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Essays on complexity in economics: perspectives and metaphors

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Tese de Doutoramento em Economia

Orientada por

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Biographical Note

Diogo Jorge Vieira Alves was born in March 25, 1985, in Aveiro, Portugal, where he still lives.

He concluded his undergraduate studies in economics in 2007, from the University of Aveiro. He spent that same year abroad as an Erasmus student in the Facoltà di Economia "Tor Vergata", in Rome.

The following year, in September 2008, he enrolled on an Erasmus Mundus Master Programme, QEM - Models and Methods of Quantitative Economics, and spent the following two years in Paris, Bielefeld and Barcelona getting acquainted with people from all over the world and cutting-edge Mathematical Methods in Economics, growing more and more suspicious of the latter. Before joining the Doctoral Programme at FEP in Semptember 2012, he worked in Data Mining as an Analytical Consultant for NeoMetrics in Barcelona, and as a Research Assistant for a project of development of a new Metaheuristic with applications to Machine Scheduling problems (SearchCol - Metaheuristic Search by Column Generation). During the program, he has attended several Summer Schools to supplement his knowledge on the burgeoning field of Complexity Theory: the 4thPh.D. Summer School - conference on "Mathematical Modeling of Complex Systems", in Athens, and the 5th Summer School of the European Social Simulation Association (ESSA) in Barcelona, both in 2014, stand out in this regard.

Throughout this time, as today, he remains an avid reader and part-time backpacker and explorer of the world.

Acknowledgements

No one that has studied Chaos Theory, even if at a relatively shallow level as undoubtedly my case was, can be immune to its rich epistemology and philosophical consequences. One of its richest insights is that things are the way they are today because events in the past were exactly as they were. But go back in time, change one event, and the causality chain unfolding will see to it that the alternative world is very different. But while I still recognize the importance of small causes, this page is mainly dedicated to the large ones.

First and foremost, I would like to thank my parents. This thesis would never be were it not for them, in so many more ways than the obvious one, that I fear this document would not be enough to list all their contributions. Gratitude is also owed to my supervisors, Professors Sandra Silva and José Matos. In particular, more than the technical aspects and supervision, I thank them for granting me what I think was more academic freedom than usual, by letting my mind wander off and tackle the questions I thought were important, while still being my anchor and making an effort to keep my feet on solid ground, always making me aware that in real life, deadlines are important. I thank Professor Ernesto Costa of the University of Coimbra, for an insightful talk I had with him in mid-2012 that marked my decision to embark on the rich field of Complexity Theory. Before that, all I had was a very hazy idea of what it was. That situation has not changed much as regards my knowledge, but I am extremely glad of having taken up an object of study that is not there for the taking, but is very challenging and humbling. Professors Pedro Campos and Pavel Brazdil introduced me to serious Agent-Based Modelling through their efforts to divulge Netlogo to FEP. Financial support in the form of a doctoral grant from FCT (Fundação para a Ciência e Tecnologia), reference SFRH/BD/85453/2012, is gratefully acknowledged. The chapters in this thesis benefitted from the direct input of several people. In particular, I thank Rodrigo L. Izquierdo for support and help with the Netlogo code for the agent-based model, that my learning-by-doing process made somewhat muddled (analyse it at your own peril). Diogo Lourenço, a colleague and a friend, gave helpful suggestions on the philosophically-oriented chapter. His own thesis was a good source of inspiration and got me started on the subtleties of the thought of Donald Davidson and Wittgenstein, as well as other philosophers of language. With the benefit of hindsight, I look back with fondness to the growing pains of the PhD program, especially some days that seemed very bleak indeed in the first year. Without the support given to me by my colleagues and friends, this experience would have been far less enjoyable. I will begin by my most direct "battle colleagues" that went through those growing pains with me: Elias Pereira, Rita Bastião, and José Gaspar. I am also heavily indebted to my friends of the infamous 252 room and beyond: Miguel Gärtner, Filipa Mota, António Neto, Francisco Pereira, Duarte Leite, Sandra Oliveira, Vera Rocha, Ana Sá, Alex Pinto, Carlos Pinto, Joana Costa, José Morais, Isabel Araújo, Nélson Vieira, Tiago Pereira, Sandeney Cabral and Tânia Pinto.

A final note of thank you to the small cause. If you picked up this thesis because you recognized my name but don´t see your own name in here, know that most likely the reason you are not here is due to the availability bias. Were it not for you and the small contingence your action had on me, this thesis would not exist.

Diogo Alves, Aveiro, January 2017

Resumo

Esta tese é composta por 4 capítulos (uma introdução e três ensaios). Não é fácil localizá-la no âmbito da Economia. Isto decorre, por um lado, do facto de que o tema dentro do qual ela se insere corresponder a um paradigma vasto e relativamente novo. Por outro lado, pelo facto de ela descrever três níveis, que correspondem, *grossu modu*, ao que se convencionou chamar em Economia micro, meso e macro. O paradigma é o da Complexidade - quando a interação não-linear de elementos de um sistema produz resultados emergentes. Isto resulta da existência de um efeito de feedback sistema - componentes. Embora grande parte dos estudos em Complexidade tenham a física como objeto de estudo, é minha convicção que estes efeitos são especialmente importantes no domínio social. Esta tese explora aplicações da Complexidade à Economia. Cada ensaio debruça-se sobre um nível de descrição distinto, por ordem crescente de abstração. O primeiro ensaio aplica técnicas de Física Estatística e Teoria do Caos a séries temporais económicas portuguesas. A premissa é que tais ferramentas providenciam "insights" epistemológicos complementares às abordagens económicas tradicionais. As primeiras diferenças das três séries não-lineares do PIB possuem caos, e a série mais significativa do crescimento do PIB, o Consumo Privado, tem uma assinatura de um processo de Criticalidade Auto-Organizada (CAO). O segundo ensaio desenvolve um modelo baseado em Agentes construído sobre uma metáfora biológica para ilustrar um possível caminho para a CAO. No modelo, este caminho é determinado por variações da taxa de poupança das empresas e da topologia de rede que as liga. Finalmente, o terceiro ensaio o elemento humano é introduzido. Com base num meta-modelo esquemático representativo da formalização neoclássica, demonstra-se que o campo de estudo da economia é na realidade um conjunto de sistemas complexos interligados. Este facto tem consequências metodológicas e epistemológicas profundas.

Abstract

This thesis is composed by 4 chapters (an introduction and three essays). It is not easy to place it in the scope of Economics. On one hand, this is so since its central theme corresponds to a vast and relatively new paradigm. On the other, by the fact that it describes three levels, corresponding roughly to the micro, meso and macro aggregation levels in Economics. The paradigm is that of Complexity - when nonlinear interaction of elements of a system produces emergent results. This is so since there is a feedback effect between the system and its components. While a good deal of the studies in Complexity Science have Physics as their object of study, it is my conviction that these effects are especially important on the social domain. This thesis explores applications of Complexity to Economics. Each essay deals with a different description level, in increasing order of abstraction. The first essay applies techniques of Statistical Physics and Chaos Theory to portuguese economic time series. The premise is that such tools provide epistemic insights that are complementary to traditional economic approaches. The first-differenced nonlinear series of the GDP all possess chaos, and the most significative of them in terms of GDP growth, Private Consumption, has the signature of a process of Self-Organized Criticality (SOC). The second essay develops an Agent-Based model built on a biological metaphor to show a possible path of an economy towards SOC. In the model, this path is determined by varying the saving rate of firms, and the network topology connecting them. Finally, the third essay introduces the human element. Based on a schematic meta-model that is representative of Neoclassical formalization, it is shown that the field of study of economics is in reality a set of interconnected complex systems. This fact has profound methodological and epistemic consequences.

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Chapter 1

Introduction

> "You need a story to displace a story."
> Nassim Nicholas Taleb, The Black Swan: The Impact of the Highly Improbable

Any scientific endeavour must be born out of an amazement - and in that sense, no phenomenon is more enticing than the remarkable degree of coordination that is achieved in economies by actively subjective minds. If and when I need a carton of milk, I know it will be available in my seller of choice, or any other, for that matter. As I am walking down the aisle, I find the carton of milk along with several other things I did not know I wanted. Most of these will have been assembled by people who, taken individually, have no idea how to assemble it from beginning to end. Some minutes later I am taking out of the wallet assembled by people I will never know a piece of paper with very little intrinsic value beyond that which people have somehow agreed to confer to it. All my actions on this tiny snippet of life, even the most mundane ones, seem to be charged with meaning. As I hold the carton of milk, I am not simply counteracting the force of gravity by holding a mass of liquid bounded by a carton: what I am holding is a key ingredient of my breakfast, as I project the future on my mind. And amazingly, you, a completely different person, will most certainly know what I mean, and possibly recognize yourself in these images even if you never lived them. If one takes a step back and think about this gigantic dance of minds, things and plans, one could almost be forgiven for thinking that there is some sort of Hegelian supermind floating about, that not only gives my actions a purpose, but also aligns all minds according to a plan. We, of course, know better: an economy is the product of human action, but not of human design, and is a petri dish of innovation, adaptation, unforeseen circumstances, matching and unmatching plans, failure and success. It mirrors our own lives as a mix of different reasons, reason and emotion, chance encounters and well thought-out plans.

This thesis revisits this age-old theme. It moves in increasing order of abstraction, from considerations spanning the dynamic nature of Portuguese economic time series, to a possible road to complexity in economic systems by an experiment with an agent-based model built around a biological metaphor, to a paper of a more reflexive and Philosophical nature connecting the nature of our thoughts to the emergence of novelty in economics. The thesis is organized around three main claims coincident with the last three of the four chapters contained therein.

First, that the collective dynamics generated by such a process, as measured by Macroeconomic Aggregates, are complex in the sense of being a strange mix between order and chaos. The second chapter, thus, tackles the macro-level, and discusses the application of some tools of complexity and chaos theory to Portuguese economic growth. It is a proposal to expose the degree of Knightian uncertainty we face as participants of a complex system. We use techniques of nonlinear time series analysis to show the existence of chaotic dynamics on the first differences of 3 of the 5 components of the Portuguese GDP. Moreover, we claim that complexity can be understood as "Pink Noise", the dynamical signature of some natural processes, and that the series of Private Consumption, upon which most of the Portuguese GDP growth is based on, is the more complex of the three in this sense, as well as in the magnitude of the empirically estimated Largest Lyapunov Exponents. Moreover, we explore the concept of permutation entropy as a natural measure of uncertainty of a process as measured by a time series to show that the Portuguese economy is becoming less complex, with complexity understood as the spontaneous capacity of a system to balance novelty emergence and adaptation. This measure is based on Gershenson and Fernandez´s proposal and is able to capture the interplay between order and chaos, or of an economy in a "corridor of stability". Throughout, the used metaphor is that of a physical system producing information that can be used for meta-purposes.

Second, having diagnosed the dynamics, it is also an interesting question to analyze how they might emerge on a simplified scenario, i.e, the meso framework of analysis, or how is it that simple rules feed back into complex dynamics. That is the purpose of the third chapter. It presents an agent-based model of economic growth featuring adaptive agents: firms and households, along with goods and labour capacities as binary strings that can be combined. Firms are populations of blueprints, i.e, a instruction sets for combining intermediate goods along with the labour supply of households to produce final goods acquired by the latter. Here, complexity is once again conceptualized as the existence of a corridor of stability, as a mix between regularity and iregular, power-law distributed endogenous shocks characteristic of Pink Noise. The model is built on a metaphor of biological evolution, following especially Stu Kauffman´s NK model; it features a fitness landscape configured by the (neg) entropy of both production methods used and final goods sold, and is adaptive in that firms learn about household preferences and these, in turn, adapt to labour search by firms. It features both a labour market and a final goods market where firms interact with households. Moreover, the model is built to be able to do a number of accessory experiments, such as percolation of production methods among firms, the influence of network topology of both firms and households on macro variables, the conditions featuring the emergence of a power law distribution on firm size, and the effect on investment of different innovation types. The paper presents some results as time series of GDP, Investment, entropy of the distribution of the populations of final goods and blueprints, and diffusion of technological standards for a baseline configuration. Moreover, we look at phase transitions on the supply side of the economy by varying, as regards the baseline configuration, the saving rate of firms and the topology of the network connecting them. The model presents features of Self-Organized Criticality in the time series behaviour of the frequency of adopters of best blueprints that emerges close to the midpoint of the spectrum of the saving rate of firms. Moreover, the effect of the fitness landscape in terms of the entropy of blueprints is large enough to compensate the different network topology diffusion rates of the best blueprint.

Finally, the fourth chapter is of a different nature and has a more personal undertone, justifying the change in the personal pronoun used throughout. Up to now, I have been claiming that the correct way to think about social, ergo economic, systems, is as a strange mix between order and chaos. The cracks in the armour of Neoclassical formalism, a methodology relying solely on the positing of order, and evident in, say, the Debreu-Sonnenschein-Mantel theorem of the 1970´s, are evident, having been treated only as a mathematical sleight of hand by researchers. I placed great hopes on the emerging field of complexity science to solve at least some of these problems. The second and third chapterss of this thesis reflect this hope. And in fact, I still think the methods of complexity are useful to some extent. But a cornerstone of the field is that one is mainly interested in examining how complexity emerges from simplicity of rules. The deliberations of individual agents take back seat on this account; it is "interactions, not individuals, are the key ingredients of behavioral complexity" (Solè, 2008, page 176). Therefore, what started out as an initial plan of analysing the individual (complexity at the micro level) to conclude the triad of macro and meso levels of complexity analysed in the previous two parts, gradually shifted towards the topics of meaning, intentionality and philosophy of action. In the fourth chapter, I delve into what it is that makes us human, and how that feeds back into the nature of economic data and the limits of our knowledge as economists. It will be shown that the key to understanding actions, and indeed thoughts, lies in the search of meaning by active minds. Moreover, no effective procedure exists to list all the possibilities of things, undermining the search for computable foundations for economics. Hence, if one compares the process of generating meaning as looking for the relevant snippets of how things can be other things as evolutionary, one obtains that, as in nature, evolution is not compressible. I stress throughout the importance of vague knowledge, and that logical formalism as expressed through the choice of an optimal plan cannot account for the way people build plans of action. Usually, such formalisms require an agent to have full knowledge of the situation and of himself, both contentious points. The way we formulate economic plans is, instead, directly a consequence of the way we think: by transferring knowledge from one domain to another, in a process that is conducive to metaphors. Moreover, it is a process of personal discovery that extends Intentionality to other human beings: in that, in a way, they are like me and we share a common, public language that evolved and which, exactly like an economy, was not planned by anyone. I suggest that Conceptual Blending can frame the thought process. Moreover, I analyze, criticise and complement a meta-model suggested by Muñoz and Encinar (2007, 2011, 2015), that is a conceptualization of Neoclassical economics, to stress the points made.

Chapter 2

Chaos and Complexity in the Portuguese Economy

2.1 Introduction

It is a well-explored theme in Heidegger and other philosophers of science that the way we question nature is not independent of the answers we expect to obtain, of the expected power that can be amassed: in this sense, the history of science is one of exploration, of subjecting things to a systematic violence. Theories are never developed in a vacuum: and recognizing the woes of modern day economics necessarily entails a small detour through the features of the neoclassical paradigm, inevitably intertwined with the policies it recommends and ultimately to the fate of economies themselves; indeed, the crisis of economics is a crisis of economic theory (Kirman, 2010). Ironically, shortly before the global meltdown in 2008, there was a widespread consensus on the "scientific principles" to be used when conducting monetary policy (Fagiolo and Roventini, 2012); today much effort is still being dedicated by the profession in extending the Dynamic Stochastic General Equilibrium workhorse along several directions, adding several frictions and contraptions to the core of the theory. The dominant paradigm is plagued by unrealistic assumptions at the micro level and by unrealistic assumptions and an additional fallacy of composition at the macro level (Al-Suwailem, 2011), the current popularity of "micro-founded" macroeconomics notwithstanding. While several research directions are departing from the Procrustean bed of mainstream economics and gaining widespread popularity and acceptance, such as behavioral and experimental economics, agent-based modeling and evolutionary game theory, there was a sizeable Kuhn-loss that went by largely unnoticed and that not even these can solve, and this is the end of the philosophical monitoring of the profession resulting in an explicit incapacity to recognize the limitations of the modeler as opposed to modeled agents inside a model. After all, imposing cognitive limitations still assumes the modeler knows the solution curtailed to the agent[1]. The evolution of economics excluded this notion of ingrained uncertainty, conflating it with quantifiable risk at the same time it spawned the new ontology from which all models and policy advice stemmed from. Hence, when forced to choose between "an uncertain reality and an unreal certainty", economists opted for the latter: the role of economics as an "elite folk science" (Ravetz, 1994) undermined its search for positive truth. This was a consequence of the evolution of the role of the economist and society itself: Hodgson (2011) points out that the funds directed to economic research are in large measure a consequence of its perceived use in forecasting and policy-making, and would not be forthcoming if the profession lingered on around philosophical doubts. However, whether they are epistemic or ontological, these limitations exist and have been wished away rather than subject to sufficient scrutiny. Frank H. Knight, an influential economist of the early XXth century, developed a notion of uncertainty that emerges when a situation cannot be classified as an example of a generic group of similar situations (Knight [1921], 2012)). Terzi (2010) and Davidson (1996) claim that Knight´s conceptualization is an epistemic theory: an order that cannot be conceptualized. In chaotic systems, the crucial property of sensitive dependence on initial conditions introduces a

[1] Muth (1961) simply eliminated the epistemic barrier between modeled agent and modeler: For all purposes, the rational agent was a calculating economist. This assumption has been relaxed since then.

fundamental epistemic limitation that can be classified as Knightian uncertainty. Chaos offers a way of integrating genuine uncertainty in the framework of economic analysis without limiting it to a statistical calculation of outcomes (Ferrari Filho, 2000). Irreducible indeterminacy is consistent with internal deterministic dynamics (Mitchel *et. al.*, 1993).

An economy is a dissipative complex system, the dynamics of which are the product, at the micro level, of the deployment of action plans by a population of heterogeneous agents. This deployment and subsequent interaction is responsible for the emergence of novelties and properties that manifest at the meso and macro levels (Muñoz and Encinar, 2007). Moreover, it is an autopoietic process (Mesjasz, 2010), the catalyst of which is the growth of distributed knowledge. Metcalfe and Ramlogan (2006) contend that there is an inherent unpredictability resulting from fundamental indeterminacy as regards human preferences, expectations, and knowledge. Indeed, the process of economic growth is one of adaptation: of a continuous tension between coordinating forces emerging in the market and novelty arising from the deployment of agents´ plans. At the macro level, the resultant dynamics can be nonlinear and away from equilibrium: and rather than featuring purely periodic and predictable motion, there is an interweaving of patterns that are repeated through time, with novelty in the form of structural breaks. The dynamical properties of a complex system appear as a mix of order and chaos to an observer: neither complete order nor complete disorder (Solè, 2008).

The aim of this paper is to obtain a deeper understanding of the qualitative dynamics of the process of Portuguese Economic Growth, at both a static and dynamic perspective, using tools from complexity, chaos theory and information theory, and to connect these properties to epistemic limits from the modeller´s point of view, by stressing the differences between observer and system intelligence in a complex system. It departs from McIntyre (1998) in that there are limits to our knowledge of some systems and aims at rehabilitating the notion of Knightian uncertainty affecting observers of the macroeconomic aggregates of the Portuguese economy and thus to answer Mesjasz´s (2010) plea for a unified approach between the methods of "hard science" and metaphor of social science. Schinckus (2009) relates the set of tools deployed by econophysics with a conception of economics that takes into account Knightian uncertainty and emergence and is methodologically akin to this work.

The topic of chaotic dynamics experienced a spurt of interest since the early 80s mainly due to the possibility of generation of endogenous business cycles. Baumol and Benhabib (1989) show how this meant a revival of a research field that had languished since the 1950´s. While some research exists that frames the topic inside the neoclassical paradigm (Benhabib and Nishimura, 1979) and Grandmont (2008), among others, show how chaotic dynamics can be generated by some parameter values inside standard macroeconomic models), I heed Faggini (2010), who claims that if anything meaningful is to be said about the real world, chaos must be sought directly in data. The search for chaos as a driving force of GDP has not received a lot of attention in the literature. A pivotal paper is Frank and Stengos (1988), who find strong evidence of nonlinearity, but weak evidence of chaos, for a sample of 4 industrialized countries for the period between 1960 and 1988. Shintani and Linton (2003) develop a statistical test of positivity of Lyapunov exponents and use it on the Frank and Stengos database, to reject the hypothesis of chaotic dynamic of GDP. However, they do recognize the same limitations as the original paper: short and noisy datasets. Shichang and Zhiwei (2010) estimate the correlation dimension of the dynamics of the economic growth of the chinese province of Liaoning. Kříž (2013) estimates the Hurst exponent, correlation dimension and largest Lyapunov exponent for the Czech GDP growth between 1995 and 2010, finding some evidence of chaos. Tash and Pahlvani (2012) investigate the existence of chaos for the

Iranian GDP. Their study indicates nonlinear dependence and positive largest Lyapunov exponent. Sanderson (2013) studies the dynamics of the HICP price index for the euro Area before and after the introduction of the Stage 3 of the ECB monetary policy aiming at price stabilization. While no evidence of chaos was found, the author reports an increase of the largest Lyapunov exponent after the policy, which may be a sign that the system became more dissipative: by constraining the volatility of prices, the ECB effectively reduced the random part of the series and may be nudging the inflationary dynamics toward a deterministic chaotic attractor. Faggini (2010) uses topological tools on the Frank and Stengos dataset to show that contrary to the conclusions of the original paper, chaotic behaviour cannot be dismissed for two of the four countries studied. Kříž. (2014) finds evidence of chaos for the Finnish GDP between 1975 and 2012. Besides the Hurst and Lyapunov exponent estimation, he estimates the correlation dimension and Fractal dimension of the series and complements the analysis with a recurrence plot showing chaotic behaviour. These tools have also been applied to study the dynamics of labour markets. Olmedo (2011) looks for chaos in the Spanish unemployment series between 1965 and 2010. The complexity of the attractor is measured through estimation of the correlation dimension and largest Lyapunov exponent, confirming the existence of chaotic dynamics. The nonlinearity of the labour market is also explored in Chen (2011). He explores the series of Taiwanese unemployment between 1978 and 2010 using topological methods to obtain insights about the underlying dynamics and is able find chaos in data. Moreover, he presents a rolling time window framework for the topological indicators derived from Recurrence Quantification Analysis, obtaining a dynamic evolution of these that allows him to detect phase transitions in data.

There are very few studies on the dynamical properties of Portuguese Economic time series. Crato (1992) analyses Portuguese economic time series using a Fractional ARIMA. He shows that the series of domestic output, market returns and inflation all have long memory. On a more recent study, Belbute and Caleiro (2009) measure the persistence (defined as the inverse of the speed with which a variable returns to its trend) of Private Consumption in the period between 1992 and 2007, distinguishing between durables and non-durables. They find that consumption of durables is more persistent and that aggregate consumption is less persistent in Portugal than in the US. As far as a literature review could reveal, this is the first paper where information theory is used directly as a measure of uncertainty of macroeconomic time series aggregates[2]. This paper attempts to fill these voids. Moreover, considerations of complexity science can shed light beyond the traditional economic tools deployed.

The rest of the paper is organized as follows. In section 2.2, we perform a nonlinearity test on the 5 first differenced series of the Portuguese GDP, and determine the scaling exponent, Hurst exponent, fractal dimension, and phase space reconstruction of the nonlinear ones. With these results, it is possible to determine empirically the Leading Lyapunov Exponent of each of them to ascertain the existence of chaos. In section 2.3, we define permutation entropy as a measure of uncertainty of a dynamical process, and apply information-theoretic measures of emergence, self-organization and complexity to each of the nonlinear time series to find that a complexity-reducing effect seems to be taking place in the series of Private Consumption. Finally, section 2.4 concludes.

[2] Although applications abound in finance, econometrics and environmental studies: see Riedl *et al.* (2013) for an overview and Dionísio, Menezes and Mendes (2006) for an application to the Portuguese stock market.

2.2 Determining the presence of chaos in the dynamics

In this section, we use standard methods to study the possible presence of chaos on portuguese macroeconomic data, collected from the INE website, covering roughly 20 years, between the first quarter of 1995 and the last quarter of 2014. Louçã (1997) warns that the aggregation effect can have spurious effects on the nonlinearity analysis of time series, and reports a case where the GDP of a country does not present evidence of nonlinearity, while the disaggregated series composing it do. The aggregation effect occurs in our dataset as well. The series of Portuguese GDP can not be concluded to be nonlinear, while three of the five disaggregated series that compose it do. Hence the need to go to the level of aggregation below that of quarterly GDP. Unfortunately, the toolset of nonlinear dynamics available has not made much headway in the study of nonstationary data and the five series all present significant nonstationarity, as measured by standard unit root tests. Since the existence of nonstationarities in data presents a significant shortcoming of the methods under discussion (Faggini, 2014), the dataset under study consists of the series of the first differences of the deseasoned, quarterly GDP under the expenditure approach: Gross public consumption (henceforth denoted as $\triangle G$), Gross private consumption ($\triangle C$), Gross private investment ($\triangle I$), Exports ($\triangle X$), and Imports ($\triangle M$). The same approach of first-differencing is followed in the seminal paper of Frank, Gençay and Stengos (1987).

2.2.1 Nonlinearity tests

Nonlinearity is a necessary, albeit not sufficient, condition for chaos. As such, the first step is to test each of the series for nonlinearity. The procedure is straightforward: I follow Faggini (2014) and fit the best ARIMA-type linear model[3] to data and perform a Brock-Dechert-Scheinkman test on the residuals thereof. The BDS test is a nonparametric, omnibus test of structure (linear or nonlinear) versus i.i.d random variables: Since the linear part has been removed by the ARIMA model, rejecting the null of i.i.d in the residuals of the linear regression is akin to the series having a nonlinear structure. The BDS test is based on the concept of correlation integral and the test statistic $W(\bullet)$ has the following functional form:

$$W(N,d,\varepsilon) = \sqrt{N}\frac{C(N,d,\varepsilon) - C(N,1,\varepsilon)^d}{\hat{\sigma}(N,d,\varepsilon)} \tag{1}$$

where $C(\bullet)$ denotes the correlation integral for a sample size N, an embedding dimension d and a tolerance level ε and $\hat{\sigma}$, the sample standard deviation of the sample.

The series that present nonlinear structure are $\triangle G$, $\triangle C$ and $\triangle M$. These are the series that will be under study henceforth, as the other 2 can be well described with a linear model. Figure 1 plots the series under study:

[3] This was done by choosing the linear model with the smallest complexity measures.

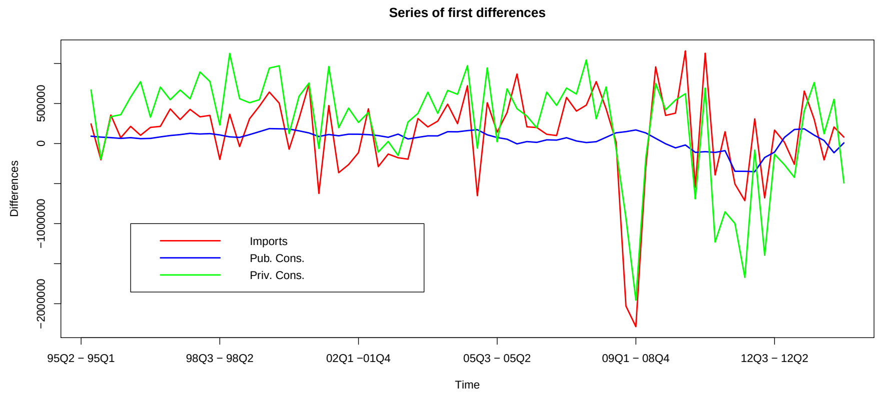
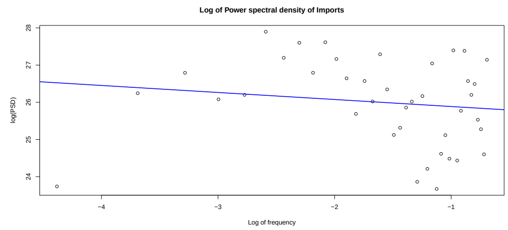


Figure 1: First differences of the series under study.

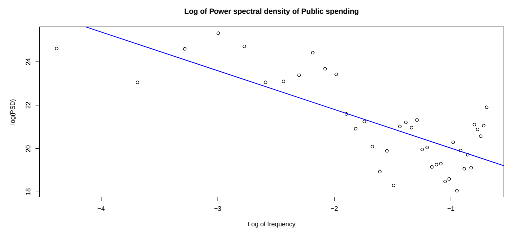
The series of Public Consumption is characterized by an apparently larger degree of stability. The structural break corresponding to the First Quarter of 2009 is evident.

2.2.2 Memory and long-range time correlation

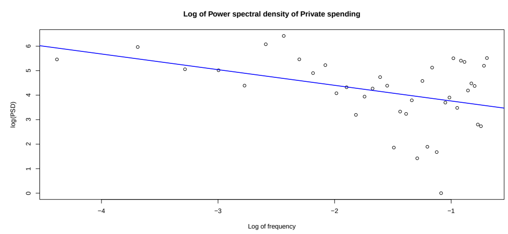
Processes with a long-range time correlation, or $1/f^{\alpha}$noise featuring a power-law distributed decay of the log of the spectral density and a scaling exponent αin a region centered around - 1, (Pink Noise) are a common feature of economic and financial data. In this type of processes, events in the past continue to have an influence in current events, and they cannot be aptly modelled by either Markov or ARIMA processes (Gao, 2013). They form the statistical signature of systems operating out of equilibrium and are an intermediate case between the unpredictability of white noise, characterized by a scaling exponent in the region $\alpha \in [-0.5, 0]$ and the less rugged series characterized by a scaling exponent smaller than - 1.5. This latter regime is known as Brown Noise and is the integration of White Noise. The estimation of the scaling exponent for real data sets can be done by plotting the logarithm of the spectral density of the process against the logarithm of the frequency. Since, by hypothesis, $S(f) = \frac{1}{f^{\alpha}}$, applying logarithms to both sides yields $log(S(f) = -\alpha log(f)$. Then, a simple linear regression can be used to estimate the scaling parameter and conclude on the nature of the noise of the process. This was the procedure applied to the three series.



(a) Imports



(b) Public Consumption



(c) Private Consumption

Figure 2: Best fit of a line to the decay of the logarithm of the Power Spectral Density of the series.

The results as regards the slope of the line are as follows:

Series	α
$\triangle G$	- 1.784
$\triangle C$	-0.639
$\triangle M$	- 0.189

Table 1: Estimated scaling exponents of the series.

As regards the scaling exponent, the scope of the dynamic behavior of the three series falls within the three types of noise. That with the highest persistence is Public spending (Brown Noise), which gives support to our intuition that those series were characterized by a larger stability. Private consumption falls under the Pink Noise regime, and the series of Imports is statistically indistinguishable from White noise. The behaviour of the series of Private Consumption is consistent with expectations. The Pink noise regime is the dynamical signature of Complex systems such as the sandpile model: the regime where regularity mixes in a nondeterministic manner with randomness. For these regimes, long-range time correlations underlying regularity can be offset by extreme events that are consistent with small causes[4], if one tracks the causality chain far enough: it is the regime of Self-Organized Criticality (Bak (1987;1993)). From an economic point of view, Private Consumption is the aggregate result of series of a myriad of individual decisions. These decisions are interlocked in complex causal chains forged by changing tastes, word-of-mouth and herding effects, interest and credit policy. By contrast, Public Spending usually is the outcome of a plan and has strategic effects that last through time in a more durable manner, besides being much less reversible. In this sense, as the result of a myriad of decisions that have weak but existing correlations through time and space, Private Consumption is the more complex of the series.

The Hurst exponent is another useful notion that captures the memory of a dynamic process (Sanderson, 2013) and the notion of path-dependence: how much the past influences the future[5]. For real time series, it is usually the case that $H > \frac{1}{2}$ (Kriz, 2014), indicating persistence: Intuitively, large values in the series are more likely to be followed by large values than by small ones. The Hurst exponent measures self-similarity of a time series: $X(at) = a^H X(t)$.

Several methods exist to estimate the Hurst exponent of a series: we use the rescaled range method, where the difference between the largest and smallest values of constructed cumulative series is rescaled by the standard deviation (Sanderson, 2013): Then, letting $R(n)$ denote the range of the series and $S(n)$ the corresponding standard deviation, the following relation holds:

$$\frac{R(n)}{S(n)} = Cn^H \tag{2}$$

The Hurst exponent can then be estimated by regressing $log(\frac{R(n)}{S(n)})$ as a function of $log(n)$. The Fractal dimension[6] F of a series measures the dimensionality of the attractor and is related to the Hurst exponent through the simple formula $F = 2 - H$. Chaotic attractors tend to have a noninteger Fractal dimension, hence the importance of estimating it. It is the property of continuous expanding (a time path never repeats itself) and folding (the attractor is bounded in size) that is at the root of the geometry of strange attractors (Barnett, Serletis and Serletis, 2012). The fractal dimension is thus a measure of strangeness of the attractor. The results for each of the series are reported on the table below:

Series	H	F
$\triangle G$	0.7093	1.2907
$\triangle C$	0.6936	1.3064
$\triangle M$	0.5887	1.4113

Table 2: Estimated Hurst Exponents and Fractal Dimension of the series

[4] For the canonical sandpile model, the addition of another grain of sand - an effect indistinguishable from all the other stimuli that preceded it - triggers an avalanche, that is a result both of the small cause - the addition of the grain - and the overall structure of the mound, i.e, the memory effect.

[5] In more technical terms, the Hurst exponent is the scaling of the variance of a stochastic process (Kriz, 2012): $\sigma^2 = \int_{-\infty}^{+\infty} y^2 f(y,t)dy = ct^{2H}$, when time-varying density f is known.

[6] Also called Hausdorff-Besicovitch dimension (Perli and Sandri, 1994).

