Weak and Strong Hysteresis in the dynamics of Labor Demand

by

Paulo Ricardo Tavares Mota

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Supervisor

José Varejão

Co-Supervisor

Paulo Beleza Vasconcelos

Faculdade Economia
Universidade do Porto
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The Author

Paulo Ricardo Tavares Mota obtained his degree in Economics from the Faculdade de Economia – Universidade do Porto (1997) and he concluded his M.Sc. in Economics in 2001 at the same university with the dissertation *O Impacto das Políticas de Gestão da Procura sobre o Desemprego de Equilíbrio (The Impact of Demand Management Policies on Equilibrium Unemployment).*

From 1997 to 1998 he worked in the Centro Regional de Segurança Social do Norte – Direcção de Serviços de Gestão Financeira, and in 1999 he was a consultant of the Bolsa de Valores de Lisboa e Porto - Instituto de Mercado de Capitais (Stock Exchange).

From 2000 to 2001 he was involved in the research of the thematic network *The Social Construction of Employment* funded by the DG Research of the European Commission. In 2001, he participated in the Conference "Rémunération et Protection Sociale: Qui Paie?"

He has been a lecturer at the Faculdade de Economia – Universidade do Porto since 1998, where he teaches International Monetary Economics and Money and Banking.

In 2007 he participated in the conference 2nd International Symposium on Economic Theory, Policy and Applications – Athens Institute for Education and Research, and in the Econophysics Colloquium and Beyond – Polytechnic University of Marche – Ancona, with the paper *Weak and Strong Hysteresis in the Dynamics of Employment.*

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ABSTRACT

Previous empirical studies have shown that when decisions are made under uncertainty and adjustment costs are fixed or linear in structure (non-convex), firms do not permanently adjust employment in order to accommodate demand shocks. Consequent to this, periods of inertia would emerge and that is sufficient to produce hysteresis.

This dissertation studies the existence of hysteresis in the dynamic path of employment at the firm and aggregate level. Firstly, we describe the path of micro-level employment and we establish its relationship with three sources of inertia: *i*) the existence of non-convex costs of adjustment; *ii*) uncertainty concerning the dynamics of aggregate product demand; *iii*) utilization of the intensive margin of adjustment of the labor input (adjustment through hours per employee). Secondly, we analyze the aggregate implications of the observed micro behavior.

If at the micro level models of hysteresis offer a good explanation for the empirical evidence, at the macro level it has been more difficult to identify the existence of hysteresis in the dynamics of employment. Aggregate series of employment tend to look smoother and, for that reason, they are apparently inconsistent with the presence of hysteresis. However, if we take into consideration the different properties of weak hysteresis (hysteresis at the micro level) and strong hysteresis (hysteresis at the macro level), and if we take into account firms' heterogeneity, i.e. if the problem of aggregation is explicitly considered as it should be in the presence of non-convex costs of adjustment, it would still be possible to uncover signs of hysteresis at the macro-level.

The empirical analysis was carried out with a monthly panel of Portuguese manufacturing firms spanning a period of eleven years. This dataset has information on both employment and hours of work as well as on a good set of other variables that may be taken as proxies for shocks. To obtain a first insight into the process of employment adjustment, we provide some descriptive statistics on net employment changes, and to test the existence of hysteresis at the micro level we estimate a model of employment asymmetric response with path dependence interpreted under the Non-Ideal Relay model of hysteresis. To test the existence of hysteresis in the aggregate employment dynamics, we apply tests constructed with the help of computational methods based on the Preisach Model and on the Linear Play Model of Hysteresis. To put our results in an

international setting, the aggregate analysis was also made with aggregate data from OECD and EUROSTAT.

We conclude that: *i*) there are strong signs of the existence of sources of employment inertia at micro level, caused by non-convex adjustment costs and by the adjustment of labor input through the number of hours per employee; *ii*) signs of hysteresis commonly found at the micro level, do not completely vanish at the macro level; *iii*) hysteresis properties are particularly discernible for small firms even if they are less so in the case of larger units; *vi*) we find strong evidence of the interrelations between the flexibility of the labor input adjustment through hours of work and the existence of aggregate employment hysteresis, but only weak evidence of the interrelations between the existence of uncertainty in the dynamics of aggregate demand and hysteresis.

These findings imply that aggregate employment is significantly shaped by lumpy adjustment at the micro level.

JEL Classification: E24, J23.

Keywords: hysteresis, adjustment costs, employment, uncertainty, hours of work

RESUMO

Estudos empíricos anteriores mostram que quando as decisões são tomadas num contexto de incerteza e quando existem custos de ajustamento lineares ou fixos (não convexos), as empresas não ajustam continuamente o nível de emprego de forma a acomodar choques da procura do seu produto. Consequentemente, emergem períodos de inércia o que é suficiente para produzir histerese.

Nesta dissertação estuda-se a existência de histerese da dinâmica do emprego ao nível da empresa e ao nível agregado. Em primeiro lugar, efectua-se uma descrição do padrão de ajustamento do emprego a nível microeconómico e estuda-se a sua relação com três fontes de inércia: *i*) a existência de custos de ajustamento não convexos; *ii*) a existência de incerteza na dinâmica da procura agregada; *iii*) a possibilidade de utilização da margem intensiva de ajustamento do factor trabalho (ajustamento através do número de horas por trabalhador). Segundo, analisamos as implicações agregadas do comportamento microeconómico observado.

Se ao nível microeconómico os modelos de histerese oferecem uma boa explicação para a observação empírica, ao nível macroeconómico tem-se revelado mais difícil identificar a existência de histerese na dinâmica do emprego. De facto, as séries agregadas do emprego tendem a ser mais alisadas, e por essa razão, aparentemente inconsistentes coma existência de histerese. No entanto, se tivermos em conta as diferentes propriedades da histerese fraca (histerese ao nível micro) e da histerese forte (histerese ao nível macro) e se considerarmos a existência de empresas heterogéneas, isto é, se o problema da agregação for explicitamente considerado, como deve ser na presença de custos de ajustamento não convexos, então deverá ser possível verificar a existência de sinais de histerese ao nível macroeconómico.

A análise empírica foi efectuada com dados mensais de empresas industriais portuguesas ao longo de um período de 11 anos. A amostra contém informação sobre o nível de emprego e sobre o nível de horas de trabalho e sobre um conjunto de outras variáveis que podem ser utilizadas como *proxies* de choques.

No sentido de obter uma primeira aproximação ao processo de ajustamento do emprego, efectuamos uma análise descritiva sobre a variação líquida do emprego e testamos a existência de histerese ao nível da empresa através da estimação de um modelo de resposta assimétrica do emprego, interpretado á luz do modelo de histerese *Non-Ideal Relay*.

De forma a testar a existência de histerese na dinâmica do emprego a nível agregado, aplicamos testes construídos com base em métodos computacionais baseados no modelo de Preisach e no *Linear Play Model* de histerese.

No sentido de comparar os resultados a nível internacional, aplicamos os testes referidos a dados agregados da OCDE e EUROSTAT de 19 países da OCDE.

Concluímos que: *i*) existem sinais claros da existência de inércia ao nível microeconómico causada pela existência de custos de ajustamento não convexos e pela possibilidade de ajustamento através da variação do número de horas de trabalho por trabalhador; *ii*) os sinais de histerese que normalmente se encontram ao nível microeconómico não se desvanecessem totalmente ao nível macroeconómico; *iii*) as propriedades de histerese são particularmente relevantes na dinâmica do emprego das empresas pequenas; *iv*) encontramos evidência significativa sobre a interacção entre a flexibilidade do ajustamento do factor trabalho através da variação do número de horas de trabalho por trabalhador e a existência de histerese no emprego. Ao contrário, não encontramos evidência significativa sobre a interacção entre a existência de incerteza na dinâmica da procura agregada e a existência de histerese no emprego.

Estes resultados mostram que a dinâmica do emprego a nível agregado é condicionada significativamente pela existência de um padrão de ajustamento discreto ao nível microeconómico.

Classificação JEL : E24, J23.

Palavras-chave: histerese, custos de ajustamento, emprego, incerteza, horas de trabalho

RÉSUMÉ

Des études empiriques précédentes ont montré que quand les décisions sont prises sous l'incertitude et les coûts d'ajustement sont fixés ou linéaires (non-convexe), les entreprises ne changent pas leur niveau l'emploi pour faire face aux chocs de demande. Par conséquent, il y a des périodes d'inertie et c'est suffisant pour produire l'hystérésis.

Cette thèse étudie l'existence d'hystérésis dans la dynamique d'emploi au niveau de l'entreprise et au niveau agrégé. Premièrement, nous décrivons la dynamique de l'emploi au niveau micro et nous établissons son relation avec trois sources d'inertie: *i*) l'existence de prix non-convexes d'ajustage; *ii*) l'incertitude concernant la dynamique de demande agrégé; *iii*) l'utilisation de la marge intensive d'ajustage de la main-d'œuvre (l'ajustage des heures par employé). Deuxièmement, nous analysons les implications agrégées de la conduite observée au niveau de l'entreprise.

Si au niveau microéconomique, les modèles d'hystérésis offrent une bonne explication de l'évidence empirique, au niveau macroéconomique il a été plus difficile d'identifier l'existence d'hystérésis dans la dynamique d'emploi. La série totale d'emploi a tendance à sembler plus lisse et, pour cette raison, ils sont apparemment inconsistants avec la présence d'hystérésis. Pourtant, si nous prenons en considération les différentes propriétés de l'hystérésis faible (l'hystérésis au niveau micro) et de l'hystérésis forte (l'hystérésis au niveau macro) et si nous tenons compte de la diversité d'entreprises, c'est-à-dire si le problème d'agrégation est explicitement considéré, comme il devrait être en présence des prix non-convexes d'ajustage, il serait toujours possible de dévoiler des signes d'hystérésis au niveau macroéconomique.

L'analyse empirique est faite avec l'information mensuelle des entreprises industrielles portugaises qui s'étendent pendant onze ans. Nous avons information sur l'emploi, sur les heures de travail, et sur un ensemble d'autres variables qui peuvent être prises comme les *proxies* pour les chocs.

Pour obtenir une première représentation dans le processus d'ajustement d'emploi, nous fournissons un peu de statistique descriptive sur les change nets d'emploi, et pour évaluer l'existence d'hystérésis au niveau micro nous estimons un modèle d'emploi de réponse asymétrique, interprétée sous le modèle d'hystérésis *Non-*

Ideal Relay. Pour évaluer l'existence d'hystérésis dans la dynamique d'emploi agrégé, nous appliquons des tests construits avec l'aide de méthodes quantificatives basées sur le Modèle de Preisach et sur le *Linear Play Model* d'Hystérésis. Pour mettre nos résultats contre un fond international, l'analyse agrégée a été aussi faite avec les données agrégées d'OECD et d'EUROSTAT.

Nous concluons que : *i*) Il y a des forts signes sur l'existence de sources d'inertie d'emploi au niveau micro, provoqués par les coûts d'ajustage non-convexes et par l'ajustage de la main-d'œuvre par le nombre d'heures par employé; *ii*) les signes d'hystérésis communément trouvés au niveau micro, ne disparaissent pas complètement au niveau macro; *iii*) les propriétés d'hystérésis sont particulièrement visibles pour de petites entreprises; *iv*) nous trouvons une forte évidence sur les corrélations entre la flexibilité de l'ajustage de la main-d'œuvre avec les heures de travail et de l'existence d'hystérésis d'emploi. Nous ne trouvons pas d'évidence significative sur les corrélations entre l'existence d'incertitude dans la dynamique de demande agrégé et d'hystérésis. Ces conclusions impliquent que l'emploi total, est de façon significative, formé par l'ajustement discret au niveau micro.

Classification de JEL : E24, J23.

Mots clé : hystérésis, cout d'ajustement, emploi, incertitude, heures de travail

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CHAPTER I

INTRODUCTION

I.1. MOTIVATION, RESEARCH OBJECTIVES AND THESIS STRUCTURE

The adjustment of the quantities of labor factor employed by firms is now widely recognized as discontinuous and lumpy as the result of the existence of non-convex costs of adjustment and/or uncertainty in the path of product demand and factor costs[1]. Technically, we can say that labor demand at the firm level exhibits hysteresis in the sense that transitory changes in the labor demand forcing variables originate permanent variations in the level of employment.

However, if at the micro level models of hysteresis offer a good explanation for the empirical evidence, at the macro level it has proven more difficult to identify hysteresis in the data. In fact, the aggregation of heterogeneous firms with asynchronic actions originates a smoother adjustment behavior of the labor factor that could seemingly be represented by a partial adjustment model, whose microeconomic foundation is the representative firm facing convex adjustment costs[2]. Moreover, the different behavior of the micro and macro series of the labor factor has contributed to the view that the existing non-convex adjustment costs at the micro level are not relevant in describing macroeconomic behavior.

The original approach to explaining the apparent contradiction between micro and macro dynamics, was conducted in the field of retail inventories, and demonstrated the importance of the cross sectional distribution of the level of inventories for aggregate dynamics, under a (S,s) adjustment model (see Blinder 1981; Caplin 1985; Caplin and Spulber 1987). A more elaborate approach linked to empirical work was initiated by Bertola and Caballero (1990). It was subsequently applied by Caballero and Engel (1993) and Caballero *et al*. (1997) to the study of the dynamics of employment, and by Caballero *et al*. (1995) to that of aggregate investment.

More recently, the Preisach Model (Cross 1995 and Piscitelli *et al*. 1999) and the Linear Play Model (Belke and Göcke 1999 and Göcke 2001) of strong hysteresis offered a different way of dealing with the aggregation of heterogeneous firms facing non-convex adjustment costs. These models have specific properties that can be tested empirically in order to verify the existence of hysteresis in the aggregate data.

[1] See Blinder (1981), Hamermesh (1989), Hamermesh and Pfann (1996) and Caballero *et al*. (1997).
[2] See Cooper (2004).

In this thesis, we contribute to this recent literature by studying the existence of hysteresis in the dynamic path of employment and hours of work at the firm and aggregate levels. The aim is to describe the path of micro-level employment in relation to the structure of the underlying costs of adjusting the labor input and to analyze the aggregate implications of the observed micro behavior.

A demand side approach to hysteresis is adopted. Furthermore, hysteresis is given its original definition inherited from physics, because this interpretation reflects more accurately the theoretical hypothesis underlying its application to the study of the dynamics of either the employment or the unemployment rate.[3]

Our empirical strategy is: firstly, we check the existence of the necessary conditions for the existence of hysteresis, i.e., the existence of non-convex costs of adjusting employment and uncertainty, and its interaction with working time legislation[4]. Secondly, we analyze the existence of hysteresis at the firm level described by the Non-Ideal Relay Model of Weak Hysteresis (Visitin 1994, Mayergoyz 2003). Thirdly, we analyze the macroeconomic consequences of the presence of weak hysteresis using the Preisach Model (see Mayergoyz 2003) and the Linear Play Model of Strong Hysteresis (Visitin 1994).

In all the empirical work, we use micro data on Portuguese manufacturing firms[5]. Portugal is a good case for studying labor demand driven hysteresis because it has one of the strictest employment protection legislation systems in Europe (OECD 2004), which is a source of non-convexities in the adjustment technology. To test the existence of hysteresis in the dynamics of employment a monthly panel of Portuguese

[3] This approach proved to be successful in the study of the dynamic behavior of a number of economic variables, from labor demand (Amable *et al.* 1994; 1995, Cross 1995; 1997; 1998 and Piscitelli *et al.* 1999; 2000), to international trade (Baldwin and Krugman 1989 and Göcke 2001) and investment (Dixit 1989; 1991; 1992 and 1997, Pindyck 1991, and Dixit and Pindyck 1994).

[4] In this thesis we only focus on the real sources of inertia. However, real inertia can interact with nominal inertia (inertia in the adjustment of nominal prices and wages) in shaping the employment adjustment process (see Andersen and Hylleberg 2000).

[5] Due to data limitations, focusing on employment in the manufacturing sector is common in the literature. We are aware that the structure of the manufacturing sector is different from that of other sectors. Major differences are related to capital intensity, nature of the demand shocks, the ability to hold inventories and differences in labor relations. These factors might influence employment flows differently. In particular we expect less volatility of the employment adjustment in the manufacturing sector when measured by the job reallocation rate. However, the existence of a strong relationship between (negative) trend manufacturing employment growth and relative gross-flow volatility, documented by Foote (1998), can offset the effects of other factors. Actually Varejão (2000) found similar quarterly job reallocation rates (5.2%) in the manufacturing and services sectors in Portugal in the period from 1991 to 1995.

manufacturing firms spanning a period of eleven years is used. This dataset has information on employment, total hours of work, earnings and sales.

To put our results in an international setting the aggregate analysis is also carried out with aggregate data from OECD and EUROSTAT.

The thesis is organized as follows:

Part I reviews the essential literature on the effects of non-convex costs of adjustment, and provides a broad characterization of the Portuguese labor market, highlighting the characteristics that could yield hysteresis.

Part II is devoted to a review of the literature on models of hysteresis with application to the dynamics of employment. In this part, we distinguish the concepts of weak (micro) and strong (macro) hysteresis and their respective properties, showing that hysteresis is different from the existence of a unit root solution to difference equations. We show that the properties of strong hysteresis are different from those of weak hysteresis. In particular, aggregation over heterogeneous firms that exhibit hysteresis increases aggregate steadiness. Yet, this does not imply that hysteresis is not important for a characterization of aggregate employment. On the contrary, aggregation reinforces the property of *remanence* of the hysteretic process, and implies that aggregate employment will contain a memory of only the non-dominated extreme values of the variables, which drive adjustment at the micro level. This distinction could be very useful in empirical investigations of the properties of aggregate time series of employment. In this part, we also extend some models, which are existent in the literature, to analyze the effects of the interaction between the presence of non-convex costs of employment adjustment and the degree of flexibility of the adjustment through hours of work on the dynamics of employment.

Part III offers preliminary evidence on the patterns of labor input adjustment that constitute an indirect test of the existence of hysteresis at the firm level. A descriptive approach is presented as well as some evidence on labor input adjustment patterns for firms in the manufacturing sector. We provide information on: *a*) job reallocation; *b*) the empirical distribution of the labor input adjustment; *c*) serial correlation of employment adjustment; *d*) interrelations between adjustments through the number of workers and through the number of hours per worker. We also implement a formal test of the existence of hysteresis at the micro level that focuses on the properties of path dependence and divergence of the linear response of employment to product demand shocks.

Part IV is dedicated to the existence of hysteresis at the aggregate level or strong hysteresis. In order to explain the aggregate dynamics of employment we apply the Preisach Model of Strong Hysteresis as a mathematical tool that could be used to reconcile the micro and macro evidence on employment adjustment. This model incorporates, explicitly, heterogeneity at firm level and asynchronous adjustment that is at the basis of the smoothness of aggregates.

To study the effect of uncertainty and the effect of the existence of the margin of adjustment hours per worker at the aggregate level, we apply the Play Model of Strong hysteresis. We also analyze the effect of uncertainty by relating the frequency of structural break, caused by hysteresis, with the existence of uncertainty.

Finally, in Part V we apply the strong hysteresis models to macro data from EUROSTAT and OECD. This part of the dissertation offers international evidence on the subject, and helps put the Portuguese case in the context of other industrialized countries.

I.2. THE CONCEPT OF HYSTERESIS AND ITS APPLICATION IN LABOR ECONOMICS

Some properties of hysteresis of economic systems were recognized earlier by Marshall (1890). Although he took a static view of a unique equilibrium between supply and demand, he also saw some limitations of this notion, at least if there were increasing returns to scale: "*… in fact under certain conceivable, though rare, conditions there can be two or more positions of real equilibrium of demand and supply, any one of which is equally consistent with the general circumstances of the market, and any one of which once reached would be stable, until some great disturbance occurred.*" (Marshall 1890, p. 665)[6].

In Labor Economics, hysteresis was first used in the 1980s to describe the fact that unemployment remains high long after the temporary shocks that originated its growth have disappeared or, more formally, to describe the path dependence of the NAIRU (Non Accelerating Inflation Rate of Unemployment) on the actual rate of unemployment.

[6] Further references to hysteresis as a property required to explain the behavior of economic systems (in the context of equilibrium analysis) are found, after the 1930s, in the work of economists such as Kaldor, Schumpeter and Georgescu-Roegen.

The use of the idea of hysteresis in the context of unemployment studies was justified on the grounds that the corresponding properties also seemed to be found in the dynamics of aggregate unemployment, especially in some European Countries. Firstly, data documented a non-linear relation between some macroeconomic shocks and unemployment. Shocks originating both on the demand and supply side of the labor market are found to have a permanent effect (*remanence effect*) on the unemployment rate, or at least to cause unemployment to return to its pre-shock level but at a very slow pace. Secondly, the experience of the 1980s and 1990s also indicates that the past values of the unemployment rate together with current macroeconomic shocks are important determinants of the current unemployment rate (a property known as *path dependence*).

Clear-cut evidence of unemployment persistence emerged as a major challenge to standard economic theory and it ultimately led to the development of hysteresis-based theories of unemployment. Theories of hysteresis offer a new characterization of aggregate time series behavior that is derived from its micro foundations, revealing that the influence of labor demand shocks on employment is more specific than previously assumed.

The notion of hysteresis, which is new to economic theory, was first introduced to the dynamics of unemployment by Edmund Phelps. Although he began by advocating that the equilibrium rate of unemployment is, in the long run, independent of the monetary policy, he later recognized that the equilibrium rate of unemployment could depend upon the actual unemployment rate rather than being stable over time (Phelps, 1972)[7].

Although in his seminal work Phelps considered that hysteresis in the unemployment rate would arise from supply side mechanisms alone, such as the presence of unions, human capital depreciation, and wealth distribution (see Phelps 1972), it is now undisputed that demand-side mechanisms, such as the presence of costs of adjustment in the labor input, may also cause hysteresis[8].

[7] As the author puts it: "*The transition from one equilibrium to the other tends to have long-lingering effects on the labor force, and these effects may be discernible in the equilibrium rate of unemployment for a long time. The natural rate of unemployment at any future date will depend upon the course of history in the interim. Such a property is sometimes called hysteresis*." (Phelps 1972, p. xxiii).

[8] The Insider-Outsider theory with its implications in terms of downward wage rigidity, and the Human Capital Theory via human capital depreciation, both offer a supply-side explanation for why there could be hysteresis in the unemployment rate. On the demand-side, the existence of costs of adjustment of the inputs, labor or other, as described by the dynamic theory of factor demand, could produce the same result. See Lindbeck and Snower (1986; 1988) and Blanchard and Summers (1986; 1987) for the

However, as economic theory seized the notion of hysteresis, it did so in a very loose way if we consider the actual properties of true hysteretic systems. For that reason, some authors consider that in most cases economics 'bastardizes' the use of the expression (Amable *et al.* 1994 and Cross 1995). In fact, in early models, hysteresis implies nothing more than *persistent* deviations of the actual unemployment rate from the natural rate. Layard *et al.* (1991) also use the word hysteresis to describe the fact that shocks could originate the departure of the actual unemployment from its level of equilibrium for some time, although the natural rate remains an attractor in the long run. Following the influential article of Blanchard and Summers (1986), hysteresis has also been used as synonymously to unit root solutions to difference equations, although in unit root processes a transient shock could leave the long run unemployment rate unchanged[9]. We stress that the true concept of hysteresis is different from some assertions currently used in economics, in particular those that approximate hysteresis with unit root processes. In contrast, unit root dynamics (frequently used as an equivalent to hysteresis) are only an approximation to describe the memory characteristics of hysteresis in a simple way that is not only different in theoretical terms but also observationally not equivalent to hysteresis.

Moreover, such a deviation from the original meaning of hysteresis was not the result of an attempt to adapt the concept to the specific nature of economic phenomena but the inevitable outcome of the need to compromise with mathematical tractability and the ability to discriminate empirically between hysteresis and other non-linear processes.

In its original formulation in the domain of physics of magnetism[10], hysteresis, from the Greek '*coming behind*', is the property of a system, in which some effects remain after the causes that originated them are removed[11]. Therefore, in this

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Insider-Outsider Theory and its explanation of the unemployment record, Becker, (1962) and Hargreaves-Heap (l980) for an application of Human Capital Theory, and for demand theories Dixit (1991; 1992) and Cross (1995; 1997) in the case of labor, and Sneessens and Drèze (1986), Layard and Nickel (1986) and Bean (1989) for investment.

[9] Blanchard and Summers (1987, p. 289) recognized that they use the term hysteresis loosely compared with the original definition in physics to denote the cases where actual employment affects equilibrium unemployment for a long time.

[10] The term hysteresis was firstly introduced by the physicist James Alfred Ewing in 1881, in the explanation of the behavior of electromagnetic fields in ferric metals (Cross 1995, p. 181).

[11] "*A mathematical modeling of hysteresis requires the consideration of a system subject to external action, i.e. an input-output system. Hysteresis is defined as a particular type of response of the system when one modifies the value of the input: the system is said to exhibit some remanence when there is a permanent effect on output after the value of the input has been modified and brought back to its initial position*." Amable *et al.* (1995, p. 155)

dissertation, hysteresis is given its original interpretation, *i.e.*, implying the properties of *remanence*, *non-linearity* and *selective memory*[12]. This original definition of hysteresis reflects more accurately the theoretical hypothesis underlying the idea of hysteresis in employment dynamics, and its apparent properties seem to fit the theoretical dynamics of employment better (Amable 1995, Cross 1995).

To illustrate the basic properties of hysteresis we assume that we can control the evolution of a scalar input variable (in its original formulation the magnetic field - X, in Figure i.1), and we consider a *black box,* which transforms this input into an output variable (the magnetic induction - Y) (Visitin 1994). Figure i.1 also shows that starting from a situation where a ferromagnetic substance is demagnetized (in the absence of electric current - X) over point A, the magnetization occurs rapidly when a magnetic force is applied, until the point where the magnetic field (Y) reaches a quasi-saturation (point B). When the magnetization force diminishes towards zero, the magnetic field diminishes to point C. The material stays permanently magnetized, i.e., it becomes a loadstone.

Hysteresis typically exhibits hysteresis loops[13], like the closed curve BCDEF (major loop) in Figure i.1. After increasing from zero to *b*, if X decreases from *b* to *a*, the pair (X, Y) moves along the curve BCDE; if, after reaching the quasi-saturation point E (for $X = a$), X increases from *a* to *b*, the pair (X, Y) moves along the curve EFB.

[12] See Visitin (1994) and Mayergoyz (2003) for more detail.

[13] Note, however, that loops are not an essential characteristic of hysteresis. The definition of hysteresis emphasizes the fact that history-dependent branching constitutes the essence of hysteresis, while looping is a particular case of branching that occurs when the input varies back and forth between two consecutive extrema, while branching takes place for arbitrary input variation (Mayergoyz 2003, p. xviii). In economics, due to the fact that it is not possible to conduct experiments where we vary the input back and forth between two consecutive extrema, it is difficult to identify loops in response to the dependent variable.

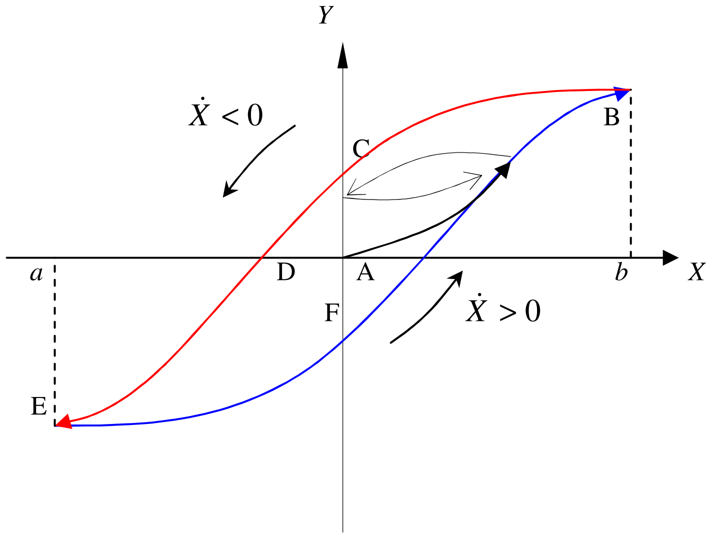

Figure i.1. The Input-Output Diagram - The Hysteresis Loop

Moreover, if X reverses its motion when X lies between *a* and *b,* the pair (X, Y) moves to the interior of the region delimited by the curve BCDE generating sub-loops (Visitin 1994). The principal characteristic of this graph is that as X increases ($\dot{X} > 0$) and subsequently decreases ($\dot{X} < 0$), a family of continuous connected curves is generated. Yet there is no single function relating the input to the output. On the contrary, a family of functions is needed to represent the X/Y relationship. Furthermore, the complicated behavior represented by the sub-loops is especially relevant to economics given the tendency of the disturbances affecting the systems to be of an irregular rather than a regular cyclical nature (Cross 1980, p. 29)

The hysteretic system described in Figure i.1 has three important characteristics: firstly, the system exhibits *remanence* and not merely persistence because after a temporary shock in the value of the input (X) the equilibrium value of the output (Y) is permanently displaced from point A to point C (in physics, the distance AC is termed the *remanence* of the electromagnetic field)[14]. Secondly, the relationship between the dependent variable and the independent variable is *non-linear*, in the sense that the trajectory followed by the value of the output as the value of the input increases (A→B) is not reversed when the value of the input starts to decrease (the value of the output now followed the path B→C). Thirdly, the system

[14] Note that, to regain the original field characteristics, a negative magnetizing force AD is required. AD is called *coercivity* – a measure of the extra force required to restore the original characteristics

has a *selective memory*, which means that only past extreme non-dominated shocks are retained in memory (see section II.4.3 for a detailed description of this property).

I.3. SOURCES OF HYSTERESIS AT THE FIRM LEVEL AND AGGREGATE IMPLICATIONS

If it exists, strong hysteresis in employment results necessarily of the aggregation of weakly hysteretic processes of employment adjustment activated by heterogeneous micro-agents (Mayergoyz 2003).

We know, at least since Oi (1962), that labor input, rather than being a variable factor, should be considered a quasi-fixed factor of production, i.e. one whose total cost is partially variable and partially fixed.

Labor adjustment costs (also named one-time fixed costs Hamermesh (1993, p. 47) are costs that are incurred at one point in time, usually when workers are hired or dismissed, and take the form of irreversible costs, *i.e.*, they are sunk costs. These costs may be *lumpy* (*fixed costs of adjustment*) or *divisible* (*variable costs of adjustment*) depending on how they vary with the size of employment adjustment (net or gross).

Adjustment costs are fixed if the costs associated with employment change are invariant to the size of the change. Examples of fixed adjustment costs include: *a)* costs of maintaining a personnel department; *b)* advertising costs; c*)* training costs that are independent of the number of workers trained; *d)* disruption in production caused by difficulties in re-scheduling the flow of workers across sites within the establishment; *e)* the fall in the firm's productivity due to reduced morale of the workforce following mass-layoff episodes (Hamermesh 1993).

Variable costs of adjustment are all adjustment costs that vary with the number of workers hired or fired (or the net variation in employment). They include: *a)* hiring costs, such as screening and interviewing applicants for the posting of job vacancies; *b)* training costs designed to enhance the productivity of the new workers; *c)* firing costs such as red-tape and severance pay. Variable costs of adjustment may be *linear*, if adjustment costs per worker are invariant to the size of employment changes, or *quadratic*, if adjustment costs per worker are increase in size employment change (Hamermesh 1993).

Adjustment costs, irrespective of their structure, are one possible source of fixity of the labor input, making firms adjust the level of employment slowly in response to shocks (Hamermesh and Pfann 1996, p. 1264).

However, the response of labor demand to an exogenous shock depends not only on the source and magnitude of adjustment costs, but also on their structure. Moreover, the structure of adjustment costs has an impact on the firms' employment path that cannot be confined to the short run.

If non-convex (i.e. linear of fixed) adjustment costs are present, employment does not change in response to small macroeconomic shocks, but it adjusts fully to its target if the shock is large enough (or following a series of small cumulative shocks). As a result, the dynamics of employment at the firm level is characterized by a high frequency of long periods of inaction followed by rare episodes of large adjustments. There is, in fact, some evidence that labor adjustment costs are at least in part non-convex, implying that at the micro-level employment proceeds in jumps (see Hamermesh 1989; 1993, Hamermesh and Pfann 1996 and Caballero *et al*. 1997, for example).

A vast literature, theoretical and empirical, shows that when decisions are made under uncertainty and adjustment costs are fixed or linear in structure, periods of inertia would emerge and this is sufficient to produce hysteresis.

The recognition of the similarities between the behavior of some economic variables and some physical phenomena (see Dixit 1992) paves the way for importing into economics the models of hysteresis as originally stated in physics.

At the micro level, some models of hysteresis easily generate an employment dynamics consistent with the empirical evidence available. At the aggregate level, however, employment series look smooth and appear to be well described by partial-adjustment like models reflecting convex adjustment costs at the firm level.

Actually, Hamermesh (1989 p. 75), in his analysis of monthly employment record of seven manufacturing plants of a large U.S. durable-goods producer between 1977 and 1987, shows that whereas the pattern of individual plant adjustment exhibits substantial inaction punctuated by periods of large adjustment, aggregation over the seven plants produces a smooth path of employment. In the same vein, Varejão (2000) and Cooper (2004) also show that while the dynamics of employment at the micro level is supportive of the importance of non-convex adjustment costs, at the aggregate level there is more evidence in favor of the convex adjustment cost model. These results, because they imply that lumpiness is a feature of individuals but not of aggregates, raise the issue of the irrelevance of micro models of employment adjustment to explain the dynamics of employment at the macro level.

In fact, aggregate series tend to look smoother and, for that reason, apparently inconsistent with the presence of hysteresis. However, if micro heterogeneity is properly accounted for, i.e. if aggregation is explicitly modeled as it should be in the presence of non-convex costs of adjustment, it will still be possible to uncover signs of hysteresis at the macro-level as well.

I.4. BRIEF CHARACTERIZATION OF THE PORTUGUESE LABOR MARKET

I.4.1. INSTITUTIONAL ASPECTS

In this section, we focus on two aspects of labor market regulations: employment protection legislation (protection of workers with permanent contracts and regulation of fixed-term contracts) and regulations on working time. We also present an overview of the major changes of legislation in our sample period from January 1995 to December 2005.

I.4.1. LEGISLATION ON DISMISSALS OF REGULAR EMPLOYMENT

To evaluate the strictness of employment protection legislation of workers with permanent contracts, we consider the following aspects of the legislation on individual and collective dismissals: *a*) reasons for dismissal; *b*) notice period; *c*) compensation; *d*) procedural obligations. These aspects were regulated by the Law of Termination of Contracts (DL 64-A/1989), by the Law of Dismissals by Failure to Adapt to Changes in the Nature of the Work (DL 400/1991) and now, they are regulated by the recent Labor Code (Law 99/2003). Except for the regulations concerning the maximum duration of fixed-term contracts, no major modifications have been introduced since 1989.

Individual dismissals

In Portugal, individual dismissals of employees with permanent contracts are permitted on disciplinary grounds in cases of employee's culpable behavior[15], and for reasons that are not the fault of the employee: *i*) extinction of the labor position[16] and; *ii*) employee's failure to adapt to changes in the nature of his work[17].

In all cases of dismissals for reasons not imputable to the worker, he or she is entitled to severance pay equal to one month's pay for each year of service[18] and a period of 60-days' advance notice.

In all cases of individual dismissal, written notice of the impending dismissal is required for the employee and for the works council and/or union. This statement must give the reasons on which the dismissal is based. The worker and his representatives are given the opportunity to dispute the employer's allegations. In case of economic redundancies, the worker may further ask for Labor Inspectorate intervention, in which case officials have to verify the validity of the arguments put forward by the employer. For all types of dismissal these procedures take at least three weeks

Collective Dismissal

Portuguese law establishes that a collective dismissal is the simultaneous dismissal or a successive dismissal within a period of three months of at least 2 workers in micro or small firms (firms with fewer than 50 workers) and at least 5 workers in medium or large firms (firms with more than 50 workers). These dismissals should be justified by the closure of the plant or an equivalent structure, or by the need to reduce the number of employees due to market, technological or structural reasons[19].

60-days' advance notification of a collective dismissal is also required for the works council or union and for the Ministry of Labor and Social Security. The written notice should include: the reason for dismissal; the number of workers being

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[15] DL- 64-A/89, No. 9. in its present version in Law 99/2003, No. 396.
[16] DL- 64-A/89, No. 26, in its present version in Law 99/2003, No. 402.
[17] DL- 400/91 No. 1, in its present version in Law 99/2003, No. 405, which emphasizes the incidental nature of the failure to adapt to changes in the nature of the work.
[18] DL- 64-A/89, No. 13, in its present version in Law 99/2003, No. 401.
[19] DL- 64-A/89, No. 16 and 26, in its present version in Law 99/2003, No. 397.

dismissed; the criterion used to select the individuals being dismissed; the method used to compute the corresponding compensation. Consultations between the three parties are mandatory within 15 days. Alternatives to redundancy, the number of dismissals, and ways to mitigate the effects of dismissal are all issues that must be addressed during this consultation process. Once an agreement is reached, each worker selected for dismissal must be notified of the impending job loss. This must be done at least 60 days before the date of dismissal. Otherwise, the worker is entitled to the corresponding pay[20].

As in the case of individual dismissals, employees are entitled to severance pay equal to one month's pay for each year of service, subject to a minimum of three months' pay[21].

In all cases of dismissals only courts may declare a dismissal unlawful, mostly on the grounds of the employer's failure to comply with mandatory dismissal procedures. Consequences of such a court decision are the employer being obliged to reinstate the worker in his previous position and pay him an amount equal to what he would have received from the time he was last paid to the moment the decision was made. The worker may choose to quit, in which case he is entitled to an indemnity corresponding to one-month's pay for each year of service (subject to a 3-month minimum).

I.4.2. LEGISLATION ON FIXED-TERM CONTRACTS

In assessing the strictness of legislation on fixed-term contracts we consider: *a*) the admissible grounds for entering into such contracts; *b*) the maximum number of contracts and the cumulative length of subsequent contract renewals; *c*) restrictions on termination of contracts.

Fixed-term contracts are permitted under a specific set of circumstances[22]: *i*) temporary replacement of permanent workers; *ii*) exceptional and temporary workload; *iii*) seasonal activity; *iv*) time limited specific projects; *v*) business start-ups; *vi*) launching of new activities of uncertain duration; *vii*) recruitment of workers

[20] DL- 64-A/89, No. 20 and 21, in its present version in the Law 99/2003, No. 398.
[21] DL- 64-A/89, No. 17, with the redaction change of the Law 32/99 and with the minor alterations introduce in the actual law (Law 99/2003, No. 419).
[22] DL 64-A/89, No. 41.

in search of their first job; *viii*) long-term unemployed. From 2003 this set of reasons should not be viewed as an exhaustive list of the objective grounds to enter into such a type of contracts. The general criterion is that the fixed-term contract is only allowed to satisfy firms' temporary needs[23].

Since 1989, the duration of fixed-term contracts, cannot exceed three years (including renewals) and cannot be renewed more than twice[24]. The new Labor Code of 2003 added that after the period of three years or after two renewals, the contract could be renewed once more if its duration were between one and three years[25].

A fixed-term contract expires only if the employer notifies the worker eight days in advance that he does not intend to renew it; otherwise it is automatically renewed. If the maximum duration of the contract is exceeded, the contract automatically becomes permanent.

If the employer terminates the contract before its term, and the termination is unlawful, the worker is entitled to compensation equal to the pay loss from the dismissal to the date of the court's decision or the term of the contract (whichever occurs first). He or she is also entitled to reinstatement if the term of the contract has not been reached.

Moreover, if, during the period of the contract the firm opens a vacancy for a permanent position, workers with fixed-term contracts who may qualify for the job are given priority over other applicants.

If, after an elapsed duration of twelve months, a contract is not renewed for reasons not imputable to the worker, he or she cannot be replaced within a period of three months.

I.4.3. WORKING TIME REGULATIONS

Working time provisions typically regulate: *a*) normal working period; *b*) medium duration of the working-week, including permitted overtime work; *c*) overtime work.

[23] Law 99/2003, No. 129.
[24] DL 64-A/89, No. 44.
[25] Law 99/2003, No. 139.

In Portugal, since 1996, the standard working hours, set by law, cannot exceed 8 hours per day or 40 hours per week. The reduction of the standard hours can be established by collective agreement but it cannot result in lower pay. The normal period of work can be defined in average terms. The maximum number of hours of work per day can be extended by a maximum of 4 hours, as long as the maximum number of hours per week does not exceed 60[26].

The maximum number of average working hours per week (including overtime) cannot exceed 48 hours. The average number of working hours is computed over a period of time that is defined by collective bargaining, and may be as long as 12 months (the legal default is 4 months)[27].

Overtime work is allowed if the firm faces a transitory increase in the work load that that does not justify hiring more workers[28]. Overtime hours are subject to a legal maximum that is equal to 175 hours per year, for micro and small firms, and 150 hours for medium or large firms. The number of overtime hours can be extended to 200 hours per year by collective agreement[29].

The overtime premium is set by law at 50% of the wage rate in the first hour, and at 75% in subsequent hours[30].

I.4.4. INTERNATIONAL COMPARISON OF LABOR MARKET REGULATION

Portugal ranks at the top in all indexes of employment protection (only Luxembourg has stricter employment protection legislation). Table i.1 reports three indexes of the strictness of the employment protection legislation: *i*) the OECD 2004 index; *ii*) the 1995 value of the time series index from Labor Market Institutions Database, version 2.00, 2001, by Stephen Nickel, and; *iii*) an index reported by

[26] Until 1996 the normal period of work was 8 hours per day and 44 hours per week (DL 409/71). The present norm (Law 99/2003, No. 163 and 164) keeps the same normal period of worked established since 1996 (Law 21/96).
[27] Until 1998 this limit was 50 hours per week on average (DL 409/71). The law currently in force (Law 99/2003, No. 169) maintains the same limit established since 1998 (Law 73/98).
[28] Law 99/2003, No. 199.
[29] Law 99/2003, No. 200. The maximum number of annual overtime hours was changed in 2003. Before 2003, the maximum number of annual overtime hours was 200 unrelated to the size of the firm (DL 421/1983).
[30] Law 99/2003, No. 258, with no change since 1983 (DL 421/83).

Nunziata (2002)[31]. The overall index of employment protection legislation published by OECD follows Grub and Well's (1993) methodology, ranking countries by the strictness of legislation concerning regular employment (including regular procedural inconveniences, notice and severance pay for no-fault individual dismissals, and difficulty of dismissal), temporary employment (including regulation on fixed-term contracts and regulations on temporary work agencies) and collective dismissals (including definition of collective dismissal, additional notification requirements, additional delays involved and other special costs to employers).

The indexes indicate great heterogeneity in the legislation and practices across countries (see Table i.1). This heterogeneity is mainly due to the regulations concerning temporary employment and less so collective dismissals. Actually Table i.1 shows that the cross-country variability of the indexes is greater in the case of temporary contracts. Nonetheless, there is a positive correlation between the strictness of the regulations concerning permanent and temporary contracts (see Table i.2). Furthermore, Employment Protection Legislation is stricter in southern European countries and less restrictive in the USA, UK and Canada. According to these indexes, Portugal is one of the countries with the strictest employment protection legislation concerning regular and temporary employment.

Concerning changes over time, Table i.1 shows a tendency for convergence in the strictness of the employment protection regulations of regular contracts between OECD countries (the variability of the indexes decreased from the late 1990s to 2003), and a tendency for divergence in the case of the strictness of the regulations concerning temporary contracts and collective dismissals. Despite this time evolution, there was little change in the relative position of the countries. Moreover, Table i.1 indicates a slight reduction in the strictness of the employment protection legislation in Portugal, due solely to the change in legislation concerning the protection of temporary employment[32].

[31] In spite of being different, the three indexes are highly rank correlated (see Table i.2).

[32] In spite of being widely used, there are problems of subjectivity in the construction of these indicators, and this could be especially relevant in the case of Portugal (Addison and Teixeira 2003). Concerning employment protection indexes, (Addison and Teixeira 2003, p. 91) consider that there are ambiguities related to: the number of categories over which one would wish to average the rankings; the implicit weighting scheme; the problem of ordinal rather than cardinal measures; the difficulty of attributing scores on the basis of the legislation that could be applied differently in practice.

Table i.1
Indicators of the Strictness of Employment Protection Legislation[1]

Country	OECD								EPL	EPI[5]
	Regular Employment		Temporary Employment		Collective Dismissals		Overall EPL		LMID[4]	
	Late 1990s	2003	Late 1990s	2003	Late 1990s	2003	Late 1990s[2]	2003[3]	1995	
Austria	2.9	2.4	1.5	1.5	3.3	3.3	2.4 (7)	2.2 (7)	1.30 (6)	12.3 (8)
Belgium	1.7	1.7	2.6	2.6	4.1	4.1	2.5 (5)	2.5 (5)	1.19 (8)	15.28 (4)
Canada	1.3	1.3	0.3	0.3	2.9	2.9	1.1 (15)	1.1 (16)	0.30 (12)	3.00 (12)
Czech Republic	3.3	3.3	0.5	0.5	2.1	2.1	1.9 (10)	1.9 (12)	-	-
Denmark	1.5	1.5	1.4	1.4	3.9	3.9	1.8 (13)	1.8 (13)	0.74 (10)	9.25 (10)
Finland	2.3	2.2	1.9	1.9	2.6	2.6	2.2 (9)	2.1 (8)	1.08 (9)	11.65 (9)
France	2.3	2.5	3.6	3.6	2.1	2.1	2.8 (3)	2.9 (4)	1.50 (2)	13.67 (6)
Germany	2.7	2.7	2.3	1.8	3.5	3.8	2.6 (4)	2.5 (6)	1.41 (3)	16.05 (3)
Hungary	1.9	1.9	0.6	1.1	2.9	2.9	1.5 (14)	1.7 (15)	-	-
Japan	2.4	2.4	1.6	1.3	1.5	1.5	1.9 (11)	1.8 (14)	1.40 (4)	14.00 (5)
Luxembourg	-	2.6	-	4.8	-	5.0	-	3.9 (1)	-	-
Netherlands	3.1	3.1	1.2	1.2	3.0	3.0	2.3 (8)	2.03 (10)	1.23 (7)	13.10 (7)
Poland	2.2	2.2	0.8	1.3	4.1	4.1	1.9 (12)	2.1 (13)	-	-
Portugal	**4.3**	**4.3**	**3.0**	**2.8**	**3.6**	**3.6**	**3.7 (1)**	**3.5 (2)**	**1.91 (1)**	**18.03 (2)**
Slovak Republic	3.6	3.5	1.1	0.4	3.3	2.5	2.5 (6)	2.0 (11)	-	-
Slovenia	-	-	-	-	-	-	-	-	-	-
Spain	2.6	2.6	3.3	3.5	3.1	3.1	3 (2)	3.1 (3)	1.32 (5)	18.65 (1)
UK	0.9	1.1	0.3	0.4	2.9	2.9	1 (16)	1.1 (17)	0.35 (11)	3.50 (11)
USA	0.2	0.2	0.3	0.3	2.9	2.9	0.7 (17)	0.7 (18)	0.10 (13)	1.00 (13)
Average	2.31	2.31	1.55	1.71	3.05	3.13	2.11	2.16	1.06	11.50
Stand. Dev.	1.02	0.96	1.08	1.29	0.71	0.85	0.76	0.82	0.53	5.73
Max.	4.30	4.30	3.60	4.80	4.10	5.00	3.70	3.90	1.91	18.65
Min.	0.20	0.20	0.30	*0.30*	1.50	1.50	0.70	0.70	0.10	1.00

1 In all cases, the more rigid the legislation, the higher the index.
2 EPL Late 90's: Employment Protection Indicator from OECD – Employment Outlook 2004 (Table 2.A.2.4).
3 EPL 2003: Employment Protection Indicator from OECD – Employment Outlook 2004 (Table 2.A.2.4). Overall indexes are calculated according to a weighted average of the scores for regular, temporary contracts and collective dismissals
4 EPL LMID: Employment Protection Indicator from Labor Market Institutions Database, version 2.00, 2001, by Stephen Nickel
5 EPI: Employment Protection Indicator [0.20] from Nunziata (2002) (Table 1, p. 38).
() ranking

Table i.2
Spearman's Rank Correlations Between Employment Protection Indicators

	EPL 2003	EPL LMID	EP	Regular Employment	Temporary Employment	Collective Dismissals
Overall EPL2003	1.00	0.80** (4.37)	0.86** 5.52	-	-	-
EPL LMID	-	1.00	0.85** 5.39	-	-	-
EP	-	-	1.00	-	-	-
Regular Employment	-	-	-	1.00	0.59** (2.95)	0.30 (1.24)
Temporary Employment	-	-	-	-	1.00	0.53* (2.48)
Collective Dismissals	-	-	-	-	-	1.00

**significant at 1%; *significant at 5%

The international comparison of strictness of the legislation concerning working time is more difficult than in the case of legislation on regular employment and fixed-terms contracts due to less availability of data. Nonetheless, we report some indicators in Table i.3. The first set of indicators is based on a survey of employers about their feelings on the strictness of legislation. This has the advantage of incorporating all potential influences, including negotiations with trade unions and political pressure, as well as legislation[33]. We also report a synthetic indicator constructed by Nunziata (2003) that uses information from OECD and EIRO.

Table i.3 shows that Portuguese working time regulation is characterized by medium strictness compared to other countries. The UK, the USA and Japan are countries with very soft legislation concerning working time, while Spain, the Netherlands, France and Germany have stricter regulations.

[33] Some caution should be taken in order to compare the results of this survey across countries. The difficulty in comparing this kind of surveys results from: *different employer's attitudes; consistency of responses when economic conditions facing firms in the same sample differ, changes in the identity of the respondent managers, and even changes in the relevant question* (Addison and Teixeira 2003, p. 120).

Table i.3
Indicators of the Strictness of Working Time Legislation[1]

Country	Limitation to Working Time Flexibility						WTR[4]
	Total Industry[2]			Service Sector[3]			
	Legal	Contractual	Readiness	Legal	Contractual	Readiness	
Austria	42	25	22	33	17	20	8.33 (7)
Belgium	42	28	37	56	38	9	0.00 (11)
Canada	-	-	-	-	-	-	10.00 (3)
Czech Republic	-	-	-	-	-	-	-
Denmark	15	53	35	-	-	-	5.00 (9)
Finland	31	44	41	68	70	72	8.33 (7)
France	57	44	31	58	55	32	10.00 (3)
Germany	30	29	18	29	26	21	10.00 (3)
Hungary	-	-	-	-	-	-	-
Japan	-	-	-	-	-	-	0.00 (11)
Luxembourg	-	-	-	-	-	-	-
Netherlands	17	16	16	-	-	-	11.67 (2)
Poland	-	-	-	-	-	-	-
Portugal	**27**	**24**	**26**	**35**	**23**	**22**	**10.00 (6)**
Slovak Republic	-	-	-	-	-	-	-
Slovenia	-	-	-	-	-	-	-
Spain	26	30	22	51	57	65	20.00 (1)
UK	12	10	6	12	10	12	0.00 (11)
USA	-	-	-	-	-	-	3.33 (10)

[1] In all cases, the more rigid the legislation, the higher the index.
[2]Limitations to Working Time Flexibility Index from European Economy – Supplement B (January 2000) (Table 6, p. 5).
[3]Limitations to Working Time Flexibility Index from European Economy – Supplement B (December 1999) (Table 6, p. 5).
- European Commission's *ad h*oc surveys of 1999 and 2000 on whether insufficient flexibility in shedding staff is an obstacle to employing more people
- The reported number is the coefficient of importance which ranges from 0 (if all respondents consider a factor to be 'not so important') to 100 (if all consider it 'very important')

[4]WTR: Working Time Regulation Index from Nunziata (2003), (Table 1, p. 38).
() rankings.

I.4.2. LABOR INPUT ADJUSTMENT AND HYSTERESIS - A SURVEY OF THE EMPIRICAL LITERATURE

To better characterize the Portuguese labor market, the analysis of the institutional framework, is, in this section, complemented by consideration of the results of the empirical studies on the outcomes of the labor market adjustment.

I.4.2.1. PATTERNS OF LABOR INPUT ADJUSTMENT

At the firm level, the empirical literature on employment adjustment shows, in general, a very unresponsive and discrete pattern of employment as a result of product demand shocks, which indicates the existence of significant non-convex costs of adjustment[34]. Examples are:

Blanchard and Portugal (2001), who, using annual and quarterly data from the *Inquérito ao Emprego Estruturado* and *Quadros de Pessoal*, show that job reallocation (especially job destruction) is lower in Portugal than in the U.S. when quarterly data is considered. This result is related to the fact that employment protection in Portugal may lead firms to smoother quarter-to-quarter movements in employment.

Varejão and Portugal (2007), who, using an unbalanced panel of 10 673 establishments from the *Inquérito ao Emprego Estruturado* (Employment Survey) from the Portuguese Ministry of Employment, provide a descriptive analysis of labor market flows that show unequivocal signs of discrete adjustment consistent with the existence of fixed employment adjustment costs. The authors also estimated a *Duration Model of Employment Adjustment* showing that when unobserved

[34] Nonetheless, Addison and Teixeira (2001b) defend that Portuguese employment shows an apparent ability to accommodate changes in output demand in spite of the strictness of employment protection legislation summarized in OECD rankings. One possible way of explaining this dissociation between the high values that the country shows in indexes of employment protection and the existence of a relatively high speed of adjustment of employment to its long run equilibrium value is the weighting scheme that does not capture the importance of a relatively more favorable ranking concerning protection on collective rather than individual dismissals (Addison and Teixeira 2001b). Moreover, the practice of collective bargaining could also originate differences between the indexes of employment protection legislation and effective employment adjustment. Actually, in Portugal, contrary to what happens in many countries, the regulations cannot in general be exceeded under collective bargaining (the principal exception being severance pay).

heterogeneity is properly accounted for, the hazard function (the instantaneous conditional probability for an establishment to abandon the inaction regime – no adjustment of the labor force) is upward sloping. An upward sloping hazard function indicates that there is a non-convex component of the adjustment costs that is important enough to shape the process of adjusting employment.

Varejão and Portugal (2000) estimated a *Switching Model of Labor Demand* in which a firm switches from inaction to action as the deviation of current employment from its equilibrium level changes in absolute terms from more than a non-negative parameter *k* positively related to the magnitude of the fixed cost of adjustment. Using data on 1 395 establishments from the *Inquérito ao Emprego Estruturado* and *Quadros de Pessoal* over the period from the first quarter of 1991 to the fourth quarter of 1995, the authors show that the switching regression performs quite well indicating the presence of important non-linearities in the employment path at the micro level. With the switching equation they obtain much higher output and wage elasticities than those estimated with the traditional partial adjusting model. These results indicate that the low elasticities associated with employment forcing variables and the very high coefficient associated with lagged employment are not the result of a slow adjustment to equilibrium level of employment with the firms closing a small fraction of the gap each quarter, but instead the result of long periods of inaction punctuated by large adjustment to match the equilibrium employment level.

Addison and Teixeira (2001a) estimated by GMM a partial adjustment model of employment demand, using an annual sample of 1 970 firms in the period 1990-97 from the *Central de Balanços do Banco de Portugal* (Balance Sheet Records of the Bank of Portugal). They obtain a lagged employment coefficient of 0.75 and comparing with aggregate results, they conclude that panel estimation with annual micro data yields more employment inertia.

Despite not focusing on hysteresis, these studies provide some evidence of the presence of the necessary condition for its occurrence, i.e. the existence of non-convex costs of employment adjustment at the micro level.

At the aggregate level, however, signs of inertia and discrete adjustment are not clearly observable.

Addison and Teixeira (2001a; 2001b) estimated a cointegrated demand equation with aggregate time series quarterly data on employment, output and relative price of energy over the period from the first quarter of 1977 to the fourth quarter of 1997 from the *Instituto Nacional de Estatística* (Portuguese Statistics Office). From the first-stage cointegrating regression they obtained a low long run output elasticity of 0.4, which is significantly low when compared with Germany (0.84), UK (0.62) and Spain (0.74). However, by estimating a one-stage error correction model Addison and Teixeira (2001a, 2001b) found a relatively high speed of adjustment in conjunction with a low employment-output elasticity, which produces a fast convergence to the long-term path.

Varejão and Portugal (2000), aggregating data on 1 395 establishments from the *Inquérito ao Emprego Estruturado* and *Quadros de Pessoal* over the period from 1991 to 1995 estimated a partial adjustment model and concluded that at annual frequencies the coefficient of the lagged dependent employment at the highest level of aggregation (0.908) is very similar to the one obtained at the establishment level (0.904). However, passing from establishment level to higher levels of aggregation seems to originate more reasonable results for the partial adjustment model, meaning that the signs of hysteresis at the micro level tend to vanish at aggregate level.

I.4.2.2. HYSTERESIS IN PORTUGUESE EMPLOYMENT

Studies regarding the existence of hysteresis in employment (or in unemployment) series in Portugal are not abundant. Moreover, the few available studies were conducted assuming a great variety of hysteresis definitions with microeconomic foundations that focus only on the supply side of the labor market.

