

FACULDADE DE ECONOMIA

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Modelling Non-Scale Effects of North-South Trade on Economic Growth and on Wage Inequality

Doctoral Thesis of

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BIOGRAPHICAL NOTE

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In spite of all these contributions, I remain solely responsible for this thesis, so that its limitations, imperfections, and possible errors, are my exclusive responsibility.

ABSTRACT

We present a dynamic, general equilibrium model of North-South technological knowledge diffusion by imitation, with endogenous scale-invariant growth. The aim is to analyse the non-scale effects of technological knowledge diffusion through international trade on economic growth and on inter and intra-country wage inequality.

The analysis is conducted sequentially under three North-South trade regimes: (i) without trade effects; (ii) trade of intermediate goods alone; and (iii) trade of intermediate and final goods. By North and South we mean two countries that differ in levels of exogenous productivity non-trade related, stocks of human capital and R&D capacity.

Growth depends on the accumulation of two types of human capital: versatile (school intensive, more productive and more abundant in the North) and specialised (on-the-job-training intensive). Growth is also driven by technological advances in the quality of available intermediate goods – Schumpeterian R&D –, regardless of the origin (innovation or imitation). Imitation is only feasible in the South once a threshold distance to the technological knowledge frontier has been attained.

If (i) the impact of openness on imitation is strong, (ii) the marginal cost of production in the South is low and (iii) international intellectual property rights protection is weak, then the world steady state growth is higher under international trade than in the autarkic North.

By allowing immediate international access of the state-of-the-art intermediate goods – through either imitation or trade –, international trade affects at once the productive structure in the South, bringing about partial convergence to the Northern structure and prices (level effect).

In addition, technological knowledge diffusion induces dynamic and steady state effects through the price channel, by which the specific types of technological knowledge and of human capital influence the direction of technological progress, economic growth and wage inequality. The paths of inter and intra-country wage inequality depend on the magnitude of the level effect and on the technology of production, of R&D and of human capital accumulation. In particular, a Schumpeterian dynamic equivalent to the static factor price equalisation result occurs.

In what concerns free trade specialisation patterns, it is shown that countries capture larger shares of world production and trade in final goods that use more intensively their abundant type of human capital.

As an extension of the framework, we analyse the timing and implications of the world diffusion of a new general purpose technology, which induces both level and growth effects. When it arises first in the production of one type of final goods in the North, the general purpose technology temporarily increases intra and inter-country wage inequality.

Keywords: North-South; Technological Knowledge Diffusion; International Trade; R&D; Human Capital; Endogenous Economic Growth; Wage Inequality; General Purpose Technology; Transitional Dynamics; Sensitivity Analysis.

JEL classification: F12, F16, F43, J24, J31, O31, O33, O34, O38, O47.

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LIST OF ABBREVIATION

BG: Balanced Growth;

CA: Comparative advantage;

CES: Constant elasticity of substitution;

FG: Final good;

FOC: First order condition;

GPT: General purpose technology;

HC: Human capital;

HOS: Heckscher-Ohlin-Samuelson;

IG: Intermediate good;

IPR: Intellectual property rights;

IT: International trade;

MC: Marginal cost;

OJT: On-the-job-training;

PTFP: Physical total factor productivity;

R&D: Research and development;

SH: Specialised human capital;

ST: Specialised technology;

TFP: Total factor productivity;

TK: Technological knowledge;

TRIPRs: Trade related intellectual property rights;

VH: Versatile human capital;

VT: Versatile technology.

CHAPTER 1

INTRODUCTION

We develop a dynamic, general equilibrium model of North-South technological knowledge diffusion by imitation, with scale-invariant endogenous growth and human capital accumulation. The main objective is to analyse the non-scale effects of technological knowledge diffusion through international trade, on economic growth and on inter and intra-country wage inequality.

International trade and mobility of factors of production are the main vehicles of technological knowledge transfer in the world. In our case, the factors of production are human capital, intermediate goods (IGs), and designs to produce intermediate goods. In the tradition of international trade theory, we assume that there are no North-South movements of assets – human capital and shares of firms that own the designs –, thus focusing on international trade of intermediate goods as the vehicle of technological knowledge transfer.

The analysis is conducted sequentially under three North-South trade regimes: (i) without trade effects; (ii) trade of intermediate goods alone; and (iii) trade of intermediate and final goods. Although with a role limited to more traditional static level effects on growth and wage inequality, the addition of trade in final goods enables the analysis of the mechanisms through which the driving forces of economic growth and technological knowledge diffusion influence the pattern of specialisation.

By North and South we mean two stylised countries that operate in the same economic environment, but that differ in the following features: (i) exogenous levels of productivity directly related with the quality of domestic institutions; (ii) initial human capital stocks; (iii) technological knowledge stocks; (iv) Research and Development (R&D) capacity; and (v) marginal cost of production of final goods.

The first feature relates to domestic institutions which affect the aggregate level of productivity, like tax laws, property rights protection and government services. We consider these institutions to be more advanced in the North; *i.e.*, the North is more developed.

As to the second feature, we assume that the initial levels of human capital are lower in the South.

The third feature refers to the aggregate domestic quality indexes measuring technological knowledge, which, in line with Bernard and Jones (1996a, b), Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999), among others, are higher in the North.

As to R&D, the North has the capacity to innovate – Schumpeterian R&D, aimed at improving the quality of intermediate goods, as in Aghion and Howitt (1992) – whereas the South's capacity is limited to the imitation of the North's innovations, as in the seminal models of Grossman and Helpman (1991a, chaps. 11-12).

Concerning the fifth distinctive feature, we assume that the constant marginal cost of production of final goods is higher in the North than in the South. Following Grossman and Helpman (1991a, chaps. 11-12) and Connolly (2003), among others, this is required for imitation by the South to take place under international trade. Otherwise, the Northern firms would always under-price the Southern ones, and trade – which must be balanced due to the assumed inexistence of international capital flows and income transfers – would be unfeasible.[1]

The differences in levels of development between North and South are assumed to have historical roots that are reflected in current institutional characteristics. Our main concern is not to explain these differences, but rather to take them as given at time zero and analyse the subsequent path of both economies under the three international trade regimes.

In terms of traditional country groupings in development economics, an appropriate taxonomy for our North and South countries would be developed *versus* developing, rather than developed *versus* underdeveloped. The South should not be too backward, otherwise it would lack the capacity of benefiting from the diffusion of technological knowledge through imitation and trade of intermediate goods – as considered in Helpman (1993).

Our general equilibrium model collects elements from various components, all closely related to the endogenous growth literature. Next, we explore those components, showing how each one fits our modelling objectives, and how we add value to the

[1] In our framework, transfers could arise from human capital mobility, and international capital flows would result from the exchange of assets of profitable firms, which are the ones that have the exclusive use of the designs for the production of intermediate goods. Both are excluded by our assumption of North-South immobility of these factors, which also implies that interest rates and wages are determined domestically.

existing literature, not only by bringing together the different elements, but also by improving on some of them.

Technological knowledge progress in models of endogenous growth

In the literature of endogenous growth, R&D activity is essential to technological knowledge progress. It is carried out or supported by private firms in search for profits, operating in competitive markets. Empirical evidence of this growth mechanism has been shown in, for example, Griliches and Lichtenberg (1984), Lichtenberg (1993) and Coe and Helpman (1995). At the theoretical level, the first-generation of comprehensive, well-articulated general equilibrium growth models based on R&D that seek to explore the role of technological knowledge change in the economic growth process, are centred on two types of R&D processes – horizontal and vertical.

In the first one, R&D is directed at developing new horizontally differentiated goods, an approach followed in prominent works by Romer (1986, 1987, 1990b), Rivera-Batiz and Romer (1991), Grossman and Helpman (1990, 1991a, chap. 3, 1991d) and Barro and Sala-i-Martin (2004, chap. 6). Since there are no quality improvements, no good ever becomes obsolete. Firms that become producers remain leaders from then on without supporting further R&D effort, since they are granted a patent that lasts forever.

In vertical process, R&D is instead directed at developing new vertically differentiated qualities of each good, an approach that was first developed by Segerstrom *et al.* (1990), Grossman and Helpman (1991a, chap. 4, b, c), and Aghion and Howitt (1992). The resulting models are called Schumpeterian, or quality ladder models, since, appealing to the Schumpeterian concept of creative destruction, it is assumed that the leadership of the firms that use the state-of-the-art qualities is only temporary – permanently subject to destruction by new qualities resulting from successful R&D.

The main advantage of horizontal R&D models is their analytical tractability. However, the vertical approach is more appropriate to study the effects of competition between leaders and followers on the process of economic growth – *e.g.*, Gancia and Zilibotti (2003). Since international trade enhances competition between producers of the same good, the Schumpeterian approach is more appropriate for our purposes.

Most vertical R&D models assume that, since all firms in each industry have access to the same R&D technology, once a firm achieves success and becomes a temporary monopoly, it does not support new R&D directed at improving its own product. Despite the coherence of the argument, the implication that leader firms do not

invest in R&D is counterfactual. Thus, some works came to assume that leader firms have R&D cost advantage over challengers, and/or that there are diminishing returns in R&D effort at the firm level, implying that they devote resources to R&D as well – *e.g.*, Segerstrom and Zolnierek (1999) and Segerstrom (2004). Others, such as Dinopoulos and Syropoulos (2001, 2004) and Sener (2003), present models where incumbents apply resources in rent-protecting activities, which increase the expected duration of their monopoly power. Aghion *et al.* (2001), for example, find that the Schumpeterian effect of product market competition is likely to be outweighed by the incentives for firms to carry out R&D in order to escape competition. In the extreme Schumpeterian case, Thompson and Waldo (1994) present a model with no entry, where R&D is conducted exclusively by the monopolistic firms.

Since our focus is not to explain the process of R&D, but rather to analyse the growth and wage inequality effects of its North-South diffusion, we choose the simplifying assumptions that there are constant returns to R&D expenditures – *e.g.*, Romer (1993) – and that only follower firms support the cost of R&D. Moreover, the empirical evidence on returns to scale of R&D efforts at the firm level seems to be inconclusive – *e.g.*, Bound *et al.* (1984), Kortum (1993) and Thompson (1996).

Based on the technology of R&D activities, Rivera-Batiz and Romer (1991) propose a different classification for the R&D growth models. When the inputs to R&D are labour (or human capital) and the level of technological knowledge, the model is called knowledge-driven – *e.g.*, Grossman and Helpman (1990, 1991a, chap. 3) and Romer (1990b). When R&D technology uses the same inputs and in the same proportion as aggregate output or, equivalently, if aggregate output itself is the input to R&D, the model is called lab-equipment – *e.g.*, Rivera-Batiz and Romer (1991) and Barro and Sala-i-Martin (2004, chaps. 6-7). Our choice is a lab-equipment model, since postulating a new production function would merely complicate the algebra without altering the relevant (for our purposes) qualitative results – *e.g.*, Papageorgiou (2002).

When integrated in endogenous growth models, R&D activities – either horizontal or vertical, knowledge-driven or lab-equipment – results in technological knowledge progress, which, in turn, is the primary determinant of growth. Two major characteristics of technological knowledge are essential for its role as an endogenous engine of growth – non-rivalry and partial non-excludability. Technological knowledge is non-rival in the sense that the marginal costs for its use by an additional firm are negligible; and it is

partially non-excludable since the returns to private investment in its production are partly private and partly public. That is, on one hand innovations are proprietary – private investment results in a temporary monopoly, if some protection (usually in the form of patents) exists. On the other hand, innovations add to the stock of public technological knowledge – technological knowledge spillovers.

In our case, endogenous technological knowledge progress takes the form of innovations in the North and to imitations in the South. Thus, non-rivalry and partial non-excludability apply to innovations as well as imitations, since they are domestically protected in the North and South, respectively.

Our framework can also accommodate exogenous innovations of the general purpose technology (GPT) type. The literature on general purpose technology – defined as innovations that have large, extensive and prolonged impacts on the economy, such as steam-engine, electricity and computers – begins with Bresnahan and Trajtenberg (1995).

The effects of the introduction of a new GPT have been typically studied in a closed-economy framework with a simplified productive structure characterised by one aggregate good – *e.g.*, Helpman and Trajtenberg (1998). This simplified structure does not allow the study of the diffusion of a GPT to other countries with different levels of development, and of its effects on world economic growth and on intra and inter-country wage inequality – issues that are in the centre of our purposes. Motivated by such issues, we broaden the scope of this literature by considering the North-South implications of the diffusion of a GPT under international trade. Since our economic structure is richer, we are also able to study intra and inter-country asymmetric impacts caused by different speeds of adoption of the GPT in the various industries.

North-South technological knowledge diffusion in models of endogenous growth

The works of Coe and Helpman (1995), Coe *et al.* (1997), Helpman and Hoffmaister (1997) and Eaton and Kortum (1999, 2001), among others, reveal that only a small number of developed countries account for most of the innovations in the world. This could suggest that the standard R&D models are only applicable to the North and not to the South. However, world productivity data reveals that the benefits of innovative R&D are much more evenly distributed in the world than expenditures on innovative R&D – *e.g.*, Coe and Helpman (1995) –, strongly suggesting that technological knowledge is diffused internationally. Thus, focusing exclusively on domestic

technological knowledge progress when the analysis considers more than one country is not appropriate.[2]

We need to consider, then, extensions of the R&D component of the standard Schumpeterian growth model so that it can be applicable to countries that innovate as well as those that imitate. We look at the North, where leading technological knowledge is created, but also at the South, where it is diffusing, and analyse in which ways and under which international trade regimes this imitation process feeds back and affects the original source of technological knowledge.

When present, under international trade, the dynamic feedback from imitation to innovation may harm the profits (and, thus, incentives to) innovative R&D. That is, in addition to the closed economy Schumpeterian creative destruction effect – challenging innovations destroy the profits of the current state-of-the-art ones –, technological knowledge diffusion with trade may bring about further business stealing – challenging imitations, coupled with lower marginal costs in the South, are traded internationally. Since diminished incentives to innovative R&D lower the rate of economic growth, these potential negative effects must be weighted against the positive static and dynamic effects of international technological knowledge diffusion with trade. We regard this as crucial in assessing the issue of whether countries – North and South – benefit from or are harmed by the commercial relationship.

The available literature considers, in some way or another, extensions of R&D endogenous growth models that are applicable to international trade and/or diffusion of technological knowledge – *e.g.*, Aghion and Howitt (1998a, chap. 11).

An important strand of that literature focuses on the effects of North-South international trade when it leads to Northern specialisation in industries that exhibit positive spillovers and Southern specialisation in industries lacking such positive externalities. This is primarily the case of models of international trade with learning-by-doing – *e.g.*, Stokey (1988),[3] Young (1991) and Lucas (1993). In these models, Southern countries experience negative dynamic effects because international trade leads to a Southern specialisation in goods in which learning-by-doing is exhausted in such a way that it could outweigh the static gains from trading with developed countries. Since each

[2] In line with this position, Stokey (1996) admits that the most important benefit for Southern countries is the transfer of technological knowledge, in contrast with her own model, where integration produces no technological knowledge spillovers.

[3] The seminal model of Stokey (1988) considers specialisation in a traditional sector with no learning-by-doing *versus* specialisation in industries with learning-by-doing, but does not consider North-South trade.

country's stock of technological knowledge depends only on its own range of production, there is no link between domestic and foreign stocks. Therefore, neither North-South technological knowledge diffusion nor the South-North feedback effects are considered by these models.

North-South diffusion is also absent in the open-economy models of Romer (1990b), and Rivera-Batiz and Romer (1991), even when international technological knowledge spillovers are present. In fact, these models derive important effects of the flows of goods and technological knowledge on economic growth and are very helpful in identifying the implications of knowledge-driven *versus* lab-equipment R&D technology – as clearly explained by Aghion and Howitt (1998a, chap. 11). However, they apply to international relations between developed countries that can benefit from each other's stocks of technological knowledge; *i.e.*, North-North in our taxonomy.

Bearing in mind our objectives, it is then useful to emphasise the models that consider North-South diffusion. Technological knowledge can be diffused internationally in various any ways, namely through:

– foreign direct investment, introduced in the literature by Vernon (1966), with endogenous growth formulations by Helpman (1999), Hejazi and Safarian (1999), Baldwin *et al.* (1999), Xu (2000) and Carr *et al.* (2001);

– international labour mobility, in the tradition of Bhagwati and Hamada (1974), and included in endogenous growth models by Galor and Stark (1994), Barro and Sala-i-Martin (2004, chap. 9) and Shea and Woodfield (1996);

– communication patterns, *e.g.*, Keller (2002a, b, 2003);

– imitation, after the seminal work of Krugman (1979), and prominently studied by Grossman and Helpman (1991a, b, c, d) in the context of endogenous growth, with important additions by Dinopoulos *et al.* (1993), Barro and Sala-i-Martin (1997) and Glass (1997).

The first two channels rely on international mobility of physical and human capital assets, which, as stated above, we are precluding in our analysis. As for the communication patterns channel, it is of little interest in our framework, since it is largely independent of international trade. That is not the case of imitation – the channel under our focus –, which is considerably enhanced by the access to state-of-the-art goods permitted by international trade. When successful, imitation allows for the diffusion of technological knowledge embodied in a good, as the imitator reverse-engineers that

good. Since, in the absence of intellectual property rights (IPRs) protection, imitative R&D is typically cheaper than innovative R&D, the successful imitator can return to international markets and under-price the original innovator.[4]

Our lab-equipment model of R&D applies to the innovative North, and is also conveniently adapted to the particularities of imitative activities in the South. Both equilibrium probabilities of successful imitation and innovation are endogenously determined, based on the maximisation of expected discounted profits. In both countries as well, R&D benefits from an autonomous positive learning effect of accumulated public technological knowledge from past successful research, as in Grossman and Helpman (1991a, chap. 12). The main specificities that we introduce in the Southern probability of successful R&D are imitation capacity and an advantage of moderate backwardness, which, together, are capable of inducing catching up with the North.

The existence of spillovers is not sufficient for North-South diffusion and catching up by the South. In fact, diffusion requires the South to absorb the technological knowledge spillovers; *i.e.*, to have the capacity to incorporate the Northern spillovers in its own R&D. In our framework, imitation capacity; *i.e.*, capacity to learn, assimilate and implement advanced technologies, is enhanced by domestic policies promoting R&D – as referred by Aghion *et al.* (2001) –, by the degree of openness and other trade policies – as suggested by Coe *et al.* (1997) –, and decreases with the human capital gap in relation to the North – in the lines of Nelson and Phelps (1966). With this specification of imitation capacity, we are able to add a new channel through which different levels of human capital impinge on the effects of technological knowledge diffusion through trade on economic growth and wage inequality.

In our framework, the advantage-of-backwardness hypothesis, first introduced by Veblen (1915) and later improved by Findlay (1978), considers that the rate of technological knowledge progress in the South is an increasing function of the gap between its own technological knowledge level and that of the North – as in Barro and Sala-i-Martin (1997) –, as long as there is imitation capacity to take advantage of the Northern spillovers and international IPRs are not fully enforced. However, we require the gap to be below a threshold distance, beyond which the cost of imitation becomes prohibitive, or, equivalently, the capacity of imitation goes to zero. In other words, the

[4] In an often-cited study, Mansfield *et al.* (1981) reports that the cost of imitation is, on the average, about 65% of the cost of innovation.

imitating South cannot be too backward, as already stated above. This is in line with Verspagen (1994), and with the survey evidence cited by Helpman (1993), according to which most imitative R&D take place in newly industrialised countries, while many less developed countries engage in this activity only marginally.

Thus, advantage of moderate backwardness in the South, together with imitation capacity, is a mechanism of catching up with the North. Thereby the model is coherent with empirical evidence suggesting that middle-income countries are converging, while low-income countries, which are far from the technological knowledge frontier, are stagnant and show no potential for rapid growth – *e.g.*, Quah (1997).

Catching up and the resulting convergence are often examined in terms of differences in total factor productivity (TFP) – Bernard and Jones (1996a, b), Hall and Jones (1999) and Acemoglu and Zilibotti (2001), among many others. Following this approach, we analyse inter and intra-country patterns of TFP resulting from technological knowledge diffusion under international trade.

Scale effects in models of endogenous growth

In order to analyse the mechanisms through which international trade affects technological knowledge progress in contexts where market size effects are not present, an essential feature of our framework is that growth is independent of the scale of the economy.

The endogenous growth literature has been debating the role of scale effects since Jones (1995a, b) criticised the first generation R&D endogenous growth models – particularly the seminal work of Romer (1990b) – on the grounds that it did unreasonably predict a very strong positive market size effect on the rate of economic growth. In Romer's (1990b) case – a knowledge-driven process –, the pool of labour employed in R&D increases the rate of technological innovation and, thus, the rate of economic growth. In lab-equipment models – *e.g.*, Rivera-Batiz and Romer (1991) – the scale effect is connected to the size of profits that, in each period, accrue to the leader producer; a larger market expands the monopolist's profits and, thus, the incentives to allocate resources to R&D, thereby increasing the rate of growth of the economy. In addition to theoretical arguments, several authors have supplied the debate with empirical evidence against strong scale effects – *e.g.*, Backus *et al.* (1992), Jones (1995a), Dinopoulos and Thompson (1999) and Zachariadis (2003).

Apart from the endogenous growth debate on the subject, we regard the independence between growth and scale effects as mainly instrumental to the isolation of the other effects of international technological knowledge diffusion under each international trade regime.

Starting with Jones (1995b) himself, who has conveniently modified Romer's (1990b) R&D specification, several modelling efforts have been made to eliminate the scale effects – see Jones (1999) and Segerstrom (1998) for detailed discussions of scale effects and different ways to avoid them. For our particular purposes, two relevant examples are Dinopoulos and Thompson (1999), which assume that an increase in scale also entail costs, and Dinopoulos and Syropoulos (2001), which consider that leader firms spend resources in rent-protecting activities that delay the next successful research.

These scale-removing efforts have focused mostly on closed economy models, or on open North-North models – Dinopoulos and Segerstrom (1999a, b), Dinopoulos and Syropoulos (2004) and Sener (2001). Little progress has been made at developing North-South models without the scale property, the exceptions (to our knowledge) being the recent work in progress of Dinopoulos and Segerstrom (2004) and Sener (2003).

In our case, we remove the scale effects by assuming that the difficulty in conducting R&D is proportional to the size of the market measured by the stock human capital. This assumption is consistent with the literature that observes that increases in scale also entail costs – *e.g.*, Becker and Murphy (1992), Quah (1998) and Alesina and Spolaore (1997), Dinopoulos and Thompson (1996, 1999) and Dinopoulos and Segerstrom (1999a). These costs involve coordination among agents, the processing of ideas, both informational and organisational, marketing, and transportation. As recently suggested by Dinopoulos and Syropoulos (2001, 2004) and Sener (2003), additional costs of scale are related to strategies, like technical barriers, undertaken by leader firms in order to increase the difficulty of R&D by follower firms, with the objective of prolonging their temporary monopoly power.

In the terminology of Dinopoulos and Segerstrom (1999a) our scale-removing assumption results in a 'permanent-effects-on-growth' specification, where both the technological knowledge progress and the rate of economic growth are endogenous – as opposed to the semi-endogenous models (*e.g.*, Jones, 1995b, and Funke and Strulik, 2000). Indeed, in these models, technological knowledge progress is endogenous but the

steady state growth rate results exogenous, which, in fact, is not that different from the neoclassical growth model.

Human capital accumulation in models of endogenous growth

In addition to Schumpeterian R&D, economic growth is also driven by endogenous accumulation of human capital – *i.e.*, when individuals improve their own skills in response to incentives, aggregate productivity rises. When decisions of skill acquisition are embedded into the model, the economy can continue to grow even if the stock of labour is limited (and even without technological knowledge progress).

The empirical evidence on this potential growth mechanism has been presented by many authors, *e.g.*, Romer (1990a), Barro (1991, 1999), Mankiw *et al.* (1992), Levine and Renelt (1992) and Barro and Sala-i-Martin (2004, chap. 12), who generally conclude that an increase in the average years of schooling of the working population, which is traditionally the main empirical measure of human capital, has positive and significant effects on growth. However, Benhabib and Spiegel (1994) point out that economic growth is strongly correlated not to the change in educational attainment, but to the level – *i.e.*, not to accumulation but to the stock of human capital.

These empirical results may, though, be biased for a number of reasons, such as measurement errors – *e.g.*, Krueger and Lindahl (2001) –, the fact that some countries, namely developing countries, are outliers – *e.g.*, Temple (1999) –, and the importance of the skilled population in the public sector – *e.g.*, Griliches (1997) and Pritchett (2001). In addition, some of the panel data estimations considering together differences over time and across countries yield a negative relation between growth and human capital – *e.g.*, De Gregorio (1992), Knight *et al.* (1993), Islam (1995) and Caselli *et al.* (1996).

Another source of bias is the role of human capital in production, which is complex. Benhabib and Spiegel (1994) and Engelbrecht (1997) obtain some improvements by considering learning variables and interaction between technological levels and human capital. Additional complexity is present in the educational process itself, which converts individual endowments into human capital. Many authors emphasise, with particular relevance for developing countries, modes of education other than formal schooling, especially on-the-job-training (OJT) – *e.g.*, Bartel and Lichtenberg (1987), Mincer (1993, 1994) and Bartel and Sicherman (1998).

At the theoretical level, Lucas (1988, 1993) is credited with the seminal analysis of endogenous growth based on human capital accumulation, to be followed by Becker *et*

al. (1990) and Stokey (1991). For example, in reference to the productivity growth in newly industrialised countries, Lucas (1993) emphasises the importance of human capital accumulation at work, with a model where OJT is enhanced by the continuous introduction of new production lines. Mincer (1993, 1994) explores further these ideas, explicitly considering that OJT, differently from learning-by-doing, is costly, relies on individual decisions in interaction with other economic decisions. In Mincer's stricter definition, the opportunity cost of OJT is the wage lost while acquiring human capital at technical, professional or specialised schools related to the particular job.

As to human capital heterogeneity, some recent R&D literature dealing with the direction of technological change and wage inequality assumes that workers are divided into two homogeneous groups, skilled and unskilled – *e.g.*, Kiley (1999), Theshmar and Thoenig (2000) and Acemoglu and Zilibotti (2001). We extend these models by endogenously specifying the supply of each type of human capital. To keep things as simple as possible, we keep the assumption that there are only two types of human capital, one more productive than the other – which we denominate versatile and specialised, respectively. Each type's accumulation function employs the inputs OJT – following Mincer's (1993, 1994) definition – and school in a specific way. We assume that versatile human capital is relatively abundant in the North, and its accumulation is school intensive, while the specialised type's is OJT intensive.

Direction of technological knowledge progress and wage inequality

We model a standard – in endogenous R&D growth theory – economic structure for the North and South. Production of perfectly competitive final goods uses quality adjusted intermediate goods as inputs, together with human capital. Intermediate goods, in turn, use innovative or imitative designs as inputs, under monopolistic competition.

Each final good can be produced by one of two substitutable technologies. The technology denominated versatile, uses versatile human capital together with a continuum set of versatile-specific intermediate goods; the other technology is called specialised, because it brings together specialised human capital and a continuum set of specialised-specific intermediate goods. That is, each set of intermediate goods complements either type of human capital, not both.

This production function, in which complementarity of inputs is coupled with substitutability between technologies, is adapted from Acemoglu and Zilibotti's (2001) horizontal R&D growth model. Although their model does not include endogenous

human capital nor technological diffusion, it proves nevertheless to be very appropriate to the analysis of the direction of technological progress and its repercussion on wage inequality. For example, in our case, when firms invest relatively more in improving the quality of versatile-specific intermediate goods, the choice of final goods' technology shifts in favour of the versatile type, thereby increasing the demand for versatile human capital. Therefore, in this framework, the direction of technological knowledge progress is closely related to the size of the wage premium of versatile human capital – *i.e.*, to the degree of intra-country wage inequality.[5]

This relationship has been receiving considerable attention in the skill-biased technological change literature – prominently by Acemoglu (1998, 2002a, b, 2003) –, although not in the context of human capital accumulation and technological knowledge diffusion. In addition, since in our framework diffusion occurs with trade, we are also to provide a link to the discussion of factor levels, factor price movements and inter-country income inequality effects, which are traditional issues in international trade analysis.

In the skill-bias literature, the incentives for firms to invest relatively more in either type of intermediate goods emerge mainly through the market size channel, by which technologies that use the more abundant type of human capital are favoured. In our case, since scale effects are removed in order to highlight the mechanisms other than market size, the focus is, instead, on the price channel – by which there are stronger incentives to improve technologies when goods produced by these technologies command higher prices; *i.e.*, technologies that use the more scarce type of human capital are favoured. In our framework, then, endogenous accumulation of human capital not only influences the path of output and wages – like in traditional endogenous human capital growth models – but also affects the direction of R&D.

Many researchers of the empirical evidence on skill-biased technological change have been interpreting the rise of the skill premium in developed countries as resulting from the market size effect in closed economies – *e.g.*, Bartel and Lichtenberg (1987), Berman *et al.* (1994), Juhn *et al.* (1993), Autor *et al.* (1998) and Bartel and Sicherman (1998). Some more recent interpretations, still in closed-economy context, attribute the increase in wage inequality to organisational and institutional change triggered by a new

[5] It is clearly not possible to discuss wage inequality issues in endogenous growth models with accumulation of aggregate human capital. This is the main reason behind our extension to heterogeneous human capital.

general purpose technology – *e.g.*, Acemoglu (1999), Aghion (2002) and Caroli and Van Reenan (2001).

Some authors, like Wood (1995) and Leamer (1994, 1996), challenge these empirical studies, on the grounds that they neglect open-economy effects. Even though these authors consider the increase in wage inequality essentially a result of static comparative advantages (in specifications without R&D and technological diffusion), this evidence may arguably be related to the price-channel effect on R&D, especially with trade between countries with distinct human capital levels.

Policies – subsidies towards R&D and international intellectual property rights protection

As a result of the close relationship between the production of intermediate goods and R&D, in our model R&D can be encouraged either by a direct subsidy or through a subsidy to the production of intermediate goods. In any of theses cases, policies also impinge on the demand for human capital. In addition, when affecting not only the level but the direction of R&D as well, policies also impact on the relative demands for the types of human capital, and, thus, on wage inequality.

Another important dimension of government intervention is the protection of IPRs, which we take for granted at the domestic level, but not internationally, as stated above. Since technological knowledge is partially non-excludable, domestic protection of IPRs increases the degree of excludability of the results of R&D in the North and in the South. In turn, international enforcement of IPRs, by protecting innovative R&D only, would prevent imitation R&D, and technological knowledge diffusion through this mechanism would not occur. This is the reason why our framework assumes that IPRs are never fully enforced internationally. Still, we allow for different degrees of enforcement, not ignoring on-going debates on the effects of strong international IPRs protection on the rates of technological knowledge progress and economic growth – in Segerstrom *et al.* (1990), Helpman (1993), Lai (1998) and Dinopoulos and Segerstrom (2004) the effect is negative, contrarily to Glass and Saggi (2002) and Sener (2003).

Our contribution to this literature results from the inclusion of human capital accumulation, which is influenced by the degree of international IPRs protection, and, in turn, affects the impact of the latter on technological knowledge progress and economic growth.

Analysis of immediate, steady state and transitional dynamic effects

Within our framework, the assessment of the non-scale effects of technological knowledge diffusion under international trade on economic growth and wage inequality, comprises three analytical stages: (i) immediate level effects, (ii) steady state effects, and (iii) transitional dynamics. By means of comparative statics and dynamics we are able to uncover the mechanisms through which these effects are influenced by initial North-South conditions, the technology of production, of R&D and of human capital accumulation, and by government policies.

Derivation of transitional dynamics is often avoided in growth models with technological knowledge diffusion. Grossman and Helpman (1991a, c), for example, analyse the North-South feedback effects only in steady state. Barro and Sala-i-Martin (1997) derive transitional dynamics for the South, but no transition path exists for the North, since their model, without international trade, does not allow for feedback effects.

Our derivation of transitional dynamics, for the North and South, requires the use of numerical methods, due to the complexity of the system of differential equations. In particular, like Funke and Strulik (2000), we use the fourth-order Runge-Kutta classical method. Sensitivity analysis is systematically conducted, not only for the purpose of comparative statics and dynamics, but also to enhance the confidence in the numerically derived results.

Summary of contributions to the literature

We build a North-South growth model, with domestic endogenous accumulation of internationally immobile assets, and with domestic but no international protection of R&D results. The model is designed in order to investigate how international trade affects the structure and the dynamics of technological knowledge progress and its North-South diffusion in contexts where market size effects are negligible, thus influencing the paths of economic growth and inter and intra-country wage inequality.

We follow and contribute to three main lines of research previously explored, notably, by: (i) technological knowledge diffusion growth models of Grossman and Helpman (1991a, chaps. 11-12) and Barro and Sala-i-Martin (1997); (ii) Jones' (1995b) scale-invariant technological knowledge progress model; and (iii) direction of technological change and wage inequality growth models of Acemoglu and Zilibotti (2001) and Acemoglu (2002a).

By considering international trade between North and South – two countries with different levels of development, but both capable of conducting R&D – we connect technological knowledge diffusion with the direction of technological change and, thus, we relate the dynamics of inter and intra-country wage inequality; in addition, we are able to analyse how trade in intermediate goods impinges on the pattern of final-goods specialisation.

Concerning, in particular, the literature on technological knowledge diffusion, our main contributions are the analysis of, on one hand, the effects of the interaction between endogenous R&D and human capital accumulation; and, on the other hand, the mechanism of international trade as the main vehicle of diffusion without scale effects. Indeed, with the possible exceptions of the work in progress of Dinopoulos and Segerstrom (2004) and Sener (2003) – still without the R&D-human capital interaction –, the literature has analysed either

– technological knowledge diffusion without international trade and with dominant scale effects, *e.g.*, Barro and Sala-i-Martin (1997);

– technological knowledge diffusion with international trade, but still with the dominance of scale effects, *e.g.*, Grossman and Helpman (1991a, chaps. 11-12);

– scale-invariant technological knowledge progress, but without diffusion nor international trade, *e.g.*, Jones (1995b).

Structure of the following chapters

In order to pursue our objectives, we model, in chapter 2, the domestic North and South economies: economic structure – final goods, intermediate goods and R&D sectors –, individuals' behaviour and human capital accumulation, and government budget and aggregate resources constraints. This characterisation highlights the North-South differences, which are crucial to the subsequent analysis.

For the purpose of subsequent comparison, we start by considering, in chapter 3, a regime of technological knowledge diffusion without trade effects. Steady state and transitional dynamics are derived for the domestic North and South economies. In each country, leader firms sell the quality-adjusted intermediate goods only domestically, but diffusion, somehow, still takes place, but does not have any feedback effects in the Northern innovative R&D.

International trade in intermediate goods is introduced in chapter 4. We analyse the immediate level effect, the steady state and transitional dynamics. The South is no longer

limited by its own ability to produce intermediate goods, since it can immediately import Northern intermediate goods that have not yet been imitated. Southern exports of some state-of-the-art intermediate goods become possible by imitating and under-pricing the Northern firms, which generates a feedback from Southern imitative R&D to Northern innovative R&D.

In chapter 5, international trade is extended to all goods, in order to enrich the analysis with the effects that the driving forces of economic growth and technological knowledge diffusion have on the pattern of final goods specialisation. Trade in final goods is driven by North-South differences in relative levels of internationally immobile factors of production, *à la* Heckscher-Ohlin.

Finally, chapter 6 offers some concluding remarks, including several paths for future research.

CHAPTER 2

MODELLING NORTH AND SOUTH ECONOMIES

2.1. Overview

To complement the generic description of North and South economies presented in chapter 1, this short overview briefly highlights some particularities of chapter 2.

As stated before, countries are subject to the same economic environment. We will therefore now present the model's key elements by characterising the domestic North and South economies in a stylised model, so that exogenous productivity, human capital (HC) and R&D differences become apparent and central to the subsequent analysis.

In line with Barro and Sala-i-Martin (1997), this chapter describes a regime without considering the direct impact of international trade (IT) on imitation. Apart from the discussion on the relative importance of embodied *versus* disembodied idea flows – *e.g.*, Romer (1993) and Wong (2004) –, ignoring for now the impact of IT is mainly instrumental, in order to compare in next chapters with the case with IT.

As far as possible, we follow the standard Schumpeterian growth model. Thus, each country possesses three productive sectors: final goods (FGs), IGs and R&D. Every FG is competitively produced by one of two substitutable technologies, versatile (VT) or specialised (ST). Moreover, the composite FG is the numeraire in each country, which can be consumed, converted into quality-adjusted IGs or devoted to R&D activities.

The R&D activity is done either by specialised firms supported by IGs firms or by the R&D department within the IGs firms. In any case, they carry out research, which can be influenced by government policies, to come up with ideas that they develop into blueprints (designs or successful research) for new (higher) quality IGs – Schumpeterian R&D, as formalised by Aghion and Howitt (1992) –, using the composite FG as the unique input – a lab-equipment model, to cite Rivera-Batiz and Romer (1991).

As in Romer (1990b), the competitive market of blueprints implies free entry into the IGs sector, which results in the elimination of profits in a present value sense. That is, IGs firms decide whether to produce a quality-adjusted IG based on the cost of the blueprint and comparing it to the discounted net revenues from sales of the quality-adjusted IG to the FGs firms. It is supposed that by investing in a blueprint, the IG

producer can buy one unit of composite FG at cost one, which is the marginal cost of production (*MC*), turn it into a new quality-adjusted IG at no cost – *i.e.*, it converts the composite FG into a quality-adjusted IG at zero cost –, and then sell it back to FGs firms.

As the IG firm has monopoly power over the sales of its quality-adjusted IG, it can charge a price above *MC*, which can also be influenced by government policies, as can, consequently, price and sales. The IGs firms cover the R&D cost by selling shares in their future profits to individuals. Hence, the shareholders of IGs firms are individuals and thus the monopoly profit flows are paid out as dividend. In a setting where IGs producers support the R&D cost in order to come up with blueprints for new quality IGs, which in principle could be produced without set-up costs by anyone with access to the blueprint, the monopoly power of the IG firm needs to be (domestically) legally protected through an infinitely-lived patent for the IPRs.

Other specificity is that lower quality-adjusted IGs firms are priced out of business in equilibrium. Each IG firm maximises its expected discounted profits, taking into account both the size of the monopoly profit flow and its likely duration or temporary monopoly power. As at all times there are follower IGs firms supporting research, this duration is finite, and each leader IG firm is driven out of business by further successful research, which eroded their profits. There is then uncertainty associated with research at the firm level, which creates jumpiness in microeconomic outcomes. However, as the probability of success across industries is independent and there is a continuum of them, we assume that this jumpiness is not transmitted to macroeconomic variables.

Due to the cost of imitation, the market for IGs will be non-competitive in the South as well, and imitator firms – which support the R&D cost in order to obtain a design in the South that is also (domestically) legally protected through an infinitely-lived patent – will adopt the same price behaviour as the innovator firms in the North.

Over time, as the quality of IGs rises, HC becomes more productive. Since individuals derive utility from consumption of a single composite FG, R&D fuels per capita consumption growth and thus utility. Individuals decide, in a structure of doing or learning, between working in a productive process and increasing their stock of HC; and, on income allocation, between consumption and savings. They have perfect foresight about the technological knowledge (TK) change over time and choose their expenditure paths accordingly to maximise their discounted utilities. In rigour, each economy has

four productive sectors: the three standard Schumpeterian and HC accumulation. This is a dynamic general equilibrium model, so all markets clear throughout time.

Each individual combines two types of education (training), at school and at OJT, which form the educational process. Lucas (1993) has suggested the importance of considering both modes of HC accumulation, in order to improve his own Lucas (1988) model. The HC accumulation in both schooling and OJT is described by functions *à la* Lucas (1988). Moreover, in line with Mincer (1993, 1994), we assume that, as in case of schooling, OJT is not an automatic process but requires time for training during which individuals do not work. Mincer (1993, 1994) states also that the two types of education may be substitutes or complements and so we write the HC accumulation as a constant elasticity of substitution (CES) function.

To keep things as simple as possible, we assume that individuals endowed with high ability accumulate versatile HC (*VH*), while those gifted with less ability fare better by accumulating specialised HC (*SH*). Thus, there is a threshold individual who is indifferent of accumulated *VH* or accumulated *SH*. Moreover, we assume that *VH* is school intensive, while *SH* is OJT intensive. Intuitively, we suppose that schooling helps in adapting to changing environments more than OJT. A crucial point is that both technologies of HC accumulation differ in their relative use of inputs, while the educational process is technologically the same for all individuals.

Our framework is, in particular, different from standard growth models that explicitly introduce HC as a separate factor of production. Generally they do so by considering more educated (skilled) workers perfect substitutes for less educated (unskilled) workers – *e.g.*, Lucas (1988) – or by considering technology a complement to both types of labour – *e.g.*, Mankiw *et al.* (1992) and Romer (1990a). Either way, there is no scope in such models for TK change to differentially affect the wages paid workers with different levels of education.

After this short overview, the chapter proceeds by characterising the Northern economy and, in the process, highlight the differences relative to the South. Sections 2.2 and 2.3 describe the FGs and the IGs sectors, respectively. Section 2.4 analyses the equilibrium for given states of TK and HC. Section 2.5 focuses on the R&D sector. First, innovation and imitation activities are modelled and, after that, the flows and duration of monopoly profits, the equilibrium probability of successful research and the behaviour of the aggregate quality indexes are computed. Section 2.6 portrays the behaviour of

individuals, stressing HC accumulation. Section 2.7 completes the description of the model, by including both the government budget and aggregate resource constraints. Section 2.8 concludes the chapter with some elements of synthesis.

2.2. Final goods sector

Final goods – Y, continuously indexed by $n \in [0, 1]$ – are produced in perfect competition. Following the Schumpeterian set-up as amply divulged by the textbooks of Barro and Sala-i-Martin (2004) and Aghion and Howitt (1998a) complemented with the contribution by Acemoglu and Zilibotti (2001),[6] we assume that each FG can be produced by one of two technologies, VT and ST.[7] VT uses *VH* complemented with a continuum set of differentiated versatile-specific IGs indexed by $j \in [0, J^{VH}]$. The ST inputs are *SH* complemented with a different continuum set of differentiated specialised-specific IGs indexed by $j \in [0, J^{SH}]$.[8] The production function of the nth FG at time t is,

$$Y_n(t) = A\left\{\left[\int_0^{J^{VH}}\left(\sum_{k=0}^{k(j,t|VH)} q^k\, x_n(k,j,t|VH)\right)^{1-\alpha} dj\right]\left[n\, vh\, VH_n(t|w)\right]^{\alpha} + \right.$$
$$\left. + \left[\int_0^{J^{SH}}\left(\sum_{k=0}^{k(j,t|SH)} q^k\, x_n(k,j,t|SH)\right)^{1-\alpha} dj\right]\left[(1-n)\, sh\, SH_n(t|w)\right]^{\alpha}\right\}. \qquad (2.1)$$

Term A is a positive exogenous variable representing the level of productivity, dependent on the country's domestic (non-IT related) institutions, namely tax laws, property rights, the maintenance of law and order, government services, cultural and geographical aspects, social conditions and others. We consider $A_{South} < A_{North}$ as the only North-South difference in the parameters of equation (2.1).[9] Thus, a source of differences between countries is likely to be caused by changes in government policies.[10]

In equation (2.1), the second and fourth expressions within square brackets represent the role of the specific HC inputs. The HC terms include the quantities

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[6] In fact, this recent contribution is based on Murphy *et al.* (1998). However, since they are interested in the effects of TK change, not its sources, they represent TK change in terms of two exogenous functions of time, which augment the services of university-educated workers and high school-educated workers.

[7] To make the idea clearer, we can presume that VT is used in FGs with industrial or modern industrial production, like computers and automobiles, while ST is used in FGs with agricultural or traditional industrial production, like textiles and footwear.

[8] The term IG is used instead of 'machine' because we treat them as flow variables, as if there were full instantaneous depreciation.

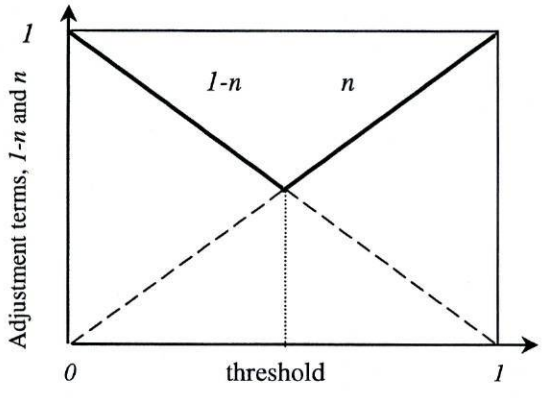
[9] Although perhaps unrealistic, we have generally assumed, by reason of tractability considerations, the simplifying assumption that the parameters are equal in the countries.

[10] The effects of these policies on outputs are analogous to those from pure differences in the levels of TK. Since A is exogenous, we omit works that focus fully on the increase of A – *e.g.*, Chipman (1970).

employed in the production of the n[th] FG, $VH_n(t|w)$ and $SH_n(t|w)$, that is, those that work and earn a wage. Consequently, index w in the HC inputs identifies the VH_n and SH_n actually employed in the production of the n[th] FG, as opposed to VH and SH in training. This implies that the output becomes determined by the stock of HC and time shares spent at work. From now on, we denote $m = \{VH, SH\}$.

The HC terms also include two types of corrective factors accounting for differential productivities. An absolute productivity advantage of VH over SH is accounted for by the joint parameters vh and sh, assuming $vh \geq sh \geq 1$; *i.e.*, assuming a technological bias in favour of VT. From now on, we denote $\overline{m} = \{vh, sh\}$. In other words, vh and sh can be called versatile-augmenting and specialised-augmenting parameters, respectively. A relative productivity advantage of either type of HC is captured by the terms n and $(1-n)$,[11] whose behaviour is represented in figure 2.1 below.

Figure 2.1. Behaviour of the adjustment terms



The use of these adjustment terms in the production function (2.1) transforms the FGs index n in a relevant ordering index. It means that VH is relatively more productive in producing FGs indexed by larger ns, and *vice-versa*. This implies that the comparative advantage (CA) of VT increases with index n. Since $n \in [0, 1]$, at each time t there is a competitive equilibrium threshold FG N, endogenously determined, where the switch from one technology to another becomes advantageous, as will become clear below.

Parameter α, where $0 < \alpha < 1$, is the m-type HC share and, therefore, measures the contribution of the respective type of HC to the FG production.[12]

[11] The adjustment terms proceed from the general adjustment function: $f(n', n) = 1 - \| n' - n \|$, where $\|...\|$ indicate absolute value. Here, we have only two types of HC and so $n' = 0$ for SH and $n' = 1$ for VH.
[12] The share parameters of VH and SH could differ, but to simplify we assume that they are equal.

In equation (2.1), the first and third expressions within square brackets sum up the contributions of the two types of IGs to production. Each of the two IGs terms includes an adjustment for quality that reflects a stylised TK change process of the quality ladder type. The size of each quality upgrade obtained with each successful research in an IG j is denoted by q, an exogenously constant over time greater than one, > 1. The particular rung of a quality ladder on which j is located is indexed by k, with higher values of k denoting higher quality. At time 0, the highest quality in each IG industry has a quality index $k = 0$.[13] One firm in each IG industry knows how to produce a $k = 0$ quality, and no firm knows how to produce any higher quality.[14] To learn how to produce higher quality IGs, R&D firms or R&D departments of IGs firms engage in R&D and each subsequent successful research will make the IG q times more productive than its predecessor.[15] Over time, IGs improve as successful researches push each IG up its quality ladder.

Thus, when the highest quality in an IG industry has index k, the next IG winner firm becomes the sole producer of the IG with quality index $k+1$. Hence, the quality index $k(j,t|m)$ of the highest quality of the IG j at time t also measures the number of successful upgrades that have occurred in that IG since time 0 up to the present. The highest quality levels in all IGs at time t define the TK state-of-the-art at that time.

The quantity $x_n(k,j,t|m)$ of the quality rung k of IG j is used, together with its HC complement – type m –, to produce the FG output at time t, $Y_n(t)$. Hence,

$$\left(\sum_{k=0}^{k(j,t|m)} q^k \, x_n(k,j,t|m) \right)^{1-\alpha}, \tag{2.2}$$

is the quality-adjusted total amount of IG j, and $(1-\alpha)$ is its share in FG production – it can inversely measure the IG monopolist's market power, as we will see later on.

In production function (2.1), we have that m and the set of IGs $[0, J^m]$ are complements, whereas VT and ST are substitutes. Due to the latter feature, it can be written instead by the following CES production function: $Y_n(t) = A\{\mathrm{VT}^{\iota} + \mathrm{ST}^{\iota}\}^{1/\iota}$, where for $\iota = 1$ VT and ST are perfect substitutes.[16] The adjustment terms in our approach slacken the perfect substitutability between VT and ST and they become

[13] Our assumption that all IGs start with the same quality is innocuous and could be perfectly slackened.
[14] We will refer types of IGs to distinguish between versatile, $[0, J^{VH}]$, and specialised, $[0, J^{SH}]$, complementary IGs, and reserve the words variety and quality to distinguish different IGs (industries) of the same type ($J^{VH} + J^{SH}$ varieties) and different upgrades of the same IG (q^0, q^1, q^2, ..., q^k, ...), respectively.
[15] As we will see later on, at each time, only the highest quality IGs are used in the production of FGs and each IG industry consists of only one IG monopolist firm.

substitutes, as if $0 < \iota < 1$, in line with the empirical evidence and theoretical literature – *e.g.*, Freeman (1986), Angrist (1995) and Acemoglu (1998, 2002a, b, 2003).[17] Moreover, the adjustment terms in our approach have the advantage of permitting that the threshold FG N be found, which will be an essential result in our analysis.

The summation in equation (2.1) only runs up through $k(j,t|m)$ because only the quality of the IGs which have been invented by time t can be used in the production of FGs at time t. Moreover, since the quality grades within an industry j are perfect substitutes as inputs to production, in equilibrium only the highest quality IGs are used because the productivity gain from using the latest generation of a given IG invariably dominates the cost differential associated with the same choice. This holds as a result of profit maximising limit pricing by monopolist producers of IGs, as we will see in section 2.3.[18] Then the production function in equation (2.1) is reduced to:

$$Y_n(t) = A\left\{\left[\int_0^{J^{VH}}\left(q^{k(j,t|_{VH})}\,x_n(j,t|VH)\right)^{1-\alpha}dj\right]\left[n\,vh\,VH_n(t|w)\right]^{\alpha} + \right.$$
$$\left. + \left[\int_0^{J^{SH}}\left(q^{k(j,t|_{SH})}\,x_n(j,t|SH)\right)^{1-\alpha}dj\right]\left[(1-n)\,sh\,SH_n(t|w)\right]^{\alpha}\right\}. \qquad (2.3)$$

Where now $x_n(j,t|m)$ is the quantity of the highest quality IG j used, together with its HC complement that is of type m, to produce the FG output at time t, $Y_n(t)$. The 'additive separability' of the IGs is an important property of this production function. It implies that discoveries of new varieties of IGs do not make any of the existing ones obsolete,[19] and that no particular IG is (totally) essential for production: each IG is useful regardless of whether other IGs are available – *i.e.*, they are perfect substitutes [*e.g.*, Easterly *et al.* (1994)]. Nevertheless, while potentially the production of FGs can employ any IG indexed by $[0, \infty[$, in our case, only the IGs $[0, J^{SH}]$ and $[0, J^{VH}]$ are used, since only these are available.

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[16] We give additional details about the CES function in section 2.6.

[17] But, Kiley (1999), *e.g.*, considers that the production with skilled workers and skilled-complementary IGs is a perfect substitute for production with unskilled workers and unskilled-complementary IGs.

[18] The assumption is not fully realistic. We should in general expect that 'the flow of services characteristics' associated with an IG depends upon the type and quantity of other IGs with which it co-operates within a production activity. If there are strong complementarities between different IGs, the best practice TK level of IG j at time t may not be the highest available. Compatibility constraints may in fact imply that it is inefficient to use very different TK levels of complementary IGs in the same activity. However, complementarities of this sort are simply ruled out in most R&D growth models.

[19] Like is the case in the expanding variety – or horizontal – R&D models of Romer (1990b) and Barro and Sala-i-Martin (1997, 2004, chap. 6), among many others.

Taking into consideration that only the highest quality IGs are used to produce FGs and that $p(j,t|m)$ denotes the price of the m-complementary highest IG from industry j at time t, relative to the price of the composite FG which we treat as the numeraire.[20] Then, since the market for FGs is competitive, the representative producer of FG n takes the price of its product, p_n, the price paid for (a unit of) m-type HC, w^{VH} and w^{SH}, and $p(j,t|m)$, as given, and maximises profits. Formally, we have:[21]

$$\underset{x_n(j|m),\, m_n}{Max}\, \Pi_n = p_n Y_n - \int_0^{J^{VH}} p(j)\, x_n(j|VH)\, dj - \int_0^{J^{SH}} p(j)\, x_n(j|SH)\, dj - w^{VH} VH_n - w^{SH} SH_n \cdot \qquad (2.4)$$

Profit maximisation by the representative producer of the FG n implies that the marginal product of each input must equal its price, and thus assures zero profits since there are constant returns to scale. It is straightforward to verify that the first order conditions (FOCs) of the above maximisation problem yield the demands for IGs by the representative producer of nth FG:[22]

$$x_n(j,t|VH) = n\, vh\, VH_n(t|w) \left[\frac{p_n(t)\, A\, (1-\alpha)}{p(j,t|VH)} \right]^{1/\alpha} q^{k(j,t|VH)[(1-\alpha)/\alpha]}; \qquad (2.5a)$$

$$x_n(j,t|SH) = (1-n)\, sh\, SH_n(t|w) \left[\frac{p_n(t)\, A\, (1-\alpha)}{p(j,t|SH)} \right]^{1/\alpha} q^{k(j,t|SH)[(1-\alpha)/\alpha]}. \qquad (2.5b)$$

Equations (2.5a, b) imply that FG firms demand more IGs when their product price, p_n, is higher, when their HC employment, $VH_n(.|w)$ or $SH_n(.|w)$, and IGs quality, $k(j,t|m)$, are greater, and when IGs prices, $p(j,t|m)$, are lower. Intuitively, a higher price for the FG increases the value of the marginal product of the factors, encouraging firms to rent more IGs. A greater level of HC employed implies more HC to use the IGs, raising demand and this feature in itself leads to a market size effect.[23] Since the demand curve for IGs is downward sloping, a higher $p(j,t|m)$ implies lower demand.

Plugging equations (2.5a, b) into equation (2.3), we obtain the supply of FG n, which then becomes given by – the time index t is now omitted:

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[20] Later on, when we introduce IT, the Northern composite FG will be used as numeraire.

[21] To simplify, here we omit the time index t, the quality rung k and the working index w. For the same reason, from now on in obtained equations, we omit indexes when there is no cause for misunderstanding.

[22] In fact, if producers of the FG n use VT, $x_n(k,j,t|VH)$ is given by equation (2.5a), while $x_n(k,j,t|SH)$ is zero, and *vice-versa* when ST is chosen. Similar considerations apply throughout.

[23] Note for now, that we are only in the scope of the standard R&D-based models in which the stock of HC is fixed and exogenously determined, and rely on scale effects to generate endogenous growth. In our case and specifically without IT effects, $VH_n(t|w)$ and $SH_n(t|w)$ are effectively the 'markets' for new technologies, since IGs monopolists can only sell IGs to domestic FGs producers employing domestic HC.

$$Y_n = A^{1/\alpha}\left[\frac{p_n(1-\alpha)}{p(j|m)}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}\left[nvh\,VH_n(.|w)\,Q^{J^{VH}} + (1-n)\,sh\,SH_n(.|w)\;Q^{J^{SH}}\right], \qquad (2.6)$$

$$\text{Where, } Q^{J^{VH}} = \int_0^{J^{VH}} q^{k(j,t|_{VH})[(1-\alpha)/\alpha]}\,dj \text{ and } Q^{J^{SH}} = \int_0^{J^{SH}} q^{k(j,t|_{SH})[(1-\alpha)/\alpha]}\,dj, \qquad (2.7)$$

are the two aggregate domestic quality indexes, measuring domestic TK in each range of IGs. Increases in the state of Q^{J^m} result from the TK progress permitted by successful researches. They improve the productivity of m and raise FGs output, and can be called versatile and specialised-augmenting technologies, respectively. Sometimes we will refer to $Q^{J^{VH}}$ and $Q^{J^{SH}}$ as VT and ST complementary since they represent the quality of IGs used by *VH* and *SH*, respectively. The ratio $Q^{J^{VH}}/Q^{J^{SH}}$ is the relative productivity of the versatile TK, which is an appropriate measure of the versatile-specialised TK bias.

We define the output aggregate – the composite FG – as:[24]

$$Y(t) \equiv \int_0^1 p_n(t)\,Y_n(t)\,dn = exp\left[\int_0^1 ln\,Y_n(t)\,dn\right], \qquad (2.8)$$

where we normalise the price of the composite FG in each period to *1*. That is, we assume (for now) that the composite FG is numeraire in each country.

2.3. Intermediate goods sector

The IGs sector supplies the FGs firms with different quality-adjusted IGs. Given that, by assumption, the production of IGs and R&D is financed by the saved resources after consumption of the composite FG, the simplest hypothesis is to consider that the production function of IGs is identical to the composite FG specified or, equivalently, that the composite FG is the input in the production of each IG.[25] Given this convenient simplification, the *MC* of producing an IG equals the *MC* of producing the composite FG, which, due to perfect competition in the FGs sector, equals the price of the composite FG (numeraire), p_Y; *i.e.*, $MC = p_Y = 1$. Hence, the *MC* of producing an IG is independent of its quality level and is identical across all domestic industries. In addition, we assume that the government can subsidise the production of IGs by paying a fraction ad-valorem, s_x, of each firm's production cost.[26] Thus, the after subsidy *MC* of IGs production is $(1-s_x)$.

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[24] As will see more explicitly later, the consumption and savings come out of the composite FG.

[25] As Papageorgiou (2002, pg. 5) remarks, "Introducing a new production function for the manufacturing of IGs would only complicate the algebra without changing the results qualitatively."

[26] Since subsidy and tax rates are relatively stable over time, we are only considering the case where they are stationary and exogenously given, since otherwise the technical complexity would be substantial.

Since all FG firms, which use the same technology, face the same input prices and choose the same input ratios, we can aggregate FG firms to obtain the aggregate demands $X(j,t|VH)$ and $X(j,t|SH)$ for the highest quality of IG j at time t:[27]

$$X(j,t\,|\,m) = X(k,\,j,t\,|\,m) = \int_0^1 x_n(j,t\,|\,m)\,dn = \int_0^1 x_n(k,j,t\,|\,m)\,dn\,. \qquad (2.9)$$

The manufacture of an IG requires a start-up cost of researching a new design. This investment in a blueprint can only be recovered if profits at each date are positive for a certain period in the future. This is guaranteed by domestically enforced patents – *i.e.*, there is a domestic system of IPRs –, which protect the leader firm's domestic monopoly of that quality IG, while at the same time, almost without costs, disseminating acquired TK to other domestic firms. Thus, IGs producers are expected to be unwilling to produce under competitive market conditions. Under these assumptions, TK of how to make an IG is (or tends to be) public (non-rival and non-excludable) within a country.

As the producer of the highest quality in each industry j of IGs is assumed to be a monopolistic competitor, it solves the following maximisation problem at each time:

$$\underset{p(k,j,t|m)}{Max}\ \Pi(k,j,t\,|\,m) = \left[\,p(k,j,t\,|\,m) - (1-s_x)\right] X(k,j,t\,|\,m)\,. \qquad (2.10)$$

Substituting the demand function given by equation (2.9) into equation (2.10) and, after that, taking the derivative with respect to price and setting it equal to zero (FOCs), it is straightforward to derive the solution of this maximisation problem as:

$$\text{Monopoly pricing} \Rightarrow\ p(k,j,t\,|\,m) = p(j,t\,|\,m) = p = \frac{1-s_x}{1-\alpha}\,. \qquad (2.11a)$$

$$\text{Limit pricing} \Rightarrow\ p(k,j,t\,|\,m) = p(j,t\,|\,m) = p = q\,(1-s_x)\,; \qquad (2.11b)$$

Equation (2.11a) shows that the profit-maximising price yields a mark-up on the after subsidy MC of manufacturing IGs, since $p > (1-s_x)$. Without change in government intervention then, like the MC, this price is a constant mark-up over time, across industries and for all quality grades, which makes the problem symmetric.[28] The closer α is to zero, the smaller the mark-up; that is, there is less room for monopoly pricing.

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[27] $X(k,j,t\,|\,m) = X(j,t\,|\,m)$ and $x(k,j,t\,|\,m) = x(j,t\,|\,m)$ since only the producer of the highest quality in each industry j sells goods. Note also and more precisely that – as we will see later in section 2.4:

$$X(j,t|SH) = \int_0^J x_n(j,t|SH)\,dn = \int_0^N x_n(j,t|SH)\,dn \text{ and } X(j,t|VH) = \int_0^J x_n(j,t|VH)\,dn = \int_N^I x_n(j,t|VH)\,dn\,.$$

This is because final producers in the range $[0, N]$ do not demand versatile-complementary IGs, whereas final producers in the range $[N, 1]$ do not demand specialised-complementary IGs.

[28] Hence, with a constant MC and a constant elasticity demand curve, the solution p to the maximisation program is independent of k and j – *i.e.*, in equilibrium all IGs firms will supply the same price, which is a

As is quite standard in the literature, in the production function (2.1), we assume that, for each IG, all quality ladders are perfect substitutes as inputs. Since the IG firm, which produces according to the latest successful research, is the only firm legally allowed to produce the highest quality IG, it will use pricing to wipe out sales of lower quality in its IG. Depending on whether $q(1-\alpha)$ is less than or greater than MC will respectively use limit or monopoly pricing to capture all the market – *e.g.*, Barro and Sala-i-Martin (2004, chap. 7). Like, for example, Grossman and Helpman (1991a, chap. 4),[29] we assume that limit pricing strategy is binding and therefore is used by all firms.

Thus, we consider that, in all IGs, they choose a price (in terms of composite FG) ε – arbitrarily small – that is lower than the constant limit price in equation (2.11b).[30] It can be called an upper boundary and reflects the fact that the leader is q times more efficient that the closest follower domestically available IG. That is, because the lowest price the producer of the second best good can charge without having negative profits is $(MC-s_x)$, the leader firm can successfully capture the entire market for this IG – it can be the only active producer in the IG market, by selling at any price slightly below $q(1-s_x)$.

In brief, suppliers to IGs firms are either successful research firms or their R&D departments, and customers are FGs firms. As is standard in the endogenous growth literature, IG firms compete in prices – Bertrand price competition – and only the best qualities are used, and the production function (2.1) reduces to equation (2.3).[31] When they earn the opportunity to produce the new quality-adjusted IGs, they produce at the after subsidy MC and sell at a limit pricing mark-up in equation (2.11b). Thus, the magnitude of quality improvements q can serve as an indicator of the market power of the incumbent firm and (imperfect) competition in IGs production. Note that, regardless of the strategy (monopoly or limit pricing), the price of IGs is taken as given by the FG producers (price takers), as we have just noted in equation (2.3).

Given equation (2.11b), we can now rewrite equations (2.5a, b) as:

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mark-up over the after subsidies MC, influenced by the elasticity of demand (that is equal to $-1/\alpha$) or, in other words, influenced by the market power common to all monopolistic competitive producers.

[29] In fact, as in Grossman and Helpman (1991a, chap. 4) $\alpha = 1$, monopoly pricing is not viable in their case.

[30] Note that, from the analysis above, for q in equation (2.11b) we have that: $1 < q \leq [1/(1-\alpha)]$.

[31] As Denicolo and Zanchettin (2002) show, this is not the case with Cournot competition, where: (i) two or more firms can be active at equilibrium, so that different quality grades of the IG can be simultaneously produced, even if older qualities are less productive; (ii) IGs firms' rents can not be terminated by the occurrence of the next successful research, although the market share and profits of the current producer decrease, and less efficient firms may be driven out of the market. We leave the implications – namely in equilibrium growth rates – of the Cournot competition in our framework for future work.

$$x_n(j,t|VH) = n\, vh\, VH_n(t|w)\left[\frac{p_n(t)\,A\,(1-\alpha)}{q(1-s_x)}\right]^{1/\alpha} q^{k(j,t|VH)[(1-\alpha)/\alpha]}; \qquad (2.12a)$$

$$x_n(j,t|SH) = (1-n)\, sh\, SH_n(t|w)\left[\frac{p_n(t)\,A\,(1-\alpha)}{q(1-s_x)}\right]^{1/\alpha} q^{k(j,t|SH)[(1-\alpha)/\alpha]}. \qquad (2.12b)$$

Equations (2.12a, b) yield the explicit industry demands for IGs.[32] Substituting these into the production function of the FGs given by equation (2.3), we obtain:

$$Y_n = A^{1/\alpha}\left[\frac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}}\left[n\, vh\, VH_n(.|w)\, Q^{JVH} + (1-n)\, sh\, SH_n(.|w)\, Q^{JSH}\right]. \qquad (2.13)$$

As in equation (2.6), equation (2.13) emphasises that the growth of FGs production is entirely driven by improvements in the domestic available quality of IGs or, in other words, by domestically available TK (versatile and specialised type), Q^{JVH} and Q^{JSH}, and by domestic HC accumulation, *VH* and *SH*, and respective utilisation in FGs sector.

2.4. Equilibrium given R&D and human capital accumulation

In this section, we obtain the domestic equilibrium values for the threshold FG *N*, the aggregate variables *Y* and *X*, the price paid for a unit of each type of HC, its growth rate as well as a (equilibrium) measure of inter and intra-country wage inequality. To this end, we proceed to characterise the equilibrium for a given state of TK – Q^{JVH} and Q^{JSH} defined by equation (2.7) – and stocks of *VH* and *SH*. After that, the R&D activities and the endogenous determination of HC accumulation together with government budget and aggregate resource constraints close the model of the domestic economy.

Generically, a feature of the equilibrium is that only one technology, VT or ST; *i.e.*, one combination of IGs of a certain type and the respective HC, will be used to produce each FG. We follow, with the due differences, Acemoglu and Zilibotti (2001).

The pattern of comparative advantage (CA) embedded in the production function (2.1) through the adjustment terms *n* and (*1–n*) makes *VH* relatively more productive in high index FGs. Together with profit maximisation, this pattern implies the existence of a threshold FG $N \in [0, 1]$, such that only ST is used to produce FGs indexed by $0 \leq n \leq N$

[32] The monopoly power of IG firms implies that the production of IGs under the decentralised (*laissez-faire* or private) case is not Pareto optimal – there is underproduction. In fact, comparing the demands for IGs in decentralised and social planner cases, it can be demonstrated that the two are equal if and only if the government in a decentralised case offsets the distortion caused by limit pricing, which occurs if $s_x = [(q-1)/q]$; that is, if government appropriately subsidised the production of IGs.

– *i.e.*, $VH_n = x_n(\cdot|VH) = 0$ for all $0 \leq n \leq N$ –, and only VT is used to produce FGs with $N < n \leq 1$ – *i.e.*, $SH_n = x_n(\cdot|SH) = 0$ for all $N < n \leq 1$. To prove this, we start noting that the profit of a representative producer of a FG n which uses m-type HC, in the period t, is

$$\Pi_n(t|m) = p_n(t)\, Y_n(t|m) - \int_0^{J^m} p(j)\, x_n(j,t|m)\, dj - w^m(t)\, m_n(t|w), \qquad (2.14)$$

Where: $Y_n(.|VH) = A^{1/\alpha}\left[\frac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} n\, vh\, VH_n(.|w)\, Q^{J^{VH}}$;

$$Y_n(.|SH) = A^{1/\alpha}\left[\frac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} (1-n)\, sh\, SH_n(.|w)\, Q^{J^{SH}}.$$

Taking equation (2.14), we can use equations (2.11), (2.12a, b) and (2.13) to write per HC profit in each period, $\pi_n(\cdot|m)$, as:

$$\pi_n(\cdot|VH) = \frac{\Pi_n(\cdot|VH)}{VH_n(\cdot|w)} = n\, vh\, (p_n A)^{1/\alpha}\left[\frac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} \alpha\, Q^{J^{VH}} - w^{VH}; \qquad (2.15a)$$

$$\pi_n(\cdot|SH) = \frac{\Pi_n(\cdot|SH)}{SH_n(\cdot|w)} = (1-n)\, sh\,(p_n A)^{1/\alpha}\left[\frac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} \alpha\, Q^{J^{SH}} - w^{SH}. \qquad (2.15b)$$

Competition implies that, in equilibrium, $\Pi_n(\cdot|m) \leq 0$, $\forall n$. That is, since the FG market is competitive, every FG producer has to make zero profit per HC employee. Moreover, we observe that the difference $\pi_n(\cdot|VH) - \pi_n(\cdot|SH)$, is given by:

$$(p_n A)^{1/\alpha}\left[\frac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} \alpha\left[n\,(vh\, Q^{J^{VH}} + sh\, Q^{J^{SH}}) - sh\, Q^{J^{SH}}\right] + w^{SH} - w^{VH}. \qquad (2.16)$$

Hence, from equation (2.16), $\pi_n(\cdot|VH) - \pi_n(\cdot|SH)$ is an increasing function of n over [*0*, *1*]. In addition, the structure of production implies that all FGs n must be produced, as results from equation (2.8), together with the assumption that individuals obtain utility from the consumption of the composite FG, as will become apparent below. Thus, for any FG n we must have either $\pi_n(\cdot|VH) = \Pi_n(\cdot|VH) = \pi_n(\cdot|SH) = \Pi_n(\cdot|SH) = 0$ or $\pi_n(\cdot|VH) = \Pi_n(\cdot|VH) = 0$ or $\pi_n(\cdot|SH) = \Pi_n(\cdot|SH) = 0$. Moreover, in equilibrium, it is impossible to have a certain m-type HC unemployed, because this would imply that the price paid for a unit of the m-type HC falls to zero. Hence, from equations (2.15a, b), a

profitable deviation would exist. Thus, a positive measure of FGs must be produced using the *m*-type HC. It follows that a threshold FG N must exist, where $0 < N < 1$, such that $\pi_n(\cdot|VH) - \pi_n(\cdot|SH) = 0$, and $\pi_n(\cdot|VH) - \pi_n(\cdot|SH) > 0$ (<0) for all $n > N$ (for $n < N$).

Then, we can rewrite the production of FG n given by equation (2.13) as:

$$Y_n = \begin{cases} A^{1/\alpha}\left[\dfrac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} (1-n)\, sh\, SH_n(.|w)\; Q^{J^{SH}} & if \quad 0 \le n \le N \\ A^{1/\alpha}\left[\dfrac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} n\, vh\, VH_n(.|w)\, Q^{J^{VH}} & if \quad N < n \le 1 \end{cases} . \tag{2.17}$$

As mentioned before, equation (2.17) shows that the production of FG n – of each type – is a linear function of the respective TK, Q^{J^m}, and the HC used, $m_n(.|w)$, and it depends also on the respective price, p_n. This is because a higher price of output increases the value of IG productivity, but not their costs, and consequently encourages firms to use more of them, thereby raising HC productivity.

In equilibrium, the marginal value product of each type of HC has to be equalised across all the relevant FGs – $n > N$ for *VH* and $n \le N$ for *SH*; that is,

$$\frac{\partial\, p_n Y_n(t)}{\partial\, m_n(t|w)} = \begin{cases} (p_n A)^{1/\alpha}\left[\dfrac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} (1-n)\, sh\, Q^{J^{SH}} & if \quad n \le N \\ (p_n A)^{1/\alpha}\left[\dfrac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} n\, vh\, Q^{J^{VH}} & if \quad n \ge N \end{cases} . \tag{2.18}$$

Equation (2.18) also represents the wage of the *m*-type HC, and accordingly the marginal value product of each type of HC is the same for all $n \in [0, N]$ and for all $n \in [N, 1]$, if $p_n^{1/\alpha}(1-n)$ and $p_n^{1/\alpha} n$, respectively, are constants.[33] Defining these constants as $p^{SH(1/\alpha)}$ and $p^{VH(1/\alpha)}$, respectively, we will have that $p_n = p^{SH}(1-n)^{-\alpha}$ and $p_n = p^{VH} n^{-\alpha}$, respectively. Substituting this result into equation (2.17), we have:

$$Y_n = \begin{cases} Y_n^{SH}(1-n)^{\alpha},\; Y_n^{SH} = A^{1/\alpha}\left[\dfrac{p^{SH}(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} sh\, SH_n(.|w)\, Q^{J^{SH}} & if \quad 0 \le n \le N \\ Y_n^{VH} n^{\alpha},\; Y_n^{VH} = A^{1/\alpha}\left[\dfrac{p^{VH}(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} vh\, VH_n(.|w)\, Q^{J^{VH}} & if \quad N < n \le 1 \end{cases} . \tag{2.19}$$

[33] *I.e.*, if they are constants since they are independent of n, but they can be dependent on the time t.

Then, since equation (2.8) implies that, in equilibrium, expenditures across FGs are equalised, that is, $p_n\, Y_n$ is constant for all n,[34] we find that $VH_n(.|w)$ and $SH_n(.|w)$ have to be constant and equal in all FGs using *VH* and *SH*, respectively.[35] To sum up,

$$p_n = p^{SH}\ (1-n)^{-\alpha},\ SH_n(.|w) = \frac{SH(.|w)}{N}\ ,\ SH(.|w) = \int_0^N SH_n(.|w)\,dn \ \text{ and } \ n \le N\,; \quad (2.20a)$$

$$p_n = p^{VH}\ n^{-\alpha},\ VH_n(.|w) = \frac{VH(.|w)}{(1-N)}\ ,\ VH(.|w) = \int_N^1 VH_n(.|w)\,dn \text{ and } n \ge N\,. \quad (2.20b)$$

The p^{VH} and p^{SH} are called two FGs price indexes. In the next chapters, we will see that they are (intra-country) different under different IT regimes and in the different countries, North and South (*i.e.*, at inter-country level), which is an important result in our analysis. Note that FGs with higher indexes produced with *SH* have a less productive technology and command higher prices. The converse is true for *VH* FGs.

To fully characterise equilibrium, we need to find the threshold FG *N*. To this end, we start noting that in FGs with $n < N$, firms using a ST obtain zero profit, while a firm using a VT would obtain negative profits. The opposite occurs for FGs $n > N$. In the threshold FG $n = N$, a firm that uses ST and a firm that uses VT should breakeven. That is, both equations (2.20a, b) should hold for $n = N$. This yields, at each moment in time, the following ratio of price indexes of FGs produced with VT and SH:

$$\frac{p^{VH}}{p^{SH}} = \left[\frac{N}{1-N}\right]^{\alpha}. \quad (2.21)$$

Equation (2.21) relates *N* to price indexes and tells us that the greater *N* and α, the greater p^{VH} is relative to p^{SH}, since $0 < \alpha < 1$. Moreover, since $p_n Y_n$ is constant for all n, then, from equation (2.19), it results that $p_n\, Y_n^{SH}\ (1-n)^{\alpha} = p_n\, Y_n^{VH}\ n^{\alpha}$, which, given equations (2.20a, b), is equivalent to writing $p^{SH}\, Y_n^{SH} = p^{VH}\, Y_n^{VH}$. Substituting Y_n^{SH} and Y_n^{VH} by the respective expressions given also in equation (2.19), and using once more

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[34] This is the standard property that factor shares are constant under Cobb-Douglas function. Take the following generic symmetric Cobb-Douglas function: $Y = x_1^{1/n}\ x_2^{1/n}\ \ldots x_n^{1/n}$. In competitive equilibrium, given symmetry: $p_1\, x_1^{1/n} = p_2\, x_2^{1/n} = p_n\, x_n^{1/n} = \frac{1}{n} Y$. The production function $Y = exp \int_0^1 ln\, Y_n\, dn$ is the limit of this case for $n \to \infty$.

[35] That is, $p_n\, Y_n = p_n\, Y_n^{SH}\,(1-n)^{\alpha} \equiv$ constant $\Rightarrow SH_n(.|w)$ is common in all industries using *SH*. From the same analyse, we have that $VH_n(.|w)$ is also common to all industries using *VH*.

equations (2.20a, b) and equation (2.21), the resulting N, as a function of the currently given variables determinants of economic viability of the technologies, VT and ST, is:[36]

$$N = \left\{ 1 + \left[\frac{Q^{J^{VH}}}{Q^{J^{SH}}} \frac{vh\,VH(.|w)}{sh\,SH(.|w)} \right]^{\frac{1}{2}} \right\}^{-1} . \qquad (2.22)$$

Equation (2.22) shows that if the technology is highly versatile-biased – *i.e.*, high $Q^{J^{VH}}/Q^{J^{SH}}$ –, and/or if there is a large relative supply of VH – *i.e.*, high $VH(.|w)/SH(.|w)$ –, and/or if there is a large productivity advantage of the VH over the SH in production – *i.e.*, high vh/sh –, the fraction of FGs employing VH and using VT is large. In other words, the relative supply of VT is large and so N is small. Thus, we can interpret N as a FGs sector bias or a technology margin. In terms of price indexes, small N implies that the relative price index of versatile FGs is also low and, conversely, the relative price index of specialised FGs is high, as equation (2.21) shows.

In this situation, the demand for specialised-specific IGs is high – as is apparent in equations (2.12a, b) and even more so in equations (2.25a, b) below –, increasing the demand for specialised-specific new designs and inducing R&D activities aimed at improving specialised-specific technologies. In summary, the structure of the stock of HC influences the direction of R&D through the FGs price channel – *i.e.*, there are stronger incentives to develop technologies when the FGs produced by these technologies command higher prices, as we will see below.[37]

In line with previous considerations, we assume that, when TK diffusion becomes possible at time t_0, the TK gap is relatively higher in the versatile-specific type, and that initial endowments of HC are such that the North is relatively VH abundant; *i.e.*,

$$\frac{Q_I^{J^{VH}}(t_0)}{Q_I^{J^{SH}}(t_0)} > \frac{Q_P^{J^{VH}}(t_0)}{Q_P^{J^{SH}}(t_0)} \quad \text{and} \quad \frac{VH_I(t_0|w)}{SH_I(t_0|w)} > \frac{VH_P(t_0|w)}{SH_P(t_0|w)} . \qquad (2.23)$$

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[36] In summary, with perfect competition in FGs, economic viability of either type of technology depends on the productivity difference (vh/sh) and on price of the specific type of HC, in addition to (as a result of complementarity) the relative productivity and prices of the specific IGs. The prices of HC depends on the quantities supplied to production, $m(.|w)$. In relative terms, the productivity-adjusted quantity of VH in production is $[vh\,VH(.|w)]/[sh\,SH(.|w)]$. As for the productivity and prices of IGs, they depend on complementarity with either type of HC, on the TK embodied, and on the elasticity of demand by the producers of FGs. These determinants are summed up in Q^{J^m} and the ratio $Q^{J^{VH}}/Q^{J^{SH}}$ is the relative productivity of the versatile-specific IGs.

[37] This price channel shows up in several papers by Acemoglu [*e.g.*, Acemoglu (2002a)], although always dominated by the market effect, which, in our case, is removed, as we will see in section 2.2.5.

From now on, whenever necessary, indexes I and P in equation (2.23) identify the countries, where index I (of 'innovator') represents the Northern country and index P (of 'Plagiarist') represents the imitator Southern country. Given the initial characteristics of the countries in equation (2.23) then, for example, – as we will see later on numerically – the North always produces more FGs with VT than the imitator country – *i.e.*, $N_P > N_I$.

Now, we want to determine the aggregate output, Y, and the total resources devoted to IGs, X. Remember that we have normalised the price of the aggregate consumption to be equal to unit (numeraire). The properties of price indexes under Cobb-Douglas technology imply that $exp \int_0^1 ln\, p_n\, dn = p_Y = 1$. Hence, with some algebra, we get that:

$$0 = \int_0^1 ln\, p_n\, dn = N\, ln(p^{SH}) + (1-N) ln(p^{VH}) + \alpha \left\{ 1 + (1-N)\, ln\left[\frac{1-N}{N}\right] + ln(N) \right\}. \qquad (2.24)$$

Then, by using equations (2.21) and (2.22), we can rewrite equation (2.24) as:

$$p^{SH} = exp(-\alpha) N^{-\alpha} = exp(-\alpha) \left\{ 1 + \left[\frac{Q^{J^{VH}}}{Q^{J^{SH}}} \frac{vh\, VH(.|w)}{sh\, SH(.|w)} \right]^{\frac{1}{2}} \right\}^{\alpha}; \qquad (2.25a)$$

$$p^{VH} = exp(-\alpha) (1-N)^{-\alpha} = exp(-\alpha) \left\{ 1 + \left[\frac{Q^{J^{VH}}}{Q^{J^{SH}}} \frac{vh\, VH(.|w)}{sh\, SH(.|w)} \right]^{-\frac{1}{2}} \right\}^{\alpha}; \qquad (2.25b)$$

$$\frac{p^{VH}}{p^{SH}} = \left[\frac{N}{1-N} \right]^{\alpha} = \left[\frac{Q^{J^{SH}} sh\, SH(.|w)}{Q^{J^{VH}} vh\, VH(.|w)} \right]^{\frac{\alpha}{2}}. \qquad (2.25c)$$

Therefore, equations (2.25a-c) ensure that the prices of *SH* specific FGs are higher and the prices of *VH* specific FGs are lower, when the relative domestic aggregate quality index or domestic measure of versatile-bias, $Q^{J^{VH}}/Q^{J^{SH}}$, is larger and when the relative supply of *VH*, $VH(.|w)/SH(.|w)$, is also large.

Now, we will determine the total resources devoted to IGs production, X, for any period. By using equations (2.9), (2.12a, b) and (2.20a, b), we have that:

$$X(j,t|SH) = \int_0^N x_n(j,t|SH)\, dn = sh\, SH(t|w) \left[\frac{p^{SH} A\, (1-\alpha)}{q(1-s_x)} \right]^{1/\alpha} q^{k(j,t|SH)[(1-\alpha)/\alpha]}; \qquad (2.26a)$$

$$X(j,t|VH) = \int_N^1 x_n(j,t|VH)\, dn = vh\, VH(t|w) \left[\frac{p^{VH} A\, (1-\alpha)}{q\, (1-s_x)} \right]^{1/\alpha} q^{k(j,t|VH)[(1-\alpha)/\alpha]}. \qquad (2.26b)$$

Thus, as a result of equations (2.25a, b) and (2.26a, b), we find that, after some calculations, the total resources devoted to IGs production are expressible as a function of the currently given aggregate domestic quality indexes and HCs at work:[38]

$$X = \int_0^{J^{SH}} X(j,t|SH)\,dj + \int_0^{J^{VH}} X(j,t|VH)\,dj =$$
$$= exp(-1)\left[\frac{A(1-\alpha)}{q(1-s_x)}\right]^{1/\alpha}\left\{\left[Q^{J^{SH}}\,sh\,SH(.|w)\right]^{\frac{1}{2}} + \left[Q^{J^{VH}}\,vh\,VH(.|w)\right]^{\frac{1}{2}}\right\}^2 . \qquad (2.27)$$

Also the equilibrium aggregate output at each time t – the composite FG from equation (2.8) – is expressible as a function of the currently given aggregate domestic quality indexes and HCs. In fact, from equations (2.17), (2.20a, b) and (2.25a, b), we obtain, after some calculations:[39]

$$Y = exp(-1)\,A^{1/\alpha}\left[\frac{1-\alpha}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}}\left\{\left[Q^{J^{SH}}\,sh\,SH(.|w)\right]^{\frac{1}{2}} + \left[Q^{J^{VH}}\,vh\,VH(.|w)\right]^{\frac{1}{2}}\right\}^2 . \qquad (2.28)$$

This representation, which features constant (equal to *2*) elasticity of substitution between the two types of HC pondered by the respective quality indexes, confirms our consideration about the type of substitutability present in production function (2.1). In fact, the value two – suggesting that ι is equal to ½ – implies substitutability between the two technologies in line with the literature – *e.g.*, Acemoglu and Zilibotti (2001) –, and will be useful to understand some differences between countries – for example, in the productive structure under different IT regimes.

Taking A as given, equation (2.28) clearly shows that the countries' growth rates are determined by the domestic (since there is no international mobility of production factors) HC accumulated and the time share spent at work – *i.e.*, by the preferences of individuals – and by the available (which without IT of IGs is the domestic) TK, accumulated either through innovation or through imitation.[40]

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[38] Or, equivalently, as explicitly dependent on the threshold N, instead of its determinants:

$$X = exp(-1)\left[\frac{A(1-\alpha)}{q(1-s_x)}\right]^{1/\alpha} Q^{J^{SH}}\,sh\;SH(.|w)\frac{1}{N^2} = exp(-1)\left[\frac{A(1-\alpha)}{q(1-s_x)}\right]^{1/\alpha} Q^{J^{VH}}\,vh\,VH(.|w)\frac{1}{(1-N)^2}$$

[39] Or, equivalently, as explicitly dependent on the threshold N, instead of its determinants:

$$Y = exp(-1)\,A^{1/\alpha}\left[\frac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} Q^{J^{SH}}\,sh\,SH(.|w)\frac{1}{N^2} = exp(-1)\,A^{1/\alpha}\left[\frac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} Q^{J^{VH}}\,vh\,VH(.|w)\frac{1}{(1-N)^2}.$$

[40] As we will see in chapter 3, in steady state equilibrium both the threshold FG N – and so FG price indexes –, and the time distribution of the HC among learning and doing will be constants.

Moreover, computing the 'net aggregate output' – *i.e.*, composite FG, Y, minus the resources devoted to IGs production, X – we observe that, all else being equal, both composite FG and net aggregate output are decreasing in the extent of monopoly power – $q \leq 1/(1-\alpha)$ – because of standard monopoly distortions.

Since the price paid for a unit of each type of HC, w^m, is equal to its marginal product, it can be presented in the following extended way:[41]

$$\frac{\partial Y}{\partial SH(.|w)} = w^{SH} = B\left[Q^{JSH}\, sh\, SH(.|w)\right]^{-\frac{1}{2}} sh\, Q^{JSH}; \tag{2.29a}$$

$$\frac{\partial Y}{\partial VH(.|w)} = w^{VH} = B\left[Q^{JVH}\, vh\, VH(.|w)\right]^{-\frac{1}{2}} vh\, Q^{JVH}; \tag{2.29b}$$

where, $B = exp(-1)\, A^{1/\alpha}\left[\frac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}}\left\{\left[Q^{JSH}\, sh\, SH(.|w)\right]^{\frac{1}{2}} + \left[Q^{JVH}\, vh\, VH(.|w)\right]^{\frac{1}{2}}\right\}$.

From equations (2.29a, b) – or from equation (2.18), and bearing in mind equations (2.20a, b) – we can conclude that the variation in the price paid for a unit of each type of HC (equilibrium wage dynamics) is given by the following equation:

$$\frac{\dot{w}^m}{w^m} = \frac{1}{\alpha}\frac{\dot{p}^m}{p^m} + \frac{\dot{Q}^{Jm}}{Q^{Jm}}. \tag{2.30}$$

From now on, we will denote a dot above a variable as the time change of that variable. Thus, equation (2.30) gives us the variation in the price paid for a unit of m-type of HC that depends on the path of domestic demand for m-type HC, which, in turn, by reason of the complementarity in FGs production, is proportional to the dynamic of both the respective FGs' price index and the aggregate domestic quality index.

As well from equations (2.29a, b), the inter-country wage inequality is given by:

$$\frac{w_P^m}{w_I^m} = \left(\frac{p_P^m A_P}{p_I^m A_I}\right)^{1/\alpha} \frac{Q_P^{Jm}}{Q_I^{Jm}}; \tag{2.31}$$

Equation (2.31) suggests that there is a higher wage in the North, as a result of the conjunction of international immobility of HC, differences in productivity, $A_I > A_P$, and differences in available TK – and also due to the value of the HC share in production.

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[41] Or, equivalently – clear from equation (2.18) –, $w^m = \left(p^m A\right)^{1/\alpha}\left[\frac{(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} \overline{m}\, Q^{Jm}$.

Also from equations (2.29a, b), we can also obtain the versatile premium, W – relative marginal product of VH or versatile wage, w^{VH}, relative to specialised wage, w^{SH}:

$$W = \frac{w^{VH}}{w^{SH}} = \left(\frac{p^{VH}}{p^{SH}}\right)^{\frac{1}{\alpha}} \frac{vh}{sh} \frac{Q^{JVH}}{Q^{JSH}} = \left(\frac{Q^{JVH}}{Q^{JSH}}\right)^{\frac{1}{2}} \left(\frac{vh\, VH(.|w)}{sh\, SH(.|w)}\right)^{-\frac{1}{2}} \frac{vh}{sh}. \qquad (2.32)$$

Equation (2.32) tells us that the versatile premium in any country (intra-country wage inequality) is greater when the parameter vh is relatively greater – *i.e.*, when the absolute productivity advantage of VH over SH is strong –, when technologies are more versatile biased – *i.e.*, when the relative versatile TK is higher –, and when VH employed in productive activity is relatively more scarce.[42] This variable, W, is of potential interest because it can be seen as a measure of the flexibility of the HC market. If it rises, it tends to make the income distribution more unequal, and *vice-versa*.

Finally, taking into account equation (2.32), the ratio of the versatile premium between countries (inter-country), $\boldsymbol{W}$, is:

$$\boldsymbol{W} = \frac{W_P}{W_I} = \left(\frac{SH_P(.|w)\, VH_I(.|w)}{SH_I(.|w)\, VH_P(.|w)}\right)^{\frac{1}{2}} \left(\frac{Q_P^{JVH}\, Q_I^{JSH}}{Q_I^{JVH}\, Q_P^{JSH}}\right)^{\frac{1}{2}}. \qquad (2.33)$$

This equation suggests that the versatile premium will be relatively greater in, for example, the imitator country, when it is relatively abundant in SH, the innovator country is relatively abundant in VH, Southern TK is more versatile-biased and Northern TK is more specialised-biased. Moreover, in two countries with the same (available) TK, which is the case of IT – as we will see in chapters 4 and 5 –, if, in addition, they have the same ratio $VH(.|w)/SH(.|w)$, the versatile premium should be the same, $\boldsymbol{W} = 1$.

In the next section, we conclude the outline of the production side of the economy, with a detailed description of the technology of R&D activities.

2.5. R&D sector

2.5.1. Overview

The R&D sector supplies IG firms with designs and R&D can be produced either by R&D specialised firms or by the R&D departments in IG firms. In particular, following Segerstrom *et al.* (1990), Grossman and Helpman (1991a, chap. 4, b, c), Aghion and Howitt (1992, 1996) and Barro and Sala-i-Martin (2004, chap. 7), among many others, R&D constitutes the search for new designs which lead to a new improved

quality of IGs. We opt for the quality ladder approach because it is more appropriate for our purposes.[43] In fact, it is more adequate to study the effects of competition between leaders and followers on the process of economic growth – *e.g.*, Gancia and Zilibotti (2003) –,[44] and IT enhances competition between producers of the same good.

In the standard models, all firms have access to the same R&D technology in the same conditions in each industry. It is implicitly assumed that top technology, once discovered, is automatically disclosed and, thereby, becomes immediately accessible to all potentially next successful researchers. That is, whilst the designs are domestically patented and the IG leader firm in each IGs industry – the one that produces according to the latest obtained patent – uses limit pricing to assure monopoly power, the TK it embodies is publicly accessible to all firms engaged in R&D activities aimed at generating further successful research. Thus, leader firms do not support R&D activities, since these cannibalise some of their own business – *i.e.*, applying resources to R&D is not profit maximising. Contrarily, the lure of monopoly rents drives potential entrants to support risky R&D projects in the search for the blueprint of a higher quality production.

However, as pointed out in Segerstrom and Zolnierek (1999) and Segerstrom (2004), among others, it is possible to contend that leader IGs firms in each industry can have R&D cost advantages over all other firms, and it is also possible that there are diminishing returns in R&D efforts for individual firms in each industry.[45] In the former case, when leader firms have a small R&D cost advantage, follower firms support all the R&D investment – the purest form of the process of 'creative destruction' results. On the other hand, when leader firms have a high R&D cost advantage, only they participate in R&D – the purest form of 'trustified capitalism' emerges. In the latter case, both leader and follower firms engage in R&D. But, when the R&D cost advantage of leader firms is large, they are responsible for a greater share of successful research, and *vice-versa*.

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[42] We can avoid that W becomes smaller than 1 considering $[vh/sh] > \{[VH(.|w)Q^{J^{SH}}]/[SH(.|w)Q^{J^{VH}}]\}$.

[43] The two approaches – vertical and horizontal – complement each other. Thus, the possibility that R&D also generates new varieties of IGs – *i.e.*, the combination between the expanding variety model and the quality ladder model – will be explored in future work.

[44] The quality ladder approach explains successful researches and growth by intentional R&D activities of private firms, formalising Schumpeter's (1934, 1942) vision of the continuing process of 'creative destruction' or Schumpeter's (1928) vision of the continuing process of 'trustified capitalism' or a mix of both Schumpeterian visions.

[45] Until the brief discussion in the 1st edition of Barro and Sala-i-Martin (2004, chap. 7), in 1995, this issue has been little explored. Recently, many works have analysed this topic of discussion assuming different points of view – *e.g.*, Aghion *et al.* (2001), Dinopoulos and Syropoulos (2001, 2004) and Etro (2004).

Considering that, in the study of Segerstrom and Zolnierek (1999) and Segerstrom (2004), the advantage resulting from TK diffusion is the same for all Southern firms, it can be proved that TK diffusion has no effect on the type of IG producers – leaders or followers. Moreover, the empirical evidence on returns to scale of R&D efforts at a firm level seems to be inconclusive – *e.g.*, Bound *et al.* (1984), Kortum (1993), Lach (1995) and Thompson (1996). Thus, given our main objective – to build a dynamic general equilibrium model to analyse the effects of international TK diffusion under different IT regimes on economic growth and on inter and intra-country wage inequality, in a set-up of no scale effects and with HC accumulation –, we analyse the case where: (i) leader and follower firms in the same country face the same (or much the same) costs, which is guaranteed due to the usual idea that the TK designs embody tends to be publicly accessible; and (ii) there are constant returns to R&D expenditures – *e.g.*, Romer (1993).

However, we consider that leader firms can use some strategies to preserve their economic revenue by delaying or retarding the successful research explored by follower firms, by reducing the flow of TK spillovers present in their successful research – *e.g.*, Dinopoulos and Syropoulos (2001, 2004) and Sener (2003). Incorporating this aspect in the R&D specification together with the assumption that the increase in scale also entails (a lot of generic) costs enables us to eliminate the scale effects; *i.e.*, allows us to analyse the mechanisms through which IT affects TK progress in contexts without scale effects.

The competitive market of blueprints implies free entry by follower firms into the IGs sector, in order to explore the next quality improvement. Moreover, the R&D supported by each follower firm is done under the same R&D technology, independently of whether it is done internally – in its R&D department – or, alternatively, externally – in R&D specialised firms. Each potential entrepreneur may target his research efforts at any of the continuum of state-of-the-art IGs; that is, it may engage in any IG. Like in most of the quality-ladder models, regardless of the country, R&D behaviour does not vary across IGs, firms or over time. Since there are a large number of small firms capable of performing R&D for either industry – *i.e.*, there is perfect competition among follower firms in each IG –, R&D expenditures of individual follower firms will be negligible.

As mentioned earlier, another specific feature of our framework is that TK change is directed. The choice will be driven by the comparison between the expected profits from producing and selling some of the highest quality of $x_n(\cdot|VH)$ or of $x_n(\cdot|SH)$ or by not entering the market. In other words, the choice will be driven by market incentives.

Thus, the degree to which new technologies are versatile-complementary – versatile-biased TK change – is determined endogenously. This also occurs in, for example, Acemoglu (1998, 2002a, b, 2003), Kiley (1999) and Acemoglu and Zilibotti (2001) but for skilled and unskilled complementary technologies and motivated by the dominance of the market effect over the price effect.

In the latter works, when there are more (exogenous given) skilled workers, the market for skill-complementary technologies is larger and this effect dominates the price effect. As a result, the IG firm that produces according to the last obtained patent is able to gain higher profits and more effort will be devoted to the innovation of skill-complementary technologies. However, since Jones (1995a, b), most of the literature suggests that the market effect must be removed in endogenous growth models. For this reason, the results obtained by Card and DiNardo (2002) (may) are not surprising. They suggest that the skill-biased technical change hypothesis present in the above works is not very helpful on its own. Ignoring the debate on the subject, we aim at understanding the effects of the imitation activity in contexts where market size effects are not crucial – *i.e.*, in a context where a larger market for TK does not lead to more research.

Furthermore, unlike Acemoglu and Zilibotti (2001) but in compliance with Bernard and Jones (1996a, b), Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999), among others, we assume that there are important differences in the available TK across countries (generally even in steady state). In fact, Northern firms devote resources to innovative R&D to discover higher quality IGs and Southern firms devote resources to imitate R&D to adopt state-of-the-art quality Northern IGs. Thus, our framework is based on the 'catching up theory', and we have formalised it by combining the relative 'backwardness hypothesis' introduced by Veblen (1915) and 'imitation capacity'.[46]

The latter captures the idea that (Southern) countries may differ in their effort and ability to imitate new technologies. The 'imitation capacity' is determined by the South's level of development, which can obstruct imitation – *e.g.*, Cohen and Levinthal (1990), Evanson and Westphal (1995) and Duffy and Papageorgiou (2000) –, by policy variables – *e.g.*, Kang (2002) and Aghion *et al.* (2000) –, by stock of HC – *e.g.*, Keller (1996), Xu (2000), Caselli and Coleman (2001) and Kneller and Stevens (2002) – and by IT – *e.g.*, Coe and Helpman (1995), Coe *et al.* (1997) and Wang and Xu (1999, 2000).

[46] Note that, without IT effects, the R&D undertaken by firms in the North is unaffected by the R&D undertaken by firms in the South.

After this overview, this section is structured according to the following plan. In subsection 2.5.2, we present the general specification for the probabilities of successful R&D – innovation and imitation. In subsection 2.5.3, we compute the flows and duration of monopoly profits in the particular case considered. In subsection 2.5.4, we describe the equilibrium probabilities of successful research and the R&D effort. In subsection 2.5.5, we conclude the section with the behaviour of the aggregate quality indexes.

2.5.2. Specification for the probabilities of successful R&D

The value of the leading-edge patent depends on the profit-yields accruing at each period t by the monopolist, and on the duration of the monopoly power. The duration, in turn, depends on the probability of a new successful research, which creatively destroys the current leading-edge design. The probability of a successful research is, thus, at the heart of R&D. We could presume a R&D specification with possible interaction across IGs or, at least, between the two types of IGs. In particular, each one could benefit either from its own stock of past successful researches, or from a combination of its own stock and stock from others. However, to simplify, we consider that, at least in innovation activity, the current R&D in an IG benefits only or is state dependent from its own stock of past successful research – *i.e.*, it builds upon its predecessors.

Let Z index the country, and $pb_Z(k,j,t|m)$ denote the instantaneous probability at time t – or a Poisson arrival rate – of successful innovation ($Z=I$) or imitation ($Z=P$) – *i.e.*, $Z \in \{I, P\}$ –,[47] in the next higher quality $[k_Z(j,t|m)+1]$ in IG industry j, which complements m-type HC.[48] In formal terms:

$$pb_Z(k,j,t|m) = rs_Z(k,j,t|m)\, ll_Z(k,j,t|m)\, lm_Z(t|m)\, cp_Z(k,j,t|m)\, btp_Z(k,j,t|m)^{\Gamma_Z}, \qquad (2.34)$$

Where:

(i) $rs_Z(k,j,t|m)$ denotes the flow of total resources devoted to R&D in country Z in IG j at time t, measured in units of the domestic composite FG, in order to get a blueprint of a new quality of the IG. The assumption that R&D inputs are in terms of composite FG run away from the lab-equipment characteristic of our framework – in the 'words' of

[47] By instantaneous probability, we mean that $pb_Z(k,j,t|m)dt$ is the probability that a certain firm will innovate or imitate during the time interval from t to $t+dt$, where dt is an infinitesimal increment of time. Alternatively stated, R&D success bears a Poisson probability distribution with an arrival rate $pb_Z(k,j,t|m)$. In any case, stochastic successful researches are the engine of quality evolution and growth.

[48] To simplify, we have omitted for now comments related with index m. However, we can still state that the role of heterogeneity between existing types of industries is not completely neglected, as suggested, especially, in the empirical evidence in the Industrial Organization literature – see Cohen *et al.* (1987).

Rivera-Batiz and Romer (1991), and instead of the knowledge driven, in which labour is the only internalised input – and serve to highlight that changes in versatile premium are not driven by changes in the level of R&D activity. Alternatively, if, *e.g.*, R&D uses more *VH*, then the versatile premium will increase in a period of high R&D activity, similarly to the versatile-biased technology adoption effect emphasised in, *e.g.*, Nelson and Phelps (1966), Galor and Tsiddon (1997) and Greenwood and Yorukoglu (1997).

(ii) $ll_Z(k, j, t \mid m) = \beta_Z \, q^{k_Z(j,t|m)}$, $\beta_I > \beta_P > 0$, $k_I \geq k_P$, represent learning by R&D in IG j at time t, in country Z, as the positive learning effect of accumulated public TK from past successful research in the IG[49] – *e.g.*, Grossman and Helpman (1991, chap. 12), Barro and Sala-i-Martin (1997) and Connolly (1999).[50] This term takes into account that laggard firms can equally participate in each R&D activity for the next quality improvement without having climbed all of the rungs of the quality ladder themselves, due to the public accessibility of the TK. Thus, R&D in IG j is (by assumption) 'state dependent' on its past successful researches, which makes future learning easier. This term is also in compliance with the 'standing on giants' shoulders' effect considered in many R&D growth models – *e.g.*, Caballero and Jaffe (1993) and Howitt (1999).[51]

The β_Z is the positive coefficients on past successful research experience. We assume that $0 < \beta_P < \beta_I$ since there should be greater learning effects in the North, which is the TK leader country, rather than the South.

For IG j, $q^{k_Z(j,t|m)}$ is the highest quality level attained through innovation or imitation. If the country has no innovative experience then $q^{k_I(j,t|m)} = 0$, and if the country has no imitative experience then $q^{k_P(j,t)} = 0$. We assume that each country has experience in at least one type of research, and if not $ll_Z(j,t|m) = 1$. Since the particular rung of a quality ladder at which IG j is located is indexed by k, without IT effects, $k_I \geq k_P$. This is because the Southern imitator firms need only imitate technologies on one quality rung (*i.e.*, one level) above the current Southern quality level, since they are only selling the imitated IGs domestically and the country is the TK follower.

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[49] Conceptually, it is important to distinguish between this learning effect and the conventional learning-by-doing. Here, learning is a process of quality improvement through past successful research, which justifies the introduction of R&D subsidies – as we will explain below –, while learning-by-doing is usually formulated as the decline of production costs induced by the cumulative experience of production.

[50] In particular, Connolly (1999) calls this positive external spillover effects induced by past successful research in an IG as learning-to-learn effect.

[51] The 'standing on giants' shoulders' effect postulates that a higher frontier TK – *i.e.*, a higher stock of existing designs – increases the probability of successful research since an investment in (intensive) R&D creates the opportunity to exploit a TK spillover from the technology frontier to the successful researchers.

(iii) $lm_Z(t|m) = m_Z(.|w)^{-\xi_Z}$, $\xi_Z > 0$, is the adverse effect of market size, capturing the idea that the difficulty of introducing new quality-adjusted IGs and replacing old ones is proportional to the size of the market measured by the respective HC employed.[52] That is, for reasons of simplicity, we reproduce the costs of scale increasing in R&D.[53]

This is due to coordination among agents, organisational and transportation costs, processing of ideas, information and marketing, as reported in works such as Becker and Murphy (1992), Quah (1998), Alesina and Spolaore (1997), Dinopoulos and Segerstrom (1999a) and Dinopoulos and Thompson (1999). The latter work, in particular, provided micro foundations for this effect in a model of growth through variety accumulation. For example, Easterly *et al.* (1994) remove similarly the scale effects by assuming that the cost of adoption new TK is proportional to the size of the market (work force).

Moreover, in its purest form, creative destruction implies that IGs firms support R&D costs in order to become leaders and, once they succeed, they sit on the laurels of their past accomplishments and do not try to protect their rents. Since this is not easily accepted, we assume that leader firms try to protect their rents by extending the expected duration of their monopoly power; *i.e.*, by making the probability of the next successful research more difficult. As the rents of leader firms are proportional to the market size, to keep the analysis tractable, we presume that *lm* also includes leader firms' strategies usually consisting of technical barriers – technical rent protecting activities –, which safeguard their economic rents by delaying the next successful research. This is possible since firms in the market gain specific TK (leading to a longer loyal customer base).

Thus, we distinguish successful researches leading to creative destruction – which (partially) drive economic growth – from some leader firms' strategies that slow down the process of creative destruction – or, in other words, that make the knowledge designs embody not immediately, totally and publicly accessible to all potentially next successful researchers. Assuming that these rent protecting activities are also included in *lm*, we think that (strong) market leaders acquire some specific TK that, once used, can – at least partially and temporarily – insulate them from the threat of leapfrogging by potential followers. In summary, the *lm* term helps us to make the strong assumption of pure creative destruction more flexible and is appropriate for our purposes. This is in

[52] Dinopoulos and Segerstrom (1999a) call this the 'permanent-effects-on-growth' specification, since it implies a steady state successful research and growth rate that is sensitive to various economic policies.
[53] As already stated, see, for example, Jones (1995b, 1999) and Segerstrom (1998) for a detailed discussion of scale effects and different ways to avoid them.

accordance with, for example, Dinopoulos and Syropoulos (2001,°2004) and Sener (2003), in which the scale effects are removed by assuming rent protecting activities.

Indeed, Levin *et al.* (1987), Eisenhardt and Brown (1998) and Cohen *et al.* (2003) identify and supply survey-based evidence on the extent of these activities. Such strategies include trade secrecy,[54] camouflaging their successful researches through the technological complexity of their goods,[55] time-pacing strategies,[56] patent blocking,[57] and employment of legal teams to litigate potential patent infringements, always with the aim of limiting the flow of TK spillovers to potential new leader firms.

We also admit that our framework is, to a certain extent, related with several recent works, like Jovanovic and Rob (1990), Stein (1997) and Canton and Uhling (1999). The former work formalises the distinction between extensive research, directed at major breakthroughs, and intensive research, aimed at refining such fundamental breakthroughs. Each product generation starts with a quality breakthrough and is then followed by a single improvement. In Stein (1997), firms compete on product quality and on distribution cost. Successful researches (in product quality) spill over to new firms, while distribution costs are taken to be firm specific. Canton and Uhling (1997) distinguish fundamental successful research leading to creative destruction from marginal successful research that decreases the creative destruction.

The parameter ξ measures the effectiveness of all items – general costs and rent protecting activities – related with increasing the difficulty of conducting R&D owing to the increase of the market size. Thus, for a given level of $m(.|w)$, lower values of ξ can be associated with a lower level of R&D difficulty. In fact, we have that the benefits of scale are over, total or partially counterbalanced by all associated items, when $\xi > 1$, $\xi = 1$, or $0 < \xi < 1$, respectively. Since there is no strong reason to believe that the set of all items is different from country to country and bearing in mind our purposes, we assume that ξ is the same in all. Moreover, to simplify, we have not considered some

[54] For example, Coca-Cola maintains the secrecy of its formula.
[55] Microsoft has been adding some features to its Windows operating system making it more complex and more difficult to understand. Intel has been producing increasingly smaller, more sophisticated microprocessors, thus making them more resistant to reverse engineering.
[56] *I.e.*, strategies aimed at expanding manufacturing capacity at regular intervals independently of the pace of new high-quality discoveries. Eisenhardt and Brown (1998) state that this is the case of Intel, Cisco Systems, Emerson Electric, Gillette, Netscape, SAP, Sony, Starbucks and 3M.
[57] *I.e.*, strategies aimed at building a fence around a major result from research by patenting several related secondary inventions without necessarily introducing them into the market, to discourage the circumvention of existing patents by potential follower firms and deter competing successful research from entering the market – *e.g.*, Cohen *et al.* (2003).

possible differences in the nature of the distinct items, and that $\xi_I = \xi_P = 1$ in order to be able to rule out scale effects. Actually, this is essential in understanding the effects resulting from international TK diffusion in contexts without market size effects.

Hence, our framework differs from earlier models of Schumpeterian growth in which, when in the presence of HC accumulation (or population growth), the rates of successful research and long run growth increase exponentially, as does the scale measured by the size of its HC (or of its population) employed – *e.g.*, Segerstrom *et al.* (1990), Grossman and Helpman (1991b, c) and Aghion and Howitt (1992).[58]

Apart from the endogenous growth debate on the subject, remember that the scale effects property embodied in equation (2.34), when *lm* is not considered, is inconsistent with, for example, the post-war time series evidence present in Jones (1995a) and the recent U.S. manufacturing industry data (1969-1988) found in Zachariadis (2003).

(iv) $cp_Z(k, j, t \mid m) = \zeta_Z^{-1} q^{k_Z(j,t|m)(-1/\alpha)}$, $0 < \alpha < 1$, $\zeta_I > \zeta_P > 0$, $k_I \geq k_P$, is the adverse effect – cost of complexity – caused by the increasing complexity of the IG upon which firms in industry j at time t are attempting to obtain the next patent in country Z. That is, as quality-adjusted IGs become more complex with each step up the quality ladder, research is progressively more difficult, implying a lower probability of success, all else being equal. Therefore, every time successful research occurs in an IG, the (next) instantaneous probabilities of R&D success decrease for any given levels of R&D effort or, in other words, the cost of R&D increases. We find this effect in a significant number of search models of R&D-based economic growth – *e.g.*, Jovanovic and Rob (1990), Barro and Sala-i-Martin (2004, chap. 7) and Kortum (1997). We can also consider that this effect is a result of 'fishing for ideas' which implies that catching fish – *i.e.*, having a new idea – becomes less likely when a large number of fish have already been caught; *i.e.*, diminishing TK opportunities – *e.g.*, Evenson (1984) and Kortum (1993).

This term further includes a country-firm specific fixed cost of research, ζ_Z. Following a common position found in the literature, we assume that, in each IG, the fixed cost of innovation is higher that the cost of imitation, $\zeta_I > \zeta_P > 0$, as new ideas gradually decrease and/or are progressively more complex to implement, in line with Mansfield *et al.* (1981) and Teece (1977), among others.

[58] Similar considerations apply to Romer (1990b) type growth models based on variety expansion.

The product resulting from the specifications for $ll_Z(k,j,t|m)$ and $cp_Z(k,j,t|m)$ corresponds to one extension of the specification $\phi(\kappa_j)$ proposed in the probability of success by Barro and Sala-i-Martin (2004, chap. 7) and Boucekkine *at al.* (2002), among many others. In fact, we add coefficient β in ll_Z, which, in our case, as well as coefficient ζ, possess different values according to the country that they report.[59] Moreover, in Barro and Sala-i-Martin (2004, chap. 7), the specification for $\phi(\kappa_j)$ is necessary for a constant growth rate of the economy over time and, in our framework, both the ll_Z and cp_Z terms seek to achieve the same objective, as will become apparent latter.

(v) $btp_P(k,j,t|m) = exp[PE(j,t|m) + IT(j,t|m)]\{g_1[\tilde{m}(t|w)]\}^{\bar{\sigma}_1}\{g_2[\tilde{Q}^{Jm}(t),d]\}^{\sigma_2(\tilde{Q}^{Jm},t)}$, with $\bar{\sigma}_1 > 0$, $\sigma_2(\tilde{Q}^{Jm},t) = -\bar{\sigma}_2 + \tilde{Q}^{Jm}(t)$ and $0 < \tilde{Q}^{Jm}(t) < 1$ is a catching up term, specific to the imitator, that sums up positive effects of 'imitation capacity' and 'backwardness hypothesis' on the instantaneous probabilities of R&D success of imitation.[60] It implies that the complexity effect can be strongly diminished in the imitator country. Parameter Γ_Z in equation (2.34) takes on the following values: $\Gamma_I = 0$ and $\Gamma_P = 1$.

The exponential of the catching up term captures two important determinants of imitation capacity, domestic policies promoting R&D – index number *PE* – and openness to IT and other IT policies – index number *IT*. From now on, we consider the simplifying assumption that the *PE* and *IT* terms in the catching up term are not specific to each IG. More specifically, index *PE* includes the correction of distortion of the foreign exchange markets and the financial development index – *e.g.*, Kang (2002) – and the creation and stimulation of national R&D organisations and other public goods like physical and institutional infrastructures – *e.g.*, Aghion *et al.* (2000, 2001).[61] And index *IT* can be instrumented by the following constant elasticity specification:[62]

$$IT(t) = M(t)^{\eta_1} FTFP(t)^{\eta_2} EX(t)^{\eta_3}, \text{ where:} \qquad (2.35)$$

– *M* denotes the imports of machinery and transport equipment per HC. It embodies innovations that are not available in local economy, and from which local researchers

[59] Our procedure enables us to highlight the differences between North-South R&D, which, in our model, is crucial. In addition, we can distinguish between learning by past R&D and learning-by-doing.

[60] Note that if we presume that it does not make much sense to think of international TK imitation without the vehicle IT – *i.e.*, in autarky – and/or another vehicle – *i.e.*, FDI and/or international mobility of production factors –, then in equation (2.34) $bpt_P = bpt_I = 1$. Furthermore, pb_P would be impossible since $\beta_P < \beta_I$ and $\zeta_P < \zeta_I$ only make sense if we consider that imitation is possible.

[61] The *btp* term is specific to R&D imitation – remember that it takes the value *1* for R&D innovation. Thus, we consider, in particular, that there are no distortions in the innovator country neither at a political level nor at IT level – so it stops having positive political effect as well as positive IT effects.

[62] Note that the influence of *IT* on the probability of successful imitation only becomes effective under IT.

may gain insights. The importance of this term has been stressed by a great number of studies – *e.g.*, DeLong and Summers (1993), Coe and Helpman (1995), Coe *et al.* (1997), Connolly (2003) and Wang and Xu (1999, 2000).

– *FTFP* is the foreign total factor productivity. It represents a qualification on the transmission of TK spillover effects from the major trade partners. Thus, for given levels of imports by a country, the effect on *pb* increases with the R&D contents of those imports, as argued by Coe *et al.* (1997). In short, total factor productivity (*TFP*) of the developed trade partners of each country is taken as a proxy to such R&D contents.