It is perhaps not surprising that the series of the differences in public spending present the highest persistence, since State spending in the period under consideration tended to be of a large and fixed character, hence with more lasting effects in time. Moreover, the period under study was one where large public works and contracting by the state took place: Between 1986 and 2010, Portugal was the EU country with the largest expansion of Public Consumption as a percentage of GDP (Barreto, 2013).

All three series have an attractor of a noninteger dimension, dubbed a "Strange attractor" in the literature. But a strange attractor is neither necessary nor sufficient for chaos (Butler, 1990). A necessary condition for ascertaining the presence of chaos in the series is phase space reconstruction; we must be able to unfold the single observable stream of values into a set of $d-histories$ (Huffaker, 2010), such that a metric between trajectories is well-defined. This is possible through the technique of time delay embedding, due to Takens (1981): the embedding method is based on mapping the dimension of the observations in a higher-dimensional space by grouping the observations of the series in lagged coordinate vectors, obtaining the orbit of a dynamical system that is topologically equivalent to the (unobserved) one (Fernandez Diaz *et al.*, 2012). The orbits are topologically equivalent in the sense that both the original and reconstructed attractors have the same dimensionality and specter of Lyapunov exponents (Perli and Sandri, 1994).

The time-delay embedding depends on two key parameters: the embedding dimension d , and the time delay τ , defining the dimension of the vectors used to reconstruct the attractor. From an original vector of observations x_t, one removes the first τ observations to obtain the vector $x_{t-\tau}$, and proceeds iteratively to obtain a matrix formed by a series of vectors of the form $X=\left[x_t, x_{t-\tau} \ldots x_{t-(d-1)\tau}\right]$. The coordinates of the attractor in the reconstructed phase space are given by the first lines of the matrix. The method requires, and is sensitive to, the choice of a proper embedding dimension and delay time. The latter is selected on the basis of the first minimum of the (Shannon) mutual information criterion between lags of the original series given by $H(x_t, x_{t-\tau}) = \sum_{x_t, x_{t-\tau}} p(x_t, x_{t-\tau}) log(\frac{p(x_t, x_{t-\tau})}{p(x_t)p(x_{t-\tau})})$, measuring the trade-off between placing too few lags and obtaining redundancies and placing too many and obtaining too scattered a dynamical process. The embedding dimension was chosen with a False Nearest Neighbour (FNN) algorithm, based on the idea that increasing the embedding dimension should not affect the neighbourhood of points in the reconstructed phase space, i.e, that the diffeomorphism between the original phase space and the reconstructed one is neighbourhood-preserving: points nearby for one embedding dimension should remain so in the next dimension. False neighbours are pairs of points close for a given embedding dimension but not for the next, their proximity being simply a consequence of the fact that the attractor is projected onto an inferior-dimensional space (Kennel, 1992). The FNN method yields the minimum dimension that retains points that are close between different embeddings, up to a tolerance level. Letting, according to *Ibid.* (1992), $R_d^2(n,r) = \sum_{k=0}^{d-1} \left[x(n+kT) - x^r(n+kT)\right]^2$ denote the distance between points x and nearest neighbor x^r in dimension d, then in dimension $d+1$, the distance between the same pair of reference points is $R_{d+1}^2(n,r) - R_d^2(n,r) = \left[x(n+dT) - x^r(n+dT)\right]^2$. A good criterion for detecting embedding errors is that the increase in distance is large when moving from one embedding dimension to the higher one. A false neighbour is one for which the distance separating itself and another point between dimensions is larger than a tolerance level, i.e, a pair of false nearest neighbors is detected when $\frac{|x(n+dT)-x^r(n+dT)|}{R_d(n,r)} > R_{tol.}$. These algorithms can be easily implemented using the package "NonlinearTseries[7]" in the statistical software R. The results for each of the series are reported in the following table:

[7] The Freedman-Diaconis rule was used when deciding upon the optimal number of bins for the determination of the mutual information criterion.

Series	Delay	Embedding Dimension
$\triangle G$	2	2
$\triangle C$	4	3
$\triangle M$	2	3

Table 3: Optimal delay and embedding dimension of the series

Trajectories on a strange attractor diverge with time. To measure the property of sensitive dependence on initial conditions, the standard way to proceed is to estimate the Leading Lyapunov Exponent (LLE) of the series. A positive LLE indicates divergence of trajectories of points that start out arbitrarily close. Denoting by $\lambda$the LLE, this divergence can be measured by$\varphi(x(t), x^r(t)) = e^{\lambda t}\varphi(x(0), x^r(0))$[8]. Several methods exist to estimate, given a phase space reconstruction based on an appropriate embedding dimension and delay time, the LLE of a series. φ behaves erratically in case of chaos: this class of algorithms studies the mean exponential rate of divergence of trajectories (Faggini, 2014). The chosen implementation estimates the LLE using Kantz´s algorithm. The results are summarized in the table below:

Series	λ
$\triangle G$	0.0142
$\triangle C$	0.0301
$\triangle M$	0.0235

Table 4: Leading Lyapunov Exponent of the series

Thus, the LLE of all series is positive and we are in the presence of weak-dimensional chaos in Portuguese economic data. These results are in the order of magnitude of the results obtained by Kriz *et al.* (2014) for several european countries. Consistently with the previous findings, the series of Private Consumption is the more chaotic of the three.

A note on the significance of these findings for the Portuguese economy is in order. Over the last 30 years, Portuguese GDP has been driven mainly by the dynamics of internal demand, comprising private and public demands along with investment. Although the aggregation effect is undoubtedly present, this sheds light on the fact that the main driving forces of GDP are driven by chaotic dynamics, and hence, so is the latter.

2.3 Information theoretic measures

2.3.1 Permutation entropy

Plans produce, and are composed of, information. Economic systems need several types of information: Amaral (1999) follows Kornai and distinguishes between a real and a control sphere, where the first comprises all productive factors and the second includes decisions. This agrees with our interpretation of the origin of complexity in economic dynamics as the interaction of plans of action: an agent deploys a plan, and "a form having

[8]Where $\varphi(x(t), x^r(t)) = \parallel x(t) - x^r(t) \parallel$represents a distance, over a suitable metric, between points x and x^rat time t.

very little energy enters into and directs a much larger energy." (*Ibid.*, page 79). The fact that plans are subjective presents no particular problem to this interpretation, as what directs the system is objective data in the form of the *result* of the interaction of plans, rather than their formation. It is, of course, not possible in either theoretical or practical terms to track the formation and evolution of plans of action in a real economy, as the last section should have made clear. However, it is possible to operationalize a notion of complexity as a balance between order and chaos, and uncertainty affecting the policymaker on the basis of information theory. That is the purpose of this section.

We begin by quickly reviewing Bandt and Pompe´s (2002) concept of permutation entropy as a measure of uncertainty. A dynamical system expressed as a time series consists of events that can be interpreted as sequences of symbols (Crutchfield, 2012). Information theory, starting with the seminal work of Shannon in the late 1940´s, is a measure of the value of the information in a message and its reverse, entropy, a measure of disorder or uncertainty surrounding a signal. As a measure of complexity, it must increase with the amount of choice of the source and hence, with the uncertainty facing the recipient of the message (Nicolis, 2012). This establishes entropy as the uncertainty facing an analyst/policymaker analysing a stream of economic data. Shannon´s entropy is operationalized as the summation over all possible outcomes of the product of the probability of the event by its logarithm[9]:

$$H(X) = -\sum_i p(x_i) log_b(p(x_i)). \tag{3}$$

Permutation entropy is the application of (3) to symbols of an alphabet representing the ordinal arrangement of the values of subsets of the series. More specifically, the series becomes an ordinal pattern of the embedding dimension d: $\pi_d(t) = (r_0 r_1 ... r_{d-1})$ such that $x_{t+r_0} \leq x_{t+r_1} \leq ... \leq x_{t+r_{d-1}}$ and $r_{s-1} < r_s$if $x_{t+r_{s-1}} = x_{t+r_s}$(Matilla-Garcia *et al.*, 2010)[10]. In other words, the series is turned into subsets of elements of an alphabet (such as the set of positive integers) of length equal to the embedding dimension, the order of which reflects the order of the values of the series. One can thus define the permutation entropy for a time series with an embedding dimension d as $h(d) = -\sum_{i=1}^{d!} p(\pi_i) log(p(\pi_i))$, where $p(\pi_i)$ indicates the occurrence of pattern ordering $\pi_i$on the subset under analysis divided by all possible patterns: this is simply the frequencist definition of probability[11]. More formally, for a series of length T, $p(\pi_i) = \frac{card.\{t\shortmid 0 \leq t \leq T-(d-1), x_d(t), \pi_i is\, of\, type\, i\}}{T-(d-1)}$. Note that the embedding dimension has a different interpretation than the one defined in the previous section, and hence is not necessarily the same: here, embedding dimension stands for the *length* of a sequence upon which permutations of values will be defined. *Ibid.* (2010) supply the following rule of thumb to select the optimal embedding: choose the largest d such that $5d! \leq T$. This allows us to pin down the embedding dimension for all three series as $d = 3$. The optimal delay time is less straightforward: I follow Riedl *et al.* (2013) and apply the common practice of selecting $\tau = 1$ in most economic and environmental studies for processes best described by discrete dynamics possessing an inherent cycle.

[9] If the base of the log, b, is two, the units are referred to as *bits*.

[10] An example may clarify this point. In *Ibid.* (2010), the authors supply the following: For a time series with seven observations, $\{3, 6, 9, 13, 5, 11, 1\}$and $t = d = 3$, as $x_{t+2} < x_{t+0} < x_{t+1}$, then the correct permutation is $\pi_3(3) = \{201\}$.

[11] i.e, the probability of event A equals the experimental number of cases where A occurs divided by all possible cases.

2.3.2 Measures of emergence, self-organization and complexity

Besides the uncertainty in economic data, which can be captured directly as permutation entropy defined above, we follow the abstract proposal of Gershenson and Fernandez (2012) and derive related[12] measures of emergence, self-organization and complexity. They describe emergence as novel information: hence, setting $E = \frac{1}{log(d!)}H(X)$, i.e, emergence as the normalized information, yields a measure of the production of novelty in data. Self-organization S reflects the adaptation to the novelty that is continuously produced in an evolutionary process: according to *ibid.* (2012), self-organization reduces information[13] and conversely, if the process produces information, S decreases. They thus define $S = 1 - E$ and we will avail ourselves of their measure. S can be seen as a measure of order and E, of variety, such that high E is interpreted as chaos in their paper. If one accepts complexity as a regime poised on the balance between order and chaos, it is clear that a high complexity can be reached only when self-organization and emergence are balanced. Hence, and according to their proposal, $C = E * S = (1 - S) * S$. DeWolf and Holvoet (2004) state, in a similar manner, that a chaotic system is not self-organising in that it doesn´t "organize itself to promote a specific function". Emergence is conceptualized as self-organization *upon which* selective pressures have been applied, and also see it as an increase in order, in that emergents become more organized.

Care should be exercised on three points. The first is not to read too much in the numerical estimates *per se* of these measures: they are to a large extent the result of a convention: we could just as well have defined emergence and self-organization in a symmetrical way[14]. Rather than numerical estimates, we are mostly interested in changes in the measures defined above over time and their interplay. We thus follow the usual approach and compute first the empirical permutation entropy, and then the resulting measures of emergence, self-organization and complexity defined above on sliding time windows of fixed length (Unakafova *et al.*, 2013). In this case, a 5-year moving window (20 quarters per window) was used for each of the series. The second is to resist the temptation of anthropomorphising the process, a jump that some of these terms can sometimes suggest, and not to interpret these results as anything that can be implemented or the result of a directing human will and intention. The third is that both chaos and order are needed along with the proper spontaneous balance between them that emerges in an economy as agents learn and adapt to novelty. One should not jump too quickly to the normative conclusion that more chaos is dangerous in and by itself: it is the *decreased balance* of them that increases fragility.

According to the theoretical machinery developed thus far, this translates as an decrease in Complexity triggered by an increase in Emergence that is not matched by a corresponding increase in Self-Organization. Kaneko (1993) argues that chaos is relevant to the emergence and maintenance of complexity and diversity; moreover, failed deployed plans are important for coordination as agents pick up debris of other people´s plans: this information goes much beyond that conveyed by prices: it also spans that conveyed by failed businesses, possible existence of a consumer base, enabling technological constraints, and so on.

[12] They derive their measures taking into account the baseline Kolmogorov-Sinai entropy: The adaptation for permutation entropy operated here presents no special difficulties, however.

[13] From a micro perspective, one can interpret this view as the diffusion and adaptation of novelty: when everyone knows or applies a rule, it is a novelty no longer.

[14] Hence, nor does this mean, for instance, that there is more chaos than self-organization, as the graphs seem to suggest.

2.3.3 Results

While open physical systems are characterized by increasing (thermodynamic) entropy over time, such a conclusion is far from warranted in the social case. According to Amaral (1999), economic systems have processes that both increase and decrease complexity[15]. The empirical results of this section corroborate this hypothesis, as no clear upward, or indeed downward, trend can be detected over the entire period.

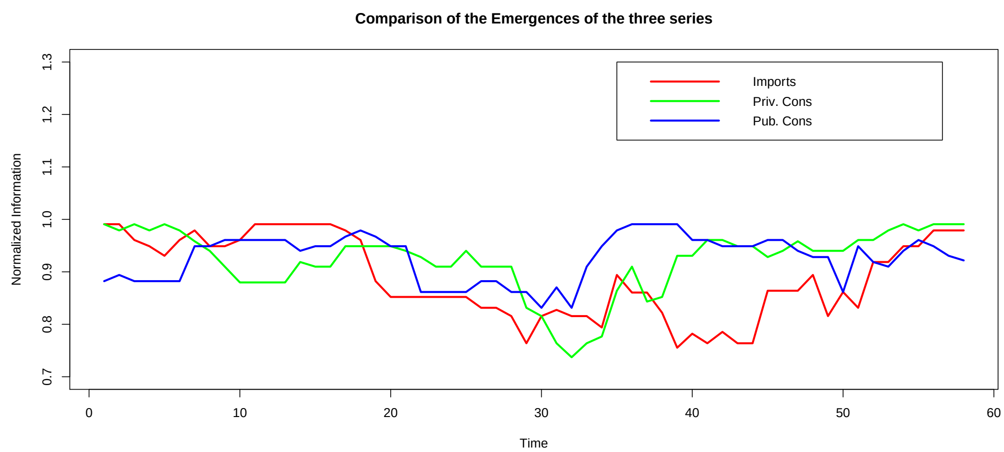


Figure 3: Emergences of the series

The graphs depicting the time evolution of self-organization, by construction, tell the symmetric part of the tale, i.e the evolution of adaptation:

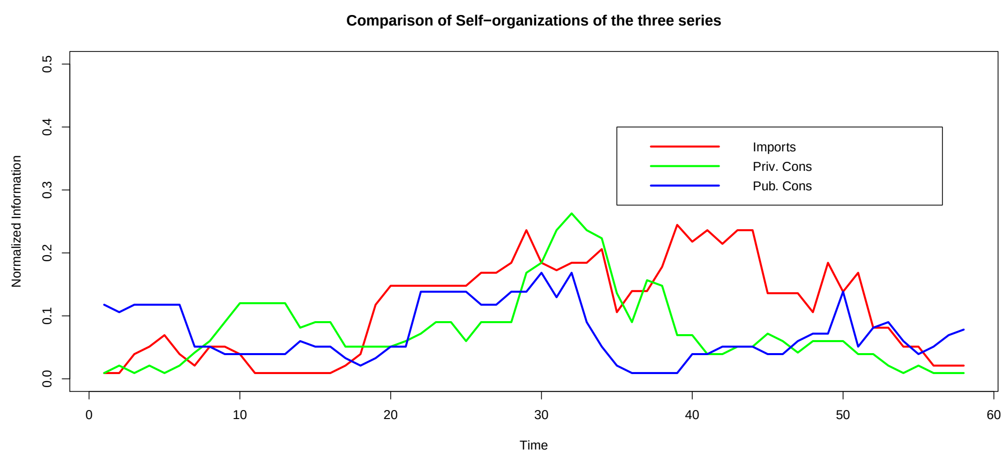


Figure 4: Self-Organization of the series

[15]He exemplifies intersectoral division of labour as a process increasing complexity and vertical integration of sectors as a process decreasing it.

Complexity encompasses both emergence (novelty/chaos) and self-organization (adaptation to novelty/order). The following graphs summarize the evolution of complexity over the last 20 years and bring together the previous two sets of graphs.

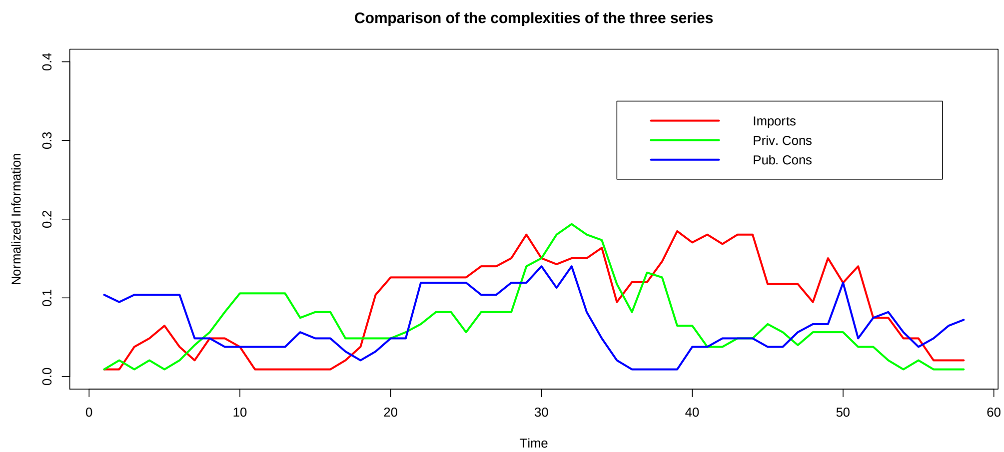


Figure 5: Evolution of the complexity of the series

Our goal is to obtain a time path of the information produced by each of the three series. We have also argued emergence to be an indicator of novel dynamic patterns and self-organization, to adaptation thereof, while complexity balances the two, and can be considered the width of a corridor of stability of an economy. While no clear trend is discernible, or that overall complexities are larger in any of the series over the entire period, the discussion of the previous sections indicates that, given the different dynamic signature of the three series, different potential consequences emerge. In particular, the Pink Noise regime underlining the evolution of Private Consumption is akin to the dynamics of Complex processes such as the sandpile model, where small causes can magnify to large consequences (Bak, 1987;1993). Indeed, this is the province of Black Swan Events. This intuition is backed by the magnitude of the Largest Lyapunov Exponent: the largest one is that of Private Consumption.

Thus, a large part of the Portuguese economy is losing complexity, in the sense of a proper balance between order and chaos: in particular, the latter is becoming more important in the dynamics, while the adaptive response is becoming smaller: i.e, more chaos and unpredictability that translate in a more uncertain and fragile economic environment, policy propaganda notwithstanding. On the other hand, Imports seem to be more resilient and not to have been affected by this complexity-reducing effect since 2011. This squares with the finding that this series has the smallest Hurst exponent and scaling parameter α, "forgets" its past sooner, and is the closest one to White noise. In this sense, randomness is resilience.

The Portuguese GDP has been driven by the dynamics of aggregate demand, especially private consumption. Barreto (2013) reports how over the last 30 years, growth of private consumption accounted for 3/4 of Portuguese GDP growth. Since the method of sliding time windows reveals a decrease in the complexity of this series, one can assume that Portuguese GDP is gaining uncertainty in the form of the increased entropy generated by it. Given this increase in the inherent uncertainty of the series, one should be wary about the announced success of policies of growth based on the expansion of private consumption. Indeed, policymakers are correct in attributing to it a pivotal role, but wrong inasmuch as they don´t consider its increasing unpredictability. The reliance on standard economic tools may help obscure the true nature of the dynamics of the series. Belbute and Carneiro (2009), for instance, claim that given the low degree of persistence of Private Consumption vis-a-vis that of the Eurozone, countercyclical policies can be more effective. We claim that more care must be paid to the novel patterns the series produces, and in this sense tools from Complexity Theory are invaluable.

2.4 Concluding remarks

Mesjasz argues for a deeper integration of complexity theory from both the “hard” and “soft” sciences viewpoint: the role of metaphor is essential in bridging the gap between these. The most appropriate one for this paper is that of a sea of tranquility on the surface upset by deep, unseen ripples in the bottom that can break through the apparent stability.

The Portuguese economy is driven by the formation, deployment and collective interaction and revision of agents´ plans of action. These are composed of decisions of several kinds that have impact on the economy - which is not to say all of these decisions are configured, or indeed gain meaning, in the economic sphere - and encapsulate goal-oriented choices, “Animal spirits”, and accidents. The formulation and revision of plans occurs on a micro-level, where subjectivity and acts of volition are crucial. Rules, institutions and the history configuring collective choices are the province of a meso-space, characterized by the ubiquity of power laws. The resulting macro dynamics of such a system are complex in a precise sense - here, as a balance between order and chaos, novelty and adaptation, that give resilience to a system while maintaining its creative capacity - and characterized by being far from equilibrium and gaining “order through fluctuations” (Prigogine and Stengers, 1985; Ramalhete Moreira, 2006): A system that follows a deterministic path (in this case, this is the interaction of plans of action) until it enters a bifurcation region (whenever a significant decision is made and acted upon), where creativity is exercised. In the neighborhood of a bifurcation, even the tiniest nudge can make the system choose one of the available paths: here, the distinction between randomness and determinism is blurred and classical laws break down. It is the province of critical decisions: in the social domain, this is what it means for a system to be on the edge of chaos. While economic growth trajectories must have a deterministic structure since economic systems are dissipative, remain structurally coherent and are irreversible in time (Lewis, 2011), there is more to it than chaos and pure determinism. As argued, the dynamics of such processes are aptly characterized by the Pink Noise behavior of the logarithm of the spectrum. We have seen how the series of Private Consumption follows such dynamics.

The first part of this essay focused on the macro level dynamics and on finding chaos for the series of first differences of imports, and private and public consumptions, the three nonlinear components of the series of first-differenced Portuguese GDP. Following standard procedures of phase space reconstruction along with the estimation of the leading Lyapunov exponent, it was found that all three nonlinear series are weakly chaotic.

Of these, the one with the behavior closest to a random process is that of imports - we have hypothesized that feature to be a sign of resilience. This was just a static picture: Hence, the second part of the essay discussed that a complex system is a hybrid of order, adaptation to novelty, self-organization, and emergence of novelty, new patterns, chaos and creativity, and applied information-theoretic measures proposed in the literature to ascribe the relative importance of each, as well as their time path. The main finding of this part is that the balance between order and chaos - which arises spontaneously - was disturbed by the onset of the symptoms of the global financial crisis, reducing the degree of self-organization with respect to emergence, especially for Private Consumption. Hence, the emergence of novelty is not being matched by an appropriate adaptation to it, a worrisome conclusion indeed. Given the methodological stance of the paper, offering policy advice extending beyond caution in the application of austerity or full reversal thereof without further study of these issues would be ill-advised. There are several topics needing further attention. First, a larger picture of the portuguese macrodynamics is needed. I focused only on three types of demand, whereas the supply side was left out. Moreover, the spectrum of economic decisions is much larger than consumption - it encompasses labour and financial markets along with all their interrelatedness. Another interesting question is whether this kind of dynamics can arise from a simulation experiment. This is the topic of the next chapter, where an agent-based setting of a simplified economy to discover a road to the "edge of chaos" from a bottom-up perspective. The objective is to tackle the meso dimension to obtain spontaneous macro dynamics with cycles of the type analysed here for a real economy. For the reasons exposed above, a proper computational tackling of the micro realm can never be achieved. However, a tentative link between intentional action, mesospace evolution and macro complex dynamics can be studied from a philosophical point of view by analyzing, complementing and criticizing the action plan approach as outlined by Muñoz and Cañibano (2007). Hence, how individual complex dynamics of cognition emerge and influence complexity at higher levels is the topic of the fourth chapter.

Chapter 3

Economic growth as a biological metaphor: an agent based approach with adaptive agents

3.1 Introduction

Economic growth is a process of growth of knowledge: and moreover, that process may be combinatorial and recombinant in nature (Weitzman, 1998), as firms and entrepreneurs learn how to combine goods to satisfy, and indeed create, demand. This process of discovery and creation is at the heart of business cycles, as agents go about inventing new products, new means of productions, and replacing methods and preferences. New goods and services borne out of entrepreneurial activity displace old ones; similarly, Metcalfe, Foster and Ramlogan (2006) state that the generation and resolution of economic diversity are the main engines of growth. This process is continuous (Huerta de Soto, 2005) and at the heart of the gales of "creative destruction" characterizing Schumpeterian growth. In the ethos of evolutionary economics, this process is akin to a biological one. In this view, more than a mechanism for the spreading of information through prices, the market is a testbed for new ideas - it is the selection environment that places a fitness on entrepreneurial decisions, in which the resulting ideas thrive or die. Moreover, markets for goods emerge endogenously as combinations of knowledge, ambition, new technologies and consumer needs in a Variation-Selection-retention environment fed by a population of rules. Microtrajectories are actualized within the boundaries of a firm (Dopfer, 2011). One must then focus on the dynamics of firms as carriers of rules (genotype) determining their behaviour (phenotype) and success or failure. Moreover, (Birchenhall, 1995) notes how Genetic Algorithms can be used to evolve new rules, as a tool to generate novelty through recombination and mutation of existing artefacts. This paper takes up on his proposal and explicitly develops a model that can simulate the emergence of complexity as the relative presence of ideas in a market - here conceived of as production methods encoded in blueprints. The innovative process of a market economy is both directional, as it is borne out of ideas of entrepreneurs related to states of the world, and "blind" in the end result of the meso-trajectory of an energy-matter actualization (Dopfer and Potts, 2004)[16]. Riechmann (1999) sees the market as a coordinator with an important role for the stability of social learning schemes, and the Genetic Algorithm as a tool capable of modeling the interplay of coordinating tendencies arising from competitive adaptations in the market, and de-coordinating tendencies caused by the introduction of novelty. This interplay between order and chaos features a dynamical signature in time series that can be measured using entropy. Another approach, the one followed here, is measuring the diversity of energy-matter actualizations as a proxy for the ruggedness of the path of an economy. Pushing further our biological metaphor, it is a measure of diversity in the population of energy-matter actualization. Modern economies, in this sense, are much like a language: we do not have evidence of increasing complexity in this strict sense.

[16] Dopfer and Potts mean by this the outcome of an idea ("energy)" that is translated into a set of actions ("matter)" leading up to its concretization and subsequent success or failure. By "meso", they mean the level of economic activity poised between the realm of individual decision-making, or "micro", and the outcomes at the aggregate level one sees in aggregate "macro" statistics. This paper also uses the same words to denote the same realities.

This model takes the biological metaphor to heart: and Stuart Kauffman´s classic NK model (Kauffman, 1993; Westhoff and Yarbrough, 1996) is in this literature, an unavoidable reference. Therein, single species are defined as collections of genes defining a genotype with an associated real number to it, its fitness. The distribution of fitness values over the space of genotypes is called the fitness landscape, - and adaptation is taken to mean exploration, or hill climbing, towards zones of the state space with higher fitness. The model is controlled by two parameters: N, the number of genes in each genotype, (hence, the fitness landscape is an N-dimensional hypercube) and K, the number of genes other genes can influence: Kauffman refers to K as epistatic interactions. Kaufmann´s species move around the hypercube if a random mechanism shows that mutating some of its genes is beneficial, in terms of supplying a higher fitness. And Kauffman also supplies an analogon to the notion of complexity, here understood as ruggedness of the fitness landscape, as the interaction between the values of N and K. With a small K, all species reach local optima. However, if N=K-1, the landscape is random. The higher the K, the more likely chaotic changes in ecosystems are, in the sense of small perturbations in the gene pool of a species triggering an avalanche of coevolutionary changes (Westhoff and Yarbrough, 1996) . This is essentially Leijohufvud´s corridor of stability hypothesis. Complex systems such as economies tend to find a compromise in this region, and indeed finding this region is one of the guidelines of the present work.

In the expanding literature on agent-based modeling applied to economic growth in the evolutionary tradition, one would be hard-pressed to find modeling exercises that both explicitly account for the biological connection, featuring selection of production methods, of goods and labour types, and with more than a mockery of the innovative process as modelled by a Markov, Poisson process or some such, but also the connection between intentionality at the micro level and adirectionality at the macro level. A notable exception is Wolfson´s XECON model (Wolfson, 1995) featuring firms, workers, a finance sector and a government. He also considers the atoms of his model to be techniques of production, as inputs required to produce one output. Firms differ in both the amount of input required to produce a unit of output and on the intensity of the use of the input. Workers supply labour and consume output based on a lexicographic ordering. However, he does not explore the dynamical traits of his model, or indeed the frequencies of populations of rules, nor does XECON feature a genetic/evolutionary basis from the outset. (Eicher, 2003), presents a model of Schumpeterian growth with evolutionary search: an evolutionary algorithm is used to turn a set of ideas into another. His model features quality ladders and rules as strings. Moreover, he also measures the relative frequency of the ideas in his artificial economy. However, he lacks the key idea of individual directionality in the process of formation of ideas. Rather, the evolution of ideas from a period to the next is modeled as a Markov process. In other words, not enough attention is paid to the idea of firms as local satisfiers, as Herbert Simon would put it (Simon, 1956). Eliasson´s model, MOSES, (Eliasson, 1984) used to analyse the industrial growth of the Swedish economy, features a changing landscape of profit and firms that supply goods with the aim of hitting a target of profit. This target controls investment decisions, and entry and exit of firms in response to perceived excess returns on capital. However, not enough attention is paid to households and the markets they form together with firms. Indeed, a key feature that is not captured in most models, with MOSES not being an exception, is the following: market results propel change, but that change cannot be imposed by the modeller directly as a changing fitness landscape: rather, it must be introduced by firms themselves as they evolve their internal structure. As McMullen and Shepherd note, "There is no market independent of the actors who create it (...) Someone, somewhere, must undergo a decision process in which action is chosen if any market "process" is to occur". (McMullen and Shepherd, 2006, page 136).

In virtue of its hybrid nature, this model can also be placed inside the more traditional literature on agent-based economic growth. And this literature is itself a maze, featuring several articles that differ greatly in how much they stray from the neoclassical paradigm. While these are richer in terms of predictive power and model features, they tend to rely on too unrealistic a picture of the individual, i.e, they trade a greater power for a greater unrealism. What follows is a review of that literature.

Dosi, Fagiolo and Roventini (2010) develop and study an agent-based model where a Schumpeterian structure of creative destruction that features entry and exit of firms is complemented by a Keynesian-like study of the effect of policies affecting demand on macro aggregates to generate long run growth with cycles. Their economy is composed of firms on two sectors, intermediate and final goods, consumers/workers that can be both employed or unemployed (supply labour in a labour market and consume from the supply of final goods producers), and a public sector that levies taxes. Innovation by capital goods firms is the motor of the long-run growth process of the economy. The labour market is Keynesian and features involuntary equilibrium unemployment and labour rationing. The wage rate depends on institutions, unemployment and average productivity. In the macro framework, they obtain endogenous growth with fluctuations for output. Their model is also able to replicate some stylized facts on the evolution of GDP, investment and consumption, as well as typical business cycle properties on the cyclical behaviour of prices, productivity, inflation and markups.

Bruun (2008) uses an agent-based model setting to translate to a computational setting the ideas of Keynes as regards the effects of fundamental uncertainty on economic behaviour and to test the adequateness of “Animal spirits” and heuristics as rules of decision. The starting point is that, as in Keynes, enterpreneurs think and make decisions in terms of monetary outcomes, rather than profit-maximizing quantities. Agents are grouped in producers of consumption goods, investment goods and consumers. The model features a credit, a labour and a financial market. The framework is one of cellular automata where consumers move in a grid and producers place consumption goods in the grid. Final good firms calculate in each period the quantity of goods to supply given last period sales and (monetary) profits following simple threshold rules. Production requires both capital goods/machines supplied by investment goods firms (by order of final goods firms, that make investments in machines following rules of thumb) and labour supplied by consumers. Consumers decide on the amount of consumption taking into account the average of the consumption of neighbours in the (Moore) neighbourhood as well as financial standing of the individual. Consumers also act in a labour market where they supply a fixed quantity of labour. Unemployment may exist if firms do not hire agents for the full amount of hours they offer. The credit market is a simple bookkeeping device that accounts for firm bankruptcies: bankrupt firms leave and are replaced. The simulation yields endogenous, i.e emergent business cycles resulting from the valuation and devaluation of firms´ stock. An increase in the price of a stock sets off investment. With a lag comes an increase in the consumption of consumption goods and an increase in the amount of supplied stocks, as investment goods firms need to finance their expenses.

Bruun (2003) and Bruun and Luna (2000) also develop a purely Schumpeterian production and consumption economy in the Swarm environment, where creative destruction and enterpreneurs operating by trial and error play a central role as a growth mechanism. Agents start out as production units on a grid where consumers move issuing demand for goods, and can evolve into workers, enterpreneurs and effecitve firms. The simplest production units are single-layer neural networks with 2 input and an output neurons. The learning process is simulated by changing the weights of each of the input neurons to try and predict the demand of moving consumers that emit two signals through an XOR[17] process. A satisfied consumer remains in the same place

[17]Exclusive OR: Boolean operation taking two inputs, that outputs a true or false truth value if one of the inputs is false, while the other

except for some mutations that are allowed for. Firms can grow out of ineffective (in terms of demand forecasting and satisfaction of consumers) production units by recruiting neighbours: i.e, they grow. As agents move on the grid and place their demands, production units both form their own forecasts and look at each of its 8 neighbours´ forecasts. With a number of correct forecasts done by a neighbour, this neighbour receives a wage offer to be recruited; the recruiting production unit becomes an enterpreneur if the neighbour accepts, adding the extra neural network to its own, thus improving its ability to satisfy demand. Thus firms can improve in their network training by this process. A firm only receives an order by a consumer, and is correspondingly paid, if it responds correctly to his signal: firms need labour to produce and pay out wages, and must contract debt from a credit sector that keeps a bookkeeping activity for bankruptcies. Thus firms that are not able to respond correctly go bankrupt and are replaced. This is the evolutionary selection, Schumpeterian feature of the model. Wages are negotiated between enterpreneurs and workers. The model is able to generate a simple growth pattern along with endogenous cycles. Naturally, the economy is constrained by the number of effective firms: once a given limit of effective firms is reached, growth stagnates. The bankruptcy mechanism, which forces firms disappearance along with renewal of production units that must restart the Neural Network training, may be the mechanism behind the observed cycles: as enterpreneurs must first contract debt before producing to train their neural network, the economy grows while building debt. After the training process and demand forecasting of consumers, wages are paid and spent on consumption goods, bringing about an upturn (ascending phase of the cycle). As more and more effective firms are created and start to concentrate, a number of firms that are unable to become effective go bust. This breakdown announces the slump of the economy and the beginning of the downturn in the cycle: the authors choose a bankruptcy rule where losses are divided among all agents. As bankruptcies spread, workers lose wages, and consumption falls. They also suggest a number of interesting extensions.