To the best of our knowledge, no study addresses the issue of hysteresis on the Portuguese labor market from a demand perspective, exploring the link between the existence of non-convex costs of employment adjustment and hysteresis at the micro level, and investigating the aggregate consequences of such behavior. The studies that focus on the issue of hysteresis for the Portuguese Labor Market, either link hysteresis with the existence of unit roots in the unemployment series or identify hysteresis on the basis of the significance of unemployment change in a Phillips Curve equation, justified by mechanisms related to the supply side of the labor market.

Insider-Outsider Explanation

From the labor supply side, and following the *Insider -Outsider* explanation of hysteresis (Blanchard and Summers 1986; 1997), Modesto *et al.* (1992), using semi-annual data from the *Instituto Nacional de Estatística* (Portuguese Statistics Office) over the period from the first semester of 1977 to the second semester of 1988, conclude that while *full hysteresis* is rejected by the data hysteresis appears as an important source of unemployment persistence[35].

Along the same line, Duarte and Andrade (2000) apply in a different way Amable's (1993) concepts of *weak* and *strong hysteresis* to characterize, respectively, the existence of a unit root and a near unit root[36] in the series of unemployment rate. They implement unit root tests for the series of unemployment, unemployment rate and employment using annual (1953-1993), semi-annual (from the first semester of 1974 to the second semester of 1998), quarterly (from the first quarter of 1983-1988:4) and monthly (from January 1983 to December 1998) data from the *Series Longas do Banco de Portugal* and from the OECD. They conclude for the existence of *strong hysteresis* based on the presence of a unit root in the unemployment series at all data frequencies.

Carneiro and Portugal (2004), using annual data over the period from 1993 to 1997 from the Social Audit (*Balanço Social*) and from the *Quadros de Pessoal* (Personnel Records), analyzed the weight of insider forces and the power of insiders in the wage negotiation. The authors conclude that insider forces, such as revenue per employee and market share have a significant weight on wage determination in all sectors. Concerning the power of insiders, when the conventional measure is used (the change in the number of insiders) in a wage equation, no robust evidence was found that insiders have an important role in wage determination. In fact, Carneiro (2004) found no evidence of membership hysteresis effect when the aggregate sample is used, but a positive and significant impact of the variation in the number of insiders on wage determination in the manufacturing sector.

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[35] According to Blanchard and Summers (1986) *full hysteresis* exists only when the insiders cause a significant impact on the determination of the wage. When this happens, the wage is not affected by the level of the unemployment rate and the traditional Phillips curve with a constant natural unemployment rate is no longer valid. On the contrary, unemployment becomes dependent on past unemployment, i.e., it follows a random walk.

[36] Stationary series, but with a long period of adjustment to the equilibrium value.

Pereira (1998) distinguishes *pure hysteresis* (when past unemployment has a permanent effect on the NAIRU implying that the unemployment rate follows a random walk) from *partial hysteresis* or persistence (when past unemployment has only a temporary effect on the NAIRU, implying that unemployment follows an autoregressive behavior with a coefficient of lagged unemployment close to but less than one)[37]. To test the existence of hysteresis, Pereira (1998) implemented unit root tests to the annual unemployment rate series over the period from 1964 to 1994 concluding for the non-rejection of the existence of a unit root.

Human Capital Depreciation Explanation

According to the human capital explanation of hysteresis, pressure over the wage is a decreasing function of the duration of unemployment. Thus, only recently unemployed workers (measured by the variation in the unemployment rate) could have a significant impact on wages.

Pereira (1998) estimated an augmented Phillips Curve that includes not only the level of the unemployment rate but also its change. In this context, *pure hysteresis* exists if the wage inflation depends negatively on the change in the unemployment rate, but not on its level. The microeconomic explanation lies in insider-outsider mechanisms or alternatively in the human capital explanation. His study reveals a significant coefficient associated with the unemployment rate level and with the unemployment rate variation, implying the rejection of *pure hysteresis* but the non-rejection of *partial hysteresis*.

In the same vein, Rosa (2004) used annual time series macro data from the *Series Longas do Banco de Portugal* over the period from 1954 to 1995 to conclude that the unemployment rate is not significant in the long run equation, but the change in the unemployment rate is significant, implying the non-rejection of the existence of a modified version of the Phillips Curve that allows for hysteresis.

Again from the supply side of the labor market, Bover *et al.* (2000) found a decreasing hazard function (based on a sample of men aged 20-64 from quarterly labor forces surveys over the period from the second quarter of 1992 to the fourth quarter of 1997) reflecting a decreasing probability for a worker to leave

[37] The author recognized that this conception of hysteresis does not follow the properties of the original definition from physics

unemployment as the time spent unemployed increased, which seems to confirm the validity of the human capital theory of hysteresis. Addison and Portugal (2003) studied the determinants of unemployment duration in a competitive risk framework with two destination states; inactivity and employment. They estimated a polynomial hazard function using data from the *Inquérito ao Emprego* (Portuguese Quarterly Employment Survey) from the *Instituto Nacional de Estatística* (Portuguese Statistics Office) and they found that the employment hazard is decreasing over a large portion of the relevant range.

I.5. CONTRIBUTION

This thesis contributes, to the best of our knowledge, with some novel input to the vast subject of employment adjustment:

- A global perspective of the employment adjustment is provided by linking the existence of hysteresis at the macro level to the discontinuous and infrequent behavior of employment demand at the micro level under the presence of non-convex costs of adjusting the number of employees and uncertainty. To do so we join economic theory, empirical appropriated data (our macro series are an aggregation of the micro data of the firms that remain in the data set from the beginning to the end reflecting only the shock) and computational methods.

- An extension of the Non-Ideal Relay Model of Weak Hysteresis to analyze the theoretical effect of the adjustment through hours of work upon the hysteresis band.

- Codes which implement strong hysteresis models in a referenced high-level technical computing language and interactive environment (MATLAB) with superior numerical properties than the few at available in the literature.

- Empirical analysis of the interaction between the adjustment of the labor input through the number of workers and through the number of hours per worker at the micro level, and its implications to the width of the band of inertia at the aggregate level, in the framework of the Linear Play model of strong hysteresis.

CHAPTER II

THE CONCEPT OF WEAK AND STRONG HYSTERESIS AND SOME MODELS WITH APPLICATION TO LABOR DEMAND

II.1. INTRODUCTION

The concept of hysteresis is a source of considerable confusion since it has been used in economics in different assertions. The word hysteresis was initially applied to unemployment rate in the assertion that a temporary disequilibrium affects the position of the equilibrium point, or creates some friction on the way back to equilibrium (Phelps 1972). Hysteresis is also used to describe persistence in deviations from equilibria. If shocks originate the deviation of unemployment rate from equilibrium rate, actual unemployment remains in disequilibrium for some time, though the equilibrium rate remains an attractor point in the long run (Layard *et al.*, 1991). Moreover, after the influential article of Blanchard and Summers (1996), hysteresis is frequently associated with the presence of a unit root in a linear dynamic system (or zero root in continuous time difference equations).

Nevertheless, hysteresis should not be mistaken with systems with zero-eigenvalues or with unit roots in discrete time series.

Firstly, hysteresis is a property of an input-output system in which the state variable is subject to an external action, while unit root process is a univariate process where shocks exert directly on the variable state.

Secondly, the unit root process cannot exhibit the property of *remanence* but merely *persistence* (Amable *et al.* 1993; 1994). As an example, let us consider the random walk process: $n_t = \rho \times n_{t-1} + \varepsilon_t$, with ρ = 1 and ε_t a white noise stochastic term. This process exhibits a *long memory* because the shocks have a permanent effect on n_t. However, whatever the magnitude of the shock that affects ε_t is, the occurrence of a first shock followed by a second one of the same intensity but of opposite sign takes the univariate equation back to its initial level. On the other hand, in the case of hysteresis the response to an impulse is not linear. The impact of a shock depends on the previous non-dominated shocks, and a transitory change in the input variable leaves the output variable permanently changed (Amable *et al.* 1993, p. 128).

Thirdly, the present value of the variable n_t keeps all the information of its trajectory over time, without showing any intrinsic dynamic of convergence to a mean value[1]. In a random walk process the shocks would cumulate over time, without

[1] Actually, if n_t follows a random walk process it can be written as: $n_t = n_0 + \sum_{j=0}^{t} \varepsilon_j$.

progressively vanishing, which implies that all innovations in ε_t have an impact over the long run best forecast of the series (actually, at any moment n_t is the best long run forecast of the series itself). Unit root processes have a long or *unselective memory of every past shock* (Göcke 2002). Differently, in a system that exhibits hysteresis the output does not depend on all past values of the input but only on the non-dominated maximums and non-dominated minimums. The system possesses a *selective memory*.

The aim of this chapter is to present the concept of hysteresis, following more closely the original definition of the term stated in physics, highlighting the properties of *remanence*, *nonlinearity* and *selective memory*.

Moreover, even though the basic idea is the same, hysteretic processe involves qualitative changes when we move from the individual firm to the macroeconomic level. This distinction also has important implications concerning the design of the tests implemented to verify the existence of hysteresis. Thus, we begin by describing the concept of *weak hysteresis* that characterizes the dynamics of employment at the micro level in the presence of non-convex costs of adjustment, and then, we present the concept of *strong hysteresis* adapted to the characterization of the dynamics of employment at the sector or the macroeconomic level.

II.2. WEAK HYSTERESIS VS. STRONG HYSTERESIS

II.2.1 DEFINITION OF HYSTERESIS

A hysteresis non-linearity is a kind of operator Γ, which relates the variable output $y(t)$ to a variable input $x(t)$. The hysteresis operator Γ is not a single function because for the same current input value $x^*(t)$ different output values $y(t)$ can be observed (see Figure i.1).

The output $y(t)$, after a certain reference time t_0, depends not only on the input value $x(t)$, with $t \geq t_0$, but also on an initial state $w(t) = w(t_0)$ of the operator Γ:

$$y(t) = \Gamma[t_0, w(t_0), x(t)](t), \qquad \forall t \in [0,T] \tag{ii.1}$$

We assume that $\Gamma[0, w(0), x(t)] = w(0)$, and that Γ must be causal, *i.e.*, it does not depend on $x\big|_{[0,T]}$. Thus, Γ is a memory *operator* (Visitin 1994). Moreover, weak hysteresis is characterized by the *rate-independence property* (Visitin 1994, p. 13). That means that the path of $[x(t), y(t)]$ is invariant with respect to any increasing homoeomorphism $\varphi : [0,T] \to [0,T]$:

$$\Gamma\left(x \circ \varphi \circ w(t^*)\right) = \Gamma\left(x \circ w(t^*)\right) \circ \varphi \text{ in } [0,T] \tag{ii.2}$$

This property allows us to represent the hysteresis operator by branching and merging curves in the (x, y) plane, without specifying the velocity of (x, y) along the curves. Therefore, a hysteresis operator is a rate-independent causal operator, and scalar hysteresis can be viewed as non-linearity with a memory which reveals itself through branching (Mayergoyz 1993, p. xiv and Visitin 1994, p. 13).

Depending on the level of aggregation considered and on the degree of heterogeneity across agents, a weak (micro) form of hysteresis and a strong (macro) one can be distinguished. Both types of hysteresis are characterized as an input-output system with very specific properties and non-linear response to shocks (Amable *et al.* 1994)

II.2.2. THE NON-IDEAL RELAY MODEL

Weak Hysteresis appears at a micro level when a variable output is related to a variable input by a hysteresis operator. Hysteresis behavior at the micro level results from the existence of non-convex (fixed or linear) costs of adjustment that induce discrete adjustment and inertia, in the response of the output variable to small continuous fluctuations of the input variable. In economics, the presence of hysteresis can also result from the existence of uncertainty in the future path of the input variable.

Among the few hysteresis operators at disposal we apply the Non-Ideal Relay Operator. The Non-Ideal Relay Operator is at the core of the classical Preisach Model of Strong Hysteresis. The Preisach Model is an aggregation procedure that allows approximation to a large class of continuous hysteresis laws, due to its phenomenological nature (Visitin 1994, Mayergoyz 2003).

The Non-Ideal Relay Model is a simple model of discontinuous weak hysteresis. Suppose that the variable output $y(t)$ can take one of two values (0 or 1), which means that the non-ideal relay operator[2] ($R_{\alpha,\beta}$) is either switched off or switched on, corresponding to $R_{\alpha,\beta}[x(t)] = 0$ or $R_{\alpha,\beta}[x(t)] = 1$ (as represented in Figure ii.1).

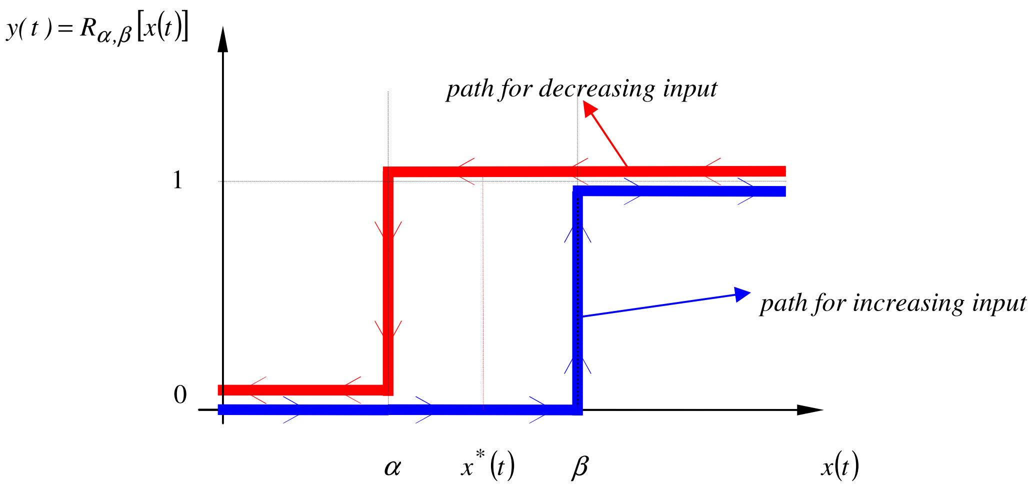


Figure ii.1: The Non-Ideal Relay Operator

The value of the variable output at a moment t can be represented by the following equation:

[2] This operator is also called the Elementary Preisach Operator or Elementary Preisach Hysteron.

$$y(t) = R_{\alpha,\beta}[t_0, w(t_0), x(t)](t) = \begin{cases} w(t_0), & \textit{if } \alpha < x(\tau) < \beta \quad \textit{for all } \tau \in [t_0, t]; \\ 1, & \textit{if there is a } t_1 \in [t_0, t] \textit{ such that} \\ & x(t_1) \geq \beta, \quad x(\tau) > \alpha \quad \textit{for all } \tau \in [t_0, t]; \\ 0, & \textit{if there is a } t_1 \in [t_0, t] \textit{ such that} \\ & x(t_1) \leq \alpha, \quad x(\tau) < \beta \quad \textit{for all } \tau \in [t_0, t] \end{cases} \tag{ii.3}$$

which means that the output depends on the input variable $x(t)$ and on the initial state $w(t_0)$ that can be either 0 or 1 (See Krasnosel'skii and Rachinskii 2003). It follows that in order to know the current value of the output it is not enough to look at the current value of the input. The previous value of the input summarized in $w(t_0)$ should also be taken into consideration.

II.2.3. The Parallel Connection Model

Macro Hysteresis, which emerges as the result of aggregation over heterogeneous micro elements that exhibit some bi-stability, is usually referred to as strong hysteresis (Amable *et al.*, 1994). This type of hysteresis requires the existence of micro units that adjust discontinuously to shocks, as the result of the presence of non-convex adjustment costs and uncertainty and an aggregation over a heterogeneous population of different micro elements.

An elementary aggregation procedure is to combine in parallel a finite number (n) of heterogeneous Non-Ideal Relays (R_{α_i,β_j}, with $1 \leq i, j \leq n$), each of them having different activation (β) and deactivation (α) triggers. If we associate some weights to the individual relays ($\mu_{i,j} \geq 0, \ \forall j$), which represent their contribution to the aggregate output, we can write $Y(t)$ as (see also Figure ii.2):

$$Y(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_{i,j} R_{\alpha_i,\beta_j} [t_0, \eta_0] x(t), \quad t \geq t_0 \quad \textit{and} \quad \beta_j \geq \alpha_i \tag{ii.4}$$

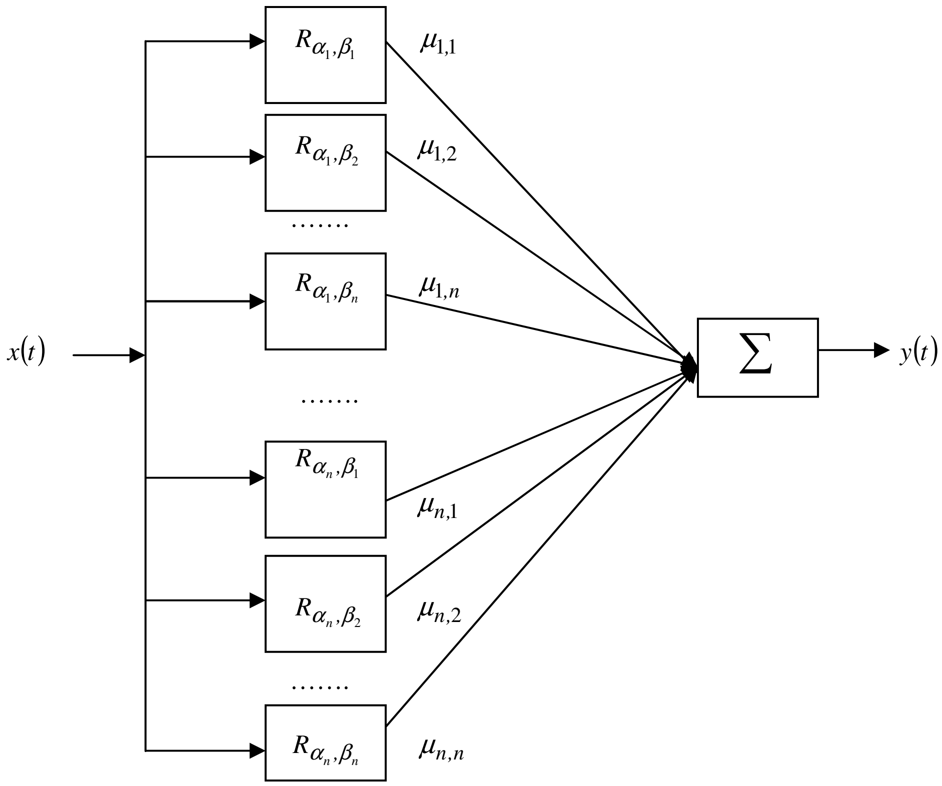


Figure ii.2. Weighted Parallel Connection of a Finite Number of Non-Ideal Relays

Figure ii.3 represents the aggregation procedure obtained by combining in parallel three individual Non-Ideal Relays. Starting from a initial value of the input variable equal to $x(0)$ and the corresponding output $Y(0)=0$, if the value of $x(t)$ increases monotonically from $x(0)$ to $x(3)$, the output of the system change to one when β_1 is reached, to two when β_2 is passed and finally to three when $x(t)>\beta_3$. If the input variable returns to its pre-shock value, $x(0)$, the output of the system remains at three. The system exhibits remanence.

Note that the aggregate output $Y(t)$ remains discontinuous, but the smoothness of the aggregate hysteresis loop increases due to heterogeneity.

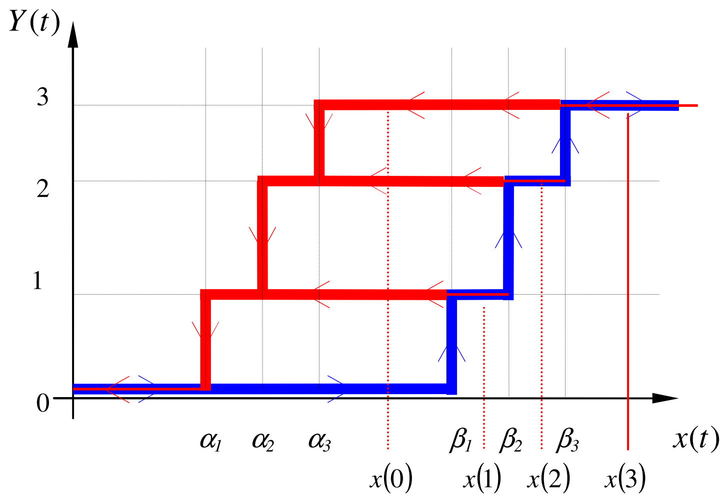


Figure ii.3. Parallel Connection Model of Strong Hysteresis

II.3 Weak Hysteresis Models of Employment Demand

II.3.1. Non-Ideal Relay Model Under Certainty

We adopt the Non-Ideal Relay Model of weak discontinuous hysteresis, described in mathematical terms in the last section, to describe the dynamics of employment at the firm level. This model is related to the firm's profit maximization goal under the existence of non-convex costs of adjustment and it is suitable to represent the dynamics of employment caused by the entry and exit of a firm, or by the employment adjustment decision of a firm that is already in operation.

To deal with the firm's decision of whether to enter, to exit the market or to stay active/inactive, we apply a simple Non-Ideal Relay Model without uncertainty. To analyze the effects of uncertainty on the decision on when it is optimal to enter or to exit the market we apply a more elaborate version of the model[3]. Finally, we extend the model to analyze the effect of the existence of adjustment through variation in the number of hours of work.

[3] We follow Cross *et al*. (1994) and Belke and Göcke (1999, 2005a).

1. Assumptions

1. The product market is perfectly competitive with *M* potential, risk neutral, (supplier) firms;
2. When active, *i.e.*, in the market, each price taker firm produces one unit of output sold at a price P_t (all the firms face a common demand schedule) and employs one unit of labor that costs w_j[4]. When out of the market, each firm produces no output and employs zero units of labor. Zero or one are the only possible values of the steady state level of firms employment[5];
3. Every individual firm must pay a fixed cost (H_j) constant in time to enter the market, which is due to the costs of hiring and training the new worker;
4. Suppose that every firm also faces a fixed cost to leave the market that is due to the cost of dismissing the worker (F_j);
5. Switching the state of activity leads to a complete depreciation of hiring or dismissal costs. Thus H_j and F_j are regarded as sunk costs;
6. The demand for labor is immediately satisfied as there is involuntary unemployment;
7. The firms are considered to be heterogeneous, in terms of the threshold values at which they hire or fire a worker. This heterogeneity is due to differences in the wage rate and in the adjustment costs and related to different technological and managerial abilities, size and maturity;
8. We assume discrete time and an infinite plan horizon;
9. We consider a discount factor $\delta = \dfrac{1}{1+i}$, where i is the risk free interest rate.

[4] We assume that the wage rate is constant in time, but could vary across firms.

[5] Firms can be viewed as an individual unit and each of them can fill one labor position only. This assumption is not as unrealistic as it would seem, it corresponds to assuming firms to be divided into elementary production units where every unit is represented individually in the model (Belke and Göcke 1998);

2. Unit cost function

Each individual firm faces variable costs (w_j) and fixed costs of hiring and training (H_j) new workers and firing costs (F_j). Then, the unit cost function to firm j in period t is:

$$C_{j,t} = \begin{cases} w_j \quad if \quad y_{j,t} = y_{j,t-1} = 1 \\ w_j + H_j \quad if \quad y_{j,t} = 1 \wedge y_{j,t-1} = 0 \\ F_j \quad if \quad y_{j,t} = 0 \wedge y_{j,t-1} = 1 \\ 0 \quad if \quad y_{j,t} = 0 \wedge y_{j,t-1} = 0 \end{cases} \qquad \text{(ii.5)}$$

where $y_{j,t}$ is the output level of firm j in period *t*.

3. Gross Profit Function

The gross profit of the firm j in period *t* (without considering hiring and firing costs) is:

$$R_{j,t} = \begin{cases} P_t - w_j \quad if \quad y_{j,t} = 1 \\ 0 \quad if \quad y_{j,t} = 0 \end{cases} \qquad \text{(ii.6)}$$

4. Supply Function (market participation condition)

Assumption 2, implies that the amount offered by an individual firm in the planned horizon t ($y_{j,t}$), is a binary variable that could assume the values 1 and 0.

The decision of whether or not the firm should enter the market, is reached by comparing the expected present values of the net returns ($V_{j,t}$) if the firm is active in period t with the expected present values of the net returns if the firm is inactive in period t. The firm should take into account: *i*) the state of activity in the preceding period; *ii*) the present net revenues; *iii*) the influence of current activity decisions on the present value of future returns. The comparison of the present value of the alternatives – activity or inactivity is carried out by assuming, firstly, a previously active firm and, secondly, a previously inactive firm.

4.1 Previously Inactive Firm – Entry Decision

A firm that immediately enters the market (in period t) will gain in period t the gross revenue less the fixed costs of entering. Since it expects to earn the same gross profit from period $t+1$ on, the net present value of an immediate enter is:

$$V_{j,t}^{entry} = -H_j + R_{j,t} + \delta V_{j,t+1} = -H_j + P_t - w_j + \delta \frac{P_t - w_j}{1-\delta} = -H_j + \frac{P_t - w_j}{1-\delta} \qquad \text{(ii.7)}$$

For a firm entering or remaining inactive is indifferent if the present value of continuous inactive (0) equals the present value of an instantaneous entry:

$$0 = -H_j + \frac{P_t - w_j}{1-\delta} \qquad \text{(ii.8)}$$

Solving Equation ii.8 for P_t, we obtain the trigger price that induces entry under certainty:

$$p_{entry,j}^{c} = w_j + (1-\delta)H_j = w_j + \frac{i}{1+i}H_j \qquad \text{(ii.9)}$$

The firm j will enter the market if the price level exceeds $p_{entry,j}^{c}$. The entry decision depends on whether the unit revenue P_t covers at least the variable cost w_j plus the interest cost of entry $\frac{i}{1+i}H_j$.

4.2 Previously Active Firm – Exit Decision

A firm which has been active in the preceding period and that will continue in activity in the future will gain:

$$V_j = \frac{P_t - w_j}{1-\delta} \tag{ii.10}$$

If a previously active firm exits in period t, it has to pay firing costs F_j, and it will receive nothing either in period t, or in the subsequent periods. The firm is indifferent between exiting in t or remaining active if:

$$V_{j,t} = \frac{P_t - w_t}{1-\delta} = -F_j \tag{ii.11}$$

Solving Equation ii.11 for P_t we obtain the trigger price that induces exit under certainty:

$$p^c_{exit,j} = w_t - (1-\delta)F_j = w_j - \frac{i}{1+i}F_j \tag{ii.12}$$

The firm will stay active if the price covers at least the variable costs less the interest cost of exit.

Under the existence of fixed costs of hiring (H_j) and firing (F_j) the employment demand function of the individual firm *j* may be represented by[6]

$$y_{j,t} = n_{j,t} = \begin{cases} 1 \quad if \quad y_{j,t-1} = n_{j,t-1} = 0 \wedge P_t \geq w_j + \frac{i}{1+i}H_j (\,entry\,) \\ 1 \quad if \quad y_{j,t-1} = n_{j,t-1} = 1 \wedge P_t > w_j - \frac{i}{1+i}F_j (\,stay \quad active\,) \\ 0 \quad if \quad y_{j,t-1} = n_{j,t-1} = 0 \wedge P_t < w_j + \frac{i}{1+i}H_j (\,stay \quad inactive\,) \\ 0 \quad if \quad y_{j,t-1} = n_{j,t-1} = 1 \wedge P_t \leq w_j - \frac{i}{1+i}F_j (\,exit\,) \end{cases} \tag{ii.13}$$

[6] See Cross *et al*. (1995) and Göcke (2002).

where $n_{j,t}$ is the level of employment of the firm *j* at time *t*[7]. It follows that the entry (expanding) trigger $P^c_{entry,j}$ is greater than the exit (contracting) trigger $P^c_{exit,j}$ and the difference between these threshold values, the band of inaction, is the interest cost of hiring and firing employees:

$$P^c_{entry,j} - P^c_{exit,j} = \frac{i}{1+i}\left(H_j + F_j\right) \tag{ii.14}$$

Thus, each firm requires an aggregate demand shock $P_t > P^c_{entry,j}$ to hire the worker, and an aggregate demand shock $P_t < P^c_{exit,j}$ to dismiss the worker. Demand shocks within the range $P^c_{exit,j} < P_t < P^c_{entry,j}$ are consistent with inaction.

The existence of a band of inaction implies that the current state of the system ($n_{j,t}$) is bi-stable and that the current value of the price (or any other shock variable) is not sufficient to determine the firm's state of employment, because the whole history of the system summarized in $n_{j,t-1}$ needs to be considered. This path dependence is caused by the remanence effect of every transitory shock that induces a change in the firm's state of activity. The employment demand behavior of the individual firm could be described by an elementary hysteretic loop (like the one represented in Figure ii.1), which is the consequence of the Non-Ideal Relay Operator (see Göcke 2002 for more detail).

II.3.2. The Effect of Uncertainty

In order to illustrate the effect of uncertainty on entry/job creation decision and on exit/job destruction decision, we assume a nonrecurring single stochastic change in the output price, which can be either positive ($+\mu$) or negative ($-\mu$) in a discrete time model. We assume that both realizations of the shock have the same probability of ½. In

[7] In this Model, the employment demand function corresponds to the product supply function and can only assume two values: 1, if the firm is in the market with one worker or 0, if the firm is outside the market employing zero workers.

this case, $P_{t+1} = P_t \pm \mu \Rightarrow E(P_{t+1}) = P_t$ and from period $t+1$ on the firm will decide under certainty again[8].

Admitting that the future path of the price is uncertain, waiting can have a positive value because it brings more information about the evolution of the price level. With uncertainty, a previously inactive/active firm has three possible strategies: *i*) stay inactive/active; *ii*) enter/exit the market; *iii*) wait and make a decision after the realization of the stochastic shock. If the firm has the possibility of delaying the decision, it faces a trade-off: waiting has the benefits mentioned above, but it also has the cost of foregoing the profits earned, if entry had occurred.

Thus, uncertainty introduces an additional cost of entering (opportunity cost) that is the value of the option to wait.

In the case of uncertainty, the labor demand function (which corresponds to the supply function) of the individual firm becomes[9]:

$$y_{j,t} = n_{j,t} = \begin{cases} 1 \quad \text{if} \quad n_{j,t-1} = 0 \wedge P_t \geq w_j + \frac{i}{1+i} H_j + \frac{1}{1+2i}\mu \ \ (\textit{entry}) \\ 1 \quad \text{if} \quad n_{j,t-1} = 1 \wedge P_t > w_j - \frac{i}{1+i} F_j \ \ (\textit{stay active}) \\ 1 \quad \text{if} \quad n_{j,t-1} = 1 \wedge w_j - \frac{i}{1+i} F_j - \frac{1}{1+2i}\mu < P_t \leq w_j - \frac{i}{1+i} F_j (\textit{wait in activity}) \\ 0 \quad \text{if} \quad n_{j,t-1} = 0 \wedge P_t < w_j + \frac{i}{1+i} H_j \ \ (\textit{stay inactive}) \\ 0 \quad \text{if} \quad n_{j,t-1} = 0 \wedge w_j + \frac{i}{1+i} H_j \leq P_t < w_t + \frac{i}{1+i} H_j + \frac{1}{1+2i}\mu \ \ (\textit{wait in inactivity}) \\ 0 \quad \text{if} \quad n_{j,t-1} = 1 \wedge P_t \leq w_j - \frac{i}{1+i} F_j - \frac{1}{1+2i}\mu \ \ (\textit{exit}) \end{cases} \qquad \text{(ii.15)}$$

Uncertainty in the future behavior of prices widens the hysteresis band (Dixit 1992, p. 121; Göcke 1999, p. 275). Combining both triggers under uncertainty, the width of the band of inaction is:

$$P^u_{entry,j} - P^u_{exit,j} = P^c_{entry,j} - P^c_{exit,j} + \frac{2\delta}{2-\delta}\mu = \frac{i}{1+i}(H_j + F_j) + \frac{2}{1+2i}\mu \qquad \text{(ii.16)}$$

[8] We introduce uncertainty by considering an expected future stochastic one-time shock, in line with Belke and Göcke (1999).

[9] See Belke and Göcke (1999; 2005a) for more details.

where $P^u_{entry,j}$ and $P^u_{exit,j}$ are the entry and the exit triggers under uncertainty respectively.

From equation (ii.16), the expansion of the inaction band is linear and separable in the anticipated absolute size of the shock (see Figure ii.4). The width of the inaction band depends positively on the fixed cost of hiring and firing, and on the degree of uncertainty (see also Bertola 1992, p. 395)[10].

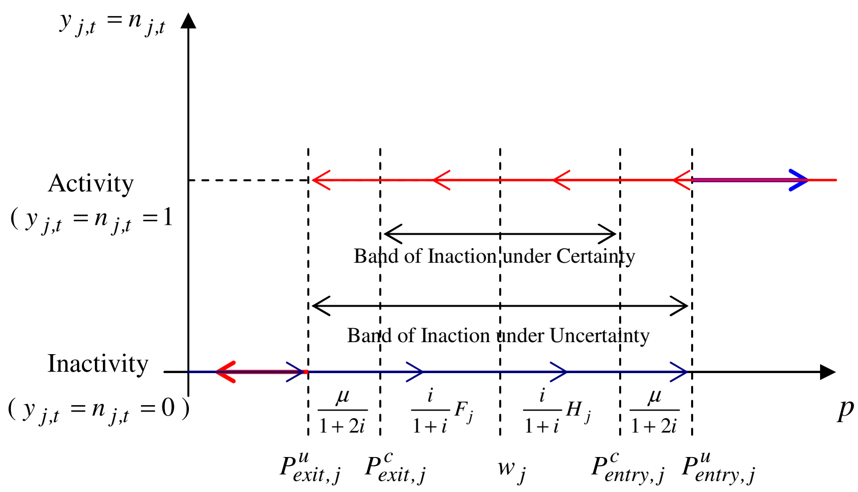


Figure ii.4. Micro Hysteresis Loop

II.3.3. THE EFFECT OF ADJUSTMENT THROUGH HOURS OF WORK

To describe the dynamics of employment under a hysteretic model, we have neglected, until now, the margin of adjustment hours of work. Nonetheless, it is more realistic to recognize that a firm can respond to a demand shock by changing the number of employees, the number or hours per worker or both.

Moreover, to the extent that there is some degree of substitutability between the adjustment of labor input through variation in hours per worker and through the variation in the number of workers, the existence of costs associated with the variation

[10] Concerning the effect of the interest rate, when $i \to 0$ the band of inaction under certainty collapses towards zero while the band of inaction under uncertainty tends to 2μ. When $i \to \infty$ the band of inaction under uncertainty tends to $H_j + F_j$. Thus, the lower the interest, rate the higher the importance of uncertainty for the width of the band of inaction (Belke and Göcke 1999, p. 266).

in the number of hours of work could interact with the employment adjustment costs and reinforce or attenuate the existence of inertia in employment dynamics.

In order to study the effect of the adjustment through the number of hours per worker in the analysis of labor input adjustment, we extended the previous model under certainty.

We keep the same basic assumptions (see section II.3.1), but now we admit that the firms could vary the number of hours of work of the single worker, within certain limits, depending on the state of product demand. We assume that the standard hours of work are equal to one and that the actual number of hours could be fixed by the firm in the interval: $h_t \in [h_d, h_u]$ with $0 < h_d < 1$ and $1 < h_u < 2$. Thus, the production function becomes: $y_{j\ t} = n_{j,t} h_{j,t}$.

We assume that changing the number of hours per worker does not involve a fixed cost of adjustment implying that there is no hysteresis in this margin of adjustment. In fact, firms can change hours more rapidly and with fewer costs than hiring or firing employees, because varying the number of hours entails no long-run commitments and the decisions related to the number of hours are easier to reverse.[11] However, we assume that firms must pay a premium for overtime work that could be legally determined or established by collective bargaining agreements.

Concerning the structure of the payment system, we assume that the wage is a function of the number of hours of work. Standard hours are fixed by law at the level of one. The decision of firms, concerning the number of effective hours of work, is influenced by the rigidity of the working time regulations. These regulations affect the upward and downward flexibility of hours, and together with employment adjustment costs determine the firms' response to demand shocks.

Regarding upward hours flexibility, we assume that the overtime premium is increasing in hours of work beyond one, and the wage premium for working less than one is also increasing in $(1 - h_t)$, which reflects the increase in the hourly wage to compensate the employee for working fewer than the standard number of hours. The structure of payment is summarized in equation (ii.17)[12]:

[11] This assumption is in line with Sargent (1978), who, using quarterly aggregate data from the United States in the period from the first quarter of 1947 to the fourth quarter of 1972, estimated straight-time adjustment costs larger than overtime adjustment costs. Shapiro (1986), using quarterly data for manufacturing from 1955 to 1980, also estimated small and insignificant adjustment costs in varying the number of hours of work of the existing workers.

[12] We follow Nickel (1978).

$$w_j^*(h_{j,t}) = \begin{cases} w_j h_{j,t} + \phi_u \left(h_{j,t} - 1\right) \ \ if \ \ h_{j,t} \geq 1 \\ w_j \Leftarrow h_{j,t} = 1 \\ w_j + \phi_d w_j \left(h_{j,t} - 1\right) \ \ if \ \ h_{j,t} \leq 1 \end{cases} \tag{ii.17}$$

with $\phi_u, \phi_d \in [0,1]$

Overtime hours that exceed one are paid at a constant rate, with ϕ_u being the overtime premium. In Equation ii.17, ϕ_u is a parameter that measures the tightness of overtime regulations concerning compensation. If the legislation is strict, ϕ_u approaches one and the firm pays the maximum amount of overtime premium. Contrarily, if legislation is soft, ϕ_u approaches zero, and each hour beyond one is paid at a standard hourly wage (w_j).

On the other hand, the wage payment for hours less than one is: $w_j + \phi_d w_j (h_{j,t} - 1)$, where ϕ_d measures downward hours flexibility. When the legislation concerning working less than standard hours is strict, ϕ_d leans closer to zero, and the firm must pay w_j. When the legislation is soft, ϕ_d approaches one, and the firm pays exactly the effective hours of work $(w_j h_{j,t})$.

Exit Decision (job destruction)

Now consider the problem of calculating the exit trigger.

A firm which has been active in the preceding period and that will continue its activity in the future will gain:

$$V_{t,j} = \frac{P_t h_{j,t} - \left(w_j + \phi_d w_j h_{j,t} - \phi_d w_j\right)}{1 - \delta} \tag{ii.18}$$

where $V_{t,j}$ is the firm's net present value if it continues to be active.

If, following a negative demand shock, the price falls to $P_t < \phi_d w_j$, the firm has an incentive to reduce the number of hours of work to the minimum possible level (h_d)[13].

The firm is indifferent between exiting in t or remaining active if:

$$V_{t,j} = \frac{P_t h_d - \left(w_j + \phi_d w_j h_d - \phi_d w_j\right)}{1-\delta} = -F_j \tag{ii.19}$$

Solving Equation ii.19 for P_t we obtain the trigger price that induces exit (fire the worker) when the firm has the possibility of reducing the number of hours of work:

$$P^h_{exit,j} = \frac{w_j\left(1 + h_d\phi_d - \phi_d\right)}{h_d} - \frac{F_j(1-\delta)}{h_d} \tag{ii.20}$$

From Equation ii.20 for ϕ_d close to one (high downward flexibility of hours) $P^h_{exit,j} = w_j - \frac{(1-\delta)F_j}{h_d} < P^c_{exit,j} = w_j - (1-\delta)F_j$. More precisely for $\phi_d > 1 - \frac{F(1-\delta)}{w_j}$ the firm will reach h_d before exit and the existence of the margin of adjustment hours of work contributes to the increases in the width of the band of inertia.

Moreover,

$$\frac{\partial p^h_{exit,j}}{\partial h_d} = \frac{F_j(1-\delta) - w_j(1-\phi_d)}{(h_d)^2} < 0, \text{ for } \phi_d > 1 - \frac{F(1-\delta)}{w_j} \tag{ii.21}$$

and

$$\frac{\partial p^h_{exit,j}}{\partial \phi_d} = \frac{w_j(h_d - 1)}{h_d} < 0 \tag{ii.22}$$

---

[13] Note that $\frac{\partial V_j}{\partial h_t} = \frac{P_t - \phi_d w_j}{1-\delta} < 0$, for $P < \phi_d w_j$.

As a lower minimum level of hours of work (h_d), and a lower penalty for reducing the number of hours of work (ϕ_d closer to one) imply a more flexible adjustment through hours of work, the width of the hysteresis band is a positive function of the downward flexibility of hours adjustment.

Job Creation Decision

In order to analyze the impact of adjustment through hours after a positive demand shock, we introduce the following additional assumptions:

i) After entering, which occurs when the price is $P^{c}_{entry,j}$, the firm has two options to adjust to a positive demand shock, i.e. to a further increase in its price level: *a*) the firm can increase the number of hours of work to a maximum of $h_u < 2$, paying an overtime premium according to equation (ii.17); *b*) the firm can open another plant, which implies hiring another worker.

ii) The production function of the second plant is the same as the first one, the firm pays the same wage w_j to the second worker, but opening another plant implies spending a fixed cost of activation of $H^{'}_{j}$ with $H^{'}_{j} > H_j$.

iii) These alternatives are mutually exclusive, and the firm cannot change the number of hours of work in the second plant (which are necessarily equal to 1).

iv) The decision to increase the number of workers is reversible, meaning that if the circumstances turn out to be very favorable, the firm can reduce the number of hours of work in the first plant to one and open a second plant.

Equation (ii.23) represents the profit of the firm if it decides to increase the number of hours of work:

$$V^{h}_{j,t}(h_u, \phi_u) = \frac{P_t h_u - \left(w_j h_u + \phi_u h_u - \phi_u\right)}{(1-\delta)} \qquad \text{(ii.23)}$$

If $P_t > w_j + \phi_u$, firm j has an incentive to increase the number of hours of work to h_u[14].

If the firm decides to open another plant it will earn:

$$V_{j,t}^{2^{nd}\ plant} = \frac{2\times\left(P_t - w_j\right)}{(1-\delta)} - H' \tag{II.24}$$

Thus, the firm will only open the second plant if the profit obtained from operating with two plants is higher than the profit obtained from the increase in the number of hours of work in the first plant. The firm will be indifferent between these alternatives for $V_{j,t}^{h}(h_u, \phi_u) = V_{j,t}^{2^{nd}\ plant}$. Solving this equation for P_t, we obtain the second plant activation trigger when the firm chooses between adjusting through the number of hours or through the number of workers:

$$P_{j,entry2}^{h} = w_j + \frac{(1-\delta)H' + \phi_u(1-h_u)}{(2-h_u)} \tag{ii.25}$$

For $\phi_u < H_j'(1-\delta)$, $P_{j,entry2}^{h} = w_j + \frac{(1-\delta)H' + \phi_u(1-h_u)}{(2-h_u)} > P_{j,entry2} = w_j + (1-\delta)H'$ meaning that the introduction of upward adjustments of labor demand, through hours of work, contributes to the delay of the activation of a second plant and enlarges the employment hysteresis band.

Moreover:

$$\frac{\partial P_{entry2,j}^{h}}{\partial h_u} = \frac{H_j'(1-\delta) - \phi_u}{(2-h_u)^2} > 0 \text{, for } \phi_u < H_j'(1-\delta) \tag{ii.26}$$

and

[14] Note that $\frac{\partial \Delta V_{j,t}^{s}}{\partial h_{j,t}} = \frac{P_t - w_j - \phi_u}{1-\delta} > 0$, for $P_t > w_j + \phi_u$.

$$\frac{\partial P^{h}_{entry2,j}}{\partial \phi_u} = \frac{-h_u}{2-h_u} < 0 \text{, for } 1 < h_u < 2 \qquad \text{(ii.27)}$$

and as a higher maximum level of hours of work (h_u) and a lower overtime premium (ϕ_d closer to zero) imply more flexible adjustment through hours of work, similarly, the width of the hysteresis band is a positive function of the upward flexibility of hours adjustment.

Figures ii.5 presents the results of a numerical simulation of the effect of the interaction between non-convex costs of adjustment and the existence of uncertainty in the future path of prices on the width of the inaction band.

We consider the model of employment demand with uncertainty presented in section II.3.2, and we set $w_j = 1$, $H_j = 1$, $F_j = 1$, $i = 0.1$ and $\mu = 0.5$. Figure ii.5 a) shows that an increase in the fixed adjustment costs (H_j, F_j) from 1 to 1.59 with increments of 0.01 originates an increase of the inaction band. The simulation shows that the inaction band is a linear positive function of the fixed costs of hiring and firing (see also Equation ii.16). The effect of uncertainty is simulated in Figure ii.5 b). We verify that an increase of uncertainty, represented by an increase in the parameter μ from 0.5 to 0.795 with increments of 0.005, originates a linear increase in the band of inaction. Figure ii.5 c) illustrates the joint effect of the existence of fixed employment adjustment costs and uncertainty. The inaction band is particularly large for high values of the fixed adjustment costs and uncertainty.

We also simulate the effect of the interaction between the presence of employment adjustment costs and the degree of flexibility in varying the number of hours per worker on the inaction band. We analyze the effect of two aspects of the flexibility of the adjustment through the number of hours per employee: *i*) the maximum number of overtime hours of work (h_u) and the minimum number of short time hours of work (h_d); *ii*) the regulation concerning compensation for overtime and short time work, measured respectively by the parameters ϕ_u and ϕ_d.

We consider the model of labor demand without uncertainty presented in section II.3.3, and we set $w_j = 1$, $H_j = 1$, $F_j = 1$, $i = 0.1$, $h_d = 0.85$, $h_u = 1.15$, $\phi_d = 1$ and $\phi_u = 0$. In Figure ii.6 we illustrate the effect on the inaction band of the downward flexibility of adjustment through hours of work. The inaction band increases exponentially with the decrease in the minimum permitted number of hours of work from $h_d = 0.85$ to $h_d = 0.3485$ with increments of -0.0085 (Figure ii.6 b) and increases linearly with the increase in the parameter ϕ_d (see Figure ii.6), which is a measure of downward hours flexibility concerning the magnitude of the hourly wage premium mandated in the case of work time reduction (downward flexibility increases as ϕ_d approaches one). From Figures ii.6 d) and e) we verify that the inaction band is especially large for combinations of large employment fixed adjustment costs with a low minimum number of hours of work and with a low value of the parameter ϕ_d. The amplification of the effects of the fixed costs of adjustment upon the inaction band, are, however, stronger for the case of the interaction with the minimum number of hours of work. Figure ii.6 f) also shows, that a larger inaction band emerges for combinations of soft legislation concerning the compensation of reduced time of work measured by the parameter ϕ_d with a low minimum number of hours of work.

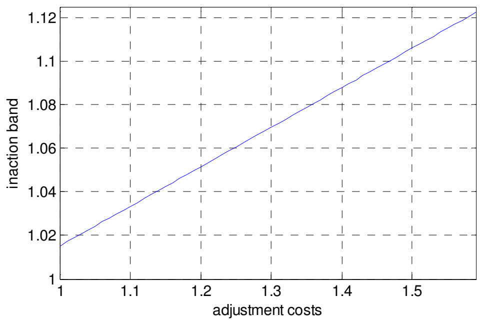
In Figure ii.7 we illustrate the effect on the inaction band of the upward flexibility of adjustment through hours of work[15]. As in the previous case, the inaction band increases exponentially with the maximum number of overtime hours of work permitted (Figure ii.7 b) and decreases linearly with the increase of the value of the parameter ϕ_u, which implies a greater rigidity of the regulations on overtime work compensation (Figure ii.7 c). Again, the effect of the employment adjustment costs upon the inaction band is amplified by the degree of flexibility of upward adjustment of hours of work. The effect of the degree of upward flexibility of the adjustment of hours of work is captured by parameter ϕ_u, which is an index of the rigidity of the legislation concerning the compensation of overtime work (Figure ii.7 e) and specially by the maximum number of overtime hours of work (Figure ii.7 d). Figure ii.7 f) shows that, as in the case of the downward adjustment, a large inaction band emerges for combinations of soft legislation concerning the compensation of overtime time work with a high maximum number of overtime hours of work.

[15] This case is more important than the previous one, as the upward adjustment in the number of hours of work is more common in practice as the way to adjust the labor factor.

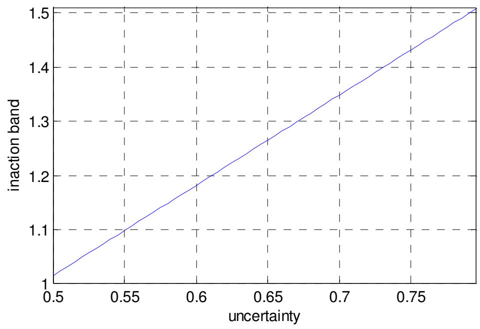
Overall, the conclusion is that there is a positive association between the width of the inaction band and the magnitude of the fixed adjustment costs, the level of uncertainty and the degree of flexibility of adjustment through hours of work.

Figure. ii.5
The effect of Employment Adjustment Costs and Uncertainty on the Inaction Band
(Model with Uncertainty)

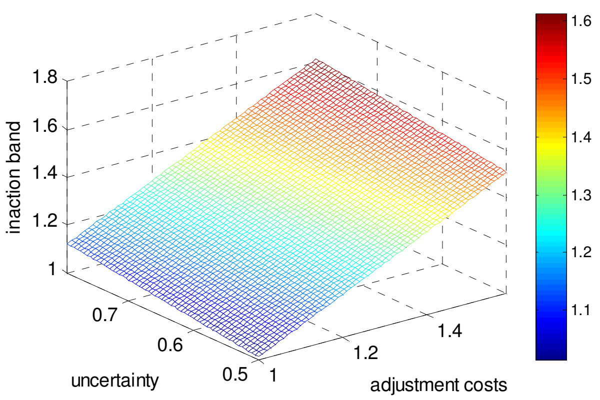
a)



b)



c)



$$Inaction \quad Band = (1-\delta)(H_j + F_j) + \frac{2}{1+2i}u$$

Basis values of the Simulation:

$w_j = 1$;

$H_j = 1$ (increment 0.01);

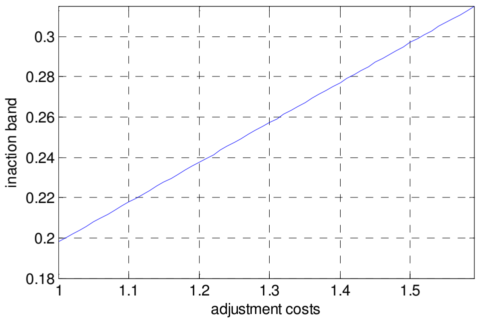
$F_j = 1$ (increment 0.01);

$i = 0.1$;

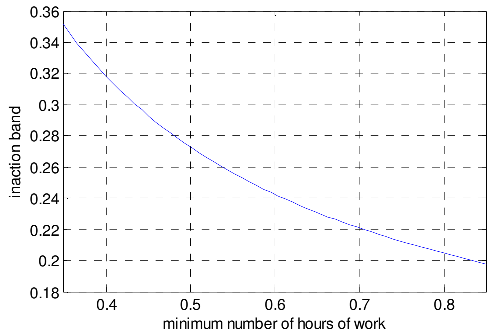
$u = 0.5$ (increment 0.005)

Figure. ii.6
The effect of Employment Adjustment Costs and Downward Hours Flexibility on the Inaction Band (Model without Uncertainty)

a)

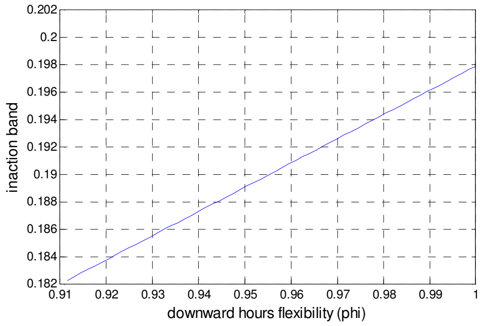


b)



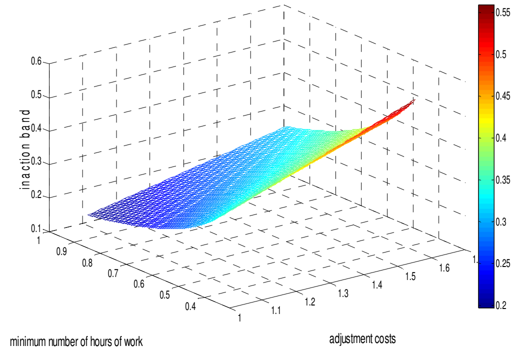
(higher values imply less flexibility in the adjustment through hours of work)

c)

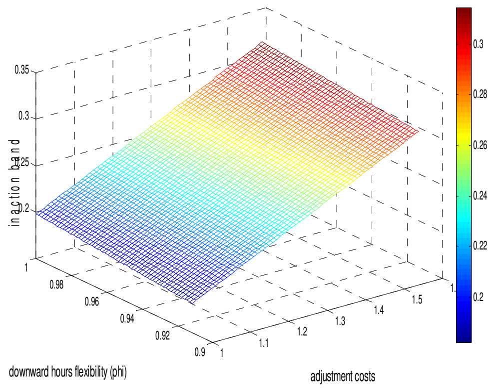


(higher values imply more flexibility in the adjustment through hours of work)

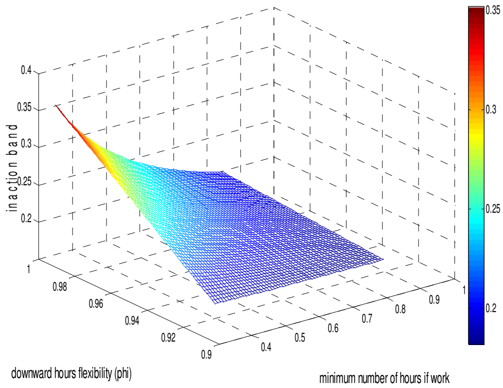
d)



e)



f)



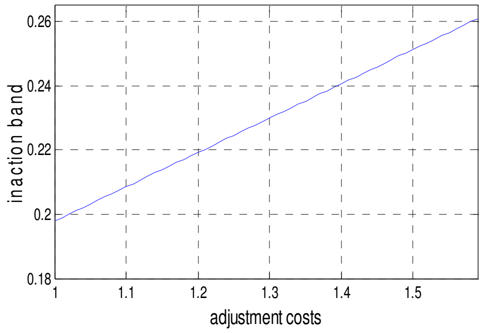
$$Inaction \quad Band = (1-\delta)\left(\frac{H_j h_d + F_j}{h_d}\right) + \frac{w\left[h_l(1-\phi_d)-1-\phi_d\right]}{h_d}$$

Basis values of the Simulation: $w_j = 1$; $H_j = 1$ (increment 0.01); $F_j = 1$ (increment 0.01); $i = 0.1$;

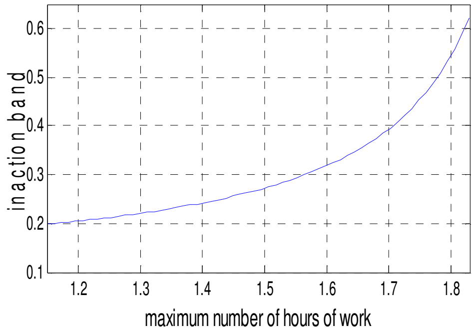
$h_d = 0.85$ (increment -0.0085); $\phi_d = 1$ (increment -0.0015)

Figure. ii.7
The effect of Employment Adjustment Costs and Upward Hours Flexibility on the Inaction Band
(Model without Uncertainty)

a)

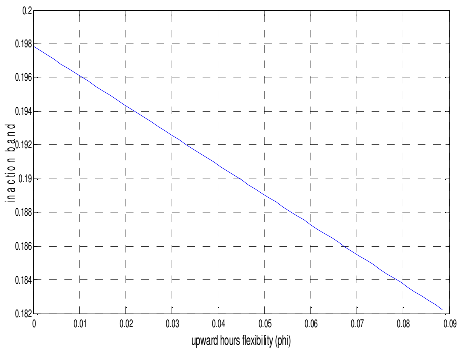


b)



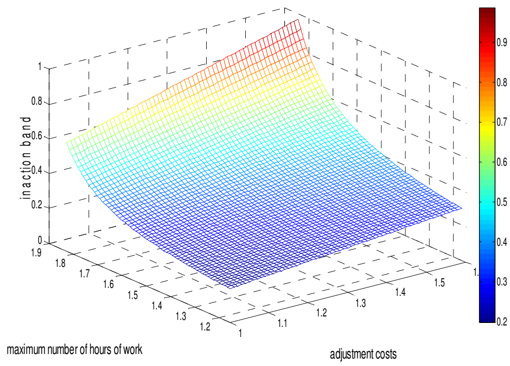
(higher values imply more flexibility in the adjustment through hours of work)

c)

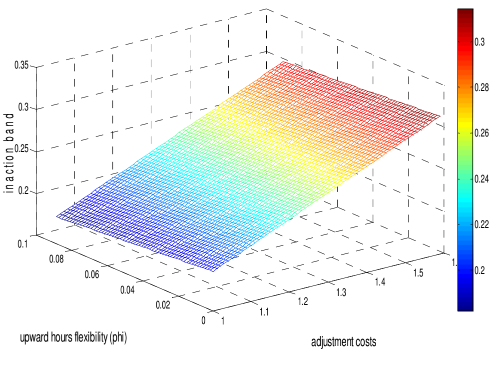


(higher values imply less flexibility in the adjustment through hours of work)

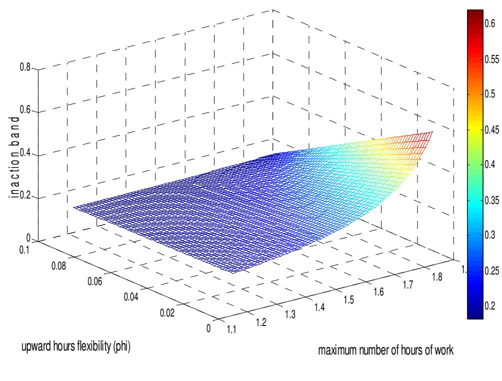
d)



e)



f)



$$Inaction \quad Band = \frac{(1-\delta)\left[H_j + (2-h_u)F_j\right] + \phi_u(1-h_u)}{2-h_u}$$

Basis values of the Simulation: $w_j = 1$; $H_j = 1$ (increment 0.01); $F_j = 1$ (increment 0.01); $i = 0.1$;

$h_u = 1.15$ (increment +0.0115); $\phi_u = 0$ (increment +0.0015)

II.3.4. PROPERTIES OF WEAK HYSTERESIS

An input-output system described by the Non-Ideal Relay Model, exhibits a weak form of hysteresis characterized by the following properties (see Amable *et al*. 1995 and Mayergoyz 2003):

i) The system exhibits *path dependence*. The history of the system matters because, for values of the input variable - $P(t)$ between $P^u_{exit,j}$ and $P^u_{entry,j}$, the system exhibits a bi-stability, in the sense that the output value associated with the same value of the input could be 0 or 1. In this case the position of the system (0 or 1) is dependent on the past trajectory of the system. More precisely, for a value of the input $P^u_{exit,j} < P^*(t) < P^u_{entry,j}$ the value of the output will be 1, if the value of the input started above $P^u_{entry,j}$, but it will be 0 if it started below $P^u_{exit,j}$ (see Figure ii.4).

ii) The *system exhibits remanence*. If the input value is initially $P^u_{exit,j} < P^*(t) < P^u_{entry,j}$ and the equilibrium value of the output is 0, and if the input value increases transitorily to $P^1(t) \geq P^u_{entry,j}$, the equilibrium value of the output changes permanently to 1. Conversely, if the value of the input is initially $P^u_{exit,j} < P^*(t) < P^u_{entry,j}$ and the output equilibrium value of the output is 1, and if the input decreases transitorily to $P^2(t) \leq P^u_{exit,j}$, the equilibrium value of the output changes permanently to 0.

iii) The *remanence effect is independent of the magnitude of the change in the input* once the values $P^u_{exit,j}$ and $P^u_{entry,j}$ are reached.

iv) The hysteresis operator is *rate-independent*, which means that the hysteretic behavior of the system is independent of how fast the input varies between two extremum points (as explained before).

v) The non-ideal relay originates a hysteretic non-linearity with *local memories*, which means that the value of the output $y(t_0)$ at some moment of time t_0, and the values of the input $P^*(t)$ at all subsequent instants of time $t \geq t_0$ uniquely predetermine the value of the output $y(t)$ for all $t > t_0$. In the example, branching occurs for any input extreme since the triggers $P^u_{exit,j}$ and $P^u_{entry,j}$ are reached.