– *EX* represents the level of exports per HC. It is a measure of international openness and other IT policies namely international integration – *e.g.*, Grossman and Helpman (1991a, chap. 11) – and so of exposure to international competition, to new international ideas and to new methods of production. It serves as a proxy of communication among agents in different countries: the higher its value, the greater the number of personal contacts and, thereby, the exchange of information between domestic and foreign individuals. Moreover, exports contribute to financing imports. Its importance for growth has been emphasised by Feder (1983), the World Bank (1987), Levine and Renelt (1992) and Clerides *et al.* (1998), among many others.[63]

Function $g_1[\tilde{m}(t\,|\,w)]$ represented in figure 2.2 below is formally given by:

$$g_1[\tilde{m}(t\,|\,w)] = 1 + \frac{exp[\tilde{m}(t\,|\,w)]}{1 + exp[\tilde{m}(t\,|\,w)]}. \tag{2.36}$$

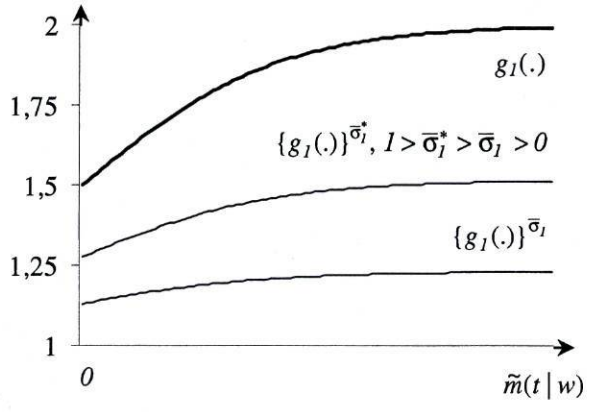
Where $\tilde{m}(t\,|\,w) \equiv m_P(t\,|\,w)/m_1(t\,|\,w)$ is the imitator's relative level of employed *m*-type HC or, in other words, the gap in *m*-type HC at work. That is, we assume that HC enhances the imitation capacity and so speeds up convergence with the innovator country – in line with Nelson and Phelps (1966) and, more recently, with Benhabib and Spiegel (1994), Aghion *et al.* (2000) and Kneller and Stevens (2002), among others.

Parameter $\overline{\sigma}_1$ in the catching up term can be interpreted as a 'semi-elasticity' of the probability of successful imitation with respect to the HC gap, $\tilde{m}(.\,|\,w)$. It indicates how quickly the probability of successful imitation rises as the HC gap narrows – see figure 2.2. It must be greater than *0* but we contend that it is not much greater than *0* in order to guarantee an adequate impact on the probability of successful imitation.

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[63] The direction of causality between economic growth and *IT*, which we omit here, has been of concern in a number of empirical studies on the subject. We quote the work of Frankel and Romer (1999), which suggests that simultaneity, when present, enhances the positive relationship.

Figure 2.2. Human capital gap function in the catching up term



Function $g_2[\tilde{Q}^{J^m}(t), d]$ – similar to Papageorgiou (2002), in a model of growth through horizontal accumulation of IGs – basically attempts to capture the benefits from relative backwardness, written formally as:

$$g_2[\tilde{Q}^{J^m}(t), d] = \begin{cases} 0 & ,if \quad 0 < \tilde{Q}^{J^m}(t) \leq d \\ -\tilde{Q}^{J^m}(t)^2 + (1+d)\tilde{Q}^{J^m}(t) - d & ,if \quad d < \tilde{Q}^{J^m}(t) < 1 \end{cases}. \qquad (2.37)$$

Where $\tilde{Q}^{J^m}(t) \equiv \frac{Q_P^{J^m}(t)}{Q_I^{J^m}(t)} = \frac{Q_P^{J^m}(t)/J^m}{Q_I^{J^m}(t)/J^m} = \frac{q^{k_P(av,t|m)}}{q^{k_I(av,t|m)}} \equiv \tilde{q}(t|m)$ is the relative TK level of the imitator's m-specific IGs;[64] provided that the gap is big – *i.e.*, if $\tilde{q}(t|m)$ is above threshold d – then the country can benefit from an advantage of backwardness, as in Barro and Sala-i-Martin (1997). When the gap is wider so that $\tilde{q}(t|m)$ is below threshold d, backwardness is no longer an advantage. In this case, the Southern country, which we call stagnant, shows no potential for rapid growth – *e.g.*, African countries. Nevertheless, equation (2.37) guarantees that $g_2(.)$ is non-negative and so economically feasible.

Thus, we do not consider the relative backwardness hypothesis in original mode, which states that the rate of TK progress in a relatively backward country is an increasing function of the gap between its own level of TK and that of the more advanced country. Our specification favours an initial quadratic (hump-shaped) imitation function – $g_2(.)$, afterwards transformed by its exponent – and also depends on the TK gap threshold,

[64] *I.e.*, we assume that the probability of successful imitation in IG j is state dependent from all past successful research in all IGs of its type in both countries, contrarily to the probability of successful innovation in IG j, which is only state dependent on its own stock of past successful research. Alternatively, we can assume that the TK gap between countries is equal in all IGs, J^{SH} and J^{VH}, and the imitator country gains in all j industries with relative backwardness or we can admit the representative domestic IG industry defined as an average of all domestic industries of its type, *av*.

$d \in (0, 1)$, which dictates whether a follower country can imitate or not, as illustrated in figure 2.3 below. The option for this function is motivated by the notion of contiguous knowledge, which asserts that TK is only diffused up to a certain point, and thereby the larger the initial TK gap is, the higher the catching up, does not apply unconditionally. Because of this property, a Southern country can imitate existing TK and grow rapidly only when it is sufficiently close to the TK frontier. Hence, the interactive process between imitation capacity and TK gap starts only if an adequate minimum development base is initially present, there being then limits to the advantages of backwardness.

The exponent of function $g_2(.)$, $\sigma_2(\tilde{Q}^{J^m}, t) = -\overline{\sigma}_2 + \tilde{Q}^{J^m}(t)$, can be interpreted as a (variable) elasticity of the probability of successful imitation with respect to $g_2(.)$. Given the small positive value of function $g_2(.)$, <1, $\sigma_2(.)$ must be negative to guarantee that the TK progress in the imitator country benefits from the relative backwardness of the country. This expected value of $\sigma_2(.)$ is obtained because of $-\overline{\sigma}_2$ and, thereby, it affects how quickly the probability of successful imitation falls as the TK gap narrows.[65]

According to figure 2.3b, for wide TK gaps, backwardness does not represent any benefit. However, after point *d*, the probability of successful imitation increases and afterwards declines, reflecting the benefits of backwardness on imitation. When the TK gap between countries reaches point *max*, the impact on the probability of successful imitation as a result of $g_2(.)$ is completely absorbed, and after this point it is 'negative' – mirroring the gradual saturation of imitation opportunities. Even so, since the country now has a TK greater than *d*, it can imitate if its imitation capacity – which tends to be improved – counterbalances the loss of benefit associated with backwardness.

Therefore, our framework is in line with Cohen and Levinthal (1990) and Verspagen (1994), among others. The former shows that a model of conditional catching up fits empirical data better than a model where the wider the initial TK gap the higher the catching up potential. The latter suggests that for a lagging country to be able to absorb TK from the leader, it must have accumulated sufficient prior TK. Our framework is also able to explain club convergence members – *e.g.*, Baumol (1986) and Quah (1997); *i.e.*, the fact that convergence is restricted to a subgroup of countries, those which

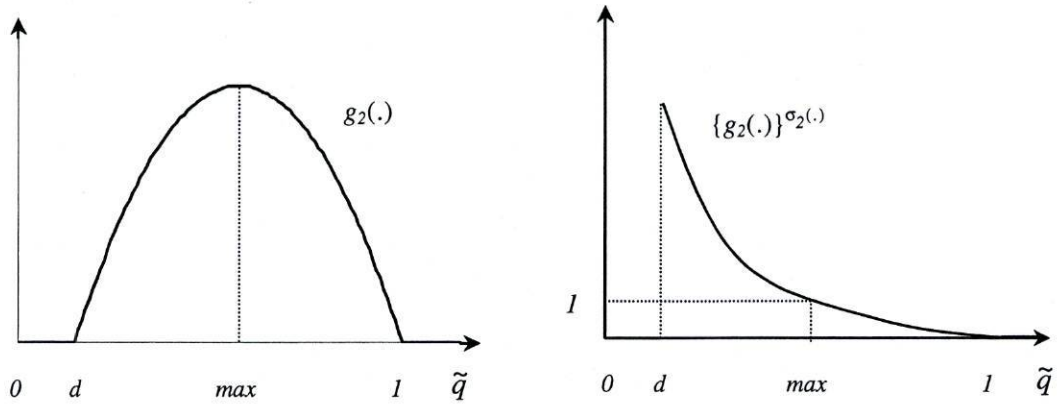
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[65] In fact, if (i) $\sigma_2(.) < 0$ the impact on pb_P owing to $g_2(.)$ is strong; *i.e.*, $\{g_2(.)\}^{\sigma_2(.)} > 1$, (ii) being completely absorbed if $\sigma_2(.) = 0$; *i.e.*, $\{g_2(.)\}^{\sigma_2(.)} = 1$, which holds when $0 = -\overline{\sigma}_2 + \tilde{Q}^{J^m}$, (iii) and if $\sigma_2(.) > 0$ is 'negative'; *i.e.*, $\{g_2(.)\}^{\sigma_2(.)} < 1$.

are able and willing to invest in innovation and imitation, whereas other southern countries will remain on the sidewalk – at least in the absence of some aid (big-push).

Figure 2.3. Technological knowledge imitation function

a) Quadratic original function **b) Transformed function by its exponential**



To sum up, in $btp_P(k,j,t|m)$, the backwardness hypothesis is specifically captured by the ratio $\tilde{Q}^{J^m}$, and the imitation capacity by terms *PE*, *IT*, $\tilde{m}(.|w)$ and d.[66]

Finally, still on the topic of the catching up term, we draw attention to the fact that initially we have considered the two alternative specifications – see Appendix 2, section A2.1. However, we drop them due to the results obtained in the transitional dynamics – in the next chapters – namely in terms of speed of convergence, which is very slow.

Bearing in mind terms (i)-(v), equation (2.34) can be re-written as follows:

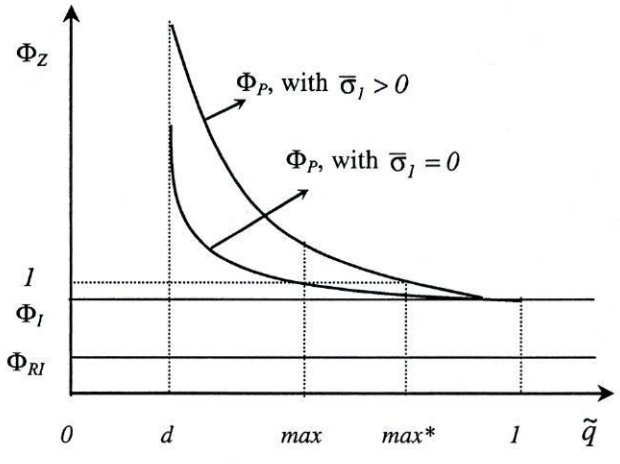
$$pb_Z(k,j,t|m) =$$

$$= rs_Z(k,j,t|m)\ \beta_Z\ \zeta_Z^{-1}\ [exp(PE+IT)\ g_1(.)^{\sigma_1}\ g_2(.)^{\sigma_2(.)}]^{\Gamma_Z}\ m_Z(t|w)^{-1}\ q^{k_Z(j,t|m)\left[\frac{\alpha-1}{\alpha}\right]} \cdot \quad (2.38)$$

Considering that $\Phi_Z \equiv \beta_Z\ \zeta_Z^{-1}\ [exp(PE+IT)\ g_1(.)^{\sigma_1}\ g_2(.)^{\sigma_2(.)}]^{\Gamma_Z}$, which can be called the experience-adjusted probability of successful research, figure 2.4 below provides and clarifies its importance – where $\beta_I/\zeta_I = \beta_P/\zeta_P$, in order to keep the depiction simpler. With regard to the relation between Φ_P and Φ_I, we have that $\Phi_P - \Phi_I \geq 0$. Moreover, the positive effect induced by backwardness and captured by $\{g_2(.)\}^{\sigma 2(.)}$ cannot be completely exhausted – *i.e.*, it cannot be equal to *1* and after that less than *1* – when the TK gap between countries reaches point *max*. It can be expanded by the imitation capacity – see points *max* and *max** for two different impacts of the gap in HC at work.

It can be further expanded by *PE* and *IT*. Thus, for the same relative level of *m*-type HC employed and relative TK level of the South's *m*-type IGs, $\Phi_P - \Phi_I$ under IT is greater since the imitation capacity is better due to term *IT*; *i.e.*, owing to openness to IT and other IT policies. Bearing in mind the relation between Φ_P and Φ_I, we can already argue that the convergence between countries in our case is driven by TK spillovers, whereas in the neoclassical model, it is entirely driven by diminishing returns to capital.

Figure 2.4. Terms Φ_Z in the probability of success



Following Barro and Sala-i-Martin (2004, chap. 7), in the next subsections, we compute the flows and duration of monopoly profits, the equilibrium probability of successful research and the behaviour of the aggregate quality indexes.

2.5.3. Explicit flow and duration of the monopoly profit

We will now proceed with the presentation of the leader firms' flow of profit at each time. Concurrently, we compare the incremental profits of follower firms taking over the leader position, $\Delta\Pi_Z(k,j,t|m) = \Pi_Z(k,j,t|m)$, with the incremental profits of leader firms replacing themselves, $\Delta\Pi_Z(k,j,t|m)$.[67] Taking into account equations (2.10), (2.11b) and (2.26a, b), we have:[68]

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[66] Thus, even if Southern countries have the same initial TK level (> *d*), some can grow faster than others.

[67] Note that since the leader firm will now be selling a product twice as productive as its nearest competitor, it will be able to use a price slightly below q^2MC. However, it will also lose its profits from its previous successful research, which was priced slightly below qMC.

[68] *I.e.*, the amount of profits at time *t* for a *j* IG monopolist using a successful research of quality *k* depends on the mark-up, after subsidies *MC*, and – without IT effects – the domestic demand for IG *j* by the FG producers. Formally, this occurs by solving the following expression:

$$\Delta\Pi_Z(k,j,t|m) = \left[p - (1 - s_x) \right] X(j,t|m) \cdot$$

$$\Delta\Pi_Z(k,j,t|m) = \overline{m}\, m_Z(t|w)\ (1-s_x)(q-1)\left[\frac{p^m A_Z\,(1-\alpha)}{q(1-s_x)}\right]^{\frac{1}{\alpha}} q^{k_Z(j,t|m)}\left[\frac{1-\alpha}{\alpha}\right]; \quad (2.39a)$$

$$\Delta\Pi_Z(k,j,t|m) = \overline{m}\, m_Z(t|w)\ (1-s_x)(q-1)\left[\frac{p^m A_Z\,(1-\alpha)}{q(1-s_x)}\right]^{\frac{1}{\alpha}} q^{k_Z(j,t|m)}\left[\frac{1-\alpha}{\alpha}\right] \cdot \left\{(q+1)\,q^{-1/\alpha} - q^{\left(\frac{\alpha-1}{\alpha}\right)}\right\} \quad (2.39b)$$

Equations (2.39a, b) give us the incremental profits of follower firms taking over the leader position and the incremental profits of leader firms replacing themselves, respectively.

Next, if we compare equations (2.39a, b), we easily conclude that the gain to a follower firm is greater so long as $q^{1/\alpha} > 1$, which is guaranteed by the assumption that $q > 1$ and $0 < \alpha < 1$. In other words, leader firms undertake no research targeted at improving the quality of their own IGs because the incremental gain of a two-step quality advantage to an incumbent is strictly smaller than the gain of a one-step quality advantage to an external successful researcher. Note that the follower firm goes from having no profits to having profits when it takes over the leader position. This is usually called the 'replacement or Arrow effect' – *e.g.*, Aghion and Howitt (1998a, chap. 2).[69]

Note that comparing net output, $Y(m)-X(m)$ or $Y-X$, and aggregate profits resulting from equation (2.39a), $\Pi_Z(m)$ or Π_Z, we can attest that that net output is greater than the respective aggregate profits. This reflects an 'appropriability effect' – *i.e.*, the monopolist IGs producers only capture a fraction of the greater productivity in the FGs sector or of the consumer surplus created by their production and productivity.[70] Moreover, we remember that, due to technological complementarity in the production

[69] Arrow (1962) was the first to establish that as long as an incumbent monopolist faces the same probability as a challenger of discovering a new process or product innovation, the former's reward to engaging in R&D investment is lower than the latter's due to the replacement effect.

[70] This imperfection suggests that the decentralised equilibrium is not Pareto optimal. In fact, in addition to the IGs production subsidy, a R&D subsidy is in order. The latter is essential since IGs firms, which support R&D costs, are concerned with stealing the incumbent's profits – which is the creative part of creative destruction or the carrot, citing Jones and Williams (2000), but they ignore the spillover on the future R&D of the successful research supported. Actually, each successful research makes the next ones easier - see the ll_Z term in equation (2.34) – and this 'imperfection' tends to increase the socially optimal intensity of R&D above the decentralised level. Moreover, as we will see below, supporter IGs firms discount profits at a rate higher than the interest rate, since they consider the probability of losing the monopoly – which is the destructive part of creative destruction or the stick, as Jones and Williams say.

function (2.1), the size of the market for m-specific IGs is the employed m-type HC. Thus, the scale effect – $m(.|t)$ – is apparent in the size of the profits in equation (2.39a).

The most relevant factor for the monopolists is not instantaneous profits, but the net present discounted value of all profits. The profit in equation (2.39a) takes place from the time of the k quality improvement, t_k, until the time of the next improvement explored by some follower firm, $t_k+dt=t_{k+1}$. Therefore, the interval over which the IG producer of the k^{th} successful research is in the forefront (or the duration of monopoly position) is $dt=T_k=t_{k+1}-t_k$. Given the equilibrium interest rate, then the expected present value of profits earned by the successful researcher, which sells a state-of-the-art quality k of the IG j, in country Z, complementary to m-type HC at time t, $V_Z(k,j,t|m)$,[71] can be explicitly written as – where E is used to represents expected value:

$$V_Z(k,j,t|m)=E\left\{\Pi_Z(k,j,t|m)\frac{1-exp[-r(t)T_{k(j,t|m)}]}{r(t)}\right\}. \qquad (2.40)$$

As a result of equation (2.40), the expected present value of profits (market, stock or shareholder value of the patent) depends positively on $T_{k(j,t|m)}$ and $\Pi_Z(k,j,t|m)$. Since we know the latter, we need to determine the former to obtain $V_Z(k,j,t|m)$. For this reason, let $PB(\tau)$ denote the cumulative probability density function for $T_{k(j,t|m)}$; *i.e.*, the probability that $T_{k(j,t|m)} \leq \tau$. Hence, the change in $PB(\tau)$ with respect to τ represents the probability per unit of time that success occurs at τ. For successful research to happen at τ, first it must not have occurred earlier, a result that has probability $[1-PB(\tau)]$. Next, depending on the condition that a discovery has not yet taken place, the probability of one occurring is $pb_Z(k,j,t|m)$ per unit of time. Thus, the derivative of $PB(\tau)$ is:

$$\frac{dPB(\tau)}{d\tau}=\left[1-PB(\tau)\right]pb_Z(k,j,t|m). \qquad (2.41)$$

We assume that over time between each successful researches in IG j, the research effort, $rs_Z(k,j,t|m)$, grows at the same rate as $m_Z(t|w)$ and, thereby, the probability of successful research, $pb_Z(k,j,t|m)$, does not vary, a situation occurring in equilibrium. Thus, $pb_Z(k,j,t|m)$ is constant over the interval $T_{k(j,t|m)}$ and we can readily solve the differential equation (2.41).[72] Using the boundary condition $PB(0)=0$ the result is:

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[71] In other words, V is the market value of the patent or the value of the monopolist firm owned by domestic consumers or individuals.

[72] See, for example, Gandolfo (1997, chap. 12) for the resolution of this differential equation.

$$PB(\tau) = 1 - exp\left[- pb_Z(k, j, t \mid m)\ \tau \right]. \qquad (2.42)$$

From equation (2.42), we can calculate the probability density function from differentiation of the cumulative density; that is, nearly

$$b(\tau) = \frac{dPB(\tau)}{d\tau} = pb_Z(k, j, t \mid m)\ exp\left[- pb_Z(k, j, t \mid m)\ \tau \right]. \qquad (2.43)$$

Therefore, the probability that the 'event' will occur sometime within the short interval between τ and $\tau + dt$ is nearly $\{ pb_Z(k, j, t \mid m)\ exp[-pb_Z(k, j, t \mid m)\ \tau]\ dt \}$. In particular, the probability that it will occur within dt from now (when $\tau = 0$) is nearly $\{ pb_Z(k, j, t \mid m)\ dt \}$. It is in this sense that $pb_Z(k, j, t \mid m)$ represents the probability per unit of time that the 'event' will occur at this point or the flow probability of the 'event'.

Now, considering equations (2.40) and (2.43), $V_Z(k, j, t \mid m)$ computed at time t is:

$$V_Z(k, j \mid m) = \frac{\Pi_Z(k, j \mid m)}{r} pb_Z(k, j \mid m) \int_0^{\infty} \left[1 - exp(-r\tau) \right] exp\left[- pb_Z(k, j \mid m)\ \tau \right] d\tau. \qquad (2.44)$$

The integral in equation (2.44) can be evaluated to get:[73]

$$V_Z(k, j, t \mid m) = \frac{\Pi_Z(k, j, t \mid m)}{r(t) + pb_Z(k, j, t \mid m)}. \qquad (2.45)$$

The denominator of equation (2.45) could be defined as the effective discount rate at the time of the successful research of rung k. It is equal to the interest rate, $r(t)$, plus the rate of 'creative destruction' – *i.e.*, the 'flow probability' of being displaced by a new successful research or the probability of successful research occurring in the IG, $pb_Z(k, j, t \mid m)$. The more research is expected to take place, the shorter the duration of the monopoly profits that will be enjoyed by the creator or imitator of the next successful research and, hence, the smaller the pay-off to innovating or imitating. In the end, if we substitute $\Pi_Z(k, j, t \mid m)$ in equation (2.45) from equations (2.39a), then we can write:

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[73] Alternatively, we could get equation (2.45) directly from a standard dynamic programming equation – *e.g.*, Aghion and Howitt (1998a, chap. 2) and Acemoglu (2003) – [or even from more general dynamic programming equations – *e.g.*, Thompson and Waldo (1994)], which states that positing that the leader will be displaced by an outsider in the next race, the value of $V_Z(k,j,t \mid m)$ is determined by the asset equation: $r(t)\ V_Z(k,j,t \mid m) = \Pi_Z(k, j, t \mid m) - pb_Z(k, j, t \mid m)\ V_Z(k,j,t \mid m)$. That is, the expected income generated by the successful research on rung k^{th} during a unit time interval, $r(t)\ V_Z(k,j,t \mid m)$, is equal to the profit flow, $\Pi_Z(k, j, t \mid m)$, being paid out continuously as dividend, minus the expected 'capital loss' that will occur when the k^{th} successful research is replaced (driving the leader's profit to zero) by a new one.

$$V_Z(k,j,t|m) = \frac{\overline{m}\, m_Z(t|w)\ (1-s_x)(q-1)\left[\dfrac{p^m A_Z(1-\alpha)}{q(1-s_x)}\right]^{\frac{1}{\alpha}} q^{k_Z(j,t|m)}\left[\dfrac{1-\alpha}{\alpha}\right]}{r(t)+pb_Z(k,j,t|m)}. \qquad (2.46)$$

Equation (2.46) shows the explicit expected reward from accomplishing the k^{th} successful research, which depends positively on the 'market size effect', $m_Z(.|t)$, on the productivity parameter, A_Z, on the FG price index, p^m, and on the mark-up over after subsidy *MC*. Conversely, it depends negatively on both the interest rate, r, and the randomness of the time of success of the $(k+1)^{\text{th}}$ research, $pb_Z(k,j,t|m)$.

2.5.4. Equilibrium probability of successful R&D and of the R&D effort

Now, in addition to determining the equilibrium probability of successful research and of the R&D effort, we seek to better understand the direction of TK progress. We start by observing how the value of the incumbent firm, $V_Z(k,j,t|m)$, determines the R&D effort, $rs_Z(k,j,t|m)$, and so the probability of successful research, $pb_Z(k,j,t|m)$.

The potential IG producer which supports R&D costs is only concerned with the expected value shown in equation (2.46) and not with the randomness of the return. This assumption can be satisfactory even if individuals are risk averse since each R&D activity is small, research outcomes in the different IGs are by assumption uncorrelated and the risks in all industries are idiosyncratic. Thus, shareholders can earn a risk-less return by holding a well-diversified portfolio of shares of firms in the continuum of IGs, whereby the portfolio rate of return equals the expected industry specific rates of return.

At each time t, the after subsidy cost of research is $[1-s_r]\,rs_Z(k,j,t|m)$, and this effort results in the probability of successful research $pb_Z(k,j,t|m)$. Hence, the expected reward for pursuing the $(k+1)^{\text{th}}$ successful research is $pb_Z(k,j,t|m)\,V_Z(k+1,j,t|m)$. Therefore, the expected present discounted flow of net profit, $\Pi_Z(k,j,t|m)$, from research in an IG j that is currently at quality k is:

$$\Pi_Z(k,j,t|m) = pb_Z(k,j,t|m)\,V_Z(k+1,j,t|m) - [1-s_r]\,rs_Z(k,j,t|m). \qquad (2.47)$$

As we assume free entry into the research business, new entrants will drive down the net present value of profits by undertaking R&D. When present value of net profits is less than zero, there would be no R&D occurring in equilibrium – *i.e.*, no rational agent invests in R&D. In equilibrium with the R&D activity carried out, the net value of a firm

cannot exceed the after subsidy cost of research, and so its expected present value of net profits equals zero. Thus, if $rs_Z(k, j, t \mid m) > 0$, $\Pi_Z(k, j, t \mid m) = 0$ must hold.[74]

Taking into account equation (2.38) and (2.46), the term $rs_Z(k, j, t \mid m)$ cancels out in equation (2.47) and the free entry condition can be written, for each time t, as:

$$pb_Z(k+1, j \mid m) = pb_Z(. \mid m) =$$

$$= \frac{\beta_Z}{\zeta_Z} \left[exp(PE+IT)\; g_1(.)^{\bar{\sigma}_1}\; g_2(.)^{\sigma_2(.)} \right]^{\Gamma_Z} \left[\frac{1-s_{xZ}}{1-s_{rZ}} \right] \left(\frac{q-1}{q} \right) \left[\frac{p^m A_Z (1-\alpha)}{1-s_x} \right]^{\frac{1}{\alpha}} \overline{m} - r_Z \quad . \tag{2.48}$$

Equation (2.48) determines the equilibrium probabilities of successful research (innovation and imitation), and specifically without IT effects – given the interest rate and the FGs' price indexes. Thus, both innovation and imitation equilibrium rates are endogenously determined based on expected discounted profit maximization. They respond negatively to the interest rate – the higher the interest rate those firms have to pay to finance R&D expenditures, the lower the equilibrium probability. They are encouraged both by an increase in the exogenous given ad-valorem subsidy rates of R&D, s_r, and of IGs production, s_x. They also respond positively to the experience-adjusted probability of innovation, Φ_I, or imitation, Φ_P, depending on the country, to the exogenous productivity variable, A, and to the FGs' price indexes, p^m.

Accordingly, there is a (dynamic) price effect, which indicates that there will be stronger incentives to develop technologies when the goods produced by these technologies command higher prices, once with $\xi = 1$ the market size influence become negligible,[75] as is apparent in equation (2.48). In fact, the adverse effect of market size as a result of the scale proportional difficulty of introducing new quality IGs in equation (2.34), is designed to offset the scale effects on profits in equation (2.39a).

A more detailed analysis of the equilibrium m-specific probability of successful research, tells us that, computing $pb_Z(VH) - pb_Z(SH)$, the incentive to improve different types of technology is affected by the FGs' price indexes. In other words, the direction taken by TK change is determined by the price channel; *i.e.*, depends on the relative price

[74] Thus, the net profits of the expected successful research coincide with its after subsidy cost or still the net value of a firm (the value of a blueprint or the expected marginal benefit of vertical R&D) equals the after subsidy cost of R&D if there are no barriers to entry in the R&D process and so $\Pi_Z(k, j, t \mid m) = 0$. If there are entry barriers in R&D, $\Pi_Z(k, j, t \mid m) > 0$. The higher the barriers, the larger $\Pi_Z(k, j, t \mid m)$.

[75] We use the word 'negligible' since the HC has indirect influence on R&D. As a matter of fact, in both countries, the stock of HC (and, consequently, also HC accumulation) influences price indexes – *e.g.*,

of the versatile-specific FGs since TK used in production of more expensive goods will be upgraded faster because it is more profitable to improve those technologies.[76] So, *e.g.*, an increase in relative price of versatile FGs expands the versatile-specialised TK bias.

Since FGs using the scarce factor command higher price – *e.g.*, equations (2.21), (2.22) and (2.25a, b) –, the price channel implies that there is more successful research directed at the scarce factor. It is similar to the impact of factor prices on TK change emphasised by the 'induced innovation literature', where the increase in the relative price of a factor induces TK progress that saves the factor. It includes theoretical works – *e.g.*, Kennedy (1964) and Samuelson (1965) –, empirical studies – *e.g.*, Schmookler (1966) and Hayami and Ruttan (1970) –, and historical works – *e.g.*, Habakkuk (1962) and David (1975).[77] Hayami and Ruttan (1970), for example, discuss the different paths of agricultural development in the U.S. and Japan. They conclude that the relative scarcity of land in Japan appears to have induced a faster rate of successful researches increasing output per acre. Habakkuk (1962) observes that the more rapid TK change in the U.S. during the nineteenth century face to British economy resulted from the relative labour scarcity in the U.S., which encouraged firms to develop labour-saving technologies.

Equilibrium m-specific probabilities of successful research in equation (2.48) turn out to be independent of IG j and quality rung k. There are two reasons behind this independence. One, specific to the equilibrium probability of successful imitation, is due to the simplifying assumption that the determinants of imitation capacity, *PE* and *IT* in the catching up term in equation (2.34)-(v), are not specific to each IG. The other and most substantial reason is due to the removal of scale of TK effects; that is, the positive influence of the quality rung on profits – see equation (2.49a) – and on the learning effect is exactly offset by its influence on the complexity cost – see equation (2.34)-(ii, iv).[78]

Thus, the probability of successful research varies with k owing to two offsetting effects. If the positive influence of the quality rung on profit effect dominates, the rate of

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equations (2.21), (2.22) and (2.25a, b) – and likewise R&D and the growth rate of the economy. Additionally, in the South, the gap in HC at work also influences the imitation capacity.

[76] This result is different from the (skill-biased technological change) literature, which does not take into consideration the endogenous accumulation of HC and which does not remove the scale effects. In this literature, the direction of the TK progress is related with the exogenous increase in the supply of skills, which induces faster upgrading of skill-complementary technologies, since under substitutability (and complementarity between types of IGs and types of workers), the market size effect dominates the price channel – *e.g.*, Acemoglu (1998, 2002a, b, 2003). This literature is, however, at least implicitly, against the Jones' critique and the post-Jones' critique literature.

[77] Since Hicks (1932) considers induced inventions as the result of a change in the relative prices of the factors, the 'induced innovation literature' in the 1960s attempted to formalise Hicks (1932)' idea.

[78] This is the technical reason for the presence of the production function parameter α in equation (2.34).

return to R&D is higher the more advanced an industry. Hence, more advanced industries grow faster than less advanced ones, and the growth rate of the economy rises over time as the average value of k increases – *i.e.*, R&D features a form of increasing returns –, and this property creates a pattern of divergence for growth rates. In contrast, when the negative influence of the quality rung on pb dominates, more advanced industries grow relatively slowly, and the growth rate of the economy falls over time – *i.e.*, R&D exhibits a form of decreasing returns. When the two forces exactly offset – as is apparent in equation (2.48) – all industries of each m-type grow at the same rate and the growth rate of the economy is constant over time – *i.e.*, R&D exhibits constant returns.

The first term on the right-hand side of equation (2.48) represents the rate of return from research or the expected flow of profit per unit of research effort. The key, however, is that a successful researcher maintains this return only until the time of the next innovation. The rate of return must therefore cover the ordinary rate of return, r, plus the premium for the probability per unit of time that a competitor will succeed, pb, and thereby drive the incumbent out of business. So, if r is constant over time, then pb is also constant, as well as for all IGs of each m-type.

Now, we can obtain the equilibrium amount of resources devoted to R&D in IG j, which uses m-type technology. Thus, first re-writing equation (2.38) as:

$$rs_Z(k,j,t\,|\,m) = \frac{pb_Z(k,j,t\,|\,m)}{ll_Z(k,j,t\,|\,m)\; lm_Z(t\,|\,m)\; cp_Z(k,j,t\,|\,m)\; btp_P(k,j,t\,|\,m)^{\Gamma_Z}}, \tag{2.49}$$

and, after that, using the expressions for the terms on the right-hand side of (2.49) from equation (2.34) for the denominator and from equation (2.48) for the numerator, then:

$$rs_Z(k,j,t\,|\,m) = q^{k_Z(j,t|m)\left[\frac{1-\alpha}{\alpha}\right]}\, m_Z(t\,|\,w) \left\{\left[\frac{1-s_{xZ}}{1-s_{rZ}}\right]\left(\frac{q-1}{q}\right)\left[\frac{p^m A_Z\,(1-\alpha)}{1-s_x}\right]^{\frac{1}{\alpha}} \overline{m} - \frac{\zeta_Z}{\beta_Z}\left[\frac{1}{exp(PE+IT)\; g_1(.)^{\bar{\sigma}_1}\, g_2(.)^{\sigma_2(.)}}\right]^{\Gamma_Z} r_Z\right\}. \tag{2.50}$$

As expected, equation (2.50) states that the more advanced and with a large market; *i.e.*, with higher k and m, invest a larger quantity of resources. However, the equilibrium probability of successful research is independent of k and m – see equation (2.48) – since more effort is required in IGs that are more advanced and have large market to generate

the same probability. Furthermore, from equation (2.50), we verify that the explicit aggregate of R&D spending in each type of IGs, denoted by $RS_Z(k,t|m)$, is given by:

$$RS_Z(k,t|m) = \int_0^{J^m} rs_Z(k,j,t|m)\,dj = Q_Z^{J^m} m_Z(t|w)$$
$$\left\{\left[\frac{1-s_{xZ}}{1-s_{rZ}}\right]\left(\frac{q-1}{q}\right)\left[\frac{1-\alpha}{1-s_{xZ}}\right]^{1/\alpha}\left(p_Z^m A_Z\right)^{1/\alpha}\overline{m} - \frac{\zeta_Z}{\beta_Z}\left[\frac{1}{exp(PE+IT)\; g_1(.)^{\bar{\sigma}_1} g_2(.)^{\sigma_2(.)}}\right]^{\Gamma_Z} r_Z\right\} \cdot \quad (2.51)$$

The equilibrium aggregate resources devoted to R&D by all follower firms in a country are $RS_Z(k,t) = RS_Z(k,t|SH) + RS_Z(k,t|VH)$. Consequently, as a result of equations (2.51) and (2.48), we can write for each time *t*:

$$RS_Z(k,t) = Q_Z^{J^{SH}}(t)\; SH_Z(t|w)\left\{\frac{\zeta_Z}{\beta_Z}\left[\frac{1}{exp(PE+IT)\; g_1(.)^{\bar{\sigma}_1} g_2(.)^{\sigma_2(.)}}\right]^{\Gamma_Z} pb_Z(SH)\right\} +$$
$$+ Q_Z^{J^{VH}}(t)\; VH_Z(t|w)\left\{\frac{\zeta_Z}{\beta_Z}\left[\frac{1}{exp(PE+IT)\; g_1(.)^{\bar{\sigma}_1} g_2(.)^{\sigma_2(.)}}\right]^{\Gamma_Z} pb_Z(VH)\right\} \cdot \quad (2.52)$$

Equation (2.52) states that the aggregate resources devoted to R&D in each country depend positively on both market profitability – *i.e.*, market size effect in *RS*, captured by $SH_Z(.|w)$ and $VH_Z(.|w)$ –, and the aggregate domestic quality indexes, $Q_Z^{J^{SH}}$ and $Q_Z^{J^{VH}}$. Conversely, they depend negatively on both the interest rate, *r*, and on the so named experience-adjusted cost of innovation or imitation, depending on the country analysed.

Finally, remember that, also as a result of equation (2.51) and, consequently, equation (2.52) together with equation (2.48), the increased resources devoted to research as the aggregate quality index and/or the HC employed rise/rises, do/does not lead to greater rates of successful research, but rather are needed to offset the greater difficulty of research as the quality and/or the levels of HC employed increase.

2.5.5. Behaviour of the aggregate quality indexes

In equilibrium, given the interest rate, the HCs and the FGs' price indexes, the probability of successful research in a single IG industry is pb_Z, as given by equation (2.48) and applies to all IGs of its *m*-type. That is, pb_Z – as a Poisson arrival rate – determines the speed of TK in a single IG, then since in equilibrium it applies to all industries of its *m*-type, it can be translated into the path of domestic *m*-type of TK.

Formally, it follows directly from Poisson distribution that the measure of IGs that are improved exactly F times at time t is $\{[pb_Z(m)\,t]^F\ exp[-pb_Z(m)\,t]\}/F!$. Hence, we can expand the two aggregate domestic quality indexes in equation (2.7) such that,

$$Q_Z^{J^m}(t) = \int_0^{J^m} q^{k_Z(j,t|m)\left(\frac{1-\alpha}{\alpha}\right)}\,dj = \sum_{F=0}^{\infty} \frac{\left[pb_Z(m)\,t\right]^F\ exp\left[-pb_Z(m)\,t\right]\ q^{F\left(\frac{1-\alpha}{\alpha}\right)}}{F!} =$$

$$= exp\left[-pb_Z(m)\,t\right] \sum_{F=0}^{\infty} \frac{\left[pb_Z(m)\,t\ q^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^F}{F!} = exp\left\{\left[pb_Z(m)\,t\right]\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]\right\} \qquad (2.53)$$

And then the equilibrium growth rate of $Q_Z^{J^m}$ at each time t is given by:[79]

$$\frac{\dot{Q}_Z^{J^m}}{Q_Z^{J^m}} = pb_Z(m)\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]. \qquad (2.54)$$

In equation (2.54), the last term on the right-hand side, $[q^{((1-\alpha)/\alpha)}-1]$, measures the impact of each vertical successful research on the stock of public TK in each country. It shows that, to determine the growth rate of $Q_Z^{J^m}$, we have to pin down the rate of return – see equation (2.48). And to determine r, we have to introduce the behaviour of individuals. A detailed description of the problem of individuals, which, to simplify, is assumed to be identical in both countries, is thus in order in the following section.

2.6. Individuals and human capital accumulation

The problem of individuals

Following Hollanders and Ter Weel (2003), we assume that countries have zero population growth and are inhabited by a large number of heterogeneous Ramsey (1928) individuals or consumers, each of them deciding on the allocation of time and income.

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[79] Alternatively, we can apply the analysis carried out by Aghion and Howitt (1992) and Barro and Sala-i-Martin (2004, chap. 7). Briefly, since as a result of the definition of aggregate quality indexes in equation (2.7), in the case of a quality improvement of IG j, the proportionate change in the quality grade is $(q^{(1-\alpha)/\alpha}-1)$, and the equilibrium probability of research per unit of time is equal for all IGs – equation (2.48) –, then the expected proportionate change in Q_Z per unit of time is $pb_Z(q^{(1-\alpha)/\alpha}-1)$. Since the number of (independent) IGs, J^m, is large enough, from the Law of Large Numbers the average growth rate of Q_Z measured over any finite time interval will be close to $pb_Z(q^{(1-\alpha)/\alpha}-1)$.

In detail, heterogeneity is present in two related characteristics of individuals. One is ability level a uniformly distributed over a range $[0, 1]$ and specific to each individual – *i.e.*, every individual in each economy has a different ability level, which can be viewed as the talent, the intelligence or the learning capacity this individual is born with and which can be developed during his/her life. The other is the type of HC – *VH* and *SH*. To simplify, we consider an exogenous threshold ability $\bar{a}$ such that individuals with high ability – greater than $\bar{a}$, $a > \bar{a}$ –, accumulate *VH*, while those with low ability – lower that $\bar{a}$, $a \leq \bar{a}$ – are only able to accumulate *SH*. Thus, each individual is indexed by a, which implicitly also defines the m-type of his/her HC.

Individuals divide their time (decide) between attending the educational process to accumulate HC, which improves their productivity, and working to earn an income, designed in composite FG, proportional to their HC.[80] Additionally, individuals (decide to) spend part of their income directly on the consumption of the composite FG and lend another part in return for future interest. In short, each individual embodies productive skills that are accumulated through endogenous, wealth maximising investment decisions that sacrifice present consumption in order to raise future productivity and income.

Individuals formally choose a plan for consumption, c, asset holdings or capital, K, and HC. We assume that they have standard, discounted, constant elasticity preferences resulting in the following infinite horizon individual lifetime utility or felicity function:

$$U(a,t) = \int_0^\infty \left[\frac{c(a,t)^{1-\theta} - 1}{1-\theta} \right] exp(-\rho t)\, dt \,. \qquad (2.55)$$

Where $c(a,t)$ is the amount of consumption of the composite FG of the individual with ability a, at time t. Parameter $\rho > 0$ is the subjective discount rate which measures how much the individual favours consumption today over consumption tomorrow. The higher the value of ρ, the less patient the individual is. Parameter $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, and $\theta \neq 1$.[81] The higher the value of θ, the more the individual will dislike deviations from a smooth consumption path. As θ approaches 0, the utility function approaches a linear form in $c(a,t)$.

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[80] Since we have not included pure unskilled labour – *i.e.*, workers without any education – as productive, we are in line with Cohen (1998), who considers that unskilled individuals who do not participate in the task upgrading efforts of society as a whole are left behind.

[81] For the special case of $\theta = 1$, the utility function is defined as $U(a,t) = \int_0^\infty ln\, c(a,t)\, exp(-\rho t)\, dt \,.$

The individual a maximises lifetime utility subject to both his/her intertemporal budget constraint – or asset holdings accumulation or still capital accumulation – and the evolution of his/her HC stock. The individual a budget constraint, at each t, equalises income earned to consumption plus savings. Savings are the accumulation of assets, *i.e.* lending takes the form of ownership of the profitable firms, which are the ones that produce IGs in monopolistic competition.[82] Therefore, the budget constraint, expressed as savings = income – consumption, for all $t \in [0, \infty[$, is given by:

$$\dot{K}(a,t) = [1 - \tau_K]\, r(t)\, K(a,t) + [1 - \tau_w]\Big[1 - u_S(a,t) - u_T(a,t)\Big]\, w^m(t)\, m(a,t) - c(a,t). \quad (2.56)$$

Where $K(a, t)$ is then physical capital or financial asset holdings of the individual with ability a, at time t,[83] and r is its return. The $u_S(a, t)$ and $u_T(a, t)$ are the fractions of (non-leisure) time t the individual with ability a spends accumulating HC at school and on-the-job-training (OJT), respectively. As a result, $u_w(a,t) = 1 - u_S(a,t) - u_T(a,t)$ is the fraction of (non-leisure) time t the individual with ability a allocates to production activity (work). Therefore, $u_S(a,t)$, $u_T(a,t)$ and $u_S(a,t) + u_T(a,t) \in [0, 1]$.[84] Thus, we implicitly assume that the amount of leisure is fixed exogenously. The $m(a, t)$ is the level of HC of the individual with ability a at time t, and so $m = SH$ if $a \leq \bar{a}$ and $m = VH$ if $a > \bar{a}$. Due to arbitrage in the domestic assets markets, the interest rate depends neither on ability nor on the type of HC, only on time t.

As a result of equation (2.56), at each time t, one part of all individual income – *i.e.*, flow of profits and the wage per unit of HC sold to the FGs sector, w^m – is paid to the government as taxes, and the rest is devoted to consumption and savings. We assume that government intervention can consist of two ad-valorem taxes at rate τ_K on assets income and τ_w on HC income, which are necessary for a balanced government budget.[85]

In the educational process, individuals accumulate *VH* or *SH* (constrained by the ability level) using both inputs – or forms of education or forms of training – schooling

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[82] The value of these firms, in turn, corresponds to the value of patents in use, as explained above.

[83] Lived patents, corresponding to the best quality IGs, are then the assets of the economy. Individuals buy them from the R&D sector – considering that there are specialised R&D firms –, and then they rent them to IGs producers – in other words, they are the owners of IGs firms – and receive the corresponding flow of profits earned (dividends). In this way, a patent is like an asset that allows individuals to save and thus to transfer present consumption to the future.

[84] That is, we normalise the lifetime of each individual to unit, and s/he allocates part of her/his lifetime to schooling, part to OJT and part to productive activity. Consequently, we interpret the fraction of time spent in HC accumulation to be the fraction of an individual's life spent in the educational process.

[85] To avoid additional distortions, the subsidies would have to be financed with a lump-sum tax. However, this kind of tax is normally difficult to find.

and OJT in any desired combination.[86] Thus, we follow a great number of models in the literature, where labour/HC is divided in two groups – *e.g.*, Acemoglu and Zilibotti (2001), Kiley (1999), Theshmar and Thoenig (2000). However, in our case, the structure of HC supply is endogenous and pure unskilled workers do not exist.

In particular, we presume that the accumulation of *VH* is relatively more school intensive (and is relatively more productive and more abundant in the North, remember), while *SH* is relatively more OJT intensive. Intuitively, schooling helps in working with versatile IGs more than OJT does, and *vice-versa*.[87] The idea that schooling provides general HC, more versatile or adaptable to changing environments, while OJT is more specific is in line with Grossman and Shapiro (1982), Mincer (1993) and Van Zon and Antonietti (2004). Besides, as the school intensive HC is accumulated by individuals with higher ability, it is also in line with Hassler and Rodriguez-Mora (2000), since they claim that ability or intelligence is crucial for adapting to new technologies.

Moreover, as noted by Mincer (1993, 1994), the inputs – schooling and OJT – may be substitutes or complements. For this reason, we consider the following CES, constant returns, production – or accumulation – function of *m*-type HC:

$$\dot{m}(a,t)=\left[\varphi^{m}\left(school(a,t)\right)^{\phi}+(1-\varphi^{m})\left(OJT(a,t)\right)^{\phi}\right]^{1/\phi}-\delta^{m}m(a,t). \qquad (2.57)$$

Where $school(a, t)$ and $OJT(a, t)$ denote, respectively, the schooling activities and OJT of an individual with ability a, at time t. Parameter $\varphi^{m} \in [0, 1]$ is the distribution or intensity parameter, which indicates how crucial the two inputs are in HC accumulation. When $\varphi^{m} = 1$, we obtain the HC accumulation function *à la* Lucas (1988); and when $\varphi^{m} = 0$, the increase in HC level is only owing to OJT. We consider that $\varphi^{VH} > \varphi^{SH}$ – *i.e.*, $(1-\varphi^{VH}) < (1-\varphi^{SH})$ – in order to obtain that *VH* accumulation is relatively school intensive and *SH* accumulation is relatively OJT intensive. To simplify, this is the unique influence of ability on HC accumulation and, as we will better understand in the next paragraphs, this also opens the way for those with more ability to accumulate more HC.

Parameter ϕ is the substitution parameter, with $\phi \in]-\infty, 0[\cup]0, 1]$, and thereby the elasticity of substitution is equal to $[1/(1-\phi)]$.[88] If $\phi > 0$, schooling and OJT tend to

[86] There is ample empirical evidence that the distinction between schooling and OJT is important in shaping and improving individuals' skill levels – *e.g.*, Van Zon and Antonietti (2004).
[87] That is, OJT helps in working with specialised IGs more than schooling does.
[88] See, for example, Varian (1992, pgs. 19-20).

be perfect substitutes, being perfect substitutes for $\phi = 1$ and the function is linear. If $\phi < 0$, schooling and OJT tend to be perfect complements, being perfect complements for $\phi \to -\infty$ and the function is Leontief since there is no substitution between the two factors. When $\phi = 0$, the function is Cobb-Douglas. Finally, parameter δ^m is the depreciation rate of the m-type HC. This rate could be thought of as the rate at which HC becomes obsolete as a result of TK progress – *e.g.*, Barro and Sala-i-Martin (2004, chap. 5) – and, for simplicity, we assume that it is always constant.

As a results of equation (2.57), a solution with a constant growth rate of $m(a,t)$, requires a constant $u_S(a,t)$ and $u_T(a,t)$. This result is driven by the idea that there is always something new to learn. Individuals intentionally keep on absorbing new information by attending training activities. In equation (2.57), we also maintain the assumption of many other models that there are constant returns in HC accumulation. In some cases, this assumption is justified by reference to externality effects, which convert diminishing returns at the individual level to constant returns at the aggregate level – *e.g.*, Tamura (1991), Blackburn and Ravn (1993) and Sjogren (1998). In other cases, it is motivated by the inclusion of a larger set of inputs in HC accumulation – *e.g.*, Roubini and Milesi-Ferretti (1998) and Pecorino (1995a, b). And in others still, it is merited by appealing to an overlapping generations economy in which offspring inherit at least some fraction of their parents' HC – *e.g.*, Azariadis and Drazen (1990), Becker *et al.* (1990), Redding (1996) and Aghion and Howitt (1998a, chap. 10).

In our case, we think in terms of the latter alternative and regard our framework as a continuous time approximation of an overlapping generations model.[89] In fact, apart from innate ability, learning capacities and, thus, the skills an individual can acquire through the educational process critically depend on the family (and social) background; that is, implicitly, we assume that altruistic parents leave everything to their children, including their knowledge. This is not difficult to believe if we accept the observation that skill acquisition is clearly a cumulative process through successive generations, and as a result children of well-educated parents often exhibit greater knowledge at birth than children of less well-educated individuals.

Schooling and OJT, in turn, are proportional to the fraction of time allocated to each and to the amount of the individual's HC; that is, the HC accumulation at school

and OJT is described *à la* Lucas (1988) or, in other words, we extend Lucas' (1988) schooling model to both schooling and OJT. In this way, we follow Mincer's (1993) view, such that the HC in OJT is modelled as a time allocation decision. Indeed, we assume that training at work may be viewed as an investment and so it is costly – *i.e.*, OJT consists in allocating a part of HC level to increasing skills.[90] Formally, we have:

$$school(a,t) = \chi_S \, m(a,t \mid school) = \chi_S \; u_S(a,t) \; m(a,t); \qquad (2.58a)$$

$$OJT\,(a,t) = \chi_T \; m(a,t \mid OJT) = \chi_T \; u_T(a,t) \; m(a,t). \qquad (2.58b)$$

As is quite standard in the literature since Lucas (1988), in equations (2.58a, b), HC begets the accumulation of more HC – schooling and OJT are intensive users of HC –, that the increase in HC also depends positively on the amount of time spent on learning and also on the productivity or efficiency of the learning activities, χ_S and χ_T.

That is, χ_S and χ_T are efficiency parameters which measure the productivity of schooling activities and the productivity of OJT, respectively. They can be understood as representing the state of TK in school (school late infrastructure) and in OJT (OJT late infrastructure), reflecting (or being proportional to) the existing 'knowledge pool' provided by the state of the General Purpose Technology (GPT).[91] We assume that the former is greater than the latter, $\chi_S > \chi_T$, because historically the knowledge accumulated at school is greater, and also since, traditionally, universities, institutes and the school teaching staff have much greater contact with scientific journals and conferences. Note that χ_S and χ_T have to be higher than δ, otherwise m can decline even if individuals spend all their time in HC accumulation. So, lower χ_S and χ_T may be a theoretical reason for the weak or negative relation between HC accumulation and growth existing in some countries – *e.g.*, Benhabib and Spiegel (1994), De Gregorio (1992), Islam (1995).

It is desirable that χ_S and χ_T can grow and at different rates across countries, generating changes in the HC structure. In particular, it would be interesting to consider

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[89] In this case, the condition for HC accumulation to display constant returns is that each new member of a family begins with a level of HC that needs only to be proportional, and not equal, to the level attained by older members. Thus, we assume that there are finitely lived individuals and infinitely lived families.

[90] Thus, OJT can be interpreted as the acquisition of HC at technical, professional or specialised schools.

[91] As we state later on, the term 'General Purpose Technologies' (GPT), introduced by Bresnahan and Trajtenberg (1995), can be defined as a technological breakthrough and is characterised by pervasiveness in use (generality of purpose) that affects an entire economic system. Examples of GPTs include the steam engine, the electric dynamo, the laser, and the recent arrival of new information technologies embodied in information and communication equipment. Associated to each new GPT there is a wave of (secondary) innovations, each of which we assume creates a new quality for IGs (the R&D component in our model).

that χ_S and χ_T are both enhanced by TK progress.[92] It would be also desirable to consider $\chi_S^{VH} > \chi_S^{SH}$ and $\chi_T^{VH} > \chi_T^{SH}$, since they are also learning rates. Then, we would broaden the influence of ability on HC accumulation, since instead of stating that individuals with ability lower than $\bar{a}$ do not have the possibility of accumulating *VH*, and *vice-versa*, the choice would result from their ability. Finally, since they mirror the quality of the educational process and are a crucial determinant of the decision to invest in HC, they open the way for government policy on HC accumulation. Then, it would also be interesting to verify the impact of financing school (and OJT) improvements, as in Lucas' model.[93] However, that would complicate the analysis and, for this reason, we assume that χ_S and χ_T are always constant; putting much emphasis on R&D activity and leaving for future work those more elaborate assumptions about these parameters.

The results of equations (2.57) and (2.58a, b) show that HC accumulation is

$$\dot{m}(a,t) = \left\{ \left[\varphi^m \left(\chi_S \, u_S(a,t) \right)^{\phi} + (1-\varphi^m) \left(\chi_T \, u_T(a,t) \right)^{\phi} \right]^{1/\phi} - \delta^m \right\} m(a,t) \,. \qquad (2.59)$$

Finally, the individual resource constraint on HC – implicit in equation (2.56) – requires that the supply of HC equalises the demand for training – *i.e.*, for school and for OJT –, and demand for production:

$$\begin{aligned} m(a,t) &= m(a,t \mid w) + m(a,t \mid school) + m(a,t \mid OJT) = \\ &= [u_w(a,t) + u_S(a,t) + u_T(a,t)] m(a,t) \end{aligned} \,. \qquad (2.60)$$

Resolving the problem of individuals and related comments

The utility maximising individual will choose how much s/he will consume and how much of his/her time is spent in school and in OJT. That is, each consumer solves an optimal control problem of maximisation of lifetime utility – equation (2.55) – subject to the constraint on the accumulation of assets – equation (2.56) – and HC – equation (2.59). Thus, $c(a,t)$, $u_S(a,t)$ and $u_T(a,t)$ are his/her control variables that serve as the (policy) instruments of optimisation, since they are subject to discretionary choice and

[92] The empirical work of Jones and Zimmer (2001), for example, follows this point of view.

[93] *E.g.*, the government should construct reward schemes, such as teachers having the right incentives to do a good job and provide a high-quality service, and/or it should put emphasis on other activities – such as school and OJT physical infrastructure – that improve both. In Lucas' model, financing school improvements have, in most cases, positive growth effects, even if they are financed by higher taxes.

this choice drives the state variables. The state variables s/he worries about are assets, $K(a,t)$, and HC, $m(a,t)$. Therefore, the individual's Hamiltonian is:[94]

$$\underset{c(a,t),\,u_S(a,t),\,u_T(a,t)}{Max\ Ham\,(a,t)} = \left[\frac{c(a,t)^{1-\theta}-1}{1-\theta}\right] exp\,(-\rho\,t) + \mu(a,t)\left\{[1-\tau_K]\,r(t)\,K(a,t) + \right.$$
$$\left. + \left[1-u_S(a,t)-u_T(a,t)\right][1-\tau_w]\,w^m(t)\,m(a,t) - c(a,t)\right\} + \lambda(a,t)\cdot \qquad (2.61)$$
$$\left\{\left\{\left[\varphi^m\left(\chi_S\,u_S(a,t)\right)^{\phi} + (1-\varphi^m)\left(\chi_T\,u_T(a,t)\right)^{\phi}\right]^{1/\phi} - \delta^m\right\} m(a,t)\right\}$$

Where $\mu(a,t)$ and $\lambda(a,t)$ are the costate variables – also called dynamic Lagrange multipliers, auxiliary, adjoint and dual variables. They measure the shadow price of associated state variables – $K(a,t)$ and $m(a,t)$, respectively – for the individual with ability a at time t. In other words, they represent the value of an increment of the associated state variables – *i.e.*, price of financial assets and price of HC, respectively – at time t in units of utility at time *0*. From the maximisation program, the FOCs are:

$$\frac{\partial\, Ham\,(a,t)}{\partial\, c(a,t)} = 0 \Leftrightarrow c(a,t)^{-\theta}\ exp\,(-\rho\,t) = \mu(a,t); \qquad (2.62a)$$

$$\frac{\partial\, Ham\,(a,t)}{\partial\, u_S(a,t)} = 0 \Leftrightarrow \mu(a,t)\,[1-\tau_w]\,w^m(t) = \lambda(a,t)\,\varphi^m\,\chi_S^{\phi}\ u_S(a,t)^{\phi-1}$$
$$\left[\varphi^m\left(\chi_S\,u_S(a,t)\right)^{\phi} + (1-\varphi^m)\left(\chi_T\,u_T(a,t)\right)^{\phi}\right]^{\frac{1-\phi}{\phi}}; \qquad (2.62b)$$

$$\frac{\partial\, Ham\,(a,t)}{\partial\, u_T(a,t)} = 0 \Leftrightarrow \mu(a,t)\,[1-\tau_w]\,w^m(t) = \lambda(a,t)\,(1-\varphi^m)\ \chi_T^{\phi}\ u_T(a,t)^{\phi-1}$$
$$\left[\varphi^m\left(\chi_S\,u_S(a,t)\right)^{\phi} + (1-\varphi^m)\left(\chi_T\,u_T(a,t)\right)^{\phi}\right]^{\frac{1-\phi}{\phi}}; \qquad (2.62c)$$

$$\frac{\partial\, Ham\,(a,t)}{\partial\, K(a,t)} = -\dot{\mu}(a,t) \Leftrightarrow [1-\tau_K]\,r(t)\,\mu(a,t) = -\dot{\mu}(a,t); \qquad (2.62d)$$

$$\frac{\partial\, Ham\,(a,t)}{\partial\, m(a,t)} = -\dot{\lambda}(a,t) \Leftrightarrow \left[1-u_S(a,t)-u_T(a,t)\right][1-\tau_w]\,w^m(t)\,\mu(a,t) +$$
$$\left\{\left[\varphi^m\left(\chi_S\,u_S(a,t)\right)^{\phi} + (1-\varphi^m)\left(\chi_T\,u_T(a,t)\right)^{\phi}\right]^{1/\phi} - \delta^m\right\}\lambda(a,t) = -\dot{\lambda}(a,t) \cdot \qquad (2.62e)$$

[94] On dynamic optimisation (application and economic interpretation) see, for example, Chiang (1992).

These relationships must satisfy the constraints in the maximisation program – *i.e.*, equations (2.56) and (2.59), together with the following transversality conditions:

$$\lim_{t\to\infty}\left[\mu(a,t)\,K(a,t)\right] = \lim_{t\to\infty}\left[\lambda(a,t)\,m(a,t)\right] = 0\,. \tag{2.63}$$

Equations (2.62a, b, c) equalise the marginal benefit and the marginal cost of consumption, of time devoted at school and of time devoted to OJT, respectively. Equations (2.62d, e), along with equation (2.63), are the optimal dynamic conditions for assets and HC accumulation, respectively.[95]

The solution for the consumption path results from the differentiation of equation (2.62a) with respect to time and taking into account equation (2.62d). We obtain the following solution for the growth rate of consumption, which turns out to be independent of the ability and type of HC that individuals accumulate:

$$\frac{\dot{c}(a,t)}{c(a,t)} = \frac{\dot{c}(t)}{c(t)} = \frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}\left[[1-\tau_K]\,r(t) - \rho\right]. \tag{2.64}$$

Where $C(t)$ represents the country's aggregate consumption and $\dot{C}/C$ its growth rate, which is included in equation (2.64) because the population does not grow. It is a well-known Euler (differential) equation, and states that the growth rate of consumption depends on the difference between the interest rate after (financial) assets income tax and the rate of time preference, and on the intertemporal elasticity of substitution. The assets income tax τ_K has a direct negative effect on consumption growth by reducing the after tax rate of return to assets, while other political variables may affect consumption growth through the interest rate. Thus, equation (2.64) expresses a positive relationship between the interest rate and the growth rate, and in the $(\dot{C}/C, r)$ space gives us an upward sloping preferences curve, which represents balanced growth (BG) paths determined by the savings decisions of individuals.

Therefore, the relation between $[1-\tau_K]\,r(t)$ and the discount rate ρ determines whether individuals choose a pattern of consumption that rises, stays constant or falls over time. We expect that $[1-\tau_K]\,r(t)$ exceeds ρ, and so individuals choose a pattern of consumption that rises over time. In other words, a higher market interest rate induces

[95] More specifically, equation (2.62a) gives the discounted marginal utility of consumption, which satisfies the dynamic optimality condition in equation (2.62d). Equations (2.62b, c) are the (static) optimality condition for the allocation of time, equating the marginal benefit and the marginal cost of an additional unit of HC devoted to working. The marginal cost involves the cost associated with future reductions in HC, as expressed by the other dynamic optimality condition in equation (2.62e).

individuals to save more now and to spend more later, resulting in increasing consumption over time. As we will see later on, in steady state, it stays constant since r stays constant. Additionally, for example, a higher value of θ – *i.e.*, a lower willingness to substitute intertemporally – implies a smaller responsiveness of the growth rate of the consumption to the gap between $[1 - \tau_K]r(t)$ and ρ.

Hence, as is quite standard in the literature – *e.g.*, Sjogren (1998), Blackburn *et al.* (2000) and Acemoglu and Zilibotti (2001) –, we can conclude that each country admits a representative consumer with the usual constant relative risk aversion preferences. This is so, even in the presence of individuals who have different ability levels and two types of HC. To this end, Caselli and Venture (2000), for example, show that the presence of skilled and unskilled workers, do not cause problems for the representative consumer assumption, since with constant relative risk aversion utility functions, these preferences can be aggregated into a constant relative risk aversion representative consumer. Also Hollanders and Ter Weel (2003), for example, consider a country that has heterogeneous individuals – with different ability levels – and work with the same utility function.

For the time allocation part of the problem, equations (2.62b) and (2.62c) give us the optimal ratio between schooling and OJT efforts, which is independent of time and individual ability, but dependent on the type of HC:

$$\frac{u_T(a,t|m)}{u_S(a,t|m)} = \frac{u_T(m)}{u_S(m)} = \left[\frac{(1-\varphi^m)\,\chi_T^{\phi}}{\varphi^m\,\chi_S^{\phi}} \right]^{\frac{1}{1-\phi}}. \tag{2.65}$$

Equation (2.65) informs us that when the two types of training – schooling and OJT – are substitutes, the optimal time allocation ratio of the training efforts is positively reported to their respective productivity and negatively when they are complementary. It makes sense. When schooling and OJT are substitutable, the increase in, for example, χ_S implies that individuals increase their allocation of HC at school, u_S. That is, with this behaviour they obtain a higher increase in HC with a given marginal effort and, then, substitute the less efficient means of HC accumulation with the more efficient. If schooling and OJT are complements, then an increase in the time allocated to the factor with higher productivity requires a greater increase in the time allocated to the lower productivity factor. In particular, given our premise about φ^m, we will always obtain that:

$$\frac{u_T(VH)}{u_S(VH)} < \frac{u_T(SH)}{u_S(SH)}. \tag{2.66}$$

Therefore, the optimal time allocation ratio between schooling and OJT tends to be different – *i.e.*, $u_T/u_S \neq 1$, according to equation (2.65) –, depending on their degree of substitution, on their productivities and on their intensity,[96] and the optimal allocation ratio of training efforts differ as to the m-type of HC accumulated – equation (2.66). Thus, considering only one type of training, we can under- or over-estimate the growth of HC. In particular, the error tends to be different (i) according the degree of substitution and (ii) in the two different types of HC accumulated, as will become apparent later on.

Generically a high degree of substitutability, coupled with the assumed higher efficiency of schooling, means that most of the HC skills necessary to the production of FGs are better obtained at school. On the other hand, strong complementarity indicates that schooling, in spite of higher efficiency, is far from providing all such necessary skills, requiring additional OJT in significant amounts. As a result, in a scenario of strong substitutability, constraining the measurement of HC to formal schooling tends to be sufficient, especially for *VH*, which uses schooling more intensively. On the contrary, the case of complementarity supports the claim in the growth-HC literature that the lack of an empirically robust relationship is partially attributable to the exclusion of OJT from the measures of HC – *e.g.*, Bartel and Lichtenberg (1987), Lucas (1993), Mincer (1993).

Given equations (2.59) and (2.65), it is also clear that when the HC accumulation is constant – it holds in steady state – the time devoted to working is constant as well.

Since the majority of the literature takes HC as a homogeneous aggregate, only the Uzawa-Lucas education sector is considered – *e.g.*, Arnold (1998), Sjogren (1998) and Garcia-Castrillo and Sanso (2000). Nevertheless, we can say now that the introduction of different HC types and a more complex educational process may substantially modify the occupational choice of training. In our case, when inputs are substitutes, individuals decide mainly to go to school, and the inverse happens when they are complementary.

Now, with equation (2.65), we can rewrite, after some calculation, equation (2.62c) in the following simplified form:

$$w^m(t) = \frac{\lambda(t|m)}{\mu(t|m)} \frac{1}{[1-\tau_w]} \varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)}\right]^{\frac{1-\phi}{\phi}} . \qquad (2.67)$$

Where then the ratio between $u_T(m)$ and $u_S(m)$ in equation (2.67) is as given by equation (2.65). And differentiating this one with respect to time, we obtain:

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[96] Moreover, from equations (2.60) and (2.65) we have that: $\frac{1-u_w(t|m)}{u_S(t|m)} = \frac{u_T(m)}{u_S(m)} + 1$.

$$\frac{\dot{w}^m(t)}{w^m(t)} = \frac{\dot{\lambda}(t|m)}{\lambda(t|m)} - \frac{\dot{\mu}(t|m)}{\mu(t|m)} \quad . \tag{2.68}$$

On the other hand, by first introducing equations (2.62b) and (2.62c) into equation (2.62e), and then equation (2.67) into the resultant, we have after some calculation:

$$\frac{\dot{\lambda}(t|m)}{\lambda(t|m)} = \frac{\dot{\lambda}(m)}{\lambda(m)} = \delta^m - \varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)}\right]^{\frac{1-\phi}{\phi}} . \tag{2.69}$$

Equation (2.69) gives us the growth in the shadow price of the state variable *m*-type HC – *i.e.*, the rate of return to additional *m*-type HC in units of utility at time *0*.

An important arbitrage condition inter-relates the returns from the different types of assets. An interior solution to the individual maximization problem requires positive amounts of both assets, *K* and *VH* (or *SH*), which is not sustainable unless their returns are equalised at all times.[97] The following resulting condition, where the optimal time allocation ratios are expressed in terms of the given wage dynamics and interest rate, ensures this; *i.e.*, given equations (2.62d) and (2.69), equation (2.68) can be written as:[98]

$$\frac{\dot{w}^m(t)}{w^m(t)} = [1 - \tau_K]\, r(t) + \delta^m - \varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)}\right]^{\frac{1-\phi}{\phi}} . \tag{2.70}$$

Equation (2.70) tells us that, given the constant optimal time allocation ratios in equation (2.65), a constant relation between wage dynamics and interest rate results – *i.e.*, the optimal time allocation ratios condition the relation between wage dynamics and interest rate.[99] Thus, if the interest rate is high today, wage dynamics are also high, and this relationship is always balanced. In this case, there will be strong investment in R&D, which improves the marginal product of HC – *i.e.*, wages, see equations (2.29a, b) – and so increases its price – see equation (2.30). The increase in wages will be adequate since individuals adjust their training effort – keeping the optimal time allocation ratio constant – to meet the demand for HC in such a way that the relationship remains constant.

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[97] Therefore, our framework is an integrated model of endogenous HC and TK accumulation where investment in *K* and in HC are two complementary activities.

[98] Note that the path of μ depends on *t*, while the path of λ depends on *m*. In fact, from equations (2.62d) and (2.69) $\frac{\dot{\mu}(a,t)}{\mu(a,t)} = \frac{\dot{\mu}(t)}{\mu(t)} = [1 - \tau_K]\, r(t)$ and $\frac{\dot{\lambda}(t|m)}{\lambda(t|m)} = \frac{\dot{\lambda}(m)}{\lambda(m)}$, respectively.

[99] Actually, equation (2.70) can be written in the following way:

$$\varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)}\right]^{\frac{1-\phi}{\phi}} - \delta^m = [1 - \tau_K]\, r(t) - \frac{\dot{w}^m(t)}{w^m(t)} .$$

Now we are able to find the equilibrium interest rate in both countries. In fact, first plugging equation (2.48) into equation (2.54), after that the resulting equation into equation (2.30) and finally equating the obtained m-type HC demand path to the necessary condition for optimisation by individuals ('supply side') – equation (2.70) – the equilibrium interest rate in the countries is reached. By Walras law, it is also the interest rate that clears the assets market – savings by individuals equal to the value of the firms. Moreover, the general equilibrium instantaneous growth rate of the countries at time t results from plugging the equilibrium interest rate into the Euler equation (2.64).

In addition to firms and individuals, the Northern and Southern economies can also be influenced by domestic government policies. A description of the government's budget is thus in order. Finally, the aggregate resource constraint must be presented in order to close the characterisation of the economies. This is the plan of the next section.

2.7. Government budget and aggregate resource constraints

2.7.1. Government budget constraint

We suppose that the government budget is balanced at each point in time; *i.e.*,

$$\tau_K\, r(t) \int_0^1 K(a,t)\, da + \tau_w\, w^m(t) \int_0^1 [u_w(a,t) m(a,t)]\, da = s_x\, X(t) + s_r\, RS(t)\,. \qquad (2.71)$$

Where $X(t)$ and $RS(t)$ are the aggregate variables indicating the aggregate resources devoted to IGs production and to R&D in equations (2.27) and (2.52), respectively. In equation (2.71), the left-hand side is government tax revenue from assets income, $\tau_K\, r(t) \int_0^1 K(a,t)\, da = \tau_K\, r(t)\, K(t)$ (where K denotes individuals' wealth and has been interpreted as assets – value of patents in use, remember) and labour income, $\tau_w\, w^m(t) \int_0^1 [u_w(a,t) m(a,t)]\, da = \tau_w\, [w^{VH}(t)\, VH(t) + w^{SH}(t)\, SH(t)]$. The right-hand side is government expenditures on subsidies for IGs, $s_x\, X(t)$, and for R&D, $s_r\, RS(t)$.

2.7.2. Aggregate resource constraints

For each country, total FG output, $Y(t)$, is allocated among total consumption, $C(t)$, production of IGs, $X(t)$, and total R&D expenditures, $RS(t)$. In other words, the aggregate FG is used for consumption, C, and savings, $\dot{K}$ – so K denotes household wealth and has been interpreted as assets, remember. The savings – or, in other words, investments – are allocated to IGs production and used as research input. Therefore, the aggregate budget constraint is given by the following equation:

$$Y(t) = C(t) + X(t) + RS(t). \qquad (2.72)$$

From the previous analysis, we verify that in equilibrium the aggregate output, Y, the demand for IGs, X, and resources devoted to R&D, RS, are all constant multiples of both domestic (since here there are no IT effects) levels of TK and domestic (since there is no international mobility of production factors) levels of HC at work. Solving equation (2.72) for the aggregate consumption, C, we get – substituting equations (2.29), (2.28) and (2.41) for Y, X and RS, respectively – that C is also a constant multiple of the aggregate quality indexes and of the levels of HC at work: $Q^{JSH}SH(.|w)$ and $Q^{JVH}VH(.|w)$.[100] In short, in equilibrium, aggregate output, consumption, IGs demand and resources devoted to research are all constant multiples of the same variables.

This completes the description of the model and in the next section we conclude the chapter with some elements of synthesis.

2.8. Some elements of synthesis

Although centred on a regime without IT effects (note that we decided to disregard the discussion on the relative importance of embodied *versus* disembodied idea flows and so we avoid the word autarky), in this chapter, we present the basic elements that characterise both economies – North and South – and which can easily be extended to both IT of IGs and IT of all goods regimes.

The North is more productive than the South by reason of the exogenous level of productivity directly related with the quality of domestic institutions, is endowed with more HC and is relatively abundant in *VH* (school intensive and relatively more productive), and its R&D activities result in innovations that improve the quality of IGs – *i.e.*, Schumpeterian R&D –, whereas the South undertakes imitative activity. These differences are assumed as given – they supposedly have historical roots –, and we want to analyse the subsequent path of both economies under the various IT regimes. Our purpose is to explain how the IT regime alone – without international mobility of factors and foreign direct investment – affects the structure and dynamics of TK diffusion, thus influencing levels, growth rates and inter and intra-country wage inequality.

As a result, we develop a North-South general equilibrium model, in such a way that productivity, R&D and HC differences become clear and central to the next analysis. As to the productive activity, we follow the conventional Schumpeterian growth model,

[100] Assuming for the imitator country that the changes in $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ are of second order.

and consider three productive sectors: FGs, IGs and R&D. In the first, each good is competitively produced by one of two substitutable technologies: either with a set of differentiated IGs and *VH* or with a different set of IGs and *SH*. Implicitly, we assume that the production function is the same for IGs and R&D sectors and so our framework is a lab equipment type model. The IGs sector consists of monopolistic producers since their production requires manufacturing costs that can be subsidised, plus start-up cost of R&D that entails the payment of dividends to the firm's shareholders. Monopolistic competition here reflects the non-rival nature of TK.

Next, we characterise the equilibrium for a given state of TK progress and HC. It becomes clear that the countries' growth rates will depend on the available TK progress, either through innovation or through imitation (or, as we will see later on, through importing when IT becomes possible), and on domestic endogenous accumulation of two types of HC, *VH* and *SH*, since there is no international mobility of HC.

It results also that only one technology, VT or ST, will in fact be used to produce a particular FG. A substitutability between them together with complementarity in the use of inputs will be particularly important for the influence of the IT regime on the direction taken by TK progress and, in turn, on wage inequality. For now, we can generically state that in cases where either the technology is highly versatile biased or there is a large relative *VH* at work, then the fraction of FGs employing *VH* and using VT will be large. In this case, the relative price of *VH* FGs will be low, which decreases the incentives to upgrade versatile TK and the relative price paid (wage) per unit of *VH*.

Not surprisingly, the versatile premium (intra-country wage inequality) is greater when technologies are more versatile biased and when *VH* at work is relatively scarcer. Thus, HC accumulation influences inequality since HC investment affects the factor proportion, which has impact on relative wages. Between countries, the versatile premium will be relatively greater in the South, when the latter is relatively abundant in *SH* and when technologies in the imitator country are relatively more versatile biased.

Moreover, together (i) differences in (non-IT related) exogenous productivity in production function, (ii) international immobility of HC, and (iii) differences in available TK suggest the maintenance of inter-country wage inequality favourable to the North.

In the R&D sector, firms carry out research to come up with ideas that they develop into blueprints for new quality IGs and which are possible from the actual state of the GPT. We can imagine the R&D firms as either the R&D department of the IGs

firm or R&D specialised firms and can be subsidised. The competitive market of blueprints implies free entry in the IGs sector, which results in the elimination of profits in a present value sense. In addition to the assumption that the North innovates and the South imitates, because in the latter country imitation is cheaper than innovation and there is domestic but no (at least full) international IPRs protection, our Schumpeterian R&D process embodies substantive differences from previous literature of TK diffusion.

In particular, we consider (together) (i) an autonomous positive learning effect of accumulated public TK from past successful research, (ii) an adverse effect of market size by considering costs of scale increasing in R&D – composed by some general costs and rent protection activities undertaken by leader firms –, which enables us to rule out the scale effects and (iii) a Southern specific catching up term, itself with some particular characteristics, as the TK gap threshold and the gap in HC at work between countries.

In fact, apart from the endogenous growth debate on the independence between economic growth and scale effects, we regard this independence as mainly instrumental in isolating the TK diffusion effects of the IT regime. In other words, we aim at emphasising the mechanisms, other than market size, by which the IT regime brings about North-South TK diffusion. In this way, a larger market for TK does not lead to more research and thereby only the price channel works – *i.e.*, there is only a greater incentive to improve technologies which produce more expensive goods.

Furthermore, since we want to focus on international TK diffusion, it seems reasonable to consider that the South in not too far behind the North. The degree of backwardness is included by making Southern imitation of existing TK conditional to the distance from the TK frontier. Hence, there is a threshold distance beyond which the cost of imitation is infinite. Therefore, when the TK gap is greater than the threshold, the South is unable to imitate and grow rapidly, whereas Southern countries with smaller gaps find it easier to imitate, grow faster and converge to the innovator's income level.

In line with the existing literature, the imitation capacity also depends on political effects, on openness to IT and other trade policies and on the gap in HC at work with the North. That is, we assume that there is no (significant) barrier to imitation and thereby the advanced Northern TK is available to some Southern countries worldwide. Due to the cost of imitation, the market for IGs will also be non-competitive in the South.

As to individuals, we assume that each country is populated by a unit mass continuum, each one with a different cognitive ability. Individuals endowed with high

ability accumulate *VH*, and those with less ability fare better by accumulating *SH*. In order to make a living, they need to generate income by selling HC on the labour market. The amount earned is in agreement with their level of HC, and is spent on consumption, on loan in return for interest or on taxes. Hence, individuals decide, in a structure of learning or doing (or about their time), to work or to increase their stock of HC. They have perfect foresight concerning the rate of TK change over time and choose their expenditure paths in order to maximise its discounted utility. Thus, we get a dynamic general equilibrium model, so all markets clear throughout time.

In the educational process, each individual combines schooling and OJT, with the aim of raising his/her HC. We assume that (i) in both inputs the HC accumulation is described by functions *à la* Lucas (1988) and is not an automatic process: it requires time of training during which individuals do not work and so entails an opportunity cost defined in terms of wage; (ii) in both, the constant productivity is proportional to the 'knowledge pool' provided by the GPT, but is greater at school; (iii) the intensity of both depends on the HC accumulated; (iv) they may be substitutes or complements, and thus a constant elasticity of substitution function is used for HC accumulation. If they are substitutes (complementary), the optimal time allocation ratio, which is independent of time and individual ability, but dependent on the type of HC, is reported positively (negatively) to their respective productivity. Substitutability also shows that schooling tends to better provide all skills for production, which is not true under complementarity.

As to each individual, the growth rate of aggregate consumption is given by the standard Euler equation (since the population does not grow), as if each country admits a representative consumer with the usual constant relative risk aversion preferences – *i.e.*, the solution for the consumption path is independent of the ability and type of HC. Thus, the economic growth rate will depend on the difference between the after assets income tax interest rate and the discount rate, and on the intertemporal elasticity of substitution.

An important arbitrage condition inter-relates the returns from the different types of assets, *K* and *HC*, through which the constant optimal time allocation ratios, $u_T(m)/u_S(m)$, can be expressed in terms of the (respective) given wage dynamics and interest rate. As a result, for example, when the interest rate is high, the wage dynamics are also high. The intuition is the following. When the interest rate is high there will be strong investment in R&D, which improves the marginal product of the HC and individuals also adjust their

training effort – keeping the optimal time allocation ratio constant – in order to meet the demand for HC in such a way that the relationship remains constant.

We assume that the government budget is always balanced. The indirect effect of government intervention in HC is not yet clear. This matter will be clarified in the next chapters. In equilibrium, resources in each country measured in aggregate FG output can be used in IGs production, in R&D, or consumed. From the aggregate resource constraint and the previous analysis, aggregate output, IGs demand, resources to R&D and to consumption are all constant multiples of the domestic (since here there are no IT effects) levels of TK and of the domestic (since there is no international mobility of production factors) levels of HC at work.

Bearing in mind the analysis developed during this chapter, it is clear that the implications resulting from applying (i) Cournot competition in the IGs sector instead of Bertrand competition; (ii) vertical and horizontal successful researches together; (iii) the world implications of R&D cost advantage of leaders over followers and in particular differences in the catching up term for Southern leaders and followers, as well as the hypothesis of diminishing returns in R&D effort possibilities at a firm level; (iv) richest assumptions about the productivity of the educational process – *i.e.*, of both training activities, schooling and OJT – and, related with that, the impact of governmental policies aimed at increasing some or both types of HC, remain for future work.

In next chapter, we conclude the analysis under the IT regime without IT effects. We investigate the steady state, paying special attention to wage inequality. In addition, we analyse other issues – *TFP* differences between countries, implications of a new GPT, transition dynamics towards the steady state through calibration and sensitivity analysis.

CHAPTER 3

NORTH-SOUTH TECHNOLOGICAL KNOWLEDGE DIFFUSION WITHOUT TRADE EFFECTS

3.1. Overview

Since the structure of the countries characterised in the last chapter is strongly oriented to the IT regime without IT effects, now we continue and conclude the analysis of this IT regime. In this case, all Northern firms involved in R&D undertake innovative research and the innovators are unaffected by potential Southern imitation. In particular, it is assumed that the North does not receive any compensation – *i.e.*, royalties –, for use by imitation of its TK in the Southern country, and it does not seek to produce abroad.

Considering that the South is sufficiently close to the TK frontier,[101] all Southern firms involved in R&D undertake imitative research so long as the cost of imitation is less than or equal to that of innovation. Moreover, in the present IT regime, Southern imitator firms need only imitate technologies on one quality rung – *i.e.*, one level – above the current Southern quality level, since they are only selling the imitated quality-adjusted IGs domestically. It is expected that the decision to imitate instead of trying to innovate on its own will greatly benefit the South.

That is, due to the potential positive effect of TK backwardness, the South must achieve higher growth rates during the transitional dynamics phase and, in steady state, both countries must experience the same economic growth rate and the same path of intra-country wage inequality, determined by the North. Thus, for example, Northern improvements which positively affect the economic growth rate also favour the South. In summary, a pattern of conditional convergence between the North and the South must emerge, which, in turn, also improves the Southern output level, the Southern total factor productivity and decreases the inter-country wage inequality.

The remainder of the chapter is organised as follows. In section 3.2, we analyse the steady state: growth rate, comparative analysis and related effects. In section 3.3, we explore two extensions of the model. First, we analyse productivity differences between countries and, then, we observe the world implications of a new GPT. In section 3.4, we

discuss the transition to and the stability of the steady state. In section 3.5, we perform a sensitivity analysis. In section 3.6, we conclude with some elements of synthesis.

3.2. The steady state

In this section, we are concerned with the steady state properties of the model.

The structure of the section is as follows. In subsection 3.2.1, we start by showing that in steady state there is a unique BG path common to both countries and we make a comparative steady state analysis. In subsection 3.2.2, we analyse other steady state effects of international TK diffusion, namely in terms of the Southern growth benefit, convergence in output levels between countries, prices and wages.

3.2.1. Steady state growth

Common steady state growth rate

Since the composite FG production has constant returns to scale in inputs Qs and $m(.|w)$ – *e.g.*, equation (2.28) –, and it is used for consumption, production of IGs and R&D – *i.e.*, Y, X, RS and C are all constant multiples of the available domestic (since there is no IT) levels of TK and of the domestic (since there is no international mobility of production factors) levels of HC at work: $Q^{JSH}SH(.|w)$ and $Q^{JVH}VH(.|w)$ –,[102] then the constant and unique steady state growth rate – which, through the Euler equation (2.64), also implies a constant steady state interest rate, r^{ss} –, designed by gr, is:[103]

$$\frac{\dot{Q}^{Jm}}{Q^{Jm}} + \frac{\dot{m}(.|w)}{m(.|w)} = \frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{RS}}{RS} = \frac{\dot{C}}{C} = \frac{\dot{c}}{c} = \frac{1}{\theta}\left[(1-\tau_K)\, r^{ss} - \rho\right] = gr\,. \qquad (3.1)$$

Clearly, steady state endogenous growth is driven by R&D and HC accumulation.[104] A higher steady state interest rate implies a lower current value of future profits and thus lower expected marginal profits, because of a discount rate effect – *e.g.*, equation (2.45). On the other hand, from equation (3.1), it also implies a higher growth rate and associate with – or due to – this, significant R&D activity. Thus, from both considerations, we have that a higher interest rate seems to induce strong R&D activity

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[101] Otherwise, the South is unable to imitate and a divergent TK pattern between the countries occurs.

[102] In the South, this is really true when $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ are constant, since the resources devoted to R&D are a function of the variables $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ – *e.g.*, equation (2.52) – and, consequently, due to the aggregate resource constraint equation (2.72), so are the resources used for consumption.

[103] For the gr in equation (3.1) to be >0 and bounded, $(1-\tau_K)\, r^{ss}$ should be greater than ρ and bounded.

[104] For example, without HC accumulation growth is only driven by R&D – see Appendix 2, section A2.2.

that shortens the period in which the IG producer can enjoy its monopoly profits; *i.e.*, it suggests a strong process of creative destruction. In steady state, as equation (3.1) states,

$$\frac{\dot{Q}^{J^{SH}}}{Q^{J^{SH}}} + \frac{\dot{SH}(.|w)}{SH(.|w)} = \frac{\dot{Q}^{J^{VH}}}{Q^{J^{VH}}} + \frac{\dot{VH}(.|w)}{VH(.|w)}, \text{ and then } \frac{\dot{p}^{SH}}{p^{SH}} = \frac{\dot{p}^{VH}}{p^{VH}} = 0. \tag{3.2}$$

Introducing the information in equation (3.2) into equation (2.30) – *i.e.*, so that the wage growth rates increase steadily, in proportion to TK progress –, we can: (i) state that improvements in *m*-specific TK create new *m*-type HC demand and bid up its wage rate, in compliance with empirical estimates of TK bias – *e.g.*, Bound and Johnson (1992) and Bartel and Sicherman (1999); (ii) easily obtain the constant steady state interest rate, following the same steps as those proposed to obtain the equilibrium interest rate. The resulting expression is – where *ss* means steady state and $\Gamma_I = 0$ and $\Gamma_P = 1$:[105]

$$r_Z^{ss} = \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - \tau_{KZ} \right]^{-1} \left\{ \frac{\beta_Z}{\zeta_Z} \left[exp(PE)\ g_1(.)^{\bar{\sigma}_1} g_2(.)^{\sigma_2(.)} \right]^{\Gamma_Z} \overline{m} \left[\frac{1-s_{xZ}}{1-s_{rZ}} \right] \left(\frac{q-1}{q} \right) \right.$$
$$\left. \left[\frac{p_Z^m\ A_Z\ (1-\alpha)}{1-s_{xZ}} \right]^{\frac{1}{\alpha}} \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right] - \delta^m + \varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)} \right]^{\frac{1-\phi}{\phi}} \right\} \tag{3.3}$$

Where p^m's, g_1 and g_2 as well as σ_2 are in steady state – *i.e.*, they are constant. The first term in large brackets depends on – or is closely related with – the rate of return from research (*i.e.*, the expected flow of profit per unit of research effort) – *e.g.*, equation (2.48), and the second term relies on the rate of return to new HC – *e.g.*, equation (2.69).

As shown in equation (3.1), the steady state growth rates result from plugging the steady state interest rate – equation (3.3) – into the growth rate of consumption derived from individuals' optimisation – Euler equation (2.64):

$$gr_Z = \left[\frac{1}{\theta} \left(\frac{1-\tau_{KZ}}{q^{\left(\frac{1-\alpha}{\alpha}\right)} - \tau_{KZ}} \right) \right] \left\{ \frac{\beta_Z}{\zeta_Z} \left[exp(PE)\ g_1(.)^{\bar{\sigma}_1} g_2(.)^{\sigma_2(.)} \right]^{\Gamma_Z} \overline{m} \left[\frac{1-s_{xZ}}{1-s_{rZ}} \right] \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right] \right.$$
$$\left. \left(\frac{q-1}{q} \right) \left[\frac{p_Z^m\ A_Z\ (1-\alpha)}{1-s_{xZ}} \right]^{\frac{1}{\alpha}} - \delta^m + \varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)} \right]^{\frac{1-\phi}{\phi}} - \rho \left(\frac{q^{\left(\frac{1-\alpha}{\alpha}\right)} - \tau_{KZ}}{1-\tau_{KZ}} \right) \right\} \tag{3.4}$$

Since from equation (3.1) the steady state growth rates are completely determined by the steady state interest rate and individual's preference parameters, the determinants of the steady state interest rate are also the determinants of the steady state growth rate.

[105] Alternatively, we can state that r^{ss} results from constant growth of wages – equation (2.70) –, which, in turn, result from constant prices of FGs and from constant growth of available TK – equation (2.30).

As explained in previous studies – *e.g.*, Rebelo (1991), Mulligan and Sala-i-Martin (1993) and Stokey and Rebelo (1995) –, in equilibrium (and so in steady state), the rates of return to all investments should be the same. As the composite FG is the only input in IGs production and R&D activities, the FGs production technology affects the equilibrium rate of return to investments and so the equilibrium interest rate. Moreover, as HC and quality-adjusted IGs are required for FGs production, the equilibrium interest rate depends on: (i) FGs production technology; (ii) HC production technology; (iii) R&D technology; and (iv) government policies – *e.g.*, equation (3.3).

As in steady state $Q^{J^{SH}} SH(.|w)$ and $Q^{J^{VH}} VH(.|w)$ must grow at the same rate, FGs price indexes stay constant – *e.g.*, equations (3.1), (3.2) and (2.25a, b).[106] The constant steady state interest rate and thus the constant steady state growth rate for the North (*i.e.*, when $Z = I$) is then a direct result of, respectively, equations (3.3) and (3.4). However, the constant steady state interest rate and thereby the constant steady state growth rate for the South (*i.e.*, when $Z = P$) requires that the South's relative m-type HC at work, $\tilde{m}(.|w)$, and the South's relative m-specific TK level, $\tilde{q}(.|m)$, must be constant as well – *i.e.*, the m-type HC employed and the m-specific TK must grow at the same rate in both countries. In fact, this is already essential and so included in equation (3.1) for the South.

Thus, we can state that, as a result of international TK diffusion, the level of $\tilde{m}(.|w)$ and $\tilde{q}(.|m) \equiv \tilde{Q}^{J^m}$ reached in steady state remain constant – *i.e.*, once in steady state both countries enjoy the same interest rate and grow at the same rate. Therefore, the common steady state growth rates of HC at work and of TK imply the maintenance of steady state North-South gaps, which tend to remain below one.[107] As there is a world BG path defined by the North, causes that improve the North's growth rate, such as better Northern institutions, also boost the South's growth rate, while Southern advances affect the steady state levels of $\tilde{m}(.|w)$ and $\tilde{Q}^{J^m}$.

As r^{ss} is constant, equations (2.48) and (2.54) suggest that $\dot{Q}^{J^{m^{ss}}}/Q^{J^{m^{ss}}}$ is also constant. Then, according to equation (3.1), $\dot{m}^{ss}(.|w)/m^{ss}(.|w)$ is also constant:

$$\frac{\dot{m}^{ss}(.|w)}{m^{ss}(.|w)} = \frac{1}{\theta}\left[(1-\tau_K)\, r^{ss} - \rho\right] - \frac{\dot{Q}^{J^{m^{ss}}}}{Q^{J^{m^{ss}}}}. \qquad (3.5)$$

[106] Thus, for example, if the steady state growth rate of Q^{JVH} is higher than that of Q^{JSH} then the steady state growth rate of $VH(.|w)$ is smaller than that of $VH(.|w)$; *i.e.*, the versatile-augmenting TK saves *VH*.
[107] This holds by assuming the starting condition (2.23), as we will see by numerically solving the model.

Since HC accumulation is dependent on m-type HC, but independent of individual ability, and as the steady state equilibrium requires that time allocation does not change over time – *i.e.*, individuals decide to devote a constant fraction of the fixed-time endowment to working and to educational process –, then from equations (2.59), (2.65) and (3.1) we have that:[108]

$$u_S^{ss}(m) = \left\{ \frac{1}{\theta} \left[(1 - \tau_K)\, r^{ss} - \rho \right] - \frac{\dot{Q}^{Jm^{ss}}}{Q^{Jm^{ss}}} + \delta^m \right\} \left\{ \varphi^{m\,[1/\phi]}\, \chi_S \left[1 + \frac{u_T(m)}{u_S(m)} \right]^{\frac{1}{\phi}} \right\}^{-1} . \qquad (3.6)$$

Given the constant value for $u_S^{ss}(m)$ in equation (3.6), we easily obtain the values for $u_T^{ss}(m)$, from equation (2.65), and for $u_w^{ss}(m)$, from the resource constraint on each type of HC: $u_w^{ss}(m) = 1 - u_S^{ss}(m) - u_T^{ss}(m)$. From equation (3.6), we further conclude that $u_S^{ss}(m)$, $u_T^{ss}(m)$ and $u_w^{ss}(m)$ are equal in both countries.

Figure 3.1 – complemented by figure 3.2 – sums up the behaviour of the North-South steady state growth rates, assuring an unique world constant growth rate defined by the North, and that – or which requires that – $\tilde{Q}^{Jm}$ and $\tilde{m}(.|w)$ remain constant.

Figure 3.1. Behaviour of the steady state growth rates

$$\begin{array}{ccccccc}
\dfrac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}} & + & \dfrac{\dot{SH}_I(.|w)}{SH_I(.|w)} & \underbrace{=}_{1)} & \dfrac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}} & + & \dfrac{\dot{VH}_I(.|w)}{VH_I(.|w)} \\
= \Big\} 2) & & = \Big\} 2) & \underbrace{=}_{1)} & = \Big\} 2) & & = \Big\} 2) \\
\dfrac{\dot{Q}_P^{J^{SH}}}{Q_P^{J^{SH}}} & + & \dfrac{\dot{SH}_P(.|w)}{SH_P(.|w)} & = & \dfrac{\dot{Q}_P^{J^{VH}}}{Q_P^{J^{VH}}} & + & \dfrac{\dot{VH}_P(.|w)}{VH_P(.|w)}
\end{array}$$

From the analyse above, the steady state equilibrium is a path where all variables either grow at a constant rate (not necessarily the same) over time – Y, C, X, RS, $m(w)$, $m(school)$, $m(OJT)$, and Q^{Jm} – or are time invariant – u_S, u_T, N, p^m, $\tilde{m}$, $\tilde{Q}^{Jm}$, r, pb_I and pb_P – such that each individual maximises his/her lifetime utility by allocating his/her time between production and HC accumulation and his/her income between consumption and saving, each (IG, FG and R&D) firm maximises its profits, and all the markets clear.

[108] It is possible to show that the condition $(1 - \tau_K)\, r^{ss} > \rho$ also assures that $0 < u_S^{ss}, u_T^{ss}, u_w^{ss} < 1$.

Comparative steady state analysis

The common steady state growth rate of both countries given by equation (3.4) for $Z = I$ – it is equal to the steady state growth rate in equation (3.4) for $Z = P$ – allows for further comparative steady state analysis. Table 3.1 sums up the qualitative results. However, only for the exogenous parameters and variables which are specifics to the North or common to both countries, because the North determines the (world) steady state growth rate. International TK diffusion highlights the importance of those Northern changes because they carry weight, not only at home but also abroad. Changes in both exogenous parameters and variables in the South – as well as in the North – affect, in particular, the South-North quality indexes, $\tilde{Q}^{Jm}$, as will be apparent below.

Table 3.1. Comparative steady state analysis

	∂gr
∂A_I	+
$\partial \overline{m}$	+
$\partial \alpha$ [a)]	–
$\partial \beta_I$	+
$\partial \zeta_I$	–

	∂gr
$\partial \theta$	–
$\partial \rho$	–
∂s_r	+
∂s_x	+
$\partial \tau_K$	–

	∂gr
$\partial \varphi^{VH}$	+
$\partial \chi_S$	+
$\partial \chi_T$	+
$\partial \delta^m$	–
$\partial \phi$	+

Note: [a)] considering the particular case $q = 1/(1-\alpha)$.

The innovator country's exogenous level of productivity, A_I, affects positively the level of output internally – *e.g.*, equation (2.28) –, and the world growth rate as well. In the North, this is due to the role of competitive monopoly profits – *e.g.*, equation (2.39a) – and in the South, this is because of international TK diffusion.[109] The m-augmenting parameter, $\overline{m}$, in the production function affects positively the growth rate also due to the role of competitive monopoly profits.

The HC share in the production function, α, has a negative impact on the growth rate. At first sight, this is because it increases the size of each quality upgrade, q, which requires more resources to obtain each successful research – *e.g.*, equation (2.50) –, and this acts as a disincentive to R&D activities. But, a greater q also increases the size of profits – see equation (2.39a). Thus, the intuition is the following. IG firm takes into account the probability of losing its monopoly, and so it discounts profits at a rate higher

[109] That is, Northern changes that improve the North's growth rate also boost the South's growth rate.

than the interest rate. The discount rate is higher when α is greater, and so it disincentives or discourages R&D activities and thereby influences negatively the growth rate.[110] This is in line with recent empirical literature – *e.g.*, Ahn (2002).

As to the R&D parameters, the higher the learning to past successful research spillover, β_I, and the smaller the fixed cost of the successful research parameter, ζ_I, the higher the incentives to do R&D become and so the higher the growth rate.

The impact of ρ and θ is consistent with the fact that if present consumption is more highly valued than future consumption (following the properties of the utility function), then this will lead to less need for private investment and so to less dispersion over time. In other words, the more patient – *i.e.*, the smaller the value of ρ – and the less keen the individuals are on consumption smoothing – *i.e.*, the smaller the value of θ – the higher the steady state growth rate.

Through both subsidies' exogenous ad-valorem s_x and s_r, government intervention can positively affect the steady state growth rate.[111] The former because it increases the monopolistic mark-up and, thereby, profits – *e.g.*, equation (2.39a) –, thus acting as an incentive to support R&D activities. The latter since it decreases the cost of R&D – *e.g.*, equation (2.47) –, thus increasing the equilibrium probability of successful research – *e.g.*, equation (2.48). In short, a rise in the subsidy rates, s_x and s_r, leads to an increase in the long run growth rate, since they promote TK progress.

An increase in assets income tax, τ_K, decreases investment in TK accumulation, as a result of the smaller expected marginal benefit of R&D, which, in turn, has direct negative effects on growth. However, taking together the set of variables, s_x, s_r and τ_K, we can state that, in fact, the collection of additional asset income taxes has a direct negative effect on the growth rate, but it also has an indirect positive effect. This is because, in this way, the government obtains resources – *e.g.*, equation (2.71) –, which contribute to lower marginal costs of production of IGs and of R&D.

Particularly interesting is the fact that the steady state growth rate is independent of the HC income tax, τ_w. The reason is that τ_w does not affect neither the Euler equation, nor the HC production technology, nor the FGs production nor the R&D technology. Generically, since τ_w is absent in equilibrium conditions, it does not affect growth.

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[110] Market power seems to be not growth enhancing – note that q serves as a measure of market power.

[111] Since common sense suggests that both R&D and IGs production subsidies should promote growth, this result also represents a check that the model is well-behaved.

To sum up, the policy implications of our results are that, in order to maintain a balanced government budget and higher growth, the HC income tax must be higher than the assets income tax. At the limit, the assets income must be free of taxes.[112] These results are in line with, for example, Zeng and Zhang (2002).

Given our particular purposes, the steady state growth rate does not exhibit scale effects. For that reason, it is in accordance with Jones' (1995a,°b) critique. However, contrarily to Jones's (1995b) influential model,[113] it is endogenous since it is determined within the framework by the existing market forces and is not completely independent of preferences and of policy variables.

In fact, in Jones's model the scale effects property is removed, but the steady state growth rate is totally determined by technological parameters and the (exogenous) rate of population growth, and so it is truly exogenous. Nevertheless, as 'innovations' are still the outcome of purposeful and costly R&D effort, he defines it as 'semi-endogenous'. As Jones's model does not explain whether or how economic policies or tastes are capable of affecting the economic growth rate, it is different from the endogenous R&D growth models in which growth usually depends upon preferences and/or policy variables.

Our results regarding the 'growth effects' of government policies are consistent with the findings in the first-generation R&D-based endogenous models with scale effect – *e.g.*, Romer (1990b), Grossman and Helpman (1991a) and Aghion and Howitt (1992, 1998b) –, those in recent R&D models without scale effects – *e.g.*, Aghion and Howitt (1992, 1998a), Howitt (1999), Zeng and Zhang (2002), and Zeng (2003) –, and with convincing evidence found by some works, like Bleaney *et al.* (2001).[114]

As to the HC production parameters – see equation (2.59) –, table 3.1 shows that the more intensive the use – in the production of *VH* and *SH* – of the more productive mode of HC production – schooling –, the higher the steady state growth rate. Also, the greater the rate of substitution between modes of HC production is – measured by ϕ –, the higher the use of the more productive one will be, and, consequently, the higher the growth rate. Productivity in the HC sector, then propagated to the economy, is also improvable directly through χ_S and χ_T. In summary, improvements in productivity in the

[112] When we investigate the effect of one of the political instruments, τ_K, s_x or s_r, we maintain the others constant and leave the task of balancing the government budget to residual changes in the HC income tax.
[113] As well as other (second-generation) R&D-based endogenous growth models without scale effects, like Segerstrom (1998) and Young (1998).
[114] However the empirical results in the literature are far from conclusive. For example, Easterly and Rebelo (1993a, b) reject the policy-effectiveness predictions.

HC sector reduce its opportunity cost and, consequently, increase its accumulation, which, in turn, increases *gr*. Moreover, if we take only one mean for HC accumulation (schooling or OJT), then we can under or over estimate the growth of HC, and so make mistakes on the value estimates for price indexes, interest rate, growth rate and so on.

It is clear that the steady state growth is affected by HC accumulation technology. We can go further by stating that it is also affected by the structure of the HC at work. In fact, its structure actuate directly in the FGs price indexes, p^m, and so on the steady state growth rate, through its impact on the direction of the R&D activity – *e.g.*, equations (2.48) and (2.54).[115] In this sense, our framework is in line with the literature considering that HC is important for long run growth and influences it by both channels, as a 'flow' and as a 'stock'. Nevertheless, as pointed out by Aghion and Howitt (1998a, chap. 10), they have starkly different implications for the effects of HC investment decisions made in labour markets on long run growth. Current investment (by raising the level of HC) leads, on the one hand, to a one time surge in output as new HC is applied in production and, on the other, has a permanent effect on TK change and hence on economic growth.

3.2.2. Steady state effects

We now come to the technological implications of steady state growth in the South, since goods, assets, HC as well as TK do not flow from South to North.[116]

Southern growth benefit with North-South technological knowledge diffusion

In order to look at the steady state effects of international TK diffusion we must investigate *gr* further. To this end, we start analysing the Southern growth benefits with imitation. Since there is a common steady state growth rate, we can compare the steady state interest rate in equation (3.3) with the one that would prevail without international flow of goods and assets, as well as TK.[117] In this case, in an optimistic scenario the probability of successful imitation collapses into an inefficient Southern probability of successful innovation, where backwardness and 'imitation capacity' disappear.[118] That is, the experience-adjusted probability of successful imitation, Φ_P, changes to $\Phi_P^* \equiv \beta_P \, \zeta_P^{-1}$. The increment in the steady state interest rate, from no imitation to imitation, depends on:

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[115] And, consequently, on intra-country wage inequality – *e.g.*, equation (2.32).

[116] Relatively to the latter – *i.e.*, the TK – it does not flow freely (from the North to the South, since there is an associated cost of imitation).

[117] For example, we can suppose that there is total international IPRs protection.

[118] That is, if there is no TK diffusion, we assume that the South can do R&D innovation activity in such away that it is less efficient than the North. Otherwise, we could assume that there is no R&D in the South.

$$\frac{\beta_I}{\zeta_I}\left(p_I^{m^{SS}} A_I\right)^{1/\alpha} - \frac{\beta_P}{\zeta_P}\left(p_{P,WP}^{m^{SS}} A_P\right)^{1/\alpha} ; \qquad (3.7)$$

where $p_{P,WP}^{m^{SS}}$ and $p_I^{m^{SS}}$ are the steady state index price of the FGs produced with m-type of HC in the South without imitation possibility and in the autarkic North, respectively. While effective evaluation of equation (3.7) requires solving for transitional dynamics through calibration and simulation, we can, however, emphasise that Southern imitation benefits the Southern steady state growth rate. In fact, previous assumptions with regard to (i) exogenous productivity variables, $A_I > A_P$, (ii) HC share in production, $1/\alpha > 1$, and (iii) learning to past successful research spillover and fixed cost of successful research parameter, $\beta_I \zeta_I^{-1} \cong \beta_P \zeta_P^{-1}$, lead to this result, considering that the influence of the steady state index price of FGs are of second order. As figure 3.2a below shows, TK diffusion benefits the (steady state) growth rate of both Southern quality indexes.

In short, without international flow of goods, assets and TK diffusion, divergence would occur between countries. International TK diffusion is sufficient to bring about steady state interest rate equalisation. Moreover, as will be apparent later on, in this case the Southern growth rate exceeds the autarkic (steady state) Northern growth rate during its transition to steady state and falls gradually toward the steady state, as was implicit in figure 3.2.[119] Thus, our framework is in line with Durlauf and Johnson (1995), Evans (1996) and Quah (1996, 1997), among many others, since they have argued that the data supports a theory of 'club convergence' – *i.e.*, that some countries are converging to a parallel growth path, while the gap between these countries and the poorest countries of the world – which are distant from the TK frontier – is steadily diverging over time.

Other comments on convergence between countries

By setting gr or r^{SS} in both countries – in equations (3.3) or (3.4) – equal to each other, we can get an equation for the steady state South-North quality index ratio, $\tilde{Q}^{J^{m^{SS}}}$:

$$\left[g_2(\tilde{Q}^{J^{m^{SS}}}, d)\right]^{-\bar{\sigma}_2+\tilde{Q}^{J^{m^{SS}}}} = \frac{\beta_I \zeta_P}{\zeta_I \beta_P} \frac{g_1[\tilde{m}(.|w)^{SS}]^{-\bar{\sigma}_1}}{exp(PE)} \left(\frac{p_I^{m^{SS}} A_I}{p_P^{m^{SS}} A_P}\right)^{1/\alpha} . \qquad (3.8)$$

As occurred in the previous case, evaluating equation (3.8) obliges us to solve for transitional dynamics. We can, however, make the following four related comments.

[119] Considering that χ_S and χ_T are both functions of the domestic TK progress, then outside – during the transitional dynamics to – the steady state, the South could accumulate more HC than the North. By this, it could speed up the process of convergence – more HC (stock) helps the imitation capacity, remember.

(i) As the differences between countries are in the respective R&D activities, $\tilde{Q}^{J^{mss}}$ is directly related with the relative profitability of R&D in both countries.

(ii) The left-hand side term decreases when $\tilde{Q}^{J^{mss}}$ increases. Consequently, a higher β_P, a good imitation capacity and a lower ζ_P implies a higher probability of successful imitation – a lower value of the right-hand side term – and so a higher $\tilde{Q}^{J^{mss}}$. Contrarily, a higher β_I or a lower ζ_I generates a higher probability of successful innovation and so a smaller $\tilde{Q}^{J^{mss}}$, although a higher common steady state growth rate. The impact of A_P and A_I is similar to β_P and β_I, respectively, but now, this occurs because they are an incentive to R&D activities by reason of their influence on monopolistic profits.[120]

(iii) If the steady state South-North quality ratios are less than one, the South will remain the imitator country and its income level will be smaller than the Northern income level, but both countries grow at the same rate. In this case, the North never has an incentive to imitate, because foreign TK to imitate never exists. Thus, there will be convergence in growth rates but not in levels – *i.e.*, conditional convergence occurs.[121] Previous assumptions with regard to exogenous levels of productivity, A_I and A_P, and in the experience-adjusted probabilities of successful research, Φ_I and Φ_P, tend to keep the North as the innovator country, as is clear in figure 3.2b below. This is consistent with, for example, the empirical findings of Barro and Sala-i-Martin (1991).

(iv) Since the South tends to reach the (world) steady state with conditional convergence, to get farther, it can resort to two potential paths, either by becoming an efficient innovator or improving its imitative activity. Given its stage of development, its innovative activity is neither very efficient nor easy to enhance. Hence, it is preferable to upgrade its imitative activity, which can be obtained with free IT – see the next chapters.

Figure 3.2 compares the steady state paths of the quality indexes in both countries. Time zero, t_0, corresponds to the South's move towards international TK diffusion. If the South is not too backward relative to the North, imitation is possible and its TK indexes growth at higher rates during its transition to the steady state and fall gradually toward a higher steady state. Thus, the TK of the South converges toward that which exists in the North, although the latter remains higher due to the Northern CA in innovation.

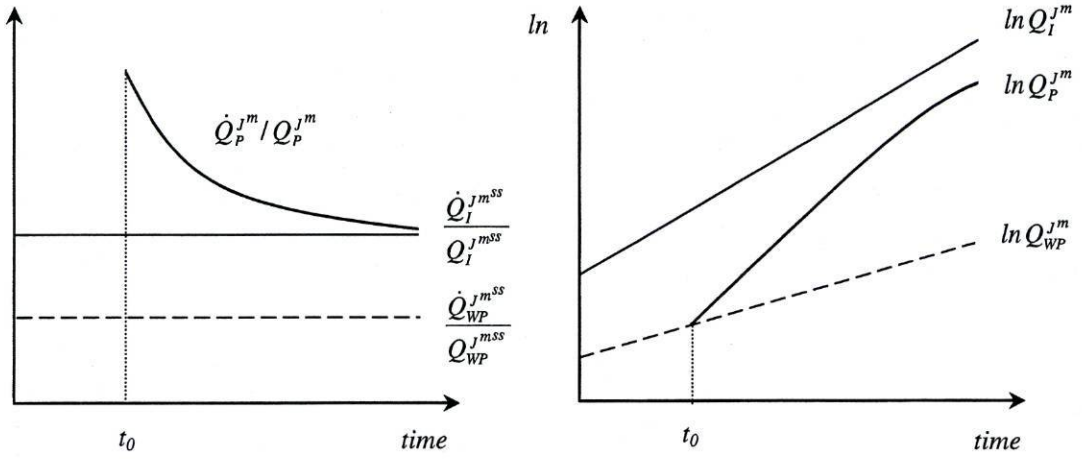
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[120] Note that changes in A_P, β_P, ζ_P, and in the imitation capacity do not affect the steady state growth rate.

[121] If the steady state South-North quality ratios are >1, the South takes the leader position once the steady state South-North quality ratios are $=1$. This will be a one-time switch with the (prior) South remaining in

Figure 3.2. Steady state behaviour of the domestic technological knowledge index, Q^{Jm}

a) Steady state growth rate **b) Steady state level**



We leave other comments on the steady state paths for the versatile-specialised TK bias, $Q^{JVH^{ss}}/Q^{JSH^{ss}}$, for the transitional dynamics. It is crucial to the steady state path of relative wages, since with the relative prices of FGs constant, the direction of TK bias is the sole determinant of the path of wage inequality – *e.g.*, equations (2.31) and (2.32).

Final goods price indexes, pattern of production and relation with technological knowledge

Due to substitutability between ST and VT and complementarity between types of IGs and types of HC in the production function (2.1), prices of FGs indexes, p^m, and the technology threshold, N, are determined by the combination of the two types of TK with the respective HC employed – *e.g.*, equations (2.22) and (2.25a-c). From equation (3.1), such steady state combinations remain constant in each country and, thereby, so do N and prices. It is now our aim to find steady state expressions for these variables.

Equalling the steady state growth rates of $Q^{JSH}SH(.|w)$ and $Q^{JVH}VH(.|w)$ in the North, we get an equation which relates the steady state price indexes, $p_I^{m^{ss}}$, as:

$$sh\left(p_I^{SH^{ss}}\right)^{1/\alpha} - vh\left(p_I^{VH^{ss}}\right)^{1/\alpha} = \Lambda_I \cdot \tag{3.9}$$

$$\text{where : } \Lambda_I = \frac{\chi_S\left\{\varphi^{VH\ [1/\phi]}\left[1+\frac{u_T(VH)}{u_S(VH)}\right]^{\frac{1-\phi}{\phi}} - \varphi^{SH\ [1/\phi]}\left[1+\frac{u_T(SH)}{u_S(SH)}\right]^{\frac{1-\phi}{\phi}}\right\} + \delta^{SH} - \delta^{VH}}{\frac{\beta_I}{\zeta_I}\left[\frac{1-s_{xI}}{1-s_{rI}}\right]\left(\frac{q-1}{q}\right)\left[\frac{A_I(1-\alpha)}{1-s_{xI}}\right]^{\frac{1}{\alpha}}\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]}.$$

the leadership position from that time on since it must have the CA in innovating so as to leapfrog the (prior) North. The world steady state growth will then be determined by the new innovator country.

Together, equations (3.9) and (2.25a, b) allow us to obtain an equation for the steady state Northern threshold FG, N_I^{ss}. Given the resulting equation and taking into consideration that in steady state both countries enjoy the same interest rate and growth rate, we can also get an equation for the steady state threshold FG gap, $\tilde{N}^{ss}$. That is,

$$N_I^{ss} = \frac{1}{2}\left\{\left(\frac{sh+vh}{\Lambda_I^*}+1\right) - \left[\left(\frac{sh+vh}{\Lambda_I^*}+1\right)^2 - \frac{4\,sh}{\Lambda_I^*}\right]^{\frac{1}{2}}\right\}; \tag{3.10a}$$

$$\tilde{N}^{ss} = \frac{N_P^{ss}}{N_I^{ss}} = N_P^{ss}\left[1-\tilde{\Omega}^{ss}\right] + \tilde{\Omega}^{ss}. \tag{3.10b}$$

Where: $\Lambda_I^* = \dfrac{\Lambda_I}{exp(-1)}$ and $\tilde{\Omega}^{ss} = \dfrac{g_1[\tilde{SH}(.|w)^{ss}]^{\bar{\sigma}_1}\left[g_2(\tilde{Q}^{JSH^{ss}}, d)\right]^{-\bar{\sigma}_2+\tilde{Q}^{JSH^{ss}}}}{g_1[\tilde{VH}(.|w)^{ss}]^{\bar{\sigma}_1}\left[g_2(\tilde{Q}^{JVH^{ss}}, d)\right]^{-\bar{\sigma}_2+\tilde{Q}^{JVH^{ss}}}}$.

In equation (3.10a), we bear in mind that N_I^{ss} is between *0* and *1*. It shows that the threshold FG in the North on the BG path is constant, as are the FGs price indexes, $p_I^{m^{ss}}$, in equation (3.9) and (2.25a-c). The outcome of equation (3.10b) demonstrates that N_P^{ss} is equal, greater and smaller than N_I^{ss} when $\tilde{\Omega}^{ss}$ is equal, greater and smaller to *1*, respectively. In any case, N_P^{ss} and $p_P^{m^{ss}}$ are also constant. Furthermore, they are affected by the North, and so, as a result of TK diffusion, the Southern productive structure and prices are conditioned by the North. Assuming that the North is already in steady state at time t_0, it remains there, since it is unaffected by imitation. If this is so, it is expected that the TK diffusion cause some convergence in the structure of production and prices between countries. In short, N_P and p_P^m are defined by both TK and HC levels in both countries,[122] but there are no immediate changes in the Southern levels of TK and of HC.

Given the steady state value of N_Z^{ss} and equation (2.22), we can write:

$$\frac{Q_Z^{JVH^{ss}}}{Q_Z^{JSH^{ss}}} = \left(\frac{1-N_Z^{ss}}{N_Z^{ss}}\right)^2 \frac{sh}{vh}\left(\frac{VH_Z(.|w)^{ss}}{SH_Z(.|w)^{ss}}\right)^{-1}. \tag{3.11}$$

Equation (3.11) tells us that the steady state domestic versatile-specialised TK bias depends on the threshold FG and on the domestic relative structure of the two types of HC. It confirms that TK progress will be endogenously biased towards the scarce HC.