Russo, Ricetti and Gallegati (2013) explore the interplay between wealth inequality and financial fragility in an agent-based macroeconomic model. Special attention is therefore paid to wealth dynamics. Their model consists of households accumulating wealth through wage and dividends received from firms, firms that accumulate profits and pay taxes and dividends to owning households, banks, also accumulating profits that can be negative or positive, through interest of loans to firms and households, a central bank and a government. The government has a current account where the side of expenditure is composed of wages paid to public workers and interest on public debt, and the revenue is obtained through income taxes and interests gained by the central bank; the central bank decides the interest rate and, consequently, the quantity of money in the economy. All the agents follow simple adaptive rules, operating in the context of a credit market, where households and firms form the demand and banks the supply of credit, charging an interest for loans and firms´ demand depends on their leverage target and net worth, banks supply depends on central bank credit, deposits, and net worth of the bank, a labour market, where both firms and the government can hire labour from households. Firms´ demand depends on total available capital. Moreover, if there is unemployment resulting from a mismatch between labour supply and the Government pays out unemployment benefits. There is also a goods market, where firms supply goods and households form the demand. Households´ desired consumption follows a simple rule of thumb that is a function of wage, the propensity to consume current income and propensity to consume wealth. The supply of firms depends on the number of hired workers. Finally, a deposit market, with banks on the demand side of this market, and households deposit money, forming the supply. Banks offer an interest rate to households for

is true.

depositing money, and hold public debt. Households choose banks ranking them on the basis of offered interest rate. These markets operate through a common decentralized matching mechanism, where agents form lists of suppliers and pick the cheapest. As a result of their simulation, endogenous business cycles appear through the interaction of the real and financial spheres of the model. When firms profits increase, they expand their output and hire more, lowering unemployment and increasing wages. This same mechanism may, however, reduce firms´ profit, both as a consequence of the increase in output and of a lowering of the hirings by firms as an attempt to curb costs. The booms and busts are mitigated by the government. A decreasing propensity to consume with respect to wealth leads to skewness of wealth distribution and to an amplifying effect of a crisis.

The model of this paper also features agents disposed in a social network, to model neighbourhood effects and the fact that these matter both for technology diffusion and spreading of consumption habits. Both households and firms are disposed in networks that can be one of four types. Schramm *et. al.* (2010) present a similar model, featuring adaptive consumer agents, brand agents, and multiple brands. Consumers adopt a brand if a threshold, dependent on an index based on price, promotion and brand density, reaches a certain value. Consumers are placed on a network to model social influence and show that diffusion of a brand among consumers is quicker in more clustered networks, as opposed to Erdos-Rényi configurations. However, in their model, only consumers are disposed in a network. There is no production or, indeed, way of studying the diffusion of technological standards among firms. Delre, Jager and Janssen (2007) study how the speed of diffusion of a product depends on the topology of the consumer network and heterogeneity. They find the speed of diffusion is large in small world networks (as it features both a small path length and high clustering), being relatively low in random and regular networks. Moreover, they also use a utility function that depends on individual preferences and on the fraction of neighbours that adopt the good.

Lee, Kim and Pak (2013) also features a network of connected consumers trading information on products, and disposed on a social network. The decision of adoption is modelled as a fuzzy set. None of these models hybridizes the social network approach with the literature on evolutionary economics.

An interesting paper that has many similarities to this is Sack, Wu and Zusman (2013). The authors study the evolutionary dynamics of literary genres on an agent-based model programmed in Netlogo. Their units of selection are also encoded as binary strings, and their consumers also have preferences based on an individual ideal type. Similarly, they also use a modified GA to model the evolution of genres. However, their model is such that consumer preferences, in the form of a fitness landscape, is programmed from the beginning, rather than emerging as time goes by in the simulation and new artifacts become available. Indeed, the success of the crossover and mutation operations depends on the configuration of this landscape.

Garibay, Hollander and O´Neall (2012) also feature an economic growth model with innovation and diffusion. They also have an analogue for species - as a set of firms with a particular technology used for production. Economic entities are resource transformers, taken from the environment, and in their model a "Government" is also used to close the monetary cycle and prevent dissipation. They explicitly use a Cobb-Douglas function, and their government can create money, thus employing monetary policy. Their structure of interaction is local in that agents configure a production network with their neighbours. Their macro variables also suggest the presence of cyclic behaviour of output.

Dahlan and Situngkir (2010) present a formalism that is similar to the one employed in this paper: the utility of a consumer is also the difference, measured as the Hamming Distance, between a supplied good and consumer preferences, both as binary strings. They also conceive of innovation as the change in memeplexes, and that triggers the change in consumer preferences as an autocatalytic process. Their model features different behaviour according to which mutation of memeplexes, along with changes of preferences, are present or not, as well as a power law on the distribution of product lifetime. However, their model does not feature a full economic cycle, money, labour, behaviour of firms or households, or changing prices. Thurner, Klimek and Hanel (2010) present a Schumpeterian economy with new goods and services emerging as combinations of existing ones in a model of self-organized criticality. It features a hypothetical production table specifying what inputs can be combined. Their model is a minimum setting where booms are followed by crashes, and turbulence is followed by restructuring. However, they do not have prices, a labour market, or indeed any measure of complexity.

The rest of the paper is as follows. Section 3.2 presents the full model. Section 3.3 presents the results of the baseline simulation along with those obtained by varying the saving rate of firms and the network connecting them. Finally, section 3.4 concludes with some remarks and further work that can be undertaken.

3.2 The model

The model is programmed in Netlogo and has the following graphical appearance:

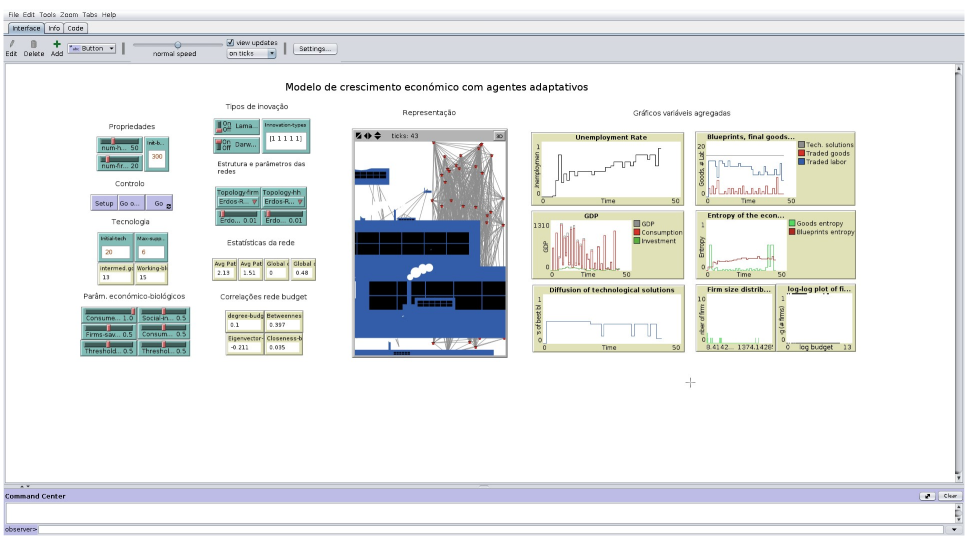


Figure 6: Graphical appearance of the model in the Netlogo GUI

3.2.1 Primitives

Let us give operational meaning to the statement that growth is really a high-dimensional search in a space of knowledge - and that knowledge is both consubstantiated and instantiated as the combination of existing goods and services into new goods and services.

I follow the insight of Dahlan and Situngkir (2010) that the primitives of the model - goods, labour types and production methods - can be seen as memeplexes to be evaluated. At the core of the model lie three types of memeplexes: intermediate goods, labour capacities, and final goods, along with a set of instructions on how to combine the first two to yield the latter. The aim of this construction is twofold. On one hand, it allows the use of Genetic Algorithms (Holland and Miller, 1991), giving intuitive sense to what it means to combine goods: combining goods and services is akin, in this model, to combining genes and letting an environment evaluate the fit of both the production method and final good. On the other hand, comparison of two goods, or of production methods, is facilitated in that the Hamming Distance between two strings can be deployed, and it serves is as a notion of similarity that informs choices of firms and households.

Both intermediate and final goods are represented here as binary strings of length 20, such as the following:

1	0	1	0	0	...	1

labour types are also encoded thus, but with a 0 on the initial position of the chromosome to distinguish them from goods:

0	1	1	1	0	...	0

These are held by households and stand for a capacity to do something. In both cases, the chromosome can encode either the presence or absence of some feature, although this is not crucial for the analysis.

The economy starts with an initial population of final goods and labour types, the number of which can increase during the simulation, with variable prices. Intermediate goods are fixed and have a fixed price throughout.

Besides the population of final and intermediate goods, there is also an initial set of production plans, dubbed henceforth blueprints, consisting on an instruction on how to combine an intermediate good and a labour type supplied by households to generate a final good. A blueprint consists on a set of chromosomes and a cutoff point that specifies how to combine them to generate a final good. The process is inspired on the operation of crossover of Genetic Algorithms: therein, two chromosomes exchange bits / genetic information from a specified bit length, a process that generates offspring that enter the gene pool for evaluation. An example of a blueprint combining the previous two, is the following:

$$\begin{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0 & 0 & ... & 1\end{bmatrix} & \begin{bmatrix}0 & 1 & 1 & 1 & 0 & ... & 0\end{bmatrix} & 3 & \begin{bmatrix}1 & 0 & 1 & 1 & 0 & ... & 0\end{bmatrix}\end{bmatrix}$$

This blueprint is really short-hand notation for a set of instructions that are unpacked by firms in the production process, and are of the following form:

$$\begin{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0 & 0 & ... & 1\end{bmatrix} & \begin{bmatrix}0 & 1 & 1 & 1 & 0 & ... & 0\end{bmatrix} & 3 & \begin{bmatrix}1 & 0 & 1 & 1 & 0 & ... & 0\end{bmatrix}\end{bmatrix}$$

$$\updownarrow$$

$$\begin{bmatrix} \begin{bmatrix} 1 & 0 & 1\vdots & 0 & 0 & ... & 1 \end{bmatrix} \\ \downarrow cross\, at\, 3 \\ \begin{bmatrix} 0 & 1 & 1\vdots & 1 & 0 & ... & 0 \end{bmatrix} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & ... & 0 \end{bmatrix}$$

The final good inherits, as it were, the bits of the labour type from point 3 onwards, and those of the intermediate good used until point 3. This is intuitive in that goods and services existing today are combinations of features existing in goods and services from before. Let $HD(String\,1 - String\,2)$ be the Hamming distance, or similarity, between two strings. The higher the Hamming distance, the more the two strings are different. Hence, goods with a relatively high Hamming Distance are very different goods.

The model features three types of agents, of which two, firms and households, are adaptive, and the third is used as a "sink" to close the model and prevent dissipation, as in Garibay, Hollander and O´Neall (2012).

It is a monetary economy: final goods and labour types have prices associated to them, that vary with supply and demand according to a Walrasian auctioneer. By this recursive reasoning, blueprints have an associated cost and profit. The latter serves as the fitness of each member of the population, following most studies on evolutionary economics.

Definition: State of the economy. The State of the economy is given by a population of intermediate goods (unchanging), a set of final goods and respective prices (initially equal), a set of labour types and associated wages (initially equal) and a population of blueprints with associated profit. This is the analogon of the fitness landscape in Kauffman´s model. Formally, we have the following set of evolving populations:

1. WA, a (set-theoretic) relation between labour types and respective wages. An element $wa^t \in WA$ is a pair associating labour type $l^t$to its wage, as$\left\{ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & ... & 0 \end{bmatrix} \rightarrow 10 \right\}$. Denote the maximal element of this relation, i.e, the labour type associated with the highest wage at t, as wa^{*t}.

2. PR, a relation between final goods and respective prices. An element $pr^t \in PR$ is a pair associating a final good to its respective price at t: $\left\{ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & ... & 0 \end{bmatrix} \rightarrow 6 \right\}$. Let the maximal element of this relation be the final good associated with the highest price at t, denoted as pr^{*t}.

3. BL a (set-theoretic) relation between blueprints and respective profits. Let $bl^t \in BL$ be a pair associating a blueprint to respective profit at t:
$\left\{ \left[\begin{bmatrix} 1 & 0 & 1 & 0 & & ... & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & ... & 0 \end{bmatrix} \quad 3 \quad \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & ... & 0 \end{bmatrix} \right] \rightarrow 1 \right\}$.
Denote the fittest blueprint by bl^{*t}.

The state of the economy is the triple $< WA, PR, BL >$. These sets will be useful as both firms and households rely on them, as well as on their maximal elements, to mutate. One can look at them as price lists of all things existing in the economy.

The following subsections specify the agent types along with their behavior.

3.2.2 Agent types

3.2.2.1 Households

Households are the first type of adaptive agent in the model. They are disposed on a network that can be one of four types specified by the user (Erdos-Renyi, Complete, Small-world and Scale-Free), specifying a set of neighbours. Each household holds an initial budget in monetary units, a labour capacity as specified above, and a reference good used as a benchmark when evaluating the goods on the market supplied by firms. Each period, each household enters the labour market to supply their unit of labour to firms and acquire one unit of final goods from them. Initially, they have the same budget. Let $Budget^{t}_{household}$ denote the budget of household j at time t, and $InitBudget_{household}$ their initial budget. Households have the capacity to adapt by changing their labour type to a more marketable one.

3.2.2.2 Firms

Firms are also connected on a network type - the four types specified above for households are available. Moreover, they also hold and initial budget in monetary units, and are carriers of blueprints: each holds a subset of all production methods in the economy. Firms are configured by blueprints, genotypes, and are hence phenotypes, or manifestations of genotypes (Hodgson and Knudsen, 2010). Hence, it is not them, but rather their composition as a set of rules, that is subject to selection. Firms hire labour from households and sell them final goods.

Firms start out with the same initial budget $InitBudget_{firm}$ and a random allotment of blueprints from those in the technological frontier of the economy. These can be repeated or not: in the first case, firms produce more than one copy of the corresponding final good. The number of blueprints serves simultaneously as the maximum production capability of firms and is a parameter that can be tuned by the user: their maximal supply. Each blueprint j has a fitness associated to it that equals its profit, given by:

$$\pi^{t}_{j} = p^{f,t}_{j} - p^{i}_{j} - w^{l,t}_{j} \tag{4}$$

(4) is standard; In words, it states that the profit of blueprint j at time t equals the price of the final good f yielded by the unpacking of j, p^{f}_{j}, minus the price paid by the firm activating j to acquire intermediate good i and wage paid to labour type l, respectively p^{i}_{j} and w^{l}_{j}. Define the cost of blueprint j as $C_j = p^{i}_{j} - w^{l}_{j}$. The budget of firms evolves as they carry out operations. Denote the budget of firm i at moment t by B^{t}_{firm}.

As both prices and agents evolve, the fitness landscape of the economy is not fixed: rather, it is in a state of flux. This is an important difference vis-a-vis most biologically inspired models of economic growth, and indeed of the NK model itself, where the landscape may be very rugged depending on the epistatic interactions, but is fixed and learnable as time goes by. Firms can adapt to it by changing their blueprint portfolio in the mutation module.

3.2.2.3 Centralized institution

As in nature and real economies, adaptation is costly. This process is simulated in this model by the fact that firms and households must pay a price in monetary units to adapt - depleting their budgets in an amount equal to the replacement cost of blueprints and labour types, respectively. In order to avoid dissipation, the amounts required are paid to a "sink agent" that is not adaptive and serves only to close the model. This agent receives the payments from both acquisition of intermediate goods[18] and reconversion costs and then redistributes the amount equally among all agents at the end of the iteration.

3.2.3 Schedule of events

The following figure is a flowchart illustrating the order of the events in the model:

[18] We may assume that this agent holds all intermediate goods. In Iran, for example, all oil wells are state-owned.

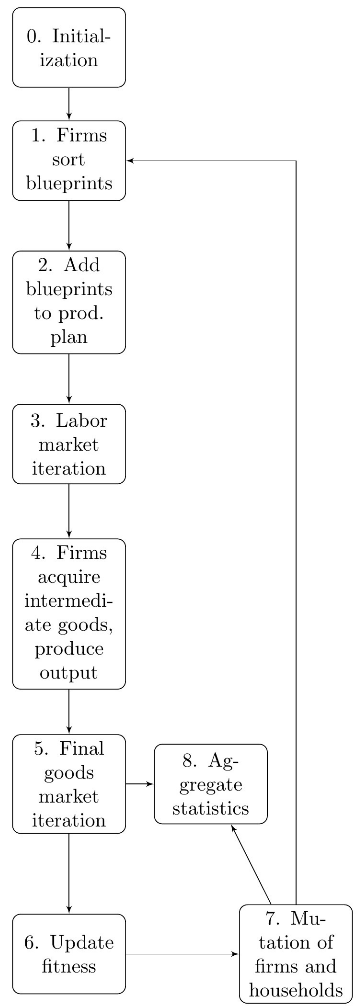


Figure 7: Flowchart of the model

Step 0. Initialization. The model first generates a series of blueprints that are the "historical inheritance" of the economy: their initial population of methods. Initially, firms are given a subset of these initial blueprints of the economy, along with an initial budget. Households are alloted a random labour type from those initially existing in the economy, a reference good with which to evaluate final goods, and an initial budget. The initial price and wage vector is set so that all blueprints have an initial profit of 0. Differences in fitness, and indeed the configuration of the fitness landscape, emerge out of the simulation and change over time. The initial fitness landscape is thus flat.

Step 1. Ordering of blueprints in decreasing fitness. Firms order blueprints in their portfolio in order of decreasing fitness.

Step 2. Firms add blueprints to a production plan. Firms place items of list obtained in Step 1. on a vector representing a production plan as a set of blueprints to activate. Firms add blueprints to their production plan until the total cost of the plan is such that condition (2) is met:

$$\sum_{j \in portfolio} C_j \leq (1 - s_{firms}) B^t_{firm} \tag{5}$$

The saving rate of firms s_{firms} is a global variable that can be tuned by the user. The production plan of each firm defines implicitly the respective demand for labour and intermediate goods. The demand of all firms for intermediate goods and labour types is thus defined.

Step 3. Labour market iteration. The labour market is the first time firms and households meet and match demand and supply. Firms place their demand for labour along with the corresponding wage in a list. Households place the labour type they supply in that period. The matching process is sequential for firms and occurs as follows. Each firm presents an ordered list of labour demands. If it finds a corresponding supply, the firm and household close the deal and that firm´s demand, along with the household, leave the list. If it does not, the firm eliminates the item from the ordered list and the iteration moves on to the next firm, that repeats the process. At the end, each labour type has an associated demand and supply, and corresponding excess demand. The model stores both the transactions carried out between pairs of households and firms and the excess demand for each labour type. Denote the excess demand of labour type l by

$$ED^t_l = Demand^t_l - Supply^t_l.$$

Unmatched households stay unemployed during that period, and unmatched firms do not activate the corresponding blueprints. Note that households can be unmatched both due to excess or insufficient demand of their particular skill set. Firms pay households the wage corresponding to the labour type supplied, and the latter update their budget, as the following figure shows:

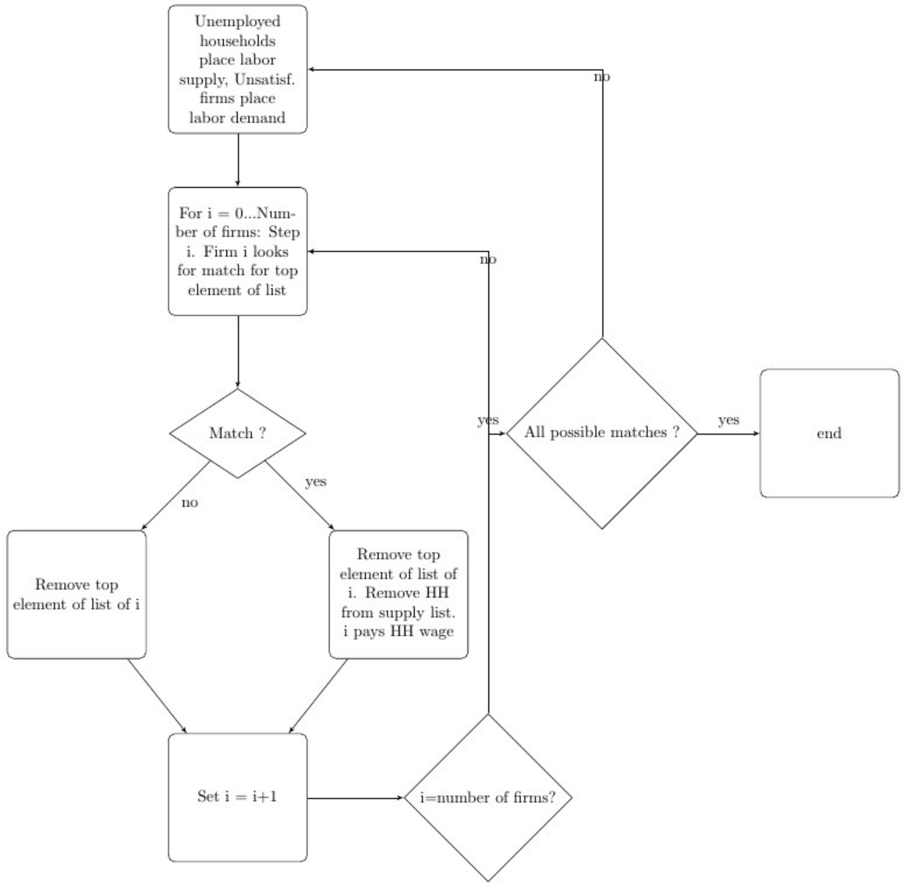


Figure 8: Labour market algorithm

Step 4. Firms acquire intermediate goods, produce corresponding final goods. Firms store the labour type they managed to contract and define the corresponding intermediate goods to complete the blueprints that will be activated according to the production plan. In the model, all intermediate goods are property of the state and have a fixed, unchanging price. Moreover, there is no capacity constraint on these goods. Further versions of the model will feature firms contracting final goods to use them as intermediate. Firms pay the state the intermediate goods they acquire and produce the final good corresponding to the blueprints they managed to complete. This yields a list of final goods to be placed on the market.

Step 5. Final goods market iteration. The final goods market is similar to the labour market, but with some key differences. Firms place the final goods they produced on a list on the market, along with their quantities and respective prices. Households first visit the goods market and store a random list of goods. Then, consult their social network for the goods stored by their neighbours. Finally, they sort the list of goods according to their utility function: the utility of household a of consuming final good f at time t is,

$$U^t_{af} = \beta(number\,of\,neighbors\,with\,good\,f\,at\,t) + (1-\beta)((length\,chromosome - (HD(good\,f - Reference\,good\,of\,a)) - p^{f,t}$$

where β is a tuneable social influence parameter. Households order the list L_a^t according to U_{af}^t and acquire one good per period. They will only visit the market if

$$(1-s_{Households})B^t_{household} \geq máx.(p^{f,t} \mid f \in L^t_a) \tag{6}$$

This process defines an excess demand for final goods that is a perfect analogon of the excess demand for labour. Hence, for final good f, $ED^t_f = Demand^t_f - Supply^t_f$. The households that were satisfied transfer to the firms the price of the corresponding acquired final good and firms update their budget. The final goods market algorithm is analogous to the one for labour, with the notable exception that firms and households trade places as supplier and demander.

Step 6. Update of fitness. Given the results on both markets, and the excess demand for both labour types and final goods, the price of all final goods and labour types is updated according to the workings of a Walrasian auctioneer. The process thus recursively defines a new profit level for each blueprint, hence a new fitness level. Firms become aware of the results and update the fitness of the blueprints in their portfolio. The set of linear difference equations accounting for this process is given by the following system:

$$\begin{cases} p^{f,t+1} = p^{f,t} + ED^t_f \\ w^{l,t+1} = w^{l,t} + ED^t_l \\ \pi^{t+1}_j = p^{f,t+1}_j - p^{i,t}_j - w^{l,t+1}_j \\ ED^t_j = Demand^t_j - Supply^t_j \end{cases} \tag{7}$$

Step 7. Mutation of firms and households. Mutation of the portfolio of blueprints by firms is triggered whenever the satisfying condition

$$B^t_{firm} \leq Init.Budget_{firm} * Threshold_{Firms}$$

is met after the iteration. Households mutate, or reset their skills, to a new labour type if the analogous condition

$$B^t_{household} \leq Init.Budget_{household} * Threshold_{HH}$$

is met.

Both $Threshold_{HH}$ and $Threshold_{Firms}$are tuneable parameters controlled by the user. Firms meeting the condition order their portfolio of blueprints in decreasing order of profit – the least successful blueprints are the candidates for mutation. The process of change is inspired on the operations of mutation and crossover of Genetic Algorithms. It generates new blueprints, new goods, and eventually demand for new labour types. Firms do not change the number of blueprints in their portfolio, only their composition.

I consider 5 operators, corresponding to different types of operations generating new blueprints. They can be categorized as Lamarckian (inheritance of acquired characteristics) and Darwinian (with crossover and recombination of existing blueprints), and roughly correspond to imitation and creation. Each operator takes as input a blueprint and replaces it with a new one. The reconversion has a cost equal to the number of bits flipped (Hamming Distance between the old and new blueprints). This is intuitive in that the closer a technology is to another, the less it costs to adapt it. The cost is paid by firms to the sink agent. Each of the methods can be coded for either presence (1) or absence (0) by the user in the model.

Firms mutate their blueprints if the following set of conditions is met:

$$\begin{array}{c} \Delta\pi \geq HD(old\,blueprint - new\,blueprint) \\ sB^t_{firm} \geq HD(old\,blueprint - new\,blueprint) \end{array} \tag{8}$$

i.e, the difference in profit makes it worthwhile to replace, and the firm has enough funds to replace the blueprint. We may call the first of these the rule of substitution. Firms replace blueprints until meeting condition:

$$Rec.Cost^t_{firm} = \sum_{k \in portfolio\,of\,firm,} HD(old\,blueprint - new\,blueprint) = sB^t_{firm}.$$

The profit of new blueprints is computed as follows. If the method yields an existing good, then take the price of that good along with the price of the intermediate factors used and compute the profit of the new blueprint. If the new blueprint generates a new good, set the price of that good equal to the average of all goods in the economy at the moment.

Households follow a similar rule as firms when deciding upon the mutation of their labour type and can be encapsulated in the following if...then statement:

$$\begin{array}{c} if\,\Delta w_i \geq HD(labortype^t_{household} - wa^{*t})\,and\,sB^t_{household} \geq \\ HD(labortype^t_{household} - wa^{*t})\,then\,mutate\,to\,labor\,type\,wa^{*t} \end{array}$$

Step 8. Aggregate statistics. In iteration t, the model computes the unemployment rate, number of blueprints and goods traded, GDP, entropy of traded goods and blueprints, diffusion of technological standards, according to the following set of formulae:

$$\begin{array}{c} Consumption^t = \sum_{f \in PR} p^{f,t} q^{f,t} \\ Investment^t = \sum_{\forall firm \in Firms} Rec.Cost_{firm} \\ GDP^t = Consumption^t + Investment^t \\ Unemployment^t = \frac{Number\,of\,Unemployed\,Households}{Number\,of\,Households} \\ Entropy^t_{Blueprints} = -\sum_{bl \in BL} prob^t_{bl} ln(prob^t_{bl}) \\ Entropy^t_{goods} = -\sum_{f \in PR} prob^{f,t} ln(prob^{f,t}) \\ Freq.Adopters^t_{bl} = \frac{Number\,of\,firms\,with\,blueprint\,bl\,in\,portfolio}{Number\,of\,firms} \end{array} \tag{9}$$

All of the formulae are self-explanatory, except the ones for entropy. I follow Hodgson and Knudsen (2010) in identifying complexity with (neg) entropy. Therein, Hodgson mentions that copies must be capable of producing novel additional components in response to new environmental conditions. Hence, complexity is the "amount of Shannon information a replicator stores about a particular environment" (*Id.*, page 122). This paper takes this as meaning the particular features of the replicators: it focuses on the distribution of goods and blueprints, rather than on the specific features. However, this approach is equivalent insofar as the fittest final goods and blueprints will conserve characteristics in terms of their genotypes and carry over to the next generation the best traits. Both $Entropy^t_{Blueprints}$ and $Entropy^t_{goods}$ measure the diversity, as Shannon´s entropy, in the population of activated blueprints and final goods sold, respectively. For both cases, $prob^t_i$ represents the relative frequency of type i among all the active types. A low level of entropy is signalled by a large presence of a single type of blueprints or final goods - there is a genotype that is dominant. A large level of entropy means more dispersion. A level of entropy of blueprints equal to 1, for example, means that all blueprints are equally represented among those activated.

The model iterates over 1-8 for a specified number of steps.

3.2.3 Mutation operators for firms

Firms are carriers of plans, and can change less fit blueprints. Hodgson and Knudsen (2010) report that generative replication has the potential to increase complexity. This section describes the types of operators available for firms when mutating their blueprint portfolio. One of the experiments this model allows is the consideration of several types of innovation with different features. They can be classed as Lamarckian or Darwinian. The first means that there is direct inheritance between new and replaced blueprints, in the sense of the replaced one being a perfect copy, hence inherits characteristics without any error term, of an existing one. This can be seen as the process of imitation, or simply reconfiguration based on existing production methods. I consider 4 types of Darwinian methods, featuring crossover and mutation of blueprints, corresponding to the process of crossing over existing methods to generate new ones. This is more akin to the process of R&D of an economy, and is, in fact, the hallmark of many a creative activity of human beings. Of these 4 types, two are adirectional and are more like the operations of a Genetic Algorithm on blueprints. Birchenhall (1995) mentions that the GA can be used as a tool to generate novelty. The other 2 consist on modifications on existing blueprints with a direction, hence are dubbed directed methods. They are more suited toward exploring the gradient of knowledge of a society. A key idea of this model is to check whether the use of blind methods has a causal impact on the entropy of goods and blueprints, versus the use of directed methods. The methods can be coded for presence or absence, as 0 or 1 in the model. All of these methods can either generate new goods or new methods of producing an existing good.

3.2.3.1 Lamarckian operator

This operator is thus dubbed in the sense that the most successful blueprints spawn offsprings that feature their acquired characteristics – in this case, a perfect copy of the most successful blueprint of the neighbours in the firm´s social network. It is a simple operation of imitation and the most suitable operator to study the process of percolation of blueprints in social networks. It does a feature of a subcritical economy in that this does not generate new types of goods or labour types. All the adaptation stems from changes in the prices of the factors. Thus, firms replace their least successful blueprint with the best blueprint of their neighbours.

3.2.3.2 Darwinian Operator I

This is the operator closest to the operation of a regular Genetic algorithm: it is blind, i.e, adirectional, but elitist. It consists on taking the set of best two blueprints of neighbours and combines them by taking the intermediate good of the first and appending to it the labour type of the second, along with a random cutoff point as in the following snippet of pseudo-code:

Algorithm 1 Pseudo-code of Darwinian Operator I

Step 1. Pick the best two blueprints of firm´s neighbours as parents, P1 and P2.
Step 2. Establish cutoff point in first item of P1 and P2.
Step 3. Build new blueprint with second item of P1, first of P2.
Step 4. Set random cutoff point in new blueprint.

Schematically,

$$\text{P1}\begin{bmatrix}[1 & 0 & 1 & 0 & 0 & ... & 1] \vdots & [0 & 1 & 1 & 1 & 0 & ... & 0] & 3 & [1 & 0 & 1 & 1 & 0 & ... & 0]\end{bmatrix}$$

$$\text{P2}\begin{bmatrix}[1 & 0 & 0 & 1 & 0 & ... & 0] \vdots & [0 & 1 & 0 & 0 & 0 & ... & 1] & 5 & [1 & 0 & 1 & 1 & 0 & ... & 0]\end{bmatrix}$$

$$\begin{bmatrix}[1 & 0 & 0 & 1 & 0 & ... & 0] \vdots & [0 & 1 & 1 & 1 & 0 & ... & 1] & 2 & [1 & 0 & 1 & 1 & 0 & ... & 0]\end{bmatrix}$$

$$\downarrow$$

$$[[1 \quad 0 \quad 0 \quad 1 \quad 0 \quad ... \quad 0] \quad [0 \quad 1 \quad 1 \quad 1 \quad 0 \quad ... \quad 1] \quad 2 \quad [1 \quad 0 \quad 1 \quad 1 \quad 0 \quad ... \quad 1]]$$

The reasoning is analogous to the roulette wheel selection method of GA: fittest parents are more likely to have fit offspring. This is akin to the process of combining fit technologies.