II.4. STRONG HYSTERESIS MODELS OF EMPLOYMENT DEMAND

II.4.1. THE PREISACH MODEL

Among the several models of strong hysteresis that closely follow the original definition of the concept, and that could be used in economics, we describe the Preisach Model.

The Preisach Model with the developments of Mayergoyz (2003), is one of the most powerful models of strong hysteresis, and is now widely recognized as a fundamental mathematical toolkit in describing a wide range of hysteretic phenomena in quite different areas (see Cross 1995 for an application to employment)[16].

The dynamics of aggregate employment (N), in response to the time evolution of the aggregate price level (shock variable) can be described by the Preisach Model of Hysteresis. The Preisach Operator is an aggregation of elementary Non-Ideal Relays (R_{α_j,β_j}) defined by the (α_j, β_j) pairs of switching values of the aggregate price level that correspond, respectively, to the exit and entry triggers ($\alpha_j = P_{exit,j}$ and $\beta_j = P_{entry,j}$). The dynamics of aggregate employment is represented by the number of active firms in the Preisach Plan, which can be defined as: $P = \{(\alpha,\beta) \in \Re^2 | \beta \geq \alpha\}$, and that is determined by the dynamics of the aggregate product demand. Each individual firm can be represented by a point in the (α,β) plan with $\beta \geq \alpha$. Assuming a continuum in P of heterogeneous hysteretic firms, each one with employment demand functions of the type of Equation ii.13, the Preisach Model of Hysteresis can be written as[17]:

$$N(t) = \iint_P u(\alpha,\beta) R_{\alpha,\beta} P(t) \; d\alpha \; d\beta \qquad \text{(ii.28)}$$

[16] The Preisach Model was originally introduced by the Hungarian Physicist Ferenc Preisach in 1935. The model was developed to represent hysteresis in ferromagnetic materials, and assumes that those substances are made of tiny magnetic particles (dipoles), which were represented by a simple hysteresis loop (see Figure ii.1).

[17] See Mayergoyz (2003).

where $N(t)$ is the aggregate employment, $u(\alpha,\beta)$ is the density function of the individual firms, also called the Preisach Function, R_{α_j,β_j} are the individual relays that represent the relation between employment and aggregate demand at the firm level, and $P(t)$ is a proxy of product aggregate demand.

Assuming $\alpha_0 = min\{P(t)|t = 1,2,...,n\}$ and $\beta_0 = max\{P(t)|t = 1,2,...,n\}$, a heterogeneous set of firm's hysteretic relays can be considered on a *limiting triangle* T defined as: $T = \{(\alpha,\beta)|\beta \geq \alpha \wedge \alpha \geq \alpha_0 \wedge \beta \leq \beta_0\}$ (see Figure 9). Moreover, the density $u(\alpha,\beta)$ is defined in T, and it is equal to zero outside this triangle.

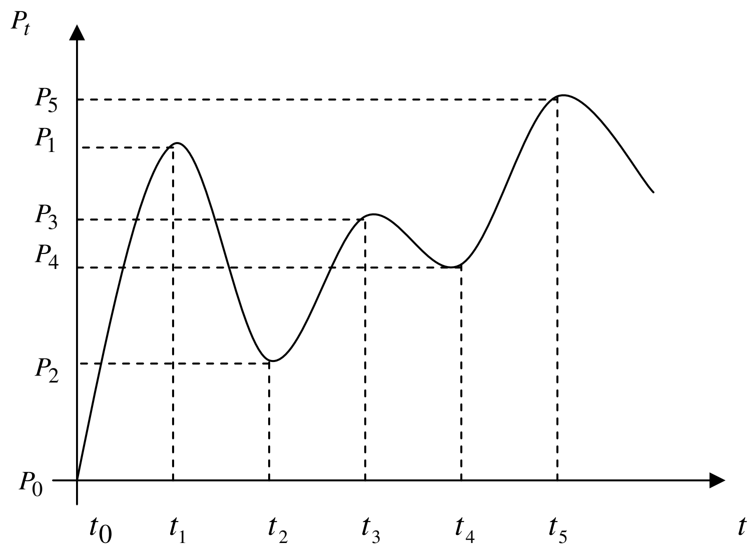


Figure ii.8. Input Function

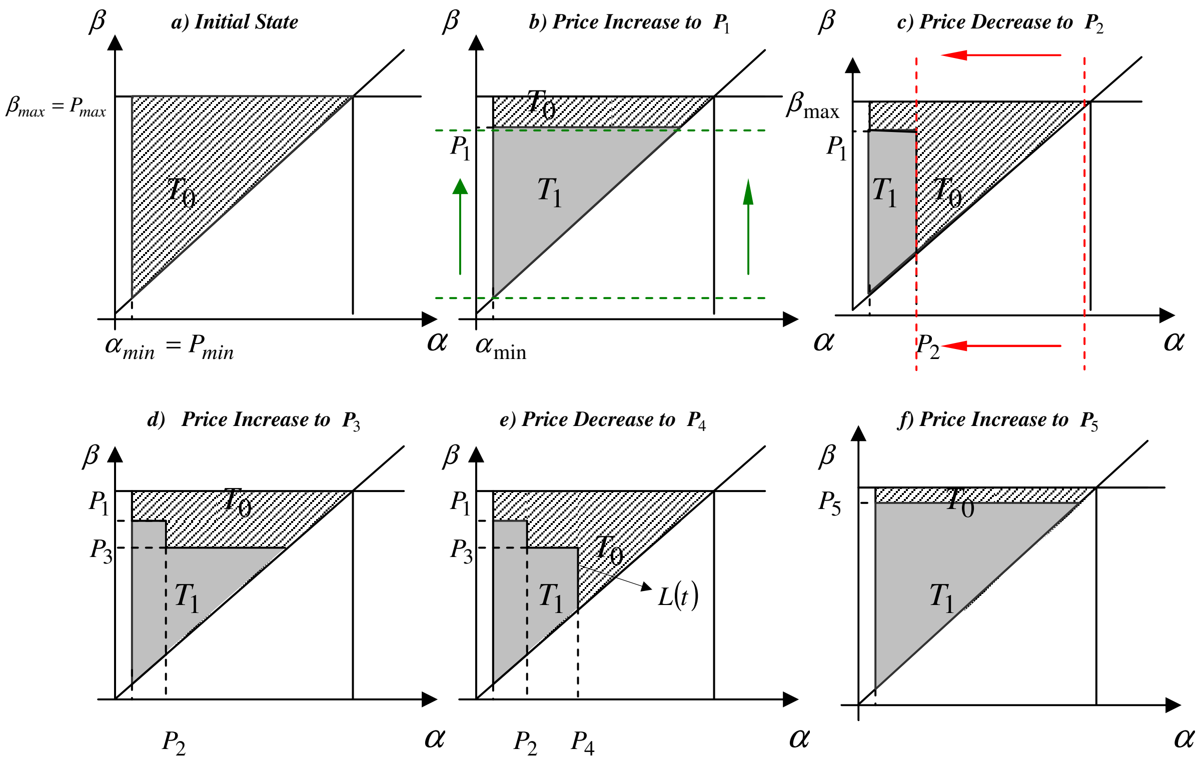


Figure ii.9. Preisach Memory Map

Consider the time sequence of values of the aggregate price level represented in Figure ii.8. The dynamics of aggregate employment in response to the cyclical variation of the price is illustrated in Figure ii.9. An increase in aggregate price level is represented by an upward displacement of a horizontal (green) line, from the position $\beta = \alpha_{min}$, to the position corresponding to $\beta = P_t$, switching relays from T_0 to T_1 (see Figure ii.9 *b*), while a decrease in the price level is represented by a leftward displacement of a (red) vertical line, from the position $\alpha = \beta_{max}$ to the position $\alpha = P_t$, switching relays from T_1 to T_0 (see Figure ii.9 *c*). At any time, triangle T is divided into two time-varying regions: T_1 and T_0 defined as:

$$
\begin{aligned}
T_1(t) &= \left\{ (\alpha,\beta) \in T \,\middle|\, \text{ output of } R_{\alpha,\beta} \text{ at } t \text{ is } 1 \right\} \\
T_0(t) &= \left\{ (\alpha,\beta) \in T \,\middle|\, \text{ output of } R_{\alpha,\beta} \text{ at } t \text{ is } 0 \right\}
\end{aligned}
\qquad \text{(ii.29)}
$$

so that $T_1(t) \cup T_0(t) = T, \forall t$.

The initial condition is that $P_t < \alpha_0$ implying that all the relays are switched off, meaning that all firms that are outside the market are employing zero workers, i.e., $n_{j,0} = 0$, $\forall j$, and the value of aggregate employment is zero ($T_0 = T$ and $T_1 = 0$). In Figure ii.9 *a)* the dashed area shows the relays that are switched off (firms that stay outside the market). Subsequently, the price starts to increase monotonically, reaching a local maximum (P_1) at the time t_1. At that time, all relays with $\beta \leq P_1$ switch on, meaning that firms with $P_{entry} \leq P_1$ start to hire workers (grey area). The relays are now divided into two sets: T_1 represents the set of the relays that are switched on (grey area), corresponding to those firms that are currently entering the market hiring employees, and T_0 represents the set of the relays that are switched off (dashed area), corresponding to those firms that are currently firing employees or deciding to stay outside the market. When the aggregate price level decreases to a local minimum P_2, those relays for which $\alpha \geq P_2$ switch off, meaning that firms with $P_{exit} \geq P_2$ start to dismiss. This dynamic serves to trace a staircase line $L(t)$, which divides the grey area where the relays are on from the dashed area where the relays are off. The vertices of the staircase line have coordinates that correspond to the sequence of the past non-dominated extremums of the input variable. Differently from what happens at the micro level, where the history of the system is summarized in $n_{j,t-1}$, history in this case is the sequence of non-dominated maximums and minimums of the aggregate price level.

II.4.2. THE EFFECT OF UNCERTAINTY AND ADJUSTMENT THROUGH HOURS OF WORK

The Preisach Model is well suited to testing the existence of hysteresis in aggregate employment dynamics but it is not adequate to analyze the effect and the relative importance of the different sources of hysteresis (existence of non-convex costs of adjustment, uncertainty and adjustment through the number of hours of work). For this purpose we apply the Linear Play Model of Hysteresis (see Visitin 1994 and Göcke 2001; 2002)[18].

The Linear Play Model of Hysteresis consists of a continuous operator P_r that can be described as a linear spring coupled in parallel with a friction element (Visitin 1994, p. 15). From Figure ii.10, we notice that the Play Operator is characterized by horizontal reversible inner branches of the same length (the *play* segment) and increasing linear limiting branches Γ_u and Γ_l (called the *spurt* segments). It can also be observed that the loops are oriented counter-clockwise. As the slope of limiting branches is fixed, the operator is characterized by only one constant - its input threshold value or the magnitude of the *play.* Together with the initial value of the operator state: the pair $(P_r(t_0), P(t_0))$, determines the value of the output $N(t)$ in dependence on the future values of the input $P(t)$.

Actually, the linear play dynamics, more typical at the firm level, could emerge at the macro level, especially when there is uncertainty concerning the future behavior of the product demand and/or input prices. Uncertainty originates a displacement of every (α_j, β_j) combination characterizing firm j to Northwest in the Preisach Triangle, as the entry/exit trigger increases/decreases by $\frac{\mu}{1+2i}$ (see section II.3.2). Consequently, the existence of uncertainty implies the emergence of a zone above and parallel to 45º - line without firms, introducing a zone of weak reaction (*play interval*) of aggregate employment (and possibly no reaction at all) with every reversal of the input variable (see Belke and Göcke 2005b, p. 199). The existence of *play intervals* originates flatter hysteresis loops. This happens because on the one hand, the play intervals have to be passed in order to originate a permanent employment impact, and

[18] The Linear Play Operator is a non-linear operator because 'linear' refers to the shape of the boundary of the hysteresis region, and not to the operator (Visitin 1994, p. 64).

on the other a small number of firms are affected by input changes (Belke and Göcke 2005b, p. 199).

In order to illustrate the Play Hysteresis dynamics under uncertainty, we assume[19]:

1. The Hysteresis loops are divided into linear partial functions with different slopes;
2. The slope of a linear section changes when a local extremum is reached;
3. Only two different slopes are considered: a small one, representing the relation between employment and the aggregate price level along a zone of relative inaction called the *play*, and a large one, representing the relation between employment and the aggregate price level along a zone of strong reaction called the *spurt*;
4. The linear sections are continuously connected resulting in a joint point called 'knot' of both adjacent sections for a local extremum;
5. A constant width of the play area ($PLAY$) is assumed.

Figure ii.10 helps to illustrate the linear play-dynamics.

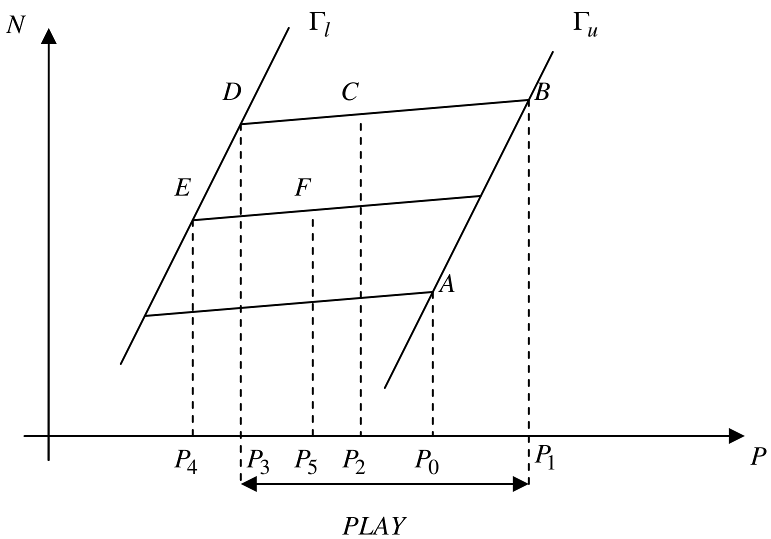


Figure ii.10. Linear Play Hysteresis-Dynamics

Suppose that starting from point A (with $P_t = P_0$) there is an increase in the aggregate price level to P_1. All the firms with $\beta < P_1$ will be hiring workers in this

[19] See Belke and Göcke (2001) and Göcke (2002).

period originating an increase of aggregate employment along the *upward spurt line* Γ_u (the system reaches pont B). From point *B* (with $P = P_1$) in the *upward spurt line*, a decrease in the price level originates an entering into the *play area*, where a weak reaction results, until the entire play area is passed. When the aggregate price level reaches P_3 employment will start to fall along the *downward spurt line* Γ_d until $P_t = P_4$ (in point E). A further increase in the price level to P_5 will induce a vertical downward displacement of *the play* area and the system reaches point F.

II.4.3. Properties of Strong Hysteresis

Focusing on the Preisach Model, according to Visitin (1994), Amable *et al.* (1995, p. 159) and Mayergoyz (2003, p. 15-20), the nontrivial aggregation of heterogeneous individual relays originates a form of hysteresis with strong properties.

Firstly, the value of the output variable depends in a more complex way on the history of the input, when compared to the weak form of hysteresis. History, in this case, is the sequence of non-dominated maximums and minimums of the input variable. The non-dominated local maxima are indicated by M_k and the non-dominated local minima by m_k. The points (M_k,m_k) are represented by the vertices of the staircase $L(t)$ (see Figure ii.9). The sequence M_1,m_1, … M_k,m_k , is known as the *reduced memory sequence* of the Preisach Operator (Visitin 1994,p. 99). Strong Hysteresis is characterized by a *memory wiping-out process*. This means that the dominated values of the input are erased from the memory bank, when the input reaches a local non-dominated extremum. The output variable retains a selective memory that is represented by the staircase (called the memory curve) formed in the Figures ii.9 *a)* to ii.9 *d)*. When the input increases to x_5, the other dominated extremums are erased from the memory (we can see this situation in Figure ii.9 *f*), in which the new extremum x_5 erases the staircase that divides the areas T_0 and T_1). Then, the past history is wiped out by price variations of sufficiently large magnitude (Visitin 1994, p.99).

Secondly, contrasting with the weak form of hysteresis, where the temporary shocks of the input variable only cause remanence if the trigger values were reached, in the strong form of hysteresis every *loading-unloading* that implies an increase/decrease of the value of the input over the last local maximum/minimum will originate remanence. This property emerges as we consider a continuum of relays on the T triangle.

Thirdly, different to the weak hysteresis, the remanence effect depends on the magnitude of the temporary disturbance occurred on the input variable. Actually, a greater positive transitory shock to the input variable will induce more relays to switch on, originating a greater remanence effect. This could be illustrated in Figure ii.3. In this case the remanence effect of a transitory shock $x(0) \to x(1) \to x(0)$ is 1, while the remanence effect of a greater transitory shock $x(0) \to x(2) \to x(0)$ is 2.

Fourthly, the Preisach Operator has the *Congruency Property,* which means that if the input varies between two extremums, regardless of what the prior history of the input is, the minor loops that are created by this cyclic behavior will be of the same shape. This means that, in spite of the position of the loops being different on the $Y(t)$axis (see Figure i.1), the coincidence of the loops can be achieved by the appropriate translation of these loops along the $Y(t)$axis.

Finally, the aggregation of individual relays that originates hysteresis behavior with local memories usually has a *non local memory*, which implies that future values of the output $Y(t)$ with $t > t_0$ depend not only on the current values of the output $Y(t_0)$, but also on the past extreme values of the input $x(t)$ (Mayergoyz 2003, p. xvii).

CHAPTER III

WEAK HYSTERESIS IN THE DYNAMICS OF LABOR DEMAND

III.1. INTRODUCTION

Previous empirical studies have shown that firms do not permanently adjust employment in order to accommodate demand shocks as they should under convex adjustment costs (see, for example, Varejão and Portugal 2007, for the case of Portugal). Hysteresis is based on the existence of non-convex costs of adjustment and implies inertia, irreversibility and occasional bursts of job creation and job destruction when the firms' product demand falls (rises) below (above) a trigger level, as described by the Non-Ideal Relay Model.

In this chapter, firstly, we offer preliminary evidence of the existence of hysteresis at the firm level by documenting the distribution of employment adjustment, in the line of Dunne (1998) and Varejão (2000). Our goal is not simply to show whether the adjustment of employment is discrete, but also to analyze how the distributions of employment adjustment vary by firm characteristics, such as the size of the firms and industry.

Secondly, we study the frequency and the size of adjustment of the labor input along its two margins: the number of employees and hours of work per employee. To analyze the pattern of labor input adjustment we focus on; *a*) the empirical evidence on job reallocation; *b*) the empirical distribution of labor input adjustment; *c*) the serial correlation of employment adjustment; *d*) the interrelation between employment and hours adjustment.

Thirdly, we implement a more direct test to the existence of hysteresis at the firm level based on the assumption that employment response to product demand shocks of the same magnitude is asymmetric, and that it depends on the difference between actual and desired level of employment, as in Parsley and Wei (1993).

III.2. DATA

The data used in this paper come from the "Inquérito Mensal à Indústria Volume de Negócios e Emprego" (IVNEI), which is a monthly survey run by the Instituto Nacional de Estatística (Portuguese Statistics Office). Data are collected by mail survey. Answering is mandatory. Its purpose is to measure the monthly evolution of four variables in manufacturing: turnover, employment, earnings and hours of work. The INVEI surveys a sample of manufacturing firms with at least 10 employees. Data are collected at the firm level and they are available for the number of employees in the firm (wage earners), the total number of man-hours (actually) worked, the total amount of earnings paid by the firms and total turnover (as measured by sales value).

The IVNEI sample we use spans over 132 months from January 1995 to December 2005. On average, 2,616 firms responded each month, making a total of 345,312 records ($firms \times months$) over the entire 11-year period (see summary characteristics of the data in Table A.1 in the appendix).

The distribution of firms by number of employees and by activity sector in the starting period is reported respectively in Table iii.1 and iii.2.

Table iii.1.
Distribution of Firms by Size (1995:01)

	Number of Firms	Proportion of Firms
$10 \leq n < 19$	299	13.45%
$20 \leq n < 49$	528	23.75%
$50 \leq n < 99$	477	21.46%
$100 \leq n < 199$	410	18.44%
$200 \leq n < 500$	374	16.82%
$n \geq 500$	135	6.07%
Total	2223	100.00%

Table iii.2.
Distribution of Firms by Activity Sector (1995:01)

	Number of Firms	Proportion of Firms
Mining	91	4.09%
Food, Tobacco and Beverages	290	13.05%
Textile, Leather and Shoes	447	20.11%
Furniture and Wood	310	13.95%
Paper and Printing	151	6.79%
Chemicals, Petroleum and Rubber and Plastic Products	182	8.19%
Non Metallic Mineral Products	184	8.28%
Primary Metals	50	2.25%
Machinery, Fabricated Metals, Motors and Cars and Other Transport Material	498	22.40%
Electricity and Gas	20	0.90%
Total	2223	1000%

The data collected were converted into two data sets, which are referred to as the pooled data set and the longitudinal data set.

The pooled data set simply pooled all the 132 monthly files. No major modifications were made to the original file, except for the records with non response or with a zero value for employment, sales and hours of work, which were deleted. This data set was used to analyze the micro pattern of labor adjustment.

The longitudinal data set resulted from merging the 132 monthly files. All the records in every month have an identification code that is unique and does not change during the whole period the firm remains in the sample. This code number served as the key for merging the original files. This was used to generate a balanced panel of 947 firms for which simultaneous information on employment, sales and total hours of work is available in each and every one of the 132 months surveyed. We use this data set to build the aggregate time series of employment, sales, hours of work and earnings, which were seasonally adjusted.

III.3. FIRM LEVEL LABOR ADJUSTMENT PATTERNS

III.3.1. EMPIRICAL EVIDENCE ON JOB REALLOCATION

This section presents preliminary indicators of employment dynamics. Table iii.3 and Figure iii.1 present the monthly average rates of job creation and destruction, net employment growth, job reallocation, excess job reallocation[1] and the minimum reallocation required to its accommodation (lower bound). The monthly average rate of net job creation and job destruction are 0.9% and 1.1% respectively, implying that the manufacturing sector's employment as a whole declined at a rate of 0.2% per month over the sample period. These findings are in line with the results of Varejão and Portugal (2007). Using quarterly manufacturing data from the period from the first quarter of 2001 to the fourth quarter of 2005, the authors reported a job creation and job destruction rates of respectively 2.3% and 3.1%, and a job reallocation rate of 5.4%, which are approximately equal to three times our calculation, but they imply less turnover than that referred to by Addison and Teixeira (2005), who, using quarterly manufacturing data from the period from the first quarter of 1977 to the fourth quarter of 1997, obtained job creation and job destruction rate estimates of respectively 3.3% and 5.2%, and a job reallocation rate equal to 8.5%.

Table iii.3
Job Flow Rates: Summary Statistics (1995:01-2005:12)[2]

	Job Creation	Job Destruction	Job Reallocation	Net Employment Change	Excess Job Reallocation	Lower Bond
Average	0.009	0.011	0.020	-0.002	0.023	0.008
Standard Dev.	0.003	0.004	0.005	0.004	0.007	0.002
Maximum	0.023	0.042	0.053	0.015	0.083	0.021
Minimum	0.005	0.007	0.012	-0.030	0.014	0.005
Correl (X,NET)	0.545	-0.700	-0.163	-	-	-

[1] Excess job reallocation is an index of simultaneous job creation and destruction.
[2] All the measures were computed according the standard Davis and Haltiwanger (1992) definitions.

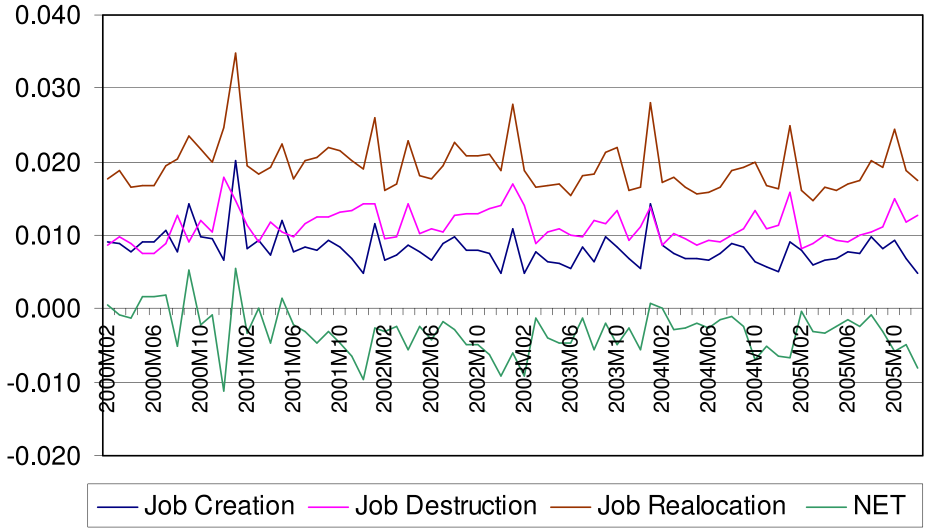


Figure iii.1. Job Flow Rates

We also computed the standard deviation of job creation and job destruction and the correlation between job reallocation and net employment changes (NET). The results we obtained indicate that job destruction is more volatile than job creation. Moreover, job creation is pro-cyclical whereas job destruction is strongly counter-cyclical. As a result, job reallocation exhibits a small negative correlation with net employment change (-0.163), meaning that job turnover is virtually acyclical. Table iii.4 further shows that job reallocation is a declining function of firm size as measured by the average number of employees (n)[3].

Table iii.4
Average Job Flow Rates by Firm Size

	Job Creation	Job Destruction	Job Reallocation	Net Employment Change	Excess Job Reallocation	Lower Bond
$10 \le n < 19$	0.017	0.023	0.040	-0.006	0.046	0.017
$20 \le n < 49$	0.014	0.017	0.031	-0.003	0.033	0.014
$50 \le n < 99$	0.011	0.014	0.025	-0.002	0.028	0.011
$100 \le n < 199$	0.010	0.012	0.023	-0.002	0.025	0.010
$200 \le n < 500$	0.009	0.010	0.019	-0.002	0.021	0.009
$n \ge 500$	0.007	0.009	0.016	-0.002	0.018	0.007

Moreover, monthly average job reallocation rates show considerable cross-industry variation ranging from 0.8% in the Electricity and Gas industry to 3% in the Food and Tobacco industry (see Table iii.5).

[3] The size classes considered are those previously defined in the data set.

Table iii.5
Average Job Flow Rates by Activity Sector

	Job Creation	Job Destruction	Job Reallocation	Net Employment Change	Excess Job Reallocation	Lower Bond
Mining	0.011	0.013	0.024	-0.001	0.025	0.011
Food, Tobacco and Beverages	0.014	0.016	0.030	-0.002	0.032	0.014
Textile, Leather and Shoes	0.006	0.010	0.016	-0.004	0.020	0.006
Furniture and Wood	0.009	0.011	0.021	-0.002	0.022	0.009
Paper and Printing	0.007	0.008	0.015	-0.001	0.016	0.007
Chemicals, Petroleum and Rubber and Plastic Products	0.006	0.009	0.015	-0.003	0.017	0.006
Non Metallic Mineral Products	0.009	0.011	0.021	-0.002	0.023	0.009
Primary Metals	0.010	0.013	0.023	-0.002	0.026	0.010
Machinery, Fabricated Metals, Motors and Cars and Other Transport Material	0.010	0.013	0.023	-0.002	0.025	0.010
Electricity and Gas	0.002	0.006	0.008	-0.005	0.013	0.002

III.3.2. THE EMPIRICAL DISTRIBUTION OF THE LABOR INPUT ADJUSTMENT

The model of employment adjustment described in section II.3.1, which incorporates non-convex costs of labor adjustment, generates inaction as an optimal response to demand fluctuation. This means that in the presence of small variations in output demand, firms should not change the number of workers. In such circumstances, a high frequency of zero net employment change episodes is expected. Besides, if irreversible costs and uncertainty are important characteristics of the adjustment technology, then firms should experience rare episodes of significant adjustment, followed by long periods of inaction, which translates into long fat tails of the empirical distribution of the net job change.

In Figure iii.2 the empirical distribution of net employment change is plotted (see also Table A.2 in the Appendix)[4]. The distribution exhibits a large mass point around zero adjustment; zero employment variation accounting for 41% of the total number of observations. This is an unequivocal sign of lumpy adjustment[5]. There is,

[4] Interval width was set at 0.05. All intervals are identified by their mid points.
[5] Our calculation of the frequency of inaction episodes is significantly less than the frequency of inaction calculated by Varejão and Portugal (2007) with quarterly data in the period 1991:01-2005:04 (73%). A possible reason is that the authors used a different data set that includes a greater percentage of small establishments in which more inaction is typically observed, although the frequency of their data is

also, some evidence of high frequency of small decreases in employment relative to small increases. However, the density function of monthly net employment changes does not exhibit significant fat tails, as expected for monthly data, and signs of smooth adjustment can be observed in the proportion of small adjustment episodes, i.e., below $\pm 5\%$ (excepting zero variation) which is approximately 44% for the whole sample.

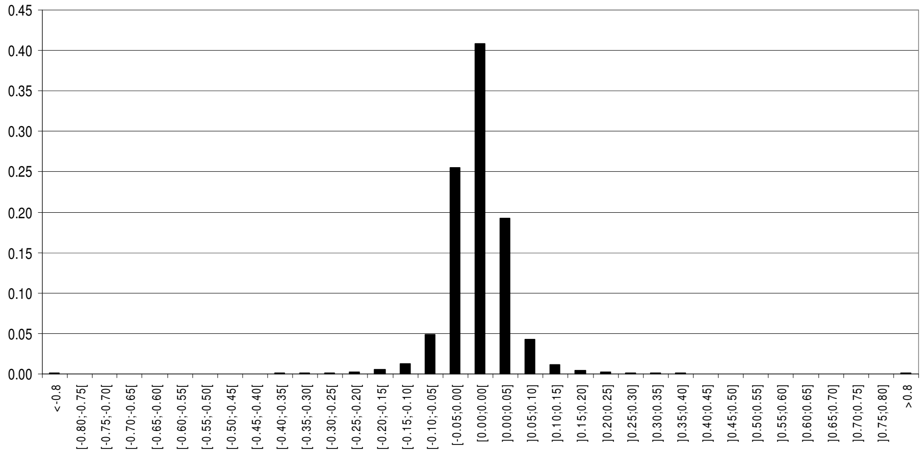


Figure iii.2. Empirical Distribution of Employment Change

Moreover, the frequency of no-adjustment episodes decreases markedly with the size of the firms. The frequency of inaction is approximately 68% for firms of 19 workers or less, 52% for firms of 20-49 workers, 38% for firms of 50-99 workers, 28% for firms of 100-199, 21% for firms of 200-499 workers and 10% for firms larger than 500 workers. The frequency of the existence of spikes[6] also decreases with the size of the firm (see Figure iii.3).

smaller.

[6] We consider a spike episode a variation greater than 20% in absolute terms.

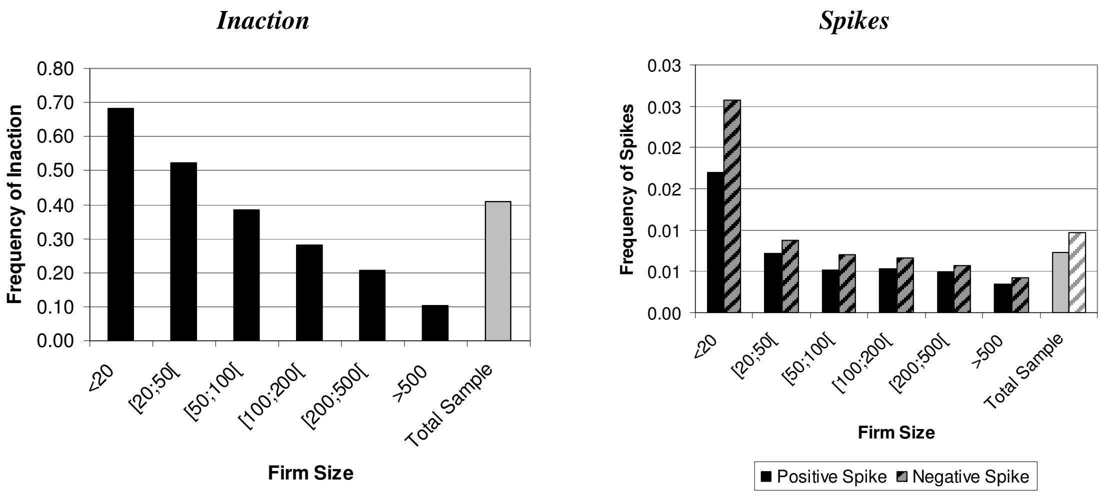


Figure iii.3. Frequency of Inaction and Spikes by Firm Size

We also observe that the differences in the incidence of inaction and the existence of spike episodes are more notorious across firms with different sizes than across sectors of activity (see Figure iii.4 and iii.5). As the observed differences across sectors are mainly determined by the sector's average firm size, the subsequent analysis will focus on firm size heterogeneity only.

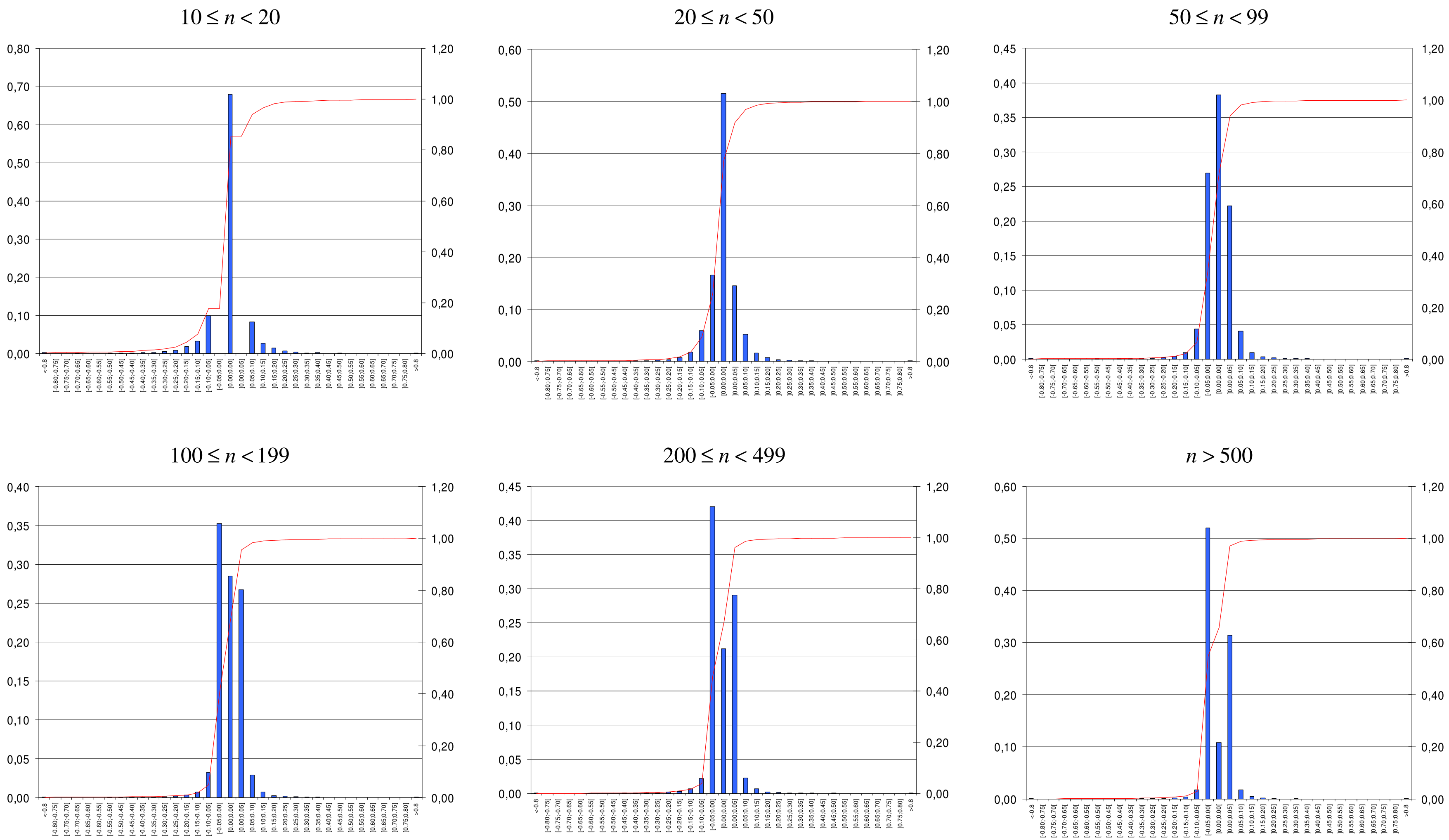


Figure iii.4. Unconditional Frequency of Monthly Net Employment Change by Firm Size

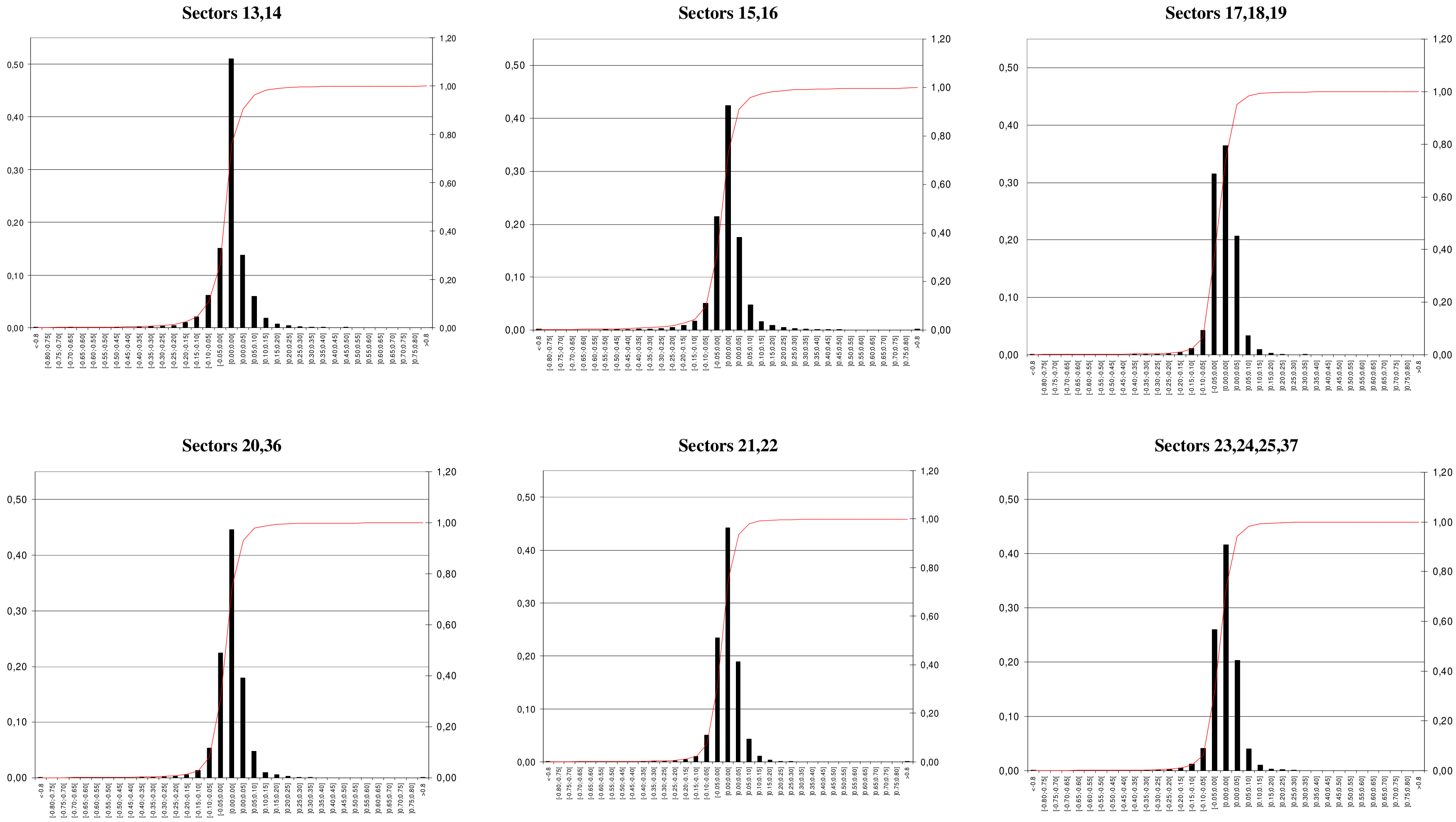


Figure iii.5. Unconditional Frequency of Monthly Employment Change by Sector Level[7]

[7] See the correspondence of the sector numbers in the Appendix.

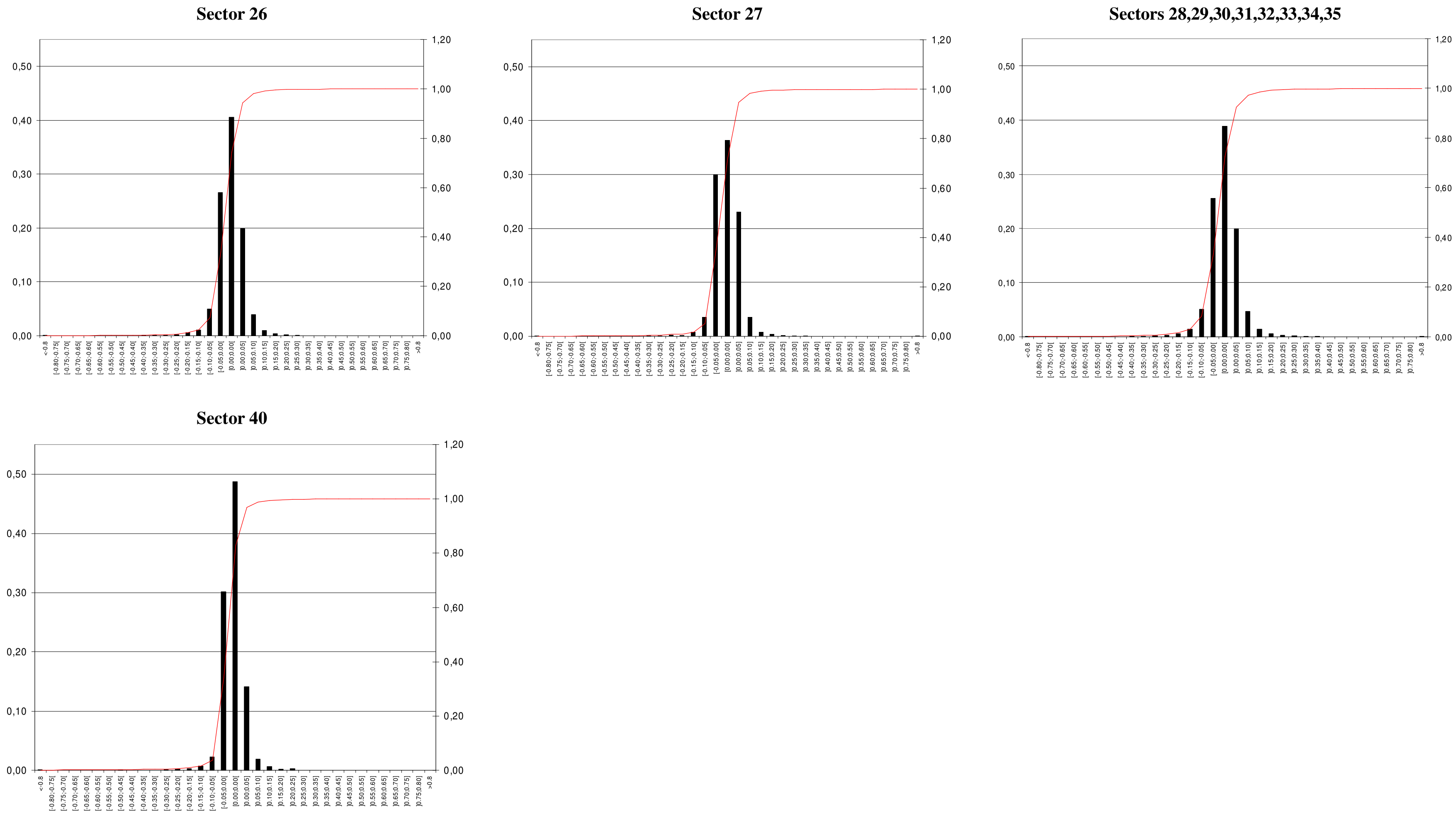


Figure iii.5. Unconditional Frequency of Monthly Employment Change by Sector Level

Figures iii.6 *a*) and *b*) show the empirical distribution of the monthly variation of earnings and sales for the entire sample of firms[8]. By simple visual inspection of these figures we can see that the shape of the two distributions differs markedly from the distribution of employment changes. In particular, monthly real sales exhibit a low frequency of episodes of zero change[9] and a greater incidence of large variations. Moreover, and differently to what happens with employment, this pattern is homogeneous across firms' size and across sector of activity (see Figure A.1 in the Appendix).

Figure iii.6. Empirical Distribution of Labor Input, Earnings and Sales Variation

In the presence of fixed employment adjustment costs it is expected that the empirical distribution of hours change exhibits high concentration in the range of small variation intervals, as firms used the number of hours per worker to respond to small variations of product demand. Figure iii.6 *c*) and *d*) show the empirical distribution of

[8] Both variables were deflated by Consumer Price Index (OECD Main-Economic Indicators); 2000=100.
[9] In the case of the sales distribution we define the no-change state as corresponding to changes between -1% and +1%.

monthly hours per worker and total hours of work changes for the entire period and for the whole sample[10]. We verify that: *a*) there is considerable inaction in the adjustment of hours but much less when compared to the adjustment of employment; *b*) there is some incidence of large and small adjustments; *c*) the empirical distribution of adjustment of total hours of work is relatively similar to the distribution of sales growth but very different from the distribution of employment adjustment. These findings indicate that the existence of inaction in the level of employment at firm level is not caused by the inexistence of shocks but by the preference of firms for the adjustment through variations in the number of hours of work per employee as opposed to the variation of employment. The previous analysis reveals that firms leave employment essentially constant but adjust hours per worker more frequently, indicating that hours are subject to much fewer adjustment costs than employment is.

III.3.3. SERIAL CORRELATION OF EMPLOYMENT ADJUSTMENT

Different structures of adjustment costs have different consequences in terms of both the serial correlation of adjustment and the dynamic interrelation between employment and hours' adjustment.

Concerning the serial correlation of the adjustment, convex adjustment costs imply that one period of small adjustment should be followed by another period of small adjustment, as firms try to spread the whole adjustment over several periods. On the contrary, non-convex adjustment costs imply that one period of adjustment should be followed by periods of inaction.

To distinguish between these two types of adjustment costs, in each period firms were classified in one employment adjustment regime: inaction ($\Delta n_t = 0$); positive growth ($\Delta n_t > 0$); negative growth ($\Delta n_t < 0$). This information was then used to compute the probabilities of transition between regimes in two consecutive periods of time (probability transition matrix). The main focus of the analysis is on the second column and on the diagonal of the matrix (see Table iii.6)[11].

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[10] We also consider zero changes in hours when the absolute value of the variation is less than |1%|. This definition is particularly adequate because not all months have the same number of working days.

[11] In order to verify if this transition pattern derives from the existence of non-convex costs of adjustment

High probabilities of transition from each regime to the inaction regime and the high probability of staying in the inaction regime, is a sign of the importance of non-convex adjustment costs that lead to hysteresis. High values on the main diagonal reveal significant serial correlation between the adjustments and should be taken as signs of smooth adjustment (except the entries that correspond to the prevalence of inaction).

Table iii.6 documents the existence of mixed signs of convex and non-convex adjustment costs. On the one hand, there is a great percentage of firms stuck in the inaction regime (the probability of a firm in the inaction regime remaining in the subsequent period is 58%) and a high probability of firms that are expanding or contracting the number of workers to move to the inaction regime in the next period. On the other hand, the main diagonal exhibits large serial correlation between positive and negative adjustments, which indicates that a large number of firms spread the adjustment over more than one period ahead.

Table iii.6 also shows very different patterns of adjustment for the different variables considered. In particular, the probability of moving from an episode of adjustment to inaction is much greater for employment than it is for hours per worker and total hours. This is again evidence of the greater importance of non-convexities in the cost of adjusting employment than in the cost of adjusting the number of hours of work.

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or instead from the time pattern of the shocks we also report identical transition probability matrix for sales and hours of work.

Table iii.6
Probability Transition Matrices

Employment

% of the total observations

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,24	0,07	0,10
$\Delta n_{t-1} > 0$	0,08	0,08	0,11
$\Delta n_{t-1} < 0$	0,10	0,11	0,11

Prob. Transition Matrix

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,58	0,17	0,25
$\Delta n_{t-1} > 0$	0,29	0,30	0,41
$\Delta n_{t-1} < 0$	0,29	0,33	0,38

Hours per Worker

% of the total observations

	$\Delta h_t = 0$	$\Delta h_t > 0$	$\Delta h_t < 0$
$\Delta h_{t-1} = 0$	0.05	0.01	0.01
$\Delta h_{t-1} > 0$	0.01	0.15	0.30
$\Delta h_{t-1} < 0$	0.01	0.30	0.16

Prob. Transition Matrix

	$\Delta h_t = 0$	$\Delta h_t > 0$	$\Delta h_t < 0$
$\Delta h_{t-1} = 0$	0,67	0,15	0,18
$\Delta h_{t-1} > 0$	0,03	0,32	0,65
$\Delta h_{t-1} < 0$	0,02	0,64	0,34

Total Hours

% of the total observations

	$\Delta th_t = 0$	$\Delta th_t > 0$	$\Delta th_t < 0$
$\Delta th_{t-1} = 0$	0,04	0,01	0,02
$\Delta th_{t-1} > 0$	0,02	0,15	0,29
$\Delta th_{t-1} < 0$	0,02	0,29	0,16

Prob. Transition Matrix

	$\Delta th_t = 0$	$\Delta th_t > 0$	$\Delta th_t < 0$
$\Delta th_{t-1} = 0$	0,54	0,20	0,26
$\Delta th_{t-1} > 0$	0,04	0,32	0,64
$\Delta th_{t-1} < 0$	0,03	0,60	0,37

Sales

% of the total observations

	$\Delta s_t = 0$	$\Delta s_t > 0$	$\Delta s_t < 0$
$\Delta s_{t-1} = 0$	0,00	0,00	0,00
$\Delta s_{t-1} > 0$	0,00	0,19	0,32
$\Delta s_{t-1} < 0$	0,00	0,32	0,17

Prob. Transition Matrix

	$\Delta s_t = 0$	$\Delta s_t > 0$	$\Delta s_t < 0$
$\Delta s_{t-1} = 0$	0,00	0,13	0,87
$\Delta s_{t-1} > 0$	0,00	0,38	0,62
$\Delta s_{t-1} < 0$	0,00	0,65	0,35

The analysis of the probability transition matrices by firm size (see Table iii.7), also reveals heterogeneity concerning the adjustment behavior of firms. For small firms, we observed not only a greater resilience of the inaction regime, but also the importance of this regime as the most likely destination of all firms that make a transition from one month to the next. Furthermore, while the resilience of the inaction regime decreases with firm size, the values of the main diagonal (with the exception of the first entry of the matrix) are low for small firms and increase with firm size. Thus, evidence of discrete adjustment is stronger for smaller firms than for larger ones.

Table iii.7
Probability Transition Matrices by Firm Size

$10 < n < 20$

% of the total observations

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,51	0,07	0,10
$\Delta n_{t-1} > 0$	0,08	0,02	0,04
$\Delta n_{t-1} < 0$	0,09	0,05	0,03

Prob. Transition Matrix

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_t = 0$	0,75	0,10	0,15
$\Delta n_t > 0$	0,57	0,15	0,28
$\Delta n_t < 0$	0,53	0,29	0,18

$20 \le n < 49$

% of the total observations

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,32	0,08	0,11
$\Delta n_{t-1} > 0$	0,09	0,05	0,08
$\Delta n_{t-1} < 0$	0,11	0,09	0,07

Prob. Transition Matrix

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,63	0,15	0,22
$\Delta n_{t-1} > 0$	0,41	0,23	0,36
$\Delta n_{t-1} < 0$	0,41	0,34	0,25

$50 \le n < 99$

% of the total observations

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,19	0,08	0,11
$\Delta n_{t-1} > 0$	0,09	0,08	0,11
$\Delta n_{t-1} < 0$	0,10	0,12	0,11

Prob. Transition Matrix

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,50	0,21	0,29
$\Delta n_{t-1} > 0$	0,30	0,29	0,40
$\Delta n_{t-1} < 0$	0,31	0,35	0,34

$100 \le n < 199$

% of the total observations

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,12	0,07	0,10
$\Delta n_{t-1} > 0$	0,07	0,10	0,14
$\Delta n_{t-1} < 0$	0,09	0,14	0,17

Prob. Transition Matrix

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,41	0,23	0,36
$\Delta n_{t-1} > 0$	0,22	0,33	0,45
$\Delta n_{t-1} < 0$	0,23	0,35	0,42

$200 \le n < 499$

% of the total observations

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,07	0,05	0,08
$\Delta n_{t-1} > 0$	0,05	0,13	0,15
$\Delta n_{t-1} < 0$	0,08	0,15	0,24

Prob. Transition Matrix

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,36	0,25	0,39
$\Delta n_{t-1} > 0$	0,16	0,38	0,46
$\Delta n_{t-1} < 0$	0,17	0,31	0,51

$n \ge 500$

% of the total observations

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,02	0,03	0,05
$\Delta n_{t-1} > 0$	0,03	0,16	0,16
$\Delta n_{t-1} < 0$	0,05	0,15	0,36

Prob. Transition Matrix

	$\Delta n_t = 0$	$\Delta n_t > 0$	$\Delta n_t < 0$
$\Delta n_{t-1} = 0$	0,36	0,25	0,39
$\Delta n_{t-1} > 0$	0,16	0,38	0,46
$\Delta n_{t-1} < 0$	0,17	0,31	0,51

III.4. INTERRELATION BETWEEN EMPLOYMENT AND HOURS ADJUSTMENT

Movements in employment and hours jointly reflect adjustment costs and the shock process. A key moment is the relative variability of hours and employment growth. Table iii.8 shows that the low variability of employment is caused mainly by the existence of costs of adjustment and not by the inexistence of shocks. In fact, the standard deviation of sales growth (0.224) is considerably greater than the standard deviation of employment variation (0.064), because firms react to shocks mainly through the variation in hours.