[122] In consequence, Southern individuals can immediately adjust the path of time allocation.

Wages: inequality and growth rates

Besides, equation (3.11) together with equation (2.32) gives the following relation between the versatile premium and the domestic relative abundance of *VH* over *SH*:

$$\frac{w_Z^{VH^{ss}}}{w_Z^{SH^{ss}}} = \left(\frac{1-N_Z^{ss}}{N_Z^{ss}}\right)\left(\frac{VH_Z(.|w)^{ss}}{SH_Z(.|w)^{ss}}\right)^{-1}. \tag{3.12}$$

Comparing equations (3.12) and (2.32), which specified the versatile premium (intra-country wage inequality measure) as a function of relative domestic HC stocks and relative domestic quality indexes, now the relation between the versatile premium and the relative domestic HC stock is less elastic: $-1 < -1/2$. That is, the demand long run curve is less elastic when the domestic quality indexes adjust – Appendix 2, section A2.3, gives us an overview of the effect a rise in *m*-type HC has on wages. We leave effective evaluation of equation (3.12) for transitional dynamics through calibration.

Next we can define the relative factor shares:

$$\frac{w_Z^{VH^{ss}} VH_Z(.|w)^{ss}}{w_Z^{SH^{ss}} SH_Z(.|w)^{ss}} = \left(\frac{1-N_Z^{ss}}{N_Z^{ss}}\right). \tag{3.13}$$

This last equation states that with endogenous TK change, the shares of a factor in the output, Y_Z, will be 'directly independent' of its abundance and are constant. If $N_P^{ss} > N_I^{ss}$ – which, as we will see later on by solving the model numerically, holds due to the starting conditions in equation (2.23) – then the *VH* share in *Y* is greater in the North.

Together, equations (2.30) and (3.1) – or figure 3.1 – indicate that the steady state rates of growth of wages are equalised between countries; *i.e.*,

$$\frac{\dot{w}_I^{m^{ss}}}{w_I^{m^{ss}}} = \frac{\dot{Q}_I^{J^{m^{ss}}}}{Q_I^{J^{m^{ss}}}} = \frac{\dot{Q}_P^{J^{m^{ss}}}}{Q_P^{J^{m^{ss}}}} = \frac{\dot{w}_P^{m^{ss}}}{w_P^{m^{ss}}} \Rightarrow \left(\frac{\dot{w}_I^{VH^{ss}}}{w_I^{VH^{ss}}} - \frac{\dot{w}_I^{SH^{ss}}}{\dot{w}_I^{SH^{ss}}}\right) = \left(\frac{\dot{w}_P^{VH^{ss}}}{w_P^{VH^{ss}}} - \frac{\dot{w}_P^{SH^{ss}}}{\dot{w}_P^{SH^{ss}}}\right) = \frac{\dot{Q}_Z^{J^{VH^{ss}}}}{Q_Z^{J^{VH^{ss}}}} - \frac{\dot{Q}_Z^{J^{SH^{ss}}}}{Q_Z^{J^{SH^{ss}}}}. \tag{3.14}$$

This can be regarded as a Schumpeterian dynamic equivalent to the static factor-price equalisation theorem – also called Samuelson theorem – due to international TK diffusion and even without IT.[123] That is, in spite of international immobility of HC, the steady state growth of wages is common to both countries because their TK indexes grow at the same rate as a result of the international TK diffusion by imitation. Thus,

(i) If the South cannot imitate since it is far from attaining the (Northern) TK frontier, then its TK change will be smaller and, thereby, its wages will decrease (diverge) relative to Northern wages (inter-country wage inequality results).

(ii) If the South can imitate, then its TK change will be greater and so its wages will benefit from the imitation activity. Moreover, since the South grows faster during the transition towards the world BG path, there is convergence in wages between countries. In any case, a wage differential remains between countries in steady state – maintenance of inter-country inequality. This is to be expected from the conjunction of international immobility of HC, differences in the exogenous productivity variables, $A_I > A_P$, HC share in production, $1/\alpha > 1$, and differences in available TK. In fact, from equation (2.31),

$$\frac{w_P^{mss}}{w_I^{mss}} = \left(\frac{p_P^{m^{ss}} A_P}{p_I^{m^{ss}} A_I} \right)^{1/\alpha} \frac{Q_P^{J^{mss}}}{Q_I^{J^{mss}}} < 1. \qquad (3.15)$$

In equation (3.15) we take into consideration that the effects of the FGs price indexes on wage differential between countries are of second order. We can evaluate this equation later on – *i.e.*, in section 3.4 – by transitional dynamics through calibration.

3.3. Some extensions

3.3.1. Overview

In this section, we proceed to incorporate two extensions into our framework. They are included here because each one is first analysed in the steady state, and then they can be evaluated numerically later on in the transitional dynamics through calibration.

The first, in subsection 3.3.2, concerns the analysis of the pattern of both the cross-FGs total factor productivity (*TFP*) and aggregate *TFP* between countries.

The second, in subsection 3.3.3, discusses the world consequences of a new GPT. In particular, we find its implications for growth, levels, inter and intra-country wage inequality, structure of production and productivity differences in two possible scenarios.

3.3.2. Final goods and aggregate productivity differences

The literature shows that there are large productivity differences between countries. However, most of this literature highlights the international disparities in the aggregate *TFP* – *e.g.*, Bernard and Jones (1996a, b), Hall and Jones (1999) and Acemoglu and Zilibotti (2001). A more detailed analysis is found less often. For example, in the three works quoted, only the last one, which here we follow as closely as possible, analyses the cross-FGs *TFP* differences and includes both Northern and Southern countries.

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[123] The static factor-price equalisation theorem states that free IT equalises factor prices between countries and thus serves as a substitute for international factor mobility – *e.g.*, Chacholiades (1990).

We find some measures for evaluating the equilibrium pattern of cross-FGs *TFP* and the equilibrium aggregate *TFP* in each country, for given TK and HC at work. After that, we use them to analyse the steady state *TFP* differences between countries. To this end, in line with the assumptions applied – *i.e.*, as a result of starting differences between countries including starting conditions in equation (2.23) –, we point out that (i) $\tilde{Q}^{J^{m\,ss}} < 1$ – *i.e.*, the South stays imitator; (ii) $\tilde{\Omega}^{ss} > 1$ – *i.e.*, $\tilde{VH}^{ss}\ \tilde{Q}^{J^{VH\,ss}} < \tilde{SH}^{ss}\ \tilde{Q}^{J^{SH\,ss}}$. Thus, $\tilde{N}^{ss} > 1$, $N_P^{ss} > N_I^{ss}$, $p_I^{SH} > p_P^{SH}$ and $p_I^{VH} < p_P^{VH}$ – *e.g.*, equations (2.22), (3.10b) and (2.25a, b).[124]

Final goods productivity differences

Starting with the equilibrium pattern of cross-FGs *TFP* becomes useful in clarifying the origins of the productivity differences in the world. The *TFP* of the representative FG firm *n*, TPF_n, is the share of its output in value terms that is imputable neither to the respective 'physical inputs' – *i.e.*, without the IGs input – nor to the individuals input *per se*. In our case, to produce the nth FG, individuals contribute in terms of embodying HC, and not in physical terms;[125] *i.e.*, if they do not embody HC they do not produce. Thus, as will be shown below, the output of the nth FG in value terms can only be decomposed into two components: (i) the *TFP* and (ii) the IG input.

In short, the *TFP* will then be particularly related to the TK embodied in the IGs and to the HC embodied in the individuals or, in other words, the *TFP* in the nth FG increases with greater TK – *i.e.*, with better quality-adjusted IGs – and with greater HC accumulation – *i.e.*, with better productivity by individuals. By definition, the IGs input used in the production of the nth FG is $X_n(t|m) \equiv \int_0^{J^m} x_n(k,j,t|m)\,dj$, where $x_n(k,j,t|m)$ is given by equations (2.12a, b). So – omitting the country and time indexes:

$$X_n^{1-\alpha} = \begin{cases} \left[\frac{p_n(t)\,A\,(1-\alpha)}{q(1-s_x)}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} \left[(1-n)\ sh\ SH_n(.|w)\ Q^{J^{SH}}\right]^{(1-\alpha)} & \text{if} \quad 0 \le n \le N \\ \left[\frac{p_n(t)\,A\,(1-\alpha)}{q(1-s_x)}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} \left[n\ vh\ VH_n(.|w)\ Q^{J^{VH}}\right]^{(1-\alpha)} & \text{if} \quad N < n \le 1 \end{cases} . \qquad (3.16)$$

---

[124] In addition, we consider that $\tilde{SH}^{ss}\ \tilde{Q}^{J^{SH\,ss}}$ – as expected – is smaller than *1* or not as great as *1*. This assumption, as well as (i) and (ii), are confirmed in transitional dynamics analysis through calibration.

[125] And, recall, $SH(t|w) \equiv \int_0^{\bar{a}} SH_n(a,t|w)\,da$ and $VH(t|w) \equiv \int_{\bar{a}}^{1} VH_n(a,t|w)\,da$ and $a \in [0, 1]$ in *I* and *P*.

With the help of equations (2.20a, b) and (3.16), we separate the output of the n[th] FG in value terms, obtained by multiplying equation (2.17) by the respective price, p_n, (i) into the TFP_n and (ii) into just one 'physical input' related to IG:

$$p_n Y_n = \begin{cases} TFP_n(.|SH)\ X_n(.|SH)^{1-\alpha} & \text{if} \quad 0 \le n \le N \\ TFP_n(.|VH)\ X_n(.|VH)^{1-\alpha} & \text{if} \quad N < n \le 1 \end{cases}; \tag{3.17}$$

$$\text{Where: } TFP_n(.|m) = A\ p^m \left[Q^{J^m}\ \overline{m}\ m_n(.|w) \right]^{\alpha}.$$

Equation (3.17) provides two main conclusions, which are shown in figure 3.3a below. In the first place, it guarantees that, in equilibrium, all FGs in each country have the same *TFP*. In fact, considering equations (2.20a, b) and (2.25a, b), we have that:

$$\frac{TFP_n(.|SH)}{TFP_n(.|VH)} = \frac{A}{A}\frac{p^{SH}}{p^{VH}}\left[\frac{Q^{J^{SH}} sh\ SH_n(.|w)}{Q^{J^{VH}} vh\ VH_n(.|w)}\right]^{\alpha} = 1. \tag{3.18}$$

In the second place, in steady state, the TFP_n is greater in the North. We arrive at this conclusion by combining our assumptions relative to the features of the countries, the first conclusion in equation (3.18) above, and equations (2.25a, b). Thus, we obtain:

$$\left[\frac{TFP_{nP}(.|SH)^{SS}}{TFP_{nI}(.|SH)^{SS}}\right]_{n \le N_I^{SS}} = \frac{A_P}{A_I}\left[\frac{1}{\tilde{N}^{SS}}\right]^{2\alpha}\left[\tilde{Q}^{J^{SH^{SS}}}\ \tilde{SH}(.|w)^{SS}\right]^{\alpha} < 1. \tag{3.19}$$

From equation (3.18) the equilibrium TPF_n is the same for all FGs in each country, and from equation (3.19) in steady state it is greater in the North, for $n \le N_I^{SS}$. The steady state TPF_n will thus be greater in the North in all *n*s, $0 \le n \le 1$. Equation (3.19) clarifies the origins of the *TFP* differences between countries, which stem from: (i) exogenous productivities, *A*s; (ii) available TK, *Q*s; (iii) HCs at work, *m*s(.|*w*); and (iv) differences in the productive structures, *N*s. *N* can be also used to measure the extent to which *VH* and *SH* are employed in FGs for which they may or may not have CA.

Furthermore, based on our previous analysis, we can define the physical *TFP* (*PTFP*), in FG *n* as $PTFP_n(.|m) = TFP_n(.|m)/p_n$, and then we can write that:

$$PTFP_n(.|m) = \begin{cases} A\frac{p^{SH}}{p_n}\left[Q^{J^{SH}} sh\ SH_n(.|w)\right]^{\alpha} = A\left[(1-n)\,Q^{J^{SH}} sh\ SH_n(.|w)\right]^{\alpha} & \text{if} \quad n \le N \\ A\frac{p^{VH}}{p_n}\left[Q^{J^{VH}} vh\ VH_n(.|w)\right]^{\alpha} = A\left[n\,Q^{J^{VH}} vh\ VH_n(.|w)\right]^{\alpha} & \text{if} \quad n > N \end{cases}. \tag{3.20}$$

In relation to the FG TFP_n, we obtained several different conclusions, which are shown in figure 3.3b below. Firstly, in order to compare the *PTFP* of the different FGs in each country, we also use equations (2.20a, b) to give:

$$\frac{PTFP_n(.|SH)}{PTFP_n(.|VH)} = \left[\frac{1-n}{n}\right]^{\alpha}\left[\frac{Q^{J^{SH}} sh\, SH(.|w)}{Q^{J^{VH}} vh\, VH(.|w)}\right]^{\alpha}\left[\frac{1-N}{N}\right]^{\alpha} = \left[\frac{1-n}{n}\right]^{\alpha}\left[\frac{N}{1-N}\right]^{\alpha}. \qquad (3.21)$$

Hence, in each country, when $n \neq N$ there are, at the most, two FGs with the same *PTFP*.[126] In the particular case when $n = N$ the ratio in equation (3.21) is equal to *1*.

Secondly, the steady state *PTFP* is also higher in the North than in the South in all FGs. Thirdly, comparing the three cases, $n \leq N_I$, $N_I < n < N_P$ and $n \geq N_P$, we find that the difference increases between $N_I < n < N_P$, and after N_P it remains constant. Formally,

$$\left[\frac{PTFP_{nP}(.|SH)^{ss}}{PTFP_{nI}(.|SH)^{ss}}\right]_{n \leq N_I^{ss}} = \frac{A_P}{A_I}\left[\frac{N_I^{ss}}{N_P^{ss}}\right]^{\alpha}\left[\frac{Q_P^{J^{SH\,ss}} SH_P(.|w)^{ss}}{Q_I^{J^{SH\,ss}} SH_I(.|w)^{ss}}\right]^{\alpha} < 1; \qquad (3.22a)$$

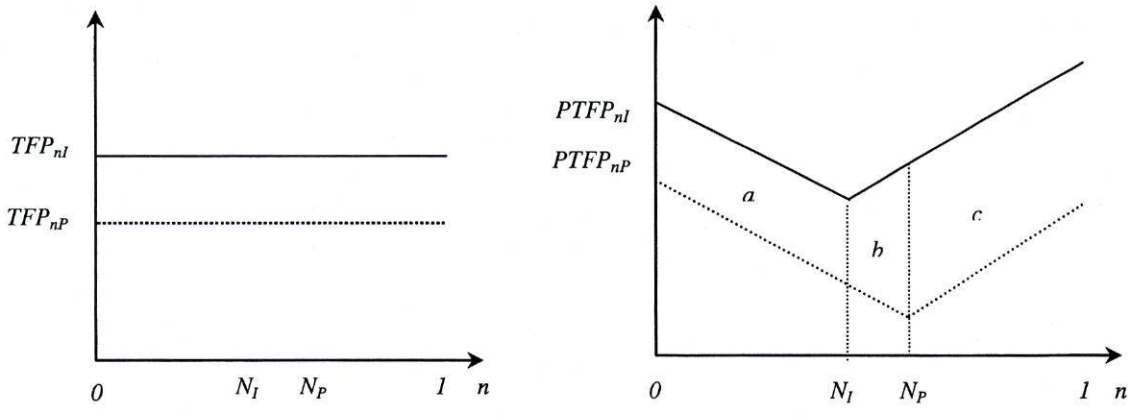
$$\left[\frac{PTFP_{nP}(.|SH)^{ss}}{PTFP_{nI}(.|VH)^{ss}}\right]_{N_I^{ss} < n < N_P^{ss}} = \frac{A_P}{A_I}\left[\frac{1-n}{n}\right]^{\alpha}\left[\frac{1-N_I^{ss}}{N_P^{ss}}\right]^{\alpha}\left[\frac{Q_P^{J^{SH\,ss}} shSH_P(.|w)^{ss}}{Q_I^{J^{VH\,ss}} vhVH_I(.|w)^{ss}}\right]^{\alpha} < 1; \qquad (3.22b)$$

$$\left[\frac{PTFP_{nP}(.|VH)^{ss}}{PTFP_{nI}(.|VH)^{ss}}\right]_{n \geq N_P^{ss}} = \frac{A_P}{A_I}\left[\frac{1-N_I^{ss}}{1-N_P^{ss}}\right]^{\alpha}\left[\frac{Q_P^{J^{VH\,ss}} VH_P(.|w)^{ss}}{Q_I^{J^{VH\,ss}} VH_I(.|w)^{ss}}\right]^{\alpha} < 1. \qquad (3.22c)$$

The last conclusion occurs because FGs between $N_I < n < N_P$ are produced by VT in the North, whereas in the South they are produced by ST.

Figure 3.3. Final goods total factor productivity (*TFP*) and physical *TFP* patterns

a) Final goods *TFP* **b) Final goods physical *TFP***



In figure 3.3b, areas $a+b$ must be equal to area c.

[126] If, for example, $N = 0.4$, then $n^{SH} = 0.3$ and $n^{VH} = 0.4(6)$ have the same *PTFP*.

Still concerning the last conclusion, we note that the result in equation (3.22a) for $n \leq N_I^{ss}$, is immediate, since $A_P < A_I$, $N_I^{ss} \leq N_P^{ss}$ and $\tilde{Q}^{J^{SH\,ss}} \tilde{SH} < 1$. Next, by computing the ratio between equations (3.22a) and (3.22b) and between equations (3.22a) and (3.22c), we find that, for $n > N_I^{ss}$ the difference increases until $n = N_P^{ss}$, where it stops. Hence, as shown in figure 3.3b, we can state that the source of this increase is the use of *SH* – and so ST – in some FGs in the South, which use *VH* – and so VT – in the North.

Aggregate productivity differences

The equilibrium FGs *TFP* can be translated into a measure of aggregate *TFP*. In view of $Y \equiv exp\left[\int_0^N ln\, p_n Y_n(.|SH)\, dn + \int_N^1 ln\, p_n Y_n(.|VH)\, dn \right]$, equations (2.20a, b), (2.25a, b) and (3.17) and using some additional algebra, the resulting expression is:

$$Y = TFP\ X(.|SH)^{(1-\alpha)N}\ X(.|VH)^{(1-\alpha)(1-N)}. \tag{3.23}$$

$$\text{Where: } TFP \equiv exp\left[\int_0^N ln TFP_n\ dn + \int_N^1 ln\, TFP_n\ dn \right] =$$

$$= A\ exp(-\alpha)\left[\left(SH(.|w)\, sh\, Q^{J^{SH}} \right)^N \left(VH(.|w)\, vh\, Q^{J^{VH}} \right)^{1-N} N^{-(2N)} (1-N)^{-2(1-N)} \right]^{\alpha}.$$

We refrain from computing the steady state ratio between the aggregate *TFP* in both countries. In fact, since they produce the same number of FGs and, in steady state, the North has higher *TFP* than the South in all FGs, then the aggregate *TFP* must also be higher in the former country than in the latter. In any case, it is apparent from (3.23) that differences between countries again stem from differences in *A*, *Q*s, *m*s and *N*s.

We can compute other – perhaps simpler – measures of aggregate productivity. We are referring to the aggregate output per individual, *Y*;[127] the aggregate output per HC occupation at work, $y \equiv \{Y[SH + VH]/[SH(.|w) + VH(.|w)]\}$, which is also an indicator of the aggregate output per individual working; and the aggregate output per efficiency unit of HC occupation at work, $y^{eff} \equiv \{Y[SH + VH]/[sh\, SH(.|w) + vh\, VH(.|w)]\}$, which is also a measure of the aggregate output per efficiency unit of individual working.

Since both economies are populated by a unit mass continuum of individuals, although each has a different cognitive ability, then equation (2.28) also gives us the product by individual and, in addition, we can easily calculate *y* and y^{eff}. The steady state

[127] As the population is stable, the growth rates of an aggregate variable and its per capita value are equal.

ratios for each of these three measures of aggregate productivity in both countries are given by the following two equations because Y_P/Y_I and y_P/y_I are equal:

$$\frac{Y_P^{ss}}{Y_I^{ss}} = \frac{y_P^{ss}}{y_I^{ss}} = \left(\frac{A_P}{A_I}\right)^{\frac{1}{\alpha}} \tilde{Q}^{J^{SH^{ss}}} \tilde{SH}(.|w)^{ss} \left(\frac{N_I^{ss}}{N_P^{ss}}\right)^2 < 1; \tag{3.24a}$$

$$\frac{y_P^{eff}}{y_I^{eff}} = \frac{Y_P^{ss}}{Y_I^{ss}} \left(\frac{SH_P^{ss} + VH_P^{ss}}{SH_I^{ss} + VH_I^{ss}}\right) \left(\frac{sh\,SH_I(.|w)^{ss} + vhVH_I(.|w)^{ss}}{sh\,SH_P(.|w)^{ss} + vhVH_P(.|w)^{ss}}\right) < 1. \tag{3.24b}$$

According to our previous considerations, equations (3.24a, b) show that aggregate productivity is greater in the North than in the South, regardless of the measure used. Thus, at FGs level and at aggregate level, the productivity is greater in the North relative to the South. This is caused by differences in the available TK, in the levels of HC at work, in the exogenous productivity, A, and in the threshold FG, N.

In the literature, most works view TK and HC differences as the most important causes of the large disparities across countries, or find evidence for this situation – *e.g.*, Romer (1993), Caselli *et al.* (1996) and Hall and Jones (1999). In our case these causes are represented by differences in Q^{J^m} and in $m(.|w)$.

In Acemoglu and Zilibotti (2001) all countries have immediate access to the same TK developed in the North. However, this region is abundant in skills and the market size channel implies that it tends to develop relatively skill-complementary technologies, which are not suited to the needs of unskilled workers performing these tasks. Importing inappropriate TK – *i.e.*, owing to the mismatch between TK and skills – leads to a lower productivity in the South. A number of other papers have emphasised the importance of the appropriateness of TK – *e.g.*, Diwan and Rodrik (1991) and Basu and Weil (1998). In our case, differences in N between countries are related to this idea.

Other works suggest that a large component of the productivity differences across countries is due to A – *e.g.*, Golub and Yeaple (2002). Bearing the last section in mind, it is clear, for example, that an exogenous improvement in A_I helps the steady state growth rate in both countries, although it does in fact expand the productivity gap between the countries. On the other hand, an exogenous improvement in A_P does not influence the world steady state growth rate, but it does depress the productivity gap between countries directly and indirectly because it increases the steady state TK in the South.

Finally, as the steady state interest rate is constant and common to both countries and the productivity differs, differences are reflected in wages – which increase steadily.

3.3.3. Consequences of a new General Purpose Technology

In this subsection, we extend the framework developed so far to analyse the effects of the introduction of the new GPT. However, contrarily to current literature, which focuses analysis on developed countries, the structure of our framework allows us to also observe the effects on less developed countries. Thus, we want to explore the world implications of the new GPT which is not contained in the literature. In order to simplify, we focus the investigation on the successive steady states that emerge.

So as to analyse some consequences of GPT, Helpman and Trajtenberg (1998) present a general equilibrium framework that incorporates the concept of a GPT in a R&D growth model directed at developing new, horizontally differentiated goods. They consider that there is one composite FG produced with both a GPT and a variety of compatible IGs. Nevertheless, when a new GPT arrives, it cannot be used immediately. In fact, as David (1990) argues, it may take several decades before major technological innovations can have a significant impact on macroeconomic activity.[128] Moreover, they only consider the existence of one economy. Hence, they cannot explain the international effects of a new GPT, such as on the world growth rate, on the productivity differences between countries, on inter and intra-country wage inequality, and so forth.[129]

In line with Helpman and Trajtenberg (1998), we assume that each FG n is also produced with the aid of an available GPT, which can be used because it is compatible with the domestic TK.[130] Then, in order to keep things as simple as possible, we consider that the switch to the new GPT occurs after the aggregate domestic quality indexes reach a certain minimum value. After this point, the new GPT can be exploited and final output goes up (boom). In short, in line with the literature, our approach to modelling the GPT has three features: (i) new GPT acts as an exogenous 'engine of growth'; (ii) it can take a large amount of time for a new GPT to be introduced; (iii) at least after a certain point in time, new GPT is beneficial to large or all FGs in each country (generality of purpose).

This last feature compels us to divide the subsection into two parts or scenarios. In the first – briefly explored –, the new GPT is beneficial to all FGs at the same time, in each country. In the second – analysed in greater detail –, first new GPT is beneficial to some FGs and only later it is beneficial to the remaining ones, in each country. Moreover, the use of a new GPT first occurs in the North. This division has formal

[128] David (1990) describes a pre-paradigm phase of twenty-five years in the case of the electric dynamo.
[129] Such as the pattern of specialisation, which will be analysed in chapter 5.
[130] Bearing in mind that the domestic TK is measured by the available aggregate quality indexes.

implications with crucial impact on the versatile premium (intra-country wage inequality) for some time, which, however, can be a long period of time, in each country.

Regardless of the number of phases considered, we assume that the model has steady state equilibrium in each one. That is, the steady state is reached more rapidly than some fresh incorporation of the new GPT in production. At the final steady state, the long run growth rate is higher in both countries, relative to the initial steady state.[131]

We begin the analysis considering that countries are in a previous steady state at time t^{ss}, producing FGs with an old GPT when a new, more productive, GPT arrives unexpectedly (exogenous). To introduce the productivity level of a new GPT, we assume that under the new GPT the (exogenous) productivity level in the production function (2.1) is given by A_Z^{NG}, where $A_Z^{NG} = A_Z\,\gamma$ and $\gamma > 1$ – from now on, the index NG represents the value of the variable or of the parameter under the new GPT. Moreover, the quality index k must be equal to or greater than k^M, where $k \geq k^M$ and k^M is the lowest rung of a quality ladder necessary for the IG j in order to use the new GPT – *i.e.*, k^M is the minimum quality rung developed in R&D which is compatible with the new GPT.

1st scenario: New GPT is beneficial to all final goods at the same time

Since the introduction of the new GPT depends on the measure of the domestic TK compatible, the switch to its use occurs only after enough successful research has taken place. Here, we assume that this occurs simultaneously for all FGs, regardless of the technology – VT or ST – used by them, in each country. Thus, the production for the nth FG under the new GPT, Y_{nZ}^{NG}, is again given by equation (2.2), but now the exogenous productivity level is $A_Z^{NG}(> A_Z)$ and $k \geq k^M$. Then, bearing chapter 2 in mind, from the time of the use of the new GPT until the arrival of a new GPT, equation (2.17) becomes:

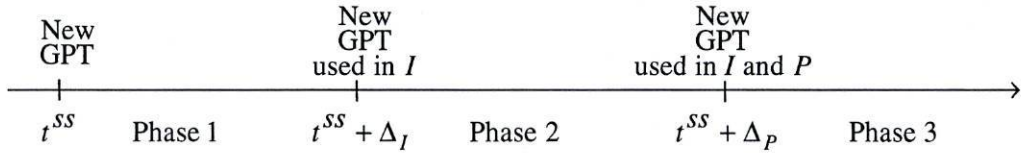
$$Y_n^{NG} = \begin{cases} A^{NG^{(1/\alpha)}} \left[\dfrac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} (1-n)\; sh\, SH_n(.|w)\; Q^{J^{SH}} & if \quad 0 \leq n \leq N \\ A^{NG^{(1/\alpha)}} \left[\dfrac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} n\, vh\, VH_n(.|w)\, Q^{J^{VH}} & if \quad N < n \leq 1 \end{cases} \cdot \qquad (3.25)$$

[131] Nevertheless, we must note from the previous analysis that the non-arrival of the new GPT does not reduce the long run growth rate to zero.

In equation (3.25), Q^{Jm} must be at least equal to (or greater than) $Q^{J_M^m}$, $Q^{Jm} \geq Q^{J_M^m}$,[132] where $Q^{J_M^m} \equiv \int_0^{J^m} q^{k^M(j,t|m)[(1-\alpha)/\alpha]}dj$ is the m-type aggregate domestic minimum quality index or TK compatible with the new GPT, and, supposedly, $Q^{J_M^{SH}}$ and $Q^{J_M^{VH}}$ occur at the same time.[133] This is true because, when comparing the production in equations (2.17) and (3.25), we take it that, as long as $Q^{J_M^m}$ occurs, the new GPT will be used in producing FGs, since A_Z^{NG} is greater than A_Z.

We let Δ_Z represent the time which is required to reach $Q^{J_M^m}$ from the previous steady state in country Z – *i.e.*, for $Q_Z^{J^m} = Q^{J_M^m}$. Thus, without any new unexpected GPT, the new GPT generates three distinct phases, as figure 3.4 shows.

Figure 3.4. Phases generated by the new GPT in the time horizon



In phase 1, during $[t^{SS}, t^{SS}+\Delta_I)$, the old GPT continues to be used in both countries. In phase 2, during $[t^{SS}+\Delta_I, t^{SS}+\Delta_P)$, the new GPT is used only in the North, since it is the TK leader country. In phase 3, from $t^{SS}+\Delta_P$ at least until new GPT arrives,[134] the previous GPT is used in both countries. In the next step, we analyse the effects of the new GPT on the steady states of each country in each phase and table 3.2 below sums up the results.

1st scenario, phase 1: the old GPT is used in both countries

Since, by assumption, in this phase the TK existing in both countries is smaller than the compatible TK associated with the new GPT, they continue to use the old GPT.

The previous steady state analysis stays valid with one single difference, since we have assumed that χ_S and χ_T are proportional to the 'knowledge pool' provided by the state of the known GPT. Thus, considering that there is a level jump in χ_S and χ_T when

[132] At the time of the introduction of the new GPT, Q^{Jm} must be at least equal to $Q^{J_M^m}$.

[133] Note that if some type of TK (for example, versatile TK) is more advanced in the previous steady state, here we implicitly assume that its TK compatible with the new GPT is also greater than that in the other type and, consequently, that they achieve TK compatibility at the same time.

[134] For simplification, we assume that some (eventual) new GPT arrives only after phase 3.

the new GPT arrives for $\chi_S^{NG} > \chi_S$ and $\chi_T^{NG} > \chi_T$,[135] respectively, the only change from the earlier analysis is that now the new temporary steady state world growth rate, $gr_{Z,Phase1}^{NG}$, is unambiguously greater that before – see equation (3.4) and table 3.1.

In short, in this phase, the new GPT confers a 'dynamic benefit' – 'growth effect' – on both countries with a positive impact on industrial – *i.e.*, on FGs – as well as on aggregate productivity. However, as the new GPT is not yet used in the production and, in particular, the R&D firms are generating new successful researches so as to reach the qualities that are compatible with the new GPT, we can state that it is a 'time to sow' (with some 'reap') for both countries, using Helpman and Trajtenberg (1998) words.

1st scenario, phase 2: the new GPT is only used in the North

Since the North is a TK leader, $Q_I^{J^m} > Q_P^{J^m}$, it reaches $Q^{J_M^m}$ at time $t = t^{ss} + \Delta_I$ and does so before the South. Thus, as at time $t = t^{ss} + \Delta_I$ the North is using a new GPT, its output rises (jumps), which can be thought of as the 'static or level gain' from new GPT – *i.e.*, the equilibrium value of Y_I in equation (2.28) jumps to Y_I^{NG} due to A_I^{NG}.

Follower IGs firms, which support the R&D costs, behave as before, except that now they expect greater profits, due to the increased productivity level of a new GPT, leading to a great rate of innovation. Therefore, the equilibrium probability of successful innovation $pb_I(m)$ in equation (2.48) increases to $pb_I^{NG}(m)$. Hence, the new Northern steady state growth rate increases again as a result of the introduction of the new GPT on the production side – *i.e.*, a new 'growth effect' occurs in the North.

In brief, relative to the successive steady state growth rates, $gr_{I,Phase2}^{NG} > gr_{I,Phase1}^{NG} > gr_I$ occurs. This happens because, when the new GPT is used in production, it confers a (new) 'dynamic benefit', in addition to the 'static benefits'. This in turn improves, with greater intensity than in the last phase, both the industrial and the aggregate *TFP* in the North. As Helpman and Trajtenberg (1998) would say, it is a 'time to reap' for the North.

As the TK in the South is distant from $Q^{J_M^m}$, it remains unable to use the new GPT. Consequently, it continues to be a 'time to sow' for the South, during which it does not benefit from a 'static gain'. However, as the TK gap between the countries is constant in steady state and the TK progress in the North increases at a higher rate than

[135] The assumption that χ_S and χ_T jump immediately the new GPT arrives implies that also u_T^{NG}/u_S^{NG} in equation (2.65) can change. This assumption is, however, a very optimistic point of view, and more so

before, the same must be true in the South.[136] In other words, the use of the new GPT by the North results in a new positive 'growth effect' in the South as well, yielding a higher common steady state growth rate. Hence, TK diffusion emphasises the importance of the new GPT in this phase, because it turns out to matter not only at home – *i.e.*, in the North –, but also abroad – *i.e.*, in the South, which expands but does not quite reach its full 'reap' capacity: $gr^{NG}_{I,Phase\,2} = gr^{NG}_{P,Phase\,2} > gr^{NG}_{I,Phase\,1} = gr^{NG}_{P,Phase\,1} > gr_I = gr_P$.

Moreover, this allows the South to attain $Q^{J^m_M}$ in less time. Nevertheless, as $A^{NG}_I > A_I$, the TK gap between countries becomes wider – *e.g.*, equation (3.8). Finally, in terms of both the industrial and aggregate productivity differences between countries as well as inter-country wage inequality, it is clear that these also widen.

1st scenario, phase 3: the new GPT is used in both countries

This phase does not affect the North and so it remains on the path reached in the previous one. But, what happens in the South? It reaches $Q^{J^m_M}$ at time $t = t^{ss} + \Delta_P$ and uses the new GPT, which confers a level jump on its equilibrium output – 'static or level benefit' – from Y_P in equation (2.28) to Y^{NG}_P. Moreover, since imitator firms behave as in the previous case, then the greater expected profits resulting from the increased productivity level of a new GPT lead to a reduction in the TK gap between countries.[137] This, jointly with the directed effect induced by the greater productivity level of the new GPT, leads to a decrease in the differential of productivity between countries as well as in the inter-country wage inequality. It is a time for a full 'reap' in the South.

Table 3.2. Main results obtained in each of the three phases

	Phase 1	Phase 2	Phase 3
Time to sow	In both, *I* and *P*	In *P*	---
Time to reap	In both, *I* and *P*	In *P*, but more in *I*	In both
Growth effects	In both, *I* and *P*	In both	---
Level effects	---	In *I*	In *P*
Inter-country wage inequality	---	↑	↓
Productivity differences	---	↑	↓

considering that this is held for both countries.

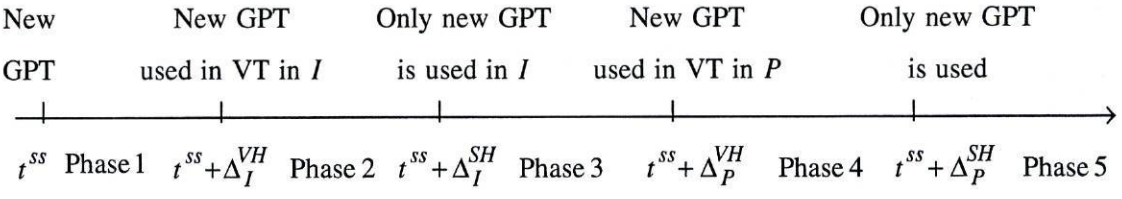
[136] *I.e.*, the South reaches a new steady state in this phase before it is TK compatible with the new GPT.

[137] Now, there is no 'growth effect' in the South – the steady state growth rate is again the Northern steady state growth rate. The use of the new GPT reduces however the TK gap between countries – *i.e.*, the positive effect of greater A^{NG}_P in gr_P is equal to the negative effect of the reduction in the TK gap in gr_P.

2nd scenario: First the new GPT is beneficial to some final goods at the same time

In line with Acemoglu (1999) and Kremer and Maskin (1999), who provide evidence indicating a number of organisational changes in the U.S. economy during the past 25 years,[138] we assume that the new GPT is introduced first in the FGs, which use VT. We let Δ_Z^m represent the time which is required to reach $Q^{J_M^m}$ from the previous steady state in country Z. Thus, until its full use in the world, the new GPT generates five potential phases with specific consequences during each one: $[t^{ss}, t^{ss}+\Delta_I^{VH})$, $[t^{ss}+\Delta_I^{VH}, t^{ss}+\Delta_I^{SH})$, $[t^{ss}+\Delta_I^{SH}, t^{ss}+\Delta_P^{VH})$, $[t^{ss}+\Delta_P^{VH}, t^{ss}+\Delta_P^{SH})$, and from $t^{ss}+\Delta_P^{SH}$ on, as figure 3.5 shows. In phase 1, the old GPT continues to be used in both countries. In phases 2 and 4, the new GPT is only used in the VT in the North and in the South, respectively. In phase 3, the new GPT is introduced in ST in the North. In phase 5, the new GPT is employed in all FGs in both countries.[139]

Figure 3.5. Phases generated by the new GPT in the time horizon



Next, we will analyse phase 2, since in phase 1 there are no differences in relation to the previous scenario. The main conclusions are summarised in table 3.3 below.

2nd scenario, phase 2: the new GPT is only used in the VT in the North

Assuming that versatile TK is greater than specialised TK, $Q_I^{JVH} > Q_I^{JSH}$, and the minimum TK compatible with the new GPT is the same for both, then the versatile TK must reach $Q^{J_M^m}$ at time $t = t^{ss}+\Delta_I^{VH}$ and before the specialised TK. Therefore, the production functions for each FG n that uses VT and ST are given, respectively, by:

$$Y_n^{NG}(t\,|VH) = A^{NG}\left\{\left[\int_0^{J^{VH}}\left(q^{k(j,t|_{VH})}\,x_n(j,t|VH)\right)^{1-\alpha}dj\right]\left[n\,vh\,VH_n(t|w)\right]^{\alpha}\right\}; \qquad (3.26a)$$

[138] That is, old style production functions are replaced by new organisational production functions. See also Bresnahan (1999), Bresnahan *et al.* (2002) and Autor *et al.* (2003).

[139] *I.e.*, by assumption the new GPT is used in the South only after it has been used in the production of all Northern FGs, since the latter is the TK leader, and any new potential GPT can only arrive after phase 5.

$$Y_n(t|SH) = A\left\{\left[\int_0^{J^{SH}} \left(q^{k(j,t|SH)}\, x_n(j,t|SH)\right)^{1-\alpha} dj\right]\left[(1-n)\, sh\, SH_n(t|w)\right]^{\alpha}\right\}. \quad (3.26b)$$

Where the quality index k is only equal to or greater than k^M, $k \geq k^M$, in all versatile-specific IGs. From equations (3.26a, b), it can be found that – where *VNG* means that the introduction of the new GPT has occurred only in VT:

$$Y_n^{VNG} = \begin{cases} A^{(1/\alpha)}\left[\dfrac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} (1-n)\, sh\, SH_n(.|w)\; Q^{J^{SH}} & if \quad 0 \leq n \leq N \\ A^{NG\,(1/\alpha)}\left[\dfrac{p_n(1-\alpha)}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} n\, vh\, VH_n(.|w)\, Q^{J^{VH}} & if \quad N < n \leq 1 \end{cases}. \quad (3.27)$$

In equation (3.27) we take it that $Q^{J^{VH}} \geq Q_M^{J^m}$.[140] Following again the analysis in section 2.4 of chapter 2, the new expression for the threshold FG $N \in [0, 1]$ is:

$$N^{VNG} = \left\{1 + \left[\frac{Q^{J^{VH}}}{Q^{J^{SH}}}\frac{vh\,VH(.|w)}{sh\,SH(.|w)}\left(\frac{A^{NG}}{A}\right)^{1/\alpha}\right]^{1/2}\right\}^{-1}. \quad (3.28)$$

Equation (3.28) tells us that at the time the new GPT is introduced in VT, N decreases. That is, FGs at the margin are moving from the use of ST to the use of VT. This is an expected result, since the introduction of the new GPT in VT makes this technology more productive and the expenditures across FGs are equalised.

There is also a level jump in the North's output, thus resulting in a 'static' gain with the use of the new GPT in this country. In fact, the equilibrium value of Y_I in equation (2.28) jumps to Y_I^{VNG} in the equation below, where $Q_I^{J^{VH}} \geq Q_M^{J^m}$:

$$Y_I^{VNG} = exp(-1)\left[\frac{1-\alpha}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}} \left\{\left[Q_I^{J^{SH}}\, sh\, SH_I(.|w)\, A_I^{(1/\alpha)}\right]^{\frac{1}{2}} + \left[Q_I^{J^{VH}}\, vh\, VH_I(.|w)\, A_I^{NG\,(1/\alpha)}\right]^{\frac{1}{2}}\right\}^2. \quad (3.29)$$

The w^m is equal to its marginal product – where below V (S) before *SH* and *VH* means the value of variables after the introduction of the new GPT only in VT (ST) –, so:

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[140] Again $Q_I^{J^{VH}} = Q_M^{J^{VH}}$ at the time the new GPT is introduced and here, by assumption, the minimum TK compatible with the new GPT is the same for both technologies and so $Q_M^{J^{VH}} = Q_M^{J^{SH}} = Q_M^{J^m}$.

$$\frac{\partial Y}{\partial SH(.|w)} = w^{VSH} = B^{VNG}\left[Q^{JSH}\, sh\, SH(.|w)\, A_I^{(1/\alpha)}\right]^{-\frac{1}{2}} sh\, Q^{JSH} A_I^{(1/\alpha)}\,; \qquad (3.30a)$$

$$\frac{\partial Y}{\partial VH(.|w)} = w^{VVH} = B^{VNG}\left[Q^{JVH}\, vh\, VH(.|w)\, A_I^{NG\,(1/\alpha)}\right]^{-\frac{1}{2}} vh\, Q^{JVH}\, A_I^{NG\,(1/\alpha)}\,; \qquad (3.30b)$$

Where: $B^{VNG} = exp(-1)\left[\frac{1-\alpha}{q(1-s_x)}\right]^{\frac{1-\alpha}{\alpha}}\left\{\left[Q_I^{JSH}\, sh\, SH_I(.|w)\, A_I^{(1/\alpha)}\right]^{\frac{1}{2}} + \left[Q_I^{JVH}\, vh\, VH_I(.|w)\, A_I^{NG\,(1/\alpha)}\right]^{\frac{1}{2}}\right\}$.

Comparing equations (3.30a, b) with equations (2.29a, b), we observe that the price paid for each unit of both types of HC increases at the time of use of the new GPT by VT. Thus, both types of workers benefit, but the benefit is greater for workers who accumulate *VH*, since the versatile premium increases and so $W_I^{VNG} > W_I$:

$$W_I^{VNG} = \frac{w_I^{VVH}}{w_I^{VSH}} = \left(\frac{Q_I^{JVH}}{Q_I^{JSH}}\right)^{\frac{1}{2}}\left(\frac{vh\, VH_I(.|w)}{sh\, SH_I(.|w)}\right)^{-\frac{1}{2}}\frac{vh}{sh}\left(\frac{A_I^{NG}}{A_I}\right)^{\frac{1}{2\alpha}}; \qquad (3.31a)$$

$$\left.\frac{W_I^{VNG}}{W_I}\right|_{t=t^{ss}+\Delta_I^{VH}} = \left(\frac{A_I^{NG}}{A_I}\right)^{\frac{1}{2\alpha}} > 1\,. \qquad (3.31b)$$

Hence, a possible explanation for the increase in demand for skills in the U.S. (and in other developed countries), more like our *VH*,[141] despite an increase in the supply of skilled workers in its (their) labour market over the past half-century,[142] is the use of new GPT only in the production of FGs which use 'skill (versatile) technology'.

Moreover, computing the new versatile premium between countries, $\boldsymbol{W}^{VNG}$, we take it that, at the time of the introduction of the new GPT, the versatile premium increases in the North – *i.e.*, $\boldsymbol{W}^{VNG} < \boldsymbol{W}$ since $A_I^{NG} > A_I$:

$$\boldsymbol{W}^{VNG} = \frac{W_P}{W_I^{VNG}} = \left(\frac{SH_P(.|w)\, VH_I(.|w)}{SH_I(.|w)\, VH_P(.|w)}\right)^{\frac{1}{2}}\left(\frac{Q_P^{JVH}\, Q_I^{JSH}}{Q_I^{JVH}\, Q_P^{JSH}}\right)^{\frac{1}{2}}\left(\frac{A_I}{A_I^{NG}}\right)^{\frac{1}{2\alpha}}; \qquad (3.32)$$

For now, we only analyse the time of the introduction of the new GPT in VT. But, what happens in the steady state, supposing that it is reached before the use of the new GPT in ST? The steady state growth rates are given by the following equations:

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[141] Note that we consider that the endogenous *VH* in our framework is more closely related with exogenous skilled workers than with the exogenous unskilled workers in the literature, and *vice-versa* for the *SH*.

[142] The increase in the relative supply of skills and the still greater increase in the demand for skills in developed countries is a stylised fact in economics – *e.g.*, Acemoglu (2002a).

$$gr^{ss}_{I,\,Phase\,2} = \left[\frac{1}{\theta}\left(\frac{1-\tau_K}{q^{\left(\frac{1-\alpha}{\alpha}\right)}-\tau_K}\right)\right]\left\{\frac{\beta_I}{\zeta_I}\,sh\left[\frac{1-s_x}{1-s_r}\right]\left(\frac{q-1}{q}\right)\left[\frac{p_I^{VSH}\,A_I\,(1-\alpha)}{1-s_{xZ}}\right]^{\frac{1}{\alpha}}\right.$$
$$\left.\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]-\delta^{SH}+\varphi^{SH\,[1/\phi]}\,\chi_S^{NG}\left[1+\frac{u_T^{NG}(t|SH)}{u_S^{NG}(t|SH)}\right]^{\frac{1-\phi}{\phi}}-\rho\left(\frac{q^{\left(\frac{1-\alpha}{\alpha}\right)}-\tau_K}{1-\tau_K}\right)\right\}; \quad (3.33a)$$

$$gr^{ss}_{I,\,Phase\,2} = \left[\frac{1}{\theta}\left(\frac{1-\tau_K}{q^{\left(\frac{1-\alpha}{\alpha}\right)}-\tau_K}\right)\right]\left\{\frac{\beta_I}{\zeta_I}vh\left[\frac{1-s_x}{1-s_r}\right]\left(\frac{q-1}{q}\right)\left[\frac{p_I^{VVH}\,A_I^{NG}\,(1-\alpha)}{1-s_x}\right]^{\frac{1}{\alpha}}\right.$$
$$\left.\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]-\delta^{VH}+\varphi^{VH\,[1/\phi]}\,\chi_S^{NG}\left[1+\frac{u_T^{NG}(t|VH)}{u_S^{NG}(t|VH)}\right]^{\frac{1-\phi}{\phi}}-\rho\left(\frac{q^{\left(\frac{1-\alpha}{\alpha}\right)}-\tau_K}{1-\tau_K}\right)\right\}. \quad (3.33b)$$

As equation (3.33b) shows, considering $m = VH$, A_I^{NG} generates greater expected profits for innovator firms in VT. However, the fall in p_I^{VVH} (it should be noted that N_I decreases) generates smaller expected profits. Therefore, for $m = VH$, there are two opposite effects on the expected profits. Considering $m = SH$, it is clear that the probability of successful innovation in specialised TK and, thereby, the growth rate, must be greater than before, since p_I^{VSH} increases. In other words, there is only one effect here. This 'growth effect' shortens then the time for the introduction of the new GPT in ST. As for a unique steady state, the growth rate for $m = VH$ – equation (3.33b) – and for $m = SH$ – equation (3.33a) – must be equal, the positive effect of A_I^{NG} must dominate the negative effect of p_I^{VVH} in the growth rate given by equation (3.33b).

To sum up, during this phase in the North there is both a 'level' effect and a 'growth' effect in the output, FGs at the margin are moving from the use of ST to the use of VT, the versatile premium increases (although the price paid per unit of *SH* also increases) and its productivity is now greater. The South benefits from the increase of the TK progress in the North ('growth' effect) and, thereby, so do its workers. This shortens the time for its use of the new GPT. Due to A_I^{NG}, productivity differences between countries widen as well as inter-country wage inequality. And, in addition, an increase in the TK gap between countries occurs too. In particular, the increase in the specialised TK gap between countries is quite obvious because p_I^{VSH} increases – see equation (3.8). Also the versatile TK gap widens since the effect of A_I^{NG} dominates the effect of p_I^{VVH}.

What happens in the other phases of the 2nd scenario

To avoid unnecessary repetition, we will now give the main results for each of the remaining three phases.

In phase 3, the new GPT is used in all FGs in the North. At this time and for the same reason as before, some FGs at the margin now shift from the use of VT to the use of ST. That is, both N_I and p_I^{SVH} increase, contrarily to p_I^{SSH}. Thus, there is again both a 'level' and a 'growth' effect on its output, and the versatile premium decreases in relation to the last phase, although the price paid per unit of *VH* also increases. There is a new improvement in the North's productivity. The effects in the South are the same as those occurring in the last phase. In particular, the increase in the versatile TK gap between countries due to the behaviour of p_I^{SVH} is now clear. However, an increase in the specialised TK gap also occurs since the effect of A_I^{NG} dominates the effect of p_I^{SSH}. The North is not affected by the introduction of the new GPT in the South. Consequently, it remains on the path reached in phase 3, as does the world growth rate.

In phase 4, the new GPT is also used in the VT in the South. At this time and for the same reason as in phase 2 in the North, some FGs at the margin move from the use of ST to the use of VT. That is, both N_P and p_P^{VVH} decrease, contrary to p_P^{VSH}. This induces a 'level effect' on its output, which increases its productivity and the versatile premium, although the price paid per unit of *SH* also increases. The inter-country wage inequality decreases. Finally, according to equation (3.8), the TK gap narrows between the countries, which compensates for the increase in productivity induced by the new GPT in the growth rate. Consequently, the 'growth effect' is as expected null.

In phase 5, all FGs in the South use the new GPT. With its introduction in ST, some FGs at the margin now move from the use of VT to the use of ST. That is, N_P increases. This again generates a 'level effect' on its output, its productivity increases and its versatile premium decreases as well as the inter-country wage inequality. At this phase, there is a new decrease in the TK gap between countries – see equation (3.8) – that counterbalances the increase in productivity due to the use of new GPT in the growth rate. In other words, the South does not experience a further 'growth effect'.

In particular, a new GPT provides a (possible) unified explanation for the changes in the structure of the price paid per type of HC and the (apparent) changes in the organisation of firms. Therefore, the above findings are in accordance with Autor *et al.*

(2003) and Aghion (2002), who argue that computers induced a change in the organisation of production by replacing unskilled tasks,[143] and with Caroli and Van Reenan (2001), who provides evidence suggesting that changes in wages have been accompanied by changes in organisational forms.[144] However, according to our extended framework, these changes are temporary, although they can occur over a long period of time. In more general terms, the concept of GPT enables us to see TK change as an irregular process that can have different effects on the economy at different times.

Table 3.3. Main results obtained in each of the five phases

	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5
Time to sow	In both	In $I<P$ [a)]	In P	In P	---
Time to reap	In both	In $I>P$ [b)]	In $I>P$ [b)]	In $I>P$ [b)]	In I and P
Growth effects	In both	In both	In both	---	---
Level effects	---	In I	In I	In P	In P
Productivity differences	---	↑	↑	↓	↓
Versatile premium	---	↑ in I	↓ in I	↑ in P	↓ in P
Inter-country wage inequality	---	↑	↑	↓	↓
Threshold final good	---	↓ in I	↑ in I	↓ in P	↑ in P
Technological knowledge gap	---	↑	↑	↓	↓

Notes: [a)] the 'sow' in greater in the South; [b)] the 'reap' is greater in the North.

Finally, note that the effect of a shock that increases the versatile-augmenting parameter *vh* has a similar impact to that which occurs with the introduction of the new GPT in VT. The main difference results from our assumption that *vh* is the same in both countries. Thus, the occurrence of this shock generates, in both countries, a positive 'level' – *e.g.*, equation (2.28) – and a 'growth' effect – *e.g.*, equation (3.4) –, increases the versatile premium – *e.g.*, equation (2.32) – and decreases the threshold FG – *e.g.*, equation (2.22). Therefore, a shock in *vh* can be an additional explanation for the increase in versatile premium, but in this case, in both countries at the same time.

[143] Note that Aghion (2002) explicitly considers computers a GPT.

[144] It can be also considered that this agrees with a series of empirical studies which have documented the rise in wage inequality in the U.S. labour market and pointed to TK change, specially the development of microcomputers, as an explanation for such a rise – *e.g.*, Bound and Johnson (1992), Levy and Murnane (1992), Katz and Murphy (1992) and Juhn *et al.* (1993).

3.4. Transition to and stability of the balanced growth path

3.4.1. Overview

Having established that in steady state there is a world growth rate common to both countries, we now need to find out whether each country converges towards that steady state (stability) or not (instability). More precisely, we wish to make a global analysis of the dynamics in order to identify the shape, slope and position of the stable arm, if such a stable arm exists, and we want to know how fast the countries converge towards the world steady state along the stable arm. Furthermore, we consider that the framework is better understood by means of the transitional dynamics analysis.

Traditionally, the study of stability follows two main approaches: the qualitative (or topological) and the quantitative. The former consists of analysing the properties of the solution to a differential equation (system) without actually knowing the solution itself or trying to approximate it.[145] The latter consists in trying to find the explicit analytical solution of the differential equation or in solving it by numerical method(s) – or still to approximate it by using power series –, and checking whether the time path of the system converges to the steady state. Given the complexity of the model,[146] we analyse the transition to and stability of the steady state numerically, displaying the model's precise time path for a set of baseline parameter values and a set of initial conditions. Moreover, we can use the numerical procedure to calibrate the model.

In the present IT regime, when the countries are off the BG path, the adjustment process or the transition to it depends on the country considered. First, as a result of market incentives, countries converge to a unique stable country specific growth path, where $Q_Z^{J^{VH}} VH_Z(.|w)$ and $Q_Z^{J^{SH}} SH_Z(.|w)$ grow at the same rate.[147] For the North the analysis ends there, since it is independent of the South – *i.e.*, the unique stable Northern growth path is also the (world) BG path. However, the transitional dynamics in the South only ends when it reaches the world steady state growth rate established by the North.

The process can be summarised as follows. Equation (3.11) can be written as:

$$\frac{Q_Z^{J^{VH^{SS}}} VH_Z^{SS}(.|w)}{Q_Z^{J^{SH^{SS}}} SH_Z^{SS}(.|w)} = \left(\frac{1-N_Z^{SS}}{N_Z^{SS}}\right)^2 \frac{sh}{vh}. \tag{3.34}$$

[145] Approach based on, *e.g.*, phase diagrams and Liapunov's 2nd method – *e.g.*, Gandolfo (1997, chap. 21).
[146] The differential equations obtained tend to be non-linear and with variable-coefficients.
[147] Once the unique country specific growth path is reached, the ratio $Q_Z^{J^{VH}} VH_Z(.|w) / Q_Z^{J^{SH}} SH_Z(.|w)$ remains constant. In this sense the uniqueness of the growth path of each country is stable.

Let us consider that the country analysed is the North. Moreover, we suppose that it starts off its unique stable BG growth path; *i.e.*, that equation (3.34) does not occur:[148]

$$\frac{Q_Z^{J^{VH}}\, VH_Z(.|w)}{Q_Z^{J^{SH}}\, SH_Z(.|w)} < \left(\frac{1-N_Z^{ss}}{N_Z^{ss}}\right)^2 \frac{sh}{vh}. \tag{3.35}$$

However, if equation (3.35) is true since, for example, $SH_Z(.|w) > SH_Z^{ss}(.|w)$, this immediately implies that equation (2.21) does not occur either. In this case,

$$\frac{p_Z^{VH}}{p_Z^{SH}} > \left(\frac{N_Z^{ss}}{1-N_Z^{ss}}\right)^{\alpha}. \tag{3.36}$$

The increase in relative price of versatile FGs expands the versatile-specialised TK bias, since the 'versatile' R&D is more profitable. Then, if equation (3.35) occurs, the market incentive will induce $Q_Z^{J^{VH}}\, VH_Z(.|w)$ to grow at a higher rate than $Q_Z^{J^{SH}}\, SH_Z(.|w)$, until equilibrium is achieved when the unique stable country specific BG growth path is obtained. Besides, off the unique country specific BG growth path, the versatile premium is greater than if the country is on its own unique growth path – *e.g.*, equation (3.12) –, and so the relative time devoted to work of *VH* tends to increase, $u_Z(w|VH)/u_Z(w|SH)$ – *e.g.*, equation (3.6) –, and likewise the relative $VH_Z(.|w)$, $VH_Z(.|w)/SH_Z(.|w)$.

In short, due to the action of market incentives, the North, which is never affected by the South, approaches its unique stable BG growth path in finite time. Without any exogenous disturbance, it remains there and so its unique growth path is also its BG path.

At initial time, t_0, when TK diffusion becomes possible, the Southern threshold FG, N_P, and so the price indexes, p_P^m, are defined taking into account both TK and HC levels in both countries – *i.e.*, due to TK diffusion the South is immediately affected by the North.[149] Specifically, as at t_0 the North is already in steady state, the South will firstly immediately converge towards a unique stable Southern growth path, such that from t_0 on, both $Q_P^{J^{VH}}\, VH_P(.|w)$ and $Q_P^{J^{SH}}\, SH_P(.|w)$ grow at the same rate – *i.e.*, from t_0 on, $\dot{N}_P/N_P = \dot{p}_P^m/p_P^m = 0$. We highlight that N_P in equation (2.22) at t_0 can be written as:

$$N_P(t_0) = \left\{1 + \left[\frac{\tilde{Q}^{J^{VH}}(t_0)\; vh\; \tilde{VH}(t_0|w)}{\tilde{Q}^{J^{SH}}(t_0)\; sh\; \tilde{SH}(t_0|w)} \frac{Q_I^{J^{VH\,ss}}(t_0)\; VH_I(t_0|w)^{ss}}{Q_I^{J^{SH\,ss}}(t_0)\; SH_I(t_0|w)^{ss}}\right]^{\frac{1}{2}}\right\}^{-1} = N_P^{ss}(t_0). \tag{3.37}$$

[148] The plan to show that a country converges to the BG path if the sign in equation (3.35) is > is identical.
[149] By assumption there is no Southern adjustment cost at t_0 or, instead, the South emerges in world at t_0.

Formally, this implies that, for example, from t_0 on and for both countries:

$$\left[\frac{Q_Z^{JVH}(t_0)\ VH_Z(t_0|w)}{Q_Z^{JSH}(t_0)\ SH_Z(t_0|w)}\right]=\left(\frac{1-N_Z^{ss}(t_0)}{N_Z^{ss}(t_0)}\right)^2\frac{sh}{vh}. \tag{3.38}$$

Nevertheless, the South's process of adjustment does not stop there. It is on its unique growth path, but out of the (world) BG path defined by the North. In line with the literature – *e.g.*, Barro and Sala-i-Martin (1997) – the South achieves higher growth rates than the North during the convergence to the world BG path. This is due to the potential proportioned by TK backwardness mirrored in the probability of successful imitation, which is greater than the probability of successful innovation off the (world) stable BG path. However, the probability of successful imitation changes with alterations in both the South-North TK gap and the gap in HC at work. Thus, there will be a transitional path for the South before reaching the unique world (or Northern) stable BG path; *i.e.*,

$$\frac{\dot{Q}_I^{Jm}(t_0^{ss})}{Q_I^{Jm}(t_0^{ss})}+\frac{\dot{m}_I(t_0^{ss}|w)}{m_I(t_0^{ss}|w)}<\frac{\dot{Q}_P^{Jm}(t_0)}{Q_P^{Jm}(t_0)}+\frac{\dot{m}_P(t_0|w)}{m_P(t_0|w)}; \tag{3.39a}$$

$$\frac{\dot{Q}_I^{Jm}(t_0^{ss})}{Q_I^{Jm}(t_0^{ss})}+\frac{\dot{m}_I(t_0^{ss}|w)}{m_I(t_0^{ss}|w)}=\frac{\dot{Q}_P^{Jm}(t^{ss})}{Q_P^{Jm}(t^{ss})}+\frac{\dot{m}_P(t^{ss}|w)}{m_P(t^{ss}|w)}; \tag{3.39b}$$

$$\frac{\dot{Q}_Z^{JVH}(.)}{Q_Z^{JVH}(.)}+\frac{\dot{VH}_Z(.|w)}{VH_Z(.|w)}=\frac{\dot{Q}_Z^{JSH}(.)}{Q_Z^{JSH}(.)}+\frac{\dot{SH}_Z(.|w)}{SH_Z(.|w)}. \tag{3.39c}$$

The rest of the section is organised as follows. In subsection 3.4.2, we describe the numerical method and present the baseline parameter values. In subsections 3.4.3 and 3.4.4, we analyse the transitional dynamics in the North and the South,[150] respectively.

3.4.2. Numerical method and choice of baseline parameter values

Numerical method

Solving ordinary differential equations numerically is a fundamental technique in continuous time dynamics, since most ordinary differential equations either cannot be solved analytically or their solutions are computationally prohibitive to develop, as is the case here. Numerical integration is thus the ultimate answer to obtaining information about the equation's behaviour. We will now begin to formulate our differential system base mathematically and then go on to briefly describe the numerical method used.

[150] That is, the Northern transition towards the world BG path and the Southern transition from its unique specific growth path until the unique stable world (or Northern) BG path.

In mathematical terms, our problems, with or without IT, are initial value first-order systems of ordinary differential (generally) non-linear equations that can be represented by $\dot{Y} = F(t,Y)$, $Y_{t_0} = Y_0$, $Y \in \Re^{nV}$, $t \in \Re_0^+$, where nV represents the number of variables, t is the time and Y_{t_ℓ} is the estimated value of the variable Y at the time t_ℓ.

Like Funke and Strulik (2000), the numerical procedure followed was the use of a fourth-order Runge-Kutta classical method,[151] which is usually considered as providing an excellent balance between precision and simplicity to program. The family of Runge-Kutta methods was designed to approximate Taylor series but with the advantage of not requiring explicit evaluations of the derivatives of $F(t,Y)$. The idea is to consider time steps, and to assume that the exact value of the slope of the step can be written as a linear combination of the function $F(t,Y)$ evaluated at certain points in the step.

For the fourth-order Runge-Kutta the slope used is the weighted mean of the slope at four different points in the step interval, with the Y values used at successive points being based on two different mid-point approximations. If an estimated value of Y, Y_{t_ℓ}, of the solution is known at a point $t_\ell = t_0 + \ell\Im$, with $\Im$ being the fixed time step size, $t_{\ell+1} - t_\ell$, the value of the solution $Y_{t_{\ell+1}}$ at time $t_{\ell+1}$ is computed by:

$$Y_{t_{\ell+1}} = Y_{t_\ell} + \frac{1}{6}\left(\aleph_1 + 2\aleph_2 + 2\aleph_3 + \aleph_4\right), \text{ where:} \tag{3.40}$$

$$\aleph_1 = \Im F(t_\ell, Y_\ell);\ \aleph_2 = \Im F(t_\ell + \frac{\Im}{2}, Y_{t_\ell} + \frac{\aleph_1}{2});\ \aleph_3 = \Im F(t_\ell + \frac{\Im}{2}, Y_{t_\ell} + \frac{\aleph_2}{2});\ \aleph_4 = \Im F(t_\ell + \Im, Y_{t_\ell} + \aleph_3).$$

Since our differential systems were solved with suitable precision with the fourth-order Runge-Kutta classical method, we need not to consider more sophisticated methods. However, other simple methods such as Euler's or lower order Runge-Kutta are not generally used in scientific computing because the truncation error per step may be large, and some of them are prone to numerical instabilities.

Baseline parameter values

The baseline parameter values are presented in table 3.4 below. It includes those which are related to the IT regimes and so only essential in the next two chapters.

Parameters are chosen to calibrate the economic steady state world growth rate around two percent under no IT effects, which approximately matches the average per

[151] The German mathematicians Runge and Kutta developed this class of methods in the latter half of the 19th Century – *e.g.*, Lambert (1973).

capita growth rate of the U.S. in the post-war period. For some parameters the choice is guided by empirical findings, while other parameter values are based on theoretical specification and set with some arbitrariness. Moreover, when the range of choice is large we prefer a value close to some critical value, the idea being to stay within a 'consensus benchmark' and to leave further discussion for the sensitivity analysis in the next section.

Table 3.4. Baseline parameter values

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
A_I	1.56	MC_P	0.50 [b)]	PE	0.25	ρ	0.03
A_P	1.00	$\xi_{Z=\{I,P\}}$	1.00 [c)]	IT	0.70 [b)]	δ^m	0.02
α [a)]	0.60	β_I	1.60	D	0.10	φ^{VH}	0.60
vh	1.20	β_P	1.00	$\bar{\sigma}_1$	0.25	χ_S	0.09
sh	1.00	ζ_I	4.00	$\bar{\sigma}_2$	0.60	χ_T	0.07
$MC_{Z=\{I,P\}}$	1.00	ζ_P	2.00	θ	1.05	ϕ	0.05

Notes: [a)] considering the particular case $q = 1/(1-\alpha)$; [b)] means that the value only makes sense under IT; [c)] this value for ξ is already considered in the deduction of the differential equations. Moreover, the values for $\sigma = 4.25$, $\sigma^*_{21} = 0.05$ and $\sigma^*_{22} = 0.232$ in the two other specifications for the catching up term – see Appendix 2 – are chosen in order to get the same steady state TK gap between countries under all specifications in the present IT regime – *i.e.*, without IT effects.

The baseline values for the exogenous productivity variables A_I and A_P are in accordance with our theoretical considerations, such that $A_I > A_P$. In other words, we assume that the North has an absolute exogenous productivity advantage over the South.

Under our assumptions, the technology parameter α has two interpretations: the HC share, α, and the mark-up ratio, $1/(1-\alpha)$. Its value is slightly lower than in its common use to calibrate models with accumulation of physical capital and fixed labour, which is $\alpha \cong 0.7$. This is due to the fact that we consider the quality-adjusted IGs as a broader concept than physical capital and, furthermore, the mark-up interpretation is central in our case. Given the value of α, the size of quality improvement, q, makes it equal to 2.5, which is greater than the ratio MC_I/MC_P. In short, its value is set in line with the mark-up estimates given by Hall (1986) and Kwan and Lai (2003).

The m-augmenting parameters sh and vh are also in line with our theoretical assumptions, such that $vh \geq sh \geq 1$. The same is true for the South's MC of production under IT; *i.e.*, $1 = MC_I > MC_P = 0.50$. This is so that under IT the Southern IGs firms, which support the R&D cost of imitation, can under-price the Northern IGs leader firms. Without IT effects, we consider that $MC_I = MC_P = 1$.

The baseline value for the elasticity of the probability of successful research, pb_Z, with respect to the market size is essential to remove the scale effects – *i.e.*, $\xi_Z = 1$.

Parameter values for learning to past successful research spillover, β, and fixed cost of successful research, ζ, are based on theoretical specifications, such that $\beta_I > \beta_P$ and $\zeta_I > \zeta_P$, and on findings by Mansfield *et al.* (1981). It is not clear what the value of the TK threshold, d, is, as no empirical estimates exist for it. We set its baseline value at *0.1*, in line with Papageorgiou (2002). The value of the direct effect of openness and other trade policies on pb_P, *IT*, is consistent with many related estimates – *e.g.*, Coe and Helpman (1995) and Coe *et al.* (1997). In our case, *IT* is perhaps over-evaluated. We ignore the existence of empirical estimates for the effect of domestic policies promoting imitation on pb_P, *PE*. Though, ascribing its importance the value *0.25* seems conservative to us, considering, among others, Kaufmann *et al.* (2002) and Dollar and Kraay (2003).

Since there are no empirical estimates for the value of both $\overline{\sigma}_1$ and $\overline{\sigma}_2$ – *i.e.*, for the 'semi-elasticity' of pb_P with respect to the gap in HC at work, $\tilde{m}(.|w)$, and for the fixed component of the (variable) elasticity of pb_P with respect to $g_2(.)$ – their values are in line with our previous considerations. Thus, $\overline{\sigma}_1$ must be greater, but not much greater, than *0* in order to guarantee a moderate – although important, see figure 2.2 – impact of the gap in HC at work on pb_P. The value for $\overline{\sigma}_2$ is set in order to guarantee that the TK progress in the South benefits from the relative backwardness of the country – see figure 2.3.

The baseline value for the inverse of the constant intertemporal elasticity of substitution, θ, is lower than that given by recent attempts at its estimation – *e.g.*, Hall (1988) and Acemoglu and Scott (1994). Our choice is, however, more in line with earlier calibrations of growth models, where it is assumed to exceed one – *e.g.*, Jones *et al.* (1993) and Mulligan and Sala-i-Martin (1993).[152] Specifically, it is chosen so as to be close to the log utility function; that is, we take into consideration the critical value $\theta = 1$. The rate of time preference or subjective discount rate, ρ, for a time period of one year, is also set in line with previous works on growth – *e.g.*, Davidson and Segerstrom (1998), Sjogren (1998), Dinopoulos and Segerstrom (1999a) and Lehmijoki (2002).

The depreciation rate of HC, δ^m, is in line with estimates by Mincer and Ofek (1982). To simplify, we consider that is the same for both types of HC. The distribution or intensity parameters in HC technology are based on the theoretical specification that

$\varphi^{VH} > \varphi^{SH}$ and $\varphi^{VH} + \varphi^{SH} = 1$. The productivity or efficiency of the learning activities' parameters are also based on a theoretical specification that $\chi_S > \chi_T$, and have been chosen in line with previous works on growth – *e.g.*, Sjogren (1998). Since, when the substitution parameter in HC technology, ϕ, is greater (smaller) than *0*, the two types of training are substitutes (complements), the baseline value that has been chosen implies that they are weak substitutes; *i.e.*, we pay heed to the critical value $\phi = 0$. We thus leave the case in which they are strong substitutes and complements for the sensitivity analysis.

Finally, we consider that there is no government intervention in the baseline case.

3.4.3. Transitional dynamics in the North

Differential equations

By considering the equilibrium in the labour market, free entry condition into R&D, individual utility maximisation with individual optimal time allocation, we will be able to characterise the transitional dynamics for the North.

Due to technological complementarity in the production of FGs, away from the steady state, the growth rate of *m*-complementary TK together with the dynamics of the prices of FGs translates into the growth of *m*-type HC demand, as equation (2.30) states. Equating this *m*-type HC demand path to the respective 'supply' – in equation (2.70) – ,[153] we get the following differential equation from the dynamics in the labour market:

$$\frac{\dot{Q}_I^{J^m}(t)}{\dot{Q}_I^{J^m}(t)} + \frac{1}{\alpha}\frac{\dot{p}_I^m(t)}{p_I^m(t)} = [1-\tau_K]\, r_I(t) + \delta^m - \varphi^{m[1/\phi]}\chi_S\left[1+\frac{u_T(t|m)}{u_S(t|m)}\right]^{\frac{1-\phi}{\phi}}. \qquad (3.41)$$

In the differential equation (3.41), we already know the growth rate of the *m*-complementary TK – it is given by equation (2.54) – and we can easily find the dynamics of the equilibrium FG price indexes from equations (2.25a, b), such that:

$$\frac{\dot{p}_I^{VH}}{p_I^{VH}} = -\frac{\alpha}{2}\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(1/2)}\right]^{-1}\frac{\dot{D}_I^1}{D_I^1} \text{ and } \frac{\dot{p}_I^{SH}}{p_I^{SH}} = \frac{\alpha}{2}\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(-1/2)}\right]^{-1}\frac{\dot{D}_I^1}{D_I^1}. \qquad (3.42)$$

Where $D_I^1 \equiv Q_I^{J^{VH}} VH_I(.|w) / Q_I^{J^{SH}} SH_I(.|w)$ must be constant in steady state; *i.e.*, due to market incentives, the North must converge to a unique stable growth path, which is the world BG path, where $Q_I^{J^{VH}} VH_I(.|w)$ and $Q_I^{J^{SH}} SH_I(.|w)$ must grow at the same rate.

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[152] Even given the great discussion in the literature about the 'true' value of θ, $\theta \in [1, 2]$ is the most common range of values considered – *e.g.*, also Attanasio and Weber (1993) and Guvenen (2001).

[153] In fact, since equation (2.70) expresses the optimal time-allocation ratios in terms of the given wages' dynamics (and interest rate), we can state that it yields the supply of effort functions for each type of HC.

Moreover, differential equation (3.41) informs us that we need to find the interest rate at each time t, which must also be constant in steady state. Thus, in order to characterise the transition path for the country, we consider the representative domestic IG, denoted by the index *av*, and defined as an average of all domestic IGs of each type, versatile or specialised. This last consideration is needed to avoid any jumpiness in quality improvements that would occur if the behaviour of an individual IG were contemplated. The expression for the expected present value of profits for the lead innovator in the *av* IG of m-complementary IG is thus given by:

$$V_I(k,av,v\,|\,m) = \Pi_I(k,av,v\,|\,m)\int_t^{\infty} exp\left[-\int_t^{s}\left(r_I(v) + pb_I(k,av,v\,|\,m)\right)dv\right]ds. \qquad (3.43)$$

In equation (3.43), $r_I(v)$ is the Northern interest rate at time v and $pb_I(k,av,v\,|\,m)$ is the probability at time v that the *av* IG of quality rung $(k+1)$ of the m-complementary TK ladder will be innovated. Since there is free entry into research, then at all times the expected present value of profits from successful innovation times the probability of successful innovation must equal the after subsidy cost of innovation; *i.e.*,

$$V_I(k+1,av,v\,|\,m)\;\; pb_I(k,av,v\,|\,m) \;=\; (1-s_{rI})\; rs_I(k,av,v\,|\,m). \qquad (3.44)$$

Hence, taking into account equation (3.43) we can rewrite equation (3.44) as:

$$\left\{\Pi_I(k+1,av,v|m)\int_t^{\infty} exp\left[-\int_t^{s}\left(r_I(v) + pb_I(k+1,av,v\,|\,m)\right)dv\right]ds\right\} \cdot \qquad (3.45)$$
$$pb_I(k,av,v|m) = (1-s_{rI})\; rs_I(k,av,v|m)$$

Differentiating both sides of equation (3.45), bearing in mind Leibniz's rule for the left-hand side of the equation, we obtain an expression for the Northern interest rate:

$$r_I(t) = \frac{\dot{rs}_I(k,av,t\,|\,m)}{rs_I(k,a,t\,|\,m)} - \frac{\dot{pb}_I(k,av,t\,|\,m)}{pb_I(k,av,t\,|\,m)} - \frac{\dot{\Pi}_I(k+1,av,t\,|\,m)}{\Pi_I(k+1,av,t\,|\,m)} + \cdot \qquad (3.46)$$
$$+\frac{pb_I(k,av,t\,|\,m)\;\Pi_I(k+1,av,t\,|\,m)}{rs_I(k,av,t\,|\,m)\;(1-s_{rI})} - pb_I(k+1,av,t\,|\,m)$$

As a result of equation (2.34) – or, instead, equation (2.38) – and (2.39a), then:

$$\frac{\dot{rs}_I(k,av,t\,|\,m)}{rs_I(k,av,t\,|\,m)} = \frac{\dot{pb}_I(k,av,t\,|\,m)}{pb_I(k,av,t\,|\,m)} + \frac{\dot{m}_I(t|w)}{m_I(t|w)} - \dot{k}_I(av,t\,|\,m)\left(\frac{\alpha-1}{\alpha}\right)\ln q; \qquad (3.47a)$$

$$\frac{\dot{\Pi}_I(k,av,t\,|\,m)}{\Pi_I(k,av,t\,|\,m)} = \frac{\dot{\Pi}_I(k+1,av,t\,|\,m)}{\Pi_I(k+1,av,t\,|\,m)} = \frac{\dot{m}_I(t\,|\,w)}{m_I(t\,|\,w)} + \dot{k}_I(av,t\,|\,m)\left(\frac{1-\alpha}{\alpha}\right) ln\, q \cdot \qquad (3.47b)$$

Hence, given equation (3.47a, b) we can rewrite equation (3.46) as:

$$r_I(t) = \frac{pb_I(k,av,t\,|\,m)\;\Pi_I(k+1,av,t\,|\,m)}{rs_I(k,av,t\,|\,m)\;(1-s_{rI})} - pb_I(k+1,av,t\,|\,m)\,. \qquad (3.48)$$

In equation (3.48), the first term represents the dividend term, and the second reflects the Schumpeterian concept of creative destruction caused by the next successful innovation, which leads to the obsolescence of the preceding one. From equations (2.39a) and (2.38) and solving, for example, for $m = VH$, we can observe that the former term becomes independent of the quality ladder rung $k_I(av|m)$,

$$\frac{pb_I(k,av,t\,|\,VH)\;\Pi_I(k+1,av,t\,|\,VH)}{rs_I(k,av,t\,|\,VH)\;(1-s_{rI})} =$$
$$= \frac{\beta_I}{\zeta_I}\left(\frac{1-s_{xI}}{1-s_{rI}}\right)\left(\frac{q-1}{q}\right)\left(A_I\,\frac{1-\alpha}{1-s_{xI}}\right)^{(1/\alpha)} vh\; exp(-1)\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(-1/2)}\right]\cdot \qquad (3.49)$$

From equation (2.54), the term $pb_I(k+1,av,t\,|\,m)$ can be written as a function of the growth rate of the respective TK. Then, first, plugging equations (3.49) and the resultant expression for $pb_I(k+1,av,t\,|\,m)$ via equation (2.54) into equation (3.48) and, subsequently, substituting $\dot{Q}_I^{J^m}/Q_I^{J^m}$ by the respective expression obtained via equation (3.41), we get two equations for the interest rate, one for each $m = \{SH, VH\}$. When these are equated, since the country's interest rate is always unique, we obtain:

$$\frac{\dot{D}_I^1}{D_I^1} = 2\,q^{\left(\frac{1-\alpha}{\alpha}\right)}\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]\left\{\frac{\beta_I}{\zeta_I}\left(\frac{1-s_{xI}}{1-s_{rI}}\right)\left(\frac{q-1}{q}\right)\left(A_I\,\frac{1-\alpha}{1-s_{xI}}\right)^{(1/\alpha)} exp(-1)\right.$$
$$\left.\left\{vh\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(-1/2)}\right]-sh\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(1/2)}\right]\right\}\right\}+2\left\{\delta^{SH}-\delta^{VH}+\cdot\right. \qquad (3.50)$$
$$\left.+\varphi^{VH^{(1/\phi)}}\chi_S\left(1+\frac{u_T(VH)}{u_S(VH)}\right)^{((1-\phi)/\phi)}-\varphi^{SH^{(1/\phi)}}\chi_S\left(1+\frac{u_T(SH)}{u_S(SH)}\right)^{((1-\phi)/\phi)}\right\}$$

Differential equation (3.50) gives us the dynamic of D_I^1, which results from the equilibrium in the labour market together with free entry conditions into R&D.

The TK indexes, $Q_I^{J^m}$, will not necessarily be constants in steady state. However, we need to know their dynamic, so as get the dynamic of $D_I^2 \equiv C_I(VH)/Q_I^{J^{VH}} VH_I(.\,|\,w)$,

$D_I^3 \equiv C_I(SH)/Q_I^{J^{VH}} VH_I(.|w)$, $D_I^4 \equiv C_I(VH)/Q_I^{J^{SH}} SH_I(.|w)$ and $D_I^5 \equiv C_I(SH)/Q_I^{J^{SH}} SH_I(.|w)$. They could be interpreted as the weight of consumption at each time t, $C_I(m)$, in relation to the large 'amount' of TK available, $Q_I^{J^m} m_I(.|w)$; *i.e.*, in relation to the TK embodied in IGs and embodied in individuals. Moreover, they must also be constant in steady state.

Once the dynamic of D_I^1 is known, we can separately obtain the dynamic of the two TK indexes, $Q_I^{J^{VH}}$ and $Q_I^{J^{SH}}$.[154] In fact, for each $m = \{SH, VH\}$, we can plug equation (3.48) – incorporating equation (3.49) and considering equation (2.54) – into equation (3.41) to give the following differential equation for, for example, the dynamics of $Q_I^{J^{VH}}$:

$$\frac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}} = \left[1+\frac{(1-\tau_K)\,q^{\left(\frac{\alpha-1}{\alpha}\right)}}{q^{\left(\frac{1-\alpha}{\alpha}\right)}-1}\right]\left\{\frac{1}{2}\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(1/2)}\right]^{-1}\frac{\dot{D}_I^1}{D_I^1}+(1-\tau_K)\,\frac{\beta_I}{\zeta_I}\left(\frac{1-s_{xI}}{1-s_{rI}}\right)\left(\frac{q-1}{q}\right)\right.$$
$$\left.\left(A_I\,\frac{1-\alpha}{1-s_{xI}}\right)^{(1/\alpha)} exp(-1)\,vh\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(-1/2)}\right]-\left[\varphi^{VH^{(1/\phi)}}\chi_S\left(1+\frac{u_T(VH)}{u_S(VH)}\right)^{((1-\phi)/\phi)}-\delta^{VH}\right]\right\} \quad (3.51)$$

In the next step, we want to find the dynamic of D_I^2, D_I^3, D_I^4 and D_I^5. From the aggregate budget constraint – see equation (2.72) – we can write that:

$$\frac{\zeta_I}{\beta_I}Q_I^{J^{VH}}\,VH_I(.|w)\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}\frac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}}+\frac{\zeta_I}{\beta_I}Q_I^{J^{SH}}\,SH_I(.|w)\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}\frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}}=$$
$$=A_I^{(1/\alpha)}\left(\frac{1-\alpha}{q\,(1-s_{xI})}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\left[1-\frac{1-\alpha}{q\,(1-s_{xI})}\right]\left\{vh\,Q_I^{J^{VH}}VH_I(.|w)\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(-1/2)}\right]+\cdot\right. \quad (3.52)$$
$$\left.+sh\,Q_I^{J^{SH}}SH_I(.|w)\left[1+\left(\frac{vh}{sh}D_I^1\right)^{(1/2)}\right]\right\}exp(-1)-C_I(.|VH)-C_I(.|SH)$$

In equation (3.52), (i) on the left-hand side, the first and the second terms represent $RS_I(.|VH)$ and $RS_I(.|SH)$, respectively – they result from equations (2.38) and (2.54);[155] (ii) on the right-hand side, the first term is $Y_I(.|VH)+Y_I(.|SH)-X_I(.|VH)-X_I(.|SH)$ – it results from equations (2.27) and (2.28) –, whereas the second and the third correspond

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[154] From equations (2.48) and (2.54), we could obtain the dynamic followed by $\dot{Q}_I^{J^{VH}}/Q_I^{J^{VH}} - \dot{Q}_I^{J^{SH}}/Q_I^{J^{SH}}$. However, it is useful to get the dynamic of each TK index separately. We can though verify numerically that the values at each t for $\dot{Q}_I^{J^{VH}}/Q_I^{J^{VH}} - \dot{Q}_I^{J^{SH}}/Q_I^{J^{SH}}$ are the same in both cases, which 'works' as a check.

[155] Since $rs_I(k,av,t|m) = pb_I(k,av,t|m)\,\beta_I^{-1}\zeta_I\,m_I(t|w)\,q^{k_I(av,t|m)\left(\frac{1-\alpha}{\alpha}\right)}$, we can go further by writing:

$$RS_I(t|m) = \int_0^{J^m} rs_I(k,av,t|m)\,dj = J^m\,rs_I(k,av,t|m) = pb_I(k,av,t|m)\;\beta_I^{-1}\zeta_I\;\; m_I(t|w)\,Q_I^{J^m}\cdot$$

to $C_I(.|VH)$ and $C_I(.|SH)$, respectively. Dividing all terms in equation (3.52) for, for example, $Q_I^{J^{SH}} SH_I(.|w)$, and after that solving them for D_I^4 and D_I^5, leads to:

$$D_I^4 + D_I^5 = -\frac{\zeta_I}{\beta_I} D_I^1 \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right]^{-1} \frac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}} - \frac{\zeta_I}{\beta_I} \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right]^{-1} \frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}} + A_I^{(1/\alpha)} \, exp(-1)$$
$$\left(\frac{1-\alpha}{q(1-s_{xI})} \right)^{\left(\frac{1-\alpha}{\alpha}\right)} \left[1 - \frac{1-\alpha}{q(1-s_{xI})} \right] \left\{ vh \, D_I^1 \left[1 + \left(\frac{vh}{sh} D_I^1 \right)^{(-1/2)} \right] + sh \left[1 + \left(\frac{vh}{sh} D_I^1 \right)^{(1/2)} \right] \right\} . \quad (3.53)$$

Equation (3.53) gives us the dynamics of D_I^4 and D_I^5, which must be stables in steady state. Since D_I^4 and D_I^5 grow at the same rate, we can write the left-hand side of equation (3.53) as $\{[D_I^4(t-dt) + D_I^5(t-dt)] \, exp[\dot{D}_I^4(t)/D_I^4(t)]\}$ instead of $[D_I^4(t) + D_I^5(t)]$.[156]

Additionally, by definition of D_I^4 and D_I^5 and considering the consumption growth rate resultant from individuals' utility maximisation – Euler equation (2.64) –, we get:

$$\frac{\dot{D}_I^4}{D_I^4} = \frac{\dot{D}_I^5}{D_I^5} = \frac{\dot{C}_I(SH)}{C_I(SH)} - \frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}} - \frac{\dot{SH}_I(.|w)}{SH_I(.|w)} = \frac{1}{\theta} \left[(1-\tau_K) \, r_I - \rho \right] - \frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}} - \frac{\dot{SH}_I(.|w)}{SH_I(.|w)} . \quad (3.54)$$

Plugging the expression for $[1-\tau_K] \, r_I(t)$ obtained in equation (3.41) – given equation (3.42) – into equation (3.54) and solving for $SH_I(t|w)$, yields:[157]

$$\frac{\dot{SH}_I(.|w)}{SH_I(.|w)} = \left(\frac{1-\theta}{\theta} \right) \frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}} + \frac{1}{\theta} \frac{1}{2} \left[1 + \left(\frac{vh}{sh} D_I^1 \right)^{(-1/2)} \right]^{-1} \frac{\dot{D}_I^1}{D_I^1} - \frac{\dot{D}_I^4}{D_I^4} +$$
$$+ \frac{1}{\theta} \left[\varphi^{SH^{(1/\phi)}} \chi_S \left(1 + \frac{u_T(SH)}{u_S(SH)} \right)^{((1-\phi)/\phi)} - \delta^{SH} \right] - \frac{\rho}{\theta} . \quad (3.55)$$

Equation (3.55) relates the dynamic of $SH_I(t|w)$. It is not necessarily constant in steady state. However, we need to know its path, to get the dynamic of the (non-leisure) time the individuals who accumulate m-type HC allocate to the production, $u_{wI}(t|m)$,[158] which must be constant in steady state. To this end, we take into consideration that:

$$m_I(t) = \underbrace{u_{wI}(t|m) \, m_I(t)}_{=m_I(t|\text{work})} + \underbrace{u_{SI}(t|m) \, m_I(t)}_{=m_I(t|\text{scholl})} + \underbrace{u_{TI}(t|m) \, m_I(t)}_{=m_I(t|\text{OJT})} \Rightarrow \frac{\dot{m}_I(t|w)}{m_I(t|w)} = \frac{\dot{u}_{wI}(t|m)}{u_{wI}(t|m)} + \frac{\dot{m}_I(t)}{m_I(t)} . \quad (3.56)$$

Given the dynamic of $m_I(t|w)$ – *e.g.*, equation (3.55), for $SH_I(t|w)$ –, and taking into account equations (2.59) and (2.65), with some additional algebra we get:

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[156] The path of D_I^2 and D_I^3 can be obtained by dividing all terms in equation (3.52) by $Q_I^{J^{VH}} VH_I(.|w)$.

[157] The path of $VH_I(t|w)$ can be obtained following the same steps or, instead, from the definition of D_I^1.

[158] Given the dynamics for $u_{wI}(t|m)$, the dynamics for $u_{SI}(t|m)$ and $u_{TI}(t|m)$ can easily be found taking into consideration equations (2.60) – or, alternatively, equation (3.56) – and (2.65).

$$\frac{\dot{u}_{wI}(t|m)}{u_{wI}(t|m)} = \frac{\dot{m}_I(.|w)}{m_I(.|w)} - \varphi^{m^{(1/\phi)}} \chi_S \left(1 + \frac{u_T(m)}{u_S(m)}\right)^{((1-\phi)/\phi)} \left(1 - u_{wI}(t|m)\right) + \delta^m . \qquad (3.57)$$

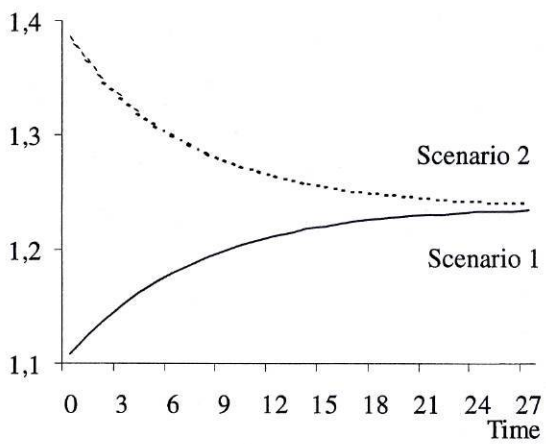
As a result of the above analysis, the stability properties of the dynamic of the North are block recursive, in the sense that we can first determine the stability of D_I^I and then recursively characterise the behaviour of the other variables.

Solving differential equations for the baseline parameter values

We are now able to analyse the transition to and stability of the steady state numerically. The behaviour of the model is displayed for the set baseline parameter values in table 3.4, for two different scenarios. In scenario 1, we consider that $D_I^I(0)$ is below its steady state value and, in scenario 2, we assume that $D_I^I(0)$ is above its steady state value.[159] In any case, the transition path lasts for *27* years, which represents a rapid speed of convergence. Table A3.1 in Appendix 3 (subsection A3.1.1), compares initial and steady state values of the relevant variables – *i.e.*, it summarises the results.

In scenario 1, $Q_I^{J^{VH}} VH_I(.|w)$ initially rises more than $Q_I^{J^{SH}} SH_I(.|w)$, but falls more rapidly towards the steady state. Thus, as figure 3.6 below shows, D_I^I is increasing, but at a slower rate until it reaches its steady state value, *1.24*. In other words, $\dot{D}_I^I / D_I^I$ is positive but declines monotonically towards zero. The converse applies in scenario 2.

Figure 3.6. Transitional dynamics of D_I^I



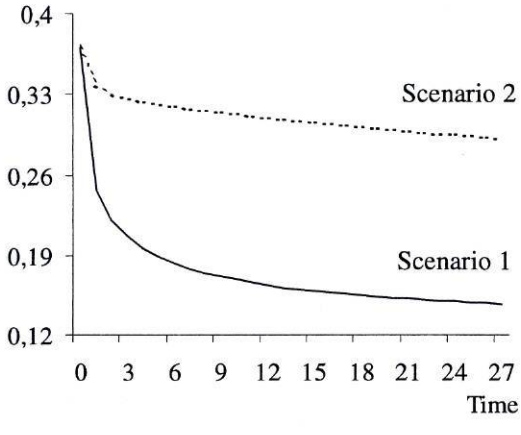
Since, regardless of the initial state of D_I^I, there is a steady state, rest or fixed point of convergence, we can state that the adjustment process of D_I^I is globally stable – *e.g.*,

Gandolfo (1997, chap. 21). Moreover, the behaviour of D_I^1 implies that, in scenario 1, the relative price of versatile FGs drops and VT gains importance in the productive process – following the same type of path as D_I^1 –, and *vice-versa* in scenario 2.

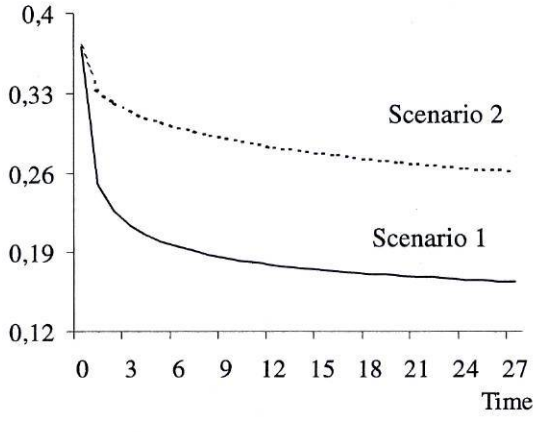
Variables D_I^2, D_I^3, D_I^4 and D_I^5 are decreasing and at a declining rate until the steady state, as figure 3.7 shows.[160] The dynamics of those variables is expected from the behaviour of $Q_I^{J^{VH}} VH_I(.|w)$, $Q_I^{J^{SH}} SH_I(.|w)$ and consumption. In particular, in scenario 1, $Q_I^{J^{VH}} VH_I(.|w)$ starts rising more than $Q_I^{J^{SH}} SH_I(.|w)$ and – as will be clearer below – both rise more than consumption. Their adjustment process is globally stable, although we have ascertained that the equilibrium point of convergence relies on the initial values.

Figure 3.7. Transitional dynamics of D_I^2 and D_I^5 in both scenarios

a) Transitional dynamics of D_I^2



b) Transitional dynamics of D_I^5



The initial position of $u_{wI}(.|m)$ becomes conditional to the initial position of D_I^1, but the steady state point is the same for all conditional initial situations. Thus, we observe that the economy starts with a relative scarcity of HC at work; that is, $u_{wI}(.|m)$ is smaller than its steady state value, and the differential is greater in scenario 1 – *e.g.*, figure 3.8. This means that the rate of return on HC is relatively high – above all in scenario 1 –, which disincentives the use of HC at work and, therefore, individuals spend a large share of their time accumulating HC. Equation (2.66) shows that *VH*, because it is school intensive, is above all accumulated in school, while *SH*, which is OJT intensive, is

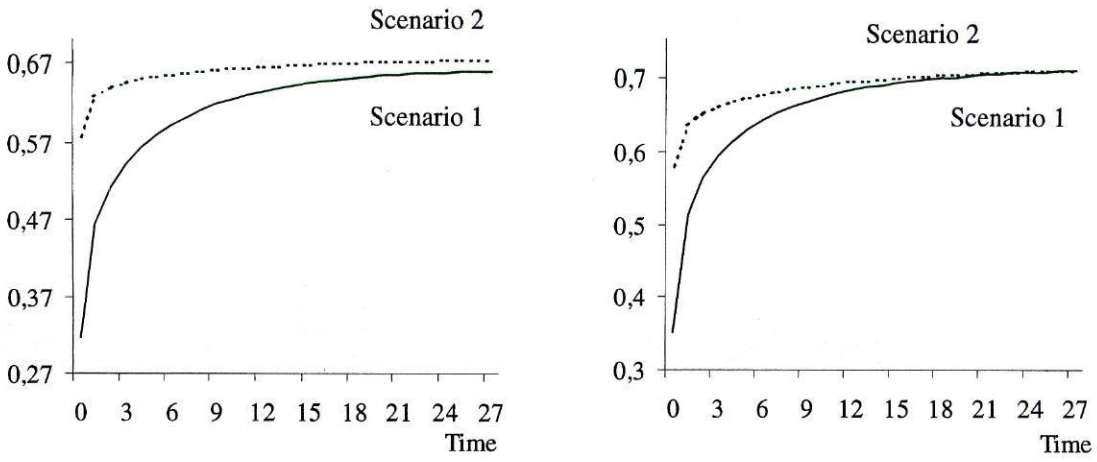
[159] The analysis of two different scenarios due to distinct initial conditions belongs to the sensitivity analysis. It is considered here in order to take advantage of the figure with the transition dynamics of D_I^1.

[160] To simplify, we present only the graphs for D_I^2 and D_I^5.

mainly accumulated in OJT. For example, from table A3.1 in Appendix 3 results for the scenario 1 that $u_{SI}(0|VH)=0.41$, $u_{TI}(0|VH)=0.27$, $u_{SI}(0|SH)=0.26$ and $u_{SI}(0|SH)=0.39$.

Figure 3.8. Transitional dynamics of the share of time spent at work

a) Transitional dynamics of $u_{wI}(.|VH)$ **b) Transitional dynamics of $u_{wI}(.|SH)$**



At the beginning of the transition, it is the growth rate of HC at work – provided by both the increase in the amount of HC and the reallocation of HC to production – that drives economic development, the growth rate of successful research being relatively low. When the economy moves toward the steady state, the time share spent in training gradually declines[161] and R&D becomes relatively more important for growth. This in line with, for example, Funke and Strulik (2000), where the HC behaviour starts to be a more important engine of economic growth than the R&D – *i.e.*, than TK progress.

As individuals initially do not have much time for work, their wage earnings are low and they can only afford to consume relatively little in relation to the steady state. Increased wage earnings, resulting from her/his higher HC stock, from a larger time share spent at work and from the increase in wage per unit of HC at work, allow the individuals to increase their level of consumption. The rise in wage earnings in scenario 1 is mainly due to both the large time share spent at work and the wage increase per unit of *VH* – as a result of TK improvements –,[162] whereas in scenario 2 it is essentially due to the wage increase per unit of *VH* – as a result of both improved TK and a rise in the versatile price index – *e.g.*, equation (2.30) –, since $u_{wI}(0|m)$ in scenario 2 is greater than $u_{wI}(0|m)$ in scenario 1. Finally, we note that the versatile premium falls in both scenarios.

[161] In scenario 1, *e.g.*, $u_{SI}(27|VH)=0.21$, $u_{TI}(27|VH)=0.13$, $u_{SI}(27|SH)=0.12$ and $u_{SI}(27|SH)=0.17$.

3.4.4. Transitional dynamics in the South

Differential equations

The transitional dynamics for the South also depart from equilibrium in the labour market, free entry condition into R&D, individual utility maximisation with individual optimal time allocation. Moreover, we consider that the North is always in steady state, since it is unaffected by Southern imitation.

We start proving, by contradiction, that at first the South immediately converges at time t_0 to a unique stable country specific growth path.

As in the North case, equating this m-type HC demand path – equation (2.30) – to the respective 'supply' – in equation (2.70) –, the following differential equation results:

$$\frac{\dot{Q}_P^{J^m}(t)}{Q_P^{J^m}(t)} + \frac{1}{\alpha}\frac{\dot{p}_P^m(t)}{p_P^m(t)} = [1-\tau_K]\, r_P(t) + \delta^m - \varphi^{m[1/\phi]}\chi_S\left[1+\frac{u_T(t|m)}{u_S(t|m)}\right]^{\frac{1-\phi}{\phi}}. \qquad (3.58)$$

Subtracting equations (3.41) – for the North – and (3.58), we get:

$$\frac{\dot{\tilde{Q}}^{J^m}(t)}{\tilde{Q}^{J^m}(t)} + \frac{1}{\alpha}\frac{\dot{p}_P^m(t)}{p_P^m(t)} = [1-\tau_K]\left[r_P(t) - r_I(t)^{ss}\right]. \qquad (3.59)$$

Where $\dot{\tilde{Q}}^{J^m}/\tilde{Q}^{J^m} \equiv \dot{Q}_P^{J^m}/Q_P^{J^m} - \dot{Q}_I^{J^m}/Q_I^{J^m}$, since by definition $\tilde{Q}^{J^m} \equiv Q_P^{J^m}/Q_I^{J^m}$. Note that as the North is, by hypothesis, always in steady state then $\dot{p}_I^{m^{ss}}/p_I^{m^{ss}} = 0$ and $r_I(t)^{ss}$ is constant. Writing equation (3.59) for each m and afterwards subtracting both, yields:

$$\frac{\dot{\tilde{Q}}^{J^{VH}}(t)}{\tilde{Q}^{J^{VH}}(t)} - \frac{\dot{\tilde{Q}}^{J^{SH}}(t)}{\tilde{Q}^{J^{SH}}(t)} = -\frac{1}{\alpha}\frac{\dot{p}_P^{VH}}{p_P^{VH}} + \frac{1}{\alpha}\frac{\dot{p}_P^{SH}}{p_P^{SH}} = \frac{1}{2}\frac{\dot{D}_P^I(t)}{D_P^I(t)}; \qquad (3.60)$$

since the dynamics of FGs price indexes comes to be given by:

$$\frac{\dot{p}_P^{VH}}{p_P^{VH}} = -\frac{\alpha}{2}\left[1+\left(\frac{vh}{sh}D_P^I\right)^{(1/2)}\right]^{-1}\frac{\dot{D}_P^I}{D_P^I} \text{ and } \frac{\dot{p}_P^{SH}}{p_P^{SH}} = \frac{\alpha}{2}\left[1+\left(\frac{vh}{sh}D_P^I\right)^{(-1/2)}\right]^{-1}\frac{\dot{D}_P^I}{D_P^I} \qquad (3.61)$$

Where, by definition, $D_P^I \equiv \frac{Q_P^{J^{VH}} VH_P(.|w)}{Q_P^{J^{SH}} SH_P(.|w)} = \frac{\tilde{Q}^{J^{VH}} \tilde{VH}(.|w)}{\tilde{Q}^{J^{SH}} \tilde{SH}(.|w)} D_I^{I^{ss}}$.[163] Thus, according to equation (3.60) when $\dot{\tilde{Q}}^{J^{VH}}/\tilde{Q}^{J^{VH}} > \dot{\tilde{Q}}^{J^{SH}}/\tilde{Q}^{J^{SH}}$ then $\dot{D}_P^I/D_P^I > 0$ or, in other words, $\frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}} + \frac{\dot{\tilde{VH}}(.|w)}{\tilde{VH}(.|w)} > \frac{\dot{\tilde{Q}}^{J^{SH}}}{\tilde{Q}^{J^{SH}}} + \frac{\dot{\tilde{SH}}(.|w)}{\tilde{SH}(.|w)}$, since $D_I^{I^{ss}}$ is constant. But this is impossible. In fact, in

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[162] Note that the fall in the price index of versatile FGs adversely affected the wage per unit of *VH*.

[163] Bearing in mind that, by definition, $\tilde{m}(.|w) \equiv m_P(.|w)/m_I(.|w)$.

that case, from equations (3.61), $\dot{p}_P^{SH}/p_P^{SH} > 0$ and $\dot{p}_P^{VH}/p_P^{VH} < 0$, which implies that there is a stronger market incentive to develop specialised TK in the South. That is, $\frac{\dot{\tilde{Q}}^{J^{SH}}}{\tilde{Q}^{J^{SH}}} + \frac{\dot{\tilde{SH}}(.|w)}{\tilde{SH}(.|w)}$ must grow at a higher rate than $\frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}} + \frac{\dot{\tilde{VH}}(.|w)}{\tilde{VH}(.|w)}$ and, if this is so, $\frac{\dot{D}_P^I}{D_P^I} < 0$.[164]

The argument to show that the conclusion is the same when $\frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}} < \frac{\dot{\tilde{Q}}^{J^{SH}}}{\tilde{Q}^{J^{SH}}}$ is identical.

Therefore, we conclude by contradiction, that, in fact, the South jumps (or defines) at time t_0 to a unique stable growth path. From this time on, $Q_P^{J^{VH}} VH_P(.|w)$ and $Q_P^{J^{SH}} SH_P(.|w)$ grow at the same rate. In short, from time t_0 on:

$$\frac{\dot{D}_P^I}{D_P^I} = \frac{\dot{p}_P^m}{p_P^m} = \frac{\dot{N}_P}{N_P} = 0, \quad \frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}} = \frac{\dot{\tilde{Q}}^{J^{SH}}}{\tilde{Q}^{J^{SH}}} \text{ and } \frac{\dot{\tilde{VH}}(.|w)}{\tilde{VH}(.|w)} = \frac{\dot{\tilde{SH}}(.|w)}{\tilde{SH}(.|w)}. \tag{3.62}$$

That is, price indexes, p_P^m, and thus the threshold FG, N_P, reach their steady state values. If this is so, then the second and third parts of equation (3.62) hold. Hence, on the demand side of the labour market, instead of equation (2.30), we actually have:

$$\frac{\dot{w}_P^m(t)}{w_P^m(t)} = \frac{\dot{Q}_P^{J^m}(t)}{Q_P^{J^m}(t)} = \frac{\dot{\tilde{Q}}^{J^m}(t)}{\tilde{Q}^{J^m}(t)} + \frac{\dot{Q}_I^{J^{mSS}}(t)}{Q_I^{J^{mSS}}(t)}. \tag{3.63}$$

As stated, the transition in the South only ends when it reaches the Northern (or world) steady state. This is due to the exploitation of TK backwardness, which implies that during the transitional phase the Southern growth rate tends to be higher than in the North. At least the Southern TK progress must be higher, as depicted in figure 3.2.

Therefore, from t_0 on, the South's transitional dynamics is fully described by a system of differential equations in the variables: $\tilde{Q}^{J^m}$, $\tilde{m}(.|w)$, D_P^2, D_P^3, D_P^4, D_P^5, $u_{wP}(t|m)$, $u_{SP}(t|m)$ and $u_{TP}(t|m)$, which must all be constants in the steady state.

It is then essential to find the value of the interest rate at each time. To this end, we again take a representative domestic IG *av* of each *m*-type. Thus, the expression for the expected present value of profits for the respective leading imitator is:

$$V_P(k,av,v|m) = \Pi_P(k,av,v|m) \int_t^\infty exp\left[-\int_t^s \left(r_P(v) + pb_P(k,av,v|m) \right) dv \right] ds. \tag{3.64}$$

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[164] At least $\dot{\tilde{Q}}^{J^{SH}}/\tilde{Q}^{J^{SH}}$ must grow at a higher rate than $\dot{\tilde{Q}}^{J^{VH}}/\tilde{Q}^{J^{VH}}$ which also implies $\dot{D}_P^I/D_P^I < 0$.

In equation (3.64), $r_P(v)$ is the Southern interest rate at time v and $pb_P(k,av,v\,|m)$ is the probability at time v that the av IG of quality rung $(k+1)$ of the m-complementary TK ladder will be imitated. Free entry into R&D determines that at all times, the probability of successful imitation times the expected present value of profits from successful imitation must equal the after subsidy cost of imitation. Thus, equations (3.44), (3.45) and (3.46) also exist here, the only difference being in the country index.

As a result of equations (2.34) or, alternatively, (2.38) and (2.39a) we have:

$$\frac{\dot{rs}_P(k,av,t\,|m)}{rs_P(k,av,t\,|m)} = \frac{\dot{pb}_P(k,av,t\,|m)}{pb_P(k,av,t\,|m)} + \frac{\dot{m}_P(t|w)}{m_P(t|w)} - \bar{\sigma}_1 \frac{\dot{g}_1[\tilde{m}(t|w)]}{g_1[\tilde{m}(t|w)]} - \dot{\sigma}_2(\tilde{Q}^{Jm}, t) \qquad ; \quad (3.65a)$$
$$ln\, g_2[\tilde{Q}^{Jm}(t), d] - \sigma_2(\tilde{Q}^{Jm}, t) \frac{\dot{g}_2[\tilde{Q}^{Jm}(t), d]}{g_2[\tilde{Q}^{Jm}(t), d]} - \dot{k}_I(av,t\,|m)\left(\frac{\alpha-1}{\alpha}\right) ln q$$

$$\frac{\dot{\Pi}_P(k,av,t\,|m)}{\Pi_P(k,av,t\,|m)} = \frac{\dot{\Pi}_P(k+1,av,t\,|m)}{\Pi_P(k+1,av,t\,|m)} = \frac{\dot{m}_P(t\,|w)}{m_P(t\,|w)} + \dot{k}_P(av,t\,|m)\left(\frac{1-\alpha}{\alpha}\right) ln q \cdot \quad (3.65b)$$

Plugging equation (3.65a, b) into the previous analysis – specifically into the equation similar to equation (3.46) – yields the following equation for the interest rate:

$$r_P = -\bar{\sigma}_1 \frac{\dot{g}[\tilde{m}(.|w)]}{g[\tilde{m}(.|w)]} - \dot{\sigma}_2(\tilde{Q}^{Jm})\, ln\, g(\tilde{Q}^{Jm}, d) - \sigma_2(\tilde{Q}^{Jm}) \frac{\dot{g}(\tilde{Q}^{Jm}, d)}{g(\tilde{Q}^{Jm}, d)} + \cdot \quad (3.66)$$
$$+ \frac{pb_P(k,av,.|m)\, \Pi_P(k+1,av,.|m)}{rs_P(k,av,.|m)\,(1-s_{rP})} - pb_P(k+1,av,.|m)$$

The Southern interest rate in equation (3.92) must be constant in steady state. The first three terms on the right-hand side represent a capital gains term, $CGT(.|m)$, which is explicitly presented in equation (3.67a) below. The fourth is the dividend term, $DT(.|m)$, which is independent of the index $k_P(av|m)$ – see equation (3.67b). The fifth reflects market losses due to creative destruction when the next successful imitation occurs.[165]

$$CGT(.|m) = -\bar{\sigma}_1 \frac{\dot{g}[\tilde{m}(.|w)]}{g[\tilde{m}(.|w)]} - \dot{\sigma}_2(\tilde{Q}^{Jm})\, ln\, g(\tilde{Q}^{Jm}, d) - \sigma_2(\tilde{Q}^{Jm}) \frac{\dot{g}(\tilde{Q}^{Jm}, d)}{g(\tilde{Q}^{Jm}, d)} =$$
$$= -\bar{\sigma}_1 \frac{\tilde{m}(.|w)\, exp[\tilde{m}(.|w)]}{\{1+exp[\tilde{m}(.|w)]\}\{1+2exp[\tilde{m}(.|w)]\}} \frac{\dot{\tilde{m}}(.|w)}{\tilde{m}(.|w)} - \dot{\tilde{Q}}^{Jm} \qquad ; \quad (3.67a)$$
$$ln\left[-\tilde{Q}^{Jm^2} + (1+d)\tilde{Q}^{Jm} - d\right] - \left(-\sigma_2 + \tilde{Q}^{Jm}\right) \frac{-2\tilde{Q}^{Jm^2} + (1+d)\tilde{Q}^{Jm}}{-\tilde{Q}^{Jm^2} + (1+d)\tilde{Q}^{Jm} - d} \frac{\dot{\tilde{Q}}^{Jm}}{\tilde{Q}^{Jm}}$$

[165] Also from equation (2.54), the impact of creative destruction on the interest rate, $pb_P(k+1,av,t\,|m)$ can be written as a function of the growth rate of the respective domestic TK.

$$DT(.|VH)=\frac{pb_P(k,av,t|VH)\ \Pi_P(k+1,av,t|VH)}{rs_P(k,av,t|VH)\ (1-s_{rP})}=\frac{\beta_P}{\zeta_P}\,exp(PE+IT)\,\{g_1[\tilde{VH}(t|w)]\}^{\bar{\sigma}_1}$$
$$\{g_2[\tilde{Q}^{JVH}(t),d]\}^{\sigma_2(\tilde{Q}^{JVH},t)}\left(\frac{1-s_{xP}}{1-s_{rP}}\right)\left(\frac{q-1}{q}\right)\left(A_P\frac{1-\alpha}{1-s_{xP}}\right)^{(1/\alpha)}vh\,exp(-1)\left[1+\left(\frac{vh}{sh}D_P^1\right)^{(-1/2)}\right] \qquad (3.67b)$$

The sign of the capital gains term in equation (3.67a) is not clear. We can, though, calculate it numerically later on. Equation (3.67b) – resulting from equations (2.38) and (2.39a) – shows the dividend term for $m = VH$ and is independent of the index $k_P(av|m)$.

From equation (3.66) we get two equations for the Southern interest rate, one for each m; equating them, since the country's interest rate is always unique,[166] we obtain:[167]

$$\begin{aligned}
\frac{\dot{\tilde{Q}}^{JVH}}{\tilde{Q}^{JVH}}=&\left\{-q^{\left(\frac{\alpha-1}{\alpha}\right)}\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}\left[\frac{\dot{Q}_I^{JSH\,ss}}{Q_I^{JSH\,ss}}-\frac{\dot{Q}_I^{JVH\,ss}}{Q_I^{JVH\,ss}}\right]+DT(.|SH)-DT(.|VH)+\frac{\dot{\tilde{VH}}(.|w)}{\tilde{VH}(.|w)}\bar{\sigma}_1\right.\\
&\left\{\tilde{VH}(.|w)\,exp\left(\tilde{VH}(.|w)\right)\left[\left(1+exp\left(\tilde{VH}(.|w)\right)\right)\left(1+2exp\left(\tilde{VH}(.|w)\right)\right)\right]^{-1}-\right.\\
&\left.\left.-\tilde{SH}(.|w)\,exp\left(\tilde{SH}(.|w)\right)\left[\left(1+exp\left(\tilde{SH}(.|w)\right)\right)\left(1+2exp\left(\tilde{SH}(.|w)\right)\right)\right]^{-1}\right\}\right\}\\
&\left\{-\tilde{Q}^{JVH}ln\left(-\tilde{Q}^{JVH^2}+(1+d)\tilde{Q}^{JVH}-d\right)-\left(-\bar{\sigma}_2+\tilde{Q}^{JVH}\right)\frac{-2\tilde{Q}^{JVH^2}+(1+d)\tilde{Q}^{JVH}}{-\tilde{Q}^{JVH^2}+(1+d)\tilde{Q}^{JVH}-d}\right.\\
&-q^{\left(\frac{\alpha-1}{\alpha}\right)}\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}+\tilde{Q}^{JSH}ln\left(-\tilde{Q}^{JSH^2}+(1+d)\tilde{Q}^{JSH}-d\right)+\\
&\left.+\left(-\bar{\sigma}_2+\tilde{Q}^{JSH}\right)\frac{-2\tilde{Q}^{JSH}+(1+d)\tilde{Q}^{JSH}}{-\tilde{Q}^{JSH^2}+(1+d)\tilde{Q}^{JSH}-d}+q^{\left(\frac{\alpha-1}{\alpha}\right)}\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}\right\}^{-1}
\end{aligned} \qquad (3.68)$$

The differential equation (3.68), in particular, incorporates information in equation (3.62) and our assumption that the North is already and always remains in steady state.

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[166] For presentation reasons, the dividend terms are not explicitly presented.