3.2.3.3 Darwinian Operator II

This operator is directed. It uses the results of the iteration to generate a new blueprint and its steps can be defined as in the following snippet of pseudo-code:

Algorithm 2 Pseudo-code of Darwinian Operator II

Step 1. Take exiting blueprint
Step 2. Consult list of sales of iteration and find good with highest price, pr^{*t}
Step 3. Compute Hamming Distances among all intermediate goods and pr^{*t}
Step 4. Compute Hamming Distances among all labour types at t and pr^{*t}
Step 5. Find minimum between **Step 3.** and **Step 4.**
Step 6. Replace item corresponding to **Step 5.** on exiting blueprint
Step 7. Adjust cutoff point so final good inherits more genes of **Step 6.**

Schematically,

$$pr^{*t} = \begin{bmatrix}1 & 0 & 1 & 1 & 1 & ... & 1\end{bmatrix}$$

$$F = min((HD[(Intermediate\, goods, pr^{*t}), (Available\, Labor\, types], pr^{*t}))$$

$$F = \begin{bmatrix}0 & 0 & 1 & 1 & 1 & ... & 0\end{bmatrix}$$

$$\begin{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0 & 0 & ... & 1\end{bmatrix} & \begin{bmatrix}0 & 0 & 1 & 1 & 1 & ... & 0\end{bmatrix} & (3 - random\, length\, chromosome) & \begin{bmatrix}1 & 0 & 1 & 1 & 1 & ... & 0\end{bmatrix}\end{bmatrix}$$

It corresponds to production using fit building blocks.

3.2.3.4 Darwinian Operator III

This operator mirrors the operation of mutation of a regular Genetic Algorithm, and corresponds to the tampering of technologies that characterizes modern economies. It flips a random bit in labour type, or changes the cutoff point randomly:

Algorithm 3 Pseudo-code of Darwinian Operator III

Step 1. Take exiting blueprint
Step 2. Choose randomly whether to mutate labour type or cutoff point
Step 3. Flip a bit in labour type and compute new final good and blueprint
Step 3´. Change to a different cutoff and compute new final good and blueprint

As in the following example:

$$\begin{bmatrix}\begin{bmatrix}1 & 0 & 1 & 0 & 0 & ... & 1\end{bmatrix}\vdots & \begin{bmatrix}0 & 1 & 1 & 1 & \color{red}{0} & ... & 1\end{bmatrix} & 3 & \begin{bmatrix}1 & 0 & 1 & 1 & 0 & ... & 1\end{bmatrix}\end{bmatrix}$$

$$\xrightarrow[Flip\, red\, bit]{}$$

$$\begin{bmatrix}[1 & 0 & 1 & 0 & 0 & ... & 1]\vdots & [0 & 1 & 1 & 1 & \color{red}{1} & ... & 1] & 3 & [1 & 0 & 1 & 1 & \color{red}{1} & ... & 1]\end{bmatrix}$$

This is the process Beinhocker (2011) refers to as “deductive tinkering”. Among all methods, it is the only one that can generate demand for new labour types.

3.2.3.5 Darwinian Operator IV

This corresponds to R&D to create new forms of producing an existing good. It takes the best good of the iteration and changes the input blueprint through trial and error until this generates a new way of producing the most sold good. This method is directed and does not generate new goods, but it may create new methods of production that are more efficient (recall the rule of substitution). The pseudo-code corresponding to this method is

Algorithm 4 Pseudo-Code of Darwinian Operator IV

Step 1. Take exiting blueprint
Step 2. Find good with highest price in iteration, pr^{*t}
Step 3. While last item of new blueprint $\neq pr^{*t}$
do [
Replace intermediate good in exiting blueprint with another existing one
Replace labour type in exiting blueprint with another existing one from WA
Randomly change cutpoint in exiting blueprint
Obtain new blueprint
Evaluate last item of new blueprint
]

When a firm replaces a blueprint, there is a random draw among all the types that are activated, so as to decide which method is used.

This concludes the presentation of the operators.

3.3 Results

3.3.1 Baseline configuration

The Netlogo BehaviourSpace tool was used to perform a series of model runs, using a baseline specification of parameters as in the following table:

Parameter	Value
Number of households	50
Number of firms	20
$InitBudget_{firms}$	300
Maximal Supply of Firms/ Nr of genotypes	6
β	0.5
s_{firms}	0.5
$s_{Households}$	0.5
$Threshold_{Firms}$	0.5
$Threshold_{HH}$	0.5
Topology of network of firms	Erdős-Rényi
Topology of network of Households	Erdős-Rényi
Innovation types	All present
Time Periods / Number of iterations	250

Table 5: Parameter configuration for baseline model run

This is a biologically-inspired model and the parameter validation is not based on any set of empirical data. Instead, all values of the tuneable parameters were set to 0.5: this is so since s_{firms}, $s_{Households}$, $Threshold_{firms}$ and $Threshold_{HH}$ all measure trade-offs between adaptive response to changing circumstances and having enough funds to act on them. This set of values was best thought to capture this trade-off, in the sense of having a model with enough infusion of novelty while still allowing for adaptation. Intuitively, this is the best configuration that allows for the emergence of the "corridor of stability". Similarly, all innovation types are present, as real economies feature mixtures between several types of innovation and imitation. The averages of the series of all simulations were stored. Moreover, for all simulations, the results of each time period were averaged to obtain representative time series of the simulation. After observation of the results, the first 50 periods were discarded as transients. Prior intuition that the setting of parameters in their midpoints would yield a stable, in a barycentric sense, economy, was confirmed. Indeed, under this configuration, all the resulting time series are stationary[19]. The number of time periods was set to 250 as in Bruun (2003).

The results for this parameter configuration are as follows.

[19] As evidenced by an Augmented Dickey-Fuller test for a p-value of 5% for all series, except the one of entropy of blueprints, where the null hypothesis of stationarity cannot be rejected only at a 10% significance level.

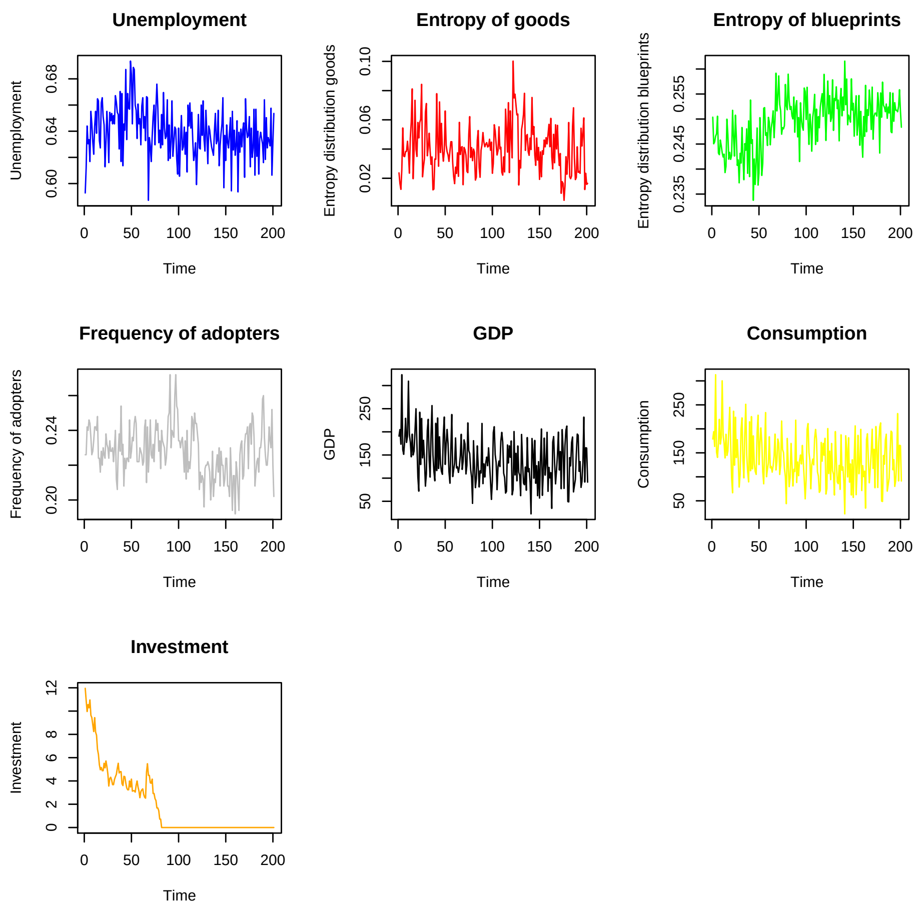


Figure 9: Time series generated by the model

The model, like several others in the literature, is able to generate cycles. By construction, the series that best reproduces failures of coordination is that of unemployment. It produces several short-run fluctuations around a high level - a result that is potentially attributable to the low number of households and the fact that firms converge on the activation of a relatively low level of blueprints.

GDP growth is clearly determined by Consumption. Investment by firms remains at a fairly low level, before tailing off to 0. This fact yields two main insights. First, the learning process of firms stabilizes. This is complemented by the rather thin band of variation of the series of the time distribution of the entropy of blueprints:

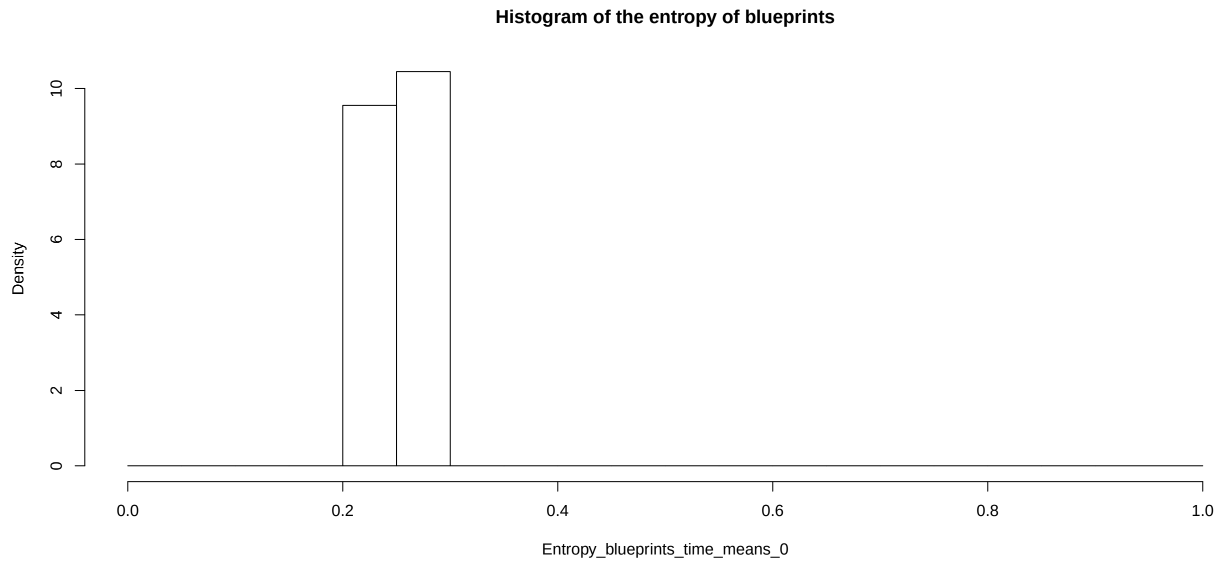


Figure 10: Variation of the entropy of blueprints for the baseline case

Second, the high rate of unemployment (average of 64%) coupled with the fact that firms can only activate up to 6 blueprints and that, as a theoretical possibility, all demand for goods and labor can be satisfied[20], implies that most blueprints are not activated. This, in turn, leads to the insight that either firms have a nonskewed size distribution and activate each very few blueprints, or a power law distribution emerged and only few firms are active, activating all their blueprints and hiring all available labour.

There is also interest in knowing whether the model can replicate the a priori relations one would expect between the series. Since all the time series are stationary, a cross-correlation plot coupled with a Granger-causality analysis suffices, with the caveat that, given the absence of cointegrating relationships, any Granger-causality will only reflect the operation of a short-term mechanism between the series. The cross-correlations and scheme of Granger causality relations are given in the following figures:

[20] Since the theoretical maximal supply equals, for each period, $NumberFirms * NumberOfGenotypes = 120$, is larger than the theoretical Maximal demand equal to the number of Households.

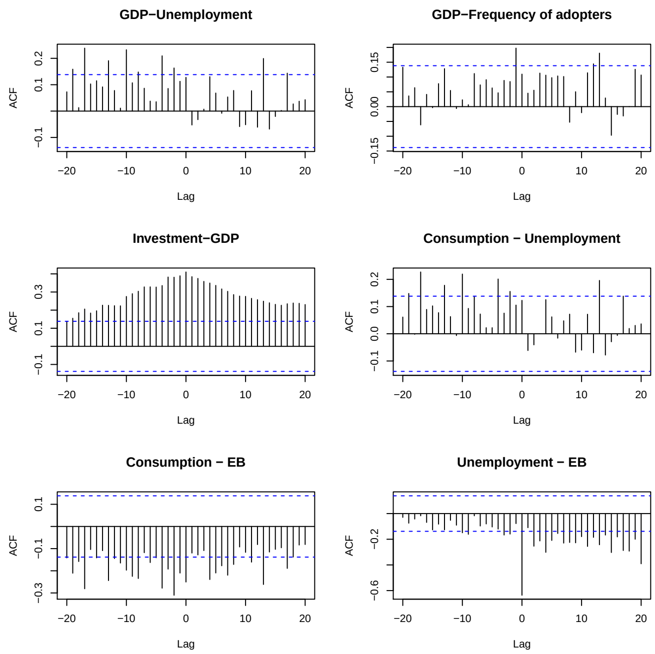


Figure 11: Cross-correlation between the series for the baseline configuration

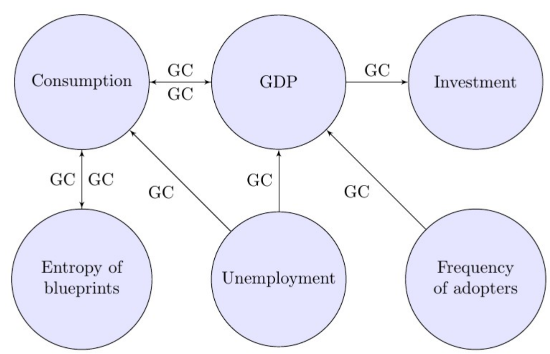


Figure 12: Granger-causality relations between the series.

Recall that on one side, firms require funds obtained from selling goods to innovate. On the other, they will only do so if their budget falls below a threshold level and they have enough budget. The positive cross-correlation between investment and GDP reveals the trade-off between these countervailing forces, i.e, the intensity of adaptation by firms and what one may name an accomodation effect. Since the series of investment tails off to 0, it is our contention that the size distribution of firms becomes skewed, with a small subset of them permanently above the threshold level, and a somewhat larger portion permanently below the threshold needed to innovate. This lends force to the Power-Law hypothesis in the distribution of firm size. The cross-correlation plot that best reveals the presence of cycles is perhaps that of the one between unemployment and GDP. They are respectively countercyclical and pro-cyclical variables, and unemployment Granger-causes GDP (and consumption). It is hypothesized that the mechanism responsible for cycles in the economy draws its force from the underpinning hypothesis of the Walrasian auctioneer, and operates thus. A high unemployment rate caused by a low supply of a labour type in high demand raises the fitness of the corresponding set of blueprints, prompting a change until the price mechanism prompts a substitution and more variety. This insight is corroborated by the observation of the high contemporaneous negative cross-correlation between unemployment and the entropy of blueprints. This is the short-run unemployment phenomenon caused by a mismatch of the demands and supplies of labour, i.e, the mechanism responsible for the fluctuations of unemployment around its long-term level. The mechanism only produces a weak effect since households are allowed to carry savings between periods, smoothing the effect of joblessness on consumption / GDP. One should also note the existence of a negative cross-correlation between the Entropy of Blueprints and Consumption. This somewhat surprising result may be explained as the result of the combination of the facts that increases in the variety of production methods increase the mismatch labor supplies and demands, and hence short-run unemployment. That the frequency of adopters should (positively) Granger-cause GDP is perhaps less intuitive. This may reflect a mechanism of preference clustering of households around a small subset of goods.

A spectral analysis of the series was also conducted. In particular, the emergence of $\frac{1}{f^{\alpha}}$ noise is a hallmark signature of the dynamical regime of many complex systems. The study is conducted by plotting on a graph the logarithm of the frequency and the logarithm of the power spectral density of the series, then adjusting and estimating the slope of a line[21]. The αparameter is a measure of the ruggedness of the series. An estimated α between - 0.5 and 0 signals the dynamical regime of a white noise process, one of unpredictability. αsmaller than - 1.5 is the statistical signature of Brown noise: a series with large time correlations between values and more predictability. Particular interest lies in the intermediate case $\alpha \in [-1.5, -0.5]$, named "Pink Noise" in the literature. This dynamical regime is characteristic of systems far from equilibrium. Perturbations may become amplified and lead to "avalanches" (Bak *et al.*, 1987), i.e, small causes leading to large effects.

Results on the estimated scaling exponent indicate no statistical difference between White noise and the signature dynamic behaviour of the series of Unemployment, Entropy of Blueprints and Entropy of goods. However, the series of Frequency of adopters falls within the regime of pink noise. The time series of investment is classified as Brown noise, although that can be an effect of its tailing off to 0 halfway through the period.

3.3.2 Variation of the saving rate of firms

The BehaviourSpace tool was also used to ascertain the model sensitivity of outcomes to variations in the s_{firm} parameter: this amounts to studying the impact of the saving rate of firms on system (9).

The tool "sweeps" the values in an interval specified by the user. In this case, between 0 and 0.9, as, by construction, the model breaks down for values of $s_{firm} = 1$, since firms do not activate blueprints. In order to ascertain the statistical significance of the results, non-parametric ANOVA (Kruskal-Wallis) tests were run on the population of the means and variances of the series under the corresponding model parameter specification, first for the entire population, and then, if statistical differences were found, a pairwise comparison between

[21] Recall the discussion of section 2.2.2.

specifications to ascertain the point of the phase transition. Thus, differences on the means (and variances) of the series were used to ascertain statistical differences between the (time-averaged) series in terms of convergence to a long-run equilibrium and fluctuations about it.

This is an important test insofar as $s_{firm}$is a parameter measuring the adaptive response of firms and the inception of novelty in the economy. Recall the discussion on the trade-off between adaptation and novelty. The initial working hypothesis is that low values of this parameter indicate that firms, contingent on their budget, will not change their blueprint portfolio by much and the economy will tend to remain in a subcritical phase. On the other hand, values of the saving rate that are too high mean that firms will change their inner structure but will not act on that change, i.e, they will not deploy new products and means of production, as the saving rate is too high for them to activate blueprints, contingent on their budget and on the pricing structure of the means of production.

The results indicate that as soon as the deployment of *some* innovation or imitation is allowed, the economy undergoes a phase transition for several statistics in (6) that is a direct consequence of the transition from a subcritical to a supercritical phase, i.e, from an economy without novelty in either final goods or blueprints to one that features these. This matches the results of Dahlan and Situngkir (2010) for their (simpler) model. The transition from $s_{firm} = 0$ to $s_{firm} = 0.1$ features phase transitions in all variables in both means and variances:

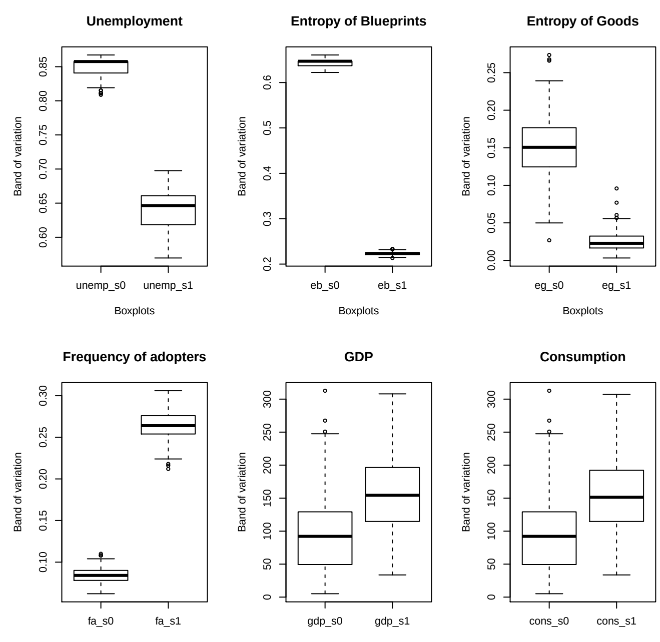


Figure 13: Phase transition from subcritical to supercritical stages

Clearly, the unemployment of the economy converges to a high long-run value. Moreover, since the fluctuations attributed to the changes in internal regime of the firms are absent, so is the variance around the long-run trend.

As hypothesized, this economy crystallizes and does so with a higher long-run value of unemployment, and lower levels of GDP, Consumption and (trivially) Investment. Thus, the introduction of novelty, even at a low level both reduces the long-run value of unemployment and adds short-run fluctuations about it. A higher rate of unemployment directly feeds back to a lower consumption, although not in a one-to-one fashion due to the presence of savings and transferences of the central agent.

The decrease and flattening of the series of Entropies is perhaps a more surprising result. This indicates that in the subcritical stage, the fitness landscape is actually more rugged than the supercritical case. This may be explained by the fact that in the subcritical phase the dynamics of the entropies are governed purely by the Walrasian workings of the economy. No mechanism exists that can dampen the fluctuations of the price of goods influencing the Excess demand and recursively directly feeding back to the entropies of goods and blueprints. This mechanism is, of course, new goods over which the preferences of households can extend to. This effect does not occur in the labour market due to the presence of the adjustment cost of labour types.

The nonparametric ANOVA tests indicate no further statistical differences in the means of the variables from $s_{firm} = 0.1$ onwards, except for the series of the Entropy of sold goods. The results indicate an approximated bell shape with fat tails until $s_{firm} = 0.7$, and a large increase in the end of the spectrum, with phase transitions in the form of statistical differences between populations at $s_{firm} = 0.4$, $s_{firm} = 0.8$ and $s_{firm} = 0.9$:

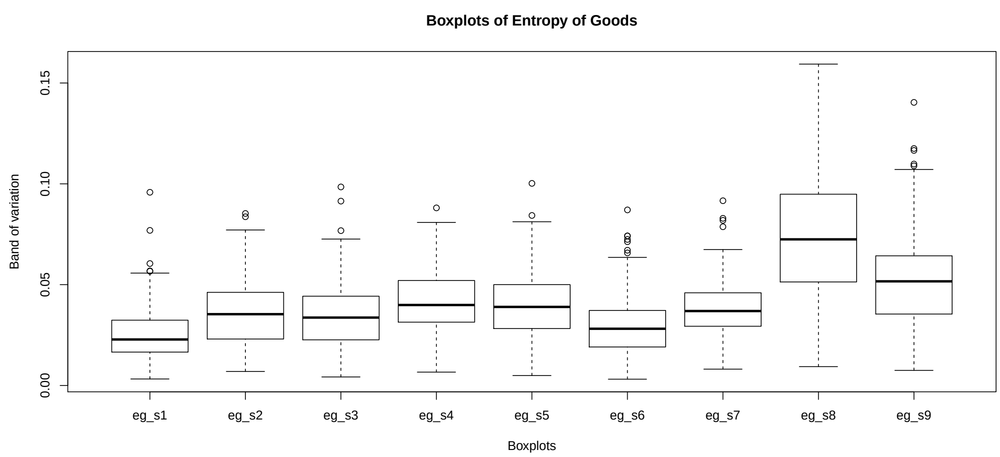


Figure 14: Boxplots of the series of Entropy of goods

This finding partially matches the results of Thurner, Klimek and Hanel (2010): therein, an exogenous innovation rate p is varied, which could be seen as the equivalent of the firms saving rate: as this rate varies, the authors find that a high p yields a lot of volatility in the variety index of produced goods, and creative destruction waves: while for an average p, they obtain a series with plateaus.

The model also features the emergence of a phase transition on the dynamical regime of the series of Frequency of adopters at $s_{firm} = 0.4$. Indeed, the signature of the series in terms of $\frac{1}{f^{\alpha}}$ noise goes from an αin the region of - 0.4 to the more interesting dynamical regime of Pink Noise, with an α consistently in the region $[-1.5, 0.5]$, namely at about - 0.7:

s_{firm}	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
α	1.074	0.9356	- 0.1336	- 0.3861	- 0.7645	- 0.605	- 0.7236	- 0.749	- 0.7894	- 0.8702

Table 6: Evolution of the scaling exponent as the control s_{firm} is varied.

One can interpret this result as the emergence of long-range correlations in this series, i.e, the emergence of memory. Note that this result is not matched by any corresponding phase transition in either means or variances of the series of the Entropy of Blueprints and hence cannot be explained by a crystallization of technologies. The underlying cause must rather be sought in the crystallization of firms in large, adaptable ones and firms that withered away. Thus, $s_{firm} = 0.4$ is the value of the control parameter above which the fate of firms starts to diverge and the skewness in the distribution of their sizes becomes more marked.

3.3.3 Varying the network topology of firms

Delre, Jager and Janssen (2007) features the sensitivity of the spread of diffusion of products to changes in the topology of the network connecting consumers. Although the model has the capability of replicating that experiment as well, here we analyse the supply side, i.e, we investigate the effect of varying the network topology of firms to the aggregated variables of the economy. Several model runs were conducted and the results compared using the same methodology as in the previous subsection. The cases covered are the Erdos-Rényi or random configuration (the baseline configuration of section 4.1), the complete network, (regular in the number of firms less 1, i.e, all firms are connected to all others); the Small-world specification, obtained from the random case by a process of rewiring that balances a short average path length and large clustering coefficient; and a Scale-Free specification, where the degree distribution follows a power law: few firms are very connected, while the majority has a small number of neighbors[22].

The Kruskal-Wallis tests reveal an interesting pattern in the form of no differences between the Erdos-Rényi and the Scale-Free configurations on one hand, and between Complete and Small-World networks, on the other, for a subset of the series. In particular, the Complete Network and Small-World produce a larger unemployment rate than the Random / Scale-Free configurations , while under the Erdos-Rényi regime both GDP and Consumption are larger.

An exception is the series of Frequency of Adopters. Results from the Kruskal-Wallis tests show statistical differences in the populations of means of the Frequency of adopters among all four specifications, although the division into the two groups is still evident and the Complete configuration remains close, in a statistical sense, to the Small-World network. Contrary to the results of Delre, Jager and Janssen, the Erdos-Rényi specification reaches the highest frequency, followed by the Scale-Free Configuration. Both the Complete and Small-World networks present downward tendencies:

[22]Formally, the fraction of nodes having x connections has, for large values, the p.d.f $P(x) \sim x^{-\gamma}$.

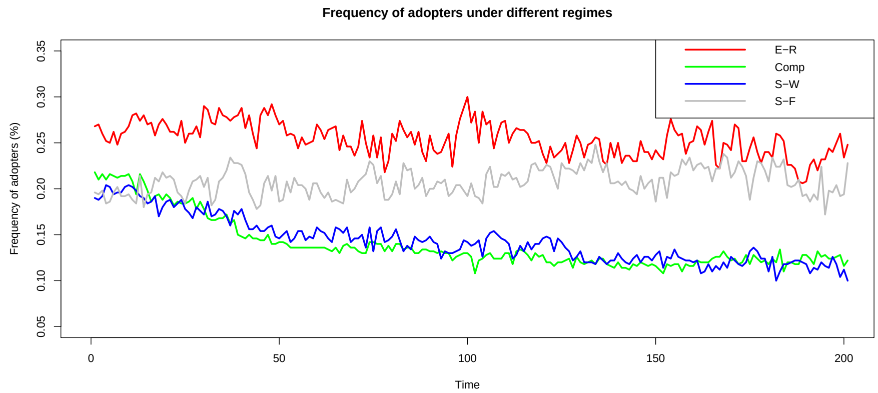


Figure 15: Comparison of the Frequency of adopters for several network specifications

This is a somewhat counterintuitive result, as one would expect the Small-World configuration to lead to higher adoption rates. However, the analysis of the frequency of adopters is less straightforward than the aforementioned paper, where only one good was introduced and tracked. The fact that the best blueprint has a volatile nature implies that the analysis must be complemented by the study of the entropy of blueprints. And this reveals that the Complete and Small-World configurations have a statistically higher level of entropy. Hence, these are characterized by more variety and their fitness landscape is more volatile. Recall that in the Small-World configuration, any two nodes are at a much shorter distance than in the Random case. The limiting case of the Small-World network is the Complete network, where any node is accessible from any other at a distance of one. This intuitive analysis must be supplemented by a formal study of the interrelations between the series. No long-term relations are present between the series of Frequency of adopters and entropy of blueprints for the Erdos-Rényi configuration. However, for both the Small-World and the Complete configurations, these series are integrated of order 1,[23] as an Augmented-Dickey Fuller test reveals. Thus, a cross-correlation analysis would yield potentially spurious results: one must think in terms of Cointegration in these cases. Any statistically meaningful cointegrating relation is, in these terms, indicator of a long-run mechanism at play. Moreover, a Granger-causality test shows that for both cases, the Entropy of Blueprints Granger-Causes the Frequency of adopters, but not the other way around.

Unsurprisingly, these two effects are of opposite sign. The implied contention is thus, that given the different topologies, for more connected networks the Entropy of blueprints effect dominates the network topology effect. More broadly, one can take the inductive step and state that industries characterized by firms connected in a Small-World fashion that exchange technologies are more complex in this limited, diversity-index based sense, although the speed of diffusion may actually render some technologies obsolete before they have the chance to spread in a significant manner. This does not imply that the series of Frequency of adopters is closer to being random in the Small-World configuration than in the Random case. In fact, it is just the opposite, as a study of the scaling exponent shows for this case:

[23]Although the series of Frequency of adopters for the Small-World specification is only mildly nonstationary.

Topology	Érdos-Rényi	Complete	Small-World	Scale-Free
α	- 0.5835	- 1.28	- 0.8335	- 0.791

Table 7: Evolution of the scaling exponent as the control Network Topology is varied.

The memory of the process for the Erdos-Rényi topology is shorter than the Small-World configuration, and closer to the dynamical signature of White Noise. The scaling exponent of the Small-World configuration, however, is richer in structure and falls inside the Pink Noise spectrum. The Scale-Free configuration occupies the middle ground between both specifications. It is hypothesized that the implied frequency of adopters inherits the power-law decay in Spectral Density from the Power law of the node degree.

Hence, the network structure and time dependent effects are still present. However, the diffusion of novelty in more connected networks overwhelms the first effect, albeit being too weak to overcomne the second. One can then specify a critical level of the network connectedness / path length interplay above which the creation of novelty effect dominates the spreading effect, though not quite strong enough to revert the differential fates of firms.

3.4 Concluding remarks

In this paper, we set out to develop an agent-based model that can replicate the emergence of the dynamics on a region of self-organization. This model is more general than others existing in the literature, in that it is a generalization of them. It is based on a biological metaphor: several types of innovations as species that emerge through combinations of existing production methods and goods and are evaluated. It features households that buy goods and supply labor, along with firms that are bundles of rules that consist in deploying combinations of production methods to adapt to the tastes of households in an economy where the main economic aggregates are determined by a Walrasian auctioneer. We have argued the Entropy of the distribution of the goods sold by firms to households, along with the entropy of the distribution of blueprints used by firms, to be a proxy of the ruggedness of the fitness landscape of the economy. Far from being a statistical curiosity, this ruggedness directly feeds back to the economy and influences its path.

We have analysed the supply side of the economy through three experiments. First, a baseline simulation shows an economy on a self-organizing corridor with the signs of a power-law distribution of firm size and where the weight of history is present in the dynamical regime of the time series of the Frequency of Adopters of the Best technology: the series has memory. The fitness landscape is directly tied to the fluctuations of unemployment around its long-run value. We have hypothesized that the corridor of stability is at its widest, in the sense of trading off variation and adaptation, when the control parameter is set at $s_{firm} = 0.5$.

Second, we have varied the saving rate of firms to indicate a possible route to this corridor. For a saving rate of 0, the preferences of households are determined in their entirety by the evolution of the prices of the factors and the fitness landscape is hence as volatile as the fluctuations of prize. For low values of this parameter, fluctuations of the landscape are dampened by the introduction of novelty in the form of other goods that can be acquired by households. We thus have an effect of reduction of volatility and a more complex economy. For intermediate values of the saving rate, however, a structure begins to emerge in the form of long-range time correlations in the series of frequency of adopters of best technology and possible emergence of a power-law distribution in firm size. Moreover, no differences emerge in the series of the entropy of blueprints - that behaviour, coupled with the fact that investment tails off in the middle of the period, is taken to mean that a convergence exists to an established set of blueprints that is able to match the preferences of households. Thus, the industrial capacity of the economy crystallizes. For larger values of the control parameter, variations in the fitness landscape are all channelled to the entropy of sold goods. In the end of the spectrum of variation, too much innovation results in a high Entropy of goods, and the economy becomes less complex.

Third, we have analyzed how the topology of the network connecting firms can impact the road to said corridor of stability. Since a working hypothesis of the model is that firms trade production methods through a network, the topology has a direct impact on the spread of technologies and indeed, the path of growth. Firms connected with a random degree distribution was taken to be the baseline case - it corresponds to the region of maximal complexity, as stated above. As we increase the connectedness of the network through random rewiring, all the while reducing the average path length, we see two forces of opposing signs at play. On one hand, the structure of a Small-World network is such that technology travels faster. On the other, the fact that firms trade more information increases the ruggedness of the fitness landscape and the turnover of technologies. We have argued the second effect to dominate the first. Since the complete network is a limiting case of the Small-World configuration, it presents the smallest spread of the four networks. This somewhat counterintuitive result is corroborated by the presence, in the case of the Complete and Small-World configurations, of a long-run equilibrium relationship between the entropy of blueprints and the frequency of adopters. This hypothesis is ascertained by the presence of a cointegrating relationship along with Granger Causality that is not present in the other two.

Work on the model continues, directed along four key directions:

1. Introduce a market for intermediate goods. This can be done by creating a combinatorial operator that turns final goods into inputs of a new blueprint. Hence, a possible emergent outcome would be firms specializing in supplying intermediate goods for other firms.

2. A meta-analysis of the robustness of the economy to variation of parameters. Several simulations have shown the economy "breaks down" at some critical parameter junctures. How general are these results, and is it possible to derive a numerical estimate of the interval of parameter combinations that characterizes the "corridor of stability" of the economy? A parallel avenue of research is the study of the qualitative dynamics of system (4).