If costs of adjustment are fixed, a negative correlation between hours growth and employment growth is expected because the firm may initially respond to relatively small profitability shocks, by changing working hours while maintaining the number of employees fixed. However, if profitability rises enough, the firm will change the number of workers and adjust average hours back to the initial level. Accordingly, this pattern of response produces a negative co-movement between hours and employment. Table iii.9 shows the simple contemporaneous correlations between employment variation, hours variation and sales growth. We found that employment variation and hours variation are weakly negative correlated with a coefficient of correlation of -0.048. This key moment indicates a potential substitutability between working hours and the number of employees. Moreover, the correlation between the variation in total hours of work and sales growth is stronger, and positive (0.266), than the one obtained between the variation in employment and sales, which that is equal to 0.062.

Table iii.8
Key Moments of Labor Input and Sales Growth*

	Employment Growth	Hours per Worker Growth	Total Hours Growth	Sales Growth
Average	0.062	0.249	0.261	0.463
Stand. Dev.	0.064	0.157	0.156	0.224
Minimum	0.000	0.000	0.000	0.003
Maximum	0.849	0.931	0.935	1.575

*The growth rates were calculated with the absolute value of the variation.

Table iii.9

Contemporaneous Correlations between Margins of adjustment and Sales

	Employment Growth	Hours Growth	Total Hours Growth	Sales Growth
Employment Growth	1.000	-0.048	0.266	0.062
Hours per Worker Growth	-	1.00	0.897	0.300
Total Hours Growth	-	-	1.000	0.312
Sales Growth	-	-	-	1.000

It is also important to analyze the correlation between employment and hours adjustment during employment inaction and during spike episodes. The question is whether inaction or spikes in one margin of adjustment increase the probability that firms adjust the other margin. Figure iii.7 shows that the empirical frequency of hours per worker variation is conditional on the existence of a positive (negative) spike episode, exhibits considerably less inaction relative to the unconditional frequency and is skewed to the left (right) reflecting the return of hours per worker to the pre-shock values when employment starts to adjust.

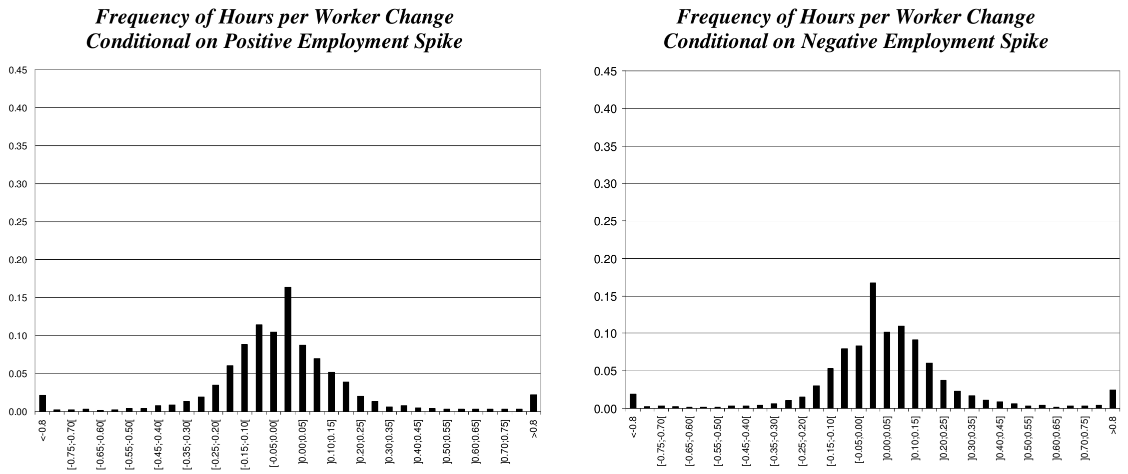

Figure iii.7. Empirical Distribution of Monthly Hours per Employee Change and Employment Adjustment

However, the empirical distribution of hours adjustment conditional on zero employment variation (see Figure iii.8) also indicates that almost 30% of firms keep the labor input stable (employment and hours per employee). It is this fact, and not the existence of convex costs of adjustment, that tends to reduce the negative correlation between employment variation and hours variation.

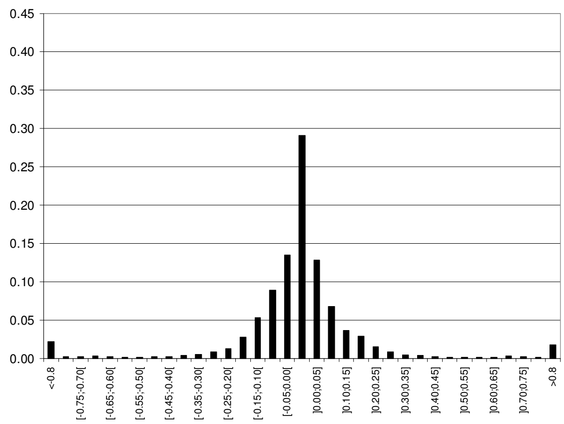

Figure iii.8. Frequency of Hours per Worker Change Conditional on Zero Employment Variation

III.5. PATH DEPENDENCE AND DIVERGENCE OF LINEAR RESPONSE

In the presence of fixed employment adjustment costs, establishments would not adjust continuously to the shocks they perceive, on the contrary, adjustment would be occasional and often large

The current employment level chosen by the firm depends on the comparison between the fixed cost of adjustment in each period and the present value of the additional profit induced by the adjustment from the old to the new equilibrium. If the cost of the change from one static equilibrium to another is higher than additional profit induced by the adjustment, the firm does not adjust the level of employment, instead the firm will adjust to the new steady sate in an instantaneous jump. In this case, the demand for labor of the optimizing firm is discontinuous and could be represented by an (S,s) type model (Hamermesh 1989):

$$n_{j,t} = \begin{cases} n_{j,t} = n_{j,t-1} + \mu_{1_{j,t}} & \Leftarrow \left| n^{*}{}_{j,t} - n_{j,t-1} \right| < k_j \\ n_{j,t} = n^{*}_{t,j} + \mu_{2_{j,t}} & \Leftarrow \left| n^{*}{}_{j,t} - n_{j,t-1} \right| \geq k_j \end{cases} \qquad \text{(iii.1)}$$

Where $n^{*}{}_{j,t}$ is the desired employment level, $n_{j,t}$ is the actual employment level, k_j, *is* the percentage deviation of last period's employment from the desired employment that is necessary to overcome fixed adjustment costs, and $\mu_{1_{j,t}}$ and $\mu_{2_{j,t}}$ are disturbances with $E\left(\mu_{1_{j,t}}, \mu_{2_{j,t}}\right) = 0$.

Figure iii.9 illustrates the relationship between employment and sales (used as a proxy of frictionless employment demand). Along the employment schedule, an increase in sales is associated with high employment. However, when hysteresis is present, employment will not change until a critical level of sales (s_j^1) is reached. Starting from point A, with $s_j = \beta_j$, if sales start to increase, the firm *j* only adjusts its employment level to n_j^*, when $\left|n^*{}_{j,t} - n_{j,t-1}\right| \geq k_j$, which happens in point B when $s_j = s_j^1$. If the sales start to decrease, returning to the initial level, the actual employment level will stay at the previous level, and only decreases to the desired level when $\left|n^*{}_{j,t} - n_{j,t-1}\right| \geq k_j$.

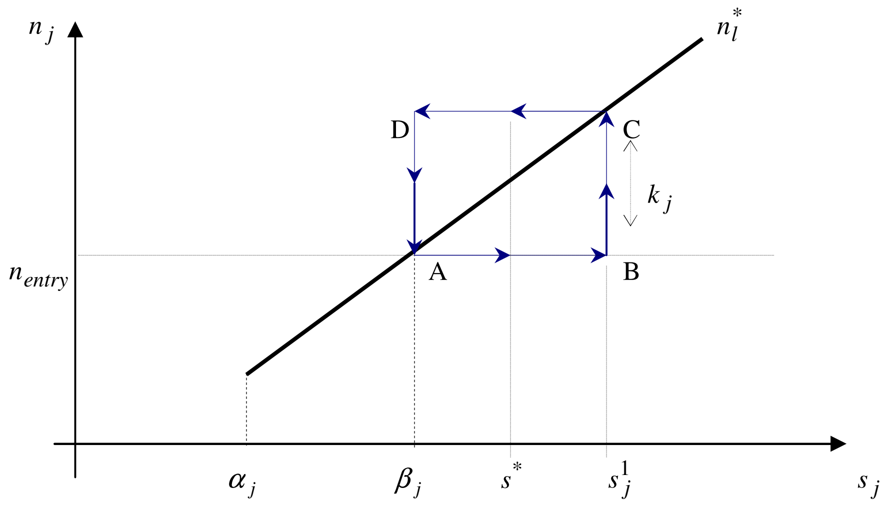

Figure iii.9. (S,s) Adjustment Policy at the Firm Level

Under hysteresis the cumulative changes in sales are important determinants of employment flows; i.e., the effect of sales changes on employment depends on the history of the past shocks as well as on the current sales variation. This implies that the same variation of the control variable could lead to different reactions of the state variable. Suppose an active firm, and consider that the control variable (sales) is far beyond the exit trigger. A small negative variation in sales will not cause any effect on the state variable (employment demand). Now, consider cumulative negative shocks in sales of the same magnitude. When the control variable is close to the exit trigger the same negative variation in sales could lead to exit. Thus, the existence of sudden jumps in the dynamics of the state variable could not be accompanied by abrupt changes in the control variable; the discontinuous behavior of the state variable could be caused by continuous and smooth behavior of the input variable.

We apply a test designed to study the existence of the hysteresis property of path dependence and divergence of the linear employment response hypothesis, in line with Parsley and Wei (1993), but applied to micro panel data.

The test is constructed on the basis of three aspects of the hysteresis hypothesis. Firstly, the change in the shock variable should be large to induce structural shifts in the relationship between the input and the output. Secondly, the history of the input matters for the determination of the aggregate actual employment. Thirdly, there is an asymmetry in the reaction of the aggregate employment to the same (small) variation of the input variable, in the sense that it depends on whether the input variable is near or far from an entry or an exit trigger.

To implement the test we assume, as in Parsley and Wei (1993), that the effect of an increase in the input value following a series of successive increases is different from the effect of an increase in the value of the input following a series of successive decreases.

Thus, we define:

$$V_{it} = \sum_{j=0}^{T} s_{it} - s_{it-j-1} \tag{iii.2}$$

as the cumulative change in sales (s_{it}) over some period *T*. We also define a dummy variable D_{it} that indicates whether the actual change in sales is in the same, or opposite, direction for the change over the previous *T* periods:

$$D_{it} = \begin{cases} 1 \Leftarrow \Delta s_{it} > 0 \wedge V_{it} > 0 \\ -1 \Leftarrow \Delta s_{it} < 0 \wedge V_{it} < 0 \\ 0 \quad otherwise \end{cases} \tag{iii.3}$$

To generate a hysteresis filtered variable of real sales we compute the measure of phase:

$$\psi s_{it} = D_{it} \Delta si_t V_{it} \tag{iii.4}$$

After computing the measure of phase with the sales variable (ψs_{it}) and with the real wages variable (ψw_{it}), we estimate an employment equation of the type:

$$\Delta n_{it} = \alpha + seasonals + \sum_{j=0}^{n} \beta_{\psi s,j} \psi s_{it-j} + \sum_{j=0}^{n} \beta_{\psi w,j} \psi w_{it-j} + \varepsilon_{it} \tag{iii.5}$$

where ε_{it} is a white noise disturbance term.

The coefficients associated with the measures of phase are expected to have the same sign as in the case of the original variables (positive in the case of ψs_{it} and negative in the case of ψw_{it}). Suppose an increase in sales after a series of successive increases. V_{it} and thus ψs_{it} will be positive and in the equation describing the behavior of employment the coefficient on ψs_{it} will be positive. However, an increase in sales after a series of successive decreases should not have an impact on employment due to hysteresis effects. V_{it} will be negative s_{it} positive and ψs_{it} will be equal to zero.

Tables 10 and 11 present the fixed effects estimates of Equation iii.5 when the order of the lag polynomials is set to zero and to three. We choose T to be 12 in the calculus of V_{it}, we assume that information about sales trends in the most recent year is sufficient to identify hysteresis effects. We compare the estimates of Equation iii.5 with the estimates of a similar employment equation with the original series of sales and real wages as independent variables.

As the preliminary evidence about the existence of hysteresis, offered in the previous sections, indicates the existence of differences in the employment adjustment process between small and large firms, in order to verify the existence of differences in the importance of the hysteresis hypothesis by firm size, we estimated equation iii.5 for the whole sample, for firms with fewer than 20 workers, and for firms with more than 500 workers.

Table iii.10 shows the point estimates of the coefficients associated with the independent variables. All the coefficients associated with the original dependent variables are significant and display the predicted sign. The estimated sales elasticity of employment is 0.0093 for the whole sample. We verify that, at the micro level, the employment of small firms is more responsive to sales than the employment of large firms (the estimated coefficients are respectively 0.0128 and 0.0060).

Concerning the existence of hysteresis the estimation of Equation iii.5 reveals that the coefficients associated with the hysteresis transformed sales variable (ψs_{it}) are significant and display the predicted positive sign. The coefficients associated with the hysteresis transformed real wages variable (ψw_{it}) display the wrong (positive) sign in the case of the small firms. In the other cases the coefficient is negative but non-significant. The goodness of fit of the employment equations increases when we include in the employment equation the hysteresis effects, in the case of the whole sample and for the sub sample of the small firms. We obtain opposite results in the case of large firms.

Table iii.10
Panel Data Employment Equation Estimates (with n=0)
Dependent Variable: employment growth rate

	Whole Sample		**Firms with Fewer than 20 Workers**		**Firms with More than 500 Workers**	
	Eq. with original independent variables: s_{it} and w_{it}	**Eq. iii.5**	**Eq. with original independent variables:** s_{it} and w_{it}	**Eq. iii.5**	**Eq. with original independent variables:** s_{it} and w_{it}	**Eq. iii.5**
Cons	-0.0223* -25.73	0.653* 42.02	-0.0403* -9.99	-0.0158 -4.09	-0.0153* -7.59	-0.0108* -5.81
$\beta_{s,0}$	0.0093* 30.98	-	0.0128* 11.79	-	0.0060* 7.92	-
$\beta_{w,0}$	-0.0226* -41.28	-	-0.0464* -18.67	-	-0.0066* -6.75	-
$\beta_{\psi s,0}$	-	0.0148* 4.86	-	0.0045* 10.89	-	0.0010* 3.17
$\beta_{\psi w,0}$	-	-0.0015 -0.41	-	0.0055* 4.49	-	-0.0007 -0.32
R^2	0.0058	0.012	0.0365	0.0595	0.4684	0.0004

We report for each variable the estimated coefficient, and the *t*-statistic respectively.

* Significant at 5%

Table 11 presents the sum of the estimated coefficients of the employment equation on the contemporaneous and first three lags of the independent variable. When we estimate the employment equation with the original variables, we verify that the cumulative effects of sales and real wages variations are higher after 3 months. In the case of the whole sample the coefficients associated with all the lags of the two independent variables are significant and display the expected sign. In the case of the sub sample of the small firms, only the second lag of the real wages display a non-expected positive sign, although, non-significant, and the sum of the estimated coefficients is negative. For the large firms, the coefficients associated with the third lag of sales are non-significant (and negative, as well as all the coefficients associated with all the lags of real wages.

Besides, only in the case of small firms, the inclusion of the transformed hysteresis variables ψs_{it} and ψw_{it}, instead of the original variables, increases the value of R-square of the employment regression. Therefore, we only can conclude for the existence of signs of hysteresis in the relationship between employment and sales and in the case of the small firms.

Table iii.11
Panel Data Employment Equation Estimates (with n=3)
Dependent Variable: employment growth rate

	Whole Sample		Firms with Fewer than 20 Workers		Firms with More than 500 Workers	
	Eq. with the original independent variables: s_{it} and w_{it}	**Eq. iii.5**	**Eq. with the original independent variables:** s_{it} and w_{it}	**Eq. iii.5**	**Eq. with the original independent variables:** s_{it} and w_{it}	**Eq. iii.5**
$Cons$	-0.02037* -10.20	-0.0111* -13.47	-0.0426* -10.39	-0.0156* -4.04	-0.0149* -7.26	-0.0109* -5.84
$\beta_{s,0}$	0.01279* 37.49	-	0.0167* 13.47	-	0.0068* 7.73	-
$\beta_{s,1}$	0.007973* 21.02	-	0.0089* 6.39	-	0.0019* 2.05	-
$\beta_{s,2}$	0.00522* 13.79	-	0.0064* 4.55	-	0.0002 0.27	-
$\beta_{s,3}$	0.00333* 9.9	-	0.0029* 2.36	-	-0.0007 -0.36	-
$\sum_0^n \beta_{sj}$	0.02924	-	0.0349	-	0.0082	-
$\beta_{w,0}$	-0.0272* -43.67	-	-0.0499* -18.15	-	-0.0070* -6.08	-
$\beta_{w,1}$	-0.0093* -13.72	-	-0.0065* -2.24	-	-0.0007 -0.55	-
$\beta_{w,2}$	-0.0055* -8.19	-	0.0003 0.13	-	-0.0003 -0.3	-
$\beta_{w,3}$	-0.0031* -5.30	-	-0.0040 -1.54	-	-0.0007 -0.64	-
$\sum_0^n \beta_{wj}$	-0.0541	-	-0.0601	-	-0.0087	-
$\beta_{\psi s,0}$	-	0.0028* 22.19	-	0.0045* 10.68	-	0.00104* 3.06
$\beta_{\psi s,1}$	-	0.0007* 5.73	-	-0.0006 -1.38	-	-0.0001 -0.54
$\beta_{\psi s,2}$		0.0006* 4.47	-	0.0003 0.79	-	-0.0004 -1.15
$\beta_{\psi s,3}$	-	0.0001 0.73	-	-0.0004 -0.96	-	-0.0002 -0.64
$\sum_0^n \beta_{\psi sj}$	-	0.0042	-	0.0065	-	0.0003
$\beta_{\psi w,0}$	-	-0.0014* -4.89	-	0.0059* 4.76	-	0.00036 1.65
$\beta_{\psi w,1}$	-	0.0061* 20.27	-	0.0059* 4.712	-	0.00048 0.22
$\beta_{\psi w,2}$	-	-0.0002 -0.70	-	0.0005 0.43	-	-0.00012 -0.56
$\beta_{\psi w,3}$	-	-0.0005 -1.69	-	0.0004 0.28	-	-0.00002 -0.11
$\sum_0^n \beta_{\psi wj}$	-	0.0055	-	0.0127	-	0.0007
R^2	0.0129	0.0012	0.0513	0.1075	0.5051	0.0008

We report for each variable the estimated coefficient, the t-statistic and the p-value respectively.

* Significant at 5%

We also estimate Equation iii.5 with the growth rate of total hours of work as the dependent variable (see the results in Table iii.12 and iii.13). As expected, labor input ($employment \times hours$) is substantially more responsive to sales variation than employment. The employment sales elasticity increases from 0.0093 to 0.1424 when we use the whole sample, from 0.0128 to 0.1141 in the case of small firms, and from 0.0060 to 0.1365, in the case of large firms (see Table iii.12).

Concerning the hysteresis effects, the results are similar to those obtained for employment. In all the cases, the coefficients associated with the hysteresis transformed sales variable are positive and significant. In the case of the hysteresis transformed real wage variables, the coefficients are significant but positive, when the whole sample and the sample of the small firms are used, and non-significant and positive when the sample of the large firms is used. Moreover, in the case on small firms the goodness of fit of the employment equations increases when the hysteresis effects are considered. The contrary happens when the whole sample and the sample of large firms are used.

Table iii.13 indicates that hysteresis is not present in the dynamics of the labor input when the whole sample is used and in the case of large firms. Only in the case of the small firms, the inclusion of the hysteresis variables instead of the original ones increased the R-squared of the employment equation, which indicate the existence of hysteresis

Table iii.12
Panel Data Labor Input Equation Estimates (with n=0)
Dependent Variable: labor input growth rate

	Whole Sample		**Firms with Fewer than 20 Workers**		**Firms with More than 500 Workers**	
	Eq. with the original independent variables: s_{it} and w_{it}	**Eq. iii.5**	**Eq. with the original independent variables:** s_{it} and w_{it}	**Eq. iii.5**	**Eq. with the original independent variables:** s_{it} and w_{it}	**Eq. iii.5**
Cons	-0.085* -24.6	-0.068* -26.58	-0.0481* -4.61	-0.0573* -5.68	-0.0604* -4.07	-0.1202 -8.54
$\beta_{s,0}$	0.1424* 117.66	-	0.1141* 40.08	-	0.1365* 23.91	-
$\beta_{w,0}$	-0.0145* -6.61	-	0.0334* 5.14	-	0.0494* 6.81	-
$\beta_{\psi s,0}$	-	0.0255* 48.29	-	0.0176* 15.36	-	0.0138* 5.12
$\beta_{\psi w,0}$	-	0.0134* 10.96	-	0.0754* 19.86	-	0.0003 0.20
R^2	0.0372	0.0116	0.1204	0.1958	0.0962	0.0076

We report for each variable the estimated coefficient, and the *t*-statistic respectively.

* Significant at 5%

Table iii.13
Panel Data Labor Input Equation Estimates (with n=3)
Dependent Variable: labor input growth rate

	Whole Sample		**Firms with Fewer than 20 Workers**		**Firms with More than 500 Workers**	
	Eq. with the original independent variables: s_{it} and w_{it}	**Eq. iii.5**	**Eq. with the original independent variables:** s_{it} and w_{it}	**Eq. iii.5**	**Eq. with the original independent variables:** s_{it} and w_{it}	**Eq. iii.5**
$Cons$	-0.00674* -8.50	-0.0897* -26.94	0.01896* -8.59	-0.0578* -5.72	-0.0732* -4.84	-0.1208* -8.58
$\beta_{s,0}$	0.1385* 100.98	-	0.1142* 32.28	-	0.1305* 19.85	-
$\beta_{s,1}$	-0.0106 -6.99	-	-0.0000 -0.02	-	-0.0177* -2.45	-
$\beta_{s,2}$	0.0093* 6.16	-	0.0115* 3.15	-	0.0135 1.88	-
$\beta_{s,3}$	0.0057* 4.31	-	0.0036 0.49	-	0.0110 1.74	-
$\sum_0^n \beta_{sj}$	0.1429	-	0.1293	-	0.1373	-
$\beta_{w,0}$	-0.0318* -12.77	-	0.0224* 3.11	-	0.0313* 3.67	-
$\beta_{w,1}$	-0.0427* -15.71	-	-0.0275* -3.61	-	-0.0406* -4.22	-
$\beta_{w,2}$	0.0107* 3.97	-	0.0218* 2.91	-	-0.0058 -0.62	-
$\beta_{w,3}$	0.0008 0.36	-	0.0059 0.87	-	-0.0049 -0.61	-
$\sum_0^n \beta_{wj}$	-0.0630	-	0.0226	-	-0.0200	-
$\beta_{\psi s,0}$	-	0.0243* 4567	-	0.0168* 14.37	-	0.0124* 4.54
$\beta_{\psi s,1}$	-	-0.0138* -24.92	-	-0.0077* -5.69	-	-0.007* -2.80
$\beta_{\psi s,2}$		0.0006 0.11	-	-0.0004 -0.36	-	-0.0030 -1.1
$\beta_{\psi s,3}$	-	-0.0008 -1.48	-	-0.0015 -1.20	-	-0.0024 -0.89
$\sum_0^n \beta_{\psi sj}$	-	0.0103	-	0.0072	-	-0.0007
$\beta_{\psi w,0}$	-	0.0137* 11.24	-	0.0771* 20.04	-	0.0003 0.19
$\beta_{\psi w,1}$	-	0.0071* 5.67	-	0.0092* 2.42	-	0.0004 0.28
$\beta_{\psi w,2}$	-	0.0005 0.38	-	0.0032 0.73	-	-0.0009 -0.56
$\beta_{\psi w,3}$	-	-0.0012 -0.90	-	-0.0030 0.60	-	0.0005 0.32
$\sum_0^n \beta_{\psi wj}$	-	0.0201	-	0.0865	-	0.0003
R^2	0.0261	0.0167	0.1123	0.18.58	0.1037	0.0047

We report for each variable the estimated coefficient, the *t*-statistic and the *p*-value respectively.

* Significant at 5%

III.4. MAIN FINDINGS

In this part, we analyze the existence of hysteresis at the firm level, based on high frequency micro data for Portugal. We start by providing some descriptive statistics on employment adjustment to check for the existence of the necessary condition for the existence of hysteresis – the existence of non-convex costs of employment adjustment. Secondly, we analyze the joint dynamics of employment and hours of work. Finally, we estimate a model with the hysteresis property of path dependence and divergence of the linear employment response to shocks that offers more direct evidence of the existence of hysteresis at the micro level.

The main conclusions are:

Firstly, the empirical distribution of net employment change exhibits clear signs of lumpy adjustment (large frequency of episodes of no adjustment and the existences of spike episodes). Moreover, the frequency of inaction and the frequency of spike episodes decrease markedly with the size of the firms. However, signs of smooth adjustment can also be observed in the proportion of small adjustment episodes.

Secondly, the shape of empirical distribution of real earnings and sales changes differs significantly from the empirical distribution of net employment changes, revealing much less inaction and greater incidence of large variations. The shape of the empirical distribution of sales is homogeneous across firms' size and across activity sectors.

Thirdly, the empirical distribution of hours' growth shows less inaction when compared with the empirical distribution of employment change and greater incidence of large adjustment episodes. Therefore, the empirical distribution of hours is more similar to the empirical distribution of sales changes than the empirical distribution of net employment changes.

Fourthly, sales growth is more volatile than total hours of work change. Hours per worker variability is significantly greater than employment variability, showing that firms react to shocks mainly through the variation in hours of work. We find a weak negative contemporaneous correlation between hours' growth and employment growth which is a sign of the presence of non-convex employment adjustment costs. These results reveal a preference of the firms for the adjustment through variations in hours of work. The firms leave employment essentially constant but adjust hours per worker more frequently, which indicate that hours are not subject to significant adjustment costs.

Finally, by estimating an employment equation that incorporates path dependence we find signs of hysteresis in the adjustment of employment, which are especially important in the case of small firms. We do not find clear signs of hysteresis in the employment adjustment of large firms. We also find that labor input is more responsive than employment to sales variation, due to the higher flexibility of adjustment through the number of hours per employee.

CHAPTER IV

STRONG HYSTERESIS IN THE DYNAMICS OF LABOR DEMAND

IV.1. INTRODUCTION

In this chapter, we address the macroeconomic consequences of discrete adjustment of employment, in the presence of non-convex costs of adjustment and uncertainty at the firm level. The key concern is the extent to which micro inertia is inherited at the macro level.

While there is vast literature that stresses the lumpy nature of employment adjustment at the micro level, given the presence of both high hiring and firing costs[1], at the aggregate level, employment series look smooth and appear to be well described by partial-adjustment like models reflecting convex adjustment costs at the firm level. This contradiction challenges the relevance of micro models of employment adjustment to explain the aggregate dynamics.

To study the macroeconomic implications of discontinuous adjustment at the firm level and reconcile the observed microeconomic behavior with aggregate evidence, it is necessary to allow for agent heterogeneity, and to consider the degree of coordination of individual firms at all points in time (Bertola and Caballero 1990, p. 253, Cross 1994, p. 213). At one extreme, if all the individual firms are identical and coordinate their actions, the aggregate employment path should be similar to the individual paths. At the other extreme, if a large group of firms are uniformly spread in the state space and their actions are uncoordinated, the aggregate employment path can look very smooth.

We apply a testing framework based on the Preisach Model of strong hysteresis, to study the dynamic behavior of employment at the aggregate level that results from the lumpy and intermittent pattern observed at the micro level[2]. This framework includes four methodological ingredients: *i*) a simple model of discontinuous behavior of employment demand at the micro level in the presence of non-convex (fixed or linear) costs of adjustment – *Non-Ideal Relay Model of Hysteresis*; *ii*) individual heterogeneity (each firm faces different fixed costs of adjusting employment; *iii*) an aggregation procedure of the

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[1] See Hamermesh (1989; 1993), Hamermesh and Pfann (1996) and Caballero *et al*. (1997).

[2] The Preisach Model is a mathematical tool that is designed and well suited to establishing the connection between micro and macro behavior, and can be very useful in the empirical investigation of the time series properties of aggregate variables. The outcome of the Preisach model, built from microeconomic units that adjust discontinuously the number of employees, is a continuous smooth series. This series exhibits strong hysteresis and cannot be described by a partial adjustment model.

individual heterogeneous behavior - *Preisach Model of Hysteresis; iv*) an estimation and testing method consistent with *i*) and *iii*).

Moreover, to study the relationship between the existence of hysteresis and uncertainty at the macro level, we apply a Linear Play Model of hysteresis and we estimate a time varying intercept employment demand equation, relating the frequency of structural break caused by hysteresis with the existence of uncertainty.

Finally, we study the effects of adjustment through hours of work upon the band of inertia, at the aggregate level, by re-estimating the Linear Play Model with a variable play segment.

In the empirical work, we used aggregate data from Portuguese manufacturing firms resulting from aggregating the series of firms that remained in our IVNEI data set from January 1995 to December 2005. Using this data set, we guarantee that the area of the potential active firms in the Preisach Triangle (see Figure ii.9) is constant, which implies that all the variation in the aggregate employment reflects the existence of shocks, and is not affected by effects related to the re-composition of the firms in the data set.

IV.2. EMPIRICAL TESTS WITH SELECTIVE MEMORY MODELS OF HYSTERESIS

IV.2.1. EMPIRICAL METHODOLOGY

To test the existence of hysteresis we proceed as follows[3]: First, we estimate a hysteresis index variable, based on a hysteretic transformation (HS_t) of the aggregate series of sales (S_t)[4] according to the Preisach Model of Strong Hysteresis and to a Linear Play Algorithm. Second, the hysteresis index variable enters as an exogenous variable in a cointegration vector that explains the dynamics of the aggregate employment. The non-

[3] This methodology follows Cross (1995), Piscitelli *et al*. (2000) and Belke and Göcke (2001).

[4] We use aggregate sales as a proxy of the state of aggregate demand represented in the models of Chapter II by P_t. Although real wages change could also be a source of hysteresis, we only test the existence of hysteresis caused by aggregate demand shocks.

linearity inherent to hysteresis is captured by the hysteresis index, while the rest of the model is kept linear. Third, we perform cointegration tests on the following regression:

$$N_t = \beta_0 + \beta_1 HS_t + \beta_2 W + \beta_3 T + \varepsilon_t \tag{iv.1}$$

where the logarithm of the aggregate employment level is explained by a hysteresis transformation of the logarithm of real sales (HS_t), by the real wage rate W, and by a time trend (*T*) to control for changes in employment not explained by output demand variation.

The existence of hysteresis in the dynamics of employment is evaluated by considering: *i*) the significance of the hysteresis variable (hysteresis implies $\beta_1 \neq 0$); *ii*) the increase of the goodness of fit of the hysteretic regression compared with the regression on the original independent variables (real sales and real wages), that is the specification of the standard labor demand function, assuming cost minimization firms that take output demand and input prices as given; *iii*) the existence of a cointegrated vector between employment and the hysteresis transformation of sales.

As the preliminary evidence on the existence of hysteresis, offered in Chapter III, indicates the existence of differences in the employment adjustment process between small and large firms, we also analyze the presence of hysteresis in these two sub samples.

IV.2.2. EMPIRICAL IMPLEMENTATION OF STRONG HYSTERESIS MODELS

IV.2.2.1. PREISACH MODEL

In this section, we describe a procedure based on the Preisach Model of Strong Hysteresis that allows us to test, empirically, the existence of hysteresis in the aggregate path of employment.

The Preisach Model of Strong Hysteresis operates a transformation on an input variable S_t in accordance with equation ii.28 that traces the dynamics of employment given by the area of the active firms (T_1) in Figure ii.9.

This procedure was implemented by writing a MATLAB program that generates the hysteresis transformation of the aggregate series of sales, used as a proxy of labor demand, following the algorithm provided in Piscitelli *et al*. (2000). We made some numerical improvements in the code such as preallocation for better memory use and 1-dim data arrays to memorize the non-dominated extremes instead of the proposed 2-dim data arrays (see Program 1 in the Appendix).

The computation of the transformed variables involves four steps:

Step 1, made at the beginning of the program, specifies α_0 and β_0 - the vertex of the Preisach Triangle. Given that we do not have information on α_0 and β_0, we assume, as in Cross (1995), that $\alpha_0 = min\{S(t)|t = 1,2,....,n\}$ and $\beta_0 = max\{S(t)|t = 1,2,....,n\}$, where $S(t)$ is the logarithm of real sales.

Step 2, requires the selection of non-dominated extreme values from the time series of the input variable $S(t)$. The maximum is given by $M(k,t) = max\, S(j)$, $j = t^{-}_{k-1,...t}$, such that $M(k,t) = S\left(t^{+}_{k}\right)$ and the minimum is given by $m(k,t) = min\, S(j)$, $j = t^{+}_{k-1,...t}$, such that $m(k,t) = S\left(t^{-}_{k}\right)$.

Step 3, involves computing the area T_1 at t, $t = 1,...,n$, updating $T_1(t) = T_1(t) + \frac{1}{2}(M(k,t) - m(k-1,t))$ to add the area of the triangle with vertex $(M(k,t), m(k-1,t))$ and $T_1(t) = T_1(t) - \frac{1}{2}(M(k,t) - m(k-1,t))$ and to subtract the area of the triangle with vertex in $(M(k,t), m(k,t))$.

Finally, Step 4 implies the specification of the Preisach Function - $u(\alpha,\beta)$ that specifies how much each (α_j, β_j) switching combination contributes to the aggregate output. In the absence of cross-section information on the distribution of (α_j, β_j) along the Preisach Triangle, we assume a uniform weight function[5].

IV.2.2.2. LINEAR PLAY MODEL

The dynamics induced by the Preisach Model can also be approximated by a Linear Play Hysteresis Operator (see Visitin 1994, Belke and Göcke 2001 and Göcke 2002 for an application to the dynamics of employment).

Based on the Play Model of hysteresis, we estimate a linear switching employment equation, with an unknown splitting factor (called the *play*), to capture the non-linear play hysteresis effects[6]. We assume that for small changes of sales there is a weak reaction of employment along a *play segment,* and for large changes of sales there is a strong reaction of the employment along a *spurt segment*. The location of the play segment is shifted vertically by movements on the spurt line in the direction of the change in employment. Thus, the realization of the aggregate employment can be expressed as a shift in the cumulate vertical displacement of the play segment, induced by past spurts, and by the change of the current state of product demand (see Figure ii.10).

The Algorithm to Compute the Play

We consider that the change in the independent variable S_t (the variable that causes hysteresis) may occur inside the play area ($PLAY$), in which case it is referred to as Δa, or on the spurt line, in which case it is referred to as $\Delta SPURT$[7]:

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[5] In fact, the results are not very sensitive to the specification of the Preisach Function, a property that is usually referred to as the *statistical stability* of the Preisach Model.

[6] The implementation of the Linear Play Model of Hysteresis follows Belke and Göcke (2001).

[7] The calculation of $\Delta SPURT_t$ is based on the assumption that in every period that the firm faces the decision to change the level of employment it must incur fixed costs of adjustment. This would happen even if the firm has located in the right spurt line and $\Delta P_t > 0$ or if the firm is located in the left spurt line and $\Delta P_t < 0$.

$$\Delta S_t = \Delta a_t + \Delta SPURT_t \text{, with } \Delta SPURT_t = \begin{cases} sign(\Delta S_t) * (|\Delta S_t - PLAY|) \Leftarrow (|\Delta S_t - PLAY|) > 0 \\ 0 \quad otherwise \end{cases}$$

(iv.2)

The change in the logarithm of aggregate employment induced by a change in the logarithm of sales (S_t), is divided in a weak reaction in the play area and in a strong reaction described by the spurt line when S_t changes sufficiently:

$$\Delta N_t = \beta_1 \Delta a_t + (\beta_1 + \beta_2) \Delta SPURT_t \text{, with } |\beta_1| < |\beta_1 + \beta_2| \quad \text{(iv.3)}$$

The location of the play line is shifted vertically by movements on the spurt line in the direction of the change in employment. The cumulate vertical displacement of the play line, induced by all previous movements on both spurt lines, is expressed as:

$$V_{t-1} = (\beta_1 + \beta_2) \sum_{t=0}^{t-1} \Delta SPURT_t \quad \text{(iv.4)}$$

Thus the realization of the dependent variable can be expressed as a shift in V induced by past spurts and the current change in the independent variable (ΔS_t):

$$N_t = C + V_{t-1} + \Delta N_t = C + (\beta_1 + \beta_2) \sum_{t=0}^{t-1} \Delta SPURT_t + \beta_1 \Delta a_t + (\beta_1 + \beta_2) \Delta SPURT_t \quad \text{(iv.5)}$$

and rearranging we have:

$$N_t = C + (\beta_1 + \beta_2) \sum_{t=0}^{t} \Delta SPURT_t + \beta_1 S_t \quad \text{(iv.6)}$$

Summing and subtracting $-\beta_1 \sum_{t=0}^{t-1} \Delta S_t$ and making $\beta_0 = C - \beta_1 \sum_{t=0}^{t-1} \Delta S_t$, we have:

$$N_t = \beta_0 + \beta_1 S_t + (\beta_1 + \beta_2) SPURT_t, \tag{iv.7}$$

$$\text{with } (\beta_1 + \beta_2) SPURT_t = (\beta_1 + \beta_2) \sum_{t=0}^{t} \Delta SPURT_t$$

The linear equation (iv.7) captures the non-linear play dynamics with the inclusion of an artificial variable $SPURT_t$ that summarizes all preceding and present movements on the spurt lines, originating a shift in the current relation between employment and input variables.

We wrote a MATLAB program (see Program 2 in the appendix) to generate the spurt variable ($SPURT_t$) following the algorithm described in Belke and Göcke (2001). The algorithm requires the estimation of the width of the play $(PLAY)$, which is assumed constant over time. The estimation of the play is executed via a grid search procedure over the values of the play $(PLAY)$[8]: *i*) given the value of the $PLAY$, the algorithm computes the spurt variable ($SPURT_t$); *ii*) given the $SPURT_t$, the *R*-square of the estimated employment equation is calculated for every play grid points; *iii*) the value of the play leading to the maximum value of the goodness of fit of Equation iv.7 is selected.

The calculus of the *R*-square requires the estimation of the β coefficients:

$$\hat{\beta} = (X' X)^{-1} X' Y \tag{iv.8}$$

As matrix inversion is a computationally expensive procedure and also a numerically unstable one, we made some improvements in the code to avoid this problem. The solution was to use a $\boldsymbol{X} = \boldsymbol{QR}$ factorization, where $\boldsymbol{Q}$ is orthogonal and R is triangular:

[8] We use a grid between 0.000 and 0.200 with increments of 0.02, and $SPURT_t$ is calculated using real sales as a proxy of employment demand.

$$\widehat{\beta} = (X'X)^{-1}X'Y = (R'Q'QR)^{-1}R'Q'Y = R^{-1}Q^{-1}Y \quad \text{(iv.9)}$$

The implementation of this procedure to calculate the *R*-square was done in an auxiliary program (see Program 3 in the Appendix)

IV.2.3. EMPIRICAL RESULTS

The test of strong hysteresis consists of checking the ability of the selected hysteresis transformed input variable, to explain the observed aggregate employment dynamics. Following the referred methodology, we estimate by OLS a cointegrated regression between employment, real sales and the hysteresis transformation of sales, according to the Preisach Model and to the Linear Play Model[9].

A first test to the existence of hysteresis consists in verifying the significance of the transformed sales variable, either when it enters alone or with the original series in the cointegrated regression[10]. We started by applying formal unit root tests to all the variables – employment (N_t), real sales (S_t), the real wage (W_t) and the hysteresis transformed sales variable (HS_t) – see Table A.9 in the Appendix. The null hypothesis is that the series are integrated of order one. In none of the cases the t_{ADF}-statistic exceeds in absolute value the critical value, meaning that we did not reject the null hypothesis. In order to check the existence of cointegration between the variables we adopt the Engle-Granger Cointegration Test and the Johansen Test Procedure (see Johansen 1988)[11]. We also

[9] We adopt the OLS estimators of the cointegrated vector, which according to Granger (1991, p. 71): "…*should give an excellent estimate of the true coefficient* …", since the OLS estimator is super-consistent when there is cointegration.

[10] When we conclude for cointegration between the variables, the estimated coefficient of an integrated regressor can be estimated in an unusually precise way, as the estimate converges for its true value at a rate T^{-1} rather than the usual $T^{-1/2}$. The estimators are superconsistent. However, the standard Gaussian asymptotic theory does not apply when there are integrated regressors meaning that the *t*-statistics do not follow the standard *t*–student distribution (Stock and Watson 1988). For that reason, we do not perform formal tests on the significance of the variables and proceed only with a broad assessment of the main performance.

[11] The Engle-Granger test is an Augment Dickey-Fuller unit-root test of the residuals of the cointegrating regression. If the series are not cointegrated then there must be a unit root in the residuals (null hypothesis of no cointegration). On the contrary, if the residuals are stationary the series are cointegrated, and the

perform a test on the increase in the goodness of fit of the original regression, when we add the hysteresis transformed variable[12].

IV.2.3.1 RESULTS OF THE PREISACH MODEL

We start by estimating by OLS an equation relating the logarithm of aggregate employment to the logarithm of real sales and a time trend (see Table iv.1, p. 109)[13]:

$$N_t = \beta_0 + \beta_1 S_t + \beta_2 T + \varepsilon_t \qquad \text{(iv.10)}$$

The estimated sales elasticity of employment is 0.254[14] and the estimated coefficient associated with the time trend shows that the employment of the manufacturing sector decreased every month at 0.2 percent. Both the logarithm of sales and the time trend are significant[15]. We also estimated equation iv.10 for firms with fewer than 20 workers, and for firms with more than 500 workers (see Table iv.1 - columns 5 and 8)[16]. We verify that the estimated coefficient associated with real sales is positive and significant for both sub samples and that aggregate employment of small firms is more responsive to sales than aggregate employment of large firms (the estimated coefficients are respectively 0.364 and 0.182).

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critical values tabulated by MacKinnon (1991) should be used. We also use the Johansen Maximum Likelihood procedure (Trace Test), to test for cointegration. The test was performed with four lags in the VAR representation and with an intercept and time trend in the cointegration equation. We report the results of testing the null hypothesis of no cointegration ($r = 0$) against the alternative of the existence of at least one cointegrated vector ($r \geq 1$).

[12] The *F*-Statistic does not follow exactly the *F*-distribution. However, a large magnitude of the *F*-Statistic relative to the standard critical value of 4.79 (for a 1% significance level) indicates that the unrestricted model is more adequate.

[13] We do not include information on earnings in the employment equation since the data set we use does not contain information on the market wage rate, but only on the unitary value of earnings paid by firms, which are already a consequence employment demand decisions. Actually, the estimated coefficient associated with the real earnings variable is non-significant and displays a positive sign, which indicates potential simultaneity problems.

[14] Not very different from 0.29, the estimate obtained by Varejão (2000) with quarterly data for the period 1991 to 1995.

[15] Note that, even considering that the *t*-values are not student-*t* distributed the *t*-statistics are three times greater than 1.96 (the 5% critical value in the case of the standard *t*-student distribution).

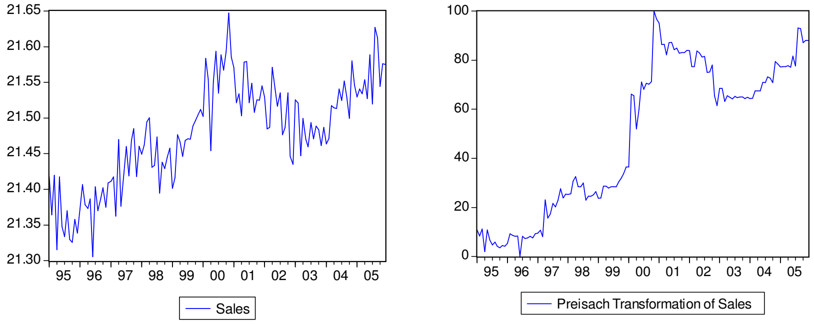
[16] The data set contains 65 firms with fewer 20 workers and 60 firms with more than 500 workers.

To test the existence of hysteresis, we replace the original series of sales (S_t) in Equation iv.10 with its hysteresis transformation (HS_t) and we estimate Equation iv.11:

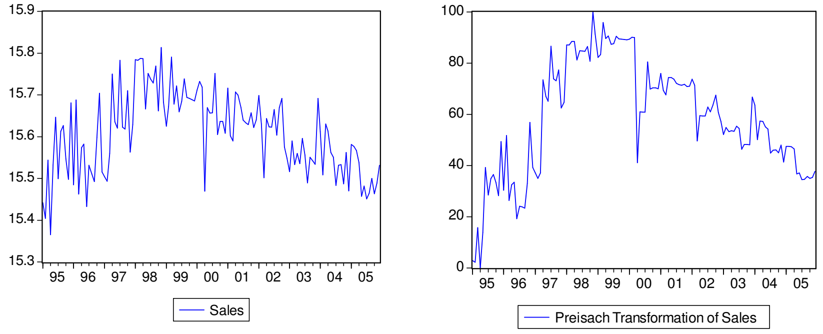
$$N_t = \beta_0 + \beta_1 HS_t + \beta_2 T + \varepsilon_t \qquad \text{(iv.11)}$$

Figure iv.1 plots the Preisach hysteresis index variable HS_t under the assumption that the Preisach Function is uniform. The Preisach hysteresis index variable computed by the previous program looks smoother than the original series of sales as it is the output of a non-linear transformation of the original variable, that retains in the memory bank only the non-dominated extremes, and combines them in a non-linear way.

Whole Sample



Firms with Fewer than 20 Workers



Firms with More than 500 Workers

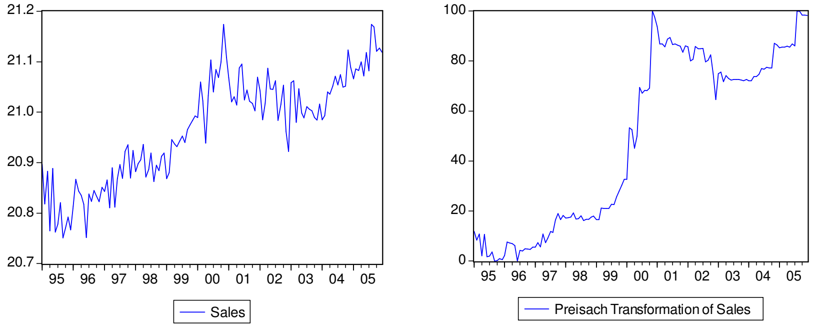


Figure iv.1. Hysteresis Transformed Real Sales Series (Preisach Model of Hysteresis)

Figure iv.2 (the plot of the hysteresis transformation of the logarithm of sales, at constant prices, against the original variable - hysteresis loop) shows the predicted dynamics of employment under hysteresis. If the Preisach Model of hysteresis offers a good description of the data, the dynamic behavior of employment at the macro level could be approximated by the hysteresis transformation of the aggregate sales series. Figure iv.2 shows that when sales vary back and forth, the response of the hysteresis transformed variable is non-linear. Moreover, we have several values of the transformed series associated with the same value of sales, which indicates that in order to know the current value of the transformed series variable it is not enough to know the current value of the sales. In other words, the historical path of the input matters. In this figure, the number of the loops is determined by the number of inflexions (the change in the sign of the first differences of the series) of the original sales variable, while the vertical distance between the increasing and the decreasing paths is determined by the magnitude of the inflexions of the original series of sales in the presence of fixed employment adjustment costs.

Figure iv.2 also shows that hysteresis properties in the relationship between aggregate employment and sales, if they are present, should be more important for the sub sample of the small firms. Actually, the relationship between the predicted dynamics captured by the hysteresis transformed variable and the original series of sales exhibits more non-linearity in the case of the small firms.

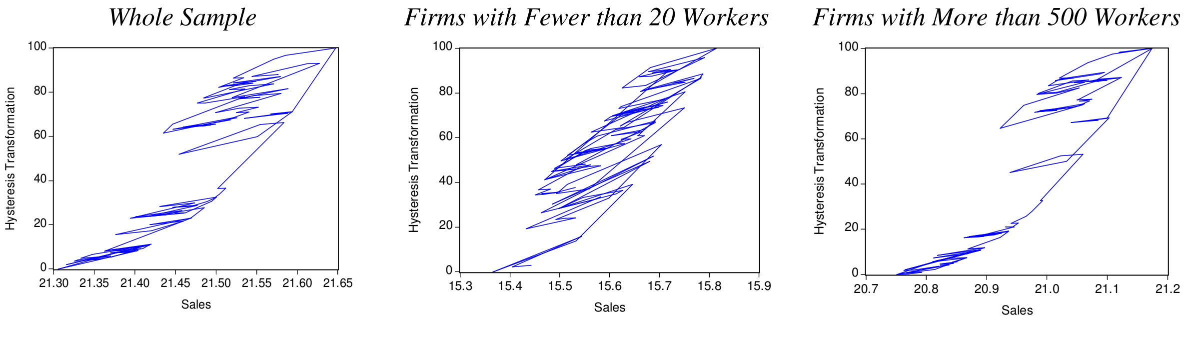


Figure iv.2. Aggregate Hysteresis Loops (Preisach Model of Hysteresis)

The results of the estimation of Equation iv.11 show an increase in the *t*-statistic associated with the estimated coefficient of sales from 6.832 to 8.616, and an increase in the goodness of fit of the regression from 84.4% to 86.5%. Nevertheless, based on cointegration tests, we fail to reject the null of no-cointegration in both regressions.

In order to obtain results that can be comparable with the results of the Linear Play Model we also estimate an equation that includes both the original variable and the transformed variable:

$$N_t = \beta_0 + \beta_1 S_t + \beta_2 HS_t + \beta_3 T + \varepsilon_t \qquad \text{(iv.12)}$$

The estimation of equation iv.12 reveals that the coefficient associated with the original series of sales becomes non-significant (at 5% level) implying that the influence of the transformed variable seems to substitute the effects of the original series of sales. Concerning the goodness of fit of the employment equation, the inclusion of the hysteresis transformation of sales significantly increases the *R*-square from 84.4% to 86.9%[17]. Based on the Trace Test, we conclude (at 5% level of significance) for the existence of a cointegrated vector relating aggregate employment, real sales and their hysteresis transformation, which is also a sign of hysteresis.

For firms with fewer than 20 workers, the goodness of fit of Equation iv.11, which relates aggregate employment to sales increased from 69.8% to 80% when we used the transformed sales variable instead of the original series. Our estimation indicates an increase of the *t*-statistics, associated with the coefficient of the transformed variable. Regarding the goodness of fit of the employment equation, the inclusion of the hysteresis transformation of the input variable significantly improved the *R*-square from 69.8% (Equation iv.10) to 81.0% (Equation iv.12)[18]. We also verify that in the case of small firms, when we estimated Equation iv.12, the coefficient associated with the original sales series turned to negative and non-significant. This means that transformed variable is more adequate to explain aggregate employment. Finally, note that based on the Trace Test we conclude for the existence of one cointegrated vector relating the variables in all equations.

[17] The *F*-statistic of the test on the increase of goodness of fit is 24.3.
[18] The F-statistic of the test on the increase of goodness of fit is 73, exceeding by far the 1% critical level (4.79) for the rejection of the hypothesis that the inclusion of the transformed variable does not significantly improve the *R*-square of the regression.

The results for large firms contrast sharply with the results for small firms. For firms with more than 500 workers the inclusion in the equation of the hysteresis variable instead of the original variable (Equation iv.11) does not originate an increase in the *R*-square and when we add the transformed variable to the equation relating employment with the original series of sales (Equation iv.12), the transformed variable appears non-significant and the increase in the goodness of fit is also not significant[19]. However, based on the Trace Test we can conclude for the existence of a cointegrated vector relating employment with both the original and the transformed series of sales[20].

In order to compare the dynamics of the labor factor with the dynamics of employment, we re-estimate Equations iv.10 to iv.11 with the logarithm of total hours of work (labor factor) as the dependent variable (see Table A.10 in the Appendix).

For the whole sample, the estimated sales elasticity of employment increases to 0.295 and the estimated coefficient associated with the time trend is again -0.002. This means that the labor factor is more responsive to sales than employment.

Concerning the hysteresis effects, for the sub sample of the small firms, we conclude that the inclusion of the hysteresis transformed variable instead of the original series of sales increases the goodness of fit of the adjustment from 0.791 (Equation iv.10) to 0.807 (Equation iv.11). Consequently, in spite of not being so evident (as in the case of aggregate employment) hysteresis is still present in the dynamics of the labor factor. When we estimate equation iv.11 for the whole sample and for the sub sample of large firms, we conclude that the explanatory power of the transformed sales variable decreases. This indicates that hysteresis do not characterize the dynamics of the labor factor adjustment of large firms.

[19] The *F*-statistic of the test on the increase of goodness of fit is 1.60.

[20] According to the Engle-Granger Cointegration test, we verify that we cannot reject the null hypothesis of no-cointegration, in all the samples. However, for the whole sample and for the sample of small firms the absolute value of the test statistics increases, when we run the regressions with the transformed hysteresis variable, meaning that we are closer to accepting the existence of cointegration.

Table iv.1
Results of the Preisach Model

Dependent Variables	Dependent variable – Logarithm of Aggregate Employment (N_t) - Sample: 1995:01-2005:12								
	Aggregate series for the whole sample			**Aggregate series for firms with fewer than 20 workers**			**Aggregate series for firms with more than 500 workers**		
	Equation iv.10	**Equation iv.11**	**Equation iv.12**	**Equation iv.10**	**Equation iv.11**	**Equation iv.12**	**Equation iv.10**	**Equation iv.11**	**Equation iv.12**
$Cons$	**6.682** 8.397 (0.000)	**12.11** 3588 (0.000)	**10.17** 9.987 (0.000)	**1.399** 2.437 (0.016)	**6.983** 775.2 (0.000)	**9.299** 9.000 (0.000)	**7.340** 8.899 (0.000)	**11.13** 2555 (0.000)	**8.215** 7.652 (0.000)
S_t	**0.254** 6.832 (0.000)	-	**0.091** 1.910 (0.058)	**0.364** 9.904 (0.000)	-	**-0.150** -2.242 (0.027)	**0.182** 4.600 (0.000)	-	**0.140** 2.723 (0.007)
HS_t	-	**0.100** 8.616 (0.000)	**0.079** 4.929 (0.000)	-	**0.185** 14.63 (0.000)	**0.242** 8.544 (0.000)	-	**0.055** 3.837 (0.000)	**0.023** 1.268 (0.207)
T	**-0.002** -22.05 (0.000)	**-0.002** -20.56 (0.000)	**-0.002** -20.43 (0.000)	**-0.001** -12.05 (0.000)	**-0.001** -18.45 (0.000)	**-0.002** -17.30 (0.000)	**-0.002** -16.12 (0.000)	**-0.002** -13.49 (0.000)	**-0.002** -14.09 (0.000)
R^2	0.844	0.865	0.869	0.698	0.800	0.810	0.826	0.819	0.829
DW	0.326	0.107	0.199	0.783	0.536	0.432	0.157	0.049	0.128
Engle Granger Cointegration Test Statistic	-1.404	-2.388	-1.834	-1.87	-2.268	-2.103	-2.079	-1.834	-2.194
MacKinnon 5% Critical Value	-3.553	-3.553	4.211	-3.553	-3.553	4.211	-3.553	-3.553	4.211
Trace Test Statistic	22.97*	20.74	44.87*	30.89*	30.06*	54.72*	21.05	18.53	57.49*
5% Critical Value	25.87	25.87	42.91	25.87	25.87	42.91	25.87	25.87	42.91

We report for each variable the estimated coefficient, the *t*-statistic and the *p*-value respectively.

* Significant at 5%

IV.2.3.2. RESULTS OF THE LINEAR PLAY MODEL

To check whether the Play Model is relevant, as the model predicts a weaker play reaction and a stronger spurt reaction, we tested the hypothesis H0: $|\beta_1| = |\beta_1 + \beta_2|$ against H1: $|\beta_1| < |\beta_1 + \beta_2|$ (see Equation iv.7). We estimated equation iv.7 with sales as an independent variable; a time trend is also included:

$$N_t = \beta_0 + \beta_1 S_t + (\beta_1 + \beta_2) SPURT_t + \beta_3 T + \varepsilon_t \tag{iv.13}$$

Through a process of grid search described in section IV.2.2.2, the values obtained for the play are: 0.106 for the whole sample; 0.170 for firms with fewer than 20 workers; 0.074 for firms with more than 500 workers. This means that the band of inaction is greater for small firms (see Figure iv.3).