[167] Alternatively we can use the aggregate budget constraint, equation (2.72). In this case, we obtain after the division by, for example, $Q_P^{JSH}\,SH_P(.|w)$ the differential equation:

$$\begin{aligned}
&\frac{\zeta_P\,D_P^1\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}}{\beta_P\,exp(PE+IT)\{g_1(.)\}^{\sigma_1}\{g_2(.)\}^{\sigma_2(.)}}\frac{\dot{\tilde{Q}}^{JVH}}{\tilde{Q}^{JVH}}+\frac{\zeta_P\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}}{\beta_P\,exp(PE+IT)\{g_1(.)\}^{\sigma_1}\{g_2(.)\}^{\sigma_2(.)}}\frac{\dot{\tilde{Q}}^{JSH}}{\tilde{Q}^{JSH}}=A_P^{(1/\alpha)}\\
&\left(\frac{1-\alpha}{q(1-s_{xP})}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\left[1-\frac{1-\alpha}{q(1-s_{xP})}\right]exp(-1)\left\{vh\,D_P^1\left[1+\left(\frac{vh}{sh}D_P^1\right)^{(-1/2)}\right]+sh\left[1+\left(\frac{vh}{sh}D_P^1\right)^{(1/2)}\right]\right\}-\\
&-D_P^4-D_P^5-\frac{\zeta_P\,D_P^1\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}}{\beta_P\,exp(PE+IT)\{g_1(.)\}^{\bar{\sigma}_1}\{g_2(.)\}^{\sigma_2(.)}}\frac{\dot{Q}_I^{JVH\,ss}}{Q_I^{JVH\,ss}}-\frac{\zeta_P\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]^{-1}}{\beta_P\,exp(PE+IT)\{g_1(.)\}^{\bar{\sigma}_1}\{g_2(.)\}^{\sigma_2(.)}}\frac{\dot{Q}_I^{JSH\,ss}}{Q_I^{JSH\,ss}}
\end{aligned}$$

Another differential equation can be obtained from the labour market. The equilibrium here, from t_0 on, implies that – see equations (3.58), (3.62) and (3.63),

$$\frac{\dot{\tilde{Q}}^{J^m}(t)}{\tilde{Q}^{J^m}(t)} + \frac{\dot{Q}_I^{J^{mSS}}(t)}{Q_I^{J^{mSS}}(t)} = [1-\tau_K]\, r_P(t) + \delta^m - \varphi^{m[1/\phi]}\, \chi_S \left[1 + \frac{u_T(t|m)}{u_S(t|m)}\right]^{\frac{1-\phi}{\phi}}. \quad (3.69)$$

Solving equation (3.69) – *i.e.*, incorporating equations (3.66) and (3.67a) into equation (3.69) –, and considering as hypothesis $m = VH$, results:

$$\begin{aligned}
\frac{\dot{\tilde{VH}}(.|w)}{\tilde{VH}(.|w)} = & \left\{ -\left[(1-\tau_K)\, q^{\left(\frac{\alpha-1}{\alpha}\right)} \left(q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right)^{-1} + 1 \right] \frac{\dot{Q}_I^{J^{VH\,SS}}}{Q_I^{J^{VH\,SS}}} - \frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}} \right. \\
& \left[1 + (1-\tau_K)\, \tilde{Q}^{J^{VH}} ln\left(-\tilde{Q}^{J^{VH2}} + (1+d)\, \tilde{Q}^{J^{VH}} - d \right) + (1-\tau_K) \left(-\bar{\sigma}_2 + \tilde{Q}^{J^{VH}} \right) \right. \\
& \left. \frac{-2\tilde{Q}^{J^{VH\,2}} + (1+d)\tilde{Q}^{J^{VH}}}{-\tilde{Q}^{J^{VH\,2}} + (1+d)\tilde{Q}^{J^{VH}} - d} + (1-\tau_K)\, q^{\left(\frac{\alpha-1}{\alpha}\right)} \left(q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right)^{-1} \right] + (1-\tau_K)\, DT(.|VH) - \\
& \left. \left[\varphi^{VH^{(1/\phi)}}\, \chi_S \left(1 + \frac{u_T(VH)}{u_S(VH)} \right)^{((1-\phi)/\phi)} - \delta^{VH} \right] \right\} \left\{ (1-\tau_K)\, \bar{\sigma}_1\, \tilde{VH}(.|w) \right. \\
& \left. exp\left(\tilde{VH}(.|w) \right) \left[\left(1 + exp\left(\tilde{VH}(.|w) \right) \right) \left(1 + 2\, exp\left(\tilde{VH}(.|w) \right) \right) \right]^{-1} \right\}^{-1}
\end{aligned} \quad (3.70)$$

The expression for the dynamics of D_P^2, D_P^3, D_P^4 and D_P^5 can be obtained in the same way as for the North. For example taking into account the definition of D_P^4 and D_P^5,

$$\begin{aligned}
\frac{\dot{D}_P^4}{D_P^4} = \frac{\dot{D}_P^5}{D_P^5} = \frac{\dot{C}_P(SH)}{C_P(SH)} - \frac{\dot{Q}_P^{J^{SH}}}{Q_P^{J^{SH}}} - \frac{\dot{SH}_P(.|w)}{SH_P(.|w)} = \frac{1}{\theta}\left[(1-\tau_K)\, r_P - \rho \right] - \\
- \frac{\tilde{Q}^{J^{SH}}}{Q^{J^{SH}}} - \frac{\dot{Q}_I^{J^{SH\,SS}}}{Q_I^{J^{SH\,SS}}} - \frac{\dot{\tilde{SH}}(.|w)}{SH(.|w)} - \frac{\dot{SH}_I(.|w)^{SS}}{SH_I(.|w)^{SS}}
\end{aligned} \quad (3.71)$$

Introducing the expression for $[1-\tau_K]\, r_P(t)$ obtained from equation (3.69) into equation (3.71), and considering the definitions of TK gap and of gap in HC at work,

$$\begin{aligned}
\frac{\dot{D}_P^4}{D_P^4} = \left(\frac{1-\theta}{\theta} \right) \left[\frac{\dot{\tilde{Q}}^{J^{SH}}}{\tilde{Q}^{J^{SH}}} + \frac{\dot{Q}_I^{J^{SH\,SS}}}{Q_I^{J^{SH\,SS}}} \right] - \left[\frac{\dot{\tilde{SH}}(.|w)}{\tilde{SH}(.|w)} + \frac{\dot{SH}_I(.|w)^{SS}}{SH_I(.|w)^{SS}} \right] - \frac{\rho}{\theta} + \\
+ \frac{1}{\theta} \left[\varphi^{SH^{(1/\phi)}}\, \chi_S \left(1 + \frac{u_T(SH)}{u_S(SH)} \right)^{((1-\phi)/\phi)} - \delta^{SH} \right]
\end{aligned} \quad (3.72)$$

Finally, we get the dynamics of the amount of (non-leisure) time individuals who accumulate m-type HC allocate the production activity, $u_{wP}(t|m)$,[168] which must be constant in steady state. To this end, we note that there is again an equation similar to (3.56). Thus, from equations (2.59) and (2.65) and the dynamics of $\tilde{m}(.|w)$, we obtain:

$$\frac{\dot{u}_{wP}(t|m)}{u_{wP}(t|m)} = \frac{\dot{\tilde{m}}(.|w)}{\tilde{m}(.|w)} + \frac{\dot{m}_I(.|w)^{ss}}{m_I(.|w)^{ss}} - \varphi^{m^{(1/\phi)}} \chi_S \left(1 + \frac{u_T(m)}{u_S(m)}\right)^{((1-\phi)/\phi)} \left(1 - u_{wP}(t|m)\right) + \delta^m . \quad (3.73)$$

As the stability of variables D_P^2, D_P^3, D_P^4, D_P^5, $u_{wP}(t|m)$, $u_{SI}(t|m)$ and $u_{TI}(t|m)$ is analysed after the stability of variables $\tilde{Q}^{Jm}$ and $\tilde{m}(.|w)$, the properties of the South's dynamics are also block recursive. The above differential equations were also deduced for the other two alternative specifications of the catching up term in Appendix 2, although they are not presented here for exposition reasons.

Solving differential equations for the baseline parameter values

The system composed by differential equations (3.62), (3.68), (3.70), (3.72) and (3.73) depicts the transitional dynamics of variables $\tilde{Q}^{JVH}$, $\tilde{Q}^{JSH}$, $\tilde{VH}(.|w)$, $\tilde{SH}(.|w)$, D_P^4, D_P^5 and $u_{wP}(t|m)$. Solving it numerically, we fully describe the South's transition path towards the world steady state, under the 'benchmark' catching up term. This is now our aim. But we go further and also consider the systems of differential equations obtained under the other two alternative catching up specifications in Appendix 2.

Table A3.2 in Appendix 3 (subsection A3.1.2) condenses the main results, by comparing initial and steady state values of the relevant variables. In addition, figures 9 and 10 below complete the set of results, by showing the path of the variables that are constant in steady state from t_0 on, under the 'benchmark' catching up term. We give less importance to the transitional dynamics under the two alternative terms, mostly because the associated speed of convergence is very slow. Moreover, under the second alternative catching up term the South supports strong 'transition costs', and so during the transition phase, its growth rate is slower than that of the North. But this is counter to the literature on TK diffusion – *e.g.*, Barro and Sala-i-Martin (1997).

It should be borne in mind that in our case – and regardless of the catching up term considered –, the idea underlying TK imitation differs from that found in the literature. That one assumes the notion of pure relative backwardness and so the rate of imitation

[168] Afterwards the dynamics for both $u_{SI}(t|m)$ and $u_{TI}(t|m)$ can easily be found.

(and growth) is a positive and monotonic function of the relative TK gap between countries. Thus, the very poor countries are the ones with most potential to imitate, while in our case the very poor countries will remain stagnant due to their inability to imitate and only middle income countries will benefit from existing Northern TK progress.

Two conclusions emerge immediately. The first is that the adjustment process is globally stable, and the second is about the speed of convergence towards the steady state. The fastest speed of convergence is observed with the benchmark catching up term, where the steady state is reached after *56* years. The speed of convergence is faster with the first alternative specification than with the second. In fact, with the first, the steady state is reached after *235* years, while with the second it takes *482* years. The prolonged time scale towards the steady state is not however surprising. For example, in the model without HC accumulation and with scale effects, proposed by Papageorgiou (2002), the steady state emerges at the end of *160* periods/years. The long time towards the steady state suggests that transitional dynamic is important and should not be neglected.

Figure 3.9a shows that both $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ increase from their initial values (*0.30* and *0.35*, respectively) towards their steady state values (*0.55* and *0.64*, respectively). As table A3.2 reports, and as expected – see notes to table 3.4 –, the steady state values of the TK gaps are independent of the catching up term. The probabilities of successful imitation rise initially. However, as the rung of quality left to be copied decreases – *i.e.*, as $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ increase –, the cost of imitation increases and the probabilities of successful imitation fall toward the steady state values: $Pb_I(.|VH) = Pb_P(.|VH) = 0.036$ and $Pb_I(.|SH) = Pb_P(.|SH) = 0.038$. Thus, $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ are increasing but at decreasing rates or, in other words, the TK in the South grows more quickly than in the North during the transitional dynamics, but slows down until steady state is reached.

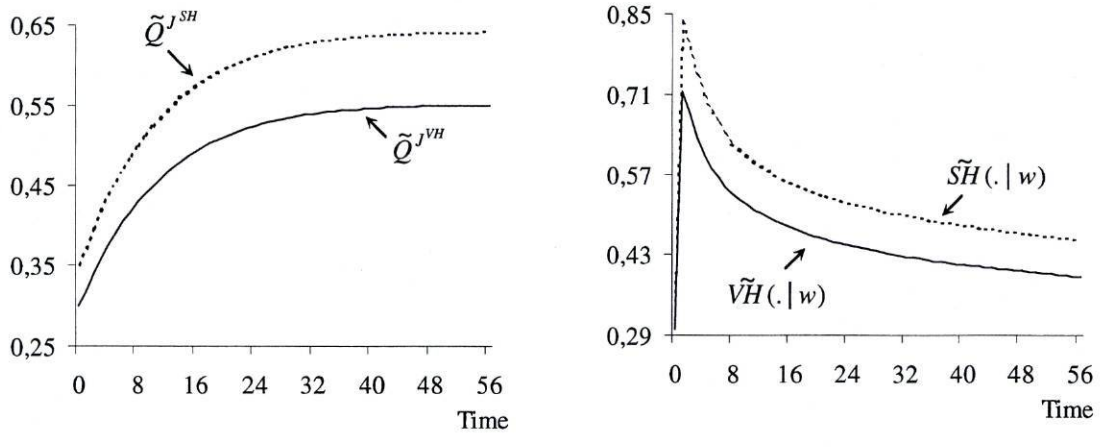
As shown in figures 3.9b (for the gap in HC at work) and 3.10a (for the share of time at work), there is an initial abrupt surge in the gaps in HC at work, $\widetilde{SH}(.|w)$ and $\widetilde{VH}(.|w)$, owing to the immediate increase in the share of time dedicated at work. After that, still during transition, they drop due to the greater share of time devoted to working in the South compared with the North – see tables A3.1 and A3.2 and figures 3.8 and 3.10a. Thus, the drop is a result of the (relative) smaller HC accumulation in the South.[169]

[169] In addition, the increase in Southern wages, due to TK diffusion, stimulates HC accumulation – *i.e.*, Southern individuals adjust smoothly the path of time, devoting more time to training and less to work.

As implied in table A3.2, the biggest drop occurs under the second alternative catching up term, which generates the smallest HC accumulation in the South. In light of the initial values, at the end of the adjustment process, a new higher steady state level of $\tilde{SH}(.\,|\,w)$ and $\tilde{VH}(.\,|\,w)$ is only reached with the benchmark catching up term.

Figure 3.9. Transitional dynamics of the TK gaps and the gaps in HC at work

a) Transitional dynamics of $\tilde{Q}^{J^m}$ **b) Transitional dynamics of $\tilde{m}(.\,|\,w)$**



The joint behaviour of $\tilde{Q}^{J^m}$ and $\tilde{m}(.\,|\,w)$ implies that initially the South grows at a higher rate than the North owing to the immediate jump in the share of time devoted to work in the South, which increases $\tilde{m}(.\,|\,w)$.[170] Afterwards, it is the magnitude of the probability of successful imitation when compared with the probability of successful innovation that accounts for the higher Southern growth rate.[171] Finally, both countries grow at the same rate due to the successive increases in the cost of imitation, which represents a form of diminishing returns.[172] In the end, both the TK gap and the gap in HC at work remain constant. That is, the interest rate and the growth rate of the South fall steadily (are monotonic decreasing) toward their (or Northern) steady state values.

If we just compare the steady state growth rates, we would conclude that imitation has improved the Southern growth rate. Under the better alternative, the country would be an inefficient innovator and its growth rate would be smaller. In fact, supposing that $p_{P,WP}^{m^{SS}} \cong p_P^{m^{SS}}$ and taking into account tables 3.4, A3.1 and A3.2, the evaluation of equation (3.7) indicates that TK diffusion favours the Southern steady state growth rate.

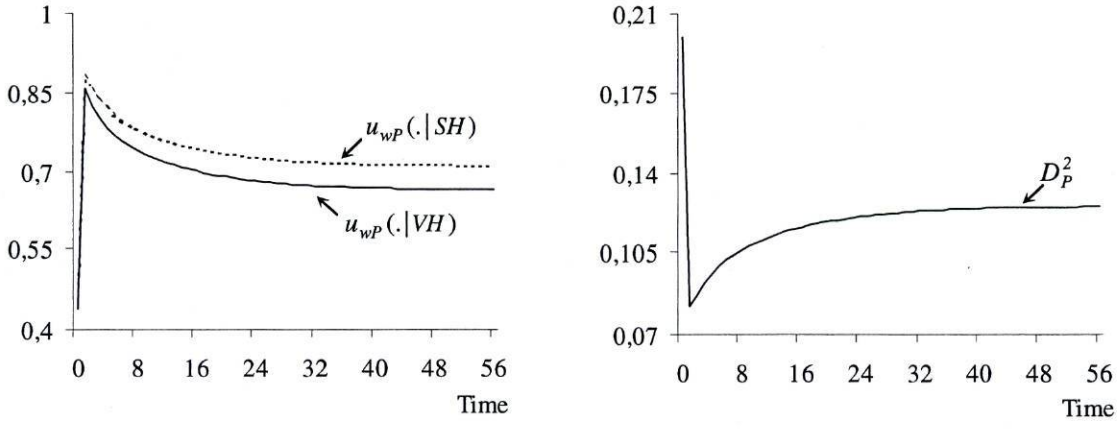
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[170] This suggests that countries have a strong potential growth – realised once its individuals reallocate time from training to work. This is mainly true if the country is poor in terms of output relative to its HC stock.
[171] In this period the higher growth rate of the South is not true under the second catching up specification.
[172] In the neoclassical growth model, the diminishing returns to capital accumulation play a similar role.

Regardless of the catching up term the South improves the TK. However, under the second alternative for the catching up term, its HC at work decreases so much after the initial immediate (and very strong) jump that its average growth rate during transition is smaller than in the North. This alternative goes against the literature that emphasises the experience of a lot of Southern countries, which had a fast initial rate of catching up and so achieved higher growth rates than the leader country, which then slowed down.

Figure 3.10. Share of time at work and weight of consumption in 'large knowledge'

a) Transitional dynamics of $u_{wP}(t|m)$ **b) Transitional dynamics of D_P^2**



The initial conditional value of each $u_{wP}(.|m)$ indicates us that the South starts with a relative scarcity of HC, and the differential is greater in the benchmark case – see table A3.2. This means that the rate of return on HC is relatively high and, therefore, that individuals spend a large share of their time accumulating HC. Thus, individuals do not have much time left over for work, their wage earnings are small and so they can only afford to consume a relatively small amount in relation to the consumption of individuals in the North – see table A3.2. As reported above, at the beginning of the transition, the growth rate of HC at work – due to both the previous increase in the amount of HC and the reallocation of HC to production – drives economic growth. After that, when the economy moves towards the steady state, R&D becomes the main engine of growth.

In steady state, the increased wage earnings achieved by the larger time share spent at work and by the wage increase per unit of HC – which here is only proportional to the growth of TK since price indexes of FGs remain the same –, allow individuals to increase their level of consumption in relation to individuals in the North – see table A3.2. We can thus state that there is an improvement in the relative welfare of the South.

In general terms, variables D_P^2, D_P^3, D_P^4 and D_P^5 are decreasing. Initially they decrease strongly, but after that they increase at a decreasing rate until the steady state – see figure 3.10b for D_P^2. The dynamics of those variables indicates that the behaviour of $Q_P^{Jm}\, m_P(.|w)$ rises (still) more than consumption. This behaviour may be conditioned by the existence of some resources shifted from consumption to imitation.

Furthermore, we confirm that capital gains do not exist in steady state and are present only in transition. Computing equation (3.67a), we observe that, for example, $CGT(t_1|VH) = 0.047$, $CGT(t_1|SH) = 0.046$, $CGT(t_{56}|VH) = CGT(t_{56}|SH) = 0$.

We can now evaluate equation (2.31) [or (3.15)]; *i.e.*, inter-country wage inequality – see table A3.2. Wages in the South, w_P^{VH} and w_P^{SH}, benefit from the imitation activity – *i.e.*, convergence – since the South grows faster than the North and improves its relative *TFP* during the transition to the world BG path. This is in line with time allocation. In fact, as during transition Southern wages grow a higher rate than in the North, Southern individuals adjust the path of time allocation, devoting gradually less time to work (after the initial increase of u_w) toward the world (or Northern) steady state value. However, even in steady state wage differentials remain, due to international immobility of HC, differences in exogenous productivity parameter, $A_I > A_P$, and differences in the available TK and in the threshold FGs. That is, from steady state on, inter-country wage inequality remains constant, because the path of productivity becomes common to both countries.

Moreover, concerning intra-country wage inequality we can state that under the baseline parameter values it is greater in the South – see calculations for time t_0 in table A3.2, and remember that after t_0 it follows the same path in both countries since $\tilde{Q}^{J^{VH}}$ and $\tilde{Q}^{J^{SH}}$ grow at the same rate. Thus, under the set of baseline parameter values and the set of initial conditions, the versatile premium decreases in both countries.[173]

We can still calculate numerically some measures of *TFP* in subsection 3.3.2 at t_0 and at steady state. The results are also shown in table A3.2 and confirm that FGs *TFP* and *PTFP* in the North are both larger than in the South. Under the benchmark catching up term – and the first alternative catching up term – the differences are smaller in steady state, as a result of advantage being taken of the opportunities provided by imitation. At the level of industrial *PTFP* the difference increases with $n \in [0, 1]$. The aggregate productivity difference is not surprising given the productivity difference in all FGs.

[173] Equation (3.14) shows that they have a common slope of intra-country wage premia path in steady state.

Breaking up the results calculated, we confirm that they are consistent with Acemoglu and Zilibotti (2001) and that the origin of the productivity differences between countries does indeed derive from differences in (i) the exogenous level of productivity, (ii) the productive structure, (iii) the TK and (iv) the HC stocks.

3.5. Sensitivity analysis

Now, we check how the position of the world's steady state, the slope of the stable arm and the convergence speed rely on the model's parameters and initial conditions. In other words, we test the robustness of the results of transitional dynamics to shocks. We shall give the results of a sensitivity analysis of the steady state and the dynamics.

The results are obtained from numerical simulations in which one parameter at a time is allowed to deviate from its baseline value, and we can thus say nothing about where the economy would end up if all parameters were altered at the same time. The first general conclusion is that the model's qualitative behaviour is similar for the ranges of parameter values tested. In fact, similar stable saddle paths to steady state are obtained, differing only slightly in the specific levels of the steady state of the variables which they approach. In short, the framework is largely robust with respect to parameter changes.

The section is structured into two subsections, each with the sensitivity analysis for the baseline results of one country: the first for the North and the second for the South.

3.5.1. Sensitivity analysis to the baseline results of the North

Taking scenario 1 in subsection 3.4.4 as a reference, table A3.3 in subsection A3.1.3 in Appendix 3 shows the effects on steady state values when allowing one parameter at a time t_0 to vary from the lower to the upper bound of a given interval; *i.e.*, it contains qualitative results. It goes further and shows how sensitive the steady state is to specific parameter changes; *i.e.*, it contains quantitative results as well.

The results in the last column of table A3.3 are consistent with the findings in table 3.1 for the steady state growth rate. The economy grows faster in steady state, the higher the exogenous level of productivity in production, A_I, and in training, χ_S and χ_T, and the lower the depreciation rate of HC, δ. A great weight of the more efficient mode of training (schooling), φ^{VH}, also leads to a higher steady state growth rate.[174] In addition, an efficient R&D sector, mirrored in a large β_I and in a smaller q and so in a smaller α,

[174] The HC growth rate rises by improving the efficiency of the HC production or the intensity of the more efficient training activity. *I.e.*, less training effort is needed to achieve the baseline steady state growth rate.

also increases the steady state growth rate.[175] Finally, the more patient – *i.e.*, the smaller the value of ρ – and the less keen individuals are on consumption smoothing – *i.e.*, the smaller the value of θ – the higher the steady state growth rate.

The economy moves faster – *i.e.*, the speed of convergence increases – towards the steady state – see column 3, table A3.3 –, the higher A_I, β_I and ϕ are and the smaller α is; that is, the more efficient the productive side of the economy is in general.

Column 4 of table A3.3 shows that the specialised technology, ST, gains weight when A_I and β_I are higher and when α is smaller. As expected, the same occurs when it is relatively easier to accumulate *SH* – *i.e.*, when ϕ, φ^{VH} and χ_S decrease and when χ_T increases. Generically, improvements in the relative production of *SH* enhance its relative accumulation, thereby increasing its relative supply and lowering its relative wages which, in turn, decreases the relative price of specialised FGs – see subsection A2.3 in Appendix 2. In this case, TK is directed towards versatile TK and so the price channel operates in favour of *VH* – case A in generic figure A3.1 shown in Appendix 3, which sums up the results on the relation TK bias-versatile premium that tends to take place.

In short, in the baseline case the price channel operates in favour of the relatively scarce HC, *SH*; whereas in case A the strong reversal of the price channel effect redirects the path of the relative wage premium in favour of *VH* – see figure A3.1 in Appendix 3 and/or the overview of the effect on wages of a rise in *m*-type HC in Appendix 2. That is, in case A the price effect is strong enough to revert to the TK bias. A greater relative versatile price of FGs implies that there is a stronger incentive to develop versatile TK.

From columns 5-8 of table A3.3, when A_I is higher output is larger, there are more resources available for R&D, the expected profits for each successful research increase and individuals devote more time to work. This leads to higher *gr*, D_I^2 and D_I^5.

When α is small, the TK progress is strong. As a consequence wages increase, which discourage the use of HC at work. Thus, individuals devote more time to OJT and schooling. In sum, this increases *gr*, but leads to small values of $u_w(m)$, D_I^2 and D_I^5.

When β_I is high the incentive to innovate is strong and this decreases D_I^2 and D_I^5 – more resources are then spent on R&D and less remain for consumption. Conversely, D_I^2 and D_I^5 are high when there are improvements in HC accumulation – that is, when, φ^{VH},

[175] We have considered the particular case $q = 1/(1-\alpha)$. Moreover, we do not consider the parameter ζ_I in table A3.3, because its influence on steady state and the dynamics is just the inverse of β_I. Note also that a

χ_S and χ_T increase and δ decreases –, which lead to higher HC levels and so to higher wage earnings and, consequently, to higher consumption.

A high δ increases the training effort needed to keep up a certain level of HC accumulation. Specifically, a high δ decreases HC accumulation, which, in turn, decreases the supply of HC. This increases wages and so disincentives the use of HC at work; *i.e.*, u_w decreases. When φ^{VH}, χ_S and χ_T increase, the supply of HC increases as well. This reduces wages which, in turn, incentives its use in FGs production. In this case, however, the results suggest that in consequence of the greater HC stock, individuals need to devote less time to work. Finally, individuals train less in steady state, the keener they are on a smooth consumption path and less patient they are.

In brief, we can state that a slight alteration of parameters θ and ρ does not seem to have a very significant effect on the steady state and dynamics, whereas A_I, α, and β_I, on the other hand, have the greatest effects. The effects due to the increase of A_I are also similar to the use of a new GPT which is useful to all FGs at the same time in the North.

Parameter ϕ prompts more specific comments. When $\phi > 0$ the two types of training are substitutable (we have chosen $\phi = 0.9$ which implies that they are highly substitutable) and for $\phi < 0$ they are complements (we have chosen $\phi = -10.0$). Table A3.3 reports that steady state growth rate is not very sensitive to the value of the degree of substitution between the two modes of training,[176] contrarily to the occupational choice between HC accumulation and production and between schooling and OJT – see table 3.5 below –, to the speed of convergence and to the use of VT and ST.

When the two types of training are complementary, they are both indispensable to HC accumulation. Thus, the time spent in school and in OJT tends to be close: the time allocated to the more efficient factor (schooling) requires more time to be allocated to the less efficient factor (OJT). Hence, if complementarity is the rule in the economy, then by considering schooling as the only means of HC accumulation we have underestimated the growth of both *VH* and *SH*, since we ignore the strong importance of OJT.

When the two types of training are highly substitutable, *VH* accumulation is made through schooling. This is because schooling is more efficient and *VH* is school

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small A_I implies a weak incentive to innovate due to its influence on monopolistic profits.

[176] It is slightly greater when the two modes of training (schooling and OJT) are substitutes, since this allows for a more extensive use of the more productive mode of HC accumulation. Conversely, when they are complementary the less productive mode gains importance. The results obtained for the case in which they are complements are quite similar for very different values ϕ, such as -10 or -1000 or still more.

intensive, which motivates individuals to substitute one means of HC accumulation with another. However, the greater part of *SH* accumulation is made through OJT, since *SH* is relatively OJT intensive and this intensity dominates the differential at efficiency level between schooling and OJT. Moreover, the more favourable situation for *VH* accumulation implies 'versatile individuals' can replace production activities by (additional) training more than 'specialised individuals' can. If substitutability is the rule, then considering schooling the only means of HC accumulation would make the estimates for the *VH* accumulation correct, but not those for *SH* accumulation.

We conclude that, under substitutability, the effort in schooling and in OJT is very different in terms of accumulated HC, but this is not true under complementarity. Thus, our numerical simulations show that if the case of *VH* accumulation under substitutability is excluded, the schooling activity is not sufficient to measure HC accumulation and growth. This would provide a theoretical explanation for the mixed evidence of empirical studies on the relation between HC accumulation and growth – *e.g.*, Mincer (1993). We therefore show that the introduction of another type of training – in addition to school – may significantly modify the occupational choice in HC accumulation.

Table 3.5. Time share spent in training and at work in steady state

Parameter	Value	$u_{SI}(VH)$	$u_{SI}(SH)$	$u_{TI}(VH)$	$u_{TI}(SH)$	$u_{wI}(VH)$	$u_{wI}(SH)$
ϕ	−10.0	0.14	0.15	0.17	0.20	0.69	0.65
	0.90	0.40	0.03	0.00	0.19	0.60	0.78

Moreover, the steady state growth rate is not too different in the two cases, contrarily to the speed of convergence, which is more rapid under substitutability. In addition, under substitutability, VT is more important in the productive process because of the behaviour of *VH* accumulation – strong accumulation – together with the strict complementarity between types of HC and types of IGs.

We will further analyse the impact of government intervention. The choice of values for parameters, s_x, s_r, and ι_K in table A3.4 in Appendix 3 comes from the analysis in section 2.3, in subsections 2.5.3 and from reality. The IGs production subsidy, s_x, is chosen assuming that government fiscal policy can offset the distortions caused by monopoly power – *i.e.*, $s_x = (q-1)/q$. The R&D subsidy, $s_r = 0.30$, is in line with Bental and Peled (2002), and is thus smaller than $s_r = s_x$ as recommended by Segerstrom and

Zolnierek (1999), if $q > 1.43$. The value for assets income tax is given for two different cases: (i) supposing the usual tax fixed at around *0.35*; (ii) assuming the value zero.

As expected, we find that under government intervention there are strong benefits to consumption. Table A3.4 presents the variation in the initial level of consumption which leads individuals to be indifferent to whether there is governmental intervention or not. We observe that in cases (i) and (ii) individuals would require an increase in the initial level of consumption greater than *0.30* and *2.55*, respectively, to be willing to end government intervention and obtain the same level of consumption at $t = 27$ – remember that at time $t = 27$ the steady state is reached in a decentralised case.

3.5.2. Sensitivity analysis to the baseline results of the South

Table A3.5 in Appendix 3 contains the effects on steady state values of the TK gaps and the gaps in HC at work and on the speed of convergence when allowing one parameter, or the initial level of the variables $\tilde{Q}^{Jm}$ and $\tilde{m}(.|w)$, at a time, to vary from the baseline case, considering the benchmark catching up term. We also observe some differences between the three alternative catching up terms – see table A3.6 in Appendix 3.[177] The first general conclusion that emerges is that the small impact of parameters and variables change on the steady state occurs with the benchmark catching up term.[178]

We observe that improvements in the successful probability of imitation, due either to higher A_P, β_P, PE, $\overline{\sigma}_1$ or $\overline{\sigma}_2$ or to smaller α – which, remember, implies a smaller q – decrease the steady state TK gap between countries, since this is equivalent to reducing the cost of imitation. This makes the relative return on R&D investments large. Moreover, the speed of convergence increases with higher $\overline{\sigma}_2$, but decreases with higher A_P, β_P and PE and is not affected by changes in α and $\overline{\sigma}_1$. Furthermore, the impact on the speed of convergence towards the steady state of A_P, β_P, PE and α – parameters common to all catching up terms – is smaller with the benchmark catching up term.

A higher β_P, PE or A_P implies that at the end of the adjustment process the gap in HC at work increases. This is because under these situations the Southern TK progress increases, which, in turn, increases wages and, as a result, individuals devote less time to

[177] In order to observe the differences across the results under the three alternative catching up terms, the variations in parameters and initial conditions considered in table A3.6 are very small.

[178] Note that the effect on the steady state growth rate – defined by the North – was described in the last subsection. Thus, for example, here we consider that the parameters that are common to both countries (α, θ, ρ, δ, φ^{VH}, χ_S, χ_T and ϕ) only change in the South.

work. Since the South does not affect the world steady state growth rate, some exogenous change that narrows the TK gap tends to be compensated in some measure by the increase in the gap in HC at work. Otherwise, the convergence in levels between countries is strong. Thus, as a smaller Southern α decreases both the TK gap and the gap in HC at work, and strongly increases the convergence in levels between countries – note that here we consider that changes in α occur only in the South; if this was not so, the Northern growth rate would increase and the gaps could increase – see equation (3.8).

A very interesting issue here is to observe the Southern implications when it is either faced with different values for the threshold parameter *d*, or it begins with different TK gaps, or even with different gaps in HC at work.

Our simulations provide us with some evidence about the way *d* influences the Southern catching up. Considering particularly the two alternative catching up terms, the main finding is that the smaller the initial value of *d*, the smaller the TK gap will be in steady state – *i.e.*, the greater $\tilde{Q}^{Jm}$ – and the greater the gap in HC at work will be – *i.e.*, the smaller $\tilde{m}(.\,|\,w)$. Moreover, the incremental gain in terms of TK compensates the loss in terms of HC. The reason for this result is that the smaller the value of *d*, the greater the probability of successful imitation and the larger the TK imitation and Southern growth in transition. In such cases the speed of convergence to the steady state growth decreases.

The effects of changes in the initial gaps between countries are different. When the initial specialised TK gap or the gap in *SH* at work increases, it also leads to a greater steady state specialised TK gap. This is primarily due to the smaller opportunities afforded by imitation, mirrored in the smaller duration of the process of transition toward the steady state. This weakens the 'specialised side' of the South, in which the country is relatively stronger. On the other hand, when the initial versatile TK gap or the gap in *VH* at work widens, it weakens the 'versatile side'; *i.e.*, it strengthens the 'specialised side', in relative terms. Consequently, this leads to a longer duration towards the steady state and a smaller TK gap. Briefly, the convergence in levels between countries is greater the smaller the initial specialised TK gap and the gap in *SH* at work between them. Hence, sensitivity analysis shows that the steady state depends on the initial conditions.[179]

The less patient and the more keen individuals are on consumption smoothing the greater the steady state TK gap and the steady state gap in HC at work, and the smaller the transition phase towards the steady state.

Southern behaviour is also sensitive to changes in the parameters of the HC production function. Of these, the substitution parameter ϕ must be pointed out. When the two types of training are complementary (remember that here this is only true in the South – the North remains in the baseline case), the South reaches a relative higher level of both TK and HC at work, and needs less time to converge towards the steady state. The opposite applies when the two types of training are substitutable. Regardless of the catching up term, this occurs because the accumulation of *SH* is greater when the two types of training are complementary, since the share of time devoted to work is smaller and so more time is devoted to training – see table 3.6 below. It is also clear that the South benefits greatly from improvements in χ_T, since *SH* is OJT intensive.

Table 3.6. Share of time devoted to working under different values for ϕ

Value	$u_{wP}(.\|VH)$		$u_{wP}(.\|SH)$	
	$t=0$	t^{ss}	$t=0$	t^{ss}
−10.0	0.43	0.69	0.46	0.66

Value	$u_{wP}(.\|VH)$		$u_{wP}(.\|SH)$	
	$t=0$	t^{ss}	$t=0$	t^{ss}
0.20	0.44	0.64	0.47	0.72

Value	$u_{wP}(.\|VH)$		$u_{wP}(.\|SH)$	
	$t=0$	t^{ss}	$t=0$	t^{ss}
0.90	0.47	0.60	0.48	0.79

It is possible to verify that Southern government intervention can enhance the steady state TK gap – and thus inter-country wage inequality – and relative consumption, $C_P(t^{ss}|m)/C_I(t^{ss}|m)$.[180] Note still that the effects resulting from the increase in A_P are also in line with the use of a new GPT which is useful to all Southern FGs at the same time.

Looking for example at equation (2.32), we have it that the path of intra-country wage inequality in both countries is determined by the North when TK diffusion starts. In fact, from time t_0 on, FGs price indexes remain constant and $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ always grow at the same rate – *e.g.*, equation (3.62). Considering for example equation (2.31), we have it that inter-country wage inequality stays constant in steady state, but it is smaller than at time t_0, since the South-North TK gap narrows. Moreover, the steady state inter-country wage inequality is clearly smaller, when: (i) A_P is greater, since it directly reduces the gap and cuts the TK gap between countries; (ii) imitation capacity is better; (iii) schooling and OJT are complementary; and (iv) government intervention occurs.

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[179] This is also true in autarkic North and, as we will see in the next chapters, in the other IT regimes.

[180] For example, a simpler $s_x = 0.20$ implies that in steady state $\tilde{Q}^{JVH} = 0.64$, $\tilde{Q}^{JSH} = 0.75$, $\tilde{VH} = 0.37$, $\tilde{SH} = 0.43$, $C_P(.|VH)/C_I(.|VH) = 0.21$ and $C_P(.|SH)/C_I(.|SH) = 0.26$.

In summary, we can state that sensitivity analysis shows that changes of parameter values and initial conditions yield similar stable paths to the steady state, differing however in terms of the steady state levels of the variables which they approach.

3.6. Some elements of synthesis

Empirical growth literature suggests that R&D and HC accumulation are the two driving forces of endogenous growth. Following this proposal, we merge the Schumpeterian and the HC growth theories in the same framework, considering that there are two substitutable technologies of production. We believe that this structure is a technical improvement on the existing R&D-based endogenous growth models, since the usual comparative static results of Schumpeterian growth theory are still valid, while, in addition, enabling us to explain various other related facts.

In accordance with the empirical evidence against scale effects (and with our particular purposes), our framework exhibits endogenous steady state growth without scale effects. A typical prediction from the first-generation R&D-based endogenous growth models is that the size of an economy exerts a positive effect on its growth rate. The positive growth scale effect predicted by the theory appears, however, to be inconsistent with some recent empirical works. These works highlight that over the last forty years, there have been important increases in R&D activity in most of the major economies, yet there has been no increase in the rate of growth. Using time-series evidence, Jones (1995a), for example, has argued that these phenomena must lead one to the rejection of standard R&D-based growth models which incorporate scale effects.

Subsequently, a new class of growth models has been developed to rectify the problem. Nevertheless, there is a potential problem associated with the new models. The long run growth becomes semi-endogenous in the sense that TK change is endogenous but the long run growth rate is exogenous, as is the case in the neoclassical growth model. Moreover, little progress has been made in developing North-South models without scale effects – in addition to our model, the exceptions seem to be the recent works of Dinopoulos and Segerstrom (2004) and Sener (2003).

Even though we ignore the endogenous growth debate on scale effects, we can state that our growth model is consistent with the empirical observations of the absence of scale effects on growth and, in addition, the long run economic growth rate remains endogenous. Basically, the economic growth rate depends on (i) government policies –

as, for example, in Howitt (1999); (ii) individual preferences; (iii) FGs production technology; (iv) HC production technology; and (v) R&D technology.

As the economic growth rate is endogenous, government policies that affect the intensity of successful quality innovations, IGs production and assets income have permanent effects on the growth rate, as well as the consumer's subjective discount rate and the inverse of the intertemporal elasticity of substitution. It is interesting to observe that in order to improve the growth rate, the resources to subsidise IGs production and R&D must be obtained from an HC income tax. This is because the steady state growth rate is independent of the HC income tax.

The constant – common to both countries – world steady state growth rate without IT is commanded by Northern TK progress, and it is not affected by the South. This means that it is affected by changes in both exogenous parameters and variables in the North, but it is not affected by changes in the exogenous parameters and variables specific to the South. For example, improvements in A_I help the world steady state growth rate, but they widen, direct and indirectly,[181] the *TFP* gap between countries and the inter-country wage inequality; improvements in A_P do not influence the world steady state growth rate, but they do depress, direct and indirectly,[182] the *TFP* gap between countries and the inter-country wage inequality. To sum up, due to TK diffusion, it is TK progress in the North and (domestic) HC accumulation that drive endogenous growth.

As to the influence of HC production parameters on the steady state world growth rate, we observe that the more intensive the use of schooling, which is the more productive mode of HC production, the higher the steady state growth rate. In addition, improvements in the productivity of both modes of HC accumulation – schooling and OJT – also increase the steady state growth rate. The greater the substitutability between modes of HC production, the higher is the use of schooling, and thus the higher the growth rate. That is to say, the intensity of the use of the two modes of HC accumulation, the respective levels of productivity and the degree of substitutability between them, have a crucial impact on the steady state growth. This underlines the importance of the HC production function in fully accounting for the role of HC accumulation in growth.

Moreover, via FGs price indexes, the steady state growth is also affected by the structure of the HC at work. We are thus in agreement with the literature, which observes the influence of HC on growth by both channels, as a flow and as a stock. Thus, HC has a

[181] Indirectly because they increase the steady state TK gap between countries.

very significant implication for our understanding of the determinants of growth. Keeping in mind that in the South, the gap in HC at work between countries still affects its ability to imitate and implement TK advances. However, whereas in the literature about skill-biased technological change, the bias that causes wage inequality is mainly induced through the market size channel, in our framework it is induced through the price channel, which is, though, affected by HC.

Finally, we must highlight that unlike some of the non-scale models – *e.g.*, Jones (1995b) and Howitt (1999) – positive population growth is not a necessary condition for positive long run growth; and differently from Arnold (1998) and Blackburn *et al.* (2000), among others, growth depends not only on individual's preferences and HC accumulation technology, but also on vertical R&D activity.

Our framework is also consistent with the existing cross-country evidence on growth rates. This issue is related to conditional convergence between some countries (upper and middle-income) and divergence between others (richest and some poorer countries). In other words, it is in accordance with the existence of 'club convergence', since the latter is restricted to a subgroup of countries, namely those who are able (and willing) to invest in R&D imitation activity. We can thus provide a theoretical explanation for the divergence in output that has occurred between some countries, as well as for the convergence that has taken place between others. The argument is thus based on the premise that the process of TK change undertaken in the North can only be easily appropriated by countries with at least some minimum level of development, relative to the TK frontier. Countries not fulfilling this requirement remain stagnant.

Consequently, unlike the neoclassical growth model, our model predictions are pessimistic for countries that are a long way from the TK frontier. Moreover, in contrast to the neoclassical model, where convergence is entirely driven by the diminishing return on capital, convergence in our framework is completely driven by TK diffusion.

As the Southern imitator country grows at a higher rate during the transition to the world steady state, the inter-country income inequality and, thus, the inter-country wage inequality decreases during this period. However, our assumptions with regard to exogenous levels of productivity related to institutions, As, and to the experience-adjusted probabilities of successful research, Φs, tend to keep the North as the TK leader,

[182] Indirectly because they decrease the steady state TK gap between countries.

and so its income level will always be higher than that of the South. In sum, there tends to be convergence in growth rates but not in levels – *i.e.*, conditional convergence occurs.

According to our framework, after a certain stage of development, (efficient) Southern innovation activity is necessary to ensure further growth (during a new process of transitional dynamics towards the steady state) and more convergence, since growth and convergence are limited in the imitative regime. Strictly speaking, more convergence is also possible due to some exogenous improvement in the imitation capacity of the South, which can be obtained by, for example, free IT. Thus, our framework agrees with the evidence in that it suggests that absolute convergence only takes place within groups of homogenous countries, which operate in similar institutional, legal and economic environments with the same technologies and tastes – *e.g.*, Quah (1996, 1997).

The distribution of the world income is then attributable to the existing *TFP* differences between countries. In particular, we observe that in steady state, both cross-FGs and aggregate *TFP* are large in the North relative to the South, because of a differences in the exogenous levels of productivity, As, available TK, Q^{Jm}s, HC composition, m, and differences in productive structures, Ns.

International TK imitation is then a window of opportunity for (some) developing countries, namely those which can imitate. It is also true that, in steady state:

(i) The productive structure and relative prices tend to be constant. In addition, the productive structure, relative prices and growth in the North are not influenced by the South, rather, the inverse is true. In particular, the path of intra-country wage inequality is defined by the North (at time t_0, when TK diffusion starts, and from this time on).

(ii) The TK and relative wages have steady state dynamics.

(iii) There is a Schumpeterian factor-price equalisation result between the North and the South, since the growth rates of wages converges between these countries. This is because their TK indexes grow at the same rate, owing to international TK diffusion (and not to IT, which does not exist in the present chapter).

(iv) Wage levels between the North and the South do not converge completely due to differences in exogenous levels of productivity, due to differences in available TK and due to the international immobility of the HC. However, since the South grows faster during the transition to the world steady state, there is some convergence in wages.

We extend the developed framework to the world implications of a new GPT. Basically, we show its consequences on world growth, levels, versatile premium,

structure of production and *TFP* differences into two different, but possible, scenarios: (i) when the new GPT is beneficial to the production of all FGs at the same time in each country (and first in the North), and (ii) when first the new GPT is beneficial to the production of some FGs in each country (and again first in the North).

In the first scenario, there are three distinct phases, which can be quite long. In the first, there is a growth effect in the world, due to the improvement in the productivity of schooling and OJT activities. In the second, the new GPT is used in the North, thereby stimulating a new growth effect. Furthermore, both the level of the output in the North and the *TFP* differences between countries increase. In the third, the new GPT is also used in the South. Then, its output level increases and the *TFP* differences decrease.

In the second scenario, there are five distinct phases, which can also be long. In the first, there is no difference relative to the last scenario. In the second, the new GPT is used only in VT in the North. This leads to a new world growth effect. In the North, there is also a level effect, the versatile premium increases and the threshold FG decreases. The *TFP* differences, TK gap and inter-country wage inequality increase. In the third, the new GPT is used in the production of all FGs in the North. Here again there is a new world growth effect. In the North, a new level effect arises, but now the versatile premium decreases and the threshold FG increases. The *TFP* differences, the TK gap and the inter-country wage inequality grow again. In the other two phases, the new GPT is introduced in the South in VT and in ST, respectively. Then, there are no world growth effects. Level effects occur in the South and the behaviour of the *TFP* differences, TK gap and inter-country wage inequality is favourable to this country. The conduct of the versatile premium and of the threshold FG in phases 4 and 5 in the South is equal to what occurred in phases 2 and 3 in the North, respectively.

We also speculate on the effects of a shock that increases the versatile-augmenting parameter *vh*. We can tell that it has a similar impact to that which occurs with the use of the new GPT in VT. The main difference results from our assumption that *vh* is the same in both countries. Thus, this shock generates a world growth effect. It induces a positive level effect, increases the versatile premium and decreases the threshold FG, also in both countries. Consequently, a shock in *vh* could be an additional explanation for the increase in versatile premium, but in both countries at the same time.

To better understand the behaviour of our framework, we have solved it numerically for reasonable parameter values using a fourth-order Runge-Kutta classical

method. The baseline parameter values were chosen based on the empirical literature and on our theoretical assumptions. Basically, considering the equilibrium in the labour market, free entry condition into R&D, individual utility maximisation with individual optimal time allocation, we are able to characterise the transitional dynamics in both countries. The time path of the system in each country converges to the steady state and stays there without any exogenous perturbation; *i.e.*, the adjustment processes are stable.

Without IT effects the North is unaffected by the South and rapidly reaches the steady state. Moreover, growth is at first driven by HC behaviour and then by R&D.

Putting it simply, in the transitional dynamics analysis of the South, we have considered that the North is already in steady state. The South converges immediately to a unique stable country specific growth path, but its adjustment process only ends when it reaches the (world) BG path established by the North. In a normal situation, it achieves higher growth rates during the transitional dynamics phase due to the TK backwardness. That is, rates of imitation are greater than rates of innovation out of the steady state, since the probability of successful imitation depends on both the South-North TK gap and gap in HC at work. Furthermore, convergence is at first driven by Southern HC behaviour and then by Southern imitation. As stated, this makes our framework similar to the neoclassical model, in the sense that in our case the decreasing probability of imitation is analogous to the diminishing returns on capital in the neoclassical model.

We confirm numerically that capital gains in the South do not exist in steady state, being only present in transition, and that the level of consumption, wages and *TFP* – in FGs and in aggregate terms – increase vis-à-vis the North. An increase in the relative welfare is thus observed. However, in steady state, wage and *TFP* differences remain between countries. Moreover, we verify that, after t_0, the TK diffusion has no influence on the versatile premium between countries or, rather, due to TK diffusion, the Northern path of intra-country wage inequality also occurs in the South. However, in our baseline case, the versatile premium level is greater in the South.

On the whole, our framework does in fact provide a dynamic argument from the point of view of the Southern imitator country in favour of TK diffusion by imitation, and this seems quite consistent with reality. The catch up pattern of TK resulting for the South – under a reasonable catching up term – seems to be in line with the experience of recently industrialised countries, such as Brazil, Greece, Ireland, Portugal, Spain, Thailand and Turkey, which had a fast initial rate of catch up that then slowed down.

Further, sensitivity analysis shows that the model behaves in a qualitatively similar way for the ranges of parameter values tested. In fact, similar stable saddle paths to the steady state are obtained, differing only in the specific levels of the steady state of the variables which they approach. We confirm that the North (and so the world) grows faster in steady state: (i) the higher the exogenous levels of productivity in the production function, which also increases the speed of convergence, and in the training sector; (ii) the more intensive is the use of the more productive mode of HC accumulation; (iii) the lower depreciation rate of HC; (iv) the more efficient R&D sector, which also moves the economy faster towards the steady state; and (v) the more patient and the less keen individuals are on consumption smoothing.

The steady state growth rate is relatively insensitive to the value of the substitution parameter in the HC production function. The opposite occurs with respect to the speed of convergence, which is greater under substitutability, as well as the importance of VT. Also the occupational choice between HC accumulation and production and between schooling and OJT is very sensitive to this parameter. For example, under substitutability the effort in schooling and in OJT is very different according the HC accumulated. Thus, the schooling activity tends not to be sufficient to measure HC accumulation and growth, and this could be a theoretical explanation for the mixed evidence of empirical studies on the relation HC accumulation-growth. This suggests that the inclusion of another type of training (OJT) in addition to schooling in HC accumulation function is in order.

We further verify numerically that the path of the versatile premium can be decreasing or increasing. The former occurs in our baseline case, where the price channel operates in favour of the relatively scarce HC, *SH*. The latter arises if ST is strongly used in the North. This takes place when A_I and β_I are higher, when α is smaller and when it is relatively easier to accumulate *SH*. Thus, in particular, intra-country wage inequality is more likely to prevail when changes in Northern production of HC relatively enhance the accumulation of the type of HC that is relatively abundant in the South. If so then the relative price of specialised FGs is small and the TK is directed to the versatile TK; *i.e.*, the price channel is strong enough to revert to the TK bias and so works in favour of *VH*.

Finally, we confirm that government intervention generates strong benefits at the level of consumption in the North.

When we consider the sensitivity analysis in the South, we confirm that changes in parameters which improve the successful probability of imitation narrow the TK gap

between countries and the inter-country wage inequality. Specifically with respect to the increase in the TK gap threshold – *i.e.*, a decrease in d –, our simulations suggest that the incremental gain in terms of TK compensates the loss in terms of HC, and that the speed of convergence to the steady state growth decreases.

It is also apparent that the effects of changes in the initial gaps between countries are distinct. If the initial specialised TK gap or the gap in *SH* at work rises, a greater steady state specialised TK gap occurs. This is due to the diminished opportunities afforded by imitation, reflected in the less time taken by the transition process towards the steady state. However, if the versatile TK gap or the gap in *VH* at work widens, it leads to a slower speed of convergence towards the steady state and a narrower TK gap.

The less patient and the more keen individuals are on consumption smoothing, the greater the TK gap and the gap in HC at work are in steady state, and the smaller the time taken towards the steady state. Southern behaviour is also sensitive to changes in the parameters of the HC accumulation technology. Thus, if the two types of training are complementary, the South reaches a higher level of both TK and HC at work, and needs less time to converge towards the steady state. The opposite applies when the two types of training are substitutable. It is also clear that the South benefits considerably from improvements in the productivity of OJT. Finally, government intervention can improve the TK gap and its consumption relative.

In the next chapters, we extend our framework to IT. In chapter 4, we consider that only IT of IGs is possible and, in chapter 5, IT is totally free. We follow again the steps taken in the previous chapters. Thus, first we briefly model the economies, underlining the differences between IT regimes. For instance, the IT of IGs by allowing immediate international access to the state-of-the-art IGs, must strongly affect the productive structure, prices and wages in the South, bringing about partial convergence to the North. Furthermore, the discussion of international IPRs – in addition to the domestic ones – can be done, as well as the analysis of the resulting world steady state – growth, prices and wages –, *TFP* differences, implications of a new GPT, transitional dynamics and sensitivity analysis. Chapters 4 and 5 differ since the dynamic growth effects of IT operate through the IT of IGs, where R&D is directly applied, while the effects of IT on FGs are limited to the somewhat more traditional – in IT theory – effects, such as the pattern of FGs specialisation of the economies.

CHAPTER 4

NORTH-SOUTH TECHNOLOGICAL KNOWLEDGE DIFFUSION WITH TRADE OF INTERMEDIATE GOODS

4.1. Overview

So far we have considered TK diffusion without IT effects. In this chapter we introduce IT of IGs, which is the relevant trade since R&D is directly applied in IGs.

This brief overview highlights some particularities induced by the IT of IGs regime, in addition to the respective draft already presented in the first chapter.

IGs embody TK progress and here we assume that they are traded without any trade costs.[183] Contrarily to the Heckscher-Ohlin-Samuelson (HOS) model, where the focus is on goods markets and IT is driven by differences in factor endowment proportions between countries, our focus here is on R&D activity, and IT is driven by differences in TK between countries. That is, a dynamic version of the static Ricardian model directly explains the IT of IGs. However, in our case, (dynamic) factor levels are also present. They affect R&D and so IT. Thus, a dynamic version of the static HOS model is also considered. In addition, R&D activity is affected by the adjustment terms and the versatile and specialised augmenting parameters in the production function. Since they affect the HC productivity, we can go further by stating that a dynamic version of the static Ricardian model is also indirectly included.

We maintain the assumption that there are two countries in the world, the Northern innovator and the Southern imitator. As a result, the South is again sufficiently close to the TK frontier. That is, its specialisation is again in the R&D imitative activity,[184] since its best R&D results are imitations of the North's innovations – as, for example, in Grossman and Helpman (1991a, chaps. 11-12).[185] Thus, once again, the North never has an incentive to imitate, since there is never foreign (state-of-the-art) TK to imitate.

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[183] The effects of IT costs are considered by, for example, Dinopoulos and Segerstrom (2004).

[184] Concerning this, Kind (2004) has developed a more general North-South trade model since Southern firms choose whether to conduct imitative R&D, innovative R&D or specialise in agricultural production.

[185] Hence, under the IT of IGs, if the South is far away from the Northern TK frontier and/or if the Southern *MC* of production is greater than or equal to the Northern *MC*, then, for lack of resources, it is not able to import IGs. As a result, there is no possibility of IT of IGs and no possibility of commercial

Without IT effects, countries produce only for their domestic market and we have assumed that the *MC* of production is equal in both. Under IT of IGs, they produce IGs for the world market and we presume that, for historical reasons, the *MC* of production is greater in the North than in the South. Specifically, we assume that the price of the composite FG in the North is always the numeraire, in the measure that we have it that $1 = MC_I > MC_P > 0$. This will be crucial for lower-priced imitation; *i.e.*, so that Southern firms that support R&D imitative activity can produce and export state-of-the-art IGs.

As stated in chapter 1, in order to achieve our objective, we further assume that: (i) countries are starting from the previous position, in which the North is in its autarkic steady state; (ii) the IT of IGs between countries is always balanced and, thereby, international capital flows do not occur – *i.e.*, the domestic interest rate is determined domestically –; and (iii) international mobility of HC is not possible as well.

The South can now immediately import higher quality Northern IGs for use in its FGs production and so it is no longer limited by its own ability to produce IGs.[186] That is, the South will import IGs that have not yet been imitated since they have better quality, and export state-of-the-art IGs that it has imitated by under-pricing the innovator leader firms. Therefore, both countries will switch to incorporating imitated IGs in their FGs production whenever they are available, and IT reflects thus the differences in TK between them. Hence, TK transfers take place either through the Southern production of IGs with the best quality after successful imitation or through the importation of higher quality Northern IGs. Consequently, our framework is now a North-South IT model where the location of IGs firms can change over time. In each IG, the new quality is initially produced in the North by the quality leader firm but then, when imitation occurs, production shifts to the South – *i.e.*, an IGs-cycle IT model results.

There is an immediate level jump in Southern output when IT of IGs starts – *i.e.*, IT of IGs confers a static benefit or level effect and this is the first effect of IT. This favour HC employed in the South by improving its productivity and induces some convergence in prices and in the pattern of FGs productive structure between countries. Thus, inter-country wage inequality decreases and, in addition, intra-South wage inequality changes as well. Furthermore, from this time on, the world growth rate will be driven by Northern

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relations between countries. If so, TK divergence must occur between countries. We do not analyse this hypothesis.

[186] In this case, we could suppose that a learning-how-to-use advanced TK cost exists but for the sake of simplicity, it will not be taken into consideration.

TK progress and by domestic HC accumulation. However, although the TK in the North is in the lead, it is affected by the South in two ways: (i) demand for IGs depends also on the Southern HC structure due to complementarity between IGs and HC inputs in the production function, and (ii) incentives to innovation depend on imitation due to the 'feedback effect' mirrored in international limit pricing.

When the two countries open up to IT, their IGs leader firms – the ones that produce according to the latest patent in each IG – will be able to sell to a larger market. However, the difficulty in conducting R&D increases because of higher scale costs. We consider that the two effects practically counterbalance each other. In addition, Northern IGs leader firms are concerned with the joint probability of losing their markets either due to the next Northern innovation or due to a lower-priced Southern imitation.

At first sight, the probability of successful imitation is now potentially better since the effect of the 'catch up theory' seems to be stronger. Indeed, the catching up term now includes the effect of openness to IT of IGs, which potentially improves the Southern imitation capacity. Accordingly, the expected profits for each Northern leader IGs firms tend to be now reduced, which can induce innovator firms to support R&D more slowly, thereby decreasing the steady state growth rate. However, the greater the probability of imitation, the faster Northern firms will need to obtain the next successful innovation in order to regain the world market. Moreover, since they produce IGs over less time, less Northern resources are spent in IGs production and, thus, more resources are available to support R&D. In short, there is a trade-off between the time in leader positions and between successful innovations: Northern leader IGs firms potentially remain less time in the market, but the time between successful innovations is also potentially smaller.

In the same way, Southern IGs firms – which support R&D imitative activity – are driven out of the market by a new successful innovation and so their expected profits depend on the probability of successful innovation. There is therefore a 'feedback effect' between Southern imitator and Northern innovator firms, which compete for the world market. This allows us to state that the IT of IGs induces a dynamic interaction between countries, which potentially drives to a higher steady state world growth rate. In fact, when the probability of successful imitation increases due to the IT of IGs – *i.e.*, the speed of TK diffusion increases –, it improves the ability of the South to benefit from innovations taking place in the North and generates a better efficient allocation of production in the world, since it becomes located in the South where *MC* is lower. This,

together with the initial level effect induced by the IT of IGs, feeds back into the North, affecting its TK through creative destruction and so the steady state world growth rate.

We can also state that there is a trade-off involving the South-North TK gap. Southern IGs firms must decide whether to simply stay out of the market or, let FGs firms import IGs initially, and then support the R&D imitative cost in order to obtain the TK necessary to produce the state-of-the-art Northern IGs. As referred above, potentially the probability of successful imitation is now higher than without IT effects. However, the IT of IGs forces Southern IGs firms to support the R&D imitative cost of state-of-the-art Northern IGs possibly several quality rungs above their own experience level.[187]

Since the Southern firms must support R&D imitative activity of all quality levels between their current domestic level and the Northern state-of-the-art IG that they are targeting before they earn profits, the total resources required to bring about successful imitation are also potentially greater than before. Thus, on the one hand, a large TK gap[188] favours the probability of successful imitation since the backwardness benefits are higher, but, on the other hand, Southern IGs firms need to spend more resources in order to support the R&D imitative activity of a large number of quality grades.[189]

Looking at wage inequality, we observe that much of the literature available has focused on measuring the influence of TK bias and IT on growing intra-country wage inequalities. The early estimates reveal a critical impact from TK change – *e.g.*, Bound and Johnson (1992), Levy and Murnane (1992), Katz and Murphy (1992), Juhn *et al.* (1993), Berman *et al.* (1994) and Berman *et al.* (1998) – and a rather weak influence from IT – *e.g.*, Katz and Murphy (1992) and Krugman and Lawrence (1994). Generally speaking, the effect of the use of a new GPT on versatile premium, as presented in phases 2 and 4 of the 2nd scenario of subsection 3.3.3 in the last chapter for the North and the South, respectively, is a theoretical explanation in line with this empirical literature.

This diagnosis has been challenged for several reasons. First, the empirical methods used in estimations were controversial – *e.g.*, Leamer (1994) and Wood (1995).

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[187] Without IT effects, the Southern IGs firms need only support imitative R&D to obtain TK up to one quality rung above the current Southern quality level, since they only sell the imitated IGs domestically.

[188] But smaller than the threshold TK gap in order for imitation to be possible.

[189] We can further imagine the extreme case in which all Southern IG firms had already imitated Northern leader IGs prior to opening to IGs free IT. Then, with the IT of IGs, Southern firms capture the IGs world market due to their *MC* production advantage. In each IG, the next innovator firm replaces the actual imitator firm, and Southern R&D imitative activity immediately seeks to imitate this new one. In this extreme case, the aggregate South-North quality level ratios will remain approximately constant and equal to *1* and, thus, there is no transition path. In any case, formally the steady state is always obtained using the equilibrium probabilities of innovation and imitations, following the same reasoning.

Second, TK bias was exogenous. Third, new estimates invalidated the first results – *e.g.*, Wood (1994). Finally, the interpretations are based on a disputable *a priori* – *i.e.*, TK bias and IT are independent of each other. To respond to these criticisms, the fundaments of the traditional explanations must be reconsidered. We believe that it is crucial to endogenise the TK change and integrate both explanations into a unified framework so that IT affects the shape of TK change. In this line, recent estimates obtained by Morrison and Siegel (2001) show that openness stimulates skill biased TK change. Thus, at least one part of the impact of TK demonstrated in the early estimates should be attributed to the IT of IGs.

The latter can account not only for accelerating TK change, but also for change in wage inequality (and not only at intra-country level). In fact, openness to the IT of IGs induces more versatile-biased Northern TK progress than under the autarkic North, by increasing the incentives to develop new TK that is complementary to *VH*. Moreover, increased wage inequality also takes place and with greater intensity – due to the level effect – in the South when the IT of IGs starts, in accordance with recent empirical literature, which finds that the demand for skills have increased in less developed (South) countries – *e.g.*, Hanson and Harrinson (1999a, b), Berman and Manchin (2000) –, but in opposition to the standard (trade) theory prediction.

In short, the IT of IGs affects both HC returns and inequality. The former is affected by induced growth effects in both countries and also by the level effect in the South. Removing the scale effects, intra-country wage inequality depends on the North-South average relative price of versatile FGs (price channel), which is, though, higher than the one in the autarkic North. More specifically, intra-country wage inequality is affected by the direction of the TK bias resulting from differences in prices of FGs in both countries and also by the level effect in the South. Inter-country wage inequality decreases due to the level effect and from then on remains constant.

The debate between the North and the South on the enforcement of international IPRs has become a problem of major concern. For this reason, according to the terms of the Uruguay Round of General Agreement on Tariffs and Trade completed in 1994, less developed countries are required to strengthen IPRs protection, a measure that resulted in the trade-related IPRs (TRIPRs) protection. Thus, interestingly, at the same time that 'globalisation' has been occurring, developed countries have been pushing for stronger international IPRs protection. The immediate question raised is: what are the long run

repercussions of this standing on the global rate of TK progress, on TK diffusion and on inter and intra-country wage inequality? We will attempt to answer this question.

We will also explore some extensions of the model under the IT of IGs regime.

First, we analyse both the cross-FGs and the aggregate *TFP* differences between countries in steady state. By allowing immediate access to state-of-the-art IGs, IT of IGs affects the productive structure in the South, bringing about partial convergence to the Northern structure and prices. Nevertheless, even when the countries have access to the same TK, differences in exogenous productivity and in HC generate differences in *TFP*. Thus, even in steady state, the cross-FGs *TFP* and aggregate *TFP* must be larger in the North, but to a lesser extent than in the previous IT regime without IT effects.

Second, we analyse the effects of the use of a new GPT. Now, the South is no longer limited by its own ability to produce the best quality IGs. Thus, all phases begin at the same time in both countries. Moreover, in view of the analysis in the last IT regime, we expect that: (i) if the new GPT is used in the production of all FGs at the same time, there is an initial growth effect, which is increased in the next phase when a level effect also occurs; (ii) if, at first, the new GPT is used in the production of versatile FGs, there are three distinct phases. In the 1st, a small growth effect occurs. In the next phases, there are both level and growth effects. In addition, the versatile premium increases in the 2nd, but decreases in the 3rd and the opposite occurs for the threshold FG.

We discuss briefly the effects of an increase in the versatile-augmenting parameter in the production function. It generates both a positive level and growth effects, increases the versatile premium and decreases the threshold FG. In other words, it induces the same type of effects as introducing the new GPT in the VT only.

Off the BG path, the TK gap between countries changes and both observe transition paths. These transition paths are dependent on the relative TK levels and on the relative levels of HC employed and, consequently, on prices of FGs in both countries. Thus, the North is clearly affected by the South, and *vice-versa*. During the transitional dynamics, the difference in prices between countries, even after the initial (static) impact, induces a directed TK effect, which affects the versatile-specialised TK bias world measure and, thus, the versatile premiums as well as the inter-country wage inequality.

Using once more the fourth-order Runge-Kutta classical numerical method, we solve our framework numerically for set baseline parameter values and set initial conditions. Due to the feedback effect between countries, now when one country is out of

the world steady state, which is influenced by the action of both, so is the other country. Thus, to analyse the stability of and convergence to the world steady state, the countries must be considered together. For the sake of simplicity, in the present chapter we will only consider the benchmark catching up specification. Next, we proceed by testing the robustness of the results to changes in parameters – *i.e.*, with the sensitivity analysis.

The remainder of the chapter is organised as follows. Sections 4.2 and 4.3 describe the FGs and the IGs sectors, respectively. Section 4.4 characterises the domestic equilibrium given the state of the world TK and the stock of HC. Section 4.5 describes the alterations resulting from the IT of IGs on the R&D sector. Section 4.6 analyses the steady state: growth rate, comparative analysis and related effects. Section 4.7 presents the extensions: *TFP* differences between countries and consequences of a new GPT. Section 4.8 investigates the transition to and the stability of the steady state. Section 4.9 performs a sensitivity analysis. Section 4.10 concludes with some elements of synthesis.

4.2. Final goods sector

Since there is no IT of FGs, each country maintains production of all of them. Once again, the production of perfectly competitive FGs uses specific IGs and specific HC. Thus, the production function of the nth FG at time t in country Z is,

$$Y_{nZ}(t) = A_Z \left\{ \left[\int_0^{J^{VH}} \left[q^{k(j,t|VH)} \underbrace{\left(x_{nZ}(k,j,t|VH,I) + x_{nZ}(k,j,t|VH,P) \right)}_{\equiv x_{nZ}(k,j,t|VH)} \right]^{1-\alpha} dj \right] \left[n\, vh\, VH_{nZ}(t|w) \right]^{\alpha} + \right.$$
$$\left. + \left[\int_0^{J^{SH}} \left[q^{k(j,t|SH)} \underbrace{\left(x_{nZ}(k,j,t|SH,I) + x_{nZ}(k,j,t|SH,P) \right)}_{\equiv x_{nZ}(k,j,t|SH)} \right]^{1-\alpha} dj \right] \left[(1-n)\, sh\, SH_{nZ}(t|w) \right]^{\alpha} \right\} \quad (4.1)$$

Once more, (i) the 1st and 3rd terms within square brackets sum up the contributions of the two types of IGs, while the 2nd and 4th represent the role of the specific HC inputs; (ii) parameters $\alpha \in]0, 1[$ and $(1-\alpha)$ are the HC and the quality-adjusted IGs shares in production, respectively; (iii) the term A_Z is a positive exogenous variable representing the level of productivity, dependent on the country's domestic institutions and $A_P < A_I$.

The HC terms include the quantities employed in the production of the nth FG – $VH_n(.|w)$ and $SH_n(.|w)$ – and two types of corrective factor accounting for differential

productivities. An absolute productivity advantage of *VH* over *SH* is accounted for by the joint parameters *vh* and *sh*, since $vh \geq sh \geq 1$. A relative productivity advantage of either type of HC is captured by the adjustment terms n and $(1-n)$. These terms transform the FGs index n in a relevant ordering index: this means that *VH* is relatively more productive in producing FGs indexed by larger *n*s, and *vice-versa*.

Each of the two IGs terms includes an adjustment for quality that reflects a stylised TK change process of the quality ladder type. As stated, q and k are the size of each quality upgrade obtained with each successful research – exogenously constant greater than one – and the rungs of the quality ladder, respectively. More specifically, we consider that q is sufficiently large, such that $q > MC_I/MC_P$. As will be clear in the next section, in this case: (i) an IG firm that supports R&D in the North captures the world market with a single quality level improvement over the IG firm in the South that produces according to the latest lower-priced imitation; and (ii) an IG firm that supports R&D in the South captures the world market by imitating the IG leader firm, since, due to lower *MC*s, it can under-price the Northern IG leader firm. Thus, production will shift to the South, whenever a successful imitation of a Northern leading quality takes place.

Production function (4.1) already cares about that the quality grades within an IG j are perfect substitutes as inputs to production and, as we will see in the next section, that, owing to profit maximising limit pricing by the monopolist producers, only the highest quality available of each IG is in fact used in the production of FG n in country Z. Thus,

$$\sum_{k=0}^{k(j,t|m)} q^k \left[x_{nZ}(k,j,t|m,I) + x_{nZ}(k,j,t|m,P)\right] \Rightarrow q^{k(j,t|m)}\left[x_{nZ}(k,j,t|m,I) + x_{nZ}(k,j,t|m,P)\right]. \quad (4.2)$$

Equations (4.1) and (4.2) take into account that the quantity of the quality rung k of the IG j used in the production of FG n in country Z, together with domestic m-type HC, can be produced by either the North, $x_{nZ}(k,j,t|m,I)$, after a successful innovation or by the South, $x_{nZ}(k,j,t|m,P)$, after a lower-priced successful imitation of the leading Northern quality level. Therefore, the quality level of IGs rises exclusively due to the R&D innovative activity – and the size of quality improvements are large –, and so do FGs output in both countries. However, both countries use the state-of-the-art IGs in their FGs production; *i.e.*, $k = k_I \geq k_P$, which can be produced domestically or not. In the latter case, countries immediately import the higher quality IG for domestic use.

We are in the presence of a Vernon-type IG cycle with production shifts from the North to the South with successful imitation and back with next successful innovation. IT directly reflects the differences in TK between countries, due to their R&D activities, and not only their factor level proportions,[190] as in the HOS model. Based on the size of the TK gap in each IG, there will be three types of IG firms in the world owing to the static Bertrand price equilibrium – *e.g.*, Grossman and Helpman (1991a, chap. 12) and table 4.1 in the next section: (i) Northern IGs firms facing Northern competition, since the quality one step below in the quality ladder has not been copied; (ii) Northern IGs firms facing Southern competition, since the quality one step below in the quality ladder has been copied; and (iii) Southern IGs firms facing Northern competition.

Let $\Xi(m)$ and $[1-\Xi(m)]$ represent the proportion of IGs with production in the North and in the South, respectively. Within $\Xi(m)$, let $\gamma(m)$ and $[1-\gamma(m)]$ represent the proportion of IGs with production in the North that face imitator competition – *i.e.*, imitator TK is less than two quality levels behind – and those that do not – *i.e.*, imitator TK is more than two quality levels behind –, respectively. In the next section, we formalise both $\Xi(m)$ and $\gamma(m)$. Given the IGs life cycle, equation (4.1) can be written as:

$$
\begin{aligned}
Y_{nZ} = A_Z \Bigg\{ \Bigg[\int_0^{J^{VH}} \Bigg[q^{k(j|VH)} \Bigg[\Xi(j|VH) \Big(\gamma(VH)\, x_{nZ}(k,j|VH,I) + [1-\gamma(VH)]\, x_{nZ}(k,j|VH,I) \Big) + \\
+ [1-\Xi(j|VH)]\, x_{nZ}(k,j|VH,P) \Bigg] \Bigg]^{1-\alpha} dj \Bigg] \Bigg[n\, vh\, VH_{nZ}(t|w) \Bigg]^{\alpha} + \\
\Bigg[\int_0^{J^{SH}} \Bigg[q^{k(j|SH)} \Bigg[\Xi(j|SH) \Big(\gamma(SH)\, x_{nZ}(k,j|SH,I) + [1-\gamma(SH)]\, x_{nZ}(k,j|SH,I) \Big) + \\
+ [1-\Xi(j|SH)]\, x_{nZ}(k,j|SH,P) \Bigg] \Bigg]^{1-\alpha} dj \Bigg] \Big[(1-n)\, sh\, SH_{nZ}(t|w) \Big]^{\alpha} \Bigg\}
\end{aligned}
\tag{4.3}
$$

Since the market for FGs is competitive, the representative producer of FG n takes the prices of their products, the price paid for (a unit of) m-type HC and the price of the IGs used as given and maximise profits. However, now there are three groups of IGs firms and, thus, there will also be three different limit prices for IGs to prevent sales of older TK on the world market – see table 4.1. Let $p(j|m,I,I)$ represent the limit price of IG j that is manoeuvred by m-type HC with production in the North, I, which competes with other Northern firms. Then, in addition, there is the price of Northern firms that face

[190] *I.e.*, IT of IGs depends on (dynamic) TK progress in the Ricardian tradition, and on (dynamic) HC and

competition from Southern firms, $p(j|m,I,P)$, and the price of Southern firms always facing competition from Northern firms, $p(j|m,P,I)$; *i.e.*, $(Z,Z)=\{(I,I),(I,P),(P,I)\}$.

The zero profit equilibrium of the (constant returns to scale perfectly competitive) FGs producers – *i.e.*, the FOCs of the maximisation problem – yields the demand for (the highest quality of) each IG by the representative producer of the n[th] FG in country *Z*:

$$x_{nZ}(k,j,t|VH,Z,Z) = n\ vh\,VH_{nZ}(t)\left[\frac{p_{nZ}(t)\,A_Z\,(1-\alpha)}{p(j,t|VH,Z,Z)}\right]^{1/\alpha} q^{k(j,t|VH)\left[\frac{1-\alpha}{\alpha}\right]}; \qquad (4.4a)$$

$$x_{nZ}(k,j,t|SH,Z,Z) = (1-n)\,sh\,SH_{nZ}(t)\left[\frac{p_{nZ}(t)\,A_Z(1-\alpha)}{p(j,t|SH,Z,Z)}\right]^{1/\alpha} q^{k(j,t|VH)\left[\frac{1-\alpha}{\alpha}\right]}. \qquad (4.4b)$$

In equations (4.4a, b), $p_{nZ}(t)$ is the price of FG n in country Z. From these equations, the demands of IGs manoeuvred by each m-type HC of the representative FG firm n in country Z are – to simplify we omit both country and time indexes:[191]

$$x_n(k,j|VH) = n\ vh\,VH_n\left[p_n\ A\,(1-\alpha)\right]^{1/\alpha} q^{k(j|VH)\left[\frac{1-\alpha}{\alpha}\right]}\left[\Xi(j|VH)\,\gamma(VH)\left(\frac{1}{p(j|VH,I,P)}\right)^{1/\alpha} + \right.$$
$$\left. + \Xi(j|VH)\,[1-\gamma(VH)]\left(\frac{1}{p(j|VH,I,I)}\right)^{1/\alpha} + [1-\Xi(j|VH)]\left(\frac{1}{p(j|VH,P,I)}\right)^{1/\alpha}\right]; \qquad (4.5a)$$

$$x_n(k,j|SH) = (1-n)\,sh\,SH_n\left[p_n\ A\,(1-\alpha)\right]^{1/\alpha} q^{k(j|SH)\left[\frac{1-\alpha}{\alpha}\right]}\left[\Xi(j|SH)\,\gamma(SH)\left(\frac{1}{p(j|SH,I,P)}\right)^{1/\alpha} + \right.$$
$$\left. + \Xi(j|SH)\,[1-\gamma(SH)]\left(\frac{1}{p(j|SH,I,I)}\right)^{1/\alpha} + [1-\Xi(j|SH)]\left(\frac{1}{p(j|SH,P,I)}\right)^{1/\alpha}\right]. \qquad (4.5b)$$

Thus, FGs firms demand more IGs when their product price, p_n, is high, when IGs prices, $p(j|m,Z,Z)$, are low, and when their HC employment, $VH_n(.|w)$ or $SH_n(.|w)$, and IGs quality, $k(j,t|m)$, are higher. As equations (4.4a, b) already reveal, even when IGs are produced in the South, their quality level is the leading innovator quality level.

Substituting equations (4.5a, b) into equation (4.1), the supply of FG n is – we omit again both country and time indexes:

TK levels in the HOS tradition – in particular, in our case the factor level proportions are dynamics.

[191] If producers of FG n use VT the demand for $x_n(k,j,t|VH)$ is given by equation (4.4a), whereas the demand for $x_n(k,j,t|SH)$ is zero, and *vice-versa* if ST is chosen. Similar considerations apply throughout.

$$Y_n(.|VH) = A^{1/\alpha}\left[p_n(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}\left[n\,vh\,VH_n(.|w)\;Q_I^{J^{VH}}\right]\left[\Xi(VH)\,\gamma(VH)\left(\frac{1}{p(j|VH,I,P)}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}+\right.$$
$$\left.+\,\Xi(VH)\,[1-\gamma(VH)]\left(\frac{1}{p(j|VH,I,I)}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}+\,[1-\Xi(VH)]\left(\frac{1}{p(j|VH,P,I)}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}\right]; \quad (4.6a)$$

$$Y_n(.|SH) = A^{1/\alpha}\left[p_n(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}\left[(1-n)\,sh\,SH_n(.|w)\;Q_I^{J^{SH}}\right]\left[\Xi(SH)\,\gamma(SH)\left(\frac{1}{p(j|SH,I,P)}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}+\right.$$
$$\left.+\,\Xi(SH)\,[1-\gamma(SH)]\left(\frac{1}{p(j|SH,I,I)}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}+\,[1-\Xi(SH)]\left(\frac{1}{p(j|SH,P,I)}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}\right]; \quad (4.6b)$$

Where:

$$Q_I^{J^{VH}} = \int_0^{J^{VH}} q^{k(j,t|VH)[(1-\alpha)/\alpha]}\,dj \text{ and } Q_I^{J^{SH}} = \int_0^{J^{SH}} q^{k(j,t|SH)[(1-\alpha)/\alpha]}\,dj \quad (4.7)$$

are again two aggregate quality indexes, which measure the TK in each specific range of IGs available in both countries. The ratio $Q_I^{J^{VH}}/Q_I^{J^{SH}}$ determines the relative productivity of the versatile TK, which is an appropriate measure of versatile-specialised TK bias in the world. Equations (4.6a, b) show that each country's aggregate output will depend directly on $Q_I^{J^{VH}}$ and $Q_I^{J^{SH}}$ and on the respective domestic HC used in production.

We define the output aggregate – the composite FG – in each country as:

$$Y_I(t) = \int_0^1 p_{nI}(t)\,Y_{nI}(t)\,dn = 1\left\{exp\left[\int_0^1 ln\,Y_{nI}(t)\,dn\right]\right\}; \quad (4.8a)$$

$$Y_P(t) = \int_0^1 p_{nP}(t)\,Y_{nP}(t)\,dn = MC_P\left\{exp\left[\int_0^1 ln\,Y_{nP}(t)\,dn\right]\right\}. \quad (4.8b)$$

where we normalise (i) the Northern price of the composite FG in each period to *1 – i.e.*, the composite FG is international numeraire in the North – and (ii) the Southern price of the composite FG in each period to MC_P. Furthermore, in equations (4.8a, b) $0 < MC_P < MC_I = 1$ since the Southern composite FG is produced at a lower *MC*. As will be clear below, the *MC* advantage of the South bears out in the production of IGs. This influences world-wide optimising limit pricing by the relevant competitive monopolists.

4.3. Intermediate goods sector

The production function of IGs is assumed to be the same as the FGs production function, since, by assumption, the production of IGs and R&D is financed by resources

saved after consumption of the composite FG or, equivalently, since the composite FG is the input in the production of each IG. The implicit assumption is that the production of IGs and R&D has the same versatile-intensity as aggregate production. Thus, the *MC* of producing an IG equals the *MC* of producing the composite FG and, consequently, its price, due to perfect competition in the FG sector. Once again, it is independent of the quality of the IGs and is identical across all domestic IGs.

All FG firms in each country that use the same technology face the same input prices and choose the same input ratios. Consequently, the world aggregate demands $X(j,t|VH,Z,Z)$ and $X(j,t|SH,Z,Z)$ for the highest quality of IG j at time t are – we omit index k since only the highest quality is produced:[192]

$$X(j,t|m,Z,Z) = \int_0^1 \left[x_{nI}(j,t|m,Z,Z) + x_{nP}(j,t|m,Z,Z) \right] dn . \qquad (4.9)$$

In equation (4.9), (Z,Z) includes then three possible cases, (I,I), (I,P), and (P,I).

The production of an IG requires a start-up cost of research, and so its profits must be positive for a certain period in the future. This is guaranteed by domestically enforced patents (IPRs),[193] since the TK of how to make an IG is public within a country.

Moreover, as the last Northern innovator is the only firm legally allowed to produce domestically the state-of-the-art quality IG, it will use pricing to wipe out sales of domestic lower quality in its IG. Depending on whether $q(1-\alpha)$ is less than or greater than MC_I it will respectively use limit or monopoly pricing to capture all the domestic market for its IG. In line with previous chapter, we assume that $q(1-\alpha) \leq MC_I$, implying that limit pricing strategy is binding and therefore is used by all innovator leader firms.[194]

Furthermore, we assume that the size of quality improvements is greater than the North-South *MC* of production ratio, $q > MC_I/MC_P$.[195] Then a Northern firm can

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[192] Since final producers in the range $[0,N]$ do not demand versatile-specific IGs, whereas final producers in the range $[N,1]$ do not demand specialised-specific IGs, then (as we will see later):

$$X(j,t|SH,I,P) = \int_0^1 \left[x_{nI}(j,t|SH,I,P) + x_{nP}(j,t|SH,I,P) \right] dn = \int_0^{N_I} x_{nI}(j,t|SH,I,P)\,dn + \int_0^{N_P} x_{nP}(j,t|SH,I,P)\,dn;$$

$$X(j,t|VH,I,P) = \int_0^1 \left[x_{nI}(j,t|VH,I,P) + x_{nP}(j,t|VH,I,P) \right] dn = \int_{N_I}^1 x_{nI}(j,t|VH,I,P)dn + \int_{N_P}^1 x_{nP}(j,t|VH,I,P)\,dn .$$

[193] In alternative to domestic patent protection, we could think that there is a cost of imitation domestically, which obstructs other firms from producing the IG. Otherwise, free-riding would prevent any R&D activity, since if firms could copy the design without costs, competition would drive ex-post rents to zero. Then, no firms would have an incentive, ex-ante, to support a sunk cost to design a new successful research. This also holds in the previous IT regime without IT effects.

[194] This is not a Southern problem, since an (imitator) entrant never competes with an (imitator) incumbent.

[195] In particular, we analyse the model's properties when $MC_I > MC_P > MC_I/q$. The 1st inequality is crucial to guarantee that production shifts from the North to the South when a successful imitation occurs

capture the world market with a single quality level improvement over a Southern imitation, and a Southern firm can acquire the world market by a successful imitation since, owing to lower *MC*s, it can under-price the Northern IG leader firm.

Now, we are going to solve the monopolist's maximization problem in order to obtain prices. As leader firms support different forms of competition, then to detain the world market with the greatest mark-up, we start noting that an entrant must consider the following *MC*s: (i) MC_I when the new leader innovator firm – the Northern entrant, I – competes with another innovator firm – the Northern incumbent, I – at the same *MC*.[196] (ii) MC_P when the Northern entrant, I, competes with a Southern incumbent, P, with a lower *MC*;[197] (iii) MC_I/q when the entrant is a Southern firm, P, with a lower *MC*, which always competes with a Northern incumbent, I, on the same quality rung.[198]

Furthermore, considering that governments can subsidise the production of IGs by paying a fraction, s_x, of each firm's production cost, the after subsidy *MC* of production for each one of the last three cases are: $(1-s_{xI})$, (MC_P-s_{xI}) and $(1-s_{xP})/q$, respectively. To keep the productive structure in both countries independent of government intervention, we assume that s_{xI} and s_{xP} are equal. Given also the respective demands for IGs in equation (4.9), formally the world-wide maximisation programs are:

$$\underset{p(j|m,I,I)}{Max}\ \Pi(j|m,I,I)=\left[p(j|m,I,I)-(MC_I-s_{xI})\right]X(j|m,I,I),\ MC_I=1; \qquad (4.10a)$$

$$\underset{p(j|m,I,P)}{Max}\ \Pi(j|m,I,P)=\left[p(j|m,I,P)-(MC_P-s_{xI})\right]X(j|m,I,P),\ MC_P<1; \qquad (4.10b)$$

$$\underset{p(j|m,P,I)}{Max}\ \Pi(j|m,P,I)=\left[p(j|m,P,I)-\frac{1}{q}(MC_I-s_{xP})\right]X(j|m,P,I). \qquad (4.10c)$$

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and the 2nd inequality is crucial to guarantee that production shifts back to the North after a successful innovation. If instead $q<MC_I/MC_P$, then a Northern entrant IG firm cannot under-price the IG producer firm in the South when it tries to explore the next successful innovation. In fact, the Northern entrant IG firm will not even be able to sell its leading IG since its production would be more expensive in cost per quality units than the imitated second best quality IG (in the world). In this case, the Northern entrant IG firm would have to innovate twice before it could market its quality-adjusted IG.

[196] In this case, the last Southern IG producer is at least two quality levels behind. Thus, the Northern entrant firm must consider the *MC* of the Northern incumbent – considering no government intervention.

[197] Otherwise, if MC_I is considered, the world market is captured by the Southern incumbent firm, which can use a price of the second best IG slightly above MC_P. That is, if the Northern entrant considers MC_I as its *MC*, the loss for FGs producers due to the increase in the price of a new quality IG is not compensated by the gain in terms of the increase in efficiency.

[198] As $MC_P > MC_I/q$ the highest mark up that the Southern entrant can use is MC_I, which corresponds to the lowest price that can be used by the Northern incumbent. In this way, the Southern entrant captures the world market and only another successful innovation can banish it.

Substituting the demand function, given by equation (4.9), into equations (4.10a-c) and after taking the derivative with respect to price and setting it equal to zero (FOCs), it is straightforward to derive the solution of the maximisation problems as:

$$p(j|m,I,I) = p(m,I,I) = p(I,I) = \frac{1}{1-\alpha}\left(MC_I - s_{xI}\right) \geq q\left(MC_I - s_{xI}\right); \qquad (4.11a)$$

$$p(j|m,I,P) = p(m,I,P) = p(I,P) = \frac{1}{1-\alpha}\left(MC_P - s_{xI}\right) \geq q\left(MC_P - s_{xI}\right); \qquad (4.11b)$$

$$p(j|m,P,I) = p(m,P,I) = p(P,I) = \frac{1}{1-\alpha}\frac{1}{q}\left(MC_I - s_{xP}\right) \geq \left(MC_I - s_{xP}\right). \qquad (4.11c)$$

The limit prices chosen by monopolists in equations (4.11a-c) represent a mark-up on the domestic after subsidy *MC* of manufacturing IGs:[199] $q(MC_I - s_{xI}) > (MC_I - s_{xI})$, $q(MC_P - s_{xI}) > (MC_I - s_{xI})$, and $(MC_I - s_{xP}) > (MC_P - s_{xP})$. Thus, (i) Northern entrants with better quality compete with a Northern incumbents at the same *MC*,[200] and they capture all the market by selling at any price slightly below $q(MC_I - s_{xI})$;[201] (ii) Northern entrants with better quality compete with Southern incumbents with lower *MC*, thus capturing all the market by selling at any price slightly below $q(MC_P - s_{xI})$;[202] and (iii) Southern entrants with lower *MC* compete on the same quality rung with a Northern incumbent, and they capture the entire market by selling the IGs at any price below $(MC_I - s_{xP})$.[203]

Table 4.1 below summarises the three possible sequences of successful R&D outcomes and their limit pricing consequences at time *t*, given quality *k* at time *t–dt*:

Table 4.1. World-wide limit pricing of the *m*-specific intermediate good *j*

t-dt	*t*	*Proportion in IGs production*	$p(j\|m, Z, Z)$
North produces and exports quality *k*	North innovates, produces and exports quality *k+1*	$\Xi(m)\,[1-\gamma(m)]$	$p(.\|.,I,I) = q\left(MC_I - s_{xI}\right)$
South produces and exports quality *k*	North innovates, produces and exports quality *k+1*	$\Xi(m)\,\gamma(m)$	$p(.\|.,I,P) = q\left(MC_P - s_{xI}\right)$
North produces and exports quality *k*	South imitates, produces and exports quality *k*	$1-\Xi(m)$	$p(.\|.,P,I) = \left(MC_I - s_{xP}\right)$

[199] *I.e.*, we assume that $(MC_I - s_x)/(MC_P - s_x) < q \leq 1/(1-\alpha)$ instead of $MC_I/MC_P < q \leq 1/(1-\alpha)$.

[200] That is, at least neither the $k(j,t) = k_I(j,t)$ innovation nor the $k_I(j,t)-1$ has yet been imitated.

[201] In this case, the lowest price the producer of the second best IG can charge is $(MC_I - s_{xI})$. The limit price reflects the fact the leader IG is *q* times more efficient than the second best domestically available IG.

[202] Remember that the lowest price the producer of the second best IG can charge is $(MC_P - s_{xP})$ when its production is located in the South and we have considered that $s_{xP} \approx s_{xI}$.

[203] Note that the lowest price the leader innovator firm can charge is $(MC_I - s_{xI})$.

These limit prices also indicate the effective market power of the IG producer in each IG. The first one is that which was derived in the previous IT regime and it is the highest. Therefore, there is a reduction in the mark-up, which can be understood as a reduction in pricing distortions due to the IT of IGs. Moreover, without introducing any changes in government intervention, the three limit prices are constant over time in each group of IGs and are independent of the quality level, which makes our problem symmetric. Finally, they are taken as given by the FG producers, which are price takers.

Plugging the limit prices in equations (4.11a-c) into equations (4.5a, b), the explicit demands of IGs manoeuvred by each *m*-type HC of the representative FG firm *n* are:

$$x_n(k,j|VH) = n\ vh\ VH_n(.|w)\left[p_n\ A\ \frac{(1-\alpha)}{q}\right]^{1/\alpha} q^{k(j|VH)\left[\frac{1-\alpha}{\alpha}\right]}\ mc_{\Xi\gamma}^{VH}; \tag{4.12a}$$

$$x_n(k,j|SH) = (1-n)\ sh\ SH_n(.|w)\left[p_n\ A\ \frac{(1-\alpha)}{q}\right]^{1/\alpha} q^{k(j|SH)\left[\frac{1-\alpha}{\alpha}\right]}\ mc_{\Xi\gamma}^{SH}. \tag{4.12b}$$

Where:

$$mc_{\Xi\gamma}^{m} \equiv \left[\Xi(m)\gamma(m)\left(\frac{1}{MC_P - s_{Ix}}\right)^{1/\alpha} + \Xi(m)[1-\gamma(m)]\left(\frac{1}{MC_I - s_{Ix}}\right)^{1/\alpha} + [1-\Xi(m)]\left(\frac{q}{MC_I - s_{Px}}\right)^{1/\alpha}\right].$$

Plugging equations (4.12a, b) into equation (4.1) we get that:

$$Y_n = A^{1/\alpha}\left[p_n\frac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}\left[(1-n)\ sh\ SH_n(.|w)\ Q_I^{J^{SH}} MC_{\Xi\gamma}^{SH} + n\ vh\ VH_n(.|w)\ Q_I^{J^{VH}} MC_{\Xi\gamma}^{VH}\right]. \tag{4.13}$$

Where: $MC_{\Xi\gamma}^{m} \equiv \Xi(m)\ \gamma_I^m + \left(\frac{q}{MC_I - s_{xP}}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}$;

$$\gamma_I^m \equiv \gamma(m)\left[\left(\frac{1}{MC_P - s_{xI}}\right)^{\left[\frac{1-\alpha}{\alpha}\right]} - \left(\frac{1}{MC_I - s_{xI}}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}\right] + \left[\left(\frac{1}{MC_I - s_{xI}}\right)^{\left[\frac{1-\alpha}{\alpha}\right]} - \left(\frac{q}{MC_I - s_{xP}}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}\right].$$

Equation (4.13) shows that the growth of FGs production in both countries is driven by improvements in the Northern quality of IGs and by the domestic employment of HC in the productive activity and, thereby, by domestic HC accumulation, *VH* and *SH*.

The term γ_I^m in the definition of $MC_{\Xi\gamma}^{m}$ is negative since $0 < \alpha < 1$, $\gamma(m) < 1$, $0 < MC_P < MC_I = 1$ and $(MC_I - s_x)/(MC_P - s_x) < q \leq 1/(1-\alpha)$. This term is useful in our modelisation of the proportion of IGs with production in the North, $\Xi(t|m)$. First, we note

that $\Xi(t|m)$ is equal to one minus the proportion of (quality-adjusted) IGs imitated at time t, $pb_P(k,av,t\,|\,m)$,[204] minus the proportion of (quality-adjusted) IGs imitated before time t, which have not been innovated again, $\Xi_1(t\,|\,m)$, plus the gains at time t due to successful innovation in the (quality-adjusted) IGs previously produced by the South, $\Xi_2(t\,|\,m)$. In formal form at time t:[205]

$$\Xi(t\,|\,m) = 1 - pb_P(k,av,t\,|\,m) - \Xi_1(t\,|\,m) + \Xi_2(t\,|\,m) \text{ and } \gamma(t|m) \equiv \tilde{Q}^{Jm}(t). \qquad (4.14)$$

In equation (4.14), $pb_P(k,av,t\,|\,m) + \Xi_1(t\,|\,m) > \Xi_2(t\,|\,m)$, $\forall t$; that is, the South produced a positive number of IGs at each t, and $\tilde{Q}^{Jm}(t)$ is a proxy for $\gamma(t|m)$. Thus, when $\gamma(t|m)$ grows, the absolute value or modulus of $\gamma_1^m(t)$; *i.e.*, $\|\gamma_1^m(t)\|$, falls. It is now essential to formalise $\Xi_1(t\,|\,m)$ and $\Xi_2(t\,|\,m)$. We do so by assuming that $\Xi(t|m)$ and $\gamma(t|m)$ are constant in steady state and the same for all IGs of each m-type. Thus,

$$\Xi_1(t\,|\,m) = \frac{\Xi_1(t) + \gamma_1^m(t)}{\gamma_1^m(t)}; \qquad (4.15a)$$

Where: $\Xi_1(t\,|\,m) \geq 0$ since $\left\|\gamma_1^m(t)\right\| > \left\|\Xi_1(t)\right\|$, $\Xi_1(t) > 0$, $\forall t$, and $\Xi_1(t^{ss}) = constant$.

$$\Xi_2(t\,|\,m) = \frac{\Xi_2(t) + \gamma_1^m(t)}{\gamma_1^m(t)} pb_1(k,av,t\,|\,m); \qquad (4.15b)$$

Where: $\left\|\gamma_1^m(t)\right\| > \left\|\Xi_2(t)\right\|$, $\Xi_2(t) \geq 0$, $\lim_{t \to t^{ss}} \Xi_2(t) = 0 \Rightarrow \lim_{t \to t^{ss}} \Xi_2(t\,|\,m) = pb_1(k,av,t\,|\,m)$

Equation (4.15a) formalises the proportion of IGs successfully imitated before time t and which are not successfully innovated again, $\Xi_1(t\,|\,m)$. It follows that it must always be greater than or equal to *0*, $\Xi_1(t\,|\,m) \geq 0$. As will be clear later on, for our purpose it is not necessary to explicitly formalise component $\Xi_1(t)$. We need only to guarantee that it is equal for m = *SH* and m = *VH*. Equation (4.15b) formalises the losses for the South due to successful innovations, $\Xi_2(t\,|\,m)$. We also consider that $\Xi_2(t)$ is equal for m = *SH* and m = *VH* and, in addition, we assume that it tends to *0* when time t tends to the steady state.[206] This is a realistic scenario, since, during the transition phase it is expected that

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[204] To obtain an accurate measure of the share of quality-adjusted IGs imitated at time t, we assume that the average of all domestic IGs of each m-type, denoted by the index *av*, is the representative domestic IG.

[205] Generally speaking, Northern IG firms support innovative R&D in all IGs and Southern IG firms support imitative R&D in the $\Xi(t|m)$; *i.e.*, in the IGs whose production is currently in the North.

[206] In equations (4.15a, b), $\Xi_2(t\,|\,m) > \Xi_1(t\,|\,m)$ since $\Xi_1(t) > \Xi_2(t)$. This is because the losses for the South due to successful innovations at each time t can be included in the set of (quality-adjusted) IGs previously produced by the South and, in addition, can also occur in the set of (quality-adjusted) IGs successful imitated at time t, when successful imitation and successful innovation occur at the same time.

initially the probability of successful imitation will be greater than the probability of successful innovation, but then falls more rapidly towards the steady state.

In short, the measure of IGs with Northern quality leaders is an increasing function of the probability of successful innovation and a decreasing function of the probability of successful imitation – the converse is true for the measure of IGs with Southern quality leaders. This result also holds in Dinopoulos and Segerstrom (2004) and allows us to state that, in fact, several (or all) IGs experience cycles, as Vernon (1966) has argued.

4.4. Equilibrium given R&D and HC accumulation

In this section, we obtain the domestic equilibrium values for the threshold FG N, the aggregate variables Y and X, the price paid for a unit of each type of HC, its growth rate as well as a measure of inter and intra-country wage inequality. The procedure is quite similar to that which was developed in section 2.4. However, now the analysis is performed for a given state of the available (Northern) TK and the domestic HC levels.

The production function (4.1) makes the *VH* relatively more productive in FGs with high indexes. Thus, if the index of the FGs decreases, the CA of *VH* also decreases, but the CA of *SH* increases. Hence, there is a threshold $N \in [0, 1]$ such that for $0 \leq n \leq N$ only ST is used and for $N < n \leq 1$ only VT is used – *e.g.*, Acemoglu and Zilibotti (2001). To derive N we start noting that the profit of the representative producer of the nth FG which uses m-type HC, in the period t, is, similarly to equation (2.14), given by:

$$\Pi_n(t\,|\,m) = p_n(t)\, Y_n(t|m) - \int_0^{J^m} p(j)\, x_n(j,t\,|\,m)\, dj - w^m(t)\, m_n(t|\,w). \qquad (4.16)$$

Where: $Y_n(.|VH) = A^{1/\alpha}\left[p_n \dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} n\, vh\, VH_n(.|\,w)\, Q_I^{J^{VH}} MC_{\Xi\gamma}^{VH}$;

$$Y_n(.|\,SH) = A^{1/\alpha}\left[p_n \frac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} (1-n)\, sh\, SH_n(.|\,w)\, Q_I^{J^{SH}} MC_{\Xi\gamma}^{SH}.$$

Taking equations (4.16), (4.12a, b) and (4.13) the per HC profit, $\pi_n(\cdot|m)$, is:

$$\pi_n(\cdot|VH) = \frac{\Pi_n(\cdot|VH)}{VH_n(.|\,w)} = n\, vh\,(p_n A)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} Q_I^{J^{VH}} MC_{\Xi\gamma}^{VH}\left[1-\frac{(1-\alpha)}{q}\right] - w^{VH}; \quad (4.17a)$$

$$\pi_n(\cdot|SH) = \frac{\Pi_n(\cdot|SH)}{SH_n(.|\,w)} = (1-n)\, sh\,(p_n A)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} Q_I^{J^{SH}} MC_{\Xi\gamma}^{SH}\left[1-\frac{(1-\alpha)}{q}\right] - w^{SH}. \quad (4.17b)$$

Since the FGs market is competitive, every FG producer obtains zero profits. Moreover, the operation $\pi_n(\cdot|VH)-\pi_n(\cdot|SH)$ is an increasing function of n over $[0, 1]$:

$$(p_n A)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}\left[n\left(vh\,Q_I^{J^{VH}}MC_{\Xi\gamma}^{VH}+sh\,Q_I^{J^{SH}}MC_{\Xi\gamma}^{SH}\right)-sh\,Q_I^{J^{SH}}MC_{\Xi\gamma}^{SH}\right]+w^{SH}-w^{VH}. \quad (4.18)$$

Besides, from the structure of production and the utility obtained with consumption of the composite FG, all FGs $n \in [0, 1]$ must be produced. Thus, for any FG n, we have either $\pi_n(\cdot|VH)=\Pi_n(\cdot|VH)=\pi_n(\cdot|SH)=\Pi_n(\cdot|SH)=0$, or $\pi_n(\cdot|VH)=\Pi_n(\cdot|VH)=0$ or $\pi_n(\cdot|SH)=\Pi_n(\cdot|SH)=0$. Furthermore, in equilibrium, no m-type HC is unemployed and so a positive measure of FGs must be produced using VH and SH. Thus, there is a N, where $0 < N < 1$, such that $\pi_n(\cdot|VH)-\pi_n(\cdot|SH)=0$, and $\pi_n(\cdot|VH)-\pi_n(\cdot|SH)>0$ (<0) for all $n > N$ (for $n < N$).[207] As a result, the production of the n[th] FG can be written as:

$$Y_n=\begin{cases}A^{1/\alpha}\left[p_n\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}MC_{\Xi\gamma}^{SH}\;(1-n)\,sh\,SH_n(.|w)\,Q_I^{J^{SH}} & \text{if}\quad 0\le n\le N\\ A^{1/\alpha}\left[p_n\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}MC_{\Xi\gamma}^{VH}\;n\,vh\,VH_n(.|w)\,Q_I^{J^{VH}} & \text{if}\quad N<n\le 1\end{cases}. \quad (4.19)$$

In equilibrium, the marginal value product of each type of HC has to be equalised across all FGs industries – $n > N$ for VH and $n \le N$ for SH; that is,

$$\frac{\partial\, p_n Y_n(t)}{\partial m_n}=\begin{cases}(p_n A)^{1/\alpha}\left[\dfrac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}MC_{\Xi\gamma}^{SH}\;(1-n)\,sh\,Q_I^{J^{SH}} & \text{if}\quad n\le N\\ (p_n A)^{1/\alpha}\left[\dfrac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}MC_{\Xi\gamma}^{VH}\;n\,vh\,Q_I^{J^{VH}} & \text{if}\quad n>N\end{cases}. \quad (4.20)$$

Equation (4.20) gives the wage of the m-type HC and shows that if $p_n^{1/\alpha}(1-n)$ and $p_n^{1/\alpha}\,n$ are constants then the marginal value product is the same for all $n \in [0, N]$ and $n \in [N, 1]$, respectively. Defining these constants as $p^{SH(1/\alpha)}$ and $p^{VH(1/\alpha)}$,[208] then $p_n = p^{SH}(1-n)^{-\alpha}$ and $p_n = p^{VH}n^{-\alpha}$, respectively,[209] and substituting in equation (4.19):

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[207] *I.e.*, in the production function $VH_n = x_n(.|vh) = 0$, $\forall\ 0 \le n \le N$, and $SH_n = x_n(.|SH) = 0$, $\forall\ N < n \le 1$.

[208] *I.e.*, p^ms are constants in the sense that they are independent of n (but they are dependent on the time t).

[209] Thus, we can write that the price paid for a unit of each type of HC, w^m, is:

$$w^{SH}=\left(p^{SH}A\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}MC_{\Xi\gamma}^{SH}\;sh\,Q_I^{J^{SH}},\quad w^{VH}=\left(p^{VH}A\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]}MC_{\Xi\gamma}^{VH}\;vh\,Q_I^{J^{VH}}.$$

$$Y_n = \begin{cases} Y_n^{SH}(1-n)^{\alpha},\ Y_n^{SH} = A^{1/\alpha}\left[p^{SH}\dfrac{(1-\alpha)}{q}\right]^{\frac{1-\alpha}{\alpha}} MC_{\Xi\gamma}^{SH}\, sh\, SH_n(.|w)\, Q_I^{J^{SH}} & \text{if } 0 \le n \le N \\ Y_n^{VH} n^{\alpha},\ Y_n^{VH} = A^{1/\alpha}\left[p^{VH}\dfrac{(1-\alpha)}{q}\right]^{\frac{1-\alpha}{\alpha}} MC_{\Xi\gamma}^{VH}\, vh\, VH_n(.|w)\, Q_I^{J^{VH}} & \text{if } N < n \le 1 \end{cases} \quad (4.21)$$

In equilibrium, expenditures across FGs, $p_n Y_n$, are equalised. Thus, we find that $VH_n(.|w)$ and $SH_n(.|w)$ are constant and equal in all ns using *VH* and *SH*, respectively.[210] To sum up, we have it – again, see equations (2.20a, b) – that:

$$p_n = p^{SH}\,(1-n)^{-\alpha},\ SH_n(.|w) = \frac{SH(.|w)}{N}\ ,\ SH(.|w) = \int_0^N SH_n(.|w)\,dn \ \text{ and } \ n \le N; \quad (4.22a)$$

$$p_n = p^{VH}\,n^{-\alpha},\ VH_n(.|w) = \frac{VH(.|w)}{(1-N)}\ ,\ VH(.|w) = \int_N^1 VH_n(.|w)\,dn \ \text{ and } \ n \ge N. \quad (4.22b)$$

Both p^{VH} and p^{SH} are then two price indexes which can again be determined. As a result FGs with higher indexes produced with *SH* have a lower productive technology and so they command higher prices. The converse is true for versatile FGs.

Next, we attempt to get the threshold FG N expression. From the previous analysis, we had it that for $n \le N$, all firms using a ST make zero profit, while firms using a VT would obtain negative profits, and *vice-versa* for $n > N$. In the threshold FG $n = N$, both a representative firm that uses ST and a representative firm that uses VT should breakeven. In short, both equations (4.22a, b) should hold for $n = N$, and, such as in equation (2.21):

$$\frac{p^{VH}}{p^{SH}} = \left[\frac{N}{1-N}\right]^{\alpha}. \quad (4.23)$$

As $p_n Y_n$ is constant for all n, then from equation (4.21) $p_n Y_n^{SH}(1-n)^{\alpha} = p_n Y_n^{VH} n^{\alpha}$ or, using equations (4.22a, b), $p^{SH} Y_n^{SH} = p^{VH} Y_n^{VH}$. Substituting Y_n^{SH} and Y_n^{VH} in equation (4.21) and using once more equations (4.22a, b) and (4.23), we obtain N as a function of the currently given variables determinants of economic viability of VT and ST:

$$N_Z = \left\{1 + \left[\frac{Q_I^{J^{VH}}}{Q_I^{J^{SH}}}\frac{vh\,VH_Z(.|w)}{sh\,SH_Z(.|w)}\right]^{\frac{1}{2}}\right\}^{-1}. \quad (4.24)$$

Equation (4.24) shows that if the TK in the North is highly versatile-biased; *i.e.*, $Q_I^{J^{VH}}/Q_I^{J^{SH}}$ is large, and/or the *VH* employed domestically is relatively greater, then the

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[210] That is, $p_n Y_n = p_n Y_n^{SH}(1-n)^{\alpha} \equiv \text{constant} \Rightarrow SH_n(.|w)$ is common in all FGs using *SH*. The same analysis implies that $VH_n(.|w)$ is also common in all FGs using *VH*.

fraction of FGs produced with VT is also greater – *i.e.*, N is small. In this case, the relative price of versatile FGs is low – equation (4.23). Thus, the demand for specialised-specific IGs is high – *e.g.*, equations (4.12a, b) – increasing the demand for specialised-specific new designs and inducing R&D aimed at improving specialised-specific TK. Hence, the structure of the stock of HC affects the direction of R&D through the FGs price channel – *i.e.*, incentives to develop specific TK are weaker when the prices of FGs produced with the TK are lower due to their use of the relatively abundant type of HC.

The static and first effect of IT of IGs affecting the South is clear in the equilibrium threshold N_P. By providing access to the state-of-the-art IGs,[211] IT affects the Southern FGs' productive structure, through the ratio Q_I^{JVH}/Q_I^{JSH}, bringing about convergence to the Northern structure and prices, which depend on N – see equation (4.23) above. Since the TK gap is always favourable to the North in either specific type of TK; *i.e.*, $Q_I^{Jm} > Q_P^{Jm}$ because the South always lags behind, the South enjoys an immediate absolute benefit relative to the North in terms of factor prices, w^m, and aggregate product, Y. This is apparent for now in equations (4.20) and (4.21) above, provided that both (i) the instantaneous jumps in the allocation of time between employment and HC accumulation, and (ii) the changes in mark-ups, are of second order. In fact, both the production of FGs and the marginal productivities of *VH* and *SH* increase with Q^{Jm}.

Let us presume that the IT regime changes at time t_0 from autarky to free IT of IGs. That is, that the jump in the available TK in the South to the measure of versatile-specialised bias in the North occurs at t_0 – see figure 4.3 below. Moreover, we consider again that equation (2.23) holds; *i.e.*, the TK gap is relatively higher in the versatile-specific type, and that initial HC levels are such that the North is relatively *VH* abundant:

$$\frac{Q_I^{JVH}(t_0)}{Q_I^{JSH}(t_0)} > \frac{Q_P^{JVH}(t_0)}{Q_P^{JSH}(t_0)} \quad \text{and} \quad \frac{VH_I(t_0 \mid w)}{SH_I(t_0 \mid w)} > \frac{VH_P(t_0 \mid w)}{SH_P(t_0 \mid w)}. \tag{4.25}$$

These assumptions imply that (the subscript *aut* denotes the autarkic regime):

$$N_P > N_I;\ N_{P,aut}(t_0) > N_P(t_0);\ \text{and}\ N_{P,aut}(t_0) - N_{I,aut}(t_0) > N_P(t_0) - N_I(t_0) > 0; \tag{4.26}$$

i.e., given the initial characteristics of the countries in equation (4.25) then – as we confirm later on numerically – the North always produces more FGs with VT than the South, although the direction of the immediate level effect is towards convergence in the structure of FGs' production and prices – see figures 4.1 and 4.2 below. Thus, contrarily

to the HOS model where the (final) goods are always intensive in a given factor regardless of the producer country – the strong factor-intensity assumption –, in our case factor-intensity reversal exists domestically and internationally. In fact, factor-intensity reversal holds at time t_0 in the South and, as we see later on, also during the transition to the steady state in both countries. Moreover, there will always be FGs that are produced with VT in the North and with ST in the South – international factor-intensity reversal.

Despite the immediate process of convergence in Southern productivity and prices of FGs and factors, the structure of FGs' production, prices and aggregate outputs remain different between countries, due to the maintenance of the gap in non-IT related exogenous productivity on FGs production, $A_I > A_P$, due to the differences in domestic HC structures and due to the international immobility of factors.

In spite of the absolute benefit relative to the North resulting from the static effect of IT on Southern individuals holding either type of HC, convergence brings about an immediate increase in inequality within the South – see figures 4.4 and 4.5 below. That is, increases in Southern wages are not balanced and, consequently, intra-Southern wage inequality occurs. In fact, against the IT theory predictions, the shift in the demand for *VH* is more pronounced, due to complementarity between IGs and HC together with the versatile-specific bias in TK, according to the first inequality in equation (4.25).

Now we are going to find the aggregate output, *Y*, and the total resources devoted to IGs, *X*. To this end, we remember that the Northern price of the composite FG equal to unit, $exp \int_0^1 ln\, p_{nI}\, dn = 1$, and in the South it is smaller than unit, $exp \int_0^1 ln\, p_{nP}\, dn = MC_P < 1$. Hence, with some algebra and equations (4.22a, b) and (4.23) we obtain:

$$p_I^{SH} = exp(-\alpha) N_I^{-\alpha},\ p_I^{VH} = exp(-\alpha)(1-N_I)^{-\alpha}; \tag{4.27a}$$

$$p_P^{SH} = exp(mc_P - \alpha) N_P^{-\alpha},\ p_P^{VH} = exp(mc_P - \alpha)(1-N_P)^{-\alpha}. \tag{4.27b}$$

In equation (4.27b), mc_P is equal to $ln MC_P$ and so smaller than *1*. From equations (4.27a, b), the prices of *SH* and *VH* specific FGs are high and low, respectively, if the Northern TK is very versatile-biased and if the domestic *VH* employed is relatively large.

From equations (4.9), (4.12a, b) and (4.22a, b), the equilibrium total resources devoted to the IGs of each type per time *t* are equal to:

$$X(t|SH) = sh\ SH(t|w) \left[p^{SH} A \frac{(1-\alpha)}{q} \right]^{1/\alpha} mc_{\Xi\gamma}^{SH}\ Q_I^{J^{SH}}; \tag{4.28a}$$

[211] Remember that the South has access to the state-of-the-art TK, either by imitation or by importing.

$$X(t|VH) = vh\ VH(t|w)\left[p^{VH} A \frac{(1-\alpha)}{q}\right]^{1/\alpha} mc_{\Xi\gamma}^{VH}\ Q_I^{J^{VH}}. \tag{4.28b}$$

Moreover, from equations (4.27a, b) and (4.28a, b) we find that:[212]

$$X_I = exp(-1)\left[A_I \frac{(1-\alpha)}{q}\right]^{1/\alpha}\left\{\left[Q_I^{J^{SH}} sh\ SH_I(.|w)\right]^{\frac{1}{2}} + \left[Q_I^{J^{VH}} vh\ VH_I(.|w)\right]^{\frac{1}{2}}\right\};$$
$$\left\{mc_{\Xi\gamma}^{SH}\left[Q_I^{J^{SH}} sh\ SH_I(.|w)\right]^{\frac{1}{2}} + mc_{\Xi\gamma}^{VH}\left[Q_I^{J^{VH}} vh\ VH_I(.|w)\right]^{\frac{1}{2}}\right\} \tag{4.29a}$$

$$X_P = exp(mc_P - \alpha)^{1/\alpha}\left[A_P \frac{(1-\alpha)}{q}\right]^{1/\alpha}\left\{\left[Q_I^{J^{SH}} sh\ SH_P(.|w)\right]^{\frac{1}{2}} + \left[Q_I^{J^{VH}} vh\ VH_P(.|w)\right]^{\frac{1}{2}}\right\}.$$
$$\left\{mc_{\Xi\gamma}^{SH}\left[Q_I^{J^{SH}} sh\ SH_P(.|w)\right]^{\frac{1}{2}} + mc_{\Xi\gamma}^{VH}\left[Q_I^{J^{VH}} vh\ VH_P(.|w)\right]^{\frac{1}{2}}\right\} \tag{4.29b}$$

From equations (4.8a, b), (4.19), (4.22a, b) and (4.27a, b), we get the equilibrium aggregate output, Y, as a function of the given Northern TK and domestic HCs at work:

$$Y_I = exp(-1)\, A_I^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\frac{1-\alpha}{\alpha}}\left\{\left[Q_I^{J^{SH}} sh\ SH_I(.|w)\right]^{\frac{1}{2}} + \left[Q_I^{J^{VH}} vh\ VH_I(.|w)\right]^{\frac{1}{2}}\right\};$$
$$\left\{MC_{\Xi\gamma}^{SH}\left[Q_I^{J^{SH}} sh\ SH_I(.|w)\right]^{\frac{1}{2}} + MC_{\Xi\gamma}^{VH}\left[Q_I^{J^{VH}} vh\ VH_I(.|w)\right]^{\frac{1}{2}}\right\} \tag{4.30a}$$

$$Y_P = exp(mc_P - \alpha)^{1/\alpha}\, A_P^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\frac{1-\alpha}{\alpha}}\left\{\left[Q_I^{J^{SH}} sh\ SH_P(.|w)\right]^{\frac{1}{2}} + \left[Q_I^{J^{VH}} vh\ VH_P(.|w)\right]^{\frac{1}{2}}\right\}.$$
$$\left\{MC_{\Xi\gamma}^{SH}\left[Q_I^{J^{SH}} sh\ SH_P(.|w)\right]^{\frac{1}{2}} + MC_{\Xi\gamma}^{VH}\left[Q_I^{J^{VH}} vh\ VH_P(.|w)\right]^{\frac{1}{2}}\right\} \tag{4.30b}$$

According to equations (4.30a, b), differences in the countries' output result from differences in A, $SH(.|w)$ and $VH(.|w)$. Thus, the countries' growth rates depend on their respective HC accumulation and on Northern TK progress.