3. The influence of the different mutation operators on the path of growth and entropy of goods. In particular, an interesting result would be to see whether there is an increase in entropy as the economy goes from a strictly Lamarckian to a Darwinian regime, and whether there is a difference in the pattern of growth switching from adirectional to directional Darwinian operators.

4. Collect information on the size of firms, as measured by their budget, and apply Clauset *et. al.*´s (2009) method to ascertain whether, and under what conditions, does a power law in firm size, as measured by their budget, emerge.

Chapter 4

Cognition, coordination and the emergence of complexity in economics

4.1 Introduction

The coordination problem, here broadly conceived as the question of how people form economic plans, imbue them with both meaning and consistent expectations about the behavior of others, being able to navigate in this sea poised between order and chaos and mostly *getting it right*, remains the oldest and proudest unsolved question in economics, one that withstood the test of time and remains as tantalizing today as when Adam Smith originally formulated it, well over 200 years ago. However, if one hopes to tackle it from the point of view of modern, i.e Neoclassical accounts of economic science, taught in most graduate programs in economics around the world, one will find something amiss - some analysis can shed light on this statement and is well worth a small digression. Indeed, there is little substance if one looks beyond the arcane mathematics used. The coordination problem has been assumed away, in that, in the richness of its original form as exposed above, it is nowhere to be found in the set of propositions forming the hard core[24] of the neoclassical formulation. While the notion of macroeconomic equilibrium from a General Equilibrium perspective encompasses a limited notion of coordination - that of behavior induced by a set of prices - the theorem[25] stating it is that a set of prices exists such that, if all agents act taking those prices as given, then the market outcomes generating those same prices will be produced. Thus, the theory is built on a *post hoc, ergo propter hoc* argument. The existence of equilibrium is equivalent to the logical possibility of pre-reconcilable choices (Weintraub, 1979): order is imposed from the very start. However, a body of knowledge whose set of propositions imposes logically the existence of coordination, albeit in both a very broad and limited sense, can not explain coordination failures like unemployment: "Systemic failures" to reach equilibrium are accomodated by the protective belt of the theory.

This is a problem that runs deep and has profound ramifications in the micro-macro divide. Since microeconomics is plagued by unrealistic behavior, macroeconomics inherited this along with an additional fallacy of composition, or simply the erroneous assumption that the behaviour of the whole matches the sum of its parts. The profession has known since the 70´s (Kirman, 2010) that the Walrasian aggregate demand function of an economy does not inherit the Weak Axiom of Revealed Preferences (WARP) property of the individual demands composing it[26]. This is, of course, the Debreu-Sonnenschein-Mantel theorem. Under these conditions, no unique equilibrium price vector is forthcoming, seriously undermining the success of the project. This conclusion did not faze researchers or policy-makers: it was business as usual, a simple mathematical sleight of

[24] In Lakatos´ sense.

[25] In this paper, I will use theorem to express a statement derivable from the axioms and rules of induction of a body of knowledge. Hence, it includes not only theorems in mathematical *strictu sensu*.

[26] For any x,$y \in B$, where B is a bundle of goods and prices $B = \left\{(p,x) \in \mathbb{R}^{\mathbb{L}}_{++} \times \mathrm{R}^{\mathrm{L}}_{+}\right\}$and x is revealed-preferred to y if $py \leq px$, the WARP states that if x is revealed-preferred to y and $x \neq y$, then y is not revealed-preferred to x.

hand (*Ibid.*, 2010). Wagner (2010) constrasts the Neo-Walrasian program, where agents have full knowledge and transitive preferences over outcomes, along with a logical account of causality, rather than a generative one, with what he calls the Neo-Mengerian programme, featuring both agents and social structure influencing one another, partial and distributed knowledge, and where meaning is derived from the interpretation of actions and plans and argues that they are incompatible. I agree with his point. Indeed, we are beings that act for a purpose: an event only becomes an action if charged with meaning derived from an interpretation of the world. It dawned upon me that it does not make sense to speak of a tension between purposeful action and the blindness of economic evolution if purposeful action is absent to begin with. And purposeful action entails with it the implication that things could have always been otherwise.

This paper is a reformulation of the coordination problem, based on a critical evaluation of an analytical model that underlies the formalism used by the economist as she deploys her models. It discusses the emergence of complexity in an economy as an ecology of plans. It focuses on three intertwined key topics: cognition, or roughly, how people perceive the world and formulate plans of action; coordination, as an account of how our mind is able to dovetail with others´ own plans: and complexity, as both the origin and the result of the interaction of said plans forming the overarching goal both within the topics discussed here. Its structure is as follows. The next section presents the analytical scheme of Muñoz and Encinar (2007, 2011, 2015). Section 3 features a set of reflections based upon the aforementioned topics along with a call for a better understanding of the cognitive part and features a desirable cognitive theory for economics should have. Section 4 presents conceptual blending (Fauconnier and Turner, 2008); Fauconnier, 1998) and argues that it fulfills the prerequisites. Finally, section 5 concludes.

4.2 Sketch of an analytical model

This section presents the main lines of Muñoz and Encinar analytical scheme (henceforth, M&E), as expounded in Muñoz and Encinar (2007, 2011, 2015). This analytical scheme underlines the way economists deploy models in the Neoclassical tradition. It is to be thought of as a generalk framework, as an axiom scheme with particulars to be filled by the modeller. Hence, if the analysis is to be thorough, the scheme must be presented, to be complemented and criticized.

M&E´s stated goal is to "analyse the apparent paradox between intentional human action and the apparent blindness of economic evolution". As intentional human action is encapsulated in the formation and deployment of action plans, they erect those as a primitive notion to be analysed. An action plan is a bundle of actions constituted with knowledge of things - goods, facts, and expectations about the behavior of others. In their own words, "a projective structure linking actions to objectives - a projective ordering of means to achieve purposeful objectives". This constitution is done in a "space of representation" and includes complex actions spread over analytical time, such as:

$$\varphi = \left\{ execute\, a^i_{th1}, a^i_{th2}\, at\, time\, t;\, a^i_{(t+1)h5}\, at\, t+1\, reach\, goal\, G^i_{(t+1)h1}, and\, goals\, G^{i*}_{(t+2)h3}, G^i_{(t+2)h2}\, at\, t+2 \right\} \tag{10}$$

Each moment in analytical time, then, is filled, as it were, with actions being undertaken and goals being reached, as the following shows:

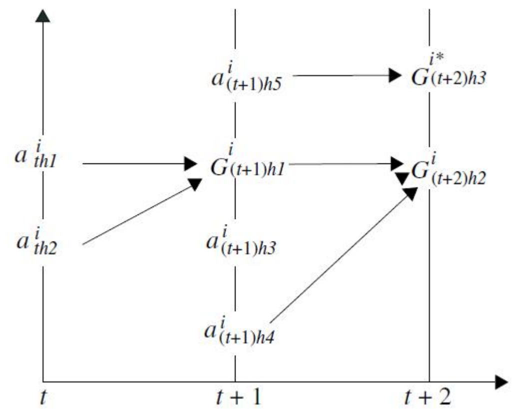


Figure 16: Plans of action unfolding in analytical time. Taken from Muñoz and Encinar (2015), page 322.

Means and goals, along with their hierarchy, emerge in the minds of individuals: they are *produced* by agents. They define meso and macroeconomics evolution as the emergence of novelty, both intended and unintended, resulting from the interactive deployment and revision of plans through learning. The key to revision, they claim, is the desire of agents to increase the feasibility of their plans. Thus, by definition a plan is planned and introduces intentionality; but the interaction begets complexity and the blindness of the process. Moreover, they define social coordination as a gain in external feasibility. Whether and how agents act is conditioned by their desires, goals and objectives: they thus set forth the notions of cognitive and ethical dynamics of individuals, along with the cultural dynamics of the society they operate in, denoted respectively by CD_t^i, ED_t^i and S_t. These three co-evolve and configure the action space of individuals. Using their notation, for each individual i, the action space at t is $G_t = f[CD_t^i, ED_t^i, S_t, u_t]$, where $u_t$denotes the state of environment. They then set out to define the analytical steps undertaken in constituting and selecting a plan of action for each individual. First, individuals constitute their set of admissible plans at time t, $P_i(t)$. They do this based on their beliefs, desires and attitudes, bundled together in M&E as agents´ "theoretical representation of reality": This generates an action space with both subjective and objective elements. Second, the selection of a plan, $p_i^*(t) \in P_i(t)$. In this step, agents obey the Principle of Economic Behavior, by choosing the $p_i^*(t)$ with the highest rank among the elements in $P_i(t)$. More formally, they set forth the following expression to indicate this selection:

$$p_i^*(t) = m\acute{a}x\{P_i(t)\} \tag{11}$$

with $P_i(t)$ shaped by an ensemble E_t^i of beliefs: this belief set being in turn defined by CD_t^i, ED_t^i and S_t. Third, the deployment of $p_i^*(t)$, triggering learning and the endogenous change in connections among elements in the plan according to a satisficing procedure, i.e, individuals search for better alternatives when things go poorly. This step is where the micro-meso link emerges. Finally, an evaluation step.

Letting s_t^i denote the state of individual i at t, one has $s_t^i \supset \{CD_t^i, ED_t^i\}$. M&E derive the following set of implications:

$$\delta^i(s_t^i, u_t) \to \{E_t^i\} \to \{p_i^*(t)\} \tag{12}$$

and

$$\alpha^i(p_i^*(t)) \rightarrow A_t^i \tag{13}$$

The δ^ioperator in (12) is an (unspecified and model-dependent) operator linking the state of an individual to his set of beliefs; this, in turn, determines the optimal plan. By (13), there is an operator mapping a plan to an action A_t^i . Implicit in their reasoning is the definition of an agent i at time t as a 2-uple of the form $< s_t^i, A_t^i >$, i.e, as a state and an action borne out of the execution of part of a plan. They define the following scheme for the evolution of the cultural dynamics of a society as determined by n agents:

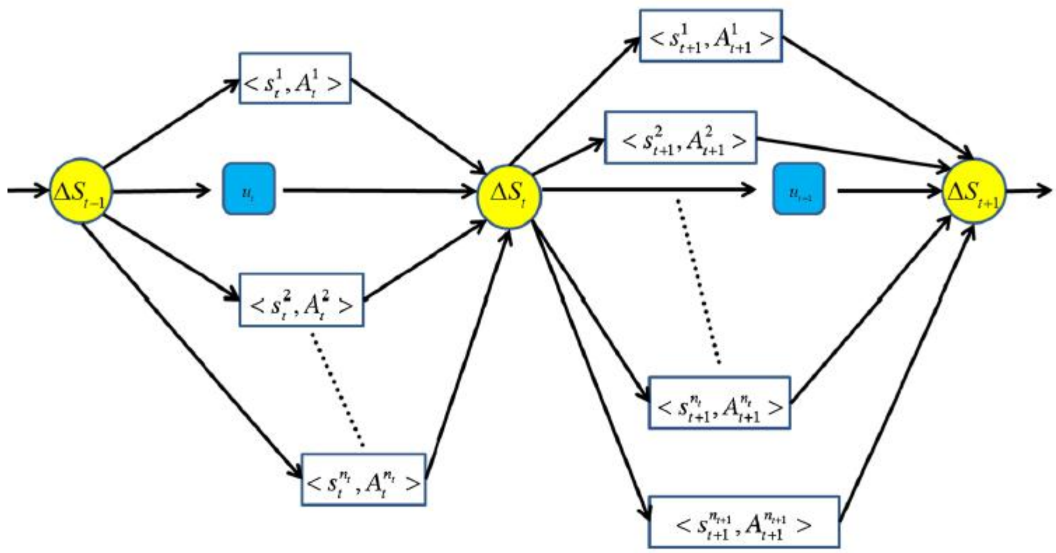


Figure 17: Evolution of cultural dynamics in M&E scheme. Taken from Muñoz and Encinar (2015), page 329.

Crucially, individuals´ plans have the following three properties:

• Feasibility. Both ex-ante, referring to the *logical* and material foundations of the plan, and ex-post, i.e, the possibility of it being accomplished when when interacting with other individuals´ plans. Denote these by $R_1$and R_2, respectively.

• Consistency. as lack of infeasibility: an individual deploys a plan that is structured consistently, both in the connection means-goals, and in not supporting contradictory goals. These are denoted respectively in M&E by $C_1$and C_2.

• Reflexivity as existence of a dynamic feedback link between constitution and revision of the plan.

They define coordination as the disappearance of inconsistencies in either C_1or C_2, hence an increase in R_1, or reconstitution of plans taking into account the result of previous deployments, i.e, an increase in R_2. In what essentially mirrors the “invisible hand” mechanism, they posit the existence - albeit, I surmise, only for explanatory purposes - of an operator over internal and external consistency and ex-ante feasibility that yields ex-post feasibility: $F[C_1, C_2, R_1] = R_2$.

M&E is flexible and general enough to encompass R_2, ergo coordination, as a tautology, as in General Equilibrium Theory, and the case where F itself changes as a result of accomodating $C_1$and C_2. They refer to the latter case as structural change: it is the result of agents eliminating rationing imposed by social over individual dynamics. They erect entrepreneurship as the "impulse and will pointing action towards generation of change" - thus, a set of intentional states configuring a goal along with the production of action consistent with such a mental image. Thus, action is configured and deployed in M&E on the basis of reason. Their account of emergence is also borne out of agents implementing their plans, as they either discover new actions, invent new goals or rearrange existing actions and goals in a new way. More importantly, they explicitly equate the *meaning* of a plan with ED_t^i ("how the world should be") and that, implicitly and in turn, with the outcome of a series of steps undertaken taking into account the state of the world as presented to them, CD_t^i. This completes the main lines of their model.

4.3 A critical analysis of M&E

The meta-model above conveys the image of an economy as an "ecology of plans" (Wagner, 2012). Recall that the stated goal is to "explain the apparent paradox between purposeful action and the blindess of economic evolution". Blindness, in this sense, is nothing more than the fact that new, unforeseen combinations emerge: this, in turn, is a reflection of the fact that very often things go on to take up roles they were not meant for, spurred by agents that, more than discovering the world and themselves, create them, in a process of dynamic feedback. We need to go deeper into these questions: this is what this section sets out to do, by presenting a critical review of M&E interwoven with a set of reflections centered around the three C´s of the title: Complexity, Cognition and Coordination.

First, I claim that M&E are correct in stressing the importance of mental acts in the formulation of plans of action, and in assigning a key role to beliefs and desires: indeed, states of mind are essential in not only creating, but also changing and replicating institutions through their practice: i.e, they are performative (Herrman-Pillath, 2010). However, their stress in the logical foundations of thought places M&E, ergo all the models built according to this scheme, as instances of what Zilhão (2010) calls Davidsonian Functionalism. This is the cognition part of the triad. I will reflect upon the way agents charge their plans and actions with intentionality and contrast it with M&E by appealing to the introduction of a cognitive scheme that is more akin to the formation of metaphors than simple assignment of meaning as a truth value to a proposition.

Second, to clarify what is meant by "Complexity". I show that an economy is really a set of interwoven complex systems, featuring complexity as indeterminacy at the semantic or interpretational level (i.e, of the individual), mapping to complexity in the logical sense, as Gödelian indeterminacy stemming essentially from the fact that the space of interpretations that could be explored by an algorithm or any recursive procedure to simulate an economy is not recursively enumerable. Finally, I advance the thesis that complexity at the dynamical level, i.e dynamics of aggregate macroeconomic variables of the type studied in the first part of the thesis, are really relational and derivative from the semantic level.

Third, to present the notion that agents manage to navigate in a sea of complexity by reasoning and performing cognitive operations that rely on the attribution of mental states to highly typified characters. This is the key to framing the coordination part of the issue.

4.4 Intentionality, economic phenomena and action

All economic phenomena are manifestations of the individual mind: valuation, preferences, utility, are all words that anchor a thinking mind to something - in other words, they express Intentional[27] realities (Cottrell, 1995). Intentionality is, according to Searle (1983), really self-referential representation - you make a mental image of the thing, and the proposition representing your Intentional state expresses its own conditions of satisfaction. A belief about something, say, has a propositional content and a psychological mode. The first is its intension; the second is the thing in the real world the belief points to, its extension or reference. Moreover, a belief does not stand on its own, but rather is supported and exists within what Searle calls a Network of related Intentional states that support one another. For example, you having the intention of buying something relies on your knowledge that you have enough money, which is in turn connected to the belief that the salesperson will accept your payment, in turn couched on the general knowledge you have of the functionings of a market economy and that you live in one. You usually also believe the person matched up against you has a comparable set of mental states - that you have some type of common ground. Eventually, if you follow through this process far enough, you will encounter Intentional states that are so deeply ingrained, you are not even aware you have them and how constitutive of your actions and thoughts they may be - that other people exist, that you are a human being, that the ground you walk on is solid, and so on. Searle calls this the Background of pre-Intentional states. I will retain both the three-tiered Searlian account of Intentionality, along with the idea of Intention as representation, thus rejecting an objectivist model of cognition - I represent the world rather than having direct access to it; the world as idea, as Schopenhauer would put it. We impose categorization to things according to three factors: our neurophysiological apparatus, the universal cognitive apparatus, and culturally determined choices applied to the input of the latter (Lakoff, 1988).

Searle´s network of Intentional states is appealing: that beliefs should support one another is intuitive enough. One can also, without loss of generality, establish an isomorphism between M&E´s conception of a plan of action as a connection between means and goals as this set of connected beliefs and other Intentional states. The considerations above state that an Intentional state expressed as a propositional content can be derived from others. This cognitive work is largely subconscious and hidden away from us. Goertzel (2013), sees our belief system as a Structured Transformation System[28] composed of elements that reinforce one another and generate a conclusion, as it were. For the previous example, using Goertzel´s notation and indexing Intentional States by

[27]I will make use of Intentional in the broad sense of mental attitude expressible in propositional form (Searle, 1983), connecting an individual or entity to the contents of her beliefs, desires, knowledge about states of the world. Hence, x Believes that p, y Knows that q, as well as intentionality in the narrow sense of wanting to do something, are all examples of Intentional states in this sense. I will follow Searle and refer to Intentionality in this sense using capital letters throughout this exposition.

[28] In Goertzel´s own words, "A structured transformation system is a transformation system with the property that, if a mind wants to make a "blueprint" telling it how to construct something from the initials using the transformations, it can often approximately do so by reasoning analogically with respect to the blueprints from other construction projects." (Goertzel, 2013, page 39.).

I_i, one has[29]

$$\{I_1, I_2, I_3, I_4, I_5\} \vdash I_6 \tag{14}$$

The theory of mind stating that mental states have causal powers on each other, along with possibly causal powers in the action of the agent holding them, is called Functionalism.

I find Goertzel´s account of our belief base as a strange attractor to be extremely convincing, in that it seems to frame perfectly some features that are evident to us upon self-reflection: That we change, but not a lot[30]; that there is something we can recognize to be invariant about us, an identity or essence through time: the immanence of a characteristic way of thinking that is constitutive of our own self. A permanence within change that mirrors the topology of a strange attractor, and indeed is a feature of many a complex system. There may actually be neurological support behind these claims. Basti and Perrone (2001) report that there may be evidence of chaotic behaviour of brain dynamics in terms of the firing rate of neurons, in that the brain rewires its connections dynamically to match modifications of a moving object, and, I dare extend, the image of a person or a belief in different circumstances. Then, the same entity[31] has a different neural correlate.

We have seen that (14) is intuitive, but what is the shape of the implication contained therein, if indeed such an implication exists, and is such implication connected in a nomical sense to the undertaking of an action, as (13) of the previous section suggests? It is to this set of questions I now turn to. Recall that M&E equate meaning with the outcome of a plan: thus, if one identifies the elements of the LHS of (13) first with a set of propositions leading to I_6, and then I_6 leading to the action of buying the thing I desire, i.e, a complex of beliefs and desires explaining the action, both (12) and (13) will have been vindicated. They are separated questions and I will tackle them in turn. I must first make a detour through the questions of meaning as explored by philosophers of language, which will take us in turn to the the modern precepts of Epistemic and Doxastic logics used in current research on Artificial Intelligence. It will be argued that a set of features exposed in M&E are enough to qualify them as Davidsonian functionalists and hence necessarily endorse a logicist view of cognition. The next subsection explores this link. Section 3.3 rebates this point of view.

4.5 The logicist program and the tracking of meaning

Lachmann (1991) stresses that observable economic phenomena are the consequence of the deployment of plans - and an action is a sequence of acts to which our mind assigns a *meaning*. Our observations as economists, and our actions as economic agents, need a setting of interpretation - a coherent story if they are to explain anything, if indeed an act is to qualify as an action. Everything begins with the hermeneutical step: hence, let us begin at the beginning, with a theory of meaning built by a mind.

One does not have to subscribe the Sapir-Whorf hypothesis to recognize the immanence of language to thought.

[29] Letting I_1 = "*I have enough money*", I_2 = "*Salesperson will accept my payment*", I_3 = "*I live in a market economy*", I_4 = "*I_3 is common knowledge among us*", I_5 = "*I desire the good*", I_6 = "*I have the intention of buying the good*"

[30] In "O Guardador de Rebanhos - Poema XXIX", Alberto Caeiro notes this perfectly as he writes, “Nem sempre sou igual no que digo e escrevo./Mudo, mas não mudo muito. /A cor das flores não é a mesma ao sol / De que quando uma nuvem passa / Ou quando entra a noite / E as flores são cor da sombra. / Mas quem olha bem vê que são as mesmas flores. / Por isso quando pareço não concordar comigo, /Reparem bem para mim...”

[31] In the philosophical literature, such an entity is called a *rigid designator*.

In that most work in the logicist tradition of thought was done by Frege, and later, either by refuting or extending his thought, and inasmuch as I will be using some elements of his work, a detour into these questions is in order.

On his classical article, Frege (1892: [1948]) distinguishes between sense and reference, and identifies the meaning of a proposition as a truth value - in particular, the correspondence between the sense of an expression - the logical form of what is being said - and its reference, i.e, the thing in the world it refers to. He conceptualizes expressions as n-ary place predicates - placeholders for things, if you will, expressing ways of combining things into expressions - of *composing* meaningful sentences. The meaning of a complex expression thus sends back to its truth value in a given ontology of things, or universe of objects, that saturate the expressions. The fact that the truth value of an expression depends on the truth value of its components is called the Compositionality Principle. Note the objectivist undertones of this: there is an external reality one refers to when assigning truth values to an expression. As regards beliefs (but not necessarily knowledge, as it is usually assumed one is always right when one knows something), a set of problems stand out as regards attribution of meaning stemming from their inherent intensionality: their truth value is independent of anything external to them. It can be true, say, that someone believes that France is a Monarchy. Letting the one-place predicate $M(x)$ denote "$x is a Monarchy$" , and if one admits x ranges over countries, clearly $M(France)$ will be assigned the truth value, ergo meaning in Fregean terms, False, as things stand in the world today. However, it may very well be the case that someone, say Ann, believes that is the case; hence, the following snippet of First-Order Logic with Identity augmented with a Belief Operator, $B_{Ann}(\exists x(M(x) \wedge x = \text{"}France\text{"}))$ is True. Incidentally, Existential Generalization (EG) is not allowed for in intensional contexts due to complications from the interchange between the Belief Operator and quantifiers. One is allowed to apply EG inside the Belief operator (or indeed, in the scope of any extensional context), whence $B_{Ann}(M(France))$ and $B_{Ann}(\exists x(M(x) \wedge x = \text{"}France\text{"})$ are equivalent as a true assignment relating as expression to a state of the world, namely that Ann believes France is a Monarchy. However, $\exists x B_{Ann}(M(x) \wedge x = \text{"}France\text{"})$ is much more contentious: such entity may exist and Ann erroneously believes it is a monarchy (such as the case of real France), or it may not exist at all beyond Ann´s belief[32]. Moreover, intensional contexts don´t authorize the substitution of co-referential terms *salva veritate*, not even considering knowledge: at least, not without further constraining the belief or knowledge set of the agent. Hence, we may very well have the following set of true implications:

- "Mary believes that Cicero is a great orator".
- Cicero is Tully.
- "Mary does not believe that Tully is a great orator".

The last sentence may be true, even though "Cicero" and "Tully" are co-referential, if Mary does not know that to be the case.

In all the cases above, there is a wedge driven between sense and reference, seriously undermining the Fregean notion of meaning - especially taking into account that meaning is a function between a sense, possibly as a complex of simpler parts, as the Compositionality principle states, and a reference as something "out there". These objections were overcome in the 60´s by making use of an updated version of the Leibnizian notion of

[32]For another example, think about the less prosaic "Plato believed Atlantis existed", written as $B_{Plato}\exists x Ax$, where A is a one-place predicate that when applied to x, maps x to Atlantis. This is undeniably true, but $\exists x B_{Plato} Ax$ ("There exists/existed an object such that Plato believed that object was Atlantis") is doubtful.

Possible Worlds. Its main proponent was Hintikka (Meyer (1982)) and the intuition is the following. That which someone believes in may not be possible in the actual world - but it is true in some possible world. Hence, that which is true is so relative to the belief base of an agent, expressed as the set of worlds he can access. Thus, truth is a matter of accessibility between worlds: propositions are true in *some* model.

Most work on modal epistemic and doxastic logics, along with their dynamic counterparts, makes use of this framework. Insofar as models in economics can be reduced to this type of formalism, albeit with the more familiar names of "states of the world" or some such, where a set of outcomes is available and agents may be uncertain about their occurrence, it is well worth going through this. The kernel of this type of logics is contained in the following definition, encapsulating the conversion of possible world semantics to the framework of First-Order Logic, along with a set of usual axioms and inference rules:

Definition: A model M comprises a set of possible worlds S with a typical element $w \in W$, N agents and binary (accessibility) relations indexed by R_i, with $R_1, R_2, ...R_N \in R$, and a valuation function V assigning the formulae composed of atoms of the language, ϕ, to a truth value in a set of possible worlds: $V : \phi \longmapsto \mathrm{P}(W)$. (Well-formed) formulae of the language, α, can then be formed with a sufficient set of logical connectives connecting the atoms. Let $K_i\alpha$ be a modal operator ("Knowledge") connecting an agent i to a formula α: Intuitively, agent i knows that α. The satisfaction relation $\models$ is defined recursively as:

- $M, w \models \phi$ iff $w \in V(\phi)$ (all atoms have a truth value assigned to them)
- $M, w \models \neg\alpha$ iff $M, w \not\models \alpha$ (either a formula or its negation can be derived: this is known as the Law of the Excluded Middle)
- $M, w \models \alpha \rightarrow \beta$ iff $M, w \not\models \alpha$ or $M, w \models \beta$ (recall the formula from Propositional Calculus stating that $(\alpha \rightarrow \beta) \leftrightarrow (\neg\alpha \vee \beta)$)
- $M, w \models K_i\alpha$ iff for all $t \in W$, wR_it implies $M, t \models \alpha$ (An agent is able to derive/know a formula, if it is accessible from his knowledge base: this is the analogon of consistency.)

This set of sets of worlds and accessibility relations among them, $< W, R >$, is called a *Kripke frame*. It is coupled with the following set of minimal rules of inference and sets of axioms, indexed by their initials:

- (PC) All tautologies of Propositional Calculus.
- (MP) $K_i\alpha \wedge K_i(\alpha \rightarrow \beta) \rightarrow K_i\beta$.
- (NEC) $\alpha \rightarrow K_i\alpha$.

This is the hardcore, read weakest, unchanged or invariant, set of propositions upon which all logics of knowledge build upon, be they individual or group, dynamic or not. These are usually supplemented by additional constraints, usually expressed as adding properties to the accessibility relations. Hence, say, adding the property of transitivity to R_i logically implies introspection, i.e, knowing something implies that one knows one knows that something, yielding $K_i\alpha \rightarrow K_iK_i\alpha$ as an additional axiom. Much recent work has addressed weaknesses of augmented versions of this type of formalism. Thus, for example, Sillari (2008) introduces the awareness function, as a way of curtailing the agent´s information set and stating he cannot be aware of every logical consequence of the statements comprising his knowledge base. But note the analogy with the amendments made in Neoclassical formalism: one defines a hardcore set of propositions and lets the protective belt, here as the augmented versions of logic, deal with inconsistencies. As we´ll see, however, there are issues with the hardcore set of propositions. Suffice it to say for now that (PC) and (NEC) hold for all well-formed formulae (wff). Since tautologies are necessarily wff, it follows from that fact along with necessitation that one usually assumes agents to know all tautologies of propositional calculus.

4.5.1 The Davidsonian approach to meaning

Donald Davidson attempted to derive the notion of meaning, rather than define it as a primitive. His approach is compositional, but in a wider sense than Frege´s: meaning arises as a correspondence, not between utterances and things, but as one between the contents of a belief of a speaker and his utterances as understood by an interpreter. The work of making sense of others´ utterances, and indeed actions, is essentially one of interpretation. He couples the notion of compositionality with Tarski´s convention T to obtain a theory of meaning superimposed on one of truth, rather than one *defined as* truth, as in Frege. Davidson outlines a theory, call it*D*, for a language *L*. He posits as axioms of *D* a set of lexical primitives, and a set of production rules of a recursive nature[33]. All of this is still essentially Fregean. But Davidson, pace Frege, does not attribute objects of either intensional or extensional nature to the linguistic expressions thus obtained. Rather, *D* yields theorems of the type "*U* means *m*" where *U* is an utterance. Meaning is only attributed as cognitive work in the mind: this is conceived of in Davidson´s schema as a translation to a meta-language *D´* that mixes both linguistic and non-linguistic cues. Thus, *D* has a translation in *D´* as *U* means *p*, where *p* is a place-holder for a sentence in the interpreter´s background that translates U. *p* is a way of tracking down meaning without reifying it[34]. Davidson´s theory is one of making sense of the utterances of others: his radical interpreter relates the sentences of a speaker to the world by assigning them truth conditions matching extra-linguistic situations. And she does this, according to Davidson, based on the key principles of Logical Coherence and Correspondence. The first implies that the interpreter looks for coherence in a *logical* sense in the speaker´s utterances, that his beliefs are structured according logically. The second, that there is a correspondence between the beliefs of an utterer and her utterances. Taken together, they imply the principle of Charity: that of maximizing agreement between an utterance and the corresponding interpretation according to the conceptual apparatus of the interpreter. Davidson´s radical interpreter places herself in the shoes of the utterer and thinks about what she would believe in that situation: it is a work of reconstructing *D´*. Davidson hits the nail on the head with this, as this empathy indeed plays a key role when trying to frame coordination in a complex economy. But there is more. The isomorphism

[33] We could see it as a Chomsky-type grammar for producing longer sentences from primitive elements

[34] A theory of meaning for German constructed by an English speaker would include as a theorem in *D´*the following: "Schnee ist weiss" means snow is white. Note I have separated the linguistic and non-linguistic cues by mentioning the sentence in German with the standard procedure of putting it in quotation marks. This theorem is an example of a T-sentence à la Tarski, and where Convention T enters the picture.

language-thought I alluded to earlier coupled the principle of Charity imply that the suitable meta-language *D´*we employ to make sense of the utterances of others, *indeed to decide whether a being is rational*, is that of First-Order Logic (henceforth, FOL) with Identity[35] (Zilhão, 2010). Davidson´s agents are "subcutaneous optimizers of truth": an intersubjective truth borne out of interpretation,i.e one of agreement. Davidson himself does not endorse the hypothesis that agents necessarily possess any type of commonality when they agree on something, commonality expressed as either a common mental state, or as a correspondence between images and corresponding neural substrates (Type-type theory)[36]. However, Zilhão (2010) reports how some authors, like Loar (1983) extended the fact that Davidson takes FOL with Identity as a suitable language for intersubjective discourse, coupled with the fact that only beings endowed with a language can be rational, to mean that the Intentional states of agents and connections among them are underlined by a neural circuitry that implements computations of FOL with identity in the formulation of thoughts and Intentional states. This is what Zilhão calls Davidsonian functionalism: there is a neurological structure capable of matching the computations of FOL with identity, or a subset thereof, complete with a definition of the primitives, rules of inference and a sufficient set of logical connectors. In this regard, Zilhão notes:

> "The main thesis of Davidsonian Functionalism is, then, the thesis according to which the idea that agents are optimizers of truth must be conceived of as en empirical truth about the functional arrangement of the cognitive apparatus of real agents (...) " (Zilhão, 2010, page 184, my translation).

and further ahead, he mentions how

"According to this point of view, then, each Intentional state of an agent, identified typologically in a functionalist manner with, for instance, a belief, given its causal connections upwards and downwards with the sensory input, other mental states and action, would likewise have its own content, identifiable in a network (...). This network would replicate, **at a physical level, a syntactical system defining a formal absolute language L of First Order** with indexicals *(...)*". (*Ibid.*, page 190, bold emphasis mine).

4.5.2 Criticisms to the logicist programme

This section focusses on the insufficiencies of the logicist programme - taken here to mean the common component of Possible-World semantics and Davidsonian Functionalism, i.e, the reliance on the methods of FOL with identity as at least an approximation of how individuals think. It will show these are untenable. The next section will show that M&E is really an instance of such programme and hence invalidate the double set of implications in (12).

[35]Davidson (2001, page 136) notes the following: "The process of devising a theory of truth for an unknown native tongue might in crude outline go as follows. First we look for the best way to fit our logic, to the extent required to get a theory satisfying Convention T, on to the new language; this may mean reading the logical structure of first-order quantification theory (plus identity) into the language, not taking the logical constants one by one, but treating this much of logic as a grid to be fitted on to the language (...) classes of sentences always held true or always held false by almost everyone almost all of the time (potential logical truths)..."

[36]Davidson (2004, page 15) writes: "The process of specifying the content of a thought or utterance does not require that we suppose there is a definite, or indeed any, object before the mind of the thinker or speaker. When we say two people have the same thought, we mean their states of mind are similar enough to enable each to interpret the other; up to a point, at least, they are able to understand each other. For two people to think alike does not require that there be things—actual entities—which are or could be identical.(...) It's the difference between putting the emphasis on identity of thoughts and putting the emphasis on acceptable interpretation."