Whole Sample | *Firms with Fewer Than 20 Workers* | *Firms with More than 500 Workers*

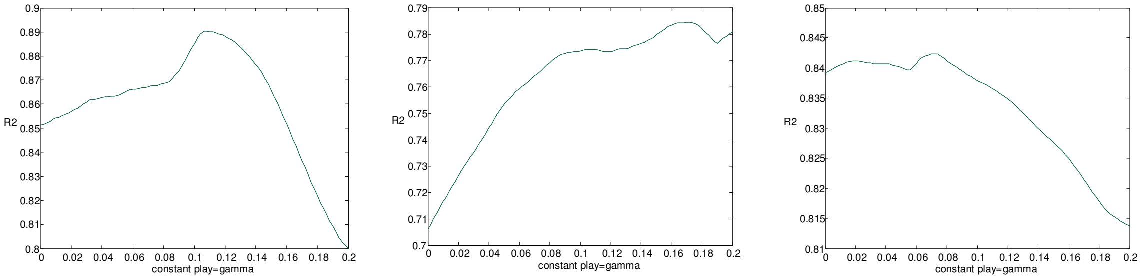
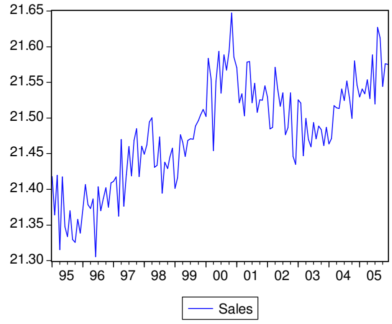
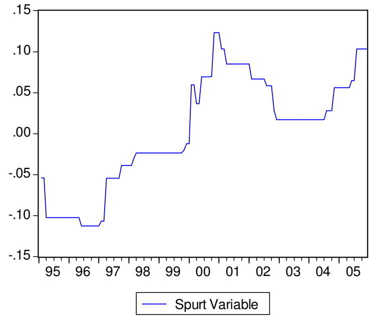


Figure iv.3. Estimation of the Constant Play Width
(Values of the *R*-square of Equation iv.13estimated for each grid play value)

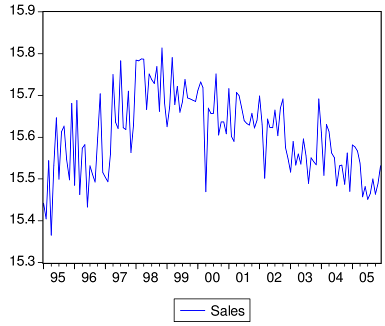
The series of the spurt variable calculated for the estimated play values are plotted in Figure iv.4 and the linear hysteresis loops are displayed in Figure iv.5. The spurt variable is a kind of filtered input, since the input variations inside the play interval are eliminated (Belke and Göcke 2001, p. 189). As the estimated play width is greater for the sub sample of the small firms, the linear play algorithm originates a transformed series, which is smoother than in the case of the large firms.

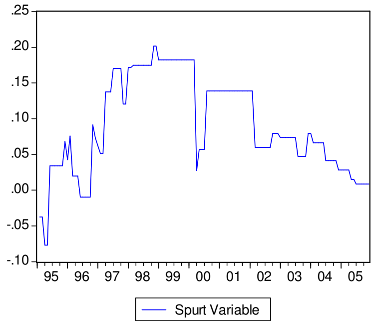
Whole Sample



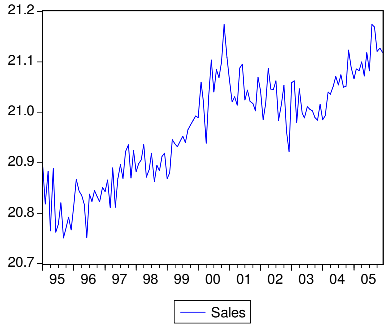


Firms with Fewer than 20 Workers





Firms with More than 500 Workers



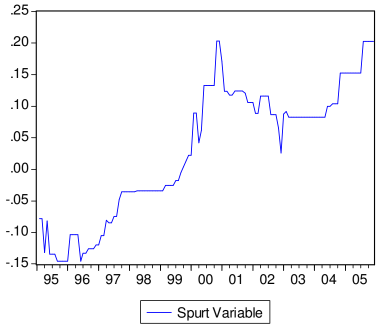


Figure iv.4. Hysteresis Transformed Real Sales Series (Linear Play Model of Hysteresis)

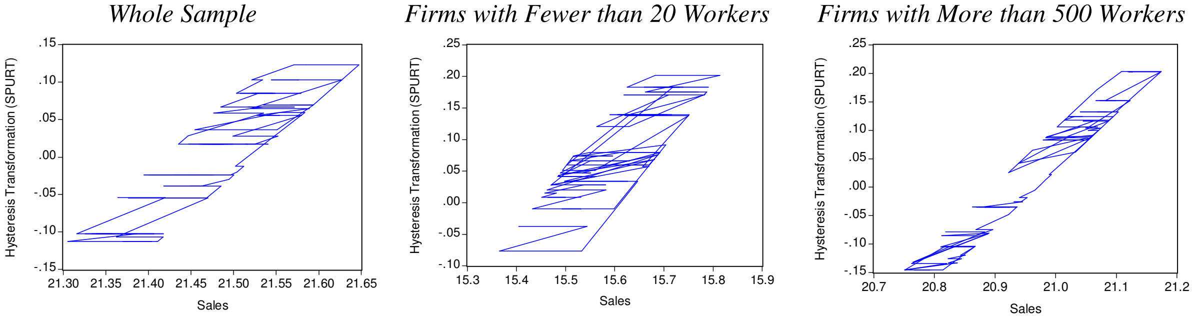


Figure iv.5. Aggregate Hysteresis Loops (Linear Play Model of hysteresis)

Table iv.2 shows that, in all cases, when we estimate Equation iv.13, which includes both the spurt and the play, the coefficient associated with the play variable is not significant while the coefficient associated with the spurt variable is significant. The t-statistics to test H_0: $|\beta_1| = |\beta_1 + \beta_2|$ against H_1: $|\beta_1| < |\beta_1 + \beta_2|$ are respectively 6.072 for the whole sample and 6.107 for the sample of the small firms, which implies that H_0 is clearly rejected. For these two samples, we conclude that the reaction along the play is weaker than the reaction along the spurt and that the influence of the hysteresis transformation of sales seems to substitute the effects of the original variable (see Table iv.2 and iv.3). In the case of large firms the t-statistic is 0.9866, which implies that we cannot reject H_0. Consequently, we did not find evidence of play hysteresis in the dynamics of aggregate employment of large firms.

Concerning the increase of the goodness of fit of the regression due to inclusion of the transformed variable, we verify that the R-square increases significantly from 84.44% (Equation iv.10) to 89.1% (Equation iv.13) for the entire sample, and from 69.8% to 78.6% for the sub-sample of small firms[21]. For large firms, the F-statistic of the test of the increase of goodness of fit is low (3.84), meaning that the inclusion of the spurt variable did not significantly increase the goodness of fit.

In addition, the Trace Test shows that, in all cases, we cannot reject the hypothesis of the existence of a cointegrated vector between aggregate employment, the original sales series and the spurt series.

[21]The F-statistic of the test on the increase of goodness of fit is respectively 45.94 and 47.69.

In order to allow a more direct comparability of the tests (based on the Preisach and on the Linear Play Models of Hysteresis) we also estimated the model with only the spurt variable:

$$N_t = \beta_0 + \beta_2 SPURT_t + \beta_3 T + \mu_t \qquad \text{(iv.14)}$$

Consequently, we assume that $\beta_1 = 0$ in equation iv.13 meaning that the play segment in Figure ii.10 is a horizontal line. Table iv.3 shows that compared with equation iv.10, the *t*-statistics of the coefficients associated with the hysteresis transformed sales variable increase for all sub samples. The results also reveal, for every class of firm size, an increase of *R*-square when the transformed variable substitutes the original one[22].

Finally, to offer more robust results on labor factor dynamics we re-estimate Equations iv.13 and iv.14 with the logarithm of total hours of work as the dependent variable (see Table A.11 in the Appendix). We do not reject H_0, when we use the whole sample and the sample for large firms, and we conclude that in the case of small firms, the difference of the reaction along the play and along the spurt is not as large as in the case of employment. These results indicate that contrary to what happens with employment, hysteresis is not so important to describe the dynamics of the labor input.

Table iv.2
Estimated Play Width and Employment Elasticities
$(PLAY_t = \gamma)$

	Whole Sample	**Small Firms**	**Large Firms**
Width of the play ($PLAY\_sp$)	0.106	0.170	0.074
Reaction along the play (β_1)	0.038 (0.829)	0.058 (1.109)	0.080 (1.116)
Reaction along the spurt ($\beta_1 + \beta_2$)	0.364* (6.779)	0.504* (6.906)	0.161 (1.961)

* Significant at 5% (*t*-statistics greater than three times the standard critical value to the rejection of non-significance)
t-Statistics in brackets

[22] Nonetheless, according to the Trace Test, we only conclude for the existence of cointegration for the case of small firms.

Table iv.3
Results of the Linear Play Model (Constant Play)

Dependent Variables	Dependent variable – Logarithm of Aggregate Employment (N_t) - Sample: 1995:01-2005:12								
	Aggregate series for the whole sample			Aggregate series for firms with fewer than 20 workers			Aggregate series for firms with more than 500 workers		
	Equation iv.10	Equation iv.13	Equation iv.14	Equation iv.10	Equation iv.13	Equation iv.14	Equation iv.10	Equation iv.13	Equation iv.14
$Cons$	**6.682** 8.397 (0.000)	**11.349** 11.64 (0.000)	**12.16** 2594 (0.000)	**1.399** 2.437 (0.016)	**6.124** 7.488 (0.000)	**7.031** 935.7 (0.000)	**7.340** 8.899 (0.000)	**9.484** 6.284 (0.000)	**11.16** 1568 (0.000)
S_t	**0.254** 6.832 (0.000)	**0.038** 0.829 (0.408)	-	**0.364** 9.904 (0.000)	**0.058** 1.109 (0.270)	-	**0.182** 4.600 (0.000)	**0.080** 1.116 (0.266)	-
$SPURT_t$	-	**0.364** 6.779 (0.000)	**0.396** 10.682 (0.000)	-	**0.504** 6.906 (0.000)	**0.569** 12.68 (0.000)	-	**0.161** 1.961 (0.052)	**0.238** 5.476 (0.000)
T	**-0.002** -22.05 (0.000)	**-0.002** -26.56 (0.000)	**-0.002** -26.73 (0.000)	**-0.001** -12.05 (0.000)	**-0.001** -15.01 (0.000)	**-0.001** -15.52 (0.000)	**-0.002** -16.12 (0.000)	**-0.002** -16.19 (0.000)	**-0.002** -16.16 (0.000)
R^2	0.844	0.891	0.890	0.698	0.786	0.784	0.826	0.844	0.842
DW	0.326	0.135	0.114	0.783	0.470	0.429	0.157	0.113	0.082
Engle Granger Cointegration Test Statistic	-1.404	-2.354	-2.488	-1.870	-2.888	-2.257	-2.079	-2.437	-2.312
MacKinnon 5% Critical Value	-3.553	4.211	-3.553	-3.553	4.211	-3.553	-3.553	4.211	-3.553
Trace Test Statistic	22.97*	51.99*	21.19	30.89*	57.24*	30.67*	21.05	45.09*	15.33
5% Critical Value	25.87	42.91	25.87	25.87	42.91	25.87	25.87	42.91	25.87

We report for each variable the estimated coefficient, the *t*-statistic and the p-value respectively.

* Significant at 5%

IV.3. HYSTERESIS, UNCERTAINTY AND ADJUSTMENT THROUGH HOURS OF WORK

Models of weak hysteresis predict that the band of inaction is a positive function of uncertainty and of the degree of flexibility of working time regulations.

In this section, we use two methods to check the effect of uncertainty upon the existence of hysteresis in the dynamics of employment at the aggregate level. One method is based on the continuous change in the relation between employment and sales, and it is implemented using Kalman Filter techniques, to estimate a time varying intercept employment equation. The other method checks for the existence of discrete changes in the relation between employment and sales and it is implemented by estimating a switching regression with a variable splitting factor (the play) that is a function of a proxy of uncertainty.

The later approach is also used to test the influence of the adjustment through hours per worker upon the existence of aggregate employment hysteresis.

IV.3.1. STRONG HYSTERESIS MODELS WITH UNCERTAINTY

IV.3.1.1. TIME VARYING INTERCEPT EMPLOYMENT MODEL

To analyze the importance of uncertainty in determining hysteretic behavior, we implement an empirical test based on the fact that the frequency of structural break hysteresis in the relationship between employment and its fundamentals is a negative function of uncertainty and the existence of fixed employment adjustment costs. This prediction emerges from Figure ii.10. The greater the play width, the less frequently changes in P_t lead to a reaction along the spurt, with back and forth movements in P_t originating a weak reaction along the play area without any structural break.

To implement this test, we estimate a stochastic time varying intercept version of the employment demand equation:

$$N_t = \beta_{0,t} + \beta_1 S_t + \beta_2 T + \varepsilon_t \qquad \text{(iv.15)}$$

where N_t, S_t and T_t are respectively the aggregate employment, real sales and a time trend.

The shifts in the intercept represent major shifts in the relation between employment and its fundamentals, while the coefficient associated with S_t represents the weak reaction along the play.

If uncertainty related to input variation determines hysteresis, the change in the time varying intercept $(\beta_{0,t})$ should be inversely related to some measure of uncertainty (see Parsley and Wei 1993):

$$\left|\Delta\hat{\beta}_{0,t}\right| = \alpha_0 + \alpha_1 \sigma_{S_t} + \omega_t \qquad \text{(iv.16)}$$

where $\hat{\beta}_{0,t}$ is a time series of the estimated time varying intercept in Equation iv.15 and σ_{S_t} is a proxy for the variability of the real sales.

Estimation of the Time Varying Intercept Version of Employment Equation

We assume a random walk structure for the time varying intercept and a maximum likelihood estimation method based upon the Kalman Filter. The specification of the model is the following:

$$\begin{cases} N_t = \beta_{0,t} + \beta_1 S_t + \beta_2 T + \varepsilon_{1_t} \\ \beta_{0,t} = \beta_{0,t-1} + \varepsilon_{2_t} \end{cases} \qquad \text{(iv.17)}$$

The Kalman Filter is a recursive procedure to calculate the optimal linear estimator of the state vector in each period t, with $t = 1,...T$, based on the available information in t, given the matrices of the system and some acceptable priors for the initial state vector and covariance matrices (see Harvey 2001).

In order to apply the Kalman Filter Algorithm the model has to be stated in *the state-space form*. The *state-space form* of the system is given by a *measurement equation* that establishes the relationship between the observables (*N*, *S*, and *T*) and the non-observable variable $(\beta_{0,t})$[23] and by a *transition* e*quation* that specifies the stochastic processes for the non-observable time varying intercept. It represents the relation between the sate vector $\boldsymbol{\alpha_t}$ and its lagged values through the transition matrix ***T*** (see Harvey 2001). Measurement

$$\begin{aligned} Y_t &= Z\alpha_t + \Sigma_t^Y \quad \text{(measurement equation)} \\ \alpha_t &= T\alpha_{t-1} + \Sigma_t^\alpha \quad \text{(transition equation)} \end{aligned} \tag{iv.18}$$

With:

$$\boldsymbol{Y}_t(1\times1) = [N_t]$$

$$Z_t(1\times3) = [1 \quad S_t \quad T]$$

$$\alpha_t(1\times4) = \begin{bmatrix} \beta_{0,t} \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

$$T(3\times3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The stochastic specification of the model is completed with the inclusion of the disturbance vector Σ_t^Y and Σ_t^α, each with mean equal to zero and covariance matrices equal to H_t and Q_t, respectively:

[23] The time varying intercept is estimated as an unobservable variable.

$$\boldsymbol{H_t}(1\times1) = \sigma_{\varepsilon}^{Y}$$

$$Q_t(4\times4) = \begin{bmatrix} \sigma_{\beta_{0,t}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The program used to estimate (iv.17) was written in GAUSS (see Program 4 in the Appendix).

Results

Table iv.4 shows the results of the estimation of the time varying intercept version of employment equation. The time varying intercept is significant for the whole sample and for the sub samples of the small and large firms, and the sales variable remain significant. The time varying intercept estimates required to estimate Equation iv.16 are shown in Figure A.3 in the Appendix.

Table iv.4
Estimates of Time Varying Intercept Version of Employment Equation
(Dependent variable np_t)

Country	β_t [24]	sp_t	T
Whole Sample	(0.000)*	**0.350*** (0.000)	**-0.002** (1.00)
Small Firms	(0.000)*	**0.154*** (0.000)	**-0.002** (0.962)
Large Firms	(0.000)*	**0.311*** (0.000)	**-0.002** (0.999)

*significant at 5%
p-values in brackets

[24] See Appendix 6 for time varying intercept estimates

We estimate Equation iv.16, firstly, for the whole sample, and then for firms with fewer than 20 workers and for firms with more than 500 (see Table iv.5). We use two forward-looking measures of uncertainty based on ex-post variability of real sales[25]:

$$\sigma_{S_t} = \frac{1}{n-1} \sum_{i=t+1}^{t+n} \left[\left(\bar{S} - S_i \right)^2 \right] \qquad \text{(iv.19)}$$

with $n = 6$ and $n = 12$

and two backward-looking measures:

$$\sigma_{S_t} = \frac{1}{n-1} \sum_{i=t-n}^{t-1} \left[\left(\bar{S} - S_i \right)^2 \right] \qquad \text{(iv.20)}$$

with $n = 6$ and $n = 12$

The estimates are in general non-significant, and when they are, they do not display the predicted negative sign. Based on this test, we do not find evidence on the effect of uncertainty upon the hysteresis band.

Table iv.5
Uncertainty Coefficient Estimates
(Dependent Variable $\Delta\hat{\beta}_{0,t}$)

	Forward Looking Measures		**Backward Looking Measures**	
	$\sigma_{s_t}\ (n=6)$	$\sigma_{s_t}\ (n=12)$	$\sigma_{s_t}\ (n=6)$	$\sigma_{s_t}\ (n=12)$
Whole Sample	**0.019** (0.202)	**0.013** (0.501)	**0.001** (0.921)	**-0.015** (0.394)
Small Firms	**0.015** (0.298)	**0.023** (0.308)	**0.037*** (0.007)	**0.060**** (0.004)
Large Firms	**-0.002** (0.886)	**-0.010** (0.643)	**0.023** (0.153)	**0.009** (0.654)

* significant at 10%; ** significant at 5%
p-values in brackets

[25] We follow Parsley and Wei (1993)

IV.3.1.2. LINEAR PLAY MODEL WITH UNCERTAINTY

We test the effect of uncertainty, at the aggregate level, by re-estimating the Linear Play Model of hysteresis allowing for a variable play width $\left(PLAY_t\right)$ that is a positive function of the standard deviation of the series of sales (σ_{S_t}), used as an uncertainty proxy variable:

$$\begin{cases} N_t = \beta_0 + \beta_1 S_t + \left(\beta_1 + \beta_2\right) SPURT_t + \beta_3 T + \omega_t \\ SPURT_t = f(\ PLAY_t\ ) \\ PLAY_t = \gamma + \delta \sigma_{S\,t} \quad with \quad \gamma, \delta \geq 0 \\ \sigma_{S_t} = \dfrac{1}{n-1} \sum\limits_{i=t-6}^{t-1} \left[\left(\overline{S} - S_i\right)^2 \right] \end{cases} \tag{iv.21}$$

Figure iv.6 plots the *R*-square of the employment equation (Equation iv.21) for each combination of grid values of γ and δ that were used to calculate the play variable[26]. As in the case of constant play, the estimated band of inertia is larger for firms with fewer than 20 workers than for firms with more than 500 (see also Figure iv.7 and Table iv.6).

We re-estimate the employment equation with the original sales variable and the spurt variables calculated on the basis of the play values (Equation 21). As in previous estimations, we also present the results on the assumption that $\beta_1 = 0$:

$$\begin{cases} N_t = \beta_0 + \beta_2 SPURT_t + \beta_3 T + \omega_t \\ SPURT_t = f(\ PLAY_t\ ) \\ PLAY_t = \gamma + \delta \sigma_{S\,t} \quad with \quad \gamma, \delta \geq 0 \\ \sigma_{S_t} = \dfrac{1}{n-1} \sum\limits_{i=t-6}^{t-1} \left[\left(\overline{S} - S_i\right)^2 \right] \end{cases} \tag{iv.22}$$

[26] We keep the same grid and increment as for the case of constant play for each parameter.

The results for the whole sample and for the sub-samples of the small and large firms are in Table iv.6 and iv.7. Concerning the magnitude of the estimates, we find, in all cases, an increase in the value of coefficients associated with the spurt variable and a decrease in the value of the coefficients associated with the play variable. These results indicate a clearer distinction between the employment reaction along the play and the reaction along the spurt, which gives even more reason for the Linear Play Model. As is the case of the estimation assuming a constant play value, the coefficient associated with the spurt variable is significant when the whole sample is used and in the case of the small firms (all the other coefficients are not significant). Moreover, the *t*-statistics of the coefficient associated with the spurt variable increase when we compare with the model with a constant play value.

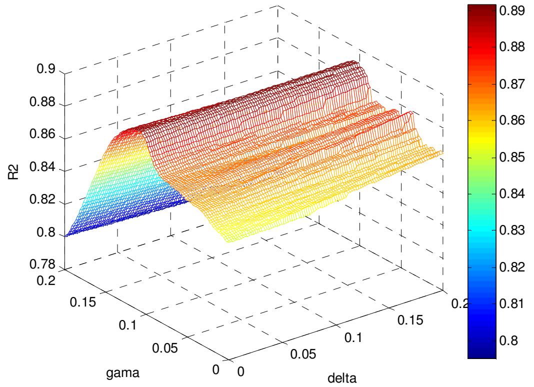
In order to assess the impact of uncertainty on the presence of hysteresis, we also test the hypothesis H_0: $\delta = 0$ against H_0: $\delta > 0$ by comparing the goodness of fit of the employment equation estimated with a constant (restricted model) and with a variable play (unrestricted model). The *F*-Statistic (for $K = 6$ parameter and $m = 1$ restriction) for a comparison of the unrestricted ($\delta > 0$) and the restricted case with $\delta = 0$ is 1.17 for the whole sample, 19.75 for small firms and 3.32 for large firms[27]. Consequently, we conclude that only in the case of small firms does uncertainty contribute to explaining the dynamics of aggregate employment through hysteresis mechanisms.

Finally, the Trace Test indicates that the variables in the Linear Play Model (Equation iv.21) are cointegrated for a 5% significance level. The existence of cointegration is verified for the case of the whole sample and also for the sub sample of the small and large firms, indicating that the previous results are not spurious.

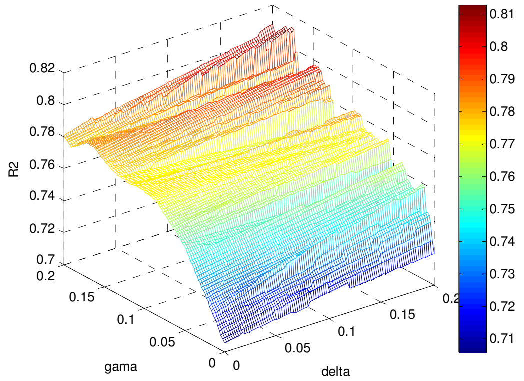
---

[27] $F(\delta = 0) = \dfrac{\left(R^2_{unrestricted} - R^2_{restricted}\right)/m}{\left(1 - R^2_{unrestricted}\right)/(N-K)}$.

***a*. Whole Sample**



***b*. Small Firms**



***c*. Large Firm**

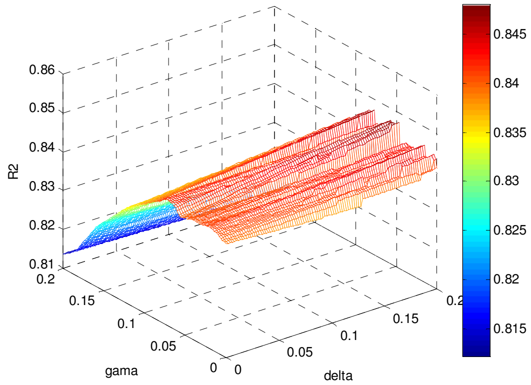


Figure iv.6. Estimation of the Variable Play Width $\left(PLAY_t = \gamma + \delta\sigma_{sp_t}\right)$

Values of the *R*-square of Equation iv.21 estimated for each combination of play parameters (γ, δ)

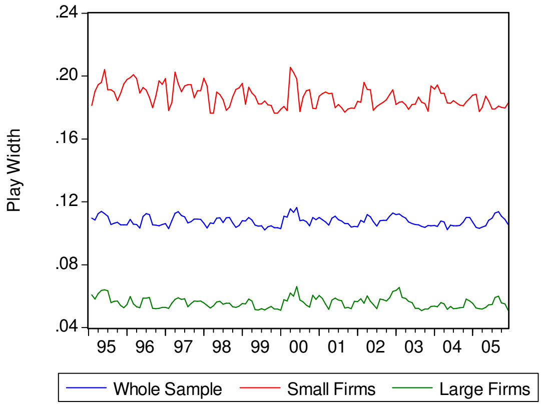


Figure iv.7 Variable Play Width $\left(PLAY_t = \gamma + \delta\sigma_{sp_t}\right)$

Table iv.6
Estimated Play Width and Employment Elasticities
$\left(PLAY_t = \gamma + \delta\sigma_{S_t}\right)$

	Whole Sample	**Small Firms**	**Large Firms**
Estimated Play Parameters	$\gamma = 0.102$ $\delta = 0.2$	$\gamma = 0.176$ $\delta = 0.2$	$\gamma = 0.05$ $\delta = 0.194$
Average Play Width	0.108	0.187	0.056
Reaction along the play (β_1)	0.035 (0.779)	0.049 (1.081)	-0.001 (-0.012)
Reaction along the spurt ($\beta_1 + \beta_2$)	0.366* (6.952)	0.632* (8.628)	0.246 (2.728)

* Significant at 5% (*t*-statistics greater than three times the standard critical value to the rejection of non-significance)
t-Statistics in brackets

Table iv.7

Results of the Linear Play Model (Variable Play: $PLAY_t = \gamma + \delta\sigma_{sp_t}$)

Dependent Variables	**Dependent variable – Logarithm of Aggregate Employment (N_t) - Sample: 1995:01-2005:12**								
	Aggregate series for the whole sample			**Aggregate series for firms with fewer than 20 workers**			**Aggregate series for firms with more than 500 workers**		
	Equation iv.10	**Equation iv.21**	**Equation iv.22**	**Equation iv.10**	**Equation iv.21**	**Equation iv.22**	**Equation iv.10**	**Equation iv.21**	**Equation iv.21**
$Cons$	**6.682** 8.397 (0.000)	**11.405** 11.825 (0.000)	**12..15** 2631 (0.000)	**1.399** 2.437 (0.016)	**6.274** 8.853 (0.000)	**7.039** 1075 (0.000)	**7.340** 8.899 (0.000)	**11.186** 6.431 (0.000)	**11.16** 1763 (0.000)
S_t	**0.254** 6.832 (0.000)	**0.035** 0.779 (0.437)	-	**0.364** 9.904 (0.000)	**0.049** 1.081 (0.280)	-	**0.182** 4.600 (0.000)	**-0.001** -0.012 (0.999)	-
$SPURT_t$	-	**0.366** 6.952 (0.000)	**0.396** 10.84 (0.000)	-	**0.632** 8.628 (0.000)	**0.692** 14.63 (0.000)	-	**0.246** 2.728 (0.008)	**0.245** 5.987 (0.000)
T	**-0.002** -22.05 (0.000)	**-0.002** -26.64 (0.000)	**-0.002** -26.77 (0.000)	**-0.001** -12.05 (0.000)	**-0.001** -17.38 (0.000)	**-0.001** -15.52 (0.000)	**-0.002** -16.12 (0.000)	**-0.002** -16.61 (0.000)	**-0.002** -16.73 (0.000)
R^2	0.844	0.892	0.892	0.698	0.815	0.813	0.826	0.848	0.848
DW	0.326	0.130	0.112	0.783	0.494	0.463	0.157	0.100	0.100
Engle Granger Cointegration Test Statistic	-1.404	-2.527	-2.485	-1.870	-2.715	-2.765	-2.079	-2.371	-2.372
MacKinnon 5%Critical Values	-3.553	4.211	-3.553	-3.553	4.211	-3.553	-3.553	4.211	-3.553
Trace Test Statistic	22.97*	52.405*	17.60	30.89*	58.81*	32.91*	21.05	48.166*	15.03
5% Critical Value	25.87	42.91	25.87	25.87	42.91	25.87	25.87	42.91	25.87

We report for each variable the estimated coefficient, the *t*-statistic and the p-value respectively.

* Significant at 5%.

IV.3.2. LINEAR PLAY MODEL WITH ADJUSTMENT THROUGH HOURS OF WORK

We test the hypothesis of influence of the adjustment through hours of work upon the band of inertia, at the macro level, by estimating the Linear Play Model of Hysteresis allowing for a variable play width that is a positive function of a proxy of the flexibility of the adjustment through hours of work:

$$\begin{cases} N_t = \beta_0 + \beta_1 S_t + (\beta_1 + \beta_2) SPURT_t + \beta_3 T + \omega_t \\ SPURT_t = f(PLAY_t) \\ PLAY_t = \gamma + \delta HI_t \quad with \quad \gamma, \varphi \geq 0 \\ HI_t = \dfrac{\sigma_{h_t}}{\sigma_{S_t}} \end{cases} \qquad \text{(iv.23)}$$

with $\sigma_{S_t} = \dfrac{1}{n-1} \sum_{i=t-6}^{t-1} \left[(\bar{S} - S_i)^2 \right]$ and $\sigma_{h_t} = \dfrac{1}{n-1} \sum_{i=t-6}^{t-1} \left[(\bar{H} - H_i)^2 \right]$

As a proxy for the flexibility of the adjustment through variation in hours per worker, we use the ratio between the standard deviation of hours per worker and the standard deviation of sales (HI_t). The idea behind this proxy is that when it is high, the working time regulations are not binding meaning that firms leave the number of workers relatively constant and adjust the number of hours per worker. When the ratio is low, the working time regulations are binding, implying that the adjustment should occur through variation in the number of workers. Therefore, the play width should be positively related to HI_t.

Figure iv.8 plots the R-square of the employment equation for each combination of grid values of γ and δ that are used to calculate the play variable. Once again, our results indicate that the estimated band of inertia is larger for small firms (see also Figure iv.9 and Table iv.8).

We re-estimate the employment equation with the original sales variable and the spurt variables calculated on the basis of the variable play values (Equation iv.23). We also present the results on the assumption that $\beta_1 = 0$:

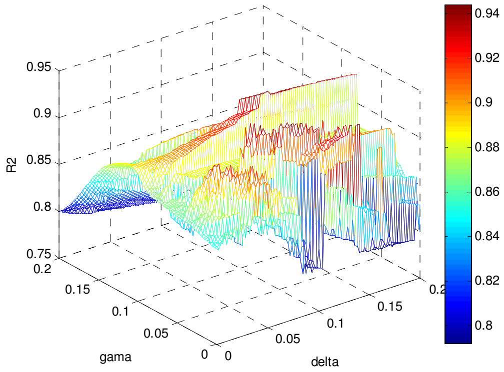
$$\begin{cases} N_t = \beta_0 + \beta_2 SPURT_t + \beta_3 T + \omega_t \\ SPURT_t = f(PLAY_t) \\ PLAY_t = \gamma + \delta HI_t \quad with \quad \gamma, \varphi \geq 0 \\ HI_t = \dfrac{\sigma_{h_t}}{\sigma_{S_t}} \end{cases} \tag{iv.24}$$

The results for the whole sample and for the sub samples of the small and large firms are in Tables iv.8 and iv.9. Concerning the magnitude of the estimates, once again, in all cases, we find an increase in the value of coefficients associated with the spurt variable and a decrease in the value of the coefficients associated with the play variable, relative to the model estimated with a constant play. These results are more robust than in the case of the estimation on the basis of variable play dependent on uncertainty, and indicate a clear distinction between the employment reaction along the play and the reaction along the spurt when we include the effect of the adjustment through hours of work. As the previous cases (estimation with a constant play and with a variable play dependent on uncertainty), the coefficient associated with the spurt variable is significant when the whole sample is used and in the case of the small firms, while all the other coefficients are not significant. Moreover, once again, the *t*-statistics of the coefficient associated with the spurt variable increases, again, when we compare with the model with a constant play value.

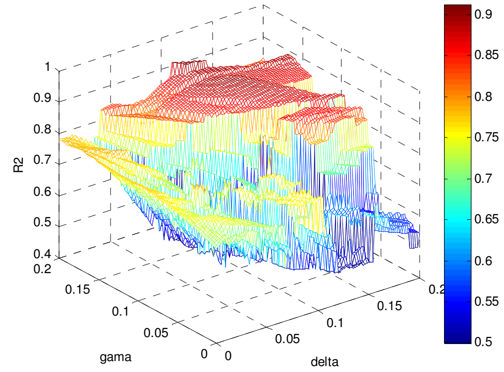
In order to assess the impact of the effect of the adjustment through the number of hours of work on the presence of hysteresis, we repeat the test on the hypothesis H_0: $\delta = 0$ against H_0: $\delta > 0$ by comparing the goodness of fit of the employment equation estimated with a constant (restricted model) and with a variable play (unrestricted model). The *F*-statistics in all cases are very high (138.12 for the whole sample, 183.93 for small firms and 105.24 for larger firms), indicating that the introduction of the hours of work margin interacts with the fixed employment adjustment costs (whose effect is captured by parameter γ) and reinforces the hysteresis mechanisms.

However, based on the Trace Test, only for the case of small firms is it possible to conclude for the existence of a cointegrated vector between the variables. Consequently, only in the case of small firms can we conclude that the possibility of adjusting the labor input by varying the number of hours of work contributes to the existence of hysteresis effects in the dynamics of aggregate employment.

Whole Sample



b. Small Firms



c. Large Firms

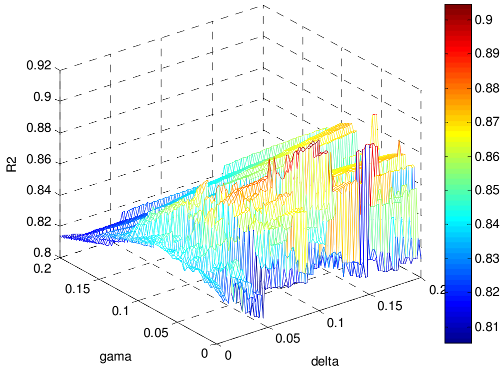


Figure iv.8. Estimation of the Play Width $\left(PLAY_t = \gamma + \delta HI_t\right)$

Values of the *R*-square of Equation 23 estimated for each combination of play parameters (γ, δ)

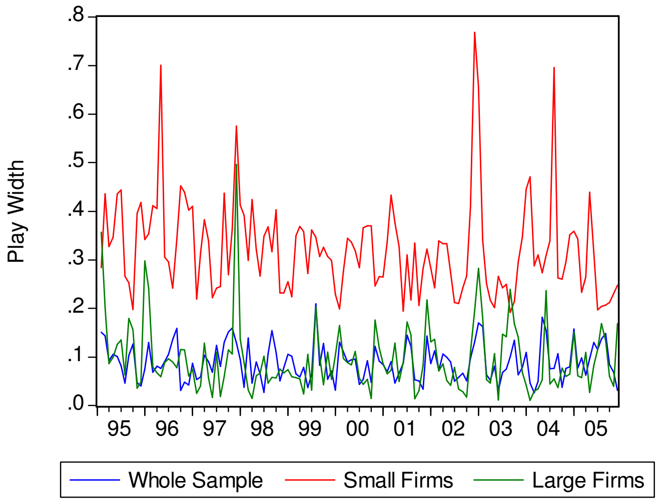


Figure iv.9 Variable Play Width $\left(PLAY_t = \gamma + \delta HI_t\right)$

Table iv.8
Estimated Play Width and Employment Elasticities
$\left(PLAY_t = \gamma + \delta HI_t\right)$

	Whole Sample	**Small Firms**	**Large Firms**
Estimated Play Parameters	$\gamma = 0.010$ $\delta = 0.108$	$\gamma = 0.190$ $\delta = 0.116$	$\gamma = 0.046$ $\delta = 1.070$
Average Play Width	0.089	0.324	0.096
Reaction along the play (β_1)	-0.098 (-3.119)	0.021 (0.750)	-0.175 (-3.873)
Reaction along the spurt ($\beta_1 + \beta_2$)	0.354* (15.45)	0.790* (17.33)	0.395* (10.615)

* Significant at 5% (*t*-statistics greater than three times the standard critical value to the rejection of non-significance)
t-Statistics in brackets

Table iv.9

Results of the Linear Play Model (Variable Play: $PLAY_t = \gamma + \delta HI_t$)

Dependent Variables	Dependent variable – Logarithm of Aggregate Employment (N_t) - Sample: 1995:01-2005:12								
	Aggregate series for the whole sample			Aggregate series for firms with fewer than 20 workers			Aggregate series for firms with more than 500 workers		
	Equation iv.10	Equation iv.18	Equation iv.19	Equation iv.10	Equation iv.18	Equation iv.19	Equation iv.10	Equation iv.18	Equation iv.19
$Cons$	**6.68** 8.397 (0.000)	**14.243** 20.996 (0.000)	**12..127** 5413 (0.000)	**1.399** 2.437 (0.016)	**6.739** 15.59 (0.000)	**7.064** 1790 (0.000)	**7.340** 8.899 (0.000)	**14.815** 15.685 (0.000)	**11.15** 3116 (0.000)
S_t	**0.254** 6.832 (0.000)	**-0.098** -3.119 (0.002)	-	**0.364** 9.904 (0.000)	**0.021** 0.750 (0.454)	-	**0.182** 4.600 (0.000)	**-0.175** -3.873 (0.000)	-
$SPURT_t$	-	**0.354** 15.45 (0.000)	**12..127** 5413 (0.000)	-	**0.790** 17.33 (0.000)	**0.813** 24.52 (0.000)	-	**0.395** 10.615 (0.000)	**0.283** 11.548 (0.000)
T	**-0.002** -22.05 (0.000)	**-0.002** -40.55 (0.000)	**-0.002** -39.34 (0.000)	**-0.001** -12.05 (0.000)	**-0.002** -17.33 (0.000)	**-0.002** -33.10 (0.000)	**-0.002** -16.12 (0.000)	**-0.002** -34.26 (0.000)	**-0.002** -27.80 (0.000)
R^2	0.844	0.948	0.944	0.698	0.913	0.912	0.826	0.915	0.904
DW	0.326	0.285	0.227	0.783	0.733	0.726	0.157	0.290	0.156
Engle Granger Cointegration Test Statistic	-1.404	-3.485	-3.05	-1.870	-3.970	-4.06*	-2.079	-3.712	-2.89
MacKinnon 5% ***Critical Values***	-3.553	4.211	-3.553	-3.553	4.211	-3.553	-3.553	4.211	-3.553
Trace Test Statistic	22.97*	35.71	24.64	30.89*	51.97*	32.891*	21.05	38.77	15.22
5% Critical Value	25.87	42.91	25.87	25.87	42.91	25.87	25.87	42.91	25.87

We report for each variable the estimated coefficient, the *t*-statistic and the p-value respectively.

*Significant at 5%.

IV.4. MAIN FINDINGS

In this chapter, we analyzed the existence of hysteresis at the macro level and we studied the influence of uncertainty and the adjustment through hours of work upon the hysteresis band. The main conclusions are stated below.

Existence of Hysteresis

The results of the Preisach Model reveal that the hysteresis transformation of sales offers a better explanation of the aggregate employment dynamics than the original variable. However, the results are distinct concerning the size of the firms. While the inclusion of the hysteresis variable significantly increases the goodness of fit of the employment equations in the case of small firms, for large ones the original series of sales offers a better explanation of employment dynamics, meaning that hysteresis is not important in this case.

Results of the Linear Model of Strong Hysteresis indicate the existence of large periods of inaction implying that a large play width is necessary to achieve the maximum goodness to fit of the employment equations. Overall, the non-linear model, which includes the hysteresis transformation of the variables, performs better than the standard linear model. Furthermore, the estimated play interval is larger for small firms than for large ones (the estimated play for the whole sample lies in the middle of these values). While for small firms the results indicate the existence of a zone of weak employment reaction (the play) and a zone of strong employment reaction (the spurt), for large firms the difference between these zones of different reaction of employment is not so clear.

The Effect of Uncertainty

We find weak evidence regarding the effect of uncertainty upon hysteresis at the aggregate level. The test, based on the estimation of a time varying intercept version of the employment equation, does not reveal signs of the influence of uncertainty.

By estimating the Linear Play Model of Hysteresis with a variable play, we verify that, except for small firms, uncertainty does not significantly affect the dynamics of employment at the macro level through hysteresis mechanisms.

The Effect of Adjustment through Hours of Work

By estimating the Linear Play Model of Hysteresis with a variable play, we conclude that, for the whole sample and also for the sub samples of small and large firms, introducing into the model the adjustment of labor factor through the number of hours of work significantly increases R-square of the employment equations. However, after carrying out cointegration tests, only in the case of small firms can we conclude clearly for the existence of an effect of the adjustment by varying the number of hours of work upon the hysteresis band.

Finally we also find that the impact of the degree of flexibility of adjusting the number of hours of work upon the hysteresis band of the small firms, at the aggregate level, is greater than the impact of uncertainty.

Distinction between Small Firms and Large Firms

We find clear differences in the adjustment of employment between small and large firms at the aggregate level. We find that, in general, hysteresis is a property of the employment dynamics of small firms. The heterogeneity concerning the existence of hysteresis by firm size can be justified by four reasons. Firstly, the fact that non-convex costs of adjusting employment represents a higher proportion of the total costs for a small firm makes it more difficult to adjusting continuously the number of employees in response to product demand shocks. Secondly, for small firms there may also be the factor of indivisibility, which means that firms cannot change the number of employees for every small labor demand shock. Thirdly, large firms have at their disposal collective dismissals as an instrument for adjusting employment downwards, which contributes to a decrease in the inaction band. In fact, in Portugal, the threshold for collective dismissals is set at 2 workers for firms with fewer than 50 employees and 5 workers for firms with more than 50 employees. This rule implies that the larger the firm is, the smaller the proportion of the threshold in the total labor force, making it easier for these firms to dismiss using this mechanism. Fourthly, in Portugal it is easier for small firms to increase the number of hours of work than for large ones. At present, the maximum number of overtime hours allowed per year is 175 hours for micro and small firms and 150 hours for medium or large firms and according to the theoretical prediction of section II.3.3 and the results of section IV.3.2, the higher the flexibility of the adjustment through the number of hours of work the greater the hysteresis effects.

CHAPTER V

INTERNATIONAL COMPARISON OF THE RESULTS

V.1. INTRODUCTION

In this part, an overview of the presence of hysteresis in different countries is presented, and its significance is discussed in relationship to labor market institutions, such as employment protection legislation and working time regulations.

In order to study the relative importance of hysteresis in the aggregate dynamics of Portuguese employment we apply the Preisach Model and the Linear Play Model of Hysteresis, already estimated for the case of Portugal, to monthly seasonally adjusted data from the manufacturing sector covering the period from January 1995 to July 2005 for nineteen OECD countries. To analyze the effect of uncertainty, we also repeat the test based on the estimation of a time varying intercept employment equation on this data set.

We use the real product as the proxy for output demand. Data sources are the EUROSTAT General Statistics – Industry Commerce and Services, for employment (number of persons employed) for the real product and for gross wages, and the EUROSTAT – Economy and Finance – Consumer prices indexes for the wage deflator. For the output (Y), we use a production index adjusted by the number of working days, for employment (N) we use the index of the number of employees, and for real wages (W), we deflate the index of gross wages by the general index of consumer prices. For the US, Japan and Canada we use data from OECD – Main Economic Indicators.

V.2. RESULTS OF THE PREISACH MODEL

We start by estimating by OLS an equation relating the logarithm of aggregate employment to the logarithm of real product, the logarithm of real wages[1] and a time trend:

$$N_t = \beta_0 + \beta_1 Y_t + \beta_1 W_t + \beta_3 T + \varepsilon_t \qquad \text{(v.1)}$$

[1] Since we don't expect the real wages series published by EUROSTAT to have the same problems as the micro series, we include real wages in the employment equation.

Table v.1 reports the results of the estimation of Equation v.1.[2] We found that the coefficient of the logarithm of the real product is significant in the majority of the countries (with the exception of Denmark, Germany, Luxembourg and Poland) and has the expected positive sign (with the exception of Denmark and Luxembourg). The coefficient of the logarithm of the real wages is also significant in the great majority of the countries, with the exception of Canada and Slovenia, but has the predicted negative sign only for Austria, Canada, the Czech Republic, Japan, Slovenia, and the United Sates[3]. The time trend is in general significant and has a negative sign. Table v.2 shows the employment-output elasticity for the countries where we have data on real wages. The estimated employment-output elasticity in Portugal is equal to 0.115, which is low when compared with the other countries.

Following the empirical methodology applied in Section IV.2.1 we computed the Preisach hysteresis transformed variable based on the original series of the output. Figure v.1 plots the Preisach hysteresis index variable for 19 OECD countries.

[2] In spite of not having data on wages for Belgium, Spain and the UK, we also analyze the presence of hysteresis in these countries. This option is justified by the importance of the countries (especial the last two) in the European context. We do not include Italy in the study due to the lack of data.

[3] Note that, even considering that the *t*-values are not *t*-student distributed, in the great majority of the cases concerning both hysteresis transformation of real product and real wages, the *t*-statistics are three times greater than 1.96 (the 5% critical value in the case of the standard *t*-student distribution).

Table v.1.
Results of the Preisach Model
Dependent variable – Logarithm of Aggregate Employment (N_t)

	Austria 1996:01-2005:06 114 obs.			**Belgium** 1995:01-2005:07 126 obs.			**Canada** 1995:04-2005:07 127 obs.			**Czech Republic** 2001:01-2005:07 55 obs.			**Denmark** 1996:01-2005:06 108 obs.			**Finland** 1995:1-2005:07 114 obs.			**France** 1996:01-2005:06 111 obs.		
	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq.v.1	Eq.v.2	Eq.v.3	Eq.v.1	Eq.v.2	Eq.v.3	Eq.v.1	Eq.v.2	Eq.v.3
$cons$	**5.273**	**5.879**	**5.896**	**1.951**	**4.658**	**3.139**	**3.504**	**3.763**	**3.205**	**4.310**	**5.856**	**4.011**	**-0.224**	**-0.178**	**2.177**	0.961	0.993	1.129	**1.893**	**-3.724**	**3.878**
	18.252	24.326	24.09	5.651	1170	8.028	3.752	3.289	3.354	13.569	15.44	11.79	-0.353	-0.256	2.261	2.668	2.955	3.315	12.918	29.29	26.71
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.724)	(0.798)	(0.026)	(0.009)	(0.004)	(0.001)	(0.000)	(0.000)	(0.000)
Y_t	**0.067**		**-0.009**	**0.605**		**0.340**	**0.513**		**0.454**	**0.307**		**0.353**	**-0.107**		**-0.382**	**0.291**		**0.125**	**0.1152**		**-0.039**
	3.448	-	-0.520	7.371	-	3.882	12.061	-	7.448	7.932	-	8.086	-1.038	-	-3.378	5.050	-	1.849	3.826	-	-2.087
	(0.000)		(0.604)	(0.000)		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.167)		(0.001)	(0.000)		(0.067)	(0.000)		(0.039)
W_t	**-0.208**	-0.277	**-0.271**				**-0.269**	**0.187**	**-0.148**	**-0.224**	-0.281	**-0.206**	**1.188**	1.071	**0.927**	**0.540**	**0.818**	**0.665**	**0.477**	**0.195**	**0.199**
	-3.095	-5.185	-4.975	-	-	-	-1.402	0.633	-0.697	-3.563	-3.076	-3.349	8.726	6.919	0.608	4.691	10.64	5.926	12.160	7.132	7.389
	(0.003)	(0.000)	(0.000)				(0.163)	(0.528)	(0.486)	(0.001)	(0.003)	(0.002)	(0.000)	(0.000)	(0.00)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
HY_t		**0.072**	**0.074**		**0.125**	**0.091**		**0.138**	**0.028**		**0.000**	**-0.028**		**0.024**	**0.117**		**0.122**	**0.096**		**0.098**	**0.105**
	-	8.995	7.884	-	8.652	5.562	-	8.026	1.374	-	1.161	-2.069	-	0.952	3.210	-	6.455	4.123	-	17.33	16.21
		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.172)		(0.113)	(0.044)		(0.344)	(0.002)		(0.000)	(0.000)		(0.000)	(0.000)
T	**0.000**	**-0.001**	**-0.001**	**-0.002**	**-0.002**	**-0.003**	**0.000**	**-0.000**	**-0.000**	**-0.002**	**-0.000**	**-0.001**	**-0.001**	**-0.001**	**-0.002**	**-0.002**	**-0.003**	**-0.003**	**0.000**	**-0.002**	**-0.002**
	-5.267	-10.65	-9.840	-13.47	-15.03	-15.98	-1.876	-0.895	-2.324	-5.034	0.178	-4.931	-6.327	-5.726	-6.638	-11.15	-12.28	-12.07	-21.40	-35.13	-35.56
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.063)	(0.373)	(0.022)	(0.000)	(0.859)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	0.551	0.713	0.714	0.785	0.807	0.827	0.854	0.792	0.857	0.802	0.579	0.818	0.706	0.703	0.733	0.652	0.687	0.696	0.842	0.954	0.956
DW	0.187	0.368	0.363	0.190	0.1345	0.228	0.127	0.056	0.116	1.436	0.609	1.617	0.304	0.319	0.417	0.270	0.305	0.297	0.188	0.628	0.566
Engle Granger Cointegration Test Statistic	-2.95	-3.316	-3.25	-2.451	-2.036	-2.11	-1.79	-1.43	-1.84	-5.322	-2.54	-5.97	-3.340	-2.88	-5.96	-3.254	-3.52	-3.58	-2.525	-4.32	-4.11
MacKinnon **5%** Critical Values	-4.22	-4.225	-4.55	-3.856	-3.856	-4.216	-4.21	-4.214	-4.54	-3.342	-3.342	-4.70	-4.231	-4.231	-4.70	-4.217	-4.217	-4.56	-4.009	-4.009	-4.56
Trace Test Statistic	47.38	25.25	61.45	23.166	9.851	36.860	44.07	37.03	69.44	33.683	37.58	78.05	48.490	45.00	64.90	38.482	38.39	66.88	45.310	40.06	71.41
5% Critical Value	42.91	42.91	63.87	25.87	42.91	25.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87

We report for each variable the estimated coefficient, the *t*-statistic and the *p*-value respectively

	Germany 1995:01-2005:06 126 obs			Hungary 1998:01-2005:06 90 obs.			Japan 1995:01-2005:06 126 obs.			Luxembourg 1995:01-2005:06 126 obs.			Netherlands 1995:01-2004:12 120 obs.			Poland 1999:05-2005:06 77 obs.		
	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3
	1.478	**2.413**	**3.262**	**2.364**	**2.444**	**2.357**	**5.600**	**6.729**	**7.394**	**2.411**	**2.486**	**2.588**	**0.990**	**2.545**	**2.588**	**1.167**	**1.454**	**2.007**
$cons$	7.112	11.65	12.59	14.615	13.69	14.051	14.831	22.91	36.026	14.246	15.14	14.02	4.988	13.69	10.91	5.474	5.767	7.195
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	0.021		**-0.149**	**0.122**		**0.128**	**0.134**		**-0.401**	**-0.005**		**-0.033**	**0.428**		**-0.017**	**-0.062**		**-0.223**
Y_t	0.690	-	-4.834	5.453	-	3.556	3.499	-	-12.20	-0.195	-	-1.203	10.238	-	-0.294	-1.171	-	-3.627
	(0.500)		(0.000	(0.000)		(0.000)	(0.001)		(0.000)	(0.846)		(0.231	(0.000)		(0.769)	(0.245)		(0.001)
	0.676	**0.493**	**0.454**	**0.457**	**0.573**	**0.453**	**-0.326**	**-0.443**	**-0.192**	**0.484**	**0.463**	**0.473**	**0.374**	**0.454**	**0.462**	**0.903**	**0.781**	**0.875**
W_t	15.508	10.900	10.712	9.317	12.652	8.356	-3.337	-6.869	-3.999	12.056	**12.68**	12.655	5.980	**11.066**	9.381	13.95	**13.672**	14.84
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	**(0.000)**	(0.000)	(0.000)	**(0.000)**	(0.000	(0.000)	**(0.000)**	(0.000)
		0.064	**0.097**		**0.057**	**-0.005**		**0.041**	**0.082**		**0.026**	**0.034**		**0.080**	**0.083**		**0.185**	**0.128**
HY_t	-	6.857	8.846	-	3.804	-0.201	-	11.609	19.858	-	1.878	2.225	-	15.999	8.824	-	6.718	4.167
		(0.000)	(0.000)		(0.000)	(0.841		(0.000)	(0.000)		(0.063)	(2.225)		(0.000)	(0.000)		(0.000)	(0.000)
	-0.001	**-0.001**	**-0.001**	**-0.003**	**-0.003**	**-0.003**	**-0.002**	**-0.002**	**-0.002**	**0.000**	**-0.000**	**-0.000**	**-0.001**	**-0.001**	**-0.001**	**-0.001**	**-0.06**	**-0.001**
T	-13.22	-16.22	-17.66	-21.76	-16.32	-15.00	-24.23	-34.44	-51.84	1.897	-0.581	-0.294	-9.511	-15.62	-15.10	-2.41	-2.247	-3.906
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	0.0601	(0.563)	(0.769)	(0.000)	(0.000)	(0.000)	(0.000)	(0.028)	(0.000)
R^2	0.947	0.962	0.968	0.915	0.901	0.916	0.969	0.984	0.993	0.822	0.827	0.829	0.870	0.921	0.921	0.964	0.965	0.971
DW	0.935	0.733	0.836	1.402	1.430	1.399	0.111	0.369	0.554	1.164	1.169	1.157	0.584	0.480	0.469	0.637	0.509	0.721
Engle Granger Cointegration Test Statistic	-3.27	-3.01	-3.27	-6.72	-6.81	-6.64	-1.58	-3.33	-2.96	-2.01	-2.58	-2.55	-2.625	-3.76	-3.71	-3.878	-3.73	-4.72
MacKinnon 5% Critical Values	-4.21	-4.21	-4.55	-4.25	-4.25	-4.59	-4.21	-4.214	-4.55	-4.215	-4.215	-4.55	-4.220	-4.220	-4.55	-4.27	-4.28	-4.62
Trace Test Statistic	41.64	28.00	57.64	63.52	60.18	91.94	44.34	49.19	69.4	41.16	34.82	62.02	23.577	30.17	58.31	49.63	55.33	66.08
5% Critical Value	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87

We report for each variable the estimated coefficient, the *t*-statistic and the *p*-value respectively