As the price paid for a unit of each type of HC, w^m, is equal to its marginal product, it can be presented in the following extended form:

$$\frac{\partial Y}{\partial SH(.|w)} = w^{SH} = B^{SH}\left[Q_I^{J^{SH}} sh\ SH(.|w)\right]^{-\frac{1}{2}} sh\ Q_I^{J^{SH}}; \tag{4.31a}$$

[212] Note that IGs can be produced domestically or not, but they need to be acquired for FGs production.

$$\frac{\partial Y}{\partial VH(.|w)} = w^{VH} = B^{VH}\left[Q_I^{J^{VH}} vh\, VH(.|w)\right]^{-\frac{1}{2}} vh\, Q_I^{J^{VH}}\ ;\ \text{where:} \qquad (4.31b)$$

$$B_I^{SH} = exp(-1)\, A_I^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} \frac{1}{2}\left\{2MC_{\Xi\gamma}^{SH}\left[Q_I^{J^{SH}} sh\, SH_I(.|w)\right]^{\frac{1}{2}} + \left[MC_{\Xi\gamma}^{SH} + MC_{\Xi\gamma}^{VH}\right]\left[Q_I^{J^{VH}} vh\, VH_I(.|w)\right]^{\frac{1}{2}}\right\};$$

$$B_P^{SH} = exp(mc_P - \alpha)^{1/\alpha} A_P^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} \frac{1}{2}\left\{2MC_{\Xi\gamma}^{SH}\left[Q_I^{J^{SH}} sh\, SH_P(.|w)\right]^{\frac{1}{2}} + \left[MC_{\Xi\gamma}^{SH} + MC_{\Xi\gamma}^{VH}\right]\left[Q_I^{J^{VH}} vh\, VH_P(.|w)\right]^{\frac{1}{2}}\right\};$$

$$B_I^{VH} = exp(-1)\, A_I^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} \frac{1}{2}\left\{\left[MC_{\Xi\gamma}^{SH} + MC_{\Xi\gamma}^{VH}\right]\left[Q_I^{J^{SH}} sh\, SH_I(.|w)\right]^{\frac{1}{2}} + 2MC_{\Xi\gamma}^{VH}\left[Q_I^{J^{VH}} vh\, VH_I(.|w)\right]^{\frac{1}{2}}\right\};$$

$$B_P^{VH} = exp(mc_P - \alpha)^{1/\alpha} A_P^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} \frac{1}{2}\left\{\left[MC_{\Xi\gamma}^{SH} + MC_{\Xi\gamma}^{VH}\right]\left[Q_I^{J^{SH}} sh\, SH_P(.|w)\right]^{\frac{1}{2}} + 2MC_{\Xi\gamma}^{VH}\left[Q_I^{J^{VH}} vh\, VH_P(.|w)\right]^{\frac{1}{2}}\right\}.$$

What is interesting here is the following comment. Comparing w^m in equations (4.31a, b) with the equivalent expressions in equation (4.20), we conclude that they are only equal if $MC_{\Xi\gamma}^{SH} = MC_{\Xi\gamma}^{VH}$. Consequently, this must be true and we also have it that also $mc_{\Xi\gamma}^{SH} = mc_{\Xi\gamma}^{VH}$ and $B_Z^{SH} = B_Z^{VH}$. Hence, the expression of the composite FGs in equations (4.30a, b) feature an elasticity of substitution constant and equal to *2* between the two types of domestic HC at work pondered by the respective Northern TK.[213]

As a result of equations (4.31a, b) – or equation (4.20) –, the variation in the price paid for a unit of each type of HC (equilibrium wage dynamics) is given by:

$$\frac{\dot{w}^m}{w^m} = \frac{1}{\alpha}\frac{\dot{p}^m}{p^m} + \frac{\dot{MC}_{\Xi\gamma}^m}{MC_{\Xi\gamma}^m} + \frac{\dot{Q}_I^{J^m}}{Q_I^{J^m}}. \qquad (4.32)$$

Hence, due to the complementarity in FGs production, the growth rate of w^m depends on the dynamic of p^m, plus the dynamic of $MC_{\Xi\gamma}^m$ and the dynamic of $Q_I^{J^m}$.[214]

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[213] Thus, for example, equations (4.30a) and (4.31a) for the North can then be written as:

$$X_I = exp(-1)\left[A_I\frac{(1-\alpha)}{q}\right]^{1/\alpha} mc_{\Xi\gamma}^{SH}\left\{\left[Q_I^{J^{SH}} sh\, SH_I(.|w)\right]^{\frac{1}{2}} + \left[Q_I^{J^{VH}} vh\, VH_I(.|w)\right]^{\frac{1}{2}}\right\}^2;$$

$$Y_I = exp(-1)\, A_I^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\frac{1-\alpha}{\alpha}} MC_{\Xi\gamma}^{SH}\left\{\left[Q_I^{J^{SH}} sh\, SH_I(.|w)\right]^{\frac{1}{2}} + \left[Q_I^{J^{VH}} vh\, VH_I(.|w)\right]^{\frac{1}{2}}\right\}^2.$$

[214] As we will see later on, in steady state both p^m and $MC_{\Xi\gamma}^m$ are constant. Then, regardless of the country, the steady state growth rate of w^m is proportional to the respective aggregate quality index in the North.

That is, equation (4.32) shows that the path of *m*-wages in each country depends on the path of domestic demand for *m*-type HC, which, in turn, depends on the evolution of (i) the domestic range of the *m*-specific technology, established by threshold FG *N*, which determines prices of (non-tradable) FGs; and (ii) the world demand for *m*-specific IGs, mirrored in international $MC^{m}_{\Xi\gamma}$ and driven by the available Northern TK,[215] since the IGs can be produced domestically or not, but they are used in FGs production.[216]

From equations (4.31a, b), the inter-country wage inequality, the versatile premium in any country, *W*, and the versatile premium between countries, ***W***, are, respectively:

$$\frac{w^{m}_{P}}{w^{m}_{I}} = \left(\frac{p^{m}_{P} A_{P}}{p^{m}_{I} A_{I}} \right)^{1/\alpha} . \tag{4.33a}$$

$$W = \frac{w^{VH}}{w^{SH}} = \left(\frac{Q_{I}^{JVH}}{Q_{I}^{JSH}} \right)^{\frac{1}{2}} \left(\frac{vh\, VH(.|w)}{sh\, SH(.|w)} \right)^{-\frac{1}{2}} \frac{vh}{sh} ; \tag{4.33b}$$

$$\boldsymbol{W} = \frac{W_{P}}{W_{I}} = \left(\frac{SH_{P}(.|w)\, VH_{I}(.|w)}{SH_{I}(.|w)\, VH_{P}(.|w)} \right)^{\frac{1}{2}} . \tag{4.33c}$$

Equation (4.33a) suggests that there is a higher wage in the North, as a result of the conjunction of international immobility of HC and differences in productivity, $A_I > A_P$. In relation to the previous IT regime in chapter 3, now the difference must be smaller, since the available (Northern) TK is the same in both countries.[217]

Equation (4.33b) shows that the versatile premium in any country is greater when the Northern TK is more versatile biased, the *VH* employed is scarcer and the absolute productivity advantage of *VH* over *SH* in production is strong. Given equation (4.25), it is clear that when IT of IGs starts, the Southern versatile premium increases. This is against the IT theory which suggests that IT between the South and the North falls the Southern skill wage premium, but is in line with empirical evidence – see Stokey (1996).

Equation (4.33c) tells us that, given the characteristics of the countries – *e.g.*, equation (4.25) for time t_0 –, the equilibrium versatile premium tends to be greater in the South, which is less clear without IT effects – see equation (2.33).

The next section is directed at the implications of the IT of IGs in the R&D sector.

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[215] Note that $MC^{m}_{\Xi\gamma}$ is a weighted average obtained from the limit prices in table 4.1.

[216] Remember that without IT effects the growth rate of w^m is only proportional to the dynamic of both the domestic respective FGs' price index and the aggregate domestic quality index – see equation (2.30).

[217] But, TK is not equalised between countries, since, at each *t*, not all innovations have been imitated.

4.5. R&D sector

4.5.1. Overview

The principles and notation underlying this section are the same as those used in section 2.5 relative to the analysis of the R&D sector without IT effects. Thus, now we will only refer to the changes resulting from the particularities induced by the IT of IGs.

In the previous IT regime there is no commercial relation between countries. As a result, there is no place for world competition in order to produce state-of-the-art IGs, and so international IPRs protection does not make sense. However, the same in not true under the IT of IGs or under totally free IT if, in addition, the South has a *MC* advantage over the North. In this case, the South can compete effectively with the North. That is, Northern firms sell IGs by supporting the discovery of the next higher-quality IG – *i.e.*, innovations provide 'better of the same' or more efficient versions of the pre-existing IGs –, and Southern firms sell IGs by imitating state-of-the-art quality IGs.

To avoid or, at least, decrease the intensity of Southern competition, international IPRs protection has become a problem of major concern, supported by the North with the argument that they encourage innovations from which both countries of the world benefit. This is reflected, for example, in the fact that one important breakthrough of the Uruguay Round of the General Agreement on Tariffs and Trade was the signing of the agreement on trade-related aspects of IPRs, TRIPRs. Its major consequence is that LDCs have to substantially strengthen the legal protection of IPRs – *e.g.*, Lai and Qiu (2003).

In line with this occurrence and in order to check the validity of the Northern argument on the need for tighter international IPRs protection, we assume that stronger international IPRs protection decreases the probability of successful imitation.

The section is organised as follows. Subsection 4.5.2 presents the specifications for the probability of successful research. Subsection 4.5.3 describes the flow and the expected present value of the monopoly profit from successful research. Subsection 4.5.4 performs the equilibrium probability of successful research and the equilibrium R&D effort. Finally, subsection 4.5.5 computes the behaviour of the aggregate quality indexes.

4.5.2. Specification for the probabilities of successful R&D

R&D activities in the North result in innovative designs for the production of IGs, which increase their quality. The South mimics the R&D process of the North, but its R&D activities result in imitation of current Northern best qualities.

The designs are domestically patented and the leader firm in each IGs – the one that produces according to the latest patent – uses limit pricing to assure monopoly. The value of the leading-edge patent depends on the profit-yield accruing at each time t to the monopolist, and on the period of the monopoly power. According the case, the duration, in turn, depends on (i) the probability of a new successful innovation, which creatively destroys the current leading-edge design – as in the Schumpeterian models introduced by Segerstrom *et al.* (1990), Grossman and Helpman (1991a, chap. 4, b, c) and Aghion and Howitt (1992) – and/or on (ii) the probability of successful imitation of the highest quality IG, which transfers the production of IGs to the South – such as in Grossman and Helpman (1991a, chaps. 11-12). The instantaneous probability of a successful research; *i.e.*, a Poisson arrival rate, denoted by $pb_Z(k,j,t|m)$, is, thus, at the centre of R&D.

Particularly, in equation (4.34) below (i) $pb_I(k,j,t|m)$ denotes the instantaneous probability of successful innovation at time t in the next higher quality of IG j – *i.e.*, at rung $k(j,t|m)+1$ –, which complements domestic m-type HC; and (ii) $pb_P(k,j,t|m)$ is the instantaneous probability of successful imitation at time t of the current higher quality of the IG j – *i.e.*, at rung $k(j,t|m)$ –, which complements domestic m-type HC:

$$pb_Z(k,j,t|m) = rs_Z(k,j,t|m)\, ll_Z(k,j,t|m)\, lm_Z(t|m)\, cp_Z(k,j,t|m)\left[btp_Z(k,j,t|m)\,\iota_Z\right]^{\Gamma_Z}, \qquad (4.34)$$

Where, in short – since the terms of $pb_Z(k,j,t|m)$ in equation (4.34) have basically the same interpretation and justification as in the previous IT regime:

(i) Γ_Z takes on value *0* for $Z = I$, $\Gamma_I = 0$, and the value *1* for $Z = P$, $\Gamma_P = 1$.

(ii) $rs_Z(k,j,t|m)$ is the flow of domestic FG resources for domestic R&D.

(iii) $ll_I(k,j,t|m) = \beta_I\, q^{k(j,t|m)}$, $ll_P(k,j,t|m) = \beta_P\, q^{k_P(j,t|m)}$, $\beta_I > \beta_P > 0$, $k \geq k_P$, is the positive learning effect of accumulated public TK from past successful research – *e.g.*, Grossman and Helpman (1991, chap. 12);

(iv) $lm_Z(t|m) = [m_I(.|w) + m_P(.|w)]^{-\xi_Z}$, where $\xi_Z > 0$ and we consider specifically that $\xi_Z = 1$. This is the adverse effect of market size – *e.g.*, Dinopoulos and Segerstrom (1999a) –, which is now measured by $[m_I(.|w) + m_P(.|w)]$. Thus, while in models that have scale effects, opening a country to IT in effect increases its size, which then leads to

a higher growth rate through the scale effect – *e.g.*, Kremer (1993), Sala-i-Martin (1997) and Connolly (1999) –; in our case, IT does not affect growth in this way.[218]

Regardless of the contexts, we ignore the endogenous growth debate on scale effects. Even so, our framework seems to be in line with some empirical evidence, which observes that models in which IT increases growth due to the scale effect receive mixed support – *e.g.*, Backus *et al.* (1992). We detect that little progress has been made at developing North-South trade models without the scale effect property. The recent works of Dinopoulos and Segerstrom (2004) and Sener (2003) seem to be the exceptions.

(v) $cp_Z(k,j,t\,|\,m) = \zeta_Z^{-1} q^{k(j,t|m)(-1/\alpha)}$, $0<\alpha<1$, $\zeta_I > \zeta_P > 0$, is the adverse effect caused by the increasing complexity of quality improvements – *e.g.*, Barro and Sala-i-Martin (2004, chap. 7) and Dinopoulos and Segerstrom (2004) –, considering that the complexity cost of imitation is lower than that of innovation – *e.g.*, Mansfield *et al.* (1981). Note that, relative to the case in section 2.5, now the South needs to imitate the best quality IGs in the North, so as to capture the whole world market.[219]

(vi) $btp_P(k,j,t\,|\,m) = exp[PE(j,t\,|\,m) + IT(j,t\,|\,m)]\{g_1[\tilde{m}(t\,|\,w)]\}^{\overline{\sigma}_1}\{g_2[\tilde{Q}^{J^m}(t), d]\}^{\sigma_2(\tilde{Q}^{J^m}, t)}$, with $\overline{\sigma}_1 > 0$, $\sigma_2(\tilde{Q}^{J^m}, t) = -\overline{\sigma}_2 + \tilde{Q}^{J^m}(t)$ and $0 < \tilde{Q}^{J^m}(t) < 1$ is a catching up term, specific to the South, that sums up positive effects of imitation capacity and backwardness.

We remember that the first exponential captures two important determinants of imitation capacity, domestic policies promoting R&D – index number *PE*; *e.g.*, Aghion *et al.* (2001) – and openness to IT and other IT policies – index number *IT*; *e.g.*, Coe *et al.* (1997). Note that now IT exercises an effective influence on $btp_P(k,j,t\,|\,m)$. For the sake of simplicity, *PE* and *IT* are not specific to each IG. The second exponential includes the functions $g_1(.)$ and $g_2(.)$, in equations (2.36) and (2.37), respectively. The first function captures the importance of HC in the imitation capacity – in line with Benhabib and Spiegel (1994), among others – and the second, basically, the backwardness advantage – in accordance with, for example, Barro and Sala-i-Martin (1997). From the function $g_2(.)$, when the TK gap between countries is wider, backwardness is no longer an advantage.

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[218] Nevertheless, we can state that the eventual increase in growth ultimately reflects a kind of scale effect – the instantaneous jump in $btp_P(k,j,t\,|\,m)$ due to the term *IT*.

[219] In the previous regime, regardless of the additional quality rungs in the North for each IG, the IGs (follower) firms in the South needed only to support the domestic R&D necessary to imitate one quality rung above that which existed domestically. In this way, they capture the whole domestic Southern market.

(vii) To allow a role for international IPRs protection, following Helpman (1993) and Glass and Saggi (2002), among others, we introduce the parameter ι_P, which captures the strength of international IPRs protection provided by Southern government, usually due to world negotiations. A higher value of ι_P means weaker protection, and *vice-versa*. So, $\iota_P = 0$ and $\iota_P = 1$ indicates full and no international IPRs protection, respectively.

At first sight, since Southern firms do not participate into the innovative process, this may create an incentive for the Southern government not to enforce international IPRs in order to speed up TK imitation. However, under this behaviour the potential Northern innovators can lose incentives to create improvements in the quality of IGs.

4.5.3. Explicit flow and duration of the monopoly profit

We can again compare the incremental profits of follower firms taking over the leader position with the incremental profits of leader firms replacing themselves. Once again we can prove that in each case the gains to follower firms are large. Thus, the identity of the quality leader in an IG changes every time successful research occurs.

When Northern leader firms are facing direct competition from other Northern (follower) firms – *i.e.*, the nearest competitor of the actual Northern leader firm are Northern follower firms –, the scenario is similar to the case in the previous regime – see subsection 2.5.5. Therefore, only Northern follower firms will support innovative R&D.

When the Northern leader IG firm which faces direct Southern competition – *i.e.*, the previous IG producer firm was located in South – supports R&D and replaces itself, it will be selling an IG twice as productive as its closest competitor and so it will use a mark-up slightly below $q^2(MC_P - s_{xI})$. But, it will also lose its profits from the previous position, where it was sold at price slightly below $q(MC_P - s_{xI})$. On the other hand, when a Northern follower IG firm supports R&D and replaces the leader, it sets its price to capture the world market from the last leader, since it is now two quality levels ahead of the last Southern producer.[220] So, a Northern entrant will sell its new better quality IG at a price slightly below $q(MC_I - s_{xI})$. This means that the incremental gain to the entrant is greater than to the incumbent replacing itself, if $q^{-1} + q^{1/\alpha} - q^{(1-\alpha)/\alpha} > 1$. This holds since by assumption $0 < \alpha < 1$ and $q > 1$, and so only Northern follower firms will support R&D.

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[220] That is, it sets its price so as to not only capture the world market from the last leader, which is one quality level behind, but also to prevent the imitator firm, now two quality levels behind. So, its limit price will be the smaller of $q(MC_I - s_{xI})$ and $q^2(MC_P - s_{xI})$. Since $q > (MC_I - s_x)/(MC_P - s_x)$ the lowest is the first.

Finally, when imitation of the (previous) Northern leader firm has already occurred – *i.e.*, at present the incumbent is located in the South –, then all firms in the North (now follower firms) can undertake or support innovative research, since they face the same costs and incentives to bring about the next successful innovation.

Taking into account equations (4.10a-c), (4.11a-c) and (4.28a, b), we have that the flow of profit of leader firms of all three types is given by – omitting time index *t*:[221]

$$\Pi(k,j,.|m,I,I)=\overline{m}\,(1-\alpha)^{\frac{1}{\alpha}}\,q^{k(j,.|m)\left[\frac{1-\alpha}{\alpha}\right]}$$
$$\left\{\left[q\left(MC_I-s_{xI}\right)-\left(MC_I-s_{xI}\right)\right]\left[m_P\left(\frac{A_P\,p_P^m}{q\,(MC_I-s_{xI})}\right)^{\frac{1}{\alpha}}+m_I\left(\frac{A_I\,p_I^m}{q\,(MC_I-s_{xI})}\right)^{\frac{1}{\alpha}}\right]\right\}; \qquad (4.35a)$$

$$\Pi(k,j,.|m,I,P)=\overline{m}\,(1-\alpha)^{\frac{1}{\alpha}}\,q^{k(j,.|m)\left[\frac{1-\alpha}{\alpha}\right]}$$
$$\left\{\left[q\left(MC_P-s_{xI}\right)-\left(MC_I-s_{xI}\right)\right]\left[m_P\left(\frac{A_P\,p_P^m}{q\,(MC_P-s_{xI})}\right)^{\frac{1}{\alpha}}+m_I\left(\frac{A_I\,p_I^m}{q\,(MC_P-s_{xI})}\right)^{\frac{1}{\alpha}}\right]\right\}; \qquad (4.35b)$$

$$\Pi(k,j,t|m,P,I)=\overline{m}\,(1-\alpha)^{\frac{1}{\alpha}}\,q^{k(j,.|m)\left[\frac{1-\alpha}{\alpha}\right]}$$
$$\left\{\left(MC_I-MC_P\right)\left[m_P\left(\frac{A_P\,p_P^m}{MC_I-s_{xP}}\right)^{\frac{1}{\alpha}}+m_I\left(\frac{A_I\,p_I^m}{MC_I-s_{xP}}\right)^{\frac{1}{\alpha}}\right]\right\}. \qquad (4.35c)$$

From equations (4.35a, b), we can obtain an aggregate expression for the flow of profit of Northern leader firms for *m*-type HC employed:

$$\Pi(k,j,t|m,I,Z)=\Pi(k,j,t|m,I)=\overline{m}\,(1-\alpha)^{\frac{1}{\alpha}}\,q^{k(j,t|m)\left[\frac{1-\alpha}{\alpha}\right]}\,q^{\left[-\frac{1}{\alpha}\right]}$$
$$\left\{\gamma(m)\left[q\left(MC_P-s_{xI}\right)-\left(MC_I-s_{xI}\right)\right]\left[m_P\left(\frac{A_P\,p_P^m}{MC_P-s_{xI}}\right)^{\frac{1}{\alpha}}+m_I\left(\frac{A_I\,p_I^m}{MC_P-s_{xI}}\right)^{\frac{1}{\alpha}}\right]+\right.$$
$$\left.+\left(1-\gamma(m)\right)\left[q\left(MC_I-s_{xI}\right)-\left(MC_I-s_{xI}\right)\right]\left[m_P\left(\frac{A_P\,p_P^m}{MC_I-s_{xI}}\right)^{\frac{1}{\alpha}}+m_I\left(\frac{A_I\,p_I^m}{MC_I-s_{xI}}\right)^{\frac{1}{\alpha}}\right]\right\} \qquad (4.36)$$

In particular, equations (4.35c) and (4.36) tell us that profits depend positively on the mark-ups and the world demand for IGs by the FGs producers. Specifically, the amount of profits at time *t* for an IG *j* depend positively on the price of the respective

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[221] *E.g.*, to get equation (4.35c) we take it that – applying the same reasoning used in obtaining (4.35a, b):

$$\Pi(k,j,t|SH,P,I)=\left[p(j|P,I)-\left(MC_P-s_{xP}\right)\right]\left[\int_0^{N_P}x_{nP}(k,j,t|SH,P,I)\,dn+\int_0^{N_I}x_{nI}(k,j,t|SH,P,I)\,dn\right];$$

$$\Pi(k,j,t|VH,P,I)=\left[p(j|P,I)-\left(MC_P-s_{xP}\right)\right]\left[\int_{N_P}^{I}x_{nP}(k,j,t|VH,P,I)\,dn+\int_{N_I}^{I}x_{nI}(k,j,t|VH,P,I)\,dn\right].$$

IGs, $p(j|m)$, on the market size, m_P and m_I, on the quality rung, $k = k_I$, on the productivity parameters, A_P and A_I, and on the price indexes, p_P^m and p_I^m. And they depend negatively on the domestic after subsidy *MCs* of the respective IG.

On the other hand, taking equation (4.34) into account, we have that:[222]

$$pb_Z(k,j,t|m) = rs_Z(k,j,t|m)\,\beta_Z\,\zeta_Z^{-1}\left[m_I(t|w)+m_P(t|w)\right]^{-1} q^{k(j,t|m)\left[\frac{\alpha-1}{\alpha}\right]} \cdot \left[exp(PE+IT)\,g_1(.)^{\bar{\sigma}_1}\,g_2(.)^{\sigma_2(.)}\,\tilde{q}^{(j,t|m)}\,\iota_P\right]^{\Gamma_Z} \quad (4.37)$$

Remember that in equation (4.37), $\Gamma_I = 0$ and $\Gamma_P = 1$. Thus, (i) when both the flow of resources devoted to R&D, rs_Z, and the coefficient on past research experience, β_Z, are high, the instantaneous probability of successful research, pb_Z, is high; (ii) when the fixed cost of research, ζ_Z, the effectiveness of all costs that capture the adverse effect of market size $[m_I(.|w)+m_P(.|w)]^{-1}$, and the complexity of research, $q^{k(j,t|m)[(\alpha-1)/\alpha]}$, are higher, the instantaneous probability of successful research is low; (iii) in addition, the probability of successful imitation is higher when the imitative capacity of the country is good, which increases with openness, and there is a weaker (or no) international IPRs protection – *i.e.*, when ι_P is higher. However, the influence of $\tilde{q}^{(j,t|m)}$ on pb_P can be smaller than before, since, with all else equal, $g_2(.)^{\sigma_2(.)}\,\tilde{q}^{(j,t|m)}$ is smaller than $g_2(.)^{\sigma_2(.)}$, given that $0 < \tilde{q}^{(j,t|m)} < 1$. This is because now, to produce the IG j, the South needs to imitate all levels between the current domestic level and the Northern state-of-the-art.

The profit in equation (4.35c) takes place from the time of the q^k (highest) quality imitation, $t_{k_I,imitation}$, until the time of the next innovation by some Northern firm, $t_{k_I,imitation} + dt = t_{(k_I+1),innovation}$. Hence, the duration of the monopoly position of the $k_I(j,t|m)^{\text{th}}$ Southern IG producer is $dt = T_{k_I,imitation} = t_{(k_I+1),innovation} - t_{k_I,imitation}$. The profit in equation (4.36) exists from the time of the q^k (highest) quality innovation, $t_{k_I,innovation}$, until the time of its imitation, $t_{k_I,innovation} + dt = t_{k_I,imitation}$, or the next innovation, $t_{k_I,innovation} + dt = t_{(k_I+1),innovation}$. Thus, the interval during which the k^{th} innovator is in the forefront is $dt = T_{k_I,innovation} = [\,t_{(k_I+1),innovation} \cup t_{k_I,imitation}\,] - t_{k_I,innovation}$.[223] In short, while

---

[222] To derive equation (4.37), we note that $\tilde{q}^{(j,t|m)} = q^{k_P(j,t|m)} / q^{k_I(j,t|m)}$ and so $q^{k_P(j,t|m)} = \tilde{q}^{(j,t|m)} q^{k_I(j,t|m)}$.

[223] The events $t_{(k_I+1),innovation}$ and $t_{k_I,imitation}$ are compatible and, consequently,

$T_{k_I,imitation}$ finishes with the next innovation, which can only be driven by some Northern firm, $T_{k_I,innovation}$ can end with its imitation or with the next innovation.

Given the equilibrium interest rate, the expected present value of profits earned by the monopolist producer of IG j – which supports a successful imitation, $V_P(k, j, t|m)$,[224] or, a successful innovation, $V_I(k, j, t|m)$,[225] and so which sells the highest quality of IG j in the world, complementary to m-type HC at time t – can be written as:

$$V_P(k, j, t|m) = E\left[\Pi_P(k, j, t|m) \frac{1 - exp\left(-r_P(t)\, T_{k_I,\, imitation}\right)}{r_P(t)} \right]; \tag{4.38a}$$

$$V_I(k, j, t|m) = E\left[\Pi_I(k, j, t|m) \frac{1 - exp\left(-r_I(t)\, T_{k_I,\, innovation}\right)}{r_I(t)} \right]. \tag{4.38b}$$

In equations (4.38a, b), to obtain $V_Z(k, j, t|m)$, we need to determine first the respective T_{k_I}. Following the same analysis as in subsection 2.5.3, $V_Z(k, j, t|m)$ is:

$$V_P(k, j, t|m) = \frac{\Pi_P(k, j, t|m)}{r_P(t) + pb_I(k, j, t|m)}; \tag{4.39a}$$

$$V_I(k, j, t|m) = \frac{\Pi_I(k, j, t|m)}{r_I(t) + pb_I(k, j, t|m) + pb_P(k, j, t|m) - pb_I(k, j, t|m)\, pb_P(k, j, t|m)}. \tag{4.39b}$$

Equations (4.39a, b) show the explicit expected reward from carrying out k_I^{th} successful imitative and innovative research, $V_Z(k, j, t|m)$. They depend positively on the amount of profits in each period, $\Pi_P(k, j, t|m)$ and $\Pi_I(k, j, t|m)$, in equations (4.35c) and (4.36), respectively. Moreover, they depend negatively on both the respective interest rate, r_P and r_I, and the respective expected duration of the flow, which corresponds, respectively, to the expected duration of the imitator's and innovator's TK leadership – or, in other words, they depend negatively on the respective effective discount rate.

$pb(k, j, t_{innovation \cup imitation}|m) = pb_I(k, j, t|m) + pb_P(k, j, t|m) - pb(k, j, t_{innovation \cap imitation}|m)$. But, they are independents and so $pb(k, j, t_{innovation \cap imitation}|m) = pb_I(k, j, t|m)\, pb_P(k, j, t|m)$. Thus, $pb(k, j, t_{innovation \cup imitation}|m) = pb_I(k, j, t|m) + pb_P(k, j, t|m) - pb_I(k, j, t|m)\, pb_P(k, j, t|m)$.

[224] In other words, $V_P(k, j, t|m)$ represents the price of the $k_I(j,t|m)^{th}$ successful imitation, the market value of the patent or still the value of the monopolist firm owned by domestic consumers.

[225] In other words, $V_I(k, j, t|m)$ represents the price for the $k_I(j,t|m)^{th}$ successful innovation, the market value of the patent or still the value of the monopolist firm owned by domestic consumers.

The duration of a patented imitation depends on the probability of the $(k_I+1)^{th}$ successful innovation, $pb_I(k, j, t \mid m)$ – second row in table 4.1. In the case of the value of a patented innovation, $V_I(k, j, t \mid m)$, the challenge comes from both the $(k_I+1)^{th}$ innovation and the k_I^{th} imitation – first and third rows in table 4.1. As a result, the duration depends on $pb_I(k, j, t \mid m) + pb_P(k, j, t \mid m) - pb_I(k, j, t \mid m)\, pb_P(k, j, t \mid m)$.

Note that the 'feedback effect' between countries is clear in both $V_P(k, j, t \mid m)$, since it depends on $pb_I(k, j, t \mid m)$, and in $V_I(k, j, t \mid m)$, since it relies on $pb_P(k, j, t \mid m)$.

4.5.4. Equilibrium probability of successful R&D and of the R&D effort

From $V_Z(k, j, t \mid m)$, we can obtain the equilibrium quantity of domestic R&D effort, $rs_Z(k, j, t \mid m)$, and so the equilibrium domestic probability of successful research, $pb_Z(k, j, t \mid m)$. This is because the functional form in equation (4.37) of the probabilities of successful research depends on the resources allocated to it. At each time t, the after subsidy domestic cost of research is $[1-s_{rZ}]\, rs_Z(k, j, t \mid m)$, which incites the domestic probability of successful R&D $pb_Z(k, j, t \mid m)$. Hence, the expected reward for pursuing the k_I^{th} imitation and the $(k_I+1)^{th}$ innovation are, respectively, $pb_P(k, j, t \mid m)\; V_P(k, j, t \mid m)$ and $pb_I(k, j, t \mid m)\; V_I(k+1, j, t \mid m)$. Thus, the expected present discounted flow of net profit, $\Pi_Z(k, j, t \mid m)$, from successful research in an IG j that is currently at quality k is:

$$\Pi_P(k, j, t \mid m) = pb_P(k, j, t \mid m)\; V_P(k, j, t \mid m) - [1 - s_{rP}]\, rs_P(k, j, t \mid m); \qquad (4.40a)$$

$$\Pi_I(k, j, t \mid m) = pb_I(k, j, t \mid m)\; V_I(k+1, j, t \mid m) - [1 - s_{rI}]\, rs_I(k, j, t \mid m). \qquad (4.40b)$$

With free entry equilibrium into the research business, the expected revenue of domestic research coincides with the respective after subsidy cost. Hence, both terms $\Pi_Z(k, j, t \mid m)$ in equations (4.40a, b) are equal to zero. Then, plugging equations (4.37) for $Z = P$ and (4.39a) in equation (4.40a), and solving for pb_I, the equilibrium probability of a successful innovation, given the interest rate, the HCs and the FGs' price indexes, is:

$$pb_I(k, j, t \mid m) = pb_I(m) = \beta_P\, \zeta_P^{-1}\, exp(PE+IT)\, g_1(.)^{\bar{\sigma}_I}\, g_2(.)^{\sigma_2(.)}\, \tilde{q}^{(j,t \mid m)}\, \iota_P$$
$$\bar{m}\; (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{MC_I - MC_P}{1 - s_{rP}} \right) mAp(t) - r_P(t), \forall k \; ; \qquad (4.41)$$

$$\text{where: } mAp(t) \equiv \left[\frac{m_P(t|w)}{m_P(t|w)+m_I(t|w)} \left(\frac{A_P\, p_P^m(t)}{MC_I - s_{xP}} \right)^{\frac{1}{\alpha}} + \frac{m_I(t|w)}{m_P(t|w)+m_I(t|w)} \left(\frac{A_I\, p_I^m(t)}{MC_I - s_{xP}} \right)^{\frac{1}{\alpha}} \right].$$

Equation (4.41) indicates that the equilibrium *m*-specific pb_I for a new quality of IG *j* is higher when profits from sales, *mAp*, are higher. In turn, the profits will be higher when the price indexes, p_Z^m, are higher. It also follows that the equilibrium pb_I turns out to be independent of *j* and *k*. There are two reasons behind this independence. The first and most substantial one is the removal of scale of TK effects – the positive influence of the quality rung on profits and on the learning effect is exactly offset by its effect on the complexity cost – see the exponents of *q* in equations (4.35c) and (4.37). The other is the simplifying assumption that the determinants of imitation capacity, *PE* and *IT* in the catching up term in equation (4.34)-(vi), are not specific to each IG.

As we have said, additional scale effects could arise through market size, as has been strongly discussed in the R&D endogenous growth literature since Jones' (1995a, b) critique. In short, due to the technological complementarity in the production function (4.1), in our model, the size of the market for *m*-specific IGs is the employed *m*-type HC. Then, the scale effect is apparent in the size of the profits in equation (4.35c) – see the HC terms within square brackets. Since we aim at understanding IT effects other than market size, the removal of scale is in order. The adverse effect of market size due to the scale-proportional difficulty of introducing new quality IGs is designed to offset the scale effect on profits – see equation (4.34)-(iv). With $\xi = 1$, offsetting is such that the market size effect becomes negligible, as is clear in expression *mAp* in equation (4.41). Then, only the price channel affects the innovation activity and its direction: relatively more expensive versatile FGs broaden the versatile-specialised TK bias, and *vice-versa*.

Equation (4.41) also shows that now pb_I is affected by the imitation activity due to the feedback effect between countries.[226] By reason of this effect, for example, the positive level effect from the North to the South when IT of IGs emerges – *i.e.*, Southern access to state-of-the-art IGs increases production and so the resources available to imitation R&D – feeds back into the North, affecting its TK through creative destruction.

[226] The equilibrium *m*-specific pb_I of each IG depends on the South-North TK gap of that IG, which at first sight can differ across IGs. However, since our framework is a general equilibrium model – *i.e.*, since we are not interested in partial equilibrium –, we can consider the representative domestic IG of each type defined as the average of all domestic IGs of this type, in line with our considerations in the previous regime. Then, the equilibrium *m*-specific pb_I depends on the aggregate South-North TK ratio. Note that

We can in addition specifically state that, even without international IPRs protection, imitation has two groups of conflicting effects on innovation, one potentially slowing and the other potentially increasing the rate of innovation. In fact, Northern IGs firms' expected present values of their profits are reduced, since imitation decreases the expected duration of their monopoly position. However, after an imitation the only way in which innovator firms can regain profits is to come up with – or support – the next innovation. Moreover, a higher probability of successful imitation increases the speed of TK transfer and so improves the ability of the South to benefit from innovations and generates an efficient allocation of production in the South, where *MC* is lower, which turns out to matter not only at home but also abroad, due to the feedback effect.

Since from chapter 2 – *e.g.*, figure 2.4 – $\Phi_P - \Phi_I \geq 0$, then the experience-adjusted probability of imitation, Φ_P, in equation (4.41), positively affects the equilibrium pb_I. Moreover, IT affects positively Φ_P via $exp(IT)$ in the catching up term. However, since $d < \tilde{Q}^{Jm} < 1$ then $\Phi_P \tilde{Q}^{Jm} < \Phi_P$, which reflects the decrease in the expected present value of Northern firms due to imitation. Thus, the equilibrium pb_I tends to be greater under the IT of IGs than in the previous regime when $\Phi_P\big|_{\text{without IT effects}} < \Phi_P \ \tilde{Q}^{Jm}\big|_{\text{IT of IGs}}$. If this is so, the increase in the equilibrium pb_I incited by imitation is boosted by the availability of resources for R&D in the North.[227] In fact, imitation shifts IGs production to the South and so induces Northern IGs firms to produce in less time. Thereby, the North devotes fewer resources to IGs production and can devote more resources to R&D.

Equation (4.41) also indicates that the equilibrium pb_I can be encouraged both by the increase in the exogenous given ad-valoren subsidy rates of R&D, s_r, and of IGs production, s_x. Moreover, under the IT of IGs, a new issue emerges, which is the effects of international IPRs. A major argument in favour of tighter IPRs in the South has been that they encourage innovations from which the countries of the world benefit. However, in accordance with Grossman and Helpman (1991d) and Helpman (1993), among others,[228] equation (4.41) tells us that this argument is not convincing, since the equilibrium pb_I is lower when there is stronger protection from imitation.

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while in the previous regime only the equilibrium *m*-specific pb_P – *e.g.*, equation (2.48) – can differ across the two types of IGs, now the same holds for the equilibrium *m*-specific pb_I – *e.g.*, equations (4.41).

[227] Moreover, the increase in pb_I generates positive spillovers for imitation, since each new innovation increases pb_P. This is because pb_P depends positively on the TK gap. The feedback effect is thus clear.

[228] In the same way as, for example, Segerstrom *et al.* (1990), a lengthening of patent duration for Northern firms lowers the rate of product innovation.

Actually, there are also two counteracting effects resulting from enforcing stronger international IPRs protection. It lowers pb_P and prolongs the expected duration of monopoly of each Northern innovator, raising the returns to innovation. However, since firms produce longer in the North, they devote more resources to IGs production and less to R&D; that is, as a result of IPRs protection, the North devotes fewer resources to R&D. This lowers the equilibrium m-specific pb_I at each time. Equation (4.41) suggests that the second effect dominates the first, and the equilibrium m-specific pb_I declines. Therefore, we can counter argument that tighter international IPRs only strengthen the monopoly power of firms located in the North, to the detriment of firms in the South.

We can also find the equilibrium m-specific probability of successful imitation, by plugging equations (4.36) and (4.39b) into equation (4.40b):[229]

$$
\begin{aligned}
pb_P(m) = \Bigg\{ & \frac{\beta_I}{\zeta_I}\,\overline{m}\,(1-\alpha)^{\frac{1}{\alpha}}\,q^{-1}\left(\frac{1}{1-s_{rl}}\right)\Bigg\{\frac{m_P(t|w)}{m_P(t|w)+m_I(t|w)}\Bigg[\gamma(m)\Bigg[q\left(MC_P-s_{xI}\right)-\left(MC_I-s_{xI}\right)\Bigg] \\
& \left(\frac{A_P\,p_P^m}{MC_P-s_{xI}}\right)^{\frac{1}{\alpha}}+\left(1-\gamma(m)\right)\Bigg[q\left(MC_I-s_{xI}\right)-\left(MC_I-s_{xI}\right)\Bigg]\left(\frac{A_P\,p_P^m}{MC_I-s_{xI}}\right)^{\frac{1}{\alpha}}\Bigg]+\frac{m_I(t|w)}{m_P(t|w)+m_I(t|w)}\cdot \\
& \Bigg[\gamma(m)\Bigg[q\left(MC_P-s_{xI}\right)-\left(MC_I-s_{xI}\right)\Bigg]\left(\frac{A_I\,p_I^m}{MC_P-s_{xI}}\right)^{\frac{1}{\alpha}}+\left(1-\gamma(m)\right)\Bigg[q\left(MC_I-s_{xI}\right)-\left(MC_I-s_{xI}\right)\Bigg] \\
& \left(\frac{A_I\,p_I^m}{MC_I-s_{xI}}\right)^{\frac{1}{\alpha}}\Bigg]\Bigg\}-\left(r_I+pb_I(m)\right)\Bigg\}\div\left(1-pb_I(m)\right)
\end{aligned}
\tag{4.42}
$$

The comparison between equations (4.37) with $Z=P$ and (4.42) shows that the effect of stronger international IPRs protection is clear on pb_P – equation (4.37) – but not on its equilibrium value – equation (4.42). In fact, international IPRs protection reduces the equilibrium pb_I – equation (4.41) –, which helps to prolong the time of expected duration of Southern profits. This is present in equation (4.42) but not in equation (4.37).

Now, in order to determine the amount of resources devoted to R&D in IG j, we start by writing equation (4.34) – or equation (4.37) – in the following way:

$$
\begin{aligned}
rs_Z(k,j,t\,|\,m) = \beta_Z^{-1}\,\zeta_Z\Big[m_I(t\,|\,w)+m_P(t\,|\,w)\Big]\,q^{k(j,t|m)\left[\frac{1-\alpha}{\alpha}\right]}\,pb_Z(k,j,t\,|\,m) \\
\Big[exp\,(PE+IT)\;g_1(.)^{\overline{\sigma}_1}\,g_2(.)^{\sigma_2(.)}\;\tilde{q}^{(j,t|m)}\;\iota_P\Big]^{-\Gamma_Z}
\end{aligned}
\tag{4.43}
$$

[229] We do not incorporate the equilibrium pb_I – equation (4.41) – in equation (4.42), so as to obtain the smallest possible equation for the equilibrium pb_P. As we will see ahead, in steady state both countries face the same equilibrium interest rate, $r_p = r_I = r$, and this can be used to slightly simplify the expression.

From equation (4.43), when IGs are TK more advanced and have a large market; *i.e.*, with higher $k = k_I$ and $[m_I(t|w)+m_P(t|w)]$, then a greater amount of research resources will be devoted to them. However, the equilibrium pb_Z is invariant with k and $[m_I(t|w)+m_P(t|w)]$. Then, more effort – *i.e.*, resources – is required in more advanced TK IGs and with larger markets in order to generate the same probability of success.

Equation (4.43) with $Z = I$ shows that the resources devoted to innovation are smaller if pb_I is lower. Equation (4.43) with $Z = P$ indicates that there are also two counteracting effects of enforcing international IPRs in the resources devoted to imitation. First, they decrease due to the smaller pb_P. Second, the less efficient 'imitation capacity' generated leads to an absorption of more resources. When the second effect dominates the first one, the resources devoted to imitation increases. In this case, we can say that there is a 'resource wasting effect' on the South.

From equation (4.43), taking into account equations (4.41) and (4.42), the definition $RS_Z(k,t|m) \equiv \int_0^{J^m} rs_Z(k,j,t|m)\, dj$, and after that the aggregate resources devoted to R&D in each country – which are $RS_Z(k,t) = RS_Z(k,t|SH) + RS_Z(k,t|VH)$ –, then:

$$RS_Z = Q_I^{J^{SH}} \left[SH_P(.|w) + SH_I(.|w) \right] \left\{ \frac{\zeta_Z}{\beta_Z} \frac{pb_Z(SH)}{\left[exp(PE+IT)\, g_1(.)^{\bar{\sigma}_1}\, g_2(.)^{\sigma_2(.)}\, \tilde{q}^{(j,.|SH)}\, \iota_P \right]^{\Gamma_Z}} \right\} + \\ + Q_I^{J^{VH}} \left[VH_P(.|w) + VH_I(.|w) \right] \left\{ \frac{\zeta_Z}{\beta_Z} \frac{pb_Z(VH)}{\left[exp(PE+IT)\, g_1(.)^{\bar{\sigma}_1}\, g_2(.)^{\sigma_2(.)}\, \tilde{q}^{(j,.|VH)}\, \iota_P \right]^{\Gamma_Z}} \right\} \quad (4.44)$$

Equation (4.44) reveals that the equilibrium aggregate resources devoted to R&D in each country depend positively on market profitability ('market size effect' in *RS*), on the Northern TK and on the equilibrium pb_Z. Hence, the imitation disincentive originated by, for example, international IPRs lowers pb_P – equation (4.37) – and so it lowers the equilibrium pb_I – equation (4.41). Thus, this drives to smaller resources devoted to R&D.

Equation (4.44) also indicates that aggregate resources devoted to R&D depend negatively on Φ_I^{-1}, in the North, and on Φ_P^{-1}, in the South. Thus, it shows that the resources devoted to imitation further depend directly and positively on the value of the international IPRs, which generates a less efficient 'imitation capacity' that absorbs more Southern resources. That is, all else equal, the more intense the international IPRs protection – *i.e.*, the smaller ι_P –, the more resources the South devotes to R&D, due to the intensity of the 'resource wasting effect'.

Finally, since $Q_I^{Jm} > Q_P^{Jm}$, we expect that, all else equal, the total resources devoted to R&D in the North under IT of IGs will be greater than in the previous regime without IT effects, as a result of the assumption that imitative firms must imitate all quality levels between their current domestic levels and the leader innovator IGs they are targeting.

4.5.5. Behaviour of the aggregate quality indexes

Since the probability of successful innovation – as a Poisson arrival rate – determines the speed of TK progress, equilibrium can be translated into the path of Northern TK, from which free IT in IGs allows the South to benefit as well. The relationship comes to yield the following expression – in which equation (4.41) is plugged – for the equilibrium rate of growth of m-specific TK:[230]

$$\frac{\dot{Q}_I^{Jm}}{Q_I^{Jm}} = pb_I(m)\left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right] = \left\{\beta_P\ \zeta_P^{-1}\ exp(PE+IT)\ g_1(.)^{\bar{\sigma}_1}\ g_2(.)^{\sigma_2(.)}\ \tilde{q}^{(.|m)}\ \iota_P\right.$$
$$\left.\overline{m}\ (1-\alpha)^{\frac{1}{\alpha}}\left(\frac{MC_I - MC_P}{1 - s_{rP}}\right) mAp\ -\ r_P\right\}\left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right]; \qquad (4.45)$$

To fully characterise aggregate economies, we need to consider the behaviour of individuals as consumers. However, in our framework, the IT of IGs does not influence this behaviour and, thereby, section 2.6 remains valid here. Furthermore, the government budget, as well as aggregate resource constraints since IT is balanced, also remain the same as in section 2.7. Thus, plugging equation (4.45) into equation (4.32) and equating the resulting m-type HC demand path to the necessary condition obtained from optimisation by individuals (supply side) – equation (2.70) – the equilibrium interest rate in the countries is obtained. By Walras' law, this is also the interest rate that clears the assets market – savings by individuals are equal to the value of the firms. The general equilibrium instantaneous growth rate of the countries at time t results then from plugging the equilibrium interest rate into the Euler equation (2.64).

In the next section, we are going to analyse the steady state in full.

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[230] In simple form, we can state that, since $k(j,t|m)$ jumps up to $k(j,t|m)+1$ when successful innovation occurs in IG j, and the equilibrium pb_I is constant across m-type IGs, then the time derivative of Q_I^{Jm} is:

$$\dot{Q}_I^{Jm}(t) = \int_0^{J^m}\left[q^{[k(j,t|m)+1]\left(\frac{1-\alpha}{\alpha}\right)} - q^{k(j,t|m)\left(\frac{1-\alpha}{\alpha}\right)}\right] pb_I(k,j,t|m)\,dj = \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right] pb_I(k,j,t|m)\,Q_I^{Jm}(t).$$

4.6. The steady state

This section is directed at the analysis of the steady state properties of the model, under IT of IGs. To this end, it is divided in two subsections. In the first, we show that there is a unique and stable BG path when the countries open up to the IT of IGs and we make a comparative steady state analysis. In the second, we analyse other related steady state effects of international TK diffusion, namely in terms of the growth benefit for both countries, convergence in levels between countries, prices and wages.

4.6.1. Steady state growth

Common steady state growth rate

From the previous analysis, we know that:

(i) by assumption, countries have the same technology of production of FGs,[231] resulting that the composite FG production has constant returns to scale in the inputs $m(.|w)$ and Q_Is – see equation (4.30a, b). Inputs Q_Is are present in both countries since they have access to the same state-of-the-art IGs through the free IT of IGs. The inputs $m(.|w)$ are country specific since there is no international mobility of production factors.

(ii) Composite FG production is used for consumption and savings, which, in turn, are allocated between production of IGs and R&D – see equation (2.72). Then, Y, X, RS and C are all constant multiples of both the levels of TK in the North and the domestic levels of HC at work – *i.e.*, $Q_I^{J^{SH}} SH(.|w)$ and $Q_I^{J^{VH}} VH(.|w)$;[232]

(iii) Also, by assumption, countries have the same technology of HC accumulation.

Together, (i), (ii) and (iii) imply that the steady state growth rates – and through Euler equation (2.64), the interest rates, r^{ss} – are equalised between countries. The constant common (unique) and stable steady state growth rate, again designed by gr, is:

$$\frac{\dot{Q}_I^{J^m}}{Q_I^{J^m}} + \frac{\dot{m}(.|w)}{m(.|w)} = \frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{RS}}{RS} = \frac{\dot{C}}{C} = \frac{\dot{c}}{c} = \frac{1}{\theta}\left[(1-\tau_K)\, r^{ss} - \rho\right] = gr\,. \qquad (4.46)$$

---

[231] Except for the exogenous level of productivity, A, and the level of HC at work, $m(.|w)$, which implies differences in the levels but not in the growth rates.

[232] In fact, that this is so for RS, X and Y can be seen in equations (4.44), (4.29a, b) and (4.30a, b), respectively. The same is true for consumption by solving the aggregate resource constraint in equation (2.72) for C. This is certainly true when $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ are constant, since the resources devoted to R&D – and so to C – are functions of the variables $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$, via the equilibrium pb_Z – see equation (4.44). Thus, regardless of the country, all variables of the aggregate budget constraint are a constant multiple of the aggregate Northern quality indexes, of the level of HC employed in both, of the gap in HC at work and of the TK gap between them. The dependence on HC in the other country and on the TK gap is introduced by the action of RS or, to be clearer, by the action of the IT of IGs.

Equation (4.46) shows that steady state growth is driven by Northern TK progress and by HC accumulation.[233] Although the Northern TK commands, it is affected by Southern demand for IGs which, in turn, depends on Southern levels of HC at work, and by Southern imitation – *e.g.*, equation (4.45). From the previous section, a higher steady state interest rate implies a lower current value of future profits and thus lower expected marginal profits, due to a discount rate effect. From equation (4.46), it also leads to a higher growth rate and associate to this, a large R&D activity. Both considerations imply that a higher r^{ss} induces a strong R&D activity that shortens the duration of monopolist positions – *i.e.*, there is a strong process of creative destruction. Once in steady state:

$$\frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}}+\frac{\dot{SH}(.|w)}{SH(.|w)}=\frac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}}+\frac{\dot{VH}(.|w)}{VH(.|w)}, \text{ then } \frac{\dot{p}^{SH}}{p^{SH}}=\frac{\dot{p}^{VH}}{p^{VH}}=0. \tag{4.47}$$

Note that in equation (4.46), it is already implicit that $\tilde{m}(.|w)$ and $\tilde{Q}^{J^m} \equiv \tilde{q}(.|m)$ are constant – *i.e.*, once in steady state, equation (4.47) holds, and we also find that:

$$\frac{\dot{SH}_I(.|w)}{SH_I(.|w)}-\frac{\dot{SH}_P(.|w)}{SH_P(.|w)}=\frac{\dot{VH}_I(.|w)}{VH_I(.|w)}-\frac{\dot{VH}_P(.|w)}{VH_P(.|w)} \Leftrightarrow \frac{\dot{\tilde{SH}}(.|w)}{\tilde{SH}(.|w)}=\frac{\dot{\tilde{VH}}(.|w)}{\tilde{VH}(.|w)} \text{ and } \frac{\dot{\tilde{Q}}^{J^{SH}}}{\tilde{Q}^{J^{SH}}}=\frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}}. \tag{4.48}$$

As in steady state $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ are constant and $r_I = r_P = r^{ss}$, the steady state m-specific pb_Z will also be constant and $\dot{MC}^m_{\Xi\gamma}/MC^m_{\Xi\gamma}=0$. In fact, looking at equation (4.13) $MC^m_{\Xi\gamma}$ is constant in steady state when $\gamma(m)$ and $\Xi(m)$ are also constant, which, looking at equations (4.14), (4.15a, b) and (4.48), does in fact occur. The steady state measure of IGs with Northern production, $\Xi(m)$, is an increasing function of the steady state rate of innovation, pb_I, and a decreasing function of the steady state rate of imitation, pb_P. The converse is true for the measure of IGs with Southern production, $1-\Xi(m)$. This is similar to the result obtained by Dinopoulos and Segerstrom (2004). Bearing in mind that $\dot{MC}^m_{\Xi\gamma}/MC^m_{\Xi\gamma} = \dot{p}^m_Z / p^m_Z = 0$, then on account of equation (4.32) the wages growth rates increase steadily, and only according to Northern TK progress.

Thus, the common steady state growth rates of HC and TK imply the maintenance of steady state North-South gaps in HC and TK. While complete convergence in available (Northern) TK is instantaneous with IT (level effect), domestic levels may not converge completely, since $\tilde{Q}^{J^m}$ stays below one and $\tilde{m}(.|w)$ may also stay below one.

[233] For example, without HC accumulation endogenous growth is only driven by Northern TK progress.

Following the steps proposed to obtain the equilibrium r, the constant r^{ss} is:

$$r^{ss} = \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - \tau_{KZ} \right]^{-1} \left\{ \beta_P \, \zeta_P^{-1} \, exp(PE+IT) \, g_1(.)^{\bar{\sigma}_1} \, g_2(.)^{\sigma_2(.)} \, \tilde{q}^{(.|m)} \, \iota_P \, \bar{m} \, (1-\alpha)^{\frac{1}{\alpha}} \right.$$
$$\left. \left(\frac{MC_1 - MC_P}{1 - s_{rP}} \right) mAp \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right] - \delta^m + \varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)} \right]^{\frac{1-\phi}{\phi}} \right\} . \quad (4.49)$$

Where – remember – p^m's, g_1 and g_2 as well as σ_2 are in steady state and so are constant.

Plugging r^{ss} – equation (4.49) – into the growth rate of consumption – Euler equation (2.64) –, the (world) steady state growth rate becomes given by:

$$gr = \left[\frac{1}{\theta} \left(\frac{1 - \tau_{KZ}}{q^{\left(\frac{1-\alpha}{\alpha}\right)} - \tau_{KZ}} \right) \right] \left\{ \beta_P \, \zeta_P^{-1} \, exp(PE+IT) \, g_1(.)^{\bar{\sigma}_1} \, g_2(.)^{\sigma_2(.)} \, \tilde{q}^{(.|m)} \, \iota_P \, \bar{m} \left(\frac{MC_1 - MC_P}{1 - s_{rP}} \right) \right.$$
$$\left. (1-\alpha)^{\frac{1}{\alpha}} \, mAp \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right] - \delta^m + \varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)} \right]^{\frac{1-\phi}{\phi}} - \rho \left(\frac{q^{\left(\frac{1-\alpha}{\alpha}\right)} - \tau_{KZ}}{1 - \tau_{KZ}} \right) \right\} . \quad (4.50)$$

Hence, gr in equation (4.50) is determined by: (i) individuals' preferences; (ii) HC production technology – equal in both countries; (iii) FGs production technology in both countries; (iv) R&D activity, which links both economies; and (v) government policies.

The $\dot{Q}_I^{J^{m^{ss}}} / Q_I^{J^{m^{ss}}}$ is constant and from equation (4.46) so is $\dot{m}^{ss}(.|w)/m^{ss}(.|w)$:

$$\frac{\dot{m}^{ss}(.|w)}{m^{ss}(.|w)} = \frac{1}{\theta} \left[(1 - \tau_K) \, r^{ss} - \rho \right] - \frac{\dot{Q}_I^{J^{m^{ss}}}}{Q_I^{J^{m^{ss}}}} . \quad (4.51)$$

As HC accumulation is dependent on the m-type HC, but independent of individual ability, and as in steady state the time allocation does not change over time – *i.e.*, individuals decide to devote a constant fraction of the fixed-time endowment to working and to training –, then from equations (2.59), (2.65) and (4.46), we obtain:[234]

$$u_S^{ss}(m) = \left\{ \frac{1}{\theta} \left[(1 - \tau_K) \, r^{ss} - \rho \right] - \frac{\dot{Q}_I^{J^{m^{ss}}}}{Q_I^{J^{m^{ss}}}} + \delta^m \right\} \left\{ \varphi^{m[1/\phi]} \chi_S \left[1 + \frac{u_T(m)}{u_S(m)} \right]^{\frac{1}{\phi}} \right\}^{-1} . \quad (4.52)$$

Once the constant value for $u_S^{ss}(m)$ in equation (4.52) is known, we obtain the values for $u_T^{ss}(m)$, by equation (2.65), and for $u_w^{ss}(m)$, by the resource constraint on each

[234] Considering the assumed condition that $(1 - \tau_K) \, r^{ss} > \rho$, then it also assures that $0 < u_S^{ss}, u_T^{ss}, u_w^{ss} < 1$.

m-type HC: $u_w^{ss}(m) = 1 - u_S^{ss}(m) - u_T^{ss}(m)$. From equation (4.52), $u_S^{ss}(m)$, $u_T^{ss}(m)$ and $u_w^{ss}(m)$ are equal in both countries, as in the previous regime – see equation (3.6).

Thus, the steady state is a path where all variables either grow at a constant rate[235] or are time invariant,[236] such that individuals maximise their lifetime utility by allocating their time between production and training and their income between consumption and saving, and each (IG, FG and R&D) firm maximises its profits, and all the markets clear.

Comparative steady state analysis

The common steady state growth rate of both countries given by equation (4.50) allows further comparative steady state analysis. Table 4.2 sums up the qualitative results, only from exogenous parameters and exogenous variables in equation (4.50). The impact of other parameters, such as β_I, requires that the steady state growth rate be found considering the steady state probability of successful imitation. However, we have decided to avoid the resulting expression, since it would be very large.

Table 4.2. Comparative steady state analysis

	∂gr		∂gr		∂gr		∂gr
∂A_I	+	$\partial \beta_P$	+	∂d	–	$\partial \tau_K$	–
∂A_P	+	$\partial \zeta_P$	–	$\partial \iota_P$	+	$\partial \varphi^{VH}$	+
$\partial \overline{m}$	+	∂PE	+	$\partial \theta$	–	$\partial \chi_S$	+
$\partial \alpha^{a)}$	–	∂IT	+	$\partial \rho$	–	$\partial \chi_T$	+
∂MC_I	+	$\partial \overline{\sigma}_1$	+	∂s_r	+	$\partial \delta^m$	–
∂MC_P	–	$\partial \overline{\sigma}_2$	+	∂s_x	+	$\partial \phi$	+

Note: [a)] considering the particular case $q = 1/(1-\alpha)$.

The countries' exogenous levels of productivity, A_I and A_P, affect positively the levels of output – *e.g.*, equations (4.30a, b). Furthermore, they also affect the world growth rate, due to the role of competitive monopoly profits – *e.g.*, equations (4.35c) and (4.36). Thus, IT emphasises the importance of improvements in domestic institutions in both countries, because they become significant not only at home but also abroad.

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[235] As is the case of Y, C, X, RS, $m(w)$, $m(school)$, $m(OJT)$, and Q_I^m.

[236] As is the case of u_S, u_T, N, p^m, $\tilde{m}$, $\tilde{Q}^{Jm}$, r, pb_I and pb_P.

A higher Northern MC of production implies a higher world growth rate when $MC_P > s_r$, as occurred under the baseline parameter values in table 3.4. However, a higher Southern MC of production implies a smaller world growth rate. A higher MC_I leads to a higher mark-up for monopolist producers in the South – *e.g.*, equations (4.11c) and/or table 4.1 –, whereas a higher MC_P implies a smaller mark-up – *e.g.*, equation (4.35c). Hence, a higher MC_I provides an incentive to imitation activity, which, in turn, affects positively the equilibrium probability of successful innovation – see equation (4.41) – and thus world growth, while the inverse holds when MC_P is higher.

All else being equal, higher learning to past successful research spillover in the South, β_P, smaller fixed cost of the successful research parameter, ζ_P, better Southern policies promoting R&D, *PE*, increased South-North interaction, *IT*, higher semi-elasticity of the probability of successful imitation with respect to the gap in HC at work and with respect to $g_2(.)$, $\overline{\sigma}_1$ and $\overline{\sigma}_2$, respectively, and a small threshold TK gap, d, enhance imitation activity and, consequently, this feeds back into the North, increasing the probability of successful innovation as well as the steady state world growth rate.[237]

International IPRs protection – *i.e.*, $\iota_P < 1$ – decreases the steady state world growth rate as a result of its direct negative influence on the probability of successful imitation, which is mirrored in a negative effect on the equilibrium probability of successful innovation. Hence, the steady state world growth rate will be unambiguously greater without international protection. Therefore, international competition favours growth.

The m-augmenting parameter $\overline{m}$, the HC share in the production function α, the discount rate ρ, the inverse of the intertemporal elasticity of substitution θ, the subsidies' exogenous ad-valorem s_x and s_r; the assets income tax τ_K, the HC income tax τ_w, the distribution parameter φ^m, the substitution parameter ϕ, the productivity of schooling and OJT activities χ_S and χ_T, respectively, have the same influence on the world steady state growth rate as in the previous regime without IT of IGs and for the same reasons.

Once again, the endogeneity of the steady state growth rate is preserved since it is affected by political factors and the preferences of individuals. In short, exogenous parameters and constant variables that directly or indirectly positively affect the equilibrium probability of successful innovation and HC accumulation have a positive effect on the world growth rate, and *vice-versa*.

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[237] And each innovation lowers the cost of imitation: positive spillovers from innovation to imitation occur.

4.6.2. Steady state effects

We now come to the technological implications of steady state growth in both countries, since now, due to IT, IGs flow from North to South and South to North.

Countries' growth benefits with North-South TK diffusion and IT of IGs

Steady state endogenous growth is driven by R&D and HC accumulation – *e.g.*, equation (4.46). This feature is not, however, specific to the IT of IGs regime – *e.g.*, equation (3.1). In order to look at the steady state effects of the IT of IGs we must investigate *gr* further. To this end, we compare the world steady state interest rate in equation (4.49) with the steady state interest rate that would prevail: (i) in the South, without the international flow of goods, assets, HC and TK; and (ii) in the world, under TK diffusion without IT effects, given by equation (3.3).[238]

In the former case, (i), the probability of successful imitation collapses into an inefficient Southern probability of successful innovation, where the catching up term disappears – *i.e.*, the experience-adjusted probability of successful imitation, Φ_P, changes to $\Phi_P^* \equiv \beta_P \, \zeta_P^{-1}$, see subsection 3.2.2. The increment in the steady state interest rate, from case (i) above to IT of IGs and imitation possibility depends on the following difference:

$$\left[exp(IT+PE)\; g_1(.)^{\bar{\sigma}_1}\; g_2(.)^{\sigma_2(.)}\; \tilde{q}^{(.|m)}\; \iota_P\; (MC_I - MC_P)\; mAp \right]\Big|_{\text{IT of IGs}} -$$

$$- \left[(MC_P - s_{xP}) \left(\frac{q-1}{q} \right) \left[p_P^m A_P \left(\frac{1}{MC_P - s_{xP}} \right) \right]^{1/\alpha} \right]\Big|_{\text{without technological knowledge diffusion}} \quad ; \qquad (4.53a)$$

and the increment in the steady state interest rate, from case (ii) above to IT of IGs and imitation possibility depends on the two following differences, which are equal:

$$\left[exp(IT)\; g_1(.)^{\bar{\sigma}_1}\; g_2(.)^{\sigma_2(.)}\; \tilde{q}^{(.|m)}\; \iota_P\; (MC_I - MC_P)\; mAp \right]\Big|_{\text{IT of IGs}} -$$

$$- \left[g_1(.)^{\sigma_1}\; g_2(.)^{\sigma_2(.)}\; (MC_P - s_{xP}) \left(\frac{q-1}{q} \right) \left[p_P^m\; A_P \left(\frac{1}{MC_P - s_{xP}} \right) \right]^{1/\alpha} \right]\Big|_{\text{without IT effects}} \quad ; \qquad (4.53b)$$

$$\left[\frac{\beta_P}{\zeta_P}\; exp(IT+PE)\; g_1(.)^{\bar{\sigma}_1}\; g_2(.)^{\sigma_2(.)}\; \tilde{q}^{(.|m)}\; \iota_P\; (MC_I - MC_P)\; mAp \right]\Big|_{\text{IT of IGs}} -$$

$$- \left[\frac{\beta_I}{\zeta_I}\; (MC_I - s_{xI}) \left(\frac{q-1}{q} \right) \left[p_I^m A_I \left(\frac{1}{MC_I - s_{xI}} \right) \right]^{1/\alpha} \right]\Big|_{\text{autarkic North}} \quad . \qquad (4.53c)$$

[238] Remember that, in this case, due to the TK, the steady state growth rate is the same in both countries.

The steady state growth rate with North-South TK diffusion by imitation and IT of IGs is greater than the steady state growth rate in the other two cases when the increment in equations (4.53a-c) is positive. The evaluation of equations (4.53a-c) requires solving for transitional dynamics through calibration and simulation. We can, though, emphasise four ways, in addition to the level effects, through which the IT of IGs influences, in opposite directions, steady state growth:

(i) the first is the positive catching up effect on the probability of successful imitation. Imitation capacity increases with the index of openness, *IT*. Through the feedback effect described above, the probability of successful innovation and thus the steady state growth rate are also positively affected – see equations (4.41) and (4.45).

(ii) The second are the positive spillovers from North to South, because each new innovation by the North tends to lower the cost of Southern imitation.

(iii) The third – counteracting – channel is the monopolistic competition mark-up. In equations (4.53a-c), the Southern monopolist's mark-up under IT of IGs is $(MC_I - MC_P)$ – *i.e.*, $(1 - MC_P)$, considering equation (4.11c) –, clearly less than the mark-up in the South in the other two cases,[239] which, ignoring the ad-valorem subsidy rate to IGs production so as to simplify, is $(q - MC_P)$ – looking at equation (2.11b).[240] This loss of profits also affects the Northern monopolist: the average mark-up between the 1st and 2nd situation in table 4.1 is smaller than without IT.[241] The reason is that without IT, Northern successful innovations and Southern imitations are protected from each other's international competition. Thus, once engaged in IT and with profitable imitation – provided that the TK threshold d is overcome –,[242] profit margins in both countries are reduced – *i.e.*, the IT of IGs reduces pricing distortions –, which acts as a disincentive to R&D activities.[243]

(iv) The fourth – counteracting as well – way through which the IT of IGs influences steady state growth, is that Southern firms have to support the R&D imitative cost of

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[239] The mark-up is mirrored in the $(MC_P - s_{xP})(q-1)q^{-1}$ term of the 2nd square bracket of (4.53a, b).

[240] In equation (2.11b) we suppose that $MC_P = 1$, remember. According to table 3.4 in subsection 3.4.2, we have that: $(1 - MC_P) = (1-0.5) = 0.50 < 2.00 = (2.5-0.5) = (q - MC_P)$ – *i.e.*, considering $q = 1/(1-\alpha)$

[241] That is, bearing in mind the after subsidy MC of production, which is $(MC_I - s_{xI})$, the average between $(1 - s_{xI})(q-1)$ and $q(MC_P - s_{xI}) - (1 - s_{xI})$ is smaller than $(1 - s_{xI})(q-1)$.

[242] If the TK threshold d is not overcome when countries open up to IT, the Southern IG firms will not support imitation – the cost of imitations will be strongly higher. As only IT of IGs is possible, the South cannot export any IG. Then, for lack of resources, it cannot also import leader IGs and no commercial relations occur between countries. As a result, existence of international IPRs does not make sense. In this case, Northern IG firms need not worry about potential imitation of their IGs, for which reason they behave exactly as if they were in autarky and TK divergence tends to occur between countries.

[243] This is different from the previous models in which the reduction of margins is offset by market enlargement – *e.g.*, Rivera-Batiz and Romer (1991), Grossman and Helpman (1991a) and Connolly (1999).

state-of-the-art Northern IGs, possibly several quality rungs above (and thus more complex) their own experience level. This is reflected in the presence of the TK gap $\tilde{q}^{(\cdot|m)}$ in the first square bracket of equations.[244]

From equation (4.53b) we can broadly state that if (i) the impact of the openness index, *IT*, is strong, (ii) the Southern *MC*, MC_P, is low, (iii) there are no international IPRs or they are low – ι_P is close to *1* –,[245] and (iv) there is no change in the Southern ad-valorem subsidy rate to IGs production, s_{xP}, between changes in IT regimes and, in addition, this ad-valorem subsidy rate is higher – by assumption $s_{xP} = s_{xI}$, remember –,[246] then the steady state growth tends to be greater under IT of IGs than with TK diffusion without IT effects. In this case, the expression in equation (4.53b) is greater than *0*.[247]

Alternatively, from equation (4.53c) we can generically state that if (i) imitation capacity is strong – *i.e.*, if (i.1) the impact of the openness index, *IT*, is strong (i.2) as well as the impact of Southern policies promoting R&D, *PE*, (i.3) the coefficient on past research experience from imitation, β_P, is higher, and (i.4) the fixed cost imitation, ζ_P, is lower – (ii) the Southern *MC*, MC_P, is low, (iii) there are no international IPRs or they are low – ι_P is close to *1* –, and (iv) there is no change in the ad-valorem subsidy rate to IGs production in North, s_{xI}, between changes in IT regimes and its value is higher,[248] then the steady state growth tends to be greater under IT of IGs than with TK diffusion without IT effects and so the expression in equation (4.53c) would be greater than *0*.[249]

If the steady state growth rate under IT of IGs is higher than the steady state growth rate with TK diffusion but without IT, then considering equation (3.7) the steady state growth rate under IT of IGs will more easily be higher than the steady state growth rate without TK diffusion and thus expression (4.53a) is greater than expressions (4.53b, c).

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[244] Hence, $\tilde{q}^{(\cdot|m)}(1-MC_P)$ in the 1st square bracket, tends to be smaller than $(1-s_{xP})(q-1)q^{-1}$, in the 2nd.

[245] Comparing our results with, for example, Dinopoulos and Segerstrom (2004) and Sener (2003) we can state that strong international IPRs protection leads to: (i) in the 1st work, a decrease in the innovation rate that is only temporarily, and not in steady state; and, (ii) in the 2nd work, a decrease in the steady state rates of imitation and innovation, and an increase in North-South wage inequality – in our case international IPRs do not (directly) affect North-South wage inequality, as we will see below. They also study the effects of increasing 'globalisation' – our existence of IT. Dinopoulos and Segerstrom find that globalisation temporarily increases the innovation rate and decreases North-South wage inequality, whereas Sener finds that globalisation decreases the steady state innovation rate and increases North-South wage inequality.

[246] The impact of the ad-valorem subsidy rate, s_{xP}, is easily justified. In fact, without IT of IGs, it does not influence the world steady state growth rate – it serves only to decrease the TK gap between countries –, whereas under IT of IGs it affects the world steady state growth rate.

[247] A smaller $(q-1)q^{-1}$ favours that equations (4.53a, b) will be greater than zero.

[248] Without IT, the ad-valorem subsidy rate, s_{xI}, affects the steady state growth rate as well. But, as there is no interdependence between countries, the impact of s_{xI} on the South does not feed back into the North.

[249] Note that once again a smaller $(q-1)q^{-1}$ favours that equations (4.53c) will be greater than zero.

In short, if the improvements in the probability of successful imitation due to the IT of IGs are sufficiently strong, both countries grow more quickly under the IT of IGs with imitation than in the other two cases. This is because once the innovations are imitated, the only way in which Northern IGs firms can capture the world market is by supporting the next innovation. A higher probability of successful imitation increases the speed of TK transfer, which improves the Southern ability to benefit from Northern innovations and also induces an efficient allocation of production in the South, where *MC* is lower.

This, together with the initial level effect induced by the IT of IGs, feeds back into the North, affecting its TK through creative destruction and so the steady state world growth rate. In fact, under the IT of IGs, the improvements in one country are significant not only at home but also abroad, due to the feedback effect. In this case, we can however state that the increase in growth ultimately also reflects a kind of scale: the instantaneous jump in the catching up term due to the openness index, *IT*.

Particularly interesting is the impact of strong international IPRs protection. It moderates the effects of IT of IGs in line with Segerstrom *et al.* (1990), Helpman (1993) and Lai (1998), Dinopoulos and Segerstrom (2004), among others.[250] In negotiations on the protection of international IPRs at the WTO, developing countries have been arguing that stronger international IPRs protection would simply generate substantial rents for Northern innovators and would not stimulate faster TK progress – see Maskus (2000). Our model seems to provide support for the position taken by developing countries.

In short, if TK diffusion, with and without IT effects, drives the South towards a TK position closer to that which exists in the North in steady state – $\tilde{Q}^{J^{m_{ss}}}$ is close to *1* – and, in addition, under IT of IGs, the *IT* index is higher, the MC_P is low and the ι_P is close to *1*, then the steady state growth under the IT of IGs will be greater. In this case, we are in the presence of a very dynamic Vernon-type IG cycle model. Moreover, (i) without international flow of goods, assets and TK, divergence would occur between countries; the same holds if there were IT of IGs and imitation but the TK threshold *d* is not overcome; (ii) international TK diffusion strongly benefits the South, as we have already seen in the last chapter; (iii) benefits seem to have expanded with free IT of IGs.

In fact, under IT of IGs, in addition to the level effects, since higher quality inputs become immediately available for FGs production, if IT of IGs strongly improves the

[250] We find however other economic models where stronger international IPRs protection often promotes innovative activity, such as Horowitz and Lai (1996) and Glass and Saggi (2002).

imitation capacity, the steady state growth rate, common to both countries, tends to be greater.[251] Thus, now the North also benefits. Moreover, the growth rate exceeds the steady state growth rate during the transition to it. We are then in line with works like Durlauf and Johnson (1995), Evans (1996) and Quah (1997).

Pattern of production, prices of final goods, wages and relation with technical progress

We now come to the technological implications of the steady state growth in both countries, in terms of pattern of production, relative prices of FGs, TK bias and wages. General figures 4.1-4.6 below compare the without international TK diffusion steady state paths with the ones resulting from a move, at time t_0, to free IT of IGs. The figures are arranged in a suitable order to accompany the sequence of analytical steps followed. This analysis will be better understood later on in transitional dynamics analysis.

Setting the steady state world growth rate of $Q_I^{J^{SH}} SH_Z(.|w)$ equal to the steady state world growth rate of $Q_I^{J^{VH}} VH_Z(.|w)$ and, in addition, considering equations (4.24) and (4.27a, b), we obtain expressions which report the steady state threshold FG, N_Z^{ss}, or the steady state price indexes, $p_Z^{SH^{ss}}$ and $p_Z^{VH^{ss}}$, with $\tilde{Q}^{J^{mss}}$, $\tilde{m}(.|w)^{ss}$ and the steady state price indexes in the other country. Thus, to find the steady state value of N_Z is now more complex. For this reason, we will only solve them numerically for each country later on.

We can, however, ascertain that due to the substitutability between ST and VT and complementarity between types of IGs and types of HC in equation (4.1), the threshold FG and relative prices of FGs are determined by the ratio of the two types of TK with the respective HC employed – *e.g.*, equations (4.23) and (4.24). Due to the steady state relationships in equation (4.46), such a ratio tends to a constant in each country and, thus, so do N and relative prices – see also equation (4.47). Despite the downward level effect in the South at time zero, differences in factor levels – as assumed by equation (4.25) – dominate the steady state, such that relative prices of FGs are always higher in the South.

Figures 4.1 and 4.2 below also show a consistent drop in the steady state levels of both N and relative prices, from without TK diffusion to IT of IGs.[252] In fact, due to differences in levels, at time zero (under IT of IGs), the North-South average relative

[251] It seems highly unlikely that integration between countries can have a large adverse effect on the world growth rate, as can happen in Gancia (2003). More specifically, in Gancia's model, sometimes globalisation increases North-South wage inequality and decreases the world economic growth rate.
[252] The magnitude of the Southern level effect at time t_0 in figures 4.1 and 4.2 can be greater or smaller.

price of versatile FGs is higher than in the North alone.[253] Through the price channel this induces relative demand for versatile-specific new designs, biasing R&D innovation in that direction more than in without international TK diffusion, as shown in both scenarios A and B of figure 4.3 below. Relative to without international TK diffusion, such steady state bias increases the world-wide supply of versatile-specific IGs, thereby lowering the steady state relative price of the versatile FGs and the TK threshold in both countries.

Figure 4.1. Final good threshold

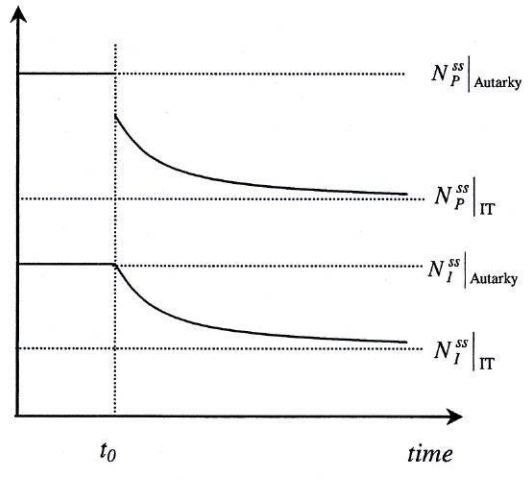
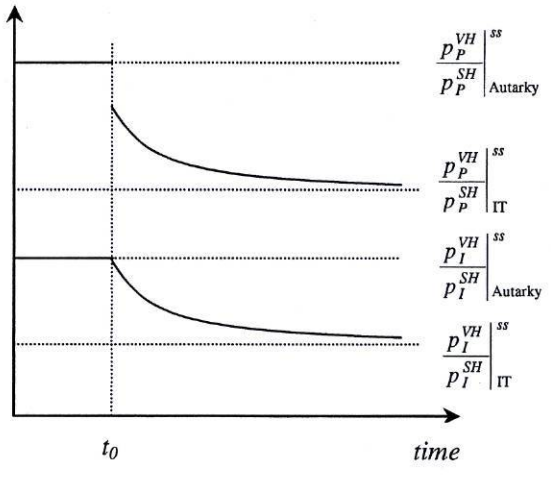


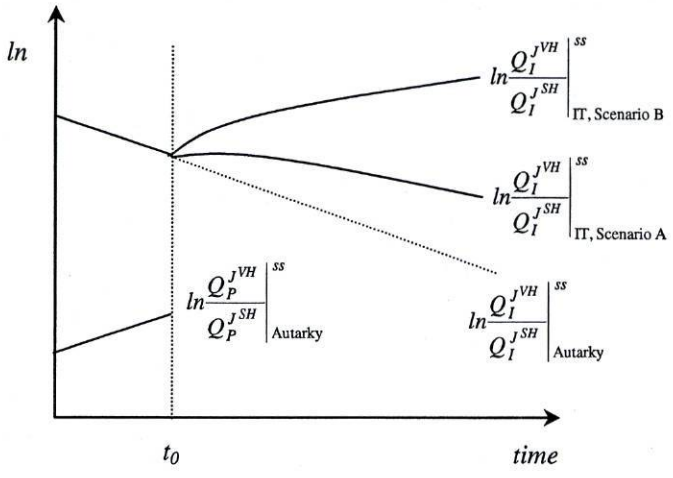
Figure 4.2. Relative price of *VH* final goods



Scenarios A and B in figure 4.3 below lead to two alternative steady state paths for the relative productivity of state-of-the-art TK.[254] Note that there is a continuum number of possible scenarios from the pattern of TK bias without TK diffusion until a pattern of a very strong versatile TK bias – stronger still than in scenario B. Scenarios A and B are chosen because they represent the two possible intra-country wage inequality paths. While the price effect on the TK bias – described above – operates in both scenarios A and B in the same direction (diminishing the rate of growth of the relative productivity of specialised TK), in case B the effect is strong enough to revert the bias.[255] Thus, in line with theoretical works like Wood (1995, 1998), Leamer (1996) and Acemoglu (1998) and with the recent estimates of Morrison and Siegel (2001) in which IT induces skill-biased TK change, in our case IT of IGs induces a greater versatile-biased TK change.

[253] That is, even after the drop at t_0 (which can be strong or not) in the relative price of Southern versatile FGs due to the level effect – access to the Northern TK induces immediate partial convergence in (non-tradable) FGs prices – North-South average relative price of versatile FGs is higher than in the North alone.
[254] The path in the South is only shown up to time zero, since its relationship with the dynamics of prices and wages is confined to the without international TK diffusion.
[255] In short, the greater relative versatile price of Southern FGs implies that now there is a stronger incentive in the North to develop versatile TK – this is a directed TK effect induced by openness to the IT of IGs.

Figure 4.3. Relative productivity of versatile TK



Furthermore, we can state that a temporary partial Schumpeterian dynamic equivalent to the static Rybczynski (1955) result holds – since it is a result of the direct action of R&D activity directed at developing new vertically differentiated IGs.[256] In fact, the IT of IGs decreases the relative productivity of specialised TK – *i.e.*, there is a relative expansion in the 'endowment' of the versatile TK –, which temporarily raises the relative output of versatile FGs – *i.e.*, $Q_I^{JVH}\, VH_Z(.|w)$ grows at a higher rate than $Q_I^{JSH}\, SH_Z(.|w)$, as is implicit in figure 4.1. Thus, there is a Rybczynski effect in the sense that there is a versatile biased expansion of the production-possibilities frontier.

Taking as a reference the analysis in the previous chapter, together with equation (4.24) and given the steady state values for the threshold FG in each country, we can find the world steady state measure of versatile-specialised-bias, which is influenced by the relative levels of the two types of HC at work in both countries; that is,

$$\frac{Q_I^{JVH}}{Q_I^{JSH}} = \left(\frac{1-N_Z^{ss}}{N_Z^{ss}}\right)^2 \frac{sh}{vh}\left(\frac{VH_Z(.|w)}{SH_Z(.|w)}\right)^{-1}. \qquad (4.54)$$

Equation (4.54) indicates that the TK progress driven by the North will be endogenously biased towards the less 'abundant' factor in the country. Furthermore, equation (4.54) together with equation (4.33a) give us a relation between the versatile premium and the domestic relative abundance of the two types of HC equal to that which was given by equation (3.12). Thus, when confronted with equation (4.33a), the resulting equation suggests that the relation between the versatile premium and the relative

[256] The static Rybczynski result states that when factor supplies are fully employed, an expansion in the endowment of one factor of production raises the output of the FG that uses the expanded factor intensively and reduces the output of the other FG – see also, for example, Chacholiades (1990).

domestic stock of HC is now less elastic. That is, as in the last chapter, the demand curve tends to become less elastic when the quality indexes 'adjust'. Finally, we can again define the relative factor shares as given by equation (3.13), and so we have once more that with endogenous TK progress, the share of a factor in the output, Y_Z, will be 'directly independent' of its 'abundance' and it is constant.

The decrease in the rate of growth of the relative productivity of the specialised TK is crucial to the steady state path of relative wages, as equation (4.32) and figures 4.4 and 4.5 show. In these figures, we take into account that the equilibrium versatile premium must be greater in the South – see equation (4.33b) and respective comments.

In the HOS model, IT affects directly the relative prices of FGs and changes in relative prices of FGs induce changes in relative wages through the Stolper-Samuelson (1941) theorem. This is because the latter states that a decline in the relative price of the imported good must reduce the return of the factor that is used intensively in its production – see also, for example, Chacholiades (1990). Thus, we can report that also a Schumpeterian dynamic equivalent to the static Stolper-Samuelson result holds in our case. In fact, our framework indicates that, by direct action of the R&D activity,[257] the IT of IGs between the North, *VH* relatively abundant, and the South puts downward pressure on the Northern relative wage of *SH* and on the Southern relative wage of *VH*.

In fact, in steady state the relative price of FGs is constant and the direction of TK bias is the sole determinant of the path of intra-country wage inequality. The immediate increase (level effect) in the relative wage of *VH* in the South ends up being reverted in steady state A (decreasing intra-country wage inequality, at a constant rate), or reinforced in steady state B (increasing intra-country wage inequality, at a constant rate), depending on the bias in favour of specialised or versatile TK, respectively. The steady state path of intra-country wage inequality, in either direction, is always more moderate than in without international TK diffusion, reflecting the weaker TK bias. More specifically,

(i) in the autarkic North, the price channel operates strongly in favour of the relatively scarce HC, *SH*. In the autarkic South, relative scarcity causes increasing inequality.[258]

(ii) in scenario A, the stimulus to the Northern demand for *VH* attenuates the path of declining inequality, relatively to what would have prevailed under autarky. In the South,

[257] Remember that due to IT of IGs, North-South average relative price of versatile FGs is higher than in the North alone and, thus, induces more versatile-biased TK progress than before, which pushes up the versatile premium – there is an adjustment process after IT, which changes innovation incentives.
[258] Increasing inequality is also reinforced by the level effect.

the strong reversal of the price channel effect – apparent in figure (4.3), comparing the autarkic bias in the South with the bias in available world TK – redirects the path of the relative wage premium in favour of declining inequality, following the same slope as the North – as equation (4.55) below shows, the common slope of the inequality path is determined in steady state by the negative growth of the versatile-specialised TK bias.

(iii) the IT of IGs causes wages in scenario B to follow a path of increasing inequality in both countries. Thus, the IT of IGs reverts autarkic decreasing inequality in the North. In the South, however, IT attenuates autarkic increasing inequality. The slope of the inequality path is determined by the positive growth of the versatile-specialised TK bias.

Figure 4.4. Decreasing inequality

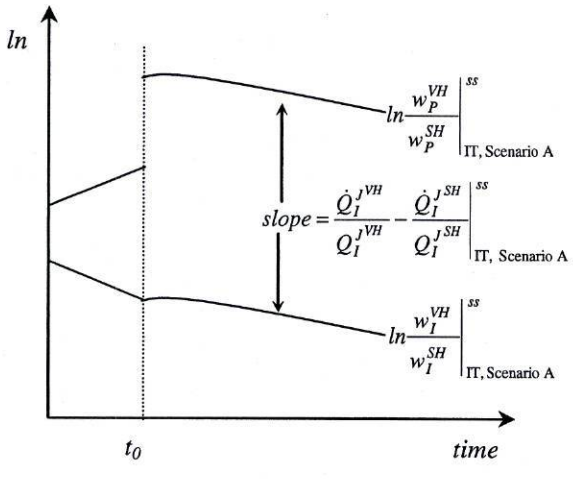
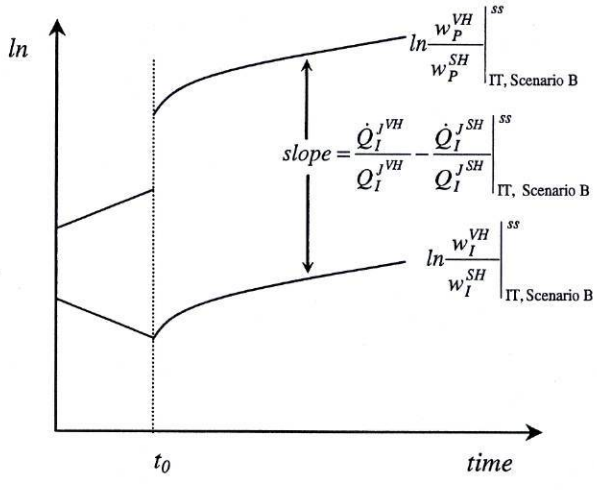


Figure 4.5. Increasing inequality



Thus, a possible result induced by the IT of IGs is the increase in intra-country wage inequality by raising the versatile premium – scenario B. This result seems to be more in line with the increasing dispersion of relatively unskilled to skilled workers wages, which is recognised as a stylised macroeconomic fact over the past twenty-five years in many OECD countries. Note that IT has accelerated over the past twenty-five years as well. Strictly speaking, our framework cannot explain this because we consider two types of HC and not workers. However, our approach provides a general idea, by considering that *VH* (*SH*) is more similar to skilled (unskilled) work.

If scenario B holds, then contrarily to the standard IT model, IT of IGs also increases the versatile premium in the South. This may explain why the demand for skills appears to have also increased in the developing (Southern) countries, as noted by Hanson and Harrison (1999a, b), Robbins (1994, 1999), Pessino (1996), Feenstra and Hanson (1996a, b, 1997), Tan and Batra (1997) and Berman and Machin (2000), among

others, unlike predictions in the standard IT model.[259] In particular, the last work quoted provides evidence of a significant increase in relative demand for skilled workers across industries in middle-income countries in the 1980s and interprets this finding as indicating the transfer of skill-biased innovations from developed (Northern) countries, in accordance with ours. Note that in recent decades, many developing countries have opened their economies to IT – *e.g.*, Wacziarg and Welch (2003).[260]

Therefore, our framework enables us to discuss the impact of the IT of IGs on the 'labour' market, explaining why both HC returns and intra-country wage inequality can increase. The former as a result of the action of induced dynamic benefits in both countries and still by the static benefit in the South. This increase should come as no surprise since HC should receive (at least) some of the benefits from IT. It is a result of the directed TK effect due to the differences in world prices in both countries and again by the static benefit in the South. After all, TK transfers are the reason for the existing impact of the IT of IGs on the 'labour' market.

Figures 4.4 and 4.5 also depict the maintenance of a wage differential between the North and the South. This is to be expected from the conjunction of assumptions of (i) international HC immobility and (ii) differences in exogenous levels of productivity (directly related with the quality of domestic institutions) and in *MC*s between the countries. But the steady state rates of growth of wages are equalised between countries, which results from the application of equation (4.32) to both types of HC in steady state,

$$\frac{\dot{w}_I^{m^{ss}}}{w_I^{m^{ss}}} = \frac{\dot{Q}_I^{J^{m^{ss}}}}{Q_I^{J^{m^{ss}}}} = \frac{\dot{Q}_P^{J^{m^{ss}}}}{Q_P^{J^{m^{ss}}}} = \frac{\dot{w}_P^{m^{ss}}}{w_P^{m^{ss}}} \Rightarrow \left(\frac{\dot{w}_I^{VH^{ss}}}{w_I^{VH^{ss}}} - \frac{\dot{w}_I^{SH^{ss}}}{\dot{w}_I^{SH^{ss}}}\right) = \left(\frac{\dot{w}_P^{VH^{ss}}}{w_P^{VH^{ss}}} - \frac{\dot{w}_P^{SH^{ss}}}{\dot{w}_P^{SH^{ss}}}\right) = \frac{\dot{Q}_I^{J^{VH\ ss}}}{Q_I^{J^{VH\ ss}}} - \frac{\dot{Q}_I^{J^{SH\ ss}}}{Q_I^{J^{SH\ ss}}} \cdot \quad (4.55)$$

This can be regarded as a Schumpeterian dynamic equivalent to the static factor-price equalisation (also called Samuelson) theorem. That is, in spite of the international immobility of HC, the steady state growth of wages is common to both countries because they share the same TK progress embodied in internationally traded IGs. Remember that in our approach, the price effect on wages works through TK progress, as in Berman *et*

[259] For example, in line with standard models, Wood (1995) argues that IT leads to a rise in the relative price of skill-intensive goods and, thus, induces Northern specialisation in high-tech goods. This, in turn, raises the relative demand for skilled labour and pushes the skill premium up, whereas the opposite development takes place in the Southern trading partners. That is, the latter export primary products or 'lower-technology goods' to the North. Consequently, IT in a Southern economy should be associated with an increase in the relative demand for unskilled labour and a narrowing of wage differentials.

[260] Wacziarg and Welch (2003) use the Sachs and Warner (1995) criterion to define all countries in the world as being either 'open' or 'closed' for each year between 1950-2000. In particular, they show that recently many developing countries have been opening up – *e.g.*, Mexico in 1986, Philippines in 1988, Turkey in 1989, Hungary in 1990, Poland in 1990, Czech Republic in 1991 and India and China after 2000.

al. (1994), Krugman and Lawrence (1994) and Davis (1998), among others, and the steady state result is given by equation (4.55). Once again,

(i) if the South is far from attaining the (Northern) TK frontier, then its TK progress will be low and so its wages will diverge relative to wages in the North.

(ii) If the South is an imitator country, then its TK change will be greater and, thus, its wages will benefit from the imitation activity. However, a steady state wage differential remains between countries, contrarily to the static factor-price equalisation theorem of the HOS model – maintenance of inter-country wage inequality, in spite of partial immediate convergence (level effect in the South).[261] That is, inter-country wage inequality in favour of the North remains constant, since the path of productivity becomes common to both countries. This is to be expected from the combination of international immobility of HC, differences in productivity, $A_I > A_P$, and in *MC*s between countries and from the HC share in production, $1/\alpha > 1$. In fact, from equation (4.33a),

$$\frac{w_P^{mss}}{w_I^{mss}} = \left(\frac{p_P^{mss} A_P}{p_I^{mss} A_I} \right)^{1/\alpha} < 1. \qquad (4.56)$$

In equation (4.56) we take into consideration that the effect of different FGs price indexes between countries on the wage differential between countries is of second order. However, we have to bear in mind that the international difference in FGs price indexes results from differences in both *MC*s of production and threshold FGs. Moreover, the international difference in threshold FGs reflects the international factor-intensity reversal, which does not occur in the HOS model. We can evaluate this equation later on by transitional dynamics through calibration.

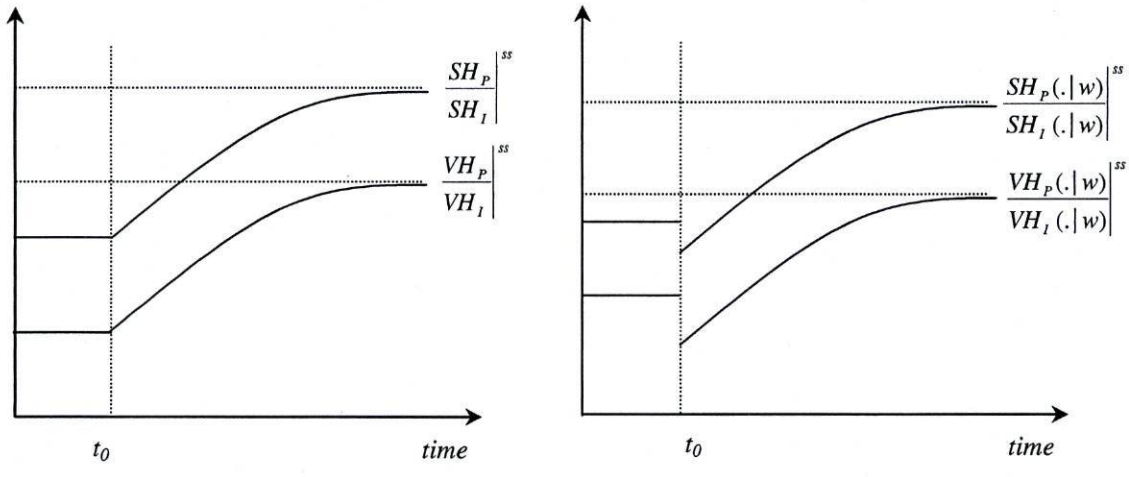
(iii) Comparing equations (3.15) and (4.56) we find that, for each type of HC, the IT of IGs unambiguously reduces the steady state degree of inter-country wage inequality, such as in Grossman and Helpman (1991d) and Dinopoulos and Segerstrom (2004). This is because with IT of IGs the South has access to state-of-the-art IGs by imports or by imitation. Here, however, we need to be very careful. In fact, strictly speaking, we cannot compare the two equations, since they depend on the FGs price indexes, which depend on *MC*s that, by assumption, are different in the South under the two IT regimes.

IT of IGs affects HC accumulation, of both types, thereby it changes inter-country gaps (also TK gaps) in favour of the South. Southern HC accumulation is immediately

[261] Remember that the HOS factor-price equalisation theorem states that free trade equalises factor rewards between countries, relatively and absolutely, and thus serves as a substitute for external factor mobility.

stimulated by higher wages, due to the level effect, and from then on continuously adjusts the levels to their higher steady states. Figure 4.6 shows the intertemporal trade-off between work and HC accumulation. Facing the permanent increase in wages brought about by IT of IGs at time t_0, Southern individuals adjust the trajectory of time allocation, devoting more time to schooling and OJT, and less time to work. That is, the ratio u_{wP}/u_{wI} falls at t_0, for both types of individuals. Then it adjusts smoothly back to its steady state level, around one. The path of the ratios of HC at work mirrors this dynamic adjustment.

Figure 4.6. Relative Southern levels of HC



Finally, one last comment associated with the effects of IGs production subsidy and R&D subsidy. When they are equal in both countries, and are increased, a permanent increase in the world steady state growth rate occurs, regardless of the IT regime – *e.g.*, tables 3.1 and 4.2 – and inter-country wage inequality in equations (3.15) and (4.56) is not directly affected. Though, when these subsidies are different between countries, then:

(i) without IT effects, when they increase in the South, the steady state world growth rate is not affected – see comments to table 3.1 – but the TK gap decreases – we can find this result by considering different subsidies between countries in the deduction of equation (3.8) – and so inter-country wage inequality as well – *e.g.*, equation (3.15);[262]

(ii) with the IT of IGs, when they increase in the South, a permanent increase in the world steady state growth rate occurs – see table 4.2 – and they have no direct influence on inter-country wage inequality – see equation (4.56).

[262] When they increase in the North, a permanent increase in the world steady state growth rate occurs and inter-country wage inequality increases.

4.7. Some Extensions

4.7.1. Overview

In this section, we explore some extensions of the model. Once again, they are considered at this stage because they are analysed in the steady state and they can be taken into account later on in the transitional dynamics through calibration.

In subsection 4.7.2, we compute and analyse the steady state pattern of both the cross-FGs *TFP* and aggregate *TFP* between countries. By allowing immediate access to state-of-the-art IGs, IT affects the productive structure in the South, bringing about partial convergence to the Northern productive structure and prices – see figures 4.1 and 4.2. However, even when the countries have access to the same TK, either through imitation or IT of IGs, differences in exogenous levels of productivity directly related with the quality of domestic institutions and in HCs imply differences in *TFP*. Thus, we expect that both the cross-FGs *TFP* and aggregate *TFP* will be larger in the North in steady state, but smaller than without IT effects.

In subsection 4.7.3, we analyse the effects of the use of the new GPT. Since now the South is no longer limited by its own ability to produce the best quality IGs, the emerging phases start at the same time in both countries. Generically, when the new GPT is beneficial to the production of all FGs at the same time, initially there is a small growth effect, which is extended in the next phase, when a level effect also occurs. When at first the new GPT is helpful in the production of versatile FGs, there are three distinct phases. In the 1st there is a small growth effect. In the 2nd and the 3rd, there are both level and growth effects. The versatile premium increases in the 2nd, and falls in the 3rd and the opposite occurs for the threshold FG, which decreases in the 2nd and increases in the 3rd.

We also discuss (very intuitively) the impact of a positive shock that increases the versatile-augmenting parameter in the production function. It generates both a positive level and growth effects, increases the versatile premium and decreases the threshold FG. That is, it induces the same type of effects as the use of the new GPT only on VT.

4.7.2. Final goods and aggregate productivity differences

In order to verify the origin of *TFP* differences between countries with IT of IGs, we begin with the steady state pattern of cross-FG *TFP* and we end up with the steady state aggregate *TFP*, bearing in mind especially figures 4.1 and 4.2.

Final goods productivity differences

From the definitions of X_n, $MC^m_{\Xi\gamma}$ and $mc^m_{\Xi\gamma}$ and from equations (4.12a, b), we have:

$$X_n^{1-\alpha} = \begin{cases} \left[\frac{p_n A (1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC^{SH}_{\Xi\gamma} \left[(1-n)\ sh\ SH_n(.|w)\ Q_I^{J^{SH}}\right]^{(1-\alpha)} & \text{if} \quad 0 \le n \le N \\ \left[\frac{p_n A (1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC^{VH}_{\Xi\gamma} \left[n\ vh\ VH_n(.|w)\ Q_I^{J^{VH}}\right]^{(1-\alpha)} & \text{if} \quad N < n \le 1 \end{cases} . \tag{4.57}$$

Taking equations (4.22a, b) and (4.57), we can decompose the output of the n[th] FG in value terms, obtained by multiplying equation (4.19) by the respective price, p_n, into two FG components: TFP_n and 'physical input' related to IG:[263]

$$p_n Y_n = \begin{cases} TFP_n(.|SH)\ X_n(.|SH)^{1-\alpha} & \text{if} \quad 0 \le n \le N \\ TFP_n(.|VH)\ X_n(.|VH)^{1-\alpha} & \text{if} \quad N \le n \le 1 \end{cases} ; \tag{4.58}$$

Where: $TFP_n(.|m) = A\ p^m \left[Q_I^{J^m}\ \overline{m}\ m_n(.|w)\right]^{\alpha}$.

Together, equations (4.22a, b), (4.27a, b) and (4.58) indicate that, in equilibrium, all FGs in each country have the same *TFP*, as figure 4.7a below shows, since:

$$\frac{TFP_n(.|SH)}{TFP_n(.|VH)} = \frac{A\ p^{SH}\ [Q_I^{J^{SH}}\ sh\ SH_n(.|w)]^{\alpha}}{A\ p^{VH}\ [Q_I^{J^{VH}}\ vh\ VH_n(.|w)]^{\alpha}} = 1. \tag{4.59}$$

In addition, considering equations (4.27a, b) and (4.59), the Northern FGs *TFP* is larger than in the South – although with a smaller differential in comparison with the previous case in subsection 3.3.2, as a direct consequence of the static benefit induced by the IT of IGs. To prove this, we start by computing the following particular situation:

$$\left[\frac{TFP_{nP}(.|SH)^{ss}}{TFP_{nI}(.|SH)^{ss}}\right]_{n \le N_I^{ss}} = \frac{A_P}{A_I}\ exp(mc_P) \left[\frac{N_I^{ss}}{N_P^{ss}}\right]^{2\alpha} \widetilde{SH}(.|w)^{ss\,\alpha} < 1. \tag{4.60}$$

Equation (4.60) is expected to be smaller than one due to differences between countries in the *A*s, in the *MC*s – which implies that $exp(mc_P) < 1$ –, and the steady state differences between countries in the *N*s – according to figure 4.1, N_I^{ss}/N_P^{ss} is smaller than *1* – and in the *SH*s – as at time t_0 $\widetilde{SH}(.|w) < 1$ then also in steady state $\widetilde{SH}(.|w) < 1$.

Comparing equation (4.60) with equation (3.19), which reports a similar situation without IT effects, now the TK gap is absent, because the South can immediately import

[263] The TPF_n is then a share of output of the n[th] FG in value terms that is imputable neither to the respective 'physical input' nor to the individual input per se, where individuals contribute in terms of embodying HC.

higher quality Northern IGs for use in FGs production. In addition, the *N*s are now closer – *i.e.*, $(N_I^{ss}/N_P^{ss})\big|_{\text{Without IT effects}} < (N_I^{ss}/N_P^{ss})\big|_{\text{With IT of IGs}} < 1$, as figure 4.1 shows. Hence, together these results now imply a smaller differential in the *TFP* for all FGs than before, since TFP_n is the same in all FGs of each country – see equation (4.59) – and $TFP_{nI}(.|SH)$ is greater than $TFP_{nP}(.|SH)$ – see equation (4.60). That is, we can generalise equation (4.60), by reporting that it is greater in all Northern FGs relative to the South – *i.e.*, $TFP_{nP}(.|m) < TFP_{nI}(.|m)$ –, but the difference is smaller than before.

When specifically comparing for each country the FG TFP_n under both without IT effects, given in equation (3.17), and the IT of IGs, given in equation (4.58), we observe for the particular situation relative to the North presented in the equation below that:

$$\frac{TFP_{nI}(.|SH)\big|_{\text{without IT effects}}}{TFP_{nI}(.|SH)\big|_{\text{with IT of IGs}}} = \left[\frac{N_I^{ss}\big|_{\text{with IT of IGs}}}{N_I^{ss}\big|_{\text{without IT effects}}}\right]^{2\alpha}\left[\frac{Q_I^{J^{SH\,ss}}SH_I^{ss}\big|_{\text{without IT effects}}}{Q_I^{J^{SH\,ss}}SH_I^{ss}\big|_{\text{with IT of IGs}}}\right]^{\alpha} < 1\cdot \quad (4.61)$$

In equation (4.61) $N_I^{ss}\big|_{\text{without IT effects}}$ is greater than $N_I^{ss}\big|_{\text{with IT of IGs}}$, due to the level effect and price effect from opening to the IT of IGs – see figure 4.1. Moreover, the latter term on the right-hand side of equation (4.61) tends also to be smaller than *1*, due to the potentially positive feedback effect, which implies that $Q_I^{J^{SH\,ss}}SH_I(.|w)^{ss}$ under IT grows at a higher rate than $Q_I^{J^{SH\,ss}}SH_I(.|w)^{ss}$ without IT effects – see comments to equation (4.53b). Since TFP_n is the same in all FGs of each country under both IT regimes – see equations (3.18) and (4.59), respectively –, equation (4.61) can be generalised to all FGs – *i.e.*, $TFP_{nI}(.|m)\big|_{\text{with IT of IGs}} > TFP_{nI}(.|m)\big|_{\text{without IT effects}}$. Besides, from equation (4.60) and its generalisation, we obtain $TFP_{nP}(.|m)\big|_{\text{with IT of IGs}} > TFP_{nP}(.|m)\big|_{\text{without IT effects}}$.

In brief, the analysis in subsection 3.3.1 of the last chapter, together with equations (4.59)-(4.61) and their generalisations, enable us to state that *TFP*: (i) is the same in all FGs in each country, $TFP_{nZ}(.|SH) = TFP_{nZ}(.|VH)$; (ii) is greater in the North, $TFP_{nI}(.|m) > TFP_{nP}(.|m)$, but the difference is now potentially smaller, $[TFP_{nI}(.|m) - TFP_{nP}(.|m)]\big|_{\text{with IT of IGs}} < [TFP_{nI}(.|m) - TFP_{nP}(.|m)]\big|_{\text{without IT effects}}$; (iii) is greater under the IT of IGs, $TFP_{nZ}(.|m)\big|_{\text{with IT of IGs}} > TFP_{nZ}(.|m)\big|_{\text{without IT effects}}$. When the IT of IGs strongly improves the imitation capacity and international IPRs protection is weak, the IT of IGs improves the FGs *TFP* in both countries, due to the strong dynamic effects generated. Moreover, the improvement is greater in the South, because it also benefits from the level effect which occurs when the IT of IGs begins.

Defining again the FG physical *TFP* as $PTFP_n(.|m) = VATFP_n(.|m)/p_n$, then:

$$PTFP_n(.|m) = \begin{cases} A\dfrac{p^{SH}}{p_n}\left[Q_I^{J^{SH}}\, sh\, SH_n(.|w)\right]^{\alpha} = A\left[(1-n)\, Q_I^{J^{SH}}\, sh\, SH_n(.|w)\right]^{\alpha} & \text{if } n < N \\ A\dfrac{p^{VH}}{p_n}\left[Q_I^{J^{VH}} vh\, VH_n(.|w)\right]^{\alpha} = A\left[n\, Q_I^{J^{VH}} vh\, VH_n(.|w)\right]^{\alpha} & \text{if } n > N \end{cases} . \quad (4.62)$$

Figure 4.7b below summarises the conclusions resulting from computing FGs *PTFP*. First, comparing the *PTFP* of the different FGs in each country, and using equations (4.22a, b), then:

$$\frac{PTFP_n(.|SH)}{PTFP_n(.|VH)} = \left[\frac{1-n}{n}\right]^{\alpha}\left[\frac{N}{1-N}\right]^{2\alpha}\left[\frac{1-N}{N}\right]^{\alpha}. \quad (4.63)$$

That is, regardless of the country, when $n \neq N$ there are two FGs with the same *PTFP*, although one of each type. In particular, when $n = N$ equation (4.63) is equal to *1*.

Second, *PTFP* is higher in the North than in the South in all FGs, as equations (4.64a-c) below show. Third, comparing the three cases, $n \leq N_I$, $N_I < n < N_P$ and $n \geq N_P$, the difference between countries increases progressively between $N_I < n < N_P$ and that after N_P it again remains constant. In fact, the result in equation (4.64a) for $n \leq N_I$ is immediate, bearing in mind the comments associated with equation (4.60). Moreover, computing the ratio between equations (4.64a) and (4.64b) and between equations (4.64a) and (4.64c), we verify that, for $n > N_I$, the difference increases progressively until $n = N_p$ and stops there. The cause for the increase is the use of *SH* – and so of ST – in some FGs in the South, which use *VH* – and so VT – in the North.

$$\left[\frac{PTFP_{nP}(.|SH)}{PTFP_{nI}(.|SH)}\right]_{n \leq N_I^{ss}} = \frac{A_P}{A_I}\left[\frac{N_I^{ss}}{N_P^{ss}}\right]^{\alpha} \widetilde{SH}(.|w)^{ss\,\alpha} < 1; \quad (4.64a)$$

$$\left[\frac{PTFP_{nP}(.|SH)}{PTFP_{nI}(.|VH)}\right]_{N_I^{ss} < n < N_P^{ss}} = \frac{A_P}{A_I}\left[\frac{1-n}{n}\right]^{\alpha}\left[\frac{1-N_I^{ss}}{N_P^{ss}}\right]^{\alpha}\left[\frac{Q_I^{J^{SH}\,ss}\, sh\, SH_P(.|w)^{ss}}{Q_I^{J^{VH}\,ss}\, vh\, VH_I(.|w)^{ss}}\right]^{\alpha} < 1; \quad (4.64b)$$

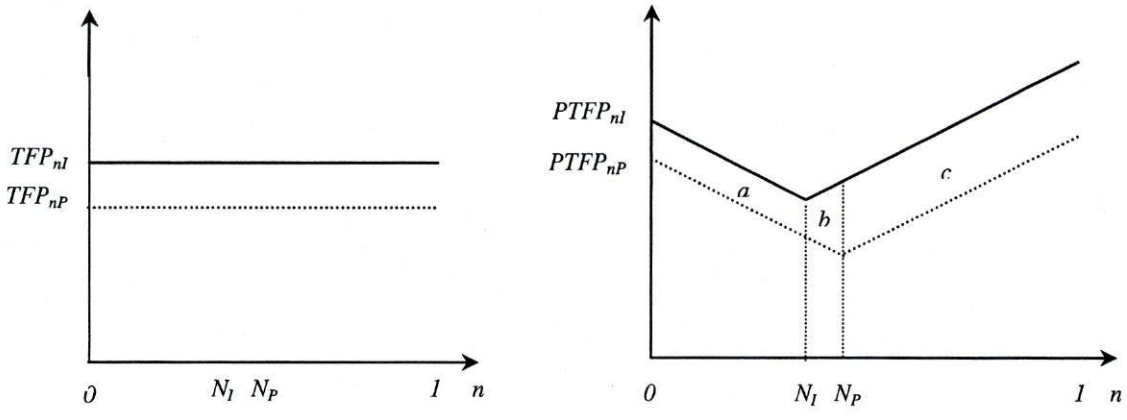
$$\left[\frac{PTFP_{nP}(.|VH)}{PTFP_{nI}(.|VH)}\right]_{n \geq N_P^{ss}} = \frac{A_P}{A_I}\left[\frac{1-N_I^{ss}}{1-N_P^{ss}}\right]^{\alpha} \widetilde{VH}(.|w)^{ss\,\alpha} < 1. \quad (4.64c)$$

Fourth, comparing equations (3.22a-c) and (4.64a-c), respectively, the differential in FGs *PTFP* is now smaller, since the TK gap is absent and the *N*s are closer. Fifth, computing the ratio between FGs *PTFP* in both IT regimes, in each country, it tends to be greater under the IT of IGs, $PTFP_{nZ}(.|m)|_{\text{with IT of IGs}} > PTFP_{nZ}(.|m)|_{\text{without IT effects}}$. The

justifications are similar to those associated with equation (4.61). In summary, the IT of IGs improves the FGs *PTFP* in both countries, but more so in the South.

Figure 4.7. Final goods total factor productivity (*TFP*) and physical *TFP* patterns

a) Final goods *TFP* **b) Final goods physical *TFP***



In figure 4.7b, area $a+b$ must be equal to area c. Relative to figure 3.4 for the previous IT regime without IT effects, we now have it that the areas are smaller.

Aggregate productivity differences

The FGs *TFP* can be translated into a measure of aggregate *TFP*. In fact, writing aggregate output as $Y \equiv exp[\int_0^N ln\, p_n Y_n(.|SH)\, dn + \int_N^1 ln\, p_n Y_n(.|VH)\, dn]$, considering equations (4.22a, b), (4.27a, b) and (4.58) and with some additional algebra, we have:

$$Y = TFP\ X(.|SH)^{(1-\alpha)N}\ X(.|VH)^{(1-\alpha)(1-N)}. \qquad (4.65)$$

$$\text{Where: } TFP = A\, exp(-\alpha)\left[\left(SH(.|w)\, sh\, Q_I^{J^{SH}}\right)^N \left(VH(.|w)\, vh\, Q_I^{J^{VH}}\right)^{1-N} N^{-(2N)}(1-N)^{-2(1-N)}\right]^{\alpha}.$$

Since both countries produce the same number of FGs and the North has higher FGs *TFP*, the Northern aggregate *TFP* is also higher than in the South. Nevertheless, from the analysis above, the difference must now be smaller than without IT effects.

Finally, computing the aggregate output per individual, Y, per individual occupation at work, $y \equiv \{Y[SH+VH]/[SH(.|w)+VH(.|w)]\}$, and per efficiency unit of individual occupation at work, $y^{eff} \equiv \{Y[SH+VH]/[sh\,SH(.|w)+vh\,VH(.|w)]\}$, between both countries, we get the following two equations/ratios, since Y_P/Y_I and y_P/y_I are equal:

$$\frac{Y_P^{ss}}{Y_I^{ss}} = \frac{y_P^{ss}}{y_I^{ss}} = \exp(mc_P)^{\frac{1}{\alpha}} \left(\frac{A_P}{A_I}\right)^{\frac{1}{\alpha}} \tilde{SH}(.|w)^{ss} \left(\frac{N_I^{ss}}{N_P^{ss}}\right)^2 < 1; \tag{4.66a}$$

$$\frac{y_P^{eff}}{y_I^{eff}} = \frac{Y_P^{ss}}{Y_I^{ss}} \left(\frac{SH_P^{ss} + VH_P^{ss}}{SH_I^{ss} + VH_I^{ss}}\right) \left(\frac{sh\,SH_I(.|w)^{ss} + vhVH_I(.|w)^{ss}}{sh\,SH_P(.|w)^{ss} + vhVH_P(.|w)^{ss}}\right) < 1. \tag{4.66b}$$

Bearing in mind the previous analysis, equations (4.66a, b) suggest that the aggregate output per individual, per individual working and per efficiency unit of individual working are greater in the North. Since under the IT of IGs the *N*s become closer between countries, the comparison between equations (4.66a, b) and equations (3.24a, b), respectively, suggests that the difference between countries is now smaller.

