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Generalized means and peaks over random thresholds value-at-risk estimation

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Abstract

In many areas of application, it is a common practice to estimate the *value at risk* at a level q (VaR_q), a value, high enough, so that the chance of an exceedance of that value is equal to q , small, often smaller than $1/n$, with n the size of available sample, $\underline{\mathbf{X}}_n = (X_{1:n} \leq \dots \leq X_{n:n})$ the associated ascending order statistics. Further assume that there exist sequences $\{a_n > 0\}_{n \geq 1}$ and $\{b_n \in R\}_{n \geq 1}$ such that the maximum, linearly normalized, i.e. $(X_{n:n} - b_n)/a_n$, converges in distribution to a non-degenerate random variable. Then (Gnedenko, 1943), the limit *cumulative distribution function* (CDF) is necessarily of the type of the general *extreme value* (EV) CDF, given by

$$G_\xi(x) = \begin{cases} \exp(-(1 + \xi x)^{-1/\xi}), & 1 + \xi x > 0, \text{ if } \xi \neq 0, \\ \exp(-\exp(-x)), & x \in R, \text{ if } \xi = 0. \end{cases}$$

The CDF F is said to belong to the max-domain of attraction of G_ξ , and we write $F \in \mathcal{D}_M(G_\xi)$. The parameter ξ is the *extreme value index* (EVI), the primary parameter of extreme events. We consider *heavy-tailed* models, i.e. *Pareto-type* underlying CDFs, with a positive EVI, working in $\mathcal{D}_M^+ := \mathcal{D}_M(\text{EV}_{\xi>0})$. These heavy-tailed models are quite common in many areas of application, like biostatistics, finance, insurance, statistical quality control and telecommunications, among others. For heavy-tailed models, the classical EVI-estimators are the Hill (H) estimators (Hill, 1975),

$$H(k) \equiv H(k; \underline{\mathbf{X}}_n) := \frac{1}{k} \sum_{i=1}^k \ln X_{n-i+1:n} - \ln X_{n-k:n}, \quad 1 \leq k < n.$$

The classical semi-parametric VaR-estimators were introduced in Weissman (1978), being given by

$$\widehat{\text{VaR}}_q(k) := X_{n-k:n} (k/(nq))^{H(k)},$$

but $H(k)$ can be replaced in the previous formula by any consistent EVI-estimator. We now suggest ways to improve the performance of the existent VaR-estimators. The estimation of high quantiles depends strongly on the EVI-estimation, and recently, new classes of reliable EVI-estimators based on adequate generalized means (GM), and dependent on a tuning real parameter q have appeared in the literature (see Caeiro *et al.*, 2016, Penalva *et al.*, 2016, Paulauskas and Vaičiulis, 2017, and references therein), and will be used for the estimation of VaR_q , $q < 1/n$. Due to the fact that the GM VaR-estimators do not enjoy the adequate behaviour, in the sense that they do not suffer the appropriate linear shift in the presence of linear transformations of the data, as does any theoretical quantile and

the PORT-Weissman-Hill VaR-estimators introduced in Araújo-Santos *et al.* (2006), with PORT stating for *peaks over random thresholds*, we further discuss the so-called PORT GM VaR-estimators, more general than the class in Figueiredo *et al.* (2017).

Keywords: Generalized means; heavy-tailed parents; semi-parametric estimation, value-at-risk.

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