	Portugal 1995:01-2005:06 126 obs.			**Slovak Republic** 1998:01-2005:06 90 obs.			**Slovenia** 1998:01-2005:06 90 obs.			**Spain** 1995:01-2004:12 120 obs.			**UK** 1995:01-2005:06 126 obs.			**USA** 1995:01-2005:06 126 obs.		
	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3	Eq. v.1	Eq. v.2	Eq.v.3
	4.258	**3.667**	**3.507**	**4.255**	**4.578**	**4.733**	**4.684**	**5.343**	**5.778**	**1.210**	**4.424**	**4.119**	**-1.290**	**4.702**	**-0.792**	**11.45**	**15.75**	**11.75**
$cons$	30.64	26.83	15.255	36.114	273	31.84	14.737	22.37	20.123	4.312	1290.5	10.516	-2.254	878.8	0.795	16.03	10.09	18.68
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	0.0259	(0.000)	(0.427)	(0.000)	(0.000)	(0.000)
	0.115		**0.039**	**0.0582**		**-0.034**	**0.143**		**-0.102**	**0.723**		**0.067**	**1.320**		**0.859**	**0.750**		**0.636**
Y_t	3.794		0.867	2.0704	-	-1.048	4.819	-	-2.557	11.422	-	0.7067	10.506	-	3.895	42.22	-	26.139
	(0.000)		(0.387	(0.041)		(0.298)	(0.000)		(0.012)	(0.000)		(0.444)	(0.000)		(0.000)	(0.000)		(0.000)
	0.263	**0.240**	**0.237**	**0.418**	**0.325**	**0.318**	**-0.163**	**-0.161**	**-0.154**							**-1.074**	**-1.297**	**-1.029**
W_t	9.164	7.961	7.775	17.882	11.277	10.728	-2.288	-2.818	-2.767	-	-	-	-	-	-	-7.054	**-3.801**	-7.694
	(0.000)	(0.000)	(0.000	(0.000)	(0.000)	(0.000)	(0.025)	(0.006)	(0.007)							(0.000)	**(0.000)**	(0.000)
		0.035	**0.027**		**0.001**	**0.087**		**0.076**	**0.106**		**0.227**	**0.208**		**0.105**	**0.046**		**0.243**	**0.057**
HY_t	-	4.394	2.267	-	5.053	4.630	-	9.112	7.527	-	17.267	8.921	-	9.763	2.513	-	15.960	6.133
		(0.000	(0.025		(0.000)	(0.001)		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.013)		(0.000)	(0.000)
	-0.002	**-0.002**	**-0.002**	**-0.001**	**-0.001**	**-0.001**	**0.000**	**-0.001**	**-0.001**	**0.001**	**-0.000**	**0.000**	**-0.003**	**-0.003**	**-0.003**	**-0.003**	**-0.003**	**-0.003**
T	-41.88	-29.18	-28.10	-3.624	6.357	-5.949	-2.609	-6.539	-7.213	6.533	-0.034	0.125	-37.99	-36.31	-38.46	-32.01	-13.94	-35.46
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.011)	(0.000)	(0.000)	(0.000)	(0.972)	(0.906)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	0.993	0.993	0.993	0.828	0.858	0.860	0.702	0.807	0.821	0.921	0.953	0.953	0.923	0.917	0.926	0.987	0.932	0.990
DW	0.786	0.30	0.634	1.299	1.175	1.109	0.345	0.491	0.536	0.247	0.16	0.174	0.257	0.105	0.217	0.263	0.058	0.319
Engle Granger Cointegration Test Statistic	-3.28	-2.714	-3.10	-2.84	-6.11	-5.96	-2.69	-3.41	-3.63	-1.83	-2.22	-2.31	-2.11	-2.60	-2.49	-3.13	-2.75	-3.89
MacKinnon 5% Critical Values	-4.21	-4.21	-4.55	-4.22	-4.22	-4.59	-4.25	-4.25	-4.59	-3.86	-3.86	-4.11	-3.85	-3.85	-4.21	-4.21	-4.21	-4.55
Trace Test Statistic	42.00	31.66	65.10	36. 43	35.23	62.62	30.72	32.87	49.36	10.67	12.70	39.95	20.53	19.07	58.29	40.41	51.81	73.48
5% Critical Value	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	25.87	42.91	25.87	25.87	42.91	25.87	42.91	42.91	63.87

We report for each variable the estimated coefficient, the *t*-statistic and the *p*-value respectively

Table v.2
Employment–Output Elasticities

Country	Elasticity	*t*-statistic
USA	0.75*	42.22
Canada	0.513*	12.06
Netherlands	0.428*	10.23
Czech Republic	0.307*	7.93
Finland	0.291*	5.05
Slovenia	0.143	4.81
Japan	0.134	3.49
Hungary	0.122	5.45
France	0.115	3.82
Portugal	**0.115**	**3.79**
Austria	0.067	3.44
Slovak Republic	0.058	2.07
Germany	0.021	0.69
Luxembourg	-0.005	-0.19
Poland	-0.062	-1.17
Denmark	-0.107	-1.10

*significant at 5% (*t*-statistics greater than three times the standard critical value to the rejection of non-significance)

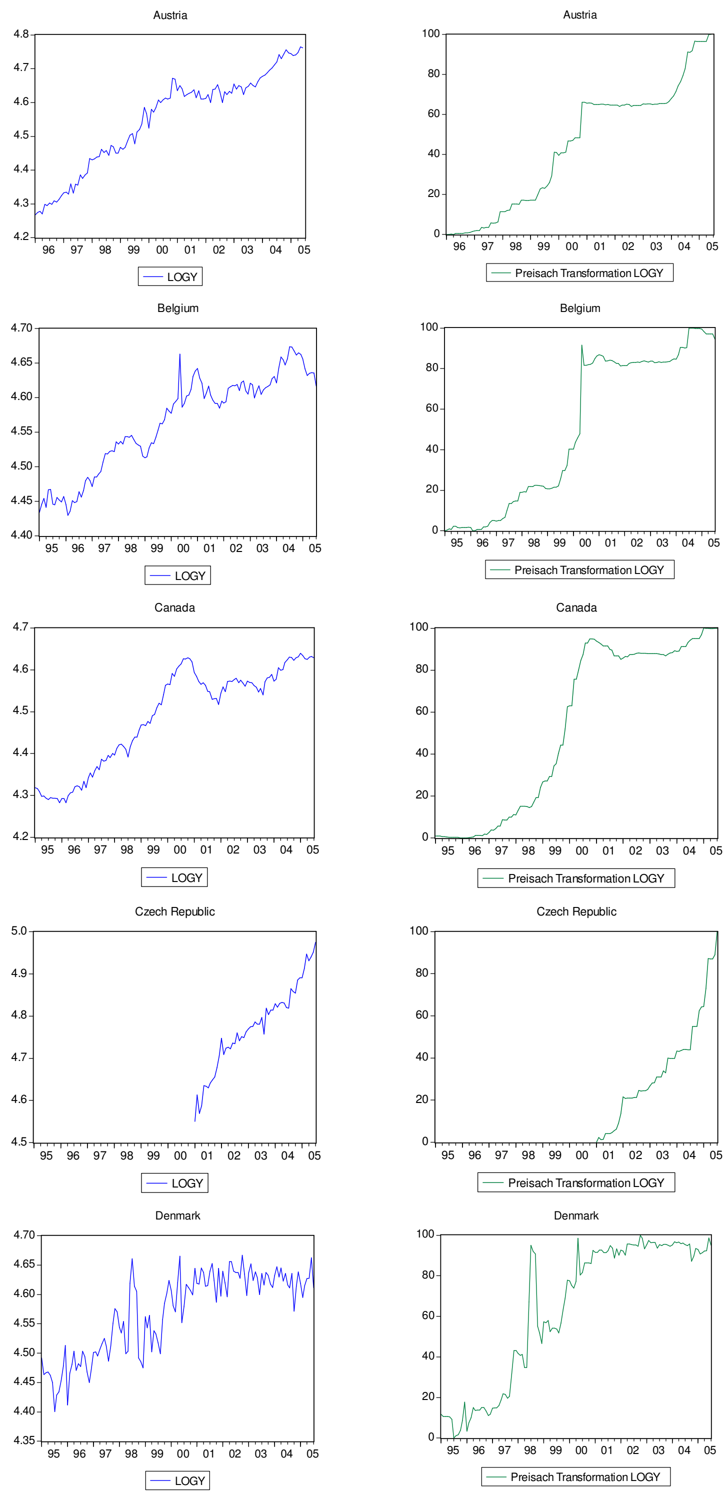


Figure v.1 Hysteresis Transformed Real Product Series (Preisach Model of Hysteresis)

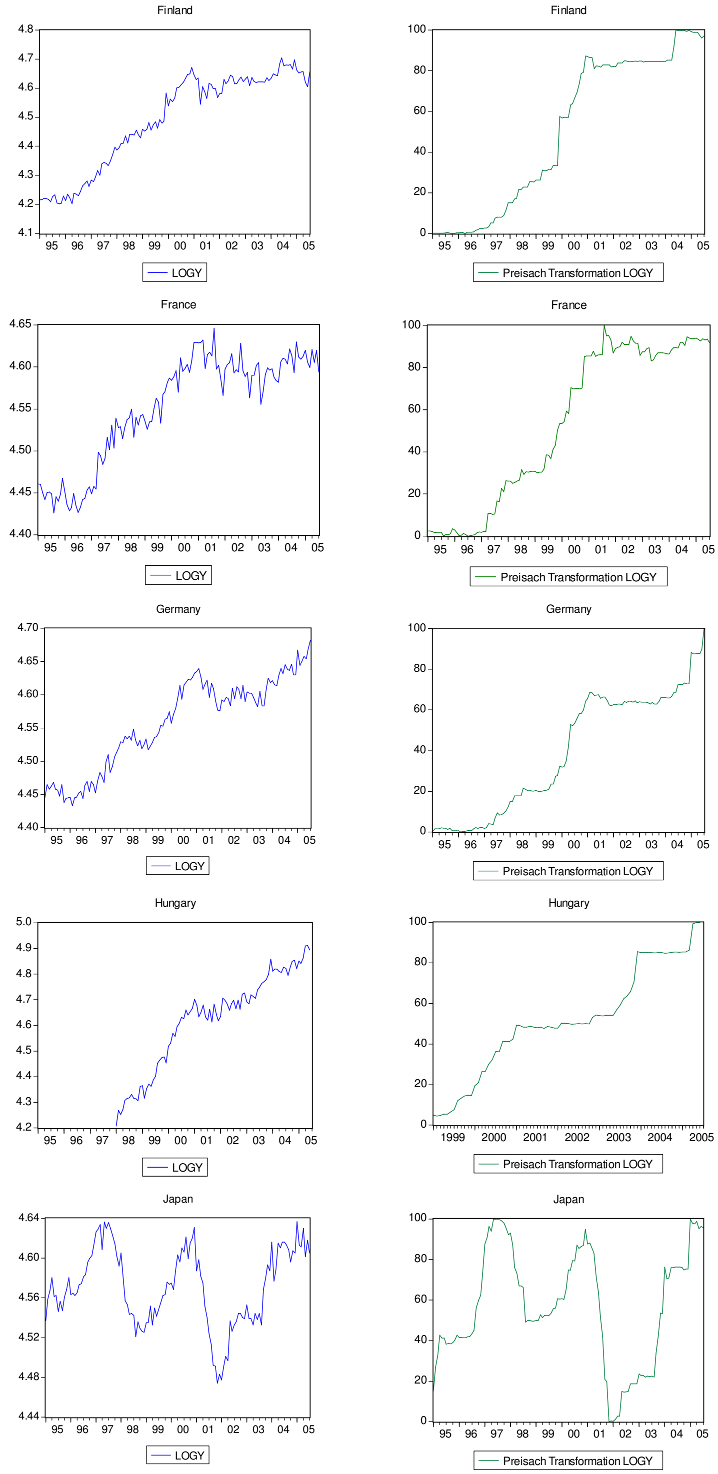


Figure v.1 Hysteresis Transformed Real Product Series (Preisach Model of Hysteresis)

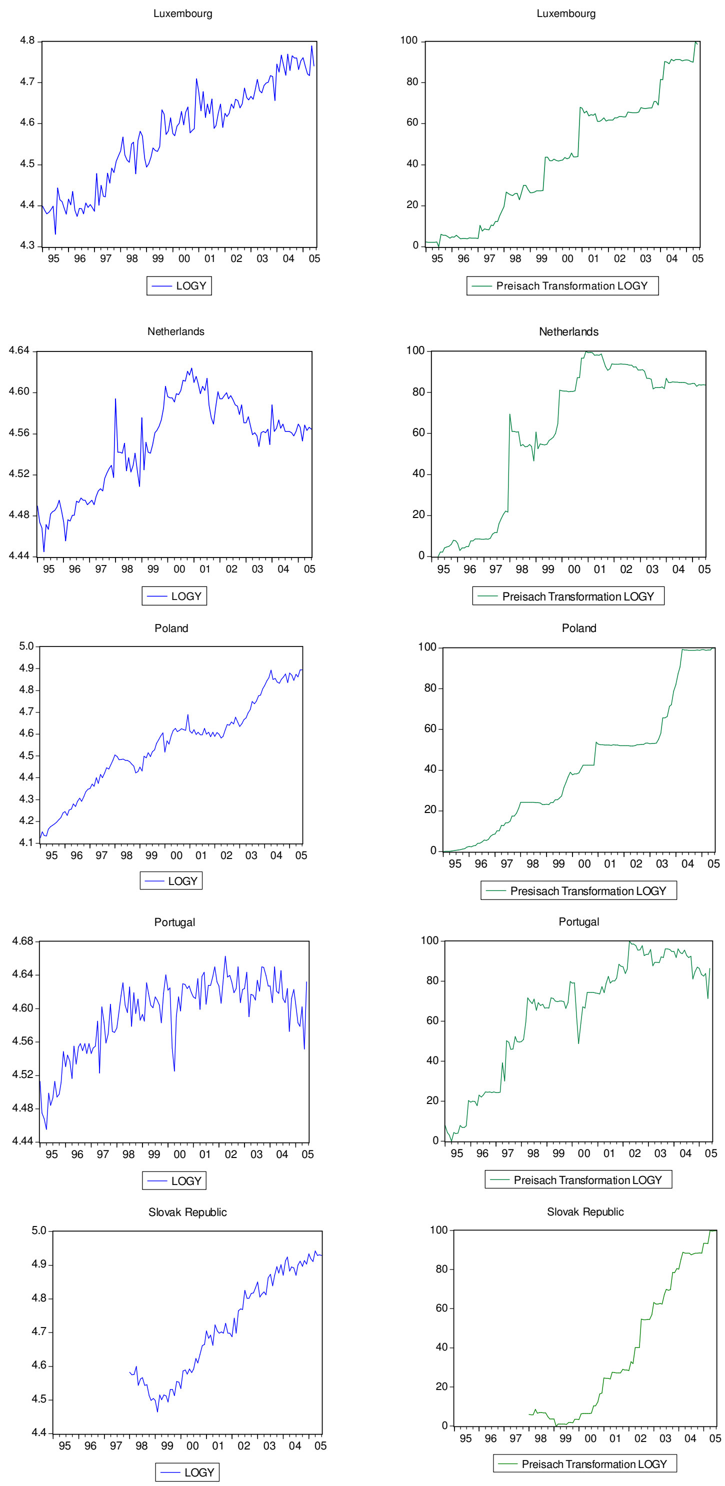


Figure v.1 Hysteresis Transformed Real Product Series (Preisach Model of Hysteresis)

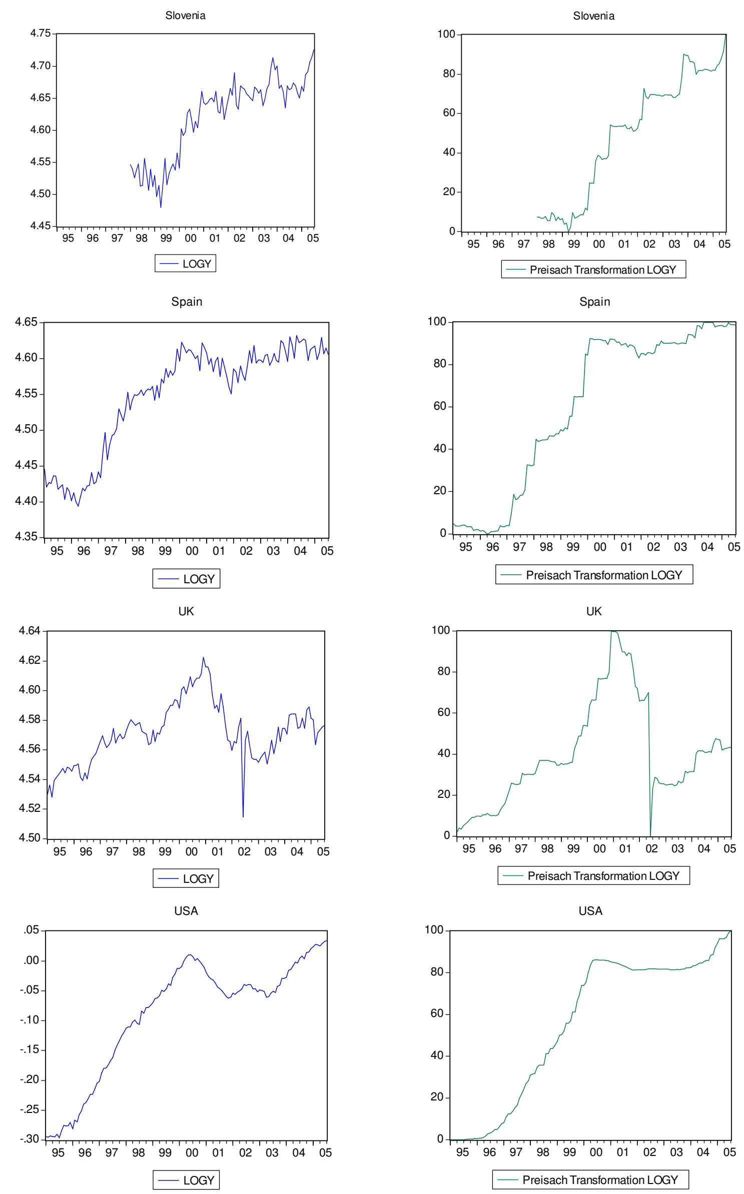


Figure v.1 Hysteresis Transformation of Real Product (Preisach Model of Hysteresis)

To test the existence of hysteresis, we replace the original series of aggregate output (Y_t) in equation v.1 with its hysteresis transformation (HY_t) and we estimate equation v.2:

$$N_t = \beta_0 + \beta_1 HY_t + \beta_2 W_t + \beta_3 T + \varepsilon_t \tag{v.2}$$

Concerning the significance of the coefficient associated with the transformed variable, we conclude, on the basis of the high value of the *t*-statistics, that it is in general significant. Exceptions are the Czech Republic, Denmark and Luxembourg.

Concerning the goodness of fit of the employment equations, and after carrying out cointegration tests, we conclude that in the majority of the countries the non-linear relation between employment and the real product captured by the inclusion of the transformed variable instead of the original one (Equation v.2), performs better than the model with the original variable (see Table v.3). The exceptions are Canada, the Czech Republic, Denmark, Hungary, the UK and the USA where the *R*-Square of the hysteretic equation is lower than the R-Square of the equation with the original sales variable.

Table v.3
Test on the Increase in Goodness of Fit (Preisach Model)

Country	R^2			$R^2_{(Eq.\ v.2)} - R^2_{(Eq.\ v.1)}$	$R^2_{(Eq.\ v.3)} - R^2_{(Eq.\ v.1)}$	*F-Stat**
	Equation v.1	Equation v.2	Equation v.3			
Austria	0.551	0.713	0.714	0.162	0.163	62.15
Belgium	0.785	0.807	0.827	0.022	0.042	30.93
Canada	0.854	0.792	0.857	-0.062	0.003	1.880
Czech Republic	0.802	0.579	0.818	-0.223	0.016	4.280
Denmark	0.706	0.703	0.733	-0.003	0.027	10.30
Finland	0.652	0.687	0.696	0.035	0.044	16.99
France	0.842	0.954	0.956	0.112	0.114	262.9
Germany	0.947	0.962	0.968	0.015	0.021	78.26
Hungary	0.915	0.901	0.916	-0.014	0.001	0.040
Japan	0.969	0.984	0.993	0.015	0.024	394.0
Luxembourg	0.822	0.827	0.829	0.005	0.007	4.948
Netherlands	0.870	0.921	0.921	0.051	0.051	77.86
Poland	0.964	0.965	0.971	0.001	0.007	17.36
Portugal	**0.993**	**0.993**	**0.993**	**0.000**	**0.000**	**5.130**
Slovak Republic	0.828	0.858	0.860	0.03	0.032	21.44
Slovenia	0.702	0.807	0.821	0.105	0.119	26.66
Spain	0.921	0.953	0.953	0.032	0.032	79.59
UK	0.923	0.917	0.926	-0.006	0.003	6.310
USA	0.987	0.932	0.990	-0.055	0.003	37.61

$$^{*}\,F-Stat=\frac{\dfrac{R^2_{Equation\_v.3}-R^2_{Equation\_v.1}}{1}}{\dfrac{1-R^2_{Equation\_v.3}}{n-5}}$$

We also estimate an equation that includes both the original variable and the transformed variable:

$$N_t = \beta_0 + \beta_1 HY_t + \beta_2 Y_t + \beta_3 W_t + \beta_4 T + \varepsilon_t \qquad \text{(v.3)}$$

When we estimate a regression that includes the original variables and the hysteresis transformation of the real product according to the Preisach Model, the coefficient associated with the original series becomes not significant or changes sign from positive to negative in fourteen of the nineteen considered countries. We conclude, in general, that the influence of the transformed variable seems to substitute the effects of the original series of the real product. Exceptions are again Canada, the Czech Republic, Hungary, the UK and the USA where the explicative power of the original sales series is greater than the explicative power of the transformed variable.

We also performed a test on the increase in the goodness of fit of the original regression when we include in the equation both the original and the hysteresis transformed series of the aggregate output (Equation v.3). We found that in general the *R*-square increases significantly in the majority of the countries with the exceptions of Canada, Czech Republic and Hungary (see Table v.3).

Table iv.1 shows the results of the Engle-Granger cointegration test[4]. When we estimate employment against real product, real wage and a time trend, we fail in general to reject the hypothesis of no-cointegration for the majority of countries (exceptions are the Czech Republic and Hungary). When we apply the Engle-Granger Cointegration Test to the models that include the transformed variables, either isolated or with the original variables, we identify the existence of a cointegrated vector in four more countries (Denmark, France, Poland and the Slovak Republic). Moreover, we observed in eight of the remaining thirteen countries an increase in the absolute value of the test statistic, meaning that the minimum significance level for accepting cointegration decreases.

These results are reinforced when we apply the Johansen cointegration test methodology. Actually, the Trace Test indicates that when we estimate the model with

[4] Firstly, we studied the stationary of the series carrying the Augmented Dickey Fuller Unit Root Test (see Table A.12 in the Appendix).

only the original variables, we find the existence of at least one cointegrated vector in seven countries, but adding to the model the Preisach transformations of the input variables (Equation v.3), we conclude for the rejection of the null of no-cointegration in 13 of the nineteen considered countries.

The tests based on the Preisach Model of hysteresis indicate that we do not find hysteresis in the dynamics of the aggregate employment in Canada, the Czech Republic, Denmark, Hungary, the UK and the USA. On the contrary, hysteresis is present in the path of employment in France, Finland, Spain, the Slovak Republic, Belgium, Japan and Poland. In the cases of Austria, Slovenia, the Netherlands, Germany and Luxembourg, in spite of the increase in of the goodness of fit of the employment equation when we replace the series of sales with its hysteretic transformation, we do not reject the inexistence of cointegration between the variables. In the case of Portugal based on the Preisach Model and with this data set, we do not find clear signs of hysteresis. The *t*-statistics of the coefficient associated with the transformed variable is higher than the *t*-statistics of the coefficient associated with the original sales variable (Equation v.3) meaning that the transformed variable has more explanatory power. However, the *R*-square does not increase significantly when we replace the series of sales with the hysteresis transformed variable.

IV.3 RESULTS OF THE LINEAR PLAY MODEL

Table v.4 (column 2) shows the estimated play using the real product in the manufacturing sector calculated as explained in section IV.2.2.2 (see also Figure A.4 in the appendix), and Figure v.2 plots the hysteresis transformed variable (the spurt variable).

Table v.4
Estimated Play and Employment Elasticities

Country	**Play Width**	**Reaction along the play** (β_1)	**Reaction along the spurt** ($\beta_1 + \beta_2$)	***t*-Statistic for testing** $H_0: \lvert\beta_1\rvert = \lvert\beta_1 + \beta_2\rvert$ against $H_1: \lvert\beta_1\rvert < \lvert\beta_1 + \beta_2\rvert$
Austria	0.102	-0.026 (-1.265)	0.238* (7.723)	8.57
Belgium	0.062	0.127 (1.492)	0.746* (8.770)	7.28
Canada	0.020	0.232 (0.925)	0.302 (1.144)	0.26
Czech Republic	0.032	0.159 (1.532)	0.166 (1.496)	0.11
Denmark	0.186	-0.193 (-2.986)	0.712* (7.368)	9.36
Finland	0.096	0.119 (1.325)	0.346 (2.995)	1.96
France	0.074	-0.021 (-1.297)	0.702* (17.84)	19.81
Germany	0.074	-0.072 (-3.128)	0.306* (9.782)	12.12
Hungary	0.054	0.048 (-1.115)	0.201 (4.404)	2.85
Japan	0.068	0.044 (1.235)	0.266* (6.625)	5.75
Luxembourg	0.122	-0.073 (-3.161)	0.232* (7.327)	9.65
Netherlands	0.000	0.437* (10.23)	-	-
Poland	0.032	-0.342 (-2.350)	0.310 (2.059)	4.33
Portugal	**0.044**	**-0.030 (-0.719)**	**0.231 (4.832)**	**5.46**
Slovak Republic	0.168	-0.028 (-0.945)	0.330 (4.916)	13.98
Slovenia	0.070	-0.044 (-1.378)	0.357* (8.069)	9.11
Spain	0.072	0.084 (1.066)	1.108* (10.167)	9.39
UK	0.024	-0.178 (-0.126)	1.106* (7.43)	9.04
USA	0.024	0.024 (0.034)	0.756* (10.368)	10.17

t-statistics in brackets
*significant at 5% (*t*-statistics greater than three times the standard critical value to the rejection of non-significance)

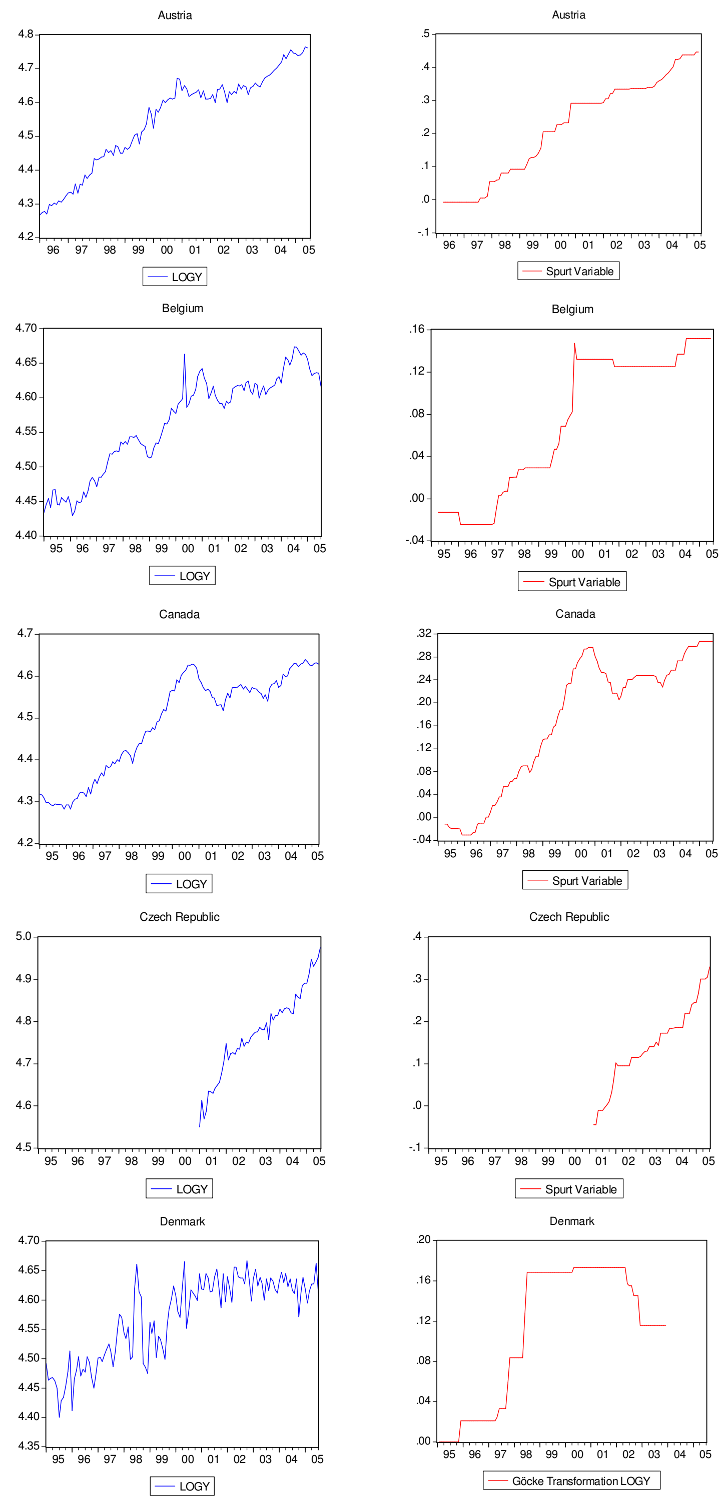


Figure v.2 Hysteresis Transformed Real Product Series (Linear Play Model of Hysteresis)

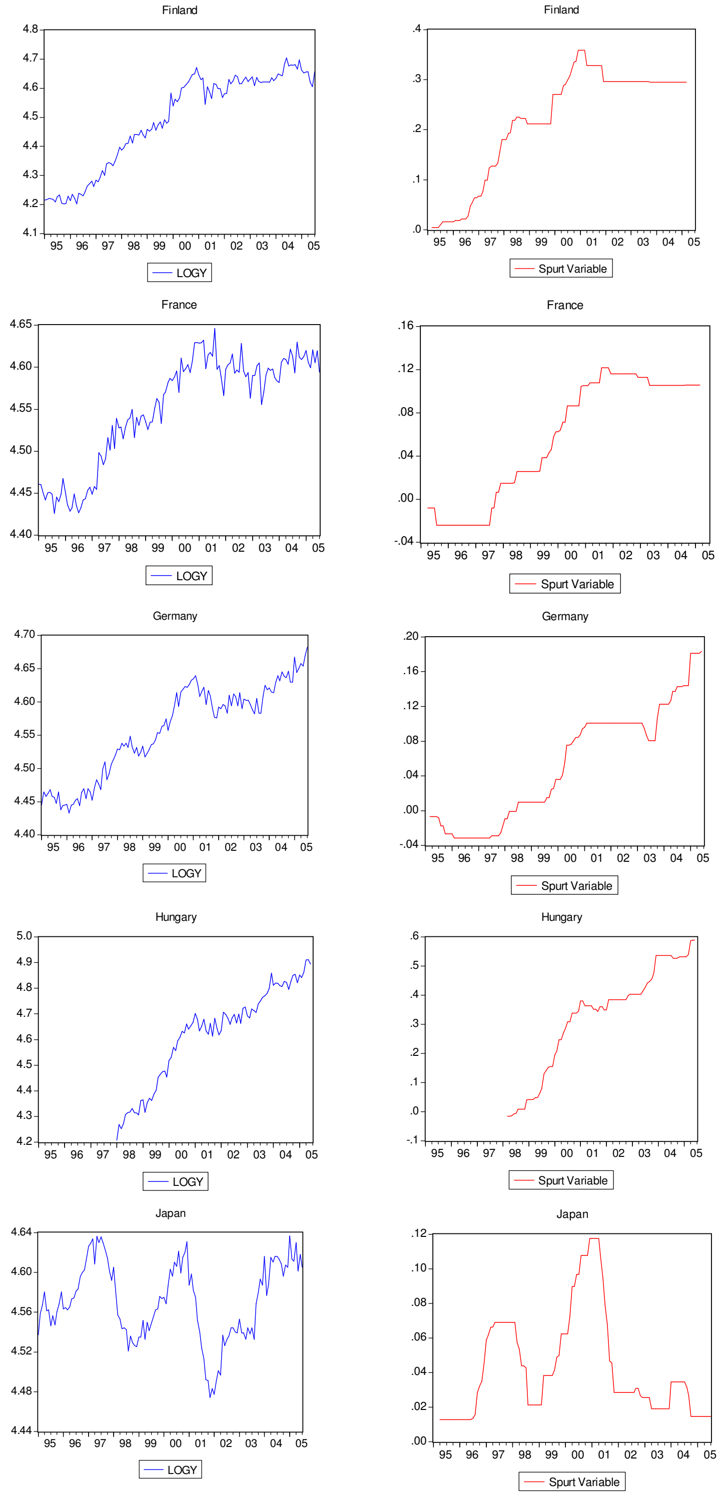


Figure v.2 Hysteresis Transformed Real Product Series (Linear Play Model of Hysteresis)

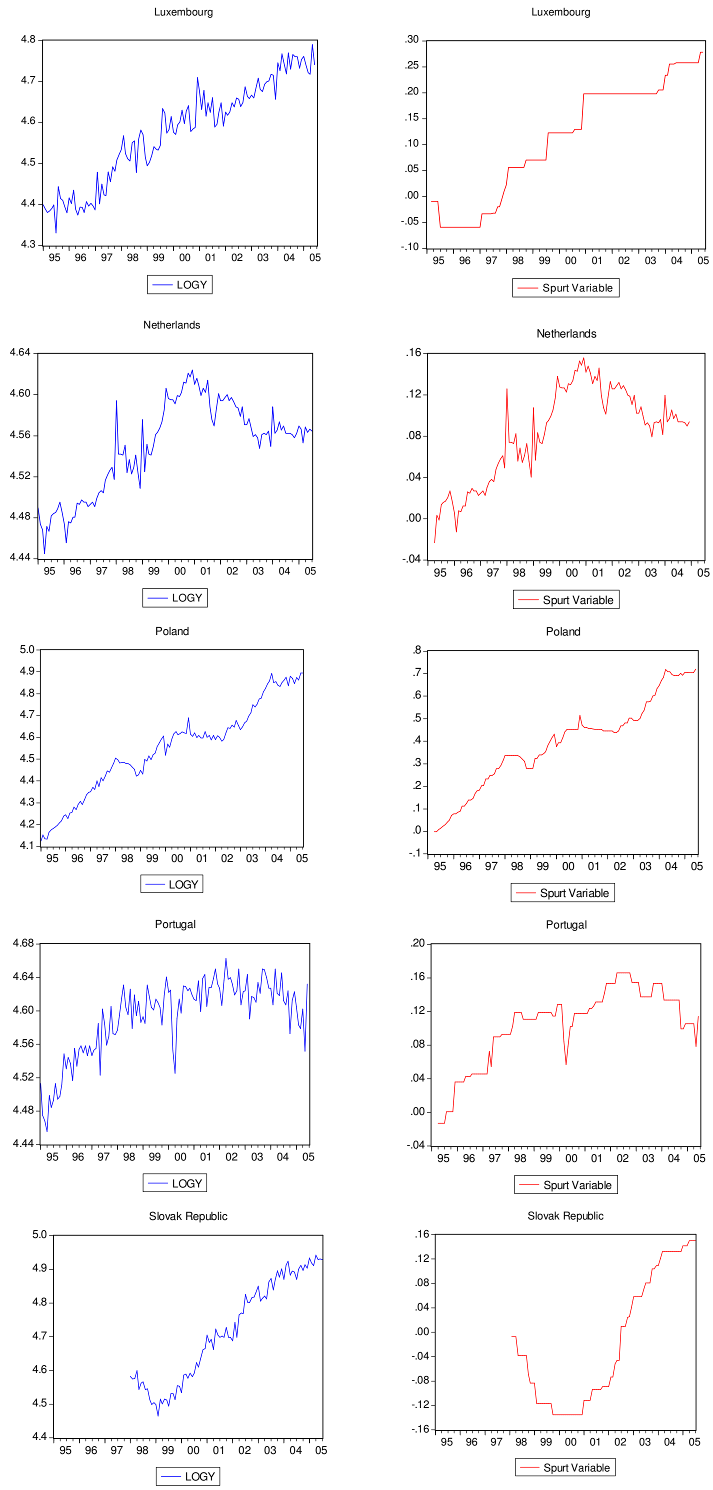


Figure v.2 Hysteresis Transformed Real Product Series (Linear Play Model of Hysteresis)

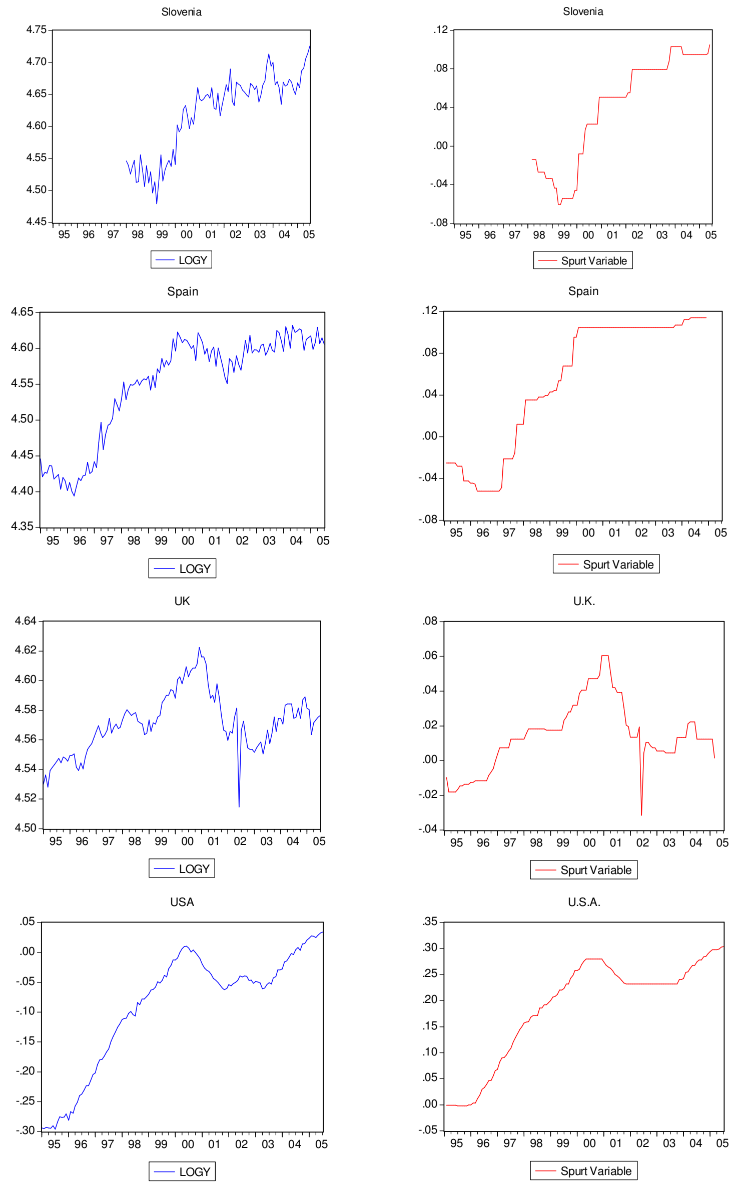


Figure v.2 Hysteresis Transformation of Real Product (Linear Play Model of Hysteresis)

To test the existence of hysteresis we start by estimating an equation that includes both the spurt and the play (see Table v.5, p. 152):

$$N_t = \beta_0 + \beta_1 S_t + (\beta_1 + \beta_2) SPURT_t + \beta_3 W + \beta_4 T + \varepsilon_t \tag{v.4}$$

To test the existence of two zones of different employment reaction that characterizes hysteresis according to the Linear Play Model, we tested the hypothesis H_0: $|\beta_1| = |\beta_1 + \beta_2|$ against H_1: $|\beta_1| < |\beta_1 + \beta_2|$. Table v.4 (column 4) reports the employment elasticities along the spurt segment. The coefficient associated with the spurt variable is significant in the great majority of the countries. Exceptions are Canada, the Czech Republic, Hungary, and the Netherlands[5]. Table v.4 (column 5) also shows the *t*-statistics for testing the hypothesis $H_{0.}$ The null hypothesis is in general rejected with the exceptions of Canada, the Czech Republic, Finland, Hungary, and the Netherlands. We conclude that in the other fourteen countries, the reaction along the play is weaker (and in most cases non-significant) than the reaction along the spurt, and this is a sign of the existence of hysteresis.

The estimated play values reflect large periods of inaction meaning that in many cases a large play width is necessary to achieve the maximum goodness of fit of the employment equations. The larger the width of the play is, the larger the increase in the *R*-square of the employment regressions. Table v.5 shows the *F-Statistic* for testing the increase of the goodness of fit of the model that includes both the spurt and the play (Equation v.4). For most countries the estimated statistic exceeds by far the 1% critical level (4.79) for the rejection of the hypothesis that the inclusion of the transformed variable (the spurt) does not significantly increases the *R*-square of the regression. Exceptions are Canada, the Czech Republic, the Netherlands and Poland.

We also repeat the cointegration tests applied in the previous section now using the hysteresis transformation of input variable, in accordance with the Linear Play Model. Table v.5 shows the results of the Engle-Granger test. The probability of the existence of cointegration in the employment regression increases when we estimate the regression with the hysteresis transformation of the independent variables according to the Linear Play Model. When we study the complete model (Equation v.4), we reject the null of no cointegration in five of the nineteen analyzed countries[6], and even when we do not reject the null of no cointegration, there is an increase in the absolute value of the observed *t*-

[5] Due to the low value of the *t*-statistic, the empirical evidence is not clear in the case of Finland, Portugal, Poland and the Slovak Republic.
[6] We reject the null hypothesis of no cointegration for Denmark, Germany, Hungary, Poland and for the Slovak Republic

statistic of the Engle-Granger test in eight more countries, meaning that variables are closer to being cointegrated for a 10% significance level. These results are confirmed when we use the Johansen cointegration test procedure. Using the Trace Test, we conclude that for a 5% significance level, we only reject the null of no-cointegration in six of the nineteen countries, in regression with only the original independent variables (Equation v.1). When we include the hysteresis transformation of the variables (Equation v.4), we reject the null of no-cointegration in eleven countries[7].

Due to the fact that, with the exception of the Netherlands, all the coefficients associated with the play variable are not significant, we also estimated Equation v.4 assuming that the $\beta_1 = 0$ (see Table v.5):

$$N_t = \beta_0 + \beta_2 SPURT_t + \beta_3 W + \beta_4 T + \mu_t \qquad \text{(v.5)}$$

We found that the coefficient associated with the spurt variable is significant in all the countries with the exception of Poland, and that the estimation of the employment equation with the inclusion of the spurt variable ($SPURT_t$) instead of the original variable (Y_t) originates an increase in the goodness of fit in all the countries except in Poland. We also found a positive correlation of 0.6 (and a 5% significant rank correlation of 0.53) between the increase of the goodness of fit of the employment equation when the output is transformed according the Preisach Model and when it is transformed according to the Linear Play Model.

Overall, considering the results of the cointegration tests and the increase in the goodness of fit of the regressions, the non-linear model that includes the hysteresis transformation of the variables displays a better performance than the standard linear model. Exceptions are Canada, the Czech Republic, the Netherlands and Poland, and for that reason, we do not find traces of hysteresis in these countries. The Linear Play Model identifies clear signs of hysteresis in Austria, France, Denmark, Germany, Hungary, Japan, Luxembourg, Portugal, the Slovak Republic and even in the USA. In the cases of

[7] We reject the null hypothesis of no cointegration for Austria, Canada, Denmark, France, Germany, Hungary, Japan, Poland, Portugal, the Slovak Republic and the USA.

Finland, Belgium, Slovenia and Spain, although there is an increase in of the goodness of fit of the employment equation when we replace the series of sales with its hysteretic transformation, we do not reject the inexistence of cointegration between the variables.

Table v.5
Results of the Linear Play Model
Dependent variable: Logarithm of Aggregate Employment (N_t)

	Austria 1996:01-2005:06 114 obs.			**Belgium** 1995:01-2005:07 126 obs.			**Canada** 1995:04-2005:07 127 obs.			**Czech Republic** 2001:01-2005:07 55 obs.			**Denmark** 1996:01-2005:06 108 obs.			**Finland** 1995:1-2005:07 114 obs.		
	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5
	5.273	**6.322**	**6.194**	**1.951**	**4.108**	**4.676**	**3.504**	**4.669**	**5.308**	**4.310**	**4.879**	**5.564**	**-0.224**	**3.503**	**2.739**	0.961	3.506	4.155
cons	18.252	23.70	25.02	5.651	10.78	1138	3.752	4.263	6.249	13.569	9.506	21.794	-0.353	4.858	3.915	2.668	4.214	6.162
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.724)	(0.000)	(0.000)	(0.009)	(0.000)	(0.000)
	0.067	**-0.026**		**0.605**	**0.127**		**0.513**	**0.232**		**0.307**	**0.159**		**-0.107**	**-0.193**		**0.291**	**0.119**	
Y_t	3.448	-1.265		7.371	1.492	-	12.061	0.925	**-**	7.932	1.532	-	-1.038	-2.986		5.050	1.325	-
	(0.000)	(0.209)		(0.000)	(0.138)		(0.000)	(0.356)		(0.000)	(0.132)		(0.167)	(0.003)		(0.000)	(0.187)	
	-0.208	**-0.349**	**-0.346**				**-0.269**	**-0.257**	**-0.177**	**-0.224**	**-0.194**	**-0.181**	**1.188**	**0.457**	0.434	**0.540**	**0.120**	**0.085**
W_t	-3.095	-6.405	-6.405	-	-	-	-1.402	-1.263	-0.962	-3.563	-3.130	-2.912	8.726	0.3.07	2.816	4.691	0.767	0.549
	(0.003)	(0.000)	(0.000)				(0.163)	(0.209)	(0.338)	(0.001)	(0.003)	(0.005)	(0.000)	(0.003)	(0.006)	(0.000)	(0.445)	(0.584)
		0.238	**0.212**		**0.746**	**0.827**		**0.302**	**0.543**		**0.165**	**0.323**		**0.712**	**0.657**		**0.346**	**0.465**
$SPURT_t$	-	7.723	9.376	-	8.770	12.64	-	1.144	12.94	-	1.496	7.692	-	7.368	6.67	-	2.995	6.380
		(0.000)	(0.000)		(0.000)	(0.000)		(0.255)	(0.000)		(0.141)	(0.000)		(0.000)	(0.000)		(0.001)	(0.00)
	0.000	**-0.001**	**-0.001**	**-0.002**	**-0.003**	**-0.002**	**0.000**	**-0.000**	**-0.000**	**-0.002**	**-0.002**	**-0.002**	**-0.001**	**-0.001**	**-0.002**	**-0.002**	**-0.001**	**-0.001**
T	-5.267	-10.81	-10.70	-13.47	-19.02	-20.36	-1.876	-2.699	-2.949	-5.034	-5.699	-5.663	-6.327	--8.248	-18.66	-11.15	-5.186	-6.064
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.063)	(0.008)	(0.004)	(0.000)	(0.00)0	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	0.551	0.710	0.701	0.785	0.864	0.86	0.854	0.861	0.860	0.802	0.832	0.824	0.706	0.809	0.792	0.652	0.666	0.661
DW	0.187	0.562	0.542	0.190	0.239	0.211	0.127	0.116	0.101	1.436	1.166	0.984	0.304	0.516	0.481	0.270	0.181	0.150
Engle Granger Cointegration Test Statistic	-2.95	-4.17	-4.11	-2.451	-2.855	-2.71	-1.79	-1.50	-1.467	-5.322	-4.496	-4.14	-3.340	-7.61	-4.74	-3.254	-3.32	-3.67
MacKinnon **5%** Critical Values	-4.22	-4.56	-4.22	-3.856	-4.22	-3.85	-4.21	-4.54	-4.214	-3.342	-4.70	-3.342	-4.231	-4.57	-4.231	-4.217	-4.56	-4.217
Trace Test Statistic	47.38	74.57	40.84	23.166	40.71	12.54	44.07	66.48	39.81	33.683	60.16	38.46	48.490	86.628	72.07	38.482	54.16	35.54
5% Critical Value	42.91	63.87	42.91	25.87	42.91	25.87	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91

We report for each variable the estimated coefficient, the *t*-statistic and the *p*-value respectively

	France 1996:01-2005:06 111 obs.			Germany 1995:01-2005:06 126 obs			Hungary 1998:01-2005:06 90 obs.			Japan 1995:01-2005:06 126 obs.			Luxembourg 1995:01-2005:06 126 obs.			Poland 1999:05-2005:06 77 obs.		
	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5
$cons$	**1.893**	**4.391**	**4.294**	**1.478**	**2.572**	**2.279**	**2.364**	**3.306**	**3.099**	**5.600**	**5.471**	**5.390**	**2.411**	**3.343**	**3.053**	**1.167**	**2.317**	**1.061**
	12.918	27.82	30.80	7.112	14.351	14.40	14.615	12.806	17.23	14.831	16.881	16.94	14.246	17.486	17.55	5.474	3.888	3.907
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Y_t	**0.1152**	**-0.021**	-	**0.021**	**-0.072**	-	**0.122**	**-0.048**	-	**0.134**	**0.044**		**-0.005**	**-0.073**		**-0.062**	**-0.342**	
	3.826	-1.297		0.690	-3.128		5.453	-1.115		3.499	1.235		-0.195	-3.161		-1.171	-2.350	
	(0.000)	(0.197		(0.500)	(0.002)		(0.000)	(0.268)		(0.001)	(0.219)		(0.846)	(0.002)		(0.245)	(0.022)	
W_t	**0.477**	**0.076**	**0.076**	**0.676**	**0.531**	**0.525**	**0.457**	**0.407**	**0.406**	**-0.326**	**-0.209**	**0.274**	**0.484**	**0.347**	**0.339**	**0.903**	**0.907**	**0.868**
	12.160	2.547	2.530	15.508	15.919	15..21	9.317	8.969	8.937	-3.337	-2.441	7.633	12.056	9.332	8.839	13.95	14.310	13.77
	(0.000)	(0.012	(0.000	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$SPURT_t$	-	**0.702**	**0.679**	-	**0.306**	**0.268**	-	**0.201**	**0.156**	-	**0.256**	**0.256**	-	**0.232**	**0.193**	-	**0.310**	**-0.021**
		17.84	19.32		9.782	8.974		4.404	7.424		6.625	6.625		7.327	6.383		2.059	-0.377
		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.707)
T	**0.000**	**-0.002**	**-0.002**	**-0.001**	**-0.001**	**-0.001**	**-0.003**	**-0.003**	**-0.003**	**-0.002**	**-0.002**	**-0.002**	**0.000**	**-0.000**	**-0.000**	**-0.001**	**-0.001**	**-0.001**
	-21.40	-46.60	-40.34	-13.22	-19.98	-21.45	-21.76	-22.29	-24.81	-24.23	-29.23	-29.23	1.897	-2.81	-4.349	-2.41	-2.824	-3.090
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	0.0601	(0.473)	(0.000)	(0.000)	(0.006)	(0.003)
R^2	0.842	0.962	0.961	0.947	0.974	0.972	0.915	0.934	0.932	0.969	0.977	0.978	0.822	0.874	0.864	0.964	0.966	0.963
DW	0.188	0.500	0.515	0.935	1.255	1.1145	1.402	1.269	1.275	0.111	0.084	0.084	1.164	0.903	0.945	0.637	0.711	0.592
Engle Granger Cointegration Test Statistic	-2.525	-3.854	-3.902	-3.27	-4.74	-4.302	-6.72	-6.533	-6.534	-1.58	-0.47	0.11	-2.01	-4.213	-6.36	-3.878	-4.96	3.86
MacKinnon 5% Critical Values	-4.009	-4.56	-4.009	-4.21	-4.55	-4.215	-4.25	-4.59	-4.25	-4.21	-4.55	-4.21	-4.215	-4.55	-4.215	-4.27	-4.62	-4.27
Trace Test Statistic	45.310	73.2	42.74	41.64	71.33	41.51	63.52	85.3	62.36	44.34	100.70	45.59	41.16	63.26	38.16	49.63	80.20	54.39
5% Critical Value	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91

We report for each variable the estimated coefficient, the *t*-statistic and the *p*-value respectively

	Portugal 1995:01-2005:06 126 obs.			Slovak Republic 1998:01-2005:06 90 obs.			Slovenia 1998:01-2005:06 90 obs.			Spain 1995:01-2004:12 120 obs.			UK 1995:01-2005:06 126 obs.			USA 1995:01-2005:06 126 obs.		
	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5	Eq. v.1	Eq. v.4	Eq.v.5
	4.258	**4.908**	**4.744**	**4.255**	**4.692**	**4.568**	**4.684**	**5.379**	**5.215**	**1.210**	**4.098**	**4.472**	**-1.290**	**5.550**	**4.739**	**11.45**	**13.642**	**13.724**
$cons$	30.64	25.91	1979.8	36.114	35.51	320.07	14.737	21.067	22.97	4.312	11.683	18.593	-2.254	7.703	1082	16.03	24.30	27.09
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	0.0259	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	0.115	**-0.030**		**0.0582**	**-0.028**		**0.143**	**-0.044**		**0.723**	**0.084**		**1.320**	**-0.178**		**0.750**	**0.024**	
Y_t	3.794	-0.719	-	2.0704	-0.945	-	4.819	-1.378	-	11.422	1.066	-	10.506	-1.126	-	42.22	0.0345	-
	(0.000)	(0.473)		(0.041)	(0.347)		(0.000)	(0.172		(0.000)	(0.288)		(0.000)	(0.263		(0.000)	(0.731)	
	0.263	**0.202**	0.200	**0.418**	**0.301**	**0.307**	**-0.163**	**-0.120**	**-0.129**							**-1.074**	**-0.864**	**-0.859**
W_t	9.164	6.800	6.777	17.882	10.062	10.415	-2.288	-2.203	-2.371	-	-	-	-	-	-	-7.054	-7.710	-7.762
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.025)	(0.000)	(0.020)							(0.000)	(0.000)	(0.000)
		0.232	**0.206**		**0.330**	**0.119**		**0.357**	**0.314**		**1.108**	**1.202**		**1.106**	**0.969**		**0.756**	**0.780**
$SPURT_t$	-	4.832	6.322	-	4.916	5.142	-	8.069	10.05	-	10.167	18.593	-	7.43	12.847	-	10.368	58.83
		(0.000	(0.000		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.000)		(0.000)	(0.000		(0.000)	(0.000
	-0.002	**-0.002**	**-0.002**	**-0.001**	**-0.000**	**-0.001**	**0.000**	**-0.001**	**-0.001**	**0.001**	**0.000**	**0.000**	**-0.003**	**-0.002**	**-0.003**	**-0.003**	**-0.003**	**-0.003**
T	-41.88	-39.62	-39.80	-3.624	-2.623	-7.267	-2.609	-6.097	-5.989	6.533	2.177	2.1098	-37.99	-33.21	-47.10	-32.01	-45.37	-46.04
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.011)	(0.000)	(0.000)	(0.000)	(0.032)	(0.029)	(0.000)	(0.000	(0.010)	(0.000)	(0.000	(0.003)
R^2	0.993	0.994	0.994	0.828	0.857	0.856	0.702	0.823	0.819	0.921	0.958	0.957	0.923	0.973	0.973	0.987	0.993	0.993
DW	0.786	0.540	0.565	1.299	1.072	1.123	0.345	0.457	0.450	0.247	0.173	0.163	0.257	0.144	0.173	0.263	0.280	0.278
Engle Granger Cointegration Test Statistic	-3.28	-3.142	-3.15	-2.84	-5.642	-5.766	-2.69	-3.32	-3.22	-1.83	-2.47	-2.472	-2.11	-3.204	-3.21	-3.13	-4.13	-4.17
MacKinnon 5% Critical Values	-4.21	-4.55	-4.21	-4.22	-4.59	-4.215	-4.25	-4.59	-4.25	-3.86	-4.22	-4.21	-3.85	-4.22	-4.21	-4.21	-4.22	-4.275
Trace Test Statistic	42.00	69.30	33.44	36. 43	78.43	35.91	30.72	54.46	35.33	10.67	39.47	14.95	20.53	29.45	18.64	40.41	84.06	55.4
5% Critical Value	42.91	63.87	42.91	42.91	63.87	42.91	42.91	63.87	42.91	25.87	42.91	25.87	25.87	42.91	25.87	42.91	63.87	42.91

We report for each variable the estimated coefficient, the *t*-statistic and the *p*-value respectively

Table v.6
Test on the Increase of Goodness of Fit (Linear Play Model)

Country	R^2 Equation v.1	R^2 Equation v.4	$R^2_{(Eq. v.4)}$ - $R^2_{(Eq. v.1)}$	F-Stat*	R^2 Equation v.5	$R^2_{(Eq. v.5)}$ - $R^2_{(Eq. v.1)}$
Austria	0.551	0.710	0.159	8.315	0.701	0.150
Belgium	0.785	0.864	0.079	76.910	0.860	0.075
Canada	0.854	0.861	0.007	0.132	0.860	0.006
Czech Republic	0.802	0.832	0.030	2.238	0.824	0.022
Denmark	0.706	0.809	0.103	54.287	0.792	0.086
Finland	0.652	0.666	0.014	8.970	0.661	0.009
France	0.842	0.962	0.120	318.330	0.961	0.119
Germany	0.947	0.974	0.027	95.680	0.972	0.025
Hungary	0.915	0.934	0.019	19.401	0.932	0.017
Japan+	0.969	0.977	0.008	43.893	0.978	0.009
Luxembourg	0.822	0.874	0.052	53.681	0.864	0.042
Netherlands	0.870	0.870	0.000	-	0.870	0.000
Poland	0.964	0.966	0.002	4.240	0.963	-0.001
Portugal	**0.993**	**0.994**	0.001	**23.344**	**0.994**	0.001
Slovak Republic	0.828	0.857	0.029	86.992	0.856	0.028
Slovenia	0.702	0.823	0.121	65.120	0.819	0.117
Spain	0.921	0.958	0.037	103.363	0.957	0.036
UK	0.923	0.973	0.050	59.952	0.973	0.050
USA	0.987	0.993	0.006	107.580	0.993	0.006

$$^{*}F-Stat=\frac{\dfrac{R^2_{Equation\_v.4}-R^2_{Equation\_v.1}}{1}}{\dfrac{1-R^2_{Equation\_v.4}}{n-5}}$$

V.4. RESULTS OF THE TIME VARYING INTERCEPT EMPLOYMENT MODEL

To analyze the effect of uncertainty on the existence hysteresis in the dynamics of aggregate employment, we also implement on this data set the test on the existence of structural break hysteresis in the relationship between employment and its fundamentals.