In summary, both FGs and aggregate *TFP* are large in the North. For each country, they tend to be larger under the IT of IGs than without IT effects. Moreover, the differential between countries now tends to be smaller. So, the IT of IGs improves the *TFP* of both countries but the improvement is greater in the South. Essentially, the *TFP* differences arise due to differences in productivity parameter, A, in the levels of HC at work, $m(.|w)$, and in productive structure, N. Thus, for example, an exogenous improvement in A_I helps the steady state world growth rate, although it expands the *TFP* gap between countries, whereas an exogenous improvement in A_P helps the steady state world growth rate and decreases the *TFP* gap between countries.

4.7.3. Consequences of a new General Purpose Technology

We proceed now by analysing the effects resulting from the introduction of the new GPT. Again, we focus the investigation on the successive steady states that emerge in each phase, by assuming that a temporary steady state is reached in each one. The distinct phases occur since the use of the new GPT requires a minimum compatible TK level.

For the same reasons as in chapter 3, the subsection is divided in two scenarios. In the 1st, the new GPT is beneficial to all FGs at the same time. In the 2nd, the new GPT is first beneficial to the FGs produced with VT. Since now the South is no longer limited by its own ability to produce the best quality IGs, the phases start at the same time in both countries. Thus, there are two phases in the former case and three phases in the later.

1st scenario: New GPT is beneficial to all final goods at the same time

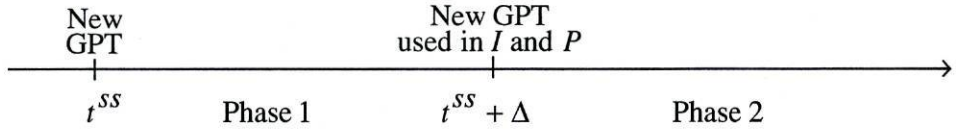
During the time from the use of the new GPT until the occurrence of another new GPT, the production function for the nth FG in country Z, Y_{nZ}^{NG}, is again given by equation

(4.1), but now the exogenous productivity level is $A_Z^{NG}(>A_Z)$ and $k \geq k^M$. Then, from the time of the use of the new GPT until the arrival of a new GPT, equation (4.19) becomes:

$$Y_n^{NG} = \begin{cases} A^{NG\,1/\alpha}\left[p_n \dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\ (1-n)\,sh\ SH_n(.|w)\ Q_I^{J^{SH}} & \text{if} \quad 0 \leq n \leq N \\ A^{NG\,1/\alpha}\left[p_n \dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\ n\,vh\ VH_n(.|w)\ Q_I^{J^{VH}} & \text{if} \quad N < n \leq 1 \end{cases} \cdot \quad (4.67)$$

In equation (4.67), $Q_I^{J^m}$ must be greater than or equal to the TK minimum $Q^{J_M^m}$, and, in the present scenario, $Q^{J_M^{SH}}$ and $Q^{J_M^{VH}}$ occur at the same time. We let Δ represent the time that is required to reach $Q^{J_M^m}$ from the previous steady state. Thus, as figure 4.8 shows, the new GPT generates two phases. In phase 1, during $[t^{ss}, t^{ss}+\Delta)$, the old GPT goes on to be used in both countries. In phase 2, from $t^{ss}+\Delta$ until a new GPT occurs, the previous new GPT is used in both countries. The effects of the new GPT in the steady state in each phase are then briefly analysed and they are summarise in table 4.3 below.

Figure 4.8. Phases generated by the new GPT in the time horizon



1st scenario, phase 1: the old GPT is used in both countries

The analysis in this phase is quite similar to the respective phase in chapter 3. Thus, in this phase the new temporary steady state world growth rate is unambiguously greater than in the previous steady state world growth rate. This is because, by assumption, there is a level jump in χ_S and χ_T when the new GPT arrives for $\chi_S^{NG} > \chi_S$ and $\chi_T^{NG} > \chi_T$ – see table 4.2. That is, the new GPT confers a 'growth effect' to both countries with positive impact on their productivity. As the new GPT has not yet been introduced on the production side, it is a 'time to sow' (with some 'reap') for both countries.

1st scenario, phase 2: the new GPT is used in both countries

At time $t = t^{ss}+\Delta$ the Northern TK reaches $Q^{J_M^m}$. Then, both countries use a new GPT and there is a level jump in their aggregate outputs ('static benefit'): Y_Z jumps to Y_Z^{NG} due to A_Z^{NG}, where, in particular, $Q_I^{J^m} \geq Q^{J_M^m}$. Moreover, the greater productivity

level associated with the new GPT, which also intensifies the feedback effect, increases the expected profits of the innovator firms. As a result, the rate of innovation increases. Hence, the new world steady state growth rate is greater than in the last phase, due to the additional growth effect generated in innovative activity with the use of the new GPT.

In short, the new GPT generates two dynamic effects. First (phase 1), in HC accumulation and after that (phase 2) in R&D innovative activity and, as a result, $gr^{NG}_{Phase\,2} > gr^{NG}_{Phase\,1} > gr$. These two dynamic effects, together with the static benefit induced by the use of a new GPT, have a positive impact on the FGs as well as on the aggregate *TFP*s of both countries. Phase 2 is thus a 'time to reap' for both countries.

Table 4.3. Main results obtained in each of the two phases

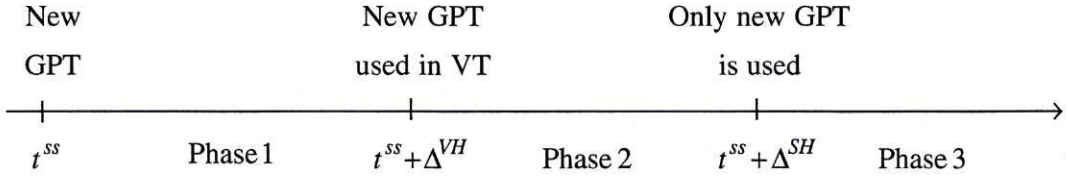
	Phase 1	Phase 2
Time to sow	In both	---
Time to reap	In both	In both
Growth effects	In both	In both
Level effects	---	In both

Note: no direct effects occur in inter and intra-country wage inequality, nor in productivity differences between countries due to the use of a new GPT.

2nd scenario: First the new GPT is beneficial to some final goods at the same time

As some recent literature recommends, we again assume that the new GPT is used first in FGs which use VT. Let Δ^m represent the time required to reach $Q^{J^m_M}$ from the previous steady state. In this case, until there is a complete use of the new GPT in the world, there are three phases with specific characteristics – see figure 4.9. In phase 1, during $[t^{ss}, t^{ss}+\Delta^{VH})$, only the old GPT is used. In phase 2, during $[t^{ss}+\Delta^{VH}, t^{ss}+\Delta^{SH})$, the new GPT is used in VT. In phase 3, from $t^{ss}+\Delta^{SH}$ on, only the new GPT is used.

Figure 4.9. Phases generated by the new GPT in the time horizon



We begin by analysing phase 2, since phase 1 remains the same as that described in the last subsection. At the end, we summarise the main conclusions in table 4.4.

2nd scenario, phase 2: the new GPT is only used in VT (but in both countries)

Considering by assumption that the versatile TK reaches the TK minimum compatible with the new GPT before the specialised TK, which corresponds to time $t = t^{ss} + \Delta^{VH}$, the production function of each nth FG is different according to the use of a VT or a ST. Then we can get an equation equivalent to equation (3.27) without IT effects, for both countries, and the threshold FG $N \in [0, 1]$ can be found as:

$$N_Z^{VNG} = \left\{ 1 + \left[\frac{Q_I^{JVH}}{Q_I^{JSH}} \frac{vh\,VH_Z(.|w)}{sh\,SH_Z(.|w)} \left(\frac{A_Z^{NG}}{A_Z} \right)^{1/\alpha} \right]^{1/2} \right\}^{-1} . \tag{4.68}$$

From equation (4.68), the use of the new GPT in VT decreases N_Z. In other words, FGs at the margin are moving from the use of ST to the use of VT.

There is also a static gain with the introduction of the new GPT in VT. The aggregate output Y_Z jumps from the value given in equation (4.30a, b) to Y_Z^{VNG} :

$$Y_I^{VNG} = exp(-1) \left[\frac{1-\alpha}{q} \right]^{\frac{1-\alpha}{\alpha}} MC_{\Xi\gamma}^{SH} \left\{ \left[Q_I^{JSH}\, sh\, SH_I(.|w)\; A_I^{1/\alpha} \right]^{\frac{1}{2}} + \left[Q_I^{JVH}\, vh\, VH_I(.|w)\; A_I^{NG\,1/\alpha} \right]^{\frac{1}{2}} \right\}^2 ; \tag{4.69a}$$

$$Y_P^{VNG} = exp\,(mc_P - \alpha)^{1/\alpha} A_P^{1/\alpha} \left[\frac{1-\alpha}{q} \right]^{\frac{1-\alpha}{\alpha}} MC_{\Xi\gamma}^{SH} \left\{ \left[Q_I^{JSH}\, sh\, SH_P(.|w)\; A_P^{1/\alpha} \right]^{\frac{1}{2}} + \left[Q_I^{JVH}\, vh\, VH_P(.|w)\, A_P^{NG\,1/\alpha} \right]^{\frac{1}{2}} \right\}^2 . \tag{4.69b}$$

The price paid per unit of each type of HC now becomes given by – where, remember, V (S) before SH and VH represents the value of variables (which changes) after the introduction of the new GPT only in VT (ST):

$$\frac{\partial Y}{\partial SH(.|w)} = w^{VSH} = A_Z^{(1/\alpha)} \left(\frac{1-\alpha}{q} \right)^{\left(\frac{1-\alpha}{\alpha}\right)} p_Z^{VSH\,(1/\alpha)}\, MC_{\Xi\gamma}^{SH}\; sh\; Q_I^{JSH} ; \tag{4.70a}$$

$$\frac{\partial Y}{\partial VH(.|w)} = w^{VVH} = A_Z^{NG\,(1/\alpha)} \left(\frac{1-\alpha}{q}\right)^{\left(\frac{1-\alpha}{\alpha}\right)} p_Z^{VVH\,(1/\alpha)} MC_{\Xi\gamma}^{SH}\ vh\ Q_I^{JVH}. \qquad (4.70b)$$

Relative to equations (4.31a, b), equations (4.70a, b) suggest that now the price paid for each unit of both types of HC increases with the use of the new GPT by VT and, contrarily to the previous IT regime in chapter 3, this is now true in both countries. Consequently, both types of workers in the world benefit. Nevertheless, workers that accumulate VH benefit more, because the versatile premium must increase, $W_Z^{VNG} > W_Z$:

$$W_Z^{VNG} = \frac{w_Z^{VVH}}{w_Z^{VSH}} = \left(\frac{Q_I^{JVH}}{Q_I^{JSH}}\right)^{\frac{1}{2}} \left(\frac{vh\ VH_Z(.|w)}{sh\ SH_Z(.|w)}\right)^{-\frac{1}{2}} \frac{vh}{sh} \left(\frac{A_Z^{NG}}{A_Z}\right)^{\frac{1}{2\alpha}}. \qquad (4.71a)$$

$$\left.\frac{W_Z^{VNG}}{W_Z}\right|_{t=t^{ss}+\Delta^{VH}} = \left(\frac{A_Z^{NG}}{A_Z}\right)^{\frac{1}{2\alpha}} > 1. \qquad (4.71b)$$

Thus, as well as the IT of IGs, the use of the new GPT in VT is another possible explanation for the increase in demand for skills in all countries, which is mirrored in the increase of intra-country wages inequality. However, these changes are temporary (phase 2), although they can occur over a long period of time.

So far, we have analysed the effects at the time of introduction of the new GPT in VT. But, we can find that, for example, the world growth rate is given by – where mAp^{VNG} is the value mAp when the new GPT is introduced only in VT:

$$gr_{\text{phase}2} = \left[\frac{1}{\theta}\left(\frac{1-\tau_{KZ}}{q^{\left(\frac{1-\alpha}{\alpha}\right)} - \tau_{KZ}}\right)\right] \left\{ \beta_P\ \zeta_P^{-1}\ exp(PE+IT)\ g_1(.)^{\bar{\sigma}_1}\ g_2(.)^{\sigma_2(.)}\ \tilde{q}^{(.|m)}\ \iota_P\ \bar{m} \left(\frac{MC_I - MC_P}{1 - s_{nP}}\right) \right.$$
$$\left. (1-\alpha)^{\frac{1}{\alpha}}\ mAp^{VNG} \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right] - \delta^m + \varphi^{m[1/\phi]}\ \chi_S^{NG} \left[1 + \frac{u_T^{NG}(t|m)}{u_S^{NG}(t|m)}\right] - \rho \left(\frac{q^{\left(\frac{1-\alpha}{\alpha}\right)} - \tau_{KZ}}{1 - \tau_{KZ}}\right) \right\} \qquad (4.72)$$

In equation (4.72), innovator firms in versatile TK benefit from A_I^{NG} and A_P^{NG} and, on the other hand, lose from the decrease in p_I^{VH} and p_P^{VH}, since N_I and N_P decrease. Also resulting from equation (4.72), the probability of innovation in specialised TK and, thereby, the growth rate must be greater than before since p_I^{SH} and p_P^{SH} increase. Hence, the growth effect shortens the time for use of the new GPT in ST. As for a unique steady state, the growth rate in equations (4.72) must be equal for $m = SH$ and for $m = VH$, the

positive effect of A_I^{NG} and A_P^{NG} dominates the negative effect of p_I^{VH} and p_P^{VH} on the growth rate, considering $m = VH$.

In brief, in this phase and for both countries, which is new relative to the previous IT regime, there are both level and growth effects, FGs at the margin are shifting from the use of ST to the use of VT, and the versatile premium increases, although the price paid per unit of *SH* also increases and the respective productivities are now greater.

2nd scenario, phase 3: only the new GPT is used in the world

In phase 3, the new GPT is introduced in all FGs in both countries. In this time and for the same reason as before, some FGs at the margin now move from the use of VT to the use of ST. In other words, N_Z increases as well as p_Z^{VH}, and inversely p_Z^{SH} decreases. The introduction of the new GPT into the production of all FGs again generates both a level and a growth effect, and the versatile premium decreases in relation to the last phase, although the price paid per unit of *VH* also increases. There is a new improvement in the countries' productivity.

Table 4.4. Main results obtained in each of the three phases

	Phase 1	Phase 2	Phase 3
Time to sow	In both	In both	---
Time to reap	In both	In both	In both
Growth effects	In both	In both	In both
Level effects	---	In both	In both
Versatile premium	---	↑ in both	↓ in both
Threshold final good	---	↓ in both	↑ in both

Note: no direct effects occur in inter-country wage inequality, in productivity differences between countries and in the TK gap.

To conclude, note that the effect of the biased HC productivity shock leads to an increase in the versatile-augmenting parameter *vh*; that is, a shock in a specific Ricardian component of our model has the same impact as the introduction of the new GPT in VT. In other words, a positive shock generates in both countries a positive level – *e.g.*, equation (4.30a, b) – and growth effect – *e.g.*, equation (4.50) –, increases the versatile premium – *e.g.*, equation (4.33b) – and decreases the threshold FG – *e.g.*, equation (4.24). Therefore, a shock in *vh* can be an additional explanation for the increase in versatile premium in both countries.

4.8. Transition to and stability of the balanced growth path

4.8.1. Overview

Under the IT of IGs, we have established that a world growth rate exists in steady state that is common to both countries – the North and the South. We need then to find out whether both countries (world) converge towards the steady state or not, using the fourth-order Runge-Kutta classical numerical method – see section 3.4 – for the set baseline parameter values – in table 3.4 – and the set initial conditions – in Appendix 3.

Under the IT of IGs, there is a feedback effect between countries. Consequently, when one is out of the world steady state – affected by the action of both countries – so is other one. Thus, in order to analyse the stability of and convergence to the world steady state, the countries must be considered together. In the next two subsections, we will analyse as fully as possible the transitional dynamics in the world. In subsection 4.8.2, we will find the differential equations and, in subsection 4.8.3, we will solve the model numerically for the baseline parameter values and initial conditions.

4.8.2. Transitional dynamics for the world: differential equations

Once again the differential equations are a result of equilibrium in the labour markets, free entry conditions into R&D – innovation and imitation –, individual utility maximisation with individual optimal time allocation.

Regardless of the country, on the demand side of the labour market, off the steady state, the Northern growth rate of m-specific TK, the dynamic of the respective FGs price index and the dynamic of $MC^{m}_{\Xi\gamma}$, which reflects international limit pricing and MCs, translate into the growth of demand m-type HC – *e.g.*, equation (4.32). Equating the wages' dynamic of m-type HC obtained on the 'demand side' with the respective wages' dynamic obtained on the 'supply side' – equation (2.70) –, we obtain:

$$\frac{\dot{Q}_I^{J^m}(t)}{Q_I^{J^m}(t)} + \frac{1}{\alpha}\frac{\dot{p}_Z^m(t)}{p_Z^m(t)} + \frac{\dot{MC}^m_{\Xi\gamma}(t)}{MC^m_{\Xi\gamma}(t)} = [1-\tau_K]\, r_Z(t) + \delta^m - \varphi^{m[1/\phi]}\chi_S\left[1+\frac{u_T(t|m)}{u_S(t|m)}\right]^{\frac{1-\phi}{\phi}}. \quad (4.73)$$

Bearing in mind the uniqueness of the country's interest rate as well as the equality between $MC^{SH}_{\Xi\gamma}$ and $MC^{VH}_{\Xi\gamma}$ – see section 4.4 –, and considering equation (4.73) for both $m = \{SH, VH\}$ and afterwards subtracting the two resulting equations, we have that:

$$\frac{\dot{D}_Z^I}{D_Z^I} = 2\left\{\frac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}} - \frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}} + \delta^{SH} - \delta^{VH} + \chi_S\left[\varphi^{VH\left[\frac{1}{\phi}\right]}\left(1+\frac{u_T(VH)}{u_S(VH)}\right)^{\left(\frac{1-\phi}{\phi}\right)} - \varphi^{SH\left[\frac{1}{\phi}\right]}\left(1+\frac{u_T(SH)}{u_S(SH)}\right)^{\left(\frac{1-\phi}{\phi}\right)}\right]\right\}. \quad (4.74)$$

Equation (4.74) gives the dynamic of D_Z^I, where $D_Z^I \equiv \frac{Q_I^{J^{VH}} VH_Z(.|w)}{Q_I^{J^{SH}} SH_Z(.|w)}$,[264] and, in addition, it provides useful information. Indeed, from equation (4.74),

$$\frac{\dot{D}_I^I}{D_I^I} = \frac{\dot{D}_P^I}{D_P^I} \Rightarrow \tag{4.75a}$$

$$\Rightarrow \frac{\dot{VH}_I(.|w)}{VH_I(.|w)} - \frac{\dot{SH}_I(.|w)}{SH_I(.|w)} = \frac{\dot{VH}_P(.|w)}{VH_P(.|w)} - \frac{\dot{SH}_P(.|w)}{SH_P(.|w)} \Rightarrow \frac{\dot{\tilde{VH}}(.|w)}{VH(.|w)} = \frac{\dot{\tilde{SH}}(.|w)}{SH(.|w)} \Rightarrow \tag{4.75b}$$

$$\Rightarrow \frac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}} - \frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}} = \frac{\dot{Q}_P^{J^{VH}}}{Q_P^{J^{VH}}} - \frac{\dot{Q}_P^{J^{SH}}}{Q_P^{J^{SH}}} \Rightarrow \frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}} = \frac{\dot{\tilde{Q}}^{J^{SH}}}{\tilde{Q}^{J^{SH}}}. \tag{4.75c}$$

Equation (4.75a) results directly from equation (4.74), and equation (4.75b) is clear from equation (4.75a) when considering the definition of D_Z^I. Equation (4.75c) results from equation (4.75b) bearing in mind the following explanation. There is a feedback effect between countries such that the equilibrium probability of successful innovation is stimulated by the equilibrium probability of successful imitation, and *vice-versa*. Thus, for example, a case such that $\dot{Q}_I^{J^{VH}} / Q_I^{J^{VH}} - \dot{Q}_I^{J^{SH}} / Q_I^{J^{SH}} < \dot{Q}_P^{J^{VH}} / Q_P^{J^{VH}} - \dot{Q}_P^{J^{SH}} / Q_P^{J^{SH}}$ is impossible to hold, since it means that imitation activity is relatively versatile and stimulates specialised innovation activity, under one scenario in which the HC does not contribute to this – *i.e.*, it is 'neutral' –, as equation (4.75b) shows.

In addition, the equality between $MC_{\Xi\gamma}^{VH}$ and $MC_{\Xi\gamma}^{SH}$ – considering the definitions of both $MC_{\Xi\gamma}^{m}$ and γ_I^m in equation (4.13) – and equation (4.14) allow us to write that:

$$\begin{aligned} &\left[1 - pb_P(k, av, t \mid SH) - \Xi_1(t \mid SH) + \Xi_2(t \mid SH)\right] \gamma_I^{SH}(t) = \\ &= \left[1 - pb_P(k, av, t \mid VH) - \Xi_1(t \mid VH) + \Xi_2(t \mid VH)\right] \gamma_I^{VH}(t) \end{aligned} \tag{4.76}$$

Plugging equations (4.15a, b) into equation (4.76), considering equations (2.54) and (4.75c) and the definition of $\tilde{Q}^{J^m}$, we get that:

$$\frac{\dot{\tilde{Q}}^{J^{VH}}(t)}{\tilde{Q}^{J^{VH}}(t)} = \frac{\dot{\tilde{Q}}^{J^{SH}}(t)}{\tilde{Q}^{J^{SH}}(t)} = \left[\frac{1}{\Xi_2(t)}\left(\gamma_I^{VH}(t) - \gamma_I^{SH}(t)\right)\right]^{-1} \left(\frac{\dot{Q}_I^{J^{VH}}(t)}{Q_I^{J^{VH}}(t)} - \frac{\dot{Q}_I^{J^{SH}}(t)}{Q_I^{J^{SH}}(t)}\right); \tag{4.77}$$

[264] Incorporating this new D_Z^I in equations (3.42) and (3.61), we obtain the dynamic of the price indexes of FGs in the new trade regime, for the North and the South, respectively.

Where: $\gamma_I^{VH}(t) - \gamma_I^{SH}(t) = \left[\tilde{Q}^{J^{VH}}(t) - \tilde{Q}^{J^{SH}}(t) \right] \left[\left(\frac{1}{MC_P - s_{xI}} \right)^{\left(\frac{1-\alpha}{\alpha}\right)} - \left(\frac{1}{MC_I - s_{xI}} \right)^{\left(\frac{1-\alpha}{\alpha}\right)} \right]$.

Since $\Xi_2(t) \geq 0$ and in steady state $\Xi_2(t) = 0$ – equation (4.15b), we assume that:

$$\Xi_2(t) = \left[\wp_1 \, exp(\wp_2 \, t) \right]^{-1}; \tag{4.78}$$

where the parameter $\wp_1 > 0$ represents the value of $\Xi_2(t_0)$ and the parameter $\wp_2 > 0$ captures its temporal evolution, so that in the steady state $\Xi_2(t^{ss}) \approx 0$. A small $\Xi_2(t)$ – *i.e.*, a greater $\wp_1$, $\wp_2$ and t – implies that the Southern losses owing to successful innovations increase. Thus, for example, when $t \to t^{ss}$ the Southern losses due to Northern R&D increase because the South produces progressively more IGs until the steady state.

We can get the dynamic of D_Z^I, $\tilde{Q}^{J^{VH}}$ and $\tilde{Q}^{J^{SH}}$ by solving the system of differential equations given by equations (4.74) and (4.77), if we know the dynamic of $\dot{Q}_I^{J^{VH}}/Q_I^{J^{VH}} - \dot{Q}_I^{J^{SH}}/Q_I^{J^{SH}}$. The easiest way to obtain the latter consists in making use of equation (4.45). Thus, solving $\dot{Q}_I^{J^{VH}}/Q_I^{J^{VH}} - \dot{Q}_I^{J^{SH}}/Q_I^{J^{SH}}$ and considering the benchmark catching up term – the sole considered now – in the probability of successful imitation,

$$\frac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}} - \frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}} = \beta_P \, \zeta_P^{-1} \, \iota_P \, exp(PE+IT) \, (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{MC_I - MC_P}{1 - s_{rP}} \right) \left(q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right)$$

$$\left\{ vh \; g_1[\tilde{VH}(.|w)]^{\bar{\sigma}_1} \, g_2(\tilde{Q}^{J^{VH}}, d)^{\sigma_2(.)} \, \tilde{q}^{(.|VH)} \, VHAp - \right. \tag{4.79}$$

$$\left. - sh \; g_1[\tilde{SH}(.|w)]^{\bar{\sigma}_1} \, g_2(\tilde{Q}^{J^{SH}}, d)^{\sigma_2(.)} \, \tilde{q}^{(.|SH)} \, SHAp \right\}$$

Where the expressions for *VHAp* and *SHAp* result from the definition of *mAp*, considering $m = \{VH, SH\}$ in equation (4.41). In equation (4.79), the price indexes – inside expressions *VHAp* and *SHAp* – are given by equations (4.27a, b), and they depend on D_Z^I. The dynamic of $\dot{Q}_I^{J^{VH}}/Q_I^{J^{VH}} - \dot{Q}_I^{J^{SH}}/Q_I^{J^{SH}}$ also depends on the dynamics of both $\tilde{VH}(.|w)$ and $\tilde{SH}(.|w)$, which according to equation (4.75b) are equal. In order to get the dynamic of $\tilde{VH}(.|w)$ – or, alternatively, of $\tilde{SH}(.|w)$ –, first we find the two expression for the Southern interest rate – one for each m – and, after that, we equal

them. Thus, considering the representative domestic IG, *av*, for each *m*-specific IG, the expression for the expected present value of profits for the lead imitator is given by:

$$V_P(k,av,v|m) = \Pi_P(k,av,v|m)\int_t^{\infty} exp\left[-\int_t^s \left(r_P(v) + pb_I(k,av,v|m)\right)dv\right]ds. \quad (4.80)$$

In equation (4.80), $r_P(v)$ is the Southern interest rate at time v and, which is crucial under IT of IGs, $pb_I(k,av,v|m)$ is the probability at time v that the *av* IG of quality rung $(k+1)$ of the *m*-specific TK ladder will be innovated. Free entry into R&D implies that at all times, the expected present value of profits from successful imitation times the probability of successful imitation must equal the after subsidy cost of imitation; *i.e.*,

$$pb_P(k,av,v|m)\ V_P(k,av,v|m) = (1-s_{rP})\ rs_P(k,av,v|m). \quad (4.81)$$

Where $pb_P(k,av,v|m)$ is the probability at time v that the *av* IG of quality rung k of the *m*-specific TK will be imitated.[265] Plugging equation (4.80) into equation (4.81),

$$pb_P(k,av,v|m)\left\{\Pi_P(k,av,v|m)\int_t^{\infty} exp\left[-\int_t^s \left(r_P(v) + pb_I(k,av,v|m)\right)dv\right]ds\right\} = \quad (4.82)$$
$$= (1-s_{rP})\ rs_P(k,av,v|m)$$

Differentiating both sides of equation (4.82), using Leibniz's rule for the left-hand side, we obtain after some algebra the following equation for the Southern interest rate:

$$r_P(t) = \frac{\dot{rs}_P(k,av,t|m)}{rs_P(k,a,t|m)} - \frac{\dot{pb}_P(k,av,t|m)}{pb_P(k,av,t|m)} - \frac{\dot{\Pi}_P(k,av,t|m)}{\Pi_P(k,av,t|m)} + \quad (4.83)$$
$$+ \frac{pb_P(k,av,t|m)\,\Pi_P(k,av,t|m)}{rs_P(k,av,t|m)\ (1-s_{rP})} - pb_I(k,av,t|m)$$

Using equations (4.37) and (4.35c) we have, respectively, that:

$$\frac{\dot{rs}_P(k,av,t|m)}{rs_P(k,av,t|m)} = \frac{\dot{pb}_P(k,av,t|m)}{pb_P(k,av,t|m)} + \frac{\dot{m}_P(t|w)+\dot{m}_I(t|w)}{m_P(t|w)+m_I(t|w)} - \bar{\sigma}_1\frac{\dot{g_1}(.)}{g_1(.)} - \quad (4.84a)$$
$$-\dot{\sigma}_2(.)\ln g_2(.) - \sigma_2(.)\frac{\dot{g_2}(.)}{g_2(.)} - \dot{k}_I(av,t|m)\left(\frac{\alpha-1}{\alpha}\right)\ln q - \frac{\dot{\tilde{Q}}^{Jm}}{\tilde{Q}^{Jm}};$$

$$\frac{\dot{\Pi}_P(k,av,t|m)}{\Pi_P(k,av,t|m)} = \frac{\dot{mAp}}{mAp} + \dot{k}_I(av,t|m)\left(\frac{1-\alpha}{\alpha}\right)\ln q. \quad (4.84b)$$

Plugging equations (4.84a, b) into (4.83) the interest rate becomes given by:

[265] Note that this definition is different from that provided in the last chapter.

$$r_P = \frac{\dot{m}_P(.|w) + \dot{m}_I(.|w)}{m_P(.|w) + m_I(.|w)} - \frac{\dot{mAp}}{mAp} - \overline{\sigma}_1 \frac{\dot{g_1}(.)}{g_1(.)} - \dot{\sigma}_2(.)\, ln\, g_2(.) - \sigma_2(.) \frac{\dot{g_2}(.)}{g_2(.)} - \\ - \frac{\dot{\tilde{Q}}^{J^m}}{\tilde{Q}^{J^m}} + \frac{pb_P(k,av,.|m)\ \Pi_P(k,av,.|m)}{rs_P(k,av,.|m)\ (1-s_{rP})} - pb_I(k,av,.|m) \qquad (4.85)$$

In equation (4.85), the first six terms represent the capital gains/losses term, the following one is a dividend term – which is independent of the quality rung – and the last reflects loss of the market when the next successful innovation occurs – which according to equation (4.45) can be written in terms of the growth rate of the respective TK.

Equating the two equations for the interest rate; that is, for $m = \{SH, VH\}$, since the country's interest rate is unique, we get, after some algebra, the following differential equation – omitting both indexes t and w; *i.e.*, the HC gaps $\tilde{m}$ must be read as $\tilde{m}(.|w)$:

$$\frac{\dot{\tilde{VH}}}{\tilde{VH}} = \frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}} \left[\tilde{Q}^{J^{VH}}\, ln\left(-\tilde{Q}^{J^{VH\,2}} + (1+d)\tilde{Q}^{J^{VH}} - d\right) - \tilde{Q}^{J^{SH}}\, ln\left(-\tilde{Q}^{J^{SH\,2}} + (1+d)\tilde{Q}^{J^{SH}} - d\right) + \right.$$
$$\left. + (-\overline{\sigma}_2 + \tilde{Q}^{J^{VH}}) \frac{-2\tilde{Q}^{J^{VH\,2}} + (1+d)\tilde{Q}^{J^{VH}}}{-\tilde{Q}^{J^{VH\,2}} + (1+d)\tilde{Q}^{J^{VH}} - d} - (-\overline{\sigma}_2 + \tilde{Q}^{J^{SH}}) \frac{-2\tilde{Q}^{J^{SH\,2}} + (1+d)\tilde{Q}^{J^{SH}}}{-\tilde{Q}^{J^{SH\,2}} + (1+d)\tilde{Q}^{J^{SH}} - d} \right]$$
$$\left\{ \left(1 + \tilde{VH}^{-1}\right)^{-1} \frac{(A_I\, p_I^{VH})^{1/\alpha} - (A_P\, p_P^{VH})^{1/\alpha}}{(A_I\, p_I^{VH})^{1/\alpha} + (A_P\, p_P^{VH})^{1/\alpha}\, \tilde{VH}} + \frac{\overline{\sigma}_1 \tilde{VH}\, exp(\tilde{VH})}{[1+exp(\tilde{VH})][1+2exp(\tilde{VH})]} \right. \qquad (4.86)$$
$$\left. - \left(1 + \tilde{SH}^{-1}\right)^{-1} \frac{(A_I\, p_I^{SH})^{1/\alpha} - (A_P\, p_P^{SH})^{1/\alpha}}{(A_I\, p_I^{SH})^{1/\alpha} + (A_P\, p_P^{SH})^{1/\alpha}\, \tilde{SH}} + \frac{\overline{\sigma}_1 \tilde{SH}\, exp(\tilde{SH})}{[1+exp(\tilde{SH})][1+2exp(\tilde{SH})]} \right\}^{-1}$$

Differential equations (4.74), (4.75a-c), (4.77), (4.79) and (4.86) form a system which describes the world transitional dynamics, such that the variables D_I^1, D_P^1, $\tilde{Q}^{J^{VH}}$, $\tilde{Q}^{J^{SH}}$, $\tilde{VH}(.|w)$ and $\tilde{SH}(.|w)$ must be constants in steady state.

We can define other important variables, which are also constant in steady state. We note that under IT of IGs: $D_Z^2 \equiv C_Z(VH)/Q_I^{J^{VH}} VH_Z(.|w)$, $D_Z^3 \equiv C_Z(SH)/Q_I^{J^{VH}} VH_Z(.|w)$, $D_Z^4 \equiv C_Z(VH)/Q_I^{J^{SH}} SH_Z(.|w)$ and $D_Z^5 \equiv C_Z(SH)/Q_I^{J^{SH}} SH_Z(.|w)$. Let $DIP^i \equiv D_I^i/D_P^i$, where $i = 2, 3, 4, 5$. Then, using equation (4.73), the dynamic followed by DIP^i is:

$$\frac{\dot{DIP}^i}{DIP^i} = \frac{1}{\theta}(1-\tau_K)(r_I - r_P) + \frac{\dot{\tilde{m}}(.|w)}{\tilde{m}(.|w)} = \frac{1}{\theta}\frac{1}{\alpha}\left[\frac{\dot{p}_I^m}{p_I^m} - \frac{\dot{p}_P^m}{p_P^m}\right] + \frac{\dot{\tilde{m}}(.|w)}{\tilde{m}(.|w)} =$$
$$= \frac{1}{2}\frac{1}{\theta}\left\{\left[1 + \left(\frac{vh}{sh} D_I^1\right)^{-(1/2)}\right]^{-1} - \left[1 + \left(\frac{vh}{sh} D_P^1\right)^{-(1/2)}\right]^{-1}\right\} \frac{\dot{D}_I^1}{D_I^1} + \frac{\dot{\tilde{m}}(.|w)}{\tilde{m}(.|w)} \qquad (4.87)$$

On the right-hand side of equation (4.87), $\tilde{m}(.|w)=\tilde{VH}(.|w)$ for $i = 2, 3$ and $\tilde{m}(.|w)=\tilde{SH}(.|w)$ for $i = 4, 5$. However, according to equation (4.75b) the dynamic of DIP^i in independent of index i. Since DIP^i can be considered a relative measure of welfare between the North and the South, its dynamic gives us an idea about the (relative) leading country at the welfare level.

We can also define $DZ^{i/i'} \equiv D_Z^i/D_Z^{i'}$, where $i = 4, 5$ and $i' = 2, 3$. Its dynamic is,

$$\frac{\dot{DZ}^i}{DZ^i}=\frac{\dot{C}_Z^{SH}}{C_Z^{SH}}-\frac{\dot{C}_Z^{VH}}{C_Z^{VH}}+\frac{\dot{Q}_I^{J^{VH}}}{Q_I^{J^{VH}}}-\frac{\dot{Q}_I^{J^{SH}}}{Q_I^{J^{SH}}}+\frac{\dot{VH}_Z(.|w)}{VH_Z(.|w)}-\frac{\dot{SH}_Z(.|w)}{SH_Z(.|w)}=\frac{\dot{D}_Z^I}{D_Z^I}. \qquad (4.88)$$

Equation (4.88) tells us that the dynamic of $DZ^{i/i'}$ is independent of both *i/i'* and of the country considered – see equation (4.75a). Since $DZ^{i/i'}$ can be called a relative measure of welfare between the specialised and versatile specific technology, its dynamic provides an explanation on the world gaining *m*-specific technology in welfare.

Additional information could also be shown, like the dynamic of D_Z^i, $i=2,3,4,5$ (in addition to the analysis in relative terms) and the dynamic of the amount of (non-leisure) time individuals allocate to production and to training. To simplify, we will only explicitly present the numerical values of $D_Z^{i\,ss}$, u_{wZ}^{ss}, u_{SZ}^{ss}, and u_{TZ}^{ss}, and in the deduction of the differential equation we avoid the use of the Northern interest rate.[266]

4.8.3. Solving differential equations for the baseline parameter values

Bearing in mind the system of differential equations which characterise the world transitional dynamics, we analyse now the transition to and stability of the steady state numerically. The dynamic is displayed for the set baseline parameter values in table 3.4. Thus, we leave for the sensitivity analysis in the next section, the impact of international IPRs protection by considering for now the baseline parameter value $\iota_P = 1$. Besides, we

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[266] To obtain this, we would have to start from free entry into R&D, which in this case implies that:

$$(1-s_H)\,rs_I(k,av,v|m)=pb_I(k,av,v|m)\left\{\Pi_I(k+1,av,v|m)\right.$$

$$\left.\int_t^{\infty} exp\left[-\int_t^s\left(r_I(v)+pb_I(k+1,av,v|m)+pb_P(k+1,av,v|m)-pb_I(k+1,av,v|m)\,pb_P(k+1,av,v|m)\right)dv\right]ds\right\},$$

where the term in large brackets on the right-hand side corresponds to the expected present value of profits for the leader innovator, $V_I(k+1,av,v|m)$. The expression for the interest rate becomes complicated due to the flow of profit of leader firms, $\Pi_I(k+1,av,v|m)$, [*e.g.*, equation (4.36)] and due to the feedback effect reflected in creative destruction. So, the analysis with the Northern interest rate is highly intractable.

assume that $\wp_1 = 5$ and $\wp_2 = 0.03$ are the baseline parameter values in equation (4.78). Table A3.8 in Appendix 3 (subsection A3.2.1) compares initial and steady state values.

We consider that IT begins at time t_0, when the North reaches the autarkic steady state – *i.e.*, the Northern autarkic steady state values are also its initial values at t_0. The adjustment process is globally stable since paths converge to the stable world steady state, which is reached at the end of 110 years. This means that under the IT of IGs, the speed of convergence – common to both countries – falls.[267] This seems to be a strong result since the speed of convergence is not very sensitive to the parameters in $\Xi_2(t)$, as we will see in the next section. The existing feedback effect between countries is accountable for this result. The prolonged time necessary to reach the steady state indicates that the transitional dynamic is important and should not be neglected.

Figure 4.10 below reveals that both D_I^I and D_P^I increase from their initial values – 1.24 and 1.06, respectively –,[268] but at a decreasing rate until they reach their steady state values – 1.29 and 1.11, respectively. That is, $Q_I^{J^{VH}} VH_Z(.|w)$ rises more initially than $Q_I^{J^{SH}} SH_Z(.|w)$ but falls more rapidly towards its steady state value. In other words, during the transition, $\dot{D}_Z^I / D_Z^I$ is positive but declines monotonically towards zero. This result confirms that a temporary partial Schumpeterian dynamic equivalent to the static Rybczynski theorem holds and that factor-intensity reversal exists.

This occurs because, despite the downward level effect in the South at time t_0, differences in HC levels imply that relative prices of versatile FGs are always higher in the South – see figure 4.13 below. Through the price channel, this induces relative demand for versatile-specific new designs, biasing R&D in that direction more than in other IT regime – autarky or TK diffusion without IT effects, as figure 4.14a exhibits.

Although through different channels, the path of D_Z^I suggest that bias of the progress in our framework is in line with findings by Acemoglu – *e.g.*, Acemoglu (2002a). In fact, in the works of Acemoglu, TK progress is biased towards the TK used by skilled workers due to the dominance of the market size channel over the price channel. Measuring the global progress as that which is embodied in IGs and in the HC,

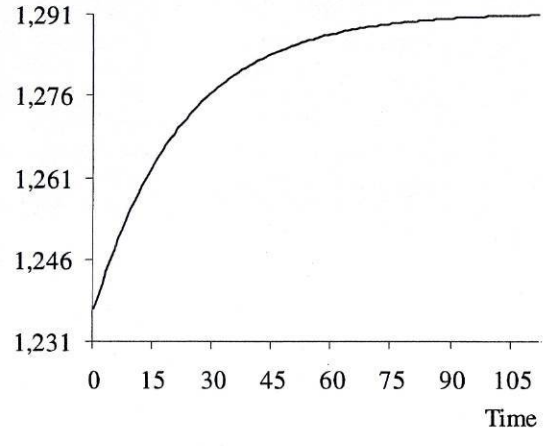
[267] Remember that in the previous regime the South reached the steady state under the benchmark catching up specification (the one considered here) at the end of 56 years.

[268] The initial value for D_I^I is taken directly from table A3.1 and the initial value for D_P^I results from:

we find that, due to the price channel – as a consequence of the IT of IGs – during transitional dynamics, it is biased towards VT, according to the path of D_Z^I.

Figure 4.10. Transitional dynamics of D_Z^I

a) In the innovator country; *i.e.*, D_I^I



b) In the imitator country; *i.e.*, D_P^I

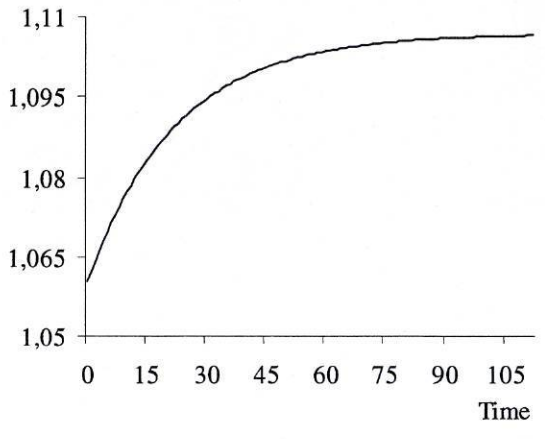


Figure 4.11 below points out that both $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ increase from their initial values (0.30 and 0.35, respectively) towards their steady state values (0.41 and 0.48, respectively). That is, initially the probabilities of successful imitation rise more than the probabilities of successful innovation. However, as the rung of quality left to be copied decreases – *i.e.*, as $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ increase –, the cost of imitation increases, and so the probabilities of successful imitation fall more rapidly than the probabilities of successful innovation towards their new higher steady state values. Thus, both $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ increase at a decreasing rate. Therefore, Southern TK grows more quickly than in the North during the transitional dynamics, but slows down until the steady state is reached.

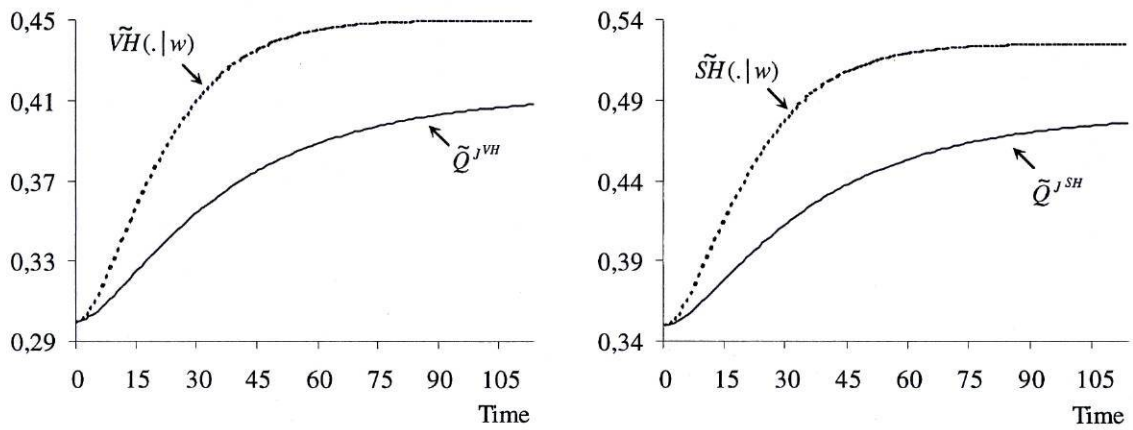
We observe that, when confronted with the previous IT regime, the steady state values for both $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ are now smaller. This is because now the transition phase is common to both countries – remember that before the North remained in steady state – and, due to the feedback effect between countries, TK in the North now increases more than in the previous IT regime. Consequently, it becomes more difficult for the South to catch up with the higher TK frontier in the North, although the South (as well as the North) now reaches higher levels of TK, as suggested by the higher growth rate.

$$D_P^I = \frac{Q_I^{JVH} VH_P(.|w)}{Q_I^{JSH} SH_P(.|w)} = \frac{\tilde{VH}_P(.|w)}{\tilde{SH}_P(.|w)} D_I^I = \frac{0.30}{0.35} \times 1.24 = 1.06$$

Figure 4.11 also presents the transitional dynamics of both gaps in HC at work, $\widetilde{VH}(.|w)$ and $\widetilde{SH}(.|w)$. They increase – *i.e.*, the gaps decrease – from the initial values 0.30 and 0.35, respectively, but at a decreasing rate until they reach their steady state values 0.45 and 0.53, respectively. Thus, both types of HC at work grow at a higher rate in the South during the transition phase, but slow down until the respective steady states are reached – see additional comments to figure 4.6 at the end of subsection 4.6.2. In steady state, both gaps in HC at work are smaller than in the previous IT regime.

Figure 4.11. Transitional dynamics of both human capitals and quality indexes gaps

a) Versatile side; *i.e.*, $\tilde{Q}^{JVH}$ and $\widetilde{VH}(.|w)$ **b) Specialised side; *i.e.*, $\tilde{Q}^{JSH}$ and $\widetilde{SH}(.|w)$**



The behaviour of $\tilde{Q}^{Jm}$ and $\tilde{m}(.|w)$ indicates that from t_0 until the steady state the South grows at a higher rate than the North, but the differential falls steadily towards the steady state. For the differential of growth, figure 4.11 suggests that initially the contribution of the increase in the HC at work is stronger and after that, the contribution of TK is greater. Finally, both countries grow at the same rate and both TK gaps and HC gaps remain constant. Thus, along the convergence path, the South narrows the gaps – *i.e.*, there is a change of inter-country gaps in favour of the South. This is consistent with the evidence, which suggests that countries close to the TK frontier – such as European countries – do not experience rapid growth but rather moderate and stable growth rates.

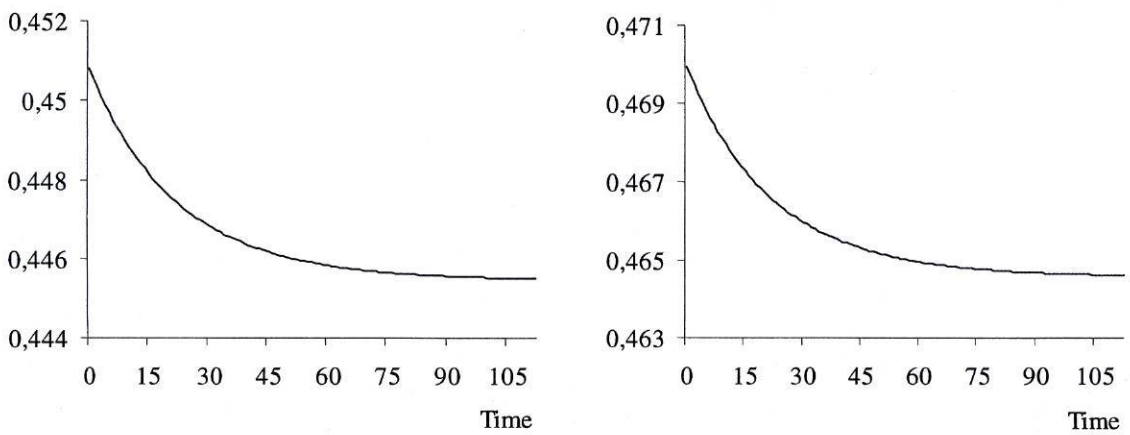
If we compare the steady state growth rates, we would conclude that imitation has improved the growth rate in both countries. According to the baseline parameter values and initial conditions, it is equal to 3%. This is because we assume that improvement in the imitation activity due to the index of openness is stronger and the differential in the *MC*s of production between countries is greater. Note that in the previous chapter,

international TK diffusion already improves the growth rate in the South, which becomes equal to that which exists in the autarkic Northern innovator country; *i.e.*, equal to 2%. Therefore, in our baseline case, the expression in equation (4.53b) is positive.

As we have said previously, despite the level effect in the South at t_0, differences in levels of HC at work – *e.g.*, equation (4.25) – imply that both threshold FG and relative prices of versatile FGs are always smaller in the North – see figures 4.12 and 4.13, table A3.8 in Appendix 3 and also equation (4.26). As expected from the path of D_Z^I in figure 4.10, figures 4.12 and 4.13 also display a drop in the levels of N and relative prices. This drop holds then because the t_0 (under IT of IGs) North-South average relative price of versatile FGs is higher than in the North alone. Through the price channel, it induces relative demand for new versatile-specific designs, biasing R&D innovation in that direction more than in the case without IT – see figure 4.14a below –, which, in turn, diminishes the perfectly competitive domestic relative prices of *VH* FGs.

Figure 4.12. Transitional dynamics of the threshold final good, N_Z

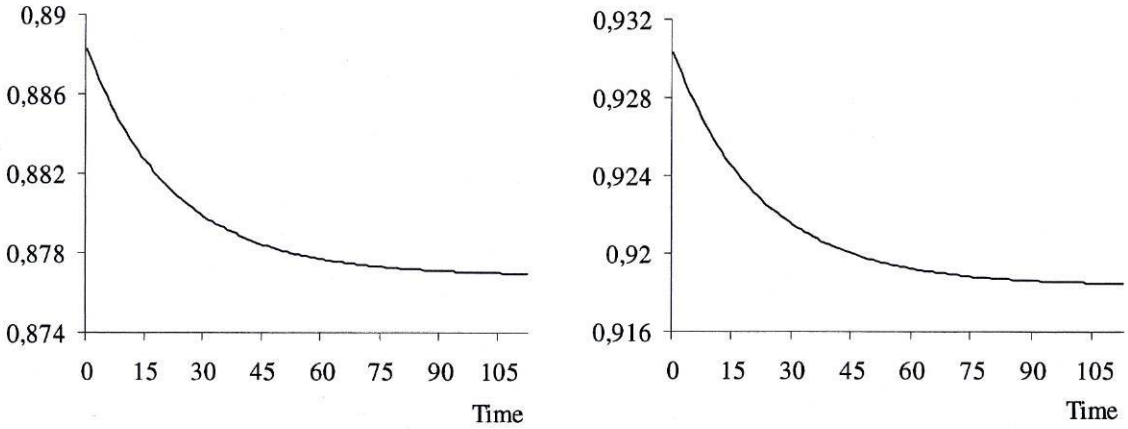
a) In the innovator country; *i.e.*, N_I **b) In the imitator country; *i.e.*, N_P**



More specifically, because of TK leadership, the relative price that operates the price channel is always the one commanding the direction of Northern TK. In the autarkic North, this relative price is $(p_I^{VH}/p_I^{SH})_{Autarky}$; *i.e.*, the relative price of *VH* FGs, which are feeding the demand for *VH*-specific IGs. With the IT of IGs, even though FGs are non-tradable, their prices in both countries are commanding the direction of Northern TK, because of tradability of the IGs necessary to produce FGs in both countries. Due to differences in initial levels – *e.g.*, equation (4.25) –, the North-South average relative price of *VH* FGs is always higher than the one prevailing in the autarkic North.

Figure 4.13. Transitional dynamics of the relative price of *VH* final goods, p_Z^{VH}/p_Z^{SH}

a) In the innovator country; *i.e.*, p_I^{VH}/p_I^{SH} **b) In the imitator country; *i.e.*, p_P^{VH}/p_P^{SH}**



Thus, through the price channel, IT bias available TK in favour of *VH*-specific IGs, as figure 4.14a shows – comparing the autarkic North bias with the bias in the available world TK. This, in turn, stimulates Northern relative demand for the complementary input, *VH*, more than under the autarkic North. In particular, figure 4.14a reveals the relative productivity of versatile TK in the baseline case, and it shows that scenario A described in subsection 4.6.2 holds. Our baseline assumption that countries are not very different in relative terms – *i.e.*, $VH_I(.|w)/SH_I(.|w) \times SH_P(.|w)/VH_P(.|w) = 0.86$ is close to one – contributed to this result. Hence, the magnitude of the level effect influences the resulting scenario, since it affects the North-South average relative price of *VH* FGs.

In summary, in terms of the case without international TK diffusion, the bias decreases the steady state threshold FG and relative price of the *VH* FGs in both countries. Thus, from t_0 until the steady state a growing share of FGs are produced with VT in both countries, which is different from the HOS model where factor-intensity reversal does not exist. Our framework allows us to thus analyse the path of the bias of TK change, and the idea is that the economic forces (profit incentives) that affect the amount of TK progress also shape its direction and, therefore, determine the TK bias.

Thus, measuring progress only by means of TK – *i.e.*, in a restrictive way in our opinion –, we have that, due to the price channel – as a consequence of IT of IGs –, it is relatively more endogenously biased towards VT than without IT. In the works of Acemoglu – *e.g.*, Acemoglu (2002a) –, the same measure of progress is biased towards the skill technology owing to the market size channel. Hence, in relative terms, the IT of

IGs in our framework biases the TK progress in the same direction as in the works of Acemoglu. This view helps us to understand the wage inequality discussion below.

Figure 4.14b shows the path of $\Xi_2(t \mid m)$, where $\Xi_2(t_0) = 0.2$ and $\Xi_2(t_{110}^{ss}) = 0.007$.

Figure 4.14. Transitional dynamics of versatile-specialised technological bias and $\Xi_2(t)$

a) Of $ln(Q_I^{VH}/Q_I^{SH})$ **b) Of $\Xi_2(t \mid m)$ in equation (4.15b)**

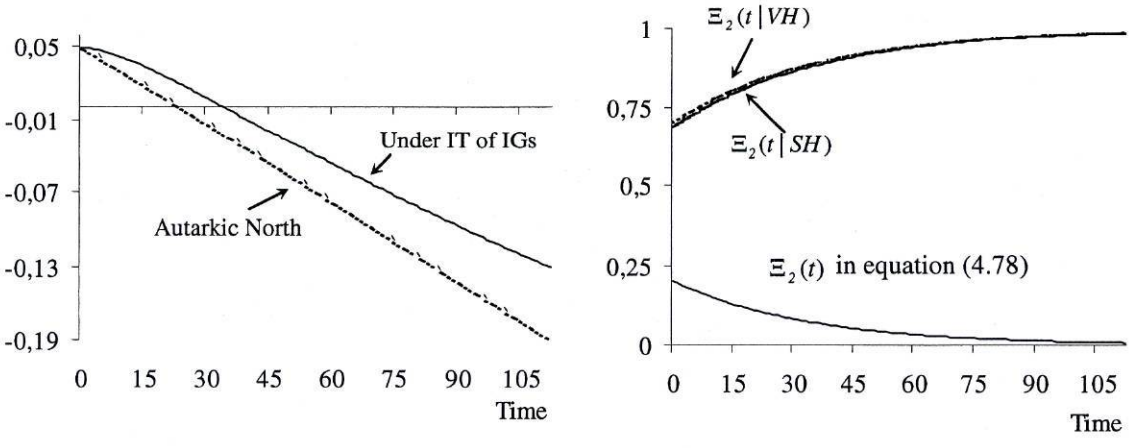


Figure 4.15a shows the path of intra-country wage inequality. In the North, the stimulus for the demand of *VH* attenuates the path of declining inequality, relative to that which would have prevailed under autarky. In this latter case the price channel operated strongly in support of the relatively scarce HC, *SH*. In autarkic South, relative scarcity causes increasing inequality, which is reinforced by the level effect – immediately increasing availability of state-of-the-art *VH*-biased IGs (see figures 4.4-4.5 in subsection 4.6.2, where the versatile premium increases at t_0 and, as a result, free IT of IGs widens the versatile premium gap between countries). However, the strong reversal of the price channel effect – apparent in figure 4.3 in subsection 4.6.2, comparing the autarkic bias in the South with the bias in the available world TK – redirects the path of the relative wage premium in favour of declining inequality, following the same slope as the North.[269]

Thus, in our baseline case, the versatile premiums decrease, but not so much as without IT effects – see figure 4.15a below. The level effect that occurs in the South is reverted and the path of wage inequality – in a specialised direction – is more moderate than without IT effects in the North. This confirms our previous statement that a Schumpeterian dynamic equivalent to the static Stolper-Samuelson result holds. Besides, our baseline case seems to be in line with some recent empirical works which find that in

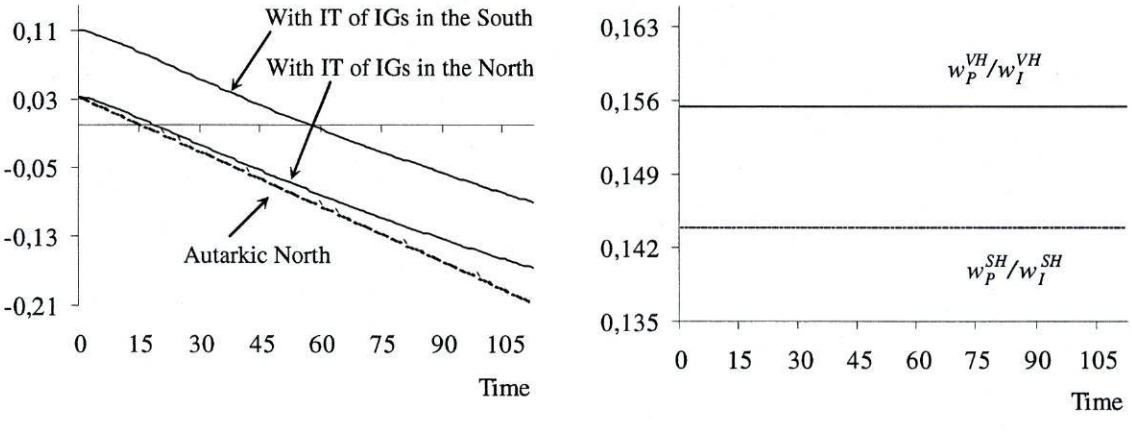
[269] As equation (4.55) shows, the common slope of the inequality path is determined, in steady state, by the negative growth of the versatile-specialised TK bias, which is depicted in figure 4.14a.

Southern countries the premium for skilled labour – more closely related with our *VH* – relative to unskilled labour – more connected with our *SH* – first widens (level effect, as a result of openness to IT and other IT policies) and then narrows – *e.g.*, Kim and Topel (1992) and Kwark and Rhee (1993). Furthermore, also in line with our results – *e.g.*, figures 4.4, 4.5, 4.15a –, Elias (1992) indicates that the skill premia in seven Latin American countries were much larger than they were in U.S. at the same time.

As for inter-country inequality, the relative level of wages jumps at t_0 in favour of the South, due to the rise in productivity brought about by newly available state-of-the-art IGs. However, from then on, inequality remains constant (maintenance of the wage differential between countries), because the path of productivity becomes common to both countries. This is shown in figure 4.15b, where, in addition, the jump is more accentuated for w_P^{VH}/w_P^{VH}, in coherence with the intra-South unbalanced gains from the level effect. In spite of Southern gains, though, international immobility of HC, and differences in domestic institutions (that we assume more productive in the North), in *MC*s and in threshold FGs sustain wage inequality in favour of the North. We cannot compare the wage differences shown in figure 4.15b with the equivalent result in the last IT regime since now the *MC*s are, by assumption, different between countries.[270]

Figure 4.15. Transitional dynamics of wages

a) Versatile premium; *i.e.*, $ln(w_Z^{VH}/w_Z^{SH})$ **b) Differences between countries, w_P^m/w_I^m**



We verify that both *DIP* and *DZ* are increasing – see figures 4.16a, b, respectively –, but at a decreasing rate until the steady state is reached. At first sight, the dynamics of *DIP* suggest that, during the transition, welfare in the North increases relative to welfare

in the South. However, from this variable we can only state that during the transitional dynamics consumption per 'large amount of TK' increases in the North. Indeed, starting from *DIP* we can find the ratio between consumption in both countries; *i.e.*,

$$DIP = \frac{C_I(.|m)}{C_P(.|m)}\tilde{m}(.|w) \Leftrightarrow \frac{C_P(.|m)}{C_I(.|m)} = \frac{\tilde{m}(.|w)}{DIP}. \tag{4.89}$$

Now, computing the ratio between consumption in both countries – since we know the dynamics of $\tilde{m}(.|w)$ and *DIP* –, there is no relevant alteration in the consumption relative of the countries during the transition phase – see table A3.8 in Appendix 3.[271] That is, both countries benefit from enhancements in the TK frontier, which are greater than in the last IT regime since the rate of successful innovation increases due to the stimulus generated by imitation, and *vice-versa* – Northern IG firms try to keep ahead of Southern IG firms and Southern IG firms try to explore the opportunities provided by backwardness, and this behaviour increases both innovation and imitation rates. In short, none of the countries benefit at a welfare level, and so this suggests that international IPRs protection does not make sense, as we will see in the next section.

At first sight, the path of *DZ* suggests that in relative terms the workers that accumulate *SH* in each economy are the ones that benefit at the welfare level. However, in rigour, we need to calculate the ratio between the consumption of each *m*-type for both countries. Thus, from the definition of *DZ*, we have that:

$$DZ = \frac{C_Z(.|SH)}{C_Z(.|VH)}D_Z^I \Leftrightarrow \frac{C_Z(.|SH)}{C_Z(.|VH)} = \frac{DZ}{D_Z^I}. \tag{4.90}$$

Computing the ratio in equation (4.90), there is no relevant change in the relative consumption between types of workers, in both countries – see table A3.8. This suggests that the share of resources spent on versatile and specialised R&D remains constant and so none of the types of workers benefit at the welfare level during the transition phase.

We conclude the section by numerically computing certain measures of *TFP* given in subsection 4.7.2 at time t_0 and in steady state, t_{110}. The results in table A3.8 and are in line with our comments about *TFP* differences between countries. Thus, the FGs *TFP* and *PTFP* are both larger in the North. The difference is, however, smaller in steady

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[270] To have an idea, if we calculate the FGs price index considering equal *MC*s, the difference will now be smaller $w_P^{VH} / w_I^{VH} = 0.49$ and $w_P^{SH} / w_I^{SH} = 0.46$.

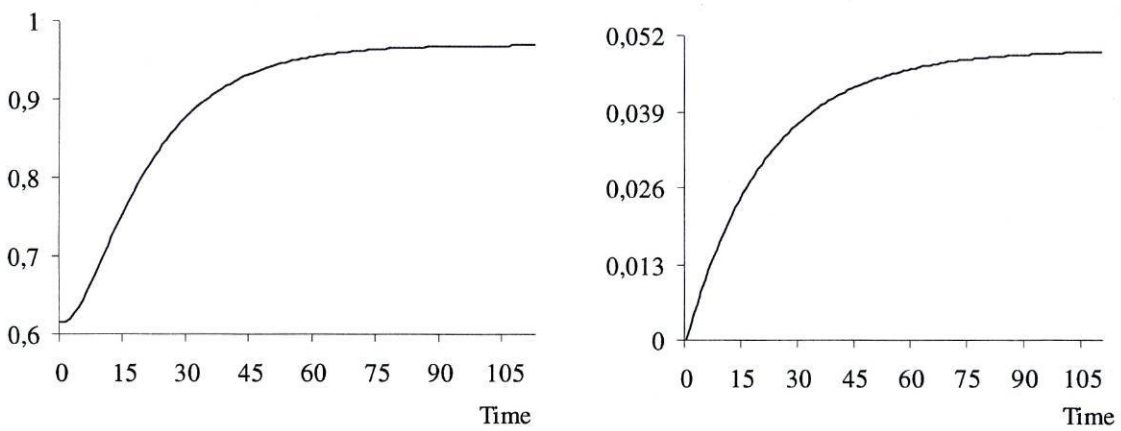
[271] The same conclusion results from the path of the interest rate in both countries. In fact, during the transition dynamics, r_P is only very slightly greater than r_I and so there is no relevant alteration in the consumption relative between countries – and in the steady state $r_P = r_I$.

state, as a result of advantage being taken of the opportunities provided by the imitation activity. At the level of the FGs *PTFP*, the difference increases slightly with n. The difference at the aggregate level is expected, bearing in mind the FGs *TFP*.

Furthermore, in comparison to the last IT regime, now the Southern FGs *PTFP*s are closer to their Northern values. This is due to international free access to state-of-the-art IGs. However, contrarily to that which could be expect at first sight, the *TFP* difference between countries in both the FGs *TFP* and the measure of aggregate level calculated are now greater. This is because the negative impact of the differences in *MC*s between countries overbalances the positive effect provided by free access to state-of-the-art IGs.

Figure 4.16. Transitional dynamics of *DIP* and *DZ*

a) Of *lnDIP* **b) Of *lnDZ***



4.9. Sensitivity analysis

We will now conduct the sensitivity analysis of the steady state and the dynamics in order to test the robustness of our results to shocks in the parameters. Numerical simulations show that the model's qualitative behaviour is similar when the value of each parameter at a time changes relative to its baseline value. Thus, the stability of the model is preserved, and slight differences are mirrored in the steady state value of the variables.

So as to comment the results summarised in table A3.9 in Appendix 3 (subsection A3.2.2), first we analyse the implications of changes in the parameters specific to the IT of IGs – *i.e.*, in the openness index, *IT*, international IPRs protection, ι_P, small marginal cost of production in the South, MC_P, and in parameters of the term of convergence $\Xi_2(t)$, $\wp_1$ and $\wp_2$. Afterwards, we discuss the effects of the other parameters and, finally, we verify the consequences of changes in the initial values of the variables.

A greater interaction between countries – *i.e.*, greater *IT* – generates both a small TK gap and a higher world growth rate, demonstrating dynamic benefits for the South of increases in IT with the North. Therefore, *IT* must be greater in order to improve the world growth rate in relation to the last IT regime. As the consumption relative between countries – equation (4.89) – remains constant, the welfare relative never changes – *i.e.*, changes in the interaction between countries do not affect the welfare relative. Thus, as freer IT of IGs improves the world growth rate, it benefits both countries. In particular, it benefits *VH* since, due to the increase in the relative price of versatile FGs, it stimulates relative demand for the complementary input, *VH*, and so the versatile premium rises.

Since international IPRs protection would be justified in the presence of welfare loss in the North, this does not make sense in our framework. In fact, analysing the consequences of the drop in parameter β_P – which is equivalent to the increase of international IPRs protection –, we verify that international IPRs protection increases the TK gap – *i.e.*, more resources are spent in R&D and in production of IGs in the North – and (slightly) the *TFP* differences between countries – inter-country wage inequality in favour of the North occurs – and decreases the world growth rate. In short, it harms both countries and, in particular, the *VH* since the versatile premium decreases.

As stated in subsection 4.6.1, we observe numerically that a small MC_P generates a greater world growth rate. This is because it represents a higher mark-up for monopolist producers in the South, which provides an incentive to imitation activity and, due to the feedback effect, also to the innovation activity. In this case, the impact of imitation on innovation is so strong that the TK gap between countries widens, as well as the HC gap, although not as markedly.[272] Thus, the *TFP* differences between countries increase.

An increase in parameters $\wp_1$ and $\wp_2$ of the term $\Xi_2(t)$ gives rise to a small world growth rate and a wider TK gap. A higher $\wp_1$ and/or $\wp_2$ implies that $\Xi_2(t)$ is small, which means that the losses for the South as a result of successful innovations are greater at each time until the steady state. This increases the TK gap and reduces the positive impact of imitation to innovation, which must be responsible for the decrease relative in the HC at work in the North. Moreover, a higher $\wp_2$ increases the speed of convergence.

Due to the role of competitive monopoly profits, an increase in one of the exogenous variables A_I and A_P generates a higher world growth rate, slightly accelerates

[272] Remember that now the path of wages, which affects the path of HC due to the TK progress, is the same in both countries – it depends on Northern TK progress.

the speed of convergence and narrows the TK gap between countries. This confirms that IT of IGs emphasises the importance of improvements in domestic institutions, since they turn out to matter not only at home but also abroad. However, numerical simulations suggest that improvements in A_I – more closely related with innovation activity – have a greater impact on the world growth rate. Moreover, while an improvement in A_I requires relatively more HC at work in the North and, thereby, widens the gap in HC at work, an improvement in A_P requires relatively more HC at work in the South and so narrows the gap in HC at work. As the paths of $\tilde{Q}^{J^m}$ and $\tilde{m}(.|w)$ follow the same direction after an improvement in A_P, the versatile premium increases in both countries. The direct impact of A_I and A_P on the exogenous productivity of the respective country implies that the *TFP* differences increase when A_I also increases and decrease when A_P also increases.

An increase in each one of parameters α, θ and ρ decreases the world growth rate – see also table 4.2. In particular, a higher α reduces the speed of convergence and the versatile premium and is a disincentive to R&D activities since it increases the discount rate of profits of IG producers, which decreases the world growth rate. Moreover, the disincentive in innovation is greater than in imitation since the TK gap decreases. A relatively small decrease in imitation implies that relatively more Southern HC at work is necessary, which also decreases the gap in HC at work. The reduction in the gaps between countries does not compensate the negative influence of a higher α in the *TFP* differences equation. The most relevant effect of an increase in θ is in the decrease of the world growth rate. In addition, an increase in ρ also increases the versatile premium.

Except for the effect on the world growth rate, changes in parameters *vh* and *sh* have opposite effects. Taking as an example an increase in *vh*, it decreases the speed of convergence and has a direct positive effect on the versatile premium in line with subsection 4.7.3. Besides, it increases the TK gap and the gap in HC at work and the *TFP* differences between countries, as well as the world growth rate. Indeed, it increases profits and so R&D activities, which improves the world growth rate. Since it represents an increase in the absolute productivity advantage of *VH* over *SH* – *i.e.*, in the HC which is always relatively more abundant in the North –, it accelerates the innovation activity more than it does imitation, which widens the TK gap. Stronger productive activity in the North needs relatively more HC at work and so the gap in HC at work broadens. The increase in *TFP* differences results from the behaviour of TK and HC at work.

Generically, an increase in parameters PE and σ_1 of the catching up term improves directly the probability of successful imitation and, due to the feedback effect, also the equilibrium probability of successful innovation. Consequently, the world growth rate increases. An increase in σ_2 also improves the imitation capacity, affecting positively the probability of successful innovation and so the world growth rate. However, contrarily to the improvement in σ_1, the increase in σ_2 has a stronger effect on innovation than on imitation, which implies that the TK gap increases. The main finding about changes in d is the greater the value of d the smaller the world growth rate. The intuition for this result is that the greater the value of d, the higher the cost of imitation – and so of innovation –, and the smaller the potential of TK imitation – and innovation – and growth.

As to the HC production parameters, the increase in δ makes HC more intensively obsolete and so reduces the world growth rate. A higher φ^m – *i.e.*, the more intensive the use in the production of *VH* and *SH* of the more productive mode of HC production, which is schooling – the higher the world growth rate. It increases in relative terms the *VH* in each country and specialised TK and so reduces the versatile premium. The effects of the increase in the substitutability of the modes of HC production – *i.e.*, in ϕ – are similar to the increase in φ^{VH}. In fact, the higher ϕ, the higher the use of the more productive one will be, and, thus, once more, the higher the world growth rate and the smaller the versatile premium. Improvements in productivity in the HC sector through χ_S and χ_T increase its accumulation, which, in turn, increases gr. A higher χ_S increases in relative terms *VH* and decreases the versatile premium, whereas a higher χ_T increases in relative terms *SH* and so the versatile premium.

In particular, a more intensive use of the more productive mode – either from increases in φ^m, χ_S, or ϕ – increases the relative advantage in accumulating *VH*, which is the type of HC that uses more intensively the more productive mode. The rise in the relative supply of *VH* lowers its relative wage, thereby increasing its use in FGs production and lowering the relative price of versatile FGs. As the relative price of specialised FGs rises with the more intensive use of schooling, TK is directed towards *SH*-specific IGs. Since the South is relatively abundant in *SH*, it benefits relatively more in terms of productivity, and thus wages, which induces the reduction in the HC gaps.

Hence, scenario B in section 4.6.2 about intra-country wage inequality is more likely to prevail when changes in production of HC relatively enhance the world accumulation of the type of HC that is relatively abundant in the South, *SH*. That is,

when φ^m, χ_S, or ϕ decreases or when χ_T increases. Indeed, improvements in the production of *SH* enhance its accumulation, thereby increasing its relative supply and lowering its relative wage, which, in turn, incentives its use in FGs production and lowers the relative price of specialized FGs. In this case, the North-South average relative price of versatile FGs is greater than in the baseline case, which, in turn, is greater than the one prevailing in the autarkic North. This accentuates the TK bias in favor of versatile-specific IGs. With sufficiently contrasting parameter values, scenario B reverts both the autarkic and baseline paths of available TK bias.

In short, in scenario B, enhanced accumulation of *SH* decreases the relative price of specialized FGs, which, in turn, strongly re-directs R&D towards designs for versatile-specific IGs. Once relative prices of FGs attain their constant steady state levels, the only source of intra-country wage inequality is the direction of TK – *e.g.*, equation (4.55). Due to complementarity in the production of FGs, the steady state increasing bias in favor of versatile-specific IGs determines the steady state path of increasing *VH* premia.

Now we analyse the effects of changes in the initial value of the constant variables in steady state. An initial small versatile TK gap increases R&D activities and so growth, but as the TK gap decreases it strongly affects imitation activity. Moreover, the *TFP* differences between countries increase due to the increase in the gap in HC at work. Opposite effects occur when the initial decrease occurs in the specialised TK gap. If the initial *SH* gap narrows, it improves imitation activity and, as a result of the feedback effect, also and strongly the innovation activity. Thus, the world growth rate increases as well as the TK gap. Strong induced innovation requires greater HC at work in the North, widening the gap between countries. Contrarily to the South, in the North the versatile premium decreases because in relative terms the *VH* increases. Opposite effects occur when the initial decrease is in the *VH* gap. Therefore, as already stated, the sensitivity analysis confirms clearly that the steady state depends on initial conditions.

Common domestic government intervention increases the gaps, *TFP* differences between countries, inter-country wage inequality in favour of the North, as well as the versatile premium and the world growth rate.[273] Thus, in particular, it generates a stronger effect in the innovation activity.

[273] In the last IT regime, Southern government intervention improves the world TK gap but does not affect the world steady state growth rate, whereas Northern government intervention increases the world steady state growth rate and must increase the world TK gap. Now, due to the feedback effect, even Southern government intervention improves the world steady state growth rate.

In summary, we can state that: (i) table A3.9 confirms numerically the information presented in table 4.2; (ii) international IPRs protection moderates positive effects of IT; (iii) greater speed of convergence is obtained with better world institutions; *i.e.*, with higher As, smaller α, smaller vh and greater $\wp_2$; (iv) a smaller TK gap is reached under a higher α and a higher initial specialised TK gap, as well as under a small initial versatile TK gap; (v) higher d, σ_2 and $\widetilde{VH}(.|w)$, and smaller $\tilde{Q}^{J^{VH}}$ and $\widetilde{SH}(.|w)$ decrease the gap in HC at work; (vi) the versatile premium is greater when α, φ^{VH}, χ_S and ϕ are small and when χ_T is great and domestic policy is implemented; (vii) *TFP* differences decrease with smaller A_I, $\tilde{Q}^{J^{VH}}$ and $\widetilde{SH}(.|w)$, and with greater A_P, $\tilde{Q}^{J^{SH}}$ and $\widetilde{VH}(.|w)$.

4.10. Some elements of synthesis

The aim of this chapter is to more clearly understand how the IT of IGs alone, which are traded without any trade costs, affects the structure and the dynamics of international TK transfer by imitation, thereby affecting the levels and growth rates of the countries, as well as prices and wages. In keeping with the tradition of growth and IT analyses, we are particularly able to study results in factor prices. These results may shed some light on issues that have been raised and discussed in the context of globalisation debates, as is the case of effects of IT (of IGs) on inter and intra-country wage inequality.

The North is the innovator that improves the quality of IGs and it is in the autarkic steady state when IT of IGs starts. It never has an incentive to imitate, since there is never better foreign TK to imitate. The South has a *MC* of production advantage and, if the TK gap between countries is smaller than the threshold gap, it is an imitator. Due to lower-priced imitation, Southern IG firms can produce and export state-of-the-art IGs. Moreover, as the South can import higher quality IGs from the North for use in its FGs production, it is no longer limited by its own ability. Thus, the South imports state-of-the-art IGs that have not yet been imitated, and export state-of-the-art IGs that it has imitated by under-pricing innovator leader firms. Hence, the TK transfers from North to South take place either through the Southern production of state-of-the-art IGs after successful imitation or through the importation of Northern higher quality IGs.

In short, since Southern TK is not equalised with the North, because, at each point in time, not all innovations have been imitated yet, there is a distinction between Southern TK and (Northern) available TK in the South. Moreover, the framework is now

a North-South IT model where the location of IGs firms can change over time. Each new quality is initially produced in the North but then, when imitation occurs, production shifts to the South – *i.e.*, countries switch to incorporating imitated IGs in their FGs production whenever they are available and, thereby, an IGs-cycle IT model results.

The IT of IGs is always balanced and international capital flows do not occur. As a result, the countries' interest rate is determined domestically. International mobility of HC is not possible as well. The IT pattern reflects the North-South differences in TK in line with the Ricardian model, but under endogenous TK progress, which makes the CA (in IGs) dynamic. That is, different R&D capacities, which, in final analysis, slide from different exogenous levels of productivity and HC stocks, cause differences in TK. As the CA also depends on the HC levels, which, in turn, rely on endogenous HC accumulation, a dynamic version of the HOS model is also considered.

When the IT of IGs begins there is an immediate level jump in Southern output (level effect), due to the available state-of-the-art IGs. This favour HC employed in the South by improving its productivity and induces some convergence in prices and in the pattern of the FGs productive structure between countries. As a result, inter-country wage inequality falls and Southern HC accumulation is immediately stimulated by higher wages. However, Southern convergence is not complete due to the maintenance of the gap in non-IT related exogenous productivity in FGs production function and due to the international immobility of factors.

As a result of the level effect, the intra-South wage inequality changes as well. According to our assumption that initially the versatile-specific TK gap is relatively wider and the North is relatively *VH* abundant, the Southern versatile premium increases. In fact, free IT of IGs widens immediately the versatile premium gap in favour of the South. This is different from the IT theory which suggests that IT between the South and the North reduce the wage premium for skilled labour in the former. However, our result seems to be is in line with empirical evidence. For example, countries that joined the European Union late – Ireland, Greece, Portugal and Spain – experienced significant increases in school enrolments rates thereafter – *e.g.*, World Development Report (1991, table 29) –, which, in turn, reflects the higher wages of the school intensive HC.

From this time on, (i) inter-country inequality remains constant, because the path of productivity becomes common to both countries and (ii) the growth rate are driven by domestic HC accumulation and Northern TK progress. Although TK in the North is in

the lead, it is influenced by the South in two ways. Due to complementarity, demand for IGs also depends on the Southern HC structure and due to the feedback effect reflected in, for example, international limit pricing, incentives to innovation depend on imitation.

The IG leader firms, which produce with the latest patent in each IG, are now able to sell to a larger market. However, the difficulty in conducting R&D (innovation or imitation) increases because of higher scale costs. As we aim at understanding the ways in which the IT of IGs affects TK transfers in contexts where market size effects are not crucial, we assume that the two effects counterbalance each other. In addition, Northern IGs leader firms are concerned with the joint probability of losing their markets either due to the next innovation in the North or due to a lower-priced imitation in the South.

The ability of the South to benefit from innovations taking place in the North increases if the positive effect of openness to the IT of IGs and other trade policies is strong. In addition, this generates a more efficient allocation of production in the world, since it becomes located in the South where *MC* of production is lower. This, together with the initial level effect induced by the IT of IGs, feeds back into the North, affecting its TK through creative destruction and, thus, the growth rates – *i.e.*, via imitation, the IT of IGs generates a positive effect on innovation. Indeed, Northern leader IGs firms remain less time in the market and the time between successful innovations is also smaller, increasing TK progress and, thus, growth. A virtuous cycle is then created since Southern IGs firms are driven out of the market by a new successful innovation.

A trade-off involving the South-North TK gap occurs. The Southern firms must support R&D imitative activity of all quality levels between their current domestic level and the Northern state-of-the-art IG that they are targeting before they earn profits. Thus, the total resources required to bring about successful imitation are potentially greater than before. Though, a wide TK gap favours the probability of successful imitation since the backwardness benefits are higher – provided that the TK threshold d is overcome.

The IT of IGs can account for the accelerating TK change and so for higher wages, but also for increased intra-country wage inequality, by raising (reducing) the versatile (specialised) premium. In fact, openness to the IT of IGs induces more versatile-biased TK change than before, by increasing the incentives to develop new technologies that are complementary to *VH*. Moreover, remember, when IT starts increased wage inequality also takes place and with greater intensity due to the level effect in the South.

Hence, IT of IGs affects both HC returns and intra-country wage inequality. The former by induced growth effects in both countries and also by the level effect in the South. The latter, by the direction of the TK progress as a result of differences in prices (price channel) between countries and also by the level effect in the South. We must point out that by removing the scale effects, we are focusing on the price channel. That is, incentives to improve specific IGs are stronger when the prices of FGs produced with these IGs are higher. Thus, under IT of IGs, even though FGs are non-tradable, their prices in both countries are commanding the direction of Northern TK, due to tradability of IGs necessary to produce FGs in both countries. Since, even after the level effect, the North-South average relative price of versatile FGs is higher than the one in autarkic North, IT of IGs bias TK in favour of versatile-specific IGs when compared with autarkic North. This, in turn, stimulates relative demand for the complementary input, *VH*.

When considering specifically the path of intra-country wage inequality, it depends in particular on the degree of the level effect and on the HC production function. If the level effect is strong, which occurs when countries are very different in factor levels when IT of IGs starts, the versatile TK bias is increasing and then the versatile premium is raising. The same occurs if the relative accumulation of *SH* is strong, which, in turn, arise when, in relative terms, the intensity and the productivity of schooling is low, the productivity of OJT is high and the degree of substitutability between schooling and OJT is low. In short, under both cases – strong level effect and strong *SH* accumulation – the North-South average relative price of versatile FGs is significantly higher, which, in turn, implies that the Northern relative productivity of versatile TK is increasing – *i.e.*, that R&D is directed towards versatile-specific IGs. Hence, the versatile premium increases.

Our case is thus different from the literature on skill-biased technological change. In this literature, the bias that causes wage inequality is mainly induced through the market size channel. In our case, changes in inter and intra-country wage inequality results similarly from TK bias, but are, instead, induced through the price channel under IT. In particular, intra-country wage inequality is more likely to prevail under IT, when changes in production of HC relatively enhance the world accumulation of the type of HC that is relatively abundant in the South, as stated above. Moreover, we can state that in the North the versatile premium always increases more under the IT of IGs than under autarky. In the South, there is a level jump in the versatile premium when IT of IGs starts, but the path can change or can be more moderate than under autarky.

Summarising the main steady state results, we observe that the Northern TK progress and the domestic HC accumulation determines the endogenous constant world growth rate. Although TK in the North commands, it is influenced by the South, as already stated. We find that when (i) the impact of openness on the probability of successful imitation is strong, (ii) the *MC* in the South is low, (iii) international IPRs protection is weak, and (iv) there is no change in the ad-valorem subsidy rate of IGs production (common to the North and South) between different IT regimes and, in addition, it is higher, then the world (common to both countries) steady state growth is higher under IT of IGs than in the autarkic North.

The levels of output, however, remain different between countries due to the maintenance of North-South gaps in exogenous levels of productivity and in levels of HC at work. The steady state TK gaps remain constant as well. In particular, IT emphasises the importance of improvements in domestic institutions, because they turn out to matter not only at home but also abroad. We also observe that there is a change in the productive structure and a corresponding drop in the relative price of versatile FGs. These effects operate through the price channel, not through the market size channel that has been removed. In particular, the change in productive structure in the versatile direction – *i.e.*, the use of VT increases temporarily – indicates that a temporary partial Schumpeterian dynamic equivalent to the static Rybczynski theorem occurs.

TK and relative wages have steady state dynamics, and the TK bias and intra-country wage inequality paths are smoothed by IT of IGs. In fact, the effect of the price channel on the R&D activity generates a Schumpeterian dynamic equivalent to the static Stolper-Samuelson theorem. Once again, there is also a dynamic Schumpeterian factor-price equalisation result – the growth rates of wages converge between countries. The degree of wage inequality between countries (inter-country wage inequality) decreases, but wage levels do not converge completely due to differences in exogenous levels of productivity (non-IT related) and due to the international immobility of HC.

Concerning government intervention, we find that the increase in subsidies to IGs production and R&D lead to a permanent increase in the world growth rate and a higher versatile premium, but do not affect inter-country wage inequality.

We also explore some extensions of the model. First, we analyse both the cross-FGs and the aggregate *TFP* differences between countries in steady state. We find that they are greater in the North, but to a lesser extent than without IT effects.

Next, we analyse the effects resulting from the use of the new GPT. Now the South is no longer limited by its own ability to produce the best quality IGs, and so the different phases begin at the same time in both countries. If the new GPT is used in production of all FGs at the same time, there is an initial growth effect, which is increased in the next phase, when there is also a level effect. If, at first, the new GPT is used in the production of versatile FGs, there are three distinct phases. In the first, a small growth effect occurs. In the next phases, there are both level and growth effects. In addition, the versatile premium increases in the second, but falls in the third and the opposite occurs for the threshold FG. An increase in the versatile-augmenting parameter in the production function induces the same type of effects as the use of the new GPT only in VT.

Outside the steady state, the TK gap between countries changes and transitional dynamics occur. These transition paths are dependent on the relative TK levels and on the relative levels of HC employed and, thus, on prices in both countries. During the transition to the steady state, the difference in prices between countries affects the versatile-specialised TK bias world measure, HC accumulation and intertemporal trade-off between work and accumulation of HC and, consequently, the versatile premia.

Therefore, to analyse the stability of and convergence to the world steady state, countries must be considered together. The system of differential equations is again the result of equilibrium in labour markets, individual utility maximisation with individual optimal time allocation and free entry conditions into R&D – innovation and imitation. Considering the benchmark catching up specification, we solve numerically the system using the fourth-order Runge-Kutta classical numerical method.

The adjustment process is globally stable since the time path of the system converges to the steady state, which is reached at the end of 110 years, and remains there without any exogenous change. The interdependence between countries decreases the speed of convergence, since in the previous chapter the South reached the steady state at the end of 56 years. Bearing in mind the long time necessary to reach the steady state, the transitional dynamic is important and should not be neglected. Moreover, transitional dynamic clarifies the working of the model and so the related mechanisms described.

Thus, it becomes clear that, despite the level effect in the South, differences in the levels of HC implies that relative prices of versatile FGs are always higher in the South. This increases relative demand for new versatile-specific designs through the price channel, biasing innovation activity in that direction more than in the previous chapter.

During the transitional phase, TK and the HC at work in the South grow more quickly than in the North, but slow down until the steady state. That is, the South grows at a higher rate than the North, but the differential falls steadily towards the steady state.

While total convergence in available TK is instantaneous with the IT of IGs (level effect), we confirm numerically that domestic levels do not converge completely. That is, the potential provided by Southern backwardness together with its imitation capacity is insufficient in providing absolute TK convergence. IT of IGs affects HC accumulation, changing gaps in favour of the South. Indeed, Southern HC accumulation is immediately stimulated by higher wages (level effect), and from then on continuously adjusts the levels to their higher steady states. When confronted with the permanent widen in wages brought about by the IT of IGs, Southern individuals adjust the path of time allocation, devoting less time to work. In other words, the South-North ratio of time spent at work falls when the IT of IGs starts. Then it adjusts smoothly back to its steady state level around one. The path of the ratios of HC at work reflects this dynamic adjustment.

Under the baseline parameter values (i) the IT of IGs with imitation improves the world growth rate; (ii) no alteration in the consumption relative of the countries occurs; (iii) regardless of the country, a winner type of workers at the welfare level does not occur; and (iv) the *TFP* difference between countries decreases until the steady state. As the world growth rate increases and the welfare relative between countries remains constant, countries unambiguously gain from TK diffusion under the IT of IGs.

Moreover, we find that (vi) the bias toward VT (temporary partial Schumpeterian dynamic equivalent to the static Rybczynski theorem) decreases the threshold FG and relative price of the versatile FGs in both countries – *i.e.*, in contrast with the HOS model, 'factor-intensity reversal' exists; (vii) the versatile premium immediately rises in the South due to the level effect and, after that, it decreases in both countries, but not as much as in without IT effects. Thus, the level effect in the South is reverted and the path of wage inequality (in a specialised direction) is more moderate than in autarkic North – a Schumpeterian dynamic equivalent to the static Stolper-Samuelson theorem occurs. Finally, we must stress that (viii) in fact the wage differential between countries remains.

The chapter finishes with the sensitivity analysis. Since numerical simulations show that the model's qualitative behaviour is similar when the value of each parameter changes at a time relative to the respective baseline value, it is stable.

Numerical simulations confirm that greater interaction between countries – greater *IT* – generates a small TK gap, a higher world growth rate, and has no influence on the welfare relative between countries. Thus, freer IT of IGs benefits both countries and, especially, the *VH* since the versatile premium increases. Conversely, since international IPRs protection decreases the world growth rate and does not affect the welfare relative, it serves only to moderate the positive effects of the IT of IGs. They also confirm that a small MC_P generates a greater world growth rate. A small MC_P implies a higher mark-up for Southern monopolist producers which provide an incentive to imitation activity and, due to the feedback effect, also to the innovation activity. The impact of imitation on innovation is so strong that the TK gap between countries widens, as well as the HC gap – although to a lesser extent. As a result, the *TFP* differences between countries increase.

Other results must be emphasised. Summarily, (i) the information in table 4.2 is confirmed; (ii) a greater speed of convergence is obtained with higher exogenous levels of productivity, a greater HC share in production, a smaller productivity advantage of *VH* over *SH* and a greater $\wp_2$; (iii) a smaller TK gap is reached under a higher HC share in production and a wider initial specialised TK gap, as well as under a smaller $\wp_1$, $\wp_2$ and an initial versatile TK gap; (iv) a smaller gap in HC at work is obtained under a higher d, $\bar{\sigma}_2$, $\tilde{Q}^{JSH}(t_0)$ and $\tilde{VH}(t_0|w)$, and a smaller initial $\tilde{Q}^{JVH}(t_0)$ and $\tilde{SH}(t_0|w)$.

At the wage inequality level, we find that (i) the versatile premium is markedly greater when α, φ^{VH}, χ_S and ϕ are small and when χ_T is great and domestic policy is implemented; (ii) inter-country wage inequality in favour of the South decrease with smaller A_I, $\tilde{Q}^{JVH}(t_0)$ and $\tilde{SH}(t_0|w)$, and with greater A_P, d, $\bar{\sigma}_2$, $\tilde{Q}^{JSH}(t_0)$ and $\tilde{VH}(t_0|w)$.

The theoretical framework developed can also be extended to explore the effects of several other dimensions of IT, which represent steps for further research. Thus, the consideration of (i) different domestic policies – in the North and South; (ii) IT costs; (iii) North and/or South import tariffs and quotas as well as export tax, subsidy and voluntary export restraint – and other IT policies; (iv) the establishment of Northern multinationals in the South; (v) HC mobility from South to North, can be readily incorporated into the model, and their effects on TK progress, growth and wage inequality can be examined; and (vi) a formal welfare analysis when IT of IGs is free could be introduced in order to analyse the optimal policies that can be used to provide policy recommendations for managing North-South international connections.

CHAPTER 5

EXTENSION TO INTERNATIONAL TRADE OF ALL GOODS

5.1. Overview

The distinction between the effects of IT of IGs and of IT of FGs is essential, since we want to underline the dynamic growth effects of IT operating through IGs – where R&D is directly applied –, while the effects of IT of FGs are limited to the somewhat more traditional (in IT theory) static level effects. Moreover, the IT of IGs is insufficient for poorer (Southern stagnant) countries to engage in IT with gains. Thus, now we add the IT of all goods – IGs and FGs – in order to analyse the pattern of FGs specialisation of both countries and to check which kind of interest poorer countries have in IT.

The pattern of FGs specialisation is determined by the relative productivity of each country in each good, which is highly influenced by the HC levels. As IT expands, the IT theories implied in our framework – dynamic versions of the Ricardian and the HOS models – gain weight and the interdependence between countries increases. However, as occurs with the CA in IGs, the CA in FGs is also dynamic and for the same reasons – endogenous accumulation of TK and HC. The dynamic CA of countries at FGs level implies that the location of FGs production can also change over time.[274]

Now, even if the South is unable to imitate, it can participate in IT. Actually, it can immediately use (Northern) state-of-the-art IGs, since imports become feasible by the chance of exporting FGs. This allows the South to enjoy the static level benefit as well as a jump to a new higher steady state growth rate; *i.e.*, immediate conditional convergence occurs. As Northern innovating firms need not worry about potential imitation, they behave as under no IT effects, except that greater expected profits due to increased market size lead to a greater rate of innovation. This, in turn, increases the world growth rate relative to the autarkic North. However, as the Southern TK stops growing, TK divergence between countries occurs. This case will only be explored as an extension.

The analysis between the North and the Southern imitator stays the most attractive and will occupy the central concerns of the chapter. Thus, the basic assumptions under IT

of IGs remain valid here, although the new issue requires small changes in the production function of FGs. In particular, we assume again that the: (i) IT starts when the North is in its autarkic steady state; (ii) South has a *MC* of production advantage; (iii) TK gap between countries is smaller that the threshold gap; (iv) price of the composite FG in the North is the numeraire; and (v) North-South IT is always balanced so that there are no international capital flows and the domestic interest rate is determined internally.[275]

In what concerns the pattern of IT, it is expected that (i) the South produces, consumes and exports a certain number of both specialised and versatile FGs, and IGs imitated previously; and (ii) the North also produces, consumes and exports a certain number of both specialised and versatile FGs, and IGs that had not yet been imitated.

Since both countries have access to the same TK, the number of FGs produced by each country will be particularly dependent on the respective levels of HC at work.

Moreover, in line with the last chapter and basically for the same reasons, the HC employed in both countries and the TK in the North will again be crucial in determining the economic growth, the direction of R&D, prices and wage inequality paths. Indeed, the difference in relation to the last chapter is that now as a result of the IT of FGs, aggregate output, prices and wages in both countries are affected by the levels of HC at work in the other, not only because the North and South levels of HC at work affect the direction of TK progress via (North-South average) FGs price indexes, but also because the levels of HC at work of one country affect directly the FGs price indexes of the other.

IT of FGs induces Southern firms to employ the available HC in the appropriate tasks and the *TFP* will be the same in each FG produced, regardless of the producer country. However, different levels of HC at work and distinct exogenous levels of productivity between countries will lead to a Southern specialisation in a lower number of FGs and a smaller aggregate *TFP*. Thus, the origin of distinct *TFP* will not be the inappropriate allocation of resources in FGs production nor the available TK, but the HC levels and the exogenous productivity. This may explain why some Southern countries, even though they have a relatively low HC and low aggregate *TFP* in comparison with the U.S., have relatively efficient industries – *e.g.*, Acemoglu and Zilibotti (2001).

The effects arising from the use of a new GPT and from a shock in the versatile-augmenting parameter in the production function are expected to be similar to the results

---

[274] Indeed, this will be really true for the production of FGs close the comparative margins.

[275] In addition, we can now assume that the imitation activity is again improved due to the stronger effect induced by a greater openness to IT and other IT policies captured by the index *IT* in the catching up term.

obtained in chapter 4. In particular, since the new GPT is used at the same time in both countries, no changes are expected in the pattern of FGs specialisation.

The chapter ends with the transitional dynamics and the sensitivity analysis, using once again the fourth-order Runge-Kutta classical numerical method for set baseline parameter values and set initial conditions. The first aim is to observe that both countries (world) converge towards the steady state. The analysis is very similar to that carried out in the previous IT regime. Indeed, due to IT, when one country is out of the world steady state so is the other, and the system of differential equations that describes the transitional dynamics will be obtained in a similar way.

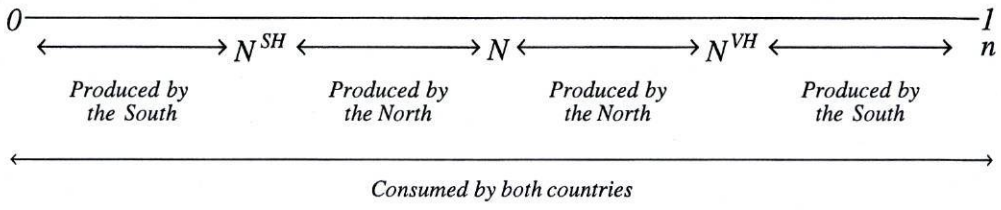
Relative to the last IT regime, the difference comes from FGs price indexes, which now reflect the pattern of FGs specialisation of the countries according to their CAs and thus the dynamic levels of HC at work. During this phase, a process of convergence characterised by the decrease in the TK gap and in the gap in HC at work between countries is expected. Thus, we point out that the number of specialised and versatile FGs produced by the South will increase, and the share will be greater when changes in parameters and initial value of the variables improve the Southern levels of HC at work.

The remainder of the chapter proceeds as follows. Sections 5.2 and 5.3 depict the FGs and the IGs sectors, respectively. Section 5.4 describes the domestic equilibrium given the state of the world TK and the stock of HC, and explains the effect of IT of FGs in the R&D sector. Section 5.5 examines the steady state. Section 5.6 analyses the South-North *TFP* differences and the consequences of IT between the North and the South which is unable to imitate. Section 5.7 examines the transition to and the stability of the steady state. Section 5.8 performs a sensitivity analysis. Section 5.9 finishes the chapter.

5.2. Final goods sector

At each time, both countries produce some versatile FGs and some specialised FGs since they have both types of HC, *SH* and *VH*, although in different proportions, and they have access to the same TK due to IT. However, now each FG is only produced by the country that enjoys the CA. We assume that the South has a CA in specialised FGs with smaller indexes, $n \in [0, N^{SH}]$, and in versatile FGs with greater indexes, $n \in [N^{VH}, 1]$. The North has a CA in the other FGs, $n \in [N^{SH}, N^{VH}]$ – *i.e.*, in FGs with intermediate indexes in the global range. These assumptions are made for analytical convenience and generate the pattern of FGs specialisation shown in figure 5.1 below:

Figure 5.1. Pattern of final goods specialisation



To get the pattern of FGs specialisation, the production function of the n[th] FG at t undertaken by perfectly competitive firms is written in the next three interrelated ways:

$$Y_{nZ}(t\,|\,SH)=\left\{\int_0^{J^{SH}}\left[q^{k(j,t|_{SH})}\left(x_{nZ}(k,j,t|SH,I)+x_{nZ}(k,j,t|SH,P)\right)\right]^{1-\alpha}dj\right\}\cdot\left\{\left[(N-n)\,sh\,SH_{nP}(t\,|w)\,A_P^{1/\alpha}\right]^{\alpha}+\left[n\,sh\,SH_{nI}(t|w)\,A_I^{1/\alpha}\right]^{\alpha}\right\}; \quad (5.1a)$$

$$Y_{nZ}(t\,|VH)=\left\{\int_0^{J^{VH}}\left[q^{k(j,t|_{VH})}\left(x_{nZ}(k,j,t|VH,I)+x_{nZ}(k,j,t|VH,P)\right)\right]^{1-\alpha}dj\right\}\cdot\left\{\left[(n-N)\,vh\,VH_{nP}(t\,|w)\,A_P^{1/\alpha}\right]^{\alpha}+\left[(1-n)\,vh\,VH_{nI}(t|w)\,A_I^{1/\alpha}\right]^{\alpha}\right\}; \quad (5.1b)$$

$$\begin{aligned}Y_{nI}(t)&=\\&=A_I\left\{\left[\int_0^{J^{VH}}\left[q^{k(j,t|_{VH})}\left(x_{nI}(k,j,t|VH,I)+x_{nI}(k,j,t|VH,P)\right)\right]^{1-\alpha}dj\right]\left[(n-N^{SH})\,vh\,VH_{nI}(t|w)\right]^{\alpha}\right.\\&\left.+\left[\int_0^{J^{SH}}\left[q^{k(j,t|_{SH})}\left(x_{nI}(k,j,t|SH,I)+x_{nI}(k,j,t|SH,P)\right)\right]^{1-\alpha}dj\right]\left[(N^{VH}-n)\,sh\,SH_{nI}(t|w)\right]^{\alpha}\right\}.\end{aligned} \quad (5.1c)$$

Equations (5.1a, b) are written such that each FG is only produced by one country. The first expression within large brackets sums up the contribution of state-of-the-art IGs to production, while the second captures the role of the productivity parameter (non-IT related) and of the specific HC input of each country. A relative productivity advantage of (the HC of) either country in production is obtained by the adjustment terms.

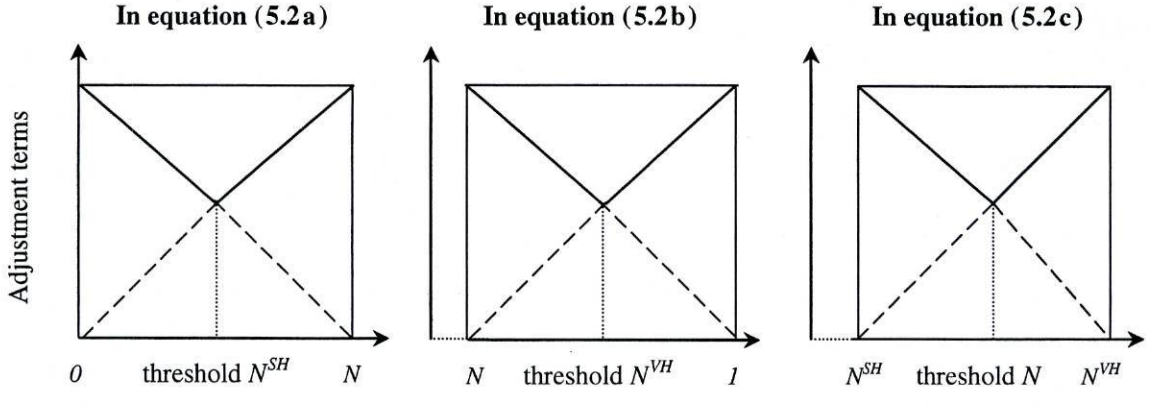
Figure 5.2 below gives us a first overview of the conduct of the adjustment terms. In equation (5.1a) they are $(N-n)$ and n for $n \in [0, N]$ – figure 5.2a –, which means that, within the FGs produced with *SH*, the South is relatively more productive in producing specialised FGs indexed by smaller *n*s, and *vice-versa*. In equation (5.1b) they are $(n-N)$ and $(1-n)$ for $n \in [N, 1]$ – figure 5.2b –, which implies that, within the FGs produced with *VH*, the South is relatively more productive in producing versatile FGs indexed by

larger *n*s, and *vice-versa*. Thus, the adjustment terms transform the FG index n in a relevant ordering index of the CA of the countries in FGs production.

Equation (5.1c) is presented so that each FG produced by the North uses only one technology, VT or ST, as occurs for each FG produced by the South. The first and third expressions within square brackets sum up the contributions of the two types of state-of-the-art IGs to production, while the second and fourth denote the role of the specific HC inputs. An absolute productivity advantage of *VH* over *SH* is accounted for by the joint parameters *vh* and *sh*, where $vh \geq sh \geq 1$. A relative productivity advantage of either type of HC is obtained by the adjustment terms $(N^{VH}-n)$ and $(n-N^{SH})$ for $n \in [N^{SH}, N^{VH}]$ – figure 5.2c –, which also transform the FGs index n in a relevant ordering index: this means that within the FGs produced by the North, *VH* is relatively more productive in producing FGs indexed by larger *n*s, and *vice-versa*.

Using the three equations above, we can find the threshold FGs N^{SH}, N and N^{VH}. As depicted by figure 5.1, they tell us that the North (South) is relatively more productive in specialised FGs with high (low) indexes, whereas the South (North) is relatively more productive in versatile FGs with high (low) indexes. In the North, the VT (ST) is relatively more productive in FGs with high (low) indexes. Given that the three threshold FGs depend on dynamic TK and on dynamic levels of HC at work, the pattern of the IT of FGs is explained by dynamic versions of the Ricardian model and the HOS model.

Figure 5.2. Behaviour of the adjustment terms



The explanation of the IGs terms is the same as in chapter 4. In short, they include an adjustment for quality that reflects a stylised TK change process of the quality ladder type, where q and k are the size of each quality improvement and the number of quality improvements, respectively. The value of q is such that IG production shifts from the

North to the South when a Southern firm imitates and shifts back to the North when a Northern firm innovates,[276] which induces a Vernon-type IG cycle. The pattern of the IT of IGs mirrors the countries' R&D activities – (dynamic) TK in the Ricardian tradition –, which also depends on their (dynamic) HC levels – in the HOS tradition.

The quality grades of each IG j are perfect substitutes as inputs to production and, due to profit maximising limit pricing by the monopolist producers, only the state-of-the-art IGs are used. Thus, the quantity of IG j with quality rung k used in the production of the nth FG in country Z, together with domestic m-type HC, can be produced by either the North, $x_{nZ}(k, j, t \mid m, I)$, after a successful innovation, or by the South, $x_{nZ}(k, j, t \mid m, P)$, after a lower-priced successful imitation of the leading Northern quality level. The term $q^{k(j,t|m)}$, where $k = k_I \geq k_P$, which corresponds to the highest quality of IG j, is useful to get the quality-adjusted amount of this IG used in the production of the nth FG.

There will also be one of three possible types of IGs producers: Northern IGs firms facing competition from other Northern firms, Northern IGs firms facing competition from Southern firms and Southern IGs firms facing competition from Northern firms. Thus, $\Xi(m)$, $[1-\Xi(m)]$, $\gamma(m)$ and $[1-\gamma(m)]$ continue to be used and they have the same meaning as in chapter 4. So, for example, equation (5.1a) can explicitly be rewritten as:

$$
\begin{aligned}
Y_n(t|SH) = \Bigg\{ & \Bigg[\int_0^{J^{SH}} \Bigg[q^{k(j|SH)} \Bigg[\Xi(j|SH) \Big(\gamma(SH)\, x_{nP}(k,j|SH,I) + [1-\gamma(SH)]\; x_{nP}(k,j|SH,I) \Big) + \\
& + [1-\Xi(j|SH)]\; x_{nP}(k,j|SH,P) \Bigg] \Bigg]^{1-\alpha} dj \Bigg] \Bigg[(N-n)\, sh\, SH_{nP}(t|w)\, A_P^{1/\alpha} \Bigg]^{\alpha} + \\
& + \Bigg[\int_0^{J^{SH}} \Bigg[q^{k(j|SH)} \Bigg[\Xi(j|SH) \Big(\gamma(SH)\, x_{nI}(k,j|SH,I) + [1-\gamma(SH)]\; x_{nI}(k,j|SH,I) \Big) + \\
& + [1-\Xi(j|SH)]\; x_{nI}(k,j|SH,P) \Bigg] \Bigg]^{1-\alpha} dj \Bigg] \Bigg[n\; sh\, SH_{nI}(t|w) A_I^{1/\alpha} \Bigg]^{\alpha} \Bigg\}
\end{aligned}
\tag{5.2}
$$

The market for FGs is competitive and under conditions of perfect competition FG producers maximise their instantaneous profits. It is straightforward to verify that the FOCs of the maximisation problem implies the next relations for (i) $n \in [0, N]$:

$$
x_{nP}(k,j|SH,Z,Z) = (N-n)\, sh\, SH_{nP} \left[\frac{p_{nP}}{p(j|SH,Z,Z)} A_P (1-\alpha) \right]^{1/\alpha} q^{k(j|SH)\left[\frac{1-\alpha}{\alpha}\right]} ; \qquad (5.3a)
$$

[276] Once again, the South can under-price the North after a successful imitation and the size of each quality upgrade is sufficiently large; *i.e.*, $(1-s_x) / (MC_P - s_x) < q \leq 1/(1-\alpha)$ – see chapter 4.

$$x_{nI}(k,j|SH,Z,Z) = n\ sh\ SH_{nI}\left[\frac{p_{nI}}{p(j|SH,Z,Z)}A_I(1-\alpha)\right]^{1/\alpha} q^{k(j|SH)\left[\frac{1-\alpha}{\alpha}\right]}; \qquad (5.3b)$$

(ii) $n \in [N, 1]$:

$$x_{nP}(k,j|VH,Z,Z) = (n-N)\ vh\ VH_{nP}\left[\frac{p_{nP}}{p(j|VH,Z,Z)}A_P(1-\alpha)\right]^{1/\alpha} q^{k(j|VH)\left[\frac{1-\alpha}{\alpha}\right]}; \qquad (5.4a)$$

$$x_{nI}(k,j|VH,Z,Z) = (1-n)\ vh\ VH_{nI}\left[\frac{p_{nI}}{p(j|VH,Z,Z)}A_I(1-\alpha)\right]^{1/\alpha} q^{k(j|VH)\left[\frac{1-\alpha}{\alpha}\right]}; \qquad (5.4b)$$

(iii) $n \in [N^{SH}, N^{VH}]$:

$$x_{nI}(k,j|SH,Z,Z) = (N^{VH}-n)\ sh\ SH_{nI}\left[\frac{p_{nI}}{p(j|SH,Z,Z)}A_I(1-\alpha)\right]^{1/\alpha} q^{k(j|SH)\left[\frac{1-\alpha}{\alpha}\right]}; \qquad (5.5a)$$

$$x_{nI}(k,j|VH,Z,Z) = (n-N^{SH})\ vh\ VH_{nI}\left[\frac{p_{nI}}{p(j|VH,Z,Z)}A_I(1-\alpha)\right]^{1/\alpha} q^{k(j|VH)\left[\frac{1-\alpha}{\alpha}\right]}. \qquad (5.5b)$$

As expected, FG firms demand more IGs when their respective product prices are high, IGs prices are low, and their HC employment and IGs quality are higher. Moreover, equations (5.3a, b), (5.4a, b) and (5.5a, b) indicates that even when the IGs are produced in the South their quality level is the leader Northern quality level.

We can easily get the demand of IGs manoeuvred by each m-type HC for $n \in [0, N]$, $n \in [N, 1]$ and $n \in [N^{SH}, N^{VH}]$. The procedure will be the same as in equations (4.5a, b). For this reason, the equations that are equivalent to equations (4.5a, b) and (4.6a, b) are omitted, and we stress that equation (4.7) remains valid here.

Finally, we define the output aggregate in each country as:

$$Y_I(t) = \int_0^1 p_{nI}(t)\, Y_{nI}(t)\, dn = 1\left\{exp\left[\int_{N^{SH}}^{N} ln\, Y_{nI}(t)\, dn + \int_{N}^{N^{VH}} ln\, Y_{nI}(t)\, dn\right]\right\}; \qquad (5.6a)$$

$$Y_P(t) = \int_0^1 p_{nP}(t)\, Y_{nP}(t)\, dn = MC_P\left\{exp\left[\int_{0}^{N^{SH}} ln\, Y_{nP}(t)\, dn + \int_{N^{VH}}^{1} ln\, Y_{nP}(t)\, dn\right]\right\}; \qquad (5.6b)$$

where we normalise (i) the Northern price of the aggregate FG in each period to *1* and (ii) the Southern price of the aggregate FG in each period to MC_P, $0 < MC_P < MC_I = 1$.

In the next section, we proceed with the novelties in the IGs sector.

5.3. Intermediate goods sector

The aggregate demands $X(k,j,t|m,Z,Z)$, where $(Z, Z) = (I, P)$, (I, I) and (P, I) – see chapter 4, section 4.2 –, for the highest quality IG j at t is given by – omitting index k:

$$X(j,t|SH,Z,Z) = \int_0^{N^{SH}} x_{nP}(j,t|SH,Z,Z)\,dn + \int_{N^{SH}}^{N} x_{nI}(j,t|SH,Z,Z)\,dn; \qquad (5.7a)$$

$$X(j,t|VH,Z,Z) = \int_N^{N^{VH}} x_{nI}(j,t|VH,Z,Z)\,dn + \int_{N^{VH}}^{1} x_{nP}(j,t|VH,Z,Z)\,dn. \qquad (5.7b)$$

The remaining considerations in last chapter about this section are still valid.

Thus, implicitly, the production function of IGs is the same as the FGs production function. Hence, the *MC* of producing an IG, which is independent of its quality and is identical across all domestic IGs, equals the *MC* of producing the composite FG that, due to perfect competition in FGs sector, corresponds to its price.