Recall that the kernel of Davidsonian functionalism and Possible-World Semantics is the reliance on the laws of FOL with identity to model the formation of thought of rational beings. In turn, these principles can be subsumed, in their minimal form, in the principles of logics of subsection 4.5. Hence, it suffices to show the inadequacy of these to invalidate the logicist programme. I will do this by taking issue with (PC). Recall that (PC) imports to epistemic logic all instances of tautologies of propositional calculus and, taken together with Necessity, imply that the agent knows all tautologies.

As regards the tautologies of propositional calculus, they seem harmless enough. But let us take a closer look at them. Some results from Computational Complexity theory tell us that evaluating a formula, in the sense of attributing truth values to its components and checking whether the complex formed evaluates to truth for all assignments, first requires that agents are able to sort out inconsistencies in their knowledge base. But this pre-problem is equivalent to the Boolean Satisfiability Problem, well within the class of NP-Hard problems when one considers more than two bits of knowledge, i.e the Boolean $KSAT$ is NP-Hard for $K > 2$, where K stands for the number of literals in each clause in Disjunctive Normal Form. And the reason is that the input for the resolvent of the clauses grows exponentially, rather than polynomially, with the number of literals. Exactly what these snippets of knowledge would look like for an economic agent is postponed to the next section, but let us unveil it for a bit and state that you, as an agent, know a certain number of things from different sources - your own experience, what you´ve been told, your conjectures about a business opportunity, and so on. They all fall under your knowledge base. And if indeed one can state it as a logical system, then it is possible to transform what you know in Disjunctive Normal Form (dnf)[37]. Your knowledge base in dnf, after an appropriate conversion, looks like $K_i((p \wedge q \wedge r) \vee (\sim p \wedge r \wedge z) \vee ...)$, where one can identify the different elements of the conjunctions as knowledge from different sources, such as when you perceive a business opportunity but someone else´s experience tells you it would be unsuccessful. You don´t know simply this (potentially very large) conjunction of disjunctions. You also know a method of resolving them, i.e, of sorting out inconsistencies before evaluating the statement in a truth table. One such way is through the logical tool of resolvents. But as your knowledge base increases and contains more literals, the size of the resolvent you need to apply to assign a truth value to the resulting dnf formula grows exponentially: In particular, while for the $2SAT$ such size is of the order $N(2N+1)$, hence polynomial, for the $3SAT$ and higher, the maximum number of literals in a clause is $3N$, and since you need to assign a truth value to all of them, your mental algorithm has to deal with a resolvent that has the size of number of all subsets of a set with 3 elements: namely, 2^{3N} Thus, the time required for a computation grows exponentially with the size of data for $K \geq 3$. For the $50SAT$ problem, the upper bound of time of the algorithm that sorts out inconsistencies performing one operation per micro-second in a knowledge base is close to *36 years*! For the $150SAT$ this upper bound is well over the age of the universe. All this before the agent builds a mental truth table to check the truthness of the knowledge base for all interpretations, i.e, actually checking tautologies. Note the meta-logical inconsistency this implies. The set of meta-propositions one appends to the theory, such as "individuals are able to assign a truth value to a complex expression", "Individuals act according to an optimal plan defined as one that maximizes the truth value of an assignment" and "Individuals do not take years to resolve inconsistencies in their knowledge base"

[37] For example, a relation of causality could be conceived of as one of logical implication: $a \to b$ is logically equivalent to $\neg a \vee b$, which is already in dnf. The problem of turning any logical expression to a dnf is in P, hence in principle realizable by a thinking mind as conceived of by Davidsonian Functionalists.

are inconsistent among themselves.

Hence, considering agents that can solve all tautologies in (PC) is generally unwarranted. One could artificially curtail the size of the tautologies agents are able to detect: research shows that agents generally do not deal with more than 7 items of knowledge committed to their short-term memory at once. But you would still have to restrict their knowledge base using some criterion, and it is unclear what that should be. One more nail in the coffin against Davidsonian functionalism: this last point can be taken up now that the formal tools have all been developed. Such a theory implies the existence of an isomorphism between the objects one sees and their logical counterparts, and between those and brain states, i.e some kind of way of identifying logical computations with the corresponding neural correlates, including a place where the brain stores at least a sufficient set of logical connectives[38]. But Basti and Perrone show that the problem of coreferentiality between an Intentional attitude of an intensional nature, and an observation of the neurological correlate of the corresponding statement in extensional logic is not solvable.

4.6 An operational approach to meaning. The building of a logical system for an economy

We´ve already seen one reason why the project of building a logical system of an economy rooted on the neurophysiological basis of individuals fails, at least if one takes this project to mean that this basis is conceived of as a subset of First-Order Logic with identity. But if we were to design a logical system based not on this, but on the knowledge we could somehow assemble of what things are, and how they stand in relation with each other, then a watered-down version of Functionalism, this time based not on mental states but rather on connections between *things*, and possuibly between things, states of the world and actions, could still stand. This will take us farther afield in the questions of meaning, coordination and complexity. Let us see how this could be done.

You would have to begin by listing, and translating in your system as symbols, all things that exist - goods, means of payment, available services, economic agents and their features, capabilities, preferences over goods they know, budget at the moment, and so on. Suppose also we have found a way of circumventing the problems arising from the fact that you would need to pin down a scale to classify things, and that such choice could be important. You now have a list of all things in your economy. But that is not enough for your purposes. The institutional guidelines that constrains human action, seem to be stable enough for us to carry out economic plans. We have a reasonable idea of what the action of the Government is, of the procedures it takes to start a business, of who our suppliers are, of what it means to do our job, of the role goods have in our plan along their functional characteristics. The role one takes up in a society comes with a script, as it were, that is largely written out for you - a set of rituals and instructions broadly specifying the actions we are to take up, along with the largely ritualized actions our counterparts are to take. This is the province of institutions: assigning individuals to roles in which their actions turn out to be consistent with the behavior of others (David (1994)). Clark (1996) provides such an example of an interaction between a seller and a buyer of goods in terms of action ladders: a set of procedures that are done in turn, where completion of each step is necessary so that agents know it is time to undertake the next step in the action complex. Indeed, institutions have a real existence in that they

[38] A sufficient set of logical connectives is a set of connectives from which all others can be derived, e.g, the set $(\neg, \wedge)$ (Hamilton, 1988).

make a difference in the way people act: they are material causes of action, although not quite efficient ones, in the sense that people can always choose to do otherwise than the precepts they state (Lewis (2000)). Be that as it may, some survive in time, up to the point they are so deeply ingrained so as to become part of the agent´s Background: in that case, they are said to have reconstitutive downward causal powers (Hodgson (2002)). In all cases, they both precede and constrain our action, which in turn is constitutive of them. Our system should account for that. This is not a fundamental difficulty. These could, in principle, be given form and added to our system by making use of Carnapian Meaning Postulates, encoding what any given role is in terms of simpler action sequences between the person slotting the role and their relational counterparts, as well as the functional properties of goods and services. To fulfill the first requirement, these would have to be supplemented with temporal elements of precedence in time. A very simple set of examples should suffice to show what I mean. Being a supplier of good x, could be translated in a meaning postulate as a 6-place unsaturated predicate taking values in our list of goods, services and persons, and analytical moments of time, such as a good x, individuals y and z, mean of payment m, and moments in time t and $t´$, such as:

$$\forall x \forall y \forall z \forall m \forall t \forall t´[S(z) \rightarrow (Supp(xyzt) \wedge Paym(yxmt´) \wedge (t < t´))]$$

This reads that for any people x and y and good z, for any object taking values from the set of objects means of payment, m, and analytical moments in time t and $t´$, being a supplier of z means that there is the (necessarily simpler) action of supplying (or giving) good x to agent y by agent z at moment t, that there is a payment meted out by agent y to agent x by using m as a means of payment at $t´$, and that the supplying of the good precedes the payment. $Supp(xyzt)$ and $Paym(xymt´)$ could then be themselves defined in terms of simpler postulates, decomposing until arriving at simple physical movements as primitives, which could have the same ontological status as goods. Similarly for the properties of goods, if one tries to capture their essence. “Screwdriverness” could be encoded in our model as, $\forall x \forall y \forall z\, [Screwdriver(x) \rightarrow Interm(x) \wedge Attach(xyz)]$: a good x is a screw-driver if it is an intermediate good and x is used to physically attach good y, say a screw, on surface z. It is easy to see how a production function could thus be encoded, and even Searlian status functions specifying the reconstitutive downward causal powers of a Government or money. These functions presuppose collective intentionality, such as when a group decides to assign the role of money to a given good, and have the following form: $< X\, counts\, as\, Y\, in\, context\, C >$. They are slightly different in that they take meaning postulates as their arguments[39]: however, it is easy to see how they could be turned into complexes of Meaning Postulates.

We have achieved an operational content to both the Searlian notion of Network as the set of semantical connections between meaning postulates. The *initial conditions* of our economy, as defined by a pooling of knowledge about what things are at this moment and how they stand in relation with each other, are then an exhaustive list of goods, services, people, firms and institutions as atoms, along with the appropriately encoded meaning postulates relating things with things and people, and people, firms and institutions among themselves. These latter will be partially or fully instantiated, or saturated, bits of knowledge. Let us follow the notation used up to now and translate these to the more mundane set of propositions $\{p, q, r\}$, and so on. Moreover, there is a method of combining them: a set of production rules of Chomskyan Grammar type. Expressions such as “Institution X allows me to carry out the production of good Y through method W and method W entails availability of intermediate good Z” is a complex of such propositions, combined in an appropriate way. Among these bits of

[39]For example, good x counts as m, a means of payment, in a context C such as a critical mass of buyers and sellers trading good x.

knowledge, some are held only by groups, some by individuals, some are common knowledge. Rescher (2005) distinguishes the following 4 types of knowledge[40]:

- Distributive: formally, $K_d G_p$iff $\exists x(x \in G \wedge K_x p)$
- Collective: $K_c G_p$ iff $\forall x(x \in G \supset K_x p)$
- Aggregative: $K_a G_p$ iff $(\exists p1)(\exists p2)(\exists p3)...(\exists pn)(\exists x1 \in G)(\exists x2 \in G)(\exists x3 \in G)...(\exists xn \in G)(K_{x1} p1 \wedge K_{x2} p2 \wedge K_{x3} p3... \wedge K_{xn} pn) \wedge (p1 \wedge p2 \wedge p3... \wedge pn) \vdash p$
- Expert: $K_e G_p$if p is accepted by some/most of those group members best informed on the relevant issue.

All four types of knowledge are present in a society, and moreover, the conjunction over all individuals is what M&E denote by "Social Dynamics"; hence, $S_t = \bigwedge_{i,p} K_i G_p$. Distributive includes private knowledge, and refers to, say, knowledge about my own preferences, condition, or (partial or full) knowledge about a production process. Collective essentially refers to what is taken as common ground among participants of a market economy. Bits of knowledge as, say, what the role of banks is, that supply and demand meet on a physical location of a market, what the steps in the action ladder are when you enter a store or sign a contract. Aggregative knowledge refers to the pooling of knowledge generating bits no individual agent knows, although possibly aprehensible through experience and induction. This is the search space of entrepreneurs: business opportunities that either are created or exist, according to whether you are a social constructivist or not. Finally, expert knowledge refers to bits of knowledge that is used by experts. Interestingly, both Collective and Expert Knowledge have the potential not only of describing bits of social reality, but actually of being constitutive, of *shaping*, those facts. This is evident in settings where expectations play a key role in defining what the facts are. Hermann-Pillath (2010) seems to have that in mind as he describes how the Black-Scholes equation for option pricing actually gave rise to a price structure consistent with the theory. I surmise that expression (4) in M&E must then be complemented by the following:

$$K_i P \rightarrow \varphi \rightarrow A_t \tag{15}$$

where P is a set of expressions of individual knowledge comprising distributive, collective, aggregative and expert knowledge (as an end or partial goals), φ stands for (10) in M&E, and A_t the action corresponding to moment t of analytical time. P will be given a more concrete form in the next subsection. Let it just be said for now that M&E implies that the first implication sign in (15) involves the compression of analytical moments in time and the separation of statements into facts and goals into a structured plan φconsisting of a complex of actions, yielding single or complex action A at moment t, and it is exactly such a complex that (10) captures.

4.7 The framing of coordination

M&E refer to coordination as a "gain in social feasibility". I see no reason to argue with their position, as long as this feasibility has a cognitive, rather than logical, connotation. But this begs the question: how is it possible that

[40] In Rescher´s notation, the subscript refers not to an individual, as up to now, but rather to the first letter of each type of knowledge. It is assumed that the four types of knowledge span groups of individuals.

in an economy with millions of agents with disparate objectives and goals, they are somehow able to dovetail their plans? How to reconcile the subjectivism of active minds with the notable degree of coordination one observes? Koppl (1998) refers to this as Lachmann´s problem. To fully appreciate his argument, a small detour through a topic of Philosophy of Language is in order. Notice the difference of the use of the word "cat" in the following two sentences.

1. My cat is called Mr. Whiskers.
2. The cat is a very common pet.

In the first sentence, "cat" has attached to it a being with real space-time coordinates: a real being existing in the world. It is a *token* of a cat. The use of "cat" in the second sentence, however, is much more interesting, in that not only are we able to understand it, but it refers, in Frege´s sense, to no cat in particular. Rescher calls this vagrant predicates - properties or beings that I know exist, yet I am unable to anchor them to something, in the sense of saturating the predicate with objects. The cat in sentence 2. is an example of a *type* - and as Koppl contends, thoughts expressed as expectations anchored to the behaviour of highly typified characters are the key to framing Lachmann´s problem. We are able to form judgments, and attribute mental states that mirror our own, to people we do not know: judgments of the type, "there will be someone (whose identity I cannot necessarily pinpoint) able to do task y", "The buyers of this good know the common ground that exists between us, as members of the same community", or "consumers have the same needs as I do". Rescher further distinguishes between Specific and General knowledge: The first is, according to Rescher, Knowledge about something that has a certain property F: formally, $\exists x K_i F x$, i.e, i knows that there is something that is F. The second, however, corresponds to the thoughts akin to types: $K_i \exists x F x$:i knows that something with property F exists, in spite of the fact she is possibly unable to adduce a *single example* of either the identity of the fact or the agent who might possess it.

If, as I surmise, aggregative knowledge plays a key role in entrepreneurial search, then the 4 types of knowledge of the previous section are instrumental in shaping a plan of action, thus giving operational content to $K_i P \rightarrow \varphi$. This type of information goes much beyond that conveyed by prices. For Schumpeter, entrepreneurial opportunities exist outside the price system, hence not as a part of the market, and entrepreneurs "act on the interface of market and science" (McMillan, Plummer, and Acs, 2007). This is true, and indeed reconducible to the types of knowledge of the previous section, but incomplete: they also act based on what they think everyone knows and on the features they think people they don´t know, possess. Goertzel refers to a proposition p of common knowledge as a hyperset of the form:

$$X = \text{Everyone in group } G \text{ recognizes both } X \text{ and } p.$$

In other words, collective knowledge is such that everyone knows both the facts stated in p and that everyone knows other people know those facts. This is the case of conventions: as David contends, conventions must attain the status of being common knowledge, such that I know that you know them, and you know that I know that you know them, and so on, if they are to serve as signposts to coordinate actions.

Both specific and General knowledge shape the previous 4, where the latter is instrumental for framing coordination. Then, the complete set of propositions P that informs the constitution of plan φas defined in (10), is:

$$P \equiv \theta[Y, X, K_i(\exists u)(\exists Z)(u \in G \wedge K_u Z)]$$

where Y contains a set of propositions comprising both private information and that in common with a (restricted) group, X is common knowledge in Goertzel´s sense, and the last refers to typified knowledge to which Rescher´s process of vagrant predication was applied: the attribution of knowledge I don´t know to people I have never met. This is an adaptation of Koppl´s framework to conceptualizing the Lachmann problem: attribution of Intentionality to highly typified characters[41]. Thus, Davidson´s interpreter is vindicated: our common language D is that of custom, of the spontaneous order that evolved and is passed on to us in the form of laws and old sayings. Watson (2005) states that, long ago, the paradox between agents acting out of self-interest and the progress of economies where plans must dovetail, where selfishness must be curtailed to an acceptable level, was stressed by Adam Smith himself, (not in *The Wealth of Nations*, but in his other, less known work among economists, *The Theory of Moral Sentiment*s). Perhaps unsurprisingly, the moral philosopher placed the emphasis on this possibility by stating that the solution to the paradox was to be found in *morals*, rather than economics: in particular, in the empathy we have towards our fellow men, of placing ourselves in their shoes; in a word, of interpreting them through the lenses of our experience, knowledge and feelings. I take the exegetic step to conclude and extend this solution to the dovetailing of plans. Hédoin (2013) also connects the question of coordination to what he calls "Common understanding", a cognitive disposition to form Intentional states underlined by the fact that we, as human beings, share a common evolutionary history and cognitive mechanism; hence, this is a capacity of the Searlian Background, thus avoiding the infinite regress. This is what he refers to as Collective Intentionality, and has mutual awareness as a consequence: following the notation used up to now, $K_i p \subseteq K_i(K_j p)$, i.e, what is evident to i is such that i knows that fact is also evident to j. It is clear this definition is contained in the one stated above for Common Knowledge.

4.8 Logical implications in M&E. Rationalities. The role of emotions in shaping action.

Let us now take stock and reflect on M&E taking into account the results of the previous sections. This part explores whether the signs of logical implication in (12) and (13), as well as expression (11), are warranted. Recall that M&E explicitly equate the meaning of a plan with the agent´s "Ethical Dynamics". Moreover, their reliance on consistency of beliefs coupled with the choice of a maximal plan, as expressed in (11), lead me to the conclusion that their analytical scheme endorses Davidsonian functionalism - a plan is a sequence of steps connected logically leading to a goal, of which the agent chooses the one that maximizes a truth value - agents as subcutaneous optimizers of truth. This is called the Principle of Economic Behaviour in M&E - and is a direct consequence of the principle of coherence in Davidson´s corpus of theory. They refer to consistency as lack of infeasibility and feasibility as the logical possibility of a plan. Thus, (12), coupled with (11), are unwarranted for the reasons adduced above, stemming essentially from inadequacy, or at least incompleteness,

[41] In turn, Koppl contends that the idea of thought in typified terms belonged originally to Theodore Schultz.

of the use of FOL with identity to model the thoughts of rational agents. There is, however, another reason why (11) must be rejected. And that is that it breaks down if one considers an open-ended definition of rationality. Hargreaves-Heap (1989) distinguishes between instrumental, procedural and expressive rationalities. The first implies the agent has an end goal, and consists in the deployment of a set of instructions, possibly encoded in an algorithm or some such, allowing her to solve the problem in an efficient manner. Agents in Rational Expectations models are typically of this type. Procedural rationality refers, instead, to pieces of knowledge handed down to you by society, developed as adaptive solutions when other members faced similar constraints in the past. It places us as historical beings: a part of the way we act is through custom. When we conform to the rules of an institution (like driving on the right side of the road), and either consciously or, in the case that is so ingrained that is subconscious, when it is in your Background of pre-intentional states, we are deploying procedural rationality. It accounts for the successes of coordination: it makes so that focal points become more salient to different people due to custom. The third type, or expressive rationality, however, is more interesting in that it entails that your goal emerges concomitantly with your action. You discover what your goal is, and more broadly, who you *are* - through your actions. Hargreaves-Heap (1989) conceives of expressive rationality as a way of discovering the world through our action, that expressively rational individuals "often act from a position of uncertainty with respect to what values they wish to entertain" (*Ibid.*, pg. 156). All three types of rationality are present as we go about our daily lives. However, if Hargreaves-Heap is correct, (11) is ill-posed if one also considers this more open-ended definition of rationality, because no maximizing principle can be forthcoming when one does not know what the goal is, as is the case when your action *is* the goal. Hargreaves-Heap seems to hold exactly such an interpretation: an act animated by expressive rationality is such that "an act is an end in itself" (pg. 176). Therefore, expressive rationality breaks down the necessity of a plan conforming to a goal, and 11) cannot account for any substantive theory of plan selection, if indeed one such exists, beyond domains that are either probably too restricted or too general for our objectives as economists.

The implication in (12) is, however, also unwarranted, even if one does not consider the open-ended definition of rationality as a guiding principle of beings featuring free will. And that has to do mainly with the role of emotions in shaping action. If one had to pinpoint an overarching principle in human, ergo economic, activity, that would be the search of pleasure. And neurological research show that peaks of dopamine in the nucleus accumbens, signalling expectations of pleasure, precede a conscient decision as devised by some plan (Carvalho, 2009). In cases like these, emotions actually trigger an action - and rationalization comes *ex post facto*. But there is another way in which emotions are important: they are filters of action. Prospect theory tells us that we have a stronger aversion to losses than to gains in the same amount. The reason for this may be evolutionary - and the theory of somatic markers states that the brain attaches, as it were, a marker to every situation and filters out those with a stronger negative one. As it turns out, emotional feelings of pleasure and loss are processed in different areas of the brain - The nucleus accumbens and the insula, respectively - and hence different subconscious evaluations may be involved, in much the same way as the knowledge you recruit when accessing the long-term memory may sometimes be inconsistent with short-term knowledge, and both inconsistent with the action actually undertaken.

4.9 The case against postulates and pinning down meaning. A complex of complexes

Let us go back to our original plan of setting up a formal model for an economy based on an exhaustive set of things, people, firms, institutions and meaning postulates connecting all of them and see how far it can take us. Let us place ourselves as policymakers that are given both the set of all things, people and meaning postulates, and all that happens in an economy as actions unfold, aggregated in statistics such as GDP, saving rates, and so on. Following figure17, our objective is to go from ΔS_t to ΔS_{t+1}.

The first thing to notice is that, up to this point, I have been talking rather loosely about meaning postulates - but such vagueness clearly would not do if we were to set up this model. And if indeed you were to set up an exhaustive list of meaning postulates, you would face a serious issue for defining a *single* one. And the reason is that meaning as a set of necessary conditions of a thing has a noticeably fleeting nature. On a famous passage in his *Philosophical Investigations*, Wittgenstein (1953, pages 31-32) makes this point about the word "games":

> "Consider for example the proceedings that we call "games". I mean board-games, card-games, ball-games, Olympic games, and so on. What is common to them all? - Don't say: "There must be something common, or they would not be called 'games' "-but look and see whether there is anything common to all. - For if you look at them you will not see something that is common to all, but similarities, relationships, and a whole series of them at that. To repeat: don't think, but look! - Look for example at board-games; here you find many correspondences with the first group, but many common features drop out, and others appear (...) we see a complicated network of similarities overlapping and criss-crossing: sometimes overall similarities, sometimes similarities of detail."

Wittgenstein´s point, applied to the formalism developed thus far, is that no common frame exists that can be subsumed by pinning down a set of necessary and sufficient conditions tied together by conjunctions that uniquely characterizes all instances of the reality expressed by the word "game". Nor do we need to do so - we simply use words and complex expressions composed of them, and we know what they mean. Such vagueness would, of course, not be allowed in any logical system where everything must be either specified as a primitive or an axiom, or derived from the transformation rules. Moreover, if we were to pin it down, we´d have to leave language to talk about it through some meta-language with which we could describe similarities and relationships, as Wittgenstein puts it. But of course, no outside vantage point exists so that one could, through an algorithm of some sort, describe in this fashion exactly what things are and how they evolve. Wittgenstein places this perfectly as he mentions words only acquire a meaning if one also reports to the language game in which they are employed, i.e, one needs a setting of context or pragmatics. On a analogy developed by Wittgenstein himself, the idea of words, and complex expressions formed from them, that can be found to exist independently of the context in which they are used is akin to an observer deducing a rule for the "mate" in check-mate of chess, independently of the rules of the game. No language is complete enough to be able to check its own consistency.

I will once more take the exegetic jump from Wittgenstein´s writings to fit my purposes. In particular, the inexistence of a vantage point from which to enumerate all the properties of things in the real world is a feature

I reiterate, with some support from the theory of Computational Complexity. The argument goes as follows. Recall the way we had defined screwdriverness in our model. As Kauffman (2013) contends, this would not suffice, since the uses of a screwdriver are not countable (of course his point holds for things in general). In slightly more technical terms, the space of all potentialities of what things are is not recursively enumerable by any algorithm. Hence, there is no way of defining an algorithm that explores the space of all interpretations, all the uses of all things in computational terms, that can be deduced to halt. This is a fundamental barrier and a first sense in which an economy can be said to be complex. This indeterminacy as uncomputability in the space of interpretations is called by Biggiero (2001) semantic complexity. When we talk and listen, we are socially conditioned to pick out the appropriate snippets of meaning, and understand, say, the correct way in which something can be something else: If you read "toxic asset" somewhere, you instinctively know what it means: an asset that is best no to hold, rather than an asset that is literally an acid or gas with certain harmful chemical properties. Hence, it is toxic in that, *like* a poisonous gas, it is better not to have contact with it, but *unlike* a poisonous gas, has no chemical properties. We do this and a plethora of other examples of everyday language use without thinking much about it or stopping to think about the marvel our mind has produced: it is really remarkable cognitive work that is being undertaken. In fact, this is exactly how metaphors are built - knowledge is taken from a familiar domain to another one and expressions take on a new meaning (Deutscher, 2010). Witt (2010) makes this point about the growth of knowledge: it is one of association, of retaining the similarities between concepts and discarding the rest. But this process is uncomputable from a logical point of view in the sense that an algorithm computing associations between a thing and all other things can be deduced not to halt. We then hit a fundamental barrier in stating the initial conditions, the axioms, of our model, for a *single thing*, let alone for the millions of interacting parts of an economy. Then, we know that no effective way exists of setting up the axioms of our model beyond a restricted sense - and leaving things out is probably unwarranted, as the proverbial story of the kingdom that was lost for the will of a nail reminds us. Pushing the linguistic metaphor a bit further, searching for the space of all meanings and stumbling upon the correct configuration is not very different from finding Shakespeare´s complete works in Jorge Luís Borges´ famous infinite library - or indeed writing them, armed only with the knowledge of the rules of syntax of the English language.

So no model that is general, read complete, enough to encompass every bit of information an economy produces is forthcoming. We should then restrict the axiom base and couple it with a set of conditions specifying what agents do upon meeting a condition or state of the world. This is the activity we face as economists when we set up our models, and is somewhat, then, more familiar ground. Let us then explore this avenue.

In this regard, Muñoz *et. al.* (2016) present a set of precepts that economists use when deploying and evaluating models. I will be using their framework for the remainder of this section for establishing the difficulty in setting up a model that associates states of the world with actions. In their meta-model, they denote S_1as axioms of a given theory - an appropriate subset of the initial conditions of the economy, as defined up to now. Hence the axioms of any economic theory can be seen as either the complete set of things and meaning postulates connecting them, or a proper subset thereof. They couple that with a set S_2 of inference rules, understood in economic models as actions of agents undertaken when meeting a certain condition: "if you meet condition a, then do b". Coupling $S_1$with S_2, one can deduce a set of theorems T - and ideally, your phenomenon of interest is among the statements of T, i.e, it is *derivable* in your theory. But in a remarkable paper, Markose (2016) shows that any model where S_2 incorporates enough need for strategic innovation, such as minority games, leads to an uncomputable fixed point. This stems essentially from an isomorphism between the need of players to introduce strategies that are innovative enough to outsmart adversaries and the emergence of Gödel´s uncomputable proposition as a meta-statement: $A \leftrightarrow \neg P(A)$: A says of itself that it is not provable. The conclusion is both easy to grasp and hard to overstate - it means that, no matter what the plan you set up for your agents as statements encapsulated in S_2 is, they will soon find out that the solution to their problem lies outside of it! In other words, your finitely axiomatizable system is necessarily incomplete, in the sense that some true statements are not provable within it. This is, of course, the famous Gödel incompleteness theorem, and one of the reasons why one sees economic models fit with all kinds of contraptions to avoid direct consequences of it. It is in this sense that Models, taken here in capital letters to stand for the set of all models that can, have, or indeed will be built and are consistent, are incomplete if consistent, or inconsistent if complete.

Much like words, where there are competing ways of uttering them that supply a language with diversity of population that is a necessary condition for evolution, so it is that the meaning of economic actions and of things can be seen as evolving from a set of competing interpretations. If you fixed an interpretation in terms of meaning postulates, soon you´d see violations of it: what you set up as an intermediate good would become a final good for some people, sales with more or less steps than envisioned, and so on. Axioms are true by definition: but agents animated with expressive rationality change those axioms as they reflect about their place in the world, about how institutions work, or about how a new use to give to an established good. But this is entrepreneurial and economic activity by excellence! Incidentally, this is why so many inventions take on new roles different from the ones they were designed for (Tuomi, 2012), and why the future is often so different from the way we collectively project it to be[42]. Lourenço and Graça Moura (2016) state that the role of things is defined by more than their functional characteristics: "whether a hammer is a means of production depends on the role it takes in someone´s plan". This process of going about our daily lives, of reproducing social institutions and the nature of things, and changing them, can be given an evolutionary framework: indeed, this is the gist of Lawson´s Population-Variation-Retention-Selection (PVRS) model, where social practices act as interactors (or selection environment, when taken as a whole), and social rules, here our finite list of meaning postulates, as replicators which are selected or modified (Lawson, 2003; Martins, 2015).

If you choose to include only those meaning postulates that are invariant, or reasons for acting that are innate to human nature, first you would have a hard time justifying exactly what those are, and second, they would probably be too weak to protect your model against fundamental indeterminacy. And the problem cannot be eliminated by simply appending the new axioms, or new bits of meaning, in a new list of axioms. For your system to do that, you would have to supply it with a set of meta-rules that would control such process. These meta-rules would also be incomplete, hence the need for meta-meta-rules, and so on ad infinitum.

From a dynamic point of view, as analysed from the perspective of the first paper in this thesis, the following leap can be made. I surmise, but have no way of ascertaining outside the framework of a working hypothesis from a philosophical point of view, that the chaos one sees in economic time series are really of a relational nature: not an intrinsic property of them, but rather a measure of the way the world deviates from what we fathom it to be. Dynamic complexity would then be both relational (between an observer and facts of the world) and derived from the other two types: as a distance between possible worlds that grows wider with time. There are, however, 2 more straightforward connections. Recall Goertzel´s point on the fact that our intentional states float around a moving fixed point, as it were, that can be interpreted as a strange attractor expressing our essence. Then,

[42] In the 1960´s, several depictions of the future in the far-flung year of 2000 involved either an apocalyptic world ravaged by nuclear war, or a marvel of technological progress where flying cars and space trips were common. The fact that those were the years of the Cold War and space landing is not innocuous: we tend to project linearly into the future (Taleb, 2012).

expressive rationality is just a way of us making sense of who we are through experimentation. Thus, a strange attractor in the mind both mirrors and generates that of a dynamical system of economies. On the other hand, every time you make a decision you generate information. Sometimes that decision can be made to conform to the structure of existing social structure - there is no new information created. However, if a critical decision is such that it alters the fitness landscape, you have created information - debris of plans, as Wagner (2010) puts it, that can be interpreted by someone else. These are the unintended consequences of economic actions - enabling constraints not only in terms of technical possibilities, but also in the spread of information, new capital combinations, and so on. They create information - decrease in entropy, and the concomitant processes of variation, retention and selection, as people try to make sense of the new reality and the opportunities opened up for them. This process translates dynamically as chaos and derives directly from the PVRS and the theory developed up to now. Then, the "corridor of stability" hypothesis of Leijonhufvud, which I analysed in the first paper for the Portuguese Economy, is given operational content - The dynamical regime of an economy between order and chaos is really the reflex of the search for meaning.

Thus, a logical system cannot account, or at least not exclusively, for the way we think. However, the semblance of order in our thoughts and speech indicates that some overarching cognitive principle may be in place. Just what that may be, is the topic of the next section.

4.10 Conceptual blending

In the previous section, we have already seen how words can acquire new meanings according to the Language Game one is playing, that these meanings are not independent of the context, and that, moreover, no Bird´s Eye view exists that can enumerate all these uses in settings where, as the social case undoubtedly is, facts are configured by the agent´s own Intentional states and all the combinatorics of their interplay, in much the same way as it is impossible to jump out of language and use it to check its own consistency. We can go deeper into the first of these questions: it underlines the theory of conceptual Blending. Look at the following two sentences, while keeping at the back of your head the discussion about meaning postulates and how there are several ways in which things can be of other types:

1. Sally is the Mother of Jenny.
2. Necessity is the Mother of Invention.

While they have the same structure, they can hardly be said to mean the same thing: and the reason why this is so, is that each of them prompts you to do very different cognitive work.

Economics requires a cognitive theory able to retain the features that are intuitively true about the way we think - these were alluded to in the previous sections. Namely, imagining possible worlds, whether as sets of propositions or not, where meaning is both emergent and compositional, albeit in a more encompassing sense than Frege´s account. We´d like to retain propositions as representations, as in Searle, in a cognitive setting that can account for at least two critical operations that are essential in the setting up of plans: that of performing compression and decompression of time, and of attributing tokens of Intentionality to people about which we know little more about other than their existence. Similarly, it should account for a product of thought that resembles metaphors and be general enough to constitute the cognitive apparatus of both a Davidsonian interpreter and of the interpreted agent. I surmise that conceptual blending can do all of that.

This section presents the principles of conceptual blending in a brief, condensed manner, and connects it to the topics analysed above. The thesis puts forward the idea that this cognitive theory should serve as a benchmark for the analysis of the deployment of plans, and indeed belong to the economists´ toolbox.

Conceptual blending is a cognitive theory, hence above the level of neurons, and below that of semantics. Think of it as describing the emergent product of assemblies of neurons. Although I will describe these with words, this is only so since language mediates the way we conceive the world, not necessarily because the end product of our thoughts have any semblance with words themselves. It underlies the construction of meaning and is largely subconscious. We blend when, say, identifying a person with an image, or when thinking about others over time and attributing them an identity that is invariant. It is our brain that attributes properties to things, and does so as prompted by cues of a communicational nature. The thinking brain is able to populate and construct a scenario and dynamically run it, even featuring things that never were, by matching features of different scenarios.