Firstly, we estimate a time varying intercept employment equation:

$$\begin{cases} N_t = \beta_{0,t} + \beta_1 Y_t + \beta_2 W + \beta_3 T + \varepsilon_{1_t} \\ \beta_{0,t} = \beta_{0,t-1} + \varepsilon_{2_t} \end{cases} \tag{v.6}$$

Table v.7 shows the results of the estimation of Equation v.6. The time varying intercept is significant in all countries. Contrasting with the original estimates of the employment demand equation, the coefficient associated with real output is only significant in the countries where the hysteresis effect is weak (Canada, Belgium, the UK and the USA).

For the other countries, these results, interpreted under the hysteresis hypothesis, postulate that the major employment reaction occurs via structural changes in the dynamics (reaction along the spurts), captured in Equation v.6 by the time varying intercept $\beta_{0,t}$, while the reaction along the play (captured by β_1) is close to zero, which makes the coefficient associated with real output to be non-significant.

Secondly, if uncertainty associated with the future input values determines hysteresis, the change in the time varying intercept $(\beta_{0,t})$ should be inversely related to a measure of uncertainty:

$$\left|\Delta\hat{\beta}_{0,t}\right| = \alpha_0 + \alpha_1\sigma_{Y_t} + \omega_t \qquad \text{(iv.7)}$$

where $\hat{\beta}_{0,t}$ is a time series of the estimated time varying intercept in Equation v.6 and σ_{Y_t} is a proxy for the variability of the real output. The test on the uncertainty hypothesis is a test to H_0: $\alpha_1 < 0$ against H_1: $\alpha_1 = 0$.

Table v.7
Estimates of Time Varying Intercept Version of Employment Equation

Country	β_t [8]	y_t	w_t	T
Austria	(0.000)	**0.0162** (0.152)	**-0.0731** (0.985)	**-0.001** (0.936)
Belgium	(0.000)	**0.029***** (0.003)	-	-0.001 (0.999)
Canada	(0.000)	**0.068**** (0.018)	**0.037** (0.275	**0.001*** (0.064)
Czech Republic	(0.000)	**0.019** (0.265)	**0.029** (0.170)	**-0.0004** (0.746)
Denmark	(0.000)	**-0.072** (0.966)	**0.153*** (0.074)	**-0.001** (0.792)
Finland	(0.000)	**-0.013** (0.687)	**0.021** (0.345)	**0.001** (0.998)
France	(0.000)	**0.008** (0.161)	**0.024** (0.082)*	**-0.001*** (0.011)
Germany	(0.000)	**0.016** (0.161)	**0.025**** (0.047)	**-0.001** (0.999)
Hungary	(0.000)	**0.011** (0.171)	**0.059**** (0.015)	**-0.001** (0.822)
Japan	(0.000)	**0.115** (0.171)	**-0.017** (0.897)	**-0.002*** (0.000)
Luxembourg	(0.000)	**-0.005** (0.843)	**0.018**** (0.039)	**0.000*** (0.065)
Netherlands	(0.000)	**0.006** (0.246)	**0.199***** (0.001	**-0.001*** (0.000)
Poland	(0.000)	**0.043**** (0.038)	**0.025**** (0.046)	**-0.003** (0.000)
Portugal	(0.000)	**-0.004** (0.630)	**0.025**** (0.046)	**-0.003** (0.682)
Slovak Republic	(0.000)	**0.009** (0.362)	**0.028** (0.165)	**0.0003** (0.682)
Slovenia	(0.000)	**0.019** (0.121)	**-0.019** (0.826)	**-0.001***** (0.001)
Spain	(0.000)	**-0.012** (0.896)	-	**0.002***** (0.001)
UK	(0.000)	**0.0315***** (0.001)	-	**-0.002** (0.000)***
USA	(0.000)	**0.369***** (0.000)	**0.250***** (0.000)	**-0.003** (0.999)

* significant at 10%
** significant at 5%
***significant at 2%
p-value in brackets

To analyze the effects of uncertainty, firstly, we estimate Equation v.5 from country to country (see Table v.8) and, secondly, we estimate a random effects model (see Table v.9). We use the two forward-looking and two backward-looking measures of

[8] See Figure A.10 in the Appendix for time varying intercept estimates

uncertainty, based on ex-post variability of real output (in line with Equations iv.19 and iv.20):

When we estimate Equation v.6 country by country, we find, in the majority of the cases, the negative predicted value for α_1, although in general these estimates are non-significant.

When we estimate α_1 with a fixed effects specification, we obtain a negative significant estimate when the forward-looking measures of uncertainty are used, and mixed results, when we use the backward-looking measures of hysteresis[9].

Based on this test, we conclude for the existence of weak evidence concerning the effects of uncertainty upon the hysteresis band.

[9] The inclusion in the panel specification of a proxy of the magnitude of the costs of adjusting employment does not change the results concerning uncertainty. Moreover, we find a negative coefficient associated with the fixed adjustment costs proxy, which is in line with the theory. However, possibly due to lack of variability of this variable, the estimates are not significant.

Table v.8
Uncertainty Coefficient Estimates – Country Estimates
(Dependent Variable $\Delta\hat{\beta}_{0_{t,j}}$)

Country	σ_{y_t} (n=6)	σ_{y_t} (n=12)	σ_{y_t} (n=6)	σ_{y_t} (n=1)
Austria	**-0.015**	**-0.016**	**-0.003**	**-0.035**
	(0.034)	(0.028)	(0.034)	(0.028)
Belgium	**-0.016**	**-0.020**	**-0.026**	-0.047
	(0.046)	(0.047)	(0.051)	(0.053)
Canada	**-0.144****	**-0.124*****	**-0.055**	**-0.082****
	(0.056)	(0.036)	(0.056)	(0.038)
Czech Republic	**0.024**	**-0.097*****	**-0.005**	**-0.018**
	(0.041)	(0.037)	(0.053)	(0.043)
Denmark	**-0.048**	**0.187**	**-0.048**	**-0.070**
	(0.088)	(0.127)	(0.086)	(0.109)
Finland	**-0.018**	**-0.060**	**-0.073**	**-0.030**
	(0.050)	(0.049)	(0.051)	(0.053)
France	**-0.035****	**-0.013**	**0.017**	**0.013**
	(0.015)	(0.027)	(0.015)	(0.026)
Germany	**0.001**	**0.043**	**-0.043**	**-0.025**
	(0.036)	(0.030)	(0.036)	(0.03)
Hungary	**-0.019**	**-0.018**	**-0.046**	**0.007**
	(0.053)	(0.036)	(0.052)	(0.036)
Japan	**0.023**	**0.044****	**0.021**	**-0.013**
	(0.023)	(0.165)	(0.023)	(0.016)
Luxembourg	**-0.008**	**0.003**	**-0.013**	**-0.016**
	(0.011)	(0.014)	(0.012)	(0.014)
Netherlands	**-0.013**	**-0.021**	**-0.015**	**0.003**
	(0.016)	(0.021)	(0.015)	(0.020)
Poland	**0.022**	**0.050**	**0.041**	**0.034**
	(0.047	(0.034)	(0.050)	(0.037)
Portugal	**-0.046**	**0.035**	**-0.017**	**-0.041**
	(0.035)	(0.026)	(0.030)	(0.041)
Slovak Republic	**0.042**	**-0.022**	**-0.012**	**-0.015**
	(0.059)	(0.045)	(0.057)	(0.052)
Slovenia	**-0.017**	**-0.034**	**0.053**	**-0.025**
	(0.047)	(0.047)	(0.047)	(0.046)
Spain	**-0.003**	**0.077**	**-0.068**	**-0.068**
	(0.053)	(0.049)	(0.051)	(0.049)
UK	**-0.092**	**-0.082**	**-0.139****	**-0.204****
	(0.0650)	(0.053)	(0.068)	(0.077)
USA	**-0.013**	**-0.007**	**0.029**	**-0.068***
	(0.059)	(0.038)	(0.063)	(0.034)

* significant at 10%
** significant at 5%
***significant at 2%
p-values in brackets

Table v.9
Uncertainty Coefficient Estimates – Panel Estimates
(Dependent Variable $\Delta\hat{\beta}_{0_{t,j}}$)

	EPL	σ_{y_t} (n=6)	σ_{y_t} (n=12)	σ_{y_t} (n=6)	σ_{y_t} (n=1)
Eq.1		**-0.004** (0.009)			
Eq.2	**-0.0004** (0.0005)	**-0.001** (0.001)			
Eq.3			**0.025**** (0.010)		
Eq.4	**-0.001** (0.001)		**0.027**** (0.011)		
Eq.5				**-0.040***** (0.010)	
Eq.6	**-0.0003** (0.001)			**-0.045***** (0.011)	
Eq.7					**-0.028***** (0.010)
Eq.8	**-0.0003** (0.001)				**-0.028***** (0.010)

** significant at 5%
***significant at 2%
Standard Errors in Brackets

V.5. PATTERN OF HYSTERESIS AND LABOR MARKET INSTITUTIONS

After computing the transformed variables according to the Preisach Model and to the Linear Play Model of Strong Hysteresis, we evaluated the existence of hysteresis by looking into three indicators: *i*) the substitution effect of the significance of the transformed variable; *ii*) the increase in the goodness of fit of the regressions; *iii*) the increase inhe absolute value of the statistics of the cointegration tests.

Tables v.10 and v.11 summarize the results of the tests. We verify that the inclusion of the hysteresis transformed variables increases, in most cases, the quality of the employment equations.

Table v.10
Summary of the Results of the Preisach Model

Number of Cases	**Substitution Effect** (original variable non-significant and hysteresis variable significant)	**Increase in the *R*-Square** (regression with the transformed variable relative to regression with the original variable)	**(*F*-Test)** (Increase in the *R*-Square with the inclusion of the Transformed Variable)	**Increase in Cointegration Tests Statistics**	
				Engle Granger Test	**Trace test**
In Favor of Hysteresis	11	15	12	14	14
Against Hysteresis	2	4	5	5	5
Difficult Conclusion	6	0	2	0	0

Note: 19 countries analyzed (for real wages we only have data for 16 countries)

Table iv.11
Summary of the Results of the Linear Play Model

Number of Cases	**Substitution Effect** (original variable non-significant and hysteresis variable significant)	**Increase in the *R*-Square** (regression with the transformed variable relative to regression with the original variable)	**(*F*-Test)** (Increase in the *R*-Square with the inclusion of the Transformed Variable)	**Increase In Cointegration Tests Statistics**	
				Engle Granger Test	**Trace test**
In Favor of Hysteresis	12	18	15	13	14
Against Hysteresis	0	1	4	6	5
Difficult Conclusion	7	0	0	0	0

Note: 19 countries analyzed

We also provide two indicators of the importance of hysteresis (see Table v.12). The Preisach indicator is simply the rank position of the country concerning the increase in the goodness of fit of the cointegrated regression. The play indicator is the rank position of the country concerning the width of the play estimated with the series of real product (see also Figure v.3)[10]. Overall, considering the information in Table v.12 and the results of the cointegration tests, the two indicators agree in identifying important signs of

[10] The comparison of the results on the existence of hysteresis in the dynamics of employment implies the existence of similar economic cycles in the considered countries, otherwise the inexistence of shocks or inflections of sales of sufficient magnitude could erroneously indicate the inexistence of hysteresis. The length of the sample is, however, sufficient to rule out this possibility.

hysteresis in the dynamics of employment in France, Finland, the Slovak Republic and Japan. Traces of hysteresis are also found in Austria, Belgium, Germany, Luxembourg, Portugal, Slovenia and Spain. However, in these countries we only found the existence of cointegration between the variables in one test we. We found that hysteresis is not relevant in Canada, the Czech Republic, the UK or in the USA.

Concerning the case of Portugal, from Table v.12 and Figure v.3, we conclude that in spite of its reputation as a country with high employment protection, it does not rank between the countries where macro employment inertia is high[11].

Table v.12
Hysteresis Indicators

Preisach Indicator $R^2_{\text{(Eq. v.2)}} - R^2_{\text{(Eq. v.1)}}$		**Linear Play Indicator** (Play Width)	
Austria	0.162	Denmark	0.186
France	0.112	Slovak Republic	0.168
Slovenia	0.105	Luxembourg	0.122
Netherlands	0.051	Austria	0.102
Finland	0.035	Finland	0.096
Spain	0.032	France	0.074
Slovak Republic	0.030	Germany	0.074
Belgium	0.022	Spain	0.072
Germany	0.015	Slovenia	0.07
Japan	0.015	Japan	0.068
Luxembourg	0.005	Belgium	0.062
Poland	0.001	Hungary	0.054
Portugal	**0.000**	**Portugal**	**0.044**
Denmark	-0.003	Czech Republic	0.032
UK	-0.006	Poland	0.032
Hungary	-0.014	UK	0.024
USA	-0.055	USA	0.024
Canada	-0.062	Canada	0.02
Czech Republic	-0.223	Netherlands	0.00

* Increase in the *R*-square when we estimate the model with the transformed series of real output instead of the original series.

[11] The fact that the outcome of the labor market adjustment does not totally reflect the very rigid employment protection legislation in Portugal is stated in Addison and Teixeira (2003, 2005). Actually, Addison and Teixeira (2005) compared the speed of the adjustment of Portuguese and the German aggregate employment, to its long run value, in the period from the first quarter of 1977 to the fourth quarter of 1997, and they concluded that it is higher in Portugal than in Germany, in spite of Portugal being a country with a more rigid labor market reputation.

***a*. Preisach Indicator** — $R^2_{\text{(Eq. v.2)}} - R^2_{\text{(Eq. v.1)}}$

***b*. Linear Play Indicator** — Play Width

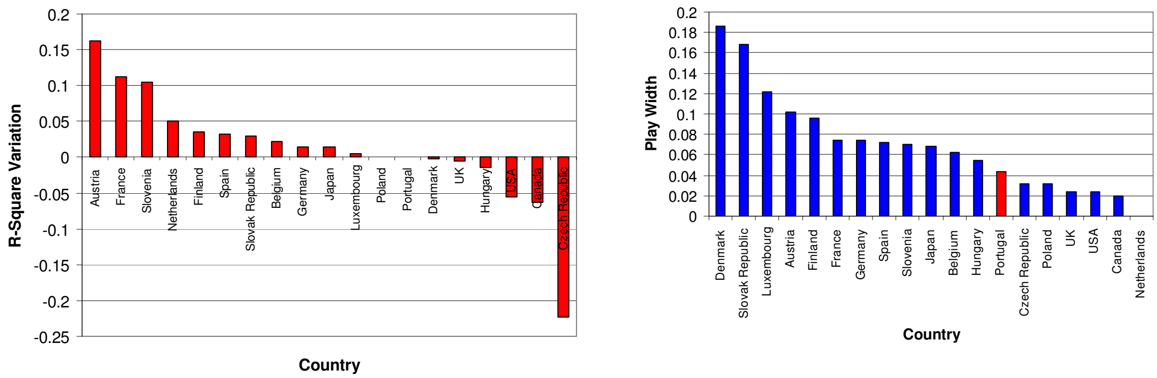


Figure v.3. Hysteresis Indicators

The theoretical models of hysteresis postulate that the band of inaction is a positive function of the magnitude of the fixed costs of adjusting employment and uncertainty, and a negative function of the magnitude of the cost of adjusting the number of hours per worker (see section II.3). Benefiting from our empirical results, we finally verify the empirical association between hysteresis and its main causes.

Table v.13 reports the rank correlations between indicators of hysteresis and proxies of employment adjustment costs, costs of adjusting hours of work and uncertainty.

In line with the predictions of the hysteresis models, we find: *i*) a positive and significant correlation between the hysteresis indicators and the strictness of employment protection legislation; *ii*) mixed and non-significant results concerning the correlation between hysteresis indicators and the proxy of strictness of working time regulations; *iii*) a positive and significant correlation (for the indicator calculated from the Linear Play Model) between the hysteresis indicator and the proxy of uncertainty.

These results indicate that labor market institutions and especially employment protection legislation affect the aggregate dynamic of employment through hysteresis mechanisms.

Table v.13
Spearman's Rank Correlation between
Labor Market Institutions Indicators and Hysteresis Indicators

Hysteresis Indicator	**Employment Adjustment Costs**				**Working Time Regulations**	**Proxy of Uncertainty**
	$EPL\_LATE90s$ [1]	EPL_2003 [2]	EPL_LMID [3]	EPI [4]	WTR [5]	σ_{y_t} (n=6)
$(PLAY)$	**0.48*** (2.14)	**0.57*** (2.80)	**0.29** (0.99)	**0.19** (0.63)	**-0.08** (0.27)	**0.63**** (3.36)
(ΔR^2) (Preisach Model)	**0.67**** (3.53)	**0.75**** (4.48)	**0.55*** (2.18)	**0.45** (1.65)	**0.35** (1.25)	**0.18** (0.77)

* Significant at 5%; **significant at 1%
t-statistics in brackets

[1] Employment Protection Indicator from OECD – Employment Outlook 2004 (Table 2.A.2.4).
[2] Employment Protection Indicator from OECD – Employment Outlook 2004 (Table 2.A.2.4).
[3] Employment Protection Indicator from Labor Market Institutions Database, version 2.00, 2001, by Stephen Nickel
[4] Employment Protection Indicator [0.20] from Nunziata (2002) (Table 1, p. 38).
[5] Working Time Regulation Index from Nunziata (2003), (Table 1, p. 38).

V.6. MAIN FINDINGS

This part was dedicated to an international comparison of the importance of hysteresis in shaping the dynamics of employment at the macro level. By applying the models of strong hysteresis to aggregate data from EUROSTAT and OECD we conclude:

Firstly, in general the non-linear model that allows for hysteresis performs better than the linear model with the original variables.

Secondly, by estimating a time varying intercept employment equation, we find only weak evidence regarding the effect of uncertainty upon the hysteresis band at the macro level.

Thirdly, we find a significant correlation between the rigidity of some labor market regulations and the importance of hysteresis, meaning that labor market institutions affect the dynamics of employment through hysteresis mechanisms.

Finally, the position of Portugal in the ranking of the importance of hysteresis does not totally reflect its rigid employment protection legislation.

CHAPTER VI

CONCLUSIONS AND FUTURE RESEARCH

VI.1. CONCLUSIONS

The aim of this dissertation was to analyze the existence of hysteresis in the adjustment of employment at the micro level and to study its aggregate implications.

At the firm level, the adjustment pattern of the labor input for Portuguese manufacturing firms in the period from January 1995 to December 2005 reveals unequivocal signs of lumpy adjustment, although smooth adjustment is also important. Lumpy adjustment is more important for small firms than for large ones. From the joint dynamics of employment, hours of work and sales, we verify that while there are some signs of the existence of non-convex costs of adjusting employment, reflected in the existence of a negative correlation between hours and employment adjustment, there is no significant cost in changing the number of hours per worker. This implies that the response of the labor input to variations in sales, is primarily implemented through the variation in the number of hours of work.

A more formal test for the existence of hysteresis was conducted, at the firm level, based on the assumption that employment response to product demand shocks of the same magnitude is asymmetric, and it depends on the differences between the actual and desired level of employment. We found that the signs of the existence of hysteresis at the micro level are stronger in the case of smaller firms.

The results of the Preisach Model reveal that the hysteresis transformation of sales offers a better explanation of the aggregate employment dynamics than the original variable. However, the results are different concerning the size of firms. While the inclusion of the hysteresis variable significantly increases the goodness of fit of the employment equations in the case of small firms, for large ones the original series of sales offers a better explanation of employment dynamics, meaning that hysteresis is not important in this case.

Results of the Linear Model of Strong Hysteresis indicate significant periods of inaction, implying that a large play width is necessary to achieve the maximum goodness of fit of employment regressions. Overall, the non-linear model, which includes the hysteresis transformation of the variables, performs better than the standard linear model. Furthermore, the estimated play interval is larger for small firms than for large ones (the estimated play for the whole sample lies in the middle of these values). While for small firms the results indicate the existence of a zone of weak employment reaction (the play) and a zone of strong employment reaction (the spurt), for large firms the differences between these zones of different reactions of employment are not as clear.

Concerning the effect of uncertainty, based on the estimation of a time varying intercept version of the employment equation, we did not find evidence regarding its influence on hysteresis at the macro level. Nonetheless, we conclude, based on the Linear Model of Hysteresis with a variable play, that uncertainty has a positive effect over the inaction band, at the macro level, for firms with fewer than 20 workers.

We found strong evidence of the existence of a relationship between the adjustment through hours of work and the existence of hysteresis at the macro level. The goodness of fit of the employment equation increased significantly, when we estimated the hysteresis variable on the basis of a variable play.

Therefore, in the case of Portugal, at the micro level, the empirical evidence indicates the existence of significant non-convex adjustment costs that originates hysteresis. The evidence is stronger for small firms than for large ones. At the aggregate level signs of hysteresis do not completely vanish. Strong hysteresis models show that, while the aggregate path of employment of firms with fewer than 20 workers is characterized by hysteresis, the hysteresis properties are not as distinguishable in the aggregate path of employment of firms with more than 500 workers.

An international comparison of the results of strong hysteresis models indicates that hysteresis in the dynamics of employment is particularly noticeable in countries such as France, Finland, the Slovak Republic and Japan. Traces of hysteresis are also found in Austria, Belgium, Germany, Luxembourg, Portugal, Slovenia and Spain. On the contrary, hysteresis is not noticeable at all in Canada, the Czech Republic, the UK or the USA. These results reveal a positive and significant correlation between the rankings of rigidities of labor market institutions and the estimated rankings of the importance of hysteresis. Thus, labor market institutions affect the dynamics of employment through hysteresis mechanisms.

In spite of being in the top rankings of the rigidity of employment protection legislation, this study places Portugal in the middle of the said extreme groups with regard the importance of hysteresis. A possible explanation for this fact could be the interaction of rigid employment protection legislation with medium rigidity concerning overtime regulations. Another explanation could be the fact that employment protection legislation indexes that are based on an interpretation of the legislation do not totally reflect the behavior of the labor market.

VI.2. BASIS FOR FUTURE RESEARCH

Naturally, several issues related to the subject in this thesis remain open.

Due to lack of available data, we only analyzed the dynamics of employment adjustment on the intensive margins (adjustment of firms that are already operating in the market). However, it is also important to analyze the dynamics of employment adjustment on the extensive margin (firms' entry and exit). This could be studied in the framework of the Preisach Model of Strong Hysteresis or with the application of a dynamic discrete choice model of a firm's participation decision.

Moreover, we ignore the influences of the investment decisions on the level of employment. Since labor and capital are interrelated and recognizing that decisions regarding physical capital adjustment are also affected by non-convex costs of adjustment, it would be important to analyze the relationship between hysteresis in the dynamics of the stock of capital and hysteresis in the dynamics of employment.

Some improvements in the programs to calculate hysteresis variables could also be implemented. In particular it would be interesting to examine the sensitivities of the results to different specifications for the Preisach Function. By applying the least square method, we should be able to estimate the density function over the Preisach Triangle that originates the higher R-square of the employment equation. We expect that in countries with more rigid employment protection legislation, a great proportion of the firms should be located in the Norwest region of the Preisach Triangle.

REFERENCES

Addison, J.T. and Paulino Teixeira, 2001a, "Employment Adjustment in Portugal: Evidence from Aggregate and Firm Data", *IZA – Discussion Paper*, No 391.

Addison, J.T. and Teixeira, Paulino, 2001b, "Employment Adjustment in a 'Sclerotic' labor market: Comparing Portugal with Germany, Spain, and the United Kingdom. *Jahrbücher für Nationalkönomie und Statistik*, No. 221, pp. 353-370.

Addison, J.T. and Teixeira, Paulino, 2003, "The Economics of Employment Protection", *Journal of Labor Research*, Vol. 24, No. 1, pp. 85-129.

Addison, J.T. and Teixeira, Paulino, 2005, "Employment Adjustment in Two Countries with Poor Reputations: Analysis of Aggregate, Firm, and Flow Data for Portugal and Germany" *International Economic and Economic Policy*, No. 1, pp. 329-348.

Addison, John T. and Portugal, Pedro, 2003, "Unemployment Duration – Competing and Defective Risks", *Journal of Human Resources*, Vol. 38, No.1, pp. 157-191.

Amable, Bruno; Henry, Jérôme; Lordon, Frédéric and Topol, Richard, 1993, "Unit_root in the Wage-Price Spiral Is Not Hysteresis in Unemployment", *Journal of Economic Studies*, Vol. 20, No. 1-2, pp. 123-135.

Amable, Bruno; Henry, Jerôme; Lordon, Frédéric; Topol, Richard, 1994, "Strong Hysteresis versus Zero-Root Dynamics, *Economics Letters*, Vol. 44, No. 1-2, pp. 43-7.

Amable, Bruno; Henry, Jérôme; Lordon, Frédéric and Topol, Richard, 1995, "Hysteresis Revisited: A Methodological Approach", in Rod Cross (ed.) *The Natural Rate of Unemployment: Reflections on 25 Years of the Hypothesis*, Cambridge University Press, pp. 181-200.

Andersen, Torben M. and Hylleberg Svend, 2000, "Sources of Persistence in Employment Adjustment – Denmark 1974-93", *Oxford Economic Papers*, No. 52, pp. 72-95.

Baldwin, Richard and Krugman, Paul, 1989, "Persistent Trade Effects of Large Exchange Rate Shocks", *Quarterly Journal of Economics*, Vol 104, No. 4, pp. 635.

Bean, Charles, 1989, "Capital shortages and persistent unemployment", *Economic Policy*, Vol. 4, No. 1, p11-53.

Becker, Gary S., 1962, "Investment in Human Beings", *Journal of Political Economy*, Vol. 70, No. 5, pp. 9-49.

Belke, Ansgar and Göcke Mathias, 2001, "Exchange rate Uncertainty and Employment: an Algorithm Describing the ´Pay´", *Applied Stochastic Models in Business and Industry*, Vol. 17, No. 2. pp. 181-204.

Belke, Ansgar and Göcke, Mathias, 2005a, "Institutional Uncertainty and European Social Union: Impacts on Job Creation and Destruction in the CEECs", Journal of Policy Modeling, No. 27, pp. 345-354.

Belke, Ansgar and Göcke, Mathias, 2005b, "Real Options Effects on Employment: Does Exchange Rate Uncertainty Matter for Aggregation*?", German Economic Review*, Vol. 6, No. 2, pp. 185-203.

Belke, Ansgar and Göcke, Matthias, 1998, Micro- and Macro-Hysteresis in Employment under Exchange Rate Uncertainty", *Institut für Europäische Wirtsschaft* - Working paper, No 23.

Belke, Ansgar and Göcke, Matthias, 1999, "A Simple Model of Hysteresis in Employment under Exchange Rate Uncertainty*", Scottish Journal of Political Economy*, Vol. 46, No 3, pp. 260-286.

Bertola, Giusep, 1992, "Labor Turnover Costs and Average Labor Demand", *Journal of Labor Economics*, Vol. 10, No. 4, pp. 389-411.

Bertola, Giuseppe and Caballero, Ricardo J., 1990, "Kinked Adjustment Costs and Aggregate Dynamics", *NBER – Macroeconomics Annual*, p. 237-288.

Blanchard, Oliver and Portugal, Pedro, 2001, "What Hides Behind Unemployment Rate: Comparing Portuguese and U.S. Labor Markets", *American Economics Review*, Vol. 91, No 1, pp. 187

Blanchard, Olivier J. and Summers, Lawrence H., 1986, "Hysteresis and the European Unemployment Problem", *NBER Macroeconomics Annual*, Cambridge: MIT Press, pp. 15-78.

Blanchard, Olivier J. e Summers, Lawrence H., 1987, "Hysteresis in Unemployment", *European Economic Review*, Vol. 31, No. 1-2, pp. 288-95.

Blinder, Alan S., 1981, "Retail Inventory Behavior and Business Fluctuations", *Brooking Papers on Economic Activity*, No. 2, pp. 443-505.

Bover, Olympia, García-Perea and Portugal, Pedro, 2000, "Labour Markets Outliers: Lessons from Portugal and Spain, *Economic Policy*, Vol. 15, No. 31, pp. 379-428.

Caballero, Richard J. and Engel, Eduardo M. R. A., 1993, "Microeconomic Adjustment Hazards Aggregate Dynamics", *Quarterly Journal of Economics*, Vol. 108, No. 2, pp. 359.

Caballero, Richard J.; Engel, Eduardo M. R. A., and Haltiwanger, John, 1995, "Plant Level Adjustment and Aggregate Investment Dynamics", *Brooking Papers on Economic Activity*, No. 2, pp. 1-39.

Caballero, Richard J.; Engel, Eduardo M. R. A., and Haltiwanger, John, 1997 "Aggregate Employment Dynamics: Building from Microeconomic Evidence", *American Economic Review*, Vol. 87, No. 1, pp. 115-37.

Caplin, Andrew S. and Spulber, Daniel F., 1987, "Menu Costs and the Neutrality of Money", *Quarterly Journal of Economics*, Vol. 102, No. 4, pp. 703-725.

Caplin, Andrew S., 1985, "The Variability of Aggregate Demand with (S,s) Inventory Policies", *Econometrica*, Vol. 53, No.6, pp. 1395-1409.

Carneiro, Anabela and Portugal, Pedro, 2004, "Market Power, Dismissal Threat, and Rent Sharing: the Role of Insider and Outsider Forces in Wage Bargaining, *CETE* – DP 2004-03.

Cooper, Russell; Willis, Jonathan L., 2004 "A Comment on the Economics of Labor Adjustment: Mind the Gap", *American Economic Review*, Vol. 94, No. 4, pp. 1223-1237.

Cross, Rod B., 1980, "On the History of Hysteresis", in Rod Cross (ed.), 1988, *Unemployment Hysteresis and the Natural Rate Hypothesis*, Blackwell.

Cross, Rod B., 1994, "The Macroeconomic Consequences of Discontinuous Adjustment: Selective Memory of Non-Dominated Extrema", *Scottish Journal of Political Economy*, Vol. 41, No. 2, pp. 212-221.

Cross, Rod B., 1995, "Is the Natural Rate Hypothesis Consistent with Hysteresis?", in Rod Cross (ed.) *The Natural Rate of Unemployment: Reflections on 25 Years of the Hypothesis*, Cambridge University Press, pp. 181-200.

Cross, Rod B., 1997, "Force Fields, Hysteresis and Economic Equilibria", *Journal of Economics Studies*, Vol. 24, No. 4, pp. 257-266.

Cross, Rod; Darby, Julia; Ireland, Jonathan; Piscitelli, Laura, 1998, "Hysteresis and Unemployment a Preliminary Investigation", *University of Strathclyde - Working Paper*.

Davis, Steven J. and Haltiwanger John, 1992, “Gross Job Creation, Gross Job Destruction, and Employment Reallocation”, *The Quarterly Journal of Economics*, Vol. 107, No. 3, pp. 819-863.

Dixit, Avinash, 1989, “Entry and Exit under Uncertainty”, *Journal of Political Economy*, Vol. 97, No. 3, pp. 620-39.

Dixit, Avinash, 1991, “Analytical Approximations in Models of Hysteresis*”, Review of Economic Studies*, Vol. 58, No. 193, pp. 141-151.

Dixit, Avinash, 1992, “Investment and Hysteresis”, *Journal of Economic Perspectives*, Vol. 6 No. 1, pp. 107-32.

Dixit, Avinash, 1997, “Investment and Employment Dynamics in the Short Run and in the Long Run”, *Oxford Economic Papers*, Vol. 49, pp. 1-20.

Dixit, Avinash, and Pindyck, Robert, 1994, *Investment under Uncertainty*, Princeton University Press.

Duarte, Adelaide and Andrade, J.S., 2000, “Histerese da Taxa de Desemprego de Equilíbrio – Uma Aplicação ao Caso Português”, *Notas Económicas*, No. 14.

Dunne, Timothy, 1998, “Capital Adjustment Patterns in Manufacturing Plants”, *Review of Economic Dynamics*, No. 1, pp. 409-429.

Foote, Christopher L., 1998, “Trend Employment Growth and Bunching of Job Creation and Destruction”, *The Quarterly Journal of Economics*, No. 113, pp. 809-834.

Göcke, Mathias, 1999, “Types of Hysteresis Applied in Economics”, *Westfälische Wihelms-Universität Münster Volhwirtschafliche Diskussions Bliträge*.

Göcke, Matthias, 2001, “A Macroeconomic Model with Hysteresis in Foreign Trade”, *Metroeconomica*, Vol. 52, No. 4, pp. 449-473.

Göcke, Matthias, 2002, "Various Concepts of Hysteresis Applied in Economics", *Journal of Economic Surveys*, Vol. 16, No 2, pp. 167-188.

Granger, C. W. J., 1991, "Developments in the Study of Cointegrated Economic Variables", in *Long-Run Economic Relationships – Readings in Cointegration*, Ed. Engle, R.F. and Granger, C. W. J., Oxford University Press.

Grubb, D. and Wells, W., 1993, "Employment Regulation and Patterns of Work in EC Countries", *OECD Economic Studies*, No. 21, Winter, pp. 7-58.

Hamermesh, Daniel S. and Pfann, Gerard A., 1996, "Adjustment Costs in Factor Demand", *Journal of Economic Literature*, Vol. 34, No. 3, pp. 1264-92.

Hamermesh, Daniel S., 1989, "Labor Demand and the Structure of Adjustment Costs", *American Economic Review*, Vol. 79, No. 4, pp. 674-89

Hamermesh, Daniel S., 1993, *Labor Demand*, Princeton University Press.

Hargreaves Heap, Shaun P., 1980, "Choosing the Wrong Natural Rate: Accelerating Inflation or Decelerating Employment and Growth", *Economic Journal*, Vol. 90, No. 359, pp. 611-620.

Harvey, Andrew C., 2001, *Forecasting, Structural Time Series and the Kalman Filter*, Cambridge University Press.

Johansen, Soren, 1988, "Statistical Analysis of Cointegrated Vectors", *Journal of Economic Dynamics and Control*, Vol. 12, No. 2-3, pp. 231-54.

Krasnosel'skii, A.M. and Rachinskii, D.I., 2003, "On a Nonlocal Condition for Existence of Cycles of Hysteresis Systems", *Automation and Remote Control*, Vol. 64, No. 2, pp. 231-251.

Layard, R and Nickell, S.J, 1986, "Unemployment in Britain", *Economica*, Vol. 53, pp. S121-69.

Layard, Richard; Nickell, Stephan and Jackman, Richard, 1991, *Unemployment Macroeconomic Performance and the Labour Market*, Oxford University Press, New York.

Lindbeck, Assar and Snower, Dennis, 1986, "Wage Setting, Unemployment, and Insider-Outsider Relations", *American Economic Review,* Vol. 76 (2), May, pp. 235-39.

Lindbeck, Assar and Snower, Dennis, 1988, *The Insider-Outsider Theory of Employment and Unemployment*, The MIT Press, pp. XXI-285.

MacKinonnon, James G., 1991, "Critical Values for Cointegration Testes", in *Long-Run Economic Relationships – Readings in Cointegration*, Oxford University Press.

Marshall, Alfred, 1890, *Principles of Economics*, Macmillan, London, 8th ed.

Mayergoyz, I.D., 2003, *Mathematical Models of Hysteresis and Their Aplications*, Elsevier.

Modesto, Leonor, Leite, Manuel and Neves, João César, 1992, "Some Aspects of the Portuguese Labour Market 1977-1988: Neutrality, Hysteresis and Wage Gap", in *The Portuguese Economy Towards* 1992, Ed. João Ferreira do Amaral, Diogo Lucena and António S. Mello, Kluwer Academic Publishers.

Nickel, S. J., 1978, "Employment and Labour Demand", *Economica*, Vol. 45, No. 180, pp. 329-345.

Nunziata, Luca, 2003, "Labour Market Institutions and Cyclical Dynamics of Employment", *Labour Economics*, Vol. 10, No. 1, pp. 31-53.

OECD, Employment Outlook, 1998.

OECD, Employment Outlook, 2004.

Oi, Walter Y., 1962, “Labor as a Quasi-Fixed Factor”, *Journal of Political Economy*”, Vol. 70, December, pp. 538-55.

Parsley, David C., and Wei Shang-Jin, 1993, “Insignificant and Inconsequential Hysteresis: The case of U.S. Bilateral Trade”, *Review of Economics and Statistics*, Vol. 75, No. 4, pp. 606-13.

Pereira, João 1998, “Histerese na Taxa de Desemprego em Portugal*”, Estudos de Economia*,Vol. 18, No. 3.

Phelps, Edmund S., 1972, *Inflation Policy and Unemployment Theory, The Cost-Benefit Approach to Monetary Planning,* Macmillan.

Pindyck, Robert, 1991, “Irreversibility, Uncertainty, and Investment”, *Journal of Economic Literature*, Vol. 29, No. 3, pp. 1110-48.

Piscitelli, Laura; Cross, Rod; Grinfeld, Michael and Lamba, Harbir, 2000, “A Test for Strong Hysteresis”, *Computational Economics*, Vo. 15, No. 1-2, pp. 59-78.

Piscitelli, Laura; Grinfeld, Michael; Lamba, Harbir, and Cross, Rod, 1999, “On Entry and Exit in Response to Aggregate Shocks”, *Applied Economics Letters*, Vol. 6, No. 9, pp. 569-572.

Rosa, Agostinho S., 2004, “Uma Estimação da Curva de Phillips para Portugal”, *Universidade de Évora – Working Paper* No. 2004/8.

Sargent, Thomas J., 1978, “Estimation of Dynamic Labor Demand Schedules under Rational Expectations”, *Journal of Political Economy*, Vol. 86, No. 6, pp. 1009-1044.

Shapiro, Matthew D., 1986, “The Dynamic Demand for Capital and Labor”, *Quarterly Journal of Economics*, Vol. 101, No. 3, pp. 513-542.

Sneessens, Henry R. and Drèze, Jaques H., 1986, "A Discussion of Belgian Unemployment, Combining Traditional Concepts and Disequilibrium Econometrics", *Economica*, Vol. 53, No. 210 (Supplement), pp. 89-119.

Stock, James and Watson, Mark, 1988, "Variable Trends in Economic Time Series", *Journal of Economic Perspectives*", Vol. 2, No. 3, pp. 147-174.

Varejão, José and Portugal, Pedro, 2007, "Employment Dynamics and the Structure of Labor Adjustment Costs", *Journal of Labor Economics*, Vol. 25, No. 1, pp. 137-165.

Varejão, José M. J., 2000, *Three Essays on Labor Demand*, Unpublished Ph.D. Dissertation. Faculty of Economics – University of Porto.

Visitin, Augusto, 1994, "*Diferential Models of Hysteresis*", Springer.

APPENDIX

Figure A.1
Unconditional Frequency of Sales Change by Firm Size

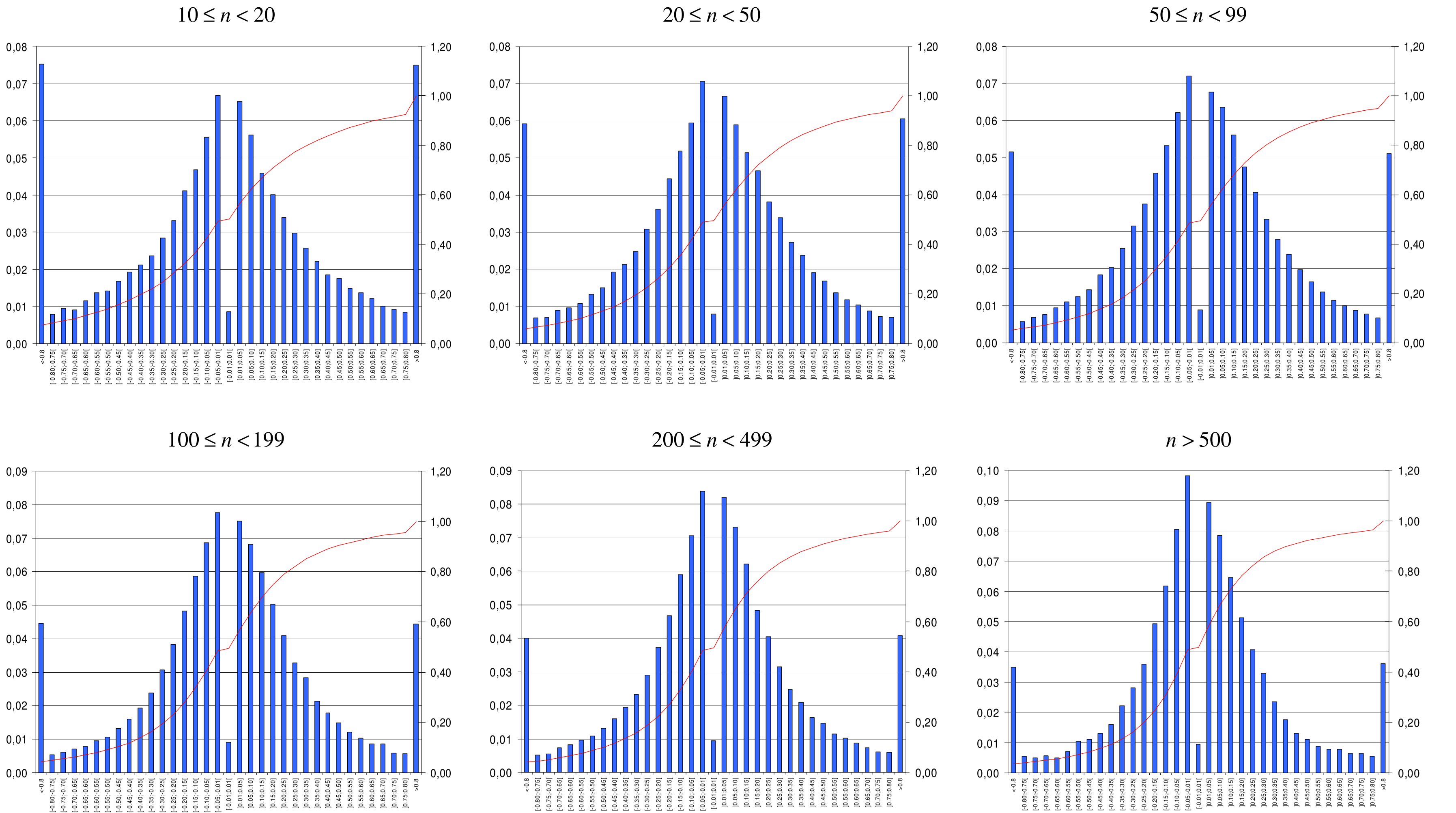
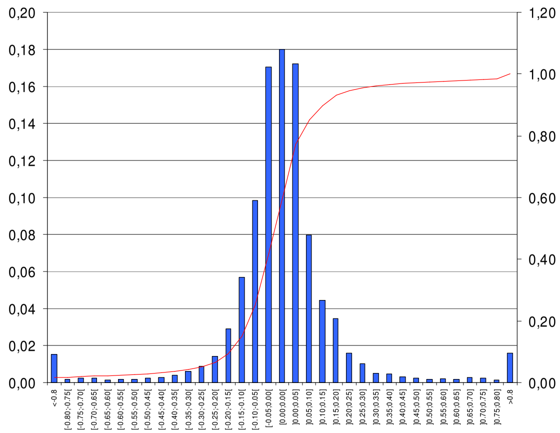
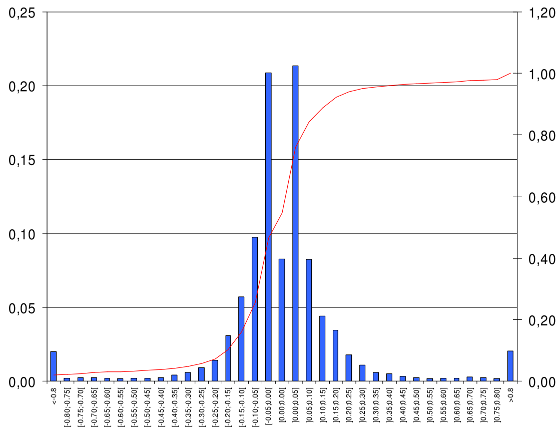


Figure A.2
Unconditional Frequency of Hours per Worker Change by Firm Size

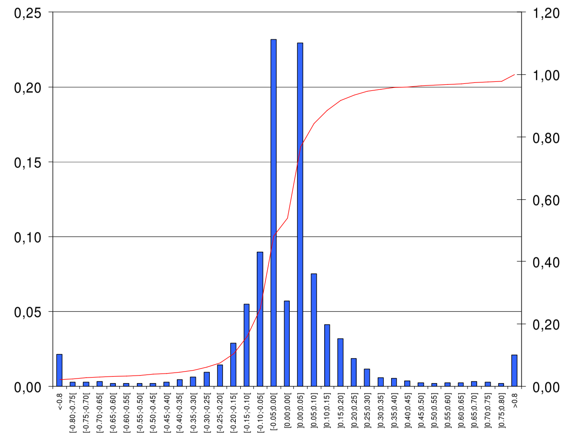
$10 \leq n < 20$



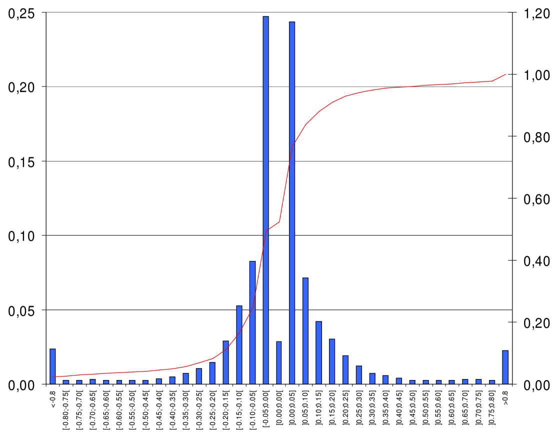
$20 \leq n < 50$



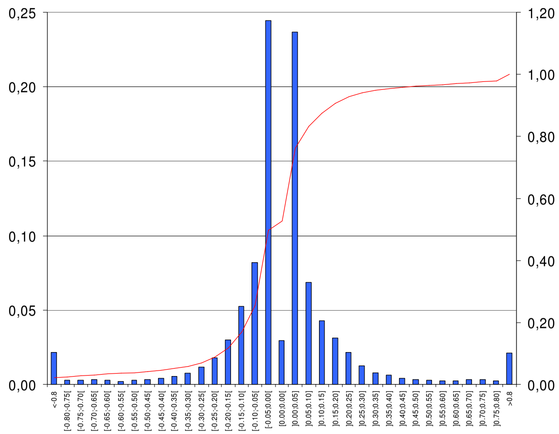
$50 \leq n < 99$



$100 \leq n < 199$



$200 \leq n < 499$



$n > 500$

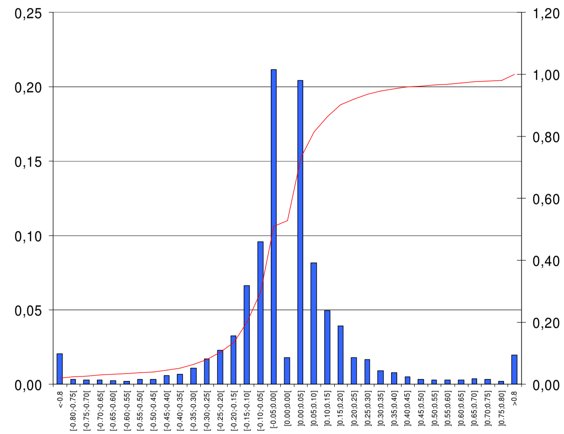
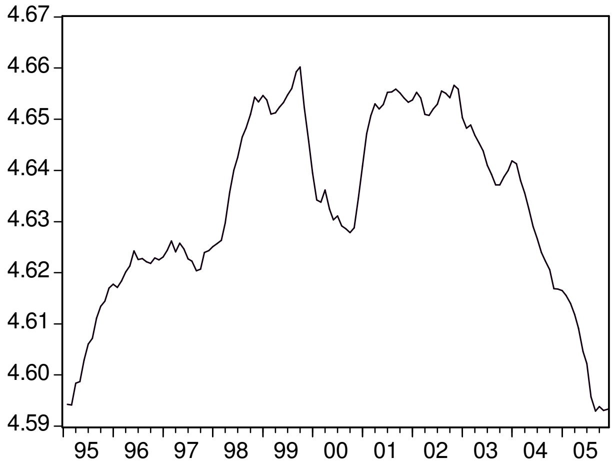
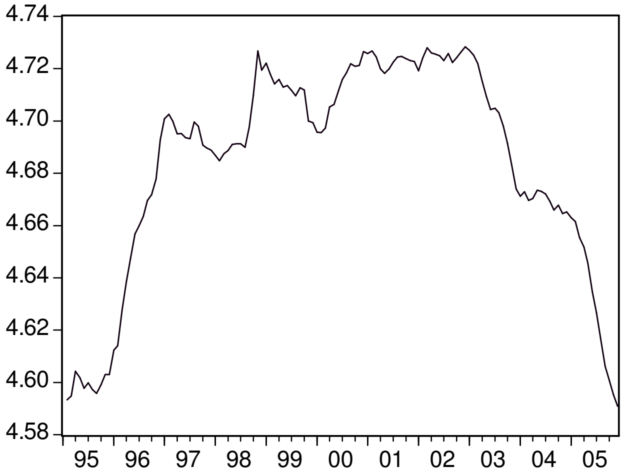


Figure A.3
Time Varying Intercept Estimates of Labor Demand Equation with Kalman Filter
(with micro data from Portuguese industrial firms)

Whole Sample



Small Firms



Large Firms

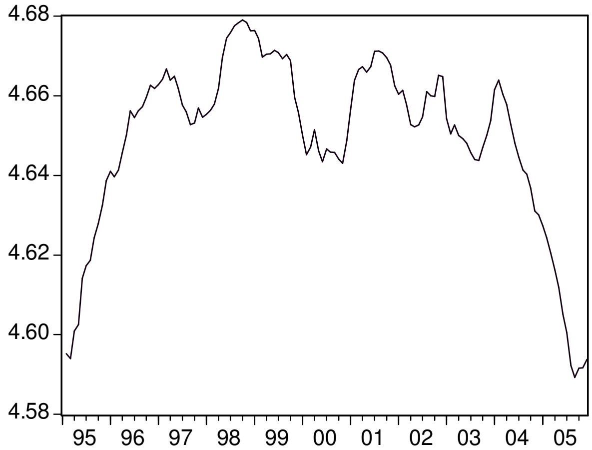
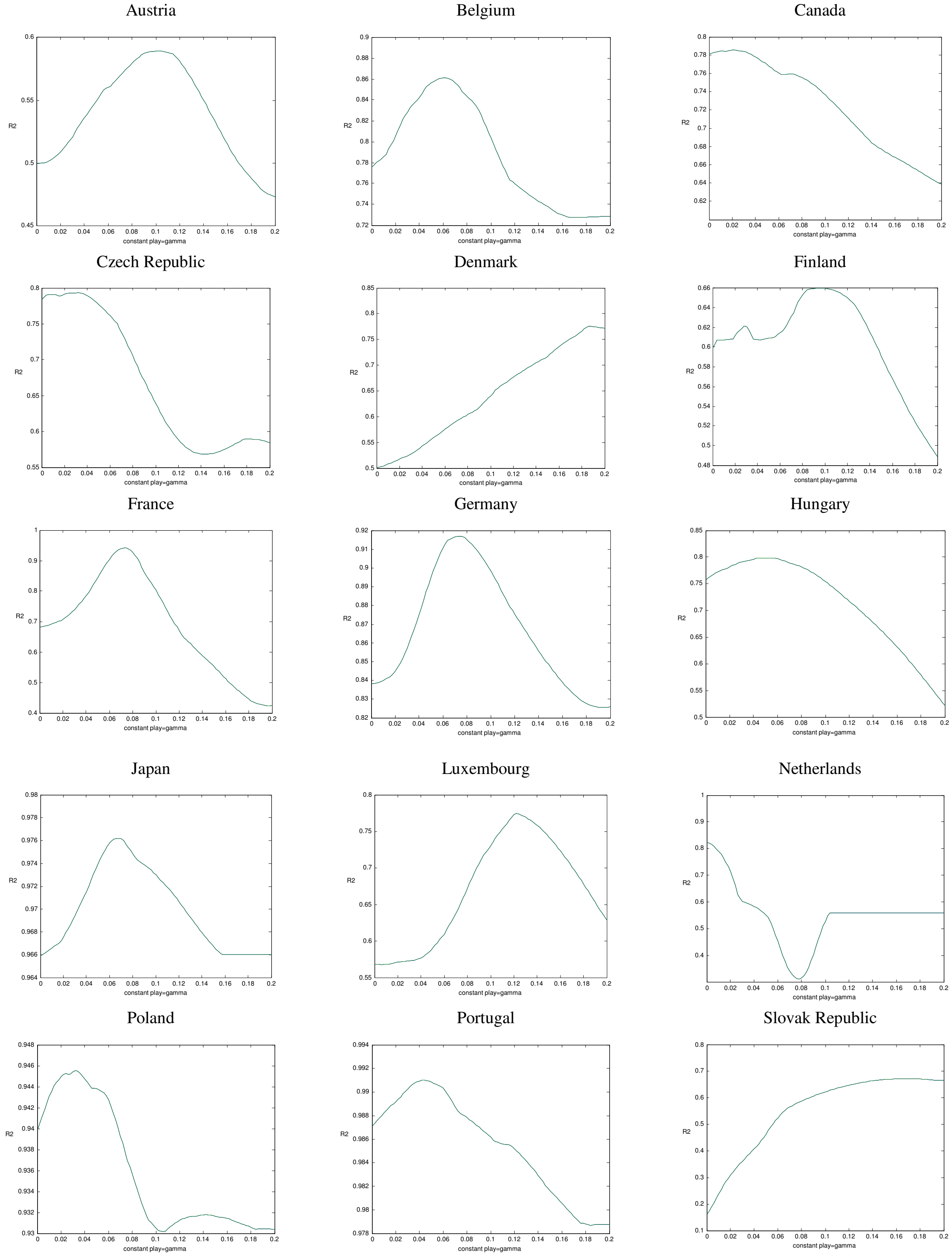
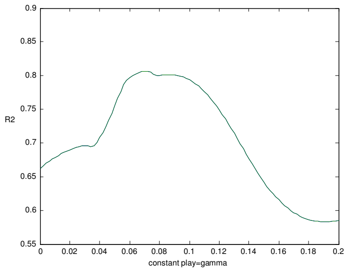


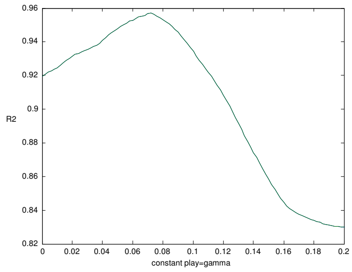
Figure A.4
Estimation of the Play Width
(with manufacturing aggregate data from EUROSTAT and OECD – dependent variable: real product)



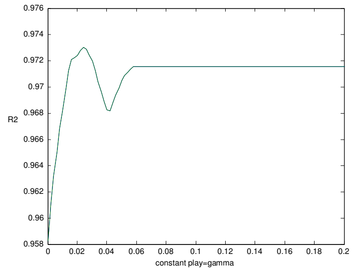
Slovenia



Spain



U.K.