The maximisation program of the monopolist which has acquired a patent to produce the highest quality in each one of the three groups of IGs is similar to the maximisation program in equations (4.10a-c). The FOCs lead to the profit-maximising prices in equations (4.11a-c). As in the previous chapters, we consider that limit pricing strategy is binding and so used by all innovator leader firms. Thus, table 4.2 remains valid, and we can rewrite – see $mc_{\Xi\gamma}^{m}$ in equation (4.12a, b) – (i) equations (5.3a, b) as:

$$x_{nP}(k,j|SH) = (N-n)\,sh\,SH_{nP}\left[\frac{p_{nP}}{q}A_P(1-\alpha)\right]^{1/\alpha} q^{k(j|SH)\left[\frac{1-\alpha}{\alpha}\right]} mc_{\Xi\gamma}^{SH}; \qquad (5.8a)$$

$$x_{nI}(k,j|SH) = n\,sh\,SH_{nI}\left[\frac{p_{nI}}{q}A_I(1-\alpha)\right]^{1/\alpha} q^{k(j|SH)\left[\frac{1-\alpha}{\alpha}\right]} mc_{\Xi\gamma}^{SH}; \qquad (5.8b)$$

(ii) equations (5.4a, b) as:

$$x_{nP}(k,j|VH) = (n-N)\,vh\,VH_{nP}\left[\frac{p_{nP}}{q}A_P(1-\alpha)\right]^{1/\alpha} q^{k(j|VH)\left[\frac{1-\alpha}{\alpha}\right]} mc_{\Xi\gamma}^{VH}; \qquad (5.9a)$$

$$x_{nI}(k,j|VH) = (1-n)\,vh\,VH_{nI}\left[\frac{p_{nI}}{q}A_I(1-\alpha)\right]^{1/\alpha} q^{k(j|VH)\left[\frac{1-\alpha}{\alpha}\right]} mc_{\Xi\gamma}^{VH}; \qquad (5.9b)$$

and (iii) equations (5.5a, b) as:

$$x_{nI}(k,j|SH) = (N^{VH}-n)\,sh\,SH_{nI}\left[\frac{p_{nI}}{q}A_I(1-\alpha)\right]^{1/\alpha} q^{k(j|SH)\left[\frac{1-\alpha}{\alpha}\right]} mc_{\Xi\gamma}^{SH}; \qquad (5.10a)$$

$$x_{nI}(k, j|VH) = (n - N^{SH})\ vh\ VH_{nI}\left[\frac{p_{nI}}{q} A_I (1-\alpha)\right]^{1/\alpha} q^{k(j|_{VH})\left[\frac{1-\alpha}{\alpha}\right]} mc_{\Xi\gamma}^{VH}. \qquad (5.10b)$$

Equations (5.8a, b), (5.9a, b), (5.10a, b) yield the explicit demands for IGs. Substituting these into equations (5.1a-c), respectively, we obtain for (i) $n \in [0, N]$:

$$Y_{nP}(t|SH) = A_P^{1/\alpha}\left[\frac{p_{nP}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}(N-n)\ sh\ SH_{nP}\ MC_{\Xi\gamma}^{SH}\ Q_I^{J^{SH}}; \qquad (5.11a)$$

$$Y_{nI}(t|SH) = A_I^{1/\alpha}\left[\frac{p_{nI}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]} n\ sh\ SH_{nI}\ MC_{\Xi\gamma}^{SH}\ Q_I^{J^{SH}}; \qquad (5.11b)$$

(ii) $n \in [N, 1]$:

$$Y_{nP}(t|VH) = A_P^{1/\alpha}\left[\frac{p_{nP}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}(n - N)\ vh\ VH_{nP}\ MC_{\Xi\gamma}^{VH}\ Q_I^{J^{VH}}; \qquad (5.12a)$$

$$Y_{nI}(t|VH) = A_I^{1/\alpha}\left[\frac{p_{nI}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}(1-n)\ vh\ VH_{nI}\ MC_{\Xi\gamma}^{VH}\ Q_I^{J^{VH}}; \qquad (5.12b)$$

and (iii) $n \in [N^{SH}, N^{VH}]$:

$$Y_{nI}(t|SH) = A_I^{1/\alpha}\left[\frac{p_{nI}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}(N^{VH} - n)\ sh\ SH_{nI}\ MC_{\Xi\gamma}^{SH}\ Q_I^{J^{SH}}; \qquad (5.13a)$$

$$Y_{nI}(t|VH) = A_I^{1/\alpha}\left[\frac{p_{nI}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}(n - N^{SH})\ vh\ VH_{nI}\ MC_{\Xi\gamma}^{VH}\ Q_I^{J^{VH}}. \qquad (5.13b)$$

The term $MC_{\Xi\gamma}^{m}$ in equations (5.11a, b), (5.12a, b) and (5.13a, b) is defined in equation (4.13). These equations can be summarised in the following way:

$$Y_{nP}(t) = A_P^{1/\alpha}\left[\frac{p_{nP}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}\left[(N-n)\ sh\ SH_{nP}\ MC_{\Xi\gamma}^{SH}\ Q_I^{J^{SH}} + (n-N)\ vh\ VH_{nP}\ MC_{\Xi\gamma}^{VH}\ Q_I^{J^{VH}}\right]; \qquad (5.14a)$$

$$Y_{nI}(t) = A_I^{1/\alpha}\left[\frac{p_{nI}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}\left[n\ sh\ SH_{nI}\ MC_{\Xi\gamma}^{SH}\ Q_I^{J^{SH}} + (1-n)\ vh\ VH_{nI}\ MC_{\Xi\gamma}^{VH}\ Q_I^{J^{VH}}\right]; \text{ or,} \qquad (5.14b)$$

$$Y_{nI}(t) = A_I^{1/\alpha}\left[\frac{p_{nI}}{q}(1-\alpha)\right]^{\left[\frac{1-\alpha}{\alpha}\right]}\left[(N^{VH} - n)\ sh\ SH_{nI}\ MC_{\Xi\gamma}^{SH}\ Q_I^{J^{SH}} + (n - N^{SH})\ vh\ VH_{nI}\ MC_{\Xi\gamma}^{VH}\ Q_I^{J^{VH}}\right]. \qquad (5.14c)$$

Equations (5.14a-c) show that in equilibrium the growth of FGs production is driven by Northern TK, $Q_I^{J^{VH}}$ and $Q_I^{J^{SH}}$, and by domestic HC employed, $VH_Z(.|w)$ and $SH_Z(.|w)$. Finally, the terms $\Xi(m)$ and $\gamma(m)$ of $MC_{\Xi\gamma}^{m}$ are as defined by equation (4.14).

5.4. Equilibrium given R&D and HC accumulation

In comparison to the equivalent section in the previous IT regimes, now there are two related novelties. First, the IT of FGs instigates an international pattern of FGs production and, second, it increases the interdependence between countries. Thus, we are going to obtain the equilibrium values for the threshold variables N, N^{SH} and N^{VH}, for the aggregate variables Y_Z and X_Z, for the price paid per unit of each type of HC at work, w^m, for its growth rate, for the versatile premium, w^{VH}/w^{SH}, and for the inter-country wage inequality, w_P^m / w_I^m, given the Northern TK levels and the domestic levels of HC at work.

First, we must prove that each FG is produced using only one technology, VT or ST, and only by one country, North (I) or South (P). The production functions in equations (5.1a-c) make I (P) relatively more productive in specialised FGs with high (low) indexes, P (I) relatively more productive in versatile FGs with high (low) indexes, and the VT (ST) more productive in FGs with high (low) indexes. As figure 5.1 shows, it must be a threshold $N^{SH} \in [0, N]$, $N^{VH} \in [N, 1]$ and $N \in [N^{SH}, N^{VH}]$ such that for:

(i) $n \leq N^{SH} < N$ the FG producer is located in P and uses ST;

(ii) $N^{SH} < n < N$ the FG producer is located in I and uses ST;

(iii) $N < n < N^{VH}$ the FG producer is located in I and uses VT;

(iv) $n \geq N^{VH} > N$ the FG producer is located in P and uses VT.

To find N^{SH}, N^{VH} and N, we start by noting that the profit of the representative producer of FG n, which uses m-type HC, at time t, regardless of the country, is still given by equations (4.16), although with differences in the values for the variables.[277]

Considering equation (4.16) and in pairs equations (5.8a) and (5.11a), (5.8b) and (5.11b), we have for $n \in [0, N]$, respectively – where $\pi_{nZ}(\cdot|m) \equiv \Pi_{nZ}(\cdot|m)/m_{nZ}(\cdot|w)$:

$$\pi_{nP}(\cdot|SH) = (N-n)\,sh\left(p_{nP}A_P\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} Q_I^{JSH} MC_{\Xi\gamma}^{SH}\left[1-\frac{(1-\alpha)}{q}\right] - w_P^{SH}; \qquad (5.15a)$$

$$\pi_{nI}(\cdot|SH) = n\,sh\left(p_{nI}A_I\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} Q_I^{JSH} MC_{\Xi\gamma}^{SH}\left[1-\frac{(1-\alpha)}{q}\right] - w_I^{SH}. \qquad (5.15b)$$

Taking into consideration equation (4.16) and in pairs equations (5.9a) and (5.12a), (5.9b) and (5.12b), we get for $n \in [N, 1]$, respectively:

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[277] If the FG producer is located in P and uses ST [VT], Y_{nP} will be given by equation (5.11a) [(5.12a)] and x_{nP} by equation (5.8a) [(5.9a)]. If the FG producer is located in I and uses ST [VT], Y_{nI} will be given by equation (5.11b) or (5.13a) [(5.12b) or (5.13b)] and x_{nI} by equation (5.8b) or (5.10a) [(5.9b) or (5.10b)].

$$\pi_{nP}(\cdot|VH) = (n-N)\, vh\left(p_{nP}A_P\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} Q_I^{J^{VH}} MC_{\Xi\gamma}^{VH}\left[1-\frac{(1-\alpha)}{q}\right] - w_P^{VH}; \qquad (5.16a)$$

$$\pi_{nI}(\cdot|VH) = (1-n)\, vh\left(p_{nI}A_I\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} Q_I^{J^{VH}} MC_{\Xi\gamma}^{VH}\left[1-\frac{(1-\alpha)}{q}\right] - w_I^{VH}. \qquad (5.16b)$$

And again bearing in mind equation (4.16) and in pairs equations (5.10a) and (5.13a), (5.10b) and (5.13b), we obtain from $n \in [N^{SH}, N^{VH}]$, respectively:

$$\pi_{nI}(\cdot|SH) = (N^{VH}-n)\, sh\left(p_{nI}A_I\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} Q_I^{J^{SH}} MC_{\Xi\gamma}^{SH}\left[1-\frac{(1-\alpha)}{q}\right] - w_I^{SH}; \qquad (5.17a)$$

$$\pi_{nI}(\cdot|VH) = (n-N^{SH})\, vh\left(p_{nI}A_I\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} Q_I^{J^{VH}} MC_{\Xi\gamma}^{VH}\left[1-\frac{(1-\alpha)}{q}\right] - w_I^{VH}. \qquad (5.17b)$$

The FGs market is competitive and so FG producers have to make zero profit per HC employee. Moreover, considering equations (5.15a, b), (5.16a, b) and (5.17a, b), the operations $\pi_{nI}(\cdot|SH) - \pi_{nP}(\cdot|SH)$, $\pi_{nP}(\cdot|VH) - \pi_{nI}(\cdot|VH)$ and $\pi_{nI}(\cdot|VH) - \pi_{nI}(\cdot|SH)$ are increasing functions of n over $n \in [0, N]$, $n \in [N, 1]$ and $n \in [N^{SH}, N^{VH}]$, respectively:

$$\Re^{SH}\left[n\left(A_I^{1/\alpha}\, p_{nI}^{1/\alpha} + A_P^{1/\alpha}\, p_{nP}^{1/\alpha}\right) - N\, A_P^{1/\alpha}\, p_{nP}^{1/\alpha}\right] - w_I^{SH} + w_P^{SH}; \qquad (5.18a)$$

$$\Re^{VH}\left[n\left(A_I^{1/\alpha}\, p_{nI}^{1/\alpha} + A_P^{1/\alpha}\, p_{nP}^{1/\alpha}\right) - N\, A_P^{1/\alpha}\, p_{nP}^{1/\alpha} - A_I^{1/\alpha}\, p_{nI}^{1/\alpha}\right] - w_P^{VH} + w_I^{VH}; \qquad (5.18b)$$

$$\Im\left[n\left(vh\, MC_{\Xi\gamma}^{VH}\, Q_I^{J^{VH}} + sh\, MC_{\Xi\gamma}^{SH}\, Q_I^{J^{SH}}\right) - N^{SH} vh\, Q_I^{J^{VH}} - N^{VH} sh\, Q_I^{J^{SH}}\right] - w_I^{VH} + w_I^{SH}; \qquad (5.18c)$$

where: $\Re^m \equiv \left(\frac{1-\alpha}{q}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}\left[1-\frac{1-\alpha}{q}\right]\overline{m}\, MC_{\Xi\gamma}^{m} Q_I^{J^m}$ and $\Im \equiv \left(A_I p_{nI}\right)^{1/\alpha}\left(\frac{1-\alpha}{q}\right)^{\left[\frac{1-\alpha}{\alpha}\right]}\left[1-\frac{1-\alpha}{q}\right]$.

Since individuals prefer variety they consume a composite FG composed by all FGs, either produced domestically or imported,[278] all FGs are produced. As both countries have the two types of HC that, in equilibrium, must all be employed, both countries produce a certain number of FGs, according to their CAs. Thus, for each FG

(i) $n \in [0, N]$, either $\pi_{nI}(\cdot|SH) = 0$, or $\pi_{nP}(\cdot|SH) = 0$, or still $\pi_{nI}(\cdot|SH) = \pi_{nP}(\cdot|SH) = 0$;

(ii) $n \in [N, 1]$, either $\pi_{nP}(\cdot|VH) = 0$, or $\pi_{nI}(\cdot|VH) = 0$, or still $\pi_{nP}(\cdot|VH) = \pi_{nI}(\cdot|VH) = 0$;

[278] As the production of FGs, as well as the production of IGs, can be located in the foreign country, there is also IT of FGs. Nevertheless, IT must be balanced.

(iii) $n \in [N^{SH}, N^{VH}]$, either $\pi_{nI}(\cdot|VH) = 0$, or $\pi_{nI}(\cdot|SH) = 0$, or still $\pi_{nI}(\cdot|VH) = \pi_{nI}(\cdot|SH) = 0$.

Hence, there is a

(i) $N^{SH} \in [0, N]$ such that $\pi_{nP}(\cdot|SH) - \pi_{nI}(\cdot|SH) = 0$ and $\pi_{nP}(\cdot|SH) - \pi_{nI}(\cdot|SH) > 0$ (<0) for all $n < N^{SH}$ (for $n > N^{SH}$);

(ii) $N^{VH} \in [N, 1]$ such that $\pi_{nP}(\cdot|VH) - \pi_{nI}(\cdot|VH) = 0$ and $\pi_{nP}(\cdot|VH) - \pi_{nI}(\cdot|VH) > 0$ (<0) for all $n > N^{VH}$ (for $n < N^{VH}$);

(iii) $N \in [N^{SH}, N^{VH}]$ such that $\pi_{nI}(\cdot|VH) - \pi_{nI}(\cdot|SH) = 0$ and $\pi_{nI}(\cdot|VH) - \pi_{nI}(\cdot|SH) > 0$ (<0) for all $n > N$ (for $n < N$).

Thus,

$$Y_n = \begin{cases} \begin{cases} A_P^{1/\alpha}\left[p_{nP}\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\,(N-n)\,sh\,SH_{nP}(.|w)\,Q_I^{J^{SH}} & if \quad 0 \le n \le N^{SH} \\ A_I^{1/\alpha}\left[p_{nI}\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\,n\,sh\,SH_{nI}(.|w)\,Q_I^{J^{SH}} & if \quad N^{SH} < n \le N \end{cases} \\ \begin{cases} A_I^{1/\alpha}\left[p_{nI}\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\,(1-n)\,vh\,VH_{nI}(.|w)\,Q_I^{J^{VH}} & if \quad N \le n \le N^{VH} \\ A_P^{1/\alpha}\left[p_{nP}\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\,(n-N)\,vh\,VH_{nP}(.|w)\,Q_I^{J^{VH}} & if \quad N^{VH} < n \le 1 \end{cases} \\ \begin{cases} A_I^{1/\alpha}\left[p_{nI}\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\,(N^{VH}-n)\,sh\,SH_{nI}(.|w)\,Q_I^{J^{SH}} & if \quad N^{SH} \le n \le N \\ A_I^{1/\alpha}\left[p_{nI}\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\,(n-N^{SH})\,vh\,VH_{nI}(.|w)\,Q_I^{J^{VH}} & if \quad N < n \le N^{VH} \end{cases} \end{cases} . \tag{5.19}$$

The equilibrium marginal value product has to be equalised across all m-type FGs,

$$\frac{\partial p_n Y_n}{\partial m_n} = \begin{cases} \begin{cases} A_P^{1/\alpha}\,p_{nP}^{1/\alpha}\left[\dfrac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\,(N-n)\,sh\,Q_I^{J^{SH}} & if \quad 0 \le n \le N^{SH} \\ A_I^{1/\alpha}\,p_{nI}^{1/\alpha}\left[\dfrac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\,n\,sh\,Q_I^{J^{SH}} & if \quad N^{SH} < n \le N \end{cases} \\ \begin{cases} A_I^{1/\alpha}\,p_{nI}^{1/\alpha}\left[\dfrac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\,(1-n)\,vh\,Q_I^{J^{VH}} & if \quad N \le n \le N^{VH} \\ A_P^{1/\alpha}\,p_{nP}^{1/\alpha}\left[\dfrac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\,(n-N)\,vh\,Q_I^{J^{VH}} & if \quad N^{VH} < n \le 1 \end{cases} \\ \begin{cases} A_I^{1/\alpha}\,p_{nI}^{1/\alpha}\left[\dfrac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\,(N^{VH}-n)\,sh\,Q_I^{J^{SH}} & if \quad N^{SH} \le n \le N \\ A_I^{1/\alpha}\,p_{nI}^{1/\alpha}\left[\dfrac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\,(n-N^{SH})\,vh\,Q_I^{J^{VH}} & if \quad N < n \le N^{VH} \end{cases} \end{cases} . \tag{5.20}$$

Equation (5.20) gives the wage of the m-type HC and shows that if $p_{nP}^{1/\alpha}(N-n)$, $p_{nI}^{1/\alpha}\, n$, $p_{nI}^{1/\alpha}(1-n)$, $p_{nP}^{1/\alpha}(n-N)$, $p_{nI}^{1/\alpha}(N^{VH}-n)$ and $p_{nI}^{1/\alpha}(n-N^{SH})$ are independent of n the marginal value product will be the same for all $n \in [0, N^{SH}]$, $n \in [N^{SH}, N]$, $n \in [N, N^{VH}]$, $n \in [N^{VH}, 1]$, $n \in [N^{SH}, N]$ and $n \in [N, N^{VH}]$, respectively. Defining these price indexes as $p_P^{SH\,(1/\alpha)}$, $p_I^{SH\,(1/\alpha)}$, $p_I^{VH\,(1/\alpha)}$, $p_P^{VH\,(1/\alpha)}$, $p_I^{SH\,(1/\alpha)}$ and $p_I^{VH\,(1/\alpha)}$, respectively, which can be determined, equation (5.19) can be written as:[279]

$$
Y_n = \begin{cases}
\begin{cases}
Y_{nP}(SH)\,(N-n)^{\alpha},\; Y_{nP}(SH) = A_P^{1/\alpha}\left[p_P^{SH}\,\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\; sh\; SH_{nP}(.\,|\,w)\; Q_I^{J^{SH}} & \text{if } \; 0 \le n \le N^{SH} \\
Y_{nI}(SH)\,n^{\alpha},\; Y_{nI}(SH) = A_I^{1/\alpha}\left[p_I^{SH}\,\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\; sh\; SH_{nI}(.\,|\,w)\; Q_I^{J^{SH}} & \text{if } \; N^{SH} < n \le N
\end{cases} \\
\begin{cases}
Y_{nI}(VH)\,(1-n)^{\alpha},\; Y_{nI}(VH) = A_I^{1/\alpha}\left[p_I^{VH}\,\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\; vh\; VH_{nI}(.\,|\,w)\; Q_I^{J^{VH}} & \text{if } \; N \le n \le N^{VH} \\
Y_{nP}(VH)(n-N)^{\alpha},\; Y_{nP}(VH) = A_P^{1/\alpha}\left[p_P^{VH}\,\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\; vh\; VH_{nP}(.\,|\,w)\; Q_I^{J^{VH}} & \text{if } \; N^{VH} < n \le 1
\end{cases} \\
\begin{cases}
Y_{nI}(SH)\,(N^{VH}-n)^{\alpha},\; Y_{nI}(SH) = A_I^{1/\alpha}\left[p_I^{SH}\,\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{SH}\; sh\; SH_{nI}(.\,|\,w)\; Q_I^{J^{SH}} & \text{if } \; N^{SH} \le n \le N \\
Y_{nI}(VH)\,(n-N^{SH})^{\alpha},\; Y_{nI}(VH) = A_I^{1/\alpha}\left[p_I^{VH}\,\dfrac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{VH}\; vh\; VH_{nI}(.\,|\,w)\; Q_I^{J^{VH}} & \text{if } \; N < n \le N^{VH}
\end{cases}
\end{cases}
\tag{5.21}
$$

Since in equilibrium expenditures across FGs, $p_n Y_n$, are equalised,[280] VH_n and SH_n have to be constant and equal in all FGs using VH and SH, in the respective country:

$$\int_0^{N^{SH}} SH_{nP}(.\,|\,w)\; dn \;=\; SH_P(.\,|\,w) \;\Rightarrow\; SH_{nP}(.\,|\,w) = \frac{SH_P(.\,|\,w)}{N^{SH}}; \tag{5.22a}$$

$$\int_{N^{SH}}^{N} SH_{nI}(.\,|\,w)\; dn \;=\; SH_I(.\,|\,w) \;\Rightarrow\; SH_{nI}(.\,|\,w) = \frac{SH_I(.\,|\,w)}{N - N^{SH}}; \tag{5.22b}$$

$$\int_{N}^{N^{VH}} VH_{nI}(.\,|\,w)\; dn \;=\; VH_I(.\,|\,w) \;\Rightarrow\; VH_{nI}(.\,|\,w) = \frac{VH_I(.\,|\,w)}{N^{VH} - N}; \tag{5.22c}$$

$$\int_{N^{VH}}^{1} VH_{nP}(.\,|\,w)\; dn \;=\; VH_P(.\,|\,w) \;\Rightarrow\; VH_{nP}(.\,|\,w) = \frac{VH_P(.\,|\,w)}{1 - N^{VH}}. \tag{5.22d}$$

---

[279] Thus, the price paid for a unit of m-type HC at work in Z, w_Z^m, can be written as:

$$w_Z^m = \left(p_Z^m\, A_Z\right)^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC_{\Xi\gamma}^{m}\; \overline{m}\; Q_I^{J^m}.$$

[280] This is because the Cobb-Douglas structure in equations (5.6a, b), the consumption of all FGs by each individual, and the equilibrium balanced IT. This implies that expenditures across all FGs, either produced domestically or imported, are equalised. That is, the value of the production of each FG is equal (and more generally, the value consumed domestically of each FG is equal). Therefore, regardless of the localisation of production, the value used domestically plus the value exported is the same for all FGs.

Table 5.1 sums up the profits of the representative n[th] FG firm, in all cases.

Table 5.1. Profits of the representative final good firm *n*

Final good firms	Profits of the final good firm *n*			
	$n \in [0, N^{SH}]$ [a)]	$n \in [N^{SH}, N]$	$n \in [N, N^{VH}]$	$n \in [N^{VH}, 1]$
Using ST in *P*	$= 0$	< 0	< 0	< 0
Using ST in *I*	< 0	$= 0$	< 0	< 0
Using VT in *I*	< 0	< 0	$= 0$	< 0
Using VT in *P*	< 0	< 0	< 0	$= 0$

Note: [a)] means that, for example, for $n \in [0, N^{SH}]$ all FG firms in *P* using ST make zero profits, while all the others in the world (that is, in *P* using VT and in *I* using either one) obtain negative profits.

To find the threshold FGs, note that (i) N^{SH} can be produced by either ST in country *P* or ST in country *I*, (ii) N^{VH} can be produced by either VT in country *I* or VT in country *P*,[281] and (iii) *N* can be produced by either ST or VT in country *I*, and must yield zero profit in either case. Thus, for example, when $n \in [0, N]$, $n \in [N, 1]$ and $n \in [N^{SH}, N^{VH}]$, both respective price indexes definitions must hold for $n = N^{SH}$, $n = N^{VH}$, $n = N$, respectively, implying the ratios in table 5.2. N^{SH} and N^{VH} can be called 'comparative margins', since FGs can be produced in both countries.

Table 5.2. Price indexes – summary and ratios for threshold final goods

Range of the n[th] final good		Price indexes	
		By definition	Ratio for threshold final goods
$n \in [0, N]$	$n \in [0, N^{SH}]$	$p_P^{SH} = p_{nP}(N-n)^{\alpha}$	$\frac{p_P^{SH}}{p_I^{SH}} = \frac{MC_P}{MC_I}\left(\frac{N-N^{SH}}{N^{SH}}\right)^{\alpha}$ [a)]
	$n \in [N^{SH}, N]$	$p_I^{SH} = p_{nI}\, n^{\alpha}$	
$n \in [N, 1]$	$n \in [N, N^{VH}]$	$p_I^{VH} = p_{nI}(1-n)^{\alpha}$	$\frac{p_P^{VH}}{p_I^{VH}} = \frac{MC_P}{MC_I}\left(\frac{N^{VH}-N}{1-N^{VH}}\right)^{\alpha}$ [a)]
	$n \in [N^{VH}, 1]$	$p_P^{VH} = p_{nP}(n-N)^{\alpha}$	
$n \in [N^{SH}, N^{VH}]$	$n \in [N^{SH}, N]$	$p_I^{SH} = p_{nI}(N^{VH}-n)^{\alpha}$	$\frac{p_I^{VH}}{p_I^{SH}} = \left(\frac{N-N^{SH}}{N^{VH}-N}\right)^{\alpha}$
	$n \in [N, N^{VH}]$	$p_I^{VH} = p_{nI}(n-N^{SH})^{\alpha}$	

Note: [a)] imply that we have assumed for $n = N^{SH}$ and $n = N^{VH}$ that $\frac{p_{N^{SH}P}}{p_{N^{SH}I}} = \frac{p_{Y_P}}{p_{Y_I}} = \frac{MC_P}{MC_I} = \frac{p_{N^{VH}P}}{p_{N^{VH}I}}$.

[281] Note that (i) and (ii) hold because we assume that there are no IT barriers or transportation costs.

Since IT results in a pattern of FGs specialisation according to the countries' CAs mirrored in the threshold FGs, then, as shown in the last column of table 5.2, IT influences the FGs price indexes. Moreover, as $p_n Y_n$ is constant for all n, together with equation (5.21) and the additional information in table 5.2, we obtain:

$$N^{SH} = \frac{R_1 R_2 R_3}{1 + R_2(1 + R_3) - R_1(1 + R_2)}; \tag{5.23a}$$

$$N^{VH} = \frac{R_2(1 + R_3 - R_1)}{1 + R_2(1 + R_3) - R_1(1 + R_2)}; \tag{5.23b}$$

$$N = \frac{R_2 R_3}{1 + R_2(1 + R_3) - R_1(1 + R_2)}. \tag{5.23c}$$

Where:

$$R_1 \equiv \left\{1 + \left[\left(\frac{A_1 MC_1}{A_P MC_P}\right)^{1/\alpha} \frac{SH_1(.|w)}{SH_P(.|w)}\right]^{\frac{1}{2}}\right\}^{-1}, \quad R_2 \equiv \left[\left(\frac{A_1 MC_1}{A_P MC_P}\right)^{1/\alpha} \frac{VH_1(.|w)}{VH_P(.|w)}\right]^{\frac{1}{2}}, \quad R_3 \equiv \left[\frac{Q_1^{JSH}}{Q_1^{JVH}} \frac{sh\, SH_1(.|w)}{vh\, VH_1(.|w)}\right]^{\frac{1}{2}}.$$

In particular, as we will see later on, equations (5.23a-c) imply that the three threshold FGs are constant in the steady state.[282] Moreover, as table 5.3 below sums up:

(i) as A_Z is crucial in the production of both types of FGs, specialised and versatile, an improvement in A_Z increases the fraction of FGs using ST and VT in country Z. In other words, the better the exogenous productivity the country has, the greater the number of FGs that it produces. This result implies that the number of FGs produced by one country increases when a new GPT is used in the country. Furthermore, concerning this, the effects of an increase in A_Z are similar to the effects of an increase in MC_Z.

(ii) The CA of country Z in specialised FGs increases when its $SH_Z(.|w)$ also increases. As the number of FGs remains constant, this implies that the number of FGs produced with VT decreases in both countries, as well as the number of FGs produced with ST in the foreign country. In summary, a greater number of FGs are produced with ST in country Z and, as a result, in the world since N increases.

(iii) When $VH_Z(.|w)$ increases, more FGs are produced with VT in country Z and less FGs are produced with ST in the South. Furthermore, when the increase is in $VH_I(.|w)$ this also decreases the number of FGs produced with VT in the South.

(iv) If Q_I^{JSH} or sh increases, the Southern number of FGs produced with ST increases and the number of FGs produced with VT decreases in both countries. In this case the

South must capture a relatively greater share of the world FGs production. However, if it is Q_I^{JVH} or vh that increases, the number of FGs produced with VT increases in both countries and the Southern number of FGs produced with ST decreases. Consequently, the North must capture a relatively greater share of the world FGs production.

(v) The explanation of the pattern of FGs specialisation is in line with the Ricardian tradition due to (i) and (iv), and is in line with the HOS tradition owing to (ii) and (iii).

Hence, in particular, each country will export relatively more FGs of the type that use more intensively the relatively abundant type of HC, and import relatively more FGs of the type that use more intensively the relatively scarce type of HC.

As we will confirm later on, the previous analysis enables us to state that the South produces (and so exports) more FGs at the end of the adjustment process. This is because it is expected that the South improve its economic development level measured by the TK gap and by the gap in HC at work in relation to the North. In addition, parameters and variables that affect the thresholds N, N^{SH} and N^{VH} affect price indexes – see table 5.2 – and, consequently, the direction of R&D through the price channel. Thus, the HC at work in both countries and the Northern TK are particularly important in determining the direction of R&D and intra-country wage inequality.

Table 5.3. Comparative statics – comparative, N^{SH} and N^{VH}, and technology margins, N

	$N^{SH}-0$	$N-N^{SH}$	$N^{VH}-N$	$1-N^{VH}$		$N^{SH}-0$	$N-N^{SH}$	$N^{VH}-N$	$1-N^{VH}$
$\frac{\partial}{\partial A_P}$	+	−	−	+	$\frac{\partial}{\partial Q_I^{JSH}}$	+	=	−	−
$\frac{\partial}{\partial A_I}$	−	+	+	−	$\frac{\partial}{\partial Q_I^{JVH}}$	−	=	+	+
$\frac{\partial}{\partial SH_P(w)}$	+	−	−	−	$\frac{\partial}{\partial sh}$	+	=	−	−
$\frac{\partial}{\partial SH_I(w)}$	−	+	−	−	$\frac{\partial}{\partial vh}$	−	=	+	+
$\frac{\partial}{\partial VH_P(w)}$	−	=	=	+	$\frac{\partial}{\partial MC_P}$	+	−	−	+
$\frac{\partial}{\partial VH_I(w)}$	−	=	+	−	$\frac{\partial}{\partial MC_I}$	−	+	+	−

Notes: $\frac{\partial N}{\partial \wp} > 0$ if $\wp = \{A_P, SH_P(.|w), SH_I(.|w), sh, Q_I^{JSH}$ and $MC_I\}$ and $\frac{\partial N}{\partial \wp} < 0$ if not.

[282] In short and taking into consideration the previous chapter, this is because in steady state $Q_I^{JSH} SH_I(.|w)$, $Q_I^{JSH} SH_P(.|w)$, $Q_I^{JVH} VH_P(.|w)$ and $Q_I^{JVH} VH_I(.|w)$ grow at the same rate.

In summary, in what concerns the pattern of IT (i) the South produces and exports specialised FGs with smaller indexes, $n \in [0, N^{SH}]$, versatile FGs with greater indexes, $n \in [N^{VH}, 1]$, and IGs imitated previously; and (ii) the North produces and exports FGs with intermediate indexes, $n \in [N^{SH}, N^{VH}]$, and IGs that had not yet been imitated.

To proceed, the value of the FGs price indexes, p_Z^m, are required. Considering the numeraire as the Northern price of the aggregate FG, $exp \int_{N^{SH}}^{N^{VH}} ln\, p_{nI}\, dn = 1$, then:

$$\int_{N^{SH}}^{N^{VH}} ln\, p_{nI}\, dn = \int_{N^{SH}}^{N} ln\, [p_I^{SH}(N^{VH} - n)]\, dn + \int_{N}^{N^{VH}} ln\, [p_I^{VH}(n - N^{SH})]\, dn = 0. \qquad (5.24)$$

From equation (5.24), with the expression for the relative versatile price in the North presented in table 5.2 and with some additional algebra, we find that:

$$p_I^{VH} = exp(-\alpha)\left[N^{VH} - N\right]^{-\alpha}\left[N^{VH} - N^{SH}\right]^{2\alpha}; \qquad (5.25a)$$

$$p_I^{SH} = exp(-\alpha)\left[N - N^{SH}\right]^{-\alpha}\left[N^{VH} - N^{SH}\right]^{2\alpha}; \qquad (5.25b)$$

$$\frac{p_I^{VH}}{p_I^{SH}} = \left[\frac{N - N^{SH}}{N^{VH} - N}\right]^{\alpha} = \left[\frac{Q_I^{J^{SH}}}{Q_I^{J^{VH}}}\frac{sh\ SH_I(.\,|w)}{vh\ VH_I(.\,|w)}\right]^{\frac{\alpha}{2}}. \qquad (5.25c)$$

Equation (5.25c) suggests that when the Northern TK is highly versatile-biased and/or its level of *VH* at work is relatively greater, the Northern relative price of versatile FGs will be low. Furthermore, comparing equations (2.24c) and (5.25c) we observe that the expression for the ratio p_I^{VH}/p_I^{SH} does not change at time t_0 when IT begins – *i.e.*, when the IT regime changes (but now) from autarky to free IT of all goods. This is because the adjustment process in world production of FGs does not affect p_I^{VH}/p_I^{SH}.

Now, using equations (5.25a, b) and the value of the ratios p_P^{SH}/p_I^{SH} and p_P^{VH}/p_I^{VH} into table 5.2, the FGs price indexes and the Southern relative price of versatile FGs are:

$$p_P^{VH} = exp(-\alpha)\frac{MC_P}{MC_I}\left[1 - N^{VH}\right]^{-\alpha}\left[N^{VH} - N^{SH}\right]^{2\alpha}; \qquad (5.26a)$$

$$p_P^{SH} = exp(-\alpha)\frac{MC_P}{MC_I}N^{SH\,-\alpha}\left[N^{VH} - N^{SH}\right]^{2\alpha}; \qquad (5.26b)$$

$$\frac{p_P^{VH}}{p_P^{SH}} = \left[\frac{N^{SH}}{1 - N^{VH}}\right]^{\alpha} = \left[\frac{Q_I^{J^{SH}}}{Q_I^{J^{VH}}}\frac{sh\ SH_P(.\,|w)}{vh\ VH_P(.\,|w)}\right]^{\frac{\alpha}{2}}. \qquad (5.26c)$$

Equation (5.26c) indicates that when the Northern TK is highly versatile-biased and/or the Southern level of *VH* at work is relatively greater, the Southern relative price of versatile FGs will be low. Hence, the ratio p_P^{VH}/p_P^{SH} changes at time t_0 when IT starts owing to the level effect induced by the IT of IGs – as explained in chapter 4.

In short, at time t_0 the South has access to the Northern TK and since $Q_I^{Jm} > Q_P^{Jm}$ it enjoys an immediate benefit relative to the North in terms of aggregate output, Y, and factor prices, w^m. This is clear in equations (5.20) and (5.21), considering that (i) the HC instantaneous jump in the allocation of time between employment and accumulation and (ii) the changes in mark-ups are of second order. That is, inter-country wage inequality falls. Besides, due to the Southern jump to the Northern measure of versatile-specialised-bias – and assuming the starting conditions stated in equation (4.25) –, the Southern relative price of versatile FGs drops at t_0 – equation (5.26c).[283] As the shift in the demand for *VH* is stronger, due to complementarity between IGs and HC together with the versatile-specific bias in TK, an immediate jump in intra-South wage inequality occurs.

As we will see later on numerically, the characteristics of the countries at t_0 implies that the North always produces relatively more versatile FGs than the South, although the level effect brings the countries closer in terms of prices and of the number of FGs produced with each technology, VT or ST. Thus, factor-intensity reversal exists, contrarily to the stated by the HOS model. However, since countries are different in terms of HC levels and HC is not internationally mobile, the convergence in prices between countries is only relative.[284] Indeed, although the drop in the Southern relative price of versatile FGs at time t_0, it remains greater than in the North. This will induce more R&D directed at improving Northern versatile TK than in the autarkic North.

In the new equilibrium, the relative price of versatile FGs and the use of ST in the world fall, while relative (relative to the autarkic North) versatile-biased TK progress and the use of VT in the world increase. Due to the increase in the use of VT, a temporary partial Schumpeterian dynamic equivalent to the static Rybczynski result arises.

To close the section, we are going to determine the total resources devoted to IGs, X, and after that the aggregate output, Y, in each country. From equations (5.8a) and (5.9a), the Southern total resources devoted to IGs production of each type at time t are:

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[283] The measure of Northern versatile-specialised-bias also affects the world pattern of FGs specialisation, as is clear from the three thresholds, N^{SH}, N^{VH} and N, given by equations (5.23a-c).

[284] Usually we talk only of distinct HC levels between countries, but we need to bear in mind that these differences also reflect different HC structures.

$$X_P(.|SH) = \int_0^{N^{SH}} \int_0^{J^{SH}} x_{nP}(k,j|SH)\, dj\, dn = sh\ SH_P(.|w) \left[p_P^{SH}\, A_P \frac{(1-\alpha)}{q} \right]^{1/\alpha} mc_{\Xi\gamma}^{SH}\, Q_I^{J^{SH}}; \quad (5.27a)$$

$$X_P(.|VH) = \int_0^{N^{VH}} \int_0^{J^{VH}} x_{nP}(k,j|VH)\, dj\, dn = vh\ VH_P(.|w) \left[p_P^{VH}\, A_P \frac{(1-\alpha)}{q} \right]^{1/\alpha} mc_{\Xi\gamma}^{VH}\, Q_I^{J^{VH}}. \quad (5.27b)$$

And from equations (5.8b) and (5.9b) [or, alternatively, from equations (5.10a) and (5.10b)], the Northern total resources devoted to each type of IGs at time t are:

$$X_I(.|SH) = \int_0^{N^{SH}} \int_0^{J^{SH}} x_{nI}(k,j|SH)\, dj\, dn = sh\ SH_I(.|w) \left[p_I^{SH}\, A_I \frac{(1-\alpha)}{q} \right]^{1/\alpha} mc_{\Xi\gamma}^{SH}\, Q_I^{J^{SH}}; \quad (5.28a)$$

$$X_I(.|VH) = \int_0^{N^{VH}} \int_0^{J^{VH}} x_{nI}(k,j|VH)\, dj\, dn = vh\ VH_I(.|w) \left[p_I^{VH}\, A_I \frac{(1-\alpha)}{q} \right]^{1/\alpha} mc_{\Xi\gamma}^{VH}\, Q_I^{J^{VH}}. \quad (5.28b)$$

Thus, from equations (5.26a, b) and (5.27a, b), on the one hand, and (5.25a, b) and (5.28a, b), on the other, the resources devoted to IGs in the South and in the North are:

$$X_P = \Theta_P^X \left\{ sh\ SH_P(.|w)\, N^{SH(-1)} mc_{\Xi\gamma}^{SH}\, Q_I^{J^{SH}} + vh\ VH_P(.|w) \left(1 - N^{VH} \right)^{-1} mc_{\Xi\gamma}^{VH}\, Q_I^{J^{VH}} \right\}; \quad (5.29a)$$

$$X_I = \Theta_I^X \left\{ sh\ SH_I(.|w) \left(N - N^{SH} \right)^{-1} mc_{\Xi\gamma}^{SH}\, Q_I^{J^{SH}} + vh\ VH_I(.|w) \left(N^{VH} - N \right)^{-1} mc_{\Xi\gamma}^{VH}\, Q_I^{J^{VH}} \right\}; \quad (5.29b)$$

$$\text{where: } \Theta_P^X \equiv exp(-1) \left[A_P \frac{(1-\alpha)}{q} \frac{MC_P}{MC_I} \right]^{1/\alpha} \left(N^{VH} - N^{SH} \right)^2 \text{ and } \Theta_I^X \equiv \Theta_P^X \left[\frac{A_I}{A_P} \frac{MC_I}{MC_P} \right]^{1/\alpha}$$

In particular, equations (5.29a, b) show clearly that due to the IT of FGs the total resources devoted to IGs in each country depends directly on the levels of HC at work of the other, via comparative and technology margins – see equations (5.23a-c).

Following the same reasoning for the aggregate output; that is, taking into account its definition, equations (5.11a) and (5.12a) [or also equation (5.14a)] for the South and equations (5.11b) and (5.12b) or equations (5.13a, b) [or still equations (5.14b, c)] for the North and equations (5.25a, b) and (5.26a, b) for the FGs price indexes, yields:

$$Y_P = \Theta_P^Y \left\{ sh\ SH_P(.|w)\, N^{SH(-1)} MC_{\Xi\gamma}^{SH}\, Q_I^{J^{SH}} + vh\ VH_P(.|w) \left(1 - N^{VH} \right)^{-1} MC_{\Xi\gamma}^{VH}\, Q_I^{J^{VH}} \right\}; \quad (5.30a)$$

$$Y_I = \Theta_I^Y \left\{ sh\ SH_I(.|w) \left(N - N^{SH} \right)^{-1} MC_{\Xi\gamma}^{SH}\, Q_I^{J^{SH}} + vh\ VH_I(.|w) \left(N^{VH} - N \right)^{-1} MC_{\Xi\gamma}^{VH}\, Q_I^{J^{VH}} \right\}. \quad (5.30b)$$

$$\text{where: } \Theta_P^Y \equiv exp(-1) \left[A_P \frac{MC_P}{MC_I} \right]^{1/\alpha} \left[\frac{1-\alpha}{q} \right]^{\frac{1-\alpha}{\alpha}} \left(N^{VH} - N^{SH} \right)^2 \text{ and } \Theta_I^Y \equiv \Theta_P^Y \left[\frac{A_I}{A_P} \frac{MC_I}{MC_P} \right]^{1/\alpha}$$

Equations (5.30a, b) show that differences in the countries' equilibrium aggregate output results directly from differences in A, $SH(.|w)$ and $VH(.|w)$, since they have

access to the same TK, due to the IT of IGs. However, owing to the IT of FGs, countries now dispute the production of FGs, which is mirrored in the three threshold FGs.

This implies that they become more interdependent, since the three threshold FGs depend on $SH(.|w)$ and $VH(.|w)$ [and on MCs] of either country. That is, when IT of FGs starts at time t_0, there is an adjustment process in the production of FGs in both countries. From then on, each FG is produced by one country according to the CAs and the output of each country is affected by the levels of HC of the other via FGs price indexes or, in other words, via the three thresholds FGs. This adjustment almost certainly increases the Southern aggregate output; *i.e.*, it presumably benefits from the greater Northern levels of HC at work and respective structure, and *vice-versa*. If this is so, we can state that a new (considering the IT of IGs alone) level effect occurs in the South and a negative level effect takes place in the North, which is negatively affected by smaller Southern HC levels and the respective structure. We will verify this later on through calibration.

Again, the price paid for a unit of each m-type HC, w^m, obtained from equations (5.30a, b) is equal to that obtained from equation (5.20) if $MC_{\Xi\gamma}^{SH} = MC_{\Xi\gamma}^{VH}$.[285] Thus,

$$w_Z^m = \left[A_Z\, p_Z^m\right]^{1/\alpha} \left[\frac{1-\alpha}{q}\right]^{\left(\frac{1-\alpha}{\alpha}\right)} MC_{\Xi\gamma}^m\, \overline{m}\; Q_I^{Jm}, \text{ where } MC_{\Xi\gamma}^{SH} = MC_{\Xi\gamma}^{VH}. \qquad (5.31)$$

Due to the IT of FGs, wages in each country are affected by the HC levels in the other, not only because the HC levels of both countries influence the direction of TK progress, Q_I^{Jm}, as in chapter 4, but also because the HC levels of both countries affect the FGs price indexes – see equations (5.23a-c), (5.25a-c) and (5.26a-c). As a result, if, as expected, the South enjoys a new positive level effect, Southern wages will be enhanced.

Bearing in mind equation (5.31), the equilibrium growth rate of w_Z^m is also given by equation (4.32). Hence, the potentially higher growth rate of Northern TK progress under IT favours workers in both countries.[286] In view of equation (5.30), inter and intra-country wage inequality, and the versatile premium between countries, W, is still given by the expressions in equations (4.33a-c), respectively.

In the next section, we are going to analyse the steady state equilibrium. Before that, we highlight that now the analysis of the R&D sector is very similar to the analysis in chapter 4, section 4.5. Therefore, we will only refer to the alterations resulting from

[285] Moreover, with $MC_{\Xi\gamma}^{SH} = MC_{\Xi\gamma}^{VH}$, equations (5.30a, b) again show an elasticity of substitution constant (= 2) between both types of domestic HC at work pondered by the associated Northern TK.

the IT of FGs, which are basically related to the manner of calculating the flow of profits from successful research firms. Thus, the expression for the flow of profit of successful research firms of all types and for each time t is still given by equations (4.35a-c). However, to get equation (4.35c), for example, we now take into account that:

$$\Pi(k,j,t|SH,P,I) = \\ = \left[p(j|P,I) - \left(MC_P - s_{xP}\right)\right]\left[\int_0^{N^{SH}} x_{nP}(k,j,t|SH,P,I)\,dn + \int_{N^{SH}}^{N} x_{nI}(k,j,t|SH,P,I)\,dn\right]; \quad (5.32a)$$

$$\Pi(k,j,t|VH,P,I) = \\ = \left[p(j|P,I) - \left(MC_P - s_{xP}\right)\right]\left[\int_N^{N^{VH}} x_{nI}(k,j,t|VH,P,I)\,dn + \int_{N^{VH}}^{J} x_{nP}(k,j,t|VH,P,I)\,dn\right]; \quad (5.32b)$$

and after that we use equations (4.11a-c), (5.3a, b) and (5.22a, b) – with the same reasoning we can derive equations (4.35a, b).[287] Hence, in these new equations, FGs price indexes are given by equations (5.25a, b) and (5.26a, b). That is, the type of IT that affects the R&D activity is the IT of IGs. Consequently, section 4.5 of the previous chapter remains valid here. This suggests that in the expressions for the equilibrium interest rate and world growth rate, IT of FGs only affects FGs price indexes – as we will see explicitly in the next section for the steady state equilibrium.

To complete the characterisation of the aggregate economies, the behaviour of individuals must be considered. Since this behaviour in not affected by IT, section 2.6 continues valid here. The government budget and aggregate resource constraints also remain the same as in section 2.7, since IT is always balanced.[288] Thus, plugging equation (4.45) – where, due to IT of FGs, FGs price indexes are now different – into equation (4.32) and equating the resulting m-type HC demand path to the necessary condition for optimisation by individuals – equation (2.70) – the equilibrium interest rate in the countries is obtained. By Walras' law, it is also the interest rate that clears the assets market. The general equilibrium instantaneous growth rate of the countries at time t results from plugging the equilibrium interest rate into the Euler equation (2.64).

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[286] Bearing in mind the static benefit(s) generated, IT favours Southern workers even more.

[287] Comparing the incremental profits of follower firms taking over the leader position with the incremental profits of leaders replacing themselves, it can be proved that the gain to a follower firm is always greater.

[288] The equilibrium aggregate output – equations (5.30a, b) –, the demand for IGs – equations (5.29a, b) – and the resources devoted to R&D – equation (4.44), considering the FGs price indexes under IT of FGs and that changes in $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ are of 2nd order – are all constant multiples of the Northern TK (since there is IT of IGs) and of the domestic level of HC at work (since there is no international mobility of production factors) – in both (to be more precisely) due to the new FGs price indexes as a result of the IT of FGs. Thus, by the aggregate resource constraint, the same takes place for the aggregate consumption.

5.5. The steady state

This section is centred on the analysis of the North-South steady state properties of our model and it is organised as follows. In subsection 5.5.1, we will point out that, for the same reasons as those presented in the previous IT regime, a unique and stable world BG path exists when countries open up to IT and conclude that the comparative steady state analysis of the previous chapter remains valid. In subsection 5.5.2, we will investigate other related steady state effects, namely the world growth benefits, convergence in levels between countries, FGs specialisation pattern, prices and wages.

5.5.1. Steady state growth

Generically, the analysis accomplished in section 4.6 of the last chapter remains valid. Once again,

(i) by assumption, countries have the same technology of production of FGs,[289] and, thereby, the aggregate output has constant returns to scale in the inputs $m(.|w)$ and Q_Is;

(ii) the aggregate output is used for consumption and savings, which, in turn, are allocated between production of IGs and R&D, since IT is balanced; then, Y, X, RS and C are all constant multiples of the Northern TK and the domestic HC at work;[290]

(iii) also, by assumption, countries have the same technology of HC accumulation.

Together, (i), (ii) and (iii) imply that the steady state growth rates – and through the Euler equation (2.64), the interest rates – are equalised between countries. The common (unique) and stable steady state growth rate, gr, is still given by equation (4.46), which, in short, suggests that a higher interest rate increases the process of creative destruction. Since in equation (4.46) it is implicit that, in steady state, $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ are constants, equations (4.47) and (4.48) also remain valid here. In addition, in steady state, both the equilibrium probabilities of successful research and $MC^{m}_{\Xi\gamma}$ are constants, which, in turn, implies that $\gamma(m)$ and $\Xi(m)$ are also constant, and that wage growth rates increase steadily according to TK progress – see equation (4.32).

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[289] Except for the level of productivity A, the adjustment terms and the level of HC at work $m(.|w)$, which implies differences in the levels but not in the growth rates. Bearing in mind that adjustment terms serve only to transform the FGs index n in an ordering index.

[290] This is really true when $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ are constant, since the resources devoted to R&D and to C are functions of the variables $\tilde{m}(.|w)$ and $\tilde{q}(.|m)$ as a result of IT of IGs. This implies that in steady state the m-type HC at work and the TK must grow at the same rate in both countries. Thus, regardless of the country, all variables of the aggregate budget constraint are in fact a constant multiple of the Northern (and Southern) TK, the levels of HC at work in both, the gap in HC at work and the TK gap between them.

Following the same steps as those used to obtain the equilibrium interest rate, we obtain the constant steady state interest rate as given by equation (4.49). After that, plugging equation (4.49) into the Euler equation (2.64), the steady state growth rate becomes given by equation (4.50) – equations (4.51) and (4.52) are also valid here. Excluding the FGs price indexes, the expressions for the equilibrium interest rate and the world growth rate are the same under IT of IGs alone and IT of all goods. Indeed, since economic growth is driven by HC accumulation and by TK advances in the quality of available IGs, the IT regime which directly affects the growth rate is the IT of IGs.

Hence, the CA in production of FGs plays no direct role in fostering growth. Thus, if changes in prices of FGs from IT of IGs alone to IT of all goods are of 2nd order and there is no increase in R&D, there is also no change in the world growth rate. This is in line with, *e.g.*, Rivera-Batiz and Romer (1991) and Grossman and Helpman (1991a).[291]

Given the previous analysis, the qualitative results of the comparative steady state analysis are then those shown in table 4.2.

5.5.2. Steady state effects

In this subsection, other related steady state effects are analysed, namely possible world growth changes induced by the IT of FGs, convergence in levels between South-North countries, pattern of FGs specialisation, prices and wages.

Countries' growth benefits with North-South TK diffusion and IT of all goods

In the last subsection, we found that, excluding FGs price indexes, the expressions for the steady state interest rate and endogenous growth rate are the same as in chapter 4, and are given by equations (4.49) and (4.50). Consequently, the increment in the steady state interest rate from (i) the South without international flow of goods, assets, HC and TK, and from (ii) the world under TK diffusion without IT effects, given by equation (3.3), to the North-South TK diffusion and IT of all goods depends also on the differences stated in equations (4.53a) and (4.53b, c), respectively.

Therefore, in generic terms the comments made in subsection 4.6.2 are still valid. Indeed, the new IT regime only affects the expressions of the FGs price indexes. Evaluation of the resulting changes in FGs price indexes requires solving for transitional dynamics through calibration and simulation. However, if, as expected, the impact of

[291] In fact, Rivera-Batiz and Romer (1991, pg. 544) claim that "... free trade in goods can affect the level of output and can therefore affect welfare, but it does not affect long-run growth rates." Similarly, Grossman

these changes on the steady state interest rate and world growth rate is not very strong, the countries' growth benefits with North-South TK diffusion under IT of all goods tends to be greater than without IT. Moreover, we can admit that imitation capacity is (again) improved due to the increase in the interdependence between countries – *i.e.*, that the index of openness, *IT*, is now higher. If so, the increase in the steady state world growth rate ultimately reflects a kind of scale: the jump in the catching up term due to *IT*.

Pattern of production, prices of final goods, wages and relation with technical progress

We now compare the without international TK diffusion steady state paths of the pattern of FGs production, FGs price indexes, wages and TK bias with the ones obtained from a move, at time zero, t_0, to free IT of all goods. The analysis made will be better understood – and so complemented – later on in the transitional dynamics analysis.[292]

In spite of the downward level effect in the Southern relative price of versatile FGs at time t_0 explained in section 5.4, differences in factor levels, as assumed by equation (4.25), dominate the steady state, such that Southern relative prices of versatile FGs are always higher – see equations (5.25c) and (5.26c) and figure 4.2 which remains valid.

As the North-South average relative price of versatile FGs is higher than in the autarkic North, then, due to the price channel, the relative demand for versatile-specific new designs increases, biasing R&D innovation in that direction more than without international TK diffusion, as shown in both scenarios A and B of figure 4.3. Relative to without international TK diffusion, such steady state bias increases the worldwide supply of versatile-specific IGs, thus decreasing the steady state relative price of versatile FGs.

From equations (5.23a-c) and considering the steady state values for the threshold FGs, we can get an available world steady state measure of versatile-specialised-bias, which depends directly on the Northern relative abundance of the two types of HC,[293]

$$\frac{Q_I^{J^{VH}}}{Q_I^{J^{SH}}} = \left(\frac{N^{VH} - N}{N - N^{SH}} \right)^2 \frac{sh}{vh} \left(\frac{VH_I(.|w)}{SH_I(.|w)} \right)^{-1} . \tag{5.33}$$

Equation (5.33) suggests that the Northern TK progress is (directly) endogenously biased towards the less abundant HC. Furthermore, equation (5.33) together with equation (4.33b) gives a direct relation between the versatile premium and the domestic relative abundance of the two types of HC in both countries:

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and Helpman (1991a, pg. 245) assume that "... the integration of product markets will have no effect on the long-run growth rate in either country."

[292] See also the analysis made in the last chapter – subsection 4.6.2 and transitional dynamics.

$$\frac{w_I^{VH}}{w_I^{SH}} = \left(\frac{N^{VH} - N}{N - N^{SH}}\right)\left(\frac{VH_I(.|w)}{SH_I(.|w)}\right)^{-1}; \tag{5.34a}$$

$$\frac{w_P^{VH}}{w_P^{SH}} = \left(\frac{N^{VH} - N}{N - N^{SH}}\right)\left(\frac{VH_I(.|w)}{SH_I(.|w)}\frac{VH_P(.|w)}{SH_P(.|w)}\right)^{-\frac{1}{2}}. \tag{5.34b}$$

Taking both equations (5.34a, b), then equation (4.33c) remains valid. In addition, equation (5.34a) suggests that the relation between the Northern versatile premium and the Northern relative stock of HC at work is now less elastic.[294]

Finally, we can define the relative factor shares as:

$$\frac{w_I^{VH} VH_I(.|w)}{w_I^{SH} SH_I(.|w)} = \left(\frac{N^{VH} - N}{N - N^{SH}}\right); \tag{5.35a}$$

$$\frac{w_P^{VH} VH_P(.|w)}{w_P^{SH} SH_P(.|w)} = \left(\frac{N^{VH} - N}{N - N^{SH}}\right)\left(\frac{\tilde{VH}_P(.|w)}{\tilde{SH}_P(.|w)}\right)^{\frac{1}{2}}. \tag{5.35b}$$

Since it is expected that $\tilde{SH}_P(.|w) > \tilde{VH}_P(.|w)$, from equations (5.35a, b) we have that the share of the *VH* in the output, Y_Z, tends to be greater in the North.

At time t_0, the IT of FGs leads to a FGs specialisation of the countries according to their CAs. Since due to the IT of IGs they have access to the same TK, each country will export (import) relatively more FGs that more intensively use the relatively abundant (scarce) HC. The path of each margin between t_0 and the steady state is shown in figure 5.3 below, which replaces figure 4.2 specific to the IT of IGs alone. The paths are primarily explained by the effect of IT on HC accumulation of both types, *VH* and *SH*. As figure 4.6 shows, IT changes inter-country gaps in favour of the South. Remember that Southern HC accumulation is immediately stimulated by higher wages, due to the level effect, and from then on continuously adjusts the levels to their higher steady states.

Since the South improves its economic development level between t_0 and the new steady state – measured by the TK gap and by the gap in HC at work relative to the North –, it must produce more FGs in steady state. In addition, considering table 5.3, N^{SH} must increase, whereas N and N^{VH} must decrease. That is, between time t_0 and the steady state

(i) the number of FGs produced with ST decreases and, thus, the number of FGs produced with VT increases;

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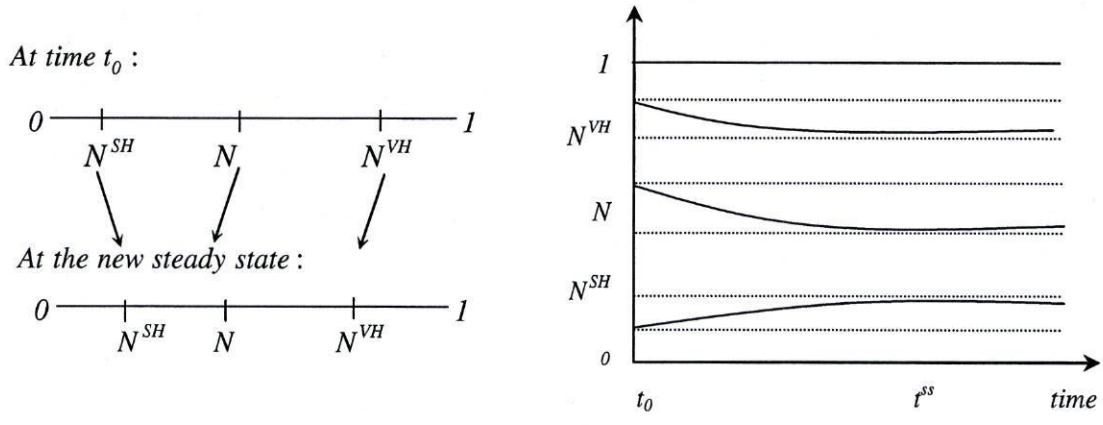
[293] Note that the Southern HC levels also affect the world steady state measure of versatile-specialised-bias.
[294] The same must be true in the South, since the path of each type of HC is the same in both countries.

(ii) the South produces and exports more FGs;

(iii) the North produces and exports a lower number of FGs.

Figure 5.3. Final goods thresholds – comparative and technology margins

a) t_0 and t^{ss} situation **b) Paths**



Due to the price channel and not because of the market size channel, in steady state

(i) the world productive structure or, in other words, the world pattern of FGs specialisation, N^{SH} N and N^{VH}, and relative prices tend to be constant;

(ii) the use of ST in the world decreases, whereas the use of VT increases – a partial Schumpeterian dynamic similar to the static Rybczynski result occurs;

(iii) relative price of versatile FGs are smaller than at time t_0;

(iv) the TK bias and relative wages have steady state dynamics;

(v) Northern versatile-biased TK increases.

The path of the versatile-biased TK is crucial to the steady state path of relative wages – see equation (5.33) and figures 4.4 and 4.5. In brief, it attenuates the path of declining intra-North wage inequality, relative to what would have prevailed under autarky, and, after the level effect, redirects the path of the intra-South wage inequality in favour of declining inequality – following the same slope as the North. That is, the steady state TK bias and wage inequality paths are more moderate with IT – a consequence of a Schumpeterian dynamic equivalent to the static Stolper-Samuelson result.[295]

Finally, we highlight that (i) a Schumpeterian dynamic equivalent to the static factor-price equalisation (also called Samuelson) theorem holds because the North-South

[295] Note that the static Stolper-Samuelson theorem does not involve any comparison between countries and that the IT of FGs itself (*i.e.*, relative to the previous IT regime with IT of IGs alone) does not affect the intra-countries FGs price indexes in relative terms when IT starts at time t_0 – *e.g.*, equations (2.25c),

steady state rates of growth in wages are equalised – see equations (3.14) and (4.55); and that (ii) North-South wage levels do not converge completely – *i.e.*, the static factor-price equalisation theorem does not hold or, in other words, inter-country wage inequality remains –, due to differences in the non-IT related exogenous productivity and due to the international immobility of the HC – see equation (4.56).

In the next section, we explore some extensions to the model.

5.6. Some Extensions

5.6.1. Overview

We proceed to investigate two extensions to the model. In subsection 5.6.2, we will discuss the steady state pattern of both the cross-FGs *TFP* and aggregate *TFP* differences between countries. In subsection 5.6.3, a short analysis of the steady state when countries in the world are the North and the Southern stagnant will be dealt with.

Since, due to the IT of IGs, the use of a new GPT in the production of FGs occurs at the same time in both countries, the main results are again condensed in tables 4.4 and 4.5, according the scenario considered – see Appendix 4, section A4.1. Also the effects of a positive shock that increases the versatile-augmenting parameter in the production function are similar to that stated in the previous IT regime – see Appendix 4, section A4.2. In short, there is no change in the pattern of FGs specialisation in relative terms.

As to the *TFP* differences analysis, the North – relatively more HC abundant – produces a greater number of FGs, but the *TFP* is the same in each FG produced, regardless of the producer country. Thus, FGs specialisation improves Southern *TFP*, because the IT of FGs induces Southern firms to employ the available HC in appropriate tasks. However, differences in the HC levels and in the productivity variable *A* lead to a Southern specialisation in a lower number of FGs and a smaller Southern aggregate *TFP*.

Under the IT of FGs, even when the South does not have access to imitation, it can import higher quality Northern IGs by exporting FGs, where it has a CA. Thus, in a new extension to the model, we show that the Southern stagnant country enjoys a static level benefit as well as a jump to a new higher steady state growth rate – *i.e.*, immediate conditional convergence occurs. Besides, now Northern innovating firms need not worry about potential imitation. They behave as in autarky, except that the greater expected profits due to increased market size lead to a greater rate of innovation, thus increasing

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(5.25c) and (5.26c). Thus, the paths of wages derive from a Schumpeterian dynamic equivalent to the static

the world growth rate in relation to the autarkic North. As the Southern TK stops to grow – R&D activities cease –, TK divergence between countries unambiguously occurs.

5.6.2. Final goods and aggregate productivity differences

Once again, so as to find the origin of the *TFP* differences between countries, we first analyse the steady state pattern of cross-FGs *TFP*.

Final goods productivity differences

By definition of $MC^{m}_{\Xi\gamma}$, of $mc^{m}_{\Xi\gamma}$ and of the IGs input in the production of the n[th] FG – $X_n(t|m) \equiv \int_0^{J^m} x_n(k,j,t|m)\,dj$ –, and equations (5.8a, b) and (5.9a, b),[296] we have:

$$X_n^{1-\alpha}(\overline{m}\,m_n)^{\alpha} = \begin{cases} \left[A_P\, p_{nP}\,\frac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC^{SH}_{\Xi\gamma}\,(N-n)^{(1-\alpha)}\, sh\,SH_{nP}(.|w)\,Q_I^{J^{SH}(1-\alpha)} & \text{if } 0 \le n \le N^{SH} \\ \left[A_I\, p_{nI}\,\frac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC^{SH}_{\Xi\gamma}\, n^{(1-\alpha)}\, sh\,SH_{nI}(.|w)\,Q_I^{J^{SH}(1-\alpha)} & \text{if } N^{SH} < n \le N \\ \left[A_I\, p_{nI}\,\frac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC^{VH}_{\Xi\gamma}\,(1-n)^{(1-\alpha)}\, vh\,VH_{nI}(.|w)\,Q_I^{J^{VH}(1-\alpha)} & \text{if } N \le n \le N^{VH} \\ \left[A_P\, p_{nP}\,\frac{(1-\alpha)}{q}\right]^{\left[\frac{1-\alpha}{\alpha}\right]} MC^{VH}_{\Xi\gamma}\,(n-N)^{(1-\alpha)}\, vh\,VH_{nP}(.|w)\,Q_I^{J^{VH}(1-\alpha)} & \text{if } N^{VH} < n \le 1 \end{cases} . \quad (5.36)$$

Taking into consideration equation (5.36), the output of the n[th] FG in value terms, obtained by multiplying equation (5.19) by FG prices, can be decomposed in two components: *TFPn* and 'physical input' related to IG:

$$p_n Y_n = \begin{cases} TFP_{nP}(.|SH)\; X_{nP}(.|SH)^{1-\alpha} & \text{if } 0 \le n \le N^{SH} \\ TFP_{nI}(.|SH)\; X_{nI}(.|SH)^{1-\alpha} & \text{if } N^{SH} < n \le N \\ TFP_{nI}(.|VH)\; X_{nI}(.|VH)^{1-\alpha} & \text{if } N \le n \le N^{VH} \\ TFP_{nP}(.|VH)\; X_{nP}(.|VH)^{1-\alpha} & \text{if } N^{VH} < n \le 1 \end{cases} ; \quad (5.37)$$

where: $TFP_{nZ}(.|m) = A_Z\; p_Z^m \left[Q_I^{J^m}\,\overline{m}\,m_{nZ}(.|w)\right]^{\alpha}$.

Equation (5.37) together with equations (5.22a-d), (5.25c) and (5.26c) imply that, on a BG path, all FGs in each country have the same *TFP*:

Stolper-Samuelson result – *i.e.*, from the endogenous paths of prices which affect the R&D activity.

[296] We can also use equations (5.10a, b) instead of (5.8b) and (5.9b), respectively.

$$\frac{TFP_{nZ}(.|SH)}{TFP_{nZ}(.|VH)} = 1. \qquad (5.38)$$

Additionally, now the *TFP* is the same in all FGs, regardless of the producer country – remember that in the last IT regimes both countries produce all FGs domestically and the *TFP* was greater in the FGs produced in the North:

$$\frac{TFP_{nP}(.|m)}{TFP_{nI}(.|m)} = 1 \text{ and so by equation (5.40) } \frac{TFP_{nP}(.|SH)}{TFP_{nI}(.|VH)} = 1. \qquad (5.39)$$

In short, the North – relatively more HC abundant – can produce a greater number of FGs, but the *TFP* is the same in all FG, regardless of the localisation of its production.

The physical *TFP* of the nth FG, $PTFP_n$, can also be obtained. Computing the relation between the *PTFP* of the two comparative margins in both countries, since they are the unique FGs that can be produced in both countries, it is possible to show that:

$$\frac{PTFP_{N^{SH}P}}{PTFP_{N^{SH}I}} = \frac{PTFP_{N^{VH}P}}{PTFP_{N^{VH}I}} = \frac{MC_I}{MC_P} > 1 \ [PTFP_n(.|m) \equiv TFP_n(.|m)/p_n, \text{ remember}] \qquad (5.40)$$

Thus, as a result of the Southern *MC* of production advantage, the Southern *PTFP* in the comparative margins is greater than in the North.

Aggregate productivity differences

Bearing in mind equations (5.6a, b), (5.22a- c) and (5.37), it can be shown that:

$$Y_P = PTFP_P \ X_P(.|SH)^{(1-\alpha)N^{SH}} X_P(.|VH)^{(1-\alpha)(1-N^{VH})}; \qquad (5.41a)$$

$$Y_I = PTFP_I \ X_I(.|SH)^{(1-\alpha)(N-N^{SH})} X_I(.|VH)^{(1-\alpha)(N^{VH}-N)}; \qquad (5.41b)$$

where: $TFP_P = \left[A_P\left(N^{VH}-N^{SH}\right)^{2\alpha}\right]^{(1-N^{VH}+N^{SH})} exp\left[-\alpha(1-N^{VH}+N^{SH})\right]$

$$\left[\left(SH_P(.|w)\, sh\ Q_I^{J^{SH}}\right)^{N^{SH}}\left(VH_P(.|w)\, vh\ Q_I^{J^{VH}}\right)^{1-N^{VH}} N^{SH-(2N^{SH})}(1-N^{VH})^{-2(1-N^{VH})}\right]^{\alpha}$$

; and,

$$TFP_I = \left[A_I\left(N^{VH}-N^{SH}\right)^{2\alpha}\right]^{(N^{VH}-N^{SH})} exp\left[-\alpha(N^{VH}-N^{SH})\right]$$

$$\left[\left(SH_I(.|w)\, sh\ Q_I^{J^{SH}}\right)^{(N-N^{SH})}\left(VH_I(.|w)\, vh\ Q_I^{J^{VH}}\right)^{(N^{VH}-N)}(N-N^{SH})^{-2(N-N^{SH})}(N^{VH}-N)^{-2(N^{VH}-N)}\right]^{\alpha}.$$

The comparison between TFP_P and TFP_I requires solving for transitional dynamics through calibration. However, since $A_I > A_P$, $m_I(.|w) > m_P(.|w)$ and, in addition and by consequence, the North produces more FGs, the expressions suggest that TFP_I is greater than TFP_P, despite the results at the *PTFP* level in the two comparative margins.

Moreover, computing the aggregate output per individual, Y, per HC occupation at work, $y = \{Y\,[SH+VH]/[SH(.|w)+VH(.|w)]\}$, and per efficiency unit of HC occupation at work, $y^{eff} = \{Y\,[SH+VH]/[shSH(.|w)+vhVH(.|w)]\}$, yields:

$$\frac{Y_P}{Y_I} = \frac{y_P}{y_I} = \left(\frac{A_P\,MC_P}{A_I\,MC_I}\right)^{\left(\frac{1}{\alpha}\right)} \left\{ \frac{SH_P(.|w)\,sh\,Q_I^{J^{SH}} N^{SH^{-1}} + VH_P(.|w)\,vh\,Q_I^{J^{VH}} (1-N^{VH})^{-1}}{SH_I(.|w)\,sh\,Q_I^{J^{SH}} (N-N^{SH})^{-1} + VH_I(.|w)\,vh\,Q_I^{J^{VH}} (N^{VH}-N)^{-1}} \right\}; \quad (5.42a)$$

$$\frac{y_P^{eff}}{y_I^{eff}} = \frac{Y_P}{Y_I} \left(\frac{SH_P + VH_P}{SH_I + VH_I}\right) \left(\frac{shSH_I(.|w) + vhVH_I(.|w)}{shSH_P(.|w) + vhVH_P(.|w)}\right). \quad (5.42b)$$

Equations (5.42a, b) also suggest that the aggregate output per individual, per individual working and per efficiency unit of individual working are greater in the North.

In short, FGs specialisation adds to Southern FGs productivity. This is because the IT of FGs induces Southern firms to employ the HC in appropriate tasks when faced with the Northern TK. However, differences in HC levels and in exogenous levels of productivity lead a smaller Southern aggregate *TFP*. In particular, improvements in A_I increase the steady state world growth rate and widen the aggregate North-South *TFP* differences, and improvements in A_P increase the steady state world growth rate and reduce the aggregate North-South *TFP* differences. This may explain why some Southern countries, even though they have a low HC level and low aggregate *TFP* in comparison with the U.S., have relatively efficient industries – *e.g.*, Acemoglu and Zilibotti (2001).

5.6.3. World common steady state growth rate without Southern imitation

If $d > \tilde{Q}^{J^m}$ when countries open up to IT of all goods, Southern firms know that the expected profits from successful imitation of leader IGs are less than their research expenditures – the cost of imitation in very expensive. However, contrarily to the case with the IT of IGs in chapter 4, where the Southern country cannot benefit from static as well as dynamic gains engendered by IT,[297] now the Southern country can import and use higher quality innovator IGs by producing and exporting some FGs, where it has a CA.

As a result, it can benefit from static gains – *i.e.*, when IT of all goods starts, the available TK in the South jumps from $Q_P^{J^m}$ to $Q_I^{J^m}$ – bearing in mind the distinction between Southern TK, $Q_P^{J^m}$, and available TK under IT, $Q_I^{J^m}$.

Assuming that without IT the Southern stagnant conducted some R&D – its reduced TK is however greater than 0 –, when IT starts it stops all activities in R&D and

[297] *I.e.*, the Southern country cannot export any goods and so, for lack of resources, it cannot import.

only becomes a producer of some FGs. That is, as Southern firms are no longer imitating, the TK developed domestically will stop growing and Northern firms need not worry about potential imitation of their IGs in the South. For this reason, innovating firms behave exactly as in the autarky case, except that they expected greater profits as a result of the increased market size. In other words, the South's market size positively affects expected profits, which become greater than in equation (2.39a), and has no influence on the probability of innovation, which is again given by equation (2.38).

Consequently, there is now a positive market size effect of the Southern market on innovation activity, which leads to a greater rate of innovation than in equation (2.54) – *i.e.*, comparatively to the innovation rate in the autarkic North. This leads to an unambiguously greater growth rate common to both countries than under the autarkic North in equation (3.4). Although TK divergence occurs between countries, the new situation is not adverse to the South since IT allows it to immediately enjoy the benefits of the higher quality IGs existing in the North and permits conditional convergence with the North at a new higher steady state growth rate.

In summary, our analysis shows that the IT of IGs can raise the growth rates of all trading countries. However, only when there is also IT of FGs, can all the countries in the world participate in IT and benefit from it. Thus, an immediate policy implication occurs: if the South wants to be on the growth rate frontier it should drop all barriers to IT.

5.7. Transition to and stability of the balanced growth path

5.7.1. Overview

Once again, a steady state world growth rate exists and it is common to both countries – see section 5.5. We aim at finding the path of convergence towards the steady state, using the fourth-order Runge-Kutta classical numerical method for a set baseline parameter values and a set initial conditions. The analysis is very similar to that made in section 4.8. Indeed, there is again interdependence between countries, which implies that when one is out of the world steady state, so is the other. Thus, they must again be analysed together. The difference relative to section 4.8 results from the FGs price indexes, which now also reflect the pattern of FGs specialisation. That is, as a result of the IT of FGs, now we are also able to analyse the pattern of FGs specialisation.

In the next subsection, we will find the differential equations and, after that (in subsection 5.7.3), we will solve the model numerically.

5.7.2. Transitional dynamics for the world: differential equations

In short, again the differential equations that depict the transitional dynamics to the steady state derive from equilibrium in the labour markets, free entry condition into R&D, individual utility maximisation with individual optimal time allocation.

The analysis made between equations (4.73) and (4.79) of subsection 4.8.2 of the last chapter remain valid here. That is, the differential equations which describe the transitional dynamic of D_Z^1, $\tilde{Q}^{J^{VH}}$, $\tilde{Q}^{J^{SH}}$ and $\dot{Q}_I^{J^{VH}}/Q_I^{J^{VH}} - \dot{Q}_I^{J^{SH}}/Q_I^{J^{SH}}$ are the same. However, the FGs price indexes are now given by equations (5.25a, b) for the North and by equations (5.26a, b) for the South, and not by equations (4.27a, b).

In order to obtain the dynamic of $\dot{Q}_I^{J^{VH}}/Q_I^{J^{VH}} - \dot{Q}_I^{J^{SH}}/Q_I^{J^{SH}}$ we also need to calculate the dynamics of $V\tilde{H}(.|w)$ and $S\tilde{H}(.|w)$, which according to equation (4.75b) follow the same path. To this end, we again find first the Southern m-type interest rate and afterwards, since the country's interest rate is unique, we equal them. This implies that the analysis made between equations (4.80) and (4.86) remains valid as well. Since the differential equation (4.86) depends on the FGs price indexes in both countries, here we can rewrite it in the following way – omitting for the sake of simplicity both indexes t and w [so the HC gaps $\tilde{m}$ must be read as $\tilde{m}(.|w)$], and bearing in mind that $MC_I = 1$:

$$\frac{\dot{V}\tilde{H}}{V\tilde{H}} = \frac{\dot{\tilde{Q}}^{J^{VH}}}{\tilde{Q}^{J^{VH}}}\left[\tilde{Q}^{J^{VH}} ln\left(-\tilde{Q}^{J^{VH\,2}} + (1+d)\tilde{Q}^{J^{VH}} - d\right) - \tilde{Q}^{J^{SH}} ln\left(-\tilde{Q}^{J^{SH\,2}} + (1+d)\tilde{Q}^{J^{SH}} - d\right) + \right.$$
$$+ (-\bar{\sigma}_2 + \tilde{Q}^{J^{VH}})\frac{-2\tilde{Q}^{J^{VH\,2}} + (1+d)\tilde{Q}^{J^{VH}}}{-\tilde{Q}^{J^{VH\,2}} + (1+d)\tilde{Q}^{J^{VH}} - d} - (-\bar{\sigma}_2 + \tilde{Q}^{J^{SH}})\frac{-2\tilde{Q}^{J^{SH\,2}} + (1+d)\tilde{Q}^{J^{SH}}}{-\tilde{Q}^{J^{SH\,2}} + (1+d)\tilde{Q}^{J^{SH}} - d}\Bigg] \cdot$$
$$\left\{\left(1 + V\tilde{H}^{-1}\right)^{-1}\frac{A_I^{1/\alpha} - (A_P MC_P)^{1/\alpha} R_2}{A_I^{1/\alpha} + (A_P MC_P)^{1/\alpha} R_2 V\tilde{H}} + \frac{\bar{\sigma}_1 V\tilde{H}\, exp(V\tilde{H})}{[1+exp(V\tilde{H})][1+2exp(V\tilde{H})]}\right. \qquad (5.43)$$
$$\left. - \left(1 + S\tilde{H}^{-1}\right)^{-1}\frac{A_I^{1/\alpha} - (A_P MC_P)^{1/\alpha}\left(\frac{1-R_1}{R_1}\right)}{A_I^{1/\alpha} + (A_P MC_P)^{1/\alpha}\left(\frac{1-R_1}{R_1}\right) S\tilde{H}} + \frac{\bar{\sigma}_1 S\tilde{H}\, exp(S\tilde{H})}{[1+exp(S\tilde{H})][1+2exp(S\tilde{H})]}\right\}^{-1}$$

Where R_1 and R_2 are defined in equations (5.23a-c).

Therefore, differential equations (4.74), (4.75a-c), (4.77), (4.79) and (5.43) form a system that describes the world transitional dynamics, such that the variables D_I^1, D_P^1, $\tilde{Q}^{J^{VH}}$, $\tilde{Q}^{J^{SH}}$, $V\tilde{H}(.|w)$ and $S\tilde{H}(.|w)$ must be constants in steady state and, consequently, so are FGs price indexes and margins – N^{SH}, N and N^{VH}. In addition, we can also easily

verify the dynamic followed by $DIP^i \equiv D_I^i/D_P^i$, where $i = 2, 3, 4, 5$. Using equation (4.73) for both countries and taking into consideration the FGs price indexes we obtain – with some algebra – the following simplified equation for the respective dynamic:

$$\frac{\dot{DIP}^i}{DIP^i} = \frac{1}{\theta}(1-\tau_K)(r_I - r_P) + \frac{\dot{\tilde{m}}(.|w)}{\tilde{m}(.|w)} = \left(\frac{2\theta-1}{2\theta}\right)\frac{\dot{\tilde{m}}(.|w)}{\tilde{m}(.|w)}. \qquad (5.44)$$

Finally, the dynamics of $DZ^{i/i'} \equiv D_Z^i/D_Z^{i'}$ remain given by equation (4.88).

5.7.3. Solving differential equations for the baseline parameter values

In view of the system of differential equations which characterise the world transitional dynamics, we analyse now the transition to and stability of the steady state numerically. Once again, we consider the baseline parameter values in table 3.4 and, in addition, that $\wp_1 = 5$ and $\wp_2 = 0.03$ and that IT starts at time t_0, when the autarkic North reaches the steady state. Table A3.10 in Appendix 3 (subsection A3.3.1) summarises the main results by comparing initial and steady state values for the variables.

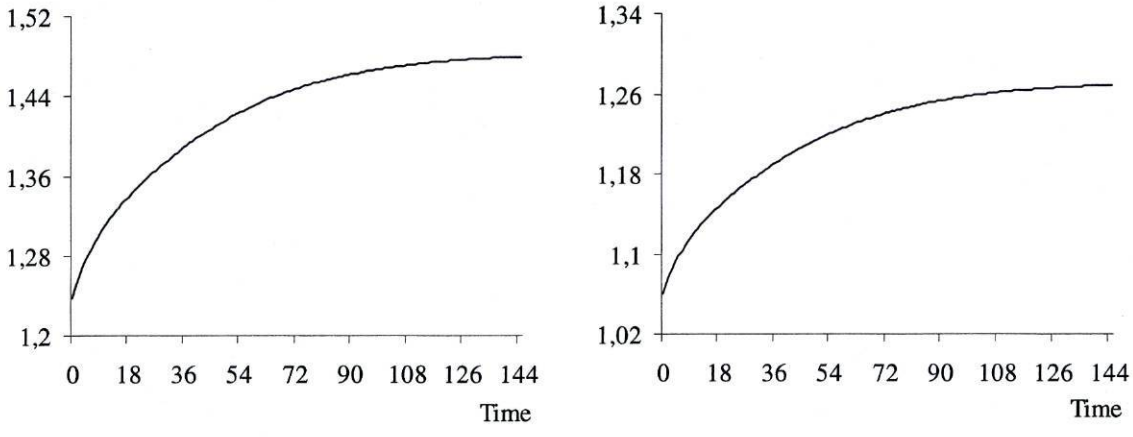
The adjustment process is globally stable, and the world steady state is attained at the end of 145 years. Hence, comparatively to the previous IT regime, the speed of convergence decreases – in the last case, the steady state was reached at the end of 110 years. This must be caused by the increasing interaction between countries that has now expanded to the IT of FGs. The lengthening in the time needed to attain the steady state suggests that the transitional dynamic should not be neglected.

As figures 5.4a, b below reveal, D_I^I and D_P^I rise from their initial values, but at a decreasing rate until they achieve their steady state values. Thus, during transitional dynamics, $Q_I^{I^{VH}} VH_Z(.|w)$ has outgrown $Q_I^{I^{SH}} SH_Z(.|w)$ or, in other words, $\dot{D}_Z^I / D_Z^I$ is positive but declines monotonically towards zero – *i.e.*, the global progress is versatile biased. The explanation for these paths is the same as that presented in the last chapter.

However, the steady state values of D_I^I and D_P^I are now greater than in the last IT regime – compare the respective values in tables A3.10 and A3.8 in Appendix 3. This is because now the FGs price indexes of one country are always influenced by the levels of HC at work (and by the *MC*) of the other country owing to the IT of FGs. Thus, the paths of FGs price indexes and, consequently, the intensity of the price channel, are different in the two IT regimes. Moreover, the paths of D_I^I and D_P^I allow us to state that a temporary partial Schumpeterian dynamic equivalent to the static Rybczynski result holds.

Figure 5.4. Transitional dynamics of D_Z^I

a) In the innovator country; *i.e.*, D_I^I **b) In the imitator country; *i.e.*, D_P^I**



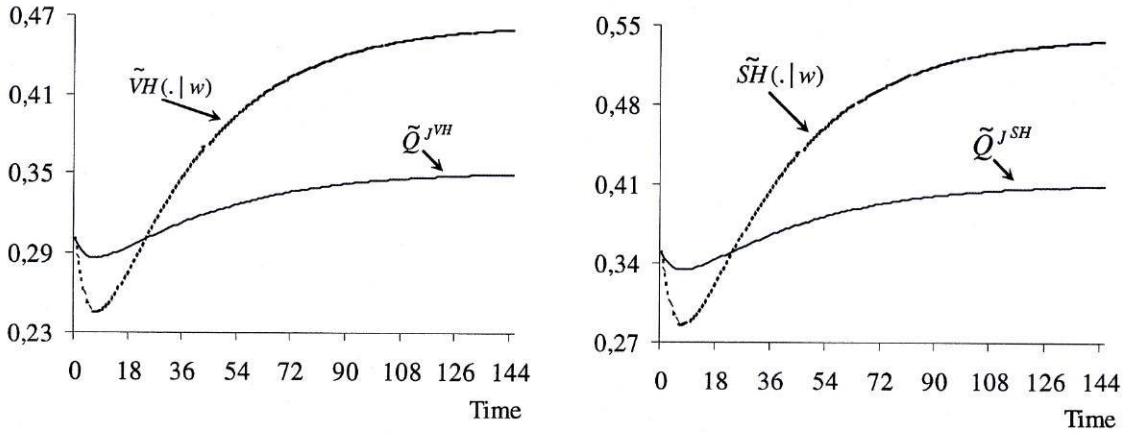
Likewise, $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ increase from their initial values (0.30 and 0.35, respectively) towards their new steady state values (0.35 and 0.41, respectively), as figure 5.5a, b below shows. That is, during the transitional dynamics, the Southern TK grows more quickly than the Northern one, but slows down until the steady state is reached. However, the introduction of the IT of FGs implies that the steady state values for both $\tilde{Q}^{JVH}$ and $\tilde{Q}^{JSH}$ are now smaller. This indicates that the opportunities offered by the imitation activity are now economically less alluring, with implications in the lower steady state growth rate – see table A3.10 in Appendix 3.[298]

Figures 5.5a, b also show that $\tilde{VH}(.|w)$ and $\tilde{SH}(.|w)$ increase from the initial values 0.30 and 0.35, respectively, but at a decreasing rate until they achieve their steady state values 0.46 and 0.54, respectively. The paths indicate that, initially and in relation to the North, the shares of time dedicated to training in the South increase due to higher wages and, thereby, the gaps in HC at work widen. Only after time t_7 do the paths change. The behaviour of the gaps in HC at work is also an indicator of the initially small incentives to do imitation. Remember that the path of wages depends directly on Northern TK progress and the gaps in HC at work affect the Southern imitation capacity.

The paths of $\tilde{Q}^{Jm}$ and $\tilde{m}(.|w)$ imply that only from t_7 until the steady state does the South grow at a higher rate than the North, and the differential falls steadily towards the steady state. Thus, from t_7 until t_{145} the South narrows the gaps and supports higher imitation costs, which pushes growth rates in the world down towards the steady state.

Figure 5.5. Transitional dynamics of both human capitals and quality indexes gaps

a) Versatile side; *i.e.*, $\tilde{Q}^{J^{VH}}$ and $\tilde{VH}(.|w)$ **b) Specialised side; *i.e.*, $\tilde{Q}^{J^{SH}}$ and $\tilde{SH}(.|w)$**



Comparing the steady state world growth rates in the three IT regimes, we can state that (i) IT with Southern imitation improves it; and (ii) a better value is reached under IT of IGs alone. Actually, their values are 2%, 3% and 2.8% in the autarkic North (and in the South with TK diffusion but without IT), IT of IGs and IT of all goods, respectively. Thus, looking only at the steady state growth rates, the desirable IT regime for both countries is the IT of IGs alone. This cannot be the case if we consider that the IT of FGs improves the imitation capacity by increasing the openness index IT – see the sensitivity analysis in the next section. Moreover, the IT of FGs allows the Southern imitator country to benefit directly from the levels of HC at work of the North, mirrored in the FGs price indexes – or, in other words, in the three threshold FGs (comparative and technology margins). In fact, computing numerically the ratios

$$\frac{Y_{P,\text{IT of IGs}}(t_0)}{Y_{P,\text{IT of all goods}}(t_0)} = \frac{N^{-2}_{P,\text{IT of IGs}}(t_0)}{\left(N^{VH}(t_0)-N^{SH}(t_0)\right)^2\left\{sh\,N^{SH(-1)}(t_0)+vh\frac{VH_P(t_0|w)}{SH_P(t_0|w)}\left(1-N^{VH}(t_0)\right)^{-1}\frac{Q_I^{J^{VH}}(t_0)}{Q_I^{J^{SH}}(t_0)}\right\}}; \quad (5.45a)$$

$$\frac{Y_{I,\text{IT of IGs}}(t_0)}{Y_{I,\text{IT of all goods}}(t_0)} = \frac{N^{-2}_{I,\text{IT of IGs}}(t_0)}{\left(N^{VH}(t_0)-N^{SH}(t_0)\right)^2\left\{sh\left(N(t_0)-N^{SH}(t_0)\right)^{-1}+vh\frac{VH_I(t_0|w)}{SH_I(t_0|w)}\left(1-N^{VH}(t_0)\right)^{-1}\frac{Q_I^{J^{VH}}(t_0)}{Q_I^{J^{SH}}(t_0)}\right\}}; \quad (5.45b)$$

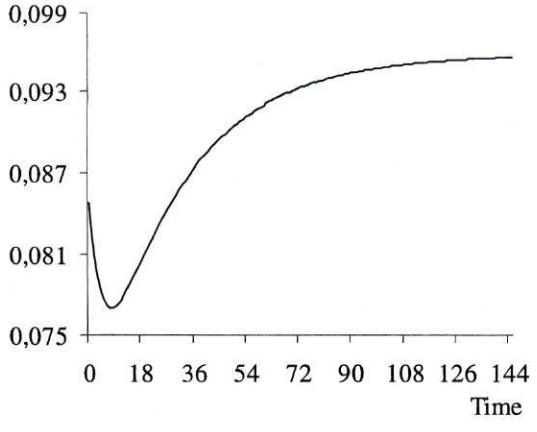
[298] This is because we consider that IT of FGs has not influence on the index IT of the catching up term.

they are equal to 0.81 and 1.16, respectively. Thus, for the South the dimension of this benefit induced by the IT of FGs, together with the long time necessary to reach the steady state, can be the desirable situation – *i.e.*, it can dominate the lower steady state growth rate. The converse seems to be true for the North, given that it supports a negative effect – *i.e.*, the levels of HC at work of the South affect the North negatively – and a lower steady state growth rate. In order to be certain, we will analyse welfare below.

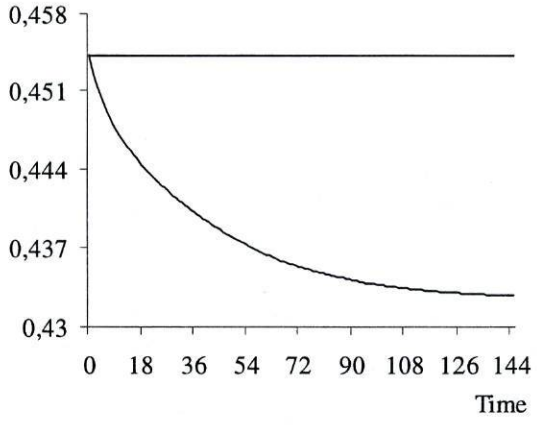
IT of FGs leads to a specialisation of the countries at a FGs level according to their CAs. Seeing that they have access to the same TK, at each time, each country will export relatively more FGs of the type that more intensively use the relatively abundant HC, and import relatively more FGs of the type that more intensively use the relatively scarce HC. The path of each margin between t_0 and the steady state is shown in figure 5.6.

Figure 5.6. Transitional dynamics of the threshold final good, N_Z

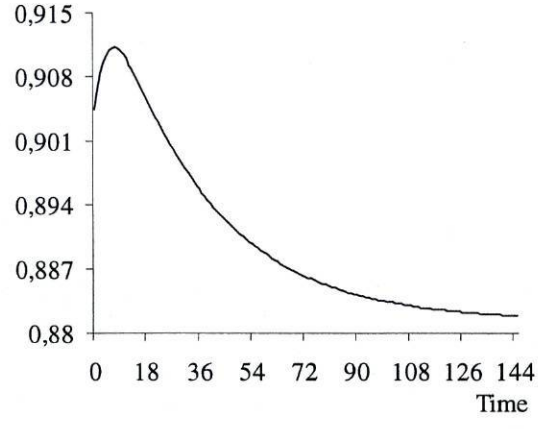
a) Path of the comparative margin N^{SH}



b) Path of the technology margin N



c) Path of the comparative margin N^{VH}



d) Path of N^{SH}, N, and N^{VH}

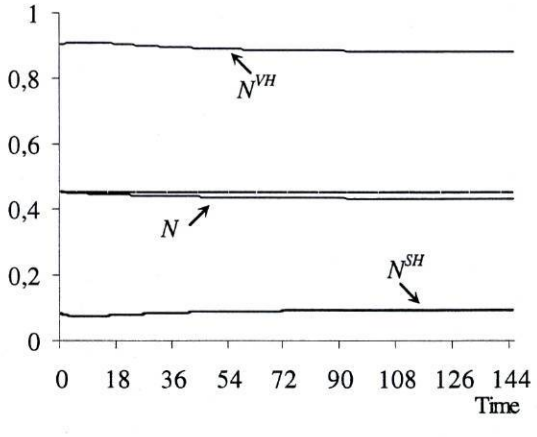


Figure 5.6 confirms numerically figure 5.3 in subsection 5.5.2. As a last resort, the paths of FGs price indexes – price channel – under international TK diffusion and IT of all goods explain them. In this way, as a direct consequence of the paths of D_I^I and D_P^I, the path of the technology margin, N, indicates that the use of ST in the world decreases, whereas the use of VT increases – see figures 5.6b, d and table A3.10 in Appendix 3. This is different from the HOS model where 'factor-intensity reversal' is absent. The share of specialised and versatile FGs produced by the South increases, shifting from 8.5% and 9.6% to 9.6% and 11.8%, respectively – see figure 5.6a, c, d and table A3.10 in Appendix 3. This is an expected result, since the South-North gaps in HC decrease.

Once again, despite the level effect in the South at time t_0, differences in levels of HC at work imply that relative prices of versatile FGs are always smaller in the North – see figures 5.8a, b and table A3.10 in Appendix 3.[299] Moreover, in line with the paths of D_I^I and D_P^I, there is a drop in the relative prices of versatile FGs in both countries. As expected, bearing in mind equations (5.25c), (5.24c), (4.24) and (4.23) and that the versatile TK bias is now also stronger than before – as we explain below –, the drop is now stronger than in the previous regime – see figures 5.7a, b and 4.13a, b.

Figure 5.7. Transitional dynamics of the relative price of *VH* final goods, p_Z^{VH}/p_Z^{SH}

a) In the innovator country; *i.e.*, p_I^{VH}/p_I^{SH} **b) In the imitator country; *i.e.*, p_P^{VH}/p_P^{SH}**

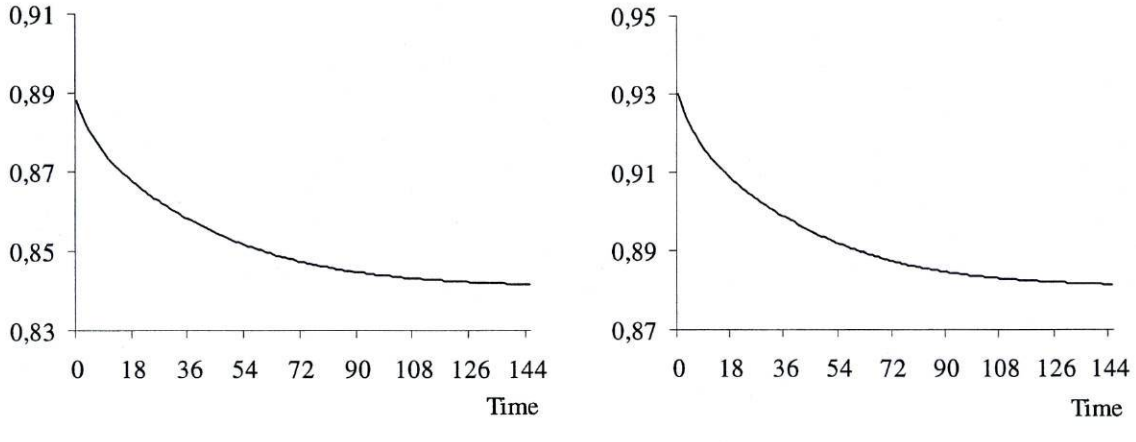
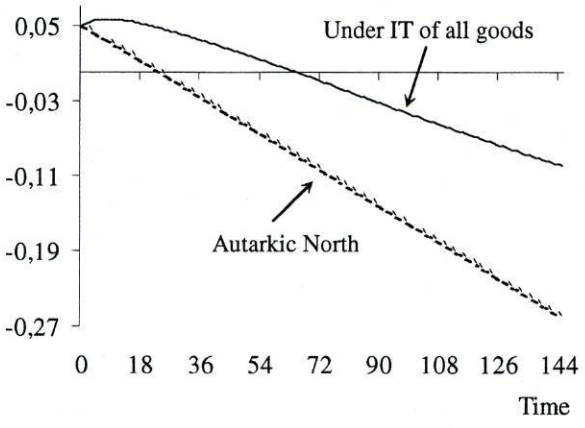


Figure 5.8 shows the path of the Northern TK bias in autarky and IT of all goods.

[299] Comparing the FGs price indexes in tables A3.8 and A3.10 in Appendix 3, we have as a result that in fact in absolute terms the FGs price indexes are now greater than before in the South and are smaller than before in the North. This is because now the levels of HC at work of one country affect the FGs price indexes of the other. Thus, the South is benefited by the higher Northern HC levels, and the North is harmed by the lower Southern HC levels.

Figure 5.8. Transitional dynamics of Versatile-specialised technological bias



In accordance with figures 4.3 and 4.14a, since at time t_0 the North-South average relative price of versatile FGs is higher than in the North alone, this increases the relative demand for versatile-specific new designs, biasing R&D innovation in that direction more than without IT – and, comparing figures 5.8 and 4.14a, even more than under IT of IGs alone. This bias increases the relative worldwide supply of versatile-specific IGs, thereby contributing to a decrease in the relative price of the versatile technology FGs.[300]

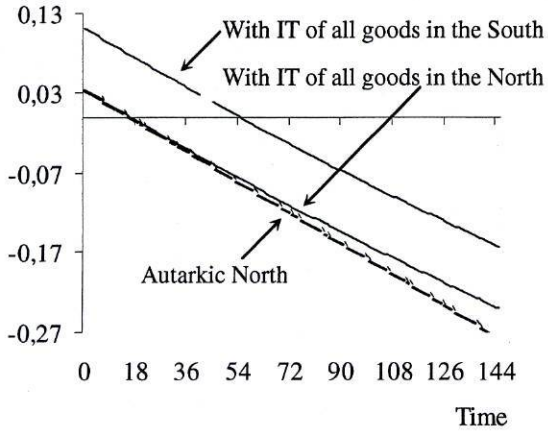
As would be expected from the previous analysis, the path of the versatile premium in both countries, as well as the respective explanation, is similar under IT of IGs alone and under IT of all goods – see figures 5.9a and 4.15a. In particular, we can state that a Schumpeterian dynamic equivalent to the static Stolper-Samuelson result occurs.

The international immobility of the HC, the differences in the exogenous levels of productivity (directly related with the quality of domestic institutions) and in the HC levels between countries imply again that there are wage differentials between them,[301] where the largest difference holds in the wages per unit of *VH* at work – see figure 5.9b and table A3.10 in Appendix 3. However, the wage differences are now smaller than in the previous IT regime. This is because of the more efficient allocation of resources – *i.e.*, HC – in FGs production. In fact, the IT of FGs induces Southern firms to employ the available HC in the appropriate tasks in comparison with the Northern TK, which implies that the *TFP* is the same in each FG produced, regardless of the producer country. Moreover and as a result, the levels of HC at work (and the *MC* of production) of one country affect the FGs price indexes, and consequently wages, of the other.

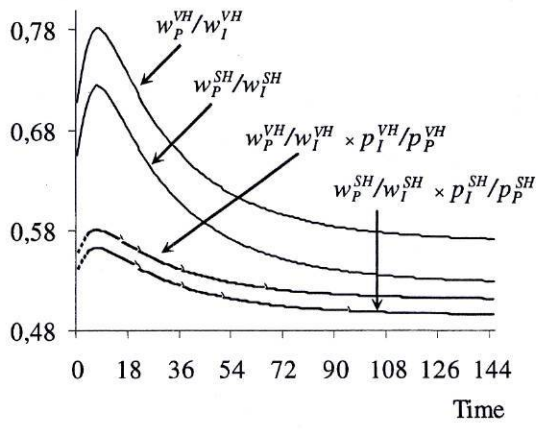
[300] As in the previous IT regime, in our baseline case the scenario A described in subsection 4.6.2 occurs.
[301] Note that more HC at work implies more FGs produced by the country.

Figure 5.9. Transitional dynamics of wages

a) Versatile premium; *i.e.*, $ln(w_Z^{VH} / w_Z^{SH})$



b) Differences between countries

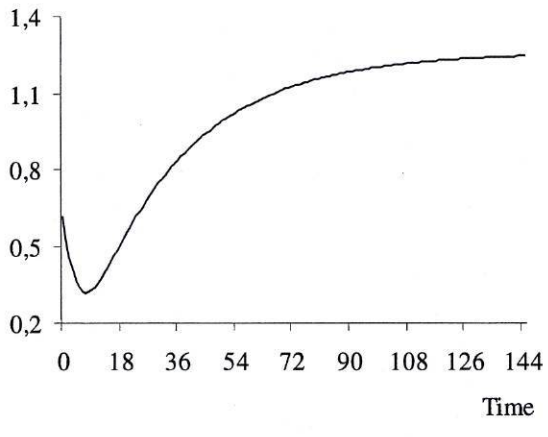


Looking at the behaviour of the relative (when compared with the North) real wages in the South – see also figure 5.9b –, we can have an idea about the welfare change during transitional dynamics. We verify that relative real wages in the South are decreasing after time t_7 and, thus, the same must hold in terms of Southern welfare.[302] It is interesting to see that, as a result of the increase in the relative wage w_P^m/w_I^m – *i.e.*, until time t_7 –, FGs producers in the South lose competitiveness and give their FG industries away to the North – see figures 5.6a, c.[303] The inverse happens after time t_7.

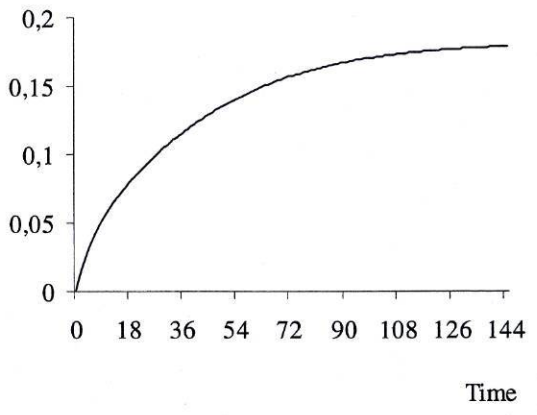
According to figures 5.10a, b we observe that *DIP* after t_7 and *DZ* are increasing, but at a decreasing rate until the steady state is reached.

Figure 5.10. Transitional dynamics of *DIP* and *DZ*

a) Of *lnDIP*



b) Of *lnDZ*



[302] Note that as the Southern interest rate must be decreasing during the transitional dynamics, the same is expected for the movement of real profits from the IGs sector.

[303] This is also in line with the path of the gaps in HC at work – see figure 5.5.

Now higher values are reached than in the last IT regime – compare figures 5.10a, b and figures 4.16a, b, respectively. The path of *DIP* shows that in the transitional dynamics, the Northern consumption per 'global or large amount of TK' rises. In addition, computing the ratio between consumption in both countries – equation (4.89) –, there is an increase in the consumption relative of the North during the transitional dynamics – see table A3.10 in Appendix 3. This confirms our statement above about the relative real wages in the South. In the previous IT regime, Southern individuals consume cheap domestic FGs; though, with FGs specialisation, Southern individuals consume expensive FGs, produced domestically or in the North. As a result, the South loses in terms of relative welfare.

Accordingly, and contradicting the first impression, it seems to be true that the IT of IGs alone is better for the South, and that the IT of all goods can be the desirable IT regime for the North. This result makes sense. Indeed, due to the increase in the price paid for FGs, the South spends a great amount of resources on consumption and, thereby, fewer resources remain for imitation. This may explain why the steady state TK of the South and the steady state world growth rate are smaller than in the previous IT regime.

Computing the ratio between the consumption of each m-type HC in each country, obtained from the definition of *DZ* – equation (4.90) –, there is no relevant change in the consumption relative between types of workers – see table A3.10 in Appendix 3.

Finally, the aggregate *TFP* difference between countries is now smaller than in the previous IT regime. This is due to the more efficient allocation of resources – *i.e.*, HC – in FGs production and because the level of HC at work of one country influences the other, even under international immobility of HC.

5.8. Sensitivity analysis

Here, we will carry out the sensitivity analysis under free IT of all goods. Again, numerical simulations show that the model's qualitative behaviour is similar if the value of each parameter at a time is changed relative to its baseline value, where slight differences are attained in the value of the variables in the new steady state. Thus, the first conclusion that becomes apparent is that the model remains stable.

The results obtained are summarised in table A3.11 in Appendix 3. We look especially at the differences between IT of IGs and IT of all goods regimes. We observe that, in addition to the effects described in the previous chapter, now changes in

parameters and initial value of variables affect the welfare relative of countries measured by the consumption relative – equation (4.89) – and, due to FGs specialisation, the pattern of FGs specialisation is also affected. This being the case, we concentrate the analysis on the changes in the welfare relative and in the pattern of FGs specialisation.

The greater interaction between countries mirrored in a greater openness index *IT* narrows slightly the gap in HC at work. This means that in relative terms the HC at work in the South increases, which provokes an increase in the number of FGs produced by the South. As expected, taking into consideration equation (4.89), the welfare of the North increases in relative terms, although only slightly.

Analysing the consequences of the reduction in parameter β_P – that is equivalent to the increase of international IPRs protection, remember –, we conclude that international IPRs protection reduces the number of FGs produced by the South, because the gap in HC at work broadens. Moreover, it decreases the welfare in the North, in relative terms. The decrease in the probability of successful imitation indicates that some IGs production shifts to the North – *i.e.*, in relative terms, it increases the measure of IGs with Northern quality leaders – so that the North spends more resources on IGs production. Thus, as in the previous IT regime, it harms both countries and, in particular, the North.

In accordance with table 5.3, a small MC_P increases the number of both specialised and versatile FGs produced by the North, but the welfare relative of this country decreases. This must be due to the increase in the amount of Northern resources spent in IGs production, as is suggested by the increase in the TK gap.

As to the parameters of the convergence term $\Xi_2(t)$, when they are high – specially parameter $\wp_2$ – the North produce more specialised and versatile FG, because its HC at work increases in relative terms, and observes an increase in its welfare relative.

In accordance with table 5.3, an increase in the exogenous variable A_I increases the number of FGs produced by the North and improves its welfare relative, whereas an increase in A_P increases the number FG produced by the South and improves its welfare relative. Improvements in the exogenous productivity decrease the share of IGs produced by the respective country – see the behaviour of the TK gaps in table A3.11 – and so it can spend more resources in consumption. This result confirms that, if possible, the use of a new GPT only in one country increases the number of FGs produced by this country.

A rise in α increases the HC at work of the South in relative terms, which implies that the number of FGs produced by the South increases. However, it benefits the North

in terms of welfare relative. In fact, this implies that the North produces a lower number of IGs and thereby it can spend more resources in consumption. Changes in θ and ρ do not affect the gap in HC at work and, for that reason, the pattern of FGs specialisation is also not affected. However, while an increase in θ favours the South in terms of welfare, changes in ρ do not affect the welfare relative.

Changes in the parameters *vh* and *sh* have again opposite effects. An increase in *vh* harms the North in terms of welfare, since it uses more resources in IGs production. This increases the number of versatile FGs produced by the North (and by the South, although only slightly) and decreases the number of specialised FGs produced by the South – table 5.3 and table A4.1 in Appendix 4. The same results occur after a decrease in *sh*.

As to the parameters *PE*, *d*, σ_1 and σ_2 of the catching up term, we observe that when each one increases, the welfare relative of the North improves. However, it is the number of FGs produced by the South that increases, due to the drop in the gap in HC at work.

As to the impact of the HC production parameters on the pattern of FGs specialisation, we observe that (i) an increase in δ does not affect it, (ii) a higher φ^{VH}, ϕ or χ_S increases the number of FGs produced by the South, due to the reduction in the gap in HC at work, and, (iii) for the same reason, an increase in χ_T decreases the number of FGs produced by the South. Apart from χ_T, increases in each one of these parameters improve the welfare relative of the North. Particularly in the case of φ^{VH}, ϕ and χ_S this is because the North uses less resources in IGs production.

With regard to changes in the initial values of the variables, we find for example that the number of FGs produced by the South increases when the initial value of (i) the versatile TK gap widens, (ii) the specialised TK decreases, (iii) the gap in *VH* at work narrows, and (iv) the gap in *SH* at work widens. In terms of the welfare relative of the North, it is for example improved if the specialised TK gap and the gap in *VH* at work fall. Thus, it is confirmed that the steady state depends on the initial conditions.

Finally, we verify that domestic government intervention widens the gap in HC at work, which implies that the number of both types of FGs produced by the North increases, but the welfare relative of the North is affected. Actually, in this case the North produces a greater share of IGs devoting small resources to consumption.

In summary, the results in table A3.11 confirm numerically those in tables 4.2 (comparative steady state analysis) and 5.3 (comparative threshold FGs analysis), and,

excluding in the welfare relative analysis, the generality of other results obtained in the sensitivity analysis of the previous chapter for the same reasons.

5.9. Some elements of synthesis

The aim of chapter 5 is to perceive how the IT of all goods, which, by assumption, are traded without any IT costs, affects the structure and the dynamics of international TK transfer by imitation, thereby influencing the levels and growth rates of the countries as well as prices, wages, welfare relative and the pattern of FGs specialisation.

Since IT of IGs remains possible, the generality of the results obtained in chapter 4 remain valid. Basically, they are only distinct in terms of degree. However, the possibility of IT of FGs applies to the analysis of the pattern of FGs specialisation and leads the way to the participation of the Southern stagnant country in IT. The former novelty occupies the central concerns of the chapter, considering then that the countries in the world are the North innovator and the South imitator, which has a *MC* advantage.

The CA of the countries in the production of FGs is dynamic, which implies that the location of FGs production can also change over time – namely the production of FGs near the comparative margins. It depends on endogenous TK progress and endogenous HC accumulation, which together determine the relative (dynamic) productivity of each country in each FG – dynamic Ricardian model –, which, in turn, is particularly dependent on the (dynamic) HC levels – dynamic HOS model.

The Southern stagnant country can now import Northern state-of-the-art IGs by exporting FGs. Hence, it can benefit from free IT, since it can benefit from the static level gain when IT starts, as well as from a jump to a world growth rate that is even greater than in the autarkic North. In fact, in this case, innovating firms need not worry about potential imitation and, thereby, they behave as in the autarkic North. However, now they have greater expected profits due to increased market size, which leads to a greater rate of innovation. Thus, the world (common) growth rate is higher than in the autarkic North and so free IT represents improvements for both countries. This is true even with world TK divergence, since the TK of the South clearly stops growing.

We find that (i) the Southern imitator country produces, consumes and exports a certain number of both specialised and versatile FGs, and IGs imitated previously that had not yet been innovated again; and (ii) the North produces, consumes and exports a certain number of specialised FGs, and IGs that had not yet been imitated. In fact, at each

time, both countries produce and export a certain number of both types of FGs since they always have both types of HC at work. Moreover, it is confirmed that:

(i) the share of FGs produced by each country increases with higher exogenous level of productivity directly related with the quality of domestic institutions;

(ii) each country exports relatively more FGs of the type that use more intensively the relatively abundant type of HC, and import relatively more FGs of the type that use more intensively the relatively scarce type of HC – in line with the HOS tradition analysis;

(iii) the share of FGs produced with ST in the South increases and the share of FGs produced with VT in both countries decreases – *i.e.*, the South captures a larger share of FGs production – with a greater specialised TK and a greater specialised-augmenting parameter – thus reflecting the influence of the Ricardian model in our framework;

(iv) the share of FGs produced with VT in both countries increases and the share of FGs produced with ST in the South decreases – *i.e.*, the North captures a larger share of FGs production – with a greater versatile TK and a greater versatile-augmenting parameter – reproducing the influence of the Ricardian model in our framework;

(v) the South produces more FGs at the end of the adjustment process – *i.e.*, in steady state than when IT starts – since it becomes more developed (the TK gap and the gap in HC at work in comparison with the North decrease).

The expansion of IT makes countries more interdependent. Now, the output of each country depends on the HC levels (and respective structure) of the other directly via FGs price indexes and indirectly via the influence of FGs price indexes in the TK progress. To sum up, output, prices and wages in one country are more affected by the HC levels in the other. When IT starts, in addition the access to Northern TK through the IT of IGs, the direct effect of FGs price indexes seems also to benefit the South – it takes advantage of the levels of HC at work of the North –, and *vice-versa* for the North.

The HC at work in both countries and the Northern TK, which is strongly affected by the South, are particularly important in determining the direction of R&D, economic growth, prices and wage inequality paths. In spite of the greater interdependence between countries, differences in HC levels and in respective structures between countries, when free IT of all goods begins, imply that the convergence in prices is only relative: the price relative of versatile FGs in the South remains higher. The interdependence between countries and the price channel – the market size channel has been removed –, induce

more R&D directed at improving versatile TK than in the autarkic North, which originates more moderate paths for the TK bias and intra-country wage inequality.

On its own, the IT of FGs does not affect the intra-countries FGs price indexes in relative terms when IT starts at time t_0, since they are equal under the IT of IGs alone. Thus, the static Stolper-Samuelson theorem cannot be applied here. In fact, the paths of wages derive from a Schumpeterian dynamic equivalent to the static Stolper-Samuelson result – *i.e.*, from the endogenous paths of prices which affect the R&D activity.

IT strongly benefits the South due to level and growth effects. However, the steady state levels of output and so of wages remain different between countries due to the gaps in the exogenous productivity and in the levels of HC at work. That is, although the path of the inter-country wage inequality favours the South, South-North wage differences remain. In fact, in steady state there is maintenance of inter-country wage inequality – *i.e.*, they grow at the same rate –, which, in turn, implies that a dynamic Schumpeterian factor-price equalisation or Samuelson result arises once more.

As the production of FGs is located in the country which has the CA, the cross-FGs *TFP* is the same regardless of the producer country. However, together, the differences in the HC levels and in the exogenous levels of productivity (directly related with the quality of domestic institutions) generate a Southern specialisation in a lower number of FGs and a greater Northern aggregate *TFP*.

As under IT some new GPT is used in both countries at the same time, it does not affect the pattern relative of FGs specialisation and, for the same reason, the same occurs as a result of a positive shock in, for example, the versatile-augmenting parameter.

Due to interdependence between countries, when one is out of the world steady state, so is the other. The system of differential equations which describes the transitional dynamics results from equilibrium in the labour markets, free entry condition into R&D, individual utility maximisation with individual optimal time allocation. Using the fourth-order Runge-Kutta classical numerical method for set baseline parameter values and set initial conditions, some interesting related conclusions can be confirmed.

First, the adjustment process is stable since the system's time path converges to the steady state, which is reached at the end of 145 years, and remains there without any exogenous perturbation. Thus, the conclusion presented in the previous chapter, that the interdependence between countries decreases the speed of convergence, is now reinforced – remember that the steady state is reached at the end of 56, 110 and 145 years

without IT effects, with IT of IGs alone and with IT of all goods, respectively. The long time necessary for the transition implies that it must be taken into consideration.

Second, during the transitional dynamics phase there is a process of convergence between countries since the TK gap and the gap in HC at work narrow – *i.e.*, the South grows at a higher rate than the North, but the differential falls steadily. As a result, the number of specialised and versatile FGs produced by the South increases by about 17%.

Third, during transitional dynamics, the versatile TK, together with the *VH*, grow at a higher rate than the specialised TK together with the *SH* – a temporary partial Schumpeterian dynamic equivalent to the static Rybczynski theorem occurs. Therefore, the use of ST in the world decreases, whereas the use of VT increases. Hence, in our framework, there is 'factor-intensity reversal' contrarily to the static HOS model.

Fourth, as the North-South average relative price of versatile FGs is higher than in the North alone, the relative demand for versatile-specific new designs increases, which boosts the relative worldwide supply of versatile-specific IGs. In our baseline case, this implies that the versatile premiums decrease, but not as much as without IT effects. Thus, the level effect in the South is reverted and the path of wage inequality – in a specialised direction – is more moderate than in the autarkic North. To sum up, a Schumpeterian dynamic equivalent to the static Stolper-Samuelson result is numerically found.

Fifth, computing the ratio between consumption in both countries, there is an increase in the consumption relative of the North during transitional dynamics. This is because FGs specialisation determines that Southern individuals consume more expensive FGs – produced domestically or in the North – than before. Therefore, it seems to be clear that the South loses in relative terms at the welfare level.