At the heart of the operation of blending lies the primitive notion of mental space: a conceptual packet that organizes our thoughts and is connected to a bundle of knowledge called organizing frame. Mental spaces are connected to each other and are represented diagrammatically in Fauconnier and Turner as circles containing single elements integrated therein:

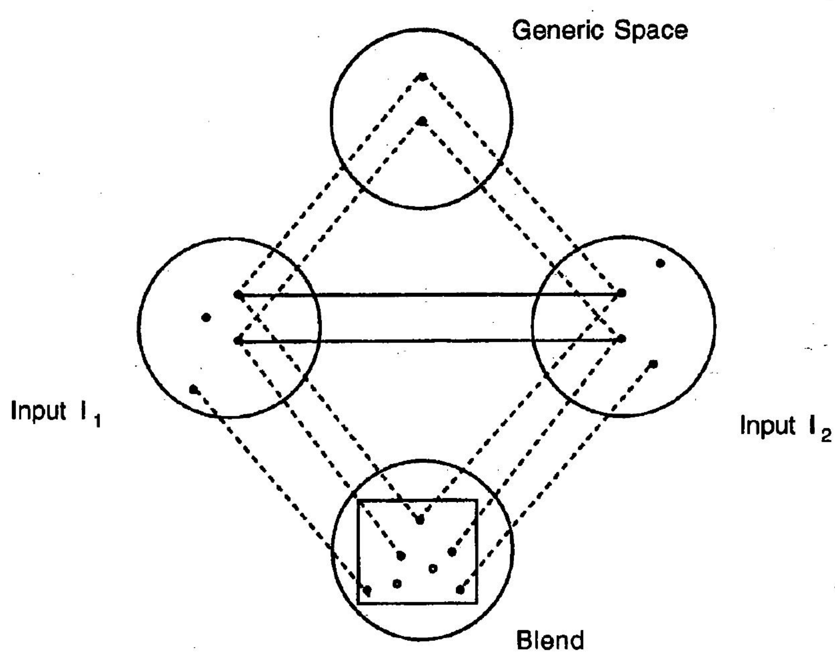


Figure 18: Basic structure of a blend (taken from Fauconnier and Turner (2008), page 46)

In the most basic setting, they consider a network model consisting of two input spaces, both contributing to the end product, a generic space, containing general terms that the inputs have in common, and the blend, featuring a selective projection of some elements of the input spaces. Crucially, the blend imports background knowledge akin to simulation where we run a cognitive version of time and events unfolding therein. The product of thought is emergent in that the blend sometimes features things that are not present in any of the input spaces. Some blends gain widespread currency: they become entrenched in a society and become cultural artifacts. Moreover, blends can be inconsistent, in the sense of a blend representing contradictory events, such as those prompted by a counterfactual statement. The cognitive work underlying the construction of the blend is also achieved by what Fauconnier and Turner call the "compression of vital relations": Change, Identity (connecting rigid designators between two spaces), Time (such as when we bring together events spread apart in time into a single scenario or imagine something some time from now), Space, Cause-Effect, Part-Whole, Representation, Role, Analogy, Disanalogy, Property, Similarity, Category, Intentionality and Uniqueness. They conceptualize meaning as the scenario that emerges in the blend after compression and decompression of vital relations. Depending on the way the elements between input spaces are recruited and the roles they take, network topologies range in increasing complexity from Simplex networks - where concepts act as placeholders taking up values, exactly as in standard Fregean semantics, to Double-Scope networks, where the blend is constructed using often contradictory input from different organizing frames. For the two sentences that opened up this section, the difference is clear. For 1., you are setting up a Simplex network, where $Mother$ is a two-place predicate Mxy assuming values $x = Sally$ and $y = Jenny$. The second case, however, is a metaphor : it prompts you to imagine an input space where you have a problem you need to solve, another where there is a solution to it, and $Mother$ is now a prompt for a setting where Necessity is an effect, something that causes you to deploy some measure to solve the problem: a Cause-Effect cross-space connector, in Fauconnier´s parlance. Hence, what we are doing is recruiting the frame "$x\, causes\, y$" of the notion, or meaning postulate, of "Mother", while abstracting away, say, actual biological links between a necessity and a matching invention as those that exist between a mother and a daughter.

For the "toxic asset" example above, the network your mind constructs is a double-scope one, since the blend recruits knowledge from both an Asset input framed by related knowledge about buying and selling of such products and the stock market, and what we may call a basic chemistry input, framed by knowledge such as that a toxic product has certain dangerous physico-chemical properties. In the generic space, you would have the organizing frame of dangerous to hold / best to avoid, that unites both input spaces. And the blend is such that you retain what is common in both inputs and understand in what respects an asset can be toxic: as something that is best to avoid, or to get rid of, lest it metaphorically poisons. Similar analogues could be found for expressions of economic content.

Fauconnier and Turner point out that the blending operation underlies the very way we think, rather than simply being a curiosity and the place where both metaphor and metonymy are born. Let us explore this statement as applied to the theory of Intentionality exposed above. According to Fauconnier and Turner, attributing a token of Intentionality to an agent, or indeed the thoughts that emerge in your mind, ergo the constitution of economic plans, are cognitively isomorphic to several operations of blending mixing different type of networks. Hence, stating that $x\ Believes\ p$ is short-hand notation for x setting up a set of mental spaces connected by cross-space mappings that map elements from one space to another performing the compression of vital relations:

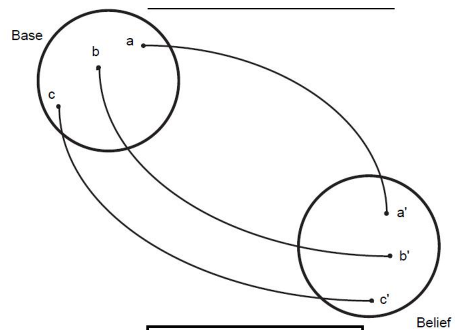


Figure 19: Mental spaces with respective elements connected by cross-space links. Taken from Fauconnier (1998), page 5.

Hence, say, the prompt "imagine yourself ten years from now" has a neural correlate to opening up a network model featuring two input spaces and cross-space mappings: where there is a base of knowledge corresponding to your perceptions of the conditions of today, another input corresponding to the world in ten years, and a cross-space mapping that connects the cognitive image you form of yourself to that future space. Crucially, projection is selective: hence, you will abstract away things like the objects around you, while imagining yourself in the place you yearn to be with the people that are cognitively salient to you. An economic plan, is invariably a sequence of imagined actions unfolding in time, linked through a cause-effect connector, where people I do not know are connected to me through a cross-space mapping linking a subjective and an intersubjective input space. Such link could carry with it the attribution of empathy while abstracting away the details of other people. Similarly, it can account for the operation of time compression as a cross-space connector between different imagined events. An economic plan features several kinds of networks and compressions. The particular type of operation depends on the scenario that is imagined, but it is clear that a plan, as envisaged by M&E, is really a set of operations of several types of networks that act as inputs on one another, where the cross-space connectors are mainly of the compression of time and what one may call empathy connector:

I have discussed a reason why First-Order Logic was unlikely to tell the full story as a mirror of human thought, namely the fact that the statements "Our mind is a logical computation device" and "The detection of tautologies is an NP-Hard problem" are inconsistent with the fact that we do not always ponder about our options for a large amount of time instead of deciding and acting, as an optimizing program would do. But it may be a part of the way we think - so much so that we are convinced by logical arguments. We are now in a position able to adduce additional reasons why Fregean compositionality is insufficient as a suitable meta-language: the scope of quantifiers occurring therein is only that of values filling up fixed Meaning Postulates. When we deploy a more complex network, either mirror, single-scope or double scope network, however, our mind is able to pick out the appropriate sense, read the adequate snippets in the respective meaning postulates, in which something can be something else and abstract away and blend only those, effectively creating new meaning postulates! And conceptual blending shows us the way we do it: we have an operational theory within which to embed triggering expressions such as "Toxic asset" of the previous section, but also "Economic activity cooled down", "The government budget is fair", "Brexit", or "Taxation is heavy". In all these cases, we are recruiting the relevant bits of knowledge from different frames and putting them together in the blend. Incidentally, if you want to get a grasp of how your mind is not like a computing machine, think about a sign cue such as "The missing chair". The blend you cognitively construct has an input space where all the chairs are present and another where no chair is present, and the blend features something that simultaneously is and is not - violating a key condition of any logical system, namely the principle of the excluded middle.

I mentioned previously how their account of interpretation and meaning, here contained in the idea of blend, is both compositional in a cognitive, albeit not truth-functional way, and emergent. Moreover, the words involved only recruit a part of what it means to be something, i.e, they take up different roles according to the language game one is playing. Some words on this regard are now in order. When you perceive the prompt "This surgeon is a butcher" (Fauconnier (1998)), the blend you construct is likely to be the same as mine: that of a surgeon who performs his work in a clumsy way, covered in blood after an operation, and so on. Butcher is only recruited to convey the frame of sloppiness. Similarly, if we invert Fauconnier´s example and state that "This butcher is a surgeon", we invoke the image of a butcher that works with the precision of a surgeon: this is the only contribution of the word. But here is the interesting feature: the frame of clumsiness in the first, or precision in the second, is present in neither of the inputs of each case! It emerges only in the blend - and the composition is meaningful in a cognitive sense. This notion of emergence underlies the way entrepreneurs combine goods and imagine a target of consumers for them, as M&E contend when they explain emergence in their model - but also a lot more: how a good can act as an institution, how an institution can become a service, in what ways a government policy is like a bottleneck, and how a mass of consumers I don´t know can become *my* customers. All of these are born out of minds animated by expressive rationality, spurred by sometimes little more than intuition, by the whim of a mind that emerges as a metaphor and that responds creatively to the circumstances it faces - and is emergence in a much wider sense than any formal model, agent-based models included, can emulate - as Darley (1994) contends, emergence is really computational irreducibility, i.e the finite analogue of formal undecidability. Our argument has come full circle.

It is worth noticing that, under this formalism, language loses much of the clout it has in the philosophy of language, to become a set of instructions to set up blends - a method of transmitting thoughts. Moreover, it is underdetermined to a significant degree, in the sense that no one-to-one correspondence between triggers and blend exists, thus vindicating semantic complexity. I reiterate the point made above: that some blends are more available than others is largely a result of social and cultural conditioning[43], of the evolution and learning of the intersubjective language that underlies our thoughts and discourse.

4.11 Concluding remarks

In this essay, I started with a critique to the neoclassical formulation of the acting individual. I placed great hopes on the burgeoning paradigm of complexity sciences to help solve the twin problem of unrealistic behaviour of individuals and the aggregation issue. I then interpreted what I took to be a paradigmatic analytical model as a starting point to connect the three topics in the title that can be expressed as an alliteration: complexity, cognition

[43] For the butcher-surgeon example above, you instinctively know I am referring to someone who undertakes his surgeon task in a clumsy way, rather than, say, a surgeon who also happens to work as a butcher on his spare time. Thus, the first interpretation has a higher cognitive availability than the second, although the first is metaphorical and the second, literal. For the second example, "Surgeon" is only recruited for meticulousness.

and coordination. The model is a hybrid insofar as it refers to emergence, coordination and complexity, whereas most in the neoclassical tradition simply shun these questions away. However, if one looks closely, the picture it paints of the acting, rational individual is dismayingly poor. I finally realized that any theoretical framework that fails to do justice to the way we think is doomed to failure on normative grounds. We must therefore pay attention to cognition, and reflect on whether a realistic picture of the thinking individual is compatible with the kind of reasoning expressed in meta-models of the type of M&E. The considerations strewn throughout the essay may seem at first sight completely out of line with what we do as economists - that thinking about logic and cognition has very little to do with the pressing issues of our profession, with the activity of building models that can help explain or find regularities and establishing economic laws. However, inasmuch as the models deployed in the Neoclassical tradition are recursive, and hence computable, they are all translatable into (potentially very large) segments of First or Second Order Logic, as, indeed, is all of mathematics, as Russell and Whitehead have shown. And thus, the discussion applies to them in a very straightforward manner. You can see this essay as a discussion of the logical foundations thereof, based on a paradigmatic analytical model - and why these are inadequate in the sense of being insufficient, both if one concentrates on the richness of emergent thought (recall the discussion on how thinking about "A missing something" violates the principle of the excluded middle) and if one focusses on the inherent limitations of logic in and of itself, stemming essentially from Gödel´s theorems. In this investigation, I found what I already knew. That we humans are a strange mix between reason and emotion, between cold, reasoned thinking that would make Descartes proud, and take place mainly in the pre-frontal cortex, and a limbic system that deals in emotions filtering options in a subconscious manner, and instinct that sometimes surprise ourselves, programmed into our very beings and transmitted by our forefathers as evolutionary adaptations to ensure the survival of the species. We navigate in a sea of critical decisions that mix creativity and rule-following in a partly planned, partly unpredictable cognitive background. It is only befitting that the result of human action shares those same features. I have attempted to show that that is precisely the case throughout this essay. That an economy is a complex of complexes but everything starts - and ends - with interpretations, with complexity from a semantic point of view. Inasmuch as our words reflect our thoughts, the connection with the theories of language presents itself quite naturally. I have shown complexity from an algorithmic point of view emerges if we deploy methods that rely solely on order, understood here as positing a priori the existence of an algorithm capable of recursively enumerating all things, if somehow the axioms of a formal theory could be fixed. Moreover, the fact that things don´t mean the same thing to all of us lies at the heart of differences of valuation that underlie chaotic dynamics of macroeconomic time series. And that this disorder is given a meaning by a thinking mind that performs cognitive operations that indeed contain, but go much beyond, Fregean semantics, as conceptual blending. This theory can frame the fact that we attribute a meaning to things in a process that is reconducive to the formation of metaphors in a given context. Moreover, it has the advantages that the key to understanding coordination, or the issue of how we are able to navigate around a maze of contradictory acts that mesh together, even though they are animated by radically different subjective minds, can be framed perfectly within it as the attribution of Intentionality to people we don´t know - we have also seen that empathy to be the solution Adam Smith proposed to his own problem, a solution to be found in the domain of ethics and that withered away with the progress of economics towards a body of knowledge with a more normative leaning. In this sense, it is also an effort to rehabilitate ethics and gnosiology in economics, along with a deeper understanding of cognition coupled with an attitude of humility - that some things are simply too difficult to understand, let alone control.

Appendix

This part presents the full Netlogo model code developed in Chapter 3 of the thesis.

```
extensions [array stats nw profiler]
breed [Households household]
breed [Firms firm]
breed [State]
globals
[
  connections-hh
  connections-firm
  Interaction-type
  number-links
  length-bit-size
goodslist
  lista-bens-corrigida
  lista-bens
  lista-bens-finais
laborlist
  lista-labor-corrigida
  lista-trabalho
  lista-trabalho-sem-duplicates
productive-capacity
  price-list-interm-goods
  prices;
  price-list
  price-update
  price-update-2
  wages
  wage-list
  wage-update
  wage-update-2
  list-of-final-goods
  labor-list
  split-points
```

blueprintermediate
  blueprint
  list-of-working-blueprints
  list-of-blueprints-profit
resultados
finalgoodz
  list-of-blueprints
  list-of-blueprints-final
  demanded-goods-hh
  demanded-goods-firm
lista-bens-interm-primitivos
  lista-bens-finais-primitivos
lista-labor-types-primitivos
  supplied-goods
  labor-demand
  labor-demand-total
  labor-supply-total
  supplied-labor
  unmatched-households
  households-that-go-shopping
  households-that-dont-shop
  size-of-shopping-list
  list-of-labor-transactions
  list-of-transactions
  unemployed-hh
  labor-market-3
  goods-market
  list-of-supplied-goods
  best-match
  new-blueprint
  new-blueprint-cost-e-profit
  reconversion-cost
  warehouse-new-goods
  warehouse-new-labor-types
  warehouse-new-blueprints
  mean-price
  mean-wage

Public-treasury
lista-com-profit-de-blueprints
lista-com-cost-de-blueprints
unemployment-rate
gdp
gdp-investment
gdp-goods
transacted-goods
transacted-labor
frequency-of-adopters-of-best
provisory-list-of-blueprints
]