U.S.A

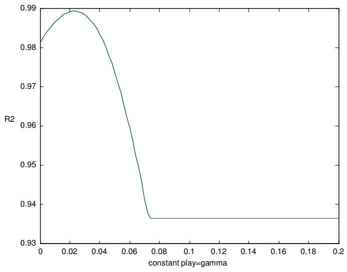
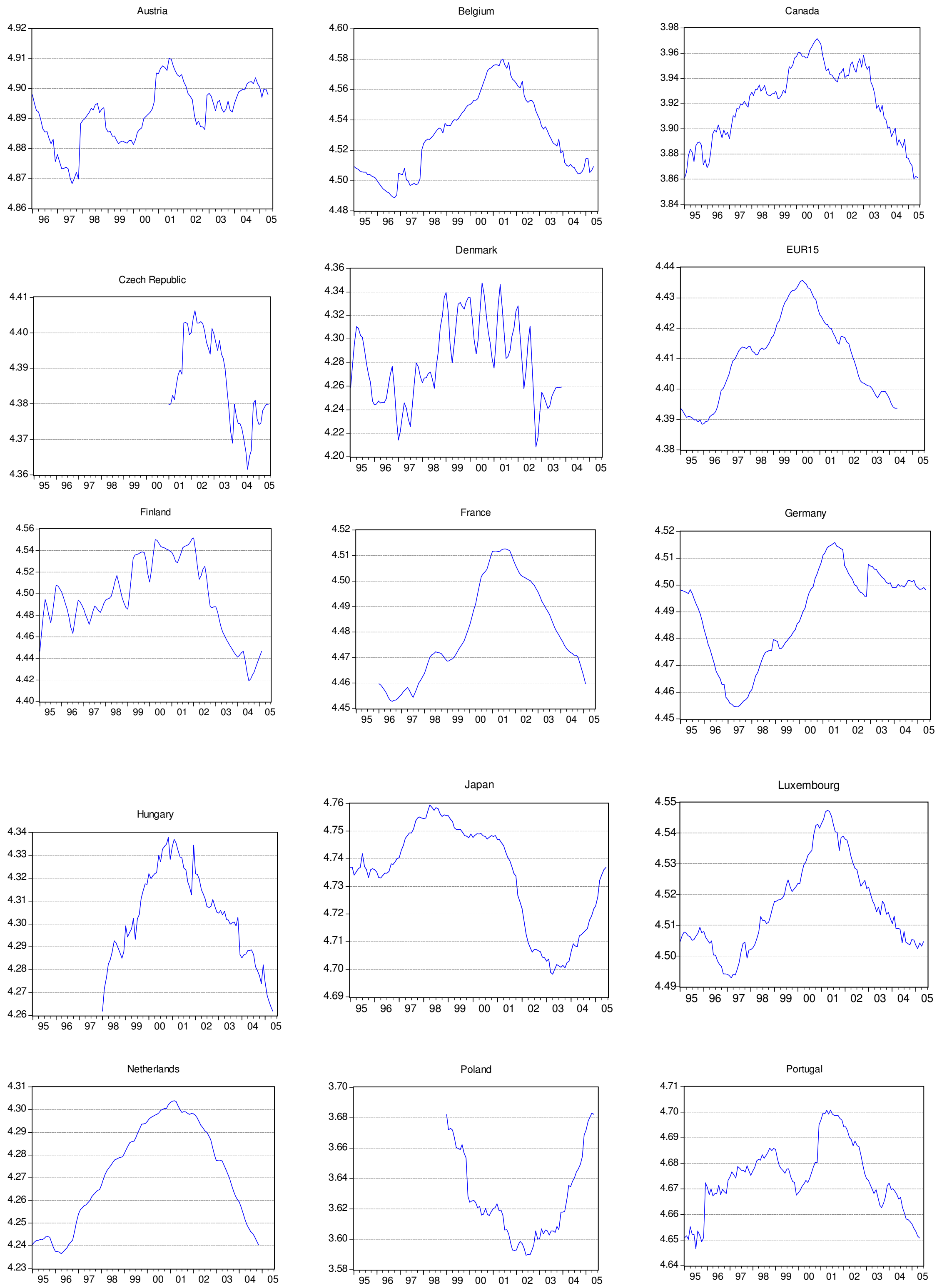


Figure A.5
Time Varying Intercept Estimates of Labor Demand Equation with Kalman Filter
(with aggregate data from EUROSTAT and OECD)



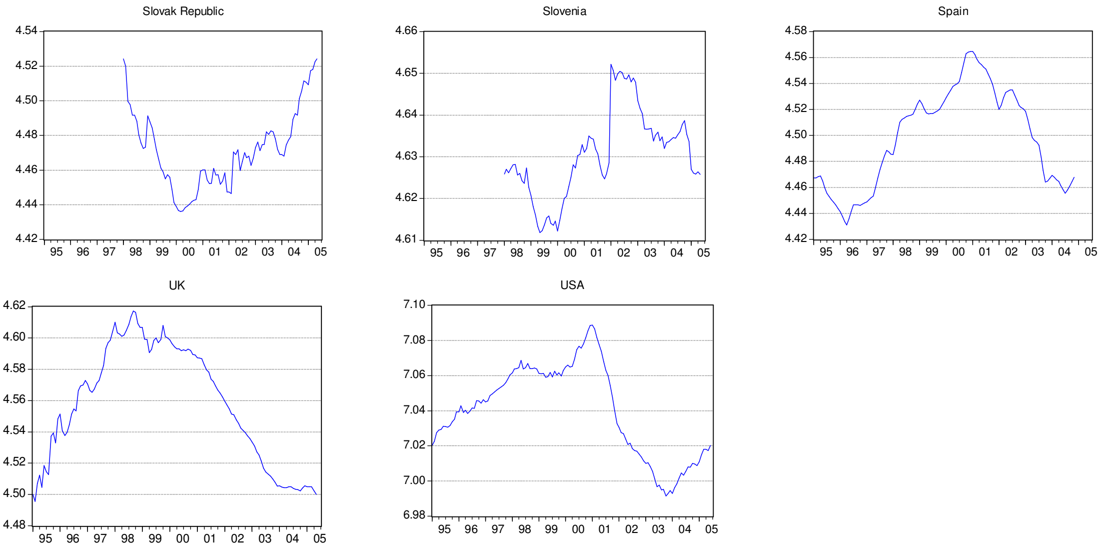

Slovak Republic
Slovenia
Spain
UK
USA

Table A.1.
Summary Characteristics of the Data

Variable	Mean	Std. Dev.	Min	Max
Employment	148.016	307.32	1	8075.00
Total Hours of Work	22346.48	46958.38	0	7707874.20
Sales	1176.26	10056.12	0	615 250. 96
Real Wages	2414.61	94266.47	0	477218282.93

Table A.2
Monthly Net Employment Changes

$\frac{\Delta n_t}{\frac{(n_{t-1}+n_t)}{2}}$ *	Number of Observations	Unconditional Frequency	Cumulated Frequency	Frequency Conditional on: $\frac{\Delta n_t}{\frac{(n_{t-1}+n_t)}{2}} \neq 0$	Frequency Conditional on: $\frac{\Delta n_t}{\frac{(n_{t-1}+n_t)}{2}} < 0$	Frequency Conditional on: $\frac{\Delta n_t}{\frac{(n_{t-1}+n_t)}{2}} > 0$
<-0.8	344	0,001	0,001	0,002	0,003	
[-0.80;-0.75[	42	0,000	0,001	0,000	0,000	
[-0.75;-0.70[	42	0,000	0,001	0,000	0,000	
[-0.70;-0.65[	94	0,000	0,002	0,001	0,001	
[-0.65;-0.60[	64	0,000	0,002	0,000	0,001	
[-0.60;-0.55[	70	0,000	0,002	0,000	0,001	
[-0.55;-0.50[	99	0,000	0,002	0,001	0,001	
[-0.50;-0.45[	130	0,000	0,003	0,001	0,001	
[-0.45;-0.40[	131	0,000	0,003	0,001	0,001	
[-0.40;-0.35[	305	0,001	0,004	0,002	0,003	
[-0.35;-0.30[	328	0,001	0,005	0,002	0,003	
[-0.30;-0.25[	525	0,002	0,007	0,003	0,005	
[-0.25;-0.20[	883	0,003	0,010	0,005	0,008	
[-0.20;-0.15[	1888	0,006	0,016	0,010	0,018	
[-0.15;-0.10[	4127	0,013	0,029	0,022	0,039	
[-0.10;-0.05[	15236	0,049	0,077	0,082	0,146	
[-0.05;0.00[	80262	0,256	0,333	0,431	0,768	
[0.00;0.00[	128058	0,408	0,741			
]0.00;0.05]	60468	0,193	0,933	0,325		0,742
]0.05;0.10]	13465	0,043	0,976	0,072		0,165
]0.10;0.15]	3686	0,012	0,988	0,020		0,045
]0.15;0.20]	1596	0,005	0,993	0,009		0,020
]0.20;0.25]	739	0,002	0,995	0,004		0,009
]0.25;0.30]	414	0,001	0,996	0,002		0,005
]0.30;0.35]	232	0,001	0,997	0,001		0,003
]0.35;0.40]	196	0,001	0,998	0,001		0,002
]0.40;0.45]	79	0,000	0,998	0,000		0,001
]0.45;0.50]	117	0,000	0,998	0,001		0,001
]0.50;0.55]	67	0,000	0,999	0,000		0,001
]0.55;0.60]	49	0,000	0,999	0,000		0,001
]0.60;0.65]	41	0,000	0,999	0,000		0,001
]0.65;0.70]	53	0,000	0,999	0,000		0,001
]0.70;0.75]	26	0,000	0,999	0,000		0,000
]0.75;0.80]	32	0,000	0,999	0,000		0,000
>0.8	231	0,001	1,000	0,001		0,003
	#Obs: 314 119**					

* Employment growth rates as percentage of the period's employment average. Extreme values (-2,2) were excluded as they do not necessarily represent shutdowns or start ups. Interval width was set at 0,05 – all intervals are identified by their mid points.
** An observation is defined as the adjustment of the firm *j* in month *t*.

Table A.3
Monthly Net Employment Changes by Firm Size

$\frac{\Delta n_t}{\frac{(n_{t-1}+n_t)}{2}}$	$n < 20$		$20 \le n < 49$		$50 \le n < 99$		$100 \le n < 199$		$200 \le n < 499$		$n \ge 500$	
	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency
<-0.8	125	0,003	81	0,001	0,001	0,001	50	0,001	20	0,000	9	0,001
[-0.80;-0.75[	7	0,000	14	0,000	0,000	0,000	6	0,000	5	0,000	1	0,000
[-0.75;-0.70[	20	0,000	9	0,000	0,000	0,000	3	0,000	4	0,000	2	0,000
[-0.70;-0.65[	55	0,001	23	0,000	0,000	0,000	9	0,000	4	0,000	1	0,000
[-0.65;-0.60[	12	0,000	16	0,000	0,000	0,000	12	0,000	8	0,000	0	0,000
[-0.60;-0.55[	17	0,000	19	0,000	0,000	0,000	15	0,000	4	0,000	3	0,000
[-0.55;-0.50[	29	0,001	12	0,000	0,000	0,000	21	0,000	12	0,000	3	0,000
[-0.50;-0.45[	56	0,001	27	0,000	0,000	0,000	13	0,000	7	0,000	5	0,000
[-0.45;-0.40[	35	0,001	30	0,000	0,000	0,000	19	0,000	18	0,000	3	0,000
[-0.40;-0.35[	124	0,003	80	0,001	0,001	0,001	32	0,001	23	0,001	3	0,000
[-0.35;-0.30[	110	0,002	88	0,001	0,001	0,001	44	0,001	27	0,001	8	0,001
[-0.30;-0.25[	197	0,004	125	0,001	0,001	0,001	58	0,001	43	0,001	11	0,001
[-0.25;-0.20[	347	0,008	239	0,003	0,002	0,002	87	0,002	69	0,002	16	0,001
[-0.20;-0.15[	784	0,018	564	0,006	0,004	0,004	166	0,003	106	0,003	20	0,001
[-0.15;-0.10[	1445	0,033	1419	0,016	0,009	0,009	345	0,006	249	0,006	52	0,003
[-0.10;-0.05[	4382	0,099	4153	0,048	0,042	0,042	1651	0,030	893	0,021	251	0,016
[-0.05;0.00[	0	0,000	14940	0,171	0,276	0,276	20027	0,362	18176	0,430	8258	0,533
[0.00;0.00[	30142	0,683	45821	0,524	0,383	0,383	15609	0,282	8741	0,207	1574	0,102
]0.00;0.05]	1	0,000	12898	0,148	0,224	0,224	14948	0,270	12427	0,294	4898	0,316
]0.05;0.10]	3698	0,084	4451	0,051	0,039	0,039	1487	0,027	895	0,021	240	0,015
]0.10;0.15]	1183	0,027	1271	0,015	0,008	0,008	361	0,007	244	0,006	60	0,004
]0.15;0.20]	620	0,014	515	0,006	0,003	0,003	132	0,002	95	0,002	23	0,001
]0.20;0.25]	254	0,006	223	0,003	0,002	0,002	78	0,001	65	0,002	10	0,001
]0.25;0.30]	149	0,003	118	0,001	0,001	0,001	50	0,001	30	0,001	7	0,000
]0.30;0.35]	71	0,002	68	0,001	0,000	0,000	25	0,000	27	0,001	10	0,001
]0.35;0.40]	86	0,002	44	0,001	0,000	0,000	22	0,000	13	0,000	3	0,000
]0.40;0.45]	14	0,000	26	0,000	0,000	0,000	12	0,000	10	0,000	4	0,000
]0.45;0.50]	43	0,001	21	0,000	0,000	0,000	14	0,000	16	0,000	4	0,000
]0.50;0.55]	16	0,000	18	0,000	0,000	0,000	14	0,000	6	0,000	2	0,000
]0.55;0.60]	11	0,000	15	0,000	0,000	0,000	7	0,000	4	0,000	2	0,000
]0.60;0.65]	10	0,000	12	0,000	0,000	0,000	8	0,000	1	0,000	2	0,000
]0.65;0.70]	16	0,000	17	0,000	0,000	0,000	7	0,000	5	0,000	0	0,000
]0.70;0.75]	8	0,000	5	0,000	0,000	0,000	5	0,000	3	0,000	0	0,000
]0.75;0.80]	8	0,000	5	0,000	0,000	0,000	9	0,000	6	0,000	0	0,000
>0.8	64	0,001	50	0,001	0,001	0,001	43	0,001	19	0,000	10	0,001
	#Obs:44 139 (14.00%)		#Obs: 88 417 (27.83%)		#Obs: 68 404 (21.78%)		#Obs: 55 389 (17.63%)		#Obs: 42 275 (13.46%)		#Obs: 15 495 (4.93%)	

Table A.4
Monthly Net Employment Changes by Sector

$\frac{\Delta n_t}{\frac{(n_{t-1}+n_t)}{2}}$	Sector									
	13,14	15,16	17,18,19	20,36	21,22	23,24,25,37	26	27	28,29,30,31, 32,33,34,35	40
<-0.8	0,001	0,002	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001
[-0.80;-0.75[	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
[-0.75;-0.70[	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
[-0.70;-0.65[	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
[-0.65;-0.60[	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
[-0.60;-0.55[	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000
[-0.55;-0.50[	0,000	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
[-0.50;-0.45[	0,001	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001
[-0.45;-0.40[	0,000	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
[-0.40;-0.35[	0,002	0,002	0,000	0,001	0,001	0,000	0,001	0,001	0,001	0,000
[-0.35;-0.30[	0,002	0,002	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,000
[-0.30;-0.25[	0,003	0,003	0,001	0,002	0,001	0,001	0,001	0,001	0,002	0,001
[-0.25;-0.20[	0,004	0,005	0,002	0,003	0,002	0,002	0,002	0,002	0,003	0,002
[-0.20;-0.15[	0,010	0,009	0,004	0,006	0,005	0,005	0,006	0,002	0,006	0,003
[-0.15;-0.10[	0,021	0,017	0,011	0,013	0,011	0,012	0,010	0,008	0,014	0,007
[-0.10;-0.05[	0,062	0,051	0,043	0,053	0,050	0,041	0,050	0,035	0,051	0,023
[-0.05;0.00[	0,151	0,215	0,315	0,224	0,234	0,260	0,266	0,300	0,256	0,302
[0.00;0.00[	0,510	0,424	0,364	0,446	0,442	0,416	0,405	0,364	0,389	0,488
]0.00;0.05]	0,137	0,175	0,207	0,180	0,189	0,203	0,198	0,231	0,199	0,141
]0.05;0.10]	0,060	0,048	0,034	0,048	0,044	0,040	0,039	0,036	0,047	0,019
]0.10;0.15]	0,019	0,016	0,009	0,010	0,011	0,010	0,010	0,008	0,014	0,006
]0.15;0.20]	0,007	0,009	0,003	0,005	0,004	0,003	0,004	0,004	0,006	0,002
]0.20;0.25]	0,004	0,005	0,001	0,002	0,001	0,002	0,002	0,002	0,003	0,002
]0.25;0.30]	0,002	0,003	0,000	0,001	0,001	0,001	0,001	0,001	0,002	0,000
]0.30;0.35]	0,001	0,002	0,001	0,000	0,000	0,000	0,000	0,001	0,001	0,000
]0.35;0.40]	0,001	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,000
]0.40;0.45]	0,000	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
]0.45;0.50]	0,001	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
]0.50;0.55]	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
]0.55;0.60]	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
]0.60;0.65]	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
]0.65;0.70]	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
]0.70;0.75]	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
]0.75;0.80]	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
>0.8	0,000	0,002	0,000	0,001	0,001	0,000	0,000	0,001	0,001	0,000
	4.36%	13.11%	20.73%	12.10%	6.20%	8.20%	8.89%	2.33%	23.29%	0.8%

Table A.5
Monthly Sales Changes

$\frac{\Delta s_t}{\frac{(s_{t-1}+s_t)}{2}}$	Number of Observations	Unconditional Frequency	Cumulated Frequency	Frequency Conditional on: $\frac{\Delta s_t}{\frac{(s_{t-1}+s_t)}{2}} \neq 0$	Frequency Conditional on: $\frac{\Delta s_t}{\frac{(s_{t-1}+s_t)}{2}} < 0$	Frequency Conditional on: $\frac{\Delta s_t}{\frac{(s_{t-1}+s_t)}{2}} > 0$
<-0.8	17408	0,055	0,055	0,055	0,113	
[-0.80;-0.75[	2048	0,007	0,062	0,007	0,013	
[-0.75;-0.70[	2264	0,007	0,069	0,007	0,015	
[-0.70;-0.65[	2536	0,008	0,077	0,008	0,016	
[-0.65;-0.60[	2938	0,009	0,087	0,009	0,019	
[-0.60;-0.55[	3437	0,011	0,098	0,011	0,022	
[-0.55;-0.50[	3967	0,013	0,110	0,013	0,026	
[-0.50;-0.45[	4595	0,015	0,125	0,015	0,030	
[-0.45;-0.40[	5627	0,018	0,143	0,018	0,037	
[-0.40;-0.35[	4424	0,014	0,157	0,014	0,029	
[-0.35;-0.30[	7749	0,025	0,181	0,025	0,050	
[-0.30;-0.25[	9615	0,031	0,212	0,031	0,063	
[-0.25;-0.20[	11602	0,037	0,249	0,037	0,075	
[-0.20;-0.15[	14394	0,046	0,295	0,046	0,094	
[-0.15;-0.10[	17150	0,055	0,349	0,055	0,112	
[-0.10;-0.05[	20160	0,064	0,414	0,064	0,131	
[-0.05;0.00[	23832	0,076	0,489	0,076	0,155	
[0.00;0.00[	171	0,001	0,490			
]0.00;0.05]	22927	0,073	0,563	0,073		0,143
]0.05;0.10]	20744	0,066	0,629	0,066		0,129
]0.10;0.15]	17917	0,057	0,686	0,057		0,112
]0.15;0.20]	15022	0,048	0,734	0,048		0,094
]0.20;0.25]	14422	0,046	0,780	0,046		0,090
]0.25;0.30]	10229	0,033	0,812	0,033		0,064
]0.30;0.35]	8336	0,027	0,839	0,027		0,052
]0.35;0.40]	6917	0,022	0,861	0,022		0,043
]0.40;0.45]	5638	0,018	0,879	0,018		0,035
]0.45;0.50]	4865	0,015	0,894	0,015		0,030
]0.50;0.55]	3951	0,013	0,907	0,013		0,025
]0.55;0.60]	3436	0,011	0,918	0,011		0,021
]0.60;0.65]	3019	0,010	0,927	0,010		0,019
]0.65;0.70]	2626	0,008	0,936	0,008		0,016
]0.70;0.75]	2196	0,007	0,943	0,007		0,014
]0.75;0.80]	2028	0,006	0,949	0,006		0,013
>0.8	15929	0,051	1,000	0,051		0,099
	#Obs: 314 119					

Table A.6
Monthly Sales Changes by Firm Size

$\frac{\Delta s_t}{\frac{(s_{t-1}+s_t)}{2}}$	$n < 20$		$20 \le n < 49$		$50 \le n < 99$		$100 \le n < 199$		$200 \le n < 499$		$n \ge 500$	
	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency
<-0.8	3051	0,0752	4820	0,0593	3262	0,0516	2288	0,0446	1575	0,0401	503	0,0346
[-0.80;-0.75[	318	0,0078	564	0,0069	361	0,0057	271	0,0053	204	0,0052	78	0,0054
[-0.75;-0.70[	385	0,0095	572	0,0070	436	0,0069	310	0,0060	218	0,0055	71	0,0049
[-0.70;-0.65[	364	0,0090	725	0,0089	484	0,0077	360	0,0070	291	0,0074	82	0,0056
[-0.65;-0.60[	467	0,0115	785	0,0097	601	0,0095	401	0,0078	324	0,0082	71	0,0049
[-0.60;-0.55[	558	0,0138	880	0,0108	696	0,0110	490	0,0095	379	0,0096	102	0,0070
[-0.55;-0.50[	573	0,0141	1073	0,0132	785	0,0124	550	0,0107	429	0,0109	250	0,0172
[-0.50;-0.45[	678	0,0167	1221	0,0150	903	0,0143	679	0,0132	519	0,0132	161	0,0111
[-0.45;-0.40[	781	0,0193	1572	0,0193	1158	0,0183	820	0,0160	630	0,0160	189	0,0130
[-0.40;-0.35[	857	0,0211	1736	0,0213	1285	0,0203	993	0,0193	765	0,0195	231	0,0159
[-0.35;-0.30[	959	0,0236	2020	0,0248	1613	0,0255	1223	0,0238	913	0,0232	319	0,0220
[-0.30;-0.25[	1154	0,0285	2512	0,0309	1991	0,0315	1580	0,0308	1140	0,0290	405	0,0279
[-0.25;-0.20[	1344	0,0331	2942	0,0362	2371	0,0375	1967	0,0383	1464	0,0372	519	0,0357
[-0.20;-0.15[	1665	0,0410	3613	0,0444	2895	0,0458	2477	0,0483	1839	0,0468	711	0,0489
[-0.15;-0.10[	1900	0,0468	4218	0,0519	3367	0,0532	3012	0,0587	2321	0,0590	891	0,0613
[-0.10;-0.05[	2251	0,0555	4831	0,0594	3927	0,0621	3528	0,0687	2774	0,0706	1160	0,0799
[-0.05;0.00[	2706	0,0667	5738	0,0706	4556	0,0720	3987	0,0777	3296	0,0838	1417	0,0975
[0.00;0.00[	351	0,0087	649	0,0080	563	0,0089	469	0,0091	372	0,0095	135	0,0093
]0.00;0.05]	2643	0,0652	5419	0,0666	4280	0,0677	3853	0,0751	3229	0,0821	1290	0,0888
]0.05;0.10]	2277	0,0561	4795	0,0590	4024	0,0636	3499	0,0682	2880	0,0732	1133	0,0780
]0.10;0.15]	1860	0,0459	4185	0,0515	3545	0,0560	3068	0,0598	2444	0,0622	931	0,0641
]0.15;0.20]	1628	0,0401	3781	0,0465	3006	0,0475	2587	0,0504	1897	0,0482	740	0,0509
]0.20;0.25]	1376	0,0339	3107	0,0382	2567	0,0406	2102	0,0410	1593	0,0405	587	0,0404
]0.25;0.30]	1206	0,0297	2752	0,0338	2111	0,0334	1683	0,0328	1237	0,0315	476	0,0328
]0.30;0.35]	1039	0,0256	2214	0,0272	1765	0,0279	1453	0,0283	973	0,0247	338	0,0233
]0.35;0.40]	896	0,0221	1933	0,0238	1510	0,0239	1093	0,0213	824	0,0210	252	0,0173
]0.40;0.45]	748	0,0184	1553	0,0191	1247	0,0197	919	0,0179	642	0,0163	189	0,0130
]0.45;0.50]	711	0,0175	1372	0,0169	1040	0,0164	766	0,0149	578	0,0147	160	0,0110
]0.50;0.55]	605	0,0149	1117	0,0137	871	0,0138	621	0,0121	451	0,0115	127	0,0087
]0.55;0.60]	554	0,0137	963	0,0118	721	0,0114	533	0,0104	400	0,0102	112	0,0077
]0.60;0.65]	493	0,0122	849	0,0104	630	0,0100	441	0,0086	344	0,0087	113	0,0078
]0.65;0.70]	406	0,0100	715	0,0088	551	0,0087	442	0,0086	289	0,0074	92	0,0063
]0.70;0.75]	374	0,0092	597	0,0073	490	0,0077	301	0,0059	242	0,0062	92	0,0063
]0.75;0.80]	343	0,0085	572	0,0070	422	0,0067	287	0,0056	236	0,0060	79	0,0054
>0.8	3040	0,0749	4917	0,0605	3229	0,0510	2277	0,0444	1606	0,0408	520	0,0358
	#Obs:40 561 (13.98%)		#Obs: 81 312 (28.02%)		#Obs: 63 263 (21.80%)		#Obs: 51 330 (17.69%)		#Obs: 39 318 (13.55%)		#Obs: 14 426 (4.97%)	

Table A.7
Monthly Hours per Employment Changes

$\frac{\Delta h_t}{\frac{(h_{t-1}+h_t)}{2}}$	Number of Observations	Unconditional Frequency	Cumulated Frequency	Frequency Conditional on: $\frac{\Delta h_t}{\frac{(h_{t-1}+h_t)}{2}} \neq 0$	Frequency Conditional on: $\frac{\Delta h_t}{\frac{(h_{t-1}+h_t)}{2}} < 0$	Frequency Conditional on: $\frac{\Delta h_t}{\frac{(h_{t-1}+h_t)}{2}} > 0$
<-0.8	6411	0,020	0,020	0,022	0,043	
[-0.80;-0.75[	763	0,002	0,023	0,003	0,005	
[-0.75;-0.70[	887	0,003	0,026	0,003	0,006	
[-0.70;-0.65[	952	0,003	0,029	0,003	0,006	
[-0.65;-0.60[	689	0,002	0,031	0,002	0,005	
[-0.60;-0.55[	626	0,002	0,033	0,002	0,004	
[-0.55;-0.50[	701	0,002	0,035	0,002	0,005	
[-0.50;-0.45[	825	0,003	0,038	0,003	0,006	
[-0.45;-0.40[	1013	0,003	0,041	0,003	0,007	
[-0.40;-0.35[	1471	0,005	0,046	0,005	0,010	
[-0.35;-0.30[	2093	0,007	0,052	0,007	0,014	
[-0.30;-0.25[	3137	0,010	0,062	0,011	0,021	
[-0.25;-0.20[	4699	0,015	0,077	0,016	0,032	
[-0.20;-0.15[	9272	0,030	0,107	0,032	0,063	
[-0.15;-0.10[	17262	0,055	0,162	0,059	0,117	
[-0.10;-0.05[	27797	0,088	0,250	0,095	0,188	
[-0.05;0.00[	69386	0,221	0,471	0,238	0,469	
[0.00;0.00[	22555	0,072	0,543			
]0.00;0.05]	70416	0,224	0,767	0,242		0,490
]0.05;0.10]	24525	0,078	0,845	0,084		0,171
]0.10;0.15]	13490	0,043	0,888	0,046		0,094
]0.15;0.20]	10011	0,032	0,920	0,034		0,070
]0.20;0.25]	5714	0,018	0,938	0,020		0,040
]0.25;0.30]	3513	0,011	0,949	0,012		0,024
]0.30;0.35]	2020	0,006	0,956	0,007		0,014
]0.35;0.40]	1662	0,005	0,961	0,006		0,012
]0.40;0.45]	1119	0,004	0,965	0,004		0,008
]0.45;0.50]	793	0,003	0,967	0,003		0,006
]0.50;0.55]	655	0,002	0,969	0,002		0,005
]0.55;0.60]	696	0,002	0,971	0,002		0,005
]0.60;0.65]	678	0,002	0,974	0,002		0,005
]0.65;0.70]	933	0,003	0,977	0,003		0,006
]0.70;0.75]	832	0,003	0,979	0,003		0,006
]0.75;0.80]	582	0,002	0,981	0,002		0,004
>0.8	5941	0,019	1,000	0,020		0,041
	#Obs: 314 119					

Table A.8
Monthly Hours per Worker Change by Firm Size

$\frac{\Delta h_t}{\frac{(h_{t-1}+h_t)}{2}}$	$n<20$		$20 \le n < 49$		$50 \le n < 99$		$100 \le n < 199$		$200 \le n < 499$		$n \ge 500$	
	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency	Number Obs.	Unconditional Frequency
<-0.8	623	0,0154	1616	0,0199	1212	0,0236	1212	0,0236	851	0,0216	295	0,0206
[-0.80;-0.75[	70	0,0017	178	0,0022	132	0,0026	132	0,0026	109	0,0028	46	0,0032
[-0.75;-0.70[	96	0,0024	208	0,0026	134	0,0026	134	0,0026	108	0,0027	39	0,0027
[-0.70;-0.65[	90	0,0022	216	0,0027	174	0,0034	174	0,0034	128	0,0033	39	0,0027
[-0.65;-0.60[	60	0,0015	154	0,0019	122	0,0024	122	0,0024	117	0,0030	38	0,0027
[-0.60;-0.55[	71	0,0018	142	0,0017	116	0,0023	116	0,0023	84	0,0021	27	0,0019
[-0.55;-0.50[	75	0,0018	155	0,0019	119	0,0023	119	0,0023	116	0,0030	47	0,0033
[-0.50;-0.45[	94	0,0023	180	0,0022	134	0,0026	134	0,0026	123	0,0031	45	0,0031
[-0.45;-0.40[	111	0,0027	219	0,0027	179	0,0035	179	0,0035	159	0,0040	85	0,0059
[-0.40;-0.35[	161	0,0040	353	0,0043	243	0,0047	243	0,0047	209	0,0053	95	0,0066
[-0.35;-0.30[	249	0,0061	467	0,0057	379	0,0074	379	0,0074	297	0,0076	158	0,0110
[-0.30;-0.25[	355	0,0088	760	0,0093	552	0,0108	552	0,0108	465	0,0118	246	0,0172
[-0.25;-0.20[	574	0,0142	1144	0,0141	754	0,0147	754	0,0147	702	0,0179	326	0,0228
[-0.20;-0.15[	1186	0,0292	2501	0,0308	1489	0,0290	1489	0,0290	1177	0,0299	466	0,0325
[-0.15;-0.10[	2312	0,0570	4639	0,0571	2707	0,0527	2707	0,0527	2062	0,0524	950	0,0663
[-0.10;-0.05[	3994	0,0985	7927	0,0975	4233	0,0825	4233	0,0825	3227	0,0821	1372	0,0958
[-0.05;0.00[	6911	0,1704	16972	0,2087	12689	0,2472	12689	0,2472	9614	0,2445	3030	0,2115
[0.00;0.00[	7304	0,1801	6728	0,0827	1476	0,0288	1476	0,0288	1164	0,0296	258	0,0180
]0.00;0.05]	6983	0,1722	17337	0,2132	12492	0,2434	12492	0,2434	9305	0,2367	2927	0,2043
]0.05;0.10]	3230	0,0796	6692	0,0823	3668	0,0715	3668	0,0715	2705	0,0688	1169	0,0816
]0.10;0.15]	1808	0,0446	3588	0,0441	2166	0,0422	2166	0,0422	1686	0,0429	708	0,0494
]0.15;0.20]	1396	0,0344	2793	0,0343	1551	0,0302	1551	0,0302	1223	0,0311	560	0,0391
]0.20;0.25]	640	0,0158	1461	0,0180	990	0,0193	990	0,0193	845	0,0215	257	0,0179
]0.25;0.30]	419	0,0103	867	0,0107	638	0,0124	638	0,0124	488	0,0124	240	0,0168
]0.30;0.35]	208	0,0051	457	0,0056	370	0,0072	370	0,0072	309	0,0079	134	0,0094
]0.35;0.40]	193	0,0048	398	0,0049	284	0,0055	284	0,0055	245	0,0062	111	0,0077
]0.40;0.45]	126	0,0031	254	0,0031	213	0,0041	213	0,0041	167	0,0042	71	0,0050
]0.45;0.50]	93	0,0023	200	0,0025	133	0,0026	133	0,0026	133	0,0034	47	0,0033
]0.50;0.55]	73	0,0018	142	0,0017	128	0,0025	128	0,0025	109	0,0028	43	0,0030
]0.55;0.60]	82	0,0020	158	0,0019	129	0,0025	129	0,0025	106	0,0027	43	0,0030
]0.60;0.65]	64	0,0016	173	0,0021	119	0,0023	119	0,0023	106	0,0027	44	0,0031
]0.65;0.70]	104	0,0026	231	0,0028	164	0,0032	164	0,0032	128	0,0033	53	0,0037
]0.70;0.75]	95	0,0023	219	0,0027	166	0,0032	166	0,0032	123	0,0031	47	0,0033
]0.75;0.80]	61	0,0015	131	0,0016	123	0,0024	123	0,0024	97	0,0025	31	0,0022
>0.8	650	0,0160	1652	0,0203	1152	0,0224	1152	0,0224	831	0,0211	279	0,0195
	#Obs: 13.98%		#Obs: 28.02%		#Obs: 21.80%		#Obs: 17.69%		#Obs: 13.55%		#Obs: 4.97%	

Table A.9
Augmented Dickey-Fuller Unit Root Tests
(Aggregate series based on micro data from Portuguese industrial firms)

	t_{ADF}-statistic*				
	N_t	S_t	W_t	HS_t	$SPURT_t$
Whole Sample	-0.489	-2.214	-2.650	-2.186	-1.689
Small Firms	-1.310	-2.701	-1.903	-2.599	-2.689
Large Firms	-1.394	-2.718	-3.219	-2.078	-2.417

Notes: The Augmented Dickey-Fuller Test is specified as:

$\Delta y_t = \alpha + \beta_0 y_{t-1} + \sum_{i=1}^{k} \beta_i \Delta y_{t-i} + \varepsilon_t$, with the number of lags k chosen according tothe Schwarz Criterion. The null hypothesis is that the series are not stationary.
* 5% Critical Value = -3.444

Table A.10
Results of the Preisach Model

Dependent Variables	Dependent variable – Logarithm of Total Hours of Work (TH_t) - Sample: 1995:01-2005:12								
	Aggregate series for all sample			Aggregate series for firms with fewer than 20 workers			Aggregate series for firms with more than 500 workers		
$Cons$	**10.850** 14.087 (0.000)	**12.281** 11.590 (0.000)	**17.148** 4586 (0.000)	**7.482** 13.845 (0.000)	**11.759** 10.293 (0.000)	**12.11** 1238 (0.000)	**11.82** 10.32 (0.000)	**11.27** 7.525 (0.000)	**16.14** 2623 (0.000)
S_t	**0.295** 8.180 (0.000)	**0.228** 4.593 (0.000)	-	**0.301** 8.699 (0.000)	**0.022** 0.303 (0.761)	-	**0.207** 3.769 (0.000)	**0.234** 3.251 (0.002)	-
HS_t	-	**0.0003** 1.944 (0.054)	**0.001** 6.645 (0.000)	-	**0.001** 4.182 (0.000)	**0.001** 10.172 (0.000)	-	**-0.0001** -0.573 (0.597)	**0.0004** 1.903 (0.059)
T	**-0.002** -27.18 (0.000)	**-0.002** -20.825 (0.000)	**-0.002** -19.961 (0.000)	**-0.002** -17.604 (0.000)	**-0.002** -18.393 (0.000)	**-0.002** -21.57 (0.000)	**-0.002** -13.724 (0.000)	**-0.002** -11.011 (0.000)	**-0.002** -10.213 (0.000)
R^2	0.893	0.896	0.879	0.781	0.807	0.807	0.779	0.779	0.761
DW	0.831	0.904	1.148	0.829	0.755	0.752	0.819	0.817	0.861
Engle Granger Cointegration Test Statistic	-3.353	-3.617	-2.774	-3.0481	-2.960	-1.922	-3.553	-3.534	-3.585
MacKinnon 5% Critical Value	-3.553	4.211	-3.553	-3.553	4.211	-3.553	-3.553	4.211	-3.553
Trace Test Statistic	13.880	31.78	13.20	18.41	42.18	15.42	20.58	46.02	16.49
5% Critical Value	25.87	42.91	25.87	25.87	42.91	25.87	25.87	42.91	25.87

We report for each variable the estimated coefficient, the *t*-statistic and the p-value respectively.

Table A.11
Results of the Linear Play Model

Dependent Variables	Dependent variable – Logarithm of Aggregate Total Hours of Work (TH_t) - Sample: 1995:01-2005:12								
	Aggregate series for all sample			**Aggregate series for firms with fewer than 20 workers**			**Aggregate series for firms with more than 500 workers**		
$Cons$	**10.850** 14.087 (0.000)	**11.628** 11.101 (0.000)	**17.205** 1825 (0.000)	**7.482** 13.845 (0.000)	**10.505** 13.137 (0.000)	**12.195** 1969 (0.000)	**11.82** 10.32 (0.000)	**8.821** 4.238 (0.000)	**17.189** 1559.5 (0.000)
S_t	**0.295** 8.180 (0.000)	**0.259** 5.325 (0.000)	-	**0.301** 8.699 (0.000)	**0.108** 2.114 (0.037)	-	**0.207** 3.769 (0.000)	**0.349** 3.532 (0.006)	-
$SPURT_t$	-	**0.070** 1.241 (0.217)	**0.290** 6.708 (0.000)	-	**0.225** 3.306 (0.001)	**0.341** 8.270 (0.000)	-	**-0.151** -1.344 (0.181)	**0.192** 4.393 (0.000)
T	**-0.002** -27.18 (0.000)	**-0.002** -25.260 (0.000)	**-0.002** -22.384 (0.000)	**-0.002** -17.604 (0.000)	**-0.001** -17.916 (0.000)	**-0.001** -18.161 (0.000)	**-0.002** -13.724 (0.000)	**-0.002** -11.917 (0.000)	**-0.002** -15.402 (0.000)
R^2	0.893	0.892	0.867	0.781	0.779	0.771	0.779	0.767	0.839
DW	0.831	1.068	1.411	0.829	0.922	0.922	0.819	0.965	1.379
Engle Granger Cointegration Test Statistic	-3.353	-4.108	-3.874	-3.0481	-3.724	-3.760	-3.553	3.534	-3.419
MacKinnon 5% Critical Value	-3.553	4.211	-3.553	-3.553	4.211	-3.553	-3.553	4.211	-3.553
Trace Test Statistic	13.880	40.20	25.504	18.41	42.85	17.08	20.58	43.83	28.189
5% Critical Value	25.87	42.91	25.87	25.87	42.91	25.87	25.87	42.91	25.87

We report for each variable the estimated coefficient, the *t*-statistic and the p-value respectively. * Significant at 5%

Table A.12
Augmented Dickey-Fuller Unit Root Tests
(Manufacturing aggregate series from EUROSTAT and OECD)

Country		N_t	S_t	W_t	HS_t	$SPURT_t$
Austria	t-statistic	-2.458	-1.506	-1.903	0.346	-0.394
	5% critical-value	-2.887	-2.887	-2.887	-2.887	-2.888
Belgium	t-statistic	1.123	-1.562	-	-0.797	-0.700
	5% critical value	-2.884	-2.884	-	-2.884	-2.885
Canada	t-statistic	-2.526	-0.850	-3.860	-1.157	-1.498
	5% critical value	-2.886	-2.884	-2.884	-2.885	-2.885
Czech Republic	t-statistic	-0.961	0.237	0.518	2.232	-0.003
	5% critical value	-2.9165	-2.917	-2.840	-2.917	-2.917
Denmark	t-statistic	-0.585	-2.922	-1.621	-1.395	-1.609
	5% critical value	-2.840	-2.884	-2.884	-2.884	-2.889
Finland	t-statistic	-1.322	-1.632	-2.258	-0.545	1.229
	5% critical value	-2.887	-2.887	-2.884	-2.884	-2.885
France	t-statistic	1.629	-1.323	-1.463	-0.875	-0.756
	5% critical value	-2.885	-2.884	-2.889	-2.884	-2.886
Germany	t-statistic	-1.590	-0.225	-0.988	1.015	0.121
	5% critical value	-2.885	-2.884	-2.884	-2.884	-2.885
Hungary	t-statistic	0.668	-1.661	-2.978	-0.177	-1.791
	5% critical value	-2.894	-2.895	-2.895	-2.899	-2.894
Japan	t-statistic	-0.672	-2.625	-0.063	-2.333	-2.31
	5% critical value	-2.885	-2.885	-2.885	-2.885	-2.885
Luxembourg	t-statistic	-1.365	-0.771	-1.437	0.011	-0.020
	5% critical value	-2.884	-2.885	-2.885	-2.884	-2.885
Netherlands	t-statistic	-0.212	-2.062	-1.464	-1.675	-
	5% critical value	-2.886	-2.885	-2.887	-2.885	-
Poland	t-statistic	-2.49	-1.037	0.314	0.629	-1.100
	5% critical value	-2.884	-2.889	-2.885	-2.899	-2.884
Portugal	t-statistic	2.195	-2.876	1.114	-1.790	-2.702
	5% critical value	-2.884	-2.885	-2.885	-2.884	-2.885
Slovak Republic	t-statistic	-2.205	0.362	-0.770	1.086	+0.853
	5% critical value	-2.893	-2.894	-2.887	-2.893	-2.895
Slovenia	t-statistic	-0.701	-0860	-0.428	-0.213	-0.449
	5% critical value	-2.894	-2.894	-2.894	-2.893	-2.895
Spain	t-statistic	-1.894	-1.510	-	-1.526	-0.832
	5% critical value	-2.887	-2.884	-	-2.884	-2.886
UK	t-statistic	2.591	-2.273	-	-1.996	-0.978
	5% critical value	-2.884	-2.884	-	-2.884	-2.886
USA	t-statistic	-0.749	-2.115	-1.243	-1.336	-2.151
	5% critical value	-2.885	-2.885	-2.884	-2.885	-2.885

Notes: The Augmented Dickey-Fuller Test is specified as: $\Delta y_t = \alpha + \beta_0 y_{t-1} + \sum_{i=1}^{k} \beta_i \Delta y_{t-i} + \varepsilon_t$, with the number of lags k chosen according to the Schwarz Criterion. The null hypothesis is that the series are not stationary.

Table. A.13
Activity Sectors and Code Numbers

Activity Sector	Code Numbers
Mining	13 and 14
Food, Tobacco and Beverages	15 and 16
Textile, Leather and Shoes	17,18 and 19
Furniture and Wood	20 and 36
Paper and Printing	21 and 22
Chemicals, Petroleum and Rubber and Plastic Products	23, 24, 25 and 37
Non Metallic Mineral Products	26
Primary Metals	27
Machinery, Fabricated Metals, Motors and Cars and Other Transport Material	28, 29, 30, 31, 32, 33, 34 and 35
Electricity and Gas	40

Table. A.14
List of Variables

Variable	Description
N_t :	Aggregate employment.
Y_t :	Aggregate real product.
S_t :	Aggregate sales.
H_t :	Average number of hours of work per employee.
TH_t :	Total number of hours of work.
P_t :	Aggregate price level.
W_t :	Aggregate wage rate.
HY_t	Hysteresis transformation of real product
HS_t	Hysteresis transformation of sales
T	Time trend.
n_t :	Firm level of employment.
y_t :	Firm level of output.
h_t :	Firm average number of hours of work per employee.
th_t :	Firm total number of hours of work.
h_d :	Firm minimum allowed number of hours per employee.
h_u :	Firm maximum allowed number of hours per employee.
w_t :	Firm wage rate.
H_j :	Fixed hiring cost.
F_j :	Fixed firing cost.
i :	Nominal interest rate.
μ :	Nonrecurring single stochastic change in price.
δ :	Time discount factor.
ψw	Measure of phase (hysteresis transformation of real wages)
ψs	Measure of phase (hysteresis transformation of sales)
ϕ_u :	Measure of the tightness of overtime regulations.
ϕ_d :	Measure of downward hours flexibility.
V_t :	Net present value.
$PLAY_t$:	Proxy of the band of inaction estimated according to algorithm in section IV.2.2.2.
$SPURT_t$	Filtered variable calculated according to algorithm in section IV.2.2.2.
σ_{S_t}	Proxy of uncertainty.
HI_t	Proxy of the flexibility of the adjustment of hours of work.

Program 1
Program to Compute Strong Hysteresis Transformation of Input Variables according to the Preisach Model (written in MATLAB)

```
%-----------------------------------------------------------------
% STRONG HYSTERESIS (FOR LABOR ECONOMICS)
%-----------------------------------------------------------------
% based on     : A test for strong hysteresis,
%                by L. Piscitelli, R. Cross, M. Grinfeld and H.
                Lamba
%                Computational Economics 15: 59-78, 2000
% written by   : Paulo Vasconcelos and Paulo Mota
% first version: 30/01/2006
% last revision: 09/03/2006
% INPUT        : v      = time series vector
% OUTPUT       : T(t)   = trapezoidal areas at period t
%                T_P(t) = T(t) in percentage
%                 at the end creates the output file "res.txt" and
                plots        the hysteresis transformation

%-----------------------------------------------------------------

% --- get data and initial information ---------------------------
-----
clear; load v.dat; n=length(v); a0=abs(max(v)); b0=abs(min(v));
M=zeros(1,n); m=M; T=M; tp=M; tm=M; % preallocating for speed

% --- compute areas -----------------------------------------------
for t=1:n
   k=1; [M(k),tp(k)]=max(v(1:t));
   if   min(v) > 0, T(t)=(M(k)-b0)^2/2;
   else T(t)=(M(k)+b0)^2/2;
   end
   tt=tp(k);
   while (tt<t),
      [m(k),tm(k)]=min(v(tp(k):t)); tm(k)=tm(k)+tp(k)-1;
      T(t)=T(t)-(M(k)-m(k))^2/2;
      if   tm(k)<t
           [M(k+1),tp(k+1)]=max(v(tm(k):t));
tp(k+1)=tp(k+1)+tm(k)-1;
           T(t)=T(t)+(M(k+1)-m(k))^2/2; tt=tp(k+1); k=k+1;
      else tt=tm(k); k=k+1;
      end
   end
end

% --- compute T in percentage and writes the result in res.txt
file ---
T_P_total=max(T); T_P=(T/T_P_total)*100;
fid = fopen('res.txt','wt'); fprintf(fid,'%12.8f\n',T_P);
fclose(fid);

% --- hysteresis plot ---------------------------------------------
plot(v,T_P); xlabel('input'); ylabel('hysteresis transformation of
input')
 %-----------------------------------------------------------------
```

Program 2
Program to Compute Strong Hysteresis Transformation of Input Variables according to the Göcke Model (Written in MATLAB)

```
%% LINEAR PLAY HYSTERESIS IN A REGRESSION FRAMEWORK (FOR LABOR
ECONOMICS)

%------------------------------------------------------------------
% based on     : Exchange rate uncertainty and employment:
%                an algorithm describing 'play',
%                by Ansgar Belke and Matthias Gocke, 2001
%                Applied Stochastic Models in Business and
               Industry 17, pp. 181-204,
% written by    : Paulo Vasconcelos and Paulo Mota
% first version: 30/06/2006
% last revision: 17/05/2007
% INPUT        : x = time series vector (log s), s=sales
%                y = time series vector (log e), e=employment
%                u = time series vector of std of log e
% OUTPUT       : spurt and play
%------------------------------------------------------------------

%% --- get data and initial information -------------------------
disp('... Reading data and initialize')

% read data
load y.dat; % employment log data
load x.dat; % sales log data
load u.dat; % std of employment log data

% variables initialization
m=size(x,1); % sample size
d_x=zeros(m,1); d_x(1)=0; for i=2:m; d_x(i)=(x(i)-x(i-1)); end;

% find first estimation quarter
figure(1); plot(x); ylabel('Sales');
sinal=sign(d_x(2)); j=2;
while sign(d_x(j))==sinal, j=j+1; end
istart=j-1; % extremum is at position j-1
if sinal<0, fprintf('minimum is %6.2f at position
%d\n',x(istart),istart);
else fprintf('maximum is %6.2f at position
%d\n',x(istart),istart); end
iend=m; n=iend-istart+1; % nb. elements in analysis
fprintf('total nb. of elements is %d and nb. of sample points is
%d\n',m,n)

% define grid for const. play
%g_prec=0.002; g_min=0; g_max=0.2; g=g_max/g_prec; % for const
play
disp(' ');
disp('introduce data for constant play:');
g_min=input('min. (default= 0.0) -> ');
if isempty(g_min); g_min=0.0; end
g_max=input('max. (default= 0.2) -> ');
if isempty(g_max); g_max=0.2; end
g_prec=input('grid precision (default= 0.002) -> ');
```

```
if isempty(g_prec); g_prec=0.002; end
g=g_max/g_prec;   % nb of points for const play

% define grid for the var. play
%h_prec=0.002; h_min=0.0; h_max=0.2; h=h_max/h_prec; % for var.
play
disp(' ');
disp('introduce data for variable play:');
disp('note: to look only for constant play, choose grid
precision=max')
h_min=input('min. for var. play (default= 0.0) -> ');
if isempty(h_min); h_min=0.0; end
h_max=input('max. for var. play (default= 0.2) -> ');
if isempty(h_max); h_max=0.2; end
h_prec=input('grid precision for var. play (default= 0.002) -> ');
if isempty(h_prec); h_prec=0.002; end
h=h_max/h_prec;   % nb of points for var. play
if h == 1, u=zeros(m,1); end; % special case to look only for
constant play

% initialize more auxiliar variables
play_const=zeros(1,g+1); play_var=zeros(1,h+1);
R2=zeros(g+1,h+1); betas=zeros(g+1,h+1,3);

%% --- Grid search ---------------------------------------------
disp('... Grid search')
for i=1:g+1      % loop for constant play
    for j=1:h+1  % loop for variable play
        gamma=g_min+(i-1)*g_prec; delta=h_min+(j-1)*h_prec;
        play(istart:iend)=gamma+delta*u(istart:iend);
        play_const(i,1)=gamma; play_var(1,j)=delta;
        spurt = play_fun(play,d_x,istart,iend,sinal);
        % perform R2 computation
        X=[ones(n,1) spurt(istart:iend) (1:n)'];
        [R2(i,j),betas(i,j,:)]=my_R2(y(istart:iend),X);
    end  % end loop for variable play
end  % end loop for constant play

%% search for highest R2
disp('... Results')
% max is at r2_max, located at i_max, j_max
[r2_max_vec,i_max_vec]=max(R2);
[r2_max,j_max]=max(r2_max_vec); i_max=i_max_vec(j_max);
% convertion from position to data
fprintf('position of max. (const,var) play =
(%d,%d)\n',i_max,j_max);
fprintf('R2 max = %d\n',R2(i_max,j_max))
play_c=g_min+(i_max-1)*g_prec; fprintf('play_constant =
%d\n',play_c);
play_v=h_min+(j_max-1)*h_prec; fprintf('play_variable =
%d\n',play_v);

%% --- output --------------------------------------------------
% run again to find spurt for better R2 (less )
d_x=zeros(m,1); d_x(1)=0; for i=2:m; d_x(i)=(x(i)-x(i-1)); end;
gamma=g_min+(i_max-1)*g_prec; delta=h_min+(j_max-1)*h_prec;
play(istart:iend)=gamma+delta*u(istart:iend);
spurt = play_fun(play,d_x,istart,iend,sinal);
```

```
%% --- output plots ----------------------------------------------
% plots
figure(2); plot(spurt); ylabel('Spurt');
figure(3); plot(play(istart:iend)); ylabel('Play');
figure(4);
if h == 1, % special case to look only for constant play
    plot(play_const(:,1),R2);
    xlabel('constant play=gama'); ylabel('R2');
else
    mesh(play_var(1,:),play_const(:,1),R2);
    xlabel('delta'); ylabel('gama'); zlabel('R2');
end;

%% --- output files ----------------------------------------------
fid = fopen('spurt.txt','wt');
fprintf(fid,'%12.8f\n',spurt(istart:iend));
fclose(fid);
fid = fopen('play.txt','wt') ;
fprintf(fid,'%12.8f\n',play(istart:iend)) ;
fclose(fid);
```

Program 3
Auxiliary Program to Compute R2 (Written in MATLAB)

```
function [R2,beta]=my_R2(y,X)
% computes R2.
% I uses QR factorization in order to insure better numerical
behavior.

ny2=size(y,1)*mean(y)^2;
[Q,R]=qr(X);
beta = R\(Q'*y);
R2=(beta'*X'*y-ny2)/(y'*y-ny2);
```

Program 4
Program to Estimate a Time-Varying Intercept Version of a Labor Demand Equation (written in GAUSS)

```
%-----------------------------------------------------------------
% Time-Varying Intercept Labor Demand
%-----------------------------------------------------------------
% written by   : Manuel M. Martins and Paulo Mota
% first version: 30/01/2006
% last revision: 09/03/2006
/*----------------------------------------------------------------
----*/
/* data file: portugal.txt (n; s; t)  */

new;
cls;
        gosub dataread;
        gosub filter;
      gosub smoothing;
        gosub results;

end;

/*****************************************************************
******/
/* SUBROUTINES
*****************************************************/
/*****************************************************************
******/

DATAREAD:

library pgraph, optmum;
#include optmum.ext;
#include gradient.ext;

optset;
         _opgtol=0.00001;
         _opstmth="newton stepbt";
         _opmdmth="bfgs stepbt";
         _opmiter=10000;
  _opusrgd=&gradre;
    _grnum=20;
         _grsca=0.4;
         _grstp=0.5;

load pu[]=portugal.txt;

pu=reshape(pu, rows(pu)/3, 3);

t=rows(pu);

e=pu[.,1];
y=pu[.,2];
ti=pu[.,3];

uc=zeros(t,3);
alphall=zeros(3,1);
Puall=zeros(3,3);
```

```
Ppall=zeros(3,3);
Trans=zeros(3,3);
smooth=0;

vs=zeros(1,t);
resids=zeros(t,1);

vas=zeros(1,t);
residas=zeros(t,1);


return;

FILTER:
param=zeros(3,1);
vparam=zeros(3,1);
gam=zeros(t,1);
vs=zeros(1,1);

hpari=  11.189334 |
       54.413693  ;

{hparf, logl, g, retcode}=optmum(&L, hpari);
save hparf;

He=hessp(&L, hparf);

Cov=inv(He);

return;


SMOOTHING:

smooth=1;

call L(hparf);

rpall=rows(puall);
cpall=cols(puall);


Puall=Puall[cpall+1:rpall,.];
Ppall=Ppall[cpall+1:rpall,.];
Alphall=Alphall[cpall+1:rpall,.];

rpall=rows(puall);

Alphas=zeros(rpall,1);

Alphas[rpall-cpall:rpall]=Alphall[rpall-cpall:rpall];

nobs=rpall/cpall;

i=nobs;
do while i > 1;
      i=i-1;
      Pstar=                                    Puall[(i-
1)*cpall+1:i*cpall,.]*Trans'*inv(Ppall[i*cpall+1:(i+1)*cpall,.]);
```

```
      Alphas[(i-1)*cpall+1:i*cpall]=            Alphall[(i-
1)*cpall+1:i*cpall]

      +Pstar*(Alphas[i*cpall+1:(i+1)*cpall]-Trans*Alphall[(i-
1)*cpall+1:i*cpall]);


endo;
Alphas=reshape(Alphas, nobs, cpall);
ucs=zeros((t-nobs),1)|Alphas[.,cpall];

return;

RESULTS:

tvalues=hparf./diag(sqrt(Cov));

print "hparf                   t-values                  p-
values="; hparf~tvalues~cdfn(tvalues);
print "param="; param;
print "param p-values="; cdfn(param./sqrt(vparam));

/* Jarque-Bera test for normality (Judge et al., pp. 890-92) */

vs=vs';
rvs=rows(vs)-1;
vs=vs[2:rvs+1,.];

u3=real(meanc(vs^3));
si2=stdc(vs)^2;
u4=real(meanc(vs^4));

b1=u3./(si2^(3/2));              /*skewness measure */
b2=u4./(si2^2);                   /*kurtosis measure */

lamb=rvs*((b1^2)./6+(b2-3)^2./24);

print "NORMALITY";
    print "Jarque-Bera statistic=   ";; lamb;
    print "Jarque-Bera significance=";; cdfchic(lamb,2);

return;

PROC(1)=L(hpar);
local   cut, logl,F, Z, Q, H, i, alphal, alphap, alphau, Tr, Pp,
Pu, Pl, slogf, sv2f, v, va,ve,lambda,vsn,
      miu, varin, vare ;

vs=zeros(1,1);  /* storing standardized residuals to compute
normality test */
vas=zeros(1,1);/* storing one-step-ahead prediction error of dinf
                    to compute MSE dinf in table 2            */

vare=(hpar[1]/1000)^2;
varin=(hpar[2]/10000)^2;

Q=zeros(3,3);
Q[1,1]=varin;

H=zeros(1,1);
```

```
H[1,1]=vare;

Tr=eye(3);

i=1;

cut=i;

alphal= 4.5552584    |
   0.35226555   |
 -0.0018051622;

Pl=0.05^2*eye(3);

slogf=0;
sv2f=0;

alphall=zeros(3,1);
Puall=zeros(3,3);
Ppall=zeros(3,3);

do while i < t;

                  i=i+1;

Z = 1~y[i]~ti[i];

                        alphap = Tr*alphal;

            Pp = Tr*Pl*Tr'+Q;

            F = Z*Pp*Z'+H;

            v=(e[i])-(Z*alphap);

                  alphau=alphap+Pp*Z'*inv(F)*v;

            Pu=Pp-Pp*Z'*inv(F)*Z*Pp;

            alphal=alphau;

            /*storing useful information: */

            uc[i,.]=alphau[1:3]';

            {va,ve} = eigv(F);
            lambda=zeros(1,1);
            lambda=diagrv(lambda, va^(-1/2));
            vsn=(ve*lambda*ve')*v;
            vs=vs~vsn;

            if smooth==1;
                  Ppall=Ppall|Pp;
                  Puall=Puall|Pu;
                  alphall=alphall|alphau;
                  Trans=Tr;
            endif;

            Pl=Pu;
            slogf=slogf+ln(det(F));
```

```
            sv2f=sv2f+v'*inv(F)*v;
vas=vas~v;

endo;

logl=0.5*(slogf+sv2f);
param=alphau;
vparam=diag(Pu);

retp(logl);
endp;
```