Sixth, international immobility of the HC, differences in the exogenous levels of productivity directly related with the quality of domestic institutions and in the HC levels between countries, imply that there are differences in the output levels and so in wages between them.

Seventh, if we compare only the steady state growth rates, we conclude that the desirable IT regime for both countries is the IT of IGs alone, where countries grow at 3% (>2.8%) a year. This cannot be the case if we consider that the IT of FGs improves the imitation capacity by increasing the openness index *IT*.

Eighth, IT of all goods benefits both countries. The South is benefited because it enjoys a strong level effect when IT starts and grows at a higher rate than without IT

effects – note that the world growth rate can be even greater than in the previous IT regime if the IT of FGs improves the openness index *IT*. The North is benefited since it enjoys a higher growth rate than in autarky and, in relative terms, its welfare increases.

Numerical simulations show that the model's qualitative behaviour is similar if the value of each parameter at a time is changed relative to its baseline value, where slight differences are attained in the value of the variables in the new steady state. This attests to the stability of the model. In particular, both the number of FGs produced by the South and the welfare of the North in relative terms increase when the openness index, the Southern *MC* of production, the specialised-augmenting parameter, the share of HC in production, the domestic policies promoting the R&D index, the TK gap threshold, the fixed elasticity of the probability of imitation with respect to g_2, the distribution parameter, the substitutability between the two modes of HC accumulation and the productivity at school also increase. The same occurs when, for example, there is an increase in the initial value of the versatile TK gap and in the gap in *SH* at work.

It would be interesting to find out, in future research, the implications of the additional extensions proposed at the end of the previous chapter for the pattern of FGs specialisation and welfare relative of countries.

CHAPTER 6

CONCLUDING REMARKS

We develop a dynamic, general equilibrium model of North-South technological knowledge diffusion by imitation, with scale-invariant endogenous growth and human capital accumulation. By North and South we mean two countries that differ in levels of the non-international trade related exogenous productivity, human capital levels and R&D capacity. Bearing in mind the initial differences between countries, which allegedly have historical roots, and the additional assumptions, the main purpose is to analyse the non-scale effects of technological knowledge diffusion under different international trade regimes – without trade effects, trade of intermediate goods alone, and trade of all goods – on economic growth and on inter and intra-country wage inequality, checking (i) the immediate level effects that arise with international trade, (ii) the steady state effects, and (iii) the transitional dynamics and sensitivity analysis results using a numerical method.

Regarding the productive activity, we follow closely the standard Schumpeterian growth model. In the final goods sector, each good is competitively produced by one of two possible technologies, versatile and specialised. The former uses versatile human capital (school intensive, more productive and more abundant in the North) complemented with a continuum set of differentiated versatile-specific intermediate goods, whereas the latter uses specialised human capital (OJT intensive) complemented with a different continuum set of differentiated specialised-specific intermediate goods. As a result of profit maximising limit pricing by the monopolists producers of intermediate goods, only the highest quality available of each intermediate good is used. Implicitly, we assume that the production function is also the same for intermediate goods and R&D sectors, which renders our framework a lab equipment model.

The intermediate goods sector is composed of monopolistic producers because their production involves (i) manufacturing costs that can be subsidised, (ii) plus the support of a start-up cost of R&D that can also be subsidised and, which must be paid to the firm's shareholders in the form of dividends. Furthermore, we assume that the North innovates, the South imitates and under international trade has a marginal cost of production advantage and there is domestic but no (at least no complete) international IPRs protection. Therefore, without international trade effects, the monopolist producers

are the ones that produce with the latest patent in each country since they sell the production only domestically, (ii) whereas under international trade, they are the ones that produce with the latest patent in the world, since they sell the production worldly.

In the R&D sector, firms do research to come up with ideas that they develop into blueprints for new quality intermediate goods – Schumpeterian R&D – and which are feasible from the actual state of the general purpose technology. Our R&D process embodies substantive differences from the previous literature of technological knowledge diffusion. We assume an autonomous positive learning effect of accumulated public technological knowledge from past successful research, an adverse effect of market size, which enables us to rule out the scale effects and a specific Southern catching up term, itself with some specific features, such as the technological knowledge gap threshold and the gap in human capital at work between countries. Due to the catching up term, imitation is cheaper than innovation.

Apart from the endogenous growth debate on the independence between economic growth and scale effects, our point of view is helpful in isolating the technological knowledge diffusion effects. In other words, we highlight the mechanisms, other than market size, by which the international trade bring about North-South technological knowledge diffusion and affects growth and wage inequality. In this manner, a larger market for technological knowledge does not promote more R&D, which is why just the price channel works – *i.e.*, there is only a greater incentive to upgrade technologies to produce more expensive goods. Otherwise, the market size channel will encourage successful researches for the more abundant type of human capital.

Similarly, as we wish to focus on international technological knowledge diffusion, it seems logical to believe that the South in not too far behind the North. The degree of backwardness is incorporated by making Southern imitation conditional to the distance from the (Northern) technological knowledge frontier. That is, there is a threshold distance beyond which the cost of imitation is infinite. Thus, we consider that only Southern countries with smaller technological knowledge gaps find it easier to imitate, grow faster and converge to the innovator's income level.

In particular, the imitation capacity depends on openness to international trade and other international trade policies. Also under international trade, a trade-off involving the South-North technological knowledge gap results. Southern firms must support R&D imitative activity of all quality levels between the current South level and the Northern

state-of-the-art intermediate good that they are targeting before they earn profits. Thus, the total resources required to bring about successful imitation are potentially greater than before. However, a wide technological knowledge gap favours the probability of successful imitation since once the threshold technological knowledge is overtaken, the backwardness benefits are higher.

Each country is populated by a unit mass continuum of individuals. In order to make a living, they have to create income by selling human capital on the labour market. The amount earned is related with their human capital level, and is spent on consumption, on loan in return for future interest or on taxes. Thus, individuals decide, in a structure of learning or doing (or about their fixed-time endowment), to work or to broaden their human capital level. They have perfect foresight about the rate of technological knowledge change over time and choose their expenditure paths in order to maximise its discounted utility. We thus get a dynamic general equilibrium model, so all markets clear throughout time.

In the educational process, individuals combine schooling and OJT. In both – schooling and OJT – (i) the human capital accumulation is described by functions *à la* Lucas (1988) and is not an automatic process: it involves time of training during which individuals do not work and so entails an opportunity cost calculated in terms of wage; (ii) the constant productivity is proportional to the knowledge pool provided by the GPT, but is greater at school; (iii) they are both necessary, but they can be substitutes or complements; (iv) substitutability indicates that schooling tends to be sufficient in providing all the essential skills for production, and *vice-versa* under complementarity. Technologies of accumulation of each m-type human capital diverge in their relative use of inputs, so that versatile human capital is relatively school intensive. Furthermore, we assume that individuals endowed with high ability accumulate versatile human capital, and those with less ability fare better by accumulating specialised human capital.

In short, we combine Schumpeterian R&D and human capital growth theories, such that technological knowledge progress – either through innovation or imitation, according to the country – and human capital accumulation are the two driving forces of endogenous growth. We contend that this technical structure represents an advancement on the standard R&D-based endogenous growth models, since the usual comparative static results of Schumpeterian growth theory are still valid, and we can explain various other related facts. Moreover, removing the scale effects and considering a production

function in which complementarity of inputs is coupled with substitutability between technologies becomes central for the effects of North-South technological knowledge diffusion on the countries' productive structure, direction of technological knowledge progress, growth and wage inequality.

For example, in cases where either technological knowledge is highly versatile biased or there is a large relative supply of versatile human capital, the share of final goods using versatile technology is large. In that case, the relative price of versatile final goods is low, which, due to the price channel, decreases the incentives to upgrade versatile technological knowledge – directed technological knowledge effect – and, thus, the growth rate of the price paid (wage) by a unit of versatile human capital. In particular, the versatile premium (intra-country wage inequality measure) is greater when technological knowledge is more versatile biased, mirroring the directed technological knowledge effect, and when versatile human capital is relatively scarcer, mirroring the traditional substitution effect.

Between countries, the versatile premium will be relatively greater in the South, when it is relatively abundant in specialised human capital and its available technological knowledge is relatively more versatile biased. Thus, we demonstrate that (dynamic) human capital composition – in the North, without international trade effects, and in the world, with international trade –, which is often neglected in endogenous growth models, is important in determining intra-country wage inequality. In addition, since the North is always more productive, inter-country wage inequality is at all times favourable to this country, but to different degrees, depending on the international trade regime.

The countries' modelisation did not focus on certain aspects, which remain for future work, such as the implications resulting from applying (i) Cournot competition in the intermediate goods sector instead of Bertrand competition; (ii) vertical and horizontal successful researches together; (iii) R&D cost advantage of leaders over followers and, in particular, differences in the catching up term for Southern leaders and followers, as well as the hypothesis of diminishing returns in R&D effort possibilities at a firm level; (iv) richer assumptions about productivity in the educational process; and (v) the impact of governmental policies aimed at increasing directly some or both types of human capital.

Level effect under international trade

Under the international trade of intermediate goods, the South is no longer limited by its own ability to produce state-of-the-art intermediate goods. It imports state-of-the-

art intermediate goods that have not yet been imitated, and exports state-of-the-art intermediate goods that it has imitated by under-pricing innovator leader firms. As a result, technological knowledge diffusion takes place either through imitation or through importation and the location of intermediate goods firms can change over time. Each new quality is initially produced in the North but then, when imitation occurs, production shifts to the South.

The international trade of intermediate goods pattern relies on technological knowledge differences between the countries, which are caused by different R&D capacities. This is in line with the Ricardian model, but in a setting of endogenous technological knowledge progress, which makes the comparative advantage in intermediate goods dynamic. Moreover, as the technological knowledge bias depends on the human capital levels, which can change because of endogenous human capital accumulation, a dynamic version of the Heckscher-Ohlin-Samuelson (HOS) model is also included. In short, the two traditional sources of the comparative advantage – technology and factor levels – are considered and they are endogenously accumulated.

Thus, when international trade of intermediate goods starts there is an instantaneous level jump in Southern output (level effect), which induces some convergence in prices and in the pattern of the final goods productive structure between countries. This favours human capital employed in the South by improving its productivity; *i.e.*, inter-country wage inequality drops, and higher wages immediately stimulate Southern human capital accumulation. However, the Southern convergence is not complete due to the maintenance of the gap in the exogenous levels of productivity directly related with the quality of domestic institutions and due to the international immobility of factors. Furthermore, intra-South wage inequality changes as well. Considering that initially the versatile-specific technological knowledge gap is relatively larger and bearing in mind that the North is relatively versatile human capital abundant, the Southern versatile premium increases.

From this time on, the growth rates are driven by domestic human capital accumulation and Northern technological knowledge progress. The latter is however strongly affected by the South. Indeed, demand for intermediate goods also depends on the Southern human capital structure and incentives to innovation depend on imitation. Thus, the positive effect of the openness to international trade of intermediate goods on imitation capacity, the lower Southern marginal cost of production and the initial level

effect induced by international trade of intermediate goods, feeds back into the North, affecting its technological knowledge through creative destruction and, thus, the growth rates. That is, international trade of intermediate goods can increase the Northern technological knowledge progress and so wages.

It can also increase intra-country wage inequality. This is because differences in human capital levels between countries when openness to international trade begins imply that the convergence in final goods price indexes is only partial: the relative price of versatile final goods remains greater in the South. Due to the interdependence between countries and to the price channel, this induces more versatile-biased Northern technological knowledge progress than in the autarkic North, thus increasing relative demand for the complementary input, which is the versatile human capital.

In addition, when international trade of final goods begins there is an adjustment process in the production of final goods in both countries, which seems to increase the Southern aggregate output; *i.e.*, it presumably benefits, in particular, from the greater Northern levels of human capital at work and respective structure, and *vice-versa*. From then on, each final good is produced by one country, and the location can change over time because the comparative advantage is also dynamic. It is essentially dependent on the (dynamic) human capital levels – dynamic HOS model.

As a result, (i) each country produces, consumes and exports a positive number of specialised and versatile final goods; (ii) the South also produces, consumes and exports intermediate goods imitated previously; (iii) the North also produces, consumes and exports intermediate goods that had not yet been imitated. Since both countries have access to the same technological knowledge, the share of final goods produced by each country is particularly dependent on the respective levels of human capital at work.

Specifically in terms of final goods production, we find that:

(i) the higher the exogenous level of productivity directly related with the quality of domestic institutions the country has, the greater the share of final goods it produces;

(ii) each country exports (imports) relatively more final goods of the type that more intensively use the relatively abundant (scarce) type of human capital;

(iii) the share of final goods produced with specialised technology in the South increases and the share of final goods produced with versatile technology in both countries decreases – *i.e.*, the South obtains a greater share – with greater specialised technological knowledge and greater specialised-augmenting parameter;

(iv) the share of final goods produced with versatile technology in both countries increases and the share of final goods produced with specialised technology in the South decreases – *i.e.*, the North captures a greater share – with greater versatile technological knowledge and greater versatile-augmenting parameter;

(v) the South must produce more final goods at the end of the adjustment process (*i.e.*, in steady state) since it is expected that it improve both technological knowledge gap and gap in human capital at work in relation to the North.

We stress that (i), (iii) and (iv) are more in line with the Ricardian tradition, whereas (ii) is more in line with the HOS tradition analysis.

International trade of final goods does not affect intra-countries final goods price indexes in relative terms when international trade starts, since they are equal to that which exists under international trade of intermediate goods alone. Thus, the static Stolper-Samuelson theorem cannot be applied here. In fact, the paths of wages derive from a Schumpeterian dynamic equivalent to the static Stolper-Samuelson result – *i.e.*, from the endogenous paths of prices which affect the R&D activity.

Interestingly, under international trade of all goods, even if the South is far away from the technological knowledge frontier and so unable to imitate, it participates in international trade. It immediately uses state-of-the-art intermediate goods, since imports become feasible due to the possibility of exporting final goods. This allows the South to enjoy the static level benefit as well as a jump to a new higher growth rate. Moreover, as Northern innovating firms need not worry about potential imitation, they behave as under autarky, except that greater expected profits due to increased market size lead to a greater rate of innovation. This, in turn, increases the world growth rate.

Steady state effects

Consistent with the majority of the recent empirical growth literature but opposed to the prediction of the first-generation R&D-based endogenous growth models, our North-South framework displays endogenous steady state growth without scale effects. Essentially, the growth rate depends on government policies, on individual preferences, and on the technology of final goods production, human capital production and R&D. Succinctly, Northern technological knowledge and domestic human capital accumulation drive steady state endogenous growth.

Under international technological knowledge diffusion but without international trade, the North commands the constant world steady state growth rate, and it is not

affected by the South. Thus, the common steady state growth rate can only be influenced by exogenous changes in Northern parameters and variables. Exogenous changes in both parameters and variables specific to the South can, however, influence the Southern transition to steady state and thus both the technological knowledge gap and gap in human capital at work between countries.

Under international trade, the North also commands the constant world steady state growth rate, but, as stated above, it is affected by Southern intermediate goods demand and by Southern R&D imitative activity. Specifically, when (i) the impact of openness on the probability of successful imitation is strong, (ii) the marginal cost of production in the South is low, (iii) international IPRs protection is weak, and (iv) there is no change in the (common to the North and South) ad-valorem subsidy rate to intermediate goods production between different international trade regimes and, in addition, it is higher, then the world steady state growth is higher under international trade.

In particular, it should be noted that

(i) international IPRs protection moderates the steady state effect of international trade on the equilibrium probability of innovation and so on the world growth rate;

(ii) subsidies to intermediate goods production and to R&D lead to a permanent increase in the world growth rate, but without international trade this is only true for Northern government intervention;

(iii) a more intensive use of the more productive mode of human capital production (school) as well as a better productivity of both learning activities lead to a higher world growth rate;

(iv) the greater the substitutability between modes of human capital production, the higher is the use of schooling and thus the higher the world growth rate;

(v) the steady state world growth rate is also affected by the structure of the human capital at work and so the influence of human capital on growth occurs through both channels, as a flow and as a stock.

Our framework is still in line with the existing cross-country evidence on growth rates. This issue is related to conditional convergence between some countries (upper and middle-income countries) and divergence between others (upper and low-income countries). Our theoretical explanation is based on the hypothesis that only Southern countries with some minimum level of development can appropriate the technological knowledge in the North since, only in that case, imitation is cheaper than innovation.

Thus, in contrast with the neoclassical growth model, where convergence is entirely driven by diminishing returns on capital, our predictions are pessimistic for countries that are distant from the technological knowledge frontier, because, in our framework, convergence is fully driven by technological knowledge diffusion.

As described above, under the international trade of final goods, the Southern stagnant country can also import Northern state-of-the-art intermediate goods by exporting final goods, which, in addition to a level effect, implies that it can benefit from a jump to an even greater world growth rate than in the autarkic North. In that case, even with world technological knowledge divergence, since the Southern technological knowledge stops growing, international trade favours both countries.

Differences in the world income between countries are in final analysis attributable to the distinct total factor productivities (TFPs). In steady state, the Northern aggregate TFP is larger than in the South. Excluding the case under international trade of final goods, the same is true for the cross-final goods TFP. Concerning the source of TFP differences, we find that (i) under international trade of all goods, they come from differences in both exogenous levels of productivity directly related with the quality of institutions and human capital levels; (ii) under international trade of intermediate goods alone, they also follow from the mismatch between Southern technological knowledge and human capital levels; and, (iii) without international trade effects, they also result from differences in the available technological knowledge. This explains why inter-country wage inequality decreases when countries are more interdependent.

In steady state it is also true that:

(i) the productive structure and relative prices tend to be constants;

(ii) the technological knowledge and relative wages have steady state dynamics;

(iii) wages are proportional to the associated available technological knowledge;

(iv) the technological knowledge bias and the versatile premia are smoothed by international trade and their paths are mainly dependent on the magnitude of the level effect and on the technology of human capital accumulation;

(v) a Schumpeterian equivalent to the static factor-price equalisation result arises, since wages grow at the same rate in both countries even without international trade;

(vi) output and wage levels between the North and the South do not converge totally since they have different exogenous levels of productivity and different human capital levels and the human capital is internationally immobile;

(vii) without international trade effects, the North is not affected by the South, rather, the inverse is true; whereas, under international trade, they are mutually affected;

(viii) international technological knowledge diffusion represents a window of opportunity for the Southern imitator.

We analyse the world implications of the use of a new GPT in two different, but promising, scenarios: (i) when the new GPT is used in all final goods production at the same time in each country, and (ii) when first the new GPT is used in versatile final goods production in each country. Under international trade the South is no longer limited by its own ability to produce state-of-the-art intermediate goods and the emerging phases begin at the same time in both countries.

In the former scenario, we find three distinct phases without international trade effects. In the first, there is a world growth effect, due to the increase in the productivity of schooling and OJT activities. In the second, the new GPT is used in the North, thus inciting a new growth effect. Moreover, both Northern output level and TFP differences between countries increase. In the third, the new GPT is also used in the South. Then, the Southern output level increases and the TFP differences between countries decrease. Under international trade, there are only two phases. In the first, there is the same world growth effect, which increases in the next phase, when there is also a world level effect.

In the latter scenario, we find five distinct phases without international trade effects. In the first, a world growth effect occurs. In the second, the new GPT is used only in Northern versatile technology, which leads to a new world growth effect. In the North, there is a level effect, the versatile premium increases and the threshold final good falls. Both TFP differences and the specialised technological knowledge gap increase. In the third, the new GPT is used in all Northern final goods production, and a new world growth effect occurs. In the North, a new level effect arises, but now the versatile premium falls and the threshold final good increases. The TFP differences and the versatile technological knowledge gap grow. In the last two phases, the new GPT is used in the South in versatile technology and in specialised technology, respectively. Then, no world growth effects emerge. Level effects occur in the South and the TFP differences decrease. The conduct of the versatile premium and of the Southern threshold final good in phases four and five is equal to that which occurred in phases two and three in the North, respectively. Finally, the behaviour of the technological knowledge gap in phases four and five is the opposite of that which existed in phases two and three, respectively.

Under international trade, there are only three distinct phases. In the first, a small growth effect occurs. In the next phases, there are both level and growth effects. In addition, the versatile premium increases in the second, but decreases in the third and the opposite occurs for the threshold final good.

We also surmise on the effects of a shock which causes a jump in the versatile-augmenting parameter in the production function. We find that it has a similar effect to that which occurs with the use of the new GPT only in versatile technology under international trade – remember that the versatile-augmenting parameter is the same in both countries. Thus, a shock in the versatile-augmenting parameter could be other explanation for the increase in versatile premium in both countries at the same time.

Transitional dynamics analysis

Regardless of the international trade regime, we solve the framework numerically by the fourth-order Runge-Kutta classical method, using realistic parameter values in view of the empirical literature and our theoretical assumptions. Given the equilibrium in the labour market, free entry condition into R&D, individual utility maximisation with individual optimal time allocation, we can depict the transitional dynamics for both North and South together under international trade or separately without international trade effects. We highlight that (i) the time path of the systems converges to the steady state and remain there without exogenous perturbations – *i.e.*, the adjustments are stable –, (ii) the interdependence between countries reduces the speed of convergence, thus increasing the relative importance of transitional dynamics, and (iii) in general, the chain of mechanisms under the different cases considered – without international trade effects, international trade of intermediate goods and international trade of all goods – are better understood and confirmed through the numerical transitional dynamics analysis.

Without international trade effects the North is unaffected by the South and quickly achieves the steady state. In Southern transitional dynamics analysis, we assume that the North is already in steady state. When international technological knowledge diffusion becomes possible, the South converges instantaneously to a unique stable country specific growth path. Nonetheless, its adjustment process only ends when it reaches the (world) balanced growth path established by the North. In a normal case, it achieves higher growth rates during the transitional phase due to the potential of technological knowledge backwardness and, thus, inter-country wage inequality decreases during this phase. This makes our framework similar to the neoclassical model, in the sense that in

our case the decreasing probability of imitation is analogous to the diminishing returns on capital in the neoclassical model.

As expected, Southern capital gains are only present in the transitional phase, and its level of consumption, wages and TFP – across-final goods and in aggregate terms – increase in comparison with the North. Therefore, an increase in welfare is observed. In any case, in steady state, wages and TFP differences remain between countries. Due to technological knowledge diffusion, the Northern path of intra-country wage inequality also occurs in the South. However, in our baseline case, the versatile premium level is greater in the South.

Under international trade, outside the world steady state, the technological knowledge gap between countries changes and they observe transitional dynamics. These transition paths depend on the relative technological knowledge levels and on the relative human capital levels employed and thus on prices in both countries. In fact, even after the initial level effect, the North-South average relative price of versatile final goods is higher than in the North alone. Through the price channel – the market size channel has been removed – this induces relative demand for versatile-specific new designs, biasing Northern R&D in that direction more than in the autarkic North, which, in turn, stimulates relative demand for the complementary input, versatile human capital.

Thus, faced with the autarkic North, the bias increases the world-wide supply of versatile-specific intermediate goods, thereby lowering during the transition to steady state both relative versatile final goods price and technological threshold in both countries – *i.e.*, (temporarily) factor-intensity reversal in the world occurs, contrarily to that which is argued by the static HOS model. As the use of versatile technology increases during the transitional phase, a temporary partial Schumpeteriam dynamic equivalent to the static Rybczynski theorem also occurs.

The versatile premium immediately rises in the South due to the initial level effect and, after that, in our baseline case it decreases in both countries, but not as much as in the autarkic North. Thus, the level effect in the South is reverted and the path of wage inequality – in a specialised direction – is more moderate than in the autarkic North. We confirm that a Schumpeterian dynamic equivalent to the static Stolper-Samuelson theorem occurs – since the price effect induced by international trade on wages works by technological knowledge change –, and that inter-country wage inequality remains.

The interdependence between countries – *i.e.*, from no international trade to international trade of intermediate goods and from international trade of intermediate goods to international trade of all goods – decreases the speed of convergence. The long time necessary to attain the steady state – 56, 110 and 145 years for the South without international trade effects, and for the world under international trade of intermediate goods alone and under international trade of all goods, respectively – implies that the transitional dynamic is important and should not be neglected. Also under international trade, the technological knowledge and the human capital at work in the South grow more quickly than in the North, but slow down until the steady state. That is, the benefit of technological knowledge backwardness together with the strong Southern human capital accumulation due to higher wages imply that the South grows at a higher rate than the North, but the differential falls steadily towards the steady state.

Under the international trade of intermediate goods, we have, in particular, that:

(i) with the baseline parameter values, trade improves the world growth rate;

(ii) no alteration in the consumption relative of the countries occurs;

(iii) countries unambiguously register gains – as a result of (i) and (ii);

(iv) as international IPRs protection decreases the world growth rate and does not influence the welfare relative, it serves only to reduce the positive effects of the trade;

(v) regardless of the country, a winner type of workers at welfare level does not occur;

(vi) the inter-country TFP difference and wage inequality fall until the steady state.

Under international trade of all goods, we find that due to the process of convergence between the South and the North the share of specialised and versatile final goods produced by the South increases, showing that the comparative advantage is dynamic. Furthermore, due to final goods specialisation, Southern individuals consume more expensive final goods – either produced domestically or not – than before. Thus, there is an increase in the consumption relative of the North during the transitional dynamics – *i.e.*, in relative terms the South loses at the welfare level. As both countries grow at a higher growth rate, the South enjoys a strong level effect and the North improves its relative welfare, the international trade of all goods benefits both countries.

Sensitivity analysis

Furthermore, the sensitivity analysis shows that, regardless of the international trade regime, the model works in a qualitatively similar way for the ranges of parameter

values tested. Actually, similar stable saddle paths to steady state are obtained, differing only in the specifics levels of the steady state of the variables that they approach.

We confirm that the world grows faster in steady state the

(i) higher the non-international trade related exogenous levels of productivity in the production function and in the training sector;

(ii) higher the exogenous ad-valorem subsidies to intermediate goods production and to R&D activity;

(iii) more efficient the R&D sector;

(iv) more patient and the less keen individuals are on consumption smoothing;

(v) more intensive the use of the more productive mode of human capital;

(vi) higher the degree of substitution between the modes of human capital production;

(vii) lower the depreciation rate of human capital.

In the autarkic North sensitivity analysis, we analyse attentively the impact of the degree of substitution between the modes of human capital accumulation in occupational choice between human capital accumulation and production and between schooling and OJT. In particular, under substitutability the effort in schooling and in OJT is very distinct according to the type of human capital accumulated. Thus, the schooling activity tends to be insufficient to evaluate human capital accumulation and growth. This could be a theoretical reason for the mixed evidence found by some empirical studies on the relation involving human capital accumulation and growth. We further verify numerically that government intervention creates strong benefits at the level of consumption in the North.

In the Southern imitator sensitivity analysis (without international trade effects), changes in parameters that improve the probability of successful imitation narrow the technological knowledge gap between countries. In particular, the Southern incremental gain in terms of technological knowledge as a result of an increase in the technological knowledge gap threshold – *i.e.*, a decrease in d –, offsets the loss in terms of human capital, and the Southern speed of convergence toward the steady state decreases.

It is also clear that the effects of changes in the initial gaps between countries are different, since the relative endowments are also different – we assume that the initial relative endowments of the South are greater in specialised technological knowledge and in specialised human capital at work. Thus, if the initial specialised technological knowledge gap or the initial gap in specialised human capital at work increases, it leads

to a greater steady state specialised technological knowledge gap. This is due to the diminished opportunities afforded by imitation, reflected in the less time taken by the process of transition towards the steady state. If the versatile technological knowledge gap or the gap in versatile human capital at work widens, it leads to a slower speed of convergence towards the steady state and a narrower technological knowledge gap. Thus, sensitivity analysis confirms that the steady state depends on the initial conditions. This is true in all situations considered; *i.e.*, in autarkic North and under international trade.

The less patient and the more keen the individuals are on consumption smoothing, the greater the technological knowledge gap and the gap in human capital at work are in steady state, and the smaller the time taken towards the steady state. Southern behaviour is also sensitive to changes in the parameters of human capital accumulation technology. Thus, if the two types of training are complementary, the South reaches a higher level of both technological knowledge and human capital at work, and needs less time to converge towards the steady state. The converse applies when the two types of training are substitutable. It is also clear that the South benefits considerably from improvements in the productivity of OJT. Finally, government intervention can improve the technological knowledge gap and its consumption relative.

Under international trade of intermediate goods, we verify that greater interaction between countries leads to a small technological knowledge gap, a higher world growth rate, and has no effect on the welfare relative between countries. Thus, international trade of intermediate goods favours both countries, and specially the versatile human capital since the versatile premium increases, while international IPRs protection has the opposite effect. Moreover, a small Southern marginal cost of production leads to a greater world growth rate, since it implies a higher mark-up for Southern monopolist producers providing an incentive for imitation activity and, due to the feedback effect, also for the innovation activity. The impact of imitation on innovation is so strong that the technological knowledge gap between countries widens, as well as the gap in human capital at work. As a result, the TFP differences between countries increase.

Other interesting results must be remarked upon:

(i) a greater speed of convergence is obtained with higher world exogenous levels of productivity directly related with the quality of domestic institutions, a greater share of human capital in production and a smaller productivity advantage of versatile human capital over specialised human capital;

(ii) the smaller technological knowledge gap is strongly reached with a higher share of human capital in production and a wider initial specialised technological knowledge gap, as well as with a narrower initially versatile technological knowledge gap;

(iii) a smaller threshold technological knowledge gap and initial gap in versatile human capital at work, as well as a higher initial specialised technological knowledge gap and initial gap in specialised human capital at work strongly reduce the gap in human capital at work between countries;

(iv) TFP differences and thus inter-country wage inequality clearly decrease with worse Northern exogenous level of productivity, better Southern exogenous level of productivity, a higher initial versatile technological knowledge gap and a higher initial gap in specialised human capital at work, and a smaller initial specialised technological knowledge gap and a smaller initial gap in versatile human capital at work;

(v) the versatile premium is markedly greater when the human capital share in production, the intensity of schooling, the productivity of schooling activities and the substitution parameter are small, and when productivity of OJT activities and the versatile-augmenting parameter in production function are great and domestic policy is implemented. Thus, in particular, this is in line with the consequences of the use of a new GPT first in versatile final goods. Moreover, also in particular and regardless of the international trade regime, intra-country wage inequality is more likely to prevail when changes in the parameters of the production of human capital relatively enhance the world accumulation of the type of human capital that is relatively abundant in the South. The mechanism is, once again, the price channel effect.

Improvements in specialised human capital production enhance its accumulation, thereby increasing its relative supply and lowering its relative wage, which, in turn, incentives its use in final goods production and lowers the relative price of specialised final goods. Accordingly, specifically under international trade, the level effect drop in the relative price of versatile final goods is in that case not very pronounced. Thus, compared with that prevailing in the autarkic North, the North-South average relative price of versatile final goods is greater. This accentuates the technological knowledge bias in favour of versatile-specific intermediate goods. Hence, with sufficiently contrasting parameter values the autarkic Northern path of available technological knowledge bias is reverted. In short, a strong improvement in specialised human capital production decreases the relative price of specialised final goods, which, in turn, strongly

re-directs R&D towards designs for versatile-specific intermediate goods, thus increasing demand for versatile human capital.

As stated, once relative prices of final goods attain their constant steady state levels, the only source of intra-country wage inequality is the direction of available technological knowledge. Due to complementarity in the production of final goods, the steady state's increasing bias in favour of versatile-specific intermediate goods determines the steady state path of increasing versatile (human capital) premia.

Our results can be interpreted in light of the literature on skill-biased technological change. In this literature, the bias that causes wage inequality is mainly induced through the market size channel. Whereas in our case, changes in the paths of inter and intra-country wage inequality result similarly from technological knowledge bias, but are induced through the price channel, conveniently insulated from market size effects.

In addition, the international trade of all goods enables us to look at the welfare relative of countries and at final goods specialisation. In particular, the share of final goods produced by the South and the welfare of the North in relative terms increase especially when the openness index, the Southern marginal cost of production, the specialised-augmenting parameter, the human capital share in production, the domestic policies promoting R&D, the technological knowledge gap threshold, the intensity of schooling, the substitutability between the modes of human capital accumulation and the productivity at school, also increase.

The theoretical framework developed in this work can also be extended to explore the effects of several other dimensions of international trade, which represent avenues for further research. The consideration of (i) different domestic policies – in the North and South; (ii) international trade costs; (iii) North and/or South import tariffs and quotas as well as export tax, subsidy and voluntary export restraint – and other international trade policies; (iv) the establishment of Northern multinationals in the South; and (v) human capital mobility from South to North, can be readily incorporated into the model, and their effects on technological knowledge progress, growth, wage inequality and pattern of final goods specialisation can be examined. Finally, (vi) a formal welfare analysis when the international trade of intermediate goods and of all goods are free could be introduced into the model in order to analyse the optimal policies that can be used to provide policy recommendations for managing North-South international connections.

APPENDIX 1

MAIN MATHEMATICAL NOTATION

In this appendix, we present the main mathematical notation. First, the generic notation is presented. Afterwards, the notation related with production of FGs and of IGs. Next, the notation related with R&D sector and, in the end, the notation related with individuals and HC accumulation.

Generic notation

gr: Economic growth rate;

I: Innovator country or the North;

P: Imitator country or the South;

r: Interest rate;

ss: Steady state;

t: Period of time;

Z: Country, $Z \in \{I, P\}$.

Final goods and intermediate goods production notation

α: HC share in production, $\alpha \in (0,1)$;

$\overline{m}$: Versatile, vh, and specialised-augmenting, sh, parameters, $\overline{m} = vh, sh$, $vh/sh > 1$;

π: Profits per HC for the producer of good;

Π: Profits for the producer of good;

A: Productivity variable dependent upon country institutions, $A > 0$;

j: Specific IG, $j \in [0, J^{VH}]$ or $j \in [0, J^{SH}]$;

J^m Number of IGs used with m;

k: Particular rung of a quality ladder at which an IG is located;

m: Type of the HC, $m = VH, SH$; *i.e.*, versatile or specialised;

MC: Marginal cost of production;

n: Final good producing firms, $n \in [0,1]$;

N: Threshold final good or comparative margin;

N^m: Comparative margin;

P^m Final goods price index;

p_n: Price of final good n;

q: Size of quality improvements with each upgrade of a particular IG;

Q^m: Domestic aggregate quality index;

RS: Total R&D expenditures;

SH: Specialised HC;

sh: Specialised-augmenting parameter;

s_r: Ad-valorem subsidy at the R&D;

s_x: Ad-valorem subsidy at the production of IGs;

VH: Versatile HC;

vh: Versatile-augmenting parameter;

w^m: Price paid for unit of m-type HC;

X: Aggregate demand for the highest quality IGs;

x: Quantity demanded of IG;

Y: Final goods output.

R&D sector notation

ζ: Fixed cost of R&D due to increasing complexity of quality improvements;

β: Positive coefficient on past research experience;

ξ: Measures the effectiveness of all costs related with scale increasing in R&D;

ξ: Measures the effectiveness of all costs related with scale increasing in R&D;

Γ_Z: Takes the values $\Gamma_Z = 0$ for $Z = I$ and $\Gamma_Z = 1$ for $Z = P$;

Φ_Z: Experience-adjusted pb_Z;

$\overline{\sigma}_1$: 'Semi-elasticity' of pb_P with respect to the gap in HC at work;

$\tilde{m}$: South's relative level of m-type HC;

$\tilde{Q}^{Jm}$: Relative TK level of the South's m-specific IGs – $\tilde{Q}^{Jm} \in (0,1)$;

$\overline{\sigma}_2$: 'Semi-elasticity' of the probability of imitation, pb_P, with respect to $g_2(.)$;

Π: Expected present discounted flow of net profit;

btp: Catching-up term – backwardness, IT and policies promoting R&D –;

cp: Adverse effect – cost of complexity – caused by the increasing complexity;

d: TK gap threshold $d \in (0,1)$;

EX: Exports per HC;

FTFP: Foreign total factor productivity;

g_1: Human capital gap function in the catching-up term;

g_2: Quadratic function that relate the backwardness hypothesis;

ll: Domestic learning by past R&D;

lm: Adverse effect of market size;

M: Imports of machinery and transports equipment per HC;

$PB(\tau)$: Cumulative probability density function for $T_{k(j,t|m)}$;

pb: Instantaneous probability of successful research;

rs: Resources devoted to R&D;

RS: Total R&D expenditures;

V_Z: Expected actual value of profits of the IG producer – market value of the patent;

$\Xi(m)$: Proportion of IGs with production in the North, under IT;

$\gamma(m)$: Proportion of IGs with production in the North that face imitator competition;

$\Xi_1(t|m)$: Share of IGs imitated before t and which are not successfully innovated again;

$\Xi_2(t|m)$: Share of losses for the South in production of IGs due to successful innovations.

Individuals and human capital accumulation notation

μ: Costate variable measuring the shadow price of the state variable K;

λ: Costate variable measuring the shadow price of the state variable m;

θ: Inverse of the intertemporal elasticity of substitution, $\theta > 0$;

ρ: Subjective discount rate, $\rho > 0$;

ϕ: Substitution parameter on HC accumulation function, $\phi \in]-\infty, 0[\cup]0, 1]$;

τ_K: Ad-valorem tax on assets income;

φ^m: Distribution parameter on HC accumulation, $\varphi > 0$ and $\varphi^{VH} > \varphi^{SH}$;

δ^m: Depreciation rate of the m-type HC;

χ_S: Efficiency parameter measuring the productivity of schooling;

χ_T: Efficiency parameter measuring the productivity of OJT activities;

τ_w: Ad-valorem tax on HC income;

a: Cognitive ability, talent, intelligence or learning capacity, $a \in [0, 1]$;

C: Aggregate consumption;

c: Individual consumption;

K: Asset holdings;

OJT: OJT activity;

$school$: Schooling activities;

u_S: Amount of time allocated in the school, $u_S \in [0, 1]$;

u_T: Amount of time allocated in the OJT, $u_T \in [0, 1]$;

u_w: Amount of time allocated at work, $u_w \in [0, 1]$.

APPENDIX 2

OTHER CONSIDERATIONS WITHOUT INTERNATIONAL TRADE EFFECTS

A2.1. Two alternative catching up specifications

On the topic of the catching-up term in equation (2.34) we presume the two following initial alternative specifications also considered in transitional dynamics:

$$btp_P(k,j,t|m)=\begin{cases}1: exp[PE+IT]\ exp\{\overline{\sigma}\, g[\tilde{m}(t|w),\tilde{Q}^{Jm}(t),d]\} \\ \text{and} \\ 2: exp[PE+IT]\{g_1[(\tilde{m}(t|w)]\}^{\overline{\sigma}_1}\ \{g_2[\tilde{Q}^{Jm}(t),d]\}^{\sigma_2^*(\tilde{Q}^{Jm},t)}\end{cases}, \qquad \text{(A2.1)}$$

Where:

$$g[\tilde{m}(t|w),\tilde{Q}^{Jm}(t),d]=$$

$$=\begin{cases}0 & \text{, if } \ 0<\tilde{Q}^{Jm}(t)\ \leq d \\ \tilde{m}(t|w)[-\tilde{Q}^{Jm}(t)^2+(1+d)\tilde{Q}^{Jm}(t)-d\,] & \text{, if } \ d<\tilde{Q}^{Jm}(t)\ <1\end{cases}$$

$$\overline{\sigma}>0,\ 0<\tilde{Q}^{Jm}(t)<1,\ \overline{\sigma}_1>0,\ \sigma_2^*[\tilde{Q}^{Jm}(t)]=\overline{\sigma}_{21}^*+\frac{\overline{\sigma}_{22}^*}{\tilde{Q}^{Jm}(t)},\ \overline{\sigma}_{21}^*>0 \text{ and } \overline{\sigma}_{22}^*>0.$$

A2.2. Steady state growth without HC accumulation

Without HC accumulation, the steady state endogenous growth is only driven by TK progress. In fact, in this case, instead of equation (3.1), we have that:

$$\frac{\dot{Q}^{Jm}}{Q^{Jm}}=\frac{\dot{Y}}{Y}=\frac{\dot{X}}{X}=\frac{\dot{RS}}{RS}=\frac{\dot{C}}{C}=\frac{\dot{c}}{c}=\frac{1}{\theta}\left[(1-\tau_K)\,r^{ss}-\rho\right]=gr. \qquad \text{(A2.2)}$$

Where $\dot{Q}^{Jm}/Q^{Jm}$ is given by equation (2.54), and the steady state South-North TK gap is constant. Thus, we set the growth rate of consumption derived from individuals' optimisation equal to that of domestic TK – since due to the aggregate budget constraint, C is a constant multiple of Q – to find the steady state interest rate in each country. Thus,

$$r_Z^{ss}=\frac{\frac{\beta_Z}{\zeta_Z}\left[exp(PE+IT)\ g_1(.)^{\overline{\sigma}_1}\ g_2(.)^{\sigma_2(.)}\right]^{\Gamma_Z}\left[\frac{1-s_{xZ}}{1-s_{rZ}}\right]\left(\frac{q-1}{q}\right)\left[\frac{p^m A_Z(1-\alpha)}{1-s_{xZ}}\right]^{\frac{1}{\alpha}}\overline{m}\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]\theta+\rho}{(1-\tau_K)+\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]\theta}. \qquad \text{(A2.3)}$$

The steady state growth rates result from plugging the steady state interest rate in equation (A2.2) into the Euler equation (2.64).

Similarly and in short, under IT and without HC accumulation, since Y in both countries is increasing in Q_I, both countries will be growing at the growth rate of Northern TK. In this case, X, RS and C in both countries are constant multiples of the Northern TK. For this reason, setting the growth rate of Q_I derived using the equilibrium probability of successful innovation equal to the growth rate of consumption from individuals' optimisation yields an expression for the steady state world interest rate. Then, the steady state world growth rates result from plugging the resulting steady state interest rate into the Euler equation (2.64).

A2.3. Overview of the effect on wages of a rise in *m*-type human capital

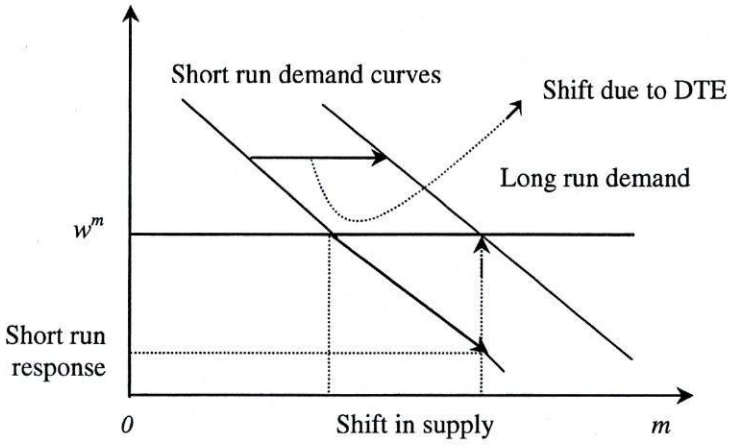
In equilibrium w^{SH} and w^{VH} depend on the square root of the ratio between the availability of the two types of HC employed pondered by the respective quality index: $\sqrt{VH(.|w)\,Q^{JVH}/SH(.|w)\,Q^{JSH}}$ and $\sqrt{SH(.|w)\,Q^{JSH}/VH(.|w)\,Q^{JVH}}$ – equations (2.29a, b). Thus, for example, an exogenous increase of $SH(.|w)$ leads to decreasing direct marginal returns. This is the substitution effect, which, all else being equal, makes the economy move along a downward sloping demand curve. This curve can be called the 'short run demand curve', as shown in figure A2.1 below. Conversely, as a fraction of $SH(.|w)$ increases, w^{VH} should increase. That is, for a given state of TK, the relative factor reward is decreasing in the relative factor supply: the less abundant one substitutes the more abundant factor, and its relative marginal product falls.

However, more $SH(.|w)$ implies that p^{SH} relative to the price index p^{VH} decreases; *i.e.*, it induces a price effect – equations (2.21) and (2.22) –, which generates a change in the direction of TK progress that, in the long run, increases Q^{JVH} by an offsetting amount. Thus, in the long run the square root of the ratio between the availability of the two types of HC employed pondered by the respective quality index remains constant.

We call this last effect induced by the price channel as directed TK effect (DTE), meaning that each firm can decide whether to direct its research effort to improving the quality of versatile IGs or to improving the quality of specialised IGs. The direction of the endogenous TK progress – *i.e.*, the versatile-specialised TK bias – depends on the relative price of the versatile-specific FGs. In the long run the substitution effect is

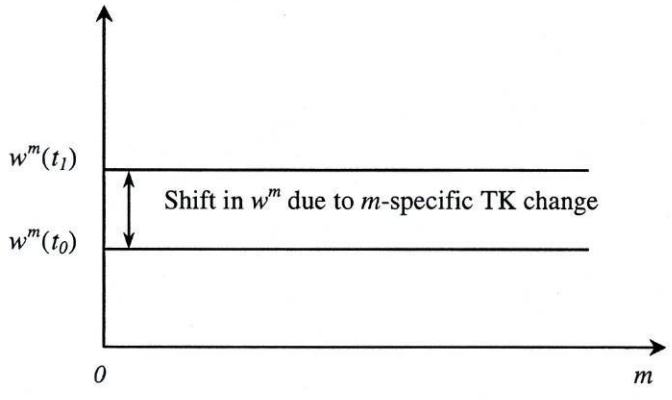
cancelled by the DTE, resulting in a constant 'long run demand curve', as the comparative static analysis in figure A2.1 shows – see also equation (2.29a, b).

Figure A2.1. Change in price paid for a unit of *m* due to a shift in *m* supply



As a result of equation (3.2) and (2.30), the FGs' price index is constant in steady state and the comparative static analysis in figure A2.2 below holds.

Figure A2.2. Long run change in price paid for a unit of *m*

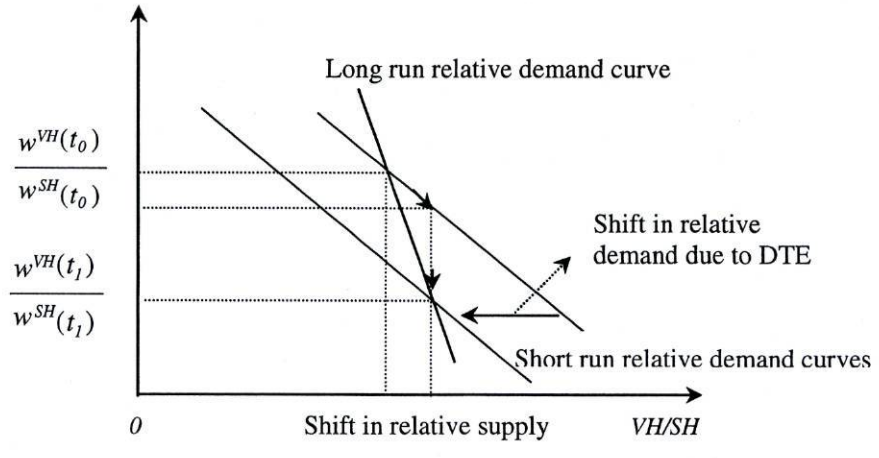


What becomes interesting in the versatile premium, W, is the fact that the substitution effect is not cancelled by the DTE. It is reinforced – see the comparative static analysis in figure A2.3 below. For example, with all else being equal, an increase in the supply of $VH(.|w)$ first moves the economy along a short run relative downward sloping demand curve, reducing W. The relative supply change also decreases the relative price p^{VH} – equations (2.21) and (2.22) – and induces a change in the direction of TK progress that increases Q^{JSH} and so a shift of the short run relative demand curve. Thus, W first falls (short run) and then falls again (long run, due to TK change). Hence, short

run relative downward sloping demand curves are more elastic (less negatively sloped curves) than the resulting long run relative downward sloping demand curve.

In brief, in the long run, w^{VH} remains constant, in line with figure A2.1, but w^{SH} increases due to the increase in Q^{JSH}, in accordance with figure A2.2, and so w^{VH}/w^{SH} falls – figure A2.3. Hence, the effect of HC accumulation on inequality occurs because HC investment affects factor proportion, which has an impact on relative wages. As *VH* accumulates, the aggregate share of *VH* rises so that the relative price of *VH* falls.

Figure A2.3. Change in the versatile premium due to shift in relative supply of versatile HC



The above analysis is true because of the value of the exponents in equation (2.32). They result from the gross substitutability between technologies, ST and VT, together with the complementarity in the use of inputs, HC and IGs. Thus, for example, the effect of Q^{JVH} and $VH(.|w)$ on W depends on the exponent in Q^{JVH}/Q^{JSH} and $VH(.|w)/SH(.|w)$, respectively. When it is greater than *0*, as is the case for Q^{JVH}/Q^{JSH}, a relative increase in Q^{JVH} increases *W*, and *vice-versa* when it is smaller than *0*, as is the case of $VH(.|w)/SH(.|w)$. When the two technologies are gross substitutes, *m*-augmenting TK change is also *m*-biased – for example, increases in Q^{JVH} increase the marginal product of *VH* more than the marginal product of *SH*.

The analysis in section 2.6 suggests that $\dot{VH}/VH > \dot{SH}/SH$, above all when schooling and OJT are substitutes. If so, in the short run, w^{VH} and w^{SH} fall due to the substitution effect – see figures A2.1, A2.4 and A2.5 –, where the drop in the former wage is greater due to the expected greater increase in $VH(.|w)$, which more strongly

decreases its direct marginal returns – see equations (2.29a, b). However, in the long run, the DTE 'works in two ways' and w^{VH} and w^{SH} rise above their initial values. That is, more $VH(.|w)$ decreases p^{VH} relative to p^{SH} – equations (2.21) and (2.22) –, which increases Q^{JSH} by an offsetting amount – w^{VH} is driven to its initial values, see figures A2.1, A2.4 and A2.5. This increase in Q^{JSH} increases w^{SH} – equations (2.29a, b) and/or equation (2.30). The same occurs with more $SH(.|w)$ and so their final values exceed the initial values – figures A2.2, A2.4 and A2.5. In the end, the W falls as in figure A2.3.

Figure A2.4. Dynamics of w^{SH}

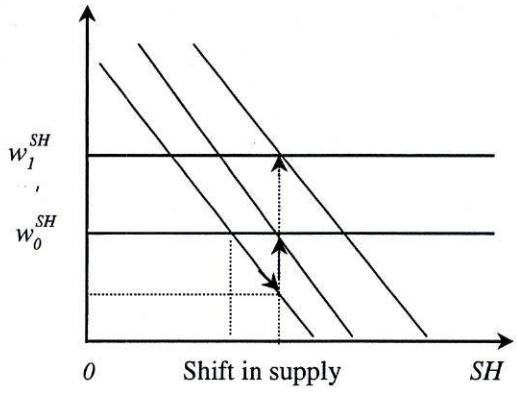
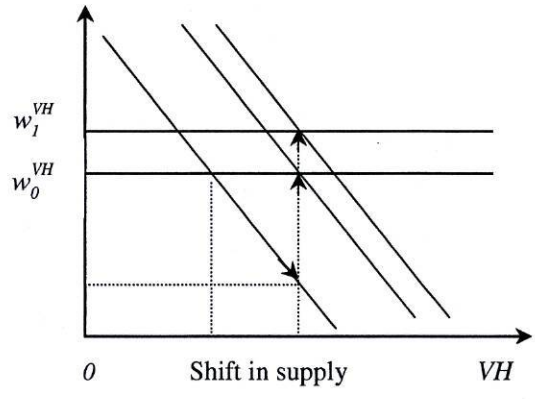


Figure A2.5. Dynamics of w^{VH}



Note that, without IT effects, if the relative increase of $m(.|w)$ holds in the South, the adjustment process induced by both the substitution and price effects will occur only in that country. However, if the relative increase of $m(.|w)$ holds in the North, it will affect the South, since the 'imitation capacity' for each m-type of IGs depends on the respective gap in HC at work between countries. Even so, at the end of the adjustment the availability of the two types of HC employed in productive activity balanced by the respective quality index remains constant in both countries.

APPENDIX 3

TABLES AND GENERIC FIGURES OF TRANSITIONAL DYNAMICS AND SENSITIVITY ANALYSIS

In this appendix, we present the tables and some figures which summarise the results of transitional dynamics and sensitivity analysis under all IT regimes – *i.e.*, no IT effects (section A3.1), IT of IGs alone (section A3.2) and IT of all goods (section A3.3).

A3.1. Transitional dynamics and sensitivity analysis under no trade effects

A3.1.1. Transitional dynamics in the North

Table A3.1. Comparing initial and steady state values of the variables

Variable	Scenario 1		Scenario 2	
	Initial	Steady state	Initial	Steady state
$D_I^1 = \frac{Q_I^{IVH} VH_I(.\|w)}{Q_I^{ISH} SH_I(.\|w)}$	$\frac{1.00\times1.00}{0.95\times0.95}=1.11$	$\frac{3.04\times1.88}{2.90\times1.59}=1.24$	$\frac{1.00\times1.00}{0.85\times0.85}=1.39$	$\frac{2.90\times0.99}{2.75\times0.84}=1.24$
D_I^2, D_I^3	0.37	0.15	0.37	0.29
D_I^4, D_I^5	0.37	0.16	0.37	0.26
$u_{wI}(.\|VH)$	0.32	0.66	0.57	0.67
$u_{wI}(.\|SH)$	0.35	0.71	0.58	0.71
p_I^{VH}	0.80	0.79	0.77	0.79
p_I^{SH}	0.87	0.89	0.90	0.89
N_I	0.46	0.45	0.44	0.45
W_I	1.098	1.031	1.089	1.037
$C_I(.\|VH)$	0.37	0.84	0.37	0.84
$C_I(.\|SH)$	0.33	0.76	0.27	0.62
$Pb_I(.\|VH)$	---	0.036	---	0.036
$Pb_I(.\|SH)$	---	0.038	---	0.038
Interest rate, r_I	---	0.05	---	0.05
Growth rate, gr	---	0.02	---	0.02

Note: the steady state is reached after 27 years.

A3.1.2. Transitional dynamics in the South

Table A3.2. Initial and steady state values of the variables under all catching up terms

Variable	Catching up specification					
	Benchmark		Alternative 1		Alternative 2	
	Initial	Steady state	Initial	Steady state	Initial	Steady state
$\tilde{Q}^{J^{VH}}$	0.30	0.55	0.30	0.55	0.30	0.55
$\tilde{Q}^{J^{SH}}$	0.35	0.64	0.35	0.64	0.35	0.64
$\tilde{VH}(.\mid w)$	0.30	0.39	0.30	0.24	0.30	0.12
$\tilde{SH}(.\mid w)$	0.35	0.45	0.35	0.28	0.35	0.14
D_P^{2} [a]	0.20	0.13	0.20	0.13	0.20	0.13
$u_{wP}(.\mid VH)$	0.44	0.66	0.51	0.66	0.49	0.66
$u_{wP}(.\mid SH)$	0.47	0.71	0.53	0.71	0.52	0.71
Time	---	56	---	235	---	482
w_P^{VH} / w_I^{VH}	0.152	0.279	0.152	0.279	0.152	0.279
w_P^{SH} / w_I^{SH}	0.151	0.277	0.151	0.277	0.151	0.277
$\frac{C_P(.\mid VH)}{C_I(.\mid VH)}$ [b]	0.05	0.19	0.05	0.11	0.05	0.06
$\frac{C_P(.\mid SH)}{C_I(.\mid SH)}$ [b]	0.07	0.23	0.07	0.15	0.07	0.07
$\frac{TFP_{nP}(.\mid SH)}{TFP_{nI}(.\mid SH)}$	0.16	0.27	0.16	0.21	0.16	0.14
$\frac{PTFP_{nP}(.\mid SH)}{PTFP_{nI}(.\mid SH)}$	0.17	0.29	0.17	0.22	0.17	0.14
$\frac{PTFP_{nP}(.\mid VH)}{PTFP_{nI}(.\mid VH)}$	0.16	0.27	0.16	0.20	0.16	0.13
$\frac{Y_P^{ss}}{Y_I^{ss}} = \frac{y_P^{ss}}{y_I^{ss}}$	0.05	0.12	0.05	0.07	0.05	0.04

Notes: [a] Considering the same initial value for D_P^2, D_P^3, D_P^4, D_P^5, then they have the same dynamic;

[b] Supposing that scenario 1 holds in the North;

Regardless of the time t, $p_P^{VH} = 0.82$, $p_P^{SH} = 0.84$ and $N_P = 0.49$;

Recall that in steady state $pb_I(m) = pb_P(m)$, $r_I = r_P$ and $grI = grP$;

Recall still that when TK diffusion starts (considering scenario 1 in table A3.1) $W_P > W_I$:

$$\frac{w_P^{VH}}{w_I^{VH}} \times \frac{w_I^{SH}}{w_P^{SH}} = \frac{0.152}{0.151} \Leftrightarrow W_P = \frac{w_P^{VH}}{w_P^{SH}} = \frac{0.152}{0.151} \times 1.031 = 1.034 > 1.031 = \frac{w_I^{VH}}{w_I^{SH}} = W_I;$$

The *TFP* differences evaluate equations (3.19), (3.22a), (3.22c) and (3.24a), respectively.

A3.1.3 Sensitivity analysis to the baseline results of the North

Table A3.3. Qualitative and quantitative results of sensitivity analysis

Parameter	Value	Time[a)]	D_I^1	D_I^2	D_I^5	$u_{wI}(VH)$	$u_{wI}(SH)$	grI
A_I	[0.05-2.5]	↓	↓	↑	↑	↑	↑	↑
	0.85	65	1.30	0.00	0.00	0.39	0.41	0.00
	1.76	25	1.23	0.40	0.45	0.85	0.87	0.03
α	[0.01-0.99]	↑	↑	↑	↑	↑	↑	↓
	0.25	20	1.21	0.02	0.02	0.47	0.47	0.06
	0.70	45	1.27	1.56	1.79	0.68	0.73	0.01
β_I	[1.00-5.00]	↓	↓	↓	↓	↓	↓	↑
	1.00	35	1.25	0.21	0.24	0.67	0.73	0.01
	5.00	16	1.21	0.10	0.11	0.66	0.71	0.10
θ	[0.50-2.50]	→	→	→	→	↑	↑	↓
	0.80	27	1.24	0.15	0.16	0.44	0.47	0.03
	2.00	27	1.24	0.15	0.16	0.86	0.82	0.01
ρ	[0.01-0.05]	→	→	→	→	↑	↑	↓
	0.01	27	1.24	0.15	0.16	0.22	0.24	0.04
	0.04	27	1.24	0.15	0.16	0.87	0.93	0.01
δ	[0.00-0.06]	→	→	↓	↓	↓	↓	↓
	0.00	27	1.24	0.28	0.31	0.77	0.81	0.03
	0.05	27	1.24	0.05	0.05	0.48	0.51	0.01
φ^{VH}	[0.51-0.99]	→	↑	↑	↑	↓	↑	↑
	0.51	27	1.20	0.14	0.15	0.70	0.68	0.02
	0.96	27	1.50	0.36	0.49	0.49	0.72	0.04
χ_S	[0.07-0.30]	→	↑	↑	↑	↓	↑	↑
	0.07	27	1.20	0.13	0.12	0.72	0.74	0.02
	0.20	27	1.45	0.29	0.38	0.52	0.66	0.03
χ_T	[0.03-0.09]	→	↓	↑	↑	↓	↓	↑
	0.03	27	1.31	0.09	0.11	0.78	0.98	0.01
	0.09	27	1.20	0.18	0.19	0.58	0.61	0.02
ϕ	$[-\infty$ -1.00][b)]	↓	↑	↑	↑	↓	↑	↑
	−10.0	30	1.20	0.14	0.15	0.69	0.65	0.02
	0.90	24	1.39	0.18	0.22	0.60	0.78	0.02

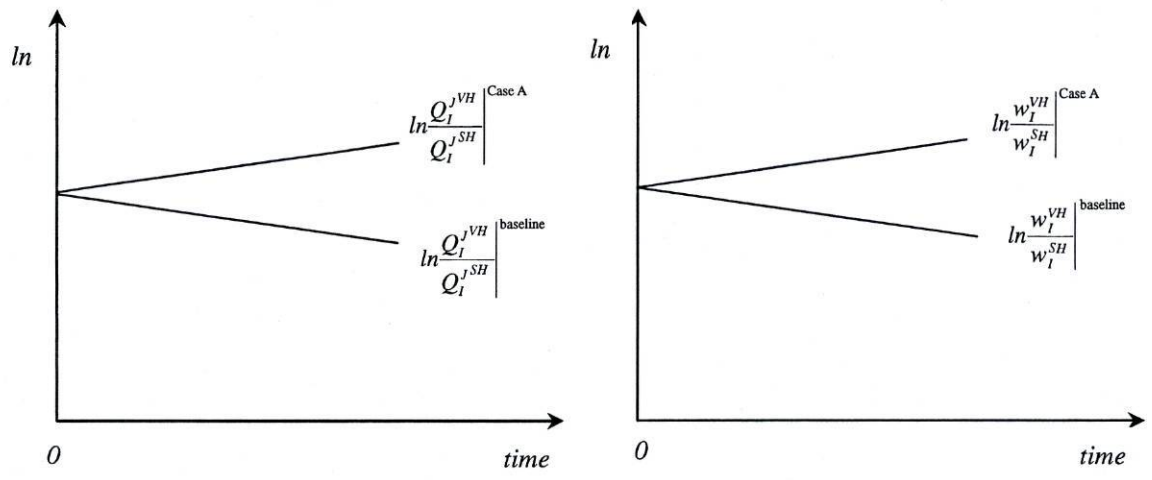
Notes: ↑, ↓, and → means that an increase in the parameter results in a monotonous increase, decrease and level off in the variable, respectively;

[a)] Column 3 gives the number of years to reach the steady state;

[b)] In true $\phi \in]-\infty, 0[\cup]0, 1]$.

Figure A3.1. Technological knowledge bias and intra-country wage inequality

a) Versatile-specialised TK bias **b) Versatile premium**



The baseline case in clear from table A3.1 by computing the versatile-specialised TK bias and by looking at term W_I. In this case, the price channel operates in favour of the relative scarce HC, *SH*.
Case A holds when A_I and β_I are higher, α is smaller and when it is relatively easier to accumulate *SH* – *i.e.*, when ϕ, φ^{VH} and χ_S decrease and when χ_T increases. Thus, in case A, the strong reversal of the price channel effect redirects the path of the relative wage premium in favour of *VH*.

Table A3.4. Consumption under post and pre-policy conditions

Situation	Parameter and its value		*Gr*	Level of consumption					
				Initial				At time $t=27$	
				$C_I(VH)$	Δ	$C_I(SH)$	Δ	$C_I(VH)$	$C_I(SH)$
Post Policy	$s_x=0.60$, $s_r=0.30$	$\tau_K=0.35$	0.06[a]	0.37	---	0.33	---	1.58	1.43
		$\tau_K=0.00$	0.08[b]					6.58	5.95
Pre Policy	$s_x=s_r=\tau_K=0$		0.02	0.70	0.33	0.63	0.30	1.58	1.43
				2.92	2.55	2.89	2.56	6.58	5.95

Notes: Δ measures the level of variation in the initial consumption necessary to get the individuals indifferent between pre and post policy conditions;
[a] and [b] the corresponding interest rates are 0.14 and 0.11, respectively. The link between growth and interest rates is not inconsistent due to the role of the ad-valorem assets income tax. Lower τ_K decreases the interest rate, and thereby accelerates the TK progress, since a lower interest rate decreases the discount rate.

A3.1.4 Sensitivity analysis to the baseline results of the South

Table A3.5. Some quantitative results of sensitivity analysis

	Value	t^{ss} b)	$\tilde{Q}^{J^{VH}}$	$\tilde{Q}^{J^{SH}}$	$\tilde{VH}$	$\tilde{SH}$		Value	t^{ss} b)	$\tilde{Q}^{J^{VH}}$	$\tilde{Q}^{J^{SH}}$	$\tilde{VH}$	$\tilde{SH}$
	0.99	56	0.54	0.63	0.41	0.47		.019	57	0.54	0.63	0.41	0.48
A_P	1.01	56	0.56	0.65	0.38	0.45	δ	.021	56	0.56	0.65	0.38	0.44
	1.10	57	0.65	0.76	0.34	0.39		.025	55	0.59	0.68	0.31	0.36
	0.59	56	0.56	0.65	0.40	0.47		0.59	56	0.55	0.64	0.39	0.46
α	0.61	56	0.55	0.64	0.38	0.45	φ^{VH}	0.61	56	0.55	0.64	0.39	0.46
	0.70	56	0.37	0.43	0.30	0.35		0.65	56	0.54	0.63	0.40	0.46
	0.99	56	0.55	0.64	0.39	0.46		.089	56	0.55	0.64	0.39	0.45
β_P	1.01	56	0.56	0.65	0.39	0.45	χ_S	.091	56	0.55	0.64	0.39	0.46
	1.10	57	0.61	0.71	0.39	0.45		.100	56	0.53	0.62	0.42	0.49
	0.24	56	0.54	0.63	0.39	0.46		.069	56	0.55	0.64	0.38	0.45
PE	0.26	56	0.56	0.65	0.39	0.45	χ_T	.071	56	0.55	0.64	0.40	0.46
	0.30	57	0.56	0.65	0.39	0.45		.080	56	0.54	0.62	0.46	0.54
	0.09	56	0.55	0.64	0.39	0.45		−10.0	55	0.57	0.66	0.40	0.46
d	0.11	55	0.55	0.64	0.39	0.45	ϕ	0.20	57	0.55	0.64	0.39	0.45
	0.15	53	0.55	0.64	0.39	0.46		0.90	58	0.49	0.57	0.39	0.45
	0.20	56	0.54	0.62	0.39	0.45		0.29	56	0.56	0.67	0.40	0.47
$\overline{\sigma}_1$	0.30	56	0.57	0.66	0.39	0.45	$\tilde{Q}^{J^{VH}}$ a)	0.31	56	0.54	0.62	0.38	0.44
	0.35	56	0.58	0.68	0.39	0.45		0.35	55	0.53	0.53	0.35	0.40
	0.25	59	0.45	0.53	0.39	0.45		0.34	56	0.55	0.62	0.38	0.45
$\overline{\sigma}_2$	0.35	52	0.65	0.76	0.39	0.45	$\tilde{Q}^{J^{SH}}$ a)	0.36	56	0.56	0.67	0.40	0.46
	0.40	35	0.76	0.88	0.40	0.47		0.40	57	0.57	0.76	0.46	0.53
	1.04	57	0.55	0.64	0.40	0.46		0.29	56	0.56	0.65	0.38	0.46
θ	1.06	56	0.55	0.64	0.38	0.45	$\tilde{VH}$ a)	0.31	56	0.55	0.64	0.40	0.45
	2.00	54	0.54	0.64	0.14	0.16		0.35	55	0.53	0.62	0.41	0.41
	.029	56	0.55	0.64	0.41	0.48		0.34	56	0.55	0.64	0.39	0.44
ρ	.031	56	0.55	0.64	0.38	0.45	$\tilde{SH}$ a)	0.36	56	0.55	0.65	0.40	0.47
	.035	55	0.54	0.64	0.30	0.35		0.40	57	0.57	0.67	0.42	0.57

Notes: a) Relates the initial values of $\tilde{Q}^{J^{VH}}$, $\tilde{Q}^{J^{SH}}$, $\tilde{VH}(.|w)$ and $\tilde{SH}(.|w)$ tested; that is, $\tilde{Q}^{J^{VH}}(0)$, $\tilde{Q}^{J^{SH}}(0)$, $\tilde{VH}(0|w)$ and $\tilde{SH}(0|w)$;

b) t^{ss} reports the effect on the number of years it takes for the economy to achieve its steady state. Remember for parameters common to both countries (α, θ, ρ, δ, φ^{VH}, χ_S, χ_T and ϕ) here we consider that they change only in the South.

Table A3.6. Quantitative results of sensitivity analysis under all catching up terms

	Value	t^{ss} b)	Benchmark				Alternative 1				Alternative 2			
			$\tilde{Q}^{J^{VH}}$	$\tilde{Q}^{J^{SH}}$	$\tilde{VH}$	$\tilde{SH}$	$\tilde{Q}^{J^{VH}}$	$\tilde{Q}^{J^{SH}}$	$\tilde{VH}$	$\tilde{SH}$	$\tilde{Q}^{J^{VH}}$	$\tilde{Q}^{J^{SH}}$	$\tilde{VH}$	$\tilde{SH}$
A_P	0.99	–	0.54	0.63	0.41	0.47	0.48	0.51	0.25	0.29	0.48	0.56	0.12	0.14
	1.01	+	0.56	0.65	0.38	0.45	0.66	0.80	0.22	0.25	0.66	0.77	0.11	0.13
α	0.59	=	0.56	0.65	0.40	0.47	0.60	0.82	0.24	0.28	0.60	0.70	0.12	0.14
	0.61	=	0.55	0.64	0.38	0.45	0.52	0.51	0.23	0.27	0.52	0.61	0.11	0.13
β_P	0.99	–	0.55	0.64	0.39	0.46	0.51	0.54	0.24	0.28	0.51	0.59	0.12	0.14
	1.01	+	0.56	0.65	0.39	0.45	0.62	0.78	0.22	0.26	0.62	0.72	0.11	0.13
PE	0.24	–	0.54	0.63	0.39	0.46	0.48	0.49	0.25	0.29	0.48	0.56	0.12	0.14
	0.26	+	0.56	0.65	0.39	0.45	0.62	0.78	0.22	0.25	0.62	0.72	0.11	0.12
d	0.09	+	0.55	0.64	0.39	0.45	0.57	0.76	0.22	0.26	0.57	0.67	0.11	0.12
	0.11	–	0.55	0.64	0.39	0.45	0.53	0.48	0.24	0.28	0.53	0.61	0.12	0.14
θ	1.04	+	0.55	0.64	0.40	0.46	0.56	0.74	0.24	0.28	0.56	0.65	0.12	0.14
	1.06	–	0.55	0.64	0.38	0.45	0.55	0.56	0.22	0.26	0.55	0.64	0.11	0.13
ρ	.029	+	0.55	0.64	0.41	0.48	0.57	0.87	0.28	0.32	0.57	0.66	0.16	0.19
	.031	–	0.55	0.64	0.38	0.45	0.54	0.53	0.22	0.26	0.54	0.63	0.09	0.11
δ	.019	+	0.54	0.63	0.41	0.48	0.49	0.76	0.27	0.31	0.49	0.57	0.16	0.19
	.021	–	0.56	0.65	0.38	0.44	0.61	0.60	0.21	0.24	0.61	0.72	0.08	0.09
φ^{VH}	0.59	=	0.55	0.64	0.39	0.46	0.57	0.68	0.22	0.26	0.57	0.67	0.12	0.13
	0.61	=	0.55	0.64	0.39	0.45	0.53	0.60	0.24	0.28	0.53	0.62	0.12	0.14
χ_S	.089	=	0.55	0.64	0.39	0.45	0.57	0.65	0.22	0.25	0.57	0.66	0.11	0.12
	.091	=	0.55	0.64	0.39	0.46	0.53	0.63	0.25	0.29	0.53	0.62	0.12	0.14
χ_T	.069	=	0.55	0.64	0.38	0.45	0.56	0.60	0.21	0.24	0.56	0.66	0.10	0.12
	.071	=	0.55	0.64	0.40	0.46	0.54	0.69	0.24	0.28	0.54	0.63	0.13	0.15
ϕ	–10.0	–	0.57	0.66	0.40	0.46	0.69	0.87	0.24	0.28	0.69	0.80	0.13	0.14
	0.20	+	0.55	0.64	0.39	0.45	0.52	0.56	0.22	0.26	0.52	0.61	0.11	0.13
$\tilde{Q}^{J^{VH}}$ a)	0.29	+	0.56	0.67	0.40	0.47	0.62	0.75	0.23	0.27	0.61	0.73	0.11	0.13
	0.31	–	0.54	0.62	0.38	0.44	0.49	0.55	0.24	0.28	0.50	0.56	0.12	0.14
$\tilde{Q}^{J^{SH}}$ a)	0.34	–	0.55	0.62	0.38	0.45	0.42	0.47	0.24	0.28	0.50	0.56	0.12	0.14
	0.36	+	0.56	0.67	0.40	0.46	0.67	0.81	0.22	0.26	0.60	0.72	0.11	0.13
$\tilde{VH}$ a)	0.29	+	0.56	0.65	0.38	0.46	0.61	0.71	0.21	0.26	0.61	0.71	0.11	0.13
	0.31	–	0.55	0.64	0.40	0.45	0.49	0.57	0.24	0.28	0.49	0.57	0.12	0.14
$\tilde{SH}$ a)	0.34	–	0.55	0.64	0.39	0.44	0.43	0.50	0.24	0.28	0.51	0.60	0.12	0.14
	0.36	+	0.55	0.65	0.40	0.47	0.67	0.78	0.22	0.27	0.60	0.70	0.11	0.13

Notes: a) Relates the initial values of $\tilde{Q}^{J^{VH}}$, $\tilde{Q}^{J^{SH}}$, $\tilde{VH}(.|w)$ and $\tilde{SH}(.|w)$ tested; that is, $\tilde{Q}^{J^{VH}}(0)$, $\tilde{Q}^{J^{SH}}(0)$, $\tilde{VH}(0|w)$ and $\tilde{SH}(0|w)$;

b) t^{ss} reports the effect on the number of years it takes for the economy to get its steady state and the symbols +, – and = means that the time to reach the steady state increases, decreases or stays equal, respectively. The column reflects the tendency in the three alternatives.

Table A3.7. Share of time devoted to working under different values for ϕ

	Value	Benchmark				Alternative 1				Alternative 2			
		$u_{wP}(.\mid VH)$		$u_{wP}(.\mid SH)$		$u_{wP}(.\mid VH)$		$u_{wP}(.\mid SH)$		$u_{wP}(.\mid VH)$		$u_{wP}(.\mid SH)$	
		$t=0$	t^{ss}	$t=0$	t^{ss}	$t=0$	t^{ss}	$t=0$	T^{ss}	$t=0$	t^{ss}	$t=0$	t^{ss}
ϕ	−10.0	0.43	0.69	0.46	0.66	0.49	0.69	0.53	0.66	0.48	0.69	0.52	0.66
	0.20	0.44	0.64	0.47	0.72	0.51	0.64	0.54	0.72	0.50	0.64	0.52	0.80

A3.2. Transitional dynamics and sensitivity analysis under IT of IGs

A3.2.1. Transitional dynamics

Table A3.8. Comparing initial and steady state values of the variables

Variable	Value at time:	
	Initial	Steady state
D_I^1	1.24	1.29
$\tilde{Q}^{J^{VH}}$	0.30	0.41
$\tilde{VH}(.\mid w)$	0.30	0.45
N_I	0.45	0.44
p_I^{VH} / p_I^{SH} b)	0.89	0.88
w_P^{VH} / w_I^{VH}	0.16	0.16
$\frac{TFP_{nP}(.\mid SH)}{TFP_{nI}(.\mid SH)}$ d)	0.16	0.21
$\frac{PTFP_{nP}(.\mid SH)}{PTFP_{nI}(.\mid SH)}$	0.33	0.43
$\frac{C_P(.\mid VH)}{C_I(.\mid VH)}$	0.16	0.16
$\frac{C_I(.\mid SH)}{C_I(.\mid VH)}$	0.81	0.81
Interest rate, r	---	.062

Variable	Value at time:	
	Initial	Steady state
D_P^1	1.06	1.11
$\tilde{Q}^{J^{SH}}$	0.35	0.48
$\tilde{SH}(.\mid w)$	0.35	0.53
N_P a)	0.47	0.46
p_P^{VH} / p_P^{VH} c)	0.93	0.92
w_P^{SH} / w_I^{SH}	0.14	0.14
$\frac{Y_P^{ss}}{Y_I^{ss}} = \frac{y_P^{ss}}{y_I^{ss}}$ e)	0.05	0.07
$\frac{PTFP_{nP}(.\mid VH)}{PTFP_{nI}(.\mid VH)}$	0.32	0.41
$\frac{C_P(.\mid SH)}{C_I(.\mid SH)}$	0.19	0.19
$\frac{C_P(.\mid SH)}{C_P(.\mid VH)}$	0.94	0.94
Growth rate, gr	---	.031

Notes: a) $N_{P,\, t_0\text{-}dt} = 0.54$;

b) we find that: $p_I^{VH}(t_0) = 0.786$, $p_I^{SH}(t_0) = 0.885$, $p_I^{VH}(t^{ss}) = 0.782$ and $p_I^{SH}(t^{ss}) = 0.891$;

c) we find that: $p_P^{VH}(t_0) = 0.402$, $p_P^{SH}(t_0) = 0.432$, $p_P^{VH}(t^{ss}) = 0.399$ and $p_P^{SH}(t^{ss}) = 0.435$;

d) Ignoring the difference in *MC*s between countries the values are 0.32 and 0.41 for t_0 and $t_{110} = t^{ss}$, respectively;

e) Omitting the difference in *MC*s between countries the values are 0.15 and 0.23 for t_0 and $t_{110} = t^{ss}$, respectively.

A3.2.2 Sensitivity analysis to the baseline results

Table A3.9. Qualitative and quantitative results of sensitivity analysis

Change Of	Value	t^{ss} a)	D_I^I b)	$\tilde{Q}^{J^{VH}}$ c)	$\tilde{VH}$ d)	DIP	DZ	W_I e)	W_P e)	PDs f)	PDs g)	gr
A_I	1.10	112	1.33	0.39	0.47	2.89	1.07	0.85	0.92	0.62	0.13	.011
	2.00	108	1.27	0.42	0.44	2.72	1.03	0.85	0.92	0.33	0.05	.054
A_P	0.60	112	1.30	0.40	0.44	2.70	1.05	0.83	0.90	0.25	0.04	.029
	1.40	108	1.28	0.41	0.47	2.89	1.04	0.87	0.95	0.61	0.13	.033
α	0.40	105	1.25	0.34	0.41	2.58	1.01	0.93	1.00	0.46	0.03	.083
	0.80	115	1.42	0.46	0.45	2.89	1.15	0.81	0.88	0.37	0.12	.004
$vh^{h)}$	1.15	105	1.24	0.44	0.46	2.87	1.00	0.84	0.91	0.43	0.08	.030
	1.30	120	1.39	0.35	0.43	2.67	1.11	0.87	0.94	0.41	0.07	.033
$sh^{h)}$	0.90	120	1.44	0.33	0.42	2.62	1.16	0.87	0.94	0.39	0.07	.029
	1.05	105	1.24	0.44	0.46	2.88	1.00	0.84	0.91	0.44	0.08	.033
MC_P	0.45	110	1.29	0.39	0.44	2.76	1.04	0.85	0.92	0.42	0.06	.035
	0.55	110	1.30	0.43	0.45	2.79	1.05	0.85	0.92	0.43	0.08	.026
β_P	0.70	110	1.32	0.40	0.45	2.78	1.06	0.84	0.91	0.42	0.07	.016
	1.30	110	1.28	0.41	0.45	2.78	1.03	0.86	0.93	0.43	0.07	.045
PE	0.15	110	1.30	0.40	0.45	2.78	1.05	0.85	0.92	0.43	0.07	.026
	0.35	110	1.29	0.41	0.45	2.78	1.04	0.85	0.92	0.43	0.07	.036
IT	0.30	110	1.32	0.40	0.45	2.78	1.07	0.84	0.91	0.43	0.07	.015
	1.00	110	1.28	0.42	0.45	2.78	1.03	0.86	0.93	0.43	0.07	.048
d	0.05	110	1.23	0.44	0.30	1.83	1.00	0.84	0.91	0.34	0.05	.035
	0.15	110	1.38	0.36	0.60	3.71	1.11	0.85	0.92	0.50	0.10	.028
σ_1	0.10	110	1.30	0.40	0.44	2.73	1.05	0.85	0.92	0.42	0.07	.027
	0.40	110	1.28	0.41	0.46	2.83	1.04	0.85	0.92	0.43	0.07	.034
σ_2	0.50	110	1.26	0.43	0.33	2.08	1.02	0.84	0.91	0.34	0.05	.027
	0.70	110	1.32	0.39	0.55	3.38	1.07	0.86	0.93	0.48	0.09	.036
θ	0.80	110	1.29	0.41	0.45	2.78	1.04	0.85	0.92	0.43	0.07	.039
	2.00	110	1.29	0.41	0.45	2.76	1.04	0.85	0.92	0.43	0.07	.017
ρ	0.02	110	1.29	0.41	0.45	2.78	1.04	0.83	0.90	0.43	0.07	.039
	0.04	110	1.29	0.41	0.45	2.78	1.04	0.88	0.95	0.43	0.07	.022
δ	0.00	110	1.29	0.41	0.45	2.78	1.04	0.85	0.92	0.43	0.07	.041
	0.04	110	1.29	0.41	0.45	2.77	1.04	0.85	0.92	0.43	0.07	.020
φ^{VH}	0.55	110	1.28	0.38	0.44	2.82	1.03	0.90	0.98	0.43	0.07	.030
	0.65	110	1.31	0.44	0.45	2.70	1.06	0.81	0.87	0.42	0.07	.032

Table A3.9. Qualitative and quantitative results of sensitivity analysis (cont.)

Change Of	Value	t^{ss} a)	D_I^1 b)	$\tilde{Q}^{J^{VH}}$ c)	$\tilde{VH}$ d)	DIP	DZ	W_I e)	W_P e)	PDs f)	PDs g)	gr
χ_S	0.08	110	1.27	0.38	0.44	2.69	1.03	0.91	0.99	0.42	0.07	.029
	0.10	110	1.30	0.44	0.45	2.80	1.05	0.80	0.86	0.43	0.07	.033
χ_T	0.06	110	1.30	0.44	0.44	2.68	1.06	0.80	0.86	0.42	0.07	.029
	0.08	110	1.29	0.38	0.45	2.82	1.03	0.90	0.98	0.43	0.07	.033
ϕ	-1,00	110	1.26	0.35	0.43	2.61	1.05	0.96	1.04	0.41	0.06	.030
	0.20	110	1.30	0.43	0.46	2.86	1.04	0.81	0.88	0.43	0.07	.031
$\wp_1$	4.00	110	1.29	0.43	0.43	2.70	1.04	0.85	0.92	0.42	0.07	.032
	6.00	110	1.29	0.39	0.46	2.85	1.04	0.85	0.92	0.43	0.08	.030
$\wp_2$	0.02	125	1.29	0.47	0.43	2.68	1.04	0.83	0.90	0.40	0.07	.034
	0.04	95	1.29	0.38	0.46	2.80	1.04	0.87	0.95	0.43	0.08	.030
D_I^{1} i, j)	1.20	110	1.29	0.38	0.44	2.75	1.08	0.85	0.91	0.42	0.07	.030
	1.25	110	1.29	0.42	0.45	2.79	1.03	0.85	0.93	0.43	0.07	.031
$\tilde{Q}^{J^{VH}}$ i)	.275	110	1.28	0.35	0.58	3.59	1.04	0.85	0.93	0.49	0.09	.027
	.325	110	1.31	0.53	0.32	1.96	1.06	0.84	0.91	0.36	0.05	.038
$\tilde{Q}^{J^{SH}}$ i)	.325	110	1.31	0.49	0.39	2.58	1.06	0.84	0.91	0.38	0.06	.035
	.375	110	1.26	0.38	0.47	2.94	1.02	0.85	0.93	0.46	0.08	.029
$\tilde{VH}$ i, j)	.275	110	1.33	0.38	0.36	2.22	1.08	0.84	0.95	0.38	0.06	.032
	.325	110	1.25	0.43	0.59	3.51	1.01	0.86	0.89	0.51	0.09	.027
$\tilde{SH}$ h, i)	.325	110	1.25	0.43	0.57	3.49	1.01	0.86	0.89	0.48	0.09	.028
	.375	110	1.33	0.38	0.37	2.28	1.08	0.84	0.95	0.40	0.06	.032
Policy	$s_x = .50$ $s_r = .26$ $t_K = .35$	110	1.25	0.30	0.31	1.91	1.01	0.92	1.04	0.34	0.05	.140

Notes: a) t^{ss} represents the number of years for the world to achieve its steady state; b) we do not present the dynamics for D_P^1 because they are similar; c) we do not present the dynamics for $\tilde{Q}^{J^{SH}}$ because they are similar; d) It represents $\tilde{VH}(.|w)$. We do not present the dynamics for $\tilde{SH}(.|w)$ since they are similar; e) In the baseline case their steady state values are: $W_I = 0.85$ and $W_P = 0.92$; f) The TFP differences, PDs, are measured by $\frac{PTFP_{nP}(.|SH)}{PTFP_{nI}(.|SH)}$, which – remember – is the measure of smaller PDs – see table A3.8 for the baseline case; g) The aggregate TFP differences, PDs, are measured by Y_P/Y_I; h) The minor changes in vh and sh are due to their strong influence on (intra-country) wage inequality; i) Relates the initial values of D_I^1, $\tilde{Q}^{J^{VH}}$, $\tilde{Q}^{J^{SH}}$, $\tilde{VH}(.|w)$ and $\tilde{SH}(.|w)$ tested; that is, $D_I^1(t_0)$, $\tilde{Q}^{J^{VH}}(t_0)$, $\tilde{Q}^{J^{SH}}(t_0)$, $\tilde{VH}(t_0|w)$ and $\tilde{SH}(t_0|w)$; j) Changes in these variables affect directly the initial value of D_P^1.

A3.3. Transitional dynamics and sensitivity analysis under IT of all goods

A3.3.1. Transitional dynamics

Table A3.10. Comparing initial and steady state values of the variables

Variable	Value at time:	
	Initial	Steady state
D_I^I	1.24	1.48
$\tilde{Q}^{J^{VH}}$	0.30	0.35
$\tilde{VH}(.\mid w)$	0.30	0.46
N^{SH}	.085	.096
N^{VH}	.904	.882
$\frac{p_I^{VH}}{p_I^{SH}}$ b)	0.89	0.84
$\frac{w_P^{VH}}{w_I^{VH}}$	0.71	0.57
$\frac{C_P(.\mid VH)}{C_I(.\mid VH)}$	0.16	0.13
$\frac{C_I(.\mid SH)}{C_I(.\mid VH)}$	0.81	0.81
Interest rate, r	---	.059

Variable	Value at time:	
	Initial	Steady state
D_P^I	1.06	1.27
$\tilde{Q}^{J^{SH}}$	0.35	0.41
$\tilde{SH}(.\mid w)$	0.35	0.54
N	.454	.433
$\frac{Y_P}{Y_I} = \frac{y_P}{y_I}$ a)	0.22	0.27
$\frac{p_P^{VH}}{p_P^{VH}}$ c)	0.93	0.88
$\frac{w_P^{SH}}{w_I^{SH}}$	0.65	0.54
$\frac{C_P(.\mid SH)}{C_I(.\mid SH)}$	0.19	0.15
$\frac{C_P(.\mid SH)}{C_P(.\mid VH)}$	0.94	0.94
Growth rate, gr	---	.028

Notes: a) Omitting the difference in *MC*s between countries the values are 0.70 and 0.86 for t_0 and $t_{145} = t^{ss}$, respectively;

b) we have that: $p_I^{VH}(t_0) = 0.698$, $p_I^{SH}(t_0) = 0.786$, $p_I^{VH}(t^{ss}) = 0.665$ and $p_I^{SH}(t^{ss}) = 0.790$;

c) we find that: $p_P^{VH}(t_0) = 0.885$, $p_P^{SH}(t_0) = 0.951$, $p_P^{VH}(t^{ss}) = 0.741$ and $p_P^{SH}(t^{ss}) = 0.841$.

A3.3.2 Sensitivity analysis to the baseline results

Table A3.11. Qualitative and quantitative results of sensitivity analysis

Change Of	Value	t^{ss} a)	D_I^1 b)	$\tilde{Q}^{JVH}$ c)	$\tilde{VH}$ d)	N^{SH}	N	N^{VH}	DIP	DZ	W_I e)	W_P e)	PDs f)	gr
A_I	1.10	155	1.64	0.32	0.39	0.11	0.42	0.86	2.76	1.32	0.79	0.85	0.34	.008
	2.00	135	1.40	0.37	0.48	0.08	0.44	0.90	3.71	1.13	0.83	0.90	0.23	.049
A_P	0.60	150	1.39	0.37	0.47	0.07	0.44	0.92	3.56	1.13	0.78	0.85	0.18	.026
	1.40	140	1.58	0.33	0.42	0.11	0.42	0.86	2.93	1.28	0.84	0.91	0.34	.028
α	0.40	130	1.23	0.35	0.43	0.07	0.45	0.93	3.16	1.00	0.85	0.93	0.16	.080
	0.80	160	1.60	0.43	0.55	0.12	0.42	0.85	4.50	1.29	0.76	0.81	0.38	.002
vh g)	1.15	135	1.40	0.39	0.52	0.10	0.45	0.88	4.20	1.13	0.81	0.87	0.29	.024
	1.30	150	1.54	0.32	0.36	0.08	0.41	0.88	2.42	1.25	0.83	0.90	0.27	.032
sh g)	0.90	150	1.53	0.32	0.39	0.09	0.42	0.88	2.72	1.24	0.85	0.99	0.25	.027
	1.05	135	1.38	0.39	0.53	0.10	0.45	0.88	4.27	1.11	0.81	0.87	0.29	.027
MC_P	0.45	145	1.46	0.34	0.44	0.09	0.43	0.89	3.24	1.18	0.82	0.88	0.24	.031
	0.55	145	1.51	0.36	0.48	0.10	0.43	0.87	3.73	1.23	0.81	0.88	0.30	.022
β_P	0.70	145	1.51	0.34	0.45	0.09	0.43	0.88	3.35	1.22	0.80	0.86	0.27	.014
	1.30	145	1.47	0.35	0.47	0.10	0.43	0.88	3.56	1.18	0.83	0.90	0.27	.040
PE	0.15	145	1.49	0.35	0.46	0.09	0.43	0.88	3.45	1.20	0.81	0.88	0.27	.022
	0.35	145	1.47	0.35	0.46	0.10	0.43	0.88	3.51	1.19	0.82	0.89	0.27	.031
IT	0.30	145	1.51	0.34	0.45	0.09	0.43	0.88	3.33	1.22	0.79	0.86	0.27	.012
	1.00	145	1.46	0.35	0.47	0.10	0.43	0.88	3.56	1.18	0.84	0.90	0.28	.042
d	0.05	145	1.33	0.39	0.37	0.09	0.45	0.90	2.40	1.09	0.81	0.88	0.24	.027
	0.15	145	1.56	0.33	0.48	0.10	0.42	0.87	3.62	1.26	0.82	0.88	0.28	.027
σ_1	0.10	145	1.48	0.35	0.44	0.09	0.43	0.88	3.31	1.20	0.81	0.88	0.27	.024
	0.40	145	1.48	0.35	0.48	0.10	0.43	0.88	3.68	1.20	0.82	0.88	0.27	.030
σ_2	0.50	145	1.39	0.37	0.40	0.09	0.44	0.89	2.74	1.13	0.81	0.87	0.25	.022
	0.70	145	1.54	0.33	0.48	0.10	0.43	0.88	3.53	1.25	0.82	0.89	0.28	.035
θ	0.80	145	1.48	0.35	0.46	0.10	0.43	0.88	3.71	1.20	0.81	0.88	0.27	.035
	2.00	145	1.48	0.35	0.46	0.10	0.43	0.88	3.16	1.20	0.81	0.88	0.27	.018
ρ	0.02	145	1.48	0.35	0.46	0.10	0.43	0.88	3.48	1.20	0.78	0.85	0.27	.036
	0.04	145	1.48	0.35	0.46	0.10	0.43	0.88	3.48	1.20	0.86	0.93	0.27	.017
δ	0.00	145	1.48	0.35	0.46	0.10	0.43	0.88	3.48	1.20	0.81	0.88	0.27	.037
	0.04	145	1.48	0.35	0.46	0.10	0.43	0.88	3.48	1.20	0.81	0.88	0.27	.016
φ^{VH}	0.55	145	1.38	0.31	0.35	0.09	0.44	0.90	2.26	1.12	0.94	1.02	0.24	.027
	0.65	145	1.57	0.39	0.53	0.10	0.43	0.87	4.27	1.27	0.69	0.75	0.29	.027

Table A3.11. Qualitative and quantitative results of sensitivity analysis (cont.)

Change Of	Value	t^{ss} a)	D_I^1 b)	$\tilde{Q}^{JVH}$ c)	$\tilde{VH}$ d)	N^{SH}	N	N^{VH}	DIP	DZ	W_I e)	W_P e)	PDs f)	gr
χ_S	0.08	145	1.38	0.31	0.35	0.09	0.44	0.90	2.29	1.11	0.94	1.02	0.24	.027
	0.10	145	1.56	0.39	0.53	0.10	0.43	0.87	4.24	1.27	0.70	0.75	0.29	.027
χ_T	0.06	145	1.56	0.39	0.53	0.10	0.43	0.87	4.23	1.27	0.70	0.76	0.29	.024
	0.08	145	1.38	0.31	0.35	0.10	0.44	0.90	2.29	1.11	0.94	1.02	0.24	.031
ϕ	-1,00	145	1.37	0.31	0.35	0.09	0.44	0.90	2.21	1.11	0.95	1.03	0.24	.029
	0.20	145	1.51	0.36	0.50	0.10	0.43	0.88	3.85	1.27	0.77	0.83	0.29	.026
$\wp_1$	4.00	145	1.38	0.37	0.48	0.10	0.44	0.88	3.50	1.21	0.81	0.88	0.27	.027
	6.00	145	1.38	0.32	0.45	0.09	0.44	0.88	3.45	1.18	0.81	0.88	0.27	.027
$\wp_2$	0.02	155	1.41	0.38	0.50	0.10	0.44	0.89	3.52	1.22	0.81	0.88	0.27	.027
	0.04	125	1.36	0.32	0.44	0.08	0.44	0.90	3.43	1.18	0.81	0.88	0.27	.029
D_I^1 h, i)	1.20	145	1.46	0.34	0.44	0.09	0.43	0.89	3.12	1.22	0.81	0.88	0.26	.027
	1.25	145	1.49	0.35	0.47	0.10	0.43	0.88	3.60	1.19	0.81	0.88	0.27	.026
$\tilde{Q}^{JVH}$ h)	.275	145	1.50	0.31	0.53	0.10	0.43	0.87	4.32	1.21	0.82	0.88	0.28	.024
	.325	145	1.47	0.42	0.39	0.09	0.43	0.89	2.62	1.19	0.81	0.88	0.25	.030
$\tilde{Q}^{JSH}$ h)	.325	145	1.50	0.38	0.42	0.09	0.43	0.90	3.04	1.21	0.81	0.88	0.25	.028
	.375	145	1.45	0.34	0.50	0.10	0.44	0.87	3.82	1.18	0.82	0.88	0.28	.026
$\tilde{VH}$ h, i)	.275	145	1.46	0.34	0.36	0.09	0.44	0.89	2.79	1.18	0.81	0.91	0.25	.029
	.325	145	1.57	0.35	0.68	0.11	0.42	0.86	5.63	1.26	0.82	0.85	0.33	.025
$\tilde{SH}$ h, i)	.325	145	1.54	0.35	0.65	0.10	0.43	0.86	5.74	1.25	0.82	0.85	0.32	.025
	.375	145	1.48	0.34	0.39	0.09	0.43	0.90	2.74	1.18	0.81	0.91	0.26	.029
Policy	$s_x = .50$ $s_r = .26$ $\iota_K = .35$	145	1.34	0.30	0.31	0.09	0.44	0.90	1.93	1.08	0.98	1.03	0.22	.133

Notes: a), b), c), d) see the respective mean in table 4.7. e) In the baseline case their steady state values are: $W_I = 0.81$ and $W_P = 0.88$; f) The productivity differences, PDs, are measured by $\frac{Y_P}{Y_I} = \frac{y_P}{y_I}$ – see table A3.10 for the baseline case; g) The minor changes in *vh* and *sh* are caused by their strong influence on wage inequality; h) Relates the $D_I^1(t_0)$, $\tilde{Q}^{JVH}(t_0)$, $\tilde{Q}^{JSH}(t_0)$, $\tilde{VH}(t_0 \mid w)$ and $\tilde{SH}(t_0 \mid w)$ tested; i) Changes in these variables affect the initial value of D_P^1.

APPENDIX 4

NEW GENERAL PURPOSE TECHNOLOGY AND HUMAN CAPITAL PRODUCTIVITY SHOCK UNDER INTERNATIONAL TRADE OF ALL GOODS

The use of a new GPT in all FGs at the same time (subsection A4.1.1) generates two phases and two associated dynamic effects: first in HC accumulation and after that in the R&D innovative activity. These effects have a positive impact on the aggregate *TFP* of both countries, which are also improved in phase 2 due to the static benefits induced by the use of a new GPT. Hence, phase 2 is a 'time to reap' for both countries. The new GPT does not affect the versatile premium or the pattern of specialisation.

The use of a new GPT first in the production of versatile FGs (subsection A4.1.2) gives rise to three distinct phases. In the 1st, a small growth effect occurs. In the 2nd and the 3rd, both level and growth effects occur. Though, in the 2nd, the versatile premium and the production of FGs using VT widen in both countries, and the inverse occur in the 3rd.

A shock in the versatile-augmenting parameter of the production function (subsection A4.2) has the same kind of effects as in the second phase when the introduction of a new GPT occurs first in the production of versatile FGs.

A4.1. Consequences of a new General Purpose Technology

In this section, we analyse briefly the consequences of a new GPT in both countries with free IT of all goods. Following the structure in the last IT regimes, we will again consider two scenarios. We focus the analysis on steady states that occur in each phase.

1st scenario: the new GPT is beneficial to all final goods at the same time

From the time of the use of the new GPT until another new GPT occurs, the production function for the nth FG in country Z, Y_{nZ}^{NG}, becomes again given by equations (5.1a-c), but with two differences. Firstly, where before A_Z appeared, $A_Z^{NG}(>A_Z)$ must now be written. Secondly, k must be equal to or greater than k^M, $k \geq k^M$. To this end, it should be noted that the Northern TK is available in both countries at the same time.

Incorporating this information in our analysis, the production of FG n in equation (5.19) remains valid, but now we write Y_{nZ}^{NG} and A_Z^{NG} instead Y_{nZ} and A_Z, respectively. This is because Q_I^{Jm} must at least be equal to the TK minimum $Q^{J_M^m}$ – remember that, by assumption, in the present scenario $Q^{J_M^{SH}}$ and $Q^{J_M^{VH}}$ occur at the same time. In this case, the new GPT generates two phases. The first one, during $[t^{ss}, t^{ss}+\Delta)$, where Δ again represents the time necessary to reach $Q^{J_M^m}$ – *i.e.*, the old GPT continues to be used in both countries. The other, from $t^{ss}+\Delta$ on, at least until new GPT occurs, during which the previous new GPT is used in both countries. Hence, figure 4.8 continues valid here.

1st scenario, phase 1: the old GPT is used in both countries

The analysis in this phase is the same as that carried out in chapter 4 in the same case. Thus, the new temporary steady state world growth rate is unambiguously greater than the previous steady state world growth rate. Therefore, the new GPT confers an immediate growth effect in both countries with positive impact on their productivity. However, until its introduction on the production side, it is above all a 'time to sow' for both countries. Given that the new GPT affects both countries at the same time in the same way, it does not affect the versatile premium and the pattern of FGs specialisation.

1st scenario, phase 2: the new GPT is used in both countries

At time $t = t^{ss}+\Delta$, the Northern TK reaches $Q^{J_M^m}$ and both countries use a new GPT. As a result, there is a level jump in their outputs: Y_Z jumps from the value given in equations (5.30a, b) to Y_Z^{NG} given by similar equations where A_Z^{NG} substitutes A_Z and Q_I^{Jm} is at least equal to $Q^{J_M^m}$ – the equals sign occurs when the new GPT is introduced.

The greater productivity level embodied in the new GPT increases the expected profits of the innovator firms and, consequently, the rate of innovation. Hence, a new growth effect arises and this is a 'time to reap' for both countries. Since in this phase the new GPT affects the specialised and the versatile side of both countries in the same way, it does not change the versatile premium and the pattern of FGs specialisation.

In summary, the main results of this scenario are once again condensed in table 4.4.

2nd scenario: first the new GPT is beneficial to some FGs at the same time

The question now is the following. What happens when IT is completely free and new GPT is introduced first in FGs that use VT? As is shown in figure 4.9, until there is

full use of the new GPT in the world, there are three different phases: during $[t^{ss}, t^{ss}+\Delta^{VH})$, $[t^{ss}+\Delta^{VH}, t^{ss}+\Delta^{SH})$ and from $t^{ss}+\Delta^{SH}$ on. In the first, only the old GPT is used in the world and so the effects are the same as in phase 1 of the last scenario. In the second, the new GPT is used in VT. In the third, only the new GPT is used in the world.

2nd scenario, phase 2: the new GPT is used only in VT (but this occurs in both countries)

In this phase, the output of the nth FG is given by $Y_{nZ}(.|SH)$ or $Y_{nZ}^{NG}(.|VH)$, according the use of ST or VT, respectively, in its production and regardless of the country considered. Thus, if the FG n is produced with ST, the production functions remain valid. However, if it is produced with VT, A_Z is substituted by A_Z^{NG}, because k has already reached k^M. Without explaining again all the steps underlying our analysis, we can state that the comparative and technology margins in equations (5.23a-c) now become given by the following expressions:

$$N^{VSH} = \frac{R_1 R_2 R_3^{VNG}}{1+R_2(1+R_3^{VNG})-R_1(1+R_2)}; \qquad \text{(A4.1a)}$$

$$N^{VVH} = \frac{R_2(1+R_3^{VNG}-R_1)}{1+R_2(1+R_3^{VNG})-R_1(1+R_2)}; \qquad \text{(A4.1b)}$$

$$N^{VNG} = \frac{R_2 R_3^{VNG}}{1+R_2(1+R_3^{VNG})-R_1(1+R_2)}. \qquad \text{(A4.1c)}$$

Where,

$$R_2 = R_2^{NG} = \left[\left(\frac{A_I^{NG}\, MC_I}{A_P^{NG}\, MC_P}\right)^{1/\alpha} \frac{VH_I(.|w)}{VH_P(.|w)}\right]^{\frac{1}{2}}, \text{since } \frac{A_I^{NG}}{A_P^{NG}} = \frac{A_I}{A_P} \text{ and } R_3^{VNG} = \left[\left(\frac{A_I}{A_I^{NG}}\right)^{1/\alpha} \frac{Q_I^{JSH}}{Q_I^{JVH}} \frac{sh\, SH_I(.|w)}{vh\, VH_I(.|w)}\right]^{\frac{1}{2}}.$$

Note that the subscript in R_3 is *VNG* because it makes the link between both technologies of production, whereas in R_2 the introduction of the new GPT is fully considered.

Comparing equations (A4.1a-c) and (5.23a-c), we obtain:

$$N^{SH} > N^{VSH},\ N^{VH} > N^{VVH} \text{ and so } (1 - N^{VH}) < (1 - N^{VVH}),\ N > N^{VNG},$$
$$(N - N^{SH}) > (N^{VNG} - N^{VSH}) \text{ and } (N^{VH} - N) < (N^{VVH} - N^{VNG}), \qquad \text{(A4.2)}$$

since $A_P^{NG} > A_P$ and $A_I^{NG} > A_I$. Thus, the new GPT unambiguously provokes a greater use of VT in the world, the specialisation of both countries in a smaller number of specialised FGs and in a higher number of versatile FGs. Moreover, a greater use of VT is equal in both countries; that is, none of them becomes relatively more competitive. In fact,

$$\frac{N^{SH}(N^{VNG}-N^{VSH})}{N^{VSH}(N-N^{SH})}=\frac{(1-N^{VH})(N^{VVH}-N^{VNG})}{(1-N^{VVH})(N^{VH}-N)}=1 \qquad \text{(A4.3)}$$

There is also a static gain with the introduction of the new GPT in VT. The Y_Z jumps from the value given in equation (5.30a, b) to Y_Z^{VNG} :

$$Y_P^{VNG}=$$
$$=\Theta_P^{Y^{VNG}}\left\{sh\ SH_P(.|w)\,N^{VSH(-1)}MC_{\Xi\gamma}^{SH}\,Q_I^{J^{SH}}\,A_P^{1/\alpha}+vh\ VH_P(.|w)\left(1-N^{VVH}\right)^{-1}MC_{\Xi\gamma}^{VH}\,Q_I^{J^{VH}}A_P^{NG(1/\alpha)}\right\}; \qquad \text{(A4.4a)}$$

$$Y_I^{VNG}=\Theta_I^{Y^{VNG}}\left\{sh\ SH_I(.|w)\left(N^{VNG}-N^{VSH}\right)^{-1}MC_{\Xi\gamma}^{SH}\,Q_I^{J^{SH}}A_I^{(1/\alpha)}+\right.$$
$$\left.+\ vh\ VH_I(.|w)\left(N^{VVH}-N^{VNG}\right)^{-1}MC_{\Xi\gamma}^{VH}\,Q_I^{J^{VH}}A_I^{NG(1/\alpha)}\right\}. \qquad \text{(A4.4b)}$$

where: $\Theta_P^{Y^{VNG}}\equiv exp(-1)\left[A_P\frac{MC_P}{MC_I}\right]^{1/\alpha}\left[\frac{1-\alpha}{q}\right]^{\frac{1-\alpha}{\alpha}}\left(N^{VVH}-N^{VSH}\right)^2$ and $\Theta_I^{Y^{VNG}}\equiv\Theta_P^{Y^{VNG}}\left[\frac{A_I}{A_P}\frac{MC_I}{MC_P}\right]^{1/\alpha}$

As a result of the jump of the aggregate output in both countries, the price paid for each unit of both types of HC increases. Hence, both types of workers in the world benefit, although workers who accumulate *VH* benefit more. Indeed, the versatile premium jumps at time $t=t^{ss}+\Delta^{VH}$ and so $W_z^{VNG}>W_z$:[304]

$$W_z^{VNG}=\frac{w_Z^{VVH}}{w_Z^{VSH}}=\left(\frac{Q_I^{J^{VH}}}{Q_I^{J^{SH}}}\right)^{\frac{1}{2}}\left(\frac{vh\ VH_Z(.|w)}{sh\ SH_Z(.|w)}\right)^{-\frac{1}{2}}\frac{vh}{sh}\left(\frac{A_Z^{NG}}{A_Z}\right)^{\frac{1}{2\alpha}}. \qquad \text{(A4.5)}$$

Thus, as in the previous chapter, the use of the new GPT in VT remains as another possible explanation for the increase in demand for skills in both countries, developed (Northern) and developing (Southern), which must be mirrored in the increase in wage inequality. Although temporary, this explanation can remain valid for a long time.

Until now we have analysed the consequences of the introduction of the new GPT at time $t=t^{ss}+\Delta^{VH}$. Moreover, we can find that the world steady state growth rate also increases. It is given by the expression in equation (4.72). Note that, when considering $m = SH$, it must be greater than before.[305] In fact, p_I^{VSH} and p_P^{VSH} must increase as a result of the greater use of VT. Therefore, we can again state that the (new) growth effect must shorten the time for the use of the new GPT in ST.

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[304] Even after $t=t^{ss}+\Delta^{VH}$ we must have $W_z^{VNG}>W_z$. That is, the A_Z^{NG} effect must dominate the changes in the endogenous TK bias and in HC at work bias.

[305] For a unique steady state, the growth rate in equations (4.72) must be equal for $m = SH$ and for $m = VH$.

In summary, there is both a positive level and growth effect for the two countries, and the use of VT as well as the versatile premium increases, although the price paid per unit of *SH* also increases. As a consequence, we can prove that there is an improvement of the productivities in both countries.

2nd scenario, phase 3: only the new GPT is used in the world

With the full introduction of the new GPT, the output of FG n in country Z becomes given by $Y_{nZ}^{NG}(.|SH)$, if it uses ST or remains given by $Y_{nZ}^{NG}(.|VH)$, if its production technology is VT. Thus, the production functions are those in equations (5.1a-c) but, since k has already reached k^M, A_Z must be replaced by A_Z^{NG}.

At this time, FGs at comparative and technology margins move from equations (A4.1a-c) to equations (5.23a-c), since $A_I^{NG}/A_P^{NG} = A_I/A_P$. That is, in comparison with the last phase, the jump in margins is exactly on the opposite side. Hence, the technology margin now shifts from the use of VT to the use of ST. Moreover, the increase in the use of ST is equal in both countries and so the countries' competitiveness is not modified. Consequently, both countries produce a higher number of specialised FGs and a small number of versatile FGs. At this time, the versatile premium decreases, despite the increase in the price paid per unit of *VH*.

Moreover, there is also a level and a growth effect. In fact, A_Z^{NG} affects both ST and VT. Thus, considering, for example, $m = VH$, the steady state growth rate is greater than before since p_I^{NVH} and p_P^{NVH} increase as a result of the greater use of ST. Finally, both level and growth effects improve the productivity of the countries once more.

Hence, the main results of this scenario are once again condensed in table 4.5.

A4.2. Consequences of a human capital productivity shock

The versatile-augmenting parameter *vh* together with the specialised-augmenting parameter *sh* and the adjustment terms emphasise the Ricardian model in our analysis. Thus, the analysis of the effects of HC productivity shocks materialised in changes in *sh* and/or *vh* on the pattern of specialisation in both countries are in order.

We assume that *vh* increases at time $t = t^{ss} + \Delta^{vh}$ for a vh^{new}, where $vh^{new} > vh$. At this moment, there is a level jump in the output of FGs that use VT to $Y_{nZ}^{new}(.|VH)$. Once again omitting some steps in the structure of our analysis, the comparative and technology margins in equations (5.23a-c) become given by:

$$N^{SH^{new}} = \frac{R_1 R_2 R_3^{new}}{1 + R_2(1 + R_3^{new}) - R_1(1 + R_2)}; \tag{A4.6a}$$

$$N^{VH^{new}} = \frac{R_2(1 + R_3^{new} - R_1)}{1 + R_2(1 + R_3^{new}) - R_1(1 + R_2)}; \tag{A4.6b}$$

$$N^{new} = \frac{R_2 R_3^{new}}{1 + R_2(1 + R_3^{new}) - R_1(1 + R_2)}. \tag{A4.6c}$$

$$\text{Where, } R_3^{new} = \left[\frac{Q_1^{J^{SH}}}{Q_1^{J^{VH}}} \frac{sh}{vh^{new}} \frac{SH_1(.|w)}{VH_1(.|w)} \right]^{\frac{1}{2}}.$$

Comparing equations (A4.6a-c) and (5.23a-c) we find that:

$$\begin{gathered} N^{SH} > N^{SH^{new}},\ N^{VH} > N^{VH^{new}},^{306}\ N > N^{new}, \\ (N - N^{SH}) > (N^{new} - N^{SH^{new}}),\ (N^{VH} - N) < (N^{VH^{new}} - N^{new}). \end{gathered} \tag{A4.7}$$

And that:

$$\frac{N^{SH}(N^{new} - N^{SH^{new}})}{N^{SH^{new}}(N - N^{SH})} = 1 = \frac{(1 - N^{VH})(N^{VH^{new}} - N^{new})}{(1 - N^{VH^{new}})(N^{VH^{new}} - N)}; \tag{A4.8}$$

That is, an increase in *vh* raises (drops) the use of VT (ST) in both countries in the same proportion when the change occurs. Hence, the competitiveness relative of the countries remains equal.

As a consequence of the jump in $Y_{nZ}(.|VH)$ to $Y_{nZ}^{new}(.|VH)$, there is also a static gain in Y_Z, which jumps from the value given in equation (5.30a, b) to Y_Z^{new} below:

$$\begin{aligned} Y_P^{new} &= \\ &= \Theta_P^{Y^{new}} \left\{ sh\ SH_P(.|w)\ N^{SH^{new}(-1)} MC_{\Xi\gamma}^{SH} Q_1^{J^{SH}} + vh^{new}\ VH_P(.|w) \left(1 - N^{VH^{new}}\right)^{-1} MC_{\Xi\gamma}^{VH} Q_1^{J^{VH}} \right\}; \end{aligned} \tag{A4.9a}$$

$$\begin{aligned} Y_1^{new} = \Theta_1^{Y^{new}} \Big\{ & sh\ SH_1(.|w) \left(N^{new} - N^{VH^{new}}\right)^{-1} \\ & MC_{\Xi\gamma}^{SH} Q_1^{J^{SH}} + vh^{new}\ VH_1(.|w) \left(N^{VH} - N^{new}\right)^{-1} MC_{\Xi\gamma}^{VH} Q_1^{J^{VH}} \Big\}; \end{aligned} \tag{A4.9b}$$

$$\text{where: } \Theta_P^{Y^{new}} \equiv exp(-1) \left[A_P \frac{MC_P}{MC_1} \right]^{1/\alpha} \left[\frac{1-\alpha}{q} \right]^{\frac{1-\alpha}{\alpha}} \left(N^{VH^{new}} - N^{SH^{new}} \right)^2 \text{ and } \Theta_1^{Y^{new}} \equiv \Theta_P^{Y^{new}} \left[\frac{A_1}{A_P} \frac{MC_1}{MC_P} \right]^{1/\alpha}.$$

The jump of the aggregate output in both countries has then a positive influence on the price paid for each unit of both types of HC. Thus, all workers in the world benefit

[306] Consequently, we have that $(1 - N^{VH}) < (1 - N^{VVH})$.

with the shock. However, workers who accumulate *VH* benefit more since the versatile premium jumps at time $t = t^{ss} + \Delta^{VH}$; that is, $W_z^{new} > W_z$:

$$W^{new} = \frac{w^{VH^{new}}}{w^{SH^{new}}} = \left(\frac{Q_I^{JVH}}{Q_I^{JSH}} \right)^{\frac{1}{2}} \left(\frac{vh^{new}}{sh} \frac{VH(.|w)}{SH(.|w)} \right)^{-\frac{1}{2}} \frac{vh^{new}}{sh}. \qquad \text{(A4.10)}$$

Therefore, an increase in *vh* is again another possible explanation for the increase in demand for skills in both countries, developed and developing.

In addition, considering, for example, $m = SH$ in equation (4.72), we conclude that the new world steady state growth rate is higher than before. This is because both $p_I^{SH^{new}}$ and $p_P^{SH^{new}}$ increase as a result of the greater use of VT. Since the growth rate must be equal for $m = SH$ and for $m = VH$, then the positive effect of the shock dominates the negative effect of $p_I^{VH^{new}}$ and $p_P^{VH^{new}}$ on the growth rate.

In the second column of table 5.4, we summarise the main results ensuing from the HC productivity shock, which increases *vh*. We go further and in additional columns include the results from other possible shocks.

Table A4.1. Main results due to HC productivity shocks

	$\uparrow vh$	$\downarrow sh$	$\downarrow vh$	$\uparrow sh$	$\uparrow vh$ and $\uparrow sh$	$\downarrow vh$ and $\downarrow sh$
Growth effect	In both	---	---	In both	In both	---
Level effect	In both	---	---	In both	In both	---
Decrease effect	---	In both	In both	---	---	In both
Negative level effect	---	In both	In both	---	---	In both
Productivity differences	---	---	---	---	---	---
Versatile premium	$\uparrow$ in both		$\downarrow$ in both		---	
Technology margin	$\downarrow$		$\uparrow$		---	
Pattern of specialisation	Versatile biased in both		Specialised biased in both		---	

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