Households-own
[
id
init-budget
budget
goods-list
utility
bliss
demand-list
avail-labor
Value
neighbors-hh
goodlisttemp
goods-list-com-id
avail-labor-com-id
labour
id-good-and-seller
network-partners
temp-list
list-of-binary-goods
network-valuation
prospective-labor-changes
]
Firms-own

```
[
  id
  size-firm
init-budget
  budget
  profit
  neighbors-firm
  Value-firm
  links-firm
blueprinttemp
firmblueprint
firmblueprint-mutated
finalgood
  prod-requirements
  requirements
  supplied-good
  demanded-labor
  labor-demand-firms
  demand-list-firm
interm-demand
  contracted-labor
  stored-labor
  supplied-final-goods
  blueprints-e-profit
  blueprints-e-cost
  list-of-blueprints-e-profit
  list-of-blueprints-e-cost
  list-of-blueprints-e-cost-e-profit
  list-of-activated-blueprints
lista-blueprints
  list-of-blueprints-e-cost-e-profit-nova
  pool-of-solutions
  total-reconversion-cost
];
State-own [
  balance-sheet
]
```

```
to-report hamming-distance [bits1 bits2]
reportlengthremovetrue (map [?1 = ?2] bits1 bits2)
end

to-report difference [l1 l2]
reportfilter [notmember? ? l2] l1
end

to-report position-of-an-element [x lst]
reportitem ((position x lst) -1) lst
end

to-report get-value [key lst]
foreachlst [
if key =first ? [
reportlast ?
   ]
  ]
report0
end

to-report get-values [key pairs]
reportreducesentencemaplast
filter [first ? = key] pairs
end

to-report get-values-2 [labortypeintermgood]
reportmapfirst
filter [last ? =labortype] intermgood
end
to-report frequency [ilst]
reportlengthfilter [? =i] lst
end
to generate-goods-labor
set length-bit-size 10
setgoodslist (list (n-values100 [n-values length-bit-size [one-of[01]]]))
setlista-bens-corrigida (one-ofgoodslist)
```

```
set lista-bens (filter [item0 ? =1] lista-bens-corrigida)
setlaborlist (list (n-values10 [n-values length-bit-size [one-of[01]]]))
setlista-labor-corrigida (one-ofgoodslist)
set lista-trabalho (filter [item0 ? =0] lista-labor-corrigida)
set lista-trabalho-sem-duplicatesremove-duplicates lista-trabalho
set split-points (list (n-valueslengthlista-bens-corrigida [randomlengthone-oflista-bens-
corrigida]));
set split-points one-of split-points
end

to setup
clear-all
set transacted-goods []
set transacted-labor []
set list-of-transactions []
ask firms [set list-of-activated-blueprints []]
  create-households num-households
  create-firms num-firms
askpatches [setpcolorwhite]
  setup-links-hh
  setup-links-firm
asklinks [ setcolorgray ]
  generate-goods-labor
  setup-households
  setup-firms
  create-state 1
ask state [
setshape"person business"
setcolorblack
  ]
ask firms [create-blueprints]
  attribute-blueprints
ask firms[set neighbors-firm find-neighbors-firm]
ask households [set avail-labor one-of labor-inic]
ask households [set avail-labor-com-id fputwho avail-labor]
ask households [set budget init-budget]
ask firms [set budget init-budget]
```

```
  set-up-prices-and-wages
  initial-profits-of-blueprints-accruing-to-firms
reset-ticks
end

to go-once
tick
  order-blueprints
  set-up-production
  agents-place-id-in-front-of-list-proc
  labor-market-sequential
  labor-market-transactions
  firms-complete-blueprints
ask firms [interm-goods-demand]
interm-goods-market-transactions
  crank-out-final-goods
  place-goods-on-market
  households-visit-goods-market
ask households with [[who] oflink-neighbors!= []] [count-number-of-goods-bin]
ask households [calculate-demand-hh]
  filter-households-that-go-to-market
  demand-sequential
  final-goods-market-transactions
  update-wages
  update-prices
  update-profit-blueprint
ask firms [firms-acknowledge-market-results]
  macro-results
  best-technology-adopters
ask firms [import-blueprint-neighbors]
ask firms [set list-of-blueprints-e-cost-e-profit sort-by [last ?1 <last ?2] list-of-blueprints-e-
cost-e-profit]
carefully [mutation-of-firms][]
carefully[increment-lists-after-mutation][]
cost-benefit-analysis-households
ask households [
set avail-labor reducesentence avail-labor
```

```
if budget <= threshold-households *init-budget and [prospective-labor-changes] ofself!= []
and budget >= hamming-distance avail-labor item0first prospective-labor-changes [
let cost hamming-distance avail-labor item0first prospective-labor-changes
set avail-labor 0
set avail-labor firstreducesentence prospective-labor-changes
set budget budget- cost
ask state [set balance-sheet balance-sheet+ cost]
   ]
 ]
ask households [
iflength avail-labor =1 [
set avail-labor reducesentence avail-labor
   ]
 ]
compute-gdp-investment
state-makes-transfers
 final-operations-on-budget-and-size
plotting
end

to setup-households;
ask households [setxyrandom-floatworld-width/2random-ycor]
ask households [setshape"house" ]
ask households [setcolorred]
ask households [setinit-budget random50]
ask households [set bliss n-values length-bit-size [one-of [01]]]
ask households [set bliss fputwho bliss]
ask households [set goods-list n-of ((lengthlista-bens) /2) lista-bens]
ask households [set neighbors-hh find-neighbors-hh]
ask households
  [
set goods-list-com-id fputwho goods-list
  ]
end

to setup-links-hh
if Topology-hh="Complete" [ask households [complete-network-hh]]
```

```
if Topology-hh="Erdos-Renyi" [ask households [Erdos-renyi-hh]]
if Topology-hh="Small-world" [ask households [small-world-network-hh]]
if Topology-hh="Scale-free" [scale-free-network-hh]
end

to setup-links-firm
if Topology-firm ="Complete" [ask firms [complete-network-firm]]
if Topology-firm ="Erdos-Renyi" [ask firms [Erdos-renyi-firm]]
if Topology-firm ="Small-world" [ask firms [small-world-network-firm]]
if Topology-firm ="Scale-free" [scale-free-network-firm]
end

to complete-network-hh; Vem da random network da library de modelos
ask households [
create-links-withother households
set number-links length [who] oflink-neighbors
  ]
end

toErdos-renyi-hh
set number-links int (0.5+ (Erdos-renyi-probability-hh* (num-households * (num-households
-1) /2)))
while [number-links >0] [
askone-of households [
if (countmy-links) < (num-households -1) [
askone-ofother households [
ifnot (link-neighbor?myself) [
create-link-withmyself [
set number-links number-links-1
          ]
        ]
      ]
    ]
  ]
  ]
end
```

```
to small-world-network-hh
letorderedlistsort households
let destinations array:from-listn-valuesnum-households [-1]
set number-links 0

setorderedlistbut-firstorderedlist
letchosenhousehold0
askfirstorderedlist [ create-link-with household chosenhousehold ]
array:set destinations number-links chosenhousehold

while [number-links < (num-households -2)] [
setchosenhousehold ((random (2* (number-links +1))) - number-links) +1
ifchosenhousehold<0 [
setchosenhousehold (array:item destinations (abschosenhousehold))
   ]
set number-links number-links+1
setorderedlistbut-firstorderedlist
askfirstorderedlist [ifchosenhousehold!= [who] offirstorderedlist [create-link-with household
chosenhousehold] ]
array:set destinations number-links chosenhousehold
  ]
end

to make-node-hh [old-node]
foreach [who] of households [
ask household ?
   [
if old-node !=selfand old-node !=nobody [
create-link-with old-node
     ]
   ]
  ]
end

to-report find-partner-hh
let total random-floatsum [countlink-neighbors] of households
let partner nobody
```

```
ask households
  [
letnccountlink-neighbors
if partner =nobody
    [
ifelsenc> total
        [ set partner self ]
        [ set total total-nc ]
    ]
  ]
report partner
end

to scale-free-network-hh
  make-node-hhnobody
  make-node-hh household 0
  make-node-hh find-partner-hh
end

to-report find-neighbors-hh
set connections-hh []
set Value Number-links +1
askother households [set Value 0]
asklink-neighborswith [Value =0][find-near-neighbors-hh]
report connections-hh
end

to find-near-neighbors-hh
set Value [Value] ofmyself-1
set connections-hhlputself connections-hh
asklink-neighborswith [Value =0][find-near-neighbors-hh]
end

to setup-firms;
ask firms [setxyrandom-floatmin-pxcorrandom-ycor]
ask firms [ setshape"factory"]
ask firms [setcolorblue]
```

```
let x random-float1
ask firms [setinit-budget init-bud-firms]
end

to complete-network-firm
ask firms [
create-links-withother firms
set number-links length [who] oflink-neighbors
  ]
end



toErdos-renyi-firm
set number-links int (0.5+ (Erdos-renyi-probability-firm * (num-firms * (num-firms -1) /2)))
while [number-links >0] [
askone-of firms [
if (countmy-links) < (num-firms -1) [
askone-ofother firms [
ifnot (link-neighbor?myself) [
create-link-withmyself [
set number-links number-links-1
           ]
          ]
        ]
      ]
    ]
  ]
end

to small-world-network-firm
letnum-agents num-firms +num-households
letorderedlistsort firms
Setnum-firms (count firms)
let destinations array:from-listn-valuesnum-firms [-1]
set number-links 0
setorderedlistbut-firstorderedlist
letchosenfirmone-of [who] of firms
```

```
askfirstorderedlist [ ifchosenfirm!= [who] offirstorderedlist [create-link-with firm chosenfirm]
]
array:set destinations number-links chosenfirm

while [number-links < (num-firms -2)] [
setchosenfirmnum-households +1+ ((random (2* (number-links +1))) - number-links)
ifchosenfirm<0 [
setchosenfirm (array:item destinations (abschosenfirm))
  ]
set number-links number-links+1
setorderedlistbut-firstorderedlist
askfirstorderedlist [if (chosenfirm!= [who] offirstorderedlistand frequency chosenfirm [who]
of households =0andchosenfirm!=num-agents) [create-link-with firm chosenfirm] ]
array:set destinations number-links chosenfirm
 ]
end

to make-node-firm [old-node]
foreach [who] of firms [
ask firm ?
  [
if old-node !=selfand old-node !=nobody [
create-link-with old-node
    ]
  ]
 ]
end

to-report find-partner-firm
let total random-floatsum [countlink-neighbors] of firms
let partner nobody
ask firms
 [
letnccountlink-neighbors
if partner =nobody
  [
ifelsenc> total
```

```
      [ set partner self ]
      [ set total total-nc ]
    ]
  ]
report partner
end

to scale-free-network-firm
  make-node-firm nobody
  make-node-firm firm num-households
  make-node-firm find-partner-firm
end

to-report find-neighbors-firm
set connections-firm []
set Value-firm Number-links +1
askother firms [set Value-firm 0]
asklink-neighborswith [Value-firm =0][find-near-neighbors-firm]
report connections-firm
end

to find-near-neighbors-firm
set Value-firm [Value-firm] ofmyself-1
if Value-firm >0 [
set connections-firm lputself connections-firm
asklink-neighborswith [Value-firm =0][find-near-neighbors-firm]]
end

to import-blueprint-neighbors
foreach [who] oflink-neighbors [
setblueprinttemp []
setblueprinttemplput [list-of-blueprints-e-cost-e-profit] of firm ? blueprinttemp
setblueprinttempreducesentenceblueprinttemp
  ]
end

to-report valuation-test [x]
```

```
report  (lengthbut-first bliss - hamming-distance x but-first bliss)
end

to-report sort-on-valuation [l]
reportsort-by [valuation-test ?1 < valuation-test ?2] l
end

to calculate-demand-hh
ask households[set demand-list sort-on-valuation list-of-binary-goods]
end

to create-blueprints
set list-of-blueprints []
while [length list-of-blueprints < Initial-tech]
  [
setblueprintermediate []
setblueprintermediatelputone-oflista-bens blueprintermediate
setblueprintermediatelputone-oflista-trabalhoblueprintermediate
setblueprintermediatelputone-of split-points blueprintermediate
set blueprint blueprintermediate
let parent1 item0 blueprint
let parent2 item1 blueprint
let split-point item2 blueprint
setresultadoslist (sentence (sublist parent1 0 split-point)
      (sublist parent2 split-point length parent2))
    (sentence (sublist parent2 0 split-point)
      (sublist parent1 split-point length parent1))
setfinalgood (filter [item0 ? =1] resultados)
setfinalgoodzone-offinalgood
set blueprint lputfinalgoodz blueprint
set list-of-blueprints lput blueprint list-of-blueprints
set list-of-working-blueprints filter [item0 ? !=item3 ?] list-of-blueprints
foreach list-of-working-blueprints[
if frequency item3 ? interm-goods-inic>0 [set list-of-working-blueprints remove ? list-of-
working-blueprints]
    ]
set list-of-working-blueprints remove-duplicates list-of-working-blueprints
```

```
  ]
end

to-report number-of-flops
let counter 0
foreach list-of-blueprints[
ifitem0 ? =item3 ? [
set counter counter+1
   ]
  ]
report counter
end

to-reportinterm-goods-inic
let a 0
setlista-bens-interm-primitivos []
foreach list-of-working-blueprints [
set a item0 ?
set lista-bens-interm-primitivos lput a lista-bens-interm-primitivos
  ]
set lista-bens-interm-primitivos remove-duplicates lista-bens-interm-primitivos
report lista-bens-interm-primitivos
end

to-report labor-inic
let a 0
setlista-labor-types-primitivos []
foreach list-of-working-blueprints [
setaitem1 ?
setlista-labor-types-primitivoslput a lista-labor-types-primitivos
  ]
setlista-labor-types-primitivosremove-duplicateslista-labor-types-primitivos
reportlista-labor-types-primitivos
end

to-report final-goods-inic
set lista-bens-finais-primitivos []
```

```
foreach list-of-working-blueprints [
set lista-bens-finais-primitivos lputitem3 ? lista-bens-finais-primitivos
  ]
set lista-bens-finais-primitivos remove-duplicates lista-bens-finais-primitivos
report lista-bens-finais-primitivos
end

to attribute-blueprints
ask firms [setfirmblueprintn-values max-supply-of-firms [one-of list-of-working-blueprints]]
end

to-report pricing-of-interm-goods
letlista []
repeatlengthinterm-goods-inic [
setlistalput (0) lista
  ]
let lista3 (map [list ?1 ?2] interm-goods-iniclista)
report lista3
end

to-report get-interm-price [good]
report get-value good pricing-of-interm-goods
end

to-report wage-setting
letlista []
repeatlength labor-inic [
setlistalput (2) lista
  ]
let lista3 (map [list ?1 ?2] labor-iniclista)
report lista3
end

to-report get-wage [labortype]
report get-value labortype wage-setting
end
to-report final-good-prices
```

```
letlista []
repeatlength final-goods-inic [
setlistalput (8) lista
  ]
let lista3 (map [list ?1 ?2] final-goods-iniclista)
report lista3
end

to-report get-price [good]
report get-value good final-good-prices
end

to-report profit-of-blueprint [blueprints]
let profit-of-a-blueprint 0
let price-of-final get-value item3 blueprints final-good-prices
let price-of-intermediate get-value item0 blueprints pricing-of-interm-goods
let wage get-value item1 blueprints wage-setting
report price-of-final - price-of-intermediate - wage
end

to-report profit-of-blueprints
set list-of-blueprints-profit []
foreach list-of-working-blueprints [
set list-of-blueprints-profit lput (list ? profit-of-blueprint ?) list-of-blueprints-profit
  ]
report list-of-blueprints-profit
end

to initial-profits-of-blueprints-accruing-to-firms
ask firms [
set blueprints-e-profit []
set blueprints-e-cost []
set list-of-blueprints-e-profit []
set list-of-blueprints-e-cost []
set list-of-blueprints-e-cost-e-profit []

foreachfirmblueprint [
```

```
set blueprints-e-profit (list ? profit-of-blueprint ?)
set list-of-blueprints-e-profit lput blueprints-e-profit list-of-blueprints-e-profit
set blueprints-e-cost (list ? cost-blueprint ?)
set list-of-blueprints-e-cost lput blueprints-e-cost list-of-blueprints-e-cost
set list-of-blueprints-e-cost-e-profit lput (list ? cost-blueprint ? profit-of-blueprint ?) list-of-
blueprints-e-cost-e-profit
    ]
  ]
end

to set-up-prices-and-wages
let list-of-working-blueprints-e-cost-e-profit []
set price-list final-good-prices
set wage-list wage-setting

foreach list-of-working-blueprints [
set list-of-working-blueprints-e-cost-e-profit lput (list ? cost-blueprint ? profit-blueprint ?)
list-of-working-blueprints-e-cost-e-profit
  ]
end

to order-blueprints
ask firms [
set list-of-blueprints-e-cost-e-profit sort-by [ last ?1 >last ?2 ] list-of-blueprints-e-cost-e-profit
let cost 0
letcumul-cost 0
set list-of-activated-blueprints []
while [cumul-cost < (1- Firms-saving-rate) * budget]
foreach [list-of-blueprints-e-cost-e-profit] ofself [
setcumul-cost (cumul-cost +item1 ?)
set list-of-activated-blueprints lput ? list-of-activated-blueprints
      ]
ifcumul-cost =0 [setcumul-cost (1- Firms-saving-rate) * budget ]
    ]
iflength list-of-activated-blueprints >length list-of-blueprints-e-cost-e-profit [
set list-of-activated-blueprints []
foreach list-of-blueprints-e-cost-e-profit [
```

```
set list-of-activated-blueprints lputfirst ? list-of-activated-blueprints
      ]
    ]
  ]
end

to set-up-production
ask firms [
setlista-blueprints []
foreach list-of-activated-blueprints [
setlista-blueprints lput ? lista-blueprints
    ]
set labor-demand-firms labor-demand-firm lista-blueprints
  ]
end
to-report labor-demand-firm [lista]
let temp-list-2 []
foreachlista [
set temp-list-2 lputitem1 ? temp-list-2
  ]
set temp-list-2 fputwho temp-list-2
report temp-list-2
end
to agents-place-id-in-front-of-list-proc
ask households [set avail-labor but-first avail-labor-com-id]
ask households [set avail-labor (list avail-labor)]

ask firms [
let counter []
set demanded-labor []
repeatlength labor-demand-firms [
set counter lputwho counter
    ]
set demanded-labor (map [list ?1 ?2] labor-demand-firms counter)
  ]
end
```

```
to-report demanders-of-labor-x [labor]
let demanders []
let list-of-firms [who] of firms
foreach list-of-firms [
ifmember? labor [labor-demand-firms] of firm ? [
set demanders lput ? demanders
   ]
  ]
report demanders
end

to labor-market-sequential
set unemployed-hh [who] of households
let unmatched-firms [who] of firms
let n n-values1 [?]
set list-of-labor-transactions []
let m []
let list-of-supplied-labor []
let list-of-demanded-labor []
let list-of-demanded-labor-2 []
let labor-market-2 []
set labor-market-3 []

ask firms [
set list-of-demanded-labor lput labor-demand-firms list-of-demanded-labor
  ]
set list-of-demanded-labor reducesentence list-of-demanded-labor
foreach n [
letlabortype ?
foreach unemployed-hh [
let household-2 ?
let labor itemlabortype [[(listfirst avail-labor)] ofturtle household-2] ofturtle household-2
set m lput labor m; para aqui vai a oferta.
let demanders demanders-of-labor-x labor
if demanders != [][
let demander one-of demanders
set list-of-labor-transactions lput (list household-2 labor demander) list-of-labor-transactions
```

```
askturtle demander [set labor-demand-firms remove-item (position labor labor-demand-firms) labor-demand-firms]
set unemployed-hhremove ? unemployed-hh
   ]                         ;
  ]
 ]
foreach m [
set list-of-supplied-labor lput (list ? frequency ? m) list-of-supplied-labor
set list-of-supplied-labor remove-duplicates list-of-supplied-labor
 ]
foreach list-of-demanded-labor [
set list-of-demanded-labor-2 lput (list ? frequency ? list-of-demanded-labor) list-of-demanded-labor-2
 ]
set list-of-demanded-labor-2 remove-duplicates list-of-demanded-labor-2
foreach bring-together-supply-and-demand-com-listas list-of-demanded-labor-2 list-of-supplied-labor [
let demand-supply item0one-ofbut-first ? -item1one-ofbut-first ?
set labor-market-2 (listfirst ? one-ofbut-first ? demand-supply)
set labor-market-3 lput labor-market-2 labor-market-3
 ]
end
to firms-complete-blueprints
ask firms [set stored-labor []]
foreach list-of-labor-transactions [
let j ?
foreach (list firm item2 j) [
ask ? [set stored-labor lputitem1 j stored-labor]
  ]
 ]
end

tointerm-goods-demand
foreach [who] of firms [
leti ?
ask firm i [setinterm-demand []]
foreach [stored-labor] of firm i [
```

```
let j ?
ask firm i [
setinterm-demand lput position-of-an-element j reducesentence [lista-blueprints] ofselfinterm-
demand
      ]
    ]
  ]
end
to crank-out-final-goods
ask firms [
set supplied-final-goods []
let matchbox []
set requirements []
let n-goods n-valueslengthinterm-demand [?]
foreach n-goods [
let good ?
set requirements lput (listitem good interm-demand item good stored-labor) requirements
    ]
foreachlista-blueprints [
leti ?
foreach requirements [
let j ?
ifsublisti02= j [
set supplied-final-goods lputitem3i supplied-final-goods
        ]
      ]
    ]
set supplied-final-goods n-valueslengthinterm-demand [remove-duplicates supplied-final-
goods]
  ]
end

to place-goods-on-market
set supplied-goods []
set list-of-supplied-goods []
ask firms [
```

```
if supplied-final-goods != [] [set list-of-supplied-goods lput [supplied-final-goods] ofself list-
of-supplied-goods]]
set list-of-supplied-goods reducesentencereducesentence list-of-supplied-goods
set supplied-goods remove-duplicates list-of-supplied-goods
end
to-report sellers-of-good-x [good]
let sellers []
let list-of-firms [who] of firms
foreach list-of-firms[
ifmember? good [supplied-final-goods] of firm ? [
set sellers lput ? sellers
  ]
 ]
report sellers
end
to-report number-of-neighbors-with-good-bin
set temp-list []
foreach (listlink-neighbors) [
leti ?
foreach [list-of-binary-goods] ofi[
let j ?
set temp-list lput j temp-list
  ]
 ]
report temp-list
end

to-report goods-own-valuation-list
set temp-list []
foreach list-of-binary-goods [
set temp-list lput valuation-test ? temp-list
 ]
report temp-list
end

to-report get-goods-prices
set temp-list []
```

```
foreach list-of-binary-goods [
set temp-list lput (get-value ? final-good-prices) temp-list
  ]
report temp-list
end

to count-number-of-goods-bin
set network-valuation []
foreach list-of-binary-goods [
set network-valuation map [frequency ? reducesentence number-of-neighbors-with-good-bin]
list-of-binary-goods
  ]
end

to-report perform-composite-list
let ab (map [list ?1 ?2] list-of-binary-goods network-valuation)
letabcsort-by [ item1 ?1 >item1 ?2 ] ab
reportabc
end

to-report perform-composite-list-2-hd
let ab (map [list ?1 ?2] list-of-binary-goods goods-own-valuation-list)
letabcsort-by [ item1 ?1 >item1 ?2 ] ab
reportabc
end

to-report get-prices-of-my-goods
let ab (map [list ?1 ?2] list-of-binary-goods get-goods-prices)
letabcsort-by [ item1 ?1 >item1 ?2 ] ab
reportabc
end

to-report list-sum [list10 list2 list3]
report (map [social-influence * ?1 + (1- social-influence) * ?2 - ?3] list10 list2 list3)
end
to-report perform-total-list
```

```
let ab (map [list ?1 ?2] list-of-binary-goods list-sum network-valuation goods-own-valuation-
list get-goods-prices)
letabcsort-by [ item1 ?1 >item1 ?2 ] ab
reportabc
end

to households-visit-goods-market
ask households [
let list-to-evaluate []
set list-to-evaluate n-ofround (Consumer-awareness *length supplied-goods) supplied-goods
set list-of-binary-goods list-to-evaluate
 ]
end

to filter-households-that-go-to-market
set households-that-go-shopping []
set households-that-dont-shop []
set size-of-shopping-list []
ask households [
let price-vector-of-my-list []
foreach [list-of-binary-goods] ofself [
set price-vector-of-my-list lput get-price-value ? price-vector-of-my-list
set size-of-shopping-list lputlength [list-of-binary-goods] ofself size-of-shopping-list
   ]
ifelse (1- consumer-saving-rate) * budget >max price-vector-of-my-list
   [set households-that-go-shopping lput [who] ofself households-that-go-shopping]
   [set households-that-dont-shop lput [who] ofself households-that-dont-shop]
 ]
set size-of-shopping-list remove-duplicates size-of-shopping-list
end

to demand-sequential
let unmatched-hh households-that-go-shopping
let unmatched-firms [who] of firms
let n n-valuesreducesentence size-of-shopping-list [?]
set list-of-transactions []
let m []
```

```
let list-of-demanded-goods []
set list-of-supplied-goods []
let list-of-supplied-goods-2 []
set goods-market []

ask firms [
if supplied-final-goods != [][
set supplied-final-goods reducesentence supplied-final-goods
set list-of-supplied-goods lput [supplied-final-goods] ofself list-of-supplied-goods
    ]
  ]
set list-of-supplied-goods reducesentence list-of-supplied-goods

foreach n [
letpref ?
foreach unmatched-hh [
let household-2 ?
let good itempref [[list-of-binary-goods] of household household-2] of household household-2
set m lputgood m; para aqui vai a procura.
let sellers sellers-of-good-x good
if sellers != [][
let seller one-of sellers
set list-of-transactions lput (list household-2 good seller) list-of-transactions
askturtle seller [set supplied-final-goods remove-item (position good supplied-final-goods)
supplied-final-goods]
set unmatched-hhremove ? unmatched-hh
      ]                          ;
    ]
  ]
foreach m [
set list-of-demanded-goods lput (list ? frequency ? m) list-of-demanded-goods
set list-of-demanded-goods remove-duplicates list-of-demanded-goods
  ]
foreach list-of-supplied-goods [
set list-of-supplied-goods-2 lput (list ? frequency ? list-of-supplied-goods) list-of-supplied-
goods-2
  ]
```

```
set list-of-supplied-goods-2 remove-duplicates list-of-supplied-goods-2
foreach bring-together-supply-and-demand-com-listas list-of-demanded-goods list-of-
supplied-goods-2 [
let demand-supply item0one-ofbut-first ? -item1one-ofbut-first ?
let market (listfirst ? one-ofbut-first ? demand-supply)
set goods-market lput market goods-market
  ]
end

to-report bring-together-supply-and-demand-com-listas [list1 list2]; procedure social, em que
conjugamos a procura e oferta de bens
let keys remove-duplicates (sentence (mapfirst list1) (mapfirst list2))
let interm-table-1 map [list ? (get-value ? list1 )] keys
let interm-table-2 map [list ? (get-value ? list2 )] keys
let table-value-demand []
let table-value-supply []
foreach interm-table-1 [
set table-value-demand lputitem1 ? table-value-demand
  ]
foreach interm-table-2 [
set table-value-supply lputitem1 ? table-value-supply
  ]
letlista-ainda-vamos-a-meio (map [list ?1 ?2] table-value-demand table-value-supply);
let lista-final (map [list ?1 ?2] keys lista-ainda-vamos-a-meio)
reportlista-final
end
to-report get-wage-value [labortype]
report get-value labortype wage-list
end

to-report get-price-value [good]
report get-value good price-list
end

to-report get-interm-price-value [intermgood]
report get-value intermgood pricing-of-interm-goods
end
```

```
to-report listing-final-goods
set list-of-final-goods []
foreach price-list [
set list-of-final-goods lputfirst ? list-of-final-goods
  ]
report list-of-final-goods
end

to-report listing-labor-types
set labor-list []
foreach wage-list [
set labor-list lputfirst ? labor-list
  ]
report labor-list
end

to update-wages
set wage-update []
set wage-update-2 []
foreach wage-list [
if frequency first ? reducesentence labor-market-3 >0 [
set wage-update lputreplace-item1 ? (item1 ? + get-values first ? labor-market-3) wage-update
    ]
  ]
set wage-list wage-update
foreach wage-list [
ifitem1 ? <0
    [set wage-update-2 lputreplace-item1 ? 0 wage-update-2]
ifitem1 ? >=0
    [set wage-update-2 lput ? wage-update-2]
  ]
set wage-list wage-update-2
end

to update-prices
set price-update []
```

```
set price-update-2 []
foreach price-list [
if frequency first ? list-of-supplied-goods >0 [
set price-update lputreplace-item1 ? (item1 ? + get-values first ? goods-market) price-update
   ]
  ]
set price-list price-update
foreach price-list [
ifitem1 ? <0
   [set price-update-2 lputreplace-item1 ? 0 price-update-2]
ifitem1 ? >=0
   [set price-update-2 lput ? price-update-2]
  ]
set price-list price-update-2
end

to-report profit-blueprint [blueprints]
let price-of-final get-value item3 blueprints price-list
let price-of-intermediate get-value item0 blueprints pricing-of-interm-goods
let wage get-value item1 blueprints wage-list
report price-of-final - price-of-intermediate - wage
end

to-report cost-blueprint [blueprints]
let price-of-intermediate get-value item0 blueprints pricing-of-interm-goods
let wage get-value item1 blueprints wage-list
report price-of-intermediate + wage
end

to update-profit-blueprint
Setlista-com-profit-de-blueprints []
foreach list-of-working-blueprints [
setlista-com-profit-de-blueprints lput (list ? cost-blueprint ? profit-blueprint ?) lista-com-
profit-de-blueprints
  ]
end
to firms-acknowledge-market-results
```

```
foreach [who] of firms [
leti ?
ask firm i [
set list-of-blueprints-e-cost-e-profit-nova []
foreach [list-of-blueprints-e-cost-e-profit] ofself [
set list-of-blueprints-e-cost-e-profit replace-item (position ? list-of-blueprints-e-cost-e-profit)
[list-of-blueprints-e-cost-e-profit] ofself (listfirst ? cost-blueprint first ? profit-blueprint first ?)
      ]
    ]
  ]
end

to labor-market-transactions
foreach list-of-labor-transactions [
ask firm item2 ? [set budget budget- get-wage-value item1 ?]
ask household item0 ? [set budget budget+ get-wage-value item1 ?]
  ]
end

tointerm-goods-market-transactions
ask state [set balance-sheet 0]
foreach [who] of firms [
leti ?
if [interm-demand] of firm i!=0 [
foreach [interm-demand] of firm i [
ask firm i [set budget budget- get-interm-price ?]
ask state [set balance-sheet balance-sheet+ get-interm-price ?]
      ]
    ]
  ]
end

to final-goods-market-transactions
foreach list-of-transactions [
ask firm item2 ? [set budget budget+ get-price-value item1 ?]
ask household item0 ? [set budget budget- get-price-value item1 ?]
  ]
```

```
end


to state-makes-transfers
letnum-agents num-households +num-firms
set public-treasury 0
ask state [set public-treasury balance-sheet]
ask state [set balance-sheet 0]
ask firms [set budget budget+ (public-treasury /num-agents) ]
ask households [set budget budget+ (public-treasury /num-agents)]
end
to-report fitness
let winner-good-price []
let winner-good-valuation []
let winner-blueprint []
let winner-labor-type-wage []
let winner-labor-type-valuation []
let process-winners []

set winner-good-price firstsort-by [last ?1 >last ?2] price-list
set winner-good-valuation firstsort-by [last ?1 >last ?2] goods-market
set winner-blueprint firstsort-by [last ?1 >last ?2] lista-com-profit-de-blueprints
set winner-labor-type-wage firstsort-by [last ?1 >last ?2] wage-list
set winner-labor-type-valuation firstsort-by [last ?1 >last ?2] labor-market-3
set process-winners (list winner-good-price winner-good-valuation winner-blueprint winner-labor-type-wage winner-labor-type-valuation)
report process-winners
end

to-reportgdp-aux-goods [good]
report frequency good reducesentence list-of-transactions * get-price-value good
end

to-reportgdp-aux-labor [labor-type]
report frequency labor-type reducesentence list-of-labor-transactions * get-wage-value labor-type
end
```

```
to macro-results
setgdp-goods 0
setgdp-investment 0
setgdp0
set transacted-goods []
set transacted-labor []
foreach list-of-transactions [
set transacted-goods lputitem1 ? transacted-goods
  ]
foreach list-of-labor-transactions [
set transacted-labor lputitem1 ? transacted-labor
  ]
set transacted-goods remove-duplicates transacted-goods
set transacted-labor remove-duplicates transacted-labor
foreach transacted-goods [
setgdp-goods gdp-goods +gdp-aux-goods ?
  ]
set unemployment-rate (length unemployed-hh/length [who] of households)
end

to best-technology-adopters
let best-adopters 0
set frequency-of-adopters-of-best 0
foreach [who] of firms [
leti ?
if frequency item2 fitness [list-of-blueprints-e-cost-e-profit] of firm i>0 [
set best-adopters best-adopters+1
    ]
  ]
set frequency-of-adopters-of-best (best-adopters /num-firms)
end
to mutate-labor [x]
while [hamming-distance avail-labor x >0]
    [
set avail-labor map [ifelse-value (random-float100.0<0.5) [1- ?] [?]] avail-labor
    ]
```

```
end
to-report produce-goods-from-incomplete-blueprints [incomp-blueprint
ifelseincomp-blueprint != [] [
letinterm-good item0incomp-blueprint
let labor-type item1incomp-blueprint
letcutpointitem2incomp-blueprint
letresultado (list (sentence (sublistinterm-good 0cutpoint)(sublist labor-type cutpointlength
labor-type)))
reportresultado
  ][report0]
end

to-report best-matching-good-for-mutation
let best-final-good []
let best-interm-good []
let list-of-interm []
set best-match []
let best-labor-type []
let list-of-labor-types []
set best-final-good item0item0 fitness
foreach wage-list [
set list-of-labor-types lputitem0 ? list-of-labor-types
  ]
set list-of-intermsort-by [hamming-distance ?1 best-final-good < hamming-distance ?2 best-
final-good] interm-goods-inic
set list-of-labor-types sort-by [hamming-distance ?1 best-final-good < hamming-distance ?2
best-final-good] list-of-labor-types
set best-interm-good first list-of-interm
set best-labor-type first list-of-labor-types
if hamming-distance best-labor-type best-final-good > hamming-distance best-interm-good
best-final-good[
set best-match best-interm-good
  ]
if hamming-distance best-interm-good best-final-good >= hamming-distance best-labor-type
best-final-good [
set best-match best-labor-type
  ]
```

```
report best-match
end
to eliminate-duds
ifelse frequency item3 new-blueprint interm-goods-inic>0 [ ; procedimento para eliminar
duds:
while [frequency item3 new-blueprint interm-goods-inic>0][
set new-blueprint []
set new-blueprint lputitem0one-ofblueprinttemp new-blueprint
set new-blueprint lputitem1one-ofblueprinttemp new-blueprint
set new-blueprint lput (1+randomlengthitem0one-ofblueprinttemp) new-blueprint
set new-blueprint lputreducesentence produce-goods-from-incomplete-blueprints new-
blueprint new-blueprint
        ]
     ]
     []
end

to-report lamarckian-copy-type-2 [old-blueprint]
ask firms [
set new-blueprint []
set new-blueprint-cost-e-profit []
letblueprinttemp-com-lucros []
set reconversion-cost 0

if [who] oflink-neighbors!= [][
setblueprinttemp-com-lucrossort-by [ last ?1 >last ?2 ] reducesentence [list-of-blueprints-e-
cost-e-profit] oflink-neighbors

let winner-blueprint reducesentencebut-lastbut-lastfirstblueprinttemp-com-lucros
set new-blueprint winner-blueprint
   ]
  ]
report new-blueprint
end

to-report darwinian-mutation-1 [old-blueprint]
ask firms [import-blueprint-neighbors]
```

```
ask firms with [lengthblueprinttemp>=2] [
letblueprinttemp-com-lucros []
let price-good 0
let profit-new-bp0
set new-blueprint-cost-e-profit []
set new-blueprint []
setblueprinttemp-com-lucrossort-by [ last ?1 >last ?2 ] blueprinttemp
let lista-provisória []
foreachblueprinttemp-com-lucros [
set lista-provisória lputfirst ? lista-provisória
  ]
setblueprinttemp lista-provisória; base de blueprints dos vizinhos.
set new-blueprint lputitem0firstblueprinttemp new-blueprint
set new-blueprint lputitem1lastblueprinttemp new-blueprint
letcuttingpointbut-firstn-valueslengthitem0firstblueprinttemp [?]
set new-blueprint lputone-ofcuttingpoint new-blueprint
set new-blueprint lputreducesentence produce-goods-from-incomplete-blueprints new-
blueprint new-blueprint
    eliminate-duds
  ]
report new-blueprint
end

to-report darwinian-mutation-2 [old-blueprint]
set new-blueprint old-blueprint
set new-blueprint-cost-e-profit []
set reconversion-cost 0
set best-match best-matching-good-for-mutation
ifitem0 best-match =1 [
set new-blueprint replace-item0 old-blueprint best-match
set new-blueprint replace-item2 old-blueprint round (length best-match *0.5+randomlength
best-match *0.5)
set new-blueprint remove-item3 new-blueprint
  ]
ifitem0 best-match =0 [
set new-blueprint replace-item1 old-blueprint best-match
```

```
set new-blueprint replace-item2 old-blueprint round (length best-match *0.5-randomlength
best-match *0.5)
set new-blueprint remove-item3 new-blueprint
   ]
set new-blueprint lputreducesentence produce-goods-from-incomplete-blueprints new-
blueprint new-blueprint
   eliminate-duds

report new-blueprint
end

to-report darwinian-mutation-3 [old-blueprint]
set new-blueprint []
set new-blueprint-cost-e-profit []
set reconversion-cost 0
let cost-new-bp0
let profit-new-bp0
let choice1 item0 old-blueprint
let choice2 item1 old-blueprint
let choice3 item2 old-blueprint
let random-choice one-of (list choice2 choice3)
let random-bit 0
if random-choice = choice2 [
letlista-possiveln-valueslength choice1 [?]
setlista-possivelbut-firstlista-possivel
set random-bit one-oflista-possivel
let random-bit2 item random-bit choice2
set choice2 replace-item random-bit choice2 (1- random-bit2)
set new-blueprint lput choice1 new-blueprint
set new-blueprint lput choice2 new-blueprint
set new-blueprint lput choice3 new-blueprint
  ];
if random-choice = choice3 [
letlista-possiveln-valueslength choice2 [?]
setlista-possivelremove choice3 lista-possivel
setlista-possivelbut-firstlista-possivel
set choice3 one-oflista-possivel
```

```
set new-blueprint lput choice1 new-blueprint
set new-blueprint lput choice2 new-blueprint
set new-blueprint lput choice3 new-blueprint
  ]
set new-blueprint lputreducesentence produce-goods-from-incomplete-blueprints new-
blueprint new-blueprint
  eliminate-duds
report new-blueprint
end

to-report darwinian-mutation-4-directed-search [old-blueprint]
let good-with-best-sales item0item0 fitness
let resulting-good []
set warehouse-new-blueprints []
set reconversion-cost 0
set new-blueprint old-blueprint
set new-blueprint-cost-e-profit []
let incomplete-blueprint []
set incomplete-blueprint lputone-ofinterm-goods-inic incomplete-blueprint
set incomplete-blueprint lputone-of listing-labor-types incomplete-blueprint
letcuttingpointbut-firstn-valueslengthone-ofinterm-goods-inic [?]
set incomplete-blueprint lputone-ofcuttingpoint incomplete-blueprint
set resulting-good reducesentence produce-goods-from-incomplete-blueprints incomplete-
blueprint
set new-blueprint lput resulting-good incomplete-blueprint
ifelseitem3 new-blueprint != good-with-best-sales [ ; procedimento para eliminar duds:
while [item3 new-blueprint != good-with-best-sales][
set new-blueprint []
set new-blueprint lputone-ofinterm-goods-inic new-blueprint
set new-blueprint lputone-of listing-labor-types new-blueprint
setcuttingpointbut-firstn-valueslengthone-ofinterm-goods-inic [?]
set new-blueprint lputone-ofcuttingpoint new-blueprint
set new-blueprint lputreducesentence produce-goods-from-incomplete-blueprints new-
blueprint new-blueprint
      ]
  ]  [];
  eliminate-duds
```

```
report new-blueprint
end
to-report replace-blueprints [old-blueprint]
set pool-of-solutions []
let solution []
ifLamarckiana [
set pool-of-solutions lamarckian-copy-type-2 old-blueprint
set solution pool-of-solutions
set reconversion-cost hamming-distance reducesentencereducesentence solution
reducesentencereducesentence old-blueprint
set solution (list solution cost-blueprint solution profit-blueprint solution reconversion-cost)
  ]
ifDarw.+Lam. [
set pool-of-solutions lput Lamarckian-copy-type-2 old-blueprint pool-of-solutions
set pool-of-solutions lput darwinian-mutation-1 old-blueprint pool-of-solutions
set pool-of-solutions lput darwinian-mutation-2 old-blueprint pool-of-solutions
set pool-of-solutions lput darwinian-mutation-3 old-blueprint pool-of-solutions
set pool-of-solutions lput darwinian-mutation-4-directed-search old-blueprint pool-of-
solutions
let weights read-from-string Innovation-types
let cum reduce [
lput (?2 + (ifelse-value (empty? ?1) [0] [last ?1])) ?1] (fput [] weights)
let x random-floatsum weights
let j  -1
let found false
while [(not found) and (j < (length cum))]
   [
set j (j +1)
if (x <=item j cum) [set found true]
   ]
set solution item j pool-of-solutions

set mean-price 0
set mean-wage 0
let list-of-prices-larger-than-0 []
let list-of-wages-larger-than-0 []
foreach price-list [
```

```
ifitem1 ? >0
    [set list-of-prices-larger-than-0 lputitem1 ? list-of-prices-larger-than-0]
  ]
foreach wage-list [
ifitem1 ? >0
    [set list-of-wages-larger-than-0 lputitem1 ? list-of-wages-larger-than-0]
  ]
ifelse list-of-prices-larger-than-0 !=[][
set mean-price mean list-of-prices-larger-than-0]
  [set mean-price 0]

ifelse list-of-wages-larger-than-0 != [][
set mean-wage mean list-of-wages-larger-than-0]
  [set mean-wage 0]
set reconversion-cost hamming-distance reducesentencereducesentence solution
reducesentencereducesentence old-blueprint

let profit-new-bp0
let cost-new-bp0
let price-new-good 0
let wage-new-labor 0
ifelse frequency solution list-of-working-blueprints =1 [
set solution (list solution cost-blueprint solution profit-blueprint solution reconversion-cost)
  ]
  [
if frequency item3 solution listing-final-goods =0and frequency item1 solution listing-labor-
types =1
    [
set cost-new-bp cost-blueprint solution
set price-new-good mean-price
set profit-new-bp mean-price - cost-new-bp
    ]

if frequency item3 solution listing-final-goods =0and frequency item1 solution listing-labor-
types =0
    [
set wage-new-labor mean-wage
```

```
set price-new-good mean-price
set cost-new-bp get-interm-price-value item0 solution + wage-new-labor
set profit-new-bp price-new-good - cost-new-bp
    ]

if frequency item3 solution listing-final-goods =1and frequency item1 solution listing-labor-
types =0
    [
set wage-new-labor mean-wage
set cost-new-bp get-interm-price-value item0 solution + wage-new-labor
set profit-new-bp get-price-value item3 solution - cost-new-bp
    ]
if frequency item3 solution listing-final-goods =1and frequency item1 solution listing-labor-
types =1
    [
set cost-new-bp cost-blueprint solution
set profit-new-bp profit-blueprint solution
    ]
set solution (list solution cost-new-bp profit-new-bp reconversion-cost)
   ]
  ]
report solution
end
to cost-benefit-analysis-households; Análise do mercado:
ask households [
set prospective-labor-changes []
set avail-labor reducesentence avail-labor
foreach wage-list [
if (get-wage-value item0 ? - get-wage-value avail-labor) > (hamming-distance avail-labor
item0 ?) [
set prospective-labor-changes lput ? prospective-labor-changes
    ]
   ]
set prospective-labor-changes sort-by [ last ?1 >last ?2 ] prospective-labor-changes
  ]
end
```

```
to mutation-of-firms
ask firms with [list-of-blueprints-e-cost-e-profit != []] [
let list-of-blueprints-e-cost-e-profit-mutated []
set total-reconversion-cost0; vai acumulando o custo total de reconversão
ifelse budget <= threshold-firms *init-budget [ ; o "trigger" da reconversão.
while [total-reconversion-cost < firms-saving-rate * budget] [; define a condição de paragem
foreach list-of-blueprints-e-cost-e-profit [
ifitem2 replace-blueprints item0 ? -item2 ? >item3 replace-blueprints item0 ?           [set list-of-blueprints-e-cost-e-profit-mutated replace-itemposition ? list-of-blueprints-e-cost-e-profit list-of-blueprints-e-cost-e-profitbut-last replace-blueprints item0 ?
set total-reconversion-cost total-reconversion-cost+last replace-blueprints item0 ?
set budget budget- total-reconversion-cost
iflength list-of-blueprints-e-cost-e-profit-mutated >=length list-of-blueprints-e-cost-e-profit [stop]
          ]
         ]
      ]
set budget budget- total-reconversion-cost
if total-reconversion-cost =0 [set total-reconversion-cost Firms-saving-rate * budget ]
   ] [set list-of-blueprints-e-cost-e-profit-mutated list-of-blueprints-e-cost-e-profit]
set list-of-blueprints-e-cost-e-profit list-of-blueprints-e-cost-e-profit-mutated
  ]
end

to increment-lists-after-mutation
ask firms [
set provisory-list-of-blueprints []
set provisory-list-of-blueprints lput list-of-blueprints-e-cost-e-profit provisory-list-of-blueprints
  ]
set provisory-list-of-blueprints remove-duplicatesreducesentence provisory-list-of-blueprints

ifDarw.+Lam. [
foreach provisory-list-of-blueprints [
if frequency first ? list-of-working-blueprints =0 [
set list-of-working-blueprints lputitem0 ? list-of-working-blueprints
   ]
```

```
  ]
foreach provisory-list-of-blueprints [
if frequency ? lista-com-profit-de-blueprints =0 [
setlista-com-profit-de-blueprints lput ? lista-com-profit-de-blueprints
   ]
  ]

foreach provisory-list-of-blueprints [
if frequency item3first ? listing-final-goods =0 [
set price-list lput (listitem3first ? mean-price) price-list
set list-of-final-goods lputitem3first ? list-of-final-goods
   ]
  ]
foreach provisory-list-of-blueprints [
if frequency item1first ? listing-labor-types =0 [
set wage-list lput (listitem1first ? mean-wage) wage-list
set labor-list lputitem1first ? labor-list
   ]
  ]
]
end

to compute-gdp-investment
setgdp-investment 0
foreach [who] of firms [
setgdp-investment gdp-investment + [total-reconversion-cost] of firm ?
  ]
setgdpgdp-goods +gdp-investment
ask state [
set balance-sheet balance-sheet+gdp-investment
  ]
end

to final-operations-on-budget-and-size
ask firms [
if budget <0 [set budget 0]
  ]
```

```
ask households [
if budget <0 [set budget 0]
  ]
ask firms [set size-firm round [budget] ofself]
ask firms [setsize (size-firm -1) *1.4/24+0.6 ]
end

to-reportneg-log-prob [p q]
report (-log (p / q) 2)
end
to-report calculate-entropy-sold-goods
letdist-total 0
letdist-entropy 0
let this-neg-log-prob0
letfreq0
let transactions []
foreach list-of-transactions [
set transactions lputitem1 ? transactions
  ]
lettamanholength transactions
iftamanho=0ortamanho=1 [settamanho2]
foreach transacted-goods
  [
setfreq frequency ? transactions
set this-neg-log-probneg-log-probfreqtamanho

setdist-entropy dist-entropy + this-neg-log-prob
]
report (precisiondist-entropy3/ (tamanho *log (tamanho) 2))
end

to-report calculate-entropy-activated-blueprints
letdist-total 0
letdist-entropy 0
let this-neg-log-prob0
letfreq0
let active-blueprints []
```

```
lettamanho0

ask firms [
ifelse list-of-activated-blueprints !=0[
foreach list-of-activated-blueprints [
set active-blueprints lput ? active-blueprints
settamanholength active-blueprints
     ]
   ]
   [settamanho2]
  ]
foreachremove-duplicates active-blueprints
 [
setfreq frequency ? active-blueprints
set this-neg-log-probneg-log-probfreqtamanho
setdist-entropy dist-entropy + this-neg-log-prob
]
report (precisiondist-entropy3/ (tamanho *log (tamanho) 2))
end
to-reportavg-path-length-firms; reporter que retorna a meanpathlength das duas redes: tem um output do lado de fora.
nw:set-context firms links
reportnw:mean-path-length
end

to-reportavg-path-length-hh
nw:set-context households links
reportnw:mean-path-length
end

to-report global-clustering-coefficient-firms
let closed-triplets sum [ nw:clustering-coefficient*countmy-links* (countmy-links-1) ] of firms
let triplets sum [ countmy-links* (countmy-links-1) ] of firms
report closed-triplets / triplets
end
```

```
to-report global-clustering-coefficient-hh
let closed-triplets sum [ nw:clustering-coefficient*countmy-links* (countmy-links-1) ] of
households
let triplets sum [ countmy-links* (countmy-links-1) ] of households
report closed-triplets / triplets
end

to-report correlation-degree-budget
lettblstats:newtable
foreach [who] of firms [
ask firm ?[
stats:addtbl (listcountlink-neighbors budget)
   ]
  ]
letcorrelstats:correlationtbl
reportitem0item1correl
end

to-report correlation-betweenness-budget
lettblstats:newtable
nw:set-context firms links
ask firms [
stats:addtbl (listnw:betweenness-centrality budget)
  ]
letcorrelstats:correlationtbl
reportitem0item1correl
end

to-report correlation-eigenvector-budget
lettblstats:newtable
nw:set-context firms links
ask firms [
stats:addtbl (listnw:eigenvector-centrality budget)
  ]
letcorrelstats:correlationtbl
reportitem0item1correl
end
```

```
to-report correlation-closeness-budget
lettblstats:newtable
nw:set-context firms links
ask firms [
stats:addtbl (listnw:closeness-centrality budget)
  ]
letcorrelstats:correlationtbl
reportitem0item1correl
end

to-report correlation-path-length-freqcy-adopters-best
lettblstats:newtable
nw:set-context firms links
stats:addtbl (listavg-path-length-firms frequency-of-adopters-of-best)
letcorrelstats:correlationtbl
reportitem0item1correl
end

to plotting
set-current-plot"Unemployment Rate"
plotxyticks unemployment-rate
set-current-plot"GDP"
set-current-plot-pen"Consumption"
plotxyticksgdp-goods
set-current-plot-pen"Investment"
plotxyticksgdp-investment
set-current-plot-pen"GDP"
plotxyticksgdp
set-current-plot"Blueprints, final goods, labor types of the economy"
set-current-plot-pen"Traded goods"
plotxytickslength transacted-goods
set-current-plot-pen"Traded Labor"
plotxytickslength transacted-labor
set-current-plot-pen"Tech. solutions"
plotxytickslength list-of-working-blueprints
set-current-plot"Diffusion of technological solutions"
```

```
plotxyticks frequency-of-adopters-of-best
set-current-plot"Entropy of the economy"
set-current-plot-pen"Goods entropy"
plotxyticks calculate-entropy-sold-goods
set-current-plot-pen"Blueprints entropy"
plotxyticks calculate-entropy-activated-blueprints
let max-budget max [budget] of firms
set-current-plot"Firm size distribution"
plot-pen-reset
set-plot-x-range1 (max-budget +1)
histogramsort [budget] of firms
end
```

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“This is not the end, this is not even the beginning of the end, this is just perhaps the end of the beginning.”

Winston S. Churchill