2	0						
1996	1	0						
1997	0	0						
1998	0	0						
1999	2	0						
2000	2	2	x		x			
2001	1	0						
2002	1	1		x				
2003	2	0						
2004	7	4	x	x		xx		
2005	5	1	x					
2006	3	1		x				
2007	7	2		x		x		
2008	5	2		x			x	
2009	4	1				x		
2010	5	2	x	x				
2011	8	4	x	x	x	x		
2012	5	1			x			
2013	10	5	x	xx	x	x		
2014	11	3		x		x	x	
2015	3	2	x			x		
2016	3	1				x		
	89		8	10	4	9	2	33
			24%	30%	12%	27%	6%	37%

Table 9: Trade unions-growth research papers

8.2 Trade unions-growth research papers

For simplification purposes, for the unions-growth research papers only the excell statistics are presented in